Partial Default*

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Abstract

We document that countries partially default often and with varying intensity, resulting in lengthy episodes and hump-shaped patterns for partial default and debt. Default episodes lead to haircuts for lenders but not to reductions in debt, because the defaulted debt accumulates and borrowing continues. We present a theory of partial default rationalizing these patterns and the heterogeneity of partial default, and partial default's comovements with spreads, debt, and output which are absent in standard sovereign default theory. We include policy counterfactuals in the form of pari passu and no-dilution clauses and debt relief policies, and their welfare implications.

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1 Introduction

Sovereign debt crises are pervasive in emerging markets. Sovereigns in these countries frequently miss payments on their debt but almost always by only a fraction of the amount due. During these *partial defaults*, sovereigns continue to pay some of the debt, continue to borrow, and accumulate the defaulted debt as arrears. Some of these partial defaults take the form of protracted episodes associated with deeper recessions and rising partial default and debt. The standard theory of sovereign default, based on the influential framework in Eaton and Gersovitz (1981), assumes that default is complete, rather than partial, and that it is followed by a period of exclusion without any borrowing, further default, or accumulation of defaulted debt. Default in that paradigm leads to a new start with reduced debt. In this paper, we propose a theory of partial default more consistent with the evidence, according to which partial default often leads to further defaults and debt increases.

A central idea in our theory is that partial default is an alternative way to effectively borrow and intertemporally transfer resources. Like standard borrowing through markets, partially defaulting raises current resources and increases future liabilities as most of the defaulted debt accumulates. Unlike standard borrowing, however, partial default does not have the acquiescence of the lenders and is associated with future resource costs. A main implication of our theory is that partial default is an amplifying force for debt crises. A country that misses payments suffers less in current consumption, but it will be in worse shape going forward because it will experience rising debt, as the defaulted payments accumulate and any new borrowing will occur at high interest rates. This theory is capable of rationalizing the large heterogeneity in partial default, partial default's comovements with economic outcomes, and rising debt and partial default during default episodes. In our theory, as in the data, large defaults are associated with higher interest rate spreads, higher debt levels, deeper recessions, and longer default episodes.

Our analysis puts at center stage emerging market panel data on debt payments, arrears, and sovereign spreads, analyzed through the lens of an accounting framework that takes a macro approach. We recover the properties of partial default and default episodes using 50 years of data for 37 emerging markets. We find that sovereigns default partially—on about 38% of the yearly amount they owe—and often—about a third of the time. The heterogeneity of partial defaults is wide, ranging from minor amounts to complete defaults. Default episodes last on average nine years, yet about 36% of episodes last fewer than two years. Partial default and debt-to-output ratios feature a hump-shaped pattern during default episodes, and these episodes are typically not associated with a net reduction in debt. On average, default episodes start with a partial default of 22% and when debt to output increases from 32% to 34%. During default episodes partial default and debt continue to rise, reaching on average 33% and 40%, respectively, in the middle of the episode. At the end of the episode, partial default stops, and average debt is 33%. We also find in time series data that partial default is systematically correlated with other outcomes of the debt crisis. Periods of larger partial defaults are associated with higher sovereign spreads, higher debt, and lower output.[1]

Our framework consists of a sovereign government in a small open economy. The government borrows in long-term bonds, can choose to partially default on its debt payments, and faces a stochastic stream of income. Partial default is a flexible yet expensive way to raise funds. Partial default is flexible because the government can choose the start of the default episode, the intensity of partial default every period, and the end of the default episode. An important aspect of our model is that the defaulted debt accumulates: when the sovereign partially defaults on a coupon, a fraction of that amount, which depends on a recovery factor parameter, is added to the total debt due next period. As partial defaults are serially correlated, the same original debt can have many of its coupons partially defaulted on. The haircuts and maturity extensions from the episode, therefore, depend on the recovery factor as well as the length and intensity of partial default during the episode. Partial default is also costly because it induces future resource costs that depend on the intensity of the default. The government can also raise funds by borrowing through markets at interest rates that compensate lenders for potential default losses. Borrowing is always possible, even during default episodes. Expected default losses, however, are more elevated during default episodes, which increase interest rates and can deter borrowing altogether.

The sovereign effectively faces a portfolio choice to intertemporally transfer resources and smooth consumption, because borrowing and partial default are distinct instruments chosen every period. We theoretically characterize the trade-offs involved in these two decisions and show that the bond price function is the key element that shapes these choices. The gains from borrowing are the increases in consumption net of the reduction in bond prices due to higher default, while its costs consist of the future debt burden evaluated at future prices. Because of default risk, the gains from borrowing are capped by a Laffer curve, as is typical of sovereign default models. The benefits to partially defaulting increase linearly with the amount defaulted on and are not subject to a Laffer curve, but are capped by the total level of debt due. The costs from partially defaulting also include the future debt burden from the accumulation of the defaulted debt, evaluated at future bond prices, plus the resource costs from the partial default.

[1] These findings are consistent with those in Cruces and Trebesch (2013) and Benjamin and Wright (2013), which document that creditors have received sizable debt recoveries after default episodes throughout history and that default episodes have not led to a reduction in debt.

We derive an optimal portfolio condition for borrowing and partial default by constructing expected returns for these instruments that summarize their benefits and costs. We show the expected returns depend on three factors: the bond price function, the level of debt, and the shape of the default costs. Borrowing is an attractive choice when the price of newly issued debt is high (so the implied interest rate on debt is low) and decreases little with a marginal increase in the amount borrowed; both of these factors tend to lower its return. Partial default is attractive only when the price of new debt is low, so that any new borrowing leads to only a small amount of current consumption, and when debt is high enough. In this case, it may be worth paying the default costs.

We obtain the parameters that characterize the default cost function as well as the discount rate, the recovery factor, the long-term bond decay parameter, and the process for the endowment in our overidentified model, using a minimum distance estimator that targets the empirical distribution of partial default and the behavior of interest rate spreads, debt, and output in emerging markets. We show that our model matches the target moments well and delivers the empirical distribution of partial default and debt.

The parameterized model contains several non-targeted implications for the properties of default episodes that resemble the data closely. Our model delivers episodes with an average length of eight years, with 42% of them lasting less than 2 years. Episodes result in sizable debt haircuts of 37% and maturity extensions of seven years. Like those in the data, the dynamics of partial default and debt during episodes in the model are hump-shaped, and default episodes do not result in a net reduction of the debt. In terms of partial default comovements, our model replicates the data in that larger partial defaults are associated with higher interest rate spreads, higher levels of debt, and deeper recessions. The resemblance between the model's patterns of default episodes and partial default comovements and those in the data provides an important validation of our model's mechanisms.

We also use the model to analyze in more detail the forces behind the lengthy default episodes. The length of the episode depends on the magnitude and persistence of the recession and on the accumulation of debt from borrowing and partial default. Default episodes generally start with a small partial default that occurs after a moderate downturn when debt is high enough. These small defaults are resolved quickly if the recession is temporary. When the recession is deeper and more persistent, however, the small but rising partial defaults amplify the debt crisis by inducing a rapid increase in debt at increasingly high interest rates. The episode ends when output recovers sufficiently to fully repay the accumulated debt. Larger debt crises require stronger output recoveries for the resolution of default, as they feature larger accumulated debt from past borrowing and partial defaults.

We study three counterfactual economies to evaluate proposals for improved resolution mechanisms for sovereign defaults. The counterfactuals compare the properties of default episodes and the sovereign's welfare across economies that have different contractual arrangements for debt. In the first counterfactual, we study more stringent pari passu clauses by eliminating the possibility of borrowing during default episodes, which we show induces more similar treatment for all the defaulted debt. In the second counterfactual, we study debt relief policies, such as the Highly Indebted Poor Countries initiative proposed by the International Monetary Fund and the World Bank, by decreasing the debt recovery factor on defaulted debt. The third counterfactual adds no-dilution covenants that have been shown to be powerful mitigators of debt crises in standard sovereign debt models. We find that economies with pari passu clauses and no-dilution covenants feature lower default frequency, shorter default episodes, and smaller haircuts. Economies with higher debt relief feature higher haircuts and lower debt levels. Overall, sovereign welfare is on average higher with pari passu clauses, while it is lower with higher debt relief and no-dilution covenants. The welfare ranking, however, depends on the level of debt–welfare in economies heavily indebted is higher with higher debt relief.

Finally, we contrast our partial default model directly to a state-of-the-art sovereign debt model with full default and renegotiation through bargaining. In this *reference model with bargaining*, the default is on the totality of the bonds but is followed by debt renegotiation, in what is a version of the setting in Benjamin and Wright (2013) or Asonuma and Joo (2020). We study this reference model because, like our benchmark model, it can also produce default episodes with a range of haircuts to lenders. We find that although this reference model is able to produce reasonable haircuts, it delivers short and stark default episodes because it lacks the amplification mechanisms of our baseline model. We highlight three important differences of the reference model. First, it fails to produce hump-shaped partial default and debt during default episodes. Second, it delivers default episode durations that are too short. Third, it gives the country a fresh start with lower debt at the end of the episode and a corresponding surge in consumption and borrowing. The comparison underscores the distinct narrative of default episodes in our theory. In standard theory, the default episode is a state of impasse, without any funds flowing between the country and its lenders, that ends with a renegotiation, after which the country enjoys a reduced debt. In our model, default episodes feature dynamics and amplification whereby partial default and debt grow, and the episode ends only when the sovereign becomes able to pay down enough of its debt.[2]

[2]Comparisons of our model against a simpler reference full-default model without bargaining are even starker.

