Essays in Corporate Finance with Machine Learning Techniques

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Statement of Originality

I, Dennis Ezimechine Iweze, confirm that the research included within this thesis is my own work or that where it has been carried out in collaboration with, or supported by others, that this is duly acknowledged below and my contribution indicated. Previously published material is also acknowledged below.

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Dennis Iweze
March 2023

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Abstract

This thesis comprises three corporate finance (entrepreneurial finance) studies with machine learning techniques.
In chapter one, I estimate the causal effect of the Venture Capital Trust (VCT) scheme on investment (change in total-asset formation) for investees in the U.K. To that end, I hand-collect data on all firms that received VCT funding (investees) from inception of the scheme till 2018. Thereafter, I adapt an unsupervised machine learning algorithm called matrix completion to estimate causal effects in settings where some investee-years are exposed to a binary treatment (VCT funding) and the goal is to estimate counterfactual outcomes for the investee-years combinations. In tandem with the hand-collected data, I use the matrix completion algorithm to estimate the causal effect of the Venture Capital Trust (VCT) scheme on the investment of investees. The estimand is the Average Treatment Effect on the Treated (ATT). I find that the VCT scheme caused a 41% increase in the investment of investees; the ATT is 41%. I also document novel insights regarding the relationship between changes to the U.K. government VCT policy, VCT fundraising and the aggregate investment of VCTs. Finally, I show that the matrix completion estimator outperforms an unconfoundedness-based estimator and alleviates the potential selection bias issue inherent in a causal study like this study.
In chapter two, I begin by highlighting the importance of VCT funding to small, young and risky firms, and the U.K. economy as whole. At a minimum, the VCT scheme is important given that it increases the supply of capital to small, young, and risky firms. Using this as a starting point, I ask whether VCT skills and the funding deal structure or luck determines the success of VCT-backed firms. Beyond the increased supply of capital to small, young and risky firms, do VCT skills and the funding deal structure determine the success of VCT-backed firms? With the aid of a Deep Neural Network Binary Classification model, a Deep Neural Network Regression model, and several attribution algorithms, I quantify the relative importance of VCT skills and the funding deal structure for the success of VCT-backed firms. I find that VCT skills and the funding deal structure are significant determinants of the success of VCT-backed firms. Specifically, prior high financial performance is the most important VCT skill determinant of the success of VC-backed firms, contributing an average

of 13% to the success of VCT-backed firms.

In chapter three, I extend the analysis in chapter two by analysing the wider U.K. VC industry, thus providing an analysis of the overarching U.K. VC industry within which the VCT scheme operates. I employ the Deep Neural Network Binary Classification model, Deep Neural Network Regression model, and several attribution algorithms from chapter two to quantify the relative importance of VC skills and the funding deal structure for the success of VC-backed firms. I find that VC specialisation in the FTSE-Industry of the firms it finances is the most important VC skill determinant of the success of VC-backed firms, contributing an average of 14% to the success of VC-backed firms.

Contents

List of Figures

List of Tables

Introduction

Venture Capital (VC) is now an established asset class that plays a critical role in the functioning of capital markets and economies worldwide. At the heart of their function is the provision of private equity financing to young, small, and financially constrained firms - that would otherwise struggle to raise capital due to their high levels of uncertain cash flows and being beset by various capital market frictions such as information asymmetry, agency issues, among many others. In the world and particularly the U.K., VCs are making substantial contributions to the fight against climate change and disease, and also making significant contributions to the U.K. economy. As per the latest British Private Equity & Venture Capital Association (BVCA) statistics - whose members represent circa 70% of U.K. VCs - 2 million people were directly employed by VC-backed firms in 2021, which represents an approximate 6% of all U.K. jobs in 2021. Additionally, 90% of VC-backed firms in 2021 were SMEs.[1] In the current era of war and cost of living crisis, VCs will clearly play a major role in charting a path to peace and economic recovery for the U.K. and global economies. VC backing is also key to the ongoing success of innovative companies tackling disease and climate change: GENinCode is a predictive genetic medicine company combining molecular tests with AI to predict cardiovascular diseases,[2] which according to the NHS,[3] is the leading cause of death and disability worldwide. ProAxsis develops diagnostic tests for lung diseases with the aid of green manufacturing techniques,[4] and Orbex is a developer of rockets for the satellite industry. Its low carbon rockets use liquid petroleum gas as a fuel source - which is kinder on the environment compared to the alternative rocket fuels commonly used.[5]

Given the continued importance of VCs (dating back to over 30 years) to firms and economies worldwide, an ever growing number of financial economics studies have sought and continue to seek to understand every aspect of the VC-entrepreneurial firm financing process. From the what perspective: what do VCs do? what capital market frictions animate the match

[1] I use the narrower term VC but the statistics also covers the private equity class in general
[2] See https://www.genincode.com/who-are-we/
[3] See https://www.nhs.uk/conditions/cardiovascular-disease/
[4] See https://proaxsis.com/about-us/
[5] See https://orbex.space/about-us

between VCs and entrepreneurial firms? what skills do VCs possess? To the how question: how do entrepreneurial firms seek out VCs? how do VCs seek out and finance entrepreneurial firms? how do they resolve issues that may arise through the investment process i.e. issues that stem from incomplete contracting? beyond the supply of capital, how do VCs add value to VC-backed firms?

In this thesis, the focus is on a U.K. government venture capital scheme called the Venture Capital Trust (VCT) scheme. I analyse the economic implications of the VCT scheme for the U.K. economy and how VCTs structure and carry out their economic function. Additionally, I analyse the structure and economic function of the wider U.K. VC industry within which the VCT scheme operates. To begin, I uncover the impact of the VCT scheme on the total-assets formation (investment) of VCT-backed firms in the U.K. Thereafter, I analyse: the skill set VCTs bring to bear on the entrepreneurial firms they finance and how it impacts their valuation, the skill set U.K. VCs bring to bear on the entrepreneurial firms they finance and how it also impacts their valuation, with all empirical analysis carried out with various machine learning approaches, which in turn helps alleviate the potential selection bias issue inherent in causal studies like this thesis.

Chapter one focuses on the impact of the VCT scheme on the total-assets formation (investment) of VCT-backed firms in the U.K. The stated aim of the scheme is to encourage entrepreneurship and stimulate the growth of young risky firms in the U.K. Using this as a starting point, I evaluate the scheme's efficacy by quantifying its impact on firm growth: total-assets formation (investment), in the U.K. I start by hand-collecting data on all firms that received VCT backing between 2003-2018. Thereafter, I adapt a Matrix Completion machine learning algorithm to estimate the average causal effect of the VCT scheme on the total-assets formation (investment) of VCT-backed firms in the U.K. The matrix completion algorithm is for imputing the counterfactual total-assets of VCT-backed firms, which then allows for the estimation of the average causal effect of the VCT scheme on the total-assets formation (investment) of VCT-backed firms in the U.K. I find that the VCT scheme led to a substantial 41% increase in the total-assets formation (investment) of VCT-backed firms in the U.K., between 2003-2018. Additionally, I analyse the impact of changes to the rules and regulations guiding the VCT scheme, on aggregate VCT fundraising and the investment patterns of VCT-backed firms.

In chapter two, I start with several facts. It is clear that the VCT scheme has had a substantial impact on entrepreneurship in the U.K. (chapter one). However, even though the VCT scheme increases the supply of capital in the U.K., it comes at a substantial cost to U.K. taxpayers in the form of tax rebates available to VCT investors. Given these facts, I turn my focus to the VCTs themselves and analyse whether their skills and the funding deal structure or luck determines the success (unrealised IRR $\geq 5\%$) of VCT-backed firms. In other words, beyond the supply of capital, do VCTs add value to VCT-backed firms? To begin, I hand-collect very detailed data on firms that received VCT backing between the periods 2014-2020. The data includes details on the current valuation of VCT-backed firms, the GBP amounts of VCT funding they received, how VCTs disbursed the funding over time, the percentage equity stake purchased by VCTs, the FTSE Industry of VCT-backed firms, and various financial and life-cycle details on VCTs and VCT-backed firms. From the hand-collected data, I estimate the unrealised IRR of each VCT's investment in a firm. The dependent variable is a binary variable equal to 1 if the investment has an unrealised IRR greater than or equal to 5% (VCT-backed firm is successful) or 0 if the investment has an unrealised IRR less than 5% (VCT-backed firm is unsuccessful). This 5% threshold for classification as a successful or unsuccessful VCT-backed firm is based on the average hurdle rate for VCT investment managers to earn their performance incentive fee, which is 5%. I then follow the prior literature in constructing various measures of VCT skill and the structure of VCT funding deals. Key among the prior literature are seminal studies such as: Sahlman (1990), to construct the duration between funding rounds. The study analyses the structure of VC firms, the dynamics between outside investors in VCs and VCs themselves, the dynamic between VCs and their firms, the agency issues that arise and how contracts have evolved to address these issues, with the top VCs particularly skilled at monitoring and advising their firms. Nahata (2008) inspires the measure of VCT reputation as a value-generating skill. The study analyses the impact of VC reputation on the successful IPO exit of VC-backed firms. Nahata (2008) constructs a primary measure of VC reputation based on the cumulative market capitalisation of IPOs backed by VCs.[6] After controlling for selection bias (i.e. the Sørensen (2008) study's concern that the performance of a VC and the firms it finances may be driven by the quality of the firms themselves as opposed to the VC's value-generating skill and reputation), controlling for syndication benefits and a variety of factors that might influence VC-backed firm performance, and also employing an adapted form of the Heckman (1979) endogeneity correction methodology, they find that reputable VCs have a higher likelihood of leading their firms to successful IPO exits.

I am also inspired by Carpenter (2000) and Barrot (2017) in constructing a measure of VCT skill that relates the option-like compensation contract of a VCT investment manager to her investment decisions. Carpenter (2000) analyses how a manager's option-like compensation contract triggers a level of risk aversion that is more complex than option pricing intuition might predict. The setting in the study centres around a risk averse fund manager compen-

[6] My measure of VCT reputation follows the Nahata (2008) secondary measure of VC reputation, which is based on a VCs share of aggregate investment in the VC industry.

sated with a call option on the assets she manages. By concavifying the objective function, Carpenter (2000) is able to derive a solution function to solve for the optimal dynamic investment policy. As the value of assets under management increases, the manager becomes more risk averse. Analogously, when the manager's option-like compensation contract is deep out of the money, the manager is incentivised to engage in excessive risk taking. Barrot (2017) analyses VC funds and finds that their option-like compensation contracts influences their investment decision. Fund managers with high prior performance invest in less innovative firms. Gompers (1996) inspires my use of the age of a VCT as a measure of VCT skill. Gompers (1996) studies VC firms and analyses the relationship between VC performance, VC fundraising and the implications for VC investment decisions. They develop a hypothesis called the "grandstanding" hypothesis. The hypothesis predicts that young VCs are motivated to "grandstand" i.e. signal their abilities to potential LP investors by taking their firms public earlier than older VCs would. With the aid of a sample of IPOs and various empirical tests, they affirm the predictions of their hypothesis. IPO underpricing is higher for firms backed by young VCs, which of course represents a transference of wealth from existing to new shareholders, and also represents an actual financial loss for the VC in question.
Kaplan and Schoar (2005) inspire my use of VCT experience at funding firms in the FTSE-Industry of a potential equity investment, as another measure of VCT skill. They study the performance of VCs as measured by the IRR of their investments, and find that on a capital weighted basis, their returns are higher than the S&P 500, with significant heterogeneity in returns across VCs and across time. Crucially, they also document a positive relationship between VC performance and VC experience, with their results remaining unchanged after controlling for selection biases, risk differences, and industry differences. This measure of VCT skill is also inspired by Sørensen (2007), who in their study on the relationship between VC experience and VC-backed firm performance, develop a two-sided matching model to separate the impact of deal flow sorting from the impact of VC experience on VC-backed firm performance. They find that firms backed by experienced VCs have a higher likelihood of success, where success is measured by IPO exit, and experienced VCs add value to their VC-backed firms through *influence*, where *influence* is the ability of experienced VCs to effectively monitor and provide several post-investment value adding services to VC-backed firms. Although, they also find that deal flow sorting - which is to say experienced VCs invest in better firms - is twice as important as *influence* for the success of VC-backed firms.
Next, I build several machine learning algorithms (Deep Neural Network Binary classification and Regression models) to analyse whether the measures of VCT skill and the structure of funding deals are determinants of the success of VCT-backed firms. The output from these machine learning algorithms are interpreted with the aid of several attribution

algorithms, which in tandem with the machine learning algorithms, take a data-driven and flexible approach to the analysis, thus helping to alleviate the potential selection-bias issue in this study. I find that VCT skills such as VCT FTSE-Industry funding specialisation, VCT with high prior financial performance, the age of a VCT, VCT with the largest market share i.e. portfolio valuation in the top quintile, and the structure of financing deals such as the total amount invested in a firm, the number of funding rounds, the number of years between funding rounds, and the number of years an investment is held, are all important determinants of the success of VCT-backed firms. Specifically, being backed by a VCT with high prior financial performance is the most important VCT skill determinant of the success of VCT-backed firms, contributing an average of 13% to the success of VCT-backed firms.

Chapter three extends the analysis in chapter two. Here, I focus on the wider U.K. VC asset class - the framework within which the VCT scheme operates - and investigate whether the skills of U.K. VCs and the funding deal structure or luck determines the success (exit IRR greater than or equal to 20%) of VC-backed firms. I start by obtaining two sets of data from the Refinitiv Workspace platform. The first dataset spans the period 2002-2022 and contains details on VC investments made by U.K. VCs into U.K. firms. The second dataset spans the period 2005-2022 and contains details on exits by U.K. VC-backed firms. By merging both sets of data, I obtain details on the equity investments made by U.K. VCs and the eventual exit of these VC-backed firms. For each firm that received VC backing, the data contains details such as the names of its VC backer/s, the GBP amount invested, the date/s the investment/s was made, the FTSE Industry of the firm, the date of exit, the firms operating stage at exit, the number of years the investment was held - from the date of first VC investment to the eventual exit, the GBP proceeds from the exit, and the type of exit (i.e. IPO, secondary sale, reverse takeover, merger etc.). Armed with the data, I estimate the realised IRR of each exit, and from this, construct a binary dependent variable equal to 1 if the exit was successful (IRR $\geq$ 20%) or 0 if unsuccessful (IRR $<$ 20%). This 20% threshold for success is inspired by the recent survey-based study of Gompers, Gornall, Kaplan, and Strebulaev (2020) wherein they show that the median net IRR that VCs market to Limited Partner's as target net IRR is 20%.

In constructing the measures of VC skill, I follow the same approach as in chapter two except for a few variables which are constructed differently due to data limitations. For instance, I follow Iliev and Lowry (2020) in employing a VCs prior exit performance (as opposed to its prior financial performance employed in chapter two) as my measure of whether the VC has a high or low prior performance. Iliev and Lowry (2020) formulate two hypothesis to test the phenomena of VCs continuing their funding of their VC-backed firms after they go public (IPO). Their Information Asymmetry Hypothesis posit that VCs continue to fund their

VC-backed firms post-IPO because, although these firms have valuable projects, they are plagued by informational problems that would otherwise limit their ability to raise external financing to fund said projects. Their second hypothesis is called The Agency Hypothesis. The hypothesis posits that post-IPO VC financiers are motivated to invest by factors other than the NPV of an investment. One of the variables with which they test this agency hypothesis is the prior exit performance of a VC financier. Across a number of empirical specifications, they find support for their information asymmetry hypothesis and no support for their agency hypothesis.

I employ the Deep Neural Network Binary Classification model, Deep Neural Network Regression model, and attribution algorithms from chapter two to quantify the relative importance of VC skills and the funding deal structure for the success of VC-backed firms. I find that VC skills and the funding deal structure are significant determinants of the success of VC-backed firms. Specifically, VC FTSE-Industry funding specialisation is the most important VC skill determinant of the success of VC-backed firms, contributing an average of 14% to the success of VC-backed firms.

Chapter 1

A Matrix Completion Approach to Policy Evaluation: Evaluating the Impact of the VCT Scheme on Investment in the U.K.

1.1 Introduction

The Venture Capital Trust (VCT) scheme, introduced in 1995, is one of three tax-based venture capital schemes, the others being the Enterprise Investment Scheme (EIS) and the Seed Enterprise Investment Scheme (SEIS). The VCT scheme is a U.K. government policy response to a perceived breakdown in financial markets and their ability to provide risky capital to risky but promising U.K. SMEs. It is designed to encourage investors to invest (indirectly) in British, unquoted, smaller, and higher risk firms - with a need for start-up, early stage or expansion capital - by investing through subscription to a VCT's shares. VCTs are U.K. publicly-quoted and closed-ended funds, and the U.K. government encourages investment in these financial intermediaries by offering tax-rebates to investors. The VCT scheme has broad base appeal, not least because at the minimum it increases the supply of finance, thus creating value for the U.K. economy, but also we know from Haltiwanger, Jarmin, and Miranda (2010) that SMEs make outsized contributions to net employment growth in an economy. Indeed, Gonzalez-Uribe and Paravisini (2019) show that the SEIS scheme caused a 10% decrease in the cost of outside equity for young firms and a 1.6% increase in the investment of young firms. They also find that conditional on the issuance of new equity under the SEIS scheme, young firms increase their investment by 8 times the equity issuance.

I find that the VCT scheme has had a significant effect on the growth of small, young firms in the U.K. It caused a 41% increase in the total-assets formation (investment) of investees between 2003-2018. Total-assets formation or investment is the change in total-assets. Based on the stated aim of the VCT scheme - which as noted earlier is to stimulate the growth of young risky firms in the U.K. - we believe our choice of total-assets formation (investment) as the measure of the efficacy of the VCT scheme, is the most relevant measure of entrepreneurial firm growth. However, data limitations prevent us from utilising additional measures such as patents and macro aggregates i.e. contribution to employment growth, contribution to GDP, to name a few.

The main objective here is to develop a selection-bias-alleviating algorithm to estimate then quantify the causal effect of the VCT scheme on total-assets formation of investees in the U.K. We adopt the Athey et al. (2018) machine learning Matrix Completion framework for estimating causal effects in a setting where some firm-years are subject to a binary shock. Specifically, the Matrix Completion framework helps to impute the counterfactual ("missing") total-assets of investees, which then allows us to estimate the average causal effect of the VCT scheme on the total-assets formation (investment) of investees (The Average Treatment Effect on the Treated; ATT). The Matrix Completion framework helps alleviate the potential selection bias and identification issues in this study. In our causal effect setting, selection bias could be due to unobserved differences across investees and non-investees control group. The selection bias could thus drive our finding of a positive causal effect, as opposed to the VCT scheme, because investees are different from the non-investees control group. Let us consider the popularly used Difference-in-Differences (DID) framework. The VCT scheme or the VCT funding of an investee becomes endogenous when selection bias causes VCTs to invest in investees that are superior to non-investees along several dimensions that are unobserved in the data. Investees with superior unobserved features, as reflected in the error term in the DID, received VCT funding. Thus, the error term is positively correlated with VCT funding (investees total-assets) and the estimated causal effect is biased upwards relative to the VCT scheme's actual causal effect on total-assets formation. This selection bias issue is even more prevalent among small, young and risky firms - the focus of the VCT scheme. These investees have very limited operating and financial histories. The Companies House reporting standards are also less strict for these small, young and risky firms - relative to bigger, older and more established firms. Additionally, and as evinced in Gompers, Gornall, Kaplan and Strebulaev (2020), VCTs emphasise numerous unobservable factors such as the management team, uniqueness of product, market and industry competition, when they screen potential investees. With the DID approach, a classic solution to the selection bias issue is the parallel trends assumption, which in this study would be that the total-assets formation (investment) of investees and non-investees control group would have evolved identically, but for the VCT

scheme. This assumption is then tested by comparing the total-assets formation of both investees and non-investees control group prior to receiving VCT funding. Unfortunately, this test does not resolve the selection-along-unobservable-characteristics issue. Not to mention, and as detailed in Athey et al. (2018), researchers have to make an ex-ante choice between exploiting cross-sectional or time-series correlation patterns or a combination of both, to create a non-investees control group. The Matrix Completion approach alleviates the selection-along-unobservable-characteristics issue and the ex-ante choice issue by allowing the data to drive what correlation patterns are exploited in the data, to construct the counterfactual outcome (total-assets). For instance, the Matrix Completion framework exploits patterns in the total-assets observations of non-investees (control group), and crucially, in the pre-VCT-funding total-assets observations of investees (treated firms), which implies that the counterfactual total-assets are constructed from a hybrid of data patterns as opposed to data patterns extracted from any one control group.
A crucial first step in this study is the hand-collection of data on all investees in the U.K. (both former and current). To our knowledge, our hand-collected data on investees is the most comprehensive VCT data available.[1] Our hand-collection efforts also allows us implement the secondary objective of this study, which is to employ hand-collected information from VCT annual reports to conduct analysis on how changes to the governmental regulations guiding VCT activities shaped VCT fundraising and the aggregate investment patterns of investees. The Athey et al. (2018) Matrix Completion framework that we adopt differs from but combines the unconfoundedness and synthetic control frameworks. Given an observed matrix of outcomes for units - which could include both treated and untreated units - they assume that the data for treated units during treatment periods is missing. The task is to impute the missing entries for treated units in the matrix. The imputed values represent the counterfactual (“missing”) outcomes. For this study, it implies imputing the counterfactual total-assets of investees. This approach to imputing counterfactual (“missing”) entries in a matrix assumes that the complete matrix has a low-rank, a rank we can implicitly realise by regularisation methods (by adding a penaliser to the objective function), and an approach that has been employed in seminal studies in the matrix completion literature such as Cai, Candes, and Shen (2008), Candes and Recht (2009), Candes et al. (2009), Candes and Plan (2009), Keshavan, Oh, and Montanari (2009). The literature on causal inference has several approaches to the problem of imputing the counterfactual (“missing”) outcomes. For instance, Imbens and Rubin (2015) take an unconfoundedness approach to the problem. This approach is akin to imputing the counterfactual (“missing”) outcomes for treated units with

[1] Popularly used platforms for data on VC Deals have very sparse coverage of VCTs that are unaffiliated with VCs.

the observed outcomes for control units - which are units that share similar pre-treatment outcome values with the treated units. Another approach is the synthetic control approach employed in studies such as Doudchenko and Imbens (2016), Ben-Michael et al. (2018). This approach is akin to imputing the counterfactual ("missing") outcomes for treated units with weighted average outcomes for control units. Here, the weights are constructed such that the weighted lagged control outcomes are equal to the lagged outcomes for treated units. Athey et al. (2018) note that whilst the unconfoundedness and synthetic control approaches are similar, they have very salient differences. They especially differ in the data-correlation patterns they exploit to impute the counterfactual ("missing") outcomes. The unconfoundedness approach assumes that the outcomes for the treated and control units follow the same trend in the pre-treatment period. Also, the typical setting in the unconfoundedness approach is one in which the treated units are assumed to be treated all at the same time, in the last period. In contrast, the synthetic control approach assumes that the correlation between outcomes for both control and treated groups are steady over time. Whereas, the typical setting in the synthetic control approach is one in which there are only one or a few treated units, significantly more control units, and a substantial number of pre-treatment periods. Athey et al. (2018) argue that, given a particular setting, both approaches are interchangeable - after some regularisation. Indeed, they show that the unconfoundedness and synthetic control approaches can also be viewed as matrix completion approaches based on matrix factorisation. However they show that the matrix completion approach has a superior performance due in part to its use of regularisation to characterise the estimator, whereas the unconfoundedness and synthetic control approaches impose restrictions on the factors in the matrix factorisation.

We now turn to re-emphasising the importance of venture capital (the wider framework within which the VCT scheme operates) funding for SMEs and by extension the economy. Kaplan and Lerner (2010) document that even though less than 0.25% of U.S. firms receive VC-backing, an estimated one-half of IPOs are VC backed, Metrick and Yasuda (2011) emphasise the positive relationship between VC funding, small firms, and innovation, and Gompers, Gornall, Kaplan and Strebulaev (2020) document: the VC-backed heritage of numerous innovative companies, their effects on the U.S. and global economy, and with the aid of survey data - explore how these VCs make decisions. However, although VCTs are analogous to VCs,[2] the specific importance of VCT funding for SMEs and in turn, the wider

[2]The main difference between VC and VCT primarily centres around the trust status and specific government regulations guiding VCTs

economy, is practically unknown in academia.[3]
The primary contribution of this paper is the quantification of the causal impact of the VCT scheme on the total-assets formation (investment) of investees in the U.K. We find that the VCT scheme has had a very discernible effect on total-assets formation (investment) in the U.K. It led to a 41% increase in the total-assets formation (investment) of investees between 2003-2018. Finally, our VCT data hand-collection efforts allow us to extract information with which we analyse the relationship between: contemporaneous changes to the VCT regulations, annual VCT fundraising, and the annual aggregate investment patterns of investees. This particularly can serve as a template for regulators to enact effective changes to the VCT regulations. For instance, we note how changes to the age criteria for first-time investees immediately affected the median size of new investees. This can inform regulators on what policies to implement to immediately affect the type of firm that receives VCT funding.
The remainder of this study is organised as follows. In section 1.2, we describe VCTs and summarise the tax benefits of investing in them. In section 1.3, we detail the investee data hand-collection process. We also present two separate summary statistics on investees and VCTs - the first is based on our hand-collected data, the second is based on Her Majesty's Revenue and Customs (HMRC) VCT data. Section 1.4 provides the framework for our estimand, the Average Treatment Effect on the Treated; ATT. In section 1.5, we present the Matrix Completion estimator. In section 1.6, we present our main results, the causal effect of the VCT scheme on the investment of investees in the U.K. (ATT). We also analyse how major VCT policy changes impacted VCT fundraising and the aggregate investment patterns of investees. Finally, we present some additional results comparing the performance of our Matrix Completion estimator with a Difference-in-Differences (DID) estimator. In section 1.7, we summarise and conclude. The appendix contains illustrations of the tax benefits from investing in VCTs, detailed steps on the closed form and numerical solution for our Matrix Completion estimator, and the major VCT policy changes between 1995-2020.

1.2 All About VCTs

Before we get into the data hand-collection, analysis and results, it is useful to provide a detailed insight into VCTs and what they are about. VCTs fall under three broad categories: generalist (VCTs that fund firms in various economic sectors), AIM (VCTs that fund firms listed on the AIM market), and specialist (VCTs that fund firms in one or a few sectors

[3]The bulk of knowledge on VCTs and their importance to macroeconomic considerations is limited to reports commissioned by various bodies such as: governmental agencies, VCTs themselves, and investment companies and their affiliates.

e.g. renewable energy infrastructure, technology, or media). A VCT appoints a regulated investment manager who invests and manages the fund on a daily basis; very few VCTs are "self-run" by their directors. The investment managers goal is to invest in firms that maximises returns to its shareholders whilst abiding by the rules and regulations guiding the VCT scheme. To that end, VCTs monitor, work with, and provide expert advice and services to their investees to help increase their value - which in turn maximises returns for VCT investors.

We will provide more details on the VCT scheme and regulations - including how these regulations have evolved - in further sections and in the appendix. But for now, the main highlights of the VCT scheme and its regulations are that VCTs: must be listed on a U.K. recognised Stock Exchange, are exempt from corporation tax on any capital gains from the disposal of an investment, can only invest in firms with a permanent establishment in the U.K., carrying on a "qualifying trade" with fewer than 250 full-time equivalent employees at the time shares are issued, and gross assets of no more than £15 million before investment and £16m immediately after investment.[4] Potential investees can receive up to £5 million in VCT financing in any 12 month period with a lifetime cap of £12 million - where these sums are also inclusive of any investment via the other two government sponsored venture capital schemes mentioned earlier: EIS and SEIS.

A VCT will typically hold an investment for a period of three to seven years before looking to sell its stake in the investee. A very high percentage of the exit proceeds - subject to the VCT's investment policy and prevailing VCT scheme regulations - are re-invested into new investees. Also, VCT regulations require tax-free dividends be paid to investors where a gain is made. In very rare instances, some VCTs are set-up with a limited lifespan. These VCTs aim to exit from all of their investees, dissolve the VCT and return all capital to their investors after a defined period e.g. seven years. These limited-life VCTs typically focus on firms with guaranteed or contractual income, thus allowing for an easy exit within a defined period. We however note that with the introduction of new risk-to-capital guidelines for the VCT scheme in 2018, limited-life VCTs are now almost if not completely non-existent.

To encourage investment in VCTs, the U.K. government offers significant tax advantages to VCT investors. An investor in VCT shares - purchased at launch, or during subsequent share class issues - receives up to 30% tax relief on their VCT share subscriptions of up to a maximum of £200,000, conditional on holding the investments for a minimum of five years. In addition to the tax-free dividends mentioned earlier, capital gains from VCT investments are also free of capital gains tax. If an investor purchases VCT shares on the secondary

[4]With very few exceptions, most trades/industries are qualifying. HMRC places restrictions on industries that Her Majesty's Treasury does not consider as in need of extra financial support e.g. agriculture, real estate, financial services, oil & gas

market i.e. after they are listed on the London Stock Exchange, there is no tax-relief on the purchase, but gains from such secondary market purchases are free of capital gains tax, in addition to any dividends from the investment being tax-free. Investors exit from VCTs by selling their shares on the London Stock Exchange or participating in any share buy-back scheme offered by VCTs or both.

Clearly, and in addition to tax-free savings from Individual Savings Accounts (ISAs) and pension allowances, VCTs are an alternative for tax-efficient investing. We illustrate this point with a simple example. Assume a company with a share price of 200p pays a 10p dividend. The dividend yield is 5%. If an investor holds the shares of said company outside an ISA or pension, the net of tax yield is 3.38% for a higher-rate taxpayer and 3.1% for an additional-rate rate taxpayer, [5] assuming the £2,000 dividend allowance has been used. Analogously, if a VCT with an initial share price of 200p pays a 10p dividend, the yield is higher than 3.38% because the VCT investor gets up to 30% income tax relief, hence the net purchase cost of the share is actually 140p. A 10p dividend from VCT shares purchased at 140p results in a tax-free yield of 7.14%. To achieve an equivalent after-tax dividend yield of 7.14% on a taxable investment , a higher-rate tax-payer would need to earn a pre-tax yield of 10.6% whilst an additional rate tax-payer would need 11.5%.

1.3 VCT Data

1.3.1 Hand-Collection of Investee Data

In this section, we detail the hand-collection and measurement of our data on U.K. firms that received VCT funding (investees). Our ultimate aim is to collate data on the annual total-assets of each investee, which we need for estimating the effect of the VCT scheme on the total-assets formation of investees: Average Treatment Effect on the Treated (ATT). There are two parts to estimating this estimand. The collected/observed total-assets values for investees and the counterfactual (“missing”) total-assets values, which we will estimate with our Matrix Completion algorithm.

Our first task was to collate the names of all investees and the date they first received VCT funding, from the inception of the VCT scheme in 1995 to 2018. Data platforms have very sparse coverage of VCT data. The VCT regulator (HMRC) does not publish this information either. We scoured the Companies House Service,[6] the London Stock Exchange (LSE) website, and the Association of Investment Companies (AIC) website - to build a list of the

[5] The tax rates are 32.5% and 38.1% respectively.

[6] A digital search service that provides free access to all public information stored on the U.K. register of companies

names of all current (62) and former VCTs. Armed with this list, we sourced and gathered every semi-annual and annual report published by every VCT from the inception of the scheme till present day (2018). From these reports, we extracted details on investees: their names, registration number, and the date they received VCT funding for the first time. We focused on the first time an investee received VCT funding because we will adopt a staggered adoption of treatment set up in our matrix completion approach, which implies that once a firm is "treated", it remains in the "treated" group forever. This meant that we did not need to track the subsequent funding rounds of each investee. Once an investee receives VCT funding (treatment), it cannot "un-receive" it, it remains in the treated group forever. Our final sample contains 1,931 unique U.K. firms.[7] The staggered adoption of treatment set up has been extensively employed in the literature on causal potential outcomes. For more on staggered adoption, see Athey and Stern (2002), Athey and Imbens (2018), and Athey et al. (2018).

We utilise the list of 1,931 investees and their registration numbers to obtain their total-assets data on the Financial Analysis Made Easy (FAME) database. FAME contains detailed financial, legal, and ownership information for public and privately incorporated firms in the U.K. and Republic of Ireland. Additionally, we collate total-assets data on 60,000 randomly selected but representative sample of the universe of U.K. firms (non-investees). This sample, in addition to our sample of 1,931 investees, will be employed in our Matrix Completion algorithm. It is also worth mentioning that our sample is free of survivorship bias - as FAME reports historical information for up to 10 years regardless of whether a firm reports financial data or not.

1.3.2 Measurement of Investee Data

Our data sample spans the time period 2003 - 2018 at an annual frequency. FAME data coverage starts from 2001, but we constricted our sample to start at 2003 because the 2001-2002 total-assets entries for a significant proportion of investees are missing. The data on U.K. non-investees is a FAME random sample which is representative of the universe of U.K. firms. These non-investees firm-year observations have no missing or zero values for total-assets between the periods 2003-2018. Our final sample consists of 1,931 investees plus 60,000 non-investees spanning the period 2003-2018, and contains information on each firm's: annual total-assets, date of incorporation, primary SIC code, company status, and

[7]The number of firms that received VCT funding for the first time is closer to 2,000. However, due to data hand-collection difficulties - especially with regard to the exact date the investee received the VCT funding - we excluded some firms from this analysis.

SME indicator.

1.3.3 Summary Statistics: Hand-Collected Investee Data

Here, we present and analyse summary statistics on our hand-collected data on investees. First thing to note in Fig.1.1. is that the median size - as measured by the total-assets - of potential investees has varied over time. It ranges from approximately £8.3m in 2008 to approximately £1.7m in 2017. Also, we note that post-2015, the median size was at its lowest in all of the sample periods (between £1.7m - £2.7m). This is as a result of the 2015 rules prohibiting VCTs from investing in potential investees older than 7 years and the mandate that the potential investee must be an entrepreneurial firm with a genuine risk of loss of capital and the objective to grow and develop. This policy change helps explain why the median pre-VCT-funding size of investees has shrunk since 2015.
We next present the number of firms that received funding per annum in Fig.1.2. We observe that VCTs invested in a record-breaking number of firms in both 2014 and 2018. What does this mean? Did VCTs fundraise a record-breaking amount in both 2014 and 2018, and by implication, invest a record-breaking amount in both years - adopting a strategy of investing this record-breaking sum across a record-breaking number of firms i.e. increase the extensive margin. Did VCTs fundraise an average amount in 2014 and 2018, and by implication, invest an average amount, but spread this across a record-breaking large number of firms, hence the record number of new investees? We can answer this by jointly analysing our Fig.1.2. with column 2 of Table 1.2. It is clear that the extensive and intensive margin both increased. We observe that more money was raised in the periods 2013-2014 and 2014-2015 relative to the last 7-8 years. Also, 2018 was record-setting in terms of the amount of funds raised by VCTs - second behind the 2005 period. This leads us to conclude that not only did VCTs raise record-breaking amounts in both 2014 and 2018, they also invested in a record-breaking number of new firms.

Fig.1.3. categorises investees according to their current Companies House status. i.e. whether they are still Active or Dissolved/In Liquidation. For example, the first set of bars (blue then red) depicts the number of firms that received first-time VCT funding in 2003, categorised according to their current status (Active or Dissolved/In Liquidation). The first thing that stands out is that the majority of investees in every single cohort are still Active. In aggregate, of the 1,931 unique firms in our sample that received VCT funding for the first time between 2003-2018, 68% of them are still Active, with the remainder 32% classed as Dissolved/In Liquidation. To put these numbers in context, the Office for National Statistics (ONS) Business Demography data on the latest five year survival rate for British firms is

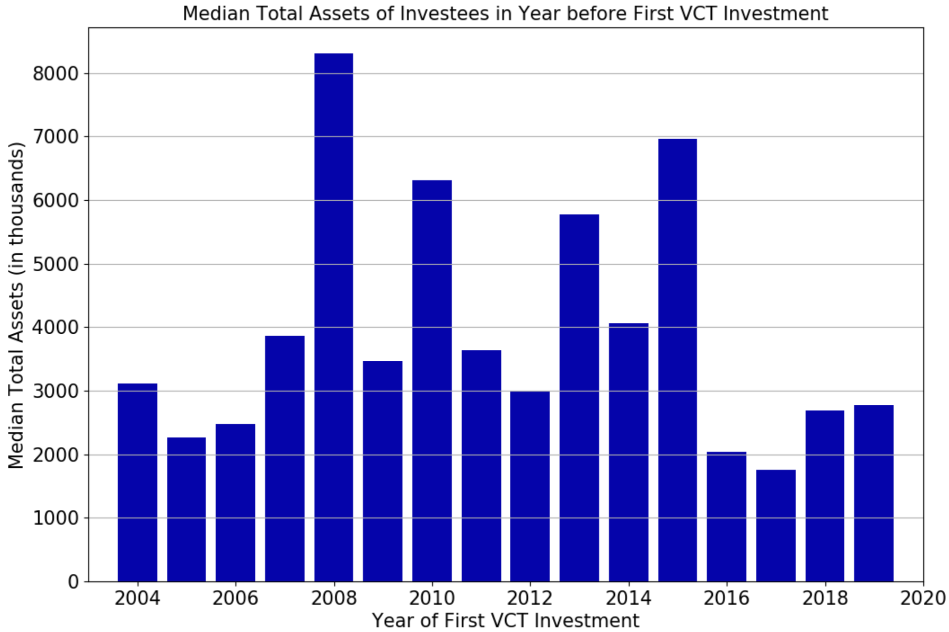

Figure 1.1. Median Total Assets of Investees in Year before VCT Investment

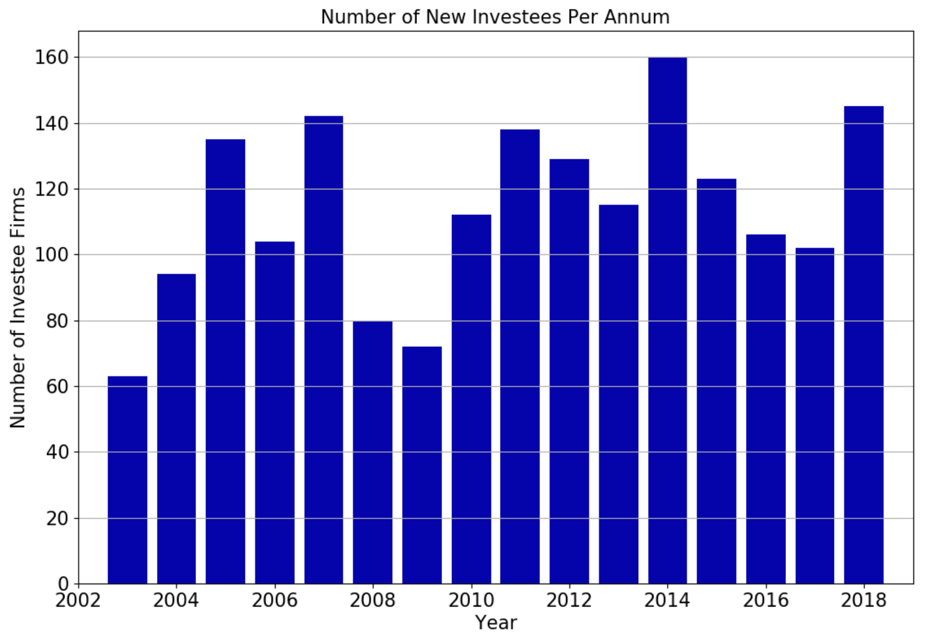

Figure 1.2. Number of New Investees Per Annum

42.5%. Investees have seemingly out-performed the national average survival rate of new firms. However, we acknowledge that the average size of investees - as measured by the range of their recent total-assets of £1.7 million - £2.7 million, is perhaps bigger than, for instance, the average startup in the Restaurants and Mobile Food Service Activities sector, and as such, using the national average survival rate to provide context might be misleading. We thus provide a more granular context by pointing out that the national average survival rate for the Computer Programming, Consultancy and Related Activities sector is 51.4%, a sector that is synonymous with large enterprises. This survival rate is still lower than that of investees at 68%. Understanding why investees have a relatively high survival rate is an important question, and will be the subject of future research.[8]

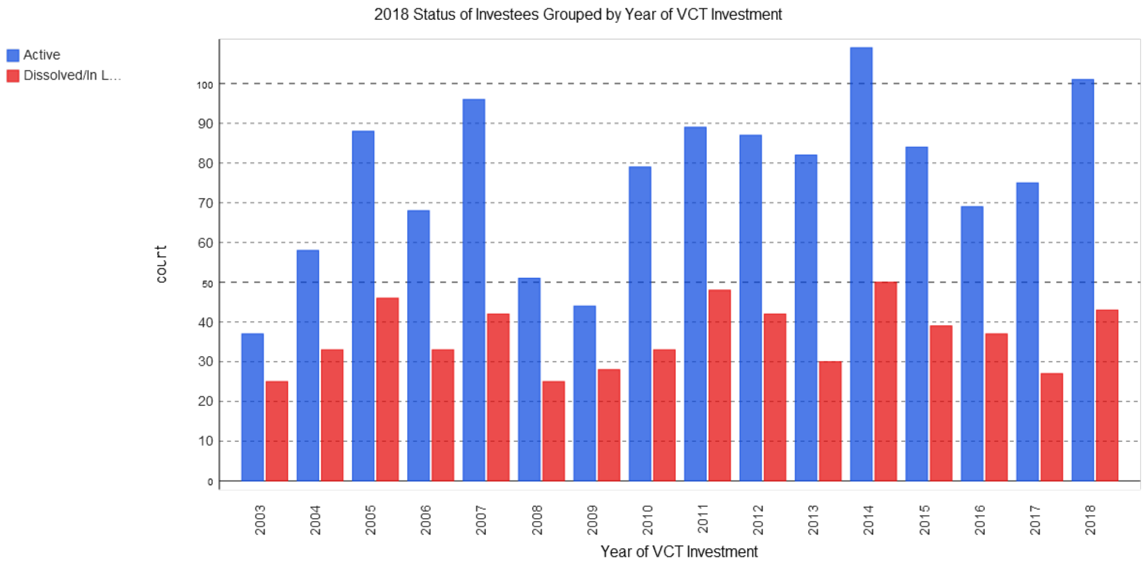

Figure 1.3. 2018 Status of Investees - Grouped by Year of VCT Investment

[8]The ONS Business Demography (2020) excel data file can be found at the following link: https://www.ons.gov.uk/businessindustryandtrade/business/activitysizeandlocation/datasets/businessdemographyreferencetable. The survival rate for Computer Programming, Consultancy and Related Activities is from Table 5.2a in the excel file.

1.3.4 Summary Statistics: HMRC Data on VCTs

Here, we present some summary statistics on VCTs raising and managing funds, collated from a recent HMRC publication (HMRC Venture Capital Trusts Statistics, 2018). In a later section, these HMRC VCT statistics will be employed in conjunction with our hand-collected data on investees, to understand how VCT policy changes and VCT fundraising drives the total-assets formation (investment) of investees.[9] In Fig.1.4., the first thing to note is that since the 2008-2009 period, the annual amount of funds raised by VCTs has been predominantly trending upwards. Between 2008-2018, there has been an almost 400% increase in the amount of funds raised - with this increase almost evenly spread across the period. In Appendix A: Major VCT Policy Changes, we detail major VCT policy changes over time and how they impacted VCT fundraising activity and of course their onward funding of SMEs. For now, the highlights are: the increased income tax relief from 20% to 40% in the 2004-2005 tax year explains the record setting amount of funds raised between 2004-2006; the 2017 Patient Capital Review and reduction in lifetime pension allowances was the major determinant of the sustained upward trend in fundraising since 2015-2016.

In Fig.1.5. the first thing to note is that the number of VCTs raising funds has almost always been less than the number of firms managing funds. VCTs do not raise funds annually. From Fig.1.5., we also note a consistently decreasing number of VCTs managing funds since the 2010-2011 tax period. This period coincided with the tightening of VCT rules i.e. VCT policy changes that limited the types and size of firms a VCT could invest in. Consequentially, VCTs started to merge in response to these changes and of course to achieve economies of scale. Additionally and as a further consequence of VCT policy changes and economies of scale, we note that the number of VCTs raising funds has been steadily declining since the 2013-2014 tax period, even though the amount of funds raised (Fig.1.4.) within the same period has been on the rise. The last thing to note is the sharp fall in the number of VCTs raising funds between 2005-2006 and 2006-2007. This was due to the decrease in the income tax relief from 40% to 30% - for VCT investors.

[9]See Section 1.6.1

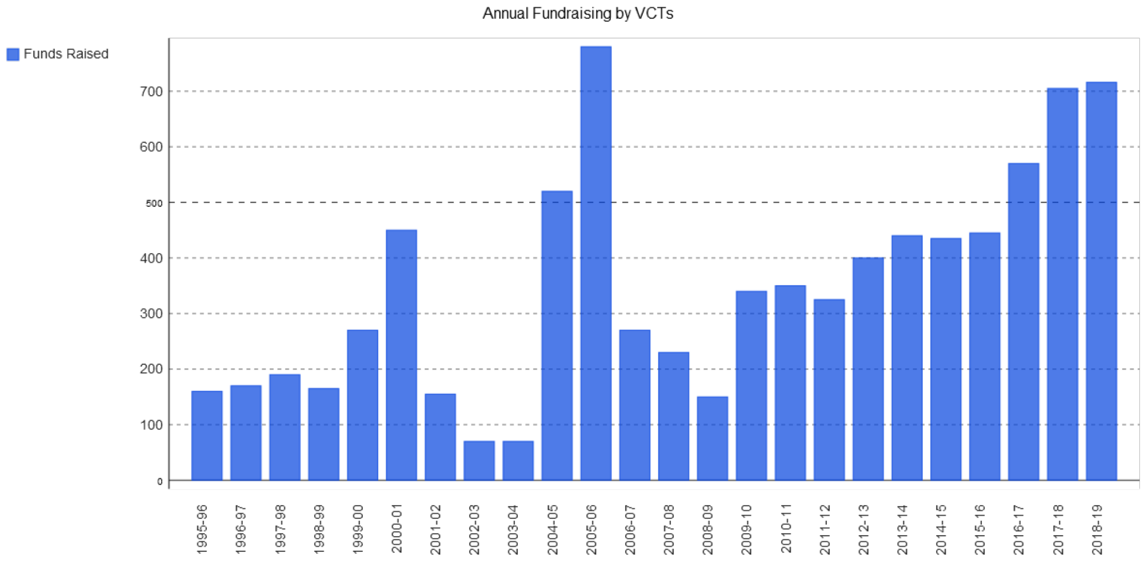

Figure 1.4. Annual Fundraising by VCTs (£ Million)

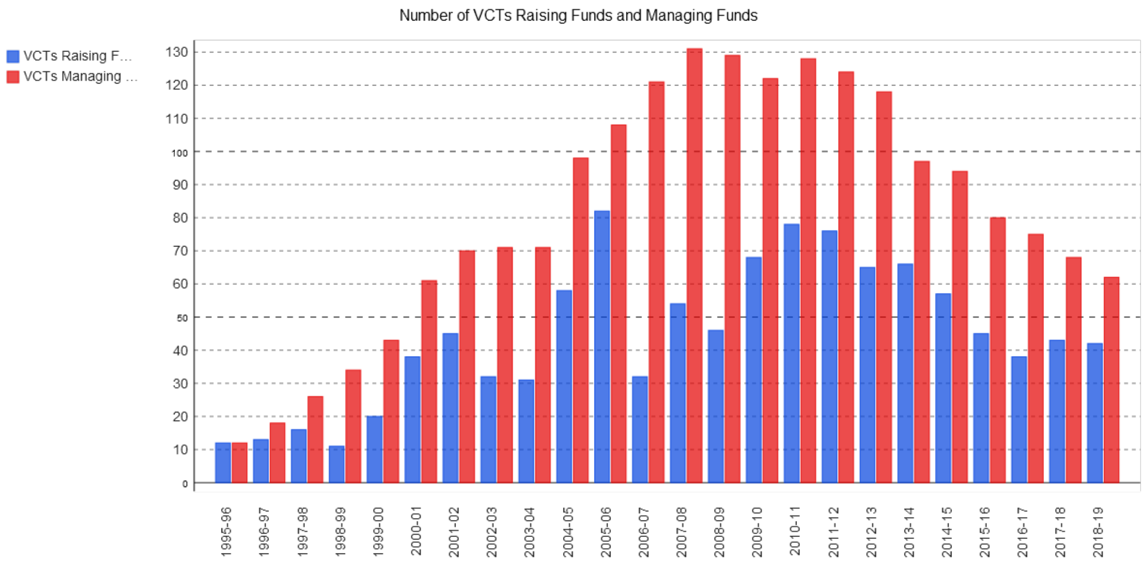

Figure 1.5. Number of VCTs Raising and Managing Funds

1.4 Average Treatment Effect on the Treated: ATT

1.4.1 Set Up

In this section, we elaborate on the causal problem, illustrate how we set up our matrix of total-assets to impute counterfactual total-assets, and also set up our estimand: the average treatment effect on the treated (ATT). Recall, the Matrix Completion algorithm allows us to impute the counterfactual total-assets for investees (which in our set up is presumed "missing"), which consequently allows us to estimate the average causal effect of the VCT scheme on the total-assets formation of investees (ATT).

