Generalized Selection Combining for Cognitive Relay Networks over Nakagami-m Fading

Yansha Deng, *Student Member, IEEE,* Lifeng Wang, *Student Member, IEEE,* Maged Elkashlan, *Member, IEEE,*
Kyeong Jin Kim, *Senior Member, IEEE,* and Trung Q. Duong, *Senior Member, IEEE*

Abstract—We consider transmit antenna selection with receive generalized selection combining (TAS/GSC) for cognitive decode-and-forward (DF) relaying in Nakagami-m fading channels. In an effort to assess the performance, the probability density function and the cumulative distribution function of the end-to-end SNR are derived using the moment generating function, from which new exact closed-form expressions for the outage probability and the symbol error rate are derived. We then derive a new closed-form expression for the ergodic capacity. More importantly, by deriving the asymptotic expressions for the outage probability and the symbol error rate, as well as the high SNR approximations of the ergodic capacity, we establish new design insights under the two distinct constraint scenarios: 1) proportional interference power constraint, and 2) fixed interference power constraint. Several pivotal conclusions are reached. For the first scenario, the full diversity order of the outage probability and the symbol error rate is achieved, and the high SNR slope of the ergodic capacity is 1/2. For the second scenario, the diversity order of the outage probability and the symbol error rate is zero with error floors, and the high SNR slope of the ergodic capacity is zero with capacity ceiling.

Index Terms—Cognitive relay network, generalized selection combining, Nakagami-m fading.

I. Introduction

The conflict between the stringent demand for high data rate and data service on the one hand, and the unbalanced spectrum occupation in time and geographic domains on the other hand, has become a challenge for future wireless systems [1]. To cope with this, cognitive radio, first coined by Mitola, has rekindled increasing interest in the efficient use of radio spectrum. In the underlay paradigm, the secondary users (SUs) are allowed to access the spectrum allocated to primary users (PUs) as long as the interference generated by the secondary transmission is restricted below a certain threshold, namely, interference temperature [2]. The constrained transmit power

at SU typically results in unstable transmission and restricted coverage, which drives the demand for robust transmission techniques suited for networks that are subject to power and interference constraints [3]. Relaying is regarded as a cost-effective approach for supporting high speed and long distance networks [4, 5].

The majority of the studies on cognitive relay networks have focused on single antenna protocols [6–8]. Multiple-input multiple-output (MIMO) techniques, well-known for their many benefits including enhanced reliability [9], spectral efficiency [10], and co-channel interference suppression [11], open up new dimensions for cognitive radio. For example, as shown in [12], multi-antennas are utilized at SU to achieve spatial multiplexing. In [13], the novel distributed antenna selection is proposed in relaying system. In [14], the effect of transmit antenna selection with receive maximal ratio combining (TAS/MRC) on the the ergodic capacity was analyzed. In [15], the outage performance of TAS/MRC and TAS/SC are examined over Nakagami-m fading channel. It is shown in [16] that the diversity order is independent of the number of PUs and the selected number of receive antennas at SU.

Different from the aforementioned works, in this paper, we consider cognitive relay networks from the viewpoint of TAS/GSC as an effective design to enhance the reliability of the secondary network and to mitigate interference to the primary network. From a power perspective, cognitive spectrum sharing with network cooperation addresses fundamental constraints on the transmit power at the SUs, while keeping the interference temperature at the PUs to a minimum [17]. On the one hand, TAS is acknowledged as a core component for uplink 4G long term evolution (LTE) and LTE Advanced systems because of its low feedback requirement compared with closed-loop transmit diversity [18]. On the other hand, with the merits of low power demand and RF cost, GSC offers a performance/implementation tradeoff between MRC and selection combining (SC) for the secondary network [19, 20][1]. Additionally, by excluding the antenna chains with weak channel powers, GSC can be more robust to channel estimation errors than MRC [21]. In [22], it is shown that GSC outperforms MRC in a non-identically distributed noise scenario.

The objective of this paper is to examine the impact of TAS/GSC in underlay cognitive relay networks over

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[1]GSC is well applied to commercial wireless networks where the receiver is subject to resource constraints, such as limited RF chains due to size and complexity limitations [26].

Nakagami-m fading. The Nakagami-m fading environment is considered due to its versatility in providing a good match to various empirically obtained measurement data [23]. In the secondary network, a single antenna which maximizes the signal-to-noise ratio (SNR) is selected at the secondary transmitter, while a subset of receive antennas with highest SNRs are combined at the secondary receiver. For coverage and reliability enhancement, a decode-and-forward (DF) relay is used in the secondary network to assist the secondary transmission. Note that the transmit powers at the secondary source (S) and the secondary relay (R) are limited by two constraints: 1) the peak interference constraint at the primary receiver, and 2) the peak transmit power constraint at S and R. It is also important to note that the performance of underlay spectrum sharing is typically restricted due to these two strict power constraints. With the help of TAS/GSC relaying, less transmit power is required at S and R, which in turn reduces the interference at the PU, allowing for high speed data services over wide area coverage. The main contributions of this paper are summarized as follows.

- We derive new exact closed-form expressions for the cumulative distribution function (CDF) of the SNR with TAS/GSC. Although the CDF expressions were presented in [19, 24] with the aid of the trapezoidal rule, they are not in closed-form and cannot be used to derive the CDF of the SNR with TAS/GSC.

- We derive new exact closed-form expressions for the outage probability and the symbol error rate (SER) to accurately assess the joint impact of antenna configuration and channel fading. We further derive the asymptotic expressions for the outage probability and the SER under the two cases: 1) proportional interference power constraint, and 2) fixed interference power constraint. We confirm that the full diversity order is achieved for the proportional interference power constraint. For the fixed interference power constraint, the diversity order is zero with error floors in the high SNR regime.

- We derive an exact closed-form expression for the ergodic capacity. Notably, this is the first closed-form expression for cognitive relay networks with TAS/GSC in Nakagami-m fading channels. More importantly, we obtain a tight high SNR approximation of the ergodic capacity for the two cases: 1) proportional interference power constraint, and 2) fixed interference power constraint. Interesting conclusions are reached. On the one hand, the high SNR slope is independent of the antenna configuration and the fading parameters, but on the other hand, the high SNR power offset is fully described by the antenna configuration and the fading parameters in the primary and secondary networks. The high SNR slope is 1/2 for the proportional interference power constraint, and is equal to zero for the fixed interference power constraint.

II. SYSTEM AND CHANNEL DESCRIPTION

We consider a dual-hop cognitive DF relay network consisting of S with N_S antennas, R with N_R antennas, D with N_D antennas, and PU with a single antenna. We assume that the PU transmitter is located far away from the secondary network. This assumption is typical in large scale networks where the interference from the PU transmitter is negligible [6, 25, 26]. We also assume there is no direct link between S and D due to long distance and deep fades [27]. Both the primary channel and the secondary channel are assumed to undergo quasi-static fading with independent and identically distributed (i.i.d.) Nakagami-m distribution. We assume perfect channel state information (CSI) between the secondary transmitter and the PU can be obtained through direct feedback from the PU [28], indirect feedback from a third party, and periodic sensing of pilot signal from the PU [29]. In the secondary network, a single transmit antenna among N_S antennas which maximizes the GSC output SNR at R is selected at S, while the L_R ($1 \leq L_R \leq N_R$) strongest receive antennas are combined at R. The signal transmitted by R is decoded and forwarded using a single transmit antenna among N_R antennas which maximizes the GSC output SNR at D, and then combined at D with the L_D ($1 \leq L_D \leq N_D$) strongest receive antennas. Let $\{g_{1ij}\}$ denote the channel coefficients of the $N_S \times N_R$ channels from S to R with $i \in \{1, \ldots, N_S\}$, $j \in \{1, \ldots, N_R\}$, and $\{g_{2jk}\}$ denote the channel coefficients of the $N_R \times N_D$ channels from R to D with $k \in \{1, \ldots, N_D\}$. Also, $\{h_{1i}\}$ denote the channel coefficients of the $N_S \times 1$ channels from S to PU, and $\{h_{2j}\}$ denote the channel coefficients of the $N_R \times 1$ channels from R to PU. The channel coefficients follow the Nakagami-m distribution with fading parameters m_{g1}, m_{g2}, m_{h1}, and m_{h2}, and average channel power gains Ω_{g1}, Ω_{g2}, Ω_{h1}, and Ω_{h2}. In the following, $\| \cdot \|$ is the Euclidean norm, $| \cdot |$ is the absolute value, and $\mathbb{E}[\cdot]$ is the expectation.

The pilot symbol block P_i, ($1 \leq i \leq N_S$), are transmitted from each transmit antenna at different time slots. Based on these pilot symbols, R perfectly estimates CSI, then arranges $\{|g_{1ij}|^2\}_{j=1}^{N_R}$ in descending order as $|g_{1i(1)}|^2 \geq |g_{1i(2)}|^2 \geq \cdots \geq |g_{1i(N_R)}|^2 \geq 0$ for the each transmit antenna i at S. Note that before the transmission process, the selected number of antenna chains L_R and L_D at the receivers are determined by the limited number of radio frequency (RF) chains due to size and complexity limitations. According to the rule of GSC, the first L_R ($1 \leq L_R \leq N_R$) received signal power(s) are combined at R to obtain $\theta_i = \sum_{j=1}^{L_R} |g_{1i(j)}|^2$. The selected transmit antenna i^* is determined at R by

$$i^* = \arg\max_{1 \leq i \leq N_S} \left\{ \theta_i = \sum_{j=1}^{L_R} |g_{1i(j)}|^2 \right\}, \qquad (1)$$

which maximizes the total received signal power. To this end, the index of the selected transmit antenna is sent back to S through the feedback channel, so that only $\lceil \log_2(N_S) \rceil$ bits needs to be sent to S. As such, the selected channel vector is denoted as $\mathbf{g}_{1i^*\theta_{i^*}} = [g_{1i^*(1)}, \cdots, g_{1i^*(L_R)}]$. Similarly, in the second hop, the index of the selected transmit antenna at R is

determined by

$$j^* = \arg\max_{1 \le j \le N_R} \left\{ \theta_j = \sum_{k=1}^{L_D} \left| g_{2j(k)} \right|^2 \right\}. \tag{2}$$

As such, we denote the selected channel vector as $\mathbf{g}_{2j^*\theta_{j^*}} = \left[g_{2j^*(1)}, \cdots, g_{2j^*(L_D)} \right]$.

According to underlay cognitive relay networks, the transmit powers at S and R are constrained as

$$P_S = \min\left(P, \frac{Q}{|h_{1i^*}|^2}\right) \quad \text{and} \quad P_R = \min\left(P, \frac{Q}{|h_{2j^*}|^2}\right), \tag{3}$$

respectively, where P is the maximum transmit power constraint at S and R, and Q is the peak interference power constraint at PU.

The instantaneous end-to-end SNR of the spectrum sharing network with TAS/GSC and DF relaying is defined as $\gamma = \min\{\gamma_1, \gamma_2\}$, where the instantaneous SNR of S $\to$ R link is

$$\gamma_1 = \min\left(\|\mathbf{g}_{1i^*\theta_{i^*}}\|^2 \bar{\gamma}_P, \frac{\|\mathbf{g}_{1i^*\theta_{i^*}}\|^2 \bar{\gamma}_Q}{|h_{1i^*}|^2} \right) \tag{4}$$

and the instantaneous SNR of R $\to$ D link is

$$\gamma_2 = \min\left(\|\mathbf{g}_{2j^*\theta_{j^*}}\|^2 \bar{\gamma}_P, \frac{\|\mathbf{g}_{2j^*\theta_{j^*}}\|^2 \bar{\gamma}_Q}{|h_{2j^*}|^2} \right). \tag{5}$$

In (4) and (5), we define $\bar{\gamma}_P = \frac{P}{N_0}$ and $\bar{\gamma}_Q = \frac{Q}{N_0}$, where N_0 is the noise power of an additive white Gaussian noise (AWGN).