Related Literature. Our work builds on the empirical and theoretical literature that studies sovereign partial default. The fact that sovereign defaults are partial has long been acknowledged by, among others, Cruces and Trebesch (2013) and Asonuma and Trebesch (2016), using detailed bond data from sovereign restructurings. We contribute to this empirical literature by developing an accounting framework to measure partial default, using widely available datasets from the World Bank, which contain aggregate measures for sovereign debt service, debt in arrears, and debt levels. An advantage of our "macro" approach is that it allows us to document the dynamics of partial default and total debt flows during default episodes for a large panel of countries.[3]

An important early contribution to the theoretical literature on partial default is Grossman and Huyck (1988), which propose a framework in which partial default is interpreted as contractually specified contingent repayment in an environment with a complete set of assets. Our work instead focuses on partial default with uncontingent debt contracts—as in quantitative sovereign debt models like, for example Arellano (2008)—but departs in essential ways from this work to address aspects of the evidence involving partial default. In most of the quantitative sovereign debt models, default can be only on the total amount of debt, is followed by a period of exclusion, and ends in a full discharge of liabilities. The sanction of complete exclusion from financial markets during default is rationalized as a commitment from lenders to collude not to lend to the sovereign. In a default episode, this model precludes any continuation of debt repayments on any coupons, further borrowing, or accumulation of the defaulted debt. By design, the theory cannot study the partial default episodes that we measure and focus on here.[4]

The standard incomplete markets sovereign default model has been extended with forms of default resolution that are less blunt, following Bulow and Rogoff's (1989) work on debt restructuring. In Yue (2010) and Asonuma and Trebesch (2016), the renegotiation is via Nash bargaining, while in Bi (2008), Benjamin and Wright (2013), and Asonuma and Joo (2020), it happens within a dynamic alternating offers setting.[5] In this work the length of the default episode and the resulting haircut and maturity extension from the episode are endogenous, yet as is the case in the earlier papers, default episodes still represent a state of impasse before resolution, with no debt repayments, borrowing, or accumulation of the defaulted debt taking place. As explained above, in drawing our conclusions, we directly compare our results with those of a reference model with bargaining.

[3]Our findings also complement those of Benjamin and Wright (2013) and Tomz and Wright (2013).

[4]See also the important contributions of Aguiar and Gopinath (2006), Hatchondo and Martinez (2009), and Chatterjee and Eyigungor (2012).

[5]Mihalache (2020) and Dvorkin et al. (2021) extend these frameworks to consider long-term bonds and endogenous maturity choice during renegotiation.

Some recent work also studies heterogeneity in defaults. Gordon and Guerron-Quintana (2018) consider a model in which when the sovereign defaults, it does so only on long-term debt coupons due. Erce and Mallucci (2018) consider selective defaults on one-period bonds based on whether the debt is issued domestically or abroad. Wicht (2021) considers selective defaults between two lenders with distinct repayment enforceability power. We note that in these papers, the defaulted debt dissipates, and default episodes are associated with reductions in the level of debt, outcomes that contrast with those in our model and the data.

Our work is also related to the literature on private defaultable debt and personal bankruptcy. As in the literature of sovereign debt, most of this work, like Chatterjee et al. (2007) and Livshits, MacGee, and Tertilt (2007), has focused on full defaults and private bankruptcy. In this context the assumption that default is a discrete action upon which debts are discharged coincides with much of bankruptcy law, in which debt is formally discharged. Recently, however, analyzing partial defaults has attracted more attention, because defaults outside formal bankruptcy procedures are substantial, as documented by Dawsey and Ausubel (2004). In Mateos-Planas and Seccia (2014), households partially default on their debts giving rise to incomplete consumption insurance in an environment with a complete set of securities. Herkenhoff and Ohanian (2015) shares with our paper the feature that borrowing continues after default, as they model foreclosures in which lenders effectively finance the borrower after payments are stopped but before the house is lost.

Section 2 describes our accounting framework and documents the evidence. Section 3 describes the model, defines the equilibrium, and discusses its main properties. Section 4 discusses the quantitative implications of the model against the evidence. Section 5 conducts counterfactual analysis of various policies discussed. Section 6 compares the implications of our model with those of a state-of-the-art reference sovereign default model with full default and renegotiation through bargaining. Section 7 concludes.

2 The Empirical Properties of Sovereign Defaults

In this section, we document the properties of sovereign partial defaults using 50 years of data from emerging markets. We first present an accounting framework to organize the analysis and then discuss salient data patterns.

2.1 Accounting Framework

We start by developing an accounting framework for a sovereign of an emerging market that borrows long-term bonds and can default.

Flow Financial Variables. Each period, the sovereign owes lenders an amount a_t, which is the sum of all the coupons from past issuances due at t. As we will see later, this amount includes not only the promised coupons at t from newly issued bonds in previous periods but also the current obligations that result from past partial defaults. We consider a flexible partial default policy that is applied to each payment due, given by d_t. A default d_t implies that the sovereign pays in period t the amount $(1 - d_t)a_t$ and does not pay $d_t a_t$. The default policies result in four variables of interest: the *debt service*, defined as the flow payment; the *defaulted coupons*, defined as the payments due defaulted on; the *debt due,* defined as the sum of the debt service and the defaulted coupons; and *partial default,* defined as the fraction defaulted on, such that

$$\text{Debt service}_t = (1 - d_t)a_t, \tag{1}$$

$$\text{Defaulted coupons}_t = d_t a_t, \tag{2}$$

$$\text{Debt due}_t = \text{Debt service}_t + \text{Defaulted coupons}_t = a_t, \tag{3}$$

$$\text{Partial default}_t = \text{Defaulted coupons/Debt due} = d_t. \tag{4}$$

Long-Term Bonds with Partial Default. We map the data into a tractable structure for long-term debt contracts that consists of perpetuity bonds with coupon payments that decay at rate δ, as in Hatchondo and Martinez (2009). A borrowing contract specifies a price q_t and a value b_t such that the sovereign receives $q_t b_t$ units in period t and promises to pay, conditional on not defaulting, $\delta^{j-1}\ b_t$ units in every future period $t + j$ for $j = 1, 2, ..., \infty$.

These contracts are tractable because they encode a rich structure of debt issuances into a single state variable, the debt due a_t, with a law of motion. A sovereign that in period t pays in full its debt due a_t and borrows b_t will have in period $t + 1$ a debt due equal to $a_{t+1} = \delta a_t + b_t$, corresponding to the coupons from the legacy debt and the new borrowing. A sovereign with defaulted coupons, however, will incur some future obligations from these defaults. We assume that any defaulted coupons $d_t a_t$ result in future obligations with present value of $\kappa d_t a_t$. The factor κ is a parameter that captures the empirical observation that during default episodes, sovereigns accumulate their defaulted debt and, in some cases, restructure their obligations with their creditors. We convert these short obligations from the defaulted coupon, $\kappa d_t a_t$, into our perpetuity contract structure by annuitizing it.[6] The accumulation of debt due, therefore, incorporates the legacy debt, the defaulted coupons, and new borrowing:

$$a_{t+1} = \delta\ a_t + (R-\delta)\ \kappa\ d_t\ a_t + b_t. \tag{5}$$

Note that while a partial default $d_t > 0$ increases consumption at t, it does not necessarily reduce debt due a_{t+1} relative to a_t. Debt due can actually increase when the recovery factor is sufficiently high, because the defaulted coupons are accumulated with interest. Also, debt due can increase if borrowing b_t is positive. This flexibility in our accounting framework will be important in interpreting the dynamics of debt during default episodes that we document below.

Debt, Duration, and Spread. To measure the debt level and its duration, we use streams of contractual payments due. At time t, the contractual payments due are the promises to pay in period $t+j$, conditional on not defaulting, $\tilde{a}_t^{t+j}$ for $j = 1, 2, ..., \infty$. These promised payments $\tilde{a}_t^{t+j}$ are deterministic sequences and, in general, will differ from the actual payments due, which are stochastic at time t because of partial default. We define the *debt level* at t as the present value of the contractual payments due and the duration of the debt as the corresponding "Macaulay duration," with flows discounted at the risk free gross interest rate R:

$$\text{Debt level}_t = \sum_{j=1}^{\infty} \frac{\tilde{a}_t^{t+j}}{R^j}, \tag{6}$$

$$\text{Duration of debt}_t = \frac{1}{\text{Debt level}_t} \sum_{j=1}^{\infty} j \frac{\tilde{a}_t^{t+j}}{R^j}. \tag{7}$$

Our bond structure implies that a sovereign with end-of-the-period debt due a_{t+1} has a sequence of contractual promises $\delta^{j-1}\ a_{t+1}$ for $j = 1, 2, ..., \infty$, and a debt level of $\frac{a_{t+1}}{R-\delta}$ with associated duration of $\frac{R}{R-\delta}$.

In practice, because of default risk, the sovereign's promises to pay the stream $\{\tilde{a}_t^{t+1}, \tilde{a}_t^{t+2}, ..., \tilde{a}_t^{\infty}\}$ will have a market value different from the debt level defined above. As is standard, we can use the market value of the debt and the streams of contractual payments to define the yield-to-maturity, which is the constant discount rate that equates these two. The sovereign spread s_t is

[6] This annuitization transforms the obligations from the short-term value of $\kappa d_t a_t$ into our long-term perpetuity contract with face value m_t, so that both have the same present value $\kappa d_t a_t = \frac{m_t}{R} + \frac{\delta m_t}{R^2} + \frac{\delta^2 m_t}{R^3} + ... = \frac{m_t}{R-\delta}$. This implies we need to include $m_t = (R-\delta)\kappa d_t a_t$ into next period's obligations a_{t+1}.

the difference between the yield-to-maturity and the risk free rate, and is therefore defined from

$$\text{Market value of debt}_t = \sum_{j=1}^{\infty} \frac{\tilde{a}_t^{t+j}}{(R+s_t)^j}. \tag{8}$$

For our perpetuity contracts, the market value of debt is $q_t a_{t+1}$ because this is the value to the promised stream δ^{j-1} a_{t+1} units in every future period $t+j$ for $j=1,2,...,\infty$. The reason why the legacy debt and accumulated defaulted coupons, implicit in such claims, carry the same price q_t as the new borrowing is that future defaults are applied uniformly across all these securities. By equation (8), the sovereign spread is inversely related to q_t and equals $s_t = \frac{1}{q_t} + \delta - R$.

