Secrecy Performance Analysis in STAR-RIS-Aided NOMA Networks

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***Abstract*—An analytical framework for physical layer security in simultaneous transmitting and reflecting reconfigurable intelligent surface (STAR-RIS) assisted non-orthogonal multiple access (NOMA) transmissions is proposed, where legitimate users and eavesdroppers are randomly deployed. To characterize system performance, the channel statistics are first provided, and the Gamma approximation is adopted for general cascaded κ-μ fading. Afterwards, the energy splitting (ES) protocol is considered and closed-form expressions of average secrecy capacity are derived. To obtain further insights, the asymptotic secrecy slope is deduced. The theoretical results show that the secrecy slope of the ES protocol is one. The numerical results demonstrate that: 1) there is an optimal resource allocation ratio of STAR-RIS to maximize the system performance; 2) the STAR-RIS-aided NOMA significantly outperforms the STAR-RIS-aided orthogonal multiple access.**

I. Introduction

Reconfigurable intelligent surfaces (RISs) have been regarded as a promising technique to support the smart radio environment and efficient secure transmissions in future communication networks [1]. One typical RIS is a uniform planer array with a large number of low-cost elements. By equipping with advanced beamforming controllers, the phase shifts of reflected signals on each RIS element can be changed independently, which helps to adjust the propagation of signals [2]. Benefiting from this feature, the RIS is able to improve the communication quality of legitimate users (LUs) while limiting eavesdropping by appropriate design on beamforming, thereby enhancing physical layer security (PLS). The authors of [3] focused on the downlink RIS-assisted secure transmission, where the design of beamforming is based on the global channel state information (CSI) of the eavesdropper (Eve) and the LU. In [4], the authors proposed a novel design on RIS beamforming to eliminate the signals received by the Eve, and hence the global CSI of the Eve is required. In [5], the secrecy outage probability (SOP) was derived under the assumption that Eve's CSI is unknown. The above works investigated PLS in the presence of fixed LUs and Eves. To capture the randomness property in wireless communication networks, stochastic geometry is a powerful tool and has been widely utilized to study the PLS in traditional communication systems [6].

For the conventional reflecting-only RIS, PLS performance within half of the space in front of the RIS can be controlled while the LUs in the other side still suffer from eavesdropping. To this end, the concept of simultaneous transmitting and reflecting RIS (STAR-RIS) has been proposed for providing full-space coverage [7], [8]. With three operation protocols, i.e., time switching (TS), energy splitting (ES), and mode switching (MS), different beamforming approaches can be implemented at both sides of the STAR-RIS, and hence the full-space PLS enhancement is realized. Note that STAR-RISs serve LUs at different sides by the same signal source, a multiple access scheme is indispensable for splitting unicast reflected and transmitted signals. Non-orthogonal multiple access (NOMA) can be a competent candidate due to its high spectral efficiency and user fairness. By employing the superposition coding at the transmitter for power multiplexing and the successive interference cancellation (SIC) at the receiver for detection, STAR-RIS-aided NOMA protects multiple LUs within the same time-frequency resource block [9]. In [10], residual hardware impairments were considered and analytical expressions of the SOP were provided for the paired NOMA LUs. In [11], the authors aimed to maximize the minimum secrecy capacity in STAR-RIS-aided uplink NOMA networks by joint secrecy beamforming design. However, these initial works considered simplified settings with fixed eavesdropping as the location of the Eve is predefined.

As we have discussed above, STAR-RIS-aided NOMA has the capability of providing security enhancement in the full space. Due to the full-space coverage introduced by STAR-RISs, the impact of full-space eavesdropping is valuable to be investigated. The study on secure STAR-RIS-aided NOMA transmissions with the consideration of randomly distributed Eves is important but has not been investigated in related works to the best of our knowledge. Motivated by this, we focus on the security performance of the STAR-RIS-aided NOMA in the presence of randomly distributed Eves in the full space in this work[1]. The main contributions are summarized as follows:

- We propose an analytical framework for STAR-RIS-aided NOMA with randomly deployed LUs and Eves in terms of PLS. The beamforming of the STAR-RIS is designed to enhance the channel gains of LUs. We employ a general κ-μ distribution to characterize the small-scale fading.
- We derive the closed-form approximations of the average secrecy capacity (ASC) for the pair of NOMA LUs. The asymptotic ASC is also derived to obtain the secrecy slope. The analytical results demonstrate that the secrecy slope of the ES protocol is one.
- We use the numerical results to validate the analysis and

[1] In this work, we only consider the impact of external Eves.