III. New Statistical Properties

In this section, we derive new statistical properties of the end-to-end SNR, which is a challenging problem due to the complex nature of TAS/GSC in Nakagami-m fading. Based on these statistical characteristics, we present the exact and asymptotic outage probability, SER, and ergodic capacity. Without loss of generality, these new statistics can be easily applied to other wireless networks with TAS/GSC.

Based on the expressions of γ_1 and γ_2 in (4) and (5), respectively, we first derive the CDF of $\|\mathbf{g}_{1i^*\theta_{i^*}}\|^2$ in the following lemmas.

A. Expressions for CDF of $\|\mathbf{g}_{1i^*\theta_{i^*}}\|^2$ in the Secondary Channel

Lemma 1. *The expressions for the CDF of $\|\mathbf{g}_{1i^*\theta_{i^*}}\|^2$ are derived as*

$$F_{\|\mathbf{g}_{1i^*\theta_{i^*}}\|^2}(x) = \left(\frac{L_R}{(m_{g1}-1)!} \binom{N_R}{L_R} \right)^{N_S} N_S!$$

$$\times \widetilde{\sum_{\mathcal{S}_R^{|\mathcal{S}_K|}}} \hbar_k x^{\theta_k} e^{-\eta_k x}, \tag{6}$$

where $\displaystyle \widetilde{\sum_{\mathcal{S}_R^{|\mathcal{S}_K|}}} \triangleq \sum_{\mathcal{S}_R} \sum_{\mathcal{S}_R^1} \cdots \sum_{\mathcal{S}_R^k} \cdots \sum_{\mathcal{S}_R^{|\mathcal{S}_K|}}, \quad \mathcal{S}_R =$

$\left\{ \left(n_{\tau,1}, \ldots, n_{\tau,|\mathcal{S}_K|} \right) \middle| \sum_{k=1}^{|\mathcal{S}_K|} n_{\tau,k} = N_S \right\}$ *with* $\{n_{\tau,k}\} \in \mathbb{Z}^+$,

$|\mathcal{S}_K|$ *is the cardinality of the set $\mathcal{S}_K$, and $\mathcal{S}_K$ denotes a set of $(2m_{g1}+1)$-tuples satisfying the following condition*

$$\mathcal{S}_K = \left\{ \left(n_{k,0}^\Phi, \ldots, n_{k,m_{g1}-1}^\Phi, n_{k,0}^F, \ldots, n_{k,m_{g1}}^F \right) \middle| \right.$$
$$\left. \sum_{i=0}^{m_{g1}-1} n_{k,i}^\Phi = L_R - 1; \sum_{j=0}^{m_{g1}} n_{k,j}^F = N_R - L_R \right\},$$

thereby $|\mathcal{S}_K| = \binom{m_{g1}+L_R-2}{m_{g1}-1}\binom{m_{g1}+N_R-L_R}{m_{g1}}$, *and* $\mathcal{S}_R^k =$

$\left\{ \left(n_{\rho_k,0}, \ldots, n_{\rho_k,m_{g1}L_R+b_k^F} \right) \middle| \sum_{n=0}^{m_{g1}L_R+b_k^F} n_{\rho_k,n} = n_{\tau,k} \right\}, \ k =$

$1, \cdots, |\mathcal{S}_K|$, *with* $\{n_{k,i}^\Phi\}$, $\{n_{k,i}^\Phi\}$, $\{n_{k,j}^F\}$, *and* $\{n_{\rho_k,n}\} \in \mathbb{Z}^+$. *In (6),* $\hbar_k$, θ_k, *and* η_k *are respectively given by*

$$\hbar_k = \prod_{k=1}^{|\mathcal{S}_K|} \left(a_k^\Phi a_k^F \frac{(n_2-1)!}{L_R^{n_2}} \right)^{n_{\tau,k}} \left(\frac{\prod_{n=0}^{m_{g1}L_R+b_k^F} \ell_k(n)^{n_{\rho_k,n}}}{\prod_{n=0}^{m_{g1}L_R+b_k^F} n_{\rho_k,n}!} \right), \tag{7}$$

$$\theta_k = \sum_{k=1}^{|\mathcal{S}_K|} \sum_{n=0}^{m_{g1}L_R+b_k^F} \mu_k(n) n_{\rho_k,n}, \tag{8}$$

and

$$\eta_k = \sum_{k=1}^{|\mathcal{S}_K|} \sum_{n=0}^{m_{g1}L_R+b_k^F} \nu_k(n) n_{\rho_k,n}, \tag{9}$$

where n_2, $\mu_k(n)$, $\nu_k(n)$, $\ell_k(n)$ a_k^Φ, a_k^F, b_k^Φ, b_k^F, Υ_{k1}, Υ_{k2}, Υ_{k3}, *and* Υ_{k4} *are defined in Appendix A.*

Proof. See Appendix A. $\square$

B. Expressions for the CDF of $\|\mathbf{g}_{2j^*\theta_{j^*}}\|^2$ in the Secondary Channel

The CDF of $\|\mathbf{g}_{2j^*\theta_{j^*}}\|^2$ follow from (6) by interchanging the parameters $m_{g1} \to m_{g2}$, $m_{h1} \to m_{h2}$, $L_R \to L_D$, $N_R \to N_D$, $N_S \to N_R$, $\mathcal{S}_R \to \mathcal{S}_D$, $|\mathcal{S}_K| \to |\mathcal{S}_T|$, $\mathcal{S}_R^{|\mathcal{S}_K|} \to \mathcal{S}_D^{|\mathcal{S}_T|}$, $\hbar_k \to \hbar_t$, $\theta_k \to \theta_t$, and $\eta_k \to \eta_t$, where $\displaystyle \widetilde{\sum_{\mathcal{S}_D^{|\mathcal{S}_T|}}} \triangleq \sum_{\mathcal{S}_D} \sum_{\mathcal{S}_D^1} \cdots \sum_{\mathcal{S}_D^t} \cdots \sum_{\mathcal{S}_D^{|\mathcal{S}_T|}}$, $\mathcal{S}_D = \left\{ \left(n_{\tau,1}, \ldots, n_{\tau,|\mathcal{S}_T|} \right) \middle| \sum_{t=1}^{|\mathcal{S}_T|} n_{\tau,t} = N_R \right\}$ with $\{n_{\tau,t}\} \in \mathbb{Z}^+$, $|\mathcal{S}_T|$ is the cardinality of the set $\mathcal{S}_T$, and $\mathcal{S}_T$ denotes a set of $(2m_{g2}+1)$-tuples satisfying the following condition

$$\mathcal{S}_T = \left\{ \left(n_{t,0}^\Phi, \ldots, n_{t,m_{g2}-1}^\Phi, n_{t,0}^F, \ldots, n_{t,m_{g2}}^F \right) \middle| \right.$$
$$\left. \sum_{i=0}^{m_{g2}-1} n_{t,i}^\Phi = L_D - 1; \sum_{j=0}^{m_{g2}} n_{t,j}^F = N_D - L_D \right\},$$

thereby $|\mathcal{S}_T| = \binom{m_{g2}+L_D-2}{m_{g2}-1}\binom{m_{g2}+N_D-L_D}{m_{g2}}$, and

$$\mathcal{S}_D^t = \left\{ \left(n_{\rho_t,0}, \ldots, n_{\rho_t,m_{g2}L_D+b_t^F} \right) \middle| \sum_{n=0}^{m_{g2}L_D+b_t^F} n_{\rho_t,n} = n_{\tau,t} \right\},$$

$t = 1, \cdots, |\mathcal{S}_T|$, with $\{n_{t,i}^\Phi\}$, $\{n_{t,i}^\Phi\}$, $\{n_{t,j}^F\}$, and $\{n_{\rho_t,n}\} \in \mathbb{Z}^+$. Note that $\hbar_t$, θ_t, η_t follow from (7), (8), (9) by interchanging the parameters $\mu_k(n) \to \mu_t(n)$, $\nu_k(n) \to \nu_t(n)$, $\ell_k(n) \to \ell_t(n)$ $a_k^\Phi \to a_t^\Phi$, $b_k^\Phi \to b_t^\Phi$, $c_k^\Phi \to c_t^\Phi$, $a_k^F \to a_t^F$,

$b_k^F \to b_t^F$, $c_k^F \to c_t^F$, $\Upsilon_{k1} \to \Upsilon_{t1}$, $\Upsilon_{k2} \to \Upsilon_{t2}$, $\Upsilon_{k3} \to \Upsilon_{t3}$, and $\Upsilon_{k4} \to \Upsilon_{t4}$. Here, $\mu_t(n)$, $\nu_t(n)$, $\ell_t(n)$ a_t^Φ, b_t^Φ, c_t^Φ, a_t^F, b_t^F, c_t^F, Υ_{t1}, Υ_{t2}, Υ_{t3}, and Υ_{t4} follow from (A.15), (A.16), (A.14), (A.6), (A.7), (A.9), (A.10), (A.17), (A.18), (A.19), and (A.20) by interchanging the parameters $m_{g1} \to m_{g2}$, $m_{h1} \to m_{h2}$, $\Omega_{h1} \to \Omega_{h2}$, $L_R \to L_D$, $N_R \to N_D$, $N_S \to N_R$, $\mathcal{S}_R \to \mathcal{S}_D$, $|\mathcal{S}_K| \to |\mathcal{S}_T|$, $\mathcal{S}_R^{|\mathcal{S}_K|} \to \mathcal{S}_D^{|\mathcal{S}_T|}$, $n_{\tau,k} \to n_{\tau,t}$, $n_{k,i}^\Phi \to n_{t,i}^\Phi$, $n_{k,i}^\Phi \to n_{t,i}^\Phi$, $n_{k,j}^F \to n_{t,j}^F$, and $n_{\rho k,n} \to n_{\rho t,n}$.

C. Expressions for the CDF of γ_1

With the help of the CDF of $\|\mathbf{g}_{1i^*\theta_{i^*}}\|^2$ and $|h_{1i^*}|^2$, the closed-form CDF of γ_1 is evaluated in the following lemma.

Lemma 2. *The expression for the CDF of γ_1 is represented as*

$$F_{\gamma_1}(x) = \left(\frac{L_R}{(m_{g1}-1)!}\binom{N_R}{L_R}\right)^{N_S} N_S! \widetilde{\sum_{\mathcal{S}_R^{|\mathcal{S}_K|}}} \hbar_k \Xi_k(x), \quad (10)$$

where

$$\Xi_k(x) = \left(1 - \frac{\Gamma\left(m_{h1}, \frac{m_{h1}Q}{\Omega_{h1}P}\right)}{\Gamma(m_{h1})}\right)\left(\frac{x}{\overline{\gamma}_P}\right)^{\theta_k} e^{-\eta_k \frac{x}{\overline{\gamma}_P}} +$$

$$\left(\frac{m_{h1}}{\Omega_{h1}}\right)^{m_{h1}}\left(\frac{x}{\overline{\gamma}_Q}\right)^{\theta_k} \frac{\Gamma\left(\theta_k + m_{h1}, \left(\frac{m_{h1}}{\Omega_{h1}} + \frac{\eta_k x}{\overline{\gamma}_Q}\right)\frac{Q}{P}\right)}{(m_{h1}-1)!\left(\frac{m_{h1}}{\Omega_{h1}} + \frac{\eta_k x}{\overline{\gamma}_Q}\right)^{\theta_k + m_{h1}}}. \quad (11)$$

Proof. See Appendix B. □

D. Expressions for the CDF of γ_2

Similarly, the CDF of γ_2 follows from (10) and (11) by interchanging the parameters $m_{g1} \to m_{g2}$, $m_{h1} \to m_{h2}$, $\Omega_{h1} \to \Omega_{h2}$, $\eta_k \to \eta_t$, and $\theta_k \to \theta_t$. Note that our expressions are valid for arbitrary fading severity parameters in all the links.