The set-up for our causal problem is adapted from Athey et al. (2018).

Consider an $N \times T$ matrix $\mathbf{Y}$ which denotes our panel data of total-assets for N investees observed over T periods, with typical observation Y_{it} $\forall$ $i \in \{1, \cdots, N\}$ and $t \in \{1, \cdots, T\}$. This adapted setup is motivated by a causal potential outcome setting (see Athey et al., 2018; Imbens and Rubin, 2015; Rubin, 1974), where at each time period, a firm is either an investee or not. We characterise this as $W_{it} \in \{0,1\}$. In other words, W_{it} is an indicator for whether a firm has received VCT funding or not. Note that in our setting, once a firm receives VCT funding, it remains in the investee group throughout the sample. Additionally, τ_{it} is an indicator for the observed and counterfactual total-assets value for an investee at period t, and is given by: $\tau_{it} \in \{0,1\}$. We now turn to laying out our estimand: the average causal effect (ATT) of the VCT scheme on the total-assets formation of investees (firms who received VCT funding). This effect is formulated as:

$$ATT = \mathbb{E}[Y_{\tau=1}|w=1] - \mathbb{E}[Y_{\tau=0}|w=1]. \tag{1.1}$$

To estimate this quantity for all investees, we need to impute the counterfactual ("missing") total-assets value for all investees. Given the form of our estimand (ATT), all the total-assets entries for $Y_{\tau=1}|w=1$ are observed. We want to impute the counterfactual ("missing") total-assets for $Y_{\tau=0}|w=1$. For ease of notation and uniformity with the matrix completion literature, we will interchangeably refer to our task as imputing the missing values of a partially observed matrix of total-assets $\mathbf{Y}$ or imputing the counterfactual total-assets of investees; the total-assets of investees had they not received VCT funding. With this task complete, we can estimate our average causal effect of the VCT scheme on the total-assets formation of investees: ATT.

Regarding the pattern of missing data, we know that investees received VCT funding in a staggered fashion. In other words, there is a staggered or time-varying adoption of treatment (Athey et al., 2018; Shaikh and Toulis, 2019). Essentially, this means investees received VCT funding at different periods, and in some cases, multiple times over several years. Nonetheless, once a firm receives VCT funding i.e. becomes an investee, we assume it remains an investee forever. This means that from the year it received VCT funding onward, we estimate its counterfactual total-assets (presume its total-assets is "missing"). We illustrate below:

$$Y_{N\times T} = \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & \cdots & T \end{array} \\ \begin{pmatrix} \checkmark & \checkmark & \checkmark & \checkmark & \cdots & \checkmark \\ \checkmark & \checkmark & \checkmark & \checkmark & \cdots & \mathsf{X} \\ \checkmark & \checkmark & \mathsf{X} & \mathsf{X} & \cdots & \mathsf{X} \\ \checkmark & \checkmark & \mathsf{X} & \mathsf{X} & \cdots & \mathsf{X} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \checkmark & \mathsf{X} & \mathsf{X} & \mathsf{X} & \cdots & \mathsf{X} \end{pmatrix} \end{array} \begin{array}{l} \\ \text{never received VCT funding} \\ \text{received VCT funding in period T} \\ \\ \text{received VCT funding in period 3} \\ \\ \text{received VCT funding in period 2} \end{array}$$

Here, the check-mark ($\checkmark$) represents observed/pre-VCT funding total-assets values whilst the X represents "missing"/post-VCT funding total-assets values. In other words, the X represents counterfactual total-assets values which we will estimate with the matrix Completion algorithm. For instance, firm N (in the last row entry) received VCT funding in period 2 - hence the X in period 2. It may or may not have received further rounds of VCT funding in subsequent periods - up to period T. Regardless, the firm remains an investee from the moment it receives VCT funding - hence the X in periods $3, 4, \cdots, T$ as well.

1.5 Matrix Completion

1.5.1 Matrix Factorisation: Singular Value Decomposition

In this section, we develop the matrix factorisation approach which underpins our Matrix Completion algorithm, which we employ to estimate the average causal effect of the VCT scheme on the total-assets formation of investees in the U.K. The matrix factorisation approach is based on a fundamental topic in unsupervised machine learning: the recovery of a low-rank matrix from high-dimensional data or data dimensionality reduction, which helps to uncover otherwise hidden information in data. This framework is widely used in far ranging fields - from economics (Athey et al., 2018) to computer vision (Candes and Plan, 2009). It is used to solve many popular machine learning tasks such as matrix completion (Candes and Tao, 2010; Athey et al., 2018) and robust principal component analysis (Candes et al., 2009). This framework has also been employed in a causal panel data settings in Economics (Athey et al., 2018), and in the building of recommender systems (Koren, Bell and Volinsky, 2009). The idea behind matrix factorisation is that the data is given in the form of a matrix Y, and we assume that the true dimensionality of the matrix (for example, the rank of the matrix) is much lower than the actual dimension of the matrix Y. This assumption can be formulated as:

$$\mathbf{Y} = WZ^{\top}, \tag{1.2}$$

for matrices $Y \in \mathbb{R}^{N\times T}$, $W \in \mathbb{R}^{N\times k}$ and $Z \in \mathbb{R}^{T\times k}$. If k is smaller than N and T, the rank of $\mathbf{Y}$ is k instead of N or T. Practically, this means we only store k(T + N) values of Y instead of NT values. The former being much smaller if k is chosen to be small. To illustrate, consider our panel data of investee-years given by $Y \in \mathbb{R}^{1931\times 16}$, where every row is the vector representation of one investee, every column represents the 16 years between 2003-2018, and assume that all 1,931 investees can (approximately) be considered linear combinations of only 10 different firms, i.e. k = 10. This means we can store the data on all firm-years with only 10 × (16 + 1,931) = 19,470 entries, as opposed to the NT=30,896 entries of the original dataset. This is approximately 63% of our original investee-years dataset.

There are numerous approaches to factorising matrices. In this paper, we focus on the singular value decomposition (SVD) approach; SVD generalises the concept of eigendecompositions of square matrices. It can be shown that every real matrix $Y \in \mathbb{R}^{N\times T}$ can be factorised into three matrices $U \in \mathbb{R}^{N\times N}$, $\Sigma \in \mathbb{R}^{N\times T}$ and $V \in \mathbb{R}^{T\times T}$ via

$$Y = U\Sigma V^{\top}, \tag{1.3}$$

where, both U and V are orthogonal matrices, i.e. $U^{\top}U = I_{N\times N}$, $UU^{\top} = I_{N\times N}$, $V^{\top}V = I_{T\times T}$ and $VV^{\top} = I_{T\times T}$, with their columns called left- and right- singular vectors of Y. In our case, where our matrix of total-assets has $N > T$, the matrix Σ is a diagonal matrix of the form:

$$\Sigma = \begin{pmatrix} \sigma_1 & 0 & \cdots & 0 \\ 0 & \sigma_2 & \cdots & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & \cdots & \sigma_n \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix}$$

The entries $\{\sigma_j\}_{j=1}^{\min(N,T)}$ are the singular values of the total-assets matrix Y, and are all non-negative i.e. $\sigma_j \geq 0 \; \forall \; j \in \{1, \cdots, \min(N,T)\}$.
Clearly, 1.2 is a special case of 1.3 i.e. $W = U\Sigma$ and $Z = V$. The SVD of our total-assets matrix Y allows us to easily compute the Frobenius norm of said matrix, given that the Frobenius norm is equivalent to the euclidean norm of the vector of singular values. Now, we can easily define our lower dimensional approximation of total-assets matrix Y, with help from its SVD.
Suppose we define a new matrix $U_k \in \mathbb{R}^{N \times T}$ as the first k columns of U. We thus have:

$$U_k U_k^\top Y = U_k U_k^\top \; U \Sigma V^\top = U_k \left(I_{k \times k} 0_{k \times (N-k)} \right) \Sigma V^\top = U \Sigma_k V^\top, \tag{1.4}$$

where $\Sigma_k \in \mathbb{R}^{N \times T}$ is defined as:

$$\Sigma_k = \begin{pmatrix} \sigma_1 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & \sigma_2 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & 0 & \ddots & 0 & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma_k & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \end{pmatrix}$$

Therefore, $Y_k = U\Sigma_k V^\top = U_k U_k^\top Y$ is a rank k approximation of our total-assets matrix Y. In actuality, it is more than a rank-k-approximation, it is the *best rank-k-approximation* in the sense of the Frobenius norm.

Theorem 1.5.1. (Best rank-k-approximation). This theorem is based on the Eckart-Young-Minsky theorem.

For any matrix $\hat{Y} \in \mathbb{R}^{N \times T}$ with rank $(\hat{Y}) = k$, we have:

$$\left\|Y - \hat{Y}\right\|_{FRO}^2 \geq \|Y - Y_k\|_{FRO}^2 = \left\|Y - U_k U_k^\top Y\right\|_{FRO}^2 = \sum_{j \geq k+1}^{\min(N,T)} \sigma_j^2.$$

Therefore, Y_k is the best rank-k-approximation in the sense of the Frobenius norm.
See Eckart and Young (1936) for a proof of this theorem.

1.5.2 The Matrix Completion Estimator

We continue with our set up adapted from Athey et al. (2018).
Given our $N \times T$ panel data/matrix of total-assets Y of investees, which we model with the form:

$$\mathbf{Y} = \mathbf{L}\ , \tag{1.5}$$

our goal is to find a low-rank approximation to said matrix. The first a-priori assumption that we want to make is that the investees in our matrix can be classified into types, and that the different types are less than N. Therefore, we assume that every investee in our matrix Y can be modelled as a linear combination of all investee types. Mathematically, this means that we assume that the matrix with all entries has a low-rank.
The task of finding a low-rank matrix approximation $\hat{L} \in \mathbb{R}^{N \times T}$ [10] of our total-assets matrix $Y \in \mathbb{R}^{N \times T}$ can be formulated as the convex optimisation problem:

$$\hat{L} = \underset{L \in \mathbb{R}^{N \times T}}{\arg\min} \left\{ \frac{1}{2} \|L - Y\|_{\text{Fro}}^2 \ + \ \alpha \|L\|_* \quad \text{subject to } P_\Omega L = P_\Omega Y \right\}, \tag{1.6}$$

where $\|\cdot\|_*$ denotes the *nuclear-norm*, which is the one-norm or the sum of the vector of singular values of Y. i.e.

$$\|L\|_* = \sum_{j=1}^{\min(N,T)} \sigma_j,$$

where $\alpha > 0$ is a regularisation parameter and $\{\sigma_j\}_{j=1}^{\min(N,T)}$ denotes the singular values of L. Effectively, the *nuclear-norm* implicitly penalises the rank of the matrix $\hat{L}$ that we wish to recover. In order to ensure that the entries for which $\hat{L}$ is known matches the observed

[10]The low-rank matrix L has rank-r where $r << \min(N,T)$ so that it is low-rank

entries, we impose the constraint $P_\Omega L = P_\Omega Y$.
$P_\Omega : \mathbb{R}^{N\times T} \to \mathbb{R}^r$ denotes the projection onto the r observed entries of our total-assets matrix Y, provided by the set Ω. $P_\Omega Y$ are the known values of our total-assets matrix at these indices. To illustrate, we characterise our orthogonal projection operator P_Ω as

$$P_\Omega(Y)_{it} = \begin{cases} Y_{it}, & \text{if } (i,t) \in \Omega \\ 0, & \text{otherwise,} \end{cases}$$

and assume our incomplete total-assets matrix Y is given as

$$\mathbf{Y} = \begin{pmatrix} 1 & 4 & ? \\ ? & 2 & 7 \end{pmatrix}.$$

We know the indices $\Omega = \{(1,1),(1,2),(2,2),(2,3)\}$, and can therefore project them, i.e.

$$P_\Omega Y = (1\ \ 4\ \ 2\ \ 7)^\top$$

Note that this operator is linear and its transpose operation $P_\Omega^\top : \mathbb{R}^r \to \mathbb{R}^{N\times T}$ is

$$P_\Omega^\top = \begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{pmatrix} = \begin{pmatrix} z_1 & z_2 & 0 \\ 0 & z_3 & z_4 \end{pmatrix},$$

for $z := P_\Omega Y$.

In appendix A, we derive a computationally efficient algorithm for the numerical solution of our optimisation problem 1.6. It is also pertinent to emphasise that 1.6 is a proximal mapping, a mapping we show has a simple closed form solution - see appendix A.

1.6 The Causal Effect of the VCT Scheme on the Total Assets Formation (Investment) of Investees

We now turn to presenting our main findings, but first we re-present our estimand, the Average Treatment effect on the Treated (ATT), for which one of its inputs (the counterfactual total-assets of investees), requires the matrix completion method of imputing missing values in an incomplete matrix. The ATT is the average causal effect of the VCT scheme on the total-assets formation (investment) of investees in the U.K. Total-assets formation or investment is the change in total-assets.

Given our approximated complete matrix (Y) of annual total-assets for all investee-years between 2003-2018, wherein we reiterate that the observed entries for investees prior to receiving VCT funding are unchanged in the approximated Y matrix, the ATT is calculated as:

$$ATT = \mathbb{E}[Y_{\tau=1}|w=1] - \mathbb{E}[Y_{\tau=0}|w=1]. \tag{1.7}$$

where for each investees, $Y_{\tau=1}|w=1$ is its observed investment (total-assets formation or change in total-assets). $Y_{\tau=0}|w=1$ is its counterfactual investment (total-assets formation or change in total-assets). W is an indicator for whether the firm is an investee or non-investee, and τ is an indicator for the observed or counterfactual investment.

Fig.1.6. is a plot of our main result - also tabulated in Table 1.1. It depicts the annual Average Treatment effect on the Treated (annual ATT). This captures the annual average difference between the observed vs. counterfactual investment for investees. As with Fig.1.6., we see in Table 1.1, that between 2004-2007, the VCT scheme caused a substantial aggregate increase in the investment of investees (increase in the total-assets formation), from 26.40% to 49.30%. 2007 heralds the beginning of a precipitous drop in the VCT-induced investment of investees. A drop that reaches its nadir in 2009 at 30.00%. Thereafter, we see a slightly sustained rise in the causal effect of the VCT scheme on investment for investees, one that peaks in both 2011 and 2014 at 50.32% and 44.90% respectively. From 2014, we have another sustained downward trend which lasts until 2016. Thereafter, the trend reverses - increasing from its 2016 value of 38.16% to 50.60% in 2018. The Average Treatment effect on the Treated (ATT): Eq.1.7, is the average of the values in column 2 of Table 1.1. The ATT is 41%. This implies the VCT scheme caused a 41% increase in the investment of investees in the U.K., between 2003-2018. An important inquiry into this 41% increase is: At what cost has this increase come? We will discuss this query in a subsequent section (Cost to Taxpayers). We now turn our attention to analysing the drivers of the aggregate annual investment (total-assets formation) of investees $Y_{\tau=1}|w=1$.

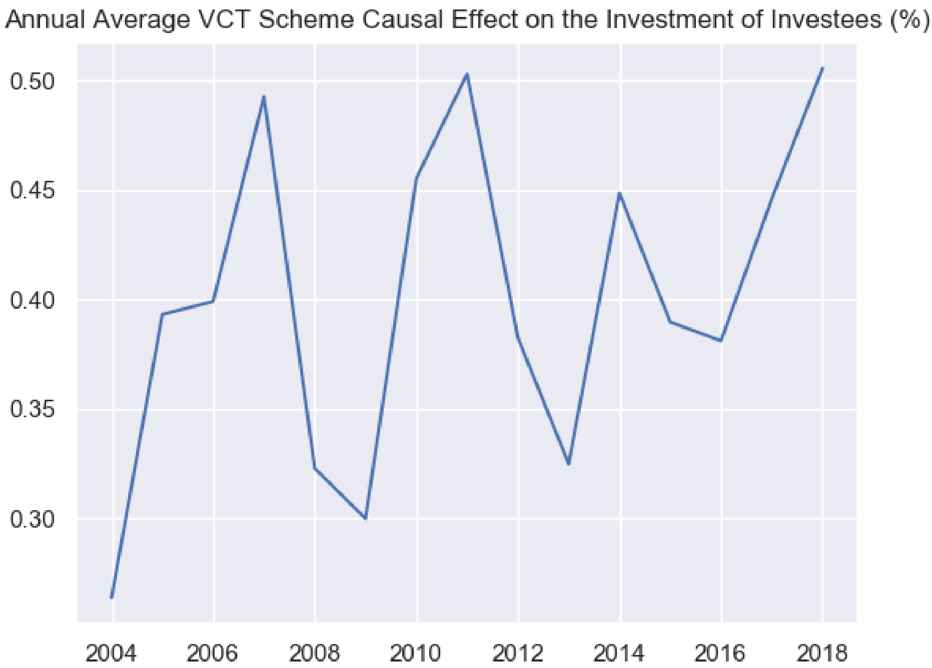

Figure 1.6. Annual Average VCT Scheme Causal Effect on the Investment of Investees (%)

Table 1.1. **Annual Average VCT Scheme Causal Effect on the Investment of Investees**

Year	ATT (%)	Number of Investees
2004	26.40	490
2005	39.33	555
2006	39.90	659
2007	49.30	726
2008	32.30	766
2009	30.00	832
2010	45.50	859
2011	50.32	900
2012	38.30	967
2013	32.50	1062
2014	44.90	1162
2015	38.98	1218
2016	38.16	1255
2017	44.61	1326
2018	50.60	1333

1.6.1 Aggregate Investment of Investees, VCT Fundraising and Major VCT Policy Changes

In this section, we turn to uncovering the drivers of a key component of our ATT result - the observed total-assets formation of investees $Y_{\tau=1}|w=1$. We see in Fig.1.7., that there is a high degree of co-movement between annual VCT fundraising (data obtained from HMRC) and the aggregate investment (total-assets formation) of investees (hand-collected data on investees) within the period 2004-2018. We now turn to linking major VCT policy changes within the period - which we document in Appendix A - to the patterns in Fig.1.7.

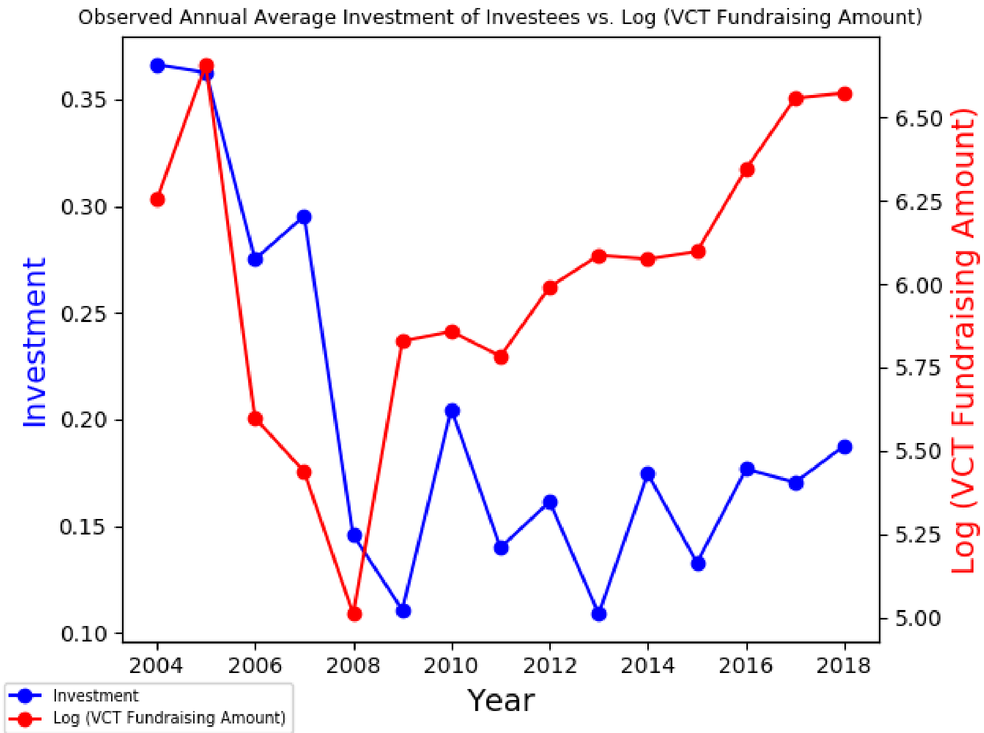

Figure 1.7. Observed Annual Average Investment of Investees vs. Log (VCT Fundraising Amount)

We start with Table 1.2., where we see a 244% aggregate increase in the amount of funds raised in 2004 and 2005 relative to the aggregate raised in the two years prior. VCT investment managers attributed this high level of fundraising and the concurrent high levels of investment by investees in the 2004-2006 period to the U.K. government's decision to raise the VCT income tax relief from 20% to 40% and increase the maximum amount individual investors could invest in VCTs to qualify for income tax relief - from £100,000 to £200,000 (6th April 2004). These expansionary changes to the VCT policy were only temporary, and in anticipation of their reversal, we see a significant drop in VCT fundraising activity and a depression in investment activity between 2005-2006. Between 2007-2009, we see a sustained downward trend in the aggregate investment of investees. This trend was not only driven by the financial crisis within the period, VCT investment managers report on how the 2007 VCT policy changes depressed their fundraising and investment activities thereafter. These policy changes mandated that VCT qualifying investees must be firms with fewer than 50 full-time employees and limited the amount of VCT funding a firm could raise, to a maximum of £2 million in any 12 month period. VCT investment managers also documented how this reduction in the size of qualifying investees increased the risk profile of potential investees and further depressed their investment activities as seen in Fig.1.7.

Between 2009-2010, VCT investment managers documented their concerns about the tightened lending conditions experienced by SMEs as a result of the lingering effects of the 2008 financial crisis. They however saw this as an opportunity to fundraise and further invest in their existing portfolios, as tightened lending conditions meant VCTs were one of the few sources of working capital and expansion capital for investees. This explains the 2009-2010 upward trend in both VCT fundraising and aggregate investment of investees. We also see in Table 1.2., that even though fundraising in the period was at a three-year high, the number of new investees that received VCT funding was the lowest it had been since 2003 (see Fig.1.2.). This means, VCTs raised more money relative to the last three years, but fewer new investees received said funds. Indeed, VCTs document how they viewed the tightened lending conditions as an opportunity to solidify their existing positions under favourable terms, hence a large proportion of the three-year-record-breaking newly raised funds went to existing investees. Between 2010-2013, Fig.1.7. depicts another downward trend in the aggregate investment activity of investees but an upward trend in VCT fundraising. VCT investment managers attribute the upward fundraising trend to the series of major VCT policy changes within the period, changes covered in Appendix A (Major VCT Policy Changes), the highlight of which centres around the reversal of the contractionary VCT policies introduced in 2007. These reversals were introduced to stimulate VCT fundraising and subsequent investment in U.K. SMEs. However, VCT investment managers were conservative in their investments. They documented their concerns about an uncertain and fragile U.K. economy. The main highlights of their concern were the sovereign debt crisis in the eurozone, upward inflationary pressures, and a sustained downward pressure on public sector spending. These reasons help explain the downward aggregate investment activity of investees trend we see in the period in Fig.1.7.

Between 2014-2015, we see a depression in the aggregate investment activity of investees and an imperceptible change in VCT fundraising. This was as a result of new legislation passed by the U.K. government in 2014 that prevented VCTs and their investors from refreshing income tax relief. However, from 2015 onward, we see a sustained upward trend in both VCT fundraising and aggregate investment activity of investees. These are as a result of the 2015 VCT policy changes introduced to bring the VCT scheme in line with the European Union's risk capital guidelines, as well as contemporaneous changes to U.K. government regulations surrounding other tax-advantaged investments. The highlight of the VCT policy changes were restrictions on investments that VCTs can make, particularly with respect to the age of potential investees, where potential investees were limited to firms that are less than 7 years old (ten years for knowledge intensive businesses). Investment managers documented their concerns that these policy changes will curtail their investment in Alternative Investment Market (AIM) shares; AIM shares form a significant proportion of VCT portfolio holdings. This line of reasoning is clearer when we consider that the London Stock Exchange requires that firms be at least 3 years old before they can registered on the AIM. VCT investment managers further interpreted these VCT policy changes as likely to reduce the scope of investments they could make, potentially increasing the risk profile of their portfolios. For instance, they claimed that replacing the shares of AIM firms with that of smaller unquoted firms will increase the risk profile of their portfolios.

However, there were two countervailing forces affecting VCT fundraising and aggregate investment activity of investees. On the one hand, the narrower set of investment opportunities documented by VCT investment managers could potentially depress investment activity. To paraphrase the sentiments of numerous investment managers "These new inhibitions will curtail significant drivers of growth in the U.K. SME ecosystem. They will curtail, as opposed to encourage, investment activity". On the other hand - and this sentiment was also explicitly expressed by VCT investment managers in their annual report - there is a high demand for VCTs to fundraise as a result of a reduction in the pension lifetime allowance from £1,250,000 to £1,000,000, the tapering away of pension tax allowances for high earners earning £110,000 a year or more, which can gradually reduce their annual allowance from the standard £40,000 to as low as £10,000,[11] and the launch of pension freedoms that allow for cash to be taken out of the pot for investment rather than buying an annuity. All of these factors caused VCTs to become more attractive to investors seeking additional tax-advantaged investments. The tax-advantage phenomena clearly dominated the narrower set of investment opportunities phenomena, and helps explain the upward trend we see in both VCT fundraising and aggregate investment activity of investees beginning in 2015 till the end of our sample in 2018.

Another crucial driver of the upward trend in VCT fundraising and aggregate investment

[11] Prior to 2009, high earners could save up to £235,000 a year in a pension and receive nearly £100,000 in tax relief. As of 6th April 2016, that sum is limited to £10,000 in a pension and just £4,000 in tax relief.

activity of investees within the latter periods of our sample, especially the 2017-2018 period, was the November 2017 Patient Capital Review, in which the U.K. Government reviewed the VCT scheme as part of its wider Patient Capital Review, which considered how to support innovative firms to access the finance they need to scale up. Her Majesty's Treasury published a consultation seeking views on how to increase the supply of capital to growing, innovative firms. The outcome was a number of proposed changes to the VCT regulations in an effort to refocus investment on potentially higher risk sectors that require capital (Her Majesty's Treasury Policy Paper, 2017).[12]

[12] See Appendix A (Major VCT Policy Changes) for a summary of the Patient Capital Review proposals.

Table 1.2. **Amount of Funds Raised and Number of VCTs. Amount:Millions. Number:Actual. Data from HMRC VCT Statistics (2018).**

Year	Funds Raised	VCTs Raising Funds in the Year	VCTs Managing Funds	Income Tax Relief
	Amount	Number	Number	(%)
1995-1996	160	12	12	20%
1996-1997	170	13	18	20%
1997-1998	190	16	26	20%
1998-1999	165	11	34	20%
1999-2000	270	20	43	20%
2000-2001	450	38	61	20%
2001-2002	155	45	70	20%
2002-2003	70	32	71	20%
2003-2004	70	31	71	20%
2004-2005	520	58	98	40%
2005-2006	780	82	108	40%
2006-2007	270	32	121	30%
2007-2008	230	54	131	30%
2008-2009	150	46	129	30%
2009-2010	340	68	122	30%
2010-2011	350	78	128	30%
2011-2012	325	76	124	30%
2012-2013	400	65	118	30%
2013-2014	440	66	97	30%
2014-2015	435	57	94	30%
2015-2016	445	45	80	30%
2016-2017	570	38	75	30%
2017-2018	705	43	68	30%
2018-2019	716	42	62	30%
Total	8,375			

1.6.2 Additional Results

In this section we employ our hand-collected data of investees and FAME data of non-investees[13] to show how the investment pattern of investees compares to that of non-investees ("control group"). The aim is to understand the patterns behind the counterfactual ("missing") total-assets value imputed by our Matrix Completion algorithm, and used in the calculation of our ATT. In Fig.1.8., we plot the observed average investment for investees vs the observed average investment for our representative random sample of 60,000 non-investees in the U.K. We observe an ostensible difference between the investment patterns of investee vs non-investees. Not only do investees - in the aggregate - invest at a much higher rate than non-investees, we also observe divergent aggregate patterns since 2009. For instance, from 2013 onward, the aggregate investment trend of investees (red line) has been steadily rising, whereas that of non-investees has steadily fallen. We however note the very similar declining investment trends for both investee and non-investees between the period 2004-2009.

We note that plotting averages can mask other patterns in the data for non-investees, especially as the non-investees range in size from the smallest firms with less than £1,000 in total-assets, to the largest with £20 billion in total-assets. To allay this concern, we repeat Fig.1.8. with one crucial change. We plot in Fig.1.9., the investment of investees vs the investment of non-investees in the top decile of investment among non-investees. We see a similar pattern in Fig.1.9., that we see in Fig.1.8., albeit with different levels of investment, where the top decile non-investees are also dis-investing but their aggregate investment remains positive, whereas the dis-investment trend in Fig.1.8., is largely negative. Between 2004-2013, the top decile non-investees had an ostensibly similar trend in their investment pattern relative to investees, although we see that the downward trend for investees is interspersed with a few periods of upward trends (2006-2007, 2009-2010). However, from 2013, we see that the aggregate investment pattern of these non-investees continues to decline - a decline that carries on to the end of the sample in 2018. On the other hand, we see an upward trend in the aggregate investment rate for investees beginning in 2013 till the end of the sample in 2018. We have already tied this increased investment rate to the major VCT policy changes and VCT fundraising in the period, so we will not belabour the point.

For completeness, we also repeat the same exercise for the non-investees in the bottom instead of the top decile of investment as depicted in Fig.1.10. This plot is also very interesting in the dynamic it depicts. There is an ostensibly similar trend in the aggregate investment pattern of investees and the bottom decile non-investees between 2003-2013, sometimes with a lag. However, and similar to the top decile non-investees, we see that the bottom decile non-investees have been continuously dis-investing from 2013 till the end of our sample in

[13] Our hand-collected data of 1,931 investees plus the FAME downloaded data of 60,000 non-investees

2018. In summary, what we see from all three figures is that from the 2013 period till the end of our sample in 2018, the aggregate investment pattern of investees was trending upward while that of non-investees (control group) was trending downward, which further emphasise the impact of the VCT scheme, and our uncovering of a significant ATT of 41%.

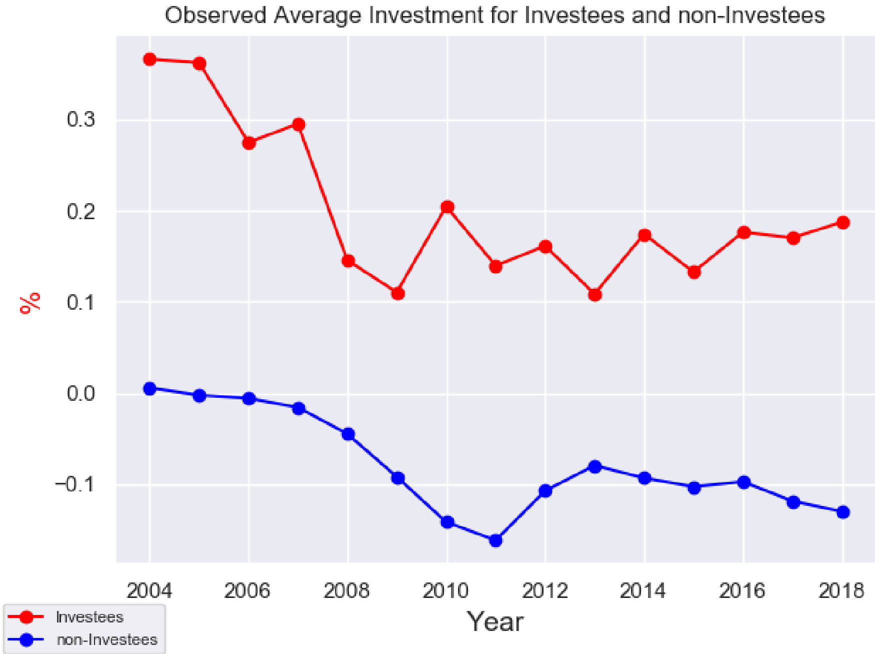

Figure 1.8. Observed Average Investment for Investees and non-Investees

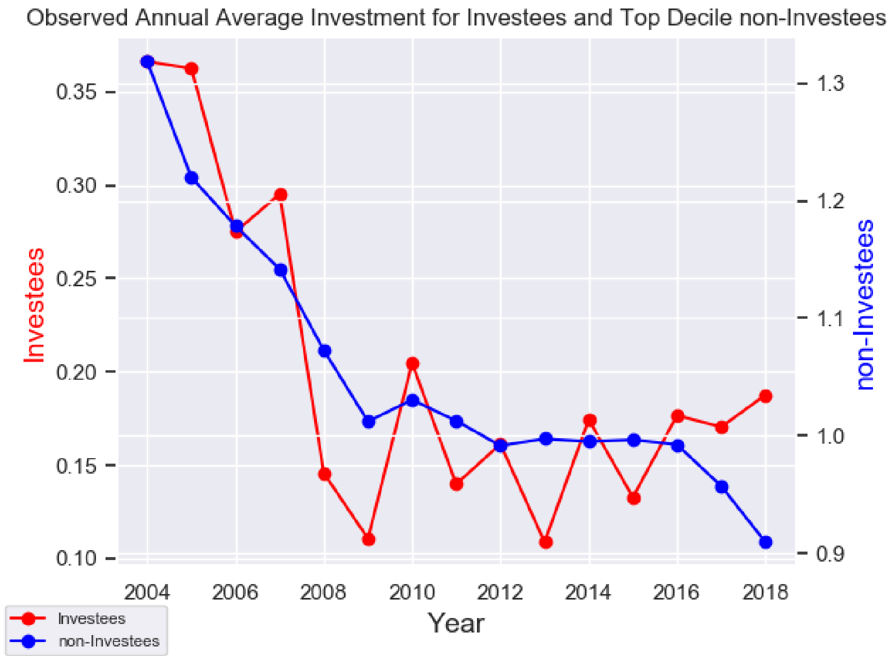

Figure 1.9. Observed Annual Average Investment for Investees and Top Decile non-Investees

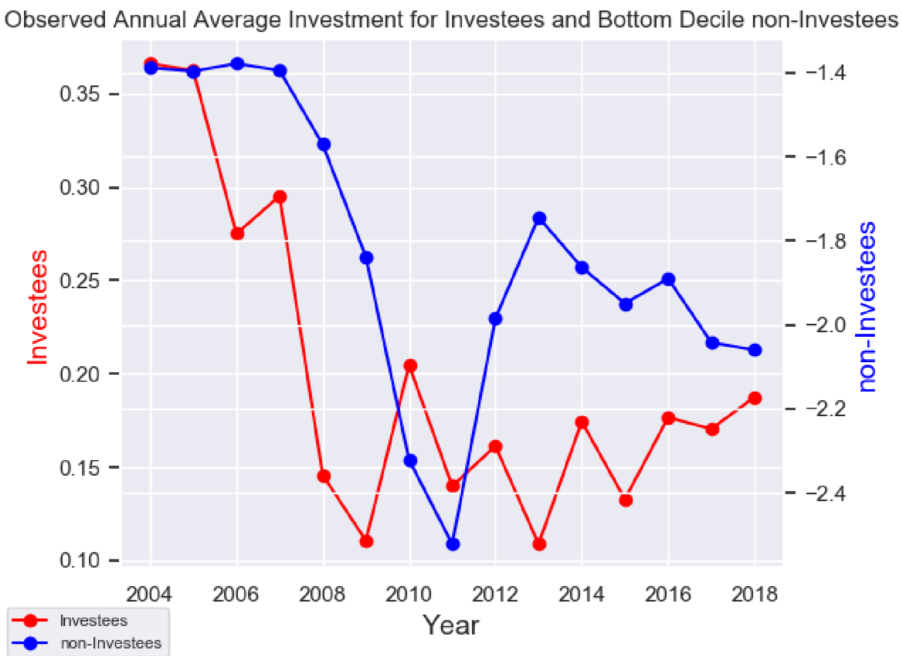

Figure 1.10. Observed Annual Average Investment for Investees and Bottom Decile non-Investees

1.6.3 Cost to Taxpayers

From the HMRC VCT data in Table 1.2., we observe that approximately £8.4 billion pounds has been raised by over 200 VCTs since inception of the VCT scheme in 1995.[14] These funds have funded the activities of SMEs and increased the total-assets formation (investment) of investees in the U.K. Specifically, we have shown that these funds have had a measurable positive impact on investment in the U.K. via its effect on investees - a 41% average increase in the investment of investees. However, we also know that VCT investors receive tax breaks such as: 30% upfront income tax relief, tax-free dividends, and exemption from capital gains tax. These tax breaks come at a non-trivial cost to the U.K. tax-payer, as it involves the actual reduction of a VCT subscribers tax bill as detailed in section (All About VCTs) and illustrated in appendix A. To further illustrate, in fiscal year 2017-2018, the subsidy expenditure for the VCT scheme was £201 million.[15] This figure is extremely conservative as it does not take into account investors making income tax relief claims through Self-Assessment nor does it consider investors making claims through other systems e.g. PAYE. Also, it does not include other tax reliefs and exemptions available through the VCT scheme such as capital gains and dividend-tax exemptions. For comparisons sake, this conservative £201 million in subsidy expenditure is over half of the total managed expenditure, [16] over the same period, for the Department for International Trade (HM Treasury Public Expenditure Statistical Analysis, 2019), which was £394 million.
In Table 1.3., we present HMRC data on the amount of investment on which relief was claimed on an annual basis between 2015-2018. We note that HMRC emphasises that the investor-level information in Table 1.3. was prepared using Self Assessment (SA) returns. Thus, the information in Table 1.3. will not cover investors making income tax relief claims through other channels (e.g. PAYE) or not making any claims. However, we know these omissions are small - because we can compare them with the amount of funds raised by VCTs in the corresponding year (Table 1.2.).

[14]To avoid duplication, we do not calculate the total for VCTs raising funds in column 3, Table 1.2. as VCTs can raise funds in multiple tax-years.

[15]£607 million - total from the last column in Table 1.3. which is the the amount of investment on which VCT investors claimed tax relief in the 2017-2018 fiscal year - multiplied by 30% tax relief.

[16]The total managed expenditure is the total amount the government spends. This is split up into: departmental budgets – the amount that government departments have been allocated to spend, also known as Departmental Expenditure Limits, and money spent in areas outside budgetary control – all spending that is not controlled by a government department and includes welfare, pensions and things such as debt interest payments, also known as Annually Managed Expenditure.

Table 1.3. **Venture Capital Trusts**
Income Tax Relief; Distribution of Investors and Amount of Investment on which Relief was Claimed from 2015-16 to 2017-18.
Data from HMRC VCT Statistics (2018).

(Upper Limit: £)	2015-2016 Investors	2015-2016 Amount of Investment (£ million)	2016-2017 Investors	2016-2017 Amount of Investment (£ million)	2017-2018 Investors	2017-2018 Amount of Investment (£ million)
1,000	1,240	1	1,075	0	1,230	1
2,500	630	1	775	1	815	1
5,000	1,365	6	1,615	7	1,720	7
10,000	2,545	22	2,800	24	3,470	29
15,000	1,195	16	1,440	19	1,700	22
20,000	1,285	24	1,490	28	1,875	36
25,000	775	18	910	21	1,140	27
50,000	2,155	83	2,550	98	3,395	130
75,000	640	40	775	48	1,020	64
100,000	635	60	670	62	995	93
150,000	330	41	410	51	535	66
200,000	620	122	725	142	1,000	194

1.6.4 Comparison of Matrix Completion and Difference-in-Differences Estimators

Our focus in this section is twofold. We illustrate in Table 1.4. and Figure 1.11., the mechanics of our Difference-in-Differences estimator, which in turn implicitly emphasises the ability of our Matrix Completion estimator at alleviating the selection bias detailed in the introduction section. But first, we compare the imputation accuracy or performance of our Matrix Completion estimator against a Difference-in-Differences (DID) estimator. We follow the general procedure in Athey et al. (2018), whereby they compare the imputation accuracy of their Matrix Completion method against four other estimators, including the DID and synthetic control estimators. For this exercise, we use the data for non-investees with N = 60,000, T = 16. Note that in the original data set there are 61,931 firms, where 1931 are investees (VCT funded), and will be excluded from this analysis. Thereafter we artificially allocate some non-investees and time periods to be VCT funded (pseudo treated/pseudo investees), and compare counterfactual/predicted total-assets values for these pseudo investee/time-periods against their actual total-assets values. Our setting is one with staggered adoption where we randomly designate non-investees as pseudo investees, with the date of VCT funding (treatment date) varying randomly among these pseudo investees. Once you receive VCT funding, you stay in the VCT funded group. In other words, once you receive treatment, you remain a treated firm. Our task is to utilise our Matrix Completion and a DID estimator to predict/impute counterfactual total-assets values for these pseudo investee/time-periods and then compare the predicted/counterfactual total-assets values for these pseudo investee/time-periods against their actual total-assets values. We compare the root-mean-squared-error (RMSE) of both algorithms on values for the pseudo investee (time, period) pairs. As with Athey et al., our aim is not necessarily to pinpoint the right or wrong algorithm. We simply want to uncover which algorithm works best in our setting where investees received VCT funding at staggered periods. We find that our Matrix Completion estimator has a superior performance with a normalised RMSE of 0.10 whereas the DID estimator has a lower performance with a normalised RMSE of 0.14. The increased performance of our MC estimator is attributable to its use of additional observations i.e. pre-VCT-funding total-assets of the pseudo investees. This finding is in line with the findings in Athey et al. (2018), where they employ several illustrations and show that their Matrix Completion estimator is superior to 4 different estimators (including DID and synthetic control) under a variety of treatment settings.

We now turn to showing in Table 1.4. and Figure 1.12., the mechanics of our Difference-in-Differences estimator, which implicitly emphasises the superior ability of our Matrix Completion algorithm at alleviating the potential selection bias detailed in the introduction.

Athey et al. (2018) detail how the Matrix Completion algorithm exploits both the patterns in the pre-VCT-funding total-assets observations of investees, and those in the entire 60,000 non-investees (control group), which implicitly alleviates the selection bias discussed earlier. The potential selection bias arises from VCTs investing in investees that are superior to non-investees along several dimensions that are unobserved in the data, which in turn causes the VCT funding of an investee to be endogenous, and thus the estimated ATT will be biased upwards relative to the VCT scheme's actual causal effect on total-assets formation. By exploiting all patterns in the data, most especially, the pattern in the pre-VCT-funding total-assets observations of investees, the Matrix Completion estimator's estimated counterfactual total-assets are not based on the pattern in any particular subset of the non-investees (control group), but on all of the patterns in all of the data - both investees and non-investees (control group). Whereas, with a simple parametric version of a Difference-in-Differences estimator, the estimator imputes the counterfactual total-assets of investees with the aid of control firms (non-investees) with identical lagged total-assets formation or investment (parallel trends). The approach centres around regressing the relevant periods total-assets on the lagged total-assets and then employing the regression estimates to predict the missing total-assets, which Athey et al. (2018) refer to as horizontal regression. In other words, with the horizontal regression, the researcher makes an ex-ante choice on what patterns in the data to exploit whereas with the Matrix Completion approach, the researcher allows the data to determine what patterns are exploited.

We now turn to illustrating how employing the Difference-in-Differences estimator which entails an ex-ante choice on what patterns in the data to exploit, potentially exacerbates the endogenous selection issue. To begin, we restrict our 60,000 non-investees (control group) to those with total-assets less than £16 million - as per VCT rules for the potential size of an investee, which reduces our non-investees (control group) dataset to 59,540 observations. We then sort and split them into 10 groups based on the following: we calculate the average growth rate of total assets (average investment) for each of our 59,540 non-investees (control group) during the sample period (2003-2018). We thereafter sort the 59,540 non-investees (control group) in ascending order of their average investment. Finally, we split them into ten groups, which means non-investees with the lowest average investment rate over the 2003-2018 period are in Decile 1, and those with the highest average investment rate over the 2003-2018 period are in Decile 10. Each group (5,954) of non-investees then serves as the control group for our 1,931 investees. We then employ our Difference-in-Differences estimator to estimate ten ATT's based on the 7,885 (1,931 + 5,954) data for investees and each decile of non-investees. Our results are depicted in both Table 1.4. and Figure 1.11. When we choose Decile 1 as our control group (non-investees with the lowest average investment rate between the period 2003-2018), we find that the VCT scheme has the greatest causal effect on investees, relative to choosing any other Decile. Conversely, when we choose Decile 10 as our control group (non-investees with the highest average investment rate between the period 2003-2018), we find that the VCT scheme has the lowest causal effect on investees, relative to choosing any other Decile. This exercise emphasises that with the Difference-in-Differences estimator, the ATT we uncover is driven by the ex-ante choice we make on what control group to employ in the estimation. By allowing the Matrix Completion algorithm choose - in a data driven manner - what patterns in the data to exploit (what control group to use), we sidestep the problem of having to ex-ante choose a control group, which implies ex-ante choosing the ATT, which potentially exacerbates the endogenous selection bias issue prevalent in a causal study like ours. For completeness, we also employ our Difference-in-Differences estimator to estimate the ATT, without splitting into deciles, our data of 59,540 observations for non-investees (control group). We find that the ATT is 26.42%.

Table 1.4. **Difference-in-Differences: Average VCT Scheme Causal Effect on the Investment of Investees.**
Each ATT estimate is estimated with a Difference-in-Differences estimator and a different control group. The data of total-assets for non-investees (control group) is first restricted to non-investees (control group) with assets less than £16 million, then discretized into 10 groups, in ascending order of average growth rate over the sample period, with each group representing a control group for the investees.

Decile	ATT (%)
Decile 1	72.01
Decile 2	46.77
Decile 3	31.04
Decile 4	23.09
Decile 5	19.14
Decile 6	18.70
Decile 7	18.66
Decile 8	17.91
Decile 9	13.78
Decile 10	1.68

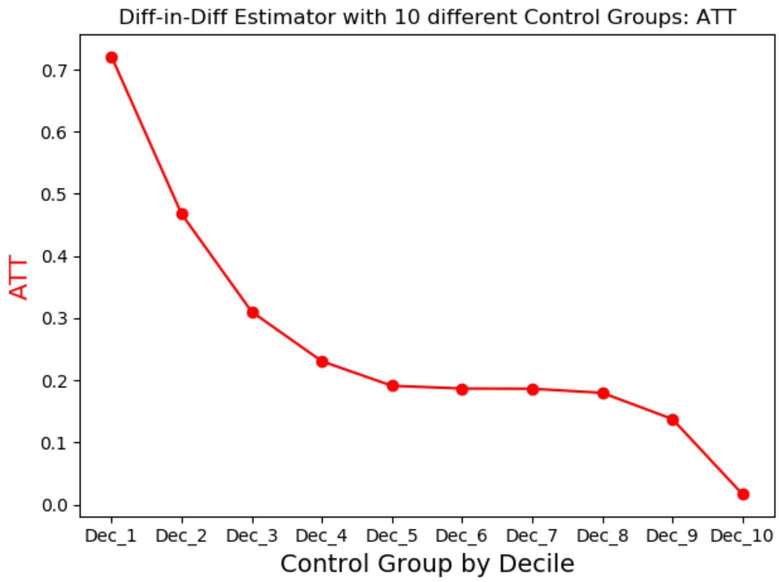

Figure 1.11. Difference-in-Differences Estimator with 10 different Control Groups: ATT

1.7 Conclusion

In this study, we sought to deepen our understanding of the VCT scheme and estimate the causal effect of the VCT scheme on the total-assets formation (investment) of investees in the U.K., between 2003-2018. We hand-collected data from all former and the current 62 VCTs operating in the U.K. Specifically, to estimate the causal effect of the VCT scheme on the total-assets formation (investment) of investees in the U.K., between 2003-2018, we hand-collected data on all investees that ever received VCT funding since inception of the VCT scheme in 1995.