Default Episodes, Haircuts, and Maturity Extensions. We also use our framework to study *default episodes*. We flag a default episode as a sequence of periods with consecutive positive partial defaults and define its *length* by the number of such periods. An episode of length $N+1$, which starts in period t and ends in period $t+N$, has $d_{t+j} > 0$ for $j = 0,1,...,N$ and $d_{t-1} = d_{t+N+1} = 0$. The sequences of debt due, debt service, defaulted coupons, and partial default in the default episode are given by $\{a_{t+j}, (1-d_{t+j})a_{t+j}, d_{t+j}a_{t+j}, d_{t+j}\}$ for $j=0,1,...,N$, respectively.[7]

Default episodes tend to give *haircuts* to creditors, as the value of the new restructured debt tends to be lower than the value of the defaulted debt, and *maturity extensions*, as the restructured debt tends to have longer maturity than the defaulted debt. In our measurement, we follow the empirical methodology in Benjamin and Wright (2013) and Cruces and Trebesch (2013) that compares present values of defaulted debt instruments to restructured debt instruments. The haircut and maturity extension are ex-post measures, as they are calculated at the end of the default episode by looking back over the defaulted and restructured debt accrued over the episode.

The *defaulted debt* for the episode is defined as the present value of the defaulted coupons:

$$\text{Defaulted debt}_t = \sum_{j=0}^{N} \frac{d_{t+j}\ a_{t+j}}{R^j}.$$

The restructured debt is constructed from the debt issuances that result from the accumulated

[7] Asonuma and Trebesch (2016) document that a third of sovereign renegotiations occur preemptively. Our accounting framework would pick up all the renegotiations (preemptive or not) that are associated with having arrears in some instruments, which they document occurs in 87% of the restructurings in their dataset.

defaulted coupons, as these are effectively new assets for creditors that arise from defaults. We construct these recursively. In the first period of the episode, the government owes the coupon a_t but pays only $(1-d_t)a_t$ in debt service. This partial default at t results in an obligation to pay $n_{t+1} = (R-\delta)\kappa d_t a_t$ at $t+1$, and $\delta^j n_{t+1}$ in all periods $t+j+1$ that follow. In period $t+1$, the government owes a_{t+1} but pays only $d_{t+1}a_{t+1}$. The accrued obligations at $t+1$ are $n_{t+2} = (R-\delta)\kappa d_{t+1}a_{t+1} + \delta n_{t+1}$, where the first term is the new obligations incurred from the $t+1$ partial default and the second term is the obligations from the partial default in period t. Proceeding recursively, we obtain the accrued obligations during the episode:

$$n_{t+j+1} = \begin{cases} (R-\delta)\kappa d_{t+j}a_{t+j} + \delta n_{t+j} & \text{for } j = 0, 1, ..., N \\ \delta^{j-N-1} n_{t+j} & \text{for } j = N+1, ..., \infty, \end{cases}$$

with $n_t = 0$ by construction. The coupons for the new restructured debt depend on this sequence as well on the partial defaults, because part of the obligations accrued is partially defaulted on during the episode, so that the lender receives only $(1-d_{t+j+1})n_{t+j+1}$. We define as the *restructured coupons* for this default episode the sequence of payments that lenders receive from the accrued defaulted coupons during the episode, $(1-d_{t+j+1})n_{t+j+1}$ for $j = 0, 1, ..., N$, and the coupons to be received after the episode $\delta^j n_{t+N+1}$ for $j = 0, 1, ..., \infty$. The new *restructured debt* is the present value of this sequence:

$$\text{Restructured debt}_t = \sum_{j=0}^{N} \frac{(1-d_{t+j})n_{t+j}}{R^j} + n_{t+N+1} \sum_{j=0}^{\infty} \frac{\delta^j}{R^{N+j+1}}.$$

With these sequences in hand, we can calculate the haircut of the default episode simply as one minus the ratio of the corresponding present values:

$$\text{Haircut}_t = 1 - \frac{\text{Restructured debt}_t}{\text{Defaulted debt}_t}. \tag{9}$$

We also define the *maturity extension* in the debt resulting from the default episode as the difference in duration between the restructured debt and the defaulted debt, applying expressions similar to (7).

2.2 Mapping Data to Accounting Model

Our accounting framework takes a "macro view" of debt crises by analyzing the totality of the reported flow payments and obligations. This approach embeds the rich details of debt crises that involve multiple creditors and restructurings into a few aggregate time series. Although this approach does not distinguish between borrowings and restructurings with banks, bond holders, or official creditors, in Appendix A we also present a narrative analysis for two default episodes in richer detail, and argue that the flexibility of our accounting framework allows us to summarize the salient features of these episodes.[8] Some advantages of our approach are that it can be combined with readily accessible aggregate data to analyze default episodes and its relation with output, total debt, and spreads, and that these data are uniformly collected for many countries.

We use the debt statistics from the World Development Indicators (WDI), International Debt Statistics (IDS), and the Debtor Reporting System, all from the World Bank, to empirically measure the variables of interest in our accounting framework. From these data, we use the debt obligations for the government, defined as public and publicly guaranteed (PPG), for both flow and stock variables. We also use bond spreads from the Global Financial Database. The dataset is annual and corresponds to a panel of 37 emerging countries from 1970 to 2019. The online Appendix contains the list of countries. Next, we discuss in detail how we map the variables available in these datasets to our accounting framework.

Mapping the flow variables from the data to our accounting framework is straightforward. Given that the World Bank datasets are of annual frequency, for simplicity, we set the time period to a year. Using data from the International Debt Statistics for debt service PPG and from the Debtor Reporting System for arrears PPG, we construct the flow variables defined in (1) through (4). We map the entry for debt service PPG in the data directly to our debt service measure in (1). PPG debt service in the data is the sum of principal repayments and interest actually paid on obligations of public debtors and obligations guaranteed by a public entity, which exactly corresponds in our accounting framework to $(1 - d_t)a_t$. To construct an empirical measure for the defaulted coupons, we sum the entries for interest in arrears and principal in arrears PPG to calculate the total payments due that are not paid. We map this sum of arrears to our measure for defaulted coupons as in (2). With these two variables, we then construct the total debt due and the partial default time series as specified in equations (3) and (4). These manipulations result in a panel dataset of debt service, defaulted coupons, debt due, and partial default.

We flag default episodes in the data using the series of partial default. These series have a large number of zeros, because countries do not have defaulted coupons all the time. In keeping with our accounting framework, a default episode begins in the period the country starts to have positive partial default and ends when the country stops having positive partial default. Its length

[8] Relatedly, Cruces and Trebesch (2013) and Asonuma and Trebesch (2016) document major restructuring events using individual bond structures.

is given by the number of periods.

We are also interested in obtaining time series for the total level of debt. Our databases report PPG debt levels in terms of face values of principal payments, which differ from the present values in (6). Dias, Richmond, and Wright (2014) analyze various measures of debt and discuss in detail these challenges. They show that the mapping between face values and present values can be close, especially for reasonable discounting rates. The reason is that although face values do not include coupon payments, they are due in the future and not discounted back; these opposing forces tend to cancel each other out. Consequently, we will simply measure debt levels with the PPG debt variable in IDS.

For haircuts and maturity extensions that result from default episodes, we rely on estimates by Cruces and Trebesch (2013), Meyer, Reinhart, and Trebesch (2018) and Fang, Schumacher, and Trebesch (2016), who define them as we do in our accounting framework. We rely on these papers because they have richer datasets that contain the entire cash flow of bonds. These papers discuss how countries in default reach arrangements with creditors during restructurings, in which the defaulted debt is exchanged with the newly restructured debt, and they measure haircuts as we do, by comparing the present values of these instruments.[9] Interestingly, they also discuss how oftentimes the restructured debt is defaulted on soon after, leading to a second round of agreements.

We measure government spreads with the spread series of the Emerging Market Bond Index (EMBI+) from the Global Financial Database for each of the countries in our sample. These spreads are the weighted average of the difference in yields between emerging market bonds issued by the government in foreign currency and a U.S. government bond of similar maturity.[10] Finally, we also measure output using real gross domestic product log and detrended with a linear trend, taken from the World Development Indicators.

2.3 Empirical Findings

We now document patterns of the variables of interest discussed in our accounting framework. We analyze distributions of partial default and default episode length and their comovement

[9]The streams of defaulted debt used by these studies might not correspond exactly to the time series of defaulted coupons from the World Bank, owing to potential differences in the treatment of accounts receivable and default acceleration clauses. However, we show in the online Appendix that applying our exact accounting framework and estimated parameters to the World Bank data on defaulted coupons leads to very comparable haircut estimates.

[10]Spreads are computed daily, so that the effect of any change in the country's maturity structure on risk free rates is netted out.

with interest rate spreads, debt-to-output ratios, and output in the data of emerging markets. These empirical regularities will provide the basis of the quantitative analysis for our model in the subsequent sections.