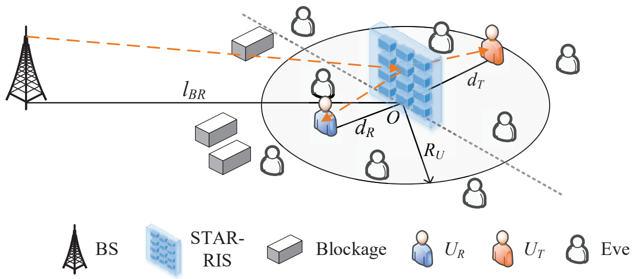

Fig. 1. System model for secure STAR-RIS-aided NOMA transmission.

to show that: 1) there is an optimal STAR-RIS resource allocation ratio to maximize the ASC; 2) NOMA is able to achieve a higher ASC than the OMA in the STAR-RIS-assisted transmission.

Notation: $(\cdot)^T$ denotes the transpose operation. $|x|$ is the amplitude of x. $\mathbb{E}[\cdot]$ denotes the expectation operator. $\mathrm{Gamma}(k,\theta)$ is the Gamma distribution with shape k and scale θ. $\Gamma(x)=\int_0^\infty t^{x-1}e^{-t}dt$ is the Gamma function. $\gamma(\alpha,x)$ is the lower incomplete Gamma function [12, eq. (8.350.1)]. ${}_pF_q(\mathbf{a}_p;\mathbf{b}_q;x)$ denotes the generalized hypergeometric function [12, eq. (9.14.1)]. We denote $[x]^+ = \max\{x,0\}$. $G_{p,q}^{m,n}\left((\cdot)\middle|\begin{matrix}(\mathbf{a}_p)\\(\mathbf{b}_q)\end{matrix}\right)$ is the Meijer G-function [12, eq. (9.301)]. For a cumulative distribution function (CDF) $F(x)$, we denote its complementary CDF as $\bar{F}(x)=1-F(x)$.

II. System Model

Consider a secure downlink transmission scenario, where a BS communicates with LUs assisted by a STAR-RIS in the presence of Eves. As shown in Fig. 1, the STAR-RIS with a random orientation is fixed at the origin of a two-dimensional plane $\mathbb{R}^2$. We fix the BS at $(-l_{BR},0)$, while the locations of LUs obey a homogeneous Poisson point process (HPPP) $\mathbf{\Phi}_u$ within a disc area with radius R_U centered at the origin. The locations of Eves obey another HPPP $\mathbf{\Phi}_e$ with the density λ_e in the considered plane $\mathbb{R}^2$. We consider that the BS, LUs, and Eves are equipped with a single antenna. The STAR-RIS consists of N elements, and these elements are capable of simultaneously transmitting and reflecting signals.

The ES protocol is employed at the STAR-RIS. Accordingly, the energy of the signal incident on each element is split into two parts for transmitting and reflecting with energy splitting ratios β_{T} and β_{R}, respectively, and we have $\beta_{\mathrm{R}}+\beta_{\mathrm{T}}=1$ according to the law of energy conservation. We consider the same β_{T} on all elements of the STAR-RIS. Those LUs located on the same side as the BS of the STAR-RIS are the reflecting LUs; otherwise are the transmitting LUs. We randomly select a reflecting LU U_{R} and a transmitting LU U_{T} to form a typical LU pair. The NOMA transmission scheme is invoked for the typical LU pair. All the Eves have powerful detection capabilities and are able to overhear the messages of all available resource blocks. The CSI of all Eves is available at the STAR-RIS and the BS. In addition, since multiuser detection techniques are adopted, the Eves can distinguish signals of different LUs when applying the NOMA scheme.

A. Channel Model

We consider an urban environment for the secure STAR-RIS-aided NOMA transmission, where direct transmission links between the BS and LUs/Eves are blocked. For the STAR-RIS-aided link, the channel model includes the path loss model and small-scale fading. We use the subscript $u=\{\mathrm{T},\mathrm{R}\}$ to denote the transmitting LU and the reflecting LU, respectively. For the paired LUs, the path loss of the STAR-RIS-aided link is related to the product of two distances, which can be expressed as $L_u=C_r\left(l_{BR}d_u\right)^{-\alpha}$, where d_u is the distance between the STAR-RIS and LU, C_r is the reference distance based intercept, and α refers to the path loss exponent. Similarly, the path loss of the Eve $i\in\mathbf{\Phi}_e$ is $L_{e,i}=C_r\left(l_{BR}d_e\right)^{-\alpha}$.

