Can "Ugly Veg" Supply Chains Reduce Food Loss?

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Abstract

The tradition of marketing only aesthetically agreeable produce by retailers contributes to a major source of food loss through "ugly veg", i.e., the produce that does not look "regular". In this paper, we examine the relations between different tiers of agri-food supply chains to study the impact of marketing ugly veg on different supply chain members and the food loss in the system. We examine and compare scenarios of a centralized supply chain, a traditional supply chain without ugly veg, an ugly veg supply chain with a single retailer offering both regular produce and ugly veg, and a two-retailer supply chain where an auxiliary retailer sells the ugly veg. We characterize the equilibrium decisions in these systems and also provide analytical results and insights on the effectiveness of different supply chain designs based on a comprehensive numerical study. We demonstrate the conditions under which the supply chain can reduce overall food loss. For sufficiently high cost of effort, selling ugly veg through the single retailer reduces food loss. Nonetheless, the grower is generally better off offering the ugly veg to an auxiliary retailer. We show that the ratio of food loss per cultivated land always decreases in the two-retailer supply chain, while the total food loss might increase for sufficiently high cost of effort.

Keywords: Agri-food supply chains, Food loss, Constraint optimization, Non-cooperative game theory.

1 Introduction

Almost half of all fruit and vegetables are lost and wasted overall (FAO, 2021b), and this occurs all along the agri-food chain, from agriculture to consumption (Beretta et al., 2013; Redlingshöfer et al., 2017; de Gorter et al., 2020; Joensuu et al., 2020). We refer to Luo et al. (2022) for a review of the research on food loss and waste from the perspective of operations management. In this article, we focus on on-farm loss that accounts for more than 20% of total fruit and vegetables production in Europe. There are many reasons for on-farm loss, including inadequate harvesting, climatic conditions, and mismatch between supply and demand (FAO, 2019). Among these, stringent specifications from buyers in terms of size, weight, and aspect constitutes a major source (House of Commons, 2017). The fruit and vegetables that do not meet the specifications are generally deemed unsalable and discarded at farms, even if they are perfectly edible. For instance, about 25% to 30% of carrots do not reach grocery stores because of physical or aesthetic defects (FAO, 2021a). It is also estimated that 5% to 25% of apples, 9% to 20% of onions, and 3% to 13% of potatoes are rejected for cosmetic reasons in England (House of Commons, 2017).

In line with the growing concerns over food waste and loss, a recent trend in practice is selling the misshaped fruit and vegetables, instead of discarding them. Misshaped fruit and vegetables are often referred to as "ugly" or "wonky", which does not necessarily reflect low nutritional quality. Initiatives to market ugly fruit and vegetables include Morrisons' "naturally wonky" product line in the UK, as well as many specialized retailers such as Wonky Veg Box (UK), Wonky Food Company (UK), Oddbox (UK), Nous Anti-gaspi (France), Etepetete (Germany), Misfits Market (USA), Imperfect Foods (USA), and Hungry Harvest (USA). A nascent stream of research aims at understanding the implications of these initiatives (Loebnitz et al., 2015; De Hooge et al., 2017; Louis and Lombart, 2018; Tu et al., 2018; van Giesen and de Hooge, 2019). Overall, these studies highlight that food shape abnormalities influence consumers' purchase intentions and that this influence depends on demographics, personality, and awareness about food waste. The literature also shows that the positioning of retailers towards sustainability and authenticity plays a role together with price discounts.

In this article, we study the supply chain of ugly fruit and vegetables, hereafter ugly veg, with an operational research lens. We consider a stylized dual channel supply chain model in which the ugly veg is proposed together with the regular produce. The demand is modeled to be price- and quality-sensitive. We examine two supply chain structures: (i) a main retailer supplying both types of produce, or regular produce only, and (ii) an auxiliary retailer marketing the ugly veg, alongside the main retailer that sells regular produce. An example of the first scenario is the UK-based retailer Morrisons and its "naturally wonky" vegetables. UK-based grocery suppliers that specialize in ugly veg, such as Wonky Veg Box, Wonky Food Company, and Oddbox, are examples of the second scenario. We study consumers', retailers', and grower's decisions and provide a characterization and interpretation of equilibrium states. We examine the impact of these two structures in terms of on-farm food loss and discuss the decisions of supply chain members. We compare these structures with the traditional supply chain without the ugly veg offering and also centralized structures where all decisions are taken unilaterally. More specifically, the paper intends to answer the following research questions: Can the supply of ugly veg to the market reduce food loss? Which supply chain structure is more efficient in reducing food loss? Under which conditions would the grower be better off selling the ugly veg, instead of discarding them?

Our contribution is four-fold:

- We provide the first analytical models for the retailing of ugly fruit and vegetables that take into account both the grower and the retailers. We consider two supply chain structures in line with practice. We account for the grower's, the retailers', and the consumers' reactions and characterize marketed quantities and on-farm food loss.

- We characterize analytically the best responses and the subgame-perfect Nash equilibria (SPNEs) under various supply chain structures studied. We derive the SPNEs for different players and their variables and discuss the features of equilibrium states.

- We show the obstacles for reducing food loss in the supply chain. This includes, among others, lack of a selling channel in the chain for the ugly veg, grower's preference for discarding ugly veg to avoid cannibalization effect, and the competition between selling channels in certain situations.

- We draw insights regarding the various possibilities in the supply chain and elaborate on situations under which the supply of ugly veg in the market results in less food loss. Our results show that the marketing of ugly veg is more likely to develop through an independent channel. However, this practice might also harm traditional retailers' revenues due to increased competition. Selling ugly veg through a dedicated retailer always reduces on-farm food loss per unit cultivated land, while it might lead to an absolute increase in the total loss under certain conditions.

The rest of the article is organized as follows. Section 2 provides a review of the related literature. In Section 3, we describe the supply chain structures considered and the sequence of decisions made. Section 4 is devoted to the study of the traditional supply chain which only caters the regular produce to the market. Section 5 explores the case where the traditional retailer markets the ugly veg. The setup with two competing retailers is studied in Section 6. Section 7 focuses on centralized settings. Section 8 is dedicated to our numerical analysis. Finally, we conclude in Section 9. All proofs appear in the Supplementary Material.

2 Literature Review

Our research falls into the field of agri-food supply chains. There is a long history of related contributions. We refer to Borodin et al. (2016), Soto-Silva et al. (2016), and Bloemhof and Soysal (2017) for in-depth reviews. Throughout this section, we focus on three streams of research that are closely related to our contributions. First, we review the supply chain literature on co-production and by-product systems. Second, we consider the interactions between farmers and retailers through contracts. Third, we discuss the literature on dual channel supply chains.

Our work is closely related to the literature on co-production that studies the simultaneous production of co-products. This occurs in various contexts such as the chemical, mineral, and semiconductor industries. We refer to Boyabatlı (2015), Liu et al. (2020) He et al. (2022) and Hilali et al. (2022) for examples of recent contributions in this field. In some settings, the co-products are vertically differentiated as they serve the same purpose but their quality varies. This applies, for instance, to the semiconductor industry. This is also particularly relevant in the agri-food industry and our focus on ugly fruit and vegetables is a noticeable example. The vast majority of the literature on co-production of vertically differentiated products focuses on fixed proportion co-production systems for which the quality of the production output is not controlled, see e.g. Bansal and Transchel (2014), Chen et al. (2017b) and Zhou et al. (2020a). Under this setting, one key decision is the classification of the production output into different grades, which are then proposed to consumers as vertically differentiated products. This practice is referred to as product line design. The contributions on product line design focus mainly on single echelon and single channel setting (Chen et al., 2013; Wang and Gutierrez, 2022), while Lu et al. (2019) is a noticeable exception. The authors study vertically differentiated co-products sold by a manufacturer to a distributor. The manufacturer determines its production, product line design, and wholesale prices while the distributor determines its purchase quantities and retail prices. They show, among others, that there exists a theoretical contract that eliminates indirect channel distortions. Our setting fundamentally differs from these studies in that we consider co-production systems with controllable proportion. Indeed, the specifications for regular produce and ugly veg are not under the control of growers, thus product line design is not relevant in our context. Instead, we assume that the grower can decide on the share of the regular produce and the ugly veg through an investment in production quality. Besides, we extend the setting from Lu et al. (2019) to a dual-channel context. Finally, one of our purposes is to study the impacts of supply chain decisions on food loss. Note that some articles recently study sustainability aspects in co-production systems. Lin et al. (2020) focus on a fixed proportion co-production system for which the low quality item can serve a market segment with environmentally-conscious consumers. They show that there are conditions under which the firm should not utilize the low quality product's environmental value. If these conditions do not hold, the firm may strategically abandon some traditional consumers and take advantage of the low quality product's environmental value. Jin et al. (2022) study co-products made of leftover materials from traditional manufacturing and they develop a game-theoretical model to investigate the economic and environmental implications of this type of co-production. They formulate the conditions under which the traditional manufacturer should introduce co-production as well as the conditions which rationalize manufacturing of co-products by a third-party. Another stream of literature studies whether it can be beneficial to sell by-products to reduce waste (Lee, 2012, 2016; Lee and Tongarlak, 2017; Suzanne et al., 2020; Zhou et al., 2020b). From a modeling perspective, by-products can be seen as co-products that serve distinct markets. The most closely related work in this stream is by Li et al. (2019). The authors focus on a fixed proportion co-production system and study whether scrapping or selling the low quality items is advisable. They assume that the low/high quality products serve two distinct markets and show that selling low quality product may be harmful for the manufacturer due to the loss of full control over both markets. We extend their analysis to a multi-echelon and multi-channel setting for vertically differentiated products.

Contracting between farmers and retailers plays a key role in agri-food supply chains. Kazaz (2004) focuses on the olive oil industry for which producers can lease farm space from farmers. This form of contract enables the producer to protect against low yield by getting a second chance to buy olives from other farmers, while benefiting from a lower olive price in case of high yield. The author provides new results that deviate from classical wisdom in the traditional yield articles. Hovelaque et al. (2009) focus on price contracts between the agri-food cooperatives and their members (farmers or growers). Tang et al. (2016) show that contracts with partially-guaranteed prices can create mutual benefits for farmers and agri-food companies and also foster sustainability if a price premium is offered. Anderson and Monjardino (2019) focus on cereal growing and study the relationship between a fertilizer supplier, cereal crop growers, and a buyer under random yield. They study a new type of contract in which the grower purchases fertilizers at a discount but agrees to reduce the price for the crop, while the buyer makes a payment to the supplier to compensate for the discount offered. Assa et al. (2021) study the impact of commodity price insurances on investments at farms, showing that the insurances based on index prices can foster investment by reducing the uncertainty of the impact. Qian and Olsen (2022) highlight that contracts with farmers often specify quality provisions. They study the quality coordination problem through conventional payment schemes and also introduce a new one. They show that their newly introduced payment scheme is better at coordinating the supply chain. A related stream of literature provides insightful guidance on how contracts can reduce the uncertainty faced by the farmers. Indeed, random yield is an important feature in the agri-food industry (Tan and Çömden, 2012) and there is substantial research on risk reduction via contract design (He and Zhang, 2010; Hu et al., 2013; Li et al., 2013; Giri and Bardhan, 2015; Hwang et al., 2018; Zare et al., 2019; Luo et al., 2021). In this article, we build on this stream of literature by studying the interactions between the farmer and the retailer(s), while we consider two interrelated types of produce, differently from the dominant focus on a single product. In a Stackelberg setting, we characterize equilibrium decisions in centralized and decentralized supply chains. We show that although supply chain coordination is economically desirable, it need not reduce food loss on its own.

There is a vast literature on dual channel supply chains. We focus here on recent contributions related to both agri-food supply chains and quality differentiation. Chen et al. (2017a) consider price and quality decisions in a dual channel supply chain. They show that supply chain performance could be improved by introducing a new channel. Lambertini (2018) studies a setting with two firms, belonging to the same supply chain, that can invest in research and development activities to increase the perceived quality of the final product. Perlman et al. (2019) investigate a dual-channel supply chain, including two suppliers that offer vertically-differentiated agricultural products. Zhang and Hezarkhani (2021) investigate the manufacturers' channel selection strategy based on a model in which two manufacturers select among three channel strategies, that is, a direct-channel strategy, a retail-channel strategy, and a dual-channel strategy, consisting of both direct and retail channels. Yu et al. (2020) study a fresh agri-food supply chain with competing retailers and explore the impacts of horizontal and vertical integration. Pu et al. (2020) study competition between a conventional grower who sells its product through an independent retailer and an organic grower who can select either the conventional farmer's retailer or an organic-only retailer. Zhang et al. (2021) assume that a manufacturer produces a high-quality and a low-quality product that can each be distributed through a direct channel or a retailer. Here, we study an agri-food supply chain with dual-channel (traditional retailer vs. dedicated retailer for ugly veg) for which the ugly veg can be viewed as a co-product that is lost at the farm level in the as-is situation. Tao et al. (2022) study the optimal channel structure for a supply chain that consists of one manufacturer and one retailer. The demand is sensitive to the green level of products and the manufacturer can invest in green technology. The authors study four channel structures and they derive analytical results and insights. Xiao et al. (2023) study a dual-channel supply chain that consists of a manufacturer selling national brand products to customers through a retailer and via a direct channel. They study the conditions under which the retailer could benefit from introducing a store brand and how would quality differentiation and power structure shape the introduction incentive and firms' profitability. From our knowledge, channel selection for vertically differentiated co-products has only been studied by Hsieh and Lai (2020). The authors consider a manufacturer that produces vertically differentiated co-products due to the sourcing of raw materials with different quality from two distinct suppliers. The production of high quality items happens at an imperfect yield such that a share of the outputs has to be downgraded to be sold as low quality items. They show that sourcing from a single supplier with high and low quality components under-perform the dual-channel strategy. The setting of the ugly veg supply chain we focus on is quite different from the setting studied by Hsieh and Lai (2020), besides, we focus on the environmental outcomes. Our analysis of the different channel strategies in terms of profit and food loss has substantial policy implications.

3 Supply Chain Structures

In this paper, we analyze several supply chain structures for marketing regular produce and ugly veg. Figure 1 depicts these structures along with the sequence of events in the non-cooperative games played among the grower and the retailer(s).

We consider three echelons in an agricultural supply chain. A (fruit/vegetable) produce is farmed by a grower (she/her), then sold to the retailer(s) (he/him), and then supplied to the

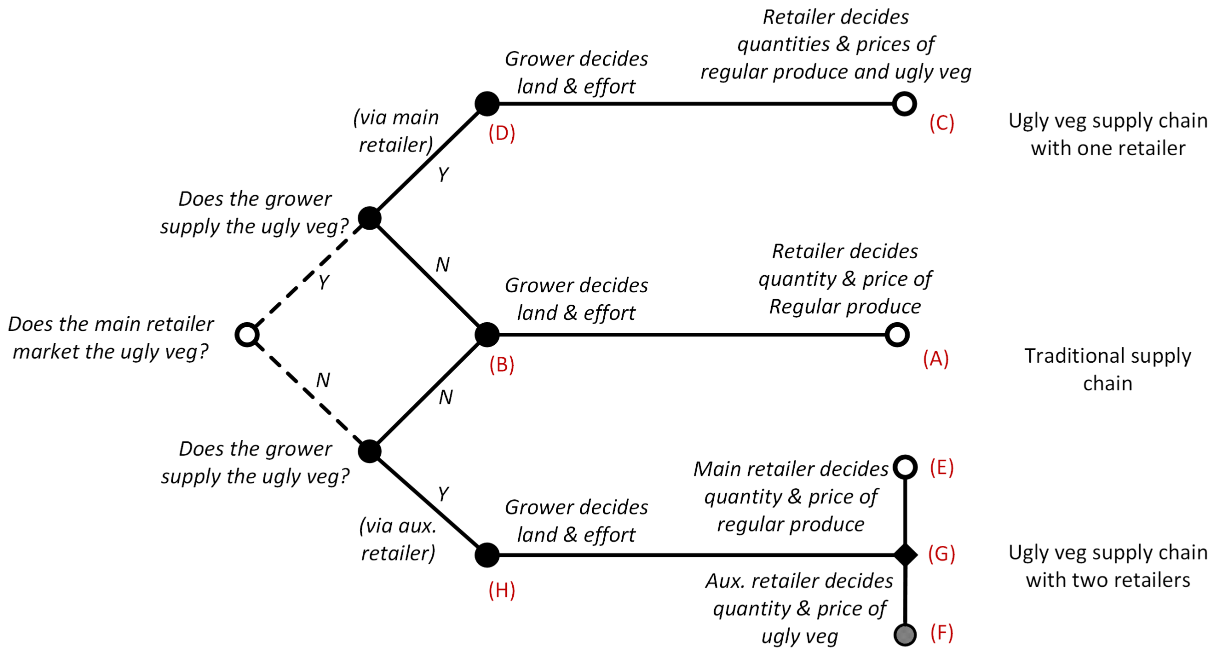

Figure 1: Supply chain structures

customers in the market. The main retailer makes the first strategic move by deciding whether to supply the ugly veg to the market or not. If he decides to market ugly veg, then it is the grower's choice whether to supply the ugly veg to the retailer or not. If either of the players decide not to incorporate the ugly veg in their business model, the outcome is the traditional supply chain where the ugly veg is lost at farm. A subgame-perfect Nash equilibrium (SPNE) of the game between the grower and the retailer in this case is indicated by (B)-(A), (B) corresponding to the grower's best response and (A) corresponding to the retailer's best response, and formulated in Section 4. If both players choose to incorporate the ugly veg, then ugly veg would be sold in the market along with the regular produce by the main retailer. An SPNE under this case is labeled with (D)-(C), corresponding to the grower's and the retailer's best responses respectively, and studied in Section 5. If the main retailer decides not to market the ugly veg, the grower can still work with an auxiliary retailer to sell the ugly veg in the market. The two retailers in this case simultaneously decide the order quantities and prices of the associated produce and a Nash equilibrium (NE) of the associated simultaneous non-cooperative game is labeled with (G), while the best responses of the main retailer and the auxiliary retailer are (E) and (F), respectively. An SPNE in this case is indicated with (H)-(G), corresponding to the grower's and the retailers' best responses respectively, and studied in Section 6.

4 Traditional Supply Chain: Inevitability of Food Loss

We now introduce the game in the traditional supply chain setup, while the full notation is presented in the Supplementary Material. The grower makes two decisions: the area of land that she cultivates, $Q \geq 0$, and the level of effort that she exerts to improve the yield, $e \in [0, 1]$. The grower's effort encompasses elements such as crop density, soil type, variety of seeds, fertilizer, and harvest technology. The combination of these two variables determines the average total yield as well as the marketable yield. The yield of regular produce (marketable yield) is random and affected by a host of factors other than the grower's effort, e.g., weather and pests, however, its expected value is linearly proportional to the effort and is calculated as Qe, while the remaining $Q(1-e)$ units of produce constitute the expected yield of ugly veg. When the grower cultivates Q units of land and exerts effort e, she incurs the production cost $c(Q, e)$. The grower's unit production cost entails an effort-dependent term and a fixed term. Here, we consider a two-part production cost that increases linearly in quantity Q and the unit production cost has a quadratic dependence on the effort, i.e., $c(Q, e) = Q(\alpha e^2 + \beta)$ with $\alpha > 0$ and $\beta > 0$ being the corresponding effort-dependent and effort-independent coefficients. The quadratic cost of effort formulation has been widely adopted in the literature. Among others, it has been used by Kaya (2011) for demand-inducing effort, Ma et al. (2013) for both quality and marketing efforts, Chen (2005) for sales force effort, Ma et al. (2019) for health care quality effort, and Liu et al. (2021) for product freshness preserving effort. To avoid trivial situations where the grower's best decision is always to exert her utmost effort, we assume $\alpha > \beta$. The unit wholesale price of the regular produce is $w \geq 0$. We assume that the wholesale price is determined by market forces beyond the control of the grower and the retailer(s). This is in line with the reality of agricultural markets and the limited power of growers. Among other references, a report from the US Department of Agriculture on pricing practices for agricultural commodities indicates that "[i]n spot markets, farmers are paid for their products at the time ownership is transferred off the farm, with prices based on prevailing market prices at the time of sale" (MacDonald et al., 2004).

We denote the market selling price with p. The market demand is price dependent. In line with the literature, we assume a linear demand function, that is, $d(p) = a - bp$ for $0 \leq p \leq a/b$ where $a > 0$ is the maximum demand, and $b > 0$ is the sensitivity coefficient of the market demand to the price of regular produce. The retailer makes two decisions: market selling price, p, and the order quantity, q. The combination of these variables, in conjunction with the wholesale prices, determines the retail profits. Without loss of generality, we normalize the variable cost of the retailer to zero. As mentioned previously, the grower first determines her production variables. After observing the grower's decisions, the retailer sets the market selling price and the order quantity of the regular produce to maximize his expected profits. In the traditional supply chain, any excess produce at the farm is lost. The regular produce is subsequently purchased by consumers and retail left-overs are wasted.

We analyse the game considering the sequential decision-making stages of the players. Following backward induction, we first examine the retailer's move to set his order quantity and selling price. Assuming that Q and e are set by the grower, for any $q \leq Qe$, the retailer's decision-making problem is as follows:

$$\max \pi(q,p) = p \min\{q, d(p)\} - wq. \tag{1}$$

The objective function in (1) is the retailer's profit due to the sales of regular produce. Given Q and e, the best choice of order quantity never exceeds the demand, since ordering excess produce only incurs costs. Furthermore, the market selling price can always be increased such that the order quantity matches the market demand. Thus, it always holds that at an SPNE the market clears, that is, we have $q = a - bp$. Therefore, the only variable to be determined is the order quantity q and the price can be calculated from $p = (a-q)/b$. The following lemma gives the best response of the retailer.

Lemma 1. *In a traditional supply chain with $w \leq a/b$, the best response order quantities of the retailer are:*

(A.i) *If $Qe \geq \frac{a-bw}{2}$ then $q_{(A.i)}(Q,e) = \frac{a-bw}{2}$.*

(A.ii) *If $Qe < \frac{a-bw}{2}$ then $q_{(A.ii)}(Q,e) = Qe$.*

The critical supply threshold is $(a-bw)/2$. The retailer orders this amount if the supply is greater than or equal to it. Otherwise, the retailer orders as much regular produce as the grower can supply. The equilibrium prices are $p_{(A.i)}(Q,e) = \frac{a+bw}{2b}$ in case (A.i) and $p_{(A.ii)}(Q,e) = \frac{a-Qe}{b}$ in case (A.ii).

Anticipating the order quantity q by the retailer, the grower makes her decisions by solving the following problem:

$$\max \Pi(Q,e) = w \min\{q, Qe\} - Q(\alpha e^2 + \beta). \tag{2}$$

The objective in (2) consists of the grower's revenue due to the supply of regular produce, minus the production cost. The next result shows the best response of the grower.

Lemma 2. *In the traditional supply chain, the grower's non-zero best response decision is* $Q_{(B)} = \frac{a-bw}{2}\sqrt{\frac{\alpha}{\beta}}$ *and* $e_{(B)} = \sqrt{\frac{\beta}{\alpha}}$.

In the traditional supply chain setting, at equilibrium the grower does not hesitate to produce the entire quantity that the retailer could sell. That is, there would be no capping of supply by the grower. Note that the above lemma excludes the possibility of grower producing zero amounts. This possibility is addressed in the next result along with the SPNEs in the traditional supply chain.

Theorem 1. *In the traditional supply chain, the condition for a unique non-zero SPNE (B)-(A.i) is* $2\sqrt{\alpha\beta} \leq w \leq \frac{a}{b}$.

The last observation regarding equilibrium strategies reveals that there is hardly any strategic friction between the parties in the traditional supply chain since all the market demand is produced and supplied by the players. This, however, is conditioned on the wholesale price being within a range which makes production profitable for the grower, while maintaining positive demand in the market.

We further analyze the magnitude of food loss and food waste. Since at equilibrium the retailer always sets the price to clear the market, there would be no food waste at the retailer. The loss at farm is $Y(Q,e) = Q - q(Q,e)$. The following lemma elaborates on the magnitude of food loss at equilibrium.

Lemma 3. *At a non-zero SPNE, there is no loss of regular produce, that is,* $Q_{(B)}e_{(B)} = q_{(A.i)}$. *However, all the ugly veg,* $Y_{(B)-(A.i)} = \left(\sqrt{\frac{\alpha}{\beta}} - 1\right)\frac{a-bw}{2}$, *would be lost.*

The food loss is inevitable in the traditional supply chain. Although higher wholesale prices result in less loss, as long as there is any demand in the market, there would be food loss. As another measure of supply chain wastefulness, consider the unit loss per cultivated area, Y/Q. In the traditional supply chain, the unit loss per cultivated area is $1 - \sqrt{\frac{\beta}{\alpha}}$ at equilibrium. The magnitude of loss thus is dependent on the relative cost of effort α/β. When exerting effort is more costly, one should expect more food loss.