We thereafter adapted and employed a Matrix Completion estimator to estimate our causal effect. This estimator is adapted from Athey et al., (2018) and has intuitive computational properties which helps alleviate the potential selection bias issue arising from estimating causal effects. We found that between 2003-2018, the causal effect of the VCT scheme on the investment of investees - the Average Treatment effect on the Treated (ATT) - was 41%. We then employed the Root Mean Square Error (RMSE) to compare the accuracy of our Matrix Completion (MC) estimator vs. a standard Difference-in-Differences (DID) estimator, at imputing missing total-assets for investees. We found that our MC estimator outperformed the DiD estimator with RMSE's of 0.10 and 0.14 respectively.

This study contributes to two broad spheres in economics. Firstly, our results add to the literature on the importance of venture capital funding for the growth of SMEs, and is consistent with findings in Gonzalez-Uribe and Paravisini (2019), Gompers, Gornall, Kaplan and Strebulaev (2020), and Iliev and Lowry (2020), who all detail the importance of VC funding for investees. Finally, our results are practically relevant for policy makers. The insights and results we provide can serve as a template to bolster the recommendations of the Patient Capital Review - in light of the current pandemic and its adverse impact on SMEs.

Chapter 2

VCT Skill and Deal Structure vs. Luck: What Drives the Success of VCT-Backed Firms

2.1 Introduction

The Venture Capital Trust (VCT) Scheme is designed to support U.K. based, young, private companies. As stated in Her Majesty's Revenue and Customs (HMRC) 2016 internal manual "The VCT scheme encourages indirect investment by individuals, through a VCT, a corporate vehicle similar to an investment trust, into small, high-risk companies or social enterprises, to help them grow and develop."[1] Crucially, the VCT scheme creates value for the U.K. economy. Classic pecking order theory implies that outside equity is the least preferred source of financing for firms. However, due to a lack of cashflows, firms might prioritise external equity as a source of financing. But, especially for start-ups, they face significant equity financing constraints as a result of capital market frictions such as information asymmetry, which inspires the need for a governmental policy intervention such as the VCT scheme. The VCT scheme, which broadens the range of financing instruments available to start-ups and increases the supply of capital to them as well, meets their demand for external equity financing, and allows for the U.K. economy to extract economic rents from entrepreneurship. As to whether VCTs skilfully deploy this increased supply of capital, Kaplan and Schoar (2005) uncover sizeable persistence and heterogeneity in VC success and attribute both results to VC skill and heterogeneity in VC skill. However, Sørensen (2007) finds that

[1]VCTs are akin to VCs but also similar to private equity investment trusts which invest in unquoted companies. However and unlike VCTs, the companies private equity trusts invest in tend to be quite big, established businesses.

endogenous selection bias is twice as important as VC skill for explaining the observed heterogeneity in VC success.[2]

We find that VCTs are skilled along several dimensions, and these skills, in addition to the funding deal structure, are significant determinants of the success of VCT-backed firms. Our goal is to develop a numerical algorithm that would allow for the quantification of the relative importance of each measure of VCT skill and the funding deal structure for the success of VCT-backed firms, whilst alleviating the obvious endogenous selection bias inherent in our study. We find that being backed by a VCT with high prior financial performance is the most important VCT skill determinant of the success of VCT-backed firms, contributing an average of 13% to the success of VCT-backed firms. High prior performing VCs have been shown to positively impact the performance of VC-backed firms. Sørensen (2007) shows that when a firm receives backing from a VC with prior success, it acts as a credible signal of unobserved firm characteristics to financial markets, and thus positively affects firm value. Nahata (2008) further reiterates how the prior success of a VC captures the VCs screening and monitoring expertise. In a VC (as opposed to VCT) context, several studies have shown that VC added services as embodied in their skills, are value generating. Kaplan and Schoar (2005) uncover sizeable persistence and heterogeneity in VC success and attribute both results to VC skill and heterogeneity in VC skill. Particularly, heterogeneity and persistence persists if new VCs cannot compete effectively with established VCs. Sørensen (2007) also finds that VC skill is an important determinant of VC success. Although, Sørensen (2007) also finds that endogeneity is twice as important as skill for explaining the observed heterogeneity in VC success. In a similar vein, Ewens, Gorbenko and Korteweg (2022) confirm the first-order importance of VC pre-investment skills (deal sourcing) for the success of VC-backed firms. They employ a dynamic search-and-matching model to deal with endogenous selection and study the impact of VC contract terms on VC-backed firm value. They find that VCs add value to their firms.

In a causal study like ours, as with the aforementioned studies as well, endogenous selection as opposed to VCT skill could drive the match between VCTs and VCT-backed firms. The following conjecture draws on the conjecture in Sørensen (2007). Consider entrepreneurial firms with high potential who also understand that VCT skills are a source of value-added for them. In this conjecture, these high potential entrepreneurial firms will seek to match with the most skilled VCTs. In turn, highly skilled VCTs will enjoy access to a proprietary deal flow of high potential entrepreneurial firms. Indeed, Sørensen (2007) reiterates how VCs consider access to proprietary deal flow as a distinct competitive advantage. Thus, and as

[2]Although the Kaplan and Schoar (2005) and Sørensen (2007) studies concern VCs, the analysis is still relevant considering VCTs are akin to VCs

discussed in Sørensen (2007), the resultant endogenous selection bias implies that VCT skills are not the sole determinant of the ex-post success of these VCT-backed firms (high potential entrepreneurial firms). Instead, these VCT-backed firms with backing from highly skilled VCTs are intrinsically better than VCT-backed firms with backing from lesser skilled VCTs. To further illustrate based on the illustration in Sørensen (2007), let us consider a standard regression model. Here, VCT skills are endogenous when selection bias causes highly skilled VCTs to invest in firms that are superior along several dimensions - unobserved in the data. VCT-backed firms with superior unobserved features i.e. a dedicated management team, as reflected in the error term, will match with skilled VCTs. Thus, the error term is positively correlated with VCT skills, and the estimated coefficient is positively biased, relative to the actual impact of VCT skills on the success of VCT-backed firms. Clearly, the ability of the most skilled VCTs to determine the success of the firms they back and the desire of intrinsically better firms to match with the most skilled VCTs are not mutually exclusive (Sørensen (2007)). Our challenge is thus to estimate the importance of VCT skills for determining the success of VCT-backed firms, whilst controlling for endogenous selection bias.

A popular approach to dealing with endogenous selection bias is to estimate a model with instrumental variables as a source of exogenous variation. However, for any instrumental variable to be valid, it must be correlated with the skill of VCTs but independent of the success of VCT-backed firms. Clearly, such instrumental variables are difficult to find. We also know that the two-stage-least-squares (2SLS) - which is a popular estimation approach that uses instrumental variables - makes strong assumptions on the causal model. For instance, the 2SLS will specify a linear relationship between various measures of VCT skills and the success of VCT-backed firms. However, as we will show in a later section, the relationship is non-linear. Also, in a study of 1309 instrumental variables regressions in thirty papers published by the journals of the American Economic Association, Young (2022) employs Monte Carlo simulations and the bootstrap to show that instrumental variable methods yield estimates that seldom outperform estimates produced by biased ordinary least squares. Another popular approach employs structural models to deal with endogenous selection bias. Nonetheless, these structural approaches are computationally intensive which necessitates the adoption of numerous simplifying assumptions to allow for model tractability (Sørensen, 2007, p. 2728).

We deal with the lack of valid instruments, assumptions of linear relationships in the 2SLS, and limitations of structural models, by developing a Deep Neural Network framework, adopting several attribution algorithms, and hand-collecting VCT data to estimate and quantify the relative importance of VCT skills and the funding deal structure for the success of VCT-backed firms. The Deep Neural Network is a flexible model that adopts a data-adaptive self-learning approach to modelling the relationship between VCT skills, the funding deal structure and the success of VCT-backed firms. It is also referred to as Deep Learning, which means that the model employs interconnected nodes and non-linear mathematical functions, in a multi-layered structure, to continuously learn and capture complex (non-linear) mappings between VCT skills, the funding deal structure and the success of VCT-backed firms. It does not suffer from the the "curse of dimensionality" problem that structural models suffer from, neither does it require the simplifying assumptions that structural models require. Nonetheless, it is widely acknowledged that the lack of interpretability of the output of Deep Neural Network models detracts from its tractability and superior performance. This is where the attribution algorithm comes in. The attribution algorithm enables us extract from the estimates of our Deep Neural Network model, the actual causal effect of each measure of VCT skill and the funding deal structure on the success of VCT-backed firms. Formally, attribution algorithms are algorithms that capture the effect of an independent variable on the output of a model (Sundararajan, Taly and Yan, 2017), which is an inherently causal task. Now, although this combination of a Neural Network and attribution algorithm allows us to sidestep in a data driven manner - the endogenous selection bias issue described earlier, we conduct a robustness check to ensure and demonstrate that our approach does indeed alleviate the endogenous selection bias issue. This robustness check draws on the work of Sørensen (2007), who finds that VC-backed firms with backing from highly experienced VCs are more successful, but that a large proportion of the success is attributable to endogenous selection bias. To that end, we exclude from the analysis, observations for the most experienced VCTs and the firms they backed, and still find that VCT skills and the funding deal structure are important determinants of the success of VCT-backed firms.

Analysing the VCT scheme is all the more important given that it creates value for the U.K. economy, particularly through the increased the supply of capital for small, young and risky firms in the U.K. In excess of £9 billion has been raised by circa 200 VCTs since inception of the scheme in 1995. These monies have had a positive measurable impact: Iweze (2020) shows that the scheme led to an aggregate increase of 41% in the investment of VCT-backed firms between 2003-2018. The Association of Investment Companies (AIC) report how the VCT scheme is associated with an average increase of 51 new employees per VCT-backed firm, post-VCT funding.[3] Media reports also enunciate the importance of VCT-backing for VCT-backed firms. For instance, when Convertr Media received £3 million in Series A funding from Albion Ventures in 2016, and as part of the funding deal, welcomed a partner from Albion Ventures to their board, they highlighted the importance of Albion

[3]Details on this statistic are available at: https://www.theaic.co.uk/system/files/search-hidden-file/AICVCTDeliveringGrowthOct15.pdf

Ventures expertise to their operations, stating "With a history of working with other tech specialists including $\cdots$, Albion Ventures will bring a depth of knowledge and experience to our operation $\cdots$ and this funding will enable us to build on and expand our offering to a wider international market".

Given the importance of VCTs to VCT-backed firms, the U.K. economy, their positive measurable impact and substantial cost to the taxpayer,[4] as reported in Iweze (2020), our research question on whether VCT skill or luck determines the success of VCT-backed firms is even more pertinent. An ever growing financial economics literature has and continues to study VCs and their skills at: pre-investment screening, deal-selection, deal contracting and post-investment monitoring and advising. Along the post-investment dimension, Lerner (1995) shows that VCs are influential in the structuring of the boards of directors of the firms they back. Also, Amornsiripanitch, Gompers and Xuan (2019) find that VCs aid in hiring outside managers and directors for their firms, with these VC-backed firms also likely to exit via relationship-based acquisitions. We also have survey evidence of post-investment value-added documented in Gompers, Gornall, Kaplan and Strebulaev (2020), where VCs enumerate the post-investment services they provide to their firms, ranging from strategic guidance, connecting them with investors and customers, operational guidance, to hiring employees and board members. As highlighted earlier, these VC skills have also been shown to be value generating. Gompers, Gornall, Kaplan and Strebulaev (2020) find that pre-investment skills are more important for VC returns relative to post-investment skills. However, they go a step further in that, in their survey, they make the distinction between deal sourcing and deal selection, with deal selection being the most important value-generating service VCs provide their firms. Sørensen (2007) finds that firms funded by highly experienced VCs are likely to go public. This result stems from the direct impact of the highly experienced VCs and sorting, which leads experienced VCs to invest in better firms. Sorting creates an endogenous selection problem. The study resolves this problem with the aid of a two-sided matching structural model to separately identify and estimate direct impact and sorting. The study finds that both effects are significant and sorting is twice as crucial for explaining the heterogeneity in exit rates across VCs. Similarly, Nanda, Samila, and Sørenson (2020) show a reverse relationship between pre-investment skills and VC performance. They analyse VC performance correlations and find that subsequent exits via IPOs are 8% higher than previous IPOs. They conclude that the reason for the subsequent increased exit performance is that the initial successful exit performance improves access to deal-flow (deal sourcing). Regarding deal structure and firm success, Hochberg, Ljungvist,

[4]For instance, Iweze (2020) details how the VCT scheme cost taxpayers a conservative £201 million in subsidy expenditure in the 2017/2018 fiscal year.

and Lu (2007) find that VC-backed firms that were financed by a syndicate of VCs have a higher likelihood of surviving to subsequent funding rounds, and highly networked VCs produce higher performance as measured by their exit (via IPO and trade sale) ratio.

Two important studies on VC performance are the Achleitner, Engel, and Reiner (2013) and Calder-Wang and Gompers (2021) study. In the former, they analyse how market factors affect VC returns. They employ a proprietary dataset on VC investments to study how market-related volatility impacts VC returns. They find that demand related factors (increase in entrepreneurial activity) initially results in higher returns. They also find that over-funding or over-reaction to the supply of VC funding destroys VC returns. In the latter study, they study whether diversity leads to improved performance of VC-backed firms. They find that when VC partners have female children, there is an increased likelihood that the VC will employ female partners. Additionally, the increased gender diversity improves overall VC performance. In summary, both studies show that VCs have an impact on the market value of VC-backed firms. The latter study especially shows how VC diversity can positively impact the success of VC-backed firms.

In this study, we quantify the relative importance of VCT skills and the funding deal structure for VCT-backed firms success. To begin, we thoroughly analyse the VCT funding of VCT-backed firms from multiple angles. In our empirical tests, we analyse the interplay between the characteristics of VCT-backed firms, the funding deal structure, and the skills of VCTs. We then quantify the relative importance of each measure of VCT skill and the funding deal structure in determining the success of VCT-backed firms. We begin with a simple univariate analysis, where we uncover an enduring relationship between VCTs and VCT-backed firms. In the aggregate, VCTs hold between 19% to 31% equity stake in VCT-backed firms, 56% of VCT-backed firms received multiple VCT funding rounds, and of those, 48% received multiple funding rounds by their original VCT-backers. We find that 44% of VCT-backed firms are successful compared to 56% unsuccessful. Pertinently, the archetypal VCT-backer of successful VCT-backed firms is remarkably more skilled along several dimensions - relative to the the archetypal VCT-backer of unsuccessful VCT-backed firms. Also, the observed financing deal structure is different for successful relative to unsuccessful VCT-backed firms. These differences are consistent with the prior literature's assertion that VCs are skilled along several pre and post investment dimensions, and these skills are value generating. With our Deep Neural Networks (binary classification and regression models) and attribution algorithms, we are able to measure and rank the relative importance of each measure of VCT skill and the funding deal structure for the success of VCT-backed firms.

Given our finding that being backed by a VCT with high prior financial performance is the most important VCT skill determinant of the success of VCT-backed firms, contributing an

average of 13% to the success of VCT-backed firms. This contributes to the Sørensen (2007) finding wherein they show that when a firm receives backing from a VC with prior success, it acts as a credible signal of unobserved firm characteristics to financial markets, and thus positively affects firm value. It also contributes to the Nahata (2008) finding where they show that the prior success of a VC captures the VCs screening and monitoring expertise. To the best of our knowledge, our study is the first to formally quantify the relative importance of various measures of VCT skill and the funding deal structure for the success of VCT-backed firms. Our finding that VCT skills impact the success of VCT-backed firms is also relatable to the Iliev and Lowry (2020) result which shows that VCs are skilled at solving the information asymmetry problem that constrains new IPO firms from accessing growth capital. We also contribute to the literature on the role of VCTs and the VCT scheme. We fill in some of the gaps in our understanding of what VCTs do. Iweze (2020) showed the importance of the VCT Scheme to the U.K. economy albeit at a substantial cost to the U.K. tax payers in the form of tax rebates enjoyed by VCT investors. Our result further justifies this cost in the sense that VCTs play an important role in the success of VCT-backed firms, through the pre and post investment skills they bring to bear on these firms, which in turn has wider implications for entrepreneurship in and growth of the U.K. economy.
The outline of our paper is as follows. In Section 2, we discuss the VCT data hand-collection process and present the VCT hand-collected data. We also present the descriptive statistics. Sections 3 and 4 introduces our machine learning approach, presents and discusses our results. Section 5 summarises and concludes.

2.2 Data and Descriptive Statistics: VCT Skill and Deal Structure

From the Companies House Database, we downloaded and manually read through VCT annual report between the periods 2014 to 2020. From these reports, we collated details on VCTs and the firms that received VCT funding (VCT-backed firms). We thereafter collected financial data on these VCT-backed firms from the FAME database. We lost approximately 13% of our hand-collected data due to a combination of non-reporting of valuations by some VCTs for some of their VCT-backed firms, and the lack of financial data for some VCT-backed firms on the FAME database. Our final sample consists of 3,629 VCT-backed firms backed by 44 VCTs, spanning the period 2014-2020. Although the VCT scheme was introduced in 1995, we began our analysis in 2015 (this reduced our sample to 1,953

VCT-backed firms) because of the structural changes, or if you will, VCT policy changes, introduced by the U.K. government in July 2015 to bring the VCT scheme in line with the European Union's risk capital guidelines. These policy changes - extensively covered in Iweze (2020) and summarised below - caused a structural break in the VCT investment landscape such that the pre-2015 period is structurally different from the post-2015 period. We illustrate below with a few key policies of the VCT scheme, at the start of the scheme in 1995 vs. 2020.

- 1995: No age restrictions on a potential VCT-backed firm.
  2020: The maximum age for a potential VCT-backed firm is 7 years old (10 years old for knowledge-intensive firms) since its first commercial sale.

- 1995: VCTs could purchase existing shares i.e. Management Buyouts were permitted.
  2020: VCTs can only invest to fund growth: VCT investment cannot be used to finance acquisitions or to buy existing shares. Companies must be able to satisfy HMRC's "risk to capital" condition [5].

- 1995: No limit to the amount of VCT funding a VCT-backed firm can receive.
  2020: VCT-backed firms are subject to an overall lifetime limit on VCT funding of £12 million (£15 million for knowledge-intensive firms).

- 1995: 70% of funds-raised by a VCT must be invested in Qualifying Investments (QI) by the third year, post-fundraise.
  2020: Non-qualifying investments can no longer be made, except for certain exemptions in managing the Company's short-term liquidity. A minimum of 80% of funds raised must be invested in QIs: 30% of cash raised must be invested in QIs[6] within the first accounting period following fundraising.

2.2.1 Data on VCTs and VCT-Backed Firms

The data hand collection entailed reading through the half-year and annual reports filed by each VCT, and SHO filings[7] by VCT-backed firms between the periods 2014-2020. From the

[5] The "risk to capital" condition requires a potential VCT-backed firm, at the time of investment, to be an entrepreneurial firm with the objective to grow and develop, and VCT investment in the firm must carry with it, a genuine risk of loss of capital. Further details on the risk to capital conditions are available at: https://www.gov.uk/hmrc-internal-manuals/venture-capital-schemes-manual/vcm8530

[6] QI is an entrepreneurial company with the objective to grow, develop and which has a genuine risk of loss of capital.

[7] Paper forms used by limited companies to notify Companies House of a change to their share capital.

reports, we extracted details on fundraising activity during the year, a breakdown on the uses of funds: what firms were invested in during the year, how the funding deal was structured (mix of equity and debt or equity only), and the equity percentage received by the VCT. We also extracted details such as the current valuation of VCT-backed firms, acquisitions and disposals during the year, and financial statement line items such as: Gains/Losses on Investment, Net Asset Value, Total Value to Paid-In (See Appendix B for a description of all variables). Our complete dataset contains 4,178 VCT-backed firms, hand-collected from VCT annual reports between 2014-2020. Next, we obtained the annual financial statements of these VCT-backed firms from the FAME database. By nature of the regulations guiding the VCT Scheme, almost every firm that receives VCT funding is a small firm [8]. And because the disclosure rules for small firms are less stringent (relative to medium or large firms), their coverage on the FAME database is spotty, with empty entries in numerous financial statement line items. As a consequence, we omitted firms without financial information on the FAME database. This, in addition to the non-reporting of valuation by some VCTs for some of their VCT-backed firms, reduced our hand-collected data by 13%, from 4,178 to 3,629 VCT-funded firms. Of the 3,629 VCT-funded firms, 98% are classed as SMEs on the FAME database. The remainder 2% comprises short-term equity investments that do not meet the QI criteria but are used by VCTs for short-term liquidity management. We also excluded from the analysis, numerous financial statement line items (i.e. CAPX, Revenue, Profits, R&D etc.) that could potentially serve as control variables. This is because of the aforementioned disclosure rules for small firms, which results in more than two-thirds of VCT-backed firms not reporting numbers for these line items. Finally, by starting our analysis in 2015, for the earlier detailed reasons, our analysis employs 1,953 VCT-backed firms, backed by 44 VCTs.

In Figure 2.1, we plot the annual aggregate funding of First-Time vs. Post-First-Time VCT-backed firms. Each year, the majority of VCT GBP investment (between 76% and 83%) goes to firms that had received VCT funding in previous years (red line). At first glance, this observation is puzzling given that the VCT scheme regulations preclude VCTs from funding firms older than 7 years (10 years for knowledge intensive firms), thus impacting a VCT's ability to multiple-fund its firms before the age restriction "bites". However, VCTs mitigate this concern by shortening the Average Time Between Successive VCT Funding Rounds

[8] The Companies House Accounting Guidance (2021) defines a small company as one with a maximum annual turnover of £10.2 million, maximum total assets of £5.1 million and an average number of employees below 50. Further details can be found at: https://www.gov.uk/government/publications/life-of-a-company-annual-requirements/life-of-a-company-part-1-accounts)

(ATBFR) to 13 months (see Table 2.1.) whereas from the Crunchbase (2018) study, we know that the Average Time Between Successive VC funding rounds (ATBFR) is 24 months. [9]

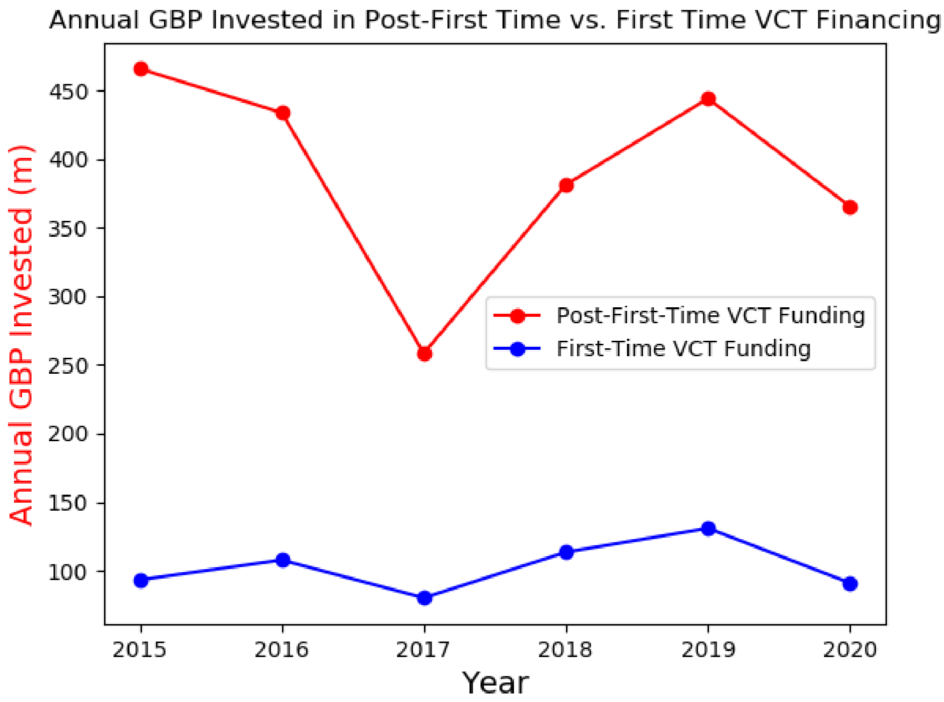

Figure 2.1. Annual GBP Invested in Post-First-Time vs. First-Time VCT Financing

2.2.2 Measuring Success of VCT-Backed Firms

We measure the success of a VCT-backed firm or if you will, a VCT investment, as an equity investment with an unrealised Internal Rate of Return (IRR) greater than or equal to 5%, which is an approximation of the average hurdle IRR at and above which VCTs earn a performance incentive fee (see Table B.1. in Appendix B). VCT investment managers earn a performance incentive fee which is supposed to align their interest with their shareholders interests. The aggregate VCT investment managers performance incentive fee is triggered if the VCTs total returns (change in NAV plus dividends paid over an accounting period) exceeds a hurdle rate of 5% (see Table B.1. in Appendix B). If the VCT investment manager achieves this 5% performance hurdle rate, they earn a performance incentive fee of between 10-20% of the excess of total returns over the 5% hurdle rate. However, if the VCT investment

[9]The ATBFR for VC is publicly available: https://news.crunchbase.com/news/the-time-between-vc-rounds-is-shrinking/

manager fails to meet the 5% performance hurdle rate in an accounting period, the deficit is carried forward to subsequent accounting periods and must be cleared before a performance incentive fee becomes due.

2.2.3 Descriptive Statistics: VCTs and VCT-Backed Firms

In Table 2.1., we present descriptive statistics for the 1,953 VCT-backed firms and for the subsets of 851 successful VCT-backed firms compared to the 1,102 unsuccessful VCT-backed firms, where the success of a VCT-backed firm is as defined in the previous section. Our descriptive statistics cover the funding deal structure, our measure of VCT-backed firm success, independent variables that proxy for VCT skill, and financial variables, with the financial variables measured in the fiscal year of each VCT-backed firm's most recent valuation. For each VCT-backed firm, the variables that proxy for the skill of each of its VCT backers are measured in the fiscal year prior to the first time the VCT-backed firm received VCT funding from the VCT whose skill we are measuring. In column (1), we report means for the 1,953 VCT-backed firms. In column (2), we report means for the 851 successful VCT-backed firms subset and in column (3), we report means for the 1,102 unsuccessful VCT-backed firms subset. In the final column, statistical significance of the differences between subset means at the 1%, 5%, and 10% levels are represented by ***, **, and * respectively.

Successful VCT-backed firms are more likely to have been backed by VCTs in the Top 5. We see from Table 2.1., that a greater number of Top 5 VCTs backed successful relative to unsuccessful VCT-backed firms. The Top 5 ranking is based on the second measure of VC reputation in the Nahata (2008) study, which is a VC's share of aggregate investment in the VC industry. Nahata (2008) motivates this measure by observing how a higher share of aggregate investment implies a higher share of funds committed by LPs who invest in a VC based on the VC's reputation. Additionally, the Nahata (2008) study further motivates this measure by noting that the Hsu (2004) study implies reputable VCs have a larger investment opportunity set and are likelier to have a higher share of aggregate investment. For each VCT, we calculate its annual Top 5 ranking based on its market share, which is the GBP valuation of all the firms it backed as a fraction of the GBP valuation of all VCT-backed firms in the VCT funding ecosystem. Top 5 VCTs are VCTs that rank in the highest quintile of VCT market share. To wit, we also see in Table 2.1., that VCTs with specialisation in funding the FTSE-Industry (FTSE-Industry Experience/Total Experience (%) of all First-Time VCT Backers) of its firms, overwhelmingly funded successful relative to unsuccessful VCT-backed firms. We also see in Table 2.1., that relative to unsuccessful VCT-backed firms, the VCT backers of successful VCT-backed firms have more experience in funding the FTSE-Industry

(Log(FTSE-Industry Experience Count of all First-Time VCT Backers)) of its firms. To construct this variable, which is the numerator in the above specialisation variable, we follow Gompers, Kovner, and Lerner (2009) in computing a VCT's experience cumulatively [10]. For instance, consider a software firm (FTSE-Industry is Technology) backed by Unicorn AIM VCT for the first time in 2020. The FTSE-Industry funding experience of Unicorn AIM VCT in 2020 is the total number of funding rounds it has participated in, in the Technology FTSE-Industry, prior to funding the software firm. The difference between this experience variable and the previously mentioned specialisation variable is that the specialisation variable is calculated using the experience variable as the numerator, and the total count of all investments made by the VCT in all industries as the denominator.
Next, we see that VCTs with High Prior Performance overwhelmingly backed successful relative to unsuccessful VCT-backed firms, where we measure a VCT's performance as the annual return on its portfolio of assets. VCTs with high prior performance are those VCTs that rank in the highest quartile of performance among the universe of VCTs in existence at the time. To reiterate, High Prior Performance, as with the other variables that proxy for VCT skill, is measured for each VCT-backed firm. So, # First-Time VCTs with High Prior Performance is the number of VCTs that backed a firm (where we only consider the first time a VCT backed a firm) where said VCT's prior performance ranked in the highest quartile of VCT performance among all VCTs. Analogously, we measure # First-Time VCTs with Low Prior performance as the number of VCTs that backed a firm where said VCT's prior performance, the first time it backed the firm, ranked in the lowest quartile of VCT performance among all VCTs. We see that VCTs with Low Prior Performance overwhelmingly backed unsuccessful relative to successful VCT-backed firms. These results are robust to the use of an alternative measure of VCT performance. This alternative measure of performance - also segmented into quartiles - is Total Value to Paid-In capital (TVPI). TVPI is the current value of outstanding investments plus the cumulative value of all distributions to date divided by the total amount of capital paid into the VCT to date.

From the descriptive statistics in Table 2.1., we also see that relative to unsuccessful VCT-backed firms, successful VCT-backed firms were backed by a greater number of young VCTs. For each VCT-backed firm , this binary measure of VCT skill is defined as the number of young VCTs that financed the firm, where young is defined as 15 years old [11] or younger at the time of the funding. This finding is surprising because of the Gompers (1996) finding of young VCs and the underpricing of their IPOs.
We will soon employ multivariate approaches to test the strength of these successful vs.

[10]The specialisation variable is also cumulated. Please see Appendix B.5 for a description of all variables.
[11]The results are robust to setting the threshold for young at 12 and 8 years old respectively.

unsuccessful VCT-backed firms differences. Nonetheless, we have thus far seen that our measures of VCT skill reinforces a success-driven-by-skill hypothesis (except for the young VCT measure). To wit, unsuccessful VCT-backed firms were mostly backed by less skilled VCTs. We now turn to the funding deal structure. We observe that relative to unsuccessful VCT-backed firms, successful VCT-backed firms received more VCT funding, were held for shorter periods, and underwent an equal number of VCT funding rounds but with a greater length of time between funding rounds (ATBFR measured in years). In summary, the funding deal structure tells us that, relative to unsuccessful VCT-backed firms, VCTs invest more money in successful VCT-backed firms, they do this over an equal number of funding rounds, and the duration between funding rounds for successful VCT-backed firms is closer to the average duration seen in the wider VC industry. Additionally, the majority (62%) of successful VCT-backed firms received multiple funding rounds (MFR), as seen in Table 2.1., whereas for unsuccessful VCT-backed firms, the minority (42%) received multiple funding rounds.[12] This also reinforces a success-driven-by-funding deal structure hypothesis. As discussed in Ewens et al. (2018), some VCs are moving toward a "spray and pray" strategy whereby they invest smaller amounts in an increased number of startups and spend less time managing and monitoring each one of them - until they startup realises some success. A single funding round indicates a lack of long-term relationship between VCT and VCT-backed firm or a lack of intermediate success by the VCT-backed firm, or both, with a single funding round VCT-backed firm losing out on the benefits of its VCT's skill. This statistic also provides suggestive evidence consistent with the Information-Asymmetry Hypothesis in Iliev and Lowry (2020): the benefits of receiving multiple funding rounds from existing VCT (VC in their case) backers are greatest among VCT-backed firms with positive NPV projects but high information asymmetry. To wit, skilled VCTs are equipped to overcome such financing friction. As with the previously discussed statistics, these differences also provide suggestive evidence in support of the funding deal structure as a key determinant of the success of VCT-backed firms.

Thus far, we have discussed how VCT backers of successful VCT-backed firms are more skilled along several dimensions relative to the VCT backers of unsuccessful VCT-backed firms. We have also seen a clear delineation between the deal structure for successful vs. unsuccessful VCT-backed firms. We now firmly turn our attention to the VCT-backed firms themselves. Relative to unsuccessful VCT-backed firms and from Table 2.1., we observe that the archetypal successful VCT-backed firm is of equal size (Total Assets), older (VCT-Backed Firm Age), has more cash-to-assets and lower debt-to-assets. In upcoming sections, and with

[12]We measure the MFR variable at the firm instead of the trust level. i.e. Downing One VCT Plc, Downing Two VCT Plc, Downing Three VCT Plc, Downing Four VCT Plc, Chrysalis VCT Plc, and Draper Espirit VCT Plc are all managed/administered by and therefore grouped as Downing LLP.

the aid of supervised machine learning techniques, we quantify the relative importance of VCT skills and the funding deal structure for the success of VCT-backed firms. But first, we present a primer on our machine learning approach.

Table 2.1. **Descriptive Statistics**

Our sample consists of 1,953 VCT-backed firms between 2015 and 2020, defined as U.K. firms that received funding from VCTs within the time period. We require all firms to have FAME data. Independent variables for VCT skill are measured in the fiscal year prior to the first time a VCT funded a firm and financial variables are calculated at the fiscal year end of each VCT-backed firm's most recent valuation. In Panel A, column (1) presents means for the 1,953 VCT-backed firms in our sample. In Column (2), we report means for the 851 successful VCT-backed firms subset and in column (3), we report means for the 1,102 unsuccessful VCT-backed firms subset, though missing FAME data means some of the financial variables are based on fewer observations. In the final column, statistical significance of the differences between sub-sample means at the 1%, 5%, and 10% levels are represented by ***, **, and * respectively. Variables are described in Appendix B. In Panel B, we present descriptive statistics on the 44 VCTs in our sample for the sample period 2015-2020, where each column reports mean values for each year of our sample period.

Panel A: Descriptive Statistics for VCT-Backed Firms

	VCT-Backed Firms	(851) Successful VCT-Backed Firms	(1,102) Unsuccessful VCT-Backed Firms	Mean-Difference
Unrealised IRR (%)	5.38	32.73	-15.74	0.485***
VCT Skill Measures				
# First-Time VCT Backers in Top 5	0.91	1.13	0.74	0.397***
FTSE-Industry Experience/Total Experience of all First-Time VCT Backers (%)	18.47	21.46	16.17	0.053***
Log(FTSE-Industry Experience Count of all First-Time VCT Backers)	3.98	4.04	3.94	0.097*
# First-Time VCTs with Low Prior Performance	0.75	0.67	0.81	-0.144***
# First-Time VCTs with High Prior Performance	1.10	1.39	0.88	0.508***
# First-Time VCTs that are Young	0.78	0.91	0.68	0.233***
Deal Structure				
# Funding Rounds	2.10	2.09	2.10	-0.0120
Log(Total VCT Funding)	6.78	7.15	6.50	0.649***
ATBFR(Years)	1.11	1.15	1.08	0.066***
Holding Period (Years)	2.12	2.10	2.14	-0.0460
VCT Equity Stake in VCT-Backed Firms (annual range: %)	19-31	18 - 32	20 - 30	
Multiple Funding Rounds (%)	55.84	62.03	41.87	-
Multiple Funding Rounds−sameVCT (%)	47.67	-	-	-
Control Variables				
VCT-Backed Firm Age	10.95	11.82	10.28	1.539**
Log(Total Assets)	11.17	11.08	11.27	-0.191
Debt-to-Assets	0.47	0.43	0.51	-0.075***
Cash-to-Assets	0.53	0.57	0.49	0.067***

(Continued)

Table 2.1. *Continued*

Panel B: Descriptive Statistics for VCTs

	2015	2016	2017	2018	2019	2020
Net Asset Value (in millions)	57.26	63.96	63.86	87.57	97.51	114.60
Investment Income / Net Asset Value (%)	3.16	2.87	2.40	2.19	2.18	2.10
Investment Management Fees / Net Asset Value (%)	2.00	2.26	2.24	1.94	2.09	2.07
Total Dividend Payout / Net Asset Value (%)	9.22	9.25	9.51	9.00	8.72	7.82
Total Value (Net Asset Value plus Cumulative Dividends Paid in millions)	63.11	71.37	98.01	95.43	105.36	122.55
Total Capital Paid in to Date (in millions)	63.83	72.65	86.60	104.03	119.92	142.84
Total Value to Paid in (TVPI in multiples)	1.66	1.70	1.65	1.74	1.74	1.83
Fundraising in the Year (in millions)	9.84	13.18	17.44	19.95	22.14	18.54
Fundraising to Net Asset Value (%)	18.89	18.94	51.21	24.39	21.03	24.83

2.3 Primer on Deep Neural Networks and Attribution Algorithms

Deep Neural Networks or Artificial Neural Networks are a series of algorithms that form an important sub-field of Machine Learning. The Deep Neural Network architecture in Figure 2.2. is composed of interconnected group of nodes called neurons or activation functions, where a neuron or activation function is a differentiable non-linear mathematical function that receives data as input, conducts a transformation on the data, and produces an output, which may be the final output or act as an input for the next neuron.

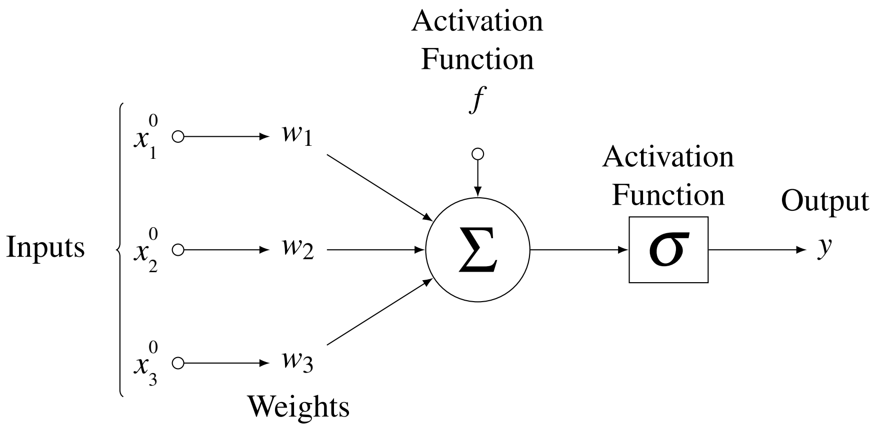

Figure 2.2. Simple Neural Network Architecture with One Hidden Layer

For illustration purposes, we have kept the Neural Network simple by drawing the architecture with one input layer, one hidden layer (weights and activation function Σ) and the output layer, whilst ignoring the addition of a bias vector to the neurons in the hidden layer, which together with the weights, are learnable parameters. Of course, the Deep Neural Network Binary Classification model and Deep Neural Network Regression model we build and employ in this study has multiple hidden layers. The $x_1^0 \cdots$ are the input data, the $w_1 \cdots$ are weight matrices connected to the neurons in the input layer and hidden layer, and are conceptually similar to coefficients in a regression. There are many non-linear activation functions. For instance, the *sigmoid* non-linear activation (which we use in this study) function allows us to uncover non-linear relationships between variables, and is given by:

$$\Sigma = \frac{1}{1+e^{-z}} \ ,$$

where z is a matrix. For a Deep Neural Network Binary Classification model, the output layer (y) is a binary output that undergoes a *softmax* (σ) operation, where *softmax* is a mathematical function that converts a vector of numbers (z) into a vector of probabilities, and is given by:

$$\sigma(\vec{z})_i = \frac{e^{z_i}}{\Sigma_{j=1}^{K} e^{z_j}} \quad ,$$

where σ is *softmax*, $(\vec{z})_i$ is an input vector, e^{z_i} is a standard exponential function for the input vector, K is the number of classes in the binary classifier, and e^{z_j} is a standard exponential function for the output vector. In conclusion, we summarise the key steps of our Deep Neural Network in Algorithm 1.

Algorithm 1: Deep Neural Network

1. **Define the Neural Network that has some learnable parameters**
   - Define the forward function to compute output tensors from input tensors.
   - Define the backward function which receives the gradient of the output tensors WRT some scalar value and computes the gradient of the input tensors WRT that same scalar value.
2. **Define a loss function (i.e. Cross-Entropy) and optimizer (i.e. ADAM - Adaptive Moment Estimation**)
3. **Randomly split the data into separate 70% training and 30% test sets**
4. **Train the Neural Network on Training Data**
   - Backpropagate the error from the loss function.
   - Update the weights of the network via ADAM:
     Weight = Weight - Learning Rate $\times$ gradient.
5. **Test the Neural Network on the Test Data**
6. **Deploy the Neural Network on Complete Dataset to make Predictions**

We now elaborate on our Deep Neural Network Binary Classification (DNNBC) model based on the steps in Algorithm 1. To begin, a tensor is a specialised data structure that is similar to a matrix, and is useful for encoding the inputs, outputs, and parameters of a model. Tensors can run on GPUs or other specialised computing hardware to speed up computing. Our DNNBC model has an architecture of 2 hidden layers, the first with 14 hidden units (corresponding to the number of independent variables) and the second with 9 hidden units (this number of hidden units is a fine-tuned hyper-parameter), each with *Sigmoid* non-linearity. The output layer performs a softmax operation and has 2 units, corresponding to the outcome of either successful VCT-funded firm (1) or unsuccessful VCT-funded firm (0). All layers are parameterised i.e. have associated weights and biases that are optimised during training of the DNNBC model. The linear component of both hidden layers applies a linear transformation on the independent variables using its stored weights and biases. The non-linear component employs a *Sigmoid* non-linear activation function to create complex mappings between the independent variables and our [1, 0] outcome variable. The final hidden layer of our DNNBC model returns raw values in $[-\infty, \infty]$, which are then processed by the *softmax* activation function to scale them to values $[0, 1]$ representing the DNNBC model's predicted probability for our [1, 0] outcome variable. In the forward propagation (forward function) part of training the DNNBC model, the DNNBC model predicts whether a VCT-funded firm was successful or unsuccessful. It achieves this by running the independent variables through each of its layers. We use the DNNBC model's prediction and a Cross-Entropy (Log-Loss) function to calculate the error (loss), then we backpropagate this error through the DNNBC model. In the backward propagation (backward function) part of training, the DNNBC model adjusts its parameters proportionate to the error in the prediction. It does this by traversing backwards from the prediction, collecting the derivatives of the error with respect to the parameters of the layers - gradients, and optimising the parameters using ADAM optimisation algorithm.[13] In summary, we train the DNNBC model by looping over our data iterator, feeding the independent variables to the DNNBC model, and optimising. After training the DNNBC model for 400 epochs (passes) over the training dataset, we apply it on the test data to predict whether each VCT-funded firm in the test data was successful or unsuccessful. We then compare these predictions against the observed success of each VCT-funded firm in the test data, to obtain a test accuracy.

We now turn to elaborating on attribution algorithms and its usefulness. Consider a linear regression model. The coefficients represent the slope between the independent variables and the dependent variable, and we fit a linear model with coefficients to minimize the residual sum of squares between the observed success of VCT-backed firms, and the success predicted by the linear approximation. However, notice that with our DNNBC model, weights are embedded in every hidden layer. They capture the relationships between the neurons in the different layers, and are continuously updated via backward propagation. This means that

[13] ADAM is a superior type of stochastic gradient descent optimisation algorithm, where stochastic gradient is a stochastic approximation of gradient descent, and gradient descent is an optimisation algorithm that follows the negative gradient of an objective function in order to locate the minimum of the function.

we don't know exactly which of the weights - hence which of the independent variables - in which of the layers were most important, or how the neurons work together to predict the final classification of successful VCT-funded firm or unsuccessful VCT-funded firm. This is referred to in the Machine Learning literature as the *black box* issue of Deep Neural Networks. Attribution algorithms were developed to resolve this issue. An attribution algorithm helps attribute or measure the contribution of each independent variable to the predictions of a Deep Neural Network - an inherently causal task. We employ three different attribution algorithms to interpret the outputs of our DNNBC model. The first is the Sundararajan, Taly and Yan (2017) *Integrated Gradients* algorithm. Their algorithm builds on the simple axiom that in a Deep Neural Network model, the gradients of the outcome variable with respect to the independent variable is analogous to the coefficients of a linear model. They thus employ the product of the gradient and the value of the independent variable as the foundation for their *Integrated Gradients* attribution algorithm. However, because gradients do not satisfy several axioms that all attribution algorithms should satisfy, they cumulate the gradients. Formally, "*Integrated Gradients* are the path integral of the gradients along the straight line path from the baseline" - defined as the starting point from which gradients are integrated - to the independent variable. (Sundararajan, Taly and Yan., 2017, p. 3). The second is the *Integrated Gradients with Smooth Gradient* algorithm, which approximates smoothing the *Integrated Gradients* method with a Gaussian Kernel.[14] The third is the Shrikumar, Greenside and Kundaje (2017) *DeepLift* algorithm, which is based on back-propagating the output of a Deep Neural Network model through each layer of the network, down to the independent variables.

We also build a Deep Neural Network Regression model for our secondary empirical specification, this model has an architecture that is conceptually similar to the DNNBC model. We use this model to carry out a regression as opposed to a binary classification task. In other words, the outcome variable (VCT success) is now a continuous as opposed to a binary variable. We will elaborate on this model in a later section, but for now we turn to showing the results from our DNNBC model and all three attribution algorithms.

[14]For more details on adding Gaussian noise, please see https://captum.ai/api/_modules/captum/attr/_core/noise_tunnel.html#NoiseTunnel

2.4 Main Result

2.4.1 Factors that Determine the Success of VCT-Backed Firms: Binary Classification Model

Capital markets phenomena such as financial constraints, asymmetric information and the separation of ownership and control, combined with the remit of the VCT scheme, suggests that (especially with the former two phenomena) VCT investment managers can ease or exacerbate these phenomena by efficiently or inefficiently allocating capital across VCT-backed firms, i.e. invest in viable projects or transfer wealth (asset substitution) from VCT shareholders to themselves by investing in negative NPV projects. We will soon elaborate on the incentives of VCT investment managers to engage in asset substitution, but for now, we note that this agency problem introduced by Jensen and Meckling (1976) has been and continues to be extensively explored in studies such as Bolton, Becht, and Roell (2002), Morck, Wolfenzon, and Yeung (2005), Eisdorfer (2008), and Iliev and Lowry (2020). Given this backdrop, we hypothesise that if the match between a VCT and a potential VCT-backed firm is not animated by capital market phenomena and the ability to resolve them, but by randomness, then in the aggregate, skilled VCTs are just as likely as unskilled (lesser skilled) VCTs to finance ex-post successful VCT-backed firms. With the same being true for skilled VCTs being just as likely as unskilled (lesser skilled) VCTs to finance ex-post unsuccessful VCT-backed firms.

To wit, in this section, we present our main results where we quantify and rank the impact of each VCT skill and the funding deal structure on the success of VCT-backed firms. Our main results are presented in Table 2.2., where we provide evidence on the importances, in descending order, of each measure of VCT skill and the funding deal structure for the success of VCT-backed firms. In each column, we report the average attribution scores from a Deep Neural Network we built and trained on our hand-collected and FAME data to carry out a binary classification task (DNNBC model), where the dependent variable is equal to 1 if the VCT-backed firm was a success (unrealised IRR $\geq 5\%$) and 0 otherwise. An attribution score from an attribution algorithm helps assess the contribution of each independent variable to the output of a Deep Neural Network model. Therefore, in each column, the reported attribution score is the average percentage contribution of each measure of VCT skill and the funding deal structure for the success of VCT-backed firms. The average attribution scores in column 1 are from the *Integrated Gradients* attribution algorithm. The second and third columns report the average attribution scores from the same Deep Neural Network Binary Classification (DNNBC) model, but estimated using the *Integrated Gradients with SmoothGrad* and *DeepLift* attribution algorithms respectively.

Our independent variables include measures of VCT skill, funding deal structure, and several control variables. For each VCT-backed firm, we measure the skill of its VCT backer in the fiscal year preceding the first-time said VCT backed the firm. Our reasoning for measuring VCT skill in the fiscal year prior to the first-time it financed a firm as opposed to say the second time or third time is simple. Studies such as Sørensen (2007) and Gompers, Gornall, Kaplan, and Strebulaev (2020) show that deal sourcing and deal selection (VCT pre-investment skills) are the most important for value creation relative to post-investment monitoring and advising (VCT post-investment skills). Also, this approach is less susceptible to the reverse causality problem because measuring VCT skill prior to the first time it finances a firm captures the VCT's skill before its first involvement with the VCT-backed firm, which of course precedes the observed success of the VCT-backed firm.

