Multihyperuniform disorder and its application on shared-aperture phased antenna arrays

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***Abstract*—The main design strategy for shared-aperture antenna arrays usually involves the stacking of patch arrays operating at different frequencies on a multilayer setup. This implies that a sophisticated feeding network needs to be designed for each stacked array and in several cases proper Frequency Selective Surfaces (FSS) need to be employed, which will be invisible for certain design frequencies and will alter the propagation characteristics of the radiated wave for other frequencies. These designs although promising are rather complex, difficult to be generalized and in most cases reach up to tri-band operation. Here, we formulate the mathematical model behind the generation of a multiple element aperiodic distribution, the multihyperuniform disordered, which has been found in several natural processes. We employ this type of disordered distribution in order to overcome the bandwidth limitations of periodic distributions and design shared aperture antenna arrays for ultra-wideband unidirectional emission applications which can be easily generalized to operate at desired operating frequencies.**

***Index Terms*—Multihyperuniform disorder, shared-aperture, phased antenna arrays.**

I. INTRODUCTION

The need for multifunctional electromagnetic designs have been of major importance to the scientific community for the past decades. Specifically, the demand for wireless airborne applications with multiple band coverage has been a subject that many antenna designers and researchers have investigated in the past, which lead to the idea of shared aperture antenna arrays. These type of antenna arrays usually consist of a number of different subarrays, each designed to cover a different operating bandwidth all within the same aperture. The first challenge with designing such an array is associated with the usage of the available real estate (i.e., the array's aperture) in an efficient manner, while the second challenge is associated with the array's performance and specifically, the radiation pattern characteristics of each of the subarrays.

Many efforts have been made in the past to design such arrays with some leading to major advancements in the field. Some examples include arrays which operate in two separate bands, such as the K/Ka-band array [1], the S/X-band array [2], the C/Ku-band array [3], the X/K-band array [4], the C/Ka-band array [5], the S/Ka-band array [6] and the UHF/S-band array [7]. Other more complicated designs can cover up to three different frequency bands with some examples including the L/S/C-band array [8] and the UHF/L/S-band array [9].

As it can be seen, the vast majority of the reported shared-aperture antenna arrays that can be found in the literature are able to cover two different operating bandwidths, whereas just a few shared-aperture antenna arrays can cover a maximum of three bands at the same time. To the authors' knowledge there are no reported shared-aperture antenna arrays that can operate on more than three operating bandwidths. This can be attributed mostly to the adopted distributions. For all reported cases, each subarray is distributed in a periodic grid with constant inter-element distances, so that at the corresponding operating frequencies the grating lobes are completely suppressed. Thus, in order to have grating lobe-free radiation patterns, as well as to satisfy the non-overlapping condition between all the elements the periodic distribution presents some limitations to the amount of frequency bands that a shared-aperture array can operate at. On the other hand, for multi-layer designs the Partially Reflective Surfaces (PRS) and the feed networks become very complex when more than two subarrays are considered in the shared-aperture array design and the complexity of the increases exponentially with the increase of the operating bands to more than three.

In order to overcome the aforementioned issues and be able to design share-aperture antenna arrays that can operate at more than three bands it becomes evident that some sort of aperiodic distribution needs to be employed which will lead to grating lobe-free radiation patterns and at the same time satisfy the element non-overlapping condition. The hyperuniform disordered distribution is a type of aperiodic element distribution which has been shown to be advantageous for active phased antenna arrays [10] compared against a periodic antenna array with the same number of elements distributed in the same aperture. Essentially, this type of distribution is optimal for sparse antenna arrays for unidirectional emission applications. As such, this type of distribution can be a very good foundation towards the design of multi-band shared-aperture antenna arrays. Towards that end, we will employ the idea of multihyperuniform disordered distributions. This type of distribution has been found in different biological processes, such as in the distribution of the receptors of a well-adapted immune system [11], or the photoreceptor pattern on avian eye retina [12]. In particular, for the latter case, the reason for which birds are known for their acute sense of vision, as well as their ability to see more colors in the optical spectrum lies on the multihyperuniform disordered distribution of the photoreceptor in their eye retinas. Inspired by this natural process, we aim to investigate the possible application of multihyperuniform disorder in the field of shared-aperture phased antenna arrays that can operate in multiple frequency bands and how the concept can be generalized in order to design such arrays for different applications.