5 Ugly Veg Supply Chain - One Retailer

We now consider a scenario wherein both regular produce and ugly veg are supplied by the grower and marketed by the same retailer. The wholesale price for the regular produce is w. Hereafter, we maintain the assumption $2\sqrt{\alpha\beta} \le w \le \frac{a}{b}$ (feasibility condition I), since otherwise the grower cannot make any profit in the traditional market (as discussed in Theorem 1). The wholesale price for the ugly veg is $w' \ge 0$ and is, due to inferior quality, less than or equal to that of the regular produce, i.e., $w' \le w$ (feasibility condition II). Similar to the previous section, we assume that both wholesale prices are determined by market forces beyond the control of the grower and the retailer. We later analyze the sensitivity of the outcomes to the changes in wholesale prices in the numerical analysis section. We denote the market selling prices of the regular produce and the ugly veg with p and p', respectively. The demands for regular produce and ugly veg are modelled, respectively, as $d(p, p') = a - bp - \lambda(p - p')$ and $d'(p, p') = \lambda(p - p')$. Here, $\lambda > 0$ is the sensitivity coefficient of the market demand for the ugly veg to the difference in the prices of the two produce types. These demand functions are special cases of linear models for substitutable products which have been widely adopted in the literature, e.g., Hsieh and Lai (2020) and Zhang et al. (2021). Essentially, our demand functions allow us to represent the demand for ugly veg as an offshoot of regular produce. In particular, the demand for ugly veg is a function of the price difference between the two types of produce, and is capped by the total demand in the traditional supply chain. The prices p and p' are within the feasible range if $a - bp - \lambda(p - p') \ge 0$ and $p \ge p'$. To ensure the non-negativity of the demand at wholesale prices, we further assume that $a - bw - \lambda(w - w') \ge 0$ (feasibility condition III).

The retailer ought to make four decisions: the market selling prices, p and p', and the order quantities, q and q', for regular produce and ugly veg, respectively. The combination of these variables, in conjunction with the wholesale prices, determines the retail profits. The sequence of events remains as before.

The retailer optimizes the prices and the quantities of both types of produce to maximize his expected profit, given the decisions of the grower. Assuming that Q and e are set by the grower, for any $q \le Qe$, the retailer's problem is as follows:

$$\max \pi(q, q', p, p') = p \min\{q, d(p, p')\} + p' \min\{q' + \max\{0, q - d(p, p')\}, Q - q, d'(p, p')\} - wq - w' \min\{q', Q - q\} \tag{3}$$

The objective function in (3) is the profit due to the sales of regular produce and ugly veg, considering constraints associated with the the market demands and the availability at the grower. This formulation allows for any excess regular produce to be sold as ugly veg, should there be demand for it.

The grower, retrospectively, optimizes her decisions, anticipating the retailer's orders q and q'. The optimization problem of the grower is as follows:

$$\max \Pi(Q,e) = w \min\{q, Qe\} + w' \min\left\{q', Q - \min\{q, Qe\}\right\} - Q(\alpha e^2 + \beta). \tag{4}$$

Equation (4) is the revenue from the supply of both types of produce, considering the retailer's order quantities, minus the production cost. We start our analysis at the retailer's tier before moving on to the grower.

5.1 The Retailer

The retailer's program can be simplified considering a feature of the SPNEs, which is outlined below.

Lemma 4. *Let q, q', p, and p' be a partial SPNE solution. They must always satisfy $q = a - bp - \lambda(p - p')$ and $q' = \lambda(p - p')$.*

According to Lemma 4, the retailer can always optimize his selling prices so that the markets for both types of produce clear. The optimization problem of the retailer thus reflects the caps on market demands for regular produce and ugly veg, and supply caps of both produce types imposed by the grower. Next, we characterize the retailer's best response in terms of order quantities, while the best response selling prices can be obtained accordingly from the market-clearing conditions in Lemma 4.

Theorem 2. *Given Q and e, the best response order quantities of the retailer fall into three categories:*

(C.i) *If $Q \geq \frac{1}{2}(a - bw)$ and $Qe \geq \frac{1}{2}[a - bw - \lambda(w - w')]$, then $q_{(C.i)}(Q,e) = \frac{a-bw-\lambda(w-w')}{2}$ and $q'_{(C.i)}(Q,e) = \frac{\lambda(w-w')}{2}$.*

(C.ii) *If $Q \geq \frac{1}{2}(a - bw)$ and $Qe < \frac{1}{2}[a - bw - \lambda(w - w')]$, then $q_{(C.ii)}(Q,e) = Qe$ and $q'_{(C.ii)}(Q,e) = \lambda\frac{a-bw'-2Qe}{2(\lambda+b)}$.*

(C.iii) *If $Q < \frac{1}{2}(a - bw)$, then $q_{(C.iii)}(Q,e) = Qe$ and $q'_{(C.iii)}(Q,e) = Q(1 - e)$.*

The conditions in the above theorem correspond to three possible scenarios with regard to the supply side, which is controlled by the grower. Case (C.i) happens when there is sufficient supply of both types of produce, which lets the retailer set order quantities without any supply constraints. This means, however, that there might be excessive supply of both types of produce at the grower's site, which would be lost. In case (C.ii), there is limited supply of the regular produce but sufficient supply of the ugly veg. Hence, there could be unused ugly veg at the grower, which would be lost. (C.iii) occurs when the supply constraints of both types of produce are active. Thus, no excess produce of any kind exists in the system, hence no food loss. Note that the situation where the grower only constrains the supply of ugly veg does not occur. As Theorem 2 indicates, at equilibrium the retailer never refrains from buying ugly veg if the grower supplies it. In the next section, we look more closely at the grower's decision-making process.

5.2 The Grower

Suppose the grower supplies the ugly veg to the market via the main retailer. Anticipating the reaction of the retailer, the grower determines the quantity Q and the effort e. The expected profit to the grower, taking into account the best response of the retailer, is formulated as follows:

$$\max \Pi(Q,e) = wq(Q,e) + w'q'(Q,e) - Q(\alpha e^2 + \beta). \tag{5}$$

For ease of exposition, let $\zeta = \frac{a-bw-\lambda(w-w')}{a-bw}$, and $\eta = w - \frac{\lambda w'}{\lambda+b}$. Our next result characterizes the best response of the grower.

Theorem 3. *The grower's non-zero best response decisions fall into two categories:*

(D.i) $e_{(D.i)} = \min\left\{\sqrt{\beta/\alpha}, \zeta\right\}$ *and* $Q_{(D.i)} = \frac{1}{2e_{(D.i)}}\left[a - bw - \lambda(w-w')\right].$

(D.ii.a) $e_{(D.ii.a)} = \frac{\eta}{2\alpha} - \sqrt{\frac{\eta^2}{4\alpha^2} - \frac{\beta}{\alpha}}$ *and* $Q_{(D.ii.a)} = \frac{1}{2}(a-bw)$ *if* $e_{(D.ii.a)} < \zeta$ *and* $\eta \geq 2\sqrt{\alpha\beta}$.

(D.ii.b) $e_{(D.ii.b)} = \frac{\eta}{2\alpha}$ *and* $Q_{(D.ii.b)} = \frac{1}{2}(a-bw)$ *if* $e_{(D.ii.b)} < \zeta$ *and* $\eta < 2\sqrt{\alpha\beta}$.

The cases (D.i) and (D.ii) are the best responses to the retailer's cases (C.i) and (C.ii), respectively. Case (D.ii) is further divided into two sub-cases based on the wholesale price of ugly veg. In case (D.ii.a), the wholesale price of the ugly veg is relatively low, while in case (D.ii.b) the opposite holds. Therefore, if the grower participates in the supply chain, it would never restrict the supply of both types of produce simultaneously, i.e., (C.iii) never occurs. Note that the case of $w = w'$ can fall either in (D.i) or (D.ii.b), which implies that this case is not the same as that of the traditional supply chain.

The best response decisions of the grower can be determined by comparing her profit under the above scenarios. Next, we determine the SPNEs in the sequential game between the grower and the retailer.

Theorem 4. *If $\eta \geq \min\{2\sqrt{\alpha\beta}, 2\alpha\zeta\}$ then the non-zero SPNE is (D.i)–(C.i). Otherwise, the non-zero SPNE is (D.ii.b)–(C.ii).*

Theorem 4 provides the conditions for the supply of regular produce to be restricted or not. The SPNE could be zero if the grower's profit is negative. We do not provide conditions for this case explicitly as we later on consider the possibility of grower not supplying the ugly veg. The case of grower not supplying ugly veg would be equivalent to the traditional supply chain, in which, under our assumption on w, the grower's profit is always non-negative. Considering the formula for η, the condition that determines the SPNE in Theorem 4 is the relationship between the wholesale prices of the regular produce and the ugly veg. When the wholesale price of the ugly veg is relatively low, that is, $w' \leq \left(w + \min\{2\sqrt{\alpha\beta}, 2\alpha\zeta\}\right)(1 + b/\lambda)$, the grower supplies sufficient amounts of both types of produce and the SPNE is (D.i)-(C.i). Otherwise, when the wholesale price of ugly veg is relatively high, the grower will restrict the supply of normal veg and the SPNE would be (D.ii.b)-(C.ii). This can be interpreted as an attempt on the grower's side to make the retailer buy more ugly veg, which is less costly to produce and relatively profitable for the grower. Therefore, when the ugly veg is valuable enough, the grower would rather supply more of it instead of the effort-intensive regular produce.

We further analyze the magnitude of food loss (which happens at the farm level) and food waste (which happens at the retail level). Considering the results of our optimization problem for the retailer, one can verify that under deterministic demand, which is the assumption of our model, the food waste is always zero; thus, the retailer always buys as much as he can sell to the market and never more than that. Food loss, however, occurs even under deterministic market demand. Formally, we denote the on-farm loss with $Y(Q,e) = Q - q(Q,e) - q'(Q,e)$. The following lemma elaborates on the volumes of food loss at equilibrium.

Lemma 5. *In the ugly veg supply chain with one retailer, at an SPNE there is no loss of regular produce. The loss of ugly veg is*

- $Y_{(D.i)-(C.i)} = \frac{a-bw-\lambda(w-w')}{2\min\left\{\sqrt{\beta/\alpha},\zeta\right\}} - \frac{a-bw}{2}$.

- $Y_{(D.ii.b)-(C.ii)} = \frac{a-bw}{2}\left(1 - \frac{b\eta}{2\alpha(\lambda+b)}\right) - \frac{\lambda(a-bw')}{2(\lambda+b)}$.

An interesting case happens when the SPNE is (D.i)-(C.i) and $\zeta \le \sqrt{\frac{\beta}{\alpha}}$. In this case, one can verify that the loss of ugly veg disappears, i.e., $Y = 0$. This would be an ideal outcome for the chain that eliminates food loss and waste completely. However, this is not always the case and food loss can be a considerable proportion of total production. The magnitude of food loss as a percentage of total production is further studied in our numerical analysis in Section 8.

5.3 Does the Grower Supply Ugly Veg?

Comparing the supply chain structures in the previous sections, we investigate the conditions under which the grower would prefer to supply the ugly veg to the main retailer, rather than discarding them. The latter happens if the grower's profit in the ugly veg supply chain is less than that in the traditional supply chain.

Theorem 5. *In the single retailer supply chain, the grower chooses to supply the ugly veg if* $(D.ii.b) - (C.ii)$ *is the non-zero SPNE of the ugly veg supply chain. Otherwise, when the non-zero SPNE of the ugly veg supply chain is* $(D.i) - (C.i)$*, the grower supplies the ugly veg whenever*

- $\eta \ge 2\alpha\zeta \ge 2\sqrt{\alpha\beta}$*, and* $w - w' \le 2\sqrt{\alpha\beta}$*, or*
- $\eta \ge 2\sqrt{\alpha\beta} \ge 2\alpha\zeta$*, and* $\lambda(w - w')^2 < (a - bw)(2\sqrt{\beta/\alpha}) - \alpha\zeta^2 - \beta$.

As Theorem 5 indicates, supply of ugly veg by the grower cannot be taken for granted. Generally, when the wholesale price of the ugly veg is high enough, the grower indeed supplies it. However, if the wholesale price of ugly veg is relatively low, the grower prefers disposing the ugly veg instead of selling it to the retailer. In such cases, the availability of ugly veg would cannibalize the market for the regular produce to the point that the grower earns less profit compared to the traditional supply chain.

5.4 Impact of Ugly Veg Supply Chain on Food Loss

Next, we compare the food loss under the ugly veg supply chain with that under the traditional supply chain. As we discussed earlier in the paper, the downside of the traditional supply chain is the on-farm food loss, which is at equilibrium is $Y = \frac{a-bw}{2}\left(\sqrt{\frac{\alpha}{\beta}} - 1\right)$. One can verify that the food loss in the traditional supply chain is never less than the loss under the case where the retailer offers both types of produce to the market under the SPNE (D.i)-(C.i). Interestingly, this does not necessarily hold under the SPNE (D.ii.b)-(C.ii), that is, in some cases the total food loss in the ugly veg supply chain can be even higher than that under the traditional supply chain. Our next result highlights the conditions under which the supply of ugly veg to the market actually reduces food loss.

Theorem 6. *In the single retailer supply chain, under the SPNE (D.i)-(C.i), the total food loss in the ugly veg supply chain is less than or equal to that in the traditional supply chain. Under the SPNE (D.ii.b)-(C.ii), a sufficient condition for having less food loss in the ugly veg supply chain is to have* $\sqrt{\beta/\alpha} \leq 3/4$.

Thus, when the ugly veg is supplied to the market, the supply chain could end up with more food loss. Nevertheless, this cannot happen if the effort is sufficiently costly.

6 Ugly Veg Supply Chain - Two Retailers

Suppose the main retailer does not market the ugly veg, but instead the grower offers the ugly veg to the market through an auxiliary retailer. We consider a three-player sequential game. Similar to the case in the previous section, the grower moves first and decides on the cultivation area of land Q and the effort e. In the second stage, the main retailer and the auxiliary retailer simultaneously decide on their purchase quantities and market prices. More specifically, the main retailer decides on p and q for the regular produce and the auxiliary retailer decides on p' and q' for the ugly veg.

Applying backward induction, we first find the NE of the non-cooperative price-quantity game between the two retailers in the second stage, both knowing the available quantities of the regular and ugly veg, that is, Qe and $Q(1-e)$, respectively. We then analyze the first stage, where the grower decides on her Q and e, anticipating the equilibrium responses in the second stage.

6.1 Main Retailer

Given the grower's decisions regarding Q and e, and assuming the auxiliary retailer's decisions q' and p', the main retailer solves the following program:

$$\max \pi_H(q,p) = p \min\{q, d\left(p,p'\right), Qe\} - wq. \tag{6}$$

The objective function is the profit due to the sales of regular produce. The sales quantity is the minimum of the main retailer's order q and the market demand for the regular produce $d\left(p,p'\right)$.

Similar to the case in the previous section, order quantity must also satisfy $q \le Qe$. One can verify, similar to previous cases, that at equilibrium the market clears, thus $q = d(p, p')$, which leads to the following best response of the main retailer.

Lemma 6. *Given Q, e, q', and p', the best response order quantity of the main retailer falls into two categories:*

(E.i) *If $w \le \frac{a+\lambda p'}{\lambda+b} \le w + 2\frac{Qe}{\lambda+b}$ then $q_{(E.i)}(Q, e, p', q') = \frac{1}{2}\left[a - bw - \lambda(w - p')\right]$.*

(E.ii) *If $w + 2\frac{Qe}{\lambda+b} < \frac{a+\lambda p'}{\lambda+b}$ then $q_{(E.ii)}(Q, e, p', q') = Qe$.*

The solution (E.i) is attained if it is better for the grower not to limit the supply of regular produce, i.e., $Qe > d(p, p')$. Then the retailer's profit is concave in the price and achieves its unconstrained maximum. This becomes infeasible for $w + 2\frac{Qe}{\lambda+b} \le \frac{a+\lambda p'}{\lambda+b}$, where the main retailer sets the price to sell all regular produce available, leading to the solution (E.ii).

6.2 Auxiliary Retailer

Given Q, e, q, and p, the auxiliary retailer solves the problem

$$\max \pi_L(q', p') = p' \min\{q', d'(p, p'), Q - q\} - w'q'. \tag{7}$$

Similar to the main retailer, the auxiliary retailer's order should not exceed the market demand for ugly veg and the grower's supply, which is formulated considering the main retailer's constraint on the regular produce and contains ugly veg and any regular produce not purchased by the main retailer. Again the market for ugly veg clears at equilibrium, i.e., $q' = d'(p - p')$, leading to the following best response of the auxiliary retailer.

Lemma 7. *Given Q, e, q, and p, the best response order quantity of the auxiliary retailer falls into two categories:*

(F.i) *If $w' \le p \le w' + \frac{2(Q-q)}{\lambda}$ then $q'_{(F.i)}(Q, e, p, q) = \frac{\lambda(p-w')}{2}$.*

(F.ii) *If $w' + \frac{2(Q-q)}{\lambda} < p$ then $q'_{(F.ii)}(Q, e, p, q) = Q - q$.*

If there is sufficient supply of ugly veg, i.e., $w' \le p < w' + \frac{2(Q-q)}{\lambda}$, the interior solution (F.i) is optimal for the auxiliary retailer, where the ugly veg price is half way from the wholesale ugly veg price to the main retailer's regular produce price. If the main retailer's price is high and the supply of ugly veg is limited, i.e., $w' + \frac{2(Q-q)}{\lambda} \le p$, then the solution is (F.ii), where all available ugly veg is bought by the auxiliary retailer.

6.3 Retailers' Equilibrium

Having obtained the retailers' best responses, we obtain the NEs of the two retailers' strategies in the simultaneous game played in the second stage. For ease of exposition, let $\xi = a(2b+3\lambda) - 2b(b+\lambda)w - b\lambda w'$, and $\chi = (b+\lambda)\left[2a - (2b+\lambda)w + \lambda w'\right]$.

Theorem 7. *Given the grower's decisions Q and e, the NE of the non-cooperative game between the main and the auxiliary retailers fall into four categories as follows:*

(G.i) *If $Q \ge \frac{\xi}{4b+3\lambda}$ and $Qe \ge \frac{\chi}{4b+3\lambda}$ then $q_{(G.i)}(Q,e) = \frac{\chi}{4b+3\lambda}$ and $q'_{(G.i)}(Q,e) = \frac{\xi-\chi}{4b+3\lambda}$.*

(G.ii) *If $Q < \frac{\xi}{4b+3\lambda}$ and $Q\left(1+\frac{be}{b+\lambda}\right) \ge a - bw$ then $q_{(G.ii)}(Q,e) = \frac{(b+\lambda)(a-bw-Q)}{b}$ and $q'_{(G.ii)}(Q,e) = \frac{(2b+\lambda)Q-(b+\lambda)(a-bw)}{b}$.*

(G.iii) *If $Q\left(1+\frac{2b(1-e)}{\lambda}\right) \ge a - bw'$ and $Qe < \frac{\chi}{4b+3\lambda}$ then $q_{(G.iii)}(Q,e) = Qe$ and $q'_{(G.iii)}(Q,e) = \lambda\frac{(a-bw'-Qe)}{2b+\lambda}$.*

(G.iv) *If $Q\left(1+\frac{2b(1-e)}{\lambda}\right) < a-bw'$ and $Q\left(1+\frac{be}{b+\lambda}\right) < a-bw$ then $q_{(G.iv)}(Q,e) = Qe$ and $q'_{(G.iv)}(Q,e) = Q(1-e)$.*

Case (G.i) happens when there is sufficient supply of both types of produce. This might lead to food loss at the grower level because both retailers are economically better off setting prices higher than those that use up all the available produce at the grower if the constraints are not-binding. In case (G.ii), there is sufficient supply of regular produce for the main retailer to set a high price and not order all the available regular produce. However, under this condition, the total supply Q is limited and the auxiliary retailer orders all the excess produce at the grower, leading to zero loss. In cases (G.i) and (G.ii), the regular produce can end up in the auxiliary channel. However, as we show below, this would not be a best response for the grower, as $q = Qe$ in all SPNEs. Case (G.iii) occurs if the regular produce supply is limited while the ugly veg supply is not. Hence, some ugly veg will be lost if the first constraint holds strictly, i.e., $Q\left(1+\frac{2b(1-e)}{\lambda}\right) > a - bw'$. In case (G.iv), the supplies of both types of produce are limited, hence there is no loss.

6.4 The Grower

In the first stage of the problem, the grower determines her strategies taking into account the retailer strategies at equilibrium in the second stage. The grower maximizes her profit by choosing Q and e, hence solves the same problem as in Eq. (5). For the categories of the grower's best responses, let us introduce the following for ease of exposition: $\rho = b\beta + (b+\lambda)w - (2b+\lambda)w'$, $\tau_1 = \frac{\xi^2 \rho}{\chi(b+\lambda)[6(b+\lambda)a-(6b+5\lambda)bw-b\lambda w']}$ and $\tau_2 = \frac{\xi^2[(2b+\lambda)w-\lambda w'-2b\beta]}{2\chi[(2b^2+6b\lambda+3\lambda^2)a-b(b+\lambda)(2b+\lambda)w-b\lambda(3b+2\lambda)w']}$.

Theorem 8. *The non-zero best response decisions for the grower falls into three categories:*

(H.i) $e_{(H.i)} = \min\left\{\sqrt{\frac{\beta}{\alpha}}, \frac{\chi}{\xi}\right\}$, *and* $Q_{(H.i)} = \frac{1}{e_{(H.i)}}\frac{\chi}{4b+3\lambda}$.

(H.ii.a) *If* $\alpha < \dfrac{\rho}{3b+2\lambda}$ *then* $e_{(H.ii.a)} = 1$ *and* $Q_{(H.ii.a)} = \dfrac{(b+\lambda)(a-bw)}{2b+\lambda}$,

(H.ii.b) *If* $\rho \geq 0$ *and* $\frac{\rho}{3b+2\lambda} \leq \alpha < \tau_1$ *then* $e_{(H.ii.b)} = \frac{b+\lambda}{b}\left(\sqrt{1+\frac{b\rho}{\alpha(b+\lambda)^2}} - 1\right)$, *and* $Q_{(H.ii.b)} = \frac{(b+\lambda)(a-bw)}{b+\lambda+be_{(H.ii.b)}}$.

(H.iii) *If* $\alpha \geq \tau_2$ *then* $e_{(H.iii)} = \frac{2b+\lambda}{2b}\left(1-\sqrt{1-\frac{2b[(2b+\lambda)(w-w')-2b(\beta-w')]}{\alpha(2b+\lambda)^2}}\right)$ *and* $Q_{(H.iii)} = \frac{\lambda(a-bw')}{\lambda+2b(1-e_{(H.iii)})}$.

The equilibrium responses of the two retailers to (H.i), (H.ii) and (H.iii) are (G.i), (G.ii), and (G.iii), respectively. As we show in the proof of the theorem, the equilibrium (G.iv) does not lead to a separate best response case, as it is either equal to or trivially dominated by one of the other best responses. Under the conditions provided for each case, we can see that the optimal effort can exceed $\sqrt{\beta/\alpha}$, potentially leading to the exertion of full effort under (H.ii.a). It is insightful to compare the case of (H.ii.a) with the main retailer's best response under the one-retailer setting. Although there is no provision of ugly veg to the market when $e = 1$, the two-retailers case does not simply reduce to the single retailer case. Under (H.ii.a)-(G.ii), the main retailer's order quantity and price are $q_{(H.ii.a)-(G.ii)} = \frac{(b+\lambda)(a-bw)}{2b+\lambda}$ and $p_{(H.ii.a)-(G.ii)} = \frac{a+(b+\lambda)w}{2b+\lambda}$, respectively. In the single-retailer supply chain structure, the main retailer's response to the grower's Q and e under (H.ii.a) would be (C.i) with $q_{(C.i)} = \frac{a-bw-\lambda(w-w')}{2}$ and $p_{(C.i)} = \frac{a+bw}{2b}$. The main retailer's price is lower under the two-retailers case, while its order quantity is higher. The sheer existence of an auxiliary retailer forces the main retailer to decrease the price and to increase the quantity.

The grower's choice among the three responses above is determined by comparing her associated profits and ensuring non-negativity, thus

$$\Pi = \max\{\Pi(Q_{(H.i)}, e_{(H.i)}), \Pi(Q_{(H.ii)}, e_{(H.ii)}), \Pi(Q_{(H.iii)}, e_{(H.iii)}), 0\}.$$

Theorem 9. *The SPNE for the two-retailers supply chain is one of (H.i)-(G.i), (H.ii.a)-(G.ii), (H.ii.b)-(G.ii), (H.iii)-(G.iii), and (B)-(A.i).*

Here, note that the last SPNE holds when the grower is better off not selling ugly veg. The complicated nature of the best response functions makes it cumbersome to derive the conditions for the SPNEs. Yet, our numerical analysis in Section 8 sheds light on more probable cases and generates insights on the grower's and retailers' profits. Here, we instead focus on the food loss.

6.5 Food Loss under Competition

We first characterize the food loss under the different SPNEs.

Lemma 8. *In the two-retailers supply chain, there is no loss of regular produce at any SPNE. The loss of ugly veg is*

- $Y_{(H.i)-(G.i)} = \frac{\chi}{4b+3\lambda}\left(\max\left\{\sqrt{\frac{\alpha}{\beta}}, \frac{\xi}{\chi}\right\} - \frac{\xi}{\chi}\right)$.
- $Y_{(H.ii.a)-(G.ii)} = Y_{(H.ii.b)-(G.ii)} = Y_{(H.iii)-(G.iii)} = 0$

As in the traditional and the single retailer ugly veg supply chain structures, there is never a loss of regular produce since $q = Qe$ in all SPNEs under the considered demand model with linear substitution. The ugly veg food loss is only positive under (H.i)-(G.i) for $\sqrt{\frac{\beta}{\alpha}} < \frac{\chi}{\xi}$. Otherwise, the ugly veg food loss is always zero as well, emphasizing the food loss reduction potential of the two-retailers ugly veg supply chain. Next, we compare the food loss under the two retailers supply chain with the traditional supply chain.