Our first measure of VCT skill is # First-Time VCTs in Top 5, which as detailed earlier, measures whether a VCT was ranked in the annual Top 5 ranking of all VCTs. This measure derives from studies such as Sahlman (1990) which discusses the skill of top VCs at post-investment value added-on, Sørensen (2007) which discusses the skill of top VCs in deal sourcing and selection, Gompers et al. (2008) which discusses the skill of top VCs at timing their exit from an investment, and most importantly, the Nahata (2008) study on VC reputation and investment performance. Nahata (2008) suggests that a VC or in our case a VCT's ranking effectively captures its pre-financing deal expertise (deal sourcing and deal selection) and post-financing deal expertise (deal monitoring and advising). The ensuing discussion is going to focus on the *Integrated Gradients* average attribution scores in column 1, but the sign on each independent variable is consistent across all three columns (all three attribution algorithms). From the average attribution score in Table 2.2., we follow the prior literature in demonstrating that being backed by a VCT ranked in the Top 5 is a significant positive determinant of the success of VCT-backed firms. The average contribution of # First-Time VCTs in Top 5 to the success of a VCT-backed firm is 13% (*Integrated Gradients*), and it is the joint most important VCT skill determinant of the success of VCT-backed firms.

Our second and third measure of VCT skill, Low Prior Performance and High Prior Performance, are motivated by Carpenter (2000), wherein the model in their study centres around a risk averse fund manager compensated with a call option on the assets she controls. They then analyse how the option compensation impacts the manager's risk appetite when she cannot hedge the option position. We are further motivated by Barrot (2017) and Nanda and Rhodes-Kropf (2018) who discuss how VCs sometimes make investment decisions based on factors unrelated to the NPV of an investment, and also by Iliev and Lowry (2020) who discuss the increased incentives of poorly performing VCs to take lottery type gambles and the analogous reduced incentives of high performing VCs to take lottery type gambles.

We follow the aforementioned studies in hypothesising that the convexity or option-like nature of a VCT investment manager's total compensation contract can incentivise said VCT investment manager to base investment decisions on her prior performance. From Table B.1. in Appendix B, we see that the average VCT investment manager's compensation contract is composed of two parts: an annual 2% management fee pegged to the Net Asset Value at fiscal year-end plus a performance incentive fee also at fiscal year-end. The average performance incentive fee [15] is a percentage of the excess Total Returns, the excess over a hurdle rate of approximately 5%.

From Table 2.2., we find that High Prior Performance is the most important and a significant positive determinant of the success of VCT-backed firms, whereas Low Prior Performance is a negative determinant of the success of VCT-backed firms. Our finding that High-Prior-Performing VCTs contribute an average of 13% to the success of VCT-backed firms is consistent with the results of corporate finance empirical studies such as Pindyck and Solimano (1993), Episcopos (1995), Caballero and Pindyck (1996), Ghosal and Loungani (1996), Leahy and Whited (1996), Bulan (2005), and Eisdorfer (2008), all of whom reinforce the axiomatically acknowledged existence of an inverse relationship between current investment and risk. High-Prior-Performing VCTs make fewer risky bets. Also, our finding that Low-Prior-Performing VCTs take more risk and as such, in the aggregate, finance unsuccessful VCT-backed firms, is consistent with the results in Carpenter (2000) who show that options that are deep out of the money seemingly incentivises excessive risk taking. Pertinently, as in the Eisdorfer (2008) study, they show that when an investor (VCT in our case) is in financial distress (Low Prior Performance in our case), risk-shifting incentives are added to its real-options consideration in determining its investment-risk assessment. As the upside of risky bets benefit the distressed investor (VCT), an increase in the risk of a project is a potential source of value for the distressed investor (VCT). They thus show how risk has diametrically opposing effects on current investment. On the one hand, the real options consideration act as a depressant of current investment - due to the option-to-delay, whereas, the risk-shifting consideration has a positive effect. This argument is the central thesis of Eisdorfer (2008), where he shows that the risk-shifting effect dominates the real-options effect - and in fact, there is a positive relationship between investment and risk - when the firm is in distress. Crucially, given that high prior performing VCTs are analogous to reputable VCs in Nahata (2008), the finding that high prior financial performance is the most important VCT skill determinant of the success of VCT-backed firms, reinforces the Nahata (2008) findings of reputable VCs adding value to VC-backed firms.

[15] Colloquially referred to as carried interest

Table 2.2. **Comparing VCT Skill and Deal Structure Importances Across Multiple Attribution Algorithms: Firm Level Deep Neural Network: Binary Classification Model: Binary Dependent Variable is the Unrealised IRR**

Our sample consists of 1,953 VCT-backed firms between 2015 and 2020 as defined in Table 2.1. We employ various attribution algorithms to interpret the results from our Deep Neural Network Binary Classification model, where the binary dependent variable is equal to 1 if the VCT-backed firm is successful. In column 1, we report the average attribution score from the *Integrated Gradients* algorithm. In columns 2 and 3, we report the average attributions scores from the *Integrated Gradients with SmoothGrad* and *DeepLift* algorithms respectively. Control variables are also included with all variables defined in Appendix B.

	Integrated Gradients	Integrated Gradients w/SmoothGrad	DeepLift
VCT Skill Measures			
# First-Time VCTs with High Prior Performance	0.13	0.10	0.11
# First-Time VCTs in Top 5	0.13	0.07	0.06
FTSE-Industry Experience/Total Experience of all First-Time VCT Backers (%)	0.09	0.23	0.06
# First-Time VCTs with Low Prior Performance	-0.02	-0.03	-0.00
# First-Time VCTs that are Young	-0.01	-0.01	-0.05
Log(FTSE-Industry Experience Count of all First-Time VCT Backers)	0.00	0.00	0.00
Deal Structure			
ATBFR(Years)	0.18	0.15	0.09
Holding Period (Years)	-0.11	-0.11	-0.05
Log(Total VCT Funding)	0.01	0.00	0.04
# Funding Rounds	0.01	0.01	0.04
Control Variables			
VCT-backed Firm Age	0.09	0.10	0.11
Log(Total Assets)	0.05	0.05	0.03
Cash-to-Assets	0.05	0.04	0.02
Debt-to-Assets	-0.01	-0.09	0.04
Training Accuracy	0.71	0.71	0.71
Test Accuracy	0.67	0.67	0.67

Our next measure of skill is the age of a VCT: "young VCT", where, as earlier discussed, a young VCT is one that is less than 15 years old at the time of funding a firm for the first time. The measure is inspired by studies such as Lee and Wahal (2004), Tian and Wang (2014), and Iliev and Lowry (2020), but primarily derives from Gompers (1996) wherein the study samples 433 IPOs, then develops and tests the hypothesis that young VCs exit (IPO) earlier, relative to older VCs. Their behaviour is motivated by their need to signal reputation, or "Grandstand" and thus successfully fundraise in the future. Also, younger VCs back younger firms and hold smaller equity stakes, all of which results in more underpricing when these firms go public - relative to firms backed by older VCs. In line with these studies, we find (presented in Table 2.2.) that being backed by a young VCT is a negative determinant of the success of VCT-backed firms, contributing an average of -1% to the success of VCT-backed firms. To the extent that innovation is synonymous with success (as defined in this study), our results are also consistent with that of Tian and Wang (2014) who show that being backed by a younger VC is related to lower innovation in the VC-backed firm.

The penultimate VCT skill measure (FTSE-Industry Experience/Total Experience of all First-Time VCT Backers) captures a VCT's specialisation in funding the FTSE-Industry of its firms and our final VCT skill measure (FTSE-Industry Experience Count of all First-Time VCT Backers) captures the sheer number of deals a VCT undertook in the FTSE-Industry of the firms it backed and thus the VCT's experience within the industry of the firms it backed. These variables are motivated by a growing body of work on VC experience and specialisation. Studies such as: Sørensen (2008) show how the VC investment decision is based on the tradeoff between specialisation, which allows them learn from past investments, and generalisation, which provides the option value of future learning. Gompers, Kovner, and Lerner (2009) analyse the relationship between specialisation at the individual and firm level at a VC and VC success. They find that the lower levels of success experienced by generalist VCs are due to poor selection of investments within industries and inefficient allocation of capital across industries.[16] Racculia (2014) shows that being financed by a specialist VC as opposed to a generalist VC results in a stronger IPO. This is due to the ability of said specialist VC to select firms with potentially innovative or disruptive technology. Our results in Table 2.2., show that VCT funding specialisation is a significant positive determinant of the success of VCT-backed firms, contributing an average of 9% to the success of VCT-backed firms, whereas funding experience is an insignificant but positive determinant of the success of VCT-backed firms, contributing an average of 0% to the success of VCT-backed firms. Our results are also in line with the results in the earlier mentioned Gompers, Kovner, and

[16]Their results also suggest that specialisation at the individual VC investment manager level is more important than specialisation at the VC organisational level

Lerner (2009) study, wherein they show that VCs with more specialisation often outperform less-specialised VCs.

Turning now to the deal structure, we see from Table 2.2., that the ATBFR (Years) is a significant positive determinant of the success of VCT-backed firms. Now we recall that the ATBFR for VC-backed firms as shown in the crunchbase (2018) study is 24 months whilst we showed in Table 2.1., that for VCT-backed firms (regardless of success) it is approximately 13 months. From this, we conjecture that the positive relationship between the ATBFR and the success of VCT-backed firms speaks to VCTs and their desire to structure financing rounds as close to the average of 24 months that we see in the larger VC ecosystem. We also know from Sahlman (1990) that staggered funding is a key control mechanism employed by VCs, where they show that VCs increase the ATBFR as the firm becomes better established. Indeed, Gompers (1995) asserts that the ATBFR should be inversely correlated with expected agency costs. We also see in Table 2.2., that the Total VCT funding of VCT-backed firms is a positive determinant of the success of VCT-backed firms, and is in line with the findings in Gompers (1995). The more successful a VCT-backed firm is, the more money a VCT invests in the VCT-backed firm. But, we can also relate this result to and analyse it in tandem with the ATBFR result. VCTs shorten the ATBFR or intensify their monitoring as VCT-backed firms realise less and less success, but they do not increase the amount of funding they provide to these unsuccessful VCT-backed firms simply because they are not realising intermediate success. In other words, and to use a colloquialism, VCTs do not "throw good money after bad". The number of funding rounds (# Funding Rounds) is a positive determinant of the success of VCT-backed firms, and is inconsistent with our finding of a positive relationship between the ATBFR and the success of VCT-backed firms. Indeed, this finding goes against the discussion in Gompers (1995), which in our case implies that VCTs who exit via non-IPO routes, would do so quickly (stage their investments over fewer funding rounds), the more success a VCT-backed firm realises. Nonetheless, the sign on the deal structure variables are mostly in line with Sahlman (1990) and Gompers (1995). VCTs invest more money, over an industry standard or increasing length of time, in successful VCT-backed firms. That they invest these monies over more funding rounds, is an inconsistent finding that we will further investigate in the next subsection. We also include the VCT holding period of VCT-backed firms, which is a significant negative determinant of the success of VCT-backed firms, and affirms the results in Gompers (1995), wherein investors (VCTs) cash in on their successes quickly - if they plan to exit via a non-IPO route - which in the case of VCTs is almost exclusively true due to the rules and regulations guiding the VCT scheme, which in turn dictates the type of firms VCTs can and cannot invest in, how long they can hold an

investment, among other concomitant restrictions.[17] With regards the controls, we include firm level variables such as VCT-backed firm financials and age.
In Table 2.3., we present results from a robustness-check exercise, in further consideration of the selection bias issue raised in Sørensen (2007), where he shows a connection between experienced VCs and selection bias, wherein the most experienced VCs tend to invest in better firms. To that end, we restrict our sample to the subset of VCT-backed firms backed by VCTs not in the top quartile of experience at funding any FTSE-Industry. Succinctly put, we exclude from our sample, VCTs in the top quartile of experience at funding firms - regardless of FTSE-Industry. Although the order of importance of the attribution scores on each measure of VCT skill and deal structure is slightly changed, the sign remains unchanged as shown in Table 2.3. We also see that VCTs with High Prior Performance and VCTs in the Top 5 remain among the most important of VCT skill measures for determining the success of VCT-backed firms, especially VCTs with High Prior Performance, which contributes an average of 34% to the success of VCT-backed firms.

[17] See Iweze (2020) for details.

Table 2.3. **Does VCT Skill and Deal Structure Still Determine the Success of VCT-Backed Firms When We Exclude Experienced VCTs from the Analysis? Comparing VCT Skill and Deal Structure Importances Across Multiple Attribution Algorithms: Binary Classification Model: Binary Dependent Variable is the Unrealised IRR**

Our sample consists of 844 VCT-backed firms between 2015 and 2020 as defined in Table 2.1, but excludes the data for VCTs in the top quartile of experience at funding firms in any FTSE-Industry. We employ various attribution algorithms to interpret the results from our Deep Neural Network Binary Classification model, where the binary dependent variable is equal to 1 if the VCT-backed firm is successful. In column 1, we report the average attribution score from the *Integrated Gradients* algorithm. In columns 2 and 3, we report the average attributions scores from the *Integrated Gradients with SmoothGrad* and *DeepLift* algorithms respectively. Control variables are also included with all variables defined in Appendix B.

	Integrated Gradients	Integrated Gradients w/SmoothGrad	DeepLift
VCT Skill Measures			
# First-Time VCTs with High Prior Performance	0.34	0.22	0.28
# First-Time VCTs that are Young	-0.25	-0.11	-0.11
# First-Time VCTs in Top 5	0.14	0.11	0.14
FTSE-Industry Experience/Total Experience of all First-Time VCT Backers (in percentages)	0.08	0.03	0.06
# First-Time VCTs with Low Prior Performance	-0.06	-0.05	-0.04
Log(FTSE-Industry Experience Count of all First-Time VCT Backers)	0.05	0.01	0.00
Deal Structure			
ATBFR(Years)	0.40	0.26	0.26
Log(Total VCT Funding)	0.25	0.12	0.19
Holding Period (Years)	0.15	0.11	0.08
# Funding Rounds	-0.02	0.05	-0.00
Control Variables			
VCT-backed Firm Age	-0.35	-0.19	-0.22
Log(Total Assets)	0.30	0.15	0.19
Cash-to-Assets	-0.11	-0.14	-0.10
Debt-to-Assets	-0.04	-0.00	0.02
Training Accuracy	0.75	0.75	0.75
Test Accuracy	0.66	0.66	0.66

2.4.2 Factors that Determine the Success of VCT-Backed Firms: Regression Model

For completeness and given that we have thus far employed the unrealised IRR, in a binary form, as our dependent variable, we employ another empirical specification, this time using the unrealised IRR in its continuous form, as the dependent variable. For this empirical specification, we build another Deep Neural Network (DNN) and train it on our hand-collected and FAME data, but this time, to carry out a regression task. This Deep Neural Network Regression (DNNR) model is conceptually similar to our Deep Neural Network Binary Classification (DNNBC) model but with an architecture more suited to a regression task.[18] The DNNR model is a four layer neural network with the non-linear component of each layer containing rectified linear activation function (ReLU).[19] We interpret the results of our DNNR model with the three different attribution algorithms described in Section 2.3 and report the results (mean attribution scores) in Table 2.4. We find further support for VCT skills and funding deal structure as determinants of the success of VCT-backed firms.

The sign on the mean attribution scores are largely consistent with that in Table 2.2. There is a slight re-ordering in terms of the ranked importance of each VCT skill measure relative to the results in Table 2.2. VCT skill: # First-Time VCTs in Top 5, is now the least important VCT skill determinant of the success of VCT-backed firms, whereas in Table 2.2., Log(FTSE-Industry Experience Count of all First-Time VCT Backers) was the least important VCT skill determinant of the success of VCT-backed firms. # First-Time VCTs with High Prior Performance is still the most important VCT skill determinant of the success of VCT-backed firms, contributing an average of 13% to the success of VCT-backed firms. Also, we see that VCT funding specialisation (FTSE-Industry Experience/Total Experience of all First-Time VCT Backers (%)) is still a top three VCT skill determinant of the success of VCT-backed firms, with an average contribution of 6% to the success of VCT-backed firms.

Turning to the deal structure, we see that the Holding Period (Years) and the number of funding rounds (# Funding Rounds) are significant and negative determinants of the success of VCT-backed firms. This finding is in line with the assertions in Gompers (1995) and Sahlman (1990) wherein they show that the Holding Period (Years) and the number of funding rounds (# Funding Rounds) are metrics for the intensity with which VCs monitor firms, and is an increasing function of expected agency costs, or in this study, should be a negative determinant of the success of VCT-backed firms. Additionally, this finding also

[18] The chosen hyper-parameters are suited to a regression task i.e. the loss function, which is the MSE loss function and the optimizer, Root Mean Squared Propagation (RMSProp), which is an enhanced form of gradient descent that employs a decaying mean of partial gradients in determining the step size for each parameter.

[19] ReLU is a non-linear activation function employed in Deep Neural Networks, and is given by: $ReLU(x) = (x)^+ = \max(0,x)$

emphasises the usefulness of employing the continuous form of our dependent variable given that we found in the previous section, that the number of funding rounds (# Funding Rounds) is a positive determinant of the success of VCT-backed firms. As detailed in the previous section, VCTs cash in on their successes quicker than they realise losses, or if you will, they hold on to unsuccessful VCT-backed firms for longer than they do successful VCT-backed firms. Cashing-in on investment successes is all the more important given that we know from our hand-collected data that in any given year, VCTs pay out a substantial average range of 8% - 10% of their Net Asset Value in dividends.[20] Dividends received from VCTs are tax exempt, which incentivises potential investors to invest in VCTs, and VCTs in turn promote their dividend payout track record when fundraising. Finally, we see in Table 2.4., that the ATBFR (Years) and Log (Total VCT Funding), are all important determinants of the success of VCT-backed firms, contributing a respective average of 9% and 7%, to the success of VCT-backed firms. In summary, the results from our DNNR model in this section, which employs the continuous form of our dependent variable (unrealised IRR), reaffirms VCTs with High Prior Performance as the most important and a significant positive determinant of the success of VCT-backed firms, contributing an average of 13% to the success of VCT-backed firms. This finding is consistent with Nahata (2008), who finds that reputable VCs add value to VC-backed firms.

[20]See Table 2.1.

Table 2.4. **Comparing VCT Skill and Deal Structure Importances Across Multiple Attribution Algorithms: Regression Model: Continuous Dependent Variable is the Unrealised IRR**

Our sample consists of 1,953 VCT-backed firms between 2015 and 2020 as defined in Table 2.1. We employ various attribution algorithms to interpret the results from our Deep Neural Network Regression Model, where the continuous dependent variable is the unrealised IRR, which proxies for the success of VCT-backed firms. In column 1, we report the average attribution score from the *Integrated Gradients* algorithm. In columns 2 and 3, we report the average attributions scores from the *Integrated Gradients with SmoothGrad* and *DeepLift* algorithms respectively. Control variables are also included with all variables defined in Appendix B.

	Integrated Gradients	Integrated Gradients w/SmoothGrad	DeepLift
VCT Skill Measures			
# First-Time VCTs with High Prior Performance	0.13	0.10	0.09
# First-Time VCTs with Low Prior Performance	-0.09	-0.01	-0.01
FTSE-Industry Experience/Total Experience of all First-Time VCT Backers (%)	0.06	0.11	0.05
Log(FTSE-Industry Experience Count of all First-Time VCT Backers)	0.04	0.03	0.04
# First-Time VCTs that are Young	0.01	0.02	0.03
# First-Time VCTs in Top 5	0.00	0.04	0.03
Deal Structure			
Holding Period (Years)	-0.31	-0.17	-0.18
# Funding Rounds	-0.20	-0.09	-0.07
ATBFR(Years)	0.09	0.03	0.02
Log(Total VCT Funding)	0.07	0.06	0.03
Control Variables			
VCT-backed Firm Age	0.18	-0.05	-0.05
Log(Total Assets)	-0.15	-0.21	-0.13
Cash-to-Assets	0.05	0.05	0.04
Debt-to-Assets	-0.06	-0.04	-0.02
RMSE	0.27	0.27	0.27

2.4.3 Factors that Determine the Success of VCT-Backed Firms: OLS Regression Model

Here, our intention is simple. We employ an OLS linear regression model with fixed effects on FTSE-Industry and standard errors clustered by year of first investment, to conduct further analysis, which in turn serves to emphasise the superior performance of our Deep Neural Network Binary Classification (DNNBC) model and Deep Neural Network Regression (DNNR) model. We start with Table 2.5., which we will compare against Table 2.1. (Descriptive Statistics), where we presented average values for both successful and unsuccessful VCT-backed firms, and their differences. To allay the concern that the differences between the average values might derive from FTSE-Industry differences,[21] we employ two OLS models to conduct our analysis, where the first exclusively employs data on the subsample of successful VCT-backed firms and the second exclusively employs data on the subsample of unsuccessful VCT-backed firms. We present both results in columns 1 and 2, with t-statistics in parentheses, and the Z test for the difference between two regression coefficients in column 3.[22] We see that the results are consistent with those in Table 2.1. # First-Time VCTs in Top 5 is positively correlated to the success of successful VCT-backed firms and negatively correlated to unsuccessful VCT-backed firms, with the coefficient for successful VCT-backed firms significant at the 5% level. This result is consistent with that of # First-Time VCTs in Top 5 in Table 2.1., where we see that Top 5 VCTs overwhelmingly backed successful relative to unsuccessful VCT-backed firms, with the difference being positive and significant at the 1% level. Again, we see in Table 2.5., that the difference in the coefficients for VCT funding specialisation (FTSE-Industry Experience/Total Experience of all First-Time VCT Backers (%)) is positive and significant. This result affirms the finding in Table 2.1. We see a consistent theme with the remaining measures of VCT skill and deal structure, where the coefficients are significant at conventional levels and the differences are in line with the differences in Table 2.1. Although, the results for VCT funding experience (Log(FTSE-Industry Experience Count of all First-Time VCT Backers)) is not in line with that of Table 2.1. Nonetheless, we now turn to employing the full sample to test the robustness of these coefficients and their differences.

[21]For instance, VCTs or VCT funds that specialise in funding firms in the renewable energy sector might enjoy success not because specialisation is a value-added skill, but due to government programmes like the Feed-in Tariffs scheme introduced by the U.K. government to encourage the production of renewable energy, a scheme that ran between 2010-2019, a period that also saw the establishment of several VCTs and VCT funds specialising in the renewable energy sector, prominent among them - Foresight Solar & Technology VCT Plc in 2010.

[22]The Z test is from "Statistical methods for comparing regression coefficients between models" by Clogg, Petkova and Haritou (1995).

In Table 2.6., we present our results from analysing the full sample (both successful and unsuccessful VCT-backed firms) using again, an OLS linear regression model with fixed effects on FTSE-Industry and standard errors clustered by year of first investment, with t-statistics in parentheses. The results are inconsistent with the results in Table 2.2. and Table 2.4. We see that VCT funding specialisation (FTSE-Industry Experience/Total Experience of all First-Time VCT Backers (%)), which is one of the most important VCT skill determinants of the success of VCT-backed firms across both Table 2.2. and Table 2.4., is also a significant and positive determinant of the success of VCT-backed firms. Additionally, # First-Time VCTs in Top 5 is also a significant and positive determinant of the success of VCT-backed firms. For the remainder results, we see that the coefficients are not significant at conventional levels. That we see no statistically significant linear dependence of the mean of the unrealised IRR (dependent variable) on X[23] emphasises the importance of our DNNBC and DNNR models. These models are built to capture complex mappings (non-linearities) between the unrealised IRR and X, non-linearities we expect to see. Recall, we constructed (as detailed in previous sections) the independent variables that proxy for VCT skill and as such we would expect a non-linear model (DNNBC and DNNR models) to be better at capturing the relationships among these constructed independent variables (VCT skill) and the dependent variable, relative to a linear model. Overall, the results indicate that VCT funding specialisation in the FTSE-Industry of the firm it backs, is crucial for the eventual success of the firm. Additionally, firms that were backed by Top 5 VCTs, are more likely to be successful, where Top 5 VCTs are VCTs that have the most expertise at screening and monitoring their VCT-backed firms. In summary, the insignificance of this OLS result emphasises that the OLS model cannot capture non-linear relationships, whereas our DNNBC and DNNR models are built to capture non-linear relationships.

[23] X is all of the independent variables except for # First-Time VCTs in Top 5 and FTSE-Industry Experience/Total Experience of all First-Time VCT Backers (in percentages).

Table 2.5. **Two OLS Models for Both Successful and Unsuccessful VCT-backed Firms with Fixed Effects on FTSE-Industry and Standard Errors Clustered by Year of First Investment: Binary Dependent Variable is the Unrealised IRR**
Our sample consists of 1,953 VCT-backed firms between 2015 and 2020 as defined in Table 2.1., where the binary dependent variable is equal to 1 if the VCT-backed firm is successful and zero otherwise. In columns (1) and (2), we report results from two OLS models with Fixed Effects on FTSE-Industry and standard errors clustered by year of investment. In Column (1), we report coefficients for the OLS model that exclusively employs data on the 851 successful VCT-backed firms subset and in column (2), we report coefficients for the OLS model that exclusively employs data on the 1,102 unsuccessful VCT-backed firms subset. t-statistics are shown in parentheses. Statistical significance of the coefficients at the 1%, 5%, and 10% levels are represented by ***, **, and * respectively. In the final column, the z test for the difference between both regression coefficients are reported. All variables are described in Appendix B.

	(726) Successful VCT-Backed Firms	(1,002) Unsuccessful VCT-Backed Firms	Difference
VCT Skill Measures			
# First-Time VCTs in Top 5	0.0150**	-0.0003	2.12
	(3.80)	(-0.05)	
FTSE-Industry Experience/Total Experience of all First-Time VCT Backers (in percentages)	0.0830*	0.0548***	0.76
	(2.37)	(3.99)	
Log(FTSE-Industry Experience Count of all First-Time VCT Backers)	-0.0100	0.0000	-2.64
	(-2.01)	(1.86)	
# First-Time VCTs with Low Prior Performance	0.0030	0.0100	-0.54
	(0.37)	(1.38)	
# First-Time VCTs with High Prior Performance	0.0030	-0.0200***	4.11
	(0.95)	(-4.24)	
# First-Time VCTs that are Young	-0.0010	0.0100**	-2.22
	(-0.23)	(2.72)	

(Continued)

Table 2.5. *Continued*

	(726) Successful VCT-Backed Firms	(1,002) Unsuccessful VCT-Backed Firms	Difference
Deal Structure			
# Funding Rounds	-0.0050	0.0000	-1.65
	(-1.37)	(1.07)	
Log(Total VCT Funding)	-0.0010	-0.0000**	1.84
	(-0.51)	(-2.85)	
ATBFR(Years)	-0.0120	-0.0300**	1.41
	(-0.90)	(-3.84)	
Holding Period (Years)	-0.0080**	-0.0076**	-0.05
	(-2.70)	(-2.68)	
Control Variables			
VCT-backed Firm Age	-0.0000*	-0.0000	-1.18
	(-2.29)	(-0.18)	
Log(Total Assets)	0.0010	0.0000	0.04
	(0.96)	(0.67)	
Debt-to-Assets	-0.0010*	-0.0036	1.12
	(-2.16)	(-1.56)	
Cash-to-Assets	0.0200	0.0100	0.67
	(1.43)	(0.91)	
Adjusted R^2	0.17	0.20	

Table 2.6. **OLS Model with Fixed Effects on FTSE-Industry and Standard Errors Clustered by Year of First Investment: Continuous Dependent Variable is the Unrealised IRR**

Our sample consists of 1,953 VCT-backed firms between 2015 and 2020 as defined in Table 2.1., where the continuous dependent variable is the unrealised IRR, which proxies for the success of VCT-backed firms. The table contains results from an OLS model with Fixed Effects on FTSE-Industry and standard errors clustered by year of investment. In the Column, we report coefficients for the OLS model. t-statistics are shown in parentheses. Statistical significance of the coefficients at the 1%, 5%, and 10% levels are represented by ***, **, and * respectively. All variables are described in Appendix B.

	(1,728) VCT-Backed Firms
VCT Skill Measures	
# First-Time VCTs in Top 5	0.0183**
	(3.72)
FTSE-Industry Experience/Total Experience of all First-Time VCT Backers (%)	0.2813***
	(6.93)
Log(FTSE-Industry Experience Count of all First-Time VCT Backers)	-0.0157
	(-1.78)
# First-Time VCTs with Low Prior Performance	0.0053
	(0.69)
# First-Time VCTs with High Prior Performance	0.0011
	(0.26)
# First-Time VCTs that are Young	0.0033
	(0.48)
Deal Structure	
# Funding Rounds	-0.0046
	(-1.03)
Log(Total VCT Funding)	0.0020
	(1.15)
ATBFR(Years)	0.0163
	(1.54)
Holding Period (Years)	-0.0087
	(-1.24)
Control Variables	
VCT-backed Firm Age	0.0001
	(0.32)
Log(Total Assets)	0.0037
	(1.85)
Debt-to-Assets	-0.0033
	(-1.23)
Cash-to-Assets	0.0340
	(1.25)
Adjusted R^2	0.11

2.5 Conclusion

Corporate finance studies have shown that in the aggregate, small, young and risky firms face financial constraints due to various capital market phenomena. These constraints are alleviated by the VCT scheme and its funding of small, young and risky U.K. firms, where we document that in any given year, the average VCT equity stake in a VCT-backed firm is a non-trivial percentage ranging from 19% to 31%. Against this backdrop, we employ several machine learning approaches and hand-collected VCT data to analyse VCTs and their equity investments. We find that VCTs are skilled along several value generating dimensions, and these skills, in addition to the financing deal structure, determine the success of VCT-backed firms. In other words, VCT skill and the funding deal structure are important determinants of the success of VCT-backed firms. Specifically, across all empirical specifications, we find that being backed by VCTs with high prior financial performance, is the most crucial VCT skill determinant of the success of VCT-backed firms, contributing an average of 13% to the success of VCT-backed firms. This result reinforces the findings in Nahata (2008), wherein they show that being backed by VCs with high prior performance (a skill that captures VC screening and monitoring expertise) is a key determinant of the success of VC-backed firms. Finally, our results offer a road-map to policy makers on future VCT policy changes. Economies all around the world including the U.K. are facing various economic challenges amidst a period of war, food and cost of living crises. These challenges will have an effect on the supply of capital to small, young and risky firms - which are the focus of the VCT scheme. These effects can be mitigated by increasing the VCT funding limit currently set at £15 million and £20 million for firms in non-knowledge and knowledge intensive industries respectively. Based on our results which show that the GBP amount of VCT funding is a significant positive determinant of the success of VCT-backed firms, allowing VCTs the flexibility to intervene and fund their firms beyond the current limits is an approach worth considering.

Chapter 3

VC Skill and Deal Structure vs Luck: What Drives the Success of VC-Backed Firms in the U.K.

3.1 Introduction

Beyond the supply of capital, VC skills are critically important because of the added services they provide to VC-backed firms. As financial intermediaries, it is axiomatically acknowledged that VCs alleviate the agency and informational asymmetry problems that plague entrepreneurial firms and constrains their access to capital (Hellman, 1998; Iliev and Lowry, 2020). In addition to their financial intermediation role, Lerner (1995) discusses how VCs help in structuring the corporate boards of VC-backed firms, Hellman and Puri (2002) show that VCs help in professionalising VC-backed firms, while Gompers, Gornall, Kaplan and Strebulaev (2020) document how VCs help improve the corporate governance of VC-backed firms. Indeed, entrepreneurial firms place a premium on the added services that VCs bring to bear on VC-backed firms. Hsu (2004) empirically shows that when faced with competing funding offers from VCs, entrepreneurial firms would sacrifice highers offers to accept lower offers from VCs with prior success and experience at funding firms in their particular industrial sector. To understand whether these VC added services are value-generating, seminal studies such as Gompers, Kovner, and Lerner (2009) measure the degree of specialisation of VCs and show a positive relationship between VC specialisation (VC skill) and the success of VC-backed firms.

We measure the skills of VCs along several dimensions and find that these skills, in addition to the funding deal structure, are determinants of the success of VC-backed firms. Our goal is to adopt a selection-bias-alleviating econometric approach to quantify the relative importance of each measure of VC skill and funding deal structure for the success of VC-backed firms. We find that the most important VC skill for determining the success of VC-backed firms are: specialisation at funding the FTSE-Industry of VC-backed firms and prior success at exiting VC-backed firms, where both skills respectively contribute 14% and 13% to the success of VC-backed firms.

Specialist VCs are better able to allocate capital within the industry they specialise in (Gompers, Kovner, and Lerner, 2009), and when a firm receives backing from a VC with prior success at exiting VC-backed firms, it acts as a credible signal of unobserved firm characteristics to financial markets, and thus positively affects firm value (Sørensen, 2007). Nahata (2008) further reiterates how a VC with prior success at exiting VC-backed firms captures the VCs screening and monitoring expertise. Concurrently, the selection bias issue, as detailed in Sørensen (2007), stems from the match between VCs and VC-backed firms. Given the documented tendency of entrepreneurial firms favouring less financially lucrative offers, so long as it comes from VCs with prior success, as evinced in Hsu (2004), this creates a proprietary deal flow for the most successful VCs, which obviously provides them proprietary access to entrepreneurial firms with the best potential. Sørensen (2007) further develops the selection bias argument by hypothesising that this access to a proprietary deal flow of high potential firms implies that ex-post successful VC-backed firms are successful because they are intrinsically better and because successful VCs add value to them. They further detail how both VC skills and the selection bias issues could concurrently determine the success of VC-backed firms, thus emphasising the need to control for the latter when analysing the impact of the former.

Our econometric approach is adopted from the Iweze (2021) study. There, they discuss the shortcomings of classic approaches (instrumental variables and structural models) to dealing with selection bias issues when estimating a model. These shortcomings inspire their and our adoption of a Deep Neural Network (DNN) model and several attribution algorithm in our econometric approach. Deep Neural Network models are a class of machine learning algorithms that consists of several interconnected layers of linear and non-linear mathematical functions that are used to model complex relationships (especially non-linear relationships) in a dataset. Our DNN model is a Deep Neural Network Regression (DNNR) model that predicts the success of VC-backed firms based on the skills of its VC-backers and the funding deal structure. Nonetheless, it is axiomatically acknowledged that Deep Neural Network models are like "black boxes" in the sense that there is no simple or direct connection between the weights and the output of the model. Because the DNNR model consists of many interconnected non-linear layers with associated weights, it is unclear which set of weights in which non-linear layer is most responsible for the final output of the DNNR model. This is where attribution algorithms come in. An attribution algorithm takes as input, the independent variables, dependent variables, and output of a Deep Neural Network model, and extracts the relative impact of each independent variable on the Deep Neural Network's output, an inherently causal task (Sundararajan, Taly and Yan, 2017). This combination of a Deep Neural Network model and attribution algorithm allows us to take a data driven approach to uncover the relative impact of each measure of VC skill and the funding deal structure on the success of VC-backed firms. Although, this data driven approach alleviates the selection bias issue detailed earlier, we conduct several tests to further alleviate the selection bias issue, and still find that VC funding specialisation is an important determinant of the success of VC-backed firms.

This study also serves as a backdrop for the Iweze (2021) study - in that we detail and analyse the wider VC environment that Venture Capital Trusts operate in. The VC industry, but specifically, the U.K. VC industry has developed as an important financial intermediary - especially over the last 20 years and even stretching back to the 1990's. It has grown into the second most significant VC hub after the U.S., and has made significant contributions to the U.K. economy in the form of economic development and entrepreneurship. From £9 billion raised in 2000 to £16.7 billion raised by 119 VC funds in 2021, this represents an increase of 86% over the period. These monies have supported investment in U.K. firms, from a total of £6.4 billion invested in 2000 to £17.3 billion in 2021 (invested in 1,320 firms out of a possible 5,000 firms), representing a 170% increase over the period, where 90% of these firms are SMEs. Furthermore, U.K. VCs directly and indirectly contributed in excess of £208 billion to the U.K. GDP in 2021.[1]

This importance of VCs to the economy further reiterates the importance of exploring whether VC skills (VC added services) determine the success of VC-backed firms. Bellucci et al. (2022) show that older, specialist, and more experienced VCs take quicker advantage of novel investment opportunities relative to younger, less-specialised VCs. Gompers, Gornall, Kaplan and Strebulaev (2020) and Fuchs et al. (2018) emphasise the importance of VC post-investment services for the success of VC-backed firms. Khanna and Matthews (2022) employ a discrete-time dynamic model to study competition among VCs with heterogeneous skills and its implications for VC-backed firms. They show an increased performance in firms backed by more skilled VCs. Kaplan and Schoar (2005) document how VC funds outperform

[1]Our statistics are from The British Private Equity & Venture Capital Association (BVCA) whose members represent approximately 70% of U.K. private equity firms. Details on the year 2021 statistics are available at: https://caphttps://www.bvca.co.uk/Portals/0/Documents/Research/2022%20Reports/Growing-Great-British-Businesses-2022.pdf
Details on the year 2000 statistics are from Mason and Harrison (2002) and is available at: https://www.jstor.org/stable/3804471

the S&P 500 and uncover significant persistence and heterogeneity in VC performance. After accounting for selection biases, risk difference, and industry differences, they conclude that heterogeneity in VC skills is the driver of their findings. We follow Kaplan and Schoar (2005) and Gompers, Kovner, and Lerner (2009) in showing that VC skills, specifically VC funding specialisation and prior success at exiting VC-backed firms, are significant determinants of the success of VC-backed firms. To the best of our knowledge, our study is the first to formally quantify the relative importance of various measures of VC skill and the funding deal structure for the success of VC-backed firms. The outline of this study is as follows. Section 2 describes the data collection process, discusses the variables constructed from the data, and presents descriptive statistics. Section 3 presents the main results where we discuss the relative importances of each VC skill and the funding deal structure for the successful exit of VC-backed firms. Section 4 concludes. A brief description of the variables constructed is in Appendix C.

3.2 Data and Descriptive Statistics: VC Skill and Deal Structure

We have two sets of data. Both are from the Refinitiv Workspace platform and cover the periods 2002 - 2022 and 2005 - 2022 respectively. The first dataset (2002 - 2022) contains 14,044 observations and covers all U.K. VC investment in U.K. firms. The second dataset (2005 - 2022) contains 3,017 observations and covers all exits by VC-backed U.K. firms. Next, we merged both dataset by matching each exit observation in the second dataset with its corresponding U.K. VC investment observation in the first dataset. This allowed us to match pertinent investment details with the eventual exit details. The merge yielded a dataset of 1,103 observations. Finally, and from the FAME database, we collected financial data on these exited U.K. firms that received VC backing from U.K. VCs. In the following enumeration, we summarise the Refinitiv Workspace data collection process and illustrate the merge that yielded the final sample:

1. To obtain the first dataset (2002 - 2022) that contains 14,044 observations and covers all U.K. VC investment in U.K. firms, go to the Refinitiv Workspace Platform, select “Search Tools” and on the new screen, select “VC Universe” from the left-hand side menu. From the “Include Focus” filter, also on the left-hand side menu, select “Investments”.

2. To obtain the second dataset (2005 - 2022) that contains 3,017 observations and covers all exits by VC-backed U.K. firms, go to the Refinitiv Workspace Platform, select

"Search Tools" and on the new screen, select "VC Universe" from the left-hand side menu. From the "Include Focus" filter, also on the left-hand side menu, select "Exits".

3. To illustrate the merge for both dataset and for simplicity, assume the second dataset (exits) contains three observations for firms A, B, and C. Also assume that the first dataset (investments) contains four observations for firms A, B, D, and E.

4. We merge both dataset to obtain a merge of all exits by U.K. firms that received backing from U.K. VCs. This merge yields a final sample of 2 exits for firms A and B. Why?

5. Firms A and B are U.K. firms that were backed by U.K. VCs who have now exited the investment.

6. Firm C is not in the merged dataset because although it is a VC-backed U.K. firm that has exited, it was backed by non-U.K. VCs.

7. Firms D and E are also not in the merged dataset because although they were backed by U.K. VCs, they had not exited as of 2022, and as such do not have exit observations.

3.2.1 Data on VCs and Exited VC-Backed Firms

From our first Refinitiv Workspace dataset on investments by U.K. VCs in U.K. firms (2002-2022), we extracted details on U.K. VC investment in the equity of U.K. firms. We will henceforth refer to a U.K. VC as VC. The data is on a VC by VC deal basis and includes pertinent details on each VC investment such as: firm name, economic sector, the date of investment, the investment stage (MBO, Expansion, Private Investment in Public Equity (PIPE), Seed, Acquisition for Expansion etc.),[2] the current VC investor's name, the amount invested by the VC, the date at which the VC investor made its first ever investment (which we use to calculate the age of the VC investor at the current investment date), and the names and amounts invested by other current and historic investors in the VC-backed firm. From the second Refinitiv Workspace dataset (2005-2022), we extracted details on exits by VC-backed U.K. firms. The majority of exits are achieved via mergers, and the data is on a VC-backed firm by VC-backed firm basis. For each exit by a VC-backed firm, our dataset contains pertinent details on the exit deal itself and the exiting firm such as: the firm's name and current operating stage, the date of the exit, the firm's current and historic investors, the GBP value of the exit and the acquiror's name, the period between the exiting firm's first investment date to the exit date (holding period), the number of funding rounds the firm

[2]Please note that there is now a blurry distinction between the investment stage at which VC firms vs. Private Equity firms invest in a firm, as noted in Iliev and Lowry (2020).

underwent, the round number at which it exited, and the total funding it received. In total, we have an exit dataset with 683 VC-backers of 3,017 exits by VC-backed U.K. firms.
We merged both dataset to obtains details on the eventual exit of U.K firms that received funding from U.K. VCs. The merge yielded a final dataset with 162 U.K. VC-backers of 1,103 exits by VC-backed U.K. firms, spanning the period 2005-2022. We thereafter extracted financial details on these exited VC-backed U.K. firms from the FAME database. The majority of these firms are young, private, SMEs, and due to their relatively permissive reporting rules - relative to public or large firms, the coverage of their financial data on the FAME database is sparse. As such, we excluded numerous financial statement line items such as CAPX, Gross Profit, Net Income, Cash Flows, Tangible and Intangible Assets etc. because they were missing entries, and ended up with four financial statement line items with no missing entries: Short-Term Liabilities, Long-Term Liabilities, Current Assets, and Total-Assets.

3.2.2 Descriptive Statistics: VCs and VC-Backed Firms

In Table 3.1., we present descriptive statistics for the 1,103 exits by VC-backed firms, including descriptive statistics on the VC-backed firms themselves, and their VC-backers. Financial variables are measured in the calendar year of the exit. For each VC-backed firm, independent variables that proxy for its VC backer's skills are measured in the calendar year preceding the first-time said VC backed the firm. We take this measurement approach because it is less susceptible to the selection bias issue detailed in the introduction. Indeed, measuring a VC's skill at alternate times such as the second or last time it financed a firm means we would be measuring the VC's skill after it has had the opportunity to observe whether the firm has realised some intermediate success, and if we conjecture that VCs would only multiple-fund firms with some intermediate success, and these firms would in turn want to be multiple-funded by the most skilled VCs because of their value added services which positively impacts firm value, then we end up with a deepening of the selection bias issue detailed in the introduction, whereby the most skilled VCs fund firms with the most potential for future successful exits. By measuring a VC's skill prior to the first time it financed a firm, we capture the VC's skill prior to the observed success of the firm in question. This means, we are better able to mitigate against the selection bias issue and capture the impact of the VC's skill on the eventual success (as measured by the exit IRR) of the firm. This approach has also been employed by studies such as Sørensen (2007) and Gompers, Gornall, Kaplan and Strebulaev (2020), especially the latter study, wherein they emphasise VC pre-investment skills as the most important value added skill VCs bring to bear on the firms they finance. We define a successful exit as an exit in which a VC realised an Internal Rate of Return (IRR) greater than or equal to 20%, which implies than an unsuccessful exit is one in which the VC realised an IRR less than 20%. This 20% value is inspired by the recent survey-based study of Gompers, Gornall, Kaplan, and Strebulaev (2020) wherein they show that the median net IRR that VCs market to Limited Partner's as target net IRR is 20%.[3] As detailed in Panel B of Table 3.1., exits include write-offs, IPOs, and everything in between.

In column (1) of Panel A, we report means for the full sample with 1,103 observation. In column (2) of Panel A, we report means for the subset of 128 successful exits by VC-backed firms and in column (3) of Panel A, we report means for the subset of 975 unsuccessful exits by VC-backed firms. In the final column of Panel A, statistical significance of the differences between subset means at the 1%, 5%, and 10% levels are represented by ***, **, and * respectively. Firstly and from Panel A of Table 3.1., we note that the average IRR for all exits by VC-backed firms, within our sample period of 2005-2022, is -57%, whereas it is 53% and -71% for success and unsuccessful VC-backed firms respectively. Successful and unsuccessful VC-backed firms were funded by VCs with differing levels of specialisation at funding the FTSE-Industry of its firms (FTSE-Industry Experience/Total Experience (%) of all First-Time VC Backers), where the average VC backer of successful firms is less specialised, relative to the average VC backer of unsuccessful firms. Also, and relative to unsuccessful VC-backed firms, successful VC-backed firms were funded by VCs with less experience at funding the FTSE-Industry of its firms (Log(FTSE-Industry Experience Count of all First-Time VC Backers)). Both variables (as with the other variables) draw from Iweze (2021) and as such, are constructed in the same vein: Log(FTSE-Industry Experience Count of all First-Time VC Backers) captures the sheer number of funding deals a VC has undertaken in the FTSE-Industry of its VC-backed firms, and for each VC-backed firm, the funding experience of its VC backer is measured in the year prior to the first time the VC financed the firm. FTSE-Industry Experience/Total Experience (%) of all First-Time VC Backers is a ratio that captures a VC's specialisation at funding the FTSE-Industry of its VC-backed firms. The numerator of the ratio is given by the Log(FTSE-Industry Experience Count of all First-Time VC Backers) measure, with the denominator given by the combined number of funding deals a VC has undertaken in all FTSE-Industries. For each VC-backed firm, the funding specialisation of its VC-backer, as with the other measures of VC skill, is measured in the year prior to the first time the VC financed the firm.