As in previous works, all subchannels of the STAR-RIS-aided transmission suffer cascaded small-scale fading. Specifically, we denote the small-scale fading vectors of the BS-RIS link and the RIS-LU/Eve link as $\mathbf{h}_{r_1}=[h_{r_1,1},...,h_{r_1,N}]^T$ and $\mathbf{h}_{r_2}=[h_{r_2,1},...,h_{r_2,N}]^T$, respectively. For LUs, the power of the overall small-scale fading for the STAR-RIS-aided cascaded channel is given by $|h_\epsilon|^2=\left|\mathbf{h}_{r_2}{}^T\tilde{\mathbf{\Theta}}_u\mathbf{h}_{r_1}\right|^2$, where $\tilde{\mathbf{\Theta}}_u=\mathrm{diag}\left(e^{j\theta_{u,1}},...,e^{j\theta_{u,N}}\right)$ is the normalized phase-shifting matrix of the STAR-RIS, where $j=\sqrt{-1}$ and $\theta_{u,n}\in[0,2\pi)$ for $n\in\{1,...,N\}$. The transmission- and reflection-coefficient matrix for LU u is $\mathbf{\Theta}_u=\sqrt{\beta_u}\tilde{\mathbf{\Theta}}_u$. To maximize the received signal power of the LUs, the STAR-RIS reconfigures the phase shifts according to the CSI so that phases of all channels can be aligned at the LUs, i.e., for $u\in\{\mathrm{R},\mathrm{T}\}$ we have

$$|h_u|^2=\left(\sum_{n=1}^{N}|h_{r_1,n}||h_{r_2,n}|\right)^2. \tag{1}$$

Different from the LUs, phases of different channels are random and independent at the Eves. The overall small-scale fading power is

$$|h_e|^2=\left(\sum_{n=1}^{N}|h_{BR,n}||h_{e,n}|e^{j\theta_n}\right)^2, \tag{2}$$

where θ_n is uniformly distributed in $[0,2\pi)$. In this work, the small-scale fading is characterized by the κ-μ distribution [13], which is a general model that includes some classical distributions such as the Rayleigh, Nakagami-m, and Rice as special cases. The transmission from the BS to the LUs through the STAR-RIS element n is the double κ-μ distribution. For $i\in\{1,2\}$, the probability density function (PDF) of the amplitude of $h_{r_i,n}$ is given by

$$f_{|h_{r_i,n}|}(x)=\frac{2\mu_i(1+\kappa_i)^{\frac{\mu_i+1}{2}}x^{\mu_i}e^{-\mu_i(1+\kappa_i)x^2}}{\kappa_i^{\frac{\mu_i-1}{2}}e^{\mu_i\kappa_i}} I_{\mu_i-1}\left(2\mu_i\sqrt{\kappa_i(1+\kappa_i)}x\right), \tag{3}$$

where κ_i and μ_i are fading parameters.

B. Signal Model

In STAR-RIS-NOMA, the SIC process is employed as in conventional NOMA systems. Without loss of generality, the SIC occurs at the LU with the better channel condition in the typical LU pair to achieve high rate performance. Let U_s and U_w denote the strong LU and the weak LU in the typical LU pair, respectively. The power allocation coefficient for U_s is a_s and that for U_w is a_w, where $a_s + a_w = 1$. For user fairness, the higher power level is allocated to U_w, i.e., $a_w > a_s$.

If the reflecting LU is the strong LU, i.e., $U_{\rm R} = U_s$, $U_{\rm R}$ decodes the massage of $U_{\rm T}$ fisrt. The signal-to-interference-plus-noise ratio (SINR) of the SIC procedure is given by

$$\gamma_{\rm SIC} = \frac{a_w \beta_{\rm R} \rho_b L_{\rm R} \left|h_{\rm R}\right|^2}{a_s \beta_{\rm R} \rho_b L_{\rm R} \left|h_{\rm R}\right|^2 + 1}, \tag{4}$$

where ρ_b is the transmit SNR for LUs.

After the successful SIC, $U_{\rm R}$ removes the messages of $U_{\rm T}$. Then $U_{\rm R}$ decodes its required messages with the following SNR

$$\gamma_{\rm R} = a_s \beta_{\rm R} \rho_b L_{\rm R} \left|h_{\rm R}\right|^2. \tag{5}$$

Since $U_{\rm T}$ decodes its message by treating the message of $U_{\rm R}$ as interference, the decoding SINR at $U_{\rm T}$ is expressed as

$$\gamma_{\rm T} = \frac{a_w \beta_{\rm T} \rho_b L_{\rm T} \left|h_{\rm T}\right|^2}{a_s \beta_{\rm T} \rho_b L_{\rm T} \left|h_{\rm T}\right|^2 + 1}. \tag{6}$$

For the case that the transmitting LU is the strong LU, the expressions can be obtained similarly, and we skip it here. Since the SIC order depends on the order of channel gains, we focus on the performance of the strong LU and the weak LU. Thus we have the subscript $u \in \{s, w\}$ in the rest of the paper.

