Explosive Cooperation in Social Dilemmas on Higher-Order Networks

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Understanding how cooperative behaviors can emerge from competitive interactions is an open problem in biology and social sciences. While interactions are usually modeled as pairwise networks, the units of many real-world systems can also interact in groups of three or more. Here, we introduce a general framework to extend pairwise games to higher-order networks. By studying social dilemmas on hypergraphs with a tunable structure, we find an explosive transition to cooperation triggered by a critical number of higher-order games. The associated bistable regime implies that an initial critical mass of cooperators is also required for the emergence of prosocial behavior. Our results show that higher-order interactions provide a novel explanation for the survival of cooperation.

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Introduction.—The pervasiveness of cooperation in our world has long puzzled researchers [1,2]. After all, the natural world, and human society is not an exception, obeys Darwinian selection, which is driven by the self-interest of individuals. In such a competitive world, costly altruistic behaviors seem inappropriate, since they do not bring any immediate advantage to the cooperators [3–6]. It is instead more profitable for self-interested individuals to defect, exploiting the benefits from the actions of cooperators who, in turn, see their sustainability jeopardized by the higher profits of free-riders [7,8].

Social dilemmas are a well-known theoretical framework for studying cooperation. In a social dilemma, each actor in a group can choose either to cooperate or to defect [9,10]. Cooperating benefits the group at an individual cost, while defectors exploit collective benefits provided by cooperators without paying any cost [11,12]. Therefore, while cooperation would be the best outcome from a group perspective, defection is the favoured strategy by selfish rational decision-makers. This tension between the two strategies defines the dilemma [13–16]. Social dilemmas are typically studied in evolutionary game theory [3,7,17–21] by implementing games, such as the Prisoner's Dilemma (PD), on structured populations [22–24]. The underlying structure of a population is usually modeled as a network, where links represent the interactions between pairs of agents [25–27]. In some cases, the structure of the network has been shown to promote prosocial behaviors through, e.g., mechanisms of network reciprocity [4,28,29], the heterogeneity of the nodes [30–33], and the presence of clustering [34]. Networks are however limited in their representation of real-world systems. The links of a network can indeed only describe pairwise interactions, while the units of a complex system can also interact in groups of more than two. Thus, networks do not allow for the accommodation of more realistic and general forms of higher-order social interactions.

In recent years, mathematical structures like hypergraphs and simplicial complexes have been used to represent interactions among three or more units [35–37]. From contagion processes [38] to synchronization [39–41] and ecological competition [42], various studies have illustrated that higher-order interactions can lead to the emergence of collective behaviors and dynamic patterns not seen in pairwise networks [36]. Since its origin [43], game theory has been formulated as an n-body problem. Therefore, it comes as no surprise that higher-order interactions have attracted attention also in the study of evolutionary game theory [11,44–50]. However, a general framework for social dilemmas on structured populations with group interactions is still missing. In fact, when higher-order payoffs for n-body games have been considered, it has been for well-mixed populations or for simple pairwise networks, such as regular lattices [12,44,45,47,51,52]. When, instead, more general interaction patterns have been considered, it has been for specific problems and payoff structures relying on strong assumptions [50,53,54]. For example, when hypergraphs were used to model group interactions at the microscopic level [49,55,56], they often employed a linear function of the number of cooperators in a group as the group payoff, limiting the general representation of social dilemma dynamics [11].

In this Letter, we introduce a general framework to extend social dilemmas to structured populations accounting for interactions in groups of variable size. In our model, the players are the nodes of a hypergraph and are involved, at the same time, in both pairwise and higher-order games as represented by hyperedges of different sizes. We do so by assigning a payoff tensor of dimension n to each hyperedge of size n. In this way, our model combines n-body games [12,44,45,47,57] with the potential of higher-order networks in representing the most general microscopic structure of the interactions [48,55,58,59]. By studying the evolutionary dynamics of the model on different types of hypergraphs, we find that the presence of higher-order interactions and their microscopic structure play an important role in the survival of cooperation in social dilemmas. In fact, above a critical number of higher-order interactions, the dynamics can show an explosive transition to a bistable state [60–62], where besides full defection (the only stable equilibrium for the pairwise PD) a cooperative stable state emerges. We provide an analytical characterization of the observed phase transition and its dependence on the parameters of the game. In particular, we found that an initial critical mass of cooperators is also needed to sustain cooperation in the long term: below this critical mass, every player becomes a defector, even if the number of higher-order interactions is above the critical threshold.