IV. OUTAGE PROBABILITY

In this section, we concentrate on the outage probability. We derive a new closed-form expresssion for the exact outage probability. In order to assess the performance at high SNRs, we derive the asymptotic outage probabilities with the proportional interference power constraint and the fixed interference power constraint.

A. Exact Analysis

In DF relaying, the end-to-end outage probability is determined by the worst link between S → R and R → D links, which is given by [30]

$$P_{out}(\gamma_{th}) = \Pr\left(\min(\gamma_1, \gamma_2) \le \gamma_{th}\right)$$
$$= F_{\gamma_1}(\gamma_{th}) + F_{\gamma_2}(\gamma_{th}) - F_{\gamma_1}(\gamma_{th})F_{\gamma_2}(\gamma_{th}). \quad (12)$$

By substituting (10) and the CDF of γ_2 into (12), the outage probability is finally derived in the following theorem.

Theorem 1. *The closed-form expression for the outage probability of spectrum sharing networks with TAS/GSC and DF*

relaying in Nakagami-m fading is derived as

$$P_{out}(\gamma_{th}) =$$
$$1 - \left(1 - \left(\frac{L_R}{(m_{g1}-1)!}\binom{N_R}{L_R}\right)^{N_S} N_S! \widetilde{\sum_{\mathcal{S}_R^{|\mathcal{S}_K|}}} \hbar_k \Xi_k(\gamma_{th})\right)$$
$$\left(1 - \left(\frac{L_D}{(m_{g2}-1)!}\binom{N_D}{L_D}\right)^{N_R} N_R! \widetilde{\sum_{\mathcal{S}_D^{|\mathcal{S}_T|}}} \hbar_t \Xi_t(\gamma_{th})\right). \quad (13)$$

Our new closed-form expression for the outage probability is valid for an arbitrary number of antennas of the secondary network and arbitrary fading severity parameters in all the links.

B. Asymptotic Analysis

1) Proportional Interference Power Constraint:
We first examine the asymptotic behavior with the proportional interference power constraint. As such, we assume that both P and Q grow large in the high SNR regime. This applies to the scenario where the PU is able to tolerate a high amount of interference from S and R. With this in mind, we take into account and study the effect of the so-called *power scaling* on the outage probability. Similar to [7, 25], we consider $Q = \mu P$, where μ is the power scaling factor and is a positive constant.

Theorem 2. *When Q scales with P, the asymptotic outage probability of cognitive spectrum sharing with TAS/GSC and DF relaying in Nakagami-m fading at high SNRs is derived as*

$$P_{out}^\infty(\gamma_{th}) = (G_c \overline{\gamma}_P)^{-G_d} + o\left(\overline{\gamma}_P^{-G_d}\right), \quad (14)$$

where the diversity order is

$$G_d = N_R \times \min\{m_{g1}N_S, m_{g2}N_D\} \quad (15)$$

and the SNR gain is

$$G_c = \begin{cases} \frac{\Delta_1(L_R)}{\gamma_{th}} & m_{g1}N_S < m_{g2}N_D \\ \frac{\Delta_2(L_R)}{\gamma_{th}} & m_{g1}N_S > m_{g2}N_D \\ \frac{\Delta_1(L_R)}{\gamma_{th}} + \frac{\Delta_2(L_R)}{\gamma_{th}} & m_{g1}N_S = m_{g2}N_D, \end{cases} \quad (16)$$

with

$$\Delta_1(L_R) = \frac{\Omega_{g1}}{m_{g1}}\left[\frac{K\left(\mathcal{S}_K^\Phi, N_R, L_R, m_{g1}, a_k^\Phi, b_k^\Phi\right)}{(m_{g1}N_R)!}\right]^{-\frac{1}{m_{g1}N_R}}$$
$$\left[\Phi(m_{h1}, \Omega_{h1}) + \frac{\Xi(m_{g1}, m_{h1}, \Omega_{h1}, N_S)}{\mu^{m_{g1}N_R N_S}}\right]^{-\frac{1}{m_{g1}N_R N_S}}, \quad (17)$$

and

$$\Delta_2(L_R) = \frac{\Omega_{g2}}{m_{g2}}\left[\frac{K\left(\mathcal{S}_T^\Phi, N_D, L_D, m_{g2}, a_t^\Phi, b_t^\Phi\right)}{(m_{g2}N_D)!}\right]^{-\frac{1}{m_{g2}N_D}}$$
$$\left[\Phi(m_{h2}, \Omega_{h2}) + \frac{\Xi(m_{g2}, m_{h2}, \Omega_{h2}, N_D)}{\mu^{m_{g2}N_R N_D}}\right]^{-\frac{1}{m_{g2}N_R N_D}}. \quad (18)$$

In (17) *and* (18)*, we have*

$$\mathrm{K}\big(\mathcal{S}^\Phi, N, L, m_g, a^\Phi, b^\Phi\big) = \frac{L\binom{N}{L}}{(m_g-1)!(m_g!)^{N-L}}$$

$$\sum_{\mathcal{S}^\Phi} a^\Phi \frac{(b^\Phi + m_g(N-L+1)-1)!}{(L)^{b^\Phi+m_g(N-L+1)}}, \tag{19}$$

$$\Phi\big(m_h, \Omega_h\big) = 1 - e^{-\mu\frac{m_h}{\Omega_h}} \sum_{j=0}^{m_h-1} \frac{\big(\mu\frac{m_h}{\Omega_h}\big)^j}{j!}, \tag{20}$$

$$\Xi\big(m_g, m_h, \Omega_h, N\big) = \frac{\Gamma(m_g N_R N + m_h, \mu\frac{m_h}{\Omega_h})}{(m_h-1)!(\frac{m_h}{\Omega_h})^{m_g N_R N}}. \tag{21}$$

Proof. See Appendix C. $\square$

Based on (15), we see that the diversity order is dominated by the fading severity parameter of the two hops and the total number of antennas at S, R, and D. Interestingly, it is independent of the fading severity parameters of the interference channel, and the selected number of antennas at R and D. The negative impact of the peak interference power constraint is reflected in the SNR gain.

Corollary 1. *The SNR gap between GSC and SC is derived as*

$$G_c = \begin{cases} -\frac{10}{m_{g1}N_R}\log(\mathrm{T}_1) & m_{g1}N_S < m_{g2}N_D \\ -\frac{10}{m_{g2}N_D}\log(\mathrm{T}_2) & m_{g1}N_S > m_{g2}N_D \\ 10\log\big(\frac{\Delta_1(L_R)+\Delta_2(L_R)}{\Delta_1(1)+\Delta_2(1)}\big) & m_{g1}N_S = m_{g2}N_D\ , \end{cases} \tag{22}$$

where

$$\mathrm{T}_1 = \frac{(m_{g1}!)^{N_R-1}(m_{g1}-1)!}{N_R(m_{g1}N_R-1)!}\mathrm{K}(S_K^\Phi, N_R, L_R, m_{g1}, a_k^\Phi, b_k^\Phi) \tag{23}$$

and

$$\mathrm{T}_2 = \frac{(m_{g2}!)^{N_D-1}(m_{g2}-1)!}{N_D(m_{g2}N_D-1)!}\mathrm{K}(S_T^\Phi, N_D, L_D, m_{g2}, a_t^\Phi, b_t^\Phi). \tag{24}$$

2) Fixed Interference Power Constraint:

Different from the proportional interference power constraint which can tolerate an extremely high peak interference power constraint and may potentially violate and harm the PU transmission [6], in this subsection, we focus on a stricter constraint where the peak interference power constraint is fixed [31]. We present the asymptotic outage probability with the fixed interference power constraint in the following theorem.

Theorem 3. *Under the fixed interference power constraint, the asymptotic outage probability of cognitive spectrum sharing with TAS/GSC and DF relaying in Nakagami-m fading at high SNRs is derived as*

$$P_{out}^\infty(\gamma_{th})$$
$$= \begin{cases} \mathrm{H}_1\big(\Phi_1\big(\frac{\gamma_{th}}{\bar{\gamma}_P}\big)^{m_{g1}N_R N_S} + \Xi_1\big(\frac{\gamma_{th}}{\bar{\gamma}_Q}\big)^{m_{g1}N_R N_S}\big) \\ \qquad\qquad\qquad\qquad m_{g1}N_S < m_{g2}N_D \\ \mathrm{H}_2\big(\Phi_2\big(\frac{\gamma_{th}}{\bar{\gamma}_P}\big)^{m_{g2}N_R N_D} + \Xi_2\big(\frac{\gamma_{th}}{\bar{\gamma}_Q}\big)^{m_{g2}N_R N_D}\big) \\ \qquad\qquad\qquad\qquad m_{g1}N_S > m_{g2}N_D \\ \big(\mathrm{H}_1\Phi_1 + \mathrm{H}_2\Phi_2\big)\big(\frac{\gamma_{th}}{\bar{\gamma}_P}\big)^{m_{g1}N_R N_S} \\ \qquad + \big(\mathrm{H}_1\Xi_1 + \mathrm{H}_2\Xi_2\big)\big(\frac{\gamma_{th}}{\bar{\gamma}_Q}\big)^{m_{g1}N_R N_S} \\ \qquad\qquad\qquad\qquad m_{g1}N_S = m_{g2}N_D\ , \end{cases} \tag{25}$$

where

$$\mathrm{H}_1 = \Big[\frac{\big(\frac{m_{g1}}{\Omega_{g1}}\big)^{m_{g1}N_R}\mathrm{K}\big(\mathcal{S}_K^\Phi, N_R, L_R, m_{g1}, a_k^\Phi, b_k^\Phi\big)}{(m_{g1}N_R)!}\Big]^{N_S} \tag{26}$$

$$\mathrm{H}_2 = \Big[\frac{\big(\frac{m_{g2}}{\Omega_{g2}}\big)^{m_{g2}N_D}\mathrm{K}\big(\mathcal{S}_T^\Phi, N_D, L_D, m_{g2}, a_t^\Phi, b_t^\Phi\big)}{(m_{g2}N_D)!}\Big]^{N_R} \tag{27}$$

$$\Phi_1 = \Phi\big(m_{h1}, \Omega_{h1}\big), \quad \Phi_2 = \Phi\big(m_{h2}, \Omega_{h2}\big), \tag{28}$$

$$\Xi_1 = \Xi\big(m_{g1}, m_{h1}, \Omega_{h1}, N_S\big), \tag{29}$$

and

$$\Xi_2 = \Xi\big(m_{g2}, m_{h2}, \Omega_{h2}, N_D\big). \tag{30}$$

Proof. The proof can be done in the same way as the proof of Theorem 2. $\square$

From (25), we see that the diversity order of the outage probability tends to zero under the fixed interference power constraint.

V. SYMBOL ERROR RATE

In this section, we focus on the SER as another important performance evaluation metric. For most modulation schemes, the SER of a conventional wireless communication system can be expressed as [32]

$$P_e = \frac{a}{2}\sqrt{\frac{b}{\pi}}\int_0^\infty \frac{e^{-b\gamma}}{\sqrt{\gamma}}F_\gamma(\gamma)d\gamma, \tag{31}$$

where a and b are modulation specific constants. For example, $a = 1$, $b = 1$ for BPSK (binary phase shift keying), $a = 2(M-1)/M$, $b = 3/(M^2-1)$ for M-PAM (M-ary pulse amplitude modulation), and $a = 2$, $b = \sin^2(\pi/M)$ for M-PSK (M-ary phase shift keying).