We consider the worst-case of the security transmission, and hence we focus on the most detrimental Eve which has the highest detecting SNR of U_u. When the most detrimental Eve is at the $\tau \in \{{\rm R}, {\rm T}\}$ side of the STAR-RIS, the instantaneous SNR of detecting the information of U_u at the Eve can be presented as

$$\gamma_{E_u} = a_u \beta_\tau \rho_e \max_{i \in \mathbf{\Phi}_e} \left\{ L_{e,i} \left|h_e\right|^2 \right\}, \tag{7}$$

where ρ_e is the transmit SNR for the Eve.

III. Secrecy Performance Analysis

In this section, we first obtain new channel statistics for STAR-RIS-aided links. Then we derive the theoretical ASC expressions of the typical LU pair in the considered networks. Finally, the asymptotic secrecy slope in the high SNR regime is provided.

A. New Channel Statistics

The STAR-RIS assisted transmission introduces cascaded small-scale fading. For the fading channel from the BS to the LU/Eve through the STAR-RIS element n, we denote $\Delta_n = |h_{r_1,n}||h_{r_2,n}|$. The PDF of Δ_n can be expressed as [13]

$$\begin{aligned} f_{\Delta_n}(x) = &\frac{2\phi_1\phi_2 x}{e^{\mu_1\kappa_1 + \mu_2\kappa_2}} \\ &\times \sum_{q=0}^{\infty}\sum_{t=0}^{\infty} \rho_{q,t} G_{0,2}^{2,0}\left(\phi_1\phi_2 x^2 \left| q+\mu_1-1, t+\mu_2-1\right.\right), \end{aligned} \tag{8}$$

where $\phi_i = \mu_i(\kappa_i+1)$ and $\rho_{q,t} = \frac{(\mu_1\kappa_1)^q(\mu_2\kappa_2)^t}{q!t!\Gamma(q+\mu_1)\Gamma(t+\mu_2)}$. The k-th order moment of the product Δ_n is given by

$$\begin{aligned} \mathbb{E}[(\Delta_n)^k] = &\frac{(\mu_1)_{\frac{k}{2}}(\mu_2)_{\frac{k}{2}}}{e^{\mu_1\kappa_1+\mu_2\kappa_2}\phi_1^{\frac{k}{2}}\phi_2^{\frac{k}{2}}} \\ &\times {}_1F_1\left(\frac{k}{2}+\mu_1;\mu_1;\kappa_1\mu_1\right){}_1F_1\left(\frac{k}{2}+\mu_2;\mu_2;\kappa_2\mu_2\right), \end{aligned} \tag{9}$$

where $(x)_m = \frac{\Gamma(x+m)}{\Gamma(x)}$ is the pochhammer symbol.

Lemma 1. *When the number of STAR-RIS elements is large enough, the overall small-scale fading power for the unordered LUs obeys a Gamma distribution*

$$|h_0|^2 \sim {\rm Gamma}\left(k_0, \theta_0\right), \tag{10}$$

where $k_0 = \frac{({m_r}^2 N + {\sigma_r}^2)^2}{4{m_r}^2{\sigma_r}^2 N + 2{\sigma_r}^4}$, $\theta_0 = \frac{4{m_r}^2{\sigma_r}^2N^2 + 2{\sigma_r}^4 N}{{m_r}^2 N + {\sigma_r}^2}$, $m_r = \mathbb{E}[\Delta_n]$ *and* ${\sigma_r}^2 = \mathbb{E}[(\Delta_n)^2] - \mathbb{E}[\Delta_n]^2$. *The overall small-scale fading power for the Eves obeys*

$$|h_e|^2 \sim {\rm Gamma}\left(1, N({m_r}^2 + {\sigma_r}^2)\right). \tag{11}$$

Proof: Based on the results in our previous work [14], if m_u and σ_u^2 are the mean and the variance of Δ_n, respectively, the overall small-scale fading power $|h_u|^2$ can be approximately fitted by a Gamma distribution ${\rm Gamma}\left(\frac{{M_0}^2}{V_0}, \frac{V_0}{M_0}\right)$, where $M_0 = {m_r}^2 N^2 + {\sigma_r}^2 N$ and $V_0 = 4{m_r}^2{\sigma_r}^2N^3 + 2{\sigma_r}^4 N^2$. Moreover, the overall small-scale fading power for the Eves obeys ${\rm Gamma}\left(1, N({m_r}^2 + {\sigma_r}^2)\right)$. According to the property of the cascaded κ-μ distribution, this lemma is proved. ∎