Turning now to VC performance as a measure of skill, we closely follow Iliev and Lowry (2020) in the construction of this measure. The annual performance of a VC is a ratio comprised of the number of investments it exited in the prior 3 years (years -3 to -1), divided

[3] In a later section and with another empirical specification, we will employ the continuous version of the IRR to allay the concern that our results are driven by our choice of 20% as the threshold for success.

by the number of investments it made in the 3 years prior to the prior 3 years (years -6 to -4). Thereafter, we create an annual ranking of VCs by discretising their annual performance into quartiles. From these rankings, we create two binary variables: # First-Time VCs with Low Prior Performance and # First-Time VCs with High Prior Performance. VCs with prior performance in the lowest quartile are low prior performing VCs (corresponding to 1 in the binary variable) whilst those with performance in the highest quartile are high prior performing VCs (corresponding to 1 in the binary variable). Finally, for each VC-backed firm, we tally up the number of low prior performing and high prior performing VCs that it received funding from. We see from Table 3.1., that successful VC-backed firms were backed by fewer low prior performing VCs and by more high prior performing VCs, relative to unsuccessful VC-backed firms. Our final measure of VC skill is a binary variable: # First-Time VCs that are Young, corresponding to 1 if a VC is young or 0 otherwise. This measure draws from the seminal Gompers (1996) study. For a VC-backed firm, its VC backer is young if in the calendar year of the backing, the VC was 15 years or younger. The number 15 is inspired by the average age of the VCs in our sample.[4] For each VC-backed firm, we calculate the age of its VC-backer by taking the difference between the calendar year the VC made its first ever investment and the calendar year it funded the VC-backed firm in question for the first time. We see in Table 3.1., that relative to successful VC-backed firms, unsuccessful VC-backed firms received significantly more funding from young VCs. Thus far, we have seen a clear difference between the skills of the VC backers of successful vs. unsuccessful VC-backed firms, with all the differences being significant at the 1% level. Consistent with Iliev and Lowry (2020), successful VC-backed firms were backed by more high prior performing and fewer low prior performing VCs, relative to unsuccessful VC-backed firms, However, and inconsistent with the predictions of Kaplan and Schoar (2005) and Sørensen (2007), the VC backers of successful VC-backed firms were less specialised and had less experience at funding the FTSE-Industry of its firms, relative to the VC backers of unsuccessful VC-backed firms. We will soon employ multivariate approaches to test the strength of these differences, but for now, we turn to the deal structure variables in Table 3.1. Here, we also see a stark difference between the VC backers of successful vs. unsuccessful VC-backed firms. Successful VC-backed firms received significantly more VC funding (Log(Total VC Funding) and underwent significantly fewer funding rounds (# Funding Rounds), relative to unsuccessful VC-backed firms. All of this suggests that VCs invest more money but intervene less, as firms realise more success. This observation is all the more important considering we know from Sahlman (1990) and Ewens et al. (2018) that staggered capital injection is a potent control mechanism employed by VCs. More GBP

[4]Our results are robust to redefining the young VC age threshold as 10, 12, 17 and 20 years old respectively.

funding over fewer funding rounds suggest a "loosening of the leash". We also see from Table 3.1., that successful VC-backed firms are held for significantly shorter periods (Holding Period (Years)). This statistic is consistent with what we know from Iweze (2021), wherein the study shows that successful VCT-backed firms are held for a shorter duration relative to unsuccessful VCT-backed firms. Additionally, the holding period of 4.55 years for successful VC-backed firms is consistent with anecdotal evidence - from the BVCA and British Business Bank - of VCs holding their VC-backed firms for between five to seven years.[5]

With regards the financial profile of VC-backed firms, we see from Table 3.1., that relative to unsuccessful VC-backed firms, successful VC-backed firms are bigger - as measured by Log(Total Assets) and have lower Debt-to-Assets and Cash-to-Assets. Turning now to Panel B in Table 3.1., we see that the top two types of exit (Exit Type) across both successful and unsuccessful VC-backed firms is in keeping with the well known fact that most VC liquidity events are achieved via acquisitions. However, and as expected, we note that the rate at which successful VC-backed firms exit via IPO is almost thrice that of unsuccessful firms. Note that the reason unsuccessful VC-backed firms have IPO exits (Panel B: 4.5% of all exits by unsuccessful VC-backed firms) is because we define an unsuccessful exit as one in which the VC realised an IRR of less than 20%. We also see for both successful and unsuccessful VC-backed firms, exits occur when the firms are either selling a product or providing a service or both. As expected, successful VC-backed firms have a profitability rate that is more than 8 times that of unsuccessful VC-backed firms.

In the next section, we adopt the machine learning techniques (Deep Neural Network Binary Classification model and Deep Neural Network Regression model) and attribution algorithms in Iweze (2021) to test the robustness of the differences between the funding deal structure and VC skills of successful vs. unsuccessful VC-backed firms. Our aim is to employ a selection-bias-alleviating approach to quantify the relative importance of VC skills and the funding deal structure for the success of VC-backed firms.

[5] This statistic is available at: https://www.bvca.co.uk/Our-Industry/Venture-Capital

Table 3.1. **Descriptive Statistics**

Our sample consists of 1,103 exits by VC-backed firms between 2005 and 2022, where a VC-backed firm is defined as a U.K. firm that received funding from a U.K. VC or U.K. VCs within the 2002-2022 period. We require all firms to have FAME financial data. For each VC-backed firm, its VC backer's skills (independent variables) are measured in the calendar year preceding the first time said VC backed the firm, and financial variables are calculated at the calendar year end of each VC-backed firm's exit. In Column (1) of Panel A, we report means for the 1,103 exits by VC-backed firms in our sample. In Column (2) of Panel A, we report means for the subset of 128 successful exits by VC-backed firms and in column (3) of Panel A, we report means for the subset of 975 unsuccessful exits by VC-backed firms. In the final column of Panel A, statistical significance of the differences between sub-sample means at the 1%, 5%, and 10% levels are represented by ***, **, and * respectively. In Column (1) of Panel B, we report proportions for the 1,103 exits by VC-backed firms in our sample. In Column (2) of Panel B, we report proportions for the subset of 128 successful exits by VC-backed firms and in column (3) of Panel B, we report proportions for the subset of 975 unsuccessful exits by VC-backed firms. All variables are described in Appendix C.

Panel A: Descriptive Statistics for Exited VC-Backed Firms

	VC-Backed Firms	(128) Successful VC-Backed Firms	(975) Unsuccessful VC-Backed Firms	Mean-Difference
IRR (in percentages)	-57.00	53.00	-71.00	1.238***
VC Skill Measures				
FTSE-Industry Experience/Total Experience (%) of all First-Time VC Backers	0.37	0.31	0.38	-0.065***
Log(FTSE-Industry Experience Count of all First-Time VC Backers)	4.15	3.53	4.24	-0.703***
# First-Time VCs with Low Prior Performance	2.18	1.27	2.30	-1.022***
# First-Time VCs with High Prior Performance	0.04	0.09	0.03	0.056***
# First-Time VCs that are Young	2.28	1.26	2.42	-1.159***
Deal Structure				
Log(Total VC Funding)	6.94	19.04	5.35	13.689***
# Funding Rounds	5.59	3.45	5.87	-2.421***
Holding Period (Years)	7.33	4.55	7.70	-3.149***
Control Variables				
VC-backed Firm Age	12.11	11.98	12.13	-0.154
Log(Total Assets)	11.51	12.58	11.16	1.423
Debt-to-Assets	0.61	0.48	0.63	-0.147
Cash-to-Assets	0.78	0.69	0.80	-0.111***

(Continued)

Table 3.1. *Continued*

Panel B: Descriptive Statistics for Exited VC-Backed Firms

	Percentage of VC-Backed Firms	Percentage of Successful VC-Backed Firms	Percentage of Unsuccessful VC-Backed Firms
Exit Type			
Merger	0.750	0.731	0.742
Secondary Sales	0.160	0.115	0.188
IPO	0.060	0.115	0.045
Write-Off	0.015	-	0.015
Reverse Takeover	0.009	0.038	0.003
Buyback	0.008	-	0.008
Current Operating Stage of Exited VC-Backed Firms			
Shipping Product or Providing Services	0.940	0.883	0.957
Clinical Trials	0.030	0.026	0.025
Profitable	0.015	0.065	0.008
Development	0.013	0.026	0.010

3.3 Main Result

3.3.1 Factors that Determine the Success of VC-Backed Firms: Binary Classification Model

Following on from the Iweze (2021) study on VCTs and whether their skills and the way they structure funding deals or luck determines the success of VCT-backed firms, we ask the same question but for VCs: Do VC skills and the funding deal structure or luck determine the success of VC-backed firms? Success as embodied in the IRR upon exit by VC-backed firms. We answer this question by employing several machine learning algorithms. In this section, we use the Deep Neural Network Binary Classification (DNNBC) model in Iweze (2021) to quantify the relative impact of VC skills and the funding deal structure on the successful exit of a VC-backed firm. This DNNBC model has 2 hidden layers, the first with 12 hidden units (corresponding to the number of independent variables) and the second with 8 hidden units (the number of hidden units is a fine-tuned hyper-parameter), each with Sigmoid non-linearity. The output layer performs a softmax operation and has 2 units, corresponding to the outcome of either successful VC-funded firm (1) or unsuccessful VC-funded firm (0). We trained the DNNBC on our Refinitiv Workspace and FAME data to carry out a binary classification task, where the dependent variable is equal to 1 if the VC-backed firm exited successfully (realised IRR $\geq 20\%$) and 0 otherwise.[6] Thereafter, we employed various attribution algorithms to interpret the results from our DNNBC model. An attribution algorithm outputs an attribution score, which can be interpreted as a percentage, and helps assess the contribution of each independent variable to the output (classification) of a Deep Neural Network model. Our main result reports the average attribution scores, which can be interpreted as percentages, in Table 3.2., where they are ranked in descending order. The average attribution scores in Columns 1, 2, and 3 are from the *Integrated Gradients*, *Integrated Gradients with SmoothGrad*, and *DeepLift* attribution algorithms respectively.
As presented earlier in Table 3.1., our independent variables include: measures of VC skill, funding deal structure, and several control variables. For each exit by a VC-backed firm, we measure the skill of its VC backer or backers in the calendar year preceding the first-time said VC backed the firm. The ensuing analysis will focus on the average attribution scores from the *Integrated Gradient* attribution algorithm (column 1), but the sign on the attribution scores are consistent across the remainder attribution algorithms (columns 2 and 3). The most important VC skill for determining the success of VC-backed firms is # First-Time VCs that are Young, which as earlier discussed, is motivated by the seminal Gompers (1996)

[6] In the next section, we will test the robustness of this 20% threshold by employing the continuous IRR as the dependent variable in our analysis.

study, wherein they show how young VCs exit their investments - relatively earlier, relative to older VCs - in a bid to signal reputation to capital markets and thus successfully fundraise in the future. An important implication of their distorted incentive is that these exits (IPOs) are more under-priced relative to exits (IPOs) by older VCs. In line with the Gompers (1996) finding, we find, as depicted in Table 3.2., that being backed by young VCs is a negative determinant of the success of VC-backed firms, contributing an average of -17% to the success of VC-backed firms.

The second most important determinant of the success of VC-backed firms is # First-Time VCs with Low Prior Performance, which is a significant negative determinant of the success of VC-backed firms, contributing an average of -14% (*Integrated Gradient*) to the success of VC-backed firms. Conversely, the fourth most important determinant of the success of VC-backed firms is # First-Time VCs with High Prior Performance , which is a positive determinant of the success of VC-backed firms, contributing an average of 1% to the success of VC-backed firms. This finding of # First-Time VCs with Low Prior Performance as a significant negative determinant and # First-Time VCs with High Prior Performance as a positive determinant of the success of VC-backed firms, is consistent with numerous financial economics studies such as Jensen and Meckling (1976), Brown, Harlow, and Starks (1996), Carpenter (2000), Iliev and Lowry (2020). In constructing both measures of VC skill, we relied on the insights in Carpenter (2000), wherein they study the optimal dynamic investment policy of a risk averse fund manager with a convex compensation structure. When the fund's value is high, the manager reduces portfolio risk and when the fund's value is low, the convex nature of the manager's compensation contract incentivises her to engage in excessive risk taking. In line with Carpenter (2000), we conjecture that the convex compensation structure (fixed management fee plus carried interest) of VC investment managers incentivises the managers of VCs with prior performance in the lowest quartile of performance for all VCs, to take excessive risk in their investment decisions, and conversely, the managers of VCs with prior performance in the highest quartile of performance for all VCs, to take less risk in their investment decisions, so as to protect their carried interest. This means that on average, VCs with low prior performance will back unsuccessful VC-backed firms, and VCs with high prior performance will back successful VC-backed firms. This result is also in line with the extant corporate finance literature such as Jensen and Meckling (1976), wherein they show that levered equity holders will prefer higher levels of asset volatility regardless of its impact on firm value. Additionally, our finding reinforces the findings in Brown, Harlow, and Starks (1996), wherein mutual fund managers engage in risk shifting, whereby the increase asset volatility when their fund is under performing.

Our fifth and least important of the VC skill measures is VC funding specialisation (FTSE-Industry Experience/Total Experience (%) of all First-Time VC Backers), where we see in Table 3.2., that VC funding specialisation is a positive determinant of VC-backed firm success, and is consistent with the findings in Gompers, Kovner, and Lerner (2009), wherein they show that VCs with the most specialisation in funding an industry are better able to allocate funding to firms within said industry, and achieve higher fund performance, relative to generalist VCs (VCs with less specialisation). Iweze (2021) also show that VCT FTSE-Industry specialisation is a positive determinant of the success of VCT-backed firms. The third most important of the VC skill measures is VC funding experience (Log(FTSE-Industry Experience Count of all First-Time VC Backers)), which is a positive determinant, contributing 6% to the success of VC-backed firms. This finding of VC experience being an important determinant of the success of VC-backed firms is in line with the findings in Sørensen (2007), where they show with the aid of a two sided matching model, that the most experienced VCs achieve higher success, as measured by the fraction of their firms that exit via IPO. This finding is driven by two distinct channels, the first is deal flow sorting, wherein the most experienced VCs enjoy access to a proprietary deal flow, and the second is the value added services they bring to bear on their firms, with deal flow sorting almost twice as important as the value added services for explaining the success of experienced VCs. Turning to our measures of deal structure, we see from Table 3.2., that these variables offer up a higher set of average attribution scores. Starting with Log(Total VC Funding), we see it is a significant positive determinant of VC-backed firm success, contributing an average of 54% to the success of VC-backed firms. VCs invest more money in successful VC-backed firms, or more specifically, they invest more money as a VC-backed firm realises more success. The # Funding Rounds and Holding Period (Years) are also significant negative determinants of the success of VC-backed firms, which is consistent with VCs "loosening the leash" the more VC-backed firms realise success, and in the case of the Holding Period (Years), VCs cash in on their successes quicker than they realise losses, a result that is also consistent with the findings in Iweze (2021).

In further consideration of the Sørensen (2007) selection bias (deal flow sorting) issue detailed in the introduction, we conduct the following empirical test: we restrict our sample to the subset of VC-backed firms backed by VCs outside the top quartile of VC funding experience (Log(FTSE-Industry Experience Count of all First-Time VC Backers)). Sørensen (2007) details how deal flow sorting creates a selection bias issue, whereby the most experienced VCs have access to proprietary deal flow and can thus choose firms with the best potential. By excluding the most experienced VCs from the analysis, we allay the Sørensen (2007) finding. Our findings, depicted in Table 3.3., are somewhat consistent with the findings in Table 3.2. However, # First-Time VCs with Low Prior Performance is now a positive determinant of the success of VC-backed firms. Additionally, VC funding specialisation is now a more important determinant of the success of VC-backed firms, relative to VC funding experience. In summary, when we exclude the most experienced VCs from the analysis, we find that heterogeneity in prior performance (# First-Time VCs with Low Prior Performance and # First-Time VCs with High Prior Performance) does not explain heterogeneity in the success of VC-backed firms. We also see in Table 3.3., that the magnitude of the average attribution scores are lower, relative to the average attribution scores in Table 3.2.

Thus far, our results are consistent with VC skill and deal structure as determinants of the success of VC-backed firms, where we find in this section that young VC is the most important VC skill determinant of the success of VC-backed firms, contributing an average of -17% to the success of VC-backed firms. One concern with the analysis thus far is our measurement of success (dependent variable) as an exit IRR $\geq 20\%$. To allay this concern, in the next section and with the aid of a Deep Neural Network Regression model, we employ the continuous form of the realised IRR as our dependent variable.

Table 3.2. **Comparing VC Skill and Deal Structure Importances Across Multiple Attribution Algorithms: Firm Level Deep Neural Network: Binary Classification Model: Binary Dependent Variable is the realised IRR.**

Our sample consists of 1,103 exits by VC-backed firms between 2005 and 2022 as defined in Table 3.1. We employ various attribution algorithms to interpret the results from our Deep Neural Network Binary Classification model, where the binary dependent variable is equal to 1 if the exit by the VC-backed firm was successful. In column 1, we report the mean attribution score from the *Integrated Gradients* algorithm. In columns 2 and 3, we report the mean attributions scores from the *Integrated Gradients with SmoothGrad* and *DeepLift* algorithms respectively. Control variables are also included with all variables defined in Appendix C.

	Integrated Gradients	Integrated Gradients w/SmoothGrad	DeepLift
VC Skill Measures			
# First-Time VCs that are Young	-0.17	-0.19	-0.21
# First-Time VCs with Low Prior Performance	-0.14	-0.16	-0.20
Log(FTSE-Industry Experience Count of all First-Time VC Backers)	0.06	0.07	0.06
# First-Time VCs with High Prior Performance	0.01	0.00	0.01
FTSE-Industry Experience/Total Experience (%) of all First-Time VC Backers	0.00	0.03	0.00
Deal Structure			
Holding Period (Years)	-0.70	-0.70	-0.81
Log(Total VC Funding)	0.54	0.58	0.53
# Funding Rounds	-0.37	-0.42	-0.54
Control Variables			
VC-backed Firm Age	0.01	0.00	-0.04
Log(Total Assets)	0.27	0.21	0.15
Cash-to-Assets	-0.14	-0.17	-0.19
Debt-to-Assets	-0.07	-0.12	-0.10
Training Accuracy	0.96	0.96	0.96
Test Accuracy	0.91	0.91	0.91

Table 3.3. **Does VC Skill and Deal Structure Still Determine the Success of VC-Backed Firms When We Exclude Experienced VCs from the Analysis? Comparing VC Skill and Deal Structure Importances Across Multiple Attribution Algorithms: Binary Classification Model: Binary Dependent Variable is the realised IRR**

Our sample consists of 828 VC-backed firms between 2005 and 2022 as defined in Table 3.1, but excludes the data for VCs in the top quartile of experience at funding firms in any FTSE-Industry. We employ various attribution algorithms to interpret the results from our Deep Neural Network Binary Classification model, where the binary dependent variable is equal to 1 if the VC-backed firm is successful. In column 1, we report the average attribution score from the *Integrated Gradients* algorithm. In columns 2 and 3, we report the average attributions scores from the *Integrated Gradients with SmoothGrad* and *DeepLift* algorithms respectively. Control variables are also included with all variables defined in Appendix C.

	Integrated Gradients	Integrated Gradients w/SmoothGrad	DeepLift
VC Skill Measures			
# First-Time VCs with Low Prior Performance	0.03	0.03	0.03
FTSE-Industry Experience/Total Experience of all First-Time VC Backers (%)	0.01	0.01	0.01
# First-Time VCs that are Young	-0.01	-0.01	-0.01
# First-Time VCs with High Prior Performance	0.01	0.00	0.01
Log(FTSE-Industry Experience Count of all First-Time VC Backers)	0.00	0.00	0.00
Deal Structure			
Log(Total VC Funding)	0.60	0.60	0.60
# Funding Rounds	-0.16	-0.18	-0.19
Holding Period (Years)	-0.06	-0.07	-0.12
Control Variables			
VC-backed Firm Age	-0.02	-0.02	-0.09
Log(Total Assets)	-0.03	-0.04	-0.03
Cash-to-Assets	-0.03	-0.03	-0.03
Debt-to-Assets	-0.00	-0.01	-0.00
Training Accuracy	0.92	0.92	0.92
Test Accuracy	0.91	0.91	0.91

3.3.2 Factors that Determine the Success of VC-Backed Firms: Regression Model

We continue our analysis by employing another machine learning algorithm from Iweze (2021) to test the robustness of our results from the Deep Neural Network Binary Classification model, presented in Table 3.2. This algorithm is also a Deep Neural Network trained on our Refinitiv Workspace and FAME data, but this time to carry out a regression task - wherein our dependent variable is still the realised IRR, but it is now a continuous as opposed to binary variable. This Deep Neural Network Regression (DNNR) model is a four layer neural network with the non-linear component of each layer containing rectified linear activation function (ReLU). We interpret the results of our DNNR model with the three different attribution algorithms (*Integrated Gradients*, *Integrated Gradients with SmoothGrad* and *DeepLift*) employed in the previous section.
Our results (mean attribution scores) are depicted in Table 3.4., where we focus on the mean attribution scores from the *Integrated Gradients* attribution algorithm in the first column. VC specialisation (FTSE-Industry Experience/Total Experience (%) of all First-Time VC Backers) is a positive and the most important VC skill determinant of the success of VC-backed firms, which is consistent with the results in Gompers, Kovner, and Lerner (2009), and Racculia (2014), wherein the former study shows a positive relationship between VC specialisation and VC success, and the latter study shows that VC specialisation improves IPO performance. VC funding experience (Log(FTSE-Industry Experience Count of all First-Time VC Backers)) is also a positive determinant of the success of VC-backed firms, wherein the sign on the mean attribution score is consistent with Kaplan and Schoar (2005), where they control for selection bias and industry differences, and still find that VCs outperform the S&P 500 on a capital weighted basis. Nonetheless, we should point out that VC funding experience is the least important VC skill determinant of the success of VC-backed firms, contributing an average of 1% to the success of VC-backed firms. # First-Time VCs that are Young is the third most important VC skill determinant of the success of VC-backed firms, contributing an average of -8% to the success of VC-backed firms. This finding reinforces the findings in the seminal Gompers (1996) study wherein they show that young VCs tend to bring their firms public (IPO) earlier than older VCs, in a bid to signal their capabilities to potential investors, establish a reputation and successfully fundraise in the future. This phenomena which Gompers (1996) calls "Grandstanding", results in greater IPO underpricing for firms taken public by younger as opposed to older VCs. VCs with low prior performance (# First-Time VCs with Low Prior Performance) is one of the least important VC skill for determining the success of VC-backed firms while VCs with high prior performance (# First-Time VCs with High Prior Performance) is the second most important VC skill for

determining the success of VC-backed firms. We see in Table 3.4., that the sign on their mean attribution scores is consistent with the findings in prior literature such as Carpenter (2000) who finds that VCs with high prior performance moderate their risk taking and VCs with low prior performance engage in excessive risk taking, which implies than on average, VCs with high prior performance will positively determine the success of VC-backed firms, and VCs with low prior performance will negatively determine the success of VC-backed firms. With the funding deal structure variables, Log (Total VC Funding) is the most important of the funding deal structure variables, contributing an average of 78% to the success of VC-backed firms, whereas # Funding Rounds is the second most important deal structure determinant of the success of VC-backed firms, contributing an average of -7% to the success of VC-backed firms. Holding Period (Years) is also an important determinant of the success of VC-backed firms, contributing an average of 3% to the success of VC-backed firms. The finding that the total VC funding is a positive determinant of the success of VC-backed firms is consistent with the findings in Gompers (1995), wherein they show that firms that yield the highest returns for VCs, received the most VC funding. Also, the finding that the # Funding Rounds is a negative determinant of the success of VC-backed firms, is consistent with Sahlman (1990), wherein they detail how staged capital commitment is a control mechanism employed by VCs to ensure they can abandon a project (VC-backed firm) if the project realises less intermediate success. In summary, we see in this Table 3.4., that the sign on the average contribution of VC skills and the funding deal structure to the success of VC-backed firms is consistent with the sign on the average contribution of VC skills and the funding deal structure to the success of VC-backed firms presented in Table 3.2.[7]

In Table 3.5., we present results from employing our DNNR model to repeat the selection-bias-alleviating empirical test detailed in the previous section. We see that the sign on the average contributions of each measure of VC skill is not fully consistent with the results from the analysis on the full sample, presented in Table 3.4. However, there is consistency in the contribution of VC funding specialisation to the success of VC-backed firms across both Table 3.4. and Table 3.5., where it is still one of the most important VC skill determinant of the success VC-backed firms.

Finally, the main theme of the results presented in this section is one of consistency - between the DNNR model in this section and the DNNBC model in the previous section - in the sign on the average contributions of the various measures of VC skill and deal structure to the success of VC-backed firms. With the DNNBC model, we highlighted the potential for measurement error in our binary dependent variable, given that we classed VC-backed firms as successful if the realised IRR from its exit was greater than or equal to 20%. This inspired

[7]Except for Holding Period (Years), which is a positive determinant of the success of VC-backed firms.

our use of the DNNR model, wherein the dependent variable was the continuous form of the realised IRR. Crucially, we find in this section, that VC specialisation is the most important VC skill determinant of the success of VC-backed firms, contributing an average of 14% to the success of VC-backed firms.

Table 3.4. **Comparing VC Skill and Deal Structure Importances Across Multiple Attribution Algorithms: Regression Model: Continuous Dependent Variable is the realised IRR**

Our sample consists of 1,103 exits by VC-backed firms between 2005 and 2022 as defined in Table 1. We employ various attribution algorithms to interpret the results from our second Deep Neural Network Regression Model, where our dependent variable is the realised IRR. In column 1, we report the mean attribution score from the *Integrated Gradients* algorithm. In columns 2 and 3, we report the mean attributions scores from the *Integrated Gradients w/SmoothGrad* and *DeepLift* algorithms respectively. Control variables are also included with all variables defined in Appendix C.

	Integrated Gradients	Integrated Gradients w/SmoothGrad	DeepLift
VC Skill Measures			
FTSE-Industry Experience/Total Experience of all First-Time VC Backers (%)	0.14	0.13	0.13
# First-Time VCs with High Prior Performance	0.13	0.12	0.10
# First-Time VCs that are Young	-0.08	-0.08	-0.08
# First-Time VCs with Low Prior Performance	-0.02	-0.02	-0.01
Log(FTSE-Industry Experience Count of all First-Time VC Backers)	0.01	0.00	0.01
Deal Structure			
Log(Total VC Funding)	0.78	0.76	0.92
# Funding Rounds	-0.07	-0.06	-0.10
Holding Period (Years)	0.03	0.02	-0.01
Control Variables			
VC-backed Firm Age	-0.02	-0.02	-0.14
Log(Total Assets)	-0.37	-0.37	-0.27
Cash-to-Assets	-0.08	-0.08	-0.09
Debt-to-Assets	-0.01	-0.02	-0.01
RMSE	0.14	0.14	0.14

Table 3.5. **Does VC Skill and Deal Structure Still Determine the Success of VC-Backed Firms When We Exclude Experienced VCs from the Analysis? Comparing VC Skill and Deal Structure Importances Across Multiple Attribution Algorithms: Regression Model: Continuous Dependent Variable is the realised IRR**

Our sample consists of 828 VC-backed firms between 2005 and 2022 as defined in Table 3.1, but excludes the data for VCs in the top quartile of experience at funding firms in any FTSE-Industry. We employ various attribution algorithms to interpret the results from our Deep Neural Network Binary Classification model, where the binary dependent variable is equal to 1 if the VC-backed firm is successful. In column 1, we report the average attribution score from the *Integrated Gradients* algorithm. In columns 2 and 3, we report the average attributions scores from the *Integrated Gradients with SmoothGrad* and *DeepLift* algorithms respectively. Control variables are also included with all variables defined in Appendix C.

	Integrated Gradients	Integrated Gradients w/SmoothGrad	DeepLift
VC Skill Measures			
# First-Time VCs with High Prior Performance	-0.08	-0.06	-0.07
FTSE-Industry Experience/Total Experience of all First-Time VC Backers (%)	0.06	0.05	0.04
# First-Time VCs that are Young	0.05	0.05	0.03
Log(FTSE-Industry Experience Count of all First-Time VC Backers)	0.01	0.01	0.01
# First-Time VCs with Low Prior Performance	0.01	0.01	0.01
Deal Structure			
Log(Total VC Funding)	0.69	0.71	0.79
# Funding Rounds	-0.06	-0.06	-0.07
Holding Period (Years)	0.01	-0.00	-0.03
Control Variables			
VC-backed Firm Age	-0.10	-0.10	-0.02
Log(Total Assets)	0.02	-0.01	-0.06
Cash-to-Assets	-0.07	-0.06	-0.08
Debt-to-Assets	-0.02	-0.02	-0.02
RMSE	0.19	0.19	0.19

3.3.3 Factors that Determine the Success of VC-Backed Firms: OLS Regression Model

In this section, and with the aid of an OLS model with fixed effects on FTSE-Industry and standard errors clustered by year of investment, we analyse the impact of VC skills and the funding deal structure on the success of VC-backed firms. Our aim is to further emphasise the superior performance of our Deep Neural Network approach. The results are presented in Table 3.6., where we see that the VC skills coefficients are insignificant at conventional levels, except for VC funding specialisation (FTSE-Industry Experience/Total Experience of all First-Time VC Backers (%)), whose coefficient is significant but negative. The negative sign on the coefficient is at odds with the findings from the DNNBC and DNNR models presented in previous sections, where we showed that VC funding specialisation is a positive determinant of the success of VC-backed firms. This finding of a negative relationship between VC funding specialisation and VC-backed firm success is also inconsistent with studies such as Gompers, Kovner, and Lerner (2009) and Racculia (2014), wherein both studies find that VC funding specialisation positively determines the success of VC-backed firms. Conversely, we note that the coefficient on each deal structure variable (# Funding Rounds, Log(Total VC Funding), and Holding Period (Years)) are significant at conventional levels. Indeed, the sign on the coefficients are also consistent with the sign on the mean attribution scores from the DNNBC and DNNR models presented in previous sections.

We now turn to explaining why the independent variables that proxy for VC skill are predominantly insignificant whilst the funding deal structure independent variables are significant. Recall, we constructed (as detailed in previous sections) the independent variables that proxy for VC skill and as such we would expect a non-linear model (DNNBC and DNNR models) to be better at capturing the relationships among these constructed independent variables (VC skill) and the dependent variable, relative to a linear model. Whereas, the funding deal structure variables are based on direct observations i.e. Log(Total VC Funding) is the logarithm of the total VC funding a VC-backed firm received, and as such, we would expect a linear model to effectively capture its relationship with the dependent variable. In summary, the insignificance of these OLS results emphasises that the OLS model cannot capture non-linear relationships, whereas our DNNBC and DNNR models are built to capture non-linear relationships, and as such, we see that the magnitude and the sign on the mean attribution scores for VC skill, in both DNNBC and DNNR models, are consistent with the earlier detailed empirical results.

Table 3.6. **OLS Model with Fixed Effects on FTSE-Industry and Standard Errors Clustered by Year of First Investment: Continuous Dependent Variable is the realised IRR**
Our sample consists of 1,103 exits by VC-backed firms between 2005 and 2022 as defined in Table 3.1., where the continuous dependent variable is the realised IRR, which proxies for the success of VC-backed firms. The table contains results from an OLS model with Fixed Effects on FTSE-Industry and standard errors clustered by year of investment. In the Column, we report coefficients for the OLS model. t-statistics are shown in parentheses. Statistical significance of the coefficients at the 1%, 5%, and 10% levels are represented by ***, **, and * respectively. All variables are described in Appendix C.

	(1,103) VC-Backed Firms
VC Skill Measures	
FTSE-Industry Experience/Total Experience of all First-Time VC Backers (%)	-0.028***
	(-3.542)
Log(FTSE-Industry Experience Count of all First-Time VC Backers)	0.000
	(1.064)
# First-Time VCs with Low Prior Performance	0.001
	(0.693)
# First-Time VCs with High Prior Performance	-0.003
	(-0.317)
# First-Time VCs that are Young	-0.001
	(-0.245)
Deal Structure	
# Funding Rounds	-0.006***
	(-7.993)
Log(Total VC Funding)	0.054***
	(8.477)
Holding Period (Years)	0.003***
	(4.441)
Control Variables	
VC-backed Firm Age	0.000
	(0.067)
Log(Total Assets)	-0.003***
	(-3.308)
Debt-to-Assets	0.002
	(1.381)
Cash-to-Assets	0.033***
	(3.685)
Adjusted R^2	0.98

3.4 Conclusion

In this study, we have analysed VCs and whether their skills and the way they structure financing deals determine the success of the firms they finance. This study provides a backdrop for the Iweze (2021) study on VCTs. It analyses the larger VC asset class within which the VCT scheme operates in the U.K. We started by noting the axiomatically acknowledged importance of VC added services for the success of VC-backed firms, which necessitates an understanding of VC skills and their impact on the success of VC-backed firms. Additionally, we acknowledged the potential selection bias issues prevalent in corporate finance studies such as ours. Specifically, we relied upon the potential selection bias issue detailed in Sørensen (2007), which would cause the most skilled VCs to match with high-potential entrepreneurial firms, and as such, selection bias and VC skills would concurrently determine the success of VC-backed firms. This potential selection bias issue inspired our use of an approach that allows us to sidestep, in a data driven manner, the potential selection bias issue. We employed a Deep Neural Network Binary Classification model, a Deep Neural Network Regression model, and several attribution algorithms to uncover the relative contribution of each measure of VC skill and the funding deal structure to the success of VC-backed firms. We found that VC skills and the funding deal structure are important determinants of the success of VC-backed firms. Based on the mean attribution scores from the Deep Neural Network Regression (DNNR) model, we found that VC funding specialisation (FTSE-Industry Experience/Total Experience of all First-Time VC Backers (%)) is the most important VC skill for determining the success of VC-backed firms, contributing an average of 14% to the success of VC-backed firms. This finding is consistent with the findings of Gompers, Kovner, and Lerner (2009), wherein they find that VC funding specialisation positively determines the success of VC-backed firms because specialist VCs are better able to efficiently allocate capital within the industry they specialise in, whereas generalist VCs realise less success because of inefficient allocation of capital within industry and across industries. Additionally, the total GBP amount invested by VCs in VC-backed firms, is the most important funding deal structure measure and a positive determinant of the success of VC-backed firms. The number of funding rounds is also a significant negative determinant of the success of VC-backed firms. In other words, ex-post successful VC-backed firms received more VC financing and underwent fewer funding rounds, relative to ex-post unsuccessful VC-backed firms. These findings are consistent with VCs employing the staging of capital infusion as a potent control mechanism, as evinced in Sahlman (1990) and Gompers (1995). Indeed, the latter study shows that the total VC funding and the number of funding rounds are all metrics for the intensity with which VCs monitor VC-backed firms. VCs intensify their monitoring of VC-backed firms as expected agency costs increases (as asset tangibility decreases), and we know from Campello (2007) that asset tangibility is positively related to firm performance, for externally financed firms. This implies that agency costs are relatively higher for unsuccessful VC-backed firms, and VCs intensely monitor ex-post unsuccessful VC-backed firms.

Finally, disaggregating VC firm-level data to fund-level data and conducting the analysis at this level is of independent interest. Consider a parent VC firm that manages both a VC fund and a VCT fund. Analysing the relative impact of its VC vs. VCT skills on the success of the firms it finances, would add another dimension to our understanding of the added services VCs bring to bear on VC-backed firms (as embodied in VC skills) and how it determines the success of VC-backed firms.

Conclusions

This thesis employs several machine learning approaches to analyse: the VCT scheme and its implications for the growth of VCT-backed firms in the U.K., VCTs and their impact on the success of the the firms they finance, U.K. VCs - the wider framework within which VCTs operate - and their impact on the success of the the firms they finance.
Chapter one focuses on adapting a Matrix Completion algorithm to estimate the causal effect of the VCT scheme on the investment of VCT-backed firms in the U.K. I find that between 2003-2018, the causal effect of the VCT scheme on the investment of VCT-backed firms was 41%. I also document the impact of changes to the VCT rules and regulation on contemporaneous VCT fundraising and the contemporaneous aggregate investment patterns of VCT-backed firms. By reading through every VCT annual report between 2003-2018, I can link changes to the VCT rules and regulations to changing VCT fundraising and aggregate investment patterns of VCT-backed firms. This exercise is particularly useful as it provides regulators with an understanding of the immediate impact of changes to the VCT scheme's rules and regulations on the aggregate investment patterns of VCT-backed firms.
Chapter two analyses the interplay between VCT skills and deal structure on the one hand, and on the other hand, the success of the firms they finance - success as measured by the unrealised IRR. I develop and train two Deep Neural Network Binary Classification and Regression models on hand-collected VCT data, and with the aid of several attribution algorithms, I quantify the relative importance of VCT skills and the funding deal structure for the success of VCT-backed firms. I find that VCT skills and the funding deal structure are significant determinants of the success of VCT-backed firms. Specifically, being backed by a VCT with high prior financial success is the most important VCT skill determinant of the success of VCT-backed firms, contributing an average of 13% to the success of VCT-backed firms.
Chapter three analyses the wider VC industry within which the VCT scheme operates. The focus is on extending the VCT analysis in chapter two to VCs, and quantifying the relative importance of VC skills and the funding deal structure for the success of VC-backed firms. I employ the Deep Neural Network Binary Classification model and Deep Neural Network

Regression model from chapter two, but this time trained on VC data from the Refinitiv Workspace platform, and with the aid of several attribution algorithms, I quantify the relative importance of VC skills and the funding deal structure for the success of VC-backed firms. I find that VC skills and the funding deal structure are significant determinants of the success of VC-backed firms. Specifically, VC funding specialisation is the most important VC skill determinant of the success of VC-backed firms, contributing an average of 14% to the success of VC-backed firms.

All three chapters in this thesis contribute to the literature on VCs, their value added services beyond the supply of capital, and their importance to the firms they finance and the economy at large. By focusing on the VCT scheme, I have demonstrated that deliberate and targeted governmental intervention can have positive effects on entrepreneurship. Although this thesis has a U.K. focus, it is complementary to numerous financial economics studies - with a U.S. or worldwide focus - that have employed various theoretical, empirical, and structural approaches to analysing VCs and the VC industry. My finding - in chapter one - of a significant positive causal effect of the VCT scheme on the investment of VCT-backed firms can be compared with the findings in Manigart, Baeyens, and Van Hyfte (2002), who in their study on the survival rate of Belgian VC-backed firms, employ survival techniques to analyse a sample of comparable VC-backed and non VC-backed firms. They find that firms backed by the two oldest government sponsored VCs have a higher survival rate. The analysis in chapters two and three are complementary to numerous studies on the value-generating impact of VCs on the firms they back. For instance, my finding that VC funding specialisation in the FTSE-Industry of the firms it finances, is a key determinant of the eventual success of said VC-backed firm, reinforces the findings in Gompers, Kovner, and Lerner (2009), and Racculia (2014). All three chapters in this thesis also demonstrate the usefulness and practical applicability of several machine learning algorithms to the field of economics. The Matrix Completion algorithm in chapter one is adapted from the Matrix Completion approach in Athey et al. (2018). It approaches the task of constructing a counterfactual outcome or selecting the control group, in a data-driven manner, thus side-stepping the potential selection bias issue inherent in a causal study like this thesis. In the same vein, the Deep Neural Network Binary Classification model and Deep Neural Network Regression model in tandem with the attribution algorithms used to interpret their outputs, take a flexible, data-driven approach to data analysis, thus helping to alleviate the potential selection bias issue in chapters two and three.

Finally, due to data limitations, chapter one employed the investment (total-assets formation) of VCT-backed firms as the dependent variable (however, I note that this choice was also inspired by the stated aim of the VCT scheme, which is to help stimulate the growth of small, young and risky firms in the U.K.). However, an interesting avenue for future research will analyse the impact of the VCT scheme on other proxies for entrepreneurship such as: employment and patents.

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Appendix A

Matrix Completion Solution

Before we detail the closed form and numerical solution to the proximal mapping/optimisation problem 1.6, we introduce a few intermediate algorithms.[1]

A.1 Bregman Proximal Method

A.1.1 Set-Up

Let $\hat{w} = \underset{w \in \mathbb{R}^N}{\arg\min} \left\{ \frac{1}{2s} \|Xw - y\|^2 \right\}$ be the compact notation for a linear regression problem. We then have the corresponding optimality condition $\nabla E(\hat{w}) = 0$ which reads as

$$X^\top X \hat{w} = X^\top y \,, \tag{A.1}$$

which is also known as the normal equation associated with $X\hat{w} = y$

A.1.2 Bregman Proximal Method

If $X^\top X$ is invertible, we can solve (A.1) for $\hat{w}$ with any of the numerous algorithms that are used to numerically solve linear systems of equations. However, if N is very large, the oft-used algorithms that solve (A.1) to exacting numerical accuracy may require substantial computational time and large memory requirements.
We can however settle on approximate solutions of (A.1) by employing iterative algorithms such as gradient descent - which is an iterative procedure of the form

[1]The derivations and solutions to come are adapted from Martin Benning "Lecture Notes in Advanced Machine Learning" (Queen Mary University of London, 2020).

$$w^{k+1} = w^k - \tau \nabla E(w^k), \tag{A.2}$$

for some energy E, step-size parameter $\tau > 0$, and an initial value $w^0 \in \mathbb{R}^N$. For example, when $E(w) = \frac{1}{2s}\|Xw - y\|^2$, gradient descent reads as

$$\begin{aligned} w^{k+1} &= w^k - \frac{\tau}{s} X^\top \left(Xw^k - y \right), \\ &= \left(I - \frac{\tau}{s} X^\top X \right) w^k + \frac{\tau}{s} X^\top y. \end{aligned} \tag{A.3}$$

(A.3) is elegant in its simplicity. Iteratively solving (A.3) simply requires the computation of matrix multiplications and simple arithmetic operations. Additionally, with an algorithm like (A.2), we can deal with minimisation problems more generic than minimising the mean squared error (MSE).

Definition A.1.1 (Sub-differential)**.** Let $E : \mathbf{C} \subset \mathbb{R}^n \to \mathbb{R}$ be a convex and continuous function. Its sub-differential ∂E is characterised as the set:

$$\partial E(\upsilon) := \{ g \in \mathbb{R}^n \mid E(w) - E(\upsilon) \geq \langle g, w - \upsilon \rangle, \ \forall \ w \in \mathbb{R}^n \}.$$

The elements $g \in \partial E(\upsilon)$ are known as sub-gradients.

Definition A.1.2 (Bregman distance)**.** Let $E : \mathbb{R}^n \to \mathbb{R}$ be a continuously differentiable function, i.e. $\nabla E(w)$ exists $\forall \ w \in \mathbb{R}^n$ and is continuous. Then its corresponding Bregman distance $D_E : \mathbb{R}^n \times \mathbb{R}^n$ is defined as

$$D_E(u, \upsilon) := E(u) - E(\upsilon) - \langle \ \nabla E(\upsilon), \ u - \upsilon \ \rangle.$$

$\forall$ arguments $u, \upsilon \in \mathbb{R}^n$. We must emphasise that Bregman distances - defined in terms of a strictly convex function - are not necessarily distances in the sense of a metric. They are a statistical distance when the points are interpreted as probability distributions i.e. data-set of observed values. Bregman distances or divergences describe the distance of a function E at point u to its linearisation around υ, and are non-negative if and only if E is convex. A common Bregman distance is the squared Euclidean distance.

We now turn to deriving when and under what conditions (A.2) actually converges, what it converges to, and how quickly it converges. To maintain generality, we will consider a generalisation of gradient descent known as Bregman proximal method (BPM). This algorithm is based on Definition A.1.1 and an iterative procedure outlined in Algorithm 2.

Algorithm 2: Bregman proximal method

Specify: Energy function E: $\mathbb{R}^N \to \mathbb{R}$, Bregman function J: $\mathbb{R}^N \to \mathbb{R}$, index K

Initialise: $w^0 \in \mathbb{R}^N$

Iterate:

for $K = 0, \cdots, K-1$ **do**

$w^{k+1} = \text{arg min}_{w\in\mathbb{R}^N} \left\{E(w) + D_J(w, w^k)\right\}$;

end

return w^K.

To understand how the BPM is supposed to help us minimise an energy E such as $E(w) = \frac{1}{2s}\|Xw-y\|^2$, we must emphasise that the choice of J is critical. For instance, if we choose $J : \mathbb{R}^N \to \mathbb{R}$ with $J(w) := \frac{1}{2}\|w\|^2$, then solving the minimisation step in Algorithm 2 is just as difficult as minimising E itself. Thus, for our choice of J, we choose

$$J(w) := \frac{1}{2\tau}\|w\|^2 - E(w),$$

where $\tau > 0$ is a positive scalar. Computing the corresponding Bregman distance yields

$$\begin{aligned} D_J(w^{k+1},\, w^k) &= \frac{1}{2\tau}\left\|w^{k+1} - w^k\right\|^2 - \, D_E(w^{k+1},\, w^k), \\ &= \frac{1}{2\tau}\left\|w^{k+1} - w^k\right\|^2 - \, E(w^{k+1}) \, + \, E(w^k) \, + \, \langle \nabla E(w^k),\, w^{k+1} - w^k\rangle. \end{aligned}$$

Inserting our computed Bregman distance into the minimisation step in Algorithm 2 yields:

$$\begin{aligned} w^{k+1} &= \underset{w\in\mathbb{R}^N}{\text{arg min}}\left\{E(w) \, + \, D_J(w,\, w^k)\right\}, \\ &= \underset{w\in\mathbb{R}^N}{\text{arg min}}\left\{E(w^k) \, + \, \langle \nabla E(w^k),\, w - w^k\rangle \, + \, \frac{1}{2\tau}\left\|w - w^k\right\|^2\right\}, \\ &= \underset{w\in\mathbb{R}^N}{\text{arg min}}\left\{\langle \nabla E(w^k),\, w\rangle \, + \, \frac{1}{2\tau}\left\|w - w^k\right\|^2\right\}. \end{aligned}$$

The objective function $L^k(w) := \langle \nabla E(w^k),\, w\rangle \, + \, \frac{1}{2\tau}\left\|w - w^k\right\|^2$ is convex and differentiable with gradient $\nabla L(w) = \nabla E(w^k) \, + \, \frac{1}{\tau}(w - w^k)$. Thus, the global minimiser can be obtained via $\nabla L(w^{k+1}) = 0$, which yields (A.2). Gradient descent is summarised in Algorithm 3.

Algorithm 3: Gradient Descent

Specify: Differentiable, convex function E: $\mathbb{R}^N \to \mathbb{R}$, step-size $\tau > 0$, index K
Initialise: $w^0 \in \mathbb{R}^N$
Iterate:
for $K = 0, \cdots, K-1$ **do**
  $w^{k+1} = w^k \; - \; \tau\nabla E(w^k)$;
end
return w^K.

The pertinent question now is this: Does Algorithm 2 (and by implication Algorithm 3) converge to a minimiser of the objective function E ? If it does, under what conditions does it converge ?

Theorem A.1.1. (Convergence of Algorithm 2). Let $E : \mathrm{C} \subset \mathbb{R}^N \to \mathbb{R}$ and $J : \mathrm{C} \subset \mathbb{R}^N \to \mathbb{R}$ be convex and continuously differentiable functions. Suppose $\hat{w}$ denotes a global minimiser of E. Thus, the iterates of Algorithm 2 satisfy

$$E(w^K) - E(\hat{w}) \leq \frac{D_J(\hat{w}, w^0) - D_J(\hat{w}, w^K)}{K}, \tag{A.4}$$

and therefore $\lim_{K\to\infty} E(w^K) = E(\hat{w})$.

Before we lay out the proof of Theorem A.1.1, we verify the following intermediate result.

Lemma A.2. We adopt the same assumptions as in Theorem A.1.1, and suppose w^* is defined as $w^* :=$ arg $\min_{w\in\mathbb{R}^N}\{E(w) + D_J(w, \overline{w})\}$. Consequentially, the following identity holds:

$$E(w^*) \; + \; D_E(w, w^*) \; + \; D_J(w, w^*) \; + \; D_J(w^*, \overline{w}) \; = \; E(w) \; + \; D_J(w, \overline{w}).$$

Proof. Assume we can characterise w^* via the optimality condition:

$$0 = \nabla E(w^*) + \nabla J(w^*) - \nabla J(\bar{w}).$$

Taking an inner product with $w^* - w$ then yields:

$$
\begin{aligned}
0 &= -\langle \nabla E(w^*), w - w^* \rangle - \langle \nabla J(w^*) - \nabla J(\bar{w}), w - w^* \rangle, \\
&= D_E(w, w^*) - E(w) + E(w^*) - \langle \nabla J(w^*), w - w^* \rangle + \langle \nabla J(\bar{w}), w - w^* \rangle, \\
&= D_E(w, w^*) - E(w) + E(w^*) + D_J(w, w^*) - J(w) + J(w^*) + \langle \nabla J(\bar{w}), w - w^* \rangle, \\
&= D_E(w, w^*) - E(w) + E(w^*) + D_J(w, w^*) - J(w) + J(w^*) + \langle \nabla J(\bar{w}), w - \bar{w} + \bar{w} - w^* \rangle, \\
&= D_E(w, w^*) - E(w) + E(w^*) + D_J(w, w^*) - D_J(w, \bar{w}) + D_J(w^*, \bar{w}),
\end{aligned}
$$

which rounds off the proof. □

Proof. **Proof of Theorem A.1.1** By employing Lemma A.2 for $w^* = w^{k+1}, \bar{w} = w^k$, and $w = \hat{w}$, we have:

$$
\begin{aligned}
E(\hat{w}) + D_J(\hat{w}, w^k) &= E(w^{k+1}) + D_E(\hat{w}, w^{k+1}) + D_J(\hat{w}, w^{k+1}) + D_J(w^{k+1}, w^k), \\
&\geq E(w^{k+1}) + D_J(\hat{w}, w^{k+1})
\end{aligned}
$$

given the convexity of E and J, which implies $D_E(\hat{w}, w^{k+1}) \geq 0$ and $D_J(w^{k+1}, w^k) \geq 0$.
Therefore, we have $E(w^{k+1}) - E(\hat{w}) \leq D_J(\hat{w}, w^k) - D_J(\hat{w}, w^{k+1})$.
Summing from $k = 0, \cdots, K-1$ then leads to

$$
\sum_{k=0}^{K-1} E(w^{k+1}) - KE(\hat{w}) \leq D_J(\hat{w}, w^0) - D_J(\hat{w}, w^K). \tag{A.5}
$$

We can also apply Lemma A.2 for $w^* = w^{k+1}, \bar{w} = w^k$, and $w = \hat{w}$ to obtain:

$$
\begin{aligned}
E(w^k) + \underbrace{D_J(w^k, w^k)}_{=0} &= E(w^{k+1}) + D_E(w^k, w^{k+1}) + D_J(w^k, w^{k+1}) + D_J(w^{k+1}, w^k), \\
&\geq E(w^{k+1}),
\end{aligned}
$$

given the convexity of E and J, which implies $D_E(w^k, w^{k+1}) \geq 0$, $D_J(w^k, w^{k+1}) \geq 0$, and $D_J(w^{k+1}, w^k) \geq 0$.
We can thus conclude $E(w^{k+1}) \leq E(w^k) \ \forall \ k = 0, \cdots, K-1$, especially $KE(w^k) \leq \sum_{k=0}^{K-1} E(w^{k+1})$.
If we plug this inequality into (A.5), it implies A.4. Given J is convex, we can also estimate:

$$
E(w^K) \ \leq \ E(\hat{w}) + \frac{D_J(\hat{w}, w^0)}{K}
$$

for a positive constant $D_J(\hat{w}, w^0)$ independent of K, thus concluding both $\lim_{K\to\infty} E(w^K) = E(\hat{w})$ and the proof.
As a little aside, it is clear that showing the convexity of E and J is sufficient to prove convergence of the objective E.

