Covert Communications in Intelligent Reflecting Surface-Assisted Two-Way Relaying Networks

Dan Deng, *Senior Member, IEEE,* Xingwang Li, *Senior Member, IEEE,* Shuping Dang, *Member, IEEE,* M. Cenk Gursoy, *Senior Member, IEEE* and Arumugam Nallanathan, *Fellow, IEEE*

***Abstract*—Covert communications is critical in many application scenarios for ensuring transmission security and privacy. Two-way protocols are widely adopted in intelligent reflecting surface (IRS) assisted relaying networks to enable covert communications and prevent wireless signals from being overheard. To quantify the performance of covert communications in this special scenario, we derive the closed-form expression of outage probability and its asymptote in this paper. Considering the worst-case of covert communications, the optimal normalized power threshold of warden's detector is analyzed under a complex Gaussian distribution approximation. To meet the requirement of the detection error probability, the ratio of the transmission power between the covert nodes is investigated. Simulation results are provided to validate the theoretical expressions as well as the asymptotic analysis with a small number of reflecting elements.**

***Index Terms*—Covert communications, detection error probability, intelligent reflecting surface, outage probability, two-way transmission.**

I. Introduction

Intelligent reflecting surface (IRS), combined with other emerging technologies, has recently attracted much interest as a future wireless communication paradigm. Through dynamic phase tuning, IRS can significantly improve the quality of the wireless links with only limited energy consumption [1]. By jointly optimizing the precoding matrix and phase-shift coefficients of IRS, the authors in [2] proposed a full-duplex multiuser scheme, which can cancel the interference effectively. This forms the theoretical foundation of IRS-assisted full-duplex relaying and proves its feasibility.

Due to the broadcast property of wireless networks, covert communication is critical for many practical wireless applications for enhancing security and privacy [3]. Subject to the detection error probability, IRS is used in [4] to assist non-orthogonal multiple access (NOMA) communications for both downlink and uplink, where the transmission power and the reflecting beamforming are jointly optimized. Benefiting from signal superposition of NOMA, two-way communications can be adopted as an enabling technology to improve the wireless security. For example, with the help of an IRS equipped relaying node, a novel user scheduler is proposed in [5] for multiuser two-way networks, which can achieve a remarkable secrecy improvement. Furthermore, the detailed performance analysis for the IRS-assisted networks is conducted for different application scenarios in [6–9].

Inspired by the advantages of IRS and two-way communications, we propose a two-way protocol for IRS-assisted relaying networks to protect covert communications from being detected by a warden. In the proposed networks, Bob always transmits public messages to Alice, which is also used as a background signal to protect covert messages. Occasionally, Alice tries to transmits its covert signal to Bob through the reverse channel. Motivated by these considerations, we investigate the combination of IRS and two-way relaying protocol to improve covert communications and reveal the effects of system parameters on the covert link and the detection error probability of the warden.

This paper aims to exploit the potentials of IRS' phase shift matrix and power allocation to enhance covert communications under the constraint of the detection error probability at warden. The analysis quantifies the fundamental performance limits and provides useful design guidelines for practical covert networks. Specifically, when the detection probability of Willie is sufficiently small, the optimal transmission power ratio of Alice to Bob is equal to the detection probability.

The contributions of this paper are listed as follows:

1) A novel IRS-assisted two-way covert communication protocol is proposed, where the IRS is used to improve the signal-to-noise ratio (SNR) of the covert link through a maximum ratio transmission scheme. Meanwhile, two-way communication is adopted to protect the private messages from being detected by a warden.
2) By using the mathematical properties of the gamma distribution, we derive the close-form and asymptotic expression of the outage probability of the covert link. Moreover, the optimal normalized power threshold of warden's detector and the optimal ratio of the transmission power between the covert nodes are investigated under a complex Gaussian distribution approximation. The obtained results can be used as the design guidelines for the parameters in practical communication systems, such as the number of the IRS reflecting elements and transmission power of the covert link.

II. System Model

As shown in Fig. 1, the considered two-way relaying network consists of four nodes: the legitimate users denoted by Alice and Bob, the warden user Willie and the two-way IRS relaying node. It is assumed that IRS is equipped with M continuous phase reflecting elements, while other

This work was partly supported by Natural Science Foundation of Guangdong Province with grant number 2022A1515010999, Scientific Research Project of Education Department of Guangdong with grant number 2021KCXTD061, Science and Technology Program of Guangzhou with grant number 202201011850, Yangcheng Scholar, Scientific Research Project of Guangzhou Education Bureau with grant number 202032761, Application Technology Collaborative Innovation Center of GZPYP with grant number 2020ZX01.

Dan Deng is with School of Information Engineering, Guangzhou Panyu Polytechnic, Guangzhou, 410630, China. (dengdan@ustc.edu). Xingwang Li is with School of Physics and Electronic Information Engineering, Henan Polytechnic University, Jiaozuo, 454150, China. (lixingwangbupt@gmail.com). Shuping Dang is with Department of Electrical and Electronic Engineering, University of Bristol, Bristol BS8 1UB, UK. (shuping.dang@bristol.ac.uk). M. Cenk Gursoy is with the Department of Electrical Engineering and Computer Science, Syracuse University, Syracuse, NY, 13244 USA (mcgursoy@syr.edu). Arumugam Nallanathan is with the School of Electronic Engineering and Computer Science, Queen Mary University of London, London E1 4NS, UK (a.nallanathan@qmul.ac.uk).