Lemma 2. *In the NOMA LU pair, CDFs of the channel power for the strong LU and the weak LU can be respectively expressed as*

$$F_{U_s}(x) = [\hat{F}_{H_0}(x)]^2, \tag{12}$$

$$F_{U_w}(x) = 2\hat{F}_{H_0}(x) - [\hat{F}_{H_0}(x)]^2, \tag{13}$$

where $\hat{F}_{H_0}(x) = \frac{\delta}{\Gamma(k_r)} G_{2,3}^{1,2}\left(\frac{{R_U}^\alpha x}{A_L \theta_r}\left| \begin{matrix} 1-\delta, 1 \\ k_r, 0, -\delta \end{matrix}\right.\right)$, $\delta = \frac{2}{\alpha}$, $A_L = C_r {l_{BR}}^{-\alpha}$, $k_r = \frac{({m_r}^2 N + {\sigma_r}^2)^2}{4{m_r}^2{\sigma_r}^2 N + 2{\sigma_r}^4}$, *and* $\theta_r = \frac{4{m_r}^2{\sigma_r}^2N^2 + 2{\sigma_r}^4 N}{{m_r}^2 N + {\sigma_r}^2}$.

Proof: See Appendix A. ∎

B. Average Secrecy Capacity Analysis

Let C_{U_u} denote the channel capacity of the pair of LUs and C_{E_u} represent the channel capacity of the most detrimental Eve with the data of U_u, respectively. Then the secrecy capacity of the NOMA LUs can be expressed as $[C_{U_u} - C_{E_u}]^+$, which is non-negative. The ASC is defined as the expectation value of secrecy capacity over the fading channel and the spatial effect, which is expressed as

$$C_u = \mathbb{E}\left([C_{U_u} - C_{E_u}]^+ \right). \quad (14)$$

Since the most detrimental Eve is either a reflecting Eve or a transmitting Eve, we first provide the channel statistics of the most detrimental reflecting/transmitting Eve.

Lemma 3. *The CDF of the received SNR $\gamma_{E_{u,\tau}}$ at the most detrimental reflecting/transmitting Eve E_τ ($\tau \in \{\mathrm{R}, \mathrm{T}\}$) in terms of the message of U_u is given by*

$$F_{\gamma_{E_{u,\tau}}}(x) = \exp\left(-m_u \left(\frac{x}{c_\tau} \right)^{-\delta} \right), \quad (15)$$

where $m_u = \frac{1}{2}\pi\delta\lambda_e(\rho_e a_u A_L W_e)^\delta \Gamma(\delta)$, $\delta = \frac{2}{\alpha}$, and $W_e = N({m_r}^2 + {\sigma_r}^2)$.

Proof: The CDF of the channel gain for the most detrimental Eve can be calculated as follows

$$F_{\gamma_{E_{u,\tau}}}(x) = \mathbb{E}_{\mathbf{\Phi}_e}\left[\prod_{\mathbf{\Phi}_e} F_{|h_e|^2}\left(\frac{{d_e}^\alpha x}{\rho_e a_u A_L c_\tau} \right) \right]. \quad (16)$$

We apply the probability generating functional [15, eq. (4.3)] and utilizing the property that $\mathrm{Gamma}(1, W_e) = W_e \mathrm{Exp}(1)$. The (16) can be rewitten as

$$\begin{aligned} &F_{\gamma_{E_{u,\tau}}}(x) \\ &= \exp\left(-\pi\lambda_e \int_0^\infty \left(1 - F_{|h_e|^2}\left(\frac{r^\alpha x}{\rho_e a_u A_L c_\tau W_e} \right) \right) r dr \right), \end{aligned} \quad (17)$$

Then the lemma is proved by applying [12, eq. (3.326.10)]. ■

When considering the ES protocol, the channel capacity of the LUs is expressed as $C_{U_u} = \log_2(1 + \gamma_u)$ and that for the Eves is $C_{E_u} = \log_2(1 + \gamma_{E_u})$. Similarly, we define the equivalent received SNR in this case as follows.

Definition 1. *For the ES protocol, we deploy an equivalent Eve of E_τ located at the same side as the U_ϵ, and the equivalent received SNR is*

$$\hat{\gamma}_{E_{\tau \to u}} = \frac{\beta_\tau}{\beta_u} \gamma_{E_u}. \quad (18)$$

Lemma 4. *For the ES protocol, the CDF of the equivalent received SNR at the U_u side for the most detrimental Eve is given by*

$$F_{\gamma_{E_u}}(x) = e^{-m_u \left(\tilde{\beta}_{u,s} x \right)^{-\delta} - m_u \left(\tilde{\beta}_{u,w} x \right)^{-\delta}}, \quad (19)$$

where $\tilde{\beta}_{u,\tau} = \frac{\beta_u}{\beta_\tau}$.