We start by describing the time series of partial default for two emerging market countries with a history of sovereign default: Argentina and Russia. In Figure 1, we plot the time series of partial default from 1970 to 2019 for these countries. The figure shows that default varies in intensity, ranging from small levels of less than 10%, as was the case in the late 2000s for Russia, to high levels of more than 90%, as in the case of Argentina in the early 2000s. In terms of default episode length, Argentina experienced two episodes with lengths equal to 10 and 18 years, and Russia experienced one episode of a length equal to 20 years. In Appendix A, we provide narratives with rich institutional details for the history of partial defaults for these two countries. We find that partial defaults extend many credit market instruments that go through several restructuring rounds. We also find that during these default episodes, sovereigns borrow in the form of new loans, unrelated to the partial default, and also as part of restructuring agreements. The macro approach of our accounting framework synthesizes these rich details into a few time series, such as the partial default series in the figure.

We now study the properties of partial default and default episodes for the 37 emerging markets, by analyzing time series of the variables of interest and dynamics within default episodes. Sovereigns partially default often, and default episodes vary in their duration. The frequency of positive partial default in the panel dataset is 36%. The varying intensity of partial default across years and countries is illustrated in Figure 2A, which plots the histogram of partial default (conditional on positive partial default) for the panel data, the year$\times$country series. The histogram shows that countries partially default at varying degrees covering the full range.

Default episodes also vary in duration. In Figure 2B, we plot the histogram of default episode length for the 70 default episodes in the dataset. Many of the default episodes are short-lived; 36% of the episodes last fewer than two years. The histogram has a long right tail, as few episodes last more than 40 years. The distribution of the default episode length in our dataset is similar to the one documented in Benjamin and Wright (2013).

Table 1 summarizes the distributions of partial default and default episode length. The mean partial default conditional on positive partial default is 38%, with a standard deviation of 22%.[11] The mean length of the default episode is equal to nine years, but a large fraction of the defaults are short. We also report in the table the estimates for haircuts and maturity extensions from Meyer, Reinhart, and Trebesch (2018) and in Fang, Schumacher, and Trebesch (2016). They find

[11]This is the time series standard deviation of positive partial default averaged across countries.

that across default episodes since 1970, haircuts average 36%, and they involve debt exchanges with maturity extensions that increase the duration of debt by six years.

The table also reports the dynamics of partial default, debt to output, and output during default episodes. We report average statistics for these variables for the period before the start of the default episode, labeled *before*, the first period of the default episode, which we label *beginning*, the middle of the default episode, which we label *middle*, and the end of the default episode, which we label *end* and which is the first period when partial default returns to zero.[12] Partial default tends to start smaller in the beginning and grow with the episode before returning to zero. On average, debt to output also features a hump-shaped pattern during default episodes, and default episodes do not lead to reductions in debt. In the period before the episode starts, the ratio of debt to output is 32%. The default episode starts with a higher debt-to-output ratio of 34%. During the default episode, the debt continues to rise, and by the middle of the episode, it is equal to 40%. Debt decreases to 33% toward the end of the episode. Output dynamics during the default episode follow a U-shaped pattern. An episode starts when output falls; it continues to fall during the episode, and it recovers somewhat at the end of the episode. These dynamics of output and debt during default episodes are consistent with previous empirical findings.[13]

Next we document the comovements of partial default with spreads, debt to output, output, and episode length. To do so, we divide the panel dataset into four bins based on the levels of partial default and report for each bin the means of partial default, spreads, debt to output, and output. The no-default bin consists of the observations with zero partial default. We partition the observations with positive partial default into three groups. The small partial default bin contains the observations below the 25th percentile; here, average partial default is 3%. The medium partial default bin contains the observations between the 25th and 75th percentiles, with an average of 28%. The large partial default bin contains the observations above the 75th percentile, with an average of 91%.

Table 2 shows that spreads, debt to output, and output have sizable differences as partial default varies. Spreads in periods of no default are on average 4%. During small and medium defaults, spreads rise modestly to an average of 6% and 8%, respectively. During large defaults, however, spreads more than double and are on average 18%. Debt to output in periods of no default is on average 24% and is higher in periods when sovereigns partially default. Debt to output rises

[12] We define the middle of the episode as the total length of the episode divided by 2, rounded to the nearest integer.

[13] Benjamin and Wright (2013) also find that default episodes are more likely to start when output is below trend and to end when output has returned to trend; they also find that debt upon exiting the default episode is no smaller than it was before the episode. Hébert and Schreger (2017) estimate substantial output costs from default episodes, using causal estimates on equity values from legal rulings.

monotonically to 33%, 43%, and 60% in periods of small, medium, and large defaults. The higher debt-to-output ratios during partial default run counter to the standard narrative that sovereign defaults reduce the debt burden. The increase in debt during default occurs because defaulted debt is accumulated and because governments continue to borrow while partial default is positive. Finally, output in periods of no default is on average 1% above trend. Output deteriorates as default rises, and it reaches -4% below trend during large defaults.[14]

We conclude this section by summarizing our findings from our emerging market data:

1. Partial default is common. Emerging markets are in partial default about one-third of the time. Partial default is on average 38% and has a large variance.

2. Higher partial default is associated with higher spreads and debt and with more depressed output.

3. Default episodes feature hump-shaped dynamics in partial default and debt and do not lead to a net reduction in debt.

3 The Model

Our environment consists of a small open economy with a stochastic endowment stream, and a sovereign that borrows in long-term bonds and can choose to partially default on the debt due. The defaulted coupons accumulate, and partial default imposes future resource costs on the economy that are increasing in the intensity of the default. Borrowing rates reflect default risk and compensate creditors for expected losses. We discuss the details of the model environment (Section 3.1) and its recursive formulation (Section 3.2). Then, we characterize the model equilibrium (Section 3.3).

3.1 Model Environment

The sovereign receives each period a stochastic endowment z_t that follows a Markov process with transition probabilities $\pi(z_{t+1}, z_t)$. The sovereign discounts the future at rate β and maximizes

[14] These comovements are related to results in Cruces and Trebesch (2013) that find that spreads are higher following default episodes with large haircuts during renegotiations and results in Tomz and Wright (2013) that document that output tends to be lower during default episodes.

expected utility over consumption sequences, c_t, with preferences given by

$$E\left\{\sum_{t=0}^{\infty}\beta^t\ u(c_t)\right\}. \tag{10}$$

The income of the sovereign, y_t, is the endowment z_t net of any costs associated with default.

The sovereign trades long-term bonds with international lenders. The long-term contracts are perpetuities that follow the structure described in our accounting framework. Each period, the borrower has total debt due, a_t, which consists of all the obligations from past borrowing and the accumulation of defaulted coupons. The sovereign chooses which fraction of these obligations to partially default on, d_t, and new borrowing b_t at price q_t. It can also buy back debt by choosing negative borrowing $b_t < 0$ at price q_t. The bond price is a function $q(a_{t+1}, d_t, z_t)$ that depends on total debt due the following period a_{t+1}, partial default today d_t, and the endowment z_t, because these variables determine future defaults, and the government internalizes that its choices affect the price. Consumption is constrained by income less total debt due net of default, $a_t(1-d_t)$, plus new borrowing:

$$c_t = y_t - a_t\ (1-d_t) + q(a_{t+1}, d_t, z_t)\ b_t. \tag{11}$$

Default carries a direct resource cost that is increasing in the intensity of the default and depends on the shock, so that income in the period after default is $y_{t+1} = z_{t+1}\Psi(d_t, z_{t+1}) \leq z_{t+1}$. We assume that $\Psi(d_t, z_{t+1})$ is decreasing and concave in d_t and that $\Psi(0, z_{t+1}) = 1$.

As explained in our accounting framework, a factor κ of the defaulted coupons, $d_t a_t$, becomes future debt obligations. The total debt due next period a_{t+1} includes the accumulation of defaulted coupons after annuitization $(R-\delta)\kappa d_t a_t$, the coupon payments due from the long-term legacy debt δa_t, and the new borrowing b_t. Its evolution follows (5).

The sovereign borrows from many identical competitive risk neutral international lenders. A lender purchases sovereign bonds by issuing securities at the risk free world gross interest rate R. In each subsequent period, the claims of the lender against the sovereign consist of the promised coupons from the initial bond and, potentially, the restructured coupons that result from a partial default on the bond.

The sovereign chooses borrowing and partial default to maximize utility, taking as given the bond price function. Free entry among international lenders determines bond prices by driving expected profits from lending to zero, taking as given the sovereign's future partial default and borrowing decisions. Since bonds issued in different periods are perfectly substitutable and part of the debt due a_{t+1}, the price of new bonds b_t is also the price of the legacy bonds.

3.2 Recursive Formulation

We focus on recursive Markov equilibria and represent the infinite horizon decision problem of the sovereign borrower as a recursive dynamic programming problem. We also represent the lenders' value as a recursive functional equation. The state vector of the model consists of three variables $\{a, y, z\}$: a is the total debt due, y is the income of the economy, and z is the endowment shock, which we need to keep track of because of the Markovian structure of shocks. In this context, the bond price is a function, $q(a', d, z)$, of what is known at the time of borrowing about the state tomorrow that will determine partial default and future borrowing. The function depends on the debt due tomorrow a' (we are now switching to standard recursive notation in which primes denote future values), the partial default decision today d, which affects output tomorrow, and today's endowment shock z, which helps predict tomorrow's shock.

Borrower. Taking as given the bond price function $q(a', d, z)$ and the borrower's decision rules in all future periods, the recursive problem of the borrower with state $\{a, y, z\}$ is to choose new borrowing b, partial default d, and consumption c to maximize its value

$$V(a, y, z) = \max_{b,d,c} \left\{ u(c) + \beta \sum_{z'} \pi(z', z) \; V(a', y', z') \right\}, \tag{12}$$

subject to the budget constraint,

$$c = y - (1 - d) \; a + q(a', d, z) \; b, \tag{13}$$

the law of motion for the evolution of debt due that incorporates defaulted debt and new borrowing,

$$a' = \delta a + (R - \delta) \; \kappa \; d \; a + b, \tag{14}$$

the evolution of income, which depends on partial default and on the shock, $y' = z' \; \Psi(d, z')$, and the constraint that default cannot exceed the debt due and is weakly positive, $0 \leq d \leq 1$.