The model.—We consider a population of N players taking part in a number M of different games, which can either be pairwise or in groups of three or more players. Such interactions are described by a hypergraph $\mathcal{H}(\mathcal{V}, \mathcal{E})$, where $\mathcal{V}$ is the set of $|\mathcal{V}| = N$ nodes representing players, and $\mathcal{E}$ is the set of $|\mathcal{E}| = M$ hyperedges [35,36]. Each hyperedge e_g, with $g \in \{1, \ldots, M\}$, is a group (a subset of $\mathcal{V}$) of two or more players interacting in game g. The hypergraph can be represented by an $N \times M$ incidence matrix B, whose entry b_{ig} is equal to 1 if player i is playing game g, and is zero otherwise. The number of games in which a player i takes part is given by the hyperdegree $k_i = \sum_{g=1}^{M} b_{ig}$, while the number of players in a game g is the size of the hyperedge $q_g = |e_g| = \sum_{i=1}^{N} b_{ig}$. We focus here on the case of hypergraphs with hyperedges of size two (2-hyperedges, or simply edges) and three (3-hyperedges), respectively, corresponding to classical pairwise games (2-games) and games played in groups of three players (3-games). Regarding payoffs, since there are q_g players involved in a symmetric game g, if we indicate as n_s the number of different strategies available, the total number of different payoffs is $n_s\binom{n_s+(q_g-1)-1}{(q_g-1)}$ (see Supplemental Material [80], SM). Here we consider only $n_s = 2$ possible strategies, cooperation (C) and defection (D), as in the classical pairwise social dilemmas, resulting in four possible different payoffs for 2-games and six for 3-games. As usual, the payoffs for 2-games can be displayed as a 2×2 matrix Π, whose element $\pi_{s_i s_j} = [\pi_{s_i}(s_j), \pi_{s_j}(s_i)]$ is the pair of payoffs for player i and j, respectively, when the first player plays strategy s_i and the second s_j. Generalizing to interactions in groups of three players, payoffs for 3-games can then be represented as a $2 \times 2 \times 2$ tensor $\mathcal{T}$, whose element $\tau_{s_i s_j s_k} = [\tau_{s_i}(s_j, s_k), \tau_{s_j}(s_i, s_k), \tau_{s_k}(s_i, s_j)]$ is now a 3-tuple with the value of the payoff for each of the three players i, j, and k, playing strategies s_i, s_j, s_k. The complete payoff structure for both 2-games ($q_g = 2$) and 3-games ($q_g = 3$) is shown in Fig. 1, using different symbols for different payoff values. As commonly done in the study of social dilemmas, without loss of generality we set the payoff for mutual cooperation equal to 1, while the payoff for mutual defection is equal to 0, for both 2-games and 3-games [53]. In a similar manner, i.e., independently from the number of players (2 or 3) in the game, with blue triangle and green square we indicate the payoffs received for unilaterally deviating from mutual cooperation and defection respectively. In this way, it is immediate to identify in blue triangle and in green square the payoffs usually denoted, in pairwise social dilemmas, as the temptation T and the sucker's payoff S. Identifying T and S is crucial for the characterization of the game. The values of T and S classify classical pairwise games into four different types, each with different Nash equilibria (NE): the Prisoner's Dilemma ($T > 1$, $S < 0$), the Chicken game ($T > 1$,