Proof: Based on **Lemma 3**, this lemma is straightforwardly proved. ■

Theorem 1. *For the ES protocol, the closed-form approximations of the ASC for the two NOMA LUs are given by*

$$C_s \approx \frac{1}{\ln 2} \sum_{m=1}^{M_s} \frac{\xi_m \bar{F}_{\gamma_{U_s}}\left(\frac{\xi_m}{\beta_s} \right) F_{\gamma_{E_s}}\left(\frac{\xi_m}{\beta_s} \right) / (1 + \xi_m)}{(M_s + 1)^2 [L_{M_s+1}(\xi_m)]^2 \exp(-\xi_m)}, \quad (20)$$

$$\begin{aligned} C_w \approx & \frac{1}{\ln 2} \sum_{m=1}^{M_w} \frac{\pi a_w \sqrt{1 - {\varphi_m}^2}}{(a_w \varphi_m + a_s + 1) M_w} \\ & \times \bar{F}_{\gamma_{U_w}}\left(\frac{a_w(\varphi_m + 1)}{2 a_s \beta_w} \right) F_{\gamma_{E_w}}\left(\frac{a_w(\varphi_m + 1)}{2 a_s \beta_w} \right), \end{aligned} \quad (21)$$

where ξ_m is the m-th root of Laguerre polynomial $L_{M_s}(x)$ and $\varphi_m = \cos\left(\frac{2m-1}{2M_w}\pi \right)$. M_s and M_w are parameters to ensure a complexity-accuracy trade-off.

Proof: See Appendix B. ■

Proposition 1. *The ASC of the typical NOMA LU pair is given by*

$$C = C_s + C_w. \quad (22)$$

C. Secrecy Slope Analysis

To gain insights into the ASC performance, the secrecy slope in the high-SNR regime is considered, which is defined as

$$\mathcal{S} = \lim_{\rho_b \to \infty} \frac{C_\infty}{\log_2(\rho_b)}, \quad (23)$$

where C_∞ is the asymptotic ASC when $\rho_b \to \infty$. The asymptotic expressions for the pair of NOMA LUs are provided in the following propositions.

Proposition 2. *For the ES protocol, the asymptotic ASC in the high-SNR regime can be expressed as*

$$\begin{aligned} C_{s,\infty} = & \log_2(a_s \beta_s \rho_b) + \sigma_s \\ & - \frac{1}{\ln 2} \sum_{m=1}^{M_s} \frac{\xi_m \bar{F}_{\gamma_{E_s}}(\xi_m / \beta_s) / (1 + \xi_m)}{(M_s + 1)^2 [L_{M_s+1}(\xi_m)]^2 \exp(-\xi_m)}, \end{aligned} \quad (24)$$

$$\begin{aligned} C_{w,\infty} = & \log_2\left(1 + \frac{a_w}{a_s} \right) \\ & - \frac{1}{\ln 2} \sum_{m=1}^{M_w} \frac{\pi a_w \sqrt{1 - {\varphi_m}^2}}{(a_w \varphi_m + a_s + 1) M_w} \bar{F}_{\gamma_{E_w}}\left(\frac{a_w(\varphi_m + 1)}{2 a_s \beta_w} \right). \end{aligned} \quad (25)$$

Proof: When $\rho_b \to \infty$, the term $C_{s,max}$ in eq. (C.1) and $C_{w,max}$ in eq. (C.2) can be expressed as $C_{s,max} \approx \mathbb{E}[\log_2(a_s \beta_s \rho_b H_s)] = \log_2(a_s \beta_s \rho_b) + \sigma_s$ and $C_{s,max} \approx \log_2\left(1 + \frac{a_w}{a_s} \right)$, respectively. ■

Remark 1. *In the STAR-RIS-aided NOMA transmission with ES protocol, the secrecy slopes of the strong LU and weak LU are $\mathcal{S}_s = 1$ and $\mathcal{S}_w = 0$. Therefore, the ASC is bounded in the high SNR regime. Moreover, the secrecy slope of the paired LUs is one.*

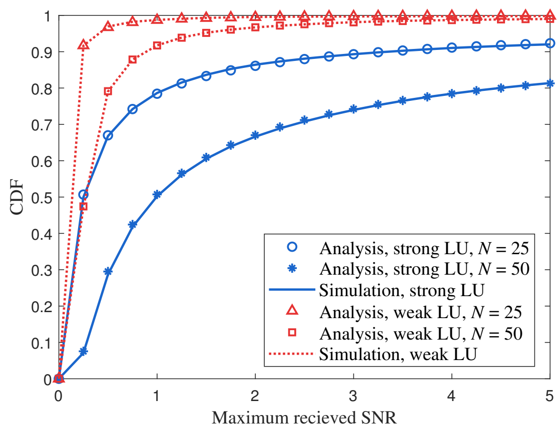

Fig. 2. CDF of the maximum received SNR at LUs with $\rho_b = 50$ dB.