A. Exact Analysis

Substituting (10) into (31), the SER of S $\to$ R link can be derived by utilizing [33, eq.8.310.1], [33, eq.8.352.2], [33, eq.9.211.4.8] and the polynomial expansion. Using the same method, P_{e2}, which is the SER of R $\to$ D link can be easily computed. Substituting the derived expressions of P_{e1} and P_{e2} into

$$P_e = 1 - \big(1 - P_{e1}\big) \times \big(1 - P_{e2}\big), \tag{32}$$

yields the SER of cognitive relay networks with TAS/GSC and DF relaying in the following theorem.

Theorem 4. *The closed-form expression for the SER of cognitive spectrum sharing with TAS/GSC and DF relaying in Nakagami-m fading is derived as*

$$P_e = 1 - \left(1 - \frac{a}{2}\sqrt{\frac{b}{\pi}}\left(\frac{L_R}{(m_{g1}-1)!}\right)\binom{N_R}{L_R}\right)^{N_S} N_S!$$

$$\widetilde{\sum_{\mathcal{S}_R^{|\mathcal{S}_K|}}} \hbar_k \Pi\left(m_{h1}, \Omega_{h1}, \theta_k, \eta_k\right)\right)$$

$$\left(1 - \frac{a}{2}\sqrt{\frac{b}{\pi}}\left(\frac{L_D}{(m_{g2}-1)!}\right)\binom{N_D}{L_D}\right)^{N_R} N_R!$$

$$\widetilde{\sum_{\mathcal{S}_D^{|\mathcal{S}_T|}}} \hbar_t \Pi\left(m_{h2}, \Omega_{h2}, \theta_t, \eta_t\right)), \tag{33}$$

where

$$\Pi\left(m_h, \Omega_h, \theta, \eta\right) = \left[\left(1 - \frac{\Gamma\left(m_h, \frac{m_h Q}{\Omega_h P}\right)}{\Gamma(m_h)}\right)\frac{\Gamma\left(\theta + \frac{1}{2}\right)}{\left(b + \frac{\eta}{\bar{\gamma}_P}\right)^{\theta+\frac{1}{2}}}\left(\frac{1}{\bar{\gamma}_P}\right)^{\theta}\right.$$

$$+ \frac{1}{(m_h-1)!}\left(\theta + m_h - 1\right)! e^{-\frac{m_h Q}{\Omega_h P}}\left(\frac{1}{\eta}\right)^{\theta+\frac{1}{2}}$$

$$\Gamma\left(\theta + \frac{1}{2}\right)\sum_{m=0}^{\theta+m_h-1}\frac{1}{m!}\left(\frac{Q}{P}\right)^m\left(\frac{m_h}{\Omega_h}\right)^{m+\frac{1}{2}}\bar{\gamma}_Q^{\frac{1}{2}}$$

$$\Psi\left(\theta + \frac{1}{2}, m - m_h + \frac{3}{2}; \left(b + \frac{\eta Q}{\bar{\gamma}_Q P}\right)\frac{\bar{\gamma}_Q m_h}{\eta \Omega_h}\right)\right]. \tag{34}$$

B. Asymptotic Analysis

1) Proportional Interference Power Constraint:
Substituting (14) into (31), together with the help of [33, eq. (3.310)], we derive the asymptotic SER under the proportional interference power constraint in the following theorem.

Theorem 5. *When Q is proportional to P, the asymptotic SER of cognitive spectrum sharing with TAS/GSC and DF relaying in Nakagami-m fading at high SNRs is derived as*

$$P_e^\infty = \left(G_c \bar{\gamma}_P\right)^{-G_d} + o\left(\bar{\gamma}_P^{-G_d}\right), \tag{35}$$

where the diversity order is

$$G_d = N_R \times \min\left\{m_{g1}N_S, m_{g2}N_D\right\} \tag{36}$$

and the SNR gain is

$$G_c = \begin{cases} \Lambda_1 \Delta_1 & m_{g1}N_S < m_{g2}N_D \\ \Lambda_2 \Delta_2 & m_{g1}N_S > m_{g2}N_D \\ \Lambda_1 \Delta_1 + \Lambda_2 \Delta_2 & m_{g1}N_S = m_{g2}N_D, \end{cases} \tag{37}$$

where

$$\Lambda_1 = \left[\frac{a}{2}\sqrt{\frac{1}{\pi}}\Gamma\left(m_{g1}N_R N_S + \frac{1}{2}\right)\right]^{-\frac{1}{m_{g1}N_R N_S}} b, \tag{38}$$

$$\Lambda_2 = \left[\frac{a}{2}\sqrt{\frac{1}{\pi}}\Gamma\left(m_{g2}N_R N_D + \frac{1}{2}\right)\right]^{-\frac{1}{m_{g2}N_R N_D}} b, \tag{39}$$

and Δ_1 and Δ_2 *are given in* (17) *and* (18), *respectively*.

Based on (35), we find that the diversity order is independent of the modulation scheme and the peak interference power constraint Q. The fading severity parameters of each hop and the antenna configuration have a direct impact on the diversity order while the interference power constraint at PU has a direct impact on the SNR gain.

2) Fixed Interference Power Constraint:
Substituting (25) into (31), we derive the asymptotic SER under the fixed interference power constraint in the following theorem.

Theorem 6. *Under the fixed interference power constraint, the asymptotic SER of cognitive spectrum sharing with TAS/GSC and DF relaying in Nakagami-m fading at high SNRs is derived as*

$$P_e^\infty$$

$$= \begin{cases} \Theta_1\left(\Phi_1\left(\frac{1}{\bar{\gamma}_P}\right)^{m_g N_R N_S} + \Xi_1\left(\frac{1}{\bar{\gamma}_Q}\right)^{m_g N_R N_S}\right) \\ \qquad\qquad\qquad m_{g1}N_S < m_{g2}N_D, \\ \Theta_2\left(\Phi_2\left(\frac{1}{\bar{\gamma}_P}\right)^{m_{g1}N_R N_D} + \Xi_2\left(\frac{1}{\bar{\gamma}_Q}\right)^{m_{g2}N_R N_D}\right) \\ \qquad\qquad\qquad m_{g1}N_S > m_{g2}N_D, \\ \left(\Theta_1\Phi_1 + \Theta_2\Phi_2\right)\left(\frac{1}{\bar{\gamma}_P}\right)^{m_g N_R N_S} \\ \quad + \left(\Theta_1\Xi_1 + \Theta_2\Xi_2\right)\left(\frac{1}{\bar{\gamma}_Q}\right)^{m_g N_R N_S} \\ \qquad\qquad\qquad m_{g1}N_S = m_{g2}N_D, \end{cases} \tag{40}$$

where

$$\Theta_1 = \frac{a\Gamma(m_{g1}N_R N_S + \frac{1}{2})}{2\sqrt{\pi}b^{m_{g1}N_R N_S}}H_1, \tag{41}$$

$$\Theta_2 = \frac{a\Gamma(m_{g2}N_R N_D + \frac{1}{2})}{2\sqrt{\pi}b^{m_{g2}N_R N_D}}H_2, \tag{42}$$

and H_1, H_2, Φ_1, Φ_2, Ξ_1, *and* Ξ_2 *are given by* (26), (27), (28), (28), (29), (30), *respectively*.

From (40), we find that the diversity order of the SER goes to zero under the fixed interference power constraint.

VI. ERGODIC CAPACITY

The ergodic capacity is an important performance indicator for cognitive underlay spectrum sharing. It is defined as the maximum achievable long-term rate, where no delay limit is taken into account. Under these assumptions, the ergodic capacity is expressed as

$$C_{erg} = \frac{1}{2}\int_0^\infty \log_2\left(1 + x\right)f_\gamma(x)dx = \frac{1}{2\ln 2}\int_0^\infty \frac{1 - F_\gamma(x)}{1 + x}dx. \tag{43}$$

To simplify (43), we define $F_{\gamma_1}(x) = 1 + \tilde{F}_{\gamma_1}(x)$ and $F_{\gamma_2}(x) = 1 + \tilde{F}_{\gamma_2}(x)$, and rewrite (43) as

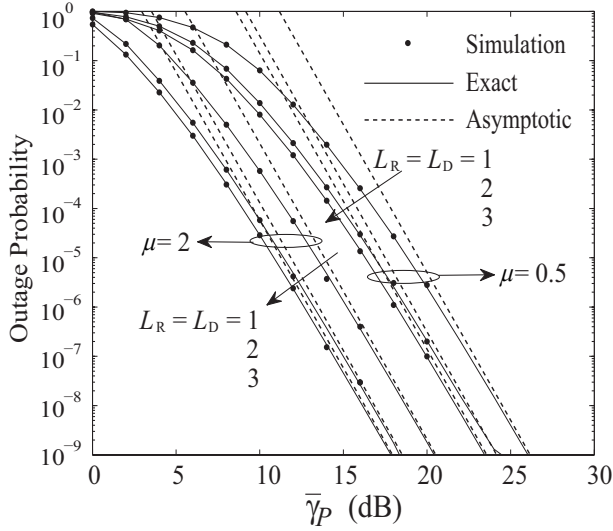$$C_{erg} = \frac{1}{2\ln 2}\int_0^\infty \frac{\tilde{F}_{\gamma_1}(x)\tilde{F}_{\gamma_2}(x)}{1 + x}dx, \tag{44}$$

Fig. 1. Cognitive spectrum sharing with TAS/GSC and DF relaying: $N_S = 2$, $N_R = 3$, $N_D = 3$, $m_{g1} = 1$, $m_{g2} = 2$, $m_{h1} = m_{h2} = 2$, and $\bar{\gamma}_Q = 2\bar{\gamma}_P$.
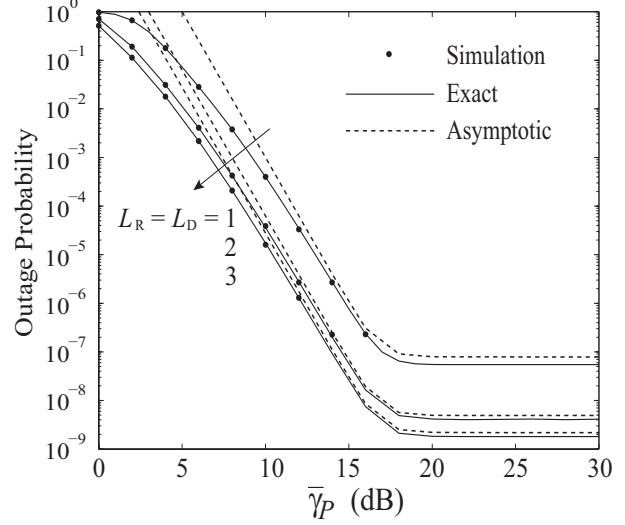

Fig. 2. Cognitive spectrum sharing with TAS/GSC and DF relaying: $N_S = 2$, $N_R = 3$, $N_D = 3$, $m_{g1} = 1$, $m_{g2} = 2$, $m_{h1} = m_{h2} = 2$, and $\bar{\gamma}_Q = 20$ dB.

where

$$\tilde{F}_{\gamma_1}(x) = \left(\frac{L_R}{(m_{g1}-1)!} \binom{N_R}{L_R} \right)^{N_S} N_S! \widetilde{\sum_{\mathcal{S}_R^{|S_K|}}} \hbar_k \text{sgn}(\eta_k) \Xi_k(x) \tag{45}$$

and

$$\tilde{F}_{\gamma_2}(x) = \left(\frac{L_D}{(m_{g2}-1)!} \binom{N_D}{L_D} \right)^{N_R} N_R! \widetilde{\sum_{\mathcal{S}_D^{|S_T|}}} \hbar_t \text{sgn}(\eta_t) \Xi_t(x). \tag{46}$$

In the following, we assume $m_{h1} = m_{h2} = m_h$ and $\Omega_{h1} = \Omega_{h2} = \Omega_h$.

A. Exact Analysis

Substituting (45) and (46) into (44), and with the help of [33, eq.8.352.2], [33, eq.9.211.4.8], and the partial fraction expression [33, eq.2.102], we obtain a general closed-form expression for the ergodic capacity in the following theorem.