□

Remark 1. It is pertinent to emphasise that Theorem A.1.1 does more than guarantee the convergence of Algorithm 2. It also gives us a rate of convergence. This rate is $1/K$, which in convex optimisation is emphasised with the big O-notation, i.e.

$$E(w^k) - E(\hat{w}) = O(\frac{1}{K}),$$

which means the left-hand-side is proportional to $1/K$. To illustrate, assume $D_J(\hat{w}, w^0) = 10$, then we will require $K = 1000$ iterations to ensure $E(w^k) - E(\hat{w}) \leq 10^{-2}$ according to Theorem A.1.1.

A.2.1 Closed Form Solution: Proximal Map

For $R(L) = \|L\|_*$ we are certain that $R(Q_1\ L\ Q_2) = R(L)$ for two orthogonal matrices $Q_1 \in \mathbb{R}^{N\times N}$ and $Q_2 \in \mathbb{R}^{T\times T}$. The intuition behind this heuristic is straightforward; the singular value decomposition (SVD) of $(Q_1\ L\ Q_2)$ is the same as the SVD of (L).
Now, let $Y = U\Sigma V^\top$ denote the SVD of Y, we can substitute $\hat{L}$ for $\tilde{L} = U^\top \hat{L} V$, with

$$\begin{aligned} \tilde{L} &= \underset{L\in\mathbb{R}^{N\times T}}{\arg\min} \left\{ \frac{1}{2}\left\| ULV^\top - U\Sigma V^\top \right\|_{\text{Fro}}^2 \ + \ \alpha \left\| ULV^\top \right\|_* \right\}, \\ &= \underset{L\in\mathbb{R}^{N\times T}}{\arg\min} \left\{ \frac{1}{2}\|L - \Sigma\|_{\text{Fro}}^2 \ + \ \alpha\|L\|_* \right\}, \end{aligned} \tag{A.6}$$

where α is a regularisation parameter that can be determined by cross-validation. Given Σ is a diagonal matrix with non-negative entries, the solution of A.6 has to be a diagonal matrix with non-negative entries as well. Consequently, A.6 simplifies to

$$\begin{aligned} \tilde{L} &= \underset{L\in\mathbb{R}_{\geq 0}^{\min(N,T)}}{\arg\min} \left\{ \frac{1}{2}\|l - \sigma\|^2 \ + \ \alpha \sum_{j=1}^{\min(N,T)} l_j \right\}, \\ &= \underset{L\in\mathbb{R}_{\geq 0}^{\min(N,T)}}{\arg\min} \left\{ \frac{1}{2}\sum_{j=1}^{\min(N,T)} (l_j - \sigma_j)^2 \ + \ \alpha \sum_{j=1}^{\min(N,T)} l_j \right\}. \end{aligned} \tag{A.7}$$

$l \in \mathbb{R}_{\geq 0}^{\min(N,T)}$ is the vector of diagonal entries of $\tilde{L}$, i.e. $\tilde{L} = \operatorname{diag}(l)$, and also the vector of singular values of $\tilde{L}$. The vector $\sigma \in \mathbb{R}_{\geq 0}^{\min(N,T)}$ denotes the singular values of Σ, i.e. $\Sigma = \operatorname{diag}(\sigma)$. Equation (A.7) has a closed-form solution - the soft-thresholding of the singular values σ! given by:

$$\tilde{l}_j = \max(\sigma_j - \alpha,\ 0), \qquad \forall j \in \{1, \cdots, \min(N,T)\}.$$

We can thus compute the solution of 1.6 via

$$\hat{L} = U\tilde{L}V^\top, \quad \text{for} \quad \tilde{L} = \operatorname{diag}(\tilde{l}). \tag{A.8}$$

As with convention, we will express the solution to A.8 in the proximal map notation as

$$\hat{L} = (I + \alpha\partial \| \,.\, \|_*)^{-1}(Y). \tag{A.9}$$

The intuitive implications of this proximal map are straightforward. Given our matrix of total-assets Y, all singular values below the threshold α will be set to zero, thus enforcing a lower rank of $\hat{L}$ compared to Y - if α is larger than at least the smallest singular value of Y. All other singular values are reduced by the factor α.

A.2.2 Numerical Solution: The Linearised Bregman Iteration

We now turn to deriving an efficient algorithm for the numerical solution of (1.6). Our algorithm is based on the following generalisation of the Bregman proximal algorithm (Algorithm 2) to non-smooth functions, otherwise known as Bregman iteration:

$$w^{k+1} = \operatorname*{arg\,min}_{w \in \mathbb{R}^N} \left\{ E(w) \; + \; D_J^{p^k}(w,\, w^k) \right\}, \tag{A.10a}$$

$$p^{k+1} = p^k - \nabla E(w^{k+1}), \tag{A.10b}$$

for initial values w^0 and $p^0 \in \partial J(w^0)$, where ∂J denotes the sub-differential of J as defined in Definition (A.1.1), and $D_J^p(w, \upsilon)$ is the generalised Bregman distance as defined in Definition (A.1.2)

$$D_J^p(w, \upsilon) = J(w) - J(\upsilon) - \langle p, w - \upsilon \rangle,$$

for $p \in \partial J(\upsilon)$. From the original Bregman method, we can derive a linearised variant for the choice $J(w) = \frac{1}{\tau}\left(\frac{1}{2}\|w\|^2 + R(w)\right) - E(w)$. Bregman iteration (A.10) then reads

$$\begin{aligned}
w^{k+1} &= \operatorname*{arg\,min}_{w \in \mathbb{R}^N} \left\{ E(w) \ + \ \frac{1}{2\tau}\left\|w - w^k\right\|^2 + \frac{1}{\tau} D_R^{q^k}(w, w^k) - E(w) + E(w^k) + \langle \nabla E(w^k), w - w^k \rangle \right\}, \\
&= \operatorname*{arg\,min}_{w \in \mathbb{R}^N} \left\{ \frac{1}{2\tau}\left\|w - w^k\right\|^2 + \frac{1}{\tau} D_R^{q^k}(w, w^k) + \langle \nabla E(w^k), w - w^k \rangle \right\}, \\
&= \operatorname*{arg\,min}_{w \in \mathbb{R}^N} \left\{ \frac{1}{2\tau}\left\|w - w^k\right\|^2 + \frac{1}{\tau} D_R^{q^k}(w, w^k) + \frac{1}{\tau}\langle \tau\nabla E(w^k), w - w^k \rangle + \frac{1}{2\tau}\left\|\tau\nabla E(w^k)\right\|^2 \right\}, \\
&= \operatorname*{arg\,min}_{w \in \mathbb{R}^N} \left\{ \frac{1}{2}\left\|w - \left(w^k - \tau\nabla E(w^k)\right)\right\|^2 + \ D_R^{q^k}(w, w^k) \right\}, \\
&= (I + \partial R)^{-1}\left(w^k + q^k - \tau\nabla E(w^k)\right), \qquad \text{(A.11a)} \\
q^{k+1} &= q^k - \left(w^{k+1} - w^k - \tau\nabla E(w^k)\right), \qquad \text{(A.11b)}
\end{aligned}$$

for sub-gradients $q^k \in \partial R(w^k)$ and the short-hand notation

$$(I + \partial R)^{-1}(z) := \operatorname*{arg\,min}_{y \in \mathbb{R}^N} \left\{ \frac{1}{2}\|y - z\|^2 + R(y) \right\}.$$

Now, we focus on the special case $E(w) = \frac{1}{2}\|Aw - b\|^2$, for a matrix $A \in \mathbb{R}^{T \times N}$ and a vector $b \in \mathbb{R}^T$. For this special case, (A.11) reads

$$\begin{aligned}
w^{k+1} &= (I + \partial R)^{-1}\left(w^k + q^k - \tau\, A^\top(Aw^k - b)\right) \qquad \text{(A.12a)} \\
q^{k+1} &= q^k - \left(w^{k+1} - w^k - \tau\, A^\top(Aw^k - b)\right). \qquad \text{(A.12b)}
\end{aligned}$$

If we assume that $(w^k + q^k)/\tau \in \mathbf{R}(A^\top)$, we can substitute
$\tau\, A^\top b^k = w^k + q^k - \tau\, A^\top(Aw^k - b)$, which modifies (A.12) to

$$w^{k+1} = (I + \partial R)^{-1} \left(\tau \, A^\top b^k \right) \tag{A.13a}$$

$$b^{k+1} = b^k - \left(A w^{k+1} - b \right). \tag{A.13b}$$

with initial value $b^0 = b$. Combining both equations of (A.13) into one yields:

$$b^{k+1} = b^k - \left(A(I + \partial R)^{-1} \left(\tau \, A^\top b^k \right) - b \right) \tag{A.14}$$

The motive behind re-characterising (A.12) to (A.14) is that (A.14) is simply gradient descent (See Algorithm 3) applied to a very specific energy that we characterise in the next section.

A.2.3 Linearised Bregman Iteration as Gradient Descent

The Linearised Bregman Iteration in the form of (A.14) is a gradient descent method with step-size one, i.e.

$$b^{k+1} = b^k - \nabla G_\tau(b^k),$$

applied to the energy

$$G_\tau(b^k) := \frac{\tau}{2} \left\| A^\top b^k \right\|^2 \; - \; \langle b^k, b \rangle \; - \; \frac{1}{\tau} \, \tilde{R}(\tau \, A^\top b^k).$$

Here, $\tilde{R}$ represents the Moreau-Yosida regularisation of the function R, i.e.

$$\begin{aligned} \tilde{R}(z) &:= \inf_{y \in \mathbb{R}^N} \left\{ R(y) + \frac{1}{2} \|y - z\|^2 \right\}, \\ &= R\left((I + \partial R)^{-1}(z) \right) \; + \; \frac{1}{2} \left\| (I + \partial R)^{-1}(z) - z \right\|^2. \end{aligned}$$

The proof is reasonably succinct if we can compute the gradient of $\tilde{R}$, since the gradient of $\frac{\tau}{2}\left\|A^\top b^k\right\|^2 - \langle b^k,\, b\rangle$ [2] simply reads $\tau\, AA^\top b^k - b$. To compute the gradient $\nabla\tilde{R}$, we start by rewriting $\tilde{R}$ as

$$
\begin{aligned}
\tilde{R}(z) &= \inf_{y\in\mathbb{R}^N}\left\{R(y)+\frac{1}{2}\|y-z\|^2\right\},\\
&= \inf_{y\in\mathbb{R}^N}\left\{R(y)+\frac{1}{2}\|y\|^2-\langle y,\, z\rangle+\frac{1}{2}\|z\|^2\right\},\\
&= \frac{1}{2}\|z\|^2-\sup_{y\in\mathbb{R}^N}\left\{\langle y,\, z\rangle-R(y)-\frac{1}{2}\|y\|^2\right\},\\
&= \frac{1}{2}\|z\|^2-\left(\frac{1}{2}\|\cdot\|^2+R\right)^*(z),
\end{aligned}
$$

where $F^*(z) := \sup_{y\in\mathbb{R}^N}\langle y,\, z\rangle - F(x)$ denotes the *convex conjugate* or *Fenchel conjugate* of a function F. We should point out that by definition, we observe that $\nabla F^*(z) = y^*$, where $y^* = argmax_{y\in\mathbb{R}^N}\{\langle y,\, z\rangle - F(y)\}$. When we have $F(y) = \frac{1}{2}\|y\|^2 + R(y)$, the problem reads as:

$$
\begin{aligned}
y^* &= \arg\max_{y\in\mathbb{R}^N}\left\{\langle y,\, z\rangle-\frac{1}{2}\|y\|^2-R(y)\right\},\\
&= \arg\max_{y\in\mathbb{R}^N}\left\{-\frac{1}{2}\|y-z\|^2-R(y)\right\},\\
&= \arg\max_{y\in\mathbb{R}^N}\left\{\frac{1}{2}\|y-z\|^2-R(y)\right\},\\
&= (I+\partial R)^{-1}(z).
\end{aligned}
$$

With regards the gradient of $\tilde{R}$, we observe

$$\nabla\tilde{R}(z) = z-(I+\partial R)^{-1}(z).$$

[2] $\langle\,\cdot\,\rangle$ denotes the inner product

As an immediate result of the chain rule, we have $\nabla\left(\left(\frac{1}{\tau}\tilde{R}\right)\circ\left(\tau A^\top\right)\right)(b^k) = A\nabla\tilde{R}(\tau A^\top b^k)$, [3] and thus conclude

$$\begin{aligned}\nabla G_\tau(b^k) &= \tau AA^\top b^k - b - \tau AA^\top b^k + A(I+\partial R)^{-1}\left(\tau A^\top b^k\right),\\ &= A(I+\partial R)^{-1}\left(\tau A^\top b^k\right) - b,\end{aligned}$$

which also serves as the proof. In the next section, we will apply Algorithm (A.14) to our optimisation problem (1.6).

A.2.4 A Bregman Algorithm for Matrix Completion

For notational convenience, let us rewrite $L-Y$ from (1.6) as S. To solve (1.6), we have $w=(L,S)$, and E and J as

$$\begin{aligned}E(L,S) &= \frac{1}{2}\| L+S-Y \|^2_{\text{Fro}} \qquad \text{and}\\ J(L,S) &= \frac{1}{\tau}\left(\frac{1}{2}\| L \|^2_{\text{Fro}} \ + \ \gamma\alpha\| L \|_* + \frac{1}{2}\| S \|^2_{\text{Fro}} + \ \gamma\| S \|_*\right) - E(L,S),\end{aligned}$$

for constants $\tau > 0$ and $\gamma > 0$.
Given our choices of E and J, we are in the exact framework of (A.12) for $A = (I \ \ I)$ and $b = Y$, thus we can numerically solve (1.6) by iterating the updates (A.13), which for our choice of E & J reads

$$L^{k+1} = (I \ + \ \gamma\,\alpha\,\partial\| \,.\, \|_*)^{-1}\left(\tau P_\Omega^\top z^k\right) \tag{A.15a}$$

$$S^{k+1} = \left(I \ + \ \gamma\partial\| \,.\, \|_{\text{Fro}}\right)^{-1}\left(\tau P_\Omega^\top z^k\right) \tag{A.15b}$$

$$z^{k+1} = z^k - \left(P_\Omega L^{k+1} - \ z\right), \tag{A.15c}$$

for $z^0 = z := P_\Omega Y$, $\tau \leq 1$ and $\alpha > 0$. Approach (A.15) is summarised in Algorithm 4 below.

[3] $\circ$ is the Hadamard product/element-wise, entry-wise or Schur product. It is a binary operation that takes two matrices of the same dimensions and produces another matrix of the same dimension as the operands, where each element i, j is the product of elements i, j of the original two matrices. It should not be confused with the more common matrix product.

Algorithm 4: Matrix Completion

Specify: parameters $\gamma > 0$, $\alpha > 0$, stopping index K

Initialise: Set of known indices of total-assets matrix Ω, $z^0 = P_\Omega Y$ and $\tau \leq 1$

Iterate:

for $K = 0, \cdots, K-1$ **do**

$L^{k+1} = (I + \gamma\, \alpha\, \partial\|\,.\,\|_*)^{-1} \left(\tau P_\Omega^\top z^k\right)$;

$S^{k+1} = (I + \gamma\, \partial\|\,.\,\|_*)^{-1} \left(\tau P_\Omega^\top z^k\right)$;

$z^{k+1} = z^k - \left(P_\Omega L^{k+1} - z\right)$;

end

return L^K, S^K.

Algorithm 4 is relatively straightforward. It entails computing the singular value decomposition of $\tau P_\Omega^\top z^k$ in every iteration, and soft-thresholding the singular values with threshold $\gamma\,\alpha$. Subsequently, we update the matrix z^k by subtracting the residual $P_\Omega L^{K+1} - z$ from it. By applying an identical procedure to the proof of Theorem A.1.1, we can prove that Algorithm 4 converges at a rate of $1/K$ for $\tau \leq 1/\|A\|^2 = 1/2$, where K denotes the number of iterations.

A.3 VCT Tax Benefits: Illustrations

In this paper, we do not highlight nor analyse the risks inherent in subscribing to the equity issue of a VCT. [4] However - in recognition of these risks - the U.K. government provides investors with a 30% income tax relief for subscriptions in new VCT fundraising. To illustrate with an illustration drawn from HMRC (2018) venture capital trust statistics, assume an investor invests £10,000 in a VCT fundraising round. This investor either receives a £3,000 cheque from the tax authority or a £3,000 reduction in her tax bill. We should emphasise that this is a tax rebate, hence restricted to the amount of income tax she paid. This means that (and given that the maximum annual VCT investment is £200,000) if she has only paid £2,000 in income tax, she would only receive a £2,000 instead of £3,000 tax rebate on her £10,000 investment. She must also hold her VCT shares for five years to permanently keep the tax rebate. Also, she does not get the rebate if she bought the shares on the secondary market. This example also illustrates the fact that the tax benefits from VCT investments are dependent on each individual investor's circumstances. We further illustrate with three more examples:

[4]VCTs are exposed to significantly higher risks than non-VCT equity investors. VCTs invest in smaller, fledgling firms, a lot of which will struggle or go into liquidation.

Example A

Francesca decides to invest £200,000 in a VCT offer for subscription. In the 2019/20 tax year she anticipates that she will pay £90,000 in income tax.

Investment	£200,000
Tax Rebate	(£60,000)
Effective Net Cost	£140,000
Tax Rebate as a percentage	30%

Example B

In the tax year 2019/20, Bukola decides to invest £10,000 in a VCT offer for subscription. She is a basic rate and non-Scottish tax-payer; she earns £30,000 annually hence will pay approximately £3,500 in income tax $([30,000 - 12,500(\text{Personal Allowance})] \times 20\%)$.

Investment	£10,000
Tax Rebate	(£3,000)
Effective Net Cost	£7,000
Tax Rebate as a percentage	30%

Example C

Adesua wants to invest £100,000 in a VCT offer for subscription. She is a higher rate and non-Scottish tax-payer; she earns £60,000 annually and has calculated that she will pay £11,500[5] in income tax in the tax year 2019/20.

Investment	£100,000
Tax Rebate	(£11,500)
Effective Net Cost	£88,500
Tax Rebate as percentage	11.5%

Adesua will not pay enough income tax to reclaim the full 30% tax rebate, hence will only receive the £11,500 in tax she paid as rebate.

[5] Her tax liability is calculated as the sum of 0% on £12,500 personal allowance, basic rate of 20% on £37,500, and higher rate of 40% on £10,000

A.4 Major VCT Policy Changes[6]

- 2004-06: 6th April 2004 - Introduction of the 40% income tax relief rate for a two-year period starting on 6 April 2004 - prior to which income tax relief was given at 20%. Also, from 6th April 2004, the maximum amount individual investors could invest in VCTs to qualify for income tax relief increased from £100,000 to £200,000. However, the holding period - to keep your income tax relief - for VCT shares held by investors increased from three to five years.
  We attribute the highest points (2004-2005) in our Fig.1.7. of aggregate annual investment to the increased income tax relief. Our assertion is backed by the 244% average increase in the amount of funds raised in both 2004 and 2005 relative to the average raised in the two years prior (See Table 1.2.) In the aggregate, VCTs attributed the high levels of funds raised and the subsequent high level of investment to the increased income tax relief.

- 2006-07: 6th April 2006 - The maximum gross assets of qualifying investees was reduced from £15 million to £7 million before investment and from £16 million to £8m immediately after investment. Also, the rate of income tax relief was reduced to 30% from 40%.

- 2007-08: 6th April 2007 - VCT qualifying investees must be firms with fewer than 50 full-time employees at the time shares are issued. 19th July 2007 - Investees can only raise a maximum of £2 million in any 12 month period under any or all of the tax-based venture capital schemes (Venture Capital Trusts, Enterprise Investment Scheme).
  Again, our analysis of the annual reports of each VCT managing funds within the 2006-2008 period reveals that the reduction in the rate of income tax relief - from 40% to 30% depressed their fundraising activities within the period. Most importantly - as explicitly reported by VCT investment managers in their annual reports - the reduction in the size of qualifying investees increased the risk profile of potential investees and further depressed their investment activities. All of this largely [7] explains the sustained downward trend in investment between 2006 - 2009 as seen in Fig.1.7. Our explanation also bears out in the numbers in Table 1.2. We see that the number of VCTs raising funds as a proportion of those managing funds drops from 68% in 2004-2006 to 34% in 2006-2008

[6] A summary of these major VCT policy changes is available at: https://octopusinvestments.com/resources/guides/venture-capital-trusts/

[7] We hedge by using the adverb "largely" because this time period also coincides with the height of the financial crisis and the attendant bear market. This however is an area we will not explore in this study.

- 2009-10: Capital raised by VCTs in a share issuance should be fully employed within two years of the issuance. However, if the issue takes place before commencement of the intended trade, then the capital raised should be fully employed within two years of commencement.
  Our analysis reveals that the 2009-10 major policy change did not drive the upward trend seen in the same period, see Fig.1.7. During the period, VCT investment managers documented their concerns about the impact of the economic downturn and tightened lending conditions on SMEs. They however saw this as an opportunity to further invest in their existing portfolios; tightened lending conditions meant VCTs were one of the few sources of financing for investees: through the provision of working capital to investees, by funding acquisitions carried out by investees, and funding the restructuring of investees.
  Thus, we see from Table 1.2. that even though fundraising in the period was at a three-year high, the number of new investees that received VCT funding was the lowest it had been since 2003 (see Fig.1.2.). This means, more money was being raised by VCTs relative to the last three years, but fewer new investees were receiving said funds. Therefore, the data in Fig.1.2. backs up the documented claim that VCTs viewed the tightened lending conditions for SMEs as an opportunity to solidify their existing positions under favourable terms, and hence, a large proportion of the three-year-record-breaking newly raised funds went to existing investees.

- 2010 - 2011: 6th April 2011 - VCTs must hold at least 70%, by VCT tax value, of its total investments (shares, securities and liquidity) in VCT qualifying holdings, within approximately three years of a fundraising. For VCTs whose accounting periods begin on or after 1 January 2020, this percentage increased to 80%. From that date, total investments also includes funds raised up to 31 December 2017.
  Also, a VCT can only invest a maximum of £1m per tax year in each of its investees, and no investment in a single investee or group of investees may constitute more than 15% (by VCT tax value) of the VCT's total investments at the date of investment.

- 2011-12: For funds raised before April 2011: at least 30% of a VCT's qualifying investment by value must be held in "eligible shares" (do not carry any preferential rights). For qualifying investments made by VCTs after 5 April 2018, together with qualifying investments made by funds raised after 5 April 2011, they must in aggregate be comprised of at least 70% by VCT tax value in "eligible shares".
  At least 10% of each investment in qualifying investees is held in eligible shares (by cost at the time of investment).

A VCT's income must come wholly or primarily from shares and securities.
VCTs must distribute sufficient dividends from their revenue available for distribution so as not to retain more than 15% of their income from shares and securities in a year.
VCTs must be listed on a U.K. recognised Stock Exchange.
The requirement that a potential investee's main trade be carried on wholly or mainly in the U.K. was cancelled, and replaced with a requirement that the investee have a permanent establishment in the U.K.
The restriction that prevented VCTs from investing more than £1m per annum in any single investee was also removed.

- 2012-13: 6th April 2012: The 2007 restriction on VCT qualifying investees having a maximum of 50 employees is increased to a maximum of 250 full time equivalent employees. Also, the 2006 reduction in gross assets of VCT qualifying investees was reversed. VCTs can once again invest in firms with maximum gross assets of £15 million before investment and £16 million after investment.
  Additionally, the rule that an investee is restricted to an annual VCT investment limit of £2m - imposed in 2007 - is increased to £5 million, with a lifetime limit of £12 million (for knowledge intensive companies the annual limit is £10 million and the lifetime limit is £20 million).
  Regarding investments made by a VCT from capital it raised on or after 6 April 2012, if an investee uses the funds to acquire shares in another company, this will not be considered as using them for a qualifying purpose.
  The main theme of the policy changes between 2010-2013 was a reversal of the 2006-2007 changes. These reversals were introduced to stimulate VCT fundraising and subsequent investment in U.K. SMEs.
  However, all of the investment managers expressed concern in their annual reports about an uncertain and fragile U.K. economy. The main highlights of their concern were the sovereign debt crisis in the eurozone, upward inflationary pressures, and a sustained downward pressure on public sector spending.
  These reasons help explain the downward trend we see in the period in Fig.1.7.

- 2014-15: From April 2014 VCTs could no longer return share capital to investors within three years of the end of the accounting period in which the VCT issued the shares.
  Additionally, legislation was introduced to prevent investors refreshing income tax relief on investments into VCTs by disposing of VCT shares and reinvesting the

proceeds in new shares. The legislation allowed new investment into VCTs to still be eligible for income tax relief. However, investments that were:

- conditional on a share buy-back or made within a six month period of a sale of shares in the same VCT would not qualify for income tax relief. The measure did not affect subscriptions for shares where the monies being subscribed represented dividends which the investor had elected to reinvest. The legislation was also changed to allow individuals to subscribe for shares in a VCT via a nominee.
  These major policy changes are responsible for the downward trend depicted in the period in Fig.1.7.

- 2015-16: 8th July 2015 - Policy changes were introduced to bring the VCT scheme in line with the European Union's risk capital guidelines:

  1. VCTs may not: offer secured loans to investees, and any returns on loan capital above 10% must only represent a commercial return on the principal; invest in investees that do not meet the new "risk to capital" condition (which requires an investee, at the time of investment, to be an entrepreneurial company with the objective to grow and develop, and where there is a genuine risk of loss of capital).
  2. Restrictions on investments that VCTs can make, particularly with respect to the age of the business. Potential investees have been limited to firms that are less than 7 years old (ten years for knowledge intensive businesses).
     Non-qualifying investments can no longer be made, except for certain exemptions in managing the Company's short-term liquidity. Exemptions are limited to investments in firms such as OEICs (Open Ended Investment Company), Investment Trusts or listed firms.
     Investment managers report that this policy change (No.2) will curtail their investment in Alternative Investment Market (AIM) shares; AIM shares form a significant proportion of VCT portfolio holdings. This line of reasoning is clearer when we consider that the London Stock Exchange requires that firms be at least 3 years old before they can registered on the AIM.
     VCTs further interpret this particular policy change as likely to reduce the scope of investments they can make, potentially increasing the risk profile of their portfolios. For instance, they claim that replacing the shares of AIM firms with that of smaller unquoted firms will increase the risk profile of their portfolios.

3. Ban on using funds raised by VCTs to finance Management Buyout (MBO), Buy-In Management Buyout (BIMBO), or company acquisitions. ***Investment managers report that this will eliminate the lower risk component of their portfolios***.
4. These policy changes were introduced with a ten-year sunset clause - providing a decade of stability with regards VCT policy changes.

In summary, we have two countervailing forces affecting VCTs. On the one hand, the narrower set of investment opportunities (No's 2, 3, 4) could potentially depress investment activity. To paraphrase the sentiments of numerous investment managers "These new inhibitions will curtail significant drivers of growth in the U.K. SME ecosystem. They will curtail, as opposed to encourage, investment activity". On the other hand - and this sentiment was also explicitly expressed by VCT investment managers in their annual report - there is a high demand for VCTs to fundraise as a result of a reduction in the pension lifetime allowance from £1,250,000 to £1,000,000, the tapering away of pension tax allowances for high earners earning £110,000 a year or more, which can gradually reduce said person's annual allowance from the standard £40,000 to as low as £10,000[8], and the launch of pension freedoms that allow for cash to be taken out of the pot for investment rather than buying an annuity. All of this has caused VCTs to become more attractive to investors seeking additional tax-advantaged investments.
The tax-advantage phenomena clearly dominated the narrower set of investment opportunities phenomena, and helps explain the upward trend we see in investment beginning in 2015 till the end of our sample in 2018.
What is clear from the major VCT policy changes between 2015-18 is the Government's desire to refocus investment towards young growth companies. We have argued that these changes have been successful in stimulating new investment, especially the ban on funding MBOs, BIMBOs and acquisitions. To reiterate the point, our reader might have noticed that prior to 2015, periods of rising growth in the rate of investment always preceded or followed periods of falling growth in investment. However, since 2015, the growth in investment has been trending upward.

- 2017-2018: Patient Capital Review:
  In the November 2017 budget, the U.K. Government reviewed the VCT scheme as

[8]Prior to 2009, high earners could save up to £235,000 a year in a pension and receive nearly £100,000 in tax relief. As of 6th April 2016, that sum is limited to £10,000 in a pension and just £4,000 in tax relief.

part of its wider "Patient Capital Review"[9]. The outcome was a number of proposed changes to the VCT regulations in an effort to refocus investment on potentially higher risk sectors that require capital (Her Majesty's Treasury Policy Paper, 2017) - summarised below:

1. Expand the VCT scheme to enable VCTs to provide follow-on investment which will help to "scale-up" investees, thus easing the transition from a dependence on VCT funding to venture funding. For instance, increasing the current Knowledge Intensive Company allowance would help increase the focus on science based firms.
2. Increasing the annual and lifetime investment limits would allow for follow on investment from VCTs, thus slowing the transition away from tax-incentivised financing (Her Majesty's Treasury Policy Paper, 2017).

Anticipation of the above changes from the Patient Capital Review also influenced the increased growth rate in investment between 2017-2018.

- April 2020: Minimum of 80% of a VCT's funds must be invested in VCT qualifying investments - up from 70%.

[9] The review considered how to support innovative firms to access the finance that they need to scale up. Her Majesty's Treasury published a consultation seeking views on how to increase the supply of capital to growing, innovative firms.

Appendix B

Distributions and Description of Variables

B.1 Distribution of VCT Skill and Deal Structure Variables

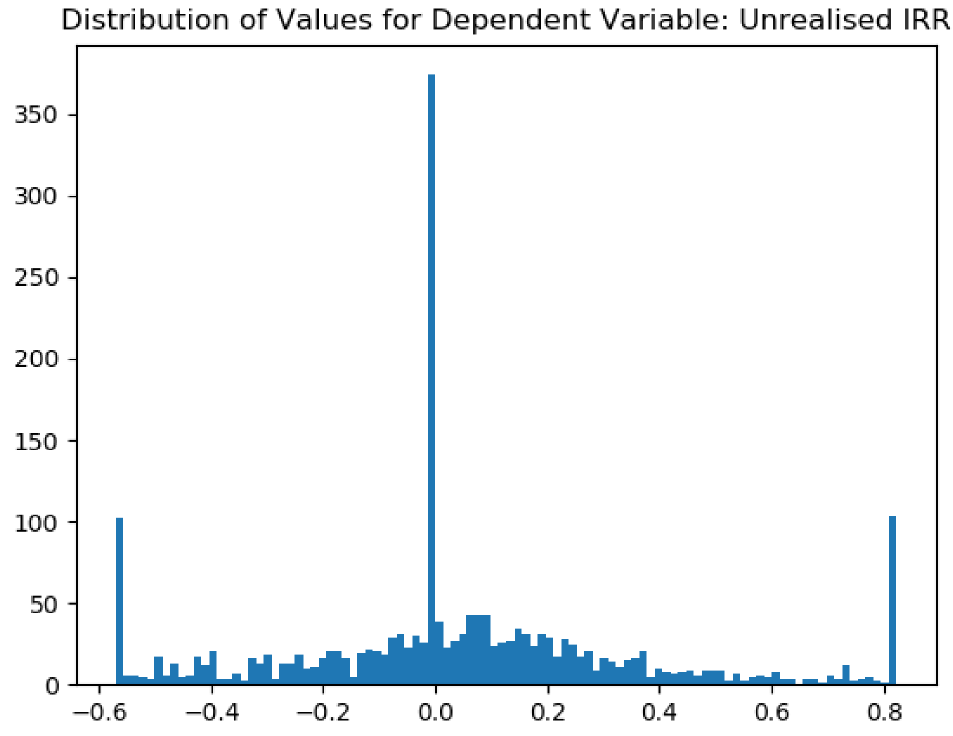

Figure B.1. Unrealised IRR

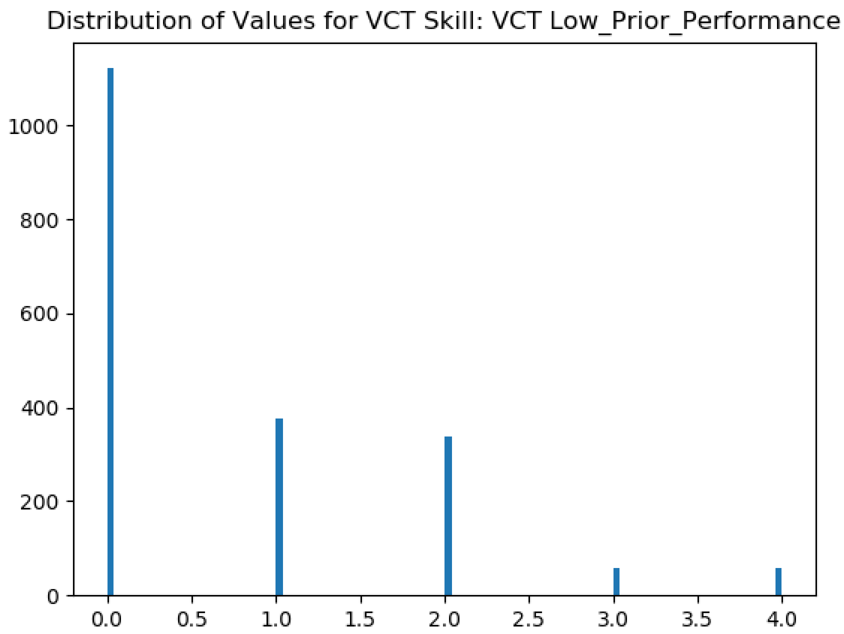

Figure B.2. Low Prior Performance

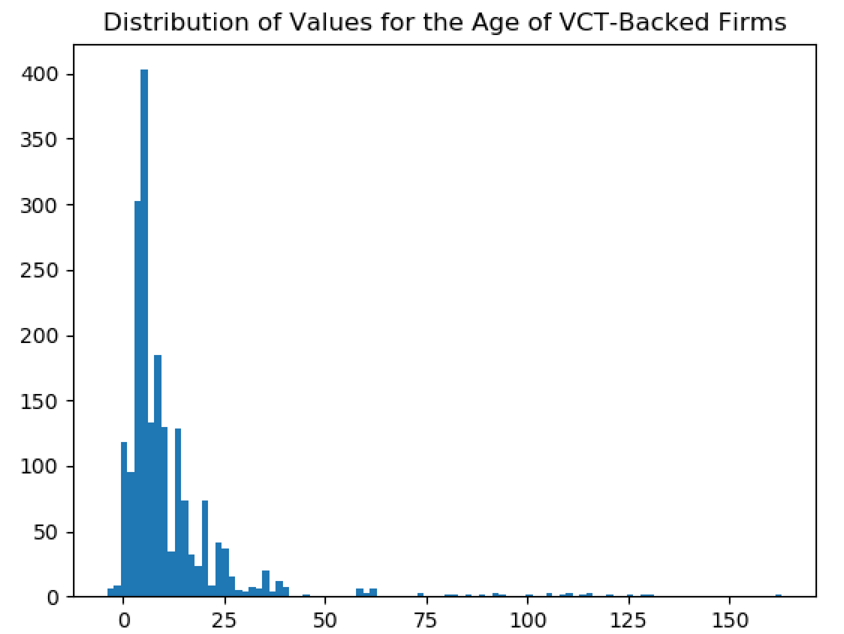

Figure B.3. Age of VCT-Backed Firms

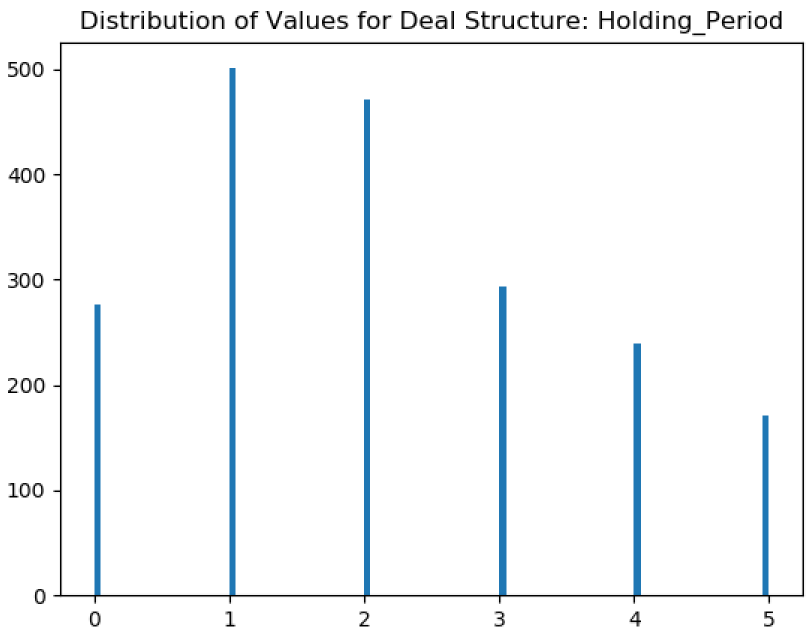

Figure B.4. Holding Period

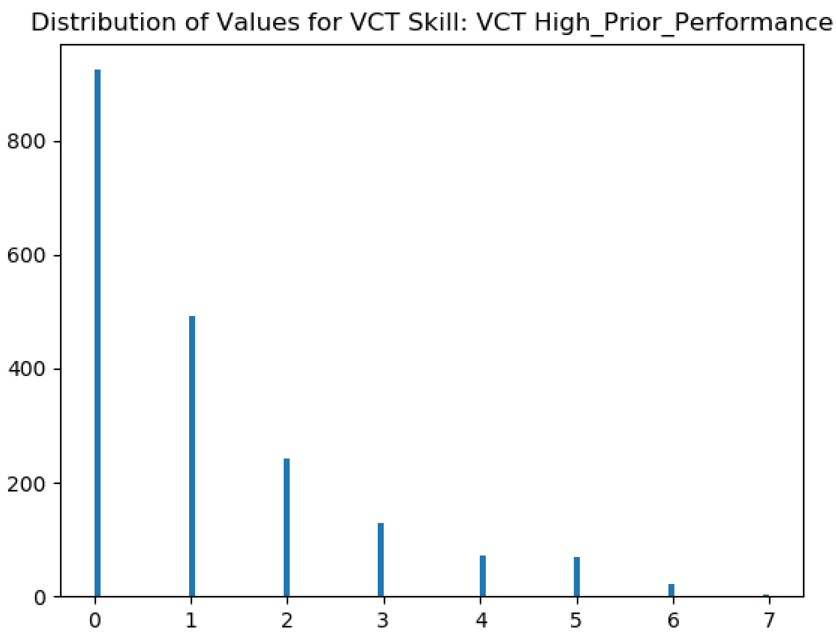

Figure B.5. High Prior Performance

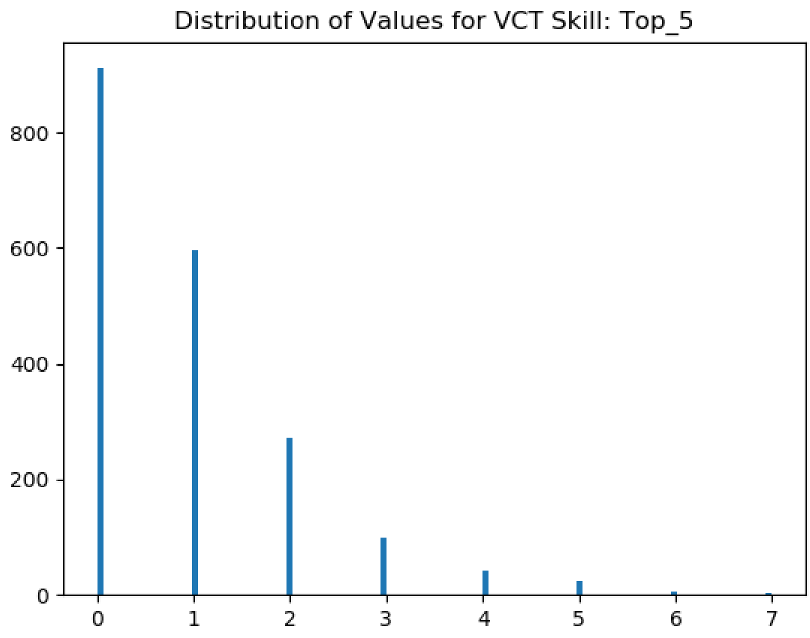

Figure B.6. Top 5

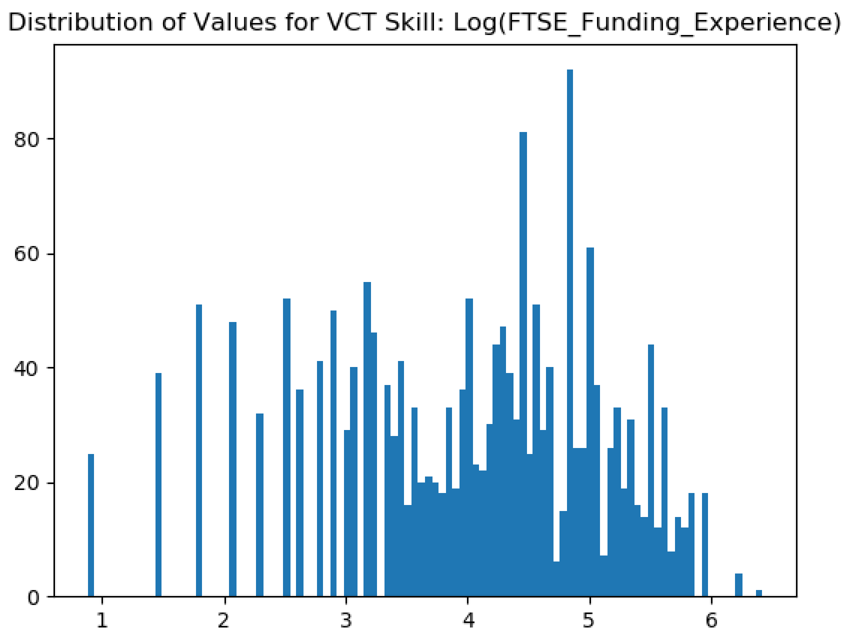

Figure B.7. Log(FTSE Funding Experience)

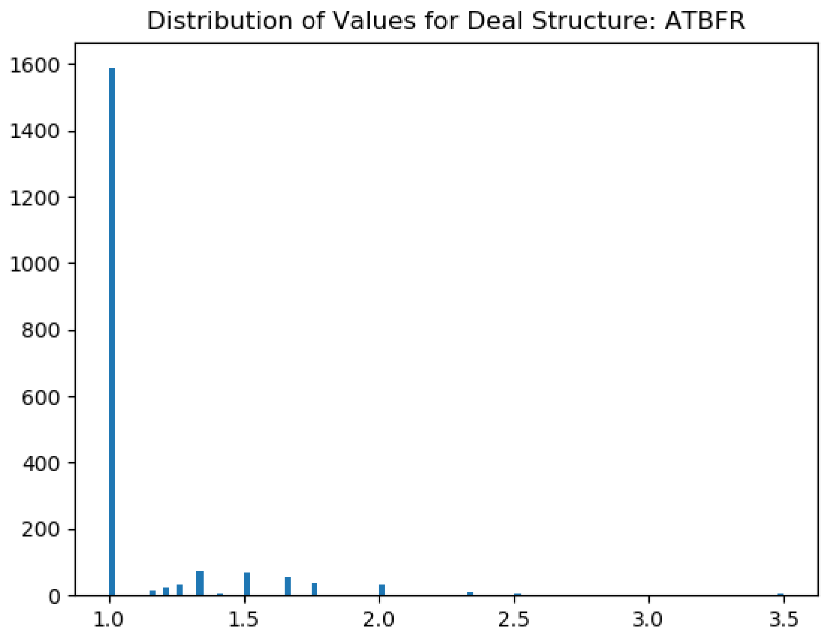

Figure B.8. ATBFR(Years)

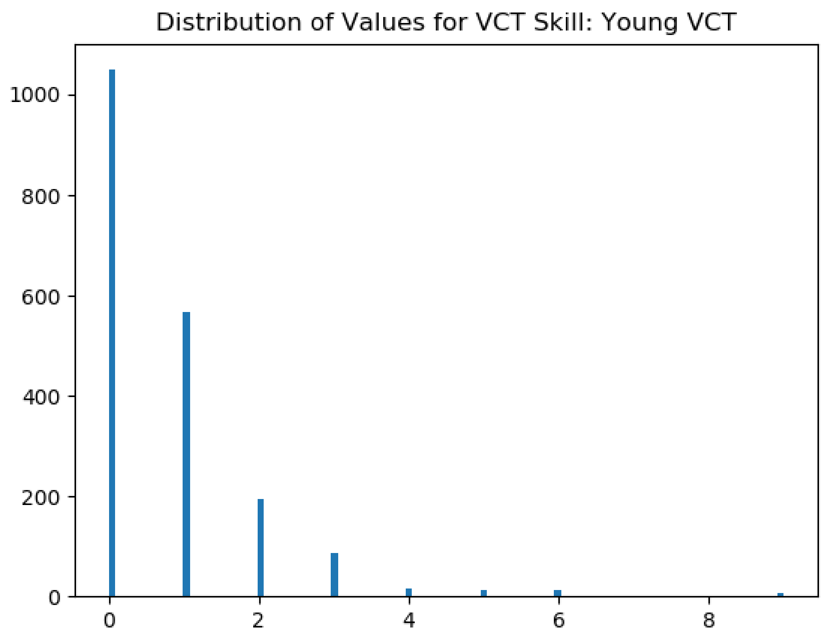

Figure B.9. Young VCT

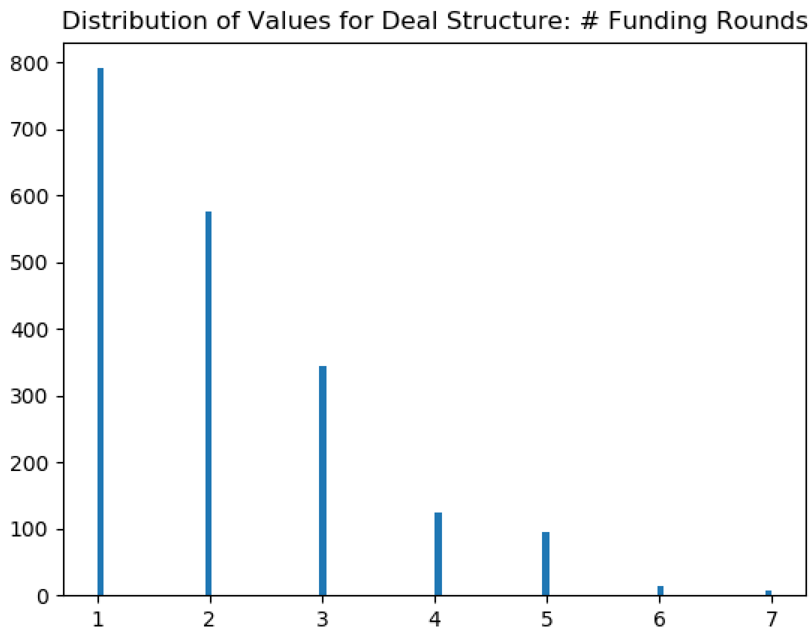

Figure B.10. Funding Rounds

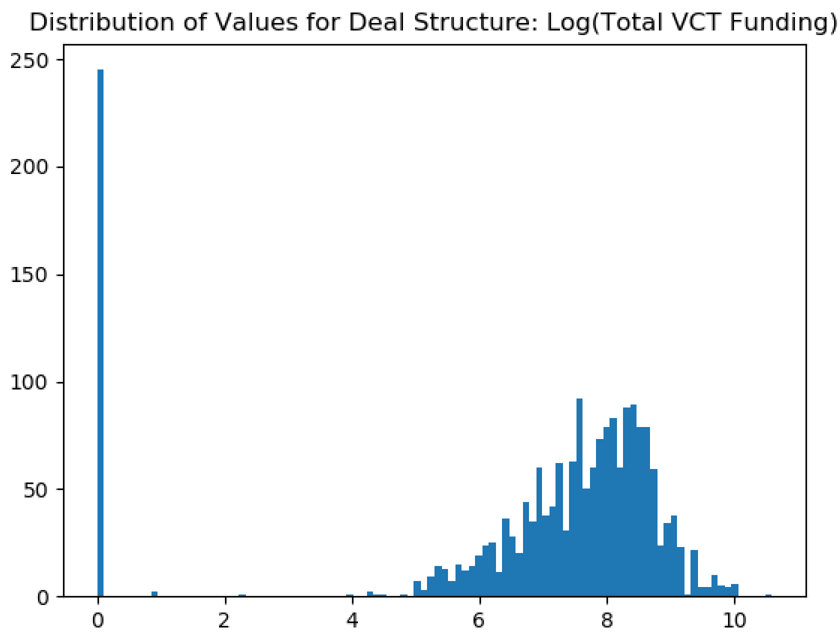

Figure B.11. Log(Total VCT Funding)