Theorem 10. *The loss per unit cultivated area, Y/Q, is always strictly less in the two-retailers supply chain than in the traditional supply chain. The total food loss, Y, in the two-retailers supply chain is greater than or equal to the loss in the traditional supply chain if the SPNE is (H.i)-(G.i) and $\sqrt{\frac{\beta}{\alpha}} \leq \frac{a-(3b+2\lambda)w+2(b+\lambda)w'}{3a-bw-2bw'}$. Otherwise, the total food loss is zero, hence strictly less under the two-retailers supply chain.*

When the SPNE is (H.i)-(G.i) with $e_{(H.i)-(G.i)} = \sqrt{\frac{\beta}{\alpha}}$, the effort in the two-retailers supply chain is the same as the equilibrium effort in the traditional supply chain, but some ugly veg now gets sold to the market via the auxiliary retailer. This leads to a net reduction of the food loss per unit cultivated area, Y/Q. By Lemma 8, the food loss is otherwise zero, hence strictly less than the positive food loss under the traditional supply chain. As the food loss per unit cultivated area is strictly less under the two retailers supply chain, we can expect the total food loss to decrease as well with the provision of ugly veg. However, when the effort is sufficiently costly, i.e., $\sqrt{\frac{\beta}{\alpha}} \leq \frac{a-(3b+2\lambda)w+2(b+\lambda)w'}{3a-bw-2bw'}$, the grower increases the cultivation area to meet the inflated demand due to lower prices thanks to the competition, keeping the effort constant. Therefore, the emergence of specialized ugly veg sellers can indeed increase the food loss if it is too costly for the grower to exert effort.

7 Centralized Ugly Veg Supply Chain

7.1 Centralized Traditional Supply Chain

We benchmark the equilibrium decisions in the decentralized scenario above with the corresponding optimal decisions of the centralized scenario where the grower and the retailer are vertically integrated. In this case, there would be no wholesale price and the optimization problem of the supply chain becomes:

$$\max \Omega(Q, e, q, p) = p \min\{q, d(p), Qe\} - Q(\alpha e^2 + \beta) \tag{8}$$

When all decisions are taken unilaterally, the obtained optimal decisions are described in the next result.

Theorem 11. *In the centralized version of the traditional supply chain, the optimal decisions are* $q_{(cT)} = \frac{a}{2} - b\sqrt{\alpha\beta}$, $e_{(cT)} = \sqrt{\frac{\beta}{\alpha}}$, *and* $Q_{(cT)} = \frac{a}{2}\sqrt{\frac{\alpha}{\beta}} - \alpha b$.

Comparing the centralized scenario with the decentralized one, we observe that in the centralized scenario the effort remains the same, market price decreases, and overall sales as well as cultivated area increase. In the centralized traditional supply chain food loss is also inevitable. Although, the unit loss per cultivated area, Y/Q, remains the same, the total loss indeed increases.

Lemma 9. *The food loss in the centralized traditional supply chain is at least as much as that in the decentralized one.*

Therefore, centralization can exacerbate the wastefulness of the traditional supply chain.

7.2 Centralized Ugly Veg Supply Chain

We now analyze the centralized version of the ugly veg supply chain and compare it with the centralized traditional supply chain. The centralized problem for the ugly veg supply chain now involves setting the price p' and the order quantity q' alongside the variables for the centralized traditional supply chain, i.e., Q, e, q, and p. The optimization problem in this case becomes:

$$\begin{aligned} \max \Omega(Q,e,q,q',p,p') =& p \min\left\{Qe, q, d(p,p')\right\} + p' \min\left\{Q-q, q' + \max\{0, q - d(p,p'), q - Qe\}, d'(p,p')\right\} \\ &- Q(\alpha e^2 + \beta) \end{aligned} \tag{9}$$

The regular produce sales quantity is the minimum of the supply from the farm, Qe, retail orders, q, and market demand, $d(p,p')$. Similarly, the ugly veg sales quantity is the minimum of the remaining supply of the farm, $Q-q$, retail ugly veg orders plus any regular produce that is available at the retail shelves and is not sold, $q' + \max\{0, q - d(p,p')\}$, and the market demand for ugly veg, $d'(p,p')$. As for the previous supply chain structures, the markets for both types of produce clear at equilibrium. Although the upward substitution is possible, this never happens in the centralized supply chain since all regular produce is sold to the intended market, that is $q = Qe$. There are two possible cases for optimal solutions, depending on whether it is economically better to produce excess ugly veg and waste some, i.e., $q' < Q(1-e)$, or not wasting any by restricting the ugly veg supply, i.e., $q' = Q(1-e)$. The following result shows the optimal solutions of these two types, respectively.

Theorem 12. *The centralized solution is:*

(cU.i) *If* $\frac{a}{b} \geq 2\sqrt{\alpha\beta}\left(1 + \frac{\lambda/b}{1-\sqrt{\beta/\alpha}}\right)$ *then* $e_{(cU.i)} = \sqrt{\frac{\beta}{\alpha}}$, $Q_{(cU.i)} = \sqrt{\frac{\alpha}{\beta}}\left(\frac{a}{2} - (b+\lambda)\sqrt{\alpha\beta}\right)$, $q_{(cU.i)} = \frac{a}{2} - (b+\lambda)\sqrt{\alpha\beta}$, $q'_{(cU.i)} = \lambda\sqrt{\alpha\beta}$.

(cU.ii) *If* $2\sqrt{\alpha\beta} < \frac{a}{b} < 2\sqrt{\alpha\beta}\left(1 + \frac{\lambda/b}{1-\sqrt{\beta/\alpha}}\right)$ *then* $e_{(cU.ii)} = \mu$, $Q_{(cU.ii)} = \frac{\alpha\lambda\mu}{1-\mu}$, $q_{(cU.ii)} = \frac{\alpha\lambda\mu^2}{1-\mu}$, $q'_{(cU.ii)} = \alpha\lambda\mu$, *where* μ *is the unique solution of* $(1-\mu)^3 + (1-\mu)\left(\frac{2\lambda}{b} + \frac{a}{b\alpha} - \frac{\beta}{\alpha} - 1\right) - \frac{2\lambda}{b} = 0$ *in the unit interval.*

When the a/b ratio is high, the optimal solution is (cU.i), according to which the central decision maker focuses on the regular veg market and tunes the effort to the unconstrained level $e_{(cU.i)} = \sqrt{\frac{\beta}{\alpha}}$. The regular produce price, $p_{(cU.i)}$ remains the same as the centralized traditional case, i.e., $p_{(cU.i)} = \frac{a}{2b} + \sqrt{\alpha\beta}$, but the ugly veg is sold at the market with a discount of $\lambda\sqrt{\alpha\beta}$. If the ratio a/b is low, it is better to decrease the effort, i.e., $e_{(cU.ii)} < \sqrt{\frac{\beta}{\alpha}}$, and to sell more ugly veg. Next, we characterize the food loss in the centralized chain.

Lemma 10. *In the centralized ugly veg supply chain, there is no loss of regular produce. There is no loss of ugly veg in the solution (cU.ii), but the loss is positive for (cU.i). In this case we have* $Y_{(cU.i)} = \left(\sqrt{\frac{\alpha}{\beta}} - 1\right)\left(\frac{a}{2} - (b + \lambda)\sqrt{\alpha\beta}\right) - \lambda\sqrt{\alpha\beta}$ *for* $\frac{a}{b} \geq 2\sqrt{\alpha\beta}\left(1 + \frac{\lambda/b}{1-\sqrt{\beta/\alpha}}\right)$.

As manifested in the above result, there is a trade-off between mainly serving the regular produce market, in which case some ugly veg gets lost (cU.i), and offering fractionally more ugly veg produce and selling all produce possible. The ugly veg gets fully utilised only when the demand ratio a/b is sufficiently high. We shall now compare the food loss in the centralized ugly veg supply chain with the loss in the traditional centralized supply chain.

Theorem 13. *The total and proportional food loss, i.e.,* Y *and* Y/Q*, respectively, is always less in the centralized ugly veg supply chain than in the centralized traditional supply chain.*

Therefore, under the centralized setting, offering the ugly veg to the market always generates a better environmental outcome, in terms of both total food loss and food loss per unit cultivated area. This is different from the decentralized supply chains both with a single retailer and with two competing retailers. Therefore, if the frictions in the supply chain can be removed, offering the ugly veg to the market reduces the food loss certainly.

8 Numerical Analysis

We numerically investigate the behaviour of the supply chain, as modeled in previous sections. We do this in two parts. First, we look at a single representative instance and discuss the equilibrium performances of the grower and the retailers as we perform sensitivity analysis on the ugly veg wholesale price. Second, we construct a sample of instances with a diverse range of parameter values and perform a descriptive statistical analysis of the equilibrium states in these situations.

8.1 An Example

In this section, we examine one exemplar instance of the supply chain. We consider a market with a maximum demand size of $a = 1,000$ units with the same price sensitivity for regular produce ($b = 25$) and ugly veg ($\lambda = 25$). We fix the wholesale price of the regular produce at $w = 20$ and consider the whole range of feasible wholesale prices for the ugly veg, i.e., $w' \in [0, 20]$. With regard to the cost function, we set the effort-dependent and effort-independent parts at $\alpha = 10$ and $\beta = 5$, respectively. With this set of parameters, we solve the models corresponding to different supply chain structures and examine the equilibrium decisions of the supply chain members.

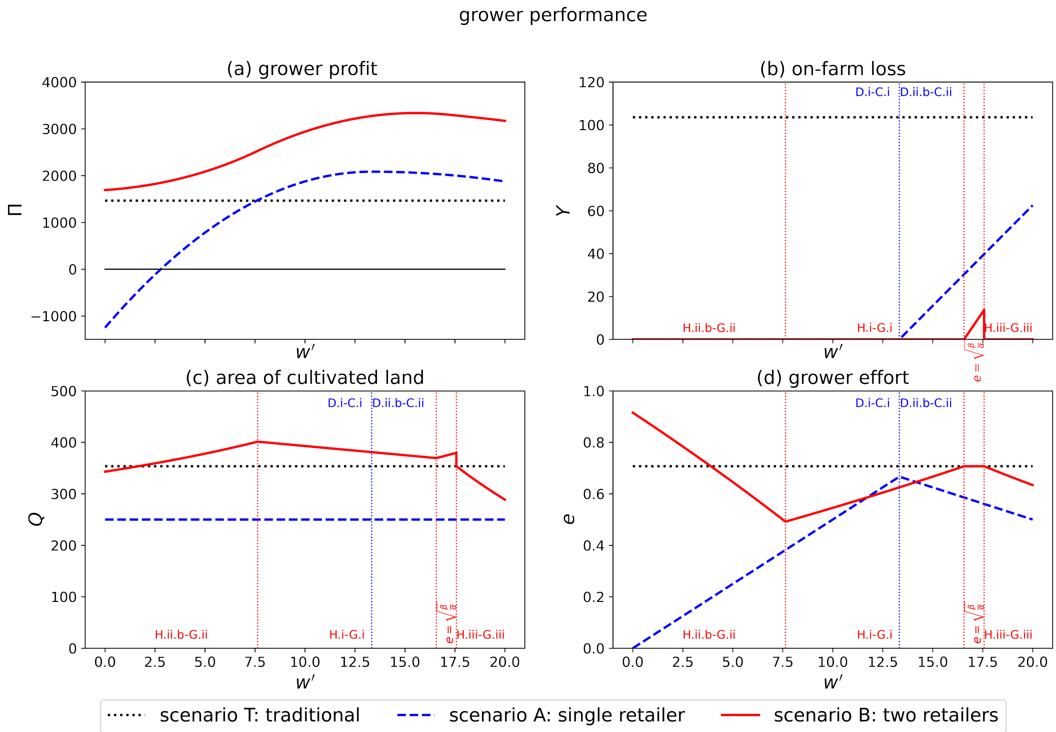

Figure 2: Grower decisions and performance under different values of w' under different supply chain structures

Figure 2 demonstrates the equilibrium performance of the grower under the aforementioned parameter values as w' is varied. In Figure 2-a, we plot the grower's profit at equilibrium in the traditional supply chain (scenario T), ugly veg supply chain with one retailer (scenario A), and ugly veg supply chain with two retailers (scenario B). The grower always prefers to sell ugly veg to an auxiliary retailer as the profit under scenario B is always higher than those under scenarios T and A. Furthermore, supplying the ugly veg to the main retailer is not profitable for the grower under low ugly veg wholesale prices ($w' < 7.59$), hence the grower would be better off by discarding the ugly veg. The possibility of supplying the ugly veg to the market drastically reduces on-farm loss, as can be seen in Figure 2-b. The loss is obliterated for sufficiently low ugly veg wholesale prices ($w' < 13.33$). The existence of an auxiliary retailer can achieve this even at high ugly veg wholesale prices prices ($w' \geq 17.57$). However, there is an intermediate range of ugly veg wholesale price ($16.57 < w' \leq 17.57$) for which the food loss under the scenario B is positive, but still substantially less than the loss under the scenario A. The loss suddenly drops to zero as the equilibrium for the scenario B switches from (H.i)-(G.i) with $e = \sqrt{\frac{\beta}{\alpha}}$ to (H.iii)-(G.iii) at $w' = 17.57$, as annotated on the figure.

The remarkable observation in Figure 2-c is that the land utilization is minimized under the scenario A for the whole range of ugly veg wholesale prices. The area of cultivated land in the two-retailers supply chain (scenario B) exceeds the area of land in the traditional supply chain (scenario T) if the ugly veg wholesale price w' is moderate ($1.67 \leq w' \leq 17.57$). If the ugly veg wholesale price is too low, in scenario B, the main retailer orders a high amount of regular produce, facilitating the grower to exert very high effort (see Figure 2-d), hence there is a very small market for ugly veg and when combined the area of the cultivated land is less under scenario B. For high ugly veg wholesale price ($w' > 17.57$), the area of cultivated land decreases as the main retailer's order quantity decreases with the narrowing margin between the two wholesale prices. Comparing the level of land utilization in Figure 2-c with the magnitude of food loss in Figure 2-b reveals that in the traditional supply chain, approximately 29% of produce is lost.

Figure 2-d highlights the striking effect of competition in the market, which can increase the grower's effort beyond the maximum level under scenarios T and B ($e > \sqrt{\frac{\beta}{\alpha}}$ for $w' \leq 3.9$) and the effort is generally higher under the two-retailers supply chain (scenario B) than the effort under the single-retailer supply chain (scenario A). With increasing ugly veg wholesale price, in scenario B, first the effort decreases as more ugly veg is supplied to the market, i.e. for $w' < 7.64$. For intermediate ugly veg wholesale prices ($7.64 \leq w' \leq 17.57$), the effort is non-decreasing as the regular produce sales increase while ugly veg sales decrease. At higher ugly veg wholesale prices, the effect changes direction and decreases again as the grower can now sell the two types of produce at similar prices, forcing the main retailer to increase its sales price at the expense of selling less. In the ugly veg supply chain with a single retailer, the grower exerts the least level of effort at low ugly veg wholesale prices since the retailer will sell ugly veg instead of regular produce in the market. The effort under Scenario A increases for $w' \leq 13.33$, but then decreases as it becomes more profitable for the grower not to limit the supply of ugly veg and instead waste some, i.e. when SPNE (D.ii.b)-(C.ii) holds. We synthesize these results with the following observation.

Observation 1. *Marketing ugly veg is beneficial for the grower under a large range of ugly veg wholesale prices. This practice can significantly reduce on-farm loss. Competition between retailers improves the grower's profit and food loss. The area of cultivated land is least for the single-retailer supply chain, while the area for the two-retailers supply chain seems to be greater than for the* *traditional supply chain when ugly veg wholesale prices is intermediate.*

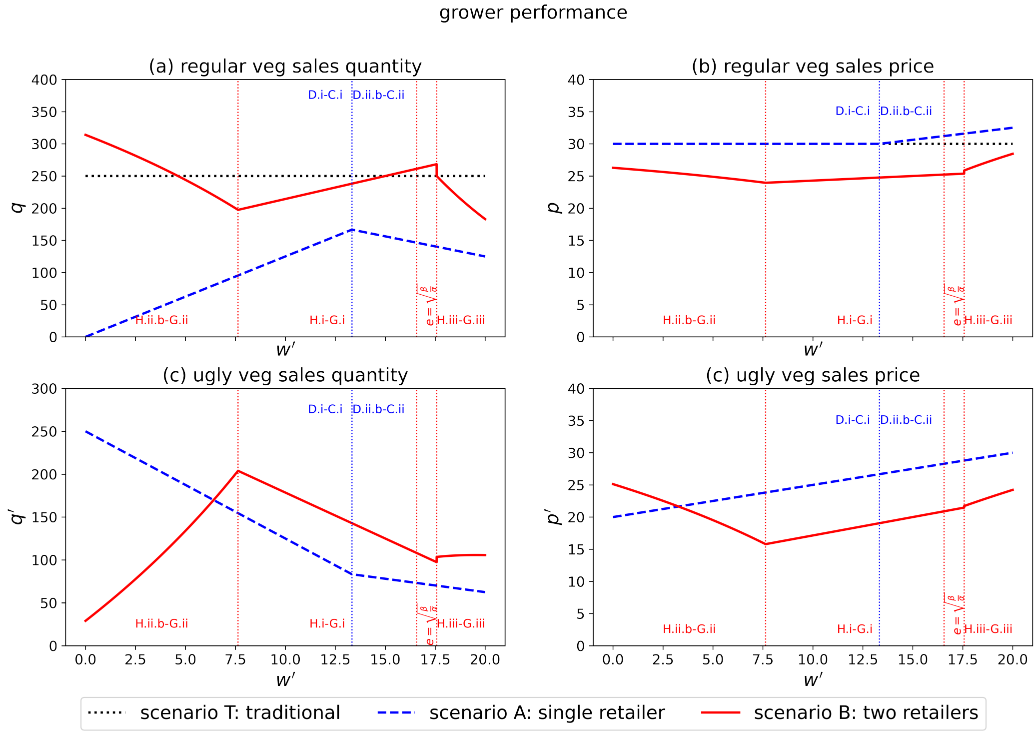

Figure 3: Retailers' decisions under different values of w' under different supply chain structures

Next, we focus on the retailers in the supply chain. Figure 3 shows the equilibrium decisions of the retailer(s) in different scenarios as functions of the ugly veg wholesale price w'. In Figure 3-a, we plot the quantity of regular produce sold in the market under different scenarios, while in Figure 3-b, we plot the corresponding selling prices. For the single-retailer supply chain, the regular veg sales price is constant for $w' < 13.33$, while the retailer substitutes ugly veg with regular produce as the wholesale price of ugly veg increases. For $w' \geq 13.33$, the retailer pushes the regular produce prices up as the ugly veg gets more costly (Figure 3-d). The regular veg sales is always lower under scenario A than under scenarios T and B due to the substitution and competition effects, respectively. The regular produce price is highest under the single-retailer scenario, as the retailer can control both prices, followed by the traditional supply chain and finally the two-retailers supply chain. Interestingly, we see that under competition (scenario B), the ugly veg and regular produce selling prices do not exhibit monotonic dependency on the ugly veg wholesale price. The selling prices simultaneously decrease with the increasing ugly veg wholesale price for $w' < 7.5$ while the auxiliary retailer increases its market share by cutting down the prices. For higher ugly veg wholesale prices, the auxiliary retailer needs to increase its selling price.

A comparison between Figure 3-a and Figure 3-c shows the opposing movements of the regular produce and ugly veg sales, while we see a non-monotonic behavior linked to the dynamics of the equilibrium prices. Remarkably, under low ugly veg wholesale prices, the single retailer would completely deprive the market of regular produce and sell only the ugly veg. This shows that if the customers' motive for buying the produce is solely price-driven, then the consolidation of supply channels for different types of produce reduces the overall supply of regular produce to the market. This, however, does not always happen in the scenario with two retailers, where the competition leads to higher overall sales with varying balances between the two channels in line with the prices.

Observation 2. *The quantity of regular produce sold can be drastically reduced if the traditional retailer markets the ugly veg, while it is more stable under competition. Marketing the ugly veg through an auxiliary retailer seems to increase the total demand and to decrease the on-farm loss. In contrast, demand cannibalization is likely to happen if the main retailer markets the ugly veg.*

Overall, we can conclude from this example that competition seems to be advisable for the marketing of ugly veg, if there are potential specialized retailers interested in selling ugly veg. In the next subsection, we aim at understanding the robustness of these results.

8.2 Statistical Sampling

We conduct an in-depth numerical analysis to derive additional observations. We aim at studying a wide variety of settings; therefore, we generate a large sample of instances. We selected 5 values for each of the 7 parameters of our model (see Table 1 for the details). We used a full factorial design, leading to $78,125$ instances. Among all of these instances, the feasibility conditions I-III stated in Section 5 are met for $21,166$ instances, which we refer to as feasible instances in what follows. We applied the analytical results of Sections 4, 5, 6 and 7 to all feasible instances. In addition, we computed equilibrium profits and volumes of on-farm loss.

We start by comparing the ugly veg supply chain with one-retailer and the traditional supply chain. At first, we can notice that the total food loss in the ugly veg supply chain is less than or equal to that in the traditional supply chain for all feasible instances. This is in line with Theorem 6 because $\sqrt{\beta/\alpha} \leq 3/4$ for all feasible instances. However, the grower supplies the ugly veg to the main retailer for $3,204$ instances, that is, only in 15.1% of all feasible instances. This is quite a surprising result at first glance, as we might expect that the option to supply the ugly veg would be exercised by the grower. However, the main retailer can increase his margin on the regular produce

Parameter	Test Values				
a	100	200	500	1000	2000
b	1	2	5	10	25
λ	1	2	5	10	25
w	1	2	5	10	20
w'	0.5	1	2	5	10
α	0.5	1	2	5	10
β	0.1	0.5	1	2	5

Table 1: Values of parameters in the simulation study

and redirect some customers towards the ugly veg when supplied with both types of produce. This creates a cannibalization effect, as observed in Subsection 8.1, and might diminish the grower's profit. The grower is more likely to benefit from supplying the ugly veg to the main retailer when $w-w'$ is relatively low. This indeed limits the grower's loss in profit if some customers are redirected towards ugly produce. Selling ugly produce is not very attractive for the main retailer either. In the instances for which the grower supplies the ugly veg via the main retailer, the retailer's profit is on average 10% less than that under the traditional supply chain. The retailer's profit in the ugly veg supply chain is less than under the traditional supply chain for 70% of these instances. This happens when the grower limits access to regular produce, that is under the SPNE $(D.ii.b)-(C.ii)$ (see Theorem 4). We deduce that both the grower and the retailer are better off when selling ugly veg for only 959 instances, that is less than 5% of the feasible instances. For these instances, the average food loss reduction is 18.9%. This shows that selling ugly produce via the main retailer can be effective in reducing on-farm loss even if the conditions for the grower and the retailer to benefit both from this initiative are quite unlikely. Thus, we derive the following insight.

Observation 3. *Selling the ugly veg through the main retailer might be an effective way to reduce on-farm food loss. However, the conditions to make both the grower and the retailer better off are not very likely to be met in practice. This explains why traditional retailers do not often propose the ugly veg together with the regular produce.*

We further assess whether supply chain coordination might help in making the supply of ugly veg through the main retailer more likely to happen. We refer to the literature on contracting in agri-food supply chains for an overview of the contracts that can help in coordinating the supply chain. We compare the results under the centralized ugly veg supply chain with the results under the traditional (decentralized) supply chain. The average increase in total profit in the centralized ugly veg supply chain is 30.8%. This is quite substantial and creates some incentive to supply the

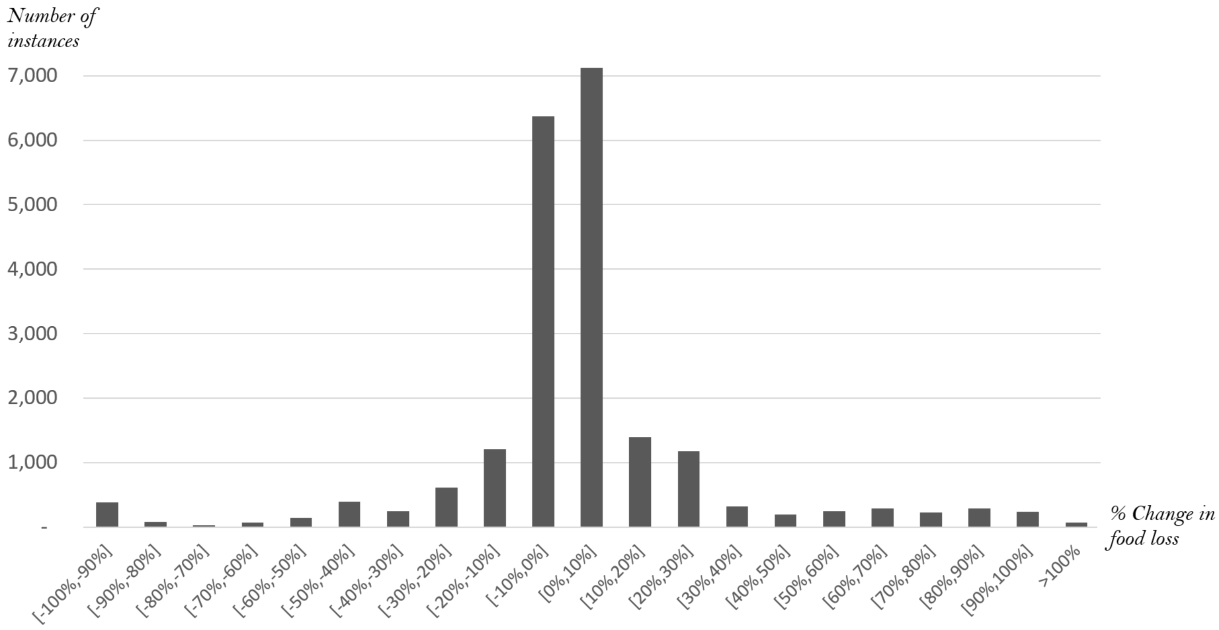

Figure 4: Distribution of on-farm loss variation due to the supply of ugly veg in the centralized scenario over the 21,166 feasible instances.

ugly veg through grower-retailer coordination. However, while there are always some ugly produce sold under the centralized ugly veg supply chain (recall that this happens for only 15.1% of all feasible instances for the decentralized ugly veg supply chain), the food loss is higher than that under the traditional supply chain by 2.9% on average. This surprising result is explained by the increase in cultivated land under the centralized ugly veg supply chain. Thus, while the unit loss per cultivated area is decreased, the increase in total cultivated land can result in an absolute increase in loss. The distribution of on-farm loss variation under the centralized ugly veg supply chain in comparison with the traditional supply chain for the $21,166$ feasible instances can be found in Figure 4.