Corresponding author: Xingwang Li.

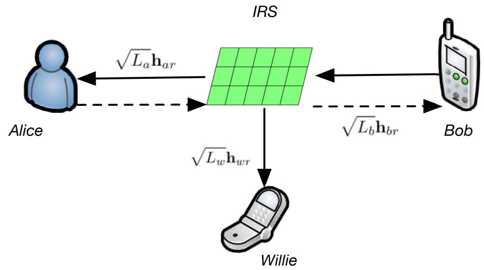

Fig. 1. Covert communications in IRS-assisted two-way relaying networks.

nodes are single-antenna devices. All the wireless links are assumed to experience quasi-static block fading. The statistical independence is assumed between adjacent blocks, and all channels are supposed to be reciprocal so that data transmissions are subject to the same amount of fading regardless of the transmission direction. Due to the severe fading introduced by shadowing, there is no direct link between Alice, Bob, and Willie. As in [5, 10], an IRS is deployed as a two-way relay to assist the signal transmission from Alice to Bob. Specifically, IRS reflects the received signals from Alice and Bob simultaneously with the same phase shift beamforming matrices. Benefiting from the two-way protocol via IRS, the signal received at Willie is superposed from Alice and Bob weighted with independent channel fading coefficients. The public signal received at Willie from Bob can be used to hide the confidential message transmitted by Alice. Generally, Bob transmits a public signal $s_b \sim \mathcal{CN}(0,1)$ to Alice with power P_b, which also acts as an artificial noise to protect the covert communications from Alice to Bob. In covert communication scenarios, with the help of the two-way IRS, Alice is also able to transmit a covert signal $s_a \sim \mathcal{CN}(0,1)$ to Bob with power P_a, simultaneously. Let d_a, d_b, and d_w be the distances from IRS to Alice, IRS to Bob, and IRS to Willie, respectively; α is the path loss exponent; L_0 is the reference path loss at the distance of $d = 1$ m; and $L_a = L_0 d_a^{-\alpha}, L_b = L_0 d_b^{-\alpha}$, and $L_w = L_0 d_w^{-\alpha}$ denote the large-scale path loss of different links; $\mathbf{h}_{ar}, \mathbf{h}_{br}$, and $\mathbf{h}_{wr} \in \mathbb{C}^{M\times 1}$ are the small-scale channel fading coefficients of the links from IRS to Alice, IRS to Bob, and IRS to Willie, respectively, with each element following standard complex Gaussian distribution, i.e., $h_{xr,m} \sim \mathcal{CN}(0,1), \forall m \in [1, M], x \in \{a, b, w\}$.

By the above nomenclature, the received signals at Willie under two hypotheses can be given by

$$\mathrm{H}_0 : y_w = \sqrt{P_b L_b L_w}\mathbf{h}_{wr}^T \mathbf{\Psi}\mathbf{h}_{br} s_b + n_{w,0}, \tag{1}$$

and

$$\begin{aligned}\mathrm{H}_1 : y_w =& \sqrt{P_a L_a L_w}\mathbf{h}_{wr}^T \mathbf{\Psi}\mathbf{h}_{ar} s_a \\ &+ \sqrt{P_b L_b L_w}\mathbf{h}_{wr}^T \mathbf{\Psi}\mathbf{h}_{br} s_b + n_{w,1},\end{aligned} \tag{2}$$

where H_0 denotes the event that only Bob is transmitting public signals, and H_1 denotes the event that both Alice and Bob are transmitting messages simultaneously; $\mathbf{\Psi} = \mathrm{diag}(e^{j\theta_1}, \cdots, e^{j\theta_M}) \in \mathbb{C}^{M\times M}$ denotes the phase shift beamforming matrix of the IRS node; $n_{w,0}, n_{w,1} \sim \mathcal{CN}(0, \sigma^2)$ are the additive white Gaussian noise (AWGN) terms at Willie.

Similarly, the received signal at Bob during the covert communication phase can be expressed as

$$\begin{aligned}y_b =& \sqrt{P_a L_a L_b}\mathbf{h}_{br}^T \mathbf{\Psi}\mathbf{h}_{ar} s_a \\ &+ \sqrt{P_b L_b L_b}\mathbf{h}_{br}^T \mathbf{\Psi}\mathbf{h}_{br} s_b + n_b,\end{aligned} \tag{3}$$

where $n_b \sim \mathcal{CN}(0, \sigma^2)$ is the AWGN received at Bob. [1]

III. Performance Analysis

In this section, we study several performance metrics of covert communications in the proposed application scenario. In particular, the detection error probability, covert capacity, and outage probability are determined in closed-form. In addition, the decision threshold optimization problem and transmission power ratio optimization problem are formulated, investigated, and solved.