This problem determines the optimal borrowing and partial default policy functions $b(a, y, z)$ and $d(a, y, z)$, the evolution of the debt due $a'(a, y, z)$, and the consumption function $c(a, y, z)$.

Lenders. There are many identical competitive risk neutral lenders that discount time at rate $1/R$. The value of one unit of debt $H(a, y, z)$ equals the expected discounted stream of payments.

It contains the amount paid today plus the expected discounted continuation value such that

$$H(a,y,z) = [1 - d(a,y,z)] + \frac{1}{R}\ [\delta + (R-\delta)\ \kappa\ d(a,y,z)] \times \sum_{z'} \pi(z',z)\ H\left[a'(a,y,z), z'\Psi[d(a,y,z),z'], z'\right]. \quad (15)$$

This expression explicitly reflects that partial default $d(a,y,z)$ reduces the value for lenders today and increases it tomorrow, as the defaulted debt accumulates and comes due in the future. Partial default also reduces tomorrow's expected output through the Ψ function. An additional channel, implicit in the equation, arises through the evolution of debt due $a'(a,y,z)$ which affects the future value $H(a',y',z')$; everything else equal, as seen in (5), higher partial default (and additional borrowing) increases total debt due tomorrow, reducing the value of debt.

Equilibrium. We now define the equilibrium for this economy. The equilibrium entails the sovereign solving its problem understanding how its behavior now and in the future affects the prices it faces and that competition among lenders leads to zero profits in expected value on loans. Formally,

Definition 1 *A recursive Markov equilibrium consists of (i) the borrower's decision rules for borrowing $b(a,y,z)$, partial default $d(a,y,z)$, and consumption $c(a,y,z)$, which induce rules of debt due $a'(a,y,z)$; (ii) the value of existing debt $H(a,y,z)$; and (iii) the bond price function $q(a',d,z)$ such that*

1. *Taking as given the bond price function $q(a',d,z)$, the borrower's decision rules satisfy its optimization problem in eq. (12).*
2. *Taking as given the borrowers' decision rules, the value of debt $H(a',d,z)$ satisfies its recursive formulation described in eq. (15).*
3. *Bond prices $q(a',d,z)$ yield expected zero profits to lenders so that*

$$q(a',d,z) = \frac{1}{R} \sum_{z'} \pi(z',z)\ H\left(a', z'\Psi(d,z'), z'\right). \quad (16)$$

3.3 Characterization of Equilibrium

In this section, we discuss the factors weighing on the sovereign's decisions to partially default and borrow. We also examine the price of bonds, an important factor in those decisions. In fact, by looking at the case in which the decision rules are differentiable, we are able to spell out the links between future decisions to borrow and partially default and the shape of the pricing function. These links allow us to characterize the first order conditions of the sovereign using only future sovereign decision rules and future prices, but not the derivatives of those future prices.

Borrowing and Partial Default To understand the trade-offs involved in the choice of borrowing b and partial default d, we analyze the optimality conditions for the sovereign. In these derivations, we follow a heuristic approach and look for a characterization in a region of the state space where the decision rules, $b(a, y, z)$ and $d(a, y, z)$, the price function $q(a', d, z)$, the value function $v(a, y, z)$, and the default cost function $\Psi(d, z')$ are all differentiable.[15] For this purpose, it is convenient to define the function $\Lambda(d', q')$ as the *debt burden*, as it captures the effective value of the debt to be paid for every unit of a':

$$\Lambda(d', q') \equiv \underbrace{1 - d'}_{\text{repayment}} + \underbrace{[\delta + (R - \delta)\ \kappa\ d']\ q'}_{\text{further repayments at price } q'} .$$

The debt burden incorporates the sum of the costs associated with current repayment at $(1 - d')$, and further repayments, the continuation amount for long-term debt δ and the accumulation of the defaulted coupons $(R - \delta)\kappa d'$, both evaluated at price q'.

The bond price function also directly relates to the debt burden. By substitution of the function H from (15) into the equilibrium condition (16), and then using (16) updated by one period to eliminate the resulting future value of H, we write the pricing function q recursively as

$$q(a', d, z) = \frac{1}{R}\ E\left\{[1 - d'] + [\delta + (R - \delta)\ \kappa\ d']\ \ q'\right\} = \frac{1}{R}\ E\left\{\Lambda(d', q')\right\}, \tag{17}$$

where the variables on the right hand side are the policy rules and bond price evaluated at the induced states: $d' = d(a', y', z')$, $q' = q(a'', d', z')$, $a'' = [\delta + (R - \delta)\kappa d']a' + b'$, $b' = b(a', y', z')$, and $y' = z'\Psi(d, z')$. The bond price function is therefore equal to the expected discounted debt burden, as it captures the expected effective value that will be paid for each unit of debt due a'.

If we use compact notation—that is, not writing the arguments of functions, denoting partial derivatives by subindexing the functions, and using primes to denote future values—the first order

[15] We know from the work of Clausen and Strub (2020) that in the short-term debt sovereign default problem, there are states in which the functions involved are not differentiable.

conditions for an interior solution for the problem in Eq. (12) with respect to b and d are

$$u_c \ [\ \underbrace{q + q_{a'}\ b}_{\text{borrowing gain}}\ ] \ = \ \beta\ E\{u'_c \quad \underbrace{\Lambda'}_{\text{debt burden}} \quad \}, \tag{18}$$

$$u_c \ \underbrace{[a + (q_{a'}\ (R-\delta)\ \kappa\ a + \ q_d)\ b]}_{\text{partial default gain}} = \ \beta\ E\{u'_c\ [\quad \underbrace{(R-\delta)\ \kappa\ a\ \Lambda'}_{\text{debt burden from defaulted coupons}} \quad - \ \underbrace{z'\ \Psi_d}_{\text{default cost}}\ ]\}. \tag{19}$$

These two optimality equations illustrate how the borrower can transfer future resources to the present by borrowing b or by partially defaulting d. In Eq. (18), the left hand side is the marginal gain from borrowing; one unit of b increases consumption by q but reduces the price as it raises the debt due, $q_{a'} < 0$. This borrowing gain, therefore, incorporates that borrowing a marginal unit changes the cost of inframarginal units by $q_{a'}b$. The right side is the marginal cost, which is the discounted expected value of increasing the debt burden, Λ'.[16] This optimality condition is similar to that arising in many dynamic sovereign default models in that the resources raised with borrowing are shaped by a Laffer curve, which limits the possibility of intertemporally transferring resources with loans.

The optimality condition for partial default d in Eq. (19) trades off partially defaulting on the debt with accumulating the defaulted coupons and experiencing the default costs. The marginal partial default gain is that consumption is increased by a, but it is discounted by the fact that the bond price q falls. This fall in the bond price is due to two factors: future debt obligations a' are increased by the defaulted coupons, $q_{a'} < 0$, and partial default reduces future resources, depressing the price, $q_d < 0$. Raising resources through partial default does not have the acquiescence of lenders, and these resources are capped not by a Laffer curve but by the total level of defaultable debt due a. The costs of partially defaulting include the increase in debt due from the accumulation of defaulted coupons, given by $(R-\delta)\kappa a \Lambda(d', q')$, and the resource cost, encoded in $\Psi_d < 0$.

These conditions illustrate that b and d are different ways to alter the debt position, each one with its own costs and benefits. Borrowing may mean higher interest rates, while partial default leads to future output losses.

By combining eqs. (17) to (19), we derive the following condition, which equates the expected returns of borrowing, R^b, and partially defaulting, R^d:

$$R^b \equiv \frac{R}{1 + q_{a'}b/q} + \text{cov}_1 = \frac{E\{z'(-\Psi_d)\}}{a(1 - q(R-\delta)\kappa) + q_d b} + \text{cov}_2 \equiv R^d, \tag{20}$$

[16] The accumulation of the defaulted coupons allows the borrower to create long-term debt even when debt contracts are short-term and $\delta = 0$.

where the terms cov_1 and cov_2 reflect covariances between marginal utilities u'_c with the debt burden Λ' and default costs $(-z'\Psi_d)$, respectively.[17] For interior solutions, these returns are equated to the expected growth rate of marginal utility, $u_c/[\beta E\{u'_c\}] = R^b = R^d$. In contrast, when partial default is zero, it must be that the partial default return is dominated by the borrowing return $R^b < R^d$, at $d = 0$—as low returns are attractive for borrowing or partial default.

The bond price function, the default costs, and the state of debt are the main determinants of the optimal portfolio of borrowing and partial default. Borrowing is an attractive choice when the price of new debt is high and this price decreases little for a marginal increase in the amount borrowed, in that the elasticity $|q_{a'}/q|$ is small. In such a case, borrowing is an attractive way to smooth consumption, as the return on this borrowing is low. In contrast, a positive partial default is attractive only when the price is low, so that any borrowing b leads to only a small increase consumption, and the price falls rapidly with borrowing. In this case, it may be worth paying the default costs encoded in $\Psi'_d < 0$, because doing so allows consumption to be increased one for one today with a partial default.

Note that with the accumulation of defaulted coupons, the level of debt due a matters only to the extent that it differs from the value of accumulated defaulted coupons $q(R-\delta)\kappa a$. A high debt a and a low recovery factor κ tend to increase the attractiveness of partial default by reducing its return. Another cost from a partial default is that since it lowers the net amount of output available for repayment tomorrow, which raises the probability of default and hence lowers the price, and which is encoded in q_d. If the government chooses a partial default, however, it can minimize these extra costs by choosing to borrow very little b.