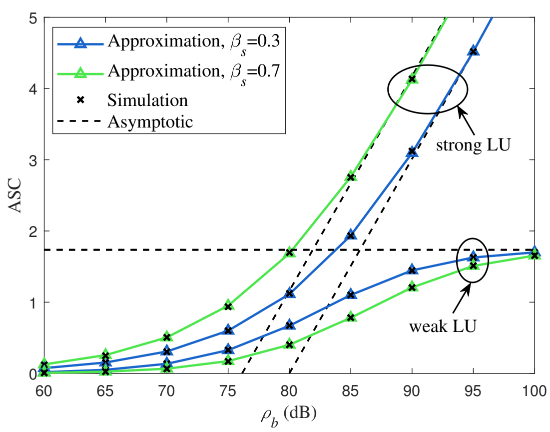

Fig. 3. Validation of the theoretical ASC.

IV. Numerical Results

In this section, we present the numerical results to demonstrate the secrecy performance of STAR-RIS-aided NOMA. Our theoretical results are validated via Monte Carlo simulations by averaging the obtained performance. Afterwards, some interesting insights are provided. Unless otherwise stated, the simulation parameters are defined as follows. For the small-scale fading, we set $\kappa_1 = \kappa_2 = 3$ and $\mu_1 = \mu_2 = 1$, hence the line-of sight (LoS) Rician channel is considered. The density of Eves is $\lambda_e = 10^{-4}$ m^{-2} and the SNR $\rho_e = 50$ dB. The path loss exponent is $\alpha = 3$. The number of elements on the STAR-RIS is $N = 25$. The radius of the disc area is $R_U = 50$ m. The power allocation coefficients for the NOMA LUs are $a_s = 0.3$ and $a_w = 0.7$.

Fig. 2 plots the CDF of the maximum received SNR of the paired LUs, where the maximum received SNR is the product of transmit SNR ρ_b and the channel power of LU $u \in \{s, w\}$. Here we set $\rho_b = 50$ dB. The analytical results fit the simulation curves quite well, and hence the derived channel statistics in **Lemma 1** and **Lemma 2** are validated. In addition, the STAR-RIS-aided channel model with a large number of elements has a higher channel power than the model with a few elements. Therefore, the enhanced received SNR at LUs can be obtained by deploying large-scale STAR-RISs.

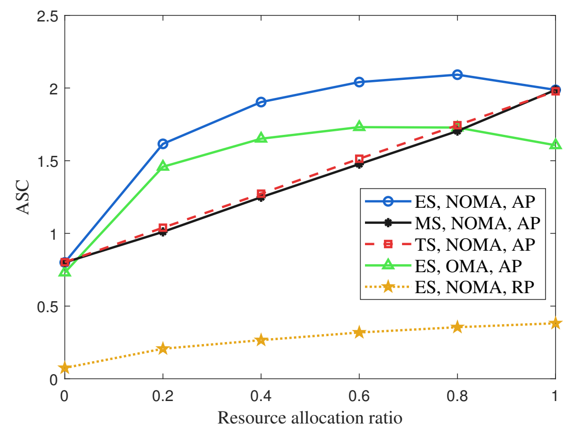

Fig. 4. ASC versus the STAR-RIS resource allocation ratio with $\rho_b = 80$ dB, where "AP" represents the aligned phase scheme in our analysis and "RP" is the random phase.

Fig. 3 verifies the theoretical ASC expressions derived in **Theorem 1**. The slope of the curves for the strong LU is asymptotically constant in the high SNR regime. For the weak LU, however, its ASC achieves an upper bound with the increase of the transmit SNR ρ_b as discussed in **Remark 1**. Furthermore, the weak LU with a small value of β_s converges to the ASC upper bound fast. Another observation is that when adjusting the energy splitting coefficient β_s, there is a tradeoff between the secrecy performance of the strong LU and the weak LU.

Fig. 4 compares the secrecy rate performance among different STAR-RIS protocols and system setups. Here, the resource allocation ratio represents the ratio of power, element, or time resource allocated to the strong LU to the total resource for ES, MS, and TS protocols, respectively. For example, the resource allocation ratio in ES protocol is β_s. We can observe that the secrecy performance of the ES protocol outperforms the MS and TS protocols. It can be explained that the ES protocol exploits an extra degree of freedom in the space domain. When the resource allocation ratio is zero or one, the performance of the three protocols is the same. In this special case, a LU on one side of STAR-RIS uses all power, element, and time resources. Moreover, by adjusting the resource allocation ratio, the highest ASC can be achieved. We also observe that the ASC of NOMA obtains a significant improvement over OMA. This illustrates the efficiency of adopting the NOMA scheme in STAR-RIS-aided systems. By comparing with the random phase setup, the results show that with appropriate design on the beamforming, the STAR-RIS can improve the secrecy rate performance remarkably.