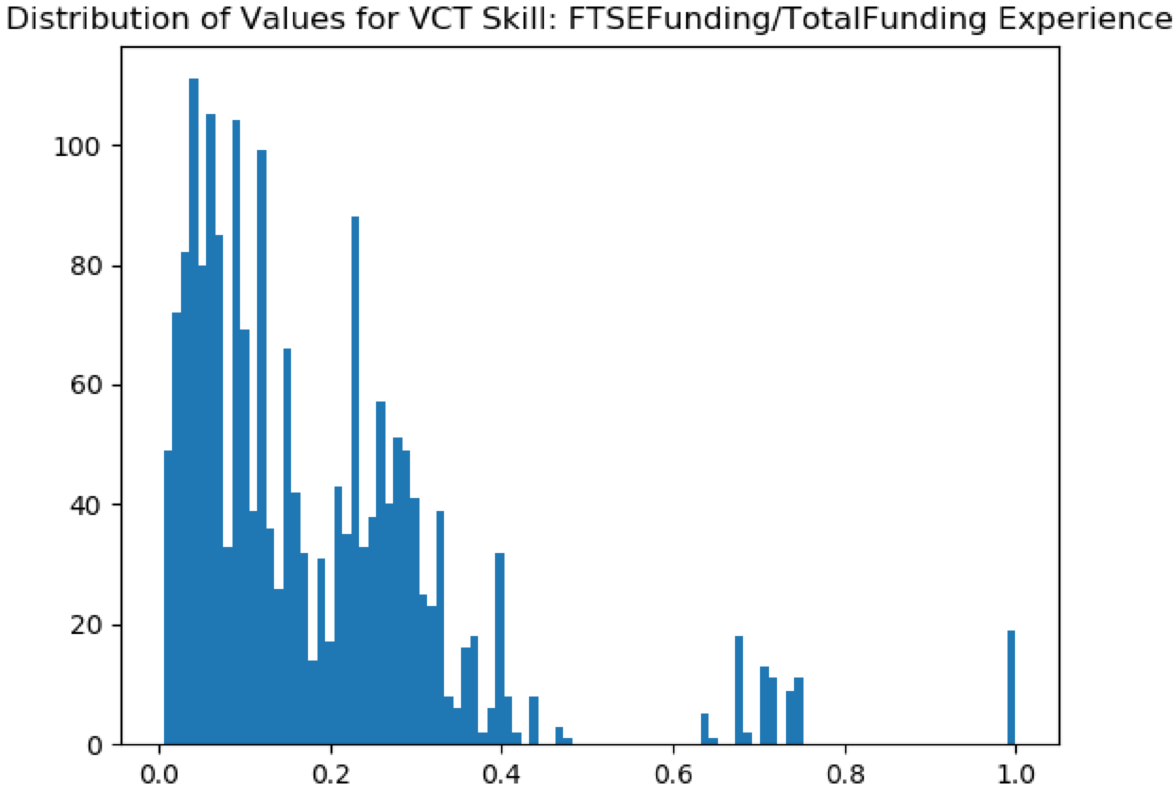

Figure B.12. FTSE-Industry Funding Specialisation

B.2 Distribution of VCT Skill and Deal Structure Variables for Successful vs. Unsuccessful VCT-Backed Firms

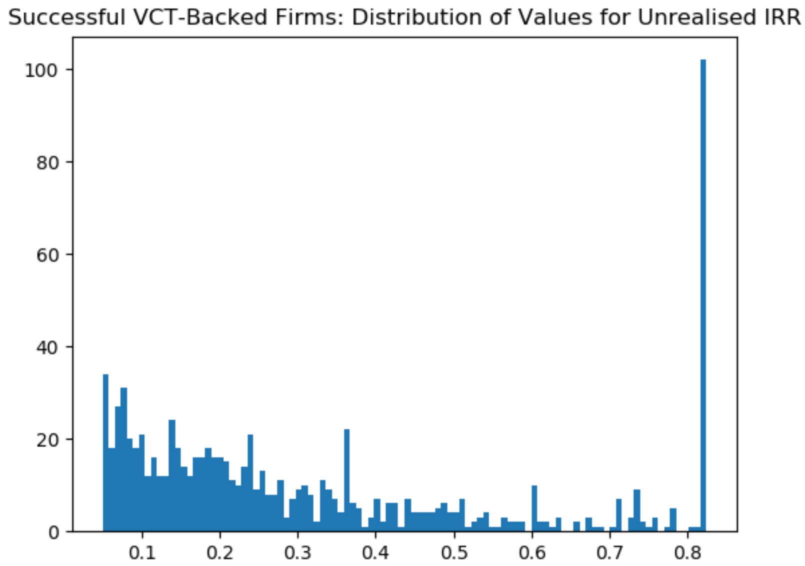

Figure B.13. Successful VCT-Backed Firms: Unrealised IRR

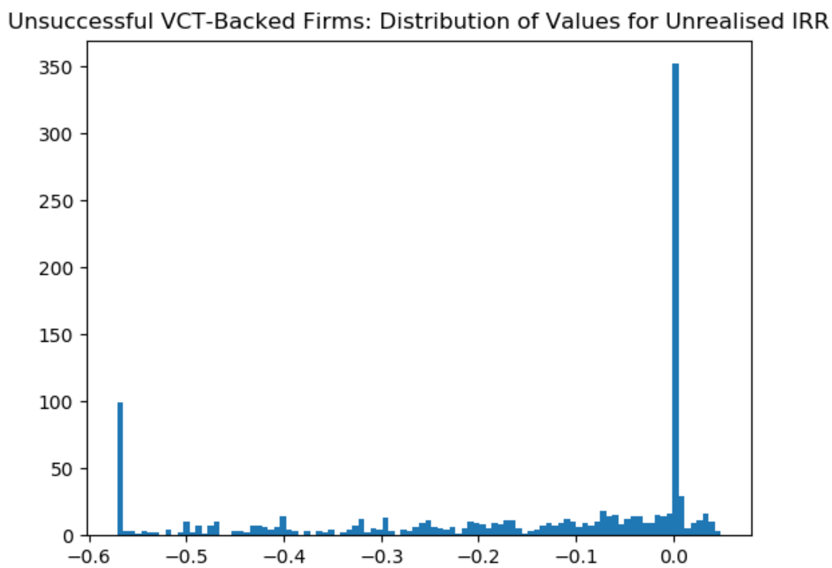

Figure B.14. Unsuccessful VCT-Backed Firms: Unrealised IRR

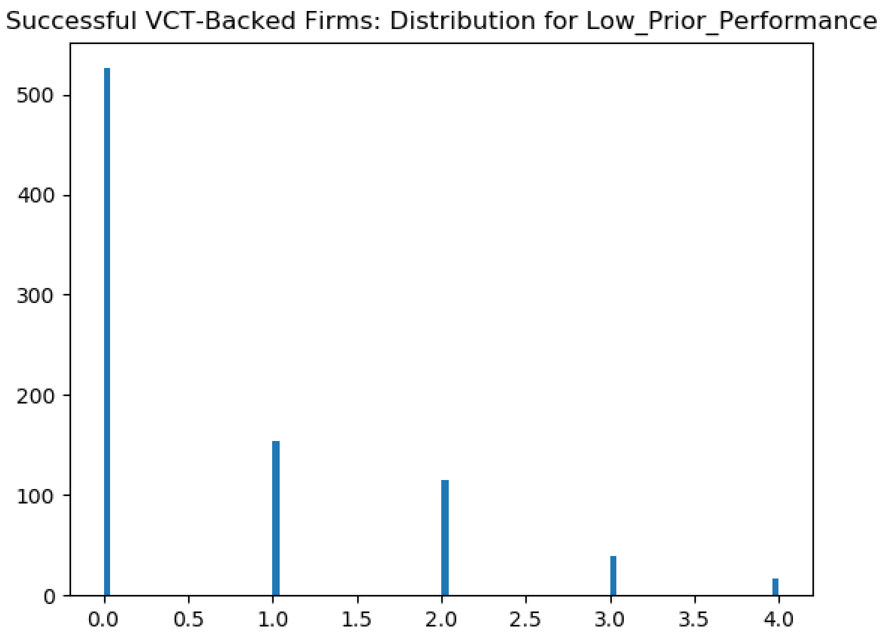

Figure B.15. Successful VCT-Backed Firms: Low Prior Performance

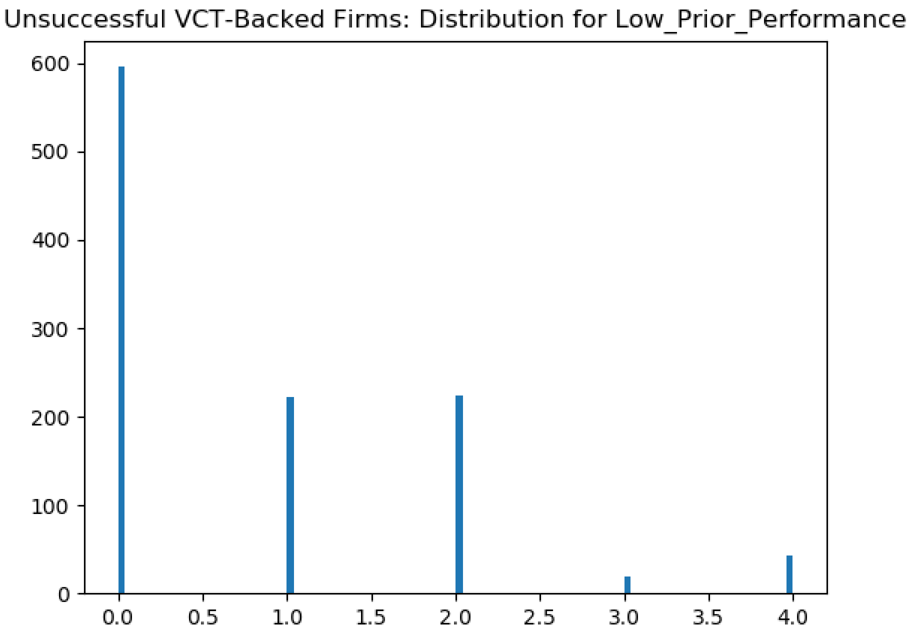

Figure B.16. Unsuccessful VCT-Backed Firms: Low Prior Performance

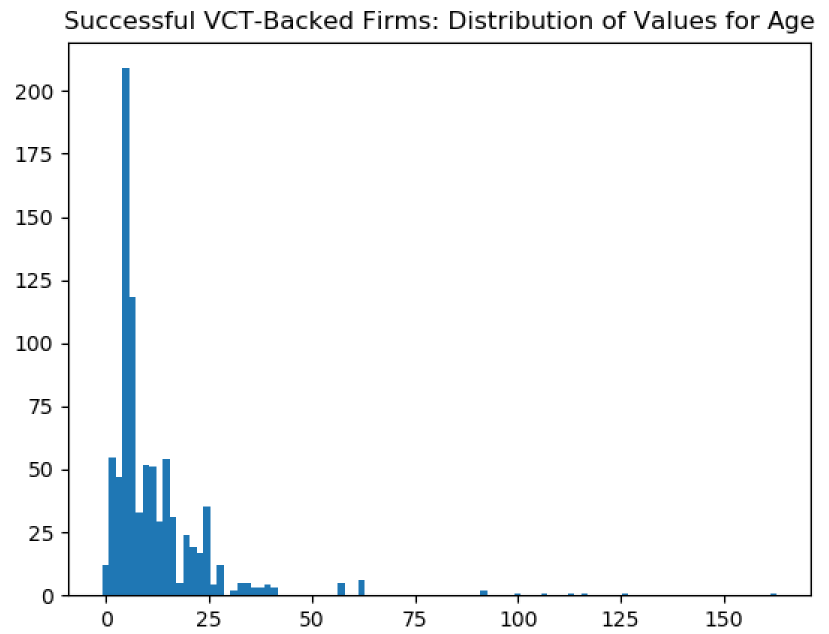

Figure B.17. Successful VCT-Backed Firms: Investee Age

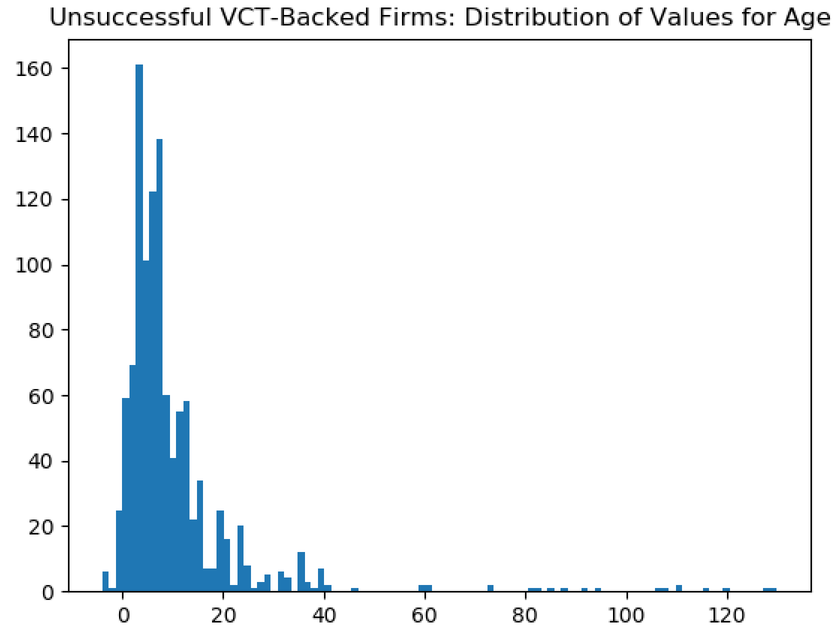

Figure B.18. Unsuccessful VCT-Backed Firms: Investee Age

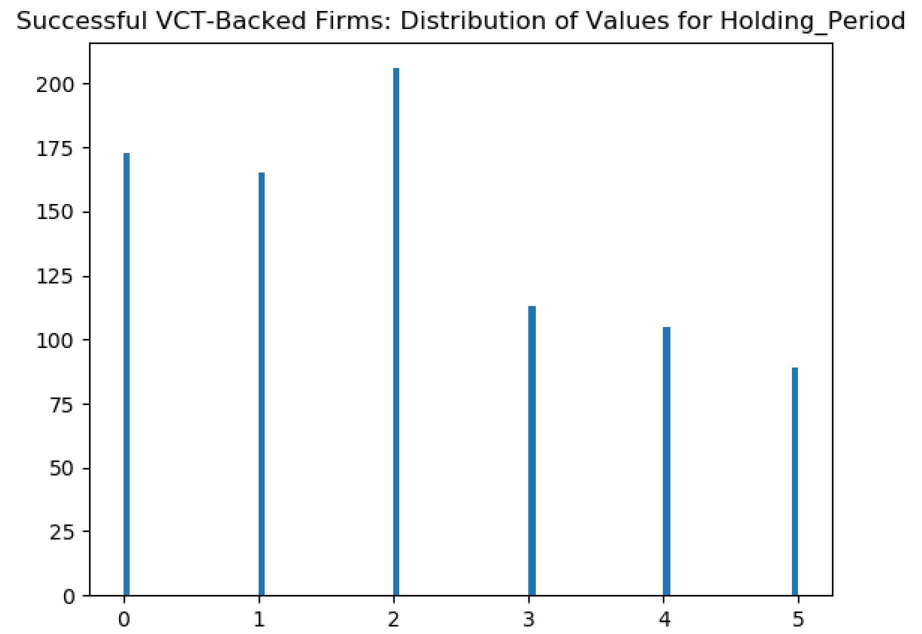

Figure B.19. Successful VCT-Backed Firms: Holding Period (Years)

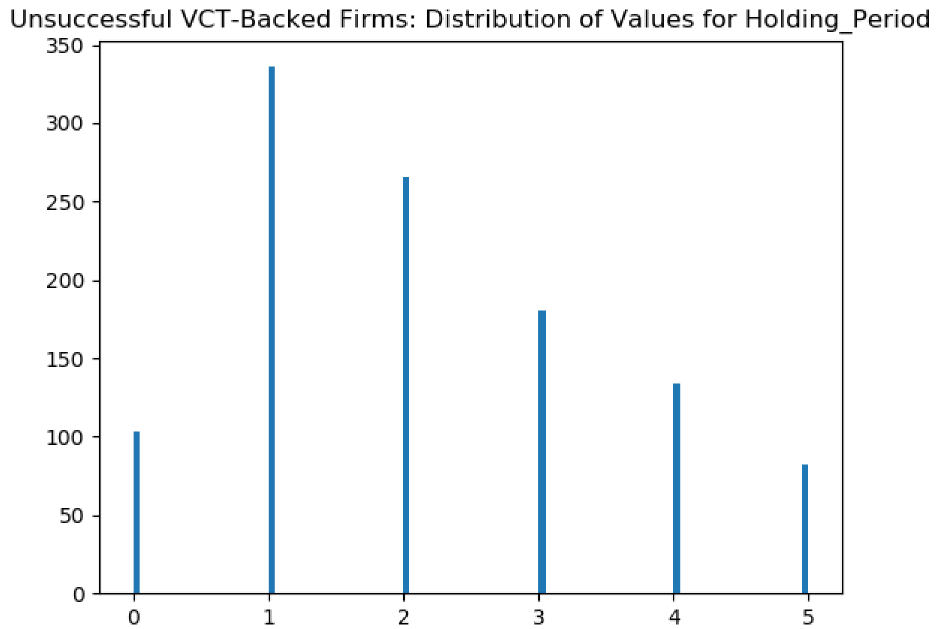

Figure B.20. Unsuccessful VCT-Backed Firms: Holding Period (Years)

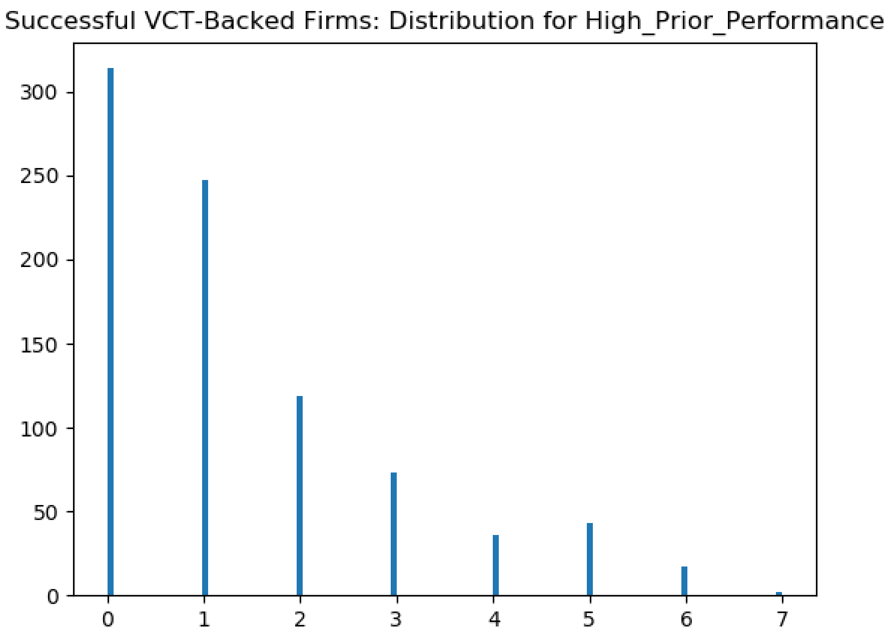

Figure B.21. Successful VCT-Backed Firms: High Prior Performance

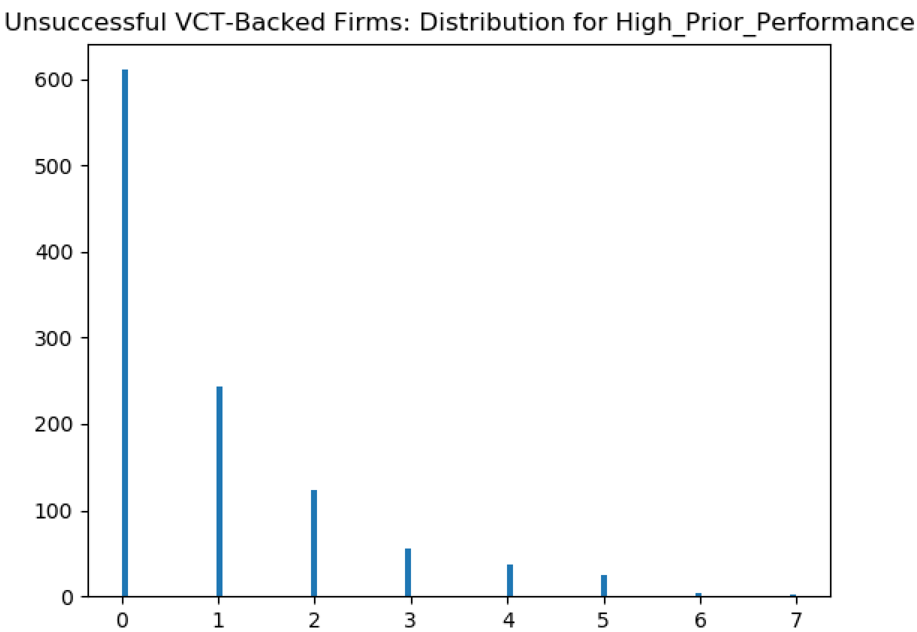

Figure B.22. Unsuccessful VCT-Backed Firms: High Prior Performance

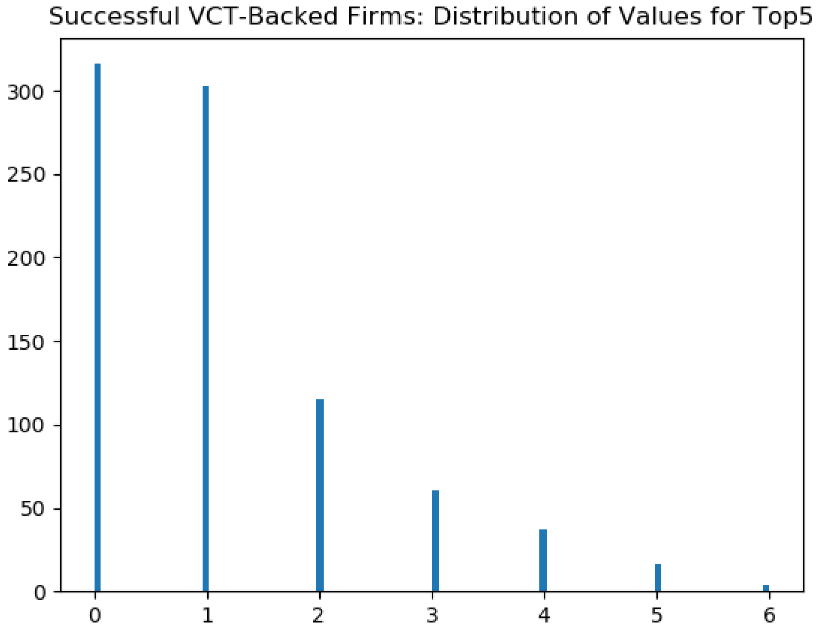

Figure B.23. Successful VCT-Backed Firms: Top 5

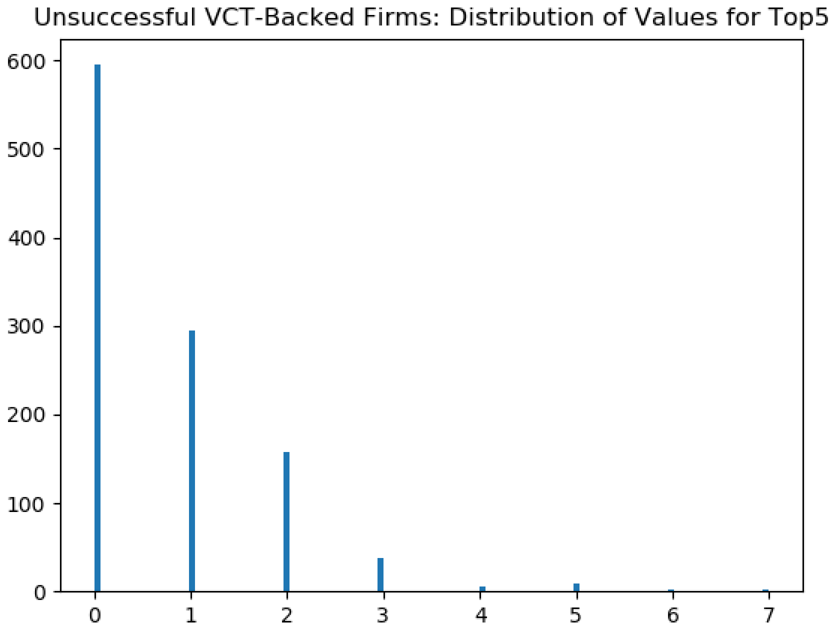

Figure B.24. Unsuccessful VCT-Backed Firms: Top 5

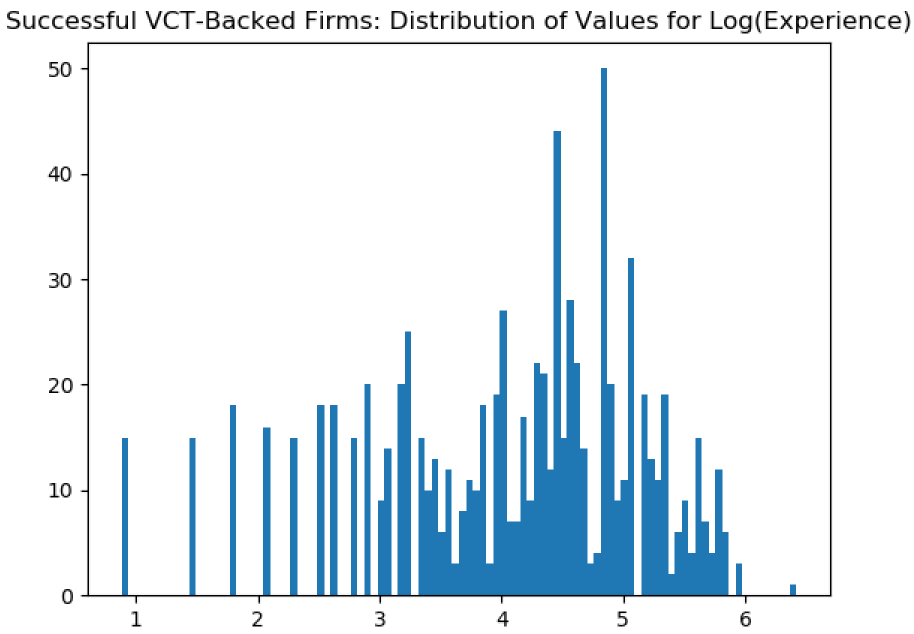

Figure B.25. Successful VCT-Backed Firms: Log(FTSE Funding Experience)

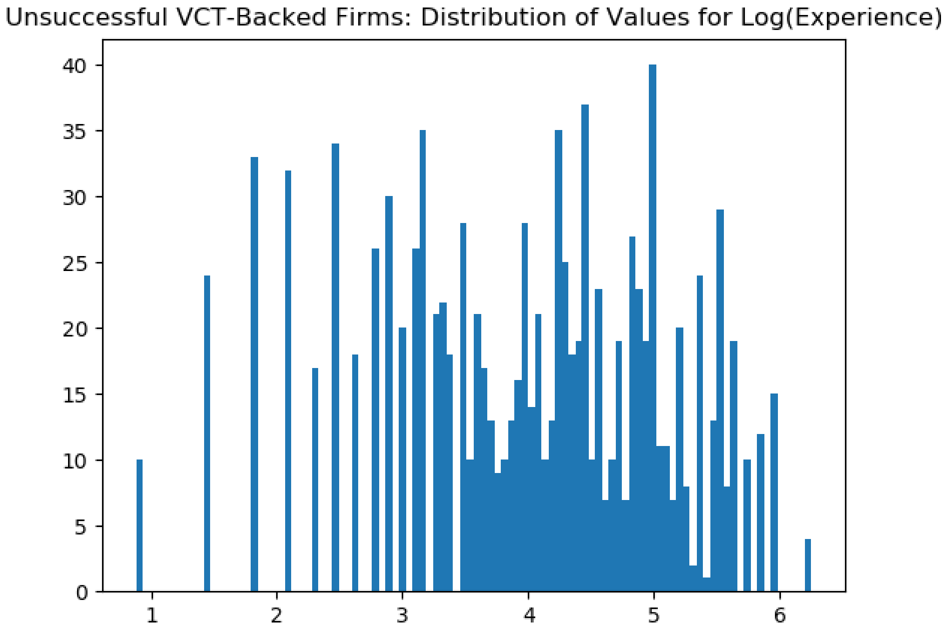

Figure B.26. Unsuccessful VCT-Backed Firms: Log(FTSE Funding Experience)

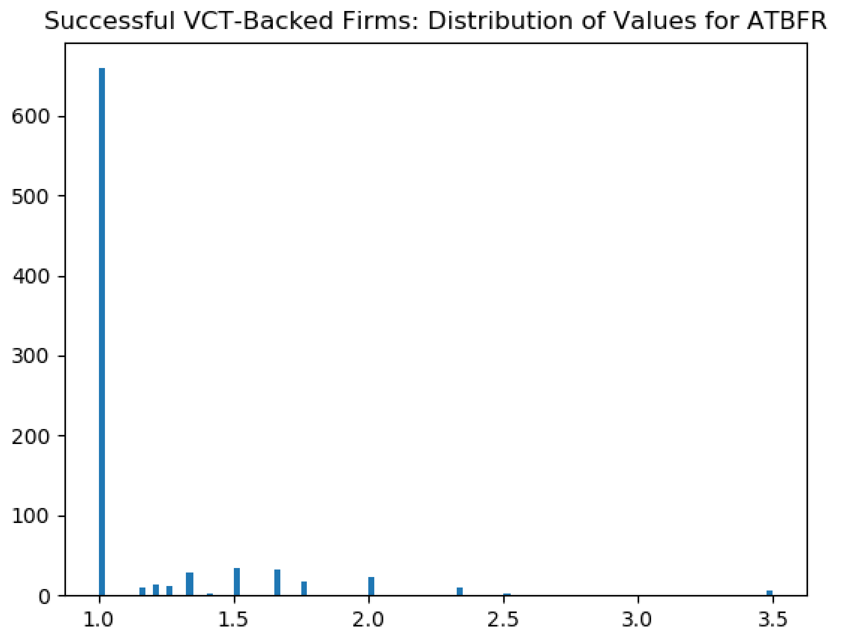

Figure B.27. Successful VCT-Backed Firms: ATBFR(Years)

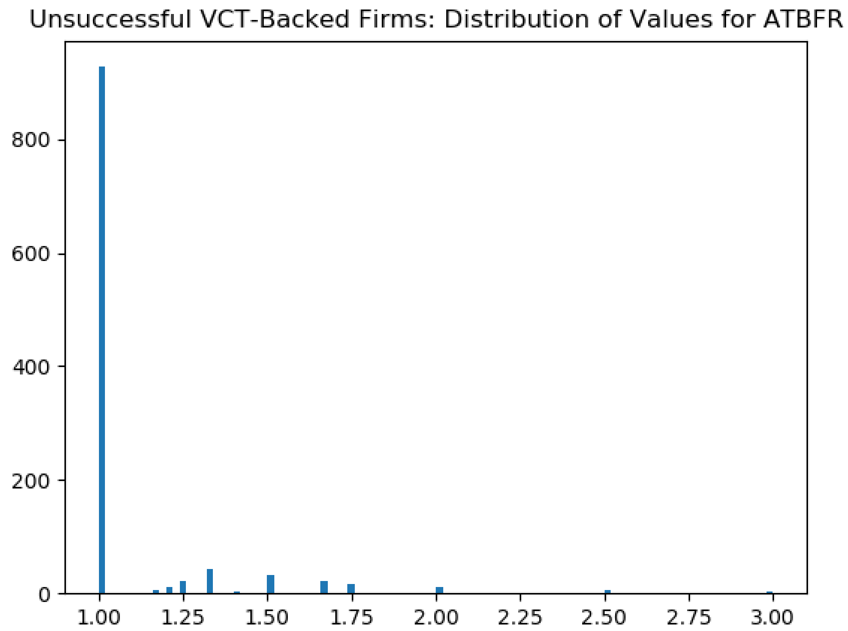

Figure B.28. Unsuccessful VCT-Backed Firms: ATBFR(Years)

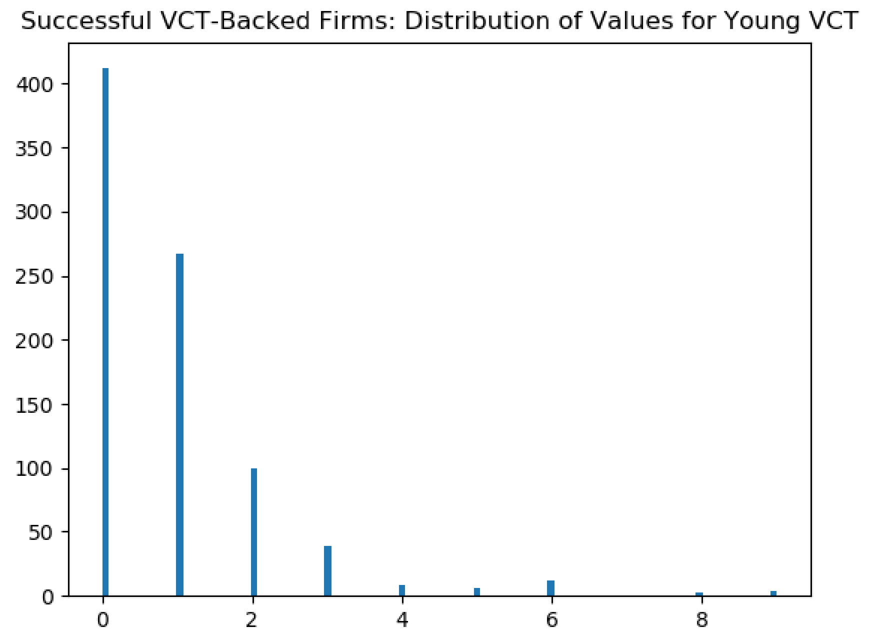

Figure B.29. Successful VCT-Backed Firms: Young VCT

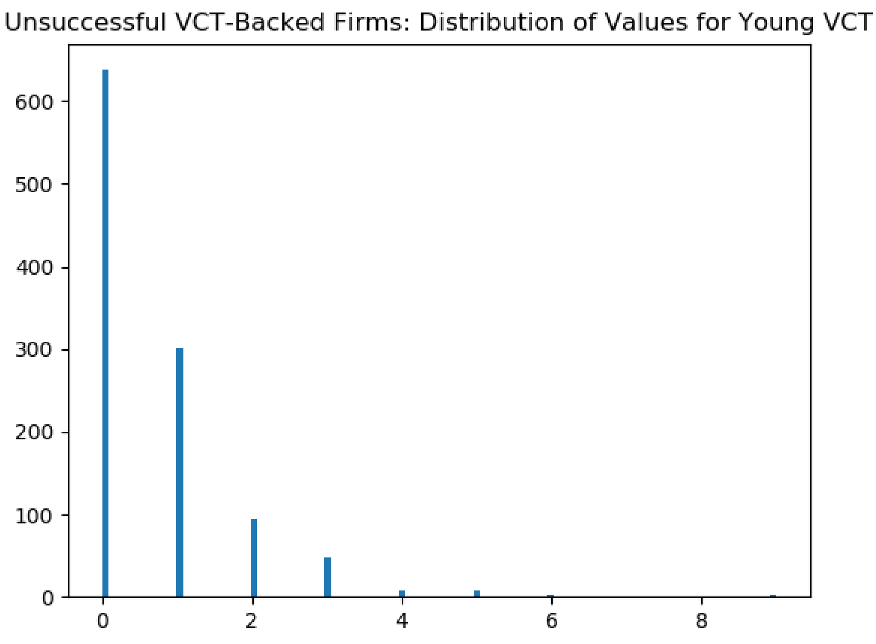

Figure B.30. Unsuccessful VCT-Backed Firms: Young VCT

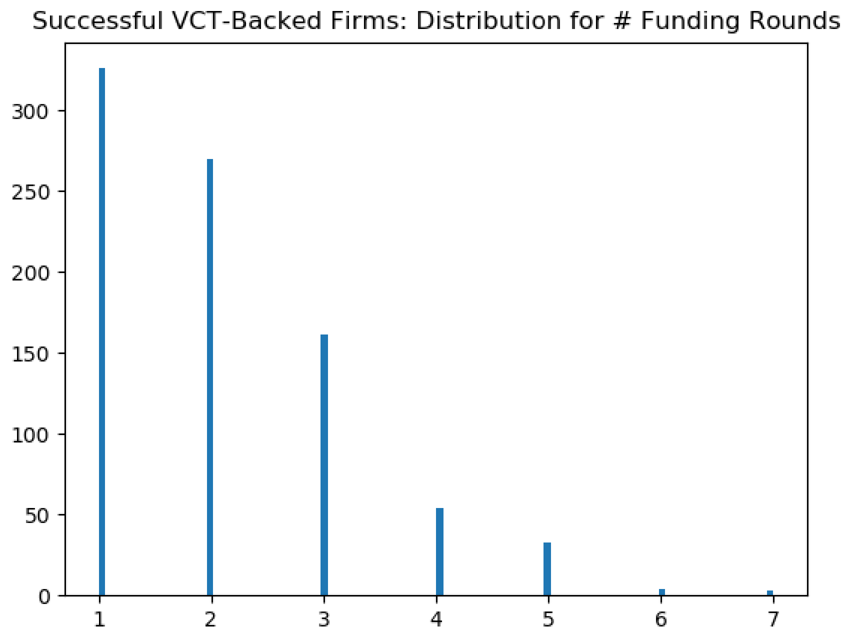

Figure B.31. Successful VCT-Backed Firms: # Funding Rounds

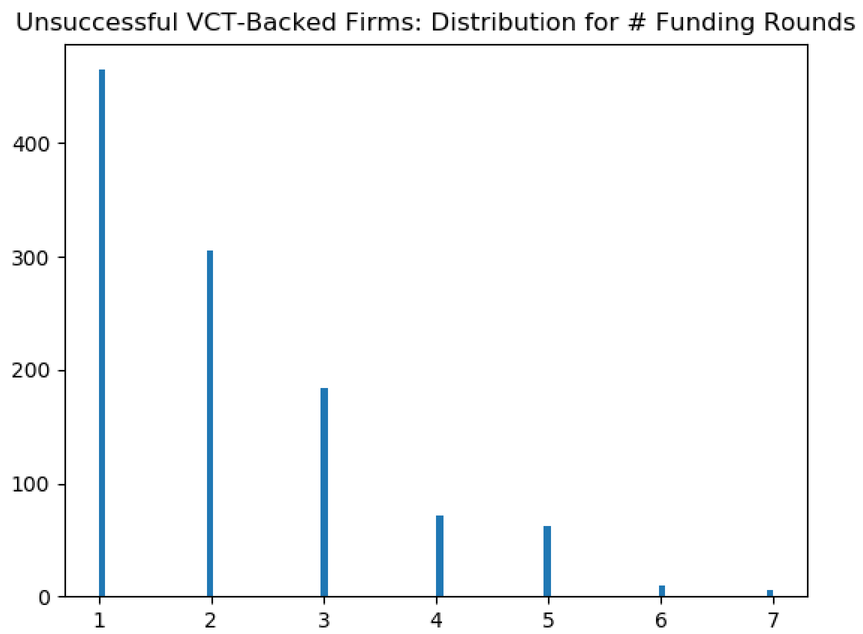

Figure B.32. Unsuccessful VCT-Backed Firms: # Funding Rounds

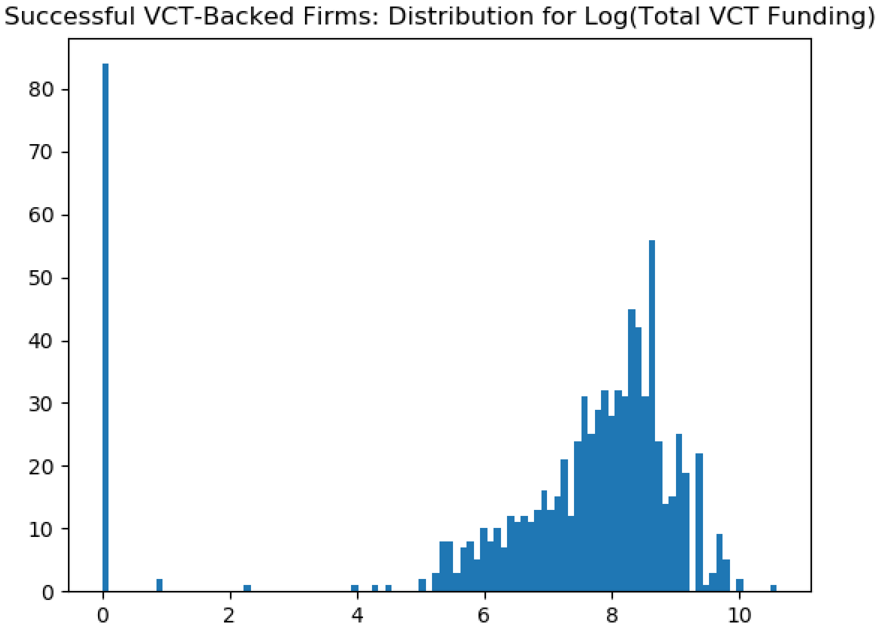

Figure B.33. Successful VCT-Backed Firms: Log(Total VCT Funding)

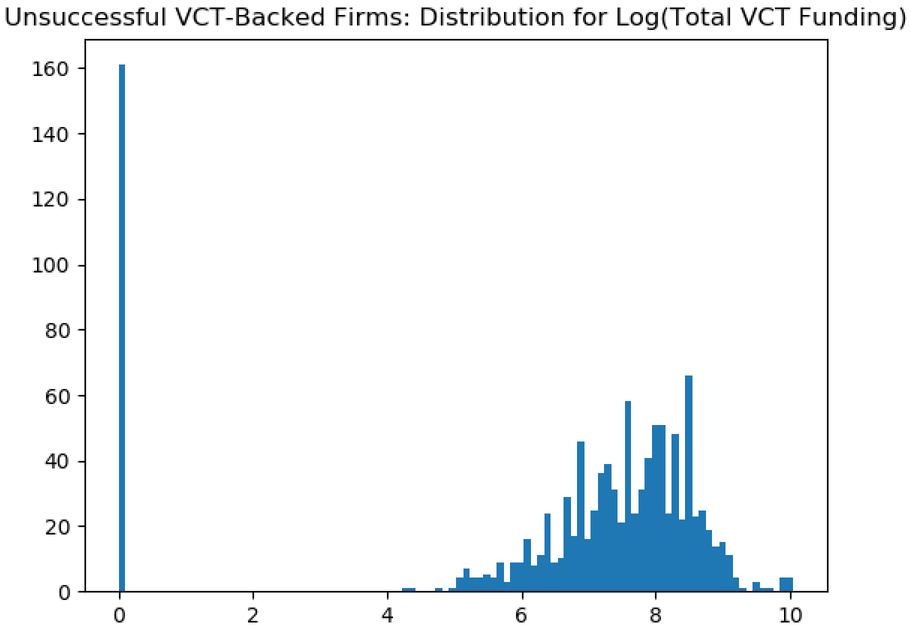

Figure B.34. Unsuccessful VCT-Backed Firms: Log(Total VCT Funding)