The portfolio condition eq. (20) also illustrates how bond prices incentivize the sovereign to exit default episodes. Consider the case in which the sovereign is in a default episode with high debt a. The sovereign can choose to remain with high debt a' for the next period, but this choice would lead to a high bond return R^b and a low partial default return R^d—arising from a low price q and highly negative elasticity of the bond price $q_{a'}/q << 0$. Instead, the sovereign chooses to reduce its debt and reduce R^b until it is equal to R^d. The deleveraging process continues until debt is low enough that bond return R^d is strictly less than that of partial default R^d, at which point the default episode ends with the sovereign choosing $d = 0$.

Bond Price Function Derivatives. The portfolio decision described above depends on the derivatives of the bond price function. Yet, these price responses, as expressed in the derivatives $q_{a'}$ and q_d, are also themselves functions of equilibrium behavior, albeit in future periods. Taking

[17] Specifically, $\text{cov}_1 = \text{cov}(\Lambda', u'_c)/(Eu'_c[q + q_{a'}b])$ and $\text{cov}_2 = \text{cov}(u'_c, (-z'\Psi'_d))/(Eu'_c[a(1-(R-\delta)\kappa q) + q_d b])$.

direct derivatives of the bond price function in equation (17) yields the following somewhat "tautological" expressions (in the sense that they depend on future derivatives of the same pricing function that we are trying to characterize):

$$q_{a'} = \frac{1}{R}\ E\Big\{\Lambda'_d\ d'_a + \Lambda'_q\left[q'_{a'}\left[\delta + b'_a + (R-\delta)\ \kappa\ \ (a'\ d'_a + d')\right] + q'_d\ d'_a\right]\Big\} \tag{21}$$

$$q_d = \frac{1}{R}\ E\Big\{\Lambda'_d\ d'_y\ \Psi'_d\ z' + \Lambda'_q\left[q'_{a'}\ \ \left[b'_y + (R-\delta)\ \kappa\ a'\ d'_y\right] + q'_d\ d'_y\right]\Psi'_d\ z'\Big\}. \tag{22}$$

Fortunately, we can eliminate the future derivatives of the pricing function, $q'_{a'}$ and q'_d by using the first order conditions. We can rearrange conditions (18) and (19) to define functions $\mathcal{B}(a, y, z)$ and $\mathcal{D}(a, y, z)$.

$$\begin{aligned} q_{a'} &= \frac{\beta\ E\{u'_c\ \Lambda' - q\ u_c\}}{u_c} && \equiv \mathcal{B}(a, y, z), \\ q_{a'}(R-\delta)\kappa a + q_d &= \frac{\beta\ E\{u_c\left[\ (R-\delta)\ \kappa\ a\ \Lambda'\ \ - z'\ \Psi'_d\right]\} - a\ u_c}{b\ u_c} && \equiv \mathcal{D}(a, y, z). \end{aligned}$$

Notice that functions $\mathcal{B}$ and $\mathcal{D}$ depend only on decision rules and the current price. We can forward those functions one period and substitute them in (21) and (22) to obtain the following. The derivative with respect to debt due a' is

$$\begin{aligned} q_{a'} =& \frac{1}{R}E\Big\{\Lambda'_d\ d'_a && \text{direct loss from not paying} \\ &+ \Lambda'_q \times && \text{continuation amount gets diluted because of the} \\ &\left[\mathcal{B}'(\delta + b'_a + (R-\delta)\ \kappa\ d') + \mathcal{D}'\ d'_a\right]\Big\} && \text{change in future prices with more debt.} \end{aligned} \tag{23}$$

It has both a direct negative effect that arises from a higher likelihood of a loss from not paying and a continuation effect, the so called dilution effect.[18] The continuation amount, $(\delta + (R-\delta)\kappa\ d')$, is diluted because future governments borrow and partially default in a way that changes future prices away from what the current government would choose. The derivative with respect to

[18]See Hatchondo, Martinez, and Sosa-Padilla (2016) for a study of dilution in the complete default model.

partial default d is

$$q_d = \frac{1}{R} E \Big\{ \Lambda'_d \ d'_y \ \Psi'_d \ z' \qquad \text{lower output tomorrow yields more default}$$

$$+ \Lambda'_q \times \qquad \text{continuation amount gets diluted because of the}$$

$$\left[(\mathcal{B}' b'_y + \mathcal{D}' \ d'_y) \ \Psi'_d \ z' \right] \Big\} \qquad \text{change in future prices with lower output.} \tag{24}$$

This derivative depends on the output costs from default $\Psi_d z'$, since lower output tomorrow has a direct effect through higher partial default tomorrow d'_y. It also contains the dilution effect on the continuation amount, which occurs because less output tomorrow induces more borrowing, (b'_y), and higher partial default, (d'_y), both of which induce changes in future prices.[19]

These first order conditions that we have described are not standard Euler equations. Indeed, they are generalized Euler equations (GEEs) in the sense that the conditions that the decision rules have to satisfy include values of the derivatives of those decision rules. This is the result of the time inconsistency of sovereign default environments. Current decision makers, if they could commit, would make choices for future actions that are different from the actual equilibrium future actions. It is possible that partial default would be chosen if they had commitment, but the actual policy rule would be in terms of previous partial default commitments.[20]

4 Quantitative Results

We now study the quantitative properties of our model and compare them with the data in emerging markets from Section 2. We map the model to the data (Sections 4.1 and 4.2) and illustrate its mechanics by analyzing the resulting decision rules (Section 4.3). To examine the implications of our model against the data, we organize our simulated data in two ways. We first look at time series properties of partial default, debt, output, and spreads: means, standard deviations and co-movements (Section 4.4). We then analyze data from default episodes: episode length, haircuts, maturity extensions, and dynamics of partial default, output, and debt during the episode (Section 4.5).

[19] Note that the derivatives of the bond price depend only on the expected value of the policy functions and their derivatives, not on the state by state values. Also, we derived the expressions for the case of positive and interior partial default the following period default (otherwise, some of the terms would be zero).

[20] See Krusell and Smith (2003) for a discussion of these issues and Mateos-Planas and Ríos-Rull (2015) for a more general characterization of equilibrium of sovereign default economies with the aid of generalized Euler equations.

4.1 Specification and Parameterization

The utility function is $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$. We posit an 11 state Tauchen (1986)-style approximation of an autoregressive process for $\log z$ with autocorrelation ρ and innovations with standard deviation σ_η.

Recall that output depends on the shock and on default costs in such a way that output next period is $y' = z'\Psi(z', d)$. Following the quantitative sovereign default literature (Arellano (2008) and Chatterjee and Eyigungor (2012)), we assume that default costs are increasing and convex in the shock—they are realized for z only above a threshold z^* and then linearly increase in z with slope ϕ_1. We extend the specifications in the literature by allowing default costs to also depend on partial default, d, through the slope and curvature parameters, $\phi_0 > 0$ and $\gamma > 1$. The functional form we consider is

$$\Psi(z', d) = (1 - \phi_0\ d^\gamma)\ (1 - \widehat{\phi}_1(z' - z^*)), \tag{25}$$

where $\widehat{\phi}_1 = \phi_1$ if $d > 0$ and $z' > z^*$, and zero otherwise. This functional form allows for a fixed cost of positive partial default, $\phi_1(z' - z^*)$, for shock levels above the threshold and also imposes that default costs are convex in partial default. These assumptions entice the borrower into small partial defaults during low shocks.[21]

A period is one year. We set the risk aversion coefficient to 2 and the risk free interest rate rate to 4%, both standard values. We use a minimum distance estimator to specify the rest of the parameters.

Estimated Parameters. The remaining nine parameters, $\Theta = \{\phi_0, \phi_1, \gamma, z^*, \beta, \kappa, \delta, \rho, \sigma_\eta\}$, are estimated by minimizing the sum of the proportional square residuals of eleven moments from emerging markets data. We focus on target moments that describe time series properties of the data. The first three moments that we target summarize the empirical distribution of partial defaults: their frequency, mean and standard deviation. Six additional moments concern the properties of debt, debt service, debt due, and sovereign spreads, three means (the debt-to-output ratio, the debt service to output ratio, and the debt due to output ratio), and three standard deviations (debt to output, debt service to output, and sovereign spreads). Finally, we also include as targets the standard deviation and persistence of output. Output is logged and

[21] The problem requires that $\gamma > 1$ for the possibility of interior partial default while borrowing. We also found that the program is better behaved, with bond prices uniformly decreasing in debt with the penalty of default based on d rather than da.

detrended using a linear trend.[22] These 11 moments are the average across the 37 emerging countries in Section 2.

We solve our model with global methods and outline the algorithm to compute the model in the online Appendix. We simulate the model for 750,000 periods and discard the initial 10% of observations. This long simulation approximates the limiting distribution across the states $\{a, y, z\}$. In measuring model variables, we use our accounting framework: partial default is d, debt is measured as in equation (6) and recorded relative to output y_t, debt service relative to output is $(1 - d_t)a_t/y_t$, debt due relative to output is a_t/y_t, and spreads are the difference in yields to maturity, which corresponds to $s_t = (1/q_t + \delta) - R$. Table 3 displays the estimated parameter values.

All parameters have consequences for all of the 11 targeted moments. However, our experience in the estimation process makes us associate some of the parameters closely with certain moments. The two parameters describing the fixed component of the penalty for default, ϕ_1 and z^*, have an impact on the frequency of defaults and the size of defaults. The parameters γ and ϕ_0, which describe the penalty for default that depends on its intensity, mainly affect the size of partial default and its volatility.[23] The discount rate β is closely connected to the mean debt variables, while the parameter that controls the accumulation of defaulted debt κ matters more for the volatilities of the debt variables and spreads. Finally, parameters controlling the z shock are related primarily to the persistence and volatility of output. The persistence and volatility for productivity have to be slightly smaller than those for the targeted output process.