These results show that coordination between the grower and the retailer might make the supply of ugly veg through the main retailer more likely to be developed in practice. However, this practice can result in an increase in absolute loss at farm.

Observation 4. *Coordination between the grower and the retailer could make the supply of ugly veg through the main retailer more beneficial. In many practical cases, coordination would be necessary for this purpose. However, this practice will not necessarily enable reducing absolute loss at farm.*

Next, we investigate the impact of including an auxiliary retailer in the system for marketing the ugly veg. We highlight that in all feasible instances studied, the grower's profit is higher with two retailers than in the corresponding situations where the main retailer sells both regular produce and ugly veg. This implies that the grower would benefit from the competition between the main and auxiliary retailers. This is another reason for scarcity of practical applications with traditional retailers selling both regular produce and ugly veg. However, the on-farm loss is not necessarily lower under the retailers' competition. Indeed, the loss is greater in the two-retailer scenario for $5,127$ instances, that is, 24.2% of the feasible instances. For these instances, the grower drastically increases the land she cultivates. On-farm loss increases subsequently, even if more ugly veg is sold.

Observation 5. *The grower would always favor selling ugly veg to an auxiliary retailer so as to benefit from the competition between the retailers. However, selling ugly veg through a dedicated channel does not necessarily reduce loss when compared with the single-retailer scenario.*

We next evaluate the implications of marketing the ugly veg through a dedicated channel, i.e., the auxiliary retailer, as compared to selling regular produce only. First, we highlight that this practice is beneficial for the grower in many cases. Indeed, the grower's profit is higher when selling the ugly veg for $20,647$ instances, that is, 98% of the feasible instances studied. This is a very encouraging result that might explain why many specialized retailers for ugly veg are currently emerging. Surprisingly, we identified $5,582$ instances in which the grower's profit is higher than that under the traditional supply chain even if there is no supply of ugly veg! This occurs under $(G.ii)(H.ii.a)$ as $e_{(H.ii.a)} = 1$ (see Theorem 8). In this setting, the mere presence of the auxiliary retailer limits the ability of the main retailer to increase selling price p. This increases the total demand which is beneficial for the grower. Overall, the presence of auxiliary retailers dedicated to ugly veg can be perceived as a risk for traditional retailers, as their profit will be subsequently diminished. Indeed, the main retailer's profit is reduced for all feasible instances studied, and the average profit reduction for the instances in which the grower is better off while selling ugly veg through a dedicated retailer is 38.5%.

Observation 6. *The grower is very likely to benefit from supplying ugly veg through a dedicated channel. The main retailer's profit could be strongly reduced in this case due to competition.*

We further study the implications of selling ugly veg via an auxiliary retailer in terms of on-farm loss. On average, the loss reduction is 45.7% compared to the traditional supply chain for the $20,647$ instances for which the grower is better off (in the remaining instances, the grower decides not to supply ugly veg). However, the loss can also increase while selling the ugly veg. This occurs for $4,595$ instances, that is, 22.3% of the instances in which the grower is better off. The variation

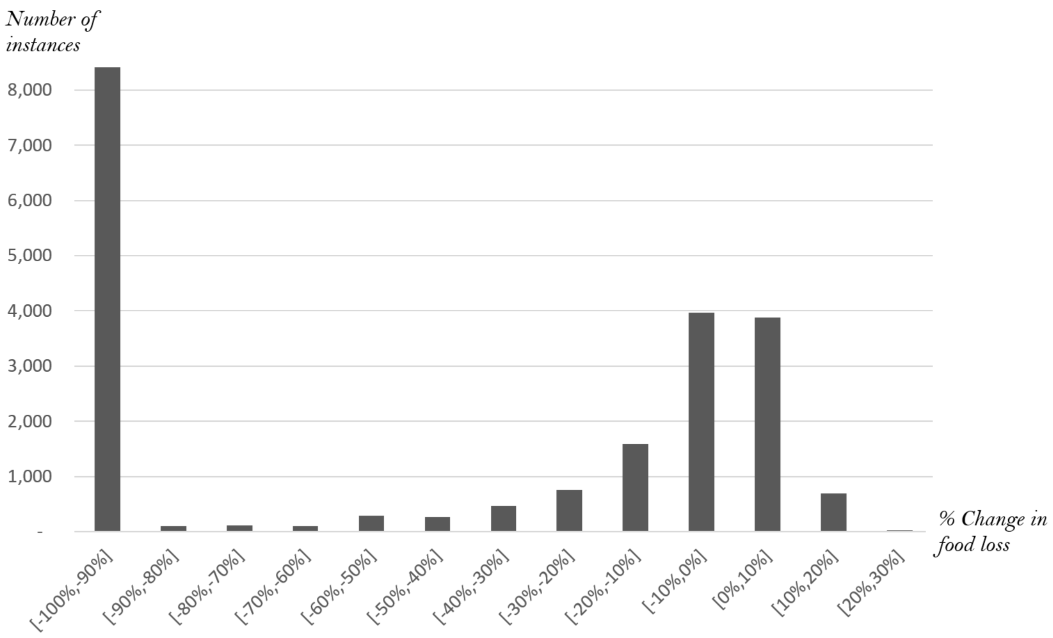

Figure 5: Distribution of on-farm loss variation due to the supply of ugly veg by an auxiliary retailer in comparison with single retailer ugly veg supply chain.

of loss over the $20,647$ instances for which the grower is better off when selling the ugly veg can be found in Figure 5.

We explain the potential increase in food loss in the ugly veg supply chain with two retailers as follows. At first, we compute the loss per unit of cultivated land as Y/Q, which is less or equal to the one of the traditional supply chain when supplying the ugly veg to the auxiliary retailer (Theorem 10). However, the grower can decrease her effort and increase the area of the cultivated land. We notice that the effort when supplying the ugly veg through the auxiliary retailer is less or equal to that of the traditional supply chain for 98% of the instances or which the grower is better off when supplying the ugly veg. Besides, in 61.7% of the instances for which the grower is better off when supplying the ugly veg, the grower increases the area of cultivated land. If we focus solely on the $4,595$ instances for which the food loss is greater when supplying the ugly veg to an auxiliary retailer, we can highlight that the effort exerted by the grower is the same as that in case of supplying the regular produce only, again as depicted by Theorem 10. Besides, the area of the cultivated land always increases. Thus, we derive the following observation.

Observation 7. *Supplying ugly veg through the auxiliary retailer might lead to an absolute increase in loss while the loss per unit of cultivated land will decrease. This happens when the grower increases the area of the cultivated land without increasing her effort.*

9 Conclusion

This article proposes some of the first analytical models to investigate the implications of retailing ugly fruit and vegetables. We study a variety of supply chain structures to understand the behaviour of the customers, the retailers, and the growers. A typical grower in the supply chain determines the area of the cultivated land as well as the effort to be exerted, which determines the yield of regular produce and ugly veg. The grower's objective is to maximize her profit under exogenous wholesale prices in a two-stage non-cooperative game. The retailers decide on the market prices and the purchase quantities so as to maximize their revenues. We consider different scenarios with regard to the supply channels of the ugly veg and also decentralized versus centralized decision making structures. Following a characterization of the subgame-perfect Nash Equilibria and optimal decisions, we analyze the implications of different supply chain structures. Our results enable us to identify the players' profits as well as the quantity of on-farm food loss.

We show that the marketing of ugly veg through the traditional retailer is likely to result in demand cannibalization. This can facilitate a reduction in on-farm food loss, but the grower's profit is likely to suffer. We conclude that the conditions for the marketing of ugly veg through the traditional retailer, i.e., when both the grower and the retailer increase their profits, are not likely to be met in practice. We also show that coordination between the grower and the retailer might help in making the distribution of ugly veg through traditional supply chains beneficial but also that this practice is likely to increase total on-farm loss due to an increase in the area of cultivated land. Our results also show that the marketing of ugly veg through a dedicated retailer is likely to increase total demand due to competition. Hence, the grower's profit is likely to increase. Therefore, we conclude that the marketing of ugly veg is more likely to develop through a dedicated channel. However, the total on-farm food loss may increase. This occurs when the grower increases the size of the cultivated land to meet additional demand for the regular produce without increasing the effort exerted. Overall, our results highlight new features that allow for a more in-depth understanding of the dynamics towards the marketing of ugly fruit and vegetables.

This research has several managerial implications for growers, retailers, consumers, and policy makers. For growers, it is beneficial to sell ugly fruit and vegetables under most conditions if there is a dedicated channel. Selling ugly veg to a traditional retailer, if he is willing to market it, can only be profitable if the ugly veg is sufficiently valuable, i.e., when the difference between the wholesale prices of regular produce and ugly veg is small, or in other types of contracts not considered here, such as whole-crop contracts (Allu and Belavina, 2020). In general, the grower would benefit from selling ugly fruit and vegetables to a dedicated retailer to take advantage of the competition between the retailers, which pushes market prices down, hence total demand up. For traditional retailers, in the conventional scenario studied here where the regular produce is substituted with ugly veg for price-sensitive customers, the offering of the ugly veg to the market constitutes mainly a risk rather than an opportunity. The traditional retailer's profit expectedly decreases if the ugly veg is offered via a dedicated channel as the demand for the regular produce decreases and also the main retailer needs to reduce the price due to competition. More interestingly, traditional retailers rarely benefit from selling the ugly veg themselves, despite controlling both prices and selling the regular produce at inflated prices, since the ugly veg must be of sufficient value so that the grower decides to offer it to the market. As the grower then restricts access to regular produce by decreasing the area of cultivated land to benefit from both types of produce, the main retailer typically ends up being economically worse off. On one hand, this can be interpreted as a reason why we do not see many traditional retailers that sell ugly fruit and vegetables. On the other hand, this does not contradict with exploring opportunities for utilizing ugly fruit and vegetables otherwise. For instance, traditional retailers can target and serve the environmentally conscious customer segment with ugly fruit and vegetables or they can design contracts to buy the whole crop from the growers—suppressing the grower's motivation to limit the supply of regular produce. Another solution for the main retailer is to ensure coordination with the growers to improve their profits while selling ugly veg, a point that we will revisit below.

From the consumers' perspective, the sales of ugly fruit and vegetables might provide access to cheaper alternatives, while generally contributing to the reduction of food loss and waste. However, we observe a trade-off for consumers between this and a potential reduction in availability, along with an increase in price, of regular produce if the ugly veg is offered by the same retailer. But competition, through a dedicated retailer for the ugly veg, can mainly address this issue. For policy makers as well as a significant number of consumers, the main question is whether the food waste and loss can be reduced by selling ugly fruit and vegetables. We show that offering the ugly fruit and vegetables has a high potential to reduce the food loss. The food loss is reduced through product substitution and demand cannibalization when ugly veg is sold via the main retailer. However, we have seen the challenges in accomplishing this through a single retailer due to reduced profits of the grower and/or the retailer. Policy makers can accordingly consider introducing incentives for selling or penalties against discarding ugly fruit and vegetables. Since we showed that economic coordination in the supply chain need not reduce food loss on its own, the role of the state is worth considering. We have also shown that the competition between retailers can reduce the food loss substantially when there is a dedicated ugly veg channel. Although the on-farm food loss per unit area always decreases in this supply chain setup, the total on-farm food loss can still increase under certain conditions due to increased land use.

We hope that our research will pave the way for future studies and we highlight three interesting future research directions. First, we highlight that the traditional retailer's profit is likely to suffer if an auxiliary retailer starts marketing ugly veg due to competition. We conclude that retailers should consider proactive strategies to safeguard against this risk. Our results show that coordination between the grower and the retailer might make the sales of ugly veg by the traditional retailer beneficial even if this practice might lead to an increase in on-farm loss. There is a vast stream of research that studies contracting in the agri-food industry. An interesting research question would consist of identifying what type of contract might help in coordinating the grower's and the retailer's decisions so as to make the marketing of ugly veg through the traditional retailer attractive and, at the same time, ensure on-farm loss reduction. Second, we believe that a better understanding of the consumers' behavior towards ugly fruit and veg might be useful. This would result in alternative modeling of the demand behavior, which could generate interesting new analytical results and insights. Third, we acknowledge that ugly veg supply chains are subject to a high level of uncertainty. Besides the lack of knowledge about customers' behavior discussed above, the yield per unit of cultivated land is affected in practice by a host of factors, such as weather conditions and pests, and there is a vast amount of literature on agri-food supply chains with random yield. In the context of ugly fruit and vegetables, there is an additional level of uncertainty related to the effect of the grower's effort on the product quality mix. In this paper, we focus on the expected values and assume away the randomness in a first attempt to understand the dynamics towards the marketing of ugly veg. In future studies, our research can be extended by analyzing the sensitivity of our insights to random yield and random product quality mix.

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List of Notations

Parameters	a	Maximum market demand
	b	Price sensitivity of regular produce demand
	λ	Price sensitivity of ugly veg demand
	w	Wholesale price of regular produce
	w'	Wholesale price of ugly veg
	α	Effort-dependent cost of cultivation coefficient
	β	Effort-independent cost of cultivation coefficient
Variables	Q	Area of cultivated land
	e	Level of effort exerted
	q	Order quantity of the regular produce
	q'	Order quantity of the ugly veg
	p	Selling price of regular produce
	p'	Selling price of ugly veg
Functions	d	Market demand for the regular produce
	d'	Market demand for the ugly veg
	Π	Grower's profit
	π	Retailer's profit
	Y	Magnitude of food loss

Table 2: List of notations

Proofs

We present a technical lemma before providing the proofs.

Lemma A1. *Let $f : \mathbb{R}^{n+1} \to \mathbb{R}$, $g^l : \mathbb{R}^n \to \mathbb{R}$, $g^u : \mathbb{R}^n \to \mathbb{R}$, $h^{(i)} : \mathbb{R}^n \to \mathbb{R}$ be differentiable. If $\frac{\partial f}{\partial x}(x, \mathbf{y}) \geq 0$ for $(x, \mathbf{y})$, where $\mathbf{y} = (y_1, y_2, \ldots, y_n)$, in the feasible region C of Eq. (10) and $\frac{\partial f}{\partial x}(x, \mathbf{y}) > 0$ for $(x, \mathbf{y}) \in C \cap \{(x', \mathbf{y}') : x' \neq g^u(\mathbf{y}')\}$. Then the problem*

$$\begin{aligned} \max \quad & f(x, \mathbf{y}) \\ s.t. \quad & g^l(\mathbf{y}) \leq x \leq g^u(\mathbf{y}) \\ & h^{(i)}(\mathbf{y}) \leq 0, i = 1, 2, \ldots, m \end{aligned} \tag{10}$$

is equivalent to the lower dimensional problem obtained by eliminating x with $x = g^u(\mathbf{y})$, i.e.

$$\begin{aligned} \max \quad & f(g^u(\mathbf{y}), \mathbf{y}) \\ s.t. \quad & h^i(\mathbf{y}) \leq 0, i = 1, 2, \ldots, m \end{aligned}$$

Similarly, if $\frac{\partial f}{\partial x}(x, \mathbf{y}) \le 0$ *for* $(x, \mathbf{y}) \in C$ *and* $\frac{\partial f}{\partial x}(x, \mathbf{y}) < 0$ *for* $(x, \mathbf{y}) \in C \cap \{(x', \mathbf{y}') : x' \neq g^l(\mathbf{y}')\}$, *the problem Eq. (10) is equivalent to*

$$\begin{aligned} \max \quad & f(g^l(\mathbf{y}), \mathbf{y}) \\ s.t. \quad & h^i(\mathbf{y}) \le 0, i = 1, 2, \ldots, m \end{aligned}$$

Proof of Lemma A1. For the first case where the objective is non-decreasing in x, let (x^*, y^*) be the optimal solution and by way of contradiction assume $x^* < g^u(\mathbf{y}^*)$. Choose $0 < \epsilon \ll 1$ such that $x + \epsilon < g^u(\mathbf{y}^*)$. Since the variable x is only involved in the first constraint, the point $(x + \epsilon, y^*)$ is feasible. As $x < g^u(\mathbf{y}^*)$, the derivative $f_x(x^*, \mathbf{y}^*)$ is positive. From first-order Taylor expansion, $f(x^* + \epsilon, \mathbf{y}^*) = f(x^*, \mathbf{y}^*) + \epsilon f_x(x^*, \mathbf{y}^*) + o(\epsilon^2)$, leading to $f(x^* + \epsilon, \mathbf{y}^*) > f(x^*, \mathbf{y}^*)$. Hence, (x^*, y^*) cannot be optimal. Since the constraint $x \le g^u(\mathbf{y})$ must be binding for the optimal solution, we replace the inequality with equality $x = g^u(\mathbf{y})$. Since $g^l(\mathbf{y}) \le g^u(\mathbf{y})$ and the variable x is not involved in any other constraints, we can eliminate x. The proof for the other case follows trivially from this. □

Proof of Lemma 1. The optimization problem can be written as $\max_{0 \le a - bp \le Qe} \pi = (p - w)(a - bp)$. The solution to the unconstrained problem is $p^* = (a + bw)/2b$ and $q^* = (a - bw)/2$. This requires that $Qe \ge (a - bw)/2$. If $Qe < (a - bw)/2$, at equilibrium we have $a - bp^* = Qe$ which obtains $p^* = (a - Qe)/b$ and $q^* = Qe$. □

Proof of Lemma 2. Given the best response function in Lemma 1, the optimization problems of the grower falls into two categories:

(B) The grower solves:

$$\begin{aligned} \max \quad \Pi(Q, e) = \quad & w\frac{a - bw}{2} - Q\left(\alpha e^2 + \beta\right) \\ s.t. \quad & Q \ge \frac{a - bw}{2e}. \end{aligned}$$

For any value of e, the best response Q attains its minimum. Thus

$$Q_{(B)-(A.i)} = \frac{a - bw}{2e_{(D.i)-(A.i)}}.$$

The best strategy for the grower solves:

$$\max_{e\in(0,1]} w\frac{a-bw}{2} - \frac{a-bw}{2}\left(\alpha e + \frac{\beta}{e}\right).$$

The best effort is $e_{(B)-(A.i)} = \sqrt{\frac{\beta}{\alpha}}$.

(B') The grower solves:

$$\begin{aligned} \max \quad \Pi(Q,e) = \quad & Q\left(we - \alpha e^2 - \beta\right) \\ s.t. \quad & Q < \frac{a-bw}{2e}. \end{aligned}$$

The problem is unbounded if $we - \alpha e^2 - \beta \geq 0$. Otherwise if $we - \alpha e^2 - \beta < 0$, we have $Q_{(B')-(A.ii)} = 0$.

□

Proof of Theorem 1. In the proof of Lemma 2, we established that the case (B) will result in retailer's choice of strategies in case (A.i). The profit of the grower in case (B), considering the reaction of the retailer in case (A.i), is

$$\Pi_{(B)-(A.i)} = \left(w - 2\sqrt{\alpha\beta}\right)\frac{a-bw}{2}.$$

In order for the SPNE to be (B)-(A.i) we must have $\Pi_{(B)-(A.i)} \geq 0$. This requires $w \geq 2\sqrt{\alpha\beta}$ and $w \leq a/b$. In order for this to happen we must have $\sqrt{\alpha\beta} \leq 2a/b$. □

Proof of Lemma 3. Since at equilibrium the market for regular produce clears, we have $q = d(p)$ so there would be no waste. At the farm level, we have $Y_{(B)-(A.i)} = \frac{a-bw}{2}\sqrt{\frac{\alpha}{\beta}} - \frac{a-bw}{2}$ which obtains the formula in the statement of the lemma. □

Proof of Lemma 4. Let q, q', p, and p' be a partial SPNE solution. We first show that $q \leq a - bp - \lambda(p-p')$ and $q' \leq \lambda(p-p')$.

Assume the contrary, that is $q > a - bp - \lambda(p-p')$. We have

$$\pi = pd(p,p') + p'\min\left\{q' + q - d(p,p')\}, d'(p,p')\right\} - wq - w'q'$$

Let $\hat{q} = d(p,p')$ and $\hat{q}' = q' + q - d(p,p')$ be alternative values. we have

$$\hat{\pi} = pd(p,p') + p' \min\left\{q' + q - d(p,p')\}, d'(p,p')\right\} - wd(p,p') - w'(q' + q - d(p,p').$$

we get $\hat{\pi} - \pi = (w - w')(q - d(p,p')) \geq 0$ by the assumption $w \geq w'$. If $w = w'$, then q and q' are alternative partial SPNE solutions. Otherwise, if $w > w'$ the above contradicts the equilibrium conditions for q and q' which is a contradiction. We can use a similar argument to prove the statement on q'.

The optimization problem can be formulated as below:

$$\begin{aligned}
\max \quad \pi(q,q',p,p'|Q,e) = \quad & (p-w)q + (p'-w')q' && (11)\\
s.t. \quad & q \leq a - bp - \lambda(p-p') && (12)\\
& q' \leq \lambda(p-p') && (13)\\
& q \leq Qe && (14)\\
& q + q' \leq Q && (15)\\
& p' \leq p && (16)\\
& q, q' \geq 0 && (17)
\end{aligned}$$

Next we show that the inequalities (12) and (13) hold strictly. First, consider the inequality (13). Suppose he contrary, that is, $q' < \lambda(p-p')$. Increasing p' would increase the objective function, and maintains inequality (12). Thus, one can find p' such that $q' = \lambda(p-p')$.

Next, consider inequality (12) and suppose the contrary, that is, $q < a - bp - \lambda(p-p')$. Let $p = \frac{a-q-\lambda(p-p')}{b}$ and $p' = p' + (p-p)$. Note that $q = a - bp - \lambda(p-p')$. The new solution q, q', p, p', satisfies all the constraints, and increases the value of the objective function which is a contradiction. Thus, the claim holds. □

Proof of Theorem 2. Considering that at equilibrium we always have $q = a - bp - \lambda(p-p')$ and

$q' = \lambda(p - p')$ the optimization problem becomes:

$$\max \quad \pi(q, q', p, p'|Q, e) = \max \quad (p - w)[a - bp - \lambda(p - p')] + (p' - w')\lambda(p - p') \tag{18}$$

$$s.t. \quad a - bp - \lambda(p - p') \leq Qe \tag{19}$$

$$a - bp \leq Q \tag{20}$$

$$a - bp - \lambda(p - p') \geq 0 \tag{21}$$

$$p \geq p' \tag{22}$$

We consider possible cases:

(C.i) Suppose the supply constraints are not binding at equilibrium. In the relaxed problem, using the first order conditions we obtain $p = \frac{a+bw}{2b}$, and $p' = \frac{a+bw'}{2b}$, and subsequently $q = \frac{a-bw-\lambda(w-w')}{2}$, and $q' = \frac{\lambda(w-w')}{2}$. For the constraints to be satisfied in this case we must have $Qe \geq [a - bw - \lambda(w - w')]/2$, and $Q \geq (a - bw)/2$.

In this case, the retailer's expected profit will be:

$$\pi_{(C.i)} = \left(\frac{a - bw}{2b}\right)\left[\frac{a - bw' - \lambda(w - w')}{2}\right] + \left(\frac{a - bw'}{2b}\right)\frac{\lambda(w - w')}{2} > 0.$$

(C.ii) This case corresponds to limited supply of regular produce only. At equilibrium of supply-relaxed problem we have $\frac{1}{2}[a - bw - \lambda(w - w')] > Qe$. This constraint must be active at equilibrium thus we get $a - bp - \lambda(p - p') = Qe$ and $p' = [(\lambda + b)p - a + Qe]/\lambda$ and $p - p' = (a - bp - Qe)/\lambda$. The optimization problem becomes:

$$\max \quad \pi(q, q', p, p'|Q, e) = \max \quad (p - w)Qe + \left(\frac{(\lambda + b)p - a + Qe}{\lambda} - w'\right)q'$$

$$s.t. \quad \lambda(p - p') \leq Q(1 - e)$$

$$p \geq p'.$$

Assuming that the phrase in braces are non-negative, the problem becomes

$$\max \quad \pi(q,q',p,p'|Q,e) = \max \quad (p-w)Qe + \left(\frac{(\lambda+b)p-a+Qe}{\lambda} - w'\right)(a-bp-Qe)$$
$$s.t. \quad p \geq \frac{a-Q}{b}$$
$$p \leq \frac{a-Qe}{b}$$
$$\frac{(\lambda+b)p-a+Qe}{\lambda} \geq w'.$$

The interior solution is
$$p = \frac{a(2b+\lambda)+\lambda bw' - 2bQe}{2b(\lambda+b)}.$$

For this solution to be feasible we must have $a \geq bw'$ and $Qe \leq \frac{1}{2}(a-bw')$ but this holds by the assumption in this case.