Theorem 7. *Our new closed-form expression for the ergodic capacity of cognitive TAS/GSC relaying in Nakagami-m fading is given in (47) at the top of the next page. In (47), we have defined the following terms*

$$\nabla(\theta) \triangleq \left(1 - \Gamma\left(m_h, \frac{Q}{P}\frac{m_h}{\Omega_h}\right) \Big/ \Gamma(m_h)\right) \left(\frac{1}{\bar{\gamma}_P}\right)^{\theta}, \tag{48}$$

$$\Delta(\theta, \eta, j, k) \triangleq \frac{(\theta + m_h - 1)!}{(m_h - 1)!} \left(\frac{1}{\bar{\gamma}_Q}\right)^{\theta} e^{-\frac{m_h Q}{\Omega_h P}} \sum_{j=0}^{\theta+m_h-1} \frac{1}{j!}$$

$$\left(\frac{Q}{P}\right)^{j} \sum_{k=0}^{j} \binom{j}{k} \left(\frac{m_h}{\Omega_h}\right)^{m_h+j-k} \left(\frac{\eta}{\bar{\gamma}_Q}\right)^{k}, \tag{49}$$

$$\tag{50}$$

$$\nu(\eta, l, k_1, k_2) \triangleq \Gamma(\tau) \left(\bar{\gamma}_Q m_h / \eta \Omega_h\right)^{\tau - l} \Psi(\tau, \tau + 1 - l;$$
$$(\eta_t + \eta_k) \bar{\gamma}_Q m_h / \bar{\gamma}_p \eta \Omega_h), \tag{51}$$

$$\partial(\eta, l) \triangleq \frac{(\bar{\gamma}_Q m_h / \eta \Omega_h - 1)^{l-1}}{(\bar{\gamma}_Q m_h / \eta_t \Omega_h - 1)^{\theta_t + m_h} (\bar{\gamma}_Q m_h / \eta_k \Omega_h - 1)^{\theta_k + m_h}},$$

$$\kappa(\theta, \eta, l, j) \triangleq \frac{(-1)^{\theta + m_h - l + 1} \binom{j + \theta + m_h - l - 1}{j - 1}}{(\bar{\gamma}_Q m_h / \Omega_h)^{j+\theta+m_h-l} (1/\eta_t - 1/\eta_k)^{j+\theta+m_h-l}}, \tag{52}$$

with $\tau = \theta_k + k_1 + \theta_t + k_2 + 1$.

Our result can be applied and simplified to the special cases of TAS/MRC and TAS/SC in Nakagami-m fading channel, as well as TAS/GSC in Rayleigh fading channels.

B. High SNR Capacity analysis

To examine the capacity performance in the high SNR regime with $\bar{\gamma}_P \to \infty$, we derive the high SNR approximation o the ergodic capacity in closed-form. With the aid of the Jensen's inequality, a tight upper bound on the ergodic capacity is given by [34]

$$C_{erg} = \frac{1}{2} \mathbb{E}\left[\log_2(1+\gamma)\right] \le \frac{1}{2}\log_2 \mathbb{E}(1+\gamma). \tag{53}$$

Thus, the tight high SNR approximation of the ergodic capacity is presented as [34, 35]

$$C_{erg}^{\infty} \approx \frac{1}{2}\log_2 \mathbb{E}(1+\gamma) \approx \frac{1}{2}\log_2 \mathbb{E}(\gamma). \tag{54}$$

$$C_{erg} = \frac{1}{2\ln 2}\left(\frac{L_R}{(m_{g1}-1)!}\binom{N_R}{L_R}\right)^{N_S} N_S! \widetilde{\sum_{\mathcal{S}_R^{|\mathcal{S}_K|}}} \hbar_k \mathrm{sgn}\,(\eta_k)\left(\frac{L_D}{(m_{g2}-1)!}\binom{N_D}{L_D}\right)^{N_R} N_R! \widetilde{\sum_{\mathcal{S}_D^{|\mathcal{S}_T|}}} \hbar_t \mathrm{sgn}\,(\eta_t)\left[\nabla\,(\theta_k)\right.$$

$$\nabla\,(\theta_t)\,\nu\,(\bar{\gamma}_Q m_h/\Omega_h, 1, 0, 0) + \Delta\,(\theta_t, \eta_t, j_2, k_2)\,\nabla\,(\theta_k)\,\frac{\left(\nu\,(\bar{\gamma}_Q m_h/\Omega_h, 1, 0, k_2) - \sum\limits_{l_2=1}^{\theta_t+m_h}\frac{\nu(\eta_t, l_2, 0, k_2)}{(\bar{\gamma}_Q m_h/\eta_t\Omega_h - 1)^{1-l_2}}\right)}{(m_h/\Omega_h - \eta_t/\bar{\gamma}_Q)^{\theta_t+m_h}}$$

$$+\frac{\left(\nu\,(\bar{\gamma}_Q m_h/\Omega_h, 1, k_1, 0) - \sum\limits_{l_1=1}^{\theta_k+m_h}\frac{\nu(\eta_k, l_1, k_1, 0)}{(\bar{\gamma}_Q m_h/\eta_k\Omega_h - 1)^{1-l_1}}\right)}{(m_h/\Omega_h - \eta_k/\bar{\gamma}_Q)^{\theta_k+m_h}}\nabla\,(\theta_t)\,\Delta\,(\theta_k, \eta_k, j_1, k_1) + \Delta\,(\theta_k, \eta_k, j_1, k_1)$$

$$\Delta\,(\theta_t, \eta_t, j_2, k_2)\left(\frac{\bar{\gamma}_Q}{\eta_k}\right)^{\theta_k+m_h}\left(\frac{\bar{\gamma}_Q}{\eta_t}\right)^{\theta_t+m_h}\left[\partial\,(\eta, 1)\,\nu\,(\bar{\gamma}_Q m_h/\Omega_h, 1, k_1, k_2) + \sum\limits_{l_1=1}^{\theta_k+m_h}\left(-\partial\,(\eta_k, l_1)+\right.\right.$$

$$\sum\limits_{j=1}^{\theta_t+m_h}\frac{\kappa\big(\theta_k, \eta_k, l_1, j\big)}{(\bar{\gamma}_Q m_h/\eta_t\Omega_h - 1)^{\theta_t+m_h-j+1}}\right)\nu\,(\eta_k, l_1, k_1, k_2) + \mathrm{sgn}\,(|\eta_k - \eta_t|)\,\nu\,(\eta_t, l_2, k_1, k_2)\sum\limits_{l_2=1}^{\theta_t+m_h}\left(-\partial\,(\eta_t, l_2)\right.$$

$$+\sum\limits_{i=1}^{\theta_k+m_h}\frac{(-1)^{i+\theta_t+m_h-l_2}}{(\bar{\gamma}_Q m_h/\eta_k\Omega_h - 1)^{\theta_k+m_h-i+1}}\kappa\,(\theta_t, \eta_t, l_2, i)\right) - \sum\limits_{l_3=\theta_k+m_h+1}^{\theta_k+\theta_t+2m_h}\frac{(1-\mathrm{sgn}\,(|\eta_k-\eta_t|))\,\nu\,(\eta_k, l_3, k_1, k_2)}{(\bar{\gamma}_Q m_h/\eta_k\Omega_h - 1)^{\theta_k+\theta_t+2m_h-m+1}}\right]\bigg]. \quad (47)$$

Therefore, we can rewrite (54) as

$$C_{erg}^{\infty} \approx \frac{1}{2}\log_2\left(\int\limits_0^{\infty} x f_\gamma\,(x)\,dx\right) = \frac{1}{2}\log_2\left(\int\limits_0^{\infty}(1 - F_\gamma\,(x))\,dx\right)$$

$$= \frac{1}{2}\log_2\int\limits_0^{\infty}\tilde{F}_{\gamma_1}\,(x)\,\tilde{F}_{\gamma_2}\,(x)\,dx. \quad (55)$$

1) Proportional Interference Power Constraint:

Based on (55), the high SNR approximation for the ergodic capacity with the proportional interference power constraint is written as

$$C_{erg}^{\infty} = \frac{1}{2}\left[\log_2\big(\bar{\gamma}_P\big) + \log_2\left(\int\limits_0^{\infty}\tilde{F}_{\gamma_1}\,(x)\,\tilde{F}_{\gamma_2}\,(x)\,d\big(\frac{x}{\bar{\gamma}_P}\big)\right)\right]. \quad (56)$$

Substituting (45) and (46) into (56), and with the help of [33, eq.8.352.2], [33, eq.9.211.4.8] and the binomial expansion, the high SNR approximation for ergodic capacity with the proportional interference power constraint is derived in the following theorem.

Theorem 8. *When Q is proportional to P, the high SNR approximation of the ergodic capacity is derived as*

$$C_{erg}^{\infty} \approx \frac{1}{2}\log_2\,(\bar{\gamma}_P) + \frac{1}{2}\log_2\,(\Upsilon), \quad (57)$$

where

$$\Upsilon = \left(\frac{L_R}{(m_{g1}-1)!}\binom{N_R}{L_R}\right)^{N_S} N_S! \widetilde{\sum_{\mathcal{S}_R^{|\mathcal{S}_K|}}}\hbar_k \mathrm{sgn}\,(\eta_k)$$

$$\left(\frac{L_D}{(m_{g2}-1)!}\binom{N_D}{L_D}\right)^{N_R} N_R! \widetilde{\sum_{\mathcal{S}_D^{|\mathcal{S}_T|}}}\hbar_t \mathrm{sgn}\,(\eta_t)$$

$$\left[\lambda^2\frac{\Gamma\,(\theta_k+\theta_t+1)}{(\eta_k+\eta_t)^{\theta_k+\theta_t+1}} + \lambda\sum\limits_{j_2=0}^{\theta_t+m_h-1}\sum\limits_{k_2=0}^{j_2}\Delta_s\,(\theta_t, j_2, k_2)\right.$$

$$\frac{\nu_s\,(\eta_t, \theta_t, 0, k_2)}{(1/\mu)^{\theta_k+j_2+1}} + \lambda\sum\limits_{j_1=0}^{\theta_k+m_h-1}\sum\limits_{k_1=0}^{j_1}\frac{\Delta_s\,(\theta_k, j_1, k_1)}{(1/\mu)^{\theta_t+j_1+1}} \quad (58)$$

$$\nu_s\,(\eta_k, \theta_k, k_1, 0) + \sum\limits_{j_1=0}^{\theta_k+m_h-1}\sum\limits_{k_1=0}^{j_1}\sum\limits_{j_2=0}^{\theta_t+m_h-1}\sum\limits_{k_2=0}^{j_2}\Omega$$

$$\left.\frac{\Delta_s\,(\theta_k, j_1, k_1)\,\Delta_s\,(\theta_t, j_2, k_2)}{(m_h/\Omega_h)^{\theta_k+\theta_t+1}(1/\mu)^{j_1+j_2+1}}\right]. \quad (59)$$