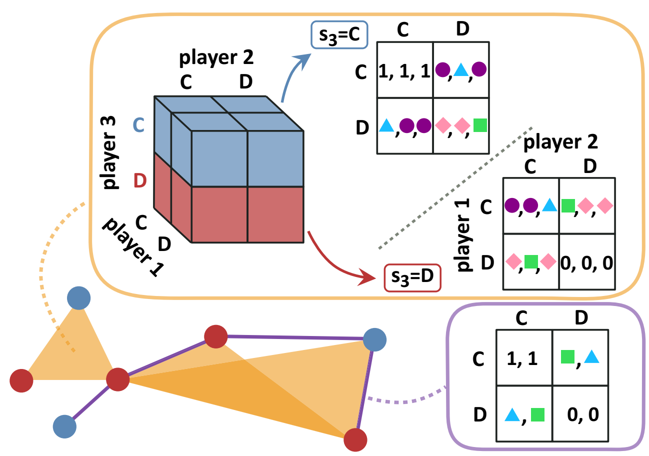

FIG. 1. Higher-order games on a hypergraph. The orange triangular areas are hyperedges of size $q_g = 3$, corresponding to games played by three players (3-games), while the purple segments are hyperedges of size $q_g = 2$, representing pairwise games (2-games). The payoff structures of symmetric 2-games and 3-games are reported in the two boxes.

$S > 0$), the Stag Hunt game ($S < 0, T < 1$), and the Harmony game ($S > 0, T < 1$) (see SM). Hence, we propose extending the same classification to 3-games. In 3-games there are two additional payoffs, for defection against a cooperator and a defector (pink diamond namely, W), and for cooperation against a cooperator and a defector (purple circle namely, G). Depending on the relative value of these two additional payoffs (if $G > W$ or $G < W$) each type of 3-games is divided into two subsets with different Nash Equilibria. As shown in the SM, by restricting G and W to the range $0 \leq G, W \leq 1$ (as we do for our results) the resulting 3-games are social dilemmas [12,63,64].

*Stochastic simulations.—*To investigate the effects of higher-order interactions on the equilibria of a system with N players, we considered the following stochastic evolutionary game dynamics. We start with a population having an initial fraction ρ_0 of cooperators. At each time step, one player (the focal) is selected at random, and a second player (the model) is chosen among the neighboring nodes on the hypergraph, connected to the focal player by hyperedges of any size. Each of the two selected players plays a 2-game with all its neighbors connected through a 2-hyperedge, and a 3-game for each 3-hyperedge it takes part in. A 2-game is completely defined by the values of the payoff matrix entries T and S, while the 3-game has the same T and S of the 2-game, but is also defined by the payoffs G and W. In each game, the focal (respectively, the model) player earns a payoff based on their strategy and the strategies of the other players involved in that particular game. The total payoff π_f of the focal player (π_m of the model player) is the sum of all the game payoffs. The focal player has then the possibility to adopt the strategy of the model player s_m, with a probability which is a nondecreasing function of the total payoff difference $\pi_m - \pi_f$, modeled as a Fermi function [9,19,65]: $p_{s_f \to s_m} = \{1 + \exp[-w(\pi_m - \pi_f)]\}^{-1}$ where w represents the strength of selection [66]. We iterate the stochastic dynamics to compute the quasistationary (QS) probability distribution [67,68] of the fraction of players adopting strategy C (cooperators). This distribution is the stationary distribution of the stochastic process conditioned on nonextinction [69], and for a wide class of stochastic processes, its properties have been shown to converge to the stationary properties when $N \to \infty$ [70,71]. In particular, in the context of evolutionary game theory, the QS distribution local maxima can converge, in unstructured populations and homogeneous random networks, to the stable fixed points ρ^* of the mean-field deterministic evolution given by the replicator dynamics [50,72]. As for the underlying structure of interactions, we have considered random hypergraphs in which we can control the number of higher-order interactions. By forming independently 2- and 3-hyperedges we have constructed random hypergraphs of N nodes with tunable average hyperdegree $\langle k \rangle = \sum_{i=1}^{N} k_i / N$ and probability $\delta = n_\Delta / (N \langle k \rangle)$ for a player to interact in a