(C.iii) This case corresponds to limited supply of ugly veg, that is $a-bp=Q$. If this is the case, it must be that the solution to the supply-relaxation of the problem obtains $a-bp>Q$ which is equivalent to $\frac{1}{2}(a-bw)>Q$.

The optimization problem becomes

$$\max \quad \pi(q,q',p,p'|Q,e) = \max \quad (p-w-p'+w')(a-bp-\lambda(p-p')) + (p'-w')Q$$
$$s.t. \quad Q \leq a-bp$$
$$a-bp-\lambda(p-p') \leq Qe$$
$$p \geq p'.$$

We check for interior solutions first. Solving the system of first order derivatives give: $p = \frac{ab-(b+2\lambda)Q-b\lambda(w-w')}{b^2}$ and $p'^* = \frac{ab-2(b+\lambda)Q-b(b+\lambda)(w-w')}{b^2}$. Upon checking the Hessian matrix we realize this is in fact a maximum to the relaxed problem.

From the expression for p we have $a-bp = (1+2\lambda/b)Q+\lambda(w-w')$ and also $p-p'^* = \frac{Q+b(w-w')}{b}$. This systems satisfies the first and third constraint but never the second constraint. Therefore the above solution is not feasible and in this case the supply of regular produce must also be capped. Therefore, at equilibrium if ugly veg is capped so is the regular veg. So we must have $a-bp-\lambda(p-p')=Qe$. This obtains $p-p'=(a-Qe-bp)/\lambda$ and $p'=[(\lambda+b)p-a+Qe]/\lambda$.

The optimization problem becomes

$$\max \quad \pi(q,q',p,p'|Q,e) = \max \quad \left[\frac{a-Qe-bp}{\lambda} - w + w'\right]Qe + \left(\frac{(\lambda+b)p-a+Qe}{\lambda} - w'\right)Q$$
$$s.t. \quad p \le \frac{a-Q}{b}$$

The objective function is increasing on p so at equilibrium the upper bound on p is active. Thus we have

$$p = \frac{a-Q}{b}, \quad p' = \frac{a-Q}{b} - \frac{Q(1-e)}{\lambda}.$$

□

Proof of Theorem 3. We analyze the grower's problem in different scenarios with regard to best response decisions of the retailer:

(D.i) Consider the reaction of the grower to the retailer best response in (C.i):

$$\max \Pi(Q,e) = w\frac{a-bw-\lambda(w-w')}{2} + w'\frac{\lambda(w-w')}{2} - Q(\beta+\alpha e^2)$$
$$s.t. \quad Q \ge \frac{1}{2}(a-bw)$$
$$Qe \ge \frac{1}{2}\left[a-bw-\lambda(w-w')\right]$$
$$Q \ge 0, \quad e \in [0,1].$$

Using the Lemma A1, there are two possibilities:

(D.i.a) If $\frac{1}{2}(a-bw) > \frac{1}{2e}\left[a-bw-\lambda(w-w')\right]$ which implies $e > (a-bw-\lambda(w-w'))/(a-bw)$, then we have $Q^* = \frac{1}{2}(a-bw)$ but the best response effort is at the boundary which is excluded so best response solution does not exist.

(D.i.b) If $\frac{1}{2}(a-bw) \le \frac{1}{2e}\left[a-bw-\lambda(w-w')\right]$ which implies $e \le (a-bw-\lambda(w-w'))/(a-bw)$, then we have $Q = \frac{1}{2e}\left[a-bw-\lambda(w-w')\right]$. The program becomes:

$$\max \Pi(Q,e) = w\frac{a-bw-\lambda(w-w')}{2} + w'\frac{\lambda(w-w')}{2} - \frac{a-bw-\lambda(w-w')}{2}\left(\frac{\beta}{e}+\alpha e\right)$$
$$s.t. \quad e \le \frac{a-bw-\lambda(w-w')}{a-bw}$$
$$e \in [0,1].$$

There are two further possibilities:

(D.i.b.1) If $\sqrt{\frac{\beta}{\alpha}} > (a-bw-\lambda(w-w'))/(a-bw)$, the best response effort happens at the boundary which is $e = \frac{a-bw-\lambda(w-w')}{a-bw}$. Thus at equilibrium we have $\frac{1}{2}(a-bw) = \frac{1}{2e}\left[a-bw-\lambda(w-w')\right]$. This solution is feasible in the program. The profit is

$$\Pi^{B.i.a}(Q^*, e^*) = \left(w - \beta\frac{a-bw}{a-bw-\lambda(w-w')} - \alpha\frac{a-bw-\lambda(w-w')}{a-bw}\right)\frac{a-bw-\lambda(w-w')}{2} + w'\lambda\frac{(w-w')}{2}.$$

There is no guarantee that this profit function is always non-negative.

(D.i.b.2) If $\sqrt{\frac{\beta}{\alpha}} \le (a-bw-\lambda(w-w'))/(a-bw)$ we have $e = \sqrt{\frac{\beta}{\alpha}}$, and $Q = \frac{1}{2}\sqrt{\frac{\alpha}{\beta}}(a-bw-\lambda(w-w'))$. Here we have $Q > \frac{1}{2}(a-bw)$ and $Qe = \frac{1}{2}\left[a-bw-\lambda(w-w')\right]$. This is a feasible solution to the program and gives the profit

$$\Pi^{B.i.b}(Q, e) = \left(w/2 - \sqrt{\alpha\beta}\right)(a-bw-\lambda(w-w')) + w'\lambda(w-w')/2.$$

With the assumption of $w \ge 2\sqrt{\alpha\beta}$, we have $\Pi^{B.i.b}(Q, e) \ge 0$.

(D.ii) This case corresponds to the best response of the retailer in case (C.ii). The optimization problem for the grower becomes:

$$\begin{aligned} \max \Pi(Q, e) = \quad & w'\lambda\frac{a-bw'}{2(\lambda+b)} + Q\left[we - \frac{w'\lambda e}{\lambda+b} - (\beta + \alpha e^2)\right] \\ s.t. \quad & Q \ge \frac{1}{2}(a-bw) \\ & Qe < \frac{1}{2}\left[a-bw-\lambda(w-w')\right] \\ & Q \ge 0, \quad e \in [0,1]. \end{aligned} \tag{23}$$

We consider three cases:

(D.ii.a) Suppose the objective function in (23) is strictly increasing on Q. If so, the SPNE quantity is obtained at the boundary $Qe = \frac{1}{2}\left[a-bw-\lambda(w-w')\right]$ which is excluded from the program.

(D.ii.b) Suppose the objective function in (23) is strictly decreasing on Q, which leads to $Q = \frac{1}{2}(a-bw)$. This requires the phrase in the braces to be negative. This is equivalent to three cases:

(D.ii.b.1) Having $w - \frac{w'\lambda}{\lambda+b} \geq 2\sqrt{\alpha\beta}$, and

$$e < \frac{(\lambda+b)w - \lambda w'}{2\alpha(\lambda+b)} - \sqrt{\left(\frac{(\lambda+b)w - \lambda w'}{2\alpha(\lambda+b)}\right)^2 - \frac{\beta}{\alpha}},$$

the program becomes:

$$\begin{aligned} \max \Pi(Q,e) = \quad & w'\lambda\frac{a-bw'}{2(\lambda+b)} + \frac{1}{2}(a-bw)\left[we - \frac{w'\lambda e}{\lambda+b} - (\beta+\alpha e^2)\right] \\ s.t. \quad & e < \frac{a-bw-\lambda(w-w')}{a-bw} \\ & e < \frac{(\lambda+b)w-\lambda w'}{2\alpha(\lambda+b)} - \sqrt{\left(\frac{(\lambda+b)w-\lambda w'}{2\alpha(\lambda+b)}\right)^2 - \frac{\beta}{\alpha}} \\ & Q \geq 0, \quad e \in [0,1]. \end{aligned}$$

The program is feasible when $\frac{(\lambda+b)w-\lambda w'}{2\alpha(\lambda+b)} - \sqrt{\left(\frac{(\lambda+b)w-\lambda w'}{2\alpha(\lambda+b)}\right)^2 - \frac{\beta}{\alpha}} \geq 0$, and the objective function is increasing on e. If $\frac{(\lambda+b)w-\lambda w'}{2\alpha(\lambda+b)} - \sqrt{\left(\frac{(\lambda+b)w-\lambda w'}{2\alpha(\lambda+b)}\right)^2 - \frac{\beta}{\alpha}} \geq \frac{a-bw-\lambda(w-w')}{a-bw}$, then the best response solution is included in the program. However, if $\frac{(\lambda+b)w-\lambda w'}{2\alpha(\lambda+b)} - \sqrt{\left(\frac{(\lambda+b)w-\lambda w'}{2\alpha(\lambda+b)}\right)^2 - \frac{\beta}{\alpha}} < \frac{a-bw-\lambda(w-w')}{a-bw}$, then the best response solution is excluded from the program.

(D.ii.b.2) We have $w - \frac{w'\lambda}{\lambda+b} \geq 2\sqrt{\alpha\beta}$, and

$$e > \frac{(\lambda+b)w - \lambda w'}{2\alpha(\lambda+b)} + \sqrt{\left(\frac{(\lambda+b)w - \lambda w'}{2\alpha(\lambda+b)}\right)^2 - \frac{\beta}{\alpha}}.$$

Using Lemma A1 we have $Q = \frac{1}{2}(a-bw)$. The program becomes:

$$\begin{aligned} \max \Pi(Q,e) = \quad & w'\lambda\frac{a-bw'}{2(\lambda+b)} + \frac{1}{2}(a-bw)\left[we - \frac{w'\lambda e}{\lambda+b} - (\beta+\alpha e^2)\right] \\ s.t. \quad & e < \frac{a-bw-\lambda(w-w')}{a-bw} \\ & e > \frac{(\lambda+b)w-\lambda w'}{2\alpha(\lambda+b)} + \sqrt{\left(\frac{(\lambda+b)w-\lambda w'}{2\alpha(\lambda+b)}\right)^2 - \frac{\beta}{\alpha}} \\ & Q \geq 0, \quad e \in [0,1]. \end{aligned}$$

If $\frac{a-bw-\lambda(w-w')}{a-bw} \leq \frac{(\lambda+b)w-\lambda w'}{2\alpha(\lambda+b)} + \sqrt{\left(\frac{(\lambda+b)w-\lambda w'}{2\alpha(\lambda+b)}\right)^2 - \frac{\beta}{\alpha}}$, then there would be no feasible solution.

However, if $\frac{a-bw-\lambda(w-w')}{a-bw} > \frac{(\lambda+b)w-\lambda w'}{2\alpha(\lambda+b)} + \sqrt{\left(\frac{(\lambda+b)w-\lambda w'}{2\alpha(\lambda+b)}\right)^2 - \frac{\beta}{\alpha}}$, then the best response solution is excluded.

(D.ii.b.3) We have $w - \frac{w'\lambda}{\lambda+b} < 2\sqrt{\alpha\beta}$. In this case we will have the interior solutions at $e^* = \frac{(\lambda+b)w-\lambda w'}{2\alpha(\lambda+b)}$. Here we must ensure (second constraint):

$$\frac{(\lambda+b)w-\lambda w'}{2\alpha(\lambda+b)} < \frac{a-bw-\lambda(w-w')}{a-bw}.$$

Note that in this case $0 < e^* < 1$. Then the profit function at equilibrium becomes:

$$\Pi^{B.ii.b.2}(Q^*, e^*) = w'\lambda\frac{a-bw'}{2(\lambda+b)} - \frac{1}{2}(a-bw)\left[\beta - \frac{1}{4\alpha}\left(w - \frac{\lambda w'}{\lambda+b}\right)^2\right].$$

(D.ii.c) The objective function is independent of Q. This happens when $w - \frac{w'\lambda}{\lambda+b} \geq 2\sqrt{\alpha\beta}$ and $e^* = \frac{(\lambda+b)w-\lambda w'}{2\alpha(\lambda+b)} \pm \sqrt{\left(\frac{(\lambda+b)w-\lambda w'}{2\alpha(\lambda+b)}\right)^2 - \frac{\beta}{\alpha}}$.

(D.ii.c.1) Suppose

$$e^* = \frac{(\lambda+b)w-\lambda w'}{2\alpha(\lambda+b)} - \sqrt{\left(\frac{(\lambda+b)w-\lambda w'}{2\alpha(\lambda+b)}\right)^2 - \frac{\beta}{\alpha}}.$$

If $e^* < \frac{a-bw-\lambda(w-w')}{a-bw}$, Then the best response is

$$Q^* \in \left[\frac{1}{2}(a-bw), \frac{1}{2}\left[a-bw-\lambda(w-w')\right]\left(\frac{(\lambda+b)w-\lambda w'}{2\alpha(\lambda+b)} - \sqrt{\left(\frac{(\lambda+b)w-\lambda w'}{2\alpha(\lambda+b)}\right)^2 - \frac{\beta}{\alpha}}\right)^{-1}\right).$$

The objective function in this case is

$$\Pi = w'\lambda\frac{a-bw'}{2(\lambda+b)}.$$

We show this case is always dominated by (D.i) if $\sqrt{\beta/\alpha} \leq \frac{a-bw-\lambda(w-w')}{a-bw}$. In this case, the choice of $e = \sqrt{\beta/\alpha}$ and $Qe = \frac{1}{2}[a-bw-\lambda(w-w')]$ which is obtained under $(D.i)$ gives the expected profit:

$$\Pi^{B.i.a}(Q^*, e^*) = \left(w/2 - \sqrt{\alpha\beta}\right)(a-bw-\lambda(w-w')) + w'\lambda(w-w')/2.$$

Since in this case $w - \frac{w'\lambda}{\lambda+b} \geq 2\sqrt{\alpha\beta}$, we have

$$\begin{aligned}\Pi^{B.i.a}(Q^*, e^*) &\geq \frac{1}{2}\left(\frac{w'\lambda}{\lambda+b}\right)(a - bw - \lambda(w - w')) + w'\lambda(w - w')/2 \\ &= w'\lambda\left(\frac{a - bw - \lambda(w - w')}{2(\lambda+b)} + \frac{w - w'}{2}\right) \\ &= w'\lambda\frac{a - bw'}{2(\lambda+b)}.\end{aligned}$$

When $\sqrt{\beta/\alpha} > \frac{a-bw-\lambda(w-w')}{a-bw}$, both (D.ii) or (D.i) might be dominant based on the other parameters. In particular, we show below when $w - \frac{w'\lambda}{\lambda+b} < \beta\frac{a-bw}{a-bw-\lambda(w-w')} + \alpha\frac{a-bw-\lambda(w-w')}{a-bw}$, then (D.ii) dominates (D.i). To have

$$w'\lambda\frac{a - bw'}{2(\lambda+b)} > \left(w - \beta\frac{a - bw}{a - bw - \lambda(w - w')} - \alpha\frac{a - bw - \lambda(w - w')}{a - bw}\right)\frac{a - bw - \lambda(w - w')}{2} + w'\lambda\frac{(w - w')}{2},$$

is equivalent to

$$w - \frac{w'\lambda}{\lambda+b} < \beta\frac{a - bw}{a - bw - \lambda(w - w')} + \alpha\frac{a - bw - \lambda(w - w')}{a - bw}.$$

Note that in this case we always have $2\sqrt{\alpha\beta} \leq \beta\frac{a-bw}{a-bw-\lambda(w-w')} + \alpha\frac{a-bw-\lambda(w-w')}{a-bw}$.

(D.ii.c.2) Suppose

$$e^* = \frac{(\lambda+b)w - \lambda w'}{2\alpha(\lambda+b)} + \sqrt{\left(\frac{(\lambda+b)w - \lambda w'}{2\alpha(\lambda+b)}\right)^2 - \frac{\beta}{\alpha}}.$$

Then the best response quantity is

$$Q^* \in \left[\frac{1}{2}(a - bw), \frac{1}{2}\left[a - bw - \lambda(w - w')\right]\left(\frac{(\lambda+b)w - \lambda w'}{2\alpha(\lambda+b)} - \sqrt{\left(\frac{(\lambda+b)w - \lambda w'}{2\alpha(\lambda+b)}\right)^2 - \frac{\beta}{\alpha}}\right)^{-1}\right).$$

The objective function in this case is

$$\Pi = w'\lambda\frac{a - bw'}{2(\lambda+b)}.$$

Note that by the assumption of this case we have $\sqrt{\beta/\alpha} \leq e < \frac{a-bw-\lambda(w-w')}{a-bw}$. By the argument in the previous case, this scenario is always dominated by (D.i).

(D.iii) Consider the reaction (C.iii) by the retailer:

$$\begin{aligned} \max \Pi(Q,e) = \quad & Q\left[we + w'(1-e) - (\beta + \alpha e^2)\right] \\ s.t. \quad & Q < \frac{1}{2}(a - bw) \\ & Q \geq 0, \quad e \in [0,1]. \end{aligned}$$

For the values of e such that the expression in braces is negative, we would have the best response Q at zero. On the other hand, for the values of e such that the expression in braces is positive, the best response solution would be at the boundary which is excluded from this case. Therefore, best response solutions cannot be happen in this scenario.

□

Proof of Theorem 4. Consider the case where $\sqrt{\beta/\alpha} \leq \zeta$, that is $2\sqrt{\alpha\beta} \leq 2\alpha\zeta$. We need to check the conditions under which we have $\Pi_{(D.ii)-(C.ii)} \geq \Pi_{(D.i)-(C.i)}$. The proof of Theorem 3 already establishes that the latter cannot hold when $\eta \geq 2\sqrt{\alpha\beta}$. So, we check if this is possible when $\eta < 2\sqrt{\alpha\beta}$. This, in light of the formulas given in the proof of Theorem 3, means having

$$w'\lambda\frac{a-bw'}{2(\lambda+b)} - \frac{1}{2}(a-bw)\left[\beta - \frac{1}{4\alpha}\left(w - \frac{\lambda w'}{\lambda+b}\right)^2\right] \geq \left(w/2 - \sqrt{\alpha\beta}\right)(a - bw - \lambda(w-w')) + w'\lambda(w-w')/2.$$

This simplifies to

$$\zeta \geq \frac{1}{2}\left[\sqrt{\frac{\beta}{\alpha}} + \frac{\eta}{2\alpha}\right],$$

which always holds in this case.

Consider the case where $\sqrt{\beta/\alpha} > \zeta$, that is $2\sqrt{\alpha\beta} > 2\alpha\zeta$. When $\eta \geq 2\sqrt{\alpha\beta}$, the proof of Theorem 3 givens the condition for having $\Pi_{(D.ii)-(C.ii)} \geq \Pi_{(D.i)-(C.i)}$ as:

$$w'\lambda\frac{a-bw'}{2(\lambda+b)} \geq \left(w - \frac{\beta}{\zeta} - \alpha\zeta\right)\frac{a - bw - \lambda(w-w')}{2} + w'\lambda\frac{(w-w')}{2}.$$

The above simplifies to $\eta \leq \alpha\zeta + \beta/\zeta$ which gives the condition in the statement of the theorem. On the other hand, from Theorem 3, it must be that $\frac{\eta}{2\alpha} - \sqrt{\frac{\eta^2}{4\alpha^2} - \frac{\beta}{\alpha}} < \zeta$. This means: $\frac{\eta}{2\alpha} - \zeta < \sqrt{\frac{\eta^2}{4\alpha^2} - \frac{\beta}{\alpha}}$.

Raising both sides to the power of two and simplifying gives $\eta > \alpha\zeta + \beta/\zeta$ which contradicts the previous condition. Therefore, this case never holds.

When $\eta < 2\sqrt{\alpha\beta}$, the proof of Theorem 3 givens the condition for having $\Pi_{(D.ii)-(C.ii)} \geq \Pi_{(D.i)-(C.i)}$ as:

$$w'\lambda\frac{a-bw'}{2(\lambda+b)} - \frac{1}{2}(a-bw)\left[\beta - \frac{1}{4\alpha}\left(w - \frac{\lambda w'}{\lambda+b}\right)^2\right] \geq \left(w - \frac{\beta}{\zeta} - \alpha\zeta\right)\frac{a-bw-\lambda(w-w')}{2} + w'\lambda\frac{(w-w')}{2}.$$

This simplifies to $(2\alpha\zeta - \eta)^2 \geq 0$, which always holds. □

Proof of Lemma 5. Considering the formula for Y and by replacing the corresponding terms, we get the formulas in the statement of the lemma. □

Proof of Theorem 5. In the following, we first compare the grower's profit under (D.i)-(C.i) and (D.ii.b)-(C.ii) solutions of the single-retailer ugly veg supply chain, $\Pi_{(D.i)-(C.i)}$, and that of the traditional supply chain, $\Pi_{(B)-(A.i)}$.

- Under the (D.i)-(C.i) solution with $e_{(D.i)} = \sqrt{\frac{\beta}{\alpha}}$ and $Q_{(D.i)} = \frac{1}{2}\sqrt{\frac{\alpha}{\beta}}\,[a - bw - \lambda(w - w')]$, the grower's profit is

$$\Pi_{(D.i)-(C.i)} = \left(w - 2\sqrt{\alpha\beta}\right)\frac{a-bw-\lambda(w-w')}{2} + w'\lambda\frac{w-w'}{2}.$$

  The grower's profit in the traditiona lsupply chain is

$$\Pi_{(B)-(C.i)} = \frac{(a-bw)(w-2\sqrt{\alpha\beta})}{2}.$$

  Therefore, the grower's profit difference is:

$$\Delta = \Pi_{(D.i)-(C.i)} - \Pi_{(B)-(C.i)} = \alpha\lambda(w-w')\left(\sqrt{\frac{\beta}{\alpha}} - \frac{w-w'}{2\alpha}\right),$$

  leading to

$$\Delta \geq 0 \iff 2\sqrt{\alpha\beta} \geq w - w'.$$

- Under the (D.i)-(C.i) solution with $e = \zeta$ we have

$$\Pi_{(D.i)-(C.i)} = (w - \beta/\zeta - \alpha\zeta)\,\frac{a-bw-\lambda(w-w')}{2} + w'\lambda\frac{(w-w')}{2}.$$

Accordingly, we have

$$\Delta = \Pi_{(D.i)-(C.i)} - \Pi_{(B)-(C.i)} = (w - \beta/\zeta - \alpha\zeta)\,\frac{a - bw - \lambda(w - w')}{2} + w'\lambda\frac{(w - w')}{2} - \frac{(a - bw)(w - 2\sqrt{\alpha\beta})}{2},$$

Thus we have

$$\Delta \geq 0 \iff \lambda(w - w')^2 < (a - bw)(2\sqrt{\beta/\alpha}) - \alpha\zeta^2 - \beta$$

Case $\eta \leq 2\sqrt{\alpha\beta}$: We want to show when $\eta \leq 2\sqrt{\alpha\beta}$, then $\Delta = \Pi_{(D.ii.b)-(C.ii)} - \Pi_{(B)} \geq 0$.

Since $w' < w$, we have

$$\begin{aligned}\Delta &= \frac{w'\lambda}{\lambda + b}\frac{a - bw'}{2} - \frac{1}{2}(a - bw)\left[\beta - \frac{1}{4\alpha}\left(w - \frac{\lambda w'}{\lambda + b}\right)^2 + w - 2\sqrt{\alpha\beta}\right] \\ &\geq \frac{1}{2}(a - bw)\left[\frac{1}{4\alpha}\left(w - \frac{\lambda w'}{\lambda + b}\right)^2 - \left(w - \frac{w'\lambda}{\lambda + b}\right) + 2\sqrt{\alpha\beta} - \beta\right].\end{aligned}$$

Consider the quadratic equation

$$\frac{1}{4\alpha}\eta^2 - \eta + 2\sqrt{\alpha\beta} - \beta.$$

We have $\Delta \geq 0$ iff $\frac{1}{4\alpha}\eta^2 - \eta + 2\sqrt{\alpha\beta} - \beta \geq 0$. This holds whenever

$$\eta \leq \hat{\eta} = 2\alpha\left(1 - \sqrt{1 - \frac{\beta}{\alpha} - 2\sqrt{\frac{\beta}{\alpha}}}\right).$$

Finally, we show by the assumption of this condition $2\sqrt{\alpha\beta} \leq \hat{\eta}$ for this is

$$\sqrt{\frac{\beta}{\alpha}} \leq 1 - \sqrt{1 - \frac{\beta}{\alpha} - 2\sqrt{\frac{\beta}{\alpha}}},$$

which is equivalent to

$$\sqrt{1 - \frac{\beta}{\alpha} - 2\sqrt{\frac{\beta}{\alpha}}} \leq 1 - \sqrt{\frac{\beta}{\alpha}}.$$

Raising the two sides to power of two:

$$1 - \frac{\beta}{\alpha} - 2\sqrt{\frac{\beta}{\alpha}} \leq 1 + \frac{\beta}{\alpha} - 2\sqrt{\frac{\beta}{\alpha}}.$$

that is $-\frac{\beta}{\alpha} \le \frac{\beta}{\alpha}$ which always holds.