In (58), we have defined

$$\Delta_s\,(\theta, j, k) \triangleq \frac{(\theta+m_h-1)!}{(m_h-1)!j!}e^{-\mu\frac{m_h}{\Omega_h}}\binom{j}{k}\left(\frac{m_h}{\Omega_h}\right)^{\theta_k+\theta_t+j+1-\theta}, \quad (60)$$

$$\kappa_s\,(\theta, \eta, l) \triangleq \frac{(-1)^{(\theta+m_h-l)}\binom{\theta_k+\theta_t+2m_h-l-1}{\theta+m_h-l}}{(1/\eta_t - 1/\eta_k)^{\theta_k+\theta_t+2m_h-l}\eta^{k_1+k_2-l}}, \quad (61)$$

$$\nu_s\,(\eta, \varepsilon, k_1, k_2) \triangleq \Gamma\,(\theta_k+k_1+\theta_t+k_2+1)\,(1/\eta)^{\theta_k+\theta_t+1}$$
$$\Psi\Big(\theta_k+k_1+\theta_t+k_2+1,\ \theta_k+k_1+\theta_t+k_2+2-m_h-\varepsilon;$$
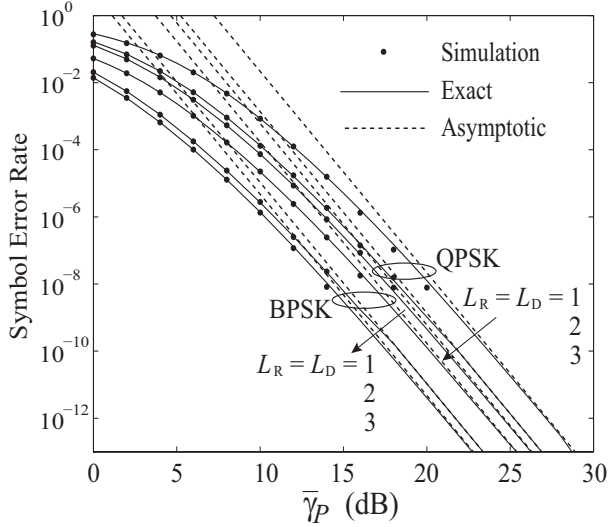$$\frac{(\eta_t+\eta_k)\,m_h\mu}{\Omega_h\eta}\Big), \quad (62)$$

and

Fig. 3. Cognitive spectrum sharing with TAS/GSC and DF relaying: $N_S = 2$, $N_R = 3$, $N_D = 3$, $m_{g1} = 1$, $m_{g2} = 2$, and $\overline{\gamma}_Q = 2\overline{\gamma}_P$.
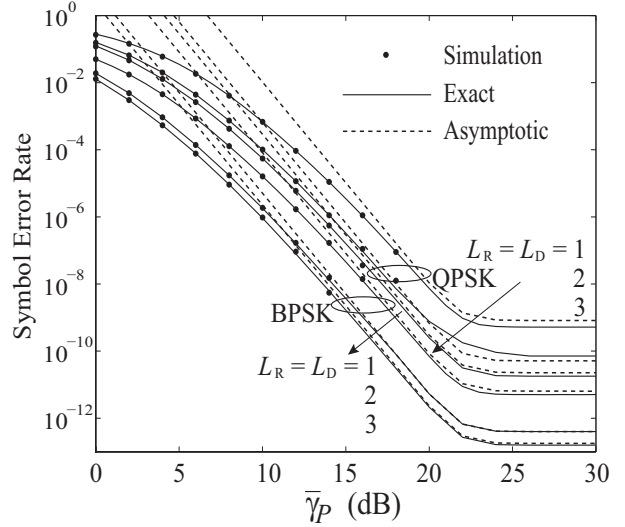

Fig. 4. Cognitive spectrum sharing with TAS/GSC and DF relaying: $N_S = 2$, $N_R = 3$, $N_D = 3$, $m_{g1} = 1$, $m_{g2} = 2$, and $\overline{\gamma}_Q = 25$ dB.

$$\Omega \overset{\triangle}{=} (1 - \text{sgn}(|\eta_k - \eta_t|)) \nu_s (\eta_k, \theta_k + \theta_t + m_h, k_1, k_2),$$

$$+ \frac{\text{sgn}(|\eta_k - \eta_t|)}{\eta_k^{\theta_k + m_h - k_1} \eta_t^{\theta_t + m_h - k_2}} \left[\sum_{l_1 = 1}^{\theta_k + m_h} \kappa_s (\theta_k, \eta_k, l_1) \right.$$

$$\nu_s (\eta_k, l_1 - m_h, k_1, k_2) + \sum_{l_2 = 1}^{\theta_t + m_h} (-1)^{\theta_k + \theta_t + 2m_h - l_2}$$

$$\left. \kappa_s (\theta_t, \eta_t, l_2) \nu_s (\eta_t, l_2 - m_h, k_1, k_2) \right]. \tag{63}$$

Note that similar as the asymptotic ergodic capacity, the tight high SNR approximations can well predict the performance behaviours in the high SNR regime. Thus, we deduce the high SNR scaling law from the high SNR approximations similar to the approach in [36] and [37]. Based on (57), we characterize two key parameters determining the affine approximation of the ergodic capacity in the high SNR regime, namely the high SNR slope and the high SNR power offset [38]. The high SNR slope is also known as the degrees of freedom or the multiplexing gain [39]. The high SNR power offset captures the joint effects of the fading model, the number of antennas at each terminal, and the interference power constraint. We represent the high SNR approximation of the ergodic capacity as [38]

$$C_{erg}^{\infty} \approx S_{\infty} (\log_2(\overline{\gamma}_P) - \mathcal{L}_{\infty}), \tag{64}$$

where S_{∞} is the high SNR slope in bits/s/Hz/(3 dB)

$$S_{\infty} = \lim_{\overline{\gamma}_P \to \infty} \frac{C_{erg}^{\infty}}{\log_2(\overline{\gamma}_P)} = \frac{1}{2} \tag{65}$$

and $\mathcal{L}_{\infty}$ is the high SNR power offset in 3 dB units

$$\mathcal{L}_{\infty} = \lim_{\overline{\gamma}_P \to \infty} \left(\log_2(\overline{\gamma}_P) - \frac{C_{erg}^{\infty}}{S_{\infty}} \right) = -\log_2(\Upsilon). \tag{66}$$

From (65), we see that the high SNR slope S_{∞} is independent of the interference power constraint, the selected number of antennas at the receiver, and the primary network. We also see that the high SNR power offset $\mathcal{L}_{\infty}$ is independent of $\overline{\gamma}_P$ from (66).

2) Fixed Interference Power Constraint:

Substituting (45) and (46) into (55), we obtain the high SNR approximation of the ergodic capacity under the fixed interference power constraint.

Theorem 9. *When Q is fixed, the high SNR approximation of the ergodic capacity is given in* (67) *at the top of the next page.*

From (67), we find that for the fixed interference power constraint, the high SNR slope collapses to zero.

VII. NUMERICAL RESULTS

In this section, we present numerical results to verify our new analytical derivations for cognitive TAS/GSC relaying in Nakagami-m fading channels. We set the threshold SNR as $\gamma_{th} = 5$ dB. All the figures clearly show that the exact curves are in precise agreement with the Monte Carlo simulations. Importantly, the asymptotic lines accurately predict the exact behaviour in the high SNR regime.

Fig. 1 plots the outage probability with the proportional interference constraint as we vary μ, L_R and L_D. The exact and asymptotic curves are plotted by using (13) and (14), respectively. For the same μ, we observe that the outage probability decreases with increasing L_R and L_D, due to an increase in the SNR gain (16). We also confirm that the diversity order is independent of L_R and L_D as reflected by the parallel slope. Another observation is that the outage probability decreases with increasing μ, which is due to the relaxed interference power constraint at the PU receiver.

$$C_{erg}^{\infty} = \frac{1}{2}\log_2\left[\left(\frac{L_R}{(m_{g1}-1)!}\binom{N_R}{L_R}\right)^{N_S}N_S!\widetilde{\sum_{\mathcal{S}_R^{|S_K|}}}\hbar_k\mathrm{sgn}\,(\eta_k)\left(\frac{L_D}{(m_{g2}-1)!}\binom{N_D}{L_D}\right)^{N_R}N_R!\widetilde{\sum_{\mathcal{S}_D^{|S_T|}}}\hbar_t\mathrm{sgn}\,(\eta_t)\left[\lambda^2\bar{\gamma}_P\right.\right.$$

$$\frac{\Gamma(\theta_k+\theta_t+1)}{(\eta_k+\eta_t)^{\theta_k+\theta_t+1}} + \bar{\gamma}_Q\left[\lambda\sum_{j_2=0}^{\theta_t+m_h-1}\sum_{k_2=0}^{j_2}\Delta_s\,(\theta_t,j_2,k_2)\nu_s\,(\eta_t,\theta_t,0,k_2)\left(\frac{\bar{\gamma}_Q}{\bar{\gamma}_P}\right)^{\theta_t+j_2} + \lambda\sum_{j_1=0}^{\theta_k+m_h-1}\sum_{k_1=0}^{j_1}\left(\frac{\bar{\gamma}_Q}{\bar{\gamma}_P}\right)^{\theta_t+j_1}\right.$$

$$\left.\left.\left.\Delta_s\,(\theta_k,j_1,k_1)\,\nu_s\,(\eta_k,\theta_k,k_1,0) + \sum_{j_1=0}^{\theta_k+m_h-1}\sum_{k_1=0}^{j_1}\sum_{j_2=0}^{\theta_t+m_h-1}\sum_{k_2=0}^{j_2}\frac{\Delta_s\,(\theta_k,j_1,k_1)\,\Delta_s\,(\theta_t,j_2,k_2)}{(m_h/\Omega_h)^{\theta_k+\theta_t+1}(P/Q)^{j_1+j_2}}\Omega\right]\right]\right]. \quad (67)$$

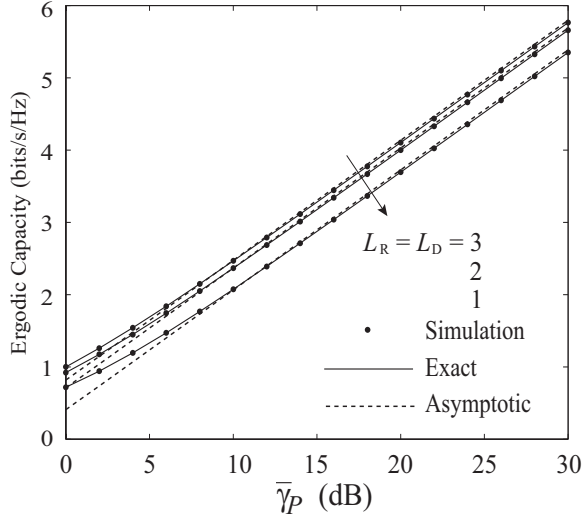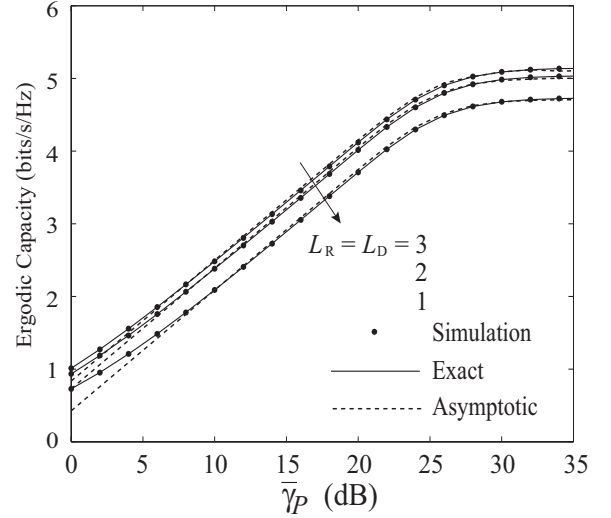Fig. 5. Cognitive spectrum sharing with TAS/GSC and DF relaying: $N_S = 2$, $N_R = 3$, $N_D = 3$, $m_{g1} = 1$, $m_{g2} = 2$, $m_{h1} = m_{h2} = 2$, and $\bar{\gamma}_Q = 2\bar{\gamma}_P$.

Fig. 6. Cognitive spectrum sharing with TAS/GSC and DF relaying: $N_S = 2$, $N_R = 3$, $N_D = 3$, $m_{g1} = 1$, $m_{g2} = 2$, $m_{h1} = m_{h2} = 2$, and $\bar{\gamma}_Q = 25$ dB.