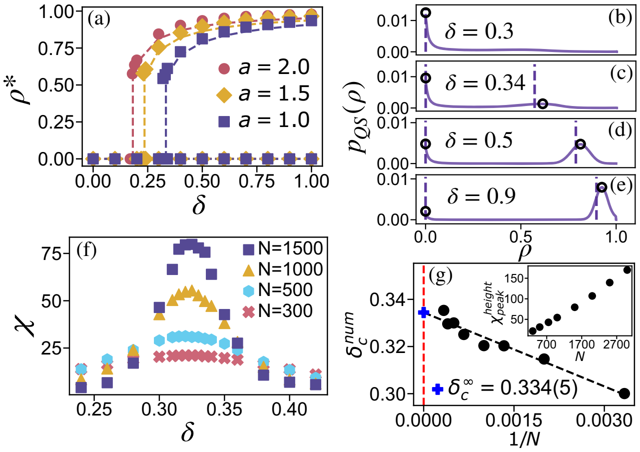

FIG. 2. (a) Fraction of cooperators at equilibrium for the PD on random hypergraphs with $N = 1500$, $\langle k \rangle = 20$ and tunable ratio δ of three-player interactions. (b)–(e) Quasistationary distributions for $a = 1$ and four values of δ. Continuous curves and symbols represent the simulation results averaged over 1500 runs, while dashed lines are the analytical mean-field predictions of Eqs. (3) and (4). (f) Susceptibility χ as a function of δ for increasing N. (g) Scaling of the position and height (inset) of the peak of χ. The black line is the fitting.

3-game. Here, $N\langle k \rangle = n_/ + n_\Delta$, where $n_/$ and n_Δ are, respectively, the number of two-player interactions (the number of 2-hyperedges multiplied by 2) and the number of three-player interactions (the number of 3-hyperedges multiplied by 3) in the system (see SM).

Figure 2 shows the results for the case of the Prisoner's Dilemma (PD). We recall that the pairwise PD is defined by payoff values $T > 1$ and $S < 0$. In particular, for our simulations we chose $T = 1.1$, $S = -0.1$ and strength of selection $w = 1/\langle k \rangle$ [73]. As for the 3-game we consider a social dilemma with the same values of T and S of the pairwise PD, and with G and W such that $0 \leq G, W \leq 1$ and $(G - W) > 0$, since in this case the one-shot 3-game has four different pure NE: full defection (D, D, D) and all the permutations of two cooperators and one defector (see SM). Figure 2(a) reports the fraction of cooperators at equilibrium as a function of the fraction δ of 3-game interactions, for different values of $a = 2(G - W)$. Symbols represent the simulation results obtained from the peaks of the QS distribution $p_{\mathrm{QS}}(\rho)$ in panels (b)–(e). A bifurcation is observed when the fraction δ exceeds a critical value $\delta_c(a)$. For $\delta < \delta_c$ the only stable fixed point, as in the pairwise PD, is full defection $\rho_D^* = 0$ [74], while for $\delta \geq \delta_c$ we have bistability, with a new stable state ρ_+^* appearing due to the effect of higher-order interactions. The observed phase transition is explosive [60–62] as, for $\delta \geq \delta_c$, the cooperative phase has $\rho_+^* \geq 0.5$. To better characterize the phase transition we have computed the susceptibility $\chi = N(\langle \rho^2 \rangle - \langle \rho \rangle^2)$ as a function of δ for different hypergraph sizes N. In first-order transitions, this