4.2 Moments in Model and Data

Table 4 shows moments of the time series for simulated data in the model and compares them with data in emerging markets. The top panel presents the results of our moment-matching exercise and shows that our model is able to generate patterns that resemble those in the data. In the model and data, partial default is positive about one-third of the time. On average, partial default is 39% in the model, close to the data counterpart of 38%. Partial default, conditional on being positive, is volatile, with a standard deviation of 19% in the model, close to the data one of 22%.

The level and volatility of debt are similar in the model and in the data. In the model, the mean

[22]In the estimation, the weighting matrix has a weight of 3 for the level of debt due, of 1.25 for the standard deviation of spread, and of 1 for the additional seven moments.

[23]As we will see below, these default cost parameters produce very minor default costs.

debt to output is 32%, with a standard deviation of 25%, which are close to the data counterparts (32% and 18%, respectively). The model also replicates the behavior of the debt service to output ratio, its mean (3.5% vs. 3.6% in the data), and its standard deviation (2.2% vs. 2.1% in the data). As shown in Chatterjee and Eyigungor (2012), models with long-term debt are able to reproduce both the debt level and the debt service; our model with long-term debt and partial default can match these, as well as the debt due. In terms of the interest rate spreads, the model produces a volatility of 3.7%, which resembles the data one of 4.1%. The model also matches well the resulting series for output in terms of its persistence and standard deviation.

Table 4 also reports some additional untargeted moments. The mean and standard deviation for defaulted coupons (conditional on positive partial default) in the model are a bit lower than in the data. In terms of mean spreads, the model generates a lower mean than the one in the data (1.6% vs. 5.3%). The discrepancy arises partly because we are modeling creditors as risk neutral.[24] Longstaff et al. (2011) find that a similar wedge arises empirically and that about 64% of sovereign spreads can be attributed to a single global factor that contains large risk premia. We could interpret their finding as stating that we should have targeted only the portion that is actually due to country-specific factors, 1.9% (36% of 5.3%).[25] The model generates a negative correlation of spreads with output and a positive correlation with debt, both of which are features of the data. The model correlations, however, are a bit stronger, as one would expect with only one shock. Finally, the table also reports the volatility of consumption and shows that in the data and the model, the volatility of consumption is close to that of output.[26]

4.3 Decision Rules

Before comparing our model with the data on partial default bins and default episodes, in this subsection, we study the model's mechanisms by analyzing the bond price functions and decision rules for partial default and borrowing. Figure 3 presents spread schedules, partial default decision rules, and the portfolio decision between borrowing and partial default. Consider first the spread schedule $spr(a', d, z)$, constructed from bond prices as in (8), presented in the left panel of the

[24] The wedge between spreads and expected default losses is related to the credit spread puzzle in corporate bonds, as explored in Chen, Collin-Dufresne, and Goldstein (2009).

[25] By the same token, it would be sensible to target a different standard deviation of the spread, corresponding only to its idiosyncratic component. For lack of better information, we have considered an alternative target of one-half of the overall standard deviation and found that the implications practically do not change.

[26] As is standard in the sovereign default literature, the series of aggregate consumption in the data includes durable consumption, which is more volatile than nondurable consumption. Alvarez-Parra, Brandao-Marques, and Toledo (2013) report an average volatility of nondurable consumption relative to output for a variety of countries of 0.93, which is closer to what the model captures.

figure. We plot the spread schedule as a function of next period's debt level $a'/(R-\delta)$ relative to mean output for two levels of the shocks z, the mean $\bar{z}$ and a lower level z_L, which is 1.6 standard deviations below the mean, and for no partial default $d=0$. We also plot the schedule for z_L and $d=1$. Recall that with long-term debt and accumulation of defaulted coupons, future debt due a' depends not only on borrowing b but also on today's debt due a and partial default d, with $a' = \delta a + b + (R-\delta)\kappa da$.[27] The figure shows that spreads increase with next period's debt because expected partial default increases in debt. The figure also shows that spreads are higher when output is low today, as this indicates a higher likelihood of low output in the future and hence of higher partial default. Finally, the spread schedule also depends on the partial default decision today d, the reason being that higher d induces a larger output loss next period and hence the possibility of higher partial default in the future. The direct output loss from partial default in the future, however, has only a very small impact on spreads, which is an indication of the small estimated output costs from default.

The middle panel in Figure 3 exhibits the partial default decision $d(a, y, z)$ as a function of the debt level relative to mean output for the same two shocks and for current output equal to the level of the shock, $y = z$. The figure shows that partial default increases with the level of debt and decreases with the level of output. When debt is small enough, partial default can be zero; as debt increases, partial default increases; and when debt is large enough, default is total and equal to 1. Comparing the two curves also illustrates that partial default decreases monotonically with output, given debt. Our model generalizes the results from models of full default in which default is more likely with high debt and low output (Arellano (2008)) to the case of partial default. In our model, not only is positive partial default more likely with high debt and low output, but also the intensity of partial default increases with high debt and low output.

The default policy rule suggests a direct impact of debt on default intensity, which tends to generate a positive correlation between these two variables. From the spread function, since the influence of default via the penalty is weak, the correlation between default intensity and spread must reflect the two common factors, output shocks and debt, driving a positive comovement between the two variables. These policy rules, however, do not account for the feedback effect of partial default on the accumulation of defaulted coupons and hence spreads, which, as we describe below, is important for the time series comovements.

We consider next the policy rules for the portfolio decision of borrowing and partial default. As discussed above, in our model the country can transfer future resources to the present by

[27] The spread schedules with $d=0$ and $d=1$ in the figure have different underlying legacy debt δa and borrowing b because we condition on the same debt due tomorrow a' across schedules.

borrowing or by partially defaulting. The optimal portfolio mix for borrowing and partial default changes with debt and income. The right panel in Figure 3 plots the total resources raised with borrowing $(qb)(a, y, z)$ and with partial default $ad(a, y, z)$ (relative to mean output) as a function of debt a for z_L and $y = z_L$. In constructing the total resources raised, we use the borrowing policy rule, $b(a, y, z)$ and also evaluate the price function with the policy rules, so that $(qb)(a, y, z) \equiv q(a'(a, y, z), d(a, y, z), z)b(a, y, z)$. The figure shows that when debt is low, the country uses only borrowing to raise external resources. As debt rises, the portfolio shifts toward partial default, with borrowing declining sharply and partial default rising toward 100%. As seen in the portfolio condition (20), partial default is more attractive when debt is high and spreads are high and steep. States with high debt due a are associated with these spread properties because of higher default risk due to elevated debt due tomorrow, $a' = \delta a + (R - \delta)\kappa da + b$. The policy rules for the optimal portfolio mix suggest that during default episodes, borrowing and default will move in opposite directions as long as debt accumulates within those episodes.

In the online Appendix, we plot impulse response functions of the variables of interest to the z shock. A low z shock increases partial default and spreads in line with the decision rules shown above. The dynamics of debt are much more persistent and hump-shaped, as defaulted coupons accumulate and new borrowing increases.

4.4 Partial Default Bins

We now highlight the quantitative implications of our model when examined against the data for periods when the economy is in partial default. We show that the model can replicate the empirical comovement of partial default with spreads, debt, and output documented for emerging markets in Section 2. We construct these statistics from the long simulation.

We first analyze the mean debt to output, spread, and output across bins based on partial default. As in Section 2, we partition the limiting distribution based on partial default into four bins. The no-default bin corresponds to the observations with zero partial default (recall that they amount to about 64% of the observations in the model and data). The bins labeled small, medium, and large correspond to periods with values for positive partial default in percentiles (0-25), (25-75), and (75-100). In Table 5, we report the averages of partial default, debt to output, spreads, and output across these partial default bins in both the data and the model.

The distributions of partial default in the model and data have a wide range, although the distribution in the model is a bit narrower. The table also shows that debt to output increases with partial default. In the model, during small defaults this ratio is about 15% higher than in the no default case, while this difference is 9% in the data. Debt to output increases further for periods with larger defaults from about 33% to 82% in the model and from 33% to 60% in the data. Spreads also tend to increase with partial default. As explained above, our model produces a lower mean spread; the comovement of spreads with partial default, however, resembles that in the data. In the data, spreads during large defaults are about 12% higher than spreads during small defaults, while they are about 7% higher in the model. Output also correlates with partial default; it tends to be more depressed as partial default rises both in the model and in the data, although this relation is stronger in the model. The comovements of spreads, debt to output, and output with partial default reflect the response of these variables to shocks and the dynamics of debt illustrated with the decision rules and the impulse response functions analyzed above.

4.5 Default Episodes

Our model has implications not only for the conditions that lead countries to partially default but also for the properties of default episodes. We now explore the model's quantitative implications for the length of default episodes and the resulting haircuts and maturity extensions. We also document the dynamics of partial default, debt to output, and output during default episodes. We compare them with the data on emerging markets and find that our model fits many of these untargeted moments well.