V. Conclusion

In this paper, the PLS of the STAR-RIS-aided NOMA system with randomly deployed LUs and Eves has been investigated, where the stochastic geometry tool has been utilized to model the locations of LUs and Eves. We have derived the analytical expressions of the ASC when the SIC order of the NOMA LUs is based on the channel gains. Then asymptotic secrecy performance has been obtained. The numerical results have provided design guidelines for the considered system: 1) the optimal secrecy performance can be achieved by adjusting the resource allocation ratio of the STAR-RIS; 2) the ES protocol has a higher secrecy performance than the TS and MS protocols.

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Appendix A: Proof of Lemma 2

In this work, the overall channel power consists of path loss and small-scale fading. We denote $H_0 = XY$, where $X = |h_u|^2$ and $Y = A_L d^{-\alpha}$ represent the power of small scale fading and path loss at the LU, respectively. According to (10), the CDF of the small-scale fading X is

$$F_X(x) = \frac{\gamma\left(k_r, x/\theta_r\right)}{\Gamma(k_r)}. \tag{A.1}$$

Noticed that the locations of LUs obey a HPPP in the disc area, the PDF of the path loss Y is given by

$$f_Y(x) = \begin{cases} \frac{2{A_L}^{2/\alpha}}{\alpha {R_U}^2} x^{-2/\alpha-1}, x > A_L {R_U}^{-\alpha} \\ 0, x \le A_L {R_U}^{-\alpha}. \end{cases} \tag{A.2}$$

For an arbitrary LU in $\mathbf{\Phi}_u$, we can formulate the CDF of the channel power H_u as follows

$$\begin{aligned} \hat{F}_{H_0}(x) &= \int_0^\infty F_X(\frac{x}{y}) f_Y(y) dy \\ &\overset{(a)}{=} \frac{2}{{R_U}^2} \int_0^{R_U} \frac{\gamma\left(k_r, \frac{x r^\alpha}{A_L \theta_r}\right)}{\Gamma(k_r)} r dr, \end{aligned} \tag{A.3}$$

where (a) is from the change of variable $r = (y/A_L)^{-1/\alpha}$. By employing the meijer G-function of lower incomplete Gamma function, we rewrite $\hat{F}_{H_0}(x)$ as

$$\begin{aligned} \hat{F}_{H_0}(x) &= \frac{2}{{R_U}^2 \Gamma(k_r)} \int_0^{R_U} r G_{1,1}^{1,2}\left(\frac{x r^\alpha}{A_L \theta_r} \middle| \begin{matrix} 1 \\ k_r, 0 \end{matrix} \right) dr \\ &\overset{(b)}{=} \frac{\delta}{\Gamma(k_r)} G_{2,3}^{1,2}\left(\frac{{R_U}^\alpha x}{A_L \theta_r} \middle| \begin{matrix} 1-\delta, 1 \\ k_r, 0, -\delta \end{matrix} \right), \end{aligned} \tag{A.4}$$

where (b) is obtained by utilizing [12, eq. (7.811.2)].

For the LU pair, according to order statistics theory, if total of K LUs have the same statitical channel characteristic, the ordered CDF of the channel power of the lth weakest LU is given by

$$F_l(x) = \sum_{k=l}^{K} \binom{K}{k} [\hat{F}_{H_0}(x)]^k [1 - \hat{F}_{H_0}(x)]^{K-k}. \tag{A.5}$$

By substuting (A.4) into (A.5), this lemma is proved.

Appendix B: Proof of Theorem 1

Based on the definition in (14), the ASC for the strong LU is expressed as

$$\begin{aligned} C_s(x) &= \int_0^\infty \int_0^x \log_2\left(\frac{1+x}{1+y}\right) f_{\gamma_{U_s}}(x/\beta_s) f_{\gamma_{E_s}}(y/\beta_s) dy dx \\ &= \frac{1}{\ln 2} \int_0^\infty \frac{\bar{F}_{\gamma_{U_s}}(x/\beta_s) F_{\gamma_{E_s}}(x/\beta_s)}{1+x} dx. \end{aligned} \tag{B.1}$$

By applying the Gauss-Laguerre quadrature, the closed-form approximation can be obtained.

For the weak LU, the ASC is zero when $a_w - \gamma_{U_s}^{\mathrm{TS}} a_s \le 0$. Thus the ASC is given by

$$C_w(x) = \frac{1}{\ln 2} \int_0^{\frac{a_w}{a_s}} \frac{\bar{F}_{\gamma_{U_w}}(x/\beta_w) F_{\gamma_{E_w}}(x/\beta_w)}{1+x} dx. \tag{B.2}$$

Then the closed-form approximation is obtained by the Chebyshev–Gauss quadrature. The proof is completed.

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