This completes the proof. □

Proof of Theorem 6. If the SPNE is (D.i)-(C.i), then the food loss is zero if $\sqrt{\beta/\alpha} > \zeta$, and equal to that in the traditional supply chain if $\sqrt{\beta/\alpha} \le \zeta$. Suppose the SPNE is (D.ii.b)-(C.ii). We need to show the conditions under which $\nabla = Y_{(D.ii.b)-(C.ii)} - Y_{(B)-(A.i)} \le 0$, that is

$$\begin{aligned}\nabla &= \frac{a-bw}{2}\left(1-\frac{b\eta}{2\alpha(\lambda+b)}\right)-\frac{\lambda(a-bw')}{2(\lambda+b)}-\left(\sqrt{\frac{\alpha}{\beta}}-1\right)\frac{a-bw}{2}\\ &= \frac{a-bw}{2}\left(2-\sqrt{\frac{\alpha}{\beta}}-\frac{b\eta}{2\alpha(\lambda+b)}\right)-\frac{\lambda}{\lambda+b}\frac{a-bw'}{2}\\ &\le \frac{a-bw}{2}\left(1-\sqrt{\frac{\alpha}{\beta}}-\frac{w}{2\alpha}\frac{b^2}{(\lambda+b)^2}+\frac{b}{\lambda+b}\right),\end{aligned}$$

where the inequality holds since $w' \le w$. To show $\nabla \le 0$ we must show

$$1-\sqrt{\frac{\alpha}{\beta}}-\frac{w}{2\alpha}\frac{b^2}{(\lambda+b)^2}+\frac{b}{\lambda+b} \le 0.$$

We know $w \ge 2\sqrt{\alpha\beta}$, thus the above holds if that is

$$\sqrt{\frac{\beta}{\alpha}}\frac{b^2}{(\lambda+b)^2}-\frac{b}{\lambda+b}-\left(1-\sqrt{\frac{\alpha}{\beta}}\right) \ge 0.$$

Letting $b/(\lambda+b) = x$, the above inequality holds whenever the quadratic equation does not have two real root which happens whenever $\Delta = 4\sqrt{\frac{\beta}{\alpha}} - 3 \le 0$, that is $\sqrt{\frac{\beta}{\alpha}} \le \frac{3}{4}$. □

Proof of Lemma 6. The main retailer solves the problem

$$\begin{aligned}\max \quad & \pi_H(q,p|Q,e,q',p') = p\min\{q, a-bp-\lambda(p-p'), Qe\} - w\min\{q, Qe\}\\ s.t. \quad & p \ge w.\end{aligned}$$

Depending on whether the grower supplies sufficient high-quality produce, we have two cases:

$$q^* = \begin{cases} a-bp-\lambda(p-p') & \text{if } Qe \ge a-bp-\lambda(p-p') \\ \{q : q \ge Qe\} & \text{o/w} \end{cases}.$$

If $Qe \ge a-bp-\lambda(p-p')$, then the high quality supply is not constrained, which leads to $\pi_H =$

$p\min\{q, a-bp-\lambda(p-p')\} - w\min\{q, Qe\}$. For $q \leq a-bp-\lambda(p-p')$, the profit becomes $\pi_H = (p-w)q$. Since $p \geq w$, the optimal is attained at the boundary $q = a - bp - \lambda(p - p')$ by Lemma A1. For $q \geq a - bp - \lambda(p - p')$ the profit function is $\pi_H = p(a - bp - \lambda(p - p')) - w\min\{q, Qe\}$, which is non-increasing in the order quantity. Hence, the optimal order quantity is $q = a - bp - \lambda(p - p')$. If $Qe \leq a - bp - \lambda(p - p')$, the high-quality supply is limited and $\pi_H = (p - w)\min\{q, Qe\}$, which is increasing in q for $q \leq Qe$ and constant otherwise. Therefore, the main retailer can set the order quantity to Qe or any higher value.

Therefore, the main retailer solves the problems:

(i) Unlimited high-quality supply:

$$\begin{aligned} \max \quad & \pi_H(p, a - bp - \lambda(p - p')|Q, e, q', p') = (p - w)\left[a - bp - \lambda(p - p')\right] \\ s.t. \quad & p \geq w \\ & a - bp - \lambda(p - p') \leq Qe. \end{aligned}$$

The objective is concave in p and has its maximum at $p = \frac{w}{2} + \frac{a+\lambda p'}{2(b+\lambda)}$. The two constraints require that $p \geq w$ and $p \geq \frac{a+\lambda p'}{\lambda+b} - \frac{Qe}{\lambda+b}$.

The interior point solution $p^* = \frac{w}{2} + \frac{a+\lambda p'}{2(b+\lambda)}$, leading to $q^* = \frac{a+\lambda p'}{2} - \frac{w(b+\lambda)}{2}$, is feasible if $w \leq \frac{a+\lambda p'}{b+\lambda} \leq w + \frac{2Qe}{b+\lambda}$. The optimal profit is $\Pi_H^* = \frac{b+\lambda}{4}\left(\frac{a+\lambda p'}{b+\lambda} - w\right)^2$.

If $\frac{a+\lambda p'}{b+\lambda} < w$ then the interior solution is infeasible. The profit is decreasing in p for $p > \frac{w}{2} + \frac{a+\lambda p'}{2(b+\lambda)}$ and $\frac{a+\lambda p'}{\lambda+b} - \frac{Qe}{\lambda+b} \leq w$. Hence, $p = w$ provides the largest profit $\Pi_H = 0$ under the condition $\frac{a+\lambda p'}{b+\lambda} < w$. However, this leads to $q = a + \lambda p' - (b + \lambda)w < 0$. Hence, this solution is not physical.

If $\frac{a+\lambda p'}{\lambda+b} \geq w + \frac{2Qe}{\lambda+b}$, then interior point solution is infeasible as $\frac{w}{2} + \frac{a+\lambda p'}{2(\lambda+b)} \leq \frac{a+\lambda p'}{\lambda+b} - \frac{Qe}{\lambda+b}$. Furthermore, $w \leq \frac{a+\lambda p'}{\lambda+b} - \frac{Qe}{\lambda+b}$. Hence, $p^* = \frac{a+\lambda p'}{\lambda+b} - \frac{Qe}{\lambda+b}$, $q^* = Qe$, $\Pi_H^* = Qe\left[\frac{a+\lambda p'}{\lambda+b} - \frac{Qe}{\lambda+b} - w\right]$.

(ii) Limited high-quality supply:

$$\begin{aligned} \max \quad & \pi_H(p, q|Q, e, q', p') = (p - w)Qe \\ s.t. \quad & p \geq w \\ & a - bp - \lambda(p - p') \geq Qe. \end{aligned}$$

The problem is feasible if $\frac{a+\lambda p'}{\lambda+b} \geq w + \frac{Qe}{\lambda+b}$. Then $p^* = \frac{a+\lambda p'}{\lambda+b} - \frac{Qe}{\lambda+b}$, because the objective is increasing in p, and q^* is any quantity that is at least Qe.

Comparing the solutions from (i) and (ii), the unlimited supply solution dominates for $w+\frac{Qe}{\lambda+b} \leq \frac{a+\lambda p'}{\lambda+b} \leq w+\frac{2Qe}{\lambda+b}$ because it assumes the interior point solution, which is better than any other price. For $\frac{a+\lambda p'}{\lambda+b} \geq w+\frac{2Qe}{\lambda+b}$, the optimal price from both problems is the same, i.e., $p^* = \frac{a+\lambda p'}{\lambda+b} - \frac{Qe}{\lambda+b}$, but the limited supply allows for the main retailer to inflate its order, knowing it will not be provided in any case.

Therefore, the main retailer's strategy is:

$$p^* = \begin{cases} \text{reject} & \text{if } \frac{a+\lambda p'}{\lambda+b} < w \\ \frac{w}{2} + \frac{a+\lambda p'}{2(b+\lambda)} & \text{if } w \leq \frac{a+\lambda p'}{\lambda+b} \leq w + 2\frac{Qe}{\lambda+b} \\ \frac{a+\lambda p'}{b+\lambda} - \frac{Qe}{b+\lambda} & \text{if } w + 2\frac{Qe}{\lambda+b} \leq \frac{a+\lambda p'}{\lambda+b} \end{cases},$$

$$q^* = \begin{cases} \text{reject} & \text{if } \frac{a+\lambda p'}{\lambda+b} < w \\ \frac{a+\lambda p'}{2} - \frac{w(b+\lambda)}{2} & \text{if } w \leq \frac{a+\lambda p'}{\lambda+b} \leq w + 2\frac{Qe}{\lambda+b} \\ \{q : q \geq Qe\} & \text{if } w + 2\frac{Qe}{\lambda+b} \leq \frac{a+\lambda p'}{\lambda+b} \end{cases}.$$

□

Proof of Lemma 7. The auxiliary retailer solves the problem

$$\begin{aligned} \max \quad & \pi_L(q', p'|Q, e, q, p) = p' \min\{q', \lambda(p-p'), \max\{Q(1-e), Q-q\}\} - w' \min\{q', \max\{Q(1-e), Q-q\}\} \\ s.t. \quad & p' \geq w'. \end{aligned}$$

There are two cases depending on whether the remaining produce for the ugly veg channel, $\max\{Q(1-e), Q-q\}$, is sufficient to meet the auxiliary retailer's demand. If there is sufficient supply for the auxiliary channel, i.e., $\max\{Q(1-e), Q-q\} \geq \lambda(p-p')$, then the profit is $\pi_L = p' \min\{q', \lambda(p-p')\} - w' \min\{q', \max\{Q(1-e), Q-q\}\}$. For order quantities up to $\lambda(p-p')$, the profit is $\pi_L = q'(p'-w')$, which is increasing in q'. If $\lambda(p-p') < q'$ then the profit becomes $\pi_L = p'\lambda(p-p') - w' \min\{q', \max\{Q(1-e), Q-q\}\}$, which is non-increasing in q'. Hence, $q'^* = \lambda(p-p')$ if the supply for the auxiliary channel is not limited. Otherwise, i.e., $\max\{Q(1-e), Q-q\} \leq \lambda(p-p')$, the profit is $\pi_L = (p'-w') \min\{q', \max\{Q(1-e), Q-q\}\}$, which is increasing for $q' \leq \max\{Q(1-e), Q-q\}$ and constant for higher order quantities.

$$q'^* = \begin{cases} \lambda(p-p') & \text{if } \max\{Q(1-e), Q-q\} \geq \lambda(p-p') \\ \{q : q \geq \max\{Q(1-e), Q-q\}\} & \text{o/w} \end{cases}.$$

Therefore, the auxiliary retailer solves the problems:

(i) Unlimited auxiliary supply:

$$\begin{aligned} \max \quad & \pi_L(p', \lambda(p-p')|Q, e, q, p) = \lambda(p'-w')(p-p') \\ s.t. \quad & p' \geq w' \\ & \lambda(p-p') \leq \max\{Q(1-e), Q-q\}. \end{aligned}$$

The objective is concave in p', attaining the maximum at $p'^* = \frac{p+w'}{2}$. The feasibility conditions are are $p' \geq w'$ and $p' \geq p - \frac{1}{\lambda}\max\{Q(1-e), Q-q\}$. The interior point solution always satisfies the first constraint for $p \geq w'$, which is required for the main retailer to participate. The interior point solution $p'^* = \frac{p+w'}{2}$, leading to $q'^* = \frac{\lambda}{2}(p-w')$ and $\Pi_L^* = \frac{\lambda}{4}(p-w')^2$, is feasible if $w' \leq p \leq w' + \frac{2}{\lambda}\max\{Q(1-e), Q-q\}$.

If $p \geq w' + \frac{2}{\lambda}\max\{Q(1-e), Q-q\}$, the interior point solution is infeasible since $\frac{\lambda(p-w')}{2} \geq \max\{Q(1-e), Q-q\}$. Since the objective function is decreasing in the ugly veg price p' for $p' > \frac{p+w'}{2}$, the optimal price is $p' = p - \frac{1}{\lambda}\max\{Q(1-e), Q-q\}$, leading to $q' = \max\{Q(1-e), Q-q\})$ and $\pi_L = \left(p - w' - \frac{1}{\lambda}\max\{Q(1-e), Q-q\}\right)\max\{Q(1-e), Q-q\}$.

(ii) Limited auxiliary supply:

$$\begin{aligned} \max \quad & \pi_L(p', \lambda(p-p')|Q, e, q, p) = (p'-w')\max\{Q(1-e), Q-q\} \\ s.t. \quad & p' \geq w' \\ & \lambda(p-p') \geq \max\{Q(1-e), Q-q\}. \end{aligned}$$

Under the feasibility condition $p \geq w' + \frac{1}{\lambda}\max\{Q(1-e), Q-q\}$, the profit is increasing in the ugly veg price p'. Hence, the optimal price is $p' = p - \frac{1}{\lambda}\max\{Q(1-e), Q-q\}$, the order quantity q' is any quantity beyond the supply to the auxiliary channel, $\max\{Q(1-e), Q-q\}$, leading to the optimal profit $\pi_L = \left(p - w' - \frac{1}{\lambda}\max\{Q(1-e), Q-q\}\right)\max\{Q(1-e), Q-q\}$.

Comparing the solutions from (i) and (ii), the unlimited supply solution dominates for $w' + \frac{1}{\lambda}\max\{Q(1-e), Q-q\} \leq p \leq w' + \frac{2}{\lambda}\max\{Q(1-e), Q-q\}$ because it is the interior point solution.

Therefore, the auxiliary retailer's strategy is:

$$p'^{*} = \begin{cases} \text{reject} & \text{if } p < w' \\ \frac{p+w'}{2} & \text{if } w' \le p \le w' + \frac{2}{\lambda}\max\{Q(1-e), Q-q\} \\ p - \frac{1}{\lambda}\max\{Q(1-e), Q-q\} & \text{if } w' + \frac{2}{\lambda}\max\{Q(1-e), Q-q\} \le p \end{cases},$$

$$q'^{*} = \begin{cases} \text{reject} & \text{if } p < w' \\ \frac{\lambda(p-w')}{2} & \text{if } w' \le p \le w' + \frac{2}{\lambda}\max\{Q(1-e), Q-q\} \\ \{q' : q' \ge \max\{Q(1-e), Q-q\}\} & \text{if } w' + \frac{2}{\lambda}\max\{Q(1-e), Q-q\} \le p \end{cases}.$$

□

Proof of Theorem 7. We analyze the equilibrium between the main retailer and the auxiliary retailer under the four possible combinations of the cases in Lemmas 6 and 7.

(G.i): For the best responses (E.i) and (F.i), the prices are $p = \frac{w}{2} + \frac{a+\lambda p'}{2(b+\lambda)}$ and $p' = \frac{p+w'}{2}$, respectively, when solved together leads to the solution (G.i). The conditions $\frac{a+\lambda p'}{\lambda+b} \ge w$ and $p \ge w'$ evaluate to

$$w \le \frac{2a + \lambda w'}{2b + \lambda} \text{ and } w \ge \frac{(2b+\lambda)w' - a}{b + \lambda}.$$

Both conditions are implied by $w \ge w'$ and $a - bw - \lambda(w - w') \ge 0$. The condition $\frac{a+\lambda p'}{\lambda+b} < w + 2\frac{Qe}{\lambda+b}$ evaluates to

$$Qe > \frac{(b+\lambda)\left[2a - (2b+\lambda)w + \lambda w'\right]}{4b + 3\lambda}.$$

The condition $p < w' + \frac{2(Q-q)}{\lambda}$ is equivalent to

$$Q > \frac{a(2b+3\lambda) - 2b(b+\lambda)w - b\lambda w'}{4b + 3\lambda}.$$

(G.ii): For (E.i) and (F.ii), we insert $p = \frac{w}{2} + \frac{a+\lambda p'}{2(b+\lambda)}$ and $q = \frac{1}{2}\left[a - bw - \lambda(w - p')\right]$ into the equation for $p' = p - \frac{Q-q}{\lambda}$, obtaining the equilibrium prices and quantities for the solution (G.ii). The condition $w \le \frac{a+\lambda p'}{b+\lambda}$ leads to $Q \le a - bw$, while $p \ge w' + \frac{2(Q-q)}{\lambda}$ evaluates to

$$Q \le \frac{a(2b+3\lambda) - 2b(b+\lambda)w - b\lambda w'}{4b + 3\lambda}.$$

The two upper bounds on Q satisfy $\frac{a(2b+3\lambda)-2b(b+\lambda)w-b\lambda w'}{4b+3\lambda} \le a - bw$ by the assumptions $w \ge w'$

and $a - bw - \lambda(w - w') \geq 0$. Hence, it is sufficient to satisfy $Q \leq \frac{a(2b+3\lambda)-2b(b+\lambda)w-b\lambda w'}{4b+3\lambda}$. The constraint $\frac{a+\lambda p'}{b+\lambda} < w + 2\frac{Qe}{b+\lambda}$ leads to

$$Q\left(1 + \frac{be}{b+\lambda}\right) > a - bw.$$

Observe that when $Q\left(1 + \frac{be}{b+\lambda}\right) > a - bw$ and $Q \leq \frac{a(2b+3\lambda)-2b(b+\lambda)w-b\lambda w'}{4b+3\lambda}$, the inequality $Qe > \frac{(b+\lambda)[2a-(2b+\lambda)w+\lambda w']}{4b+3\lambda}$ is always satisfied.

(G.iii): For (E.ii) and (F.i), We solve for p by inserting $p' = \frac{p+w'}{2}$ into $p = \frac{a+\lambda p'-Qe}{b+\lambda}$, which is then used to calculate the equilibrium levels of the remaining variables under the solution (G.iii). The condition $\frac{a+\lambda p'}{\lambda+b} \geq w + 2\frac{Qe}{\lambda+b}$ leads to

$$Qe \leq \frac{(b+\lambda)\left[2a - (2b+\lambda)w + \lambda w'\right]}{4b+3\lambda},$$

while $w' \leq p$ evaluates to

$$Qe \leq a - bw'.$$

Due to the assumptions $w \geq w'$ and $a-bw-\lambda(w-w') \geq 0$, we always have $\frac{(b+\lambda)[2a-(2b+\lambda)w+\lambda w']}{4b+3\lambda} \leq a - bw'$. The third constraint $p \leq w' + \frac{2Q(1-e)}{\lambda}$ leads to $Q\left(1 + \frac{2b(1-e)}{\lambda}\right) \geq a - bw'$.

(G.iv): For (E.ii) and (F.ii), By inserting $p' = p - \frac{Q(1-e)}{\lambda}$ into $p = \frac{a+\lambda p'}{b+\lambda} - \frac{Qe}{b+\lambda}$ and then substituting in the other equations, we obtain the equilibrium solution (G.iv). The condition $\frac{a+\lambda p'}{\lambda+b} \geq w + 2\frac{Qe}{\lambda+b}$ leads to

$$Q\left(1 + \frac{be}{b+\lambda}\right) \leq a - bw,$$

while $p \geq w' + \frac{2Q(1-e)}{\lambda}$ leads to $Q\left(1 + \frac{2b(1-e)}{\lambda}\right) \leq a - bw'$.

□

Lemma A2. $(w - w')^2 \geq 4\alpha(\beta - w')$.

Proof of Lemma A2. If $\beta - w' \leq 0$, the claim clearly holds. Suppose $\beta - w' > 0$. Taking square root from the both sides, we must show $w \geq 2\sqrt{\alpha(\beta - w')} + w'$. Note that by assumption we have $w \geq 2\sqrt{\alpha\beta}$. To complete the proof we show $2\sqrt{\alpha\beta} \geq 2\sqrt{\alpha(\beta - w')} + w'$ which is equivalent to $2\sqrt{\alpha\beta} - w' \geq 2\sqrt{\alpha(\beta - w')}$. Raising both sides to the power of two and simplification obtains $w' \geq 4\sqrt{\alpha}(\sqrt{\beta} - \sqrt{\alpha})$. The latter holds by the assumption $\beta \leq \alpha$ and $w' \geq 0$. Therefore, the claim holds. □

Proof of Theorem 8. The grower solves four problems for the four cases in Theorem 7. For each case, we replace strict inequalities with the corresponding less-than-equal-to or greater-than-equal-to constraints to make a direct comparison with the boundary conditions presented in the condition (H.v) of Theorem 7.

(GR.i): For the equilibrium responses of the two retailers in **(G.i)**, the grower solves the first-stage problem:

$$\begin{aligned} \max \quad \Pi(Q,e) = &\frac{w(b+\lambda)\left[2a-(2b+\lambda)w+\lambda w'\right]}{4b+3\lambda} + \frac{w'\lambda\left[a+(b+\lambda)w-(2b+\lambda)w'\right]}{4b+3\lambda} \\ &- Q(\alpha e^2+\beta) \\ s.t. \quad Q \geq &\frac{a(2b+3\lambda)-2b(b+\lambda)w-b\lambda w'}{4b+3\lambda} \\ Qe \geq &\frac{(b+\lambda)\left[2a-(2b+\lambda)w+\lambda w'\right]}{4b+3\lambda} \\ 0 \leq &e \leq 1. \end{aligned}$$

Since the profit is decreasing in the effort e and the second constraint sets the lower bound on e, we insert the SPNE effort

$$e^E = \frac{(b+\lambda)\left[2a-(2b+\lambda)w+\lambda w'\right]}{(4b+3\lambda)Q},$$

and apply Lemma A1 to reduce to a problem in Q. Here, note that the effort range condition, $0 \leq e^E \leq 1$, is always satisfied because

$$Q \geq \frac{a(2b+3\lambda)-2b(b+\lambda)w-b\lambda w'}{4b+3\lambda} \geq \frac{(b+\lambda)\left[2a-(2b+\lambda)w+\lambda w'\right]}{4b+3\lambda}.$$

By $w \geq w'$ and $a - bw - \lambda(w - w') \geq 0$. By inserting e^E, we solve the following problem

$$\begin{aligned} \max \quad \Pi(Q,e^E(Q)) = &\frac{w(b+\lambda)\left[2a-(2b+\lambda)w+\lambda w'\right]}{4b+3\lambda} + \frac{w'\lambda\left[a+(b+\lambda)w-(2b+\lambda)w'\right]}{4b+3\lambda} \\ &-\alpha\frac{(b+\lambda)^2\left[2a-(2b+\lambda)w+\lambda w'\right]^2}{(4b+3\lambda)^2 Q} - \beta Q \\ s.t. \quad Q \geq &\frac{a(2b+3\lambda)-2b(b+\lambda)w-b\lambda w'}{4b+3\lambda}. \end{aligned}$$

The objective is concave for $Q > 0$, as $\frac{d^2\Pi}{dQ^2} = -\frac{\alpha(b+\lambda)^2(a-bw)^2}{b^2Q^3} < 0$, and has its maximum at

$Q = \frac{(b+\lambda)[2a-(2b+\lambda)w+\lambda w']}{4b+3\lambda}\sqrt{\frac{\alpha}{\beta}}$, which leads to the profit

$$\Pi(Q^E, e^E) = \frac{(b+\lambda)(w-2\sqrt{\alpha\beta})\left[2a-(2b+\lambda)w+\lambda w'\right] + \lambda w'\left[a+(b+\lambda)w-(2b+\lambda)w'\right]}{4b+3\lambda}.$$

The profit for the interior point solution is non-negative because $w \geq 2\sqrt{\alpha\beta}$, $w \geq w'$, and $a - bw - \lambda(w - w') \geq 0$ by assumption. Depending on whether the SPNE quantity is feasible, we have two cases:

(H.i) If $\alpha \geq \left[\frac{(2b+3\lambda)a - 2(b+\lambda)bw - b\lambda w'}{(b+\lambda)\left[2a-(2b+\lambda)w+\lambda w'\right]}\right]^2 \beta$, then $Q^* = \frac{(b+\lambda)\left[2a-(2b+\lambda)w+\lambda w'\right]}{4b+3\lambda}\sqrt{\frac{\alpha}{\beta}}$ and $e^* = \sqrt{\frac{\beta}{\alpha}}$.

(H.v) If $\alpha \leq \left[\frac{(2b+3\lambda)a - 2(b+\lambda)bw - b\lambda w'}{(b+\lambda)\left[2a-(2b+\lambda)w+\lambda w'\right]}\right]^2 \beta$, then $Q^* = \frac{(2b+3\lambda)a - 2(b+\lambda)bw - b\lambda w'}{4b+3\lambda}$ and $e^* = \frac{(b+\lambda)\left[2a-(2b+\lambda)w+\lambda w'\right]}{(2b+3\lambda)a - 2(b+\lambda)bw - b\lambda w'}$.