A. Detection Error Probability

Based on the received signal power, the warden user Willie uses a long-term radio frequency power meter as the test statistic to detect whether Alice is transmitting a signal or not. Willie's average received power is given by

$$P_w = \begin{cases} P_b L_b L_w |\delta_{bw}|^2 + \sigma^2, & \mathrm{H}_0, \\ P_a L_a L_w |\delta_{aw}|^2 + P_b L_b L_w |\delta_{bw}|^2 + \sigma^2, & \mathrm{H}_1, \end{cases} \tag{4}$$

where $\delta_{aw} = \mathbf{h}_{wr}^T \mathbf{\Psi}\mathbf{h}_{ar}$ and $\delta_{bw} = \mathbf{h}_{wr}^T \mathbf{\Psi}\mathbf{h}_{br}$ denote the composite channel fading coefficients from Alice to Willie and Bob to Willie via IRS, respectively. Thus, the binary decision criterion at Willie can be written as

$$P_w \underset{\mathrm{D}_1}{\overset{\mathrm{D}_0}{\lessgtr}} \tau, \tag{5}$$

where τ is the power detection threshold of Willie; D_0 and D_1 are the decision results corresponding to hypotheses H_0 and H_1, respectively. Both $\Pr(\mathrm{H}_0)$ and $\Pr(\mathrm{H}_1)$ are assumed to be 1/2. In this case, according to the definition in [12, eq. (2)], based on (4) and (5), the detection error probability of Willie's detector can be simplified as follows:

$$P_E = P_{FA} + P_{MD} = \Pr\{\mathrm{D}_1|\mathrm{H}_0\} + \Pr\{\mathrm{D}_0|\mathrm{H}_1\}, \tag{6}$$

where P_{FA} and P_{MD} denote the false alarm probability under condition H_0 and the missed detection probability under condition H_1, respectively.

B. Capacity and Outage Probability of the Covert Link

It is assumed that the perfect self-interference cancellation (SIC) receiver is adopted by both Alice and Bob. [2] Con-

[1] Specifically, since the independent channel mode is adopted in (1)-(3), it provides additional degrees of freedom of the wireless networks. The diversity of the covert link is linear with the number of IRS reflecting elements, which will be proven in the following sections. On the contrary, if the wireless links experience correlated channels, the diversity order will be significantly reduced. The detailed discussion on the effects of the correlated channels can be found in [11].

[2] Note that neither SIC nor channel state information (CSI) can be perfect in practical wireless systems. Recent literature investigated the negative effects of the residual error of SIC [13] and CSI [7] on the network performance, which provides helpful thoughts and references to the system design. The results obtained in this paper with perfect SIC and CSI can be used as an upper bound for the design of practical communication systems. A similar research methodology and assumptions have also been adopted in existing literature such as [5, 14]. The effects of the imperfect SIC and imperfect CSI on covert communications in IRS-assisted relaying networks are worth studying as future work.

sequently, the second term in (3) can be cancelled from the received signal, and the instantaneous SNR at Bob during the covert communication can be derived as

$$\gamma_b = \frac{P_a L_a L_b}{\sigma^2} |\delta_{ab}|^2, \tag{7}$$

where $\delta_{ab} = \mathbf{h}_{br}^T \boldsymbol{\Psi} \mathbf{h}_{ar}$ denotes the composite channel fading coefficient from Alice to Bob via IRS. The corresponding covert communication capacity is obtained by

$$C_b = \log_2(1 + \gamma_b). \tag{8}$$

An outage event occurs when the covert capacity falls below a predefined capacity threshold. Therefore, the outage probability can be calculated as

$$\begin{aligned} P_O =& \Pr\{C_b < C_T\} = \Pr\{\gamma_b < \gamma_T\} \\ =& \Pr\left\{ |\delta_{ab}|^2 < \frac{\gamma_T}{\overline{\gamma_b}} \right\}, \end{aligned} \tag{9}$$

where $\overline{\gamma_b} = \frac{P_a L_a L_b}{\sigma^2}$ denotes the nominal SNR from Alice to Bob; $C_T > 0$ is the predefined capacity threshold for the covert link, and $\gamma_T = 2^{C_T} - 1$ is the required SNR subject to C_T.

In order to maximize the instantaneous SNR at Bob in (7), the maximum ratio transmission (MRT) scheme is adopted by the IRS. Generally, it is difficult for the IRS to obtain the channel state information of a passive warden. In this case, the MRT scheme, which maximizes the received SNR from Alice to Bob, is the optimal strategy under the knowledge limit of passive warden, which has been proven and also adopted in other IRS networks [5, 15]. The IRS is connected to a remote controller, which adjusts the IRS phase shift beamforming matrix in real time. On the other hand, Alice can estimate the exact channel state information through pilot signal transmitted from Bob and feeds it back to the IRS controller. Due to the channel reciprocity, the channel fading coefficients of the reverse link can also be obtained.