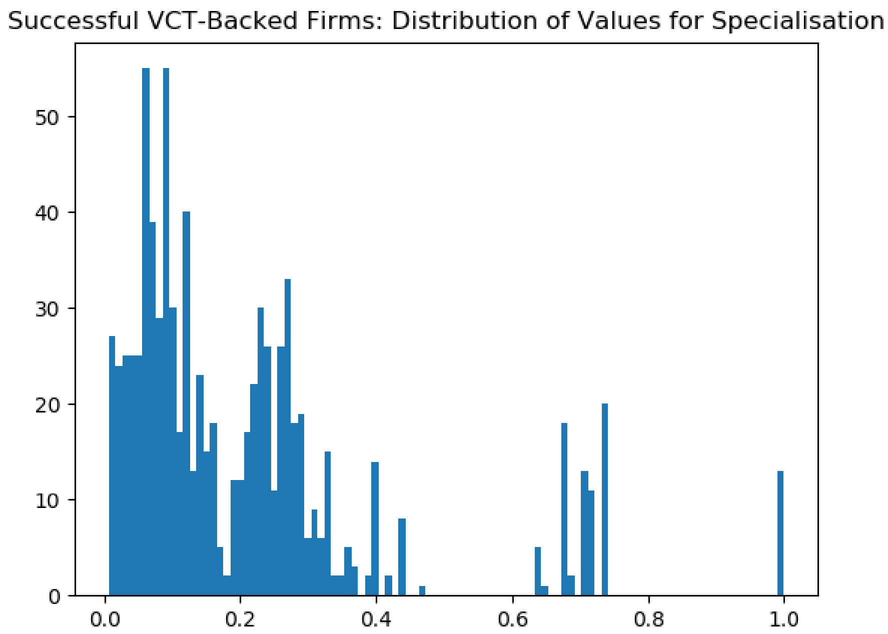

Figure B.35. Successful VCT-Backed Firms: FTSE-Industry Funding Specialisation

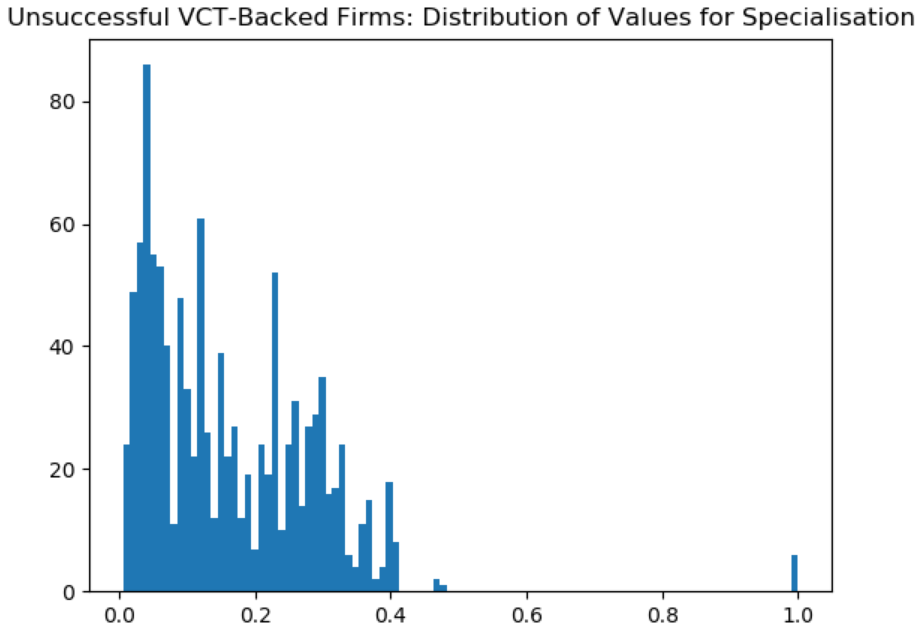

Figure B.36. Unsuccessful VCT-Backed Firms: FTSE-Industry Funding Specialisation

B.3 Distribution of Financial Data for VCT-Backed Firms

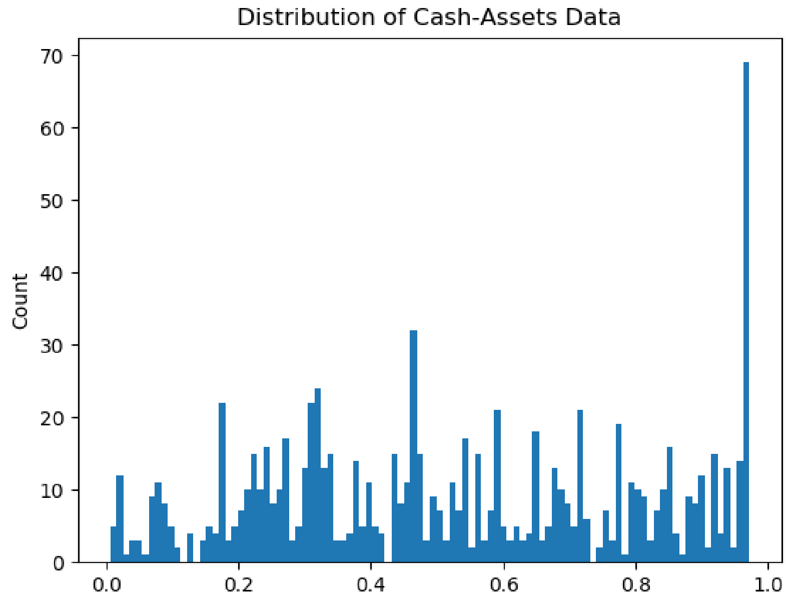

Figure B.37. Cash-Assets

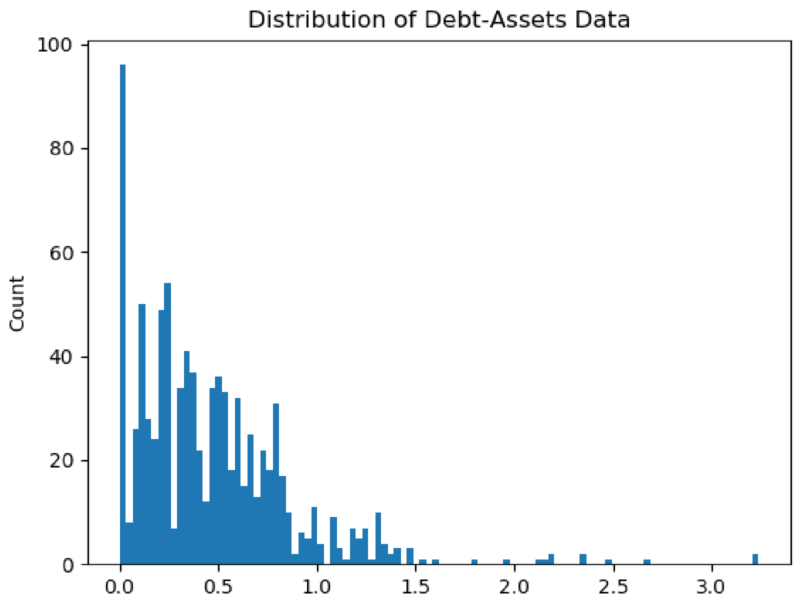

Figure B.38. Absolute Value of Debt-Assets

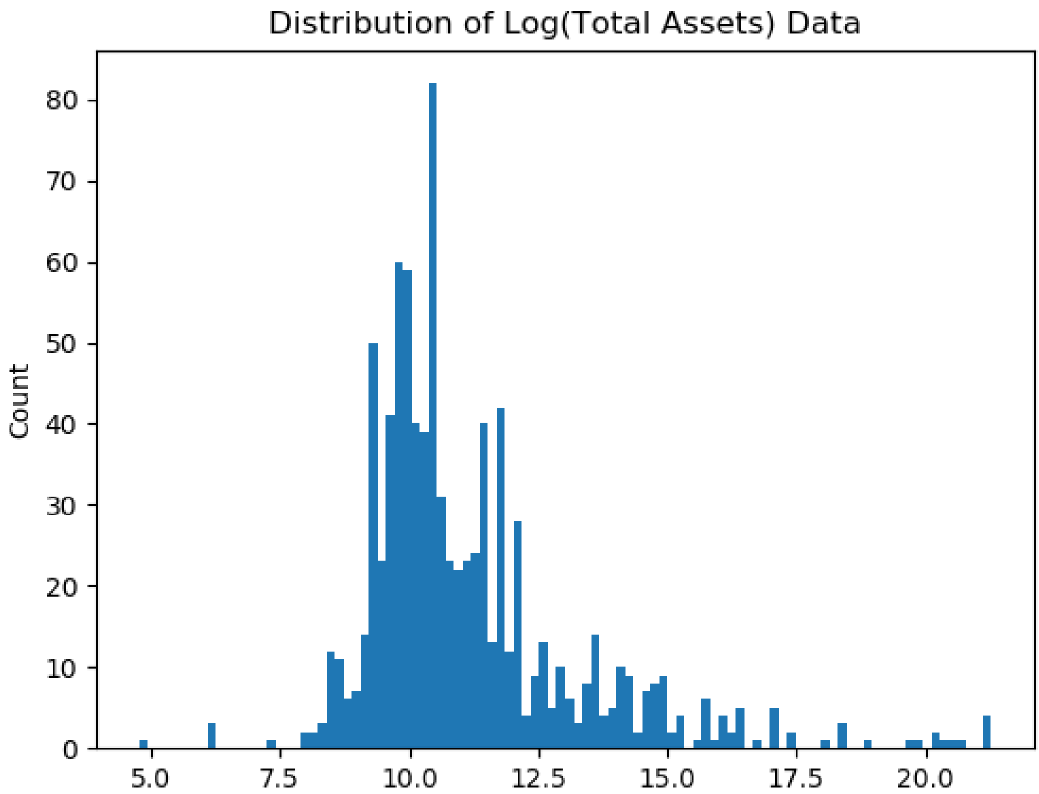

Figure B.39. Log(Total Assets)

B.4 Distribution of Financial Data for Successful vs. Unsuccessful VCT-Backed Firms

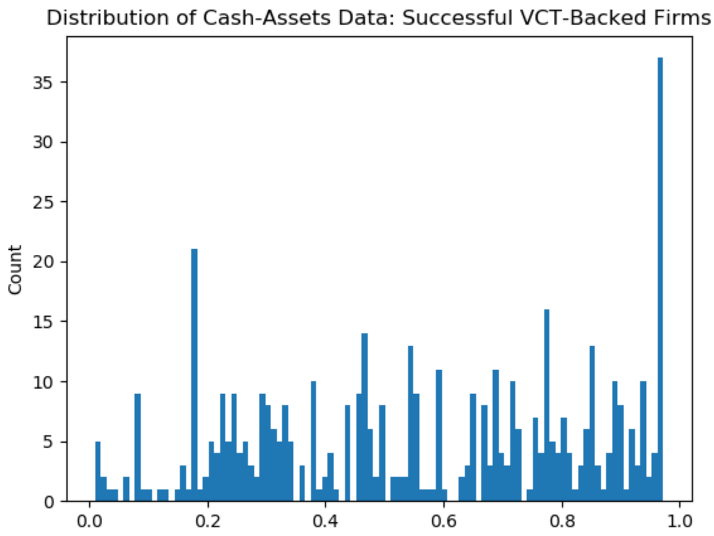

Figure B.40. Cash-Assets

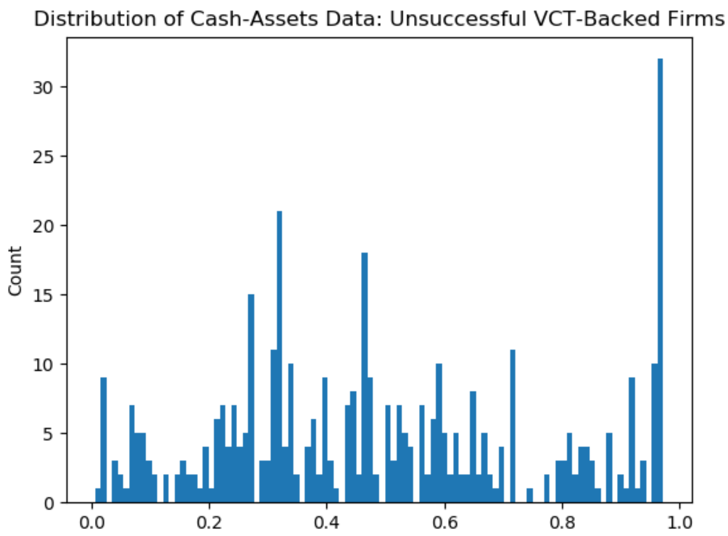

Figure B.41. Cash-Assets

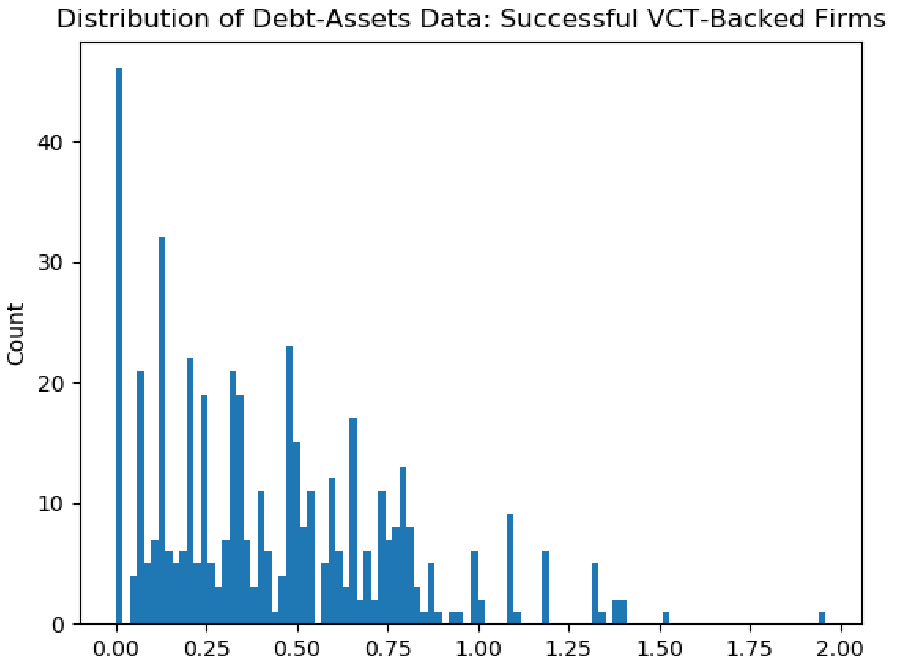

Figure B.42. Absolute Value of Debt-Assets

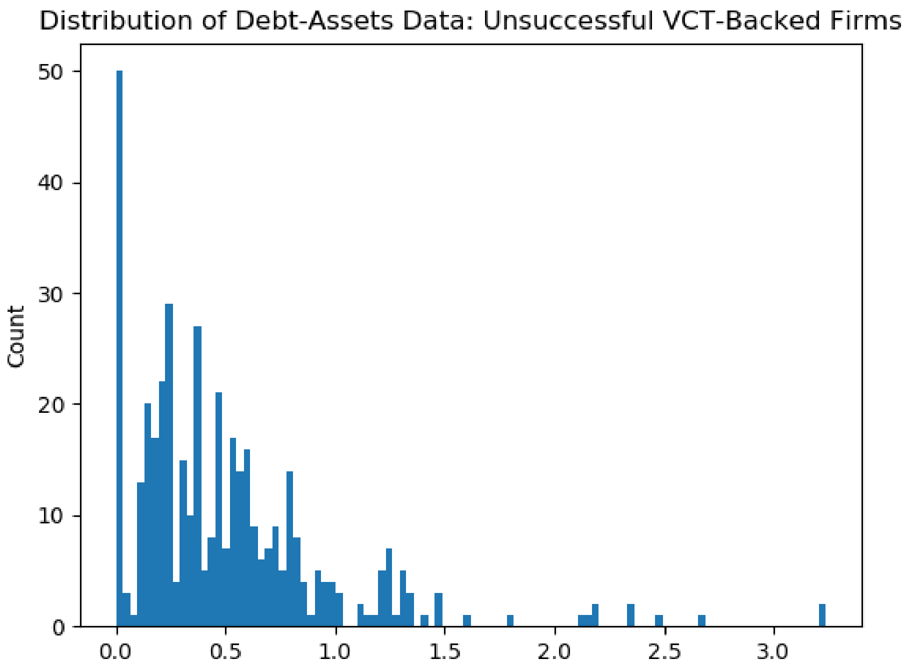

Figure B.43. Absolute Value of Debt-Assets

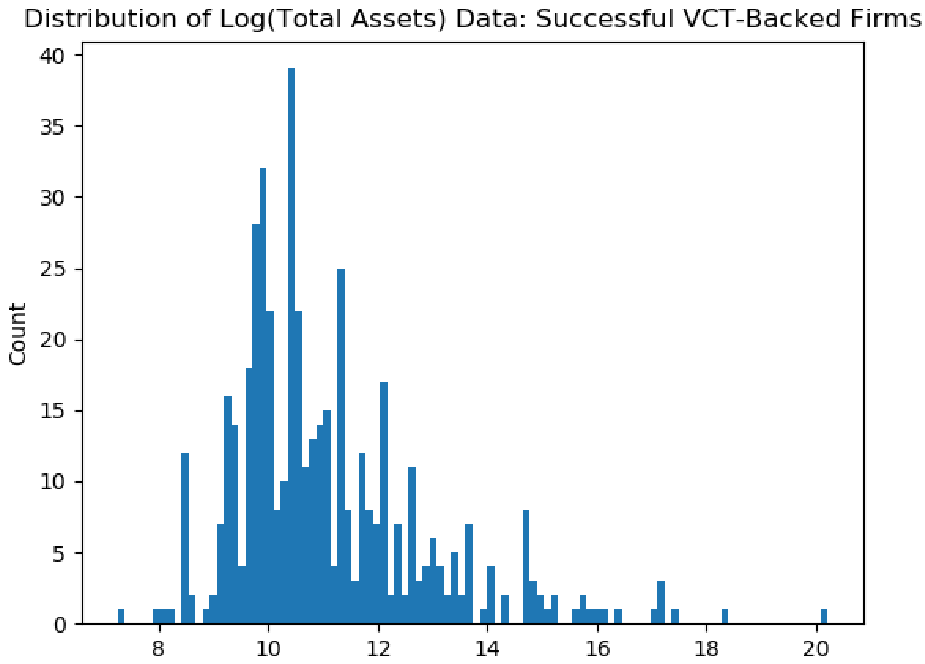

Figure B.44. Log(Total Assets)

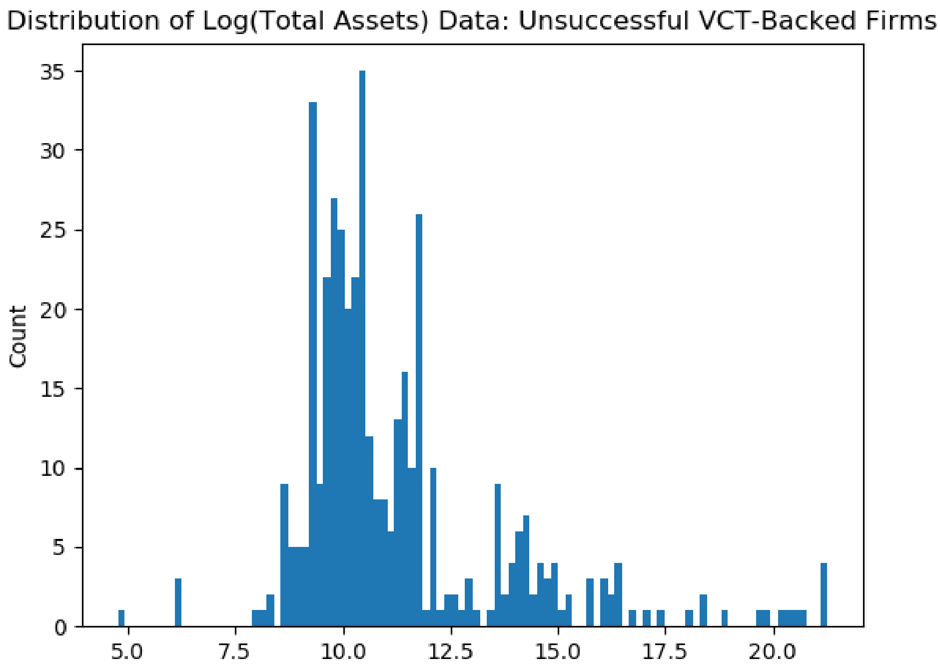

Figure B.45. Log(Total Assets)

B.5 Description of Variables

Firm's first-time VCT backer/s skill: Summed across all first-time VCT backers to form one firm level measure; hand-collected data

- #First-Time VCTs in Top 5: A binary variable equal to 1 if the first time a firm was backed by a VCT, said VCT ranked in the Top 5 of all VCTs in the prior fiscal year. Top 5 is defined according to the Nahata (2008) study.

- Log(First-Time VCTs FTSE Industry Experience Count): For a firm, this is the number of times its first time VCT backer funded firms in its FTSE Industry.

- First-Time VCTs FTSE-Industry Experience / Total Experience (%): For a firm, this is the number of times its first time VCT backer funded firms in its FTSE Industry, divided by the total number of times its first time VCT backer funded firms in all FTSE Industries.

- # First-Time VCTs with Low Prior Performance: A binary variable equal to 1 if the first time a firm was backed by a VCT, said VCT ranked in the bottom quartile of VCT performance, in the prior fiscal year. A VCTs performance is measured as the annual return on it's portfolio of assets.

- # First-Time VCTs with High Prior Performance: A binary variable equal to 1 if the first time a firm was backed by a VCT, said VCT ranked in the top quartile of VCT performance, in the prior fiscal year. A VCTs performance is measured as the annual return on it's portfolio of assets.

- # First-Time VCTs that are Young: A binary variable equal to 1 if the first time a firm was backed by a VCT, said VCT was young. For a firm, the age of each of it's VCT backers is calculated as the difference between the year of funding and the VCTs date of incorporation. Young is defined as fifteen years or younger.

VCT Funding Deal Structure

- Log(Total VCT Funding): Total GBP proceeds raised by a firm in all VCT funding rounds from all its VCT backers.

- ATBFR(Years): The average time between funding rounds, measured in years for firms with multiple funding rounds, is the average of the difference between a firms first and second, second and third, third and $\cdots$ VCT funding rounds.

- Holding Period: For a VCT, the holding period of an investment is the difference between the first time it invested and the fiscal year of the latest valuation of the investment (or the fiscal year it exited the investment).

- VCT Equity Stake: Measured on an annual basis, it is the percentage equity stake of an investee, held by its VCT backer.

- # Funding Rounds: This is the number of VCT funding rounds a firm underwent per fiscal year (max 1 funding round per fiscal year), from the first VCT funding round to the latest VCT funding round.

- Multiple Funding Rounds (%): Measured over the entire fiscal years in our sample, it is the number of investee firms that received multiple VCT funding rounds divided by the total number of investee firms that received VCT funding rounds.

Life Cycle Variable

- Investee Age: Measured from it's VCT backers perspective as the number of years from the date of incorporation to the fiscal year of its most recent valuation.

FAME Financial Variables (Calculated as of the Fiscal Year-End of the Most Recent Valuation; All Ratios are winsorised at the 0.5% and 95% Levels))

- Log(Total Assets): Total assets

- Debt-to-Assets: Long term liabilities plus current liabilities divided by total assets.

- Cash-to-Assets: Current assets divided by total assets.

Table B.1. **Comparing each VCT Investment Manager's Management Fee, Performance Incentive Fee and the Hurdle Rate for the Performance Incentive Fee**

Column 1 is a list of the 44 VCTs in our sample over the 2014-2020 period. Column 2 reports the latest (2020) management fee for each VCT investment manager. Column 3 reports the latest (2020) hurdle rate before each VCT investment manager's performance incentive fee is triggered. Column 4 is a brief description of the hurdle rate for each VCT investment manager's performance incentive fee. Total returns is calculated as the change in NAV plus Dividends paid - over an accounting period.

VCT Name	Management Fee % of NAV	Hurdle Rate (%) for Performance Incentive Fee	Brief Description: Hurdle Rate for Performance Incentive Fee
1	2	3	4
Albion VCT Plc	1.90	3.5	Total Returns Exceeds RPI Inflation plus 2 percent
Kings Arms Yard VCT Plc	2.0	3.5	Total Returns Exceeds RPI Inflation plus 2 Percent
Albion Enterprise VCT Plc	2.0	3.5	Total Returns Exceeds RPI Inflation plus 2 Percent
Crown Place VCT Plc	1.75	2.75	Total Returns Exceeds Average RBS Base Rate plus 2 Percent
Albion Development VCT Plc	2.25	3.5	Total Returns Exceeds RPI Inflation plus 2 Percent
Albion Technology & General VCT Plc	2.5	3.5	Total Returns Exceeds RPI Inflation plus 2 Percent
Downing One VCT Plc	2.75	5.0	Unrealised and/or Realised IRR of an Investment Exceeds 5 Percent
Downing Two VCT Plc (2019)	2.0	7.0	Dividend Growth of 7 Percent
Downing Three VCT Plc (2019)	2.0	7.0	Dividend Growth of 7 Percent
Downing Four VCT Plc	1.83	7.0	Dividend Growth of 7 Percent
British Smaller Companies VCT 2 Plc	2.0	N/A	-
British Smaller Companies VCT Plc	2.0	5.8	Total Returns Exceeds 5.8 Percent.
Octopus AIM VCT Plc	2.0	N/A	-
Octopus AIM VCT 2 Plc	2.0	N/A	-
Octopus Titan VCT Plc	2.0	N/A	Total Returns is Positive
Octopus Apollo VCT Plc	2.0	0.75	Total Returns Exceeds BOE Base Rate.
Chrysalis VCT Plc	1.65	5.0	Realised IRR of an Investment Exceeds 5 Percent.
Molten Ventures VCT Plc	2.0	3.5	Dividend Growth of 3.5 Percent.
Unicorn AIM VCT Plc	2.0	N/A	-
Northern Venture Trust Plc	2.06	6.0	Total Returns Exceeds 6 Percent
Northern 2 VCT Plc	2.06	6.0	Total Returns Exceeds 6 Percent
Northern 3 VCT Plc	2.06	5.7	Total Returns Exceeds 5.7 Percent

(Continued)

Table B.1. *Continued*

VCT Name	Management Fee % of NAV	Hurdle Rate (%) for Performance Incentive Fee	Brief Description: Hurdle Rate for Performance Incentive Fee
1	2	3	4
Amati AIM VCT Plc	1.75	N/A	-
Hargreave Hale AIM VCT Plc	1.70	N/A	-
Hargreave Hale AIM VCT 2 Plc	1.50	N/A	-
Mobeus Income & Growth VCT Plc	2.0	6.0	Dividend Growth of 6 Percent
Mobeus Income & Growth 2 VCT Plc	2.0	8.32	Dividend Growth of 8.32 Percent
Mobeus Income & Growth 4 VCT Plc	2.0	6.0	Dividend Growth of 6.0 Percent
The Income & Growth VCT Plc	2.4	6.0	Dividend Growth of 6.0 Percent
Foresight VCT Plc	2.0	5.5	Realised IRR of an Investment Exceeds 4 Percent plus RPI Inflation.
Foresight Enterprise VCT Plc	2.0	N/A	High Water Mark
Foresight Solar & Technology VCT Plc	1.5	5.0	Total Returns Exceeds 5 Percent.
Calculus VCT Plc	1.75	5.0	Total Returns Exceeds 5 Percent.
Pembroke VCT Plc	2.0	8.0	Total Returns Exceeds 8 Percent.
ProVen VCT Plc	2.0	N/A	-
ProVen Growth & Income VCT Plc	2.0	1.75	Total Returns Exceeds BOE Base Rate plus 1 Percent.
Maven Income & Growth VCT Plc	1.9	N/A	-
Maven Income & Growth VCT 3 Plc	2.5	N/A	Total Returns is Positive
Maven Income & Growth VCT 4 Plc	2.5	N/A	Total Returns is Positive
Maven Income & Growth VCT 5 Plc	1.675	4.0	Realised IRR of an Investment Exceeds 4 Percent.
Baronsmead Venture Trust Plc	2.0	4.0	Total Returns Exceeds 4 Percent.
Baronsmead Second Venture Trust Plc	2.50	8.0	Total Returns Exceeds 8 Percent.
Gresham House Renewable Energy VCT 1 Plc	1.15	N/A	-
Gresham House Renewable Energy VCT 2 Plc	1.15	N/A	-
Number of Observations	44	29	-
Mean	1.97	5.05	-
Median	2.0	5.00	-

Appendix C

Distributions and Description of Variables

C.1 Distribution of VC Skill and Deal Structure Variables

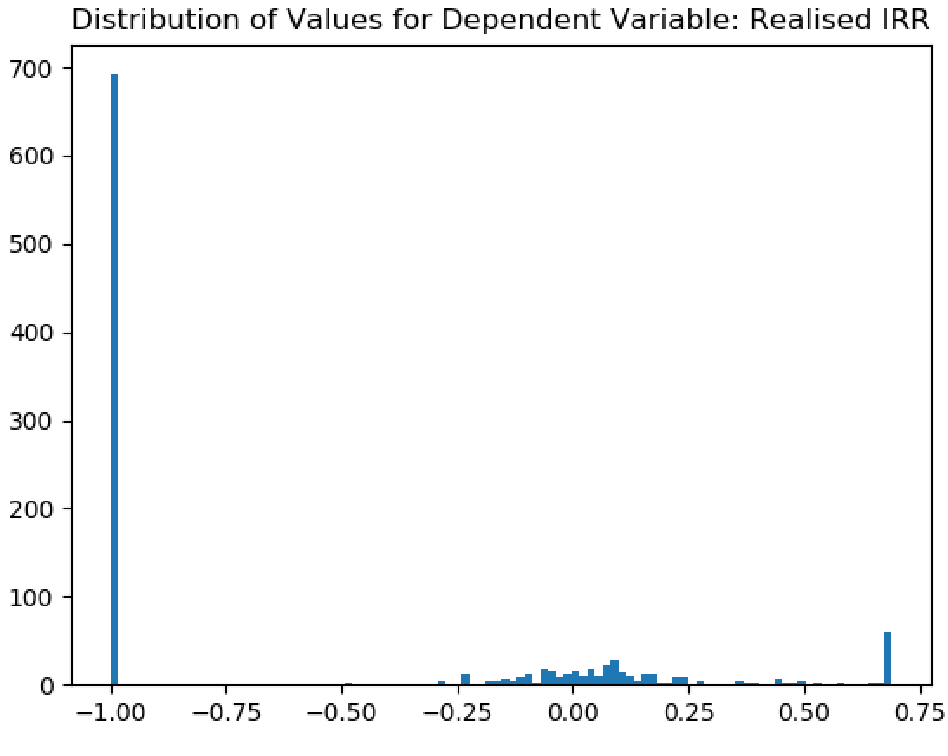

Figure C.1. Realised IRR

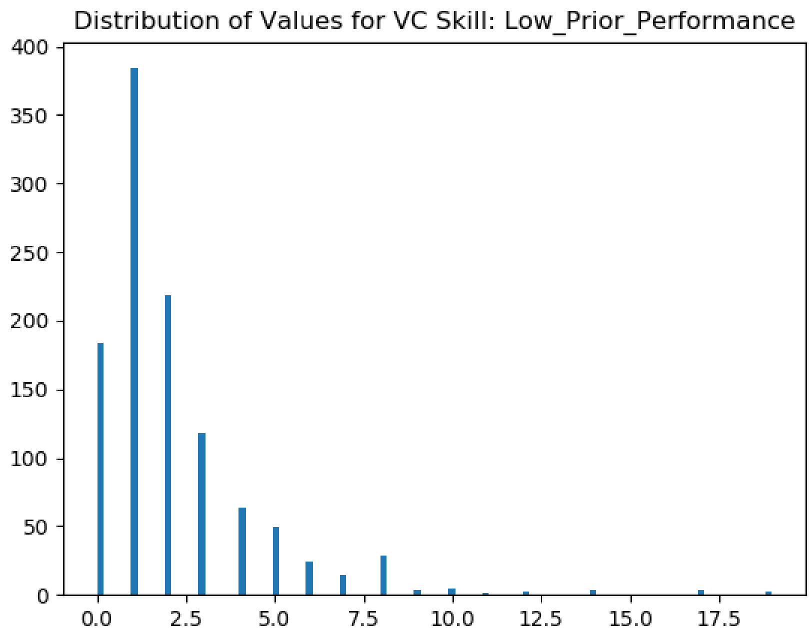

Figure C.2. Low Prior Performance

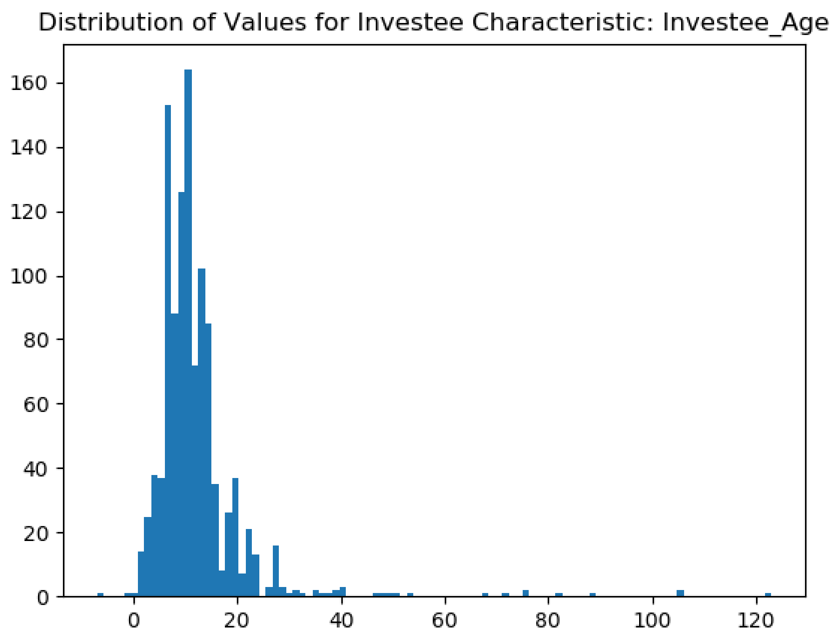

Figure C.3. Investee Age

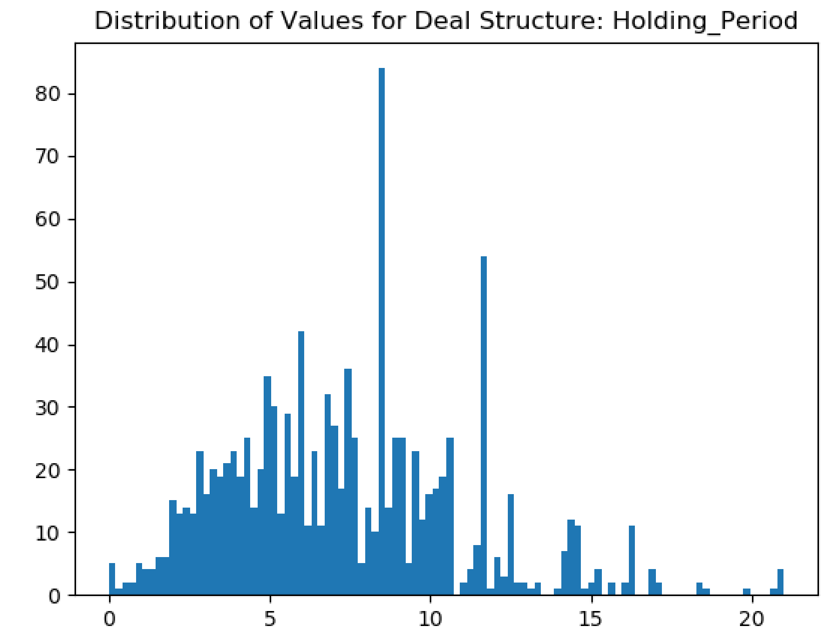

Figure C.4. Holding Period

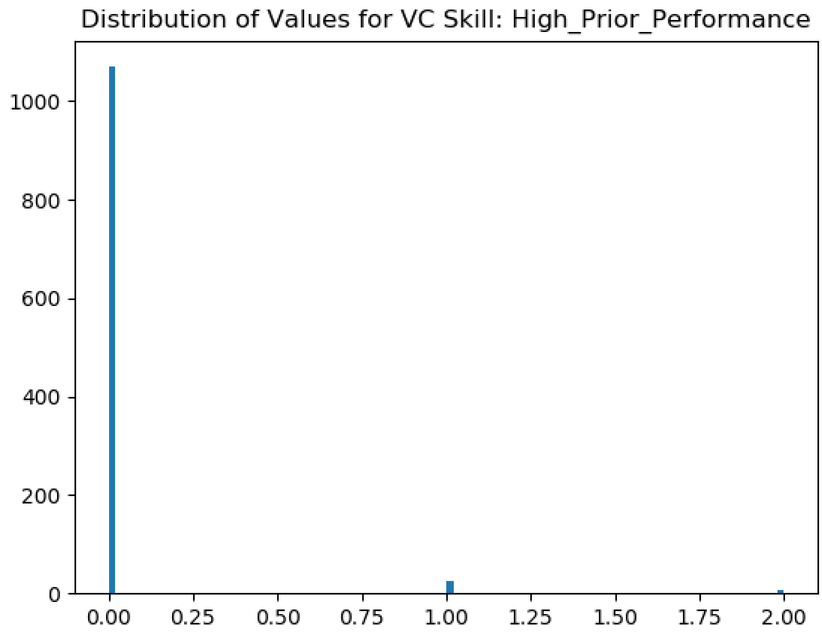

Figure C.5. High Prior Performance

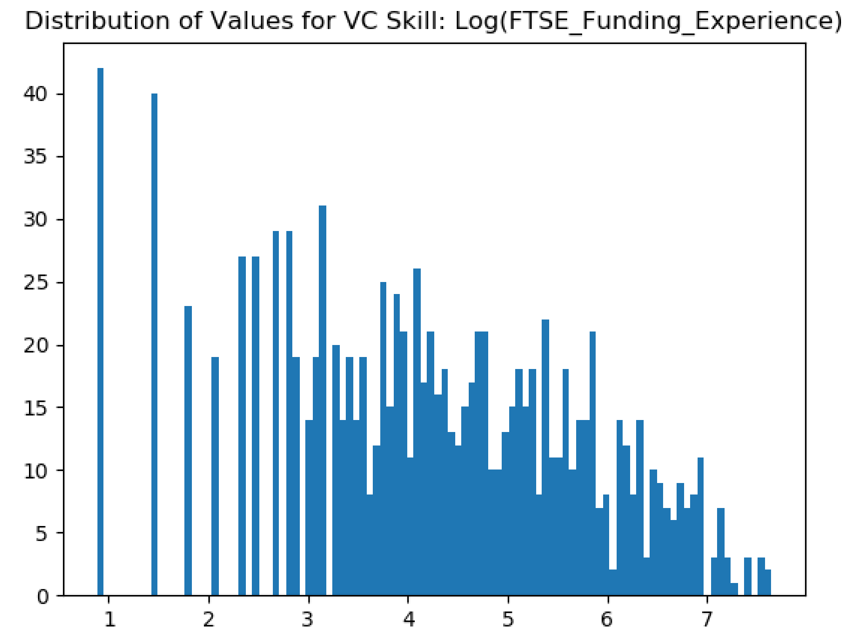

Figure C.6. Log(FTSE Funding Experience)

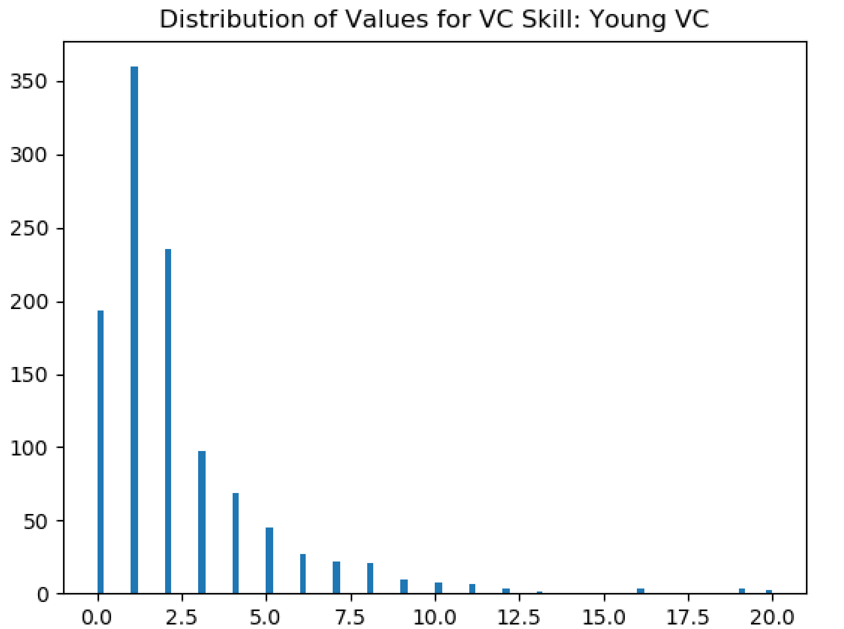

Figure C.7. Young VC

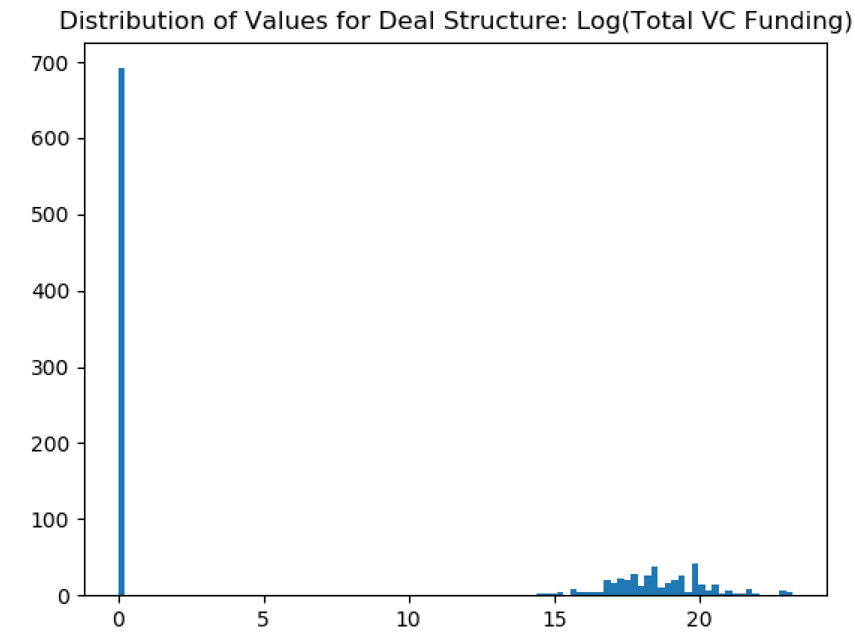

Figure C.8. Log(Total VC Funding)

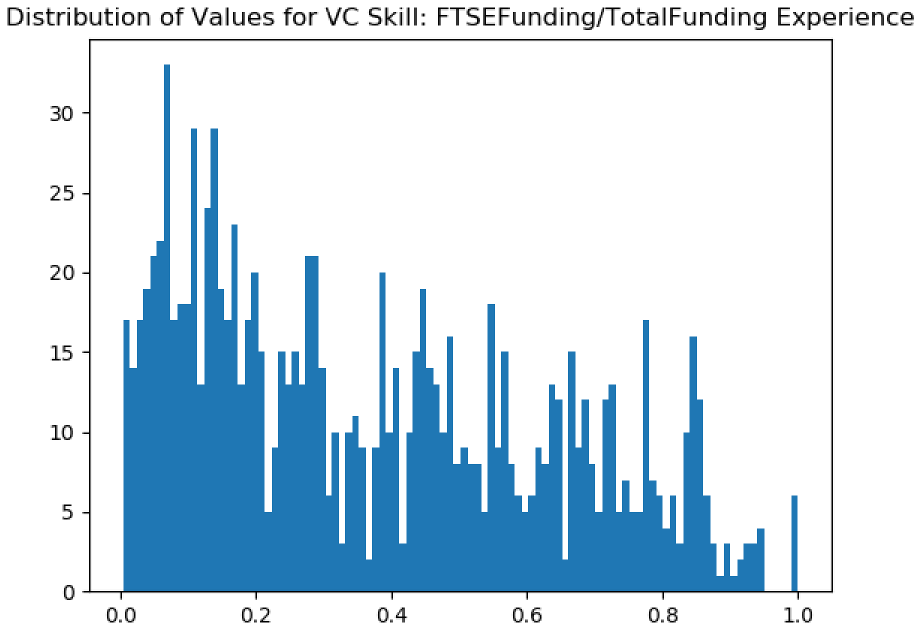

Figure C.9. FTSE-Industry Funding Specialisation

C.2 Distribution of Financial Data for VC-Backed Firms

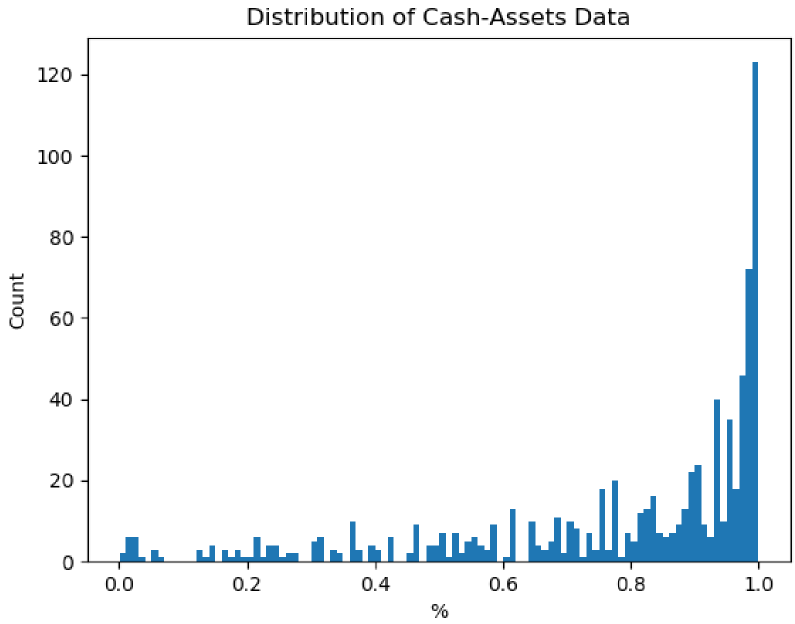

Figure C.10. Cash-Assets

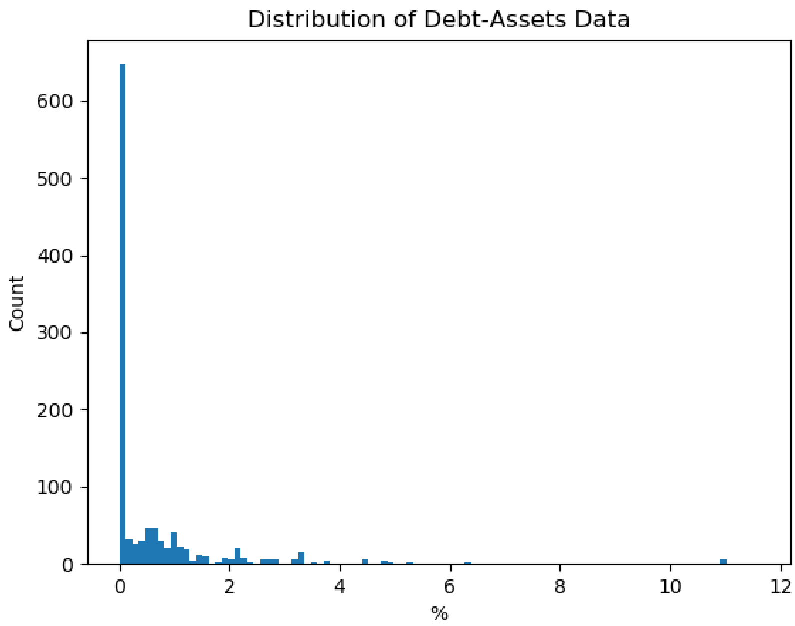

Figure C.11. Absolute Value of Debt-Assets

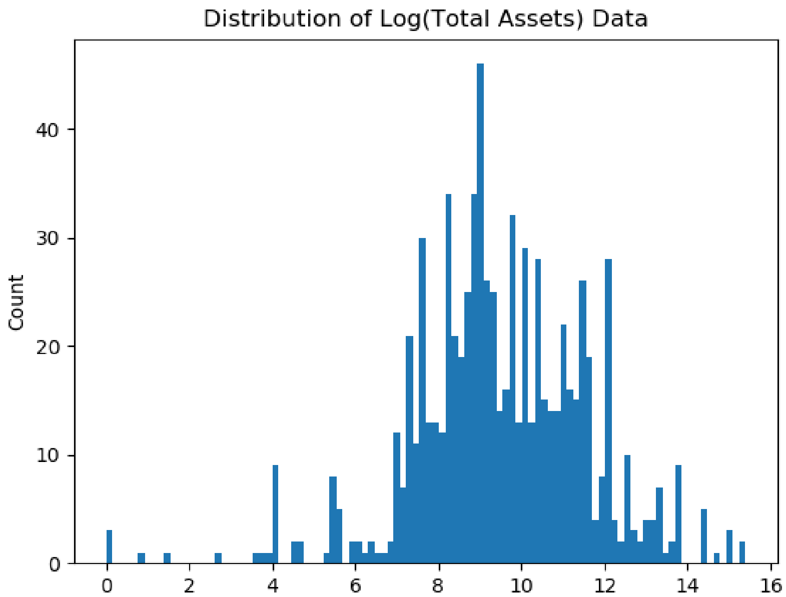

Figure C.12. Log(Total Assets)

C.3 Description of Variables

Firm's first-time VC backer/s skill: Summed across all first-time VC backers to form one firm level measure; Refinitiv Workspace data

- Log(First-Time VCs FTSE Industry Experience Count): For a firm, this is the number of times its first time VC backer, funded firms in its FTSE Industry.
- First-Time VCs FTSE-Industry Experience / Total Experience (%): For a firm, this is the number of times, its first time VC backer, funded firms in its FTSE Industry, divided by the total number of times its first time VC backer funded firms in all FTSE Industries.
- # First-Time VCs with Low Prior Performance: A binary variable equal to 1 if the first time a firm was backed by a VC, said VC ranked in the bottom quartile of VC performance, in the prior calendar year. A VCs prior performance is measured as the number of exits it had in the prior three years divided by the number of investments it made over the 3 years prior to the prior 3 years.
- # First-Time VCs with High Prior Performance: A binary variable equal to 1 if the first time a firm was backed by a VC, said VC ranked in the top quartile of VC performance, in the prior calendar year. A VCs prior performance is measured as the number of exits it had in the prior three years divided by the number of investments it made over the 3 years prior to the prior 3 years.
- # First-Time VCs that are Young: A binary variable equal to 1 if the first time a firm was backed by a VC, said VC was young. For a firm, the age of each of it's VC backer/s is calculated as the difference between the year of funding and the year the VC/s made its first investment. Young is defined as fifteen years or younger.

VC Funding Deal Structure

- Log(Total VC Funding): Total GBP proceeds raised by a firm in all VC funding rounds from all its VC backers.
- Holding Period: For a VC, the holding period of an investment is the difference between the first time it invested and the calendar year it exited the investment.
- # Funding Rounds: This is the number of VC funding rounds a firm underwent before the eventual exit.

Life Cycle Variable

- Investee Age: Measured as the number of years from the date of incorporation to the calendar year of the exit.

FAME Financial Variables (All Ratios are winsorised at the 0.5% and 95% Levels))

- Log(Total Assets): Total assets
- Debt-to-Assets: Long term liabilities plus current liabilities divided by total assets.
- Cash-to-Assets: Current assets divided by total assets.