(GR.ii): For the equilibrium responses in **(G.ii)**, the grower solves the problem

$$\begin{aligned}
\max \quad & \Pi(Q,e) = \frac{(w-w')(b+\lambda)(a-bw)}{b} - Q\left[\alpha e^2 + \beta + \frac{(b+\lambda)w - (2b+\lambda)w'}{b}\right] \\
s.t. \quad & Q \leq \frac{a(2b+3\lambda) - 2b(b+\lambda)w - b\lambda w'}{4b+3\lambda} \\
& Q\left(1 + \frac{be}{b+\lambda}\right) \geq a - bw \\
& 0 \leq e \leq 1.
\end{aligned}$$

First, observe that

$$a - bw \geq \frac{a(2b+3\lambda) - 2b(b+\lambda)w - b\lambda w'}{4b+3\lambda},$$

by $a - bw - \lambda(w - w') \geq 0$ and $w \geq w'$. Hence, $Q < a - bw$, unless $a = bw$ and $w = w'$ at the same time, and then the effort cannot be zero. Putting the constraints together, we obtain:

$$\frac{(b+\lambda)(a-bw)}{2b+\lambda} \leq \frac{(b+\lambda)(a-bw)}{b(1+e)+\lambda} \leq Q \leq \frac{a(2b+3\lambda) - 2b(b+\lambda)w - b\lambda w'}{4b+3\lambda} \leq \frac{(b+\lambda)(a-bw)}{b+\lambda}.$$

Since the profit is decreasing in the effort e, we can use Lemma A1 to obtain $e^* = \frac{(b+\lambda)(a-bw-Q)}{bQ}$.

Thus, the grower solves the following problem in Q:

$$\max \quad \Pi(Q) = \frac{(w-w')(b+\lambda)(a-bw)}{b} - \alpha\frac{(b+\lambda)^2(a-bw-Q)^2}{b^2 Q} - Q\left[\beta + \frac{(b+\lambda)w-(2b+\lambda)w'}{b}\right]$$
$$s.t. \quad \frac{(b+\lambda)(a-bw)}{2b+\lambda} \le Q \le \frac{a(2b+3\lambda)-2b(b+\lambda)w-b\lambda w'}{4b+3\lambda}.$$

The profit is concave for $Q > 0$ as $\frac{\partial^2\Pi}{\partial Q^2} = -\frac{2\alpha(b+\lambda)^2(a-bw)^2}{b^2Q^3} < 0$ and attains its unconstrained maximum at

$$Q = \sqrt{\frac{\alpha(b+\lambda)^2(a-bw)^2}{\alpha(b+\lambda)^2 + b\left[b\beta+(b+\lambda)w-(2b+\lambda)w'\right]}},$$

which satisfies the lower bound iff $\alpha(3b+2\lambda) \ge b\beta + (b+\lambda)w - (2b+\lambda)w'$. If this constraint does not hold, the profit is decreasing in the order quantity because the grower already exerts full effort. Then the best response order quantity is $Q = \frac{(b+\lambda)(a-bw)}{2b+\lambda}$, leading to $e = 1$ and $\Pi = \frac{b(b+\lambda)(a-bw)(w-\alpha-\beta)}{2b+\lambda}$. Since $\frac{b\beta+(b+\lambda)w-(2b+\lambda)w'}{3b+2\lambda} \le w-\beta$, due to $w \ge 2\sqrt{\alpha\beta} \ge 2\beta$ by assumption, the profit is always non-negative for this solution if $\alpha < \frac{b\beta+(b+\lambda)w-(2b+\lambda)w'}{3b+2\lambda}$.

If the interior point satisfies the lower bound, the upper bound condition is:

$$\alpha \le \frac{\left[b(\beta-w')+(b+\lambda)(w-w')\right]\left[(2b+3\lambda)(a-bw)+b\lambda(w-w')\right]^2}{(b+\lambda)^2\left[2(a-bw)-\lambda(w-w')\right]\left[6(b+\lambda)(a-bw)+b\lambda(w-w')\right]}.$$

If α exceeds this bound then

$$Q = \frac{(2b+3\lambda)a-2(b+\lambda)bw-b\lambda w'}{4b+3\lambda} \quad \text{and} \quad e = \frac{(b+\lambda)\left[2a-(2b+\lambda)w+\lambda w'\right]}{(2b+3\lambda)a-2(b+\lambda)bw-b\lambda w'},$$

which is the boundary condition *(H.v)*, presented above for the case *(GR.i)*.

Hence, we have three cases: *(H.ii)* If $\alpha < \frac{b\beta+(b+\lambda)w-(2b+\lambda)w'}{3b+2\lambda}$, then $Q^* = \frac{(b+\lambda)(a-bw)}{2b+\lambda}$ and $e^* = 1$.

(H.iii) If

$$\frac{b\beta+(b+\lambda)w-(2b+\lambda)w'}{3b+2\lambda} \le \alpha \le \frac{\left[(2b+3\lambda)a-2(b+\lambda)bw-b\lambda w'\right]^2\left[b\beta+(b+\lambda)w-(2b+\lambda)w'\right]}{(b+\lambda)^2\left[2a-(2b+\lambda)w+\lambda w'\right]\left[6(b+\lambda)a-(6b+5\lambda)bw-b\lambda w'\right]}$$

then

$$Q = \sqrt{\frac{\alpha(b+\lambda)^2(a-bw)^2}{\alpha(b+\lambda)^2+b^2\beta+b(b+\lambda)w-b(2b+\lambda)w'}},$$

and

$$e = \left(\sqrt{1 + \frac{b\left[b\beta + (b+\lambda)w - (2b+\lambda)w'\right]}{\alpha(b+\lambda)^2}} - 1\right)\left(\frac{b+\lambda}{b}\right).$$

(H.v) If $\alpha \geq \dfrac{\left[(2b+3\lambda)a - 2(b+\lambda)bw - b\lambda w'\right]^2 \left[b\beta + (b+\lambda)w - (2b+\lambda)w'\right]}{(b+\lambda)^2 \left[2a - (2b+\lambda)w + \lambda w'\right]\left[6(b+\lambda)a - (6b+5\lambda)bw - b\lambda w'\right]}$ then

$$Q = \frac{(2b+3\lambda)a - 2(b+\lambda)bw - b\lambda w'}{4b+3\lambda} \quad \text{and} \quad e = \frac{(b+\lambda)\left[2a - (2b+\lambda)w + \lambda w'\right]}{(2b+3\lambda)a - 2(b+\lambda)bw - b\lambda w'}.$$

(GR.iii): For the equilibrium responses in **(G.iii)**, the grower solves the problem

$$\begin{aligned}
\max \quad & \Pi(Q,e) = \frac{w'\lambda\,(a - bw')}{2b+\lambda} + Q\left[\left(\frac{(2b+\lambda)w - \lambda w'}{2b+\lambda}\right)e - \alpha e^2 - \beta\right] \\
s.t. \quad & Qe \leq \frac{(b+\lambda)\left[2a - (2b+\lambda)w + \lambda w'\right]}{4b+3\lambda} \\
& Q\left(1 + \frac{2b(1-e)}{\lambda}\right) \geq a - bw' \\
& 0 \leq e \leq \frac{(b+\lambda)\left[2a - (2b+\lambda)w + \lambda w'\right]}{(2b+3\lambda)a - 2b(b+\lambda)w - b\lambda w'}.
\end{aligned}$$

The last constraint $e \leq \frac{(b+\lambda)[2a-(2b+\lambda)w+\lambda w']}{(2b+3\lambda)a-2b(b+\lambda)w-b\lambda w'}$ is due to the effort range for the equilibrium response **(G.iii)**. We branch the problem into sub-problems depending on the sign of the multiplier of Q in the objective function. If $\left(\frac{(2b+\lambda)w-\lambda w'}{2b+\lambda}\right)e - \alpha e^2 - \beta < 0$, then Q takes the lowest feasible value $Q = \frac{\lambda(a-bw')}{2b+\lambda-2be}$. This problem is identical to *(GR.iv.b)*, which is analyed below, but with the additional constraint $\left(\frac{(2b+\lambda)w-\lambda w'}{2b+\lambda}\right)e - \alpha e^2 - \beta \leq 0$. Hence, no better solution than *(GR.iv.b)* can be achieved. Therefore, we only consider the other case with

$$\left(\frac{(2b+\lambda)w - \lambda w'}{2b+\lambda}\right)e - \alpha e^2 - \beta \geq 0,$$

where the order quantity takes the highest possible value $Q = \dfrac{(b+\lambda)\left[2(a-bw) - \lambda(w-w')\right]}{(4b+3\lambda)e}$

by Lemma A1. We obtain the following problem in e:

$$\begin{aligned}
\max \quad \Pi(Q,e) = &\frac{[2(b+\lambda)w + \lambda w'](a-bw) - \lambda[(b+\lambda)w - (2b+\lambda)w'](w-w')}{4b+3\lambda} \\
&- \frac{(b+\lambda)[2a-(2b+\lambda)w + \lambda w']}{4b+3\lambda}\left(\frac{\alpha e^2+\beta}{e}\right) \\
s.t. \quad & -\alpha e^2 + \left[\frac{(2b+\lambda)w - \lambda w'}{2b+\lambda}\right] e - \beta \geq 0 \\
& 0 \leq e \leq \frac{(b+\lambda)[2a-(2b+\lambda)w+\lambda w']}{(2b+3\lambda)a - 2b(b+\lambda)w - b\lambda w'}.
\end{aligned}$$

The interior point solution occurs at $e = \sqrt{\dfrac{\beta}{\alpha}}$, where

$$Q = \frac{(b+\lambda)[2a-(2b+\lambda)w+\lambda w']}{4b+3\lambda}\sqrt{\frac{\alpha}{\beta}}.$$

The interior point solution and the second constraint lead to the solutions *(H.i)* and *(H.v)* depending on if $\alpha \geq \beta\left[\dfrac{(2b+3\lambda)a - 2(b+\lambda)bw - b\lambda w'}{(b+\lambda)[2a-(2b+\lambda)w+\lambda w']}\right]^2$ or otherwise, respectively.

(GR.iv): For the equilibrium responses in **(G.iv)**, the grower solves the problem

$$\begin{aligned}
\max \quad & \Pi(Q,e) = Q\left[w' - \beta + (w-w')e - \alpha e^2\right] \\
s.t. \quad & Q\left(1 + \frac{be}{b+\lambda}\right) \leq a - bw \\
& Q\left(1 + \frac{2b(1-e)}{\lambda}\right) \leq a - bw' \\
& 0 \leq e \leq 1.
\end{aligned}$$

For the profit to be non-negative, $w' - \beta + (w-w')e - \alpha e^2 \geq 0$, for which a necessary condition is non-negativity at its unconstrained maximum, which occurs at $e = \frac{w-w'}{2\alpha}$. This evaluates to $(w-w')^2 \geq 4\alpha(\beta - w')$, which holds by Lemma A2. We add the non-negativity constraint explicitly to exclude the solutions with negative Q.

$$\begin{aligned} \max \quad & \Pi(Q,e) = Q\left[w' - \beta + (w - w')e - \alpha e^2\right] \\ s.t. \quad & Q\left(1 + \frac{be}{b+\lambda}\right) \le a - bw \\ & Q\left(1 + \frac{2b(1-e)}{\lambda}\right) \le a - bw' \\ & w' - \beta + (w - w')e - \alpha e^2 \ge 0 \\ & 0 \le e \le 1. \end{aligned}$$

Under the additional constraint, the objective is increasing in Q, which must take the value at its upper bound by Lemma A1. We then have two problems depending on whether the first or the second constraint is binding.

(GR.iv.a): The first constraint is active if $\frac{(b+\lambda)(a-bw)}{b+\lambda+be} \le \frac{\lambda(a-bw')}{2b+\lambda-2be}$, that is $e \ge \frac{(b+\lambda)[2a-(2b+\lambda)w+\lambda w']}{(2b+3\lambda)a-2b(b+\lambda)w-b\lambda w'}$. Then $Q = \frac{(b+\lambda)(a-bw)}{b+\lambda+be}$ and the problem in e becomes

$$\begin{aligned} \max \quad & \Pi(e) = \frac{(b+\lambda)(a-bw)}{b+\lambda+be}\left[w' - \beta + (w - w')e - \alpha e^2\right] \\ s.t. \quad & \frac{(b+\lambda)\left[2a - (2b+\lambda)w + \lambda w'\right]}{(2b+3\lambda)a - 2b(b+\lambda)w - b\lambda w'} \le e \le 1 \\ & w' - \beta + (w - w')e - \alpha e^2 \ge 0. \end{aligned}$$

The first-order and second-order derivatives are

$$\begin{aligned} \frac{\partial \Pi}{\partial e} &= \frac{(b+\lambda)(a-bw)\left[-b\alpha e^2 - 2(b+\lambda)\alpha e + (b+\lambda)w - (2b+\lambda)w' + b\beta\right]}{(b+\lambda+be)^2}, \\ \frac{\partial^2 \Pi}{\partial e^2} &= -\frac{2(b+\lambda)(a-bw)\left[\alpha(b+\lambda)^2 + \beta b^2 + b(b+\lambda)w - b(2b+\lambda)w'\right]}{(b+\lambda+be)^3}. \end{aligned}$$

The objective is concave in e if

$$\alpha(b+\lambda)^2 + \beta b^2 + b(b+\lambda)w - b(2b+\lambda)w' \ge 0.$$

Then, the unconstrained maximum is attained when the first order condition holds, that is

$$e^* = \left(\sqrt{1 + \frac{b\left[b\beta + (b+\lambda)w - (2b+\lambda)w'\right]}{\alpha(b+\lambda)^2}} - 1\right)\left(\frac{b+\lambda}{b}\right).$$

For the SPNE effort to be non-negative, we need a stricter condition:

$$b\beta + (b+\lambda)w - (2b+\lambda)w' \geq 0.$$

The profit under the interior point solution is non-negative iff $(w-w')^2 \geq 4\alpha(\beta - w')$, which holds by assumption. The interior point solution is feasible iff

$$\frac{b\beta + (b+\lambda)w - (2b+\lambda)w'}{3b+2\lambda} \leq \alpha \leq \frac{\left[(2b+3\lambda)a - 2(b+\lambda)bw - b\lambda w'\right]^2 \left[b\beta + (b+\lambda)w - (2b+\lambda)w'\right]}{(b+\lambda)^2\left[2a-(2b+\lambda)w+\lambda w'\right]\left[6(b+\lambda)a-(6b+5\lambda)bw-b\lambda w'\right]},$$

when

$$Q = \sqrt{\frac{\alpha(b+\lambda)^2(a-bw)^2}{\alpha(b+\lambda)^2 + b^2\beta + b(b+\lambda)w - b(2b+\lambda)w'}}$$

and

$$e = \left(\sqrt{1 + \frac{b\left[b\beta + (b+\lambda)w - (2b+\lambda)w'\right]}{\alpha(b+\lambda)^2}} - 1\right)\left(\frac{b+\lambda}{b}\right).$$

This solution is identical to *(H.iii)*. Otherwise, either $e = \frac{(b+\lambda)[2(a-bw)-\lambda(w-w')]}{(2b+3\lambda)(a-bw)+b\lambda(w-w')}$ or $e = 1$, which are identical to *(H.v)* and *(H.i)*, respectively. Hence, no new solutions are obtained from the solution of the problem *(GR.iv.a)*.

(GR.iv.b): The second constraint is active if $e \leq \dfrac{(b+\lambda)\left[2a-(2b+\lambda)w+\lambda w'\right]}{(2b+3\lambda)a-2b(b+\lambda)w-b\lambda w'}$, when $Q = \dfrac{\lambda(a-bw')}{2b+\lambda-2be}$. Then, the grower's problem in e becomes:

$$\begin{aligned} \max \quad & \Pi(e) = \frac{\lambda(a-bw')}{2b+\lambda-2be}\left[w' - \beta + (w-w')e - \alpha e^2\right] \\ s.t. \quad & 0 \leq e \leq \frac{(b+\lambda)\left[2a-(2b+\lambda)w+\lambda w'\right]}{(2b+3\lambda)a-2b(b+\lambda)w-b\lambda w'} \\ & w' - \beta + (w-w')e - \alpha e^2 \geq 0. \end{aligned}$$

The first-order derivative is

$$\frac{d\Pi}{de} = \frac{\lambda(a-bw')\left[2b\alpha e^2 - 2(2b+\lambda)\alpha e + (2b+\lambda)w - \lambda w' - 2b\beta\right]}{(2b+\lambda-2be)^2}$$

Observe that

$$\left.\frac{d\Pi}{de}\right|_{e=0} = \frac{\lambda(a-bw')\left[(2b+\lambda)w - \lambda w' - 2b\beta\right]}{(\lambda+2b)^2} > 0$$

since $w > 2\sqrt{\alpha\beta} > 2\beta$ and $w \geq w'$. The second-order derivative is

$$\frac{d^2\Pi}{de^2} = -\frac{2\lambda(a - bw')\left[(2b+\lambda)^2\alpha + 4b^2\beta - 2b(2b+\lambda)w + 2b\lambda w'\right]}{(2b + \lambda - 2be)^3}.$$

The objective is concave in the effort e and has an interior point solution with $e \in [0, 1]$ if $\beta \leq \frac{(2b+\lambda)w - \lambda w'}{2b}$ and $\alpha \geq \frac{2b\left[(2b+\lambda)w - \lambda w' - 2b\beta\right]}{(2b+\lambda)^2}$. Since $w \geq 2\beta$ and $w \geq w'$, the inequality $\beta \leq \frac{(2b+\lambda)w - \lambda w'}{2b}$ always holds. Therefore, if $\alpha \geq \frac{2b\left[(2b+\lambda)w - \lambda w' - 2b\beta\right]}{(2b+\lambda)^2}$ the interior point solution is

$$e^* = \left(1 - \sqrt{1 - \frac{2b\left[(2b+\lambda)w - \lambda w' - 2b\beta\right]}{\alpha(2b+\lambda)^2}}\right)\left(\frac{2b+\lambda}{2b}\right),$$

for which the profit is non-negative because $(w - w')^2 + 4\alpha(w' - \beta) \geq 0$. The interior point solution is feasible if

$$\alpha \geq \frac{\left[(2b+\lambda)w - \lambda w' - 2b\beta\right]\left[(2b+3\lambda)a - 2b(b+\lambda)w - b\lambda w'\right]^2}{2(b+\lambda)\left[2a - (2b+\lambda)w + \lambda w'\right)\left]\left[(2b^2 + 6b\lambda + 3\lambda^2)a - b(b+\lambda)(2b+\lambda)w - b\lambda(3b+2\lambda)w'\right]}.$$

For $w \geq w'$, $a - bw - \lambda(w - w') \geq 0$ and $\beta \leq \frac{(2b+\lambda)w - \lambda w'}{2b}$:

$$\frac{\left[(2b+\lambda)w - \lambda w' - 2b\beta\right]\left[(2b+3\lambda)a - 2b(b+\lambda)w - b\lambda w'\right]^2}{2(b+\lambda)\left[2a - (2b+\lambda)w + \lambda w'\right)\left]\left[(2b^2 + 6b\lambda + 3\lambda^2)a - b(b+\lambda)(2b+\lambda)w - b\lambda(3b+2\lambda)w'\right]} \geq \frac{2b\left[(2b+\lambda)w - \lambda w' - 2b\beta\right]}{(2b+\lambda)^2}.$$

Hence, the condition $\alpha \geq \frac{2b[(2b+\lambda)w - \lambda w' - 2b\beta]}{(2b+\lambda)^2}$ does not need to be imposed separately for the interior solution. If the above condition does not hold, a corner solution must prevail. Since $\frac{\partial\Pi}{\partial e}\big|_{e=0} = \frac{\lambda(a - bw')\left[2b(w-\beta) + \lambda(w - w')\right]}{(2b+\lambda)^2}$ is always positive, as $w \geq 2\sqrt{\alpha\beta} \geq \beta$ and $w > w'$, then $e = \frac{(b+\lambda)\left[2a - (2b+\lambda)w + \lambda w'\right]}{(2b+3\lambda)a - 2b(b+\lambda)w - b\lambda w'}$. Therefore, the two cases are:

(H.iv)

If $\alpha \geq \frac{\left[(2b+\lambda)w - \lambda w' - 2b\beta\right]\left[(2b+3\lambda)a - 2b(b+\lambda)w - b\lambda w'\right]^2}{2(b+\lambda)\left[2a - (2b+\lambda)w + \lambda w'\right)\left]\left[(2b^2 + 6b\lambda + 3\lambda^2)a - b(b+\lambda)(2b+\lambda)w - b\lambda(3b+2\lambda)w'\right]}$

$$\text{then} \quad Q = \sqrt{\frac{\alpha\lambda^2(a - bw')^2}{\alpha(2b+\lambda)^2 + 4b^2(\beta - w') - 2b(2b+\lambda)(w - w')}} \quad \text{and}$$

$$e = \left(1 - \sqrt{1 - \frac{2b\left[(2b+\lambda)(w-w') - 2b(\beta - w')\right]}{\alpha(2b+\lambda)^2}}\right)\left(\frac{2b+\lambda}{2b}\right).$$

(H.v)

$$\text{If } \alpha < \frac{\left[(2b+\lambda)w - \lambda w' - 2b\beta\right]\left[(2b+3\lambda)a - 2b(b+\lambda)w - b\lambda w'\right]^2}{2(b+\lambda)\left[2a - (2b+\lambda)w + \lambda w')\right]\left[(2b^2+6b\lambda+3\lambda^2)a - b(b+\lambda)(2b+\lambda)w - b\lambda(3b+2\lambda)w'\right]}$$

$$\text{then} \quad Q = \frac{(2b+3\lambda)a - 2(b+\lambda)bw - b\lambda w'}{4b+3\lambda} \quad \text{and} \quad e = \frac{(b+\lambda)\left[2a - (2b+\lambda)w + \lambda w'\right]}{(2b+3\lambda)a - 2b(b+\lambda)w - b\lambda w'}$$

□

Proof of Theorem 9. The equilibrium responses of the suppliers in Lemma 8 are derived from the mathematical programs (GR.i) to (GR.iv) above in the proof, which correspond to the retailer equilibria (G.i) to (G.iv), respectively. Hence, the cases (H.i) to (H.iii) match with the cases (G.i) to (G.iii), respectively. The grower does not offer ugly veg to the market unless it is profitable for him. Otherwise, the grower selects the traditional supply chain with no ugly veg sales, i.e., (B)-(A.i). Therefore, the SPNE is the one among the equilibria (H.i)-(G.i), (H.ii)-(G.ii), (H.iii)-(G.iii), and (B)-(A.i) in which the grower's profit is maximum. Note that (H.ii.a)-(G.ii.a) and (H.ii.b)-(G.ii.b) cannot co-exist, hence we shortly write (H.ii)-(G.ii). □

Proof of Lemma 8. We consider the different SPNEs:

(H.i)-(G.i): For this solution, $Q_{(H.i)-(G.i)} = \frac{\chi}{4b+3\lambda}\frac{1}{e_{(H.i)-(G.i)}}$ with $e_{(H.i)-(G.i)} = \min\left\{\sqrt{\frac{\beta}{\alpha}}, \frac{\chi}{\xi}\right\}$, and $q_{(H.i)-(G.i)} = \frac{\chi}{4b+3\lambda}$ and $q'_{(H.i)-(G.i)} = \frac{\xi-\chi}{4b+3\lambda}$. Therefore, $q = Qe$ holds, hence there is no regular produce loss. The ugly veg loss is $Y_{(H.i)-(G.i)} = Q\,(1-e) - q' = \frac{\chi}{4b+3\lambda}\left(\max\left\{\sqrt{\frac{\alpha}{\beta}}, \frac{\xi}{\chi}\right\} - \frac{\xi}{\chi}\right)$. The food loss is positive for $\sqrt{\frac{\beta}{\alpha}} < \frac{\chi}{\xi}$ and zero otherwise.

(H.ii.a)-(G.ii): For this solution, $e_{(H.ii.a)-(G.ii)} = 1$, $Q_{(H.ii.a)-(G.ii)} = \frac{(b+\lambda)(a-bw)}{2b+\lambda}$, $q_{(H.ii.a)-(G.ii)} = \frac{(b+\lambda)(a-bw)}{2b+\lambda}$ and $q'_{(H.ii.a)-(G.ii)} = 0$. Since $q = Qe$ and $q' = Q(1-e) = 0$ there is no food loss, i.e., $Y_{(H.ii.a)-(G.ii)} = 0$.

(H.ii.b)-(G.ii): For this solution

$$
\begin{aligned}
Q_{(H.ii.b)-(G.ii)} &= \frac{(b+\lambda)(a-bw)}{\lambda+b+be_{(H.ii.b)-(G.ii)}} \\
q_{(H.ii.b)-(G.ii)} &= \frac{(b+\lambda)(a-bw)e_{(H.ii.b)-(G.ii)}}{\lambda+b+be_{(H.ii.b)-(G.ii)}} \\
q'_{(H.ii.b)-(G.ii)} &= \frac{(b+\lambda)(a-bw)\left(1-e_{(H.ii.b)-(G.ii)}\right)}{\lambda+b+be_{(H.ii.b)-(G.ii)}}.
\end{aligned}
$$

Since both $q = Qe$ and $q' = Q(1-e) = 0$ hold, there is no food loss, i.e., $Y_{(H.ii.b)-(G.ii)} = 0$.

(H.iii)-(G.iii): For this solution

$$
\begin{aligned}
Q_{(H.iii)-(G.iii)} &= \frac{\lambda(a-bw')}{\lambda+2b-2be_{(H.iii)-(G.iii)}} \\
q_{(H.iii)-(G.iii)} &= \frac{\lambda(a-bw')e_{(H.iii)-(G.iii)}}{\lambda+2b-2be_{(H.iii)-(G.iii)}} \\
q'_{(H.iii)-(G.iii)} &= \frac{\lambda(a-bw')\left(1-e_{(H.iii)-(G.iii)}\right)}{\lambda+2b-2be_{(H.iii)-(G.iii)}}.
\end{aligned}
$$

Since both $q = Qe$ and $q' = Q(1-e) = 0$ hold, there is no food loss, i.e., $Y_{(H.iii)-(G.iii)} = 0$.