As an optimal phase shift beamforming algorithm, the phase shift of the mth element of the IRS is obtained by

$$\theta_m = -(\theta_{ar,m} + \theta_{br,m}), \forall m \in [1, M], \tag{10}$$

where $\theta_{ar,m}$ and $\theta_{br,m}$ denote the phases of mth elements of $\mathbf{h}_{ar}$ and $\mathbf{h}_{br}$, respectively. Then the composite channel fading coefficient from Alice to Bob δ_{ab} can be rewritten as

$$\delta_{ab} = \sum_{m=1}^{M} |h_{ar,m} h_{br,m}|. \tag{11}$$

Since it is difficult to derive the exact probability density function (PDF) of δ_{ab}, the Gamma distribution is adopted to approximate the actual distribution of δ_{ab}. By using [5, Lemma 1], the cumulative distribution function (CDF) of the sum of the product of two Rayleigh random variables can be approximated with that of a Gamma distribution as follows:

$$F_{\delta_{ab}}(z) = \frac{1}{\Gamma(M\mu)} \gamma(M\mu, \lambda z), \tag{12}$$

where

$$\begin{cases} \mu &= \frac{\pi^2}{16 - \pi^2}, \\ \lambda &= \frac{4\pi}{16 - \pi^2}, \end{cases} \tag{13}$$

are the parameters of the Gamma distribution; $\gamma(s, x)$ denotes the lower incomplete gamma function [16, (8.350)]; and $\Gamma(s)$ is the complete gamma function [16, (8.310)]. Note that, based on moment matching, by carefully design of the shape parameter $M\mu$ and the scale parameter λ, the gamma distribution is an accurate approximation to match the sum of multiple positive random variables in (11). The parameters $M\mu$ and λ can be determined by matching the first-order and the second-order moments of the original distribution. Moreover, the gamma distribution has been proven to be easy-to-analyze with tractable results and also adopted in a wide range of literature, such as [17, 18].

Substituting (12) into (9), yields

$$P_O = \Pr\left\{ \delta_{ab} < \sqrt{\frac{\gamma_T}{\overline{\gamma_b}}} \right\} = \frac{1}{\Gamma(M\mu)} \gamma\left(M\mu, \lambda \sqrt{\frac{\gamma_T}{\overline{\gamma_b}}} \right). \tag{14}$$

When Alice's transmission power is sufficiently large, asymptotic analysis is conducted to reveal the effects of system parameters on the outage probability. If nominal SNR $\overline{\gamma_b}$ is sufficiently large, by applying the approximation $\gamma(s, x) \to \frac{x^s}{s}$, for $x \to 0$ onto (14), we have

$$P_O \to \frac{\left(\lambda \sqrt{\frac{\gamma_T}{\overline{\gamma_b}}} \right)^{M\mu}}{\Gamma(M\mu + 1)}. \tag{15}$$

Remark 1: Based on the above asymptotic analysis, it is concluded that the diversity order of the outage probability is $M\mu/2$ with respect to the nominal SNR. To meet different requirements of quality of service in wireless networks, the number of IRS' elements M can be derived from (15), which can be used as the design criterion for practical communication systems.

C. Optimal Normalized Power Detection Threshold

To study the detection threshold, we first need to derive the PDF of the composite channel fading coefficients δ_{aw} and δ_{bw} by the following lemma.

Lemma 1 *If the number of elements in IRS M is sufficiently large, the PDF of δ_{aw} can be approximated by the PDF of a zero-mean complex Gaussian distribution with variance equal to M, i.e., $\delta_{aw} \sim \mathcal{CN}(0, M)$.*

Proof: *By using of the optimal phase shift beamforming of IRS in (10), we can rewrite δ_{aw} as*

$$\begin{aligned} \delta_{aw} &= \mathbf{h}_{wr}^T \boldsymbol{\Psi} \mathbf{h}_{ar} \\ &= \sum_{m=1}^{M} |h_{ar,m} h_{wr,m}| e^{j(\theta_{ar,m} + \theta_{wr,m} + \theta_m)} \\ &= \sum_{m=1}^{M} |h_{ar,m} h_{wr,m}| e^{j(\theta_{wr,m} - \theta_{br,m})}, \end{aligned} \tag{16}$$

where $h_{xr,m}, x \in \{a, b, w\}$, is the mth element of $\mathbf{h}_{xr,m}$, and $\theta_{xr,m}$ is the phase of $h_{xr,m}$. Note that both $\theta_{br,m}$ and $\theta_{wr,m}$ are circularly uniformly distributed on $[0, 2\pi)$, and independent of each other. We can easily prove that the difference of $\theta_{br,m}$ and $\theta_{wr,m}$ follows the uniform distribution on $[0, 2\pi)$. According to the central limit theorem (CLT), when M is sufficiently large, δ_{aw} can be approximated to be a *zero-mean complex Gaussian random variable. Since $h_{ar,m}$ and $h_{wr,m}$ are independent of each other, we can obtain the variance of δ_{aw} to be M.* ■

According to Lemma 1, the composite channel fading power $|\delta_{aw}|^2$ and $|\delta_{bw}|^2$ follow an exponential distribution with parameter M, i.e.,

$$f_{|\delta_{aw}|^2}(x) = f_{|\delta_{bw}|^2}(x) = \frac{1}{M} e^{-\frac{x}{M}}, \forall x > 0, \tag{17}$$

by which we can derive the detection error probability of Willie, i.e., P_E by the following theorem.