Fig. 2 examines the impact of the fixed interference power constraint on the outage probability as varying L_R and L_D. The exact and asymptotic curves are plotted using (13) and (25), respectively. Interestingly, the outage probability becomes saturated for $\bar{\gamma}_P > 18$ dB. This is due to the fact that when $\bar{\gamma}_P \to \infty$, $\min\left(P, Q/|h_{1i^*}|^2\right) \approx \left(Q/|h_{1i^*}|^2\right)$ and $\min\left(P, Q/|h_{2j^*}|^2\right) \approx \left(Q/|h_{2j^*}|^2\right)$, as such the fixed peak interference power constraint becomes the dominant factor. By setting $L_R = L_D = 1$ and $L_R = L_D = 3$, we also see that TAS/MRC outperforms TAS/GSC, and TAS/GSC outperforms TAS/SC.

Fig. 3 plots the exact and asymptotic SER with the proportional interference power constraint from (33) and (35), respectively. The plot confirms that the diversity order is independent of the modulation scheme, L_R, and L_D. We see that the SER decreases as L_R and L_D increase. We also see that BPSK outperforms QPSK, which is predicted from the SNR gain (16).

Fig. 4 plots the exact and asymptotic SER with the fixed interference power constraint from (33) and (40), respectively.

We see that the SER decreases as L_R and L_D increase, and BPSK outperforms QPSK. Similar to Fig. 2, the SER becomes saturated for $\bar{\gamma}_P > 22$ dB, which confirms that the diversity order goes to zero.

Fig. 5 plots the exact ergodic capacity and its high SNR approximation with the proportional interference power constraint from (47) and (57), respectively. We see that the high SNR approximations of the ergodic capacity are tight and well predict the behavior of the ergodic capacity at high SNRs. It is obvious that the ergodic capacity can be improved by increasing L_R and L_D. The parallel curves confirm that the high SNR slope is independent of L_R and L_D.

Fig. 6 examines the impact of the fixed interference power constraint on the ergodic capacity. The exact ergodic capacity and its high SNR approximation are from (47) and (67), respectively. Interestingly, we find that the capacity ceiling occurs for $\bar{\gamma}_P > 30$ dB. This is due to the fact that when $\gamma_P \to \infty$, $\min\left(P, Q/|h_{1i^*}|^2\right) \approx \left(Q/|h_{1i^*}|^2\right)$ and $\min\left(P, Q/|h_{2j^*}|^2\right) \approx \left(Q/|h_{2j^*}|^2\right)$. Once again, the fixed interference power constraint becomes the dominant factor.

By setting $L_R = L_D = 1$ and $L_R = L_D = 3$, we see that TAS/MRC outperforms TAS/GSC and TAS/GSC outperforms TAS/SC.

VIII. CONCLUSIONS

We have taken into account the cognitive DF relay network with TAS/GSC over Nakagami-m fading. This framework is well suited for the reliability enhancement of the secondary network and interference alleviation of the primary network. We derived new statistical properties of the end-to-end SNR. Based on these, we have derived closed-form expressions for the exact and asymptotic outage probability, symbol error rate, and ergodic capacity with the proportional and the fixed interference power constraints. Our results are valid for Nakagami-m fading and arbitrary number of antennas in the secondary network. Based on the relationship of the maximum transmit power constraint and peak interference power constraint, we conclude that: 1) under the proportional interference power constraint, the diversity order is determined by the fading parameter and the antenna configuration of the secondary network, and the high SNR slope is 1/2; and 2) under the fixed interference power constraint, the diversity order is zero with error floor, and the high SNR slope is zero with capacity ceiling.

APPENDIX A
A PROOF OF LEMMA 1

We first present the probability density function (PDF) and CDF for the channel power gain of a single branch of the secondary network channel with the Nakagami-m fading as [40]

$$f(x) = \frac{x^{m_{g1}-1}}{(m_{g1}-1)!}\left(\frac{m_{g1}}{\Omega_{g1}}\right)^{m_{g1}}e^{-\frac{m_{g1}}{\Omega_{g1}}x} \qquad (A.1)$$

and

$$F(x) = 1 - \frac{\Gamma\left(m_{g1}, x\frac{m_{g1}}{\Omega_{g1}}\right)}{\Gamma(m_{g1})}, \qquad (A.2)$$

respectively. The marginal moment generating function (MGF) of (A.1) is given by [19]

$$\Phi(s,x) = \left(\frac{m_{g1}}{\Omega_{g1}}\right)^{m_{g1}}\sum_{i=0}^{m_{g1}-1}\frac{x^i e^{-\left(s+\frac{m_{g1}}{\Omega_{g1}}\right)x}}{i!\left(s+\frac{m_{g1}}{\Omega_{g1}}\right)^{m_{g1}-i}}. \qquad (A.3)$$

As shown in [19, 24], the MGF expression for the channel power gain with GSC is expressed as

$$\Phi_{GSC}(s) = L_R\binom{N_R}{L_R}\int_0^\infty e^{-sx}f(x)\left(\Phi(s,x)\right)^{L_R-1}(F(x))^{N_R-L_R}dx. \qquad (A.4)$$

Here the MGF is defined as $\Phi_\gamma(s) = E[e^{-\gamma s}]$.

Based on (A.3), and using the multinomial theorem [41], we rewrite $(\Phi(s,x))^{L_R-1}$ as

$$(\Phi(s,x))^{L_R-1} = \left(\frac{m_g}{\Omega_g}\right)^{m_g(L_R-1)}\sum_{\mathcal{S}_K^\Phi}a_k^\Phi x^{b_k^\Phi}e^{-c_k^\Phi x}$$
$$\left(s+\frac{m_{g1}}{\Omega_{g1}}\right)^{b_k^\Phi - m_{g1}(L_R-1)}, \qquad (A.5)$$

where $\mathcal{S}_K^\Phi = \left\{\left(n_{k,0}^\Phi,\ldots,n_{k,m_{g1}-1}^\Phi\right)\Big|\sum_{i=0}^{m_{g1}-1}n_{k,i}^\Phi = L_R-1\right\}$ with $\left\{n_{k,i}^\Phi\right\} \in \mathbb{Z}^+$, a_k^Φ, b_k^Φ, and c_k^Φ are, respectively, given by

$$a_k^\Phi = \frac{(L_R-1)!}{\prod_{i=0}^{m_{g1}-1}n_{k,i}^\Phi!}\prod_{i=0}^{m_{g1}-1}\left(\frac{1}{i!}\right)^{n_{k,i}^\Phi}, b_k^\Phi = \sum_{i=0}^{m_{g1}-1}n_{k,i}^\Phi i, \quad (A.6)$$

and $\quad c_k^\Phi = (L_R-1)\left(s+\frac{m_{g1}}{\Omega_{g1}}\right). \qquad (A.7)$

Based on (A.2), we proceed to employ the multinomial theorem to express $(F(x))^{N_R-L_R}$ as

$$(F(x))^{N_R-L_R} = \sum_{\mathcal{S}_K^F}a_k^F x^{b_k^F}e^{-c_k^F x}, \qquad (A.8)$$

where $\mathcal{S}_K^F = \left\{\left(n_{k,0}^F,\ldots,n_{k,m_g}^F\right)\Big|\sum_{j=0}^{m_g}n_{k,j}^F = N_R-L_R\right\}$ with $\left\{n_{k,j}^F\right\} \in \mathbb{Z}^+$, a_k^F, b_k^F, and c_k^F are, respectively, given by

$$a_k^F = \frac{(N_R-L_R)!}{\prod_{j=0}^{m_{g1}}n_{k,j}^F!}\prod_{j=0}^{m_{g1}-1}\left(\frac{-1}{j!}\right)^{n_{k,j+1}^F}\left(\frac{m_{g1}}{\Omega_{g1}}\right)^{b_k^F}, \qquad (A.9)$$

$$b_k^F = \sum_{j=0}^{m_{g1}-1}jn_{k,j+1}^F, \text{and} \quad c_k^F = \frac{m_{g1}}{\Omega_{g1}}\sum_{j=1}^{m_{g1}}n_{k,j}^F. \qquad (A.10)$$

Substituting (A.1), (A.5) and (A.8) into (A.4), and applying [33, eq. (3.351.3)], $\Phi_{GSC}(s)$ is derived as

$$\Phi_{GSC}(s) =$$
$$\frac{L_R}{(m_{g1}-1)!}\binom{N_R}{L_R}\left(\frac{m_{g1}}{\Omega_{g1}}\right)^{m_{g1}L_R}\sum_{S_k^\Phi\in\mathcal{S}_K^\Phi}\sum_{S_k^F\in\mathcal{S}_K^F}a_k^\Phi a_k^F$$
$$\frac{\left(b_k^\Phi + b_k^F + m_{g1}-1\right)!\left(s+\frac{m_{g1}}{\Omega_{g1}}\right)^{b_k^\Phi - m_{g1}(L_R-1)}}{\left(s+c_k^\Phi + c_k^F + \frac{m_{g1}}{\Omega_{g1}}\right)^{b_k^\Phi + b_k^F + m_{g1}}}. \qquad (A.11)$$

Let $F_{GSC}(x)$ denote the CDF of the channel power gain of the secondary network with GSC. The Laplace transform of $F_{GSC}(x)$ is given by $\mathcal{L}[F_{GSC}(x)] = \Phi_{GSC}(s)/s$ [20].

Therefore, the Laplace transform for $F_{GSC}(x)$ is

$$\mathcal{L}\left[F_{GSC}(x)\right] = \frac{L_R}{(m_{g1}-1)!}\binom{N_R}{L_R}\left(\frac{m_{g1}}{\Omega_{g1}}\right)^{m_{g1}L_R}$$
$$\sum_{\mathcal{S}_K^{\Phi}}\sum_{\mathcal{S}_K^{F}}a_k^{\Phi}a_k^{F}\frac{(b_k^{\Phi}+b_k^{F}+m_{g1}-1)!}{(L_R)^{b_k^{\Phi}+b_k^{F}+m_{g1}}}$$
$$\frac{\left(s+\frac{m_{g1}}{\Omega_{g1}}\right)^{b_k^{\Phi}-m_{g1}(L_R-1)}}{s\left(s+\frac{c_k^{F}}{L_R}+\frac{m_{g1}}{\Omega_{g1}}\right)^{b_k^{\Phi}+b_k^{F}+m_{g1}}}. \quad \text{(A.12)}$$

Using the partial fraction expansion [33, eq. (2.102)], we can rewrite (A.12) in an equivalent form. Then, taking the inverse Laplace transform of $\mathcal{L}\left[F_{GSC}(x)\right]$, we obtain

$$F_{GSC}(x) = \frac{L_R}{(m_{g1}-1)!}\binom{N_R}{L_R}\sum_{\mathcal{S}_K}\sum_{n=0}^{m_{g1}L_R+b_k^{F}}a_k^{\Phi}a_k^{F}$$
$$\frac{(b_k^{\Phi}+b_k^{F}+m_{g1}-1)!}{L_R^{b_k^{\Phi}+b_k^{F}+m_{g1}}}\ell_k(n)x^{\mu_k(n)}e^{-\nu_k(n)x}, \quad \text{(A.13)}$$

where the set $\mathcal{S}_K$ has been defined in *Lemma 1*, $\ell_k(n)$, $\mu_k(n)$ and $\nu_k(n)$ are, respectively, given by

$\ell_k(n)$

$$=\begin{cases}
\left(\frac{m_{g1}}{\Omega_{g1}}\right)^{\mu_k(n)-b_k^{F}}\left(\frac{1}{L_R}\sum_{j=1}^{m_{g1}}n_{k,j}^{F}+1\right)^{-n_2} \\
\qquad\qquad\qquad\qquad\qquad n=0 \\
\left(\frac{m_{g1}}{\Omega_{g1}}\right)^{\mu_k(n)-b_k^{F}}\left(\Upsilon_{k1}+\Upsilon_{k2}-\frac{1-\text{sgn}(c_k^{F})}{(n-1)!}\right) \\
\qquad\qquad\qquad 1\le n \le m_{g1}(L_R-1)-b_k^{\Phi} \\
\left(\frac{m_{g1}}{\Omega_{g1}}\right)^{\mu_k(n)-b_k^{F}}\left(\Upsilon_{k3}+\Upsilon_{k4}-\frac{1-\text{sgn}(c_k^{F})}{(n-1)!}\right) \\
\qquad\quad m_{g1}(L_R-1)-b_k^{\Phi} < n \le m_{g1}L_R+b_k^{F},
\end{cases}$$
$$\text{(A.14)}$$