□

Proof of Theorem 10. Under the traditional supply chain the SPNE is (B)-(A.i) with $e_{(B)-(A.i)} = \sqrt{\frac{\beta}{\alpha}}$, $Q_{(B)-(A.i)} = \frac{a-bw}{2}\sqrt{\frac{\alpha}{\beta}}$, $q_{(B)-(A.i)} = \frac{a-bw}{2}$, and $q'_{(B)-(A.i)} = 0$. The total food loss is $Y_{(B)-(A.i)} = \frac{a-bw}{2}\left(\sqrt{\frac{\alpha}{\beta}}-1\right)$, while the food loss per unit cultivated area is $\frac{Y_{(B)-(A.i)}}{Q_{(B)-(A.i)}} = 1-\sqrt{\frac{\beta}{\alpha}}$.

By Lemma 8, the food loss is non-zero in the two-retailers supply chain only for the SPNE (H.i)-(G.i) and if $\sqrt{\frac{\beta}{\alpha}} < \frac{\chi}{\xi}$. Since the food loss is otherwise zero, it can never be greater than the food loss under the traditional supply chain if this does not hold. Therefore, we compare (H.i)-(G.i) with (B)-(A.i) for $\sqrt{\frac{\beta}{\alpha}} < \frac{\chi}{\xi}$. We have

$$
\begin{aligned}
Y_{(H.i)-(G.i)} &= \frac{\chi}{4b+3\lambda}\left(\sqrt{\frac{\alpha}{\beta}}-\frac{\xi}{\chi}\right) \\
\frac{Y_{(H.i)-(G.i)}}{Q_{(H.i)-(G.i)}} &= 1-\sqrt{\frac{\beta}{\alpha}}-\frac{(\xi-\chi)}{\chi}\sqrt{\frac{\beta}{\alpha}}.
\end{aligned}
$$

Since $\xi > \chi$, we obtain $\frac{Y_{(H.i)-(G.i)}}{Q_{(H.i)-(G.i)}} < \frac{Y_{(B)-(A.i)}}{Q_{(B)-(A.i)}}$. Hence, the food loss per unit area is always less under the two-retailers supply chain.

To compare the total food loss, let $\Delta_Y \equiv Y_{(H.i)-(G.i)} - Y_{(B)-(A.i)}$.

$$\Delta_Y = \frac{\lambda}{2(4b+3\lambda)}\left[\left(a-(3b+2\lambda)w+2(b+\lambda)w'\right)\sqrt{\frac{\alpha}{\beta}} - \left(3a-bw-2bw'\right)\right]$$

If $a-(3b+2\lambda)w+2(b+\lambda)w' \leq 0$ then $\Delta_Y \leq 0$ as $3a-bw-2bw' > 0$. Therefore, the food loss under the two-retailers supply chain's SPNE (H.i)-(G.i) is higher if and only if

$$\sqrt{\frac{\beta}{\alpha}} < \frac{a-(3b+2\lambda)w+2(b+\lambda)w'}{3a-bw-2bw' >}.$$

We finally show that $\frac{a-(3b+2\lambda)w+2(b+\lambda)w'}{3a-bw-2bw'} \leq \frac{\chi}{\xi}$, hence the condition above implies $\sqrt{\frac{\beta}{\alpha}} < \frac{\chi}{\xi}$. We check the sign of

$$\begin{aligned} A &= \chi\left(3a-bw-2bw'\right) - \xi\left(a-(3b+2\lambda)w+2(b+\lambda)w'\right) \\ &= (b+\lambda)\left[2(a-bw)-\lambda(w-w')\right]\left[3(a-bw)+2b(w-w')\right] \\ &\quad - \left[(2b+3\lambda)(a-bw)+b\lambda(w-w')\right]\left[(a-bw)-2(b+\lambda)(w-w')\right] \\ &= (4b+3)(a-bw)\left[(a-bw)+(2b+\lambda)(w-w')\right] > 0, \end{aligned}$$

which completes the proof. □

Proof of Theorem 11. The optimal decisions are derived from the first order conditions. □

Proof of Lemma 9. $Q_{(cT)} = \sqrt{\frac{\alpha}{\beta}}a/2 - b\alpha$ and $Q_{(B)} = \sqrt{\frac{\alpha}{\beta}}a/2 - (bw/2)\sqrt{\frac{\alpha}{\beta}}$. Since $w \geq 2\sqrt{\alpha\beta}$, $Q_{(B)} \leq \sqrt{\frac{\alpha}{\beta}}a/2 - b\alpha$, hence $Q_{(B)} \leq Q_{(cT)}$. □

Proof of Lemma 10. For both (cU.i) and (cU.ii), $Y^* = Q^*e^* - q^* = 0$. For $(cU.i)$, the ugly veg waste $Y_{(cU.i)} = Q_{(cU.i)}\left(1-e_{(cU.i)}\right) - q'_{(cU.i)} = \left(\sqrt{\frac{\alpha}{\beta}}-1\right)\left(\frac{a}{2}-(b+\lambda)\sqrt{\alpha\beta}\right) - \lambda\sqrt{\alpha\beta}$. For $(cU.ii)$, $Q_{(cU.ii)}\left(1-e_{(cU.ii)}\right) = \alpha\lambda e_{(cU.ii)} = q'_{(cU.ii)}$. Thus, $Y'_{(cU.ii)} = 0$. □

Proof of Theorem 12. Let q, q', p, p' be the optimal solutions of the centralized problem. We first prove that the optimal must satisfy $q = Qe$ and $q' \leq Q(1-e)$

The objective is

$$\Omega = p\min\left\{q, d(p,p'), Qe\right\} + p'\min\left\{q'+q-\min\left\{q,d(p,p'),Qe)\right\}, d'(p,p'), Q-q\right\} - Q(\alpha e^2+\beta).$$

For $p \geq p'$, selling regular produce as regular produce is more profitable. Hence, $q = \min\left\{d(p,p'), Qe\right\}$, which leads to

$$\Omega = p \min\left\{d(p,p'), Qe\right\} + p' \min\left\{q', d'(p,p'), Q - \min\left\{d(p,p'), Qe\right\}\right\} - Q(\alpha e^2 + \beta).$$

The profit is increasing in q' for $q' \leq \min\{d'(p,p'), Q - \min\{d(p,p'), Qe\}\}$ and then remains constant. Without loss of generality and by the assumption for the centralized case that the loss/waste occurs at the grower site, we obtain $q' = \min\{d'(p,p'), Q - \min\{d(p,p'), Qe\}\}$. If $Qe < d(p,p')$ at the optimum then $\Omega = pQe + p' \min\{d'(p,p'), Q(1-e)\} - Q(\alpha e^2 + \beta)$. Since $d'(p,p')$ is increasing in p, so is Ω, hence p will be increased. However, the only upper bound on p is set by $Qe < d(p,p')$, which is a strict upper bound. Hence, no optimum exists, which is a contradiction. Therefore, $Qe \geq d(p,p')$ must hold at the optimum, leading to

$$\Omega = pd(p,p') + p' \min\left\{d'(p,p'), Q - d(p,p')\right\} - Q(\alpha e^2 + \beta).$$

The objective function is decreasing in e, which appears only in the constraints $Qe \geq d(p,p')$ and $0 \leq e \leq 1$. Since $d(p,p') \geq 0$, $Qe = d(p,p')$ most hold, leading to

$$q^* = Q^* e^* = d(p^*, p'^*).$$

Inserting $d(p^*, p'^*) = a - bp^* - \lambda(p^* - p'*)$, we obtain $p^* = \frac{a - Q^* e^* + \lambda p'^*}{\lambda + b}$ and

$$\Omega = \frac{Qe\left(a - Qe + \lambda p'\right)}{\lambda + b} + p' \min\left\{\frac{\lambda}{\lambda + b}\left(a - Qe - bp'\right), Q(1-e)\right\} - Q(\alpha e^2 + \beta).$$

By way of contradiction assume $Q(1-e) < \frac{\lambda}{\lambda+b}\left(a - Qe - bp'\right)$. Then Ω is increasing in p' and it must be increased up to one of the upper bounds set by $Q(1-e) < \frac{\lambda}{\lambda+b}\left(a - Qe - bp'\right)$ or $p' \leq p$. The optimum can only occur at $p' = p$ as the other inequality is strict. Therefore, $bp' = a - Qe$ and the first leads to $Q(1-e) < 0$, which is infeasible. Hence, $Q(1-e) \geq \frac{\lambda}{\lambda+b}\left(a - Qe - bp'\right)$, resulting in $q'^* = d'(p^*, p'^*)$.

By the intermediate result $q = Qe$ and $q' \leq Q(1-e)$, the problem becomes

$$\max \quad \Omega(p', Q, e) = \frac{\lambda p'(a-bp')}{b+\lambda} + Q\left(\frac{a}{b+\lambda}e - \left(\frac{Q}{b+\lambda} + \alpha\right)e^2 - \beta\right) \tag{24}$$

$$s.t. \quad \lambda a \leq b\lambda p' + (b+\lambda-be)Q \tag{25}$$

$$Qe \leq a - bp' \tag{26}$$

$$Q, p' \geq 0 \tag{27}$$

$$0 \leq e \leq 1 \tag{28}$$

If $Q = 0$ then Eq. 25 and 26 evaluate to $p' \geq a/b$ and $p' \leq a/b$, respectively, hence $p' = a/b$, leading to $\Omega = 0$. Since the objective is always positive in the optimal solutions obtained below, there is no need to consider $Q = 0$, thus $Q > 0$. If $p' = 0$ then $\frac{d\Omega}{dp'}(p' = 0) = \frac{\lambda a}{b+\lambda} > 0$. Hence, the objective can be increased by increasing p'. Hence, $p' = 0$ can be optimal only if the constraint $Qe \leq a - bp'$ is binding, not allowing to increase p'. Therefore, $Qe = a$. Inserting this into the objective we obtain $\Omega = -\frac{a}{e}(\alpha e^2 + b)$, which is negative. This cannot be optimal, hence $p' > 0$.

We first consider the interior point solution (cU.i) and then the boundary solution (cU.ii).

(cU.i) For the interior point solution, the first order conditions must hold, that is

$$\frac{d\Omega}{dp'} = \frac{\lambda(a-2bp')}{\lambda+b} = 0$$
$$\frac{d\Omega}{dQ} = \left(\frac{a}{b+\lambda}e - \left(\frac{2Q}{b+\lambda} + \alpha\right)e^2 - \beta\right) = 0$$
$$\frac{d\Omega}{de} = Q\left(\frac{a}{b+\lambda} - 2\left(\frac{Q}{b+\lambda} + \alpha\right)e\right) = 0,$$

which leads to

$$p'_{(cU.i)} = \frac{a}{2b}, e_{(cU.i)} = \sqrt{\frac{\beta}{\alpha}}, Q_{(cU.i)} = \sqrt{\frac{\alpha}{\beta}}\left(\frac{a}{2} - (b+\lambda)\sqrt{\alpha\beta}\right). \tag{29}$$

Using Lemma 10 to find $p_{(cU.i)}$, $q_{(cU.i)}$ and $q'_{(cU.i)}$ completes the solution for (cU.i). Constraint 25 holds for the solution (cU.i) iff $\frac{a}{2b} \geq \sqrt{\alpha\beta}\left(1 + \frac{\lambda/b}{1-\sqrt{\frac{\beta}{\alpha}}}\right)$. The other constraints always hold.

We finally consider the Hessian matrix for $(cU.i)$

$$\mathcal{H} = \begin{pmatrix} \frac{d^2\Omega}{dp'^2} & \frac{d^2\Omega}{dp'dQ} & \frac{d^2\Omega}{dp'de} \\ \frac{d^2\Omega}{dp'dQ} & \frac{d^2\Omega}{dQ^2} & \frac{d^2\Omega}{dQde} \\ \frac{d^2\Omega}{dp'de} & \frac{d^2\Omega}{dedQ} & \frac{d^2\Omega}{de^2} \end{pmatrix} = \begin{pmatrix} -\frac{2b\lambda}{b+\lambda} & 0 & 0 \\ 0 & -\frac{2e^2}{b+\lambda} & \frac{a}{b+\lambda} - 2e\left(\frac{2Q}{b+\lambda} + \alpha\right) \\ 0 & \frac{a}{b+\lambda} - 2e\left(\frac{2Q}{b+\lambda} + \alpha\right) & -2Q\left(\frac{Q}{b+\lambda} + \alpha\right) \end{pmatrix}.$$

When evaluated under the (cU.i) solution,

$$\mathcal{H}^* = \begin{pmatrix} -\frac{2b\lambda}{b+\lambda} & 0 & 0 \\ 0 & -\frac{2\beta}{(b+\lambda)\alpha} & -\frac{a}{b+\lambda} + 2\sqrt{\alpha\beta} \\ 0 & -\frac{a}{b+\lambda} + 2\sqrt{\alpha\beta} & -\frac{a\alpha}{2\beta}\left(\frac{a}{b+\lambda} - 2\sqrt{\alpha\beta}\right) \end{pmatrix}.$$

The Hessian is negative definite if and only if $\frac{a}{b+\lambda}\left(\frac{a}{b+\lambda} - 2\sqrt{\alpha\beta}\right) > \left(\frac{a}{b+\lambda} - 2\sqrt{\alpha\beta}\right)^2$. This always holds in the feasible region where $\frac{a}{b} \geq 2\sqrt{\alpha\beta}\left(1 + \frac{\lambda/b}{1-\sqrt{\beta/\alpha}}\right) > \frac{2(b+\lambda)\sqrt{\alpha\beta}}{b}$. Hence, this constitutes a minimum and completes the solution for (cU-i).

(cU.ii) We now consider the boundary solution with $p' = \frac{a}{b} - \frac{(b+\lambda-be)Q}{b\lambda}$. Inserting it into the problem into Eq. 24-28, we obtain the mathematical program:

$$\begin{aligned} \max \quad \Omega(Q,e) = \quad & Q\left(\frac{a}{b} + \frac{Qe(2-e)}{\lambda} - \frac{(b+\lambda)Q}{b\lambda} - \alpha e^2 - \beta\right) \\ s.t. \quad & Q(b+\lambda-be) \leq \lambda a \\ & 0 < Q \\ & 0 \leq e \leq 1. \end{aligned}$$

To obtain the interior solution, we set $\frac{d\Omega}{de} = 2Q\left(\frac{Q(1-e)}{\lambda} - \alpha e\right)$ to zero, leading to

$$Q^* = \frac{\alpha\lambda e^*}{1-e^*}.$$

Inserting Q^* into $\frac{d\Omega}{dQ} = \frac{a}{b} - \beta + \frac{2Qe(2-e)}{\lambda} - \frac{2(b+\lambda)Q}{b\lambda} - \alpha e^2 = 0$ and re-arranging in terms of $1-e^*$, we obtain

$$(1-e^*)^3 + (1-e^*)\left(\frac{2\lambda}{b} + \frac{a}{b\alpha} - \frac{\beta}{\alpha} - 1\right) - \frac{2\lambda}{b} = 0.$$

To check concavity, we look at the Hessian matrix

$$\mathcal{H} = \begin{pmatrix} \frac{d^2\Omega}{dQ^2} & \frac{d^2\Omega}{dQde} \\ \frac{d^2\Omega}{dedQ} & \frac{d^2\Omega}{de^2} \end{pmatrix} = \begin{pmatrix} -\frac{2}{\lambda}\left[1+\frac{\lambda}{b}-e(2-e)\right] & 2\left[\frac{2Q(1-e)}{\lambda}-\alpha e\right] \\ 2\left[\frac{2Q(1-e)}{\lambda}-\alpha e\right] & -2Q\left[\frac{Q}{\lambda}+\alpha\right] \end{pmatrix}.$$

We should have $\frac{d^2\Omega}{dQ^2}\frac{d^2\Omega}{de^2} \geq \left(\frac{d^2\Omega}{dQde}\right)^2$. At the interior point, this evaluates to

$$\frac{4\alpha^2 e^*}{(1-e^*)^2}\left[1+\frac{\lambda}{b}-e(2-e)\right] \geq 4\alpha^2 e^{*2},$$

which is equivalent to $(1-e^*)^3 + \frac{\lambda}{b} \geq 0$. This always holds for $e^* \in [0,1]$.

Let $f(x) = (1-x)^3 + (1-x)\left(\frac{2\lambda}{b}+\frac{a}{b\alpha}-\frac{\beta}{\alpha}-1\right)-\frac{2\lambda}{b}$. Then $f(x=0) = \frac{a}{b\alpha}-\frac{\beta}{\alpha}$ and $f(x=1) = -\frac{2\lambda}{b}$. Since $\frac{a}{b} \geq 2\sqrt{\alpha\beta} > 2\beta$, we have $f(x=0) > 0$ and $f(x=1) < 0$. The function f is convex for $x \in [0,1]$ as $f''(x) = 6(1-x) \geq 0$. Hence, there always exists a unique $0 < x^* < 1$ such that $f(x^*) = 0$ due to the intermediate value theorem and the convexity of the function. Hence, there is always a unique $e_{(cU.ii)} \in (0,1)$ that solves the mathematical program. If $\frac{a}{b} \geq 2\sqrt{\alpha\beta}\left(1+\frac{\lambda/b}{1-\sqrt{\beta/\alpha}}\right)$ then

$$\left(1-e_{(cU.ii)}\right)\left[\left(1-e_{(cU.ii)}\right)^2-\left(1-\sqrt{\frac{\beta}{\alpha}}\right)^2\right]+\frac{2\lambda}{b}\left[\frac{1-e_{(cU.ii)}}{1-\sqrt{\frac{\beta}{\alpha}}}-1\right] \leq 0,$$

which holds iff $e_{(cU.ii)} \geq \sqrt{\frac{\beta}{\alpha}}$. Similarly, $e_{(cU.ii)} < \sqrt{\frac{\beta}{\alpha}}$ for $\frac{a}{b} < 2\sqrt{\alpha\beta}\left(1+\frac{\lambda/b}{1-\sqrt{\beta/\alpha}}\right)$.

When (cU.ii) is the unique solution, i.e., $2\sqrt{\alpha\beta} < \frac{a}{b} < 2\sqrt{\alpha\beta}\left(1+\frac{\lambda/b}{1-\sqrt{\beta/\alpha}}\right)$, we need to show it is feasible. The conditions $0 \leq e_{(cU.ii)} \leq 1$ and $Q_{(cU.ii)} = \frac{\alpha\lambda e_{(cU.ii)}}{1-e_{(cU.ii)}} > 0$ hold, as shown above. We need to show $Q_{(cU.ii)}\left(\lambda+b-be_{(cU.ii)}\right) \leq \lambda a$. The condition is equivalent to $\frac{a}{b\alpha}-2\frac{\lambda}{b}\frac{e_{(cU.ii)}}{1-e_{(cU.ii)}}+\frac{a}{b\alpha}-2e_{(cU.ii)} \geq 0$. Re-arranging the terms in the equation $\left(1-e_{(cU.ii)}\right)^3 + \left(1-e_{(cU.ii)}\right)\left(\frac{2\lambda}{b}+\frac{a}{b\alpha}-\frac{\beta}{\alpha}-1\right)-\frac{2\lambda}{b} = 0$, we obtain

$$\frac{a}{b\alpha}-2\frac{\lambda}{b}\frac{e_{(cU.ii)}}{1-e_{(cU.ii)}} = 1+\frac{\beta}{\alpha}-\left(1-e_{(cU.ii)}\right)^2.$$

Hence, $\frac{a}{b\alpha}-2\frac{\lambda}{b}\frac{e_{(cU.ii)}}{1-e_{(cU.ii)}}+\frac{a}{b\alpha}-2e_{(cU.ii)} = \frac{a}{b\alpha}+\frac{\beta}{\alpha}-e^2_{(cU.ii)}$. Since $e_{(cU.ii)} < \sqrt{\frac{\beta}{\alpha}}$ for $2\sqrt{\alpha\beta} < \frac{a}{b} < 2\sqrt{\alpha\beta}\left(1+\frac{\lambda/b}{1-\sqrt{\beta/\alpha}}\right)$, $\frac{a}{b\alpha}+\frac{\beta}{\alpha}-e^2_{(cU.ii)} > 0$, that is (cU.ii) is feasible when it is the unique solution.

We need to finally show that when the solutions (cU.i) and (cU.ii) co-exist, i.e., for $\frac{a}{b} \geq 2\sqrt{\alpha\beta}\left(1+\frac{\lambda/b}{1-\sqrt{\beta/\alpha}}\right)$, the solution (cU.ii) is dominated by (cU.i). The profits under the optimal policies are

$$\Pi_C^{(cU.i)} = \frac{1}{b+\lambda}\left[\frac{\lambda a^2}{4b} + \left(\frac{a}{2} - (b+\lambda)\sqrt{\alpha\beta}\right)^2\right] \text{ and}$$

$$\Pi_C^{(cU.ii)} = \lambda\alpha^2 e_{(cU.ii)}^2\left[1 + \frac{\lambda}{b\left(1-e_{(cU.ii)}\right)^2}\right].$$

We shall analyze $\Delta_C \equiv \Pi_C^{(cU.i)} - \Pi_C^{(cU.ii)}$ as a function of a for $a \geq 2b\sqrt{\alpha\beta}\left(1+\frac{\lambda/b}{1-\sqrt{\beta/\alpha}}\right)$.

$$\frac{d\Delta_C}{da} = \frac{\partial\Delta_C}{\partial a} + \frac{\partial\Delta_C}{\partial e_{(cU.ii)}}\frac{\partial e_{(cU.ii)}}{\partial a} = \frac{a}{2b} - \sqrt{\alpha\beta} - \frac{\alpha\lambda e_{(cU.ii)}}{b\left(1-e_{(cU.ii)}\right)}$$

$$\frac{d^2\Delta_C}{da^2} = \frac{\partial d\Delta_C/da}{\partial a} + \frac{\partial d\Delta_C/da}{\partial e_{(cU.ii)}}\frac{\partial e_{(cU.ii)}}{\partial a} = \frac{1}{2b}\left[\frac{1}{1+\frac{\lambda/b}{\left(1-e_{(cU.ii)}\right)^3}}\right].$$

If $a = 2b\sqrt{\alpha\beta}\left(1+\frac{\lambda/b}{1-\sqrt{\beta/\alpha}}\right)$ then $\left(1-e_{(cU.ii)}\right)\left[\left(1-e_{(cU.ii)}\right)^2 - \left(1-\sqrt{\frac{\beta}{\alpha}}\right)^2\right] + \frac{2\lambda}{b}\left[\frac{1-e_{(cU.ii)}}{1-\sqrt{\frac{\beta}{\alpha}}} - 1\right] = 0$. Hence, $e_{(cU.ii)} = \sqrt{\frac{\beta}{\alpha}}$, which leads to

$$\Delta_C\left(a = 2b\sqrt{\alpha\beta}\left(1+\frac{\lambda/b}{1-\sqrt{\beta/\alpha}}\right)\right) = 0,$$

$$\frac{d\Delta_C}{da}\left(a = 2b\sqrt{\alpha\beta}\left(1+\frac{\lambda/b}{1-\sqrt{\beta/\alpha}}\right)\right) = 0, \text{ and}$$

$$\frac{d^2\Delta_C}{da^2} > 0, \text{ for } \frac{a}{b} \geq 2\sqrt{\alpha\beta}\left(1+\frac{\lambda/b}{1-\sqrt{\beta/\alpha}}\right).$$

Hence, $\Delta_C \geq 0$ for $\frac{a}{b} \geq 2\sqrt{\alpha\beta}\left(1+\frac{\lambda/b}{1-\sqrt{\beta/\alpha}}\right)$, which completes the proof.

□

Proof of Theorem 13. In both cases, there can only be ugly veg food loss. In the centralized traditional supply chain, the ugly veg food loss is $Y_{(Trad)} = Q^*(1-e^*) = \left(\sqrt{\frac{\alpha}{\beta}} - 1\right)\left(\frac{a}{2} - b\sqrt{\alpha\beta}\right)$ for all a. By Lemma 10, when the optimal solution is (cU.ii), the ugly veg food loss is zero, hence lower than the traditional case. For the solution (cU.i), $Y_{(cU.i)} = \left(\sqrt{\frac{\alpha}{\beta}} - 1\right)\left(\frac{a}{2} - (b+\lambda)\sqrt{\alpha\beta}\right) - \lambda\sqrt{\alpha\beta}$.

Since all parameters are non-negative, $Y_{(cU.i)} < Y_{(Trad)}$. If we consider proportional food waste, $\frac{Y_{(Trad)}}{Q_{(Trad)}} = 1 - \sqrt{\frac{\beta}{\alpha}}$, while $\frac{Y_{(cU.i)}}{Q_{(cU.i)}} = 1 - \sqrt{\frac{\beta}{\alpha}} - \frac{\lambda\sqrt{\alpha\beta}}{Q_{(cU.i)}}$. Hence, $\frac{Y_{(cU.i)}}{Q_{(cU.i)}} < \frac{Y_{(Trad)}}{Q_{(Trad)}}$. □