Theorem 1 *Given the transmission power configuration of Alice and Bob as well as the power detection threshold of Willie, P_E is given by*

$$P_E = \begin{cases} 1, & \text{if } \xi < 0, \\ 1 - \frac{\xi}{M} e^{-\xi/M}, & \text{if } \xi \geq 0, \beta = 1, \\ 1 - \frac{\beta}{1-\beta}\left(e^{-\frac{\xi}{M}} - e^{-\frac{\xi}{M\beta}}\right), & \text{if } \xi \geq 0, \beta \neq 1, \end{cases} \tag{18}$$

where $\xi = \frac{\tau - \sigma^2}{P_b L_b L_w}$ denotes the normalized detection threshold, and $\beta = \frac{P_a L_a}{P_b L_b}$ denotes the ratio of the effective transmission power from Alice to that of Bob. With given channel fading loss L_a and L_b, β indicates the ratio of Alice's power to that of Bob.

Proof: *According to the definition of the false alarm probability, by using (4), P_{FA} can be rewritten as*

$$\begin{aligned} P_{FA} &= Pr\{D_1|H_0\} = Pr\{P_b L_b L_w |\delta_{bw}|^2 + \sigma^2 > \tau\} \\ &= Pr\{|\delta_{bw}|^2 > \xi\}. \end{aligned} \tag{19}$$

Substituting (17) into (19), yields

$$P_{FA} = \begin{cases} 1, & \text{if } \tau < \sigma^2, \\ e^{-\xi/M}, & \text{if } \tau \geq \sigma^2. \end{cases} \tag{20}$$

Similarly, following the definition of the missed detection probability, we have

$$\begin{aligned} P_{MD} &= Pr\{D_0|H_1\} \\ &= Pr\left\{P_a L_a L_w |\delta_{aw}|^2 + P_b L_b L_w |\delta_{bw}|^2 + \sigma^2 < \tau\right\} \\ &= Pr\left\{\beta|\delta_{aw}|^2 + |\delta_{bw}|^2 < \xi\right\}. \end{aligned} \tag{21}$$

By using (17) with necessary mathematical derivation, we can obtain

$$P_{MD} = \begin{cases} 0, & \text{if } \tau < \sigma^2, \\ 1 - (1 + \frac{\xi}{M}) e^{-\xi/M}, & \text{if } \tau \geq \sigma^2, \beta = 1, \\ 1 - \frac{1}{1-\beta} e^{-\frac{\xi}{M}} + \frac{\beta}{1-\beta} e^{-\frac{\xi}{M\beta}}, & \text{if } \tau \geq \sigma^2, \beta \neq 1. \end{cases} \tag{22}$$

Substituting (20) and (22) into (6) leads to the expressions on the detection error probability in (18). ■

In the worst case, Willie always tries to choose the optimal normalized power threshold to minimize P_E, i.e.,

$$\xi^* = \arg\min_{\xi}\{P_E\}. \tag{23}$$

Theorem 2 *With fixed transmission power of Alice and Bob, the optimal normalized power threshold of Willie's detector can be derived as*

$$\xi^* = \begin{cases} M, & \text{if } \beta = 1, \\ \frac{M \ln \beta}{1 - 1/\beta}, & \text{if } \beta \neq 1, \end{cases} \tag{24}$$

and the corresponding minimum detection error probability is

$$P_{E,min} = \begin{cases} 1 - e^{-1}, & \text{if } \beta = 1, \\ 1 - \beta^{\frac{1}{1-\beta}}, & \text{if } \beta \neq 1. \end{cases} \tag{25}$$

Proof: *Considering the close-form expression of P_E in (18), in the case of $\xi < 0$, $P_E = 1$, which cannot be optimized.*

In the case of $\xi \geq 0, \beta = 1$, by taking the derivative of P_E with respect to ξ, and making the derivative be zero, the optimal normalized power threshold can be solved to be $\xi^* = M$, and the minimum detection error probability is thereby $P_{E,min} = 1 - e^{-1}$.

Similarly, in the case of $\xi \geq 0, \beta \neq 1$, the optimal normalized power threshold can be obtained in the same manner as $\xi^* = \frac{M \ln \beta}{1 - 1/\beta}$, and the corresponding minimum detection error probability is $P_{E,min} = 1 - \beta^{\frac{1}{1-\beta}}$. ■

D. Transmission Power Ratio Optimization

To improve the covert communication capacity, Alice chooses to increase the transmission power. Meanwhile, in order to protect covert communications from being detected by Willie, Alice also needs to constrain its transmission power to guarantee that detection error probability falls below a predefined threshold. Therefore, there exists an optimal power ratio for the tradeoff between the transmission outage probability and the detection error probability, which can be mathematically formulated as

$$\beta^* = \arg\max_{\beta} \left\{P_{E,min} \geq (1 - \epsilon)\right\}, \tag{26}$$

where $\epsilon \in [0, 1)$ denotes the detection probability. Specifically, a smaller ϵ leads to better secrecy performance.