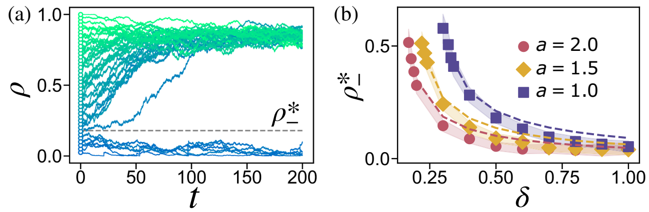

FIG. 3. Basins of attraction and critical mass of cooperators for the PD on random hypergraphs. (a) Temporal evolution of the fraction of cooperators for various initial conditions and $\delta = 0.4$, $a = 1.5$, $\langle k \rangle = 20$. (b) Unstable stationary state ρ_-^* as a function of δ for average hyperdegree $\langle k \rangle = 20$ and different values of a. Symbols show the simulation results, while the lines are the analytical mean-field predictions. The shaded areas represent the errors.

susceptibility peaks around the value of the control parameter where the new phase ρ_+^* appears, and the peak diverges in the limit $N \to \infty$ because the system oscillates between the two phases [75]. Figure 2(f) shows that, in our case, χ becomes more pronounced with increasing hypergraph size N. The critical value of δ in the thermodynamic limit can be extracted through a finite-size scaling analysis. The results are reported in Fig. 2(g), where $\delta_c^\infty = 0.334(5)$ is obtained as the y intercept of the fitting of the critical $\delta_c^{\text{num}}(N)$ measured numerically (see SM) [75]. Figure 3(a) illustrates the typical time evolution of the system. It reports the fraction of cooperators $\rho(t)$ as a function of time for 20 different initial conditions ρ_0. We notice that when ρ_0 is smaller than a given threshold ρ_-^*, the dynamics typically converges to the full defection state. Conversely, when $\rho_0 > \rho_-^*$, it converges to the stable state ρ_+^* where a finite fraction of the population are cooperators. In other words, ρ_-^* represents the initial critical mass of cooperators needed for cooperation to survive in the long term. Figure 3(b) shows that ρ_-^* is a decreasing function of δ for any value of the parameter a. This implies that smaller initial densities of cooperators are sufficient to sustain stable cooperation in systems with a larger fraction δ of 3-game interactions (see SM for further details on the stochastic simulations). Notice that the results above depend on the topology of the hypergraph, as we found that a cooperative state is still possible, but for $\delta \geq \delta_c$ we do not observe bistability in the more constrained case of regular lattices with a tunable number of higher-order interactions (see SM).

Analytical results.—To better understand the influence of higher-order interactions on the game outcome, we analytically examined the case of a well-mixed population, where each player interacts either in a 3-game with probability δ or in a 2-game with probability $1-\delta$. The dynamics of the fraction ρ of cooperators for a well-mixed population in the thermodynamic limit is described by the mean-field replicator equation (RE) [7,19,76,77]:

$$\frac{d\rho}{dt} = \rho(1-\rho)[\pi_C(\rho,\delta) - \pi_D(\rho,\delta)] \tag{1}$$

where π_C and π_D are the expected payoffs of a cooperator and a defector, functions of the density of cooperators ρ and of the fraction δ of 3-game interactions (see SM). Hence, the payoff difference is also a function of ρ and δ:

$$\pi_C - \pi_D = -\rho^2 c\delta + \rho(c\delta - b - 2S) + S \tag{2}$$

where $c = (a+b)$, $b = T - S - 1$ and $a = 2(G-W)$. Therefore, besides the two absorbing states full-defection $\rho_D^* = 0$ and full-cooperation $\rho_C^* = 1$, Eq. (1) has two nontrivial stationary states $\rho_\pm^*$ for which $\pi_C - \pi_D = 0$:

$$\rho_\pm^* = \frac{c\delta - b - 2S \pm \sqrt{(c\delta - b)^2 + 4S(b+S)}}{2c\delta}. \tag{3}$$