II. HYPERUNIFORM AND MULTIHYPERUNIFORM DISORDERED DISTRIBUTIONS

A hyperuniform point distribution of N points in d-dimensional Euclidean space is one in which the variance of the number of points $\sigma_N^2(R)$ within a spherical sampling window of radius R grows more slowly than R^d for large R. In turn, this implies that these systems do not possess infinite-wavelength density fluctuations and that the structure factor characterizing the point pattern vanishes in the vicinity of the origin in reciprocal space [13]. The structure factor is a function that is proportional to the scattered intensity of radiation from a system of points and for this reason it is also referred to as the scattering pattern. For a point configuration of N points residing within a rectangular area of side lengths L_x, L_y at positions $\mathbf{r}_1$, $\mathbf{r}_2$, ..., $\mathbf{r}_N$, the structure factor is:

$$S(\mathbf{k}) = \frac{1}{N}\left|\sum_{j=1}^{N} e^{i\mathbf{k}\cdot\mathbf{r}_j}\right|^2 , \qquad (1)$$

where $\mathbf{k}$ is an appropriate infinite set of wave vectors. Specifically, on the boundaries of the two-dimensional rectangular computational domain, periodic boundary conditions are applied. Due to this, the corresponding infinite set of wave vectors in reciprocal space is defined as $\mathbf{k} = (2\pi n_x/L_x, 2\pi n_y/L_y)$, where $n_x, n_y \in \mathbb{Z}$. In order to generate such a distribution, the collective coordinate approach is employed as described in [14], where the total potential energy of a distribution of points is minimized under a reciprocal space condition. In particular, we require that the minimization occurs within a certain region in reciprocal space defined by $(0 < |\mathbf{k}| \leq K^*)$. The potential energy of a N-particle system residing within an area Ω is defined as:

$$\Phi = \frac{1}{\Omega}\sum_{\mathbf{k}\leq K^*}\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\cos\left[\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)\right]. \qquad (2)$$

Within this region the structure factor will be minimized leading to a hyperuniform distribution of points. In order to quantify the amount of disorder/order in such a system, the resulting distribution is characterized by the so-called stealthiness parameter defined as $\chi = M(K)/N$, which ranges from 0 to 1, with 0 indicating a random structure and 1 indicating a periodic. $M(K)$ is the number of constrained degrees of freedom associated with the potential energy of the system through $M(K) = -\Phi\Omega/N$. Previous studies in the field have shown that if $\chi \geq 0.5$ the distribution belongs to the crystalline regime and for $N > 100$ the threshold value for passing to the crystalline regime increase to about 0.58. An example of this can be seen in Fig. 1. It is readily apparent that the structure factor of Eq. 1 closely resembles the radiation pattern formula of an array of radiation sources and it has been shown that an antenna array with hyperuniform disorder has a radiation pattern that closely resembles those shown in Fig. 1 for a proper selection of χ [10].

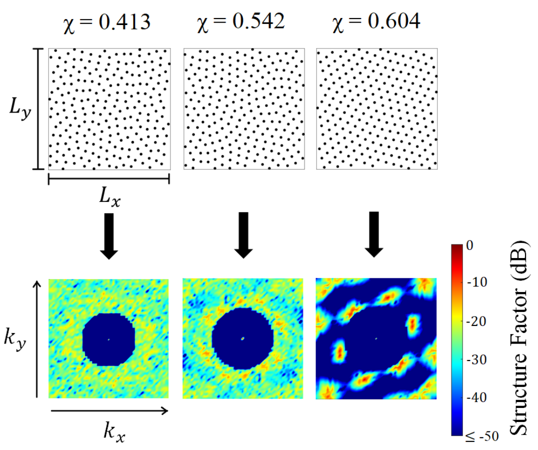

Fig. 1. The effect of the stealthiness parameter (χ) to the resulting real space element locations (top) and the corresponding reciprocal space structure factor (bottom) for an array of 225 elements.

Going further with the idea of employing hyperuniform disorder in the field of phased antenna arrays, we aim to use the idea of multihyperuniform disorder to extend the bandwidth of such an array and eventually design shared-aperture antenna arrays with multihyperunIform disorder. The idea stems from the distribution of the photoreceptors on avian eye retina. In particular, on the retinas of birds' eyes 5 different species of photoreceptors, each assigned to sample light at different frequencies, are arranged in such a manner leading to their acute sense of vision and their ability to sample lights at frequencies which other animals cannot. It becomes apparent that if we can mimic such a distribution, then we can design shared-aperture antenna arrays which can operate at different frequencies and the aperiodicity in the distribution will enable us to overcome any possible element overlap issues, while maintaining a wide-bandwidth unidirectional radiation pattern profile for the frequencies of interest.

In order to illustrate this concept we are designing a shared-aperture antenna array made of 5 different subarrays of elements which are designed to operate at specific frequencies. Furthermore, we consider that each array is made of 25 elements, resulting in an overall array of 125 elements. Then, we derive the resulting hyperuniform distributions, while taking into account the real-space constraint of non-overlapping elements for all the arrays. The result of this process can be seen in Fig. 2, where the top panel illustrates the subarray distributions, whereas the bottom panel illustrates the corresponding structure factor reciprocal space plots indicating the hyperuniformity of the designs. An interesting attribute of such a distribution is that the overall pattern of elements is also hyperuniform disordered which can be verified by looking at the rightmost bottom panel of Fig. 2.