Considering the expressions on $P_{E,min}$ in (25), when $\beta = 1$, the minimum detection error probability is fixed as $P_{E,min} = 1 - e^{-1}$ and cannot be optimized. Therefore only the case of $\beta \neq 1$ needs to be discussed.

Theorem 3 *When $\beta \neq 1$, depending on the detection probability of Willie, the optimal transmission power ratio β^* is given by*

$$\beta^* = \mathit{LambertW}(0, \epsilon \ln \epsilon) / \ln \epsilon, \tag{27}$$

where LambertW(k, x) is the kth branch of the Lambert W function [19].

Proof: *Let $\overline{P_E} = 1 - P_{E,min} = \beta^{\frac{1}{1-\beta}}$. Since $\overline{P_E}$ ranges in $(0, 1]$, by taking the natural logarithm on $\overline{P_E}$ and applying derivation with respect to β, we have*

$$\frac{\partial \ln \overline{P_E}}{\partial \beta} = \frac{1}{(1-\beta)^2}\left(\frac{1}{\beta} - 1 - \ln \frac{1}{\beta}\right), \forall \beta \neq 1. \tag{28}$$

Because $\ln(1+x) < x, \forall x > 0$, *with mathematical variable substitution, we have*

$$\begin{cases} \ln(y) < y - 1, & \text{if } y > 1, \\ \ln\left(\frac{1}{y}\right) < \frac{1}{y} - 1, & \text{if } 0 < y < 1. \end{cases} \tag{29}$$

Jointly considering (29) and (28), yields $\frac{\partial \ln \overline{P_E}}{\partial \beta} > 0, \forall \beta \neq 1$. According to the monotonicity of the composite function, *we can conclude that $P_{E,min}$ is a monotonically decreasing function with respect to β. Thus, the optimal transmission power of Alice β^* can be obtained by solving the following equation*

$$P_{E,min} = 1 - \beta^{\frac{1}{1-\beta}} = 1 - \epsilon. \tag{30}$$

With necessary variable substitution $x = \beta \ln \epsilon$, we can rewrite (30) as

$$xe^x = \epsilon \ln \epsilon. \tag{31}$$

In essence, $\mathrm{LambertW}(0, \mathrm{z})$ *is the main branch solution of $xe^x = z$ [19]. By using the definition of the Lambert W function, the expression on β^* in (27) can be proven.* ■

Specifically, when detection probability ϵ is small enough, by using the Taylor series expansion of $\mathrm{LambertW}(0, x)$ [19, (3.1)], we can obtain

$$\mathrm{LambertW}(0, x) = \sum_{n=1}^{\infty} \frac{(-n)^{n-1}}{n!} x^n \to x, \text{if } x \to 0, \tag{32}$$

In the case that $\epsilon \to 0$, substituting (32) into (27), yields

$$\beta^* \to \epsilon. \tag{33}$$

Remark 2: There is an important observation from (33) that when the detection probability of Willie is sufficiently small, the optimal transmission power ratio of Alice to Bob is equal to the detection probability, which can be used as the power design reference for the two-way relaying networks.

IV. Simulation and Discussion

Simulation results are provided to verify the theoretical analysis for the IRS-assisted two-way relaying covert communications. The system simulation parameters are listed as follows: the reference path loss is $L_0 = 10^{-3}$, and the noise power is $\sigma^2 = -106$ dBm. Without loss of generality, the symmetrical network topology is considered, and the distances from different nodes to IRS are set as $d_a = d_b = d_w = 50$ m. Note that the nominal SNR defined in (9) denotes the average received SNR at Bob, which counts in the path loss. The path loss exponent is set as $\alpha = 2.5$. Specifically, the larger path loss exponent introduces severer propagation attenuation on average, results in a lower nominal SNR and thus the system performance degradation. Similarly, the larger distances d_a, d_b introduce a higher path loss and result in an increased outage probability of the covert link. Based on (14), we can observe that d_w does not impact the outage performance.

First, the impacts of nominal SNR $\overline{\gamma_b}$ on the outage probability of the covert link is shown in Fig. 2, where the transmission power configuration is $\beta = 1$, i.e., the P_b is set the same as P_a; the predefined capacity threshold is $C_T = 1$ bps/Hz, and the number of IRS elements varies from 1 to 4. Both the exact results produced by (14) and the asymptotic results given by (15) are compared with the numerical results generated by Monte Carlo simulations in different SNR configurations. From this figure, it is observed that the simulation curves match perfectly with the theoretical analysis. In the high SNR region, the simulation curves converge to the corresponding asymptotic curves. All results validate the accuracy of the approximate analysis in (14) and (15) with different numbers of reflecting elements M. Furthermore, Fig. 2 shows that larger M leads to decrease the outage probability of the covert link via IRS. This is simply because a larger number of reflecting elements on an IRS introduces a higher degree of freedom and hence a higher diversity order, which can be exploited by the MRT beamforming algorithm in (10) to yield better outage performance. Specifically, the diversity order of the outage probability monotonically increases with M, which is consistent with *Remark 1*.