The existence of real-valued $\rho_\pm^*$ depends on the discriminant $\Delta = (c\delta - b)^2 + 4S(b+S) \geq 0$. Given that $(c\delta - b)^2$ is always positive, a sufficient condition for the existence is $4S(b+S) = 4S(T-1) > 0$, which is always satisfied for the Stag-Hunt game and Chicken game. For the Prisoner's Dilemma and the Harmony game instead $\Delta \geq 0$ holds only for certain values of the parameters. In particular, for the game we are focusing on in this Letter, namely, the PD, we have $T > 1$ and $S < 0$, hence $b = T - S - 1 > 0$. Also, $c = a + b > 0$, as we are considering a 3-game with $a = 2(G-W) > 0$. This leads to positive real-valued $\rho_\pm^*$ when

$$\delta \geq \delta_1^{\text{th}} = \frac{b + \sqrt{-4S(b+S)}}{c}. \tag{4}$$

In particular, for $\delta = \delta_1^{\text{th}}$, where the two solutions ρ_+^* and ρ_-^* appear and coincide ($\Delta = 0$), they take the value $\rho_\pm^*(\delta_1^{\text{th}}) = 0.5 - (b+2S)/[2(b + \sqrt{-4S(b+S)})]$, while for $\delta > \delta_1^{\text{th}}$ we have $0 < \rho_-^* < \rho_\pm^*(\delta_1^{\text{th}}) < \rho_+^* < 1$ (see SM). A stability analysis of the solutions reveals that, while $\rho_D^* = 0$ and ρ_+^* are stable, ρ_-^* and $\rho_C^* = 1$ are unstable stationary states. Therefore, Eq. (4) gives us the mean-field critical threshold δ_1^{th} of three-player interactions for cooperation to survive in the higher-order PD. In fact, if δ is below this critical threshold the only stable stationary state is full defection $\rho_D^* = 0$, as in the pairwise PD. If instead the fraction of 3-game interactions δ exceeds δ_1^{th}, an explosive transition to a bistable state emerges, where both $\rho_D^* = 0$ and $0 < \rho_+^* < 1$ are stable stationary states. In Figs. 2(a)–2(e) the analytical mean-field results are reported as dashed lines. In particular, the analytical predictions for the stable states ρ_+^* and ρ_D^* are in perfect agreement with the peaks of the quasistationary distributions in Figs. 2(b)–2(e) and with the symbols in panel (a) reporting the stable fixed points obtained through stochastic simulations on random hypergraphs. At the same time, the critical fraction of 3-game interactions δ_1^{th} [vertical lines in Fig. 2(a)] accurately marks the discontinuous transition to bistability observed numerically, coinciding with the appearance of

$\rho^*_{\pm}(\delta_1^{\text{th}})$. In particular, for the specific values of the parameters used in our simulations, we have $\delta_1^{\text{th}} = 0.\bar{3}$ and $\rho^*_{\pm}(\delta_1^{\text{th}}) = 0.5$, in perfect agreement with the simulation results. Figure 3 displays the unstable solution ρ^*_-, which defines the basins of attraction of the two stable stationary states ρ^*_D and ρ^*_+, showing again a good agreement between the mean-field predictions (dashed lines) and the stochastic simulations [trajectories in Fig. 3(a) and symbols in Fig. 3(b)].

Conclusions.—In this Letter, we introduce a general game theory framework to study social dilemmas when both pairwise and higher-order interactions are possible. Our main finding is that cooperation can persist even in scenarios like the PD, where pairwise interactions typically lead to full defection. The transition to a stable cooperative state is explosive when the number of higher-order interactions surpasses a critical threshold determined by game parameters. The presence of bistability, however, indicates that the survival of cooperators is not guaranteed: a critical mass of initial cooperators is needed to sustain stable prosocial behavior. This is in agreement with empirical observations regarding the critical mass of initiators required to trigger social and cultural changes [78,79]. Our findings show that higher-order interactions can foster cooperation in competitive settings, offering a novel solution to social dilemmas. While we focus on the PD in this Letter, our higher-order framework readily applies to other games. We also hope our work inspires systematic investigations into the impact of various real-world features, such as different topologies of higher-order networks and temporal changes in their connectivity, on evolutionary game dynamics.

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