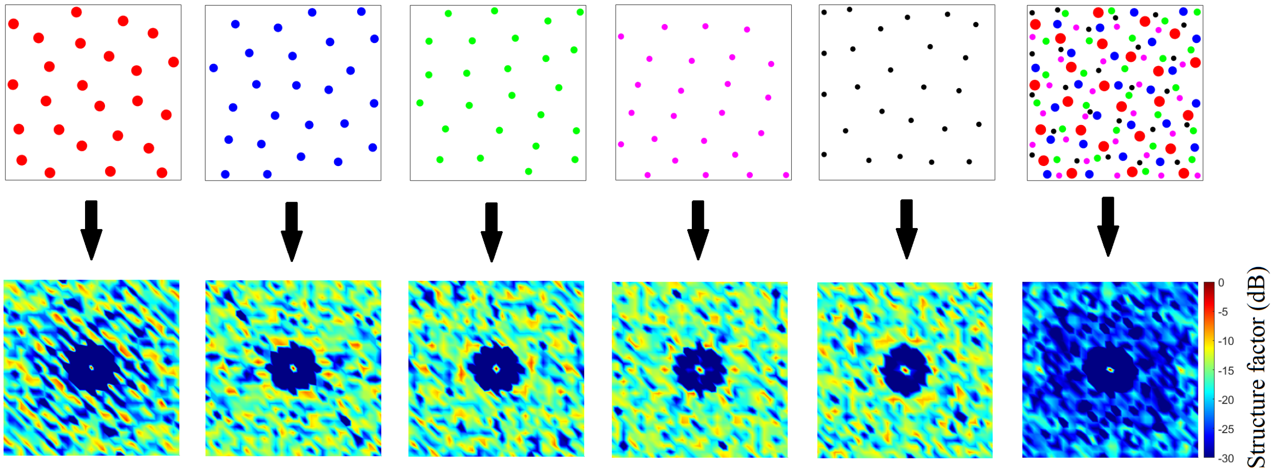

Fig. 2. A multihyperuniform disordered distribution of 5 different subarrays operating at different operating frequencies. From left to right the operating frequency of the subarray increases, as indicated by the size of the distributed circular patches. The bottom panel illustrates the corresponding structure factor of each subarray. The rightmost panel illustrates the overall shared-aperture array distribution (top) along with its corresponding structure factor plot (bottom).

The process begins by distributing the low frequency array, which is made of the largest elements and then gradually we distribute the rest of the arrays with an ascending order in terms of operating frequency. The reason behind this is can be easily understood as follows: When we are to distribute the elements in the overall aperture the space occupied from the previously distributed subarrays is already taken. Thus, the smaller the elements to be distributed the more chances that the algorithm will converge to a feasible solution (i.e., hyperuniform disorder without any element overlaps). As such, we are gradually filling the aperture area with elements and for obvious reasons it makes more sense to leave the smaller elements for last, since the available aperture area becomes smaller and smaller.

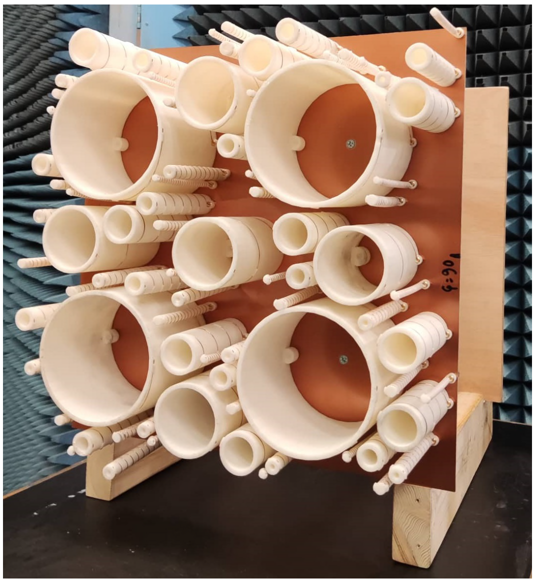

Fig. 3. The shared-aperture multihyperuniform helical antenna array made of 7 different helical subarrays. The array operates with low side lobes over a $35:1$ frequency range $(0.4-14$ GHz) with unidirectional radiation patterns.

III. CONCLUSION AND DISCUSSION

In conclusion, we have shown how the idea of multihyperuniform disorder can be applied in the field of shared-aperture antenna arrays to cover large operating bandwidths with no element overlaps. The idea stems from biological processes which can be found in nature and is an advancement of the idea of antenna arrays with hyperuniform disorder which have been shown to have grating lobe-free radiation patterns for wide bandwidth and steering angles. Fig. 3 illustrates the fabricated shared-aperture helical antenna array with multihyperuniform disorder that has unidirectional emission properties with suppressed sidelobes for an ultra-wideband frequency range $(0.4-14)$ GHz. As a future work we aim to measure the performance of the aforementioned array.

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