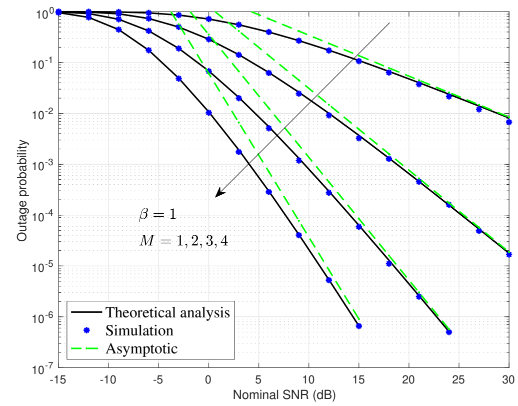

Fig. 2. Nominal SNR versus the outage probability of the covert link.

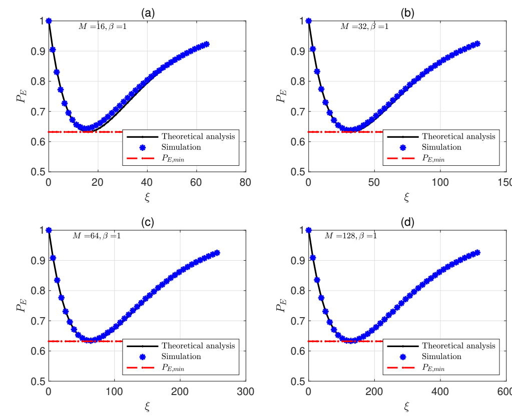

Fig. 3. Normalized power threshold versus the detection error probability.

The impact of normalized power threshold ξ on the detection error probability P_E with $\beta = 1$ is demonstrated in Fig. 3. where M changes from 16 to 128. The simulation results are compared with the theoretical expressions on P_E in (18), as well as on $P_{E,min}$ in (25). [3] From these figures, we can observe that, when the number of IRS' elements is set as $M = 16$, the theoretical expressions on P_E and $P_{E,min}$ match well with the simulation results. Moreover, with a fixed transmission power ratio β, there exists an optimal normalized power threshold ξ^*, which results in the lowest P_E. This can

[3]When the number of reflecting elements is not large enough, say $M < 16$, the approximate distribution in 17, which is derived from CLT, is not sufficiently accurate. Thus, it will result in the difference between the theoretical analysis and the simulation results.

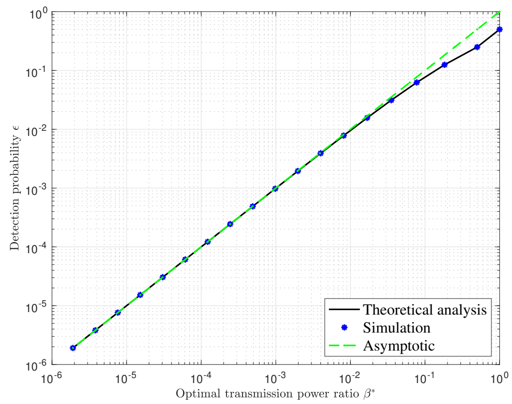

Fig. 4. Relationship between the optimal transmission power ratio and the detection probability.

be explained by the fact that a smaller ξ increases the false alarm probability, while a larger ξ raises the missed detection probability. Therefore, an optimal normalized power threshold can be chosen by Willie as per Theorem 2.

The relationship between the optimal transmission power ratio β^* and detection probability ϵ is shown in Fig. 4, under the assumption that Willie is always capable of choosing the optimal normalized power threshold ξ^* given in (24). Both the theoretical analysis on β^* in (27) and its asymptotic expression in (33) are compared with the simulation results. As clearly illustrated in this figure, detection probability of Willie ϵ changes from 10^{-6} to 1. Also, it is observed that the theoretical analysis curve matches closely with the simulation curve. Specifically, when the detection probability is small, e.g., $\epsilon < 10^{-1}$, the simulation results can be precisely predicted by the asymptotic expression. From this figure, we can also see that a larger detection probability requirement ϵ leads to a larger optimal transmission power ratio β^*, because large transmission power at Alice facilitates the detection of Willie and makes the messages prone to be overheard.

V. Conclusion

In this paper, a two-way relaying protocol was adopted in IRS-assisted full-duplex relaying networks to protect covert communications from being detected by a warden. Approximating by the Gamma distribution, we derived the close-form expression, as well as the asymptotic expression in the regime of large transmission power, for the outage probability of the covert link. Based on the asymptotic analysis, we obtained the diversity order of covert communications in the considered scenario. We studied the worst case and obtained the optimal normalized power threshold of Willie's detector under a complex Gaussian distribution approximation. To meet the requirement of the detection error probability in the worst case, the ratio of the transmission power between the covert nodes was also investigated. We proved a crucial property that, when the detection probability is sufficiently small, the optimal transmission power ratio is equal to the detection probability. Furthermore, the proposed scheme can be adopted in the future covert communications, and the obtained results can be used as the guidelines for the system design.