$\mu_k(n)$

$$=\begin{cases}
0, & n=0 \\
n-1, & 1\le n\le m_{g1}(L_R-1)-b_k^{\Phi} \\
n-\text{sgn}(c_k^{F})\left(m_{g1}(L_R-1)-b_k^{\Phi}\right)-1, \\
& m_{g1}(L_R-1)-b_k^{\Phi}<n\le m_{g1}L_R+b_k^{F},
\end{cases}$$
$$\text{(A.15)}$$

and

$\nu_k(n)$

$$=\begin{cases}
0, & n=0 \\
\frac{m_{g1}}{\Omega_{g1}}, & 1\le n\le m_{g1}(L_R-1)-b_k^{\Phi} \\
\frac{c_k^{F}}{L_R}+\frac{m_{g1}}{\Omega_{g1}}, & m_{g1}(L_R-1)-b_k^{\Phi}<n\le m_{g1}L_R+b_k^{F},
\end{cases}$$
$$\text{(A.16)}$$

with $n_2 = b_k^{\Phi}+b_k^{F}+m_{g1}$. In (A.14), Υ_{k1}, Υ_{k2}, Υ_{k3}, and Υ_{k4}

are given by

$$\Upsilon_{k1}=-\frac{\text{sgn}(c_k^{F})}{(n-1)!}\left(\frac{1}{L_R}\sum_{j=1}^{m_{g1}}n_{k,j}^{F}+1\right)^{-n_2}, \quad \text{(A.17)}$$

$$\Upsilon_{k2}=(-1)^{1-n_1}\frac{\text{sgn}(c_k^{F})}{(n-1)!}\sum_{l=1}^{n_2}\left(\frac{1}{L_R}\sum_{n=1}^{m_{g1}}n_{k,n}^{F}+1\right)^{-(n_2-l+1)}$$
$$\binom{l-n_1-1}{l-1}\left(\frac{1}{L_R}\sum_{n=1}^{m_{g1}}n_{k,n}^{F}\right)^{n_1-l}, \quad \text{(A.18)}$$

$$\Upsilon_{k3}=-\frac{\text{sgn}(c_k^{F})}{(n_1-1)!}\left(\frac{1}{L_R}\sum_{j=1}^{m_{g1}}n_{k,j}^{F}+1\right)^{-(n_2-n_1+1)}, \quad \text{(A.19)}$$

and

$$\Upsilon_{k4}=\frac{\text{sgn}(c_k^{F})}{(n_1-1)!}\sum_{l=1}^{m_{g1}(L_R-1)-b_k^{\Phi}}(-1)^{l+1}$$
$$\binom{n_2-n_1+l-1}{l-1}\left(\frac{1}{L_R}\sum_{j=1}^{m_{g1}}n_{k,j}^{F}\right)^{-(n_2-n_1+l)}, \quad \text{(A.20)}$$

where $n_1 = n - m_{g1}(L_R-1)+b_k^{\Phi}$.

The CDF of $\left\|\mathbf{g}_{1i^*\theta_{i^*}}\right\|^2$ is given by $F_{\left\|\mathbf{g}_{1i^*\theta_{i^*}}\right\|^2}(x) = (F_{GSC}(x))^{N_S}$. Based on (A.13), and employing the multinomial theorem, we can derive the CDF of $\left\|\mathbf{g}_{1i^*\theta_{i^*}}\right\|^2$ as (6).

APPENDIX B
A PROOF OF LEMMA 2

According to (4), the CDF of γ_1 can be written as

$$F_{\gamma_1}(x)=\text{Pr}\left\{\left\|\mathbf{g}_{1i^*\theta_{i^*}}\right\|^2\le\frac{x}{\bar{\gamma}_P}, \left|h_{1i^*}\right|^2\le\frac{Q}{P}\right\}$$
$$+\text{Pr}\left\{\frac{\left\|\mathbf{g}_{1i^*\theta_{i^*}}\right\|^2}{\left|h_{1i^*}\right|^2}\le\frac{x}{\bar{\gamma}_Q}, \left|h_{1i^*}\right|^2\ge\frac{Q}{P}\right\}. \quad \text{(B.1)}$$

The CDF of $\left|h_{1i^*}\right|^2$ is expressed as

$$F_{\left|h_{1i^*}\right|}(x)=1-\frac{\Gamma\left(m_{h1}, x\frac{m_{h1}}{\Omega_{h1}}\right)}{\Gamma(m_{h1})} \quad \text{(B.2)}$$

By substituting (B.2) and (6) into (B.1), we derive the closed-form expression of $F_{\gamma_1}(x)$ as (10).

APPENDIX C
A PROOF OF THEOREM 2

Based on (A.8), we consider transmission in the high SNR regime with $\bar{\gamma}_P\to\infty$. Applying the Taylor series expansion truncated to the kth order given by $e^x=\sum_{j=0}^{k}\frac{x^j}{j!}+o(x^k)$ in (A.8), the asymptotic expression for (A.8) is written as

$$\left(F(x)\right)^{N_R-L_R}=\frac{\left(\frac{m_{g1}}{\Omega_{g1}}\right)^{m_{g1}(N_R-L_R)}x^{m_{g1}(N_R-L_R)}}{(m_{g1}!)^{N_R-L_R}}. \quad \text{(C.1)}$$

Substituting (A.1), (A.5) and (C.1) into (A.4) yields

$$\Phi_{GSC}(s) = \left(\frac{m_{g1}}{\Omega_{g1}}\right)^{m_{g1}N_R} \frac{\mathrm{K}\left(\mathcal{S}_K^\Phi, N_R, L_R, m_{g1}, a_k^\Phi, b_k^\Phi\right)}{\left(s + \frac{m_{g1}}{\Omega_{g1}}\right)^{m_{g1}N_R}}.$$

(C.2)

Note that $\mathcal{L}\left[F_{GSC}(x)\right] = \Phi_{GSC}(s)/s$ [20], we derive

$$L\left[F_{GSC}(x)\right] = \mathrm{K}\left(\mathcal{S}_K^\Phi, N_R, L_R, m_{g1}, a_k^\Phi, b_k^\Phi\right)$$
$$\left(\frac{1}{s} - \sum_{r=1}^{m_{g1}N_R} \frac{\left(\frac{m_{g1}}{\Omega_{g1}}\right)^{r-1}}{\left(s + \frac{m_{g1}}{\Omega_{g1}}\right)^r}\right). \quad (C.3)$$

Taking the inverse Laplace transform of (C.3), we obtain

$$F_{GSC}(x) = \mathrm{K}\left(\mathcal{S}_K^\Phi, N_R, L_R, m_{g1}, a_k^\Phi, b_k^\Phi\right)$$
$$\left(1 - \sum_{r=1}^{m_{g1}N_R} \frac{\left(\frac{m_{g1}}{\Omega_{g1}}\right)^{r-1}}{(r-1)!} x^{r-1} e^{-\frac{m_{g1}}{\Omega_{g1}}x}\right). \quad (C.4)$$

Again, employing the Taylor series expansion truncated to the kth order given by $e^x = \sum_{j=0}^{k} \frac{x^j}{j!} + o(x^k)$ in (C.4), (C.4) can be rewritten as

$$F_{GSC}(x) = \left(\frac{m_{g1}x}{\Omega_{g1}}\right)^{m_{g1}N_R} \mathrm{K}\left(\mathcal{S}_K^\Phi, N_R, L_R, m_{g1}, a_k^\Phi, b_k^\Phi\right).$$

(C.5)

Based on (C.5), the asymptotic expression for the CDF of $\left\|\mathbf{g}_{1i^*\theta_{i^*}}\right\|^2$ is given by

$$F_{\left\|\mathbf{g}_{1i^*\theta_{i^*}}\right\|^2}(x) =$$
$$\left[\left(\frac{m_{g1}x}{\Omega_{g1}}\right)^{m_{g1}N_R} \mathrm{K}\left(\mathcal{S}_K^\Phi, N_R, L_R, m_{g1}, a_k^\Phi, b_k^\Phi\right)\right]^{N_S}. \quad (C.6)$$

By substituting (C.6) into (B.1), the first non-zero order expansion of the CDF of γ_1 is attained and yields the asymptotic outage probability of cognitive relay network with the proportional interference power constraint as (14).

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Maged Elkashlan (M'06) received the Ph.D. degree in Electrical Engineering from the University of British Columbia, Canada, 2006. From 2006 to 2007, he was with the Laboratory for Advanced Networking at University of British Columbia. From 2007 to 2011, he was with the Wireless and Networking Technologies Laboratory at Commonwealth Scientific and Industrial Research Organization (CSIRO), Australia. During this time, he held an adjunct appointment at University of Technology Sydney, Australia. In 2011, he joined the School of Electronic Engineering and Computer Science at Queen Mary University of London, UK, as an Assistant Professor. He also holds visiting faculty appointments at the University of New South Wales, Australia, and Beijing University of Posts and Telecommunications, China. His research interests fall into the broad areas of communication theory, wireless communications, and statistical signal processing for distributed data processing, millimeter wave communications, cognitive radio, and wireless security.

Yansha Deng (S'13) is currently working toward the M.S. degree at Central South University, Changsha, China. She is also currently working toward the Ph.D. degree in electronic engineering at Queen Mary University of London, London, U.K.

Her research interests include multiple-antenna systems, cognitive radio, cooperative networks, molecular communication, and physical layer security.

Lifeng Wang (S'12) is working towards his Ph.D. degree in Electronic Engineering at Queen Mary University of London. Before that, he received the M.S. degree in Electronic Engineering from the University of Electronic Science and Technology of China, in 2012.

His research interests include physical layer security, massive MIMO, millimeter-wave communications and 5G HetNets.

Kyeong Jin Kim (SM'11) received the M.S. degree from the Korea Advanced Institute of Science and Technology in 1991, and the M.S. and Ph.D. degrees in electrical and computer engineering from the University of California at Santa Barbara, Santa Barbara, CA, USA, in 2000. From 1991 to 1995, he was a Research Engineer with the Video Research Center, Daewoo Electronics, Ltd., Seoul, Korea. In 1997, he joined the Data Transmission and Networking Laboratory at the University of California at Santa Barbara. After receiving his degrees, he joined the Nokia Research Center and Nokia Inc., Dallas, TX, USA, as a Senior Research Engineer, where he was an L1 Specialist from 2005 to 2009. From 2010 to 2011, he was an Invited Professor with Inha University, Incheon, Korea. Since 2012, he has been a Senior Principal Research Staff with Mitsubishi Electric Research Laboratories, Cambridge, MA, USA. His research has been focused on the transceiver design, resource management, scheduling in the cooperative wireless communications systems, cooperative spectrum sharing systems, device-to-device communications, secrecy systems, and GPS systems.

Trung Q. Duong (S'05, M'12, SM'13) received his Ph.D. degree in Telecommunications Systems from Blekinge Institute of Technology (BTH), Sweden in 2012, and then continued working at BTH as a project manager. Since 2013, he has joined Queen's University Belfast, UK as a Lecturer (Assistant Professor). He held a visiting position at Polytechnic Institute of New York University and Singapore University of Technology and Design in 2009 and 2011, respectively. His current research interests include cooperative communications, cognitive radio networks, physical layer security, massive MIMO, cross-layer design, mm-waves communications, and localization for radios and networks.