References

[1] S. Atapattu, R. Fan, P. Dharmawansa, G. Wang, J. Evans, and T. A. Tsiftsis, "Reconfigurable intelligent surface assisted two–way communications: Performance analysis and optimization," *IEEE Transactions on Communications*, vol. 68, no. 10, pp. 6552–6567, July 2020.

[2] Z. Peng, Z. Zhang, C. Pan, L. Li, and A. L. Swindlehurst, "Multiuser full-duplex two-way communications via intelligent reflecting surface," *IEEE Transactions on Signal Processing*, vol. 69, pp. 837–851, Jan. 2021.

[3] B. A. Bash, D. Goeckel, and D. Towsley, "Limits of reliable communication with low probability of detection on AWGN channels," *IEEE Journal on Selected Areas in Communications*, vol. 31, no. 9, pp. 1921–1930, Aug. 2013.

[4] L. Lv, Q. Wu, Z. Li, Z. Ding, N. Al-Dhahir, and J. Chen, "Covert communication in intelligent reflecting surface-assisted NOMA systems: Design, analysis, and optimization," *IEEE Transactions on Wireless Communications*, vol. 21, no. 3, pp. 1735–1750, Mar. 2022.

[5] L. Lv, Q. Wu, Z. Li, N. Al-Dhahir, and J. Chen, "Secure two-way communications via intelligent reflecting surfaces," *IEEE Communications Letters*, vol. 25, no. 3, pp. 744–748, Mar. 2021.

[6] T. Wang, M.-A. Badiu, G. Chen, and J. P. Coon, "Outage probability analysis of STAR-RIS assisted noma network with correlated channels," *IEEE Communications Letters*, pp. 1–1, 2022.

[7] ——, "Outage probability analysis of RIS-assisted wireless networks with Von Mises phase errors," *IEEE Wireless Communications Letters*, vol. 10, no. 12, pp. 2737–2741, Sep. 2021.

[8] P. Xu, W. Niu, G. Chen, Y. Li, and Y. Li, "Performance analysis of RIS-assisted systems with statistical channel state information," *IEEE Transactions on Vehicular Technology*, vol. 71, no. 1, pp. 1089–1094, Nov. 2022.

[9] P. Xu, G. Chen, G. Pan, and M. D. Renzo, "Ergodic secrecy rate of RIS-assisted communication systems in the presence of discrete phase shifts and multiple eavesdroppers," *IEEE Wireless Communications Letters*, vol. 10, no. 3, pp. 629–633, Dec. 2021.

[10] S. Atapattu, R. Fan, P. Dharmawansa, G. Wang, J. Evans, and T. A. Tsiftsis, "Reconfigurable intelligent surface assisted two–way communications: performance analysis and optimization," *IEEE Transactions on Communications*, vol. 68, no. 10, pp. 6552–6567, Jul. 2020.

[11] D. Deng and et al., "Relay selection for cooperative NOMA system over correlated fading channel," *Phycom*, vol. 35, pp. 702–709, Jan. 2019.

[12] J. Hu, S. Yan, F. Shu, and J. Wang, "Covert transmission with a self-sustained relay," *IEEE Transactions on Wireless Communications*, vol. 18, no. 8, pp. 4089–4102, Jun. 2019.

[13] J. Jose, P. Shaik, and V. Bhatia, "VFD-NOMA under imperfect SIC and residual inter-relay interference over generalized Nakagami-m fading channels," *IEEE Communications Letters*, vol. 25, no. 2, pp. 646–650, Jan. 2021.

[14] L. Lv, H. Jiang, Z. Ding, L. Yang, and J. Chen, "Secrecy-enhancing design for cooperative downlink and uplink NOMA with an untrusted relay," *IEEE Transactions on Communications*, vol. 68, no. 3, pp. 1698–1715, Mar. 2020.

[15] Y. Chen, "Performance of ambient backscatter systems using reconfigurable intelligent surface," *IEEE Communications Letters*, vol. 25, no. 8, pp. 2536–2539, Sep. 2021.

[16] I. Gradshteyn and I. Ryzhik, *Table of Integrals, Series, and Products, 7th Ed.* San Diego, California 92101-4495, USA: Academic Press, Elsevier Inc, 2007.

[17] S. Al-Ahmadi and H. Yanikomeroglu, "On the approximation of the generalized-K distribution by a gamma distribution for modeling composite fading channels," *IEEE Transactions on Wireless Communications*, vol. 9, no. 2, pp. 706–713, Sep. 2010.

[18] K. P. Peppas, G. P. Efthymoglou, and A. G. Kanatas, "Approximations to the distribution of the sum of generalized normal RVs using the moments matching method and its applications in performance analysis of equal gain diversity receivers," *IEEE Transactions on Vehicular Technology*, vol. 67, no. 8, pp. 7230–7241, Apr. 2018.

[19] R. Corless, G. Gonnet, D. Hare, D. Jeffrey, and D. Knuth., "On the Lambert W function," *Advances in Computational Mathematics*, vol. 5, no. 1, pp. 329–359, Aug. 1996.