Recall that default episodes in the model and in the data correspond to sequences of periods of uninterrupted positive partial default, $d > 0$, that are preceded and followed by at least one period with zero default, $d = 0$. As we report in Table 6, default episodes tend to last many years, with an average length of eight years in the model, close to the nine years in the data. A high fraction of these episodes, however, are short, lasting two years or fewer, both in the model and in the data. The model and data feature large heterogeneity in default episode length, as reflected by the coefficient of variation of 1.5 and 1.1, respectively. We also compare haircuts and maturity extensions. Our model generates an average haircut of 37% and a maturity extension of seven years with the restructured debt. These moments are very close to the data counterparts of 36% and six years.[28] Finally, in terms of correlations, the bottom of Table 6 shows that in our model, longer default episodes feature higher levels of partial default, just as in the data. Our model's good fit against these untargeted default episode properties provides additional validation of our model's mechanisms and constitutes an important quantitative success.

We now document the dynamics of the variables of interest during default episodes. Figure 4

[28] In the online Appendix, we also report empirical haircuts using our dataset, accounting framework, and estimated parameter values. We find a median haircut of 33%, which resembles our baseline statistic.

plots the dynamics for the average default episode in the model, which lasts eight years. The typical default episode starts when the economy has sufficient debt and is hit by a low z shock. As long as the economy continues in recession, it remains in the default episode with positive partial default. Debt increases during the episode because of the accumulation of default coupons and new borrowing. Credit is restricted during the default episode, as attested to by the elevated spreads. When output recovers sufficiently, in period 9 of the figure, the economy exits the default episode by choosing zero partial default. In our model these dynamics depend on the duration and severity of the recession and the degree of debt accumulation during the recession. Contrary to standard theory, and consistent with the data, default episodes in our model do not lead to a reduction in debt.[29]

In Table 6, we report some moments that summarize these dynamics and compare them with the data by following the procedure described in Table 1. Recall that the label *before* corresponds to the period before the beginning of the default episode, *beginning* is the first period of the episode, *middle* corresponds to the period at the midpoint of the episode, and *end* corresponds to the period after the episode with zero partial default. For each episode in our simulation, we compute the relevant values and report in the table the average across the episodes.

As Table 6 shows, on average default episodes start with partial defaults of 21% and 22% in the model and in the data, respectively, which grow over the episode to 28% and 33% in both the model and the data, respectively. The dynamics of debt during default episodes in the model and data are hump-shaped. In the model, debt to output in the period before the default episode is equal to 32%. At the start of the default episode, debt rises to 35% and continues to grow. In the middle of the episode, debt is 44%, and toward the end of the episode, it falls to 42%. In the model and in the data, default episodes do not lead to reductions in debt, and debt ratios continue to rise during the default episode. The table reports the dynamics of output relative to the beginning of the default episode as U-shaped in the model and in the data. The episode starts with a contraction in output; it continues to decline during the episode, as seen by the larger contraction in the middle of the episode. The episode ends when output recovers. The magnitude of the output dynamics is more accentuated in the model than in the data.

The dynamics of default episodes illustrate the propagation and amplification mechanisms in our model. Adverse shocks tighten spread schedules, making it more costly to roll over the debt, and generating the start of a default episode when the economy is sufficiently indebted. Partial default increases to alleviate the decline in output and tight financial conditions. Borrowing also

[29]The average paths shown in Figure 4 mask the fact that in the event of large defaults, borrowing declines and spreads rise sharply. This property helps reconcile our model with the empirical findings in Cruces and Trebesch (2013) and Asonuma and Trebesch (2016) about credit access and defaults.

expands moderately at high spreads. The rise in partial default and borrowing both increases debt and creates a dynamic amplification during the default episode. As debt stays elevated, partial default and spreads remain persistently high, lengthening the default episode. These dynamics reverse when output recovers, the economy sufficiently lowers its debt, and the spread schedule is more favorable. The recovery, combined with more ample credit, leads to an end to the default episode.

These dynamics give a distinct narrative of default episodes in emerging markets to that of standard theory. In standard theory, the default episode is a state of impasse without capital flowing from the country to its lenders. The default episode ends either exogenously or with a renegotiation, after which the country gets a fresh start with lower (or zero) debt. In our theory, the default episode features dynamics with amplification. The country starts the default episode with a partial default when it is in a recession and when credit is restricted by a steep spread schedule. During the episode, the country continues to pay part of its debt and continues to borrow at increasingly high interest rates while its defaulted coupons are accumulating. These choices increase the debt during the episode, putting the country in a more vulnerable financial position with even tighter spread schedules. These conditions result in larger partial defaults. This situation continues and worsens while the country is in a recession. As the economy begins to recover and debt starts to fall, the spread schedule becomes more relaxed. These improved conditions allow the country to diminish its partial default. The default episode ends when output has recovered sufficiently and debt has declined.[30] In our theory, market forces embedded in the bond price function matter for the resolution of defaults. In the final section of the paper, we will make a direct comparison of our model with the reference model, but first we analyze counterfactuals in our baseline model.

5 Counterfactuals

We now conduct counterfactuals and gauge policies aimed at improving the resolution of defaults and debt sustainability. This exercise is especially useful in our partial default model because it features rich default dynamics that resemble the data. We compare our baseline with three alternative contractual arrangements for debt. The first counterfactual considers an economy in which the country does not have access to international markets when default is positive. As we

[30] In the online Appendix, we provide results for our baseline model with the restriction that default is binary, $d = \{0, 1\}$. The binary default model cannot deliver the empirical partial default distribution, but it can deliver a sizable length of episodes—which illustrates that accumulation of defaulted coupons and market access during default are important elements of our theory for understanding the properties of default episodes. As we show below, these are dimensions that the reference bargaining model does not deliver, as it lacks these crucial elements.

will see, such an environment is similar to one with more stringent pari passu clauses in bond contracts. The second counterfactual considers an economy with higher debt relief from default, which results from a parameterization with a lower value for the recovery factor κ. We use this counterfactual to study the debt relief initiative for Highly Indebted Poor Countries. The third counterfactual studies the implications of bond covenants that compensate long-term debt holders from debt dilution. These covenants have been shown to be quite useful in reducing debt crises in models of full default. Our results suggest that pari passu clauses and no-dilution covenants lead to shorter default episodes, lower default frequency, and lower haircuts from default episodes. Welfare for the sovereign is higher in these economies when debt is low. An economy with higher debt relief, in contrast, features higher haircuts, lower debt-to-output ratios, and has higher welfare in states of high debt.

5.1 No Market Access during Default: Pari Passu

An important feature of our model is that the economy continues to have access to financial markets and to pay part of its debt during default episodes. As we have seen, bond markets are endogenously restricted in our model because of elevated default risk during periods with positive partial default, but modest levels of new bonds continue to be issued. We now analyze a counterfactual economy in which the sovereign cannot borrow when partial default is positive. This experiment can be interpreted as adding more stringent pari passu clauses to the bond contracts because it eliminates our model's feature that bonds issued later in the default episode have better "settlements" because they carry lower bond-specific haircuts.

As explained in Olivares-Caminal (2013), bonds with pari passu clauses, sometimes called *most-favored creditor clauses*, stipulate that "during default episodes, if subsequent settlements have better terms, those terms will also be extended to the previously exchanged bonds." In one sense all bonds in our model are treated equally: in any period, all coupon payments due for all bonds are collected in a and are treated the same, as d is applied to a, so there is pari passu in coupon payments. Nevertheless, if one were to construct bond-specific haircuts, these would differ across vintages of bonds issued at different points in time during a default episode. Haircuts on bonds issued later in the episode are smaller than the haircuts on the original legacy debt the economy held at the beginning of the episode, because compared with bonds issued earlier, the later issuances experience fewer periods with positive partial default.[31] Such instances

[31] For example, average haircuts in the benchmark model during default episodes of eight years are 42% for the legacy debt outstanding at the beginning of the episode and 30% for bond issues during the last year of the episode.

of differential treatment across bonds during default episodes can be interpreted as a violation of pari passu clauses in bond markets. Therefore, by eliminating these issuances, we ameliorate pari passu concerns.

In the first two columns of Table 7 we compare the results from an economy with no market access during default to our benchmark model. The effects on default episodes are sizable. Default episodes become much shorter, lasting on average two years, and are more homogeneous. Not having access to international markets adds extra costs from maintaining positive partial default and limits the rise in debt during the episode. Both these factors encourage faster exit. Shorter episodes then lead to smaller haircuts and shorter maturity extensions.

The effects on the distribution of partial default are also significant. Without market access, partial defaults are less frequent, higher on average, and more homogeneous, as reflected by the lower standard deviation. Also in this economy, debt to output is lower, and the standard deviation of spreads is lower.

The bottom panel of the table reports consumption equivalent welfare for the sovereign relative to that in the baseline economy. These implications should be interpreted as differences in welfare for a sovereign that operates in different economies. With pari passu welfare tends to be higher when the economy has low debt or does not partially default, and it tends to be lower when the economy has high debt or partially defaults. For example, for the z_L shock, welfare is about 0.12% higher with zero debt and about 0.07% lower with a high debt of about 64% of output. When considering the overall effects, measured by the average welfare gain or loss using the stationary debt distribution of the baseline, we see that pari passu yields gains. For lenders, their ex-ante profits are always zero, but interestingly, we also find that the ex-post value given by Eq. (15) is higher with pari passu across the state space.

5.2 Larger Debt Relief: HIPC Initiative

Another important feature of our model is that the defaulted debt does not dissipate after default but instead accumulates, with a fraction κ of the defaulted debt coming due in the future. We now explore the implications of lowering the recovery factor κ from 0.70 in the benchmark to 0.60.

Since 1996, the International Monetary Fund and the World Bank have promoted the Heavily Indebted Poor Countries (HIPC) debt relief initiative. Countries that qualify for this program experience debt relief from multilateral institutions and bilateral creditors. As described in Arslanalp and Henry (2006), the purpose of this initiative was to bring debt to sustainable levels and free up resources for spending and growth. While our model abstracts from investment, it features partial defaults with associated resource costs and allows for welfare evaluations. In this context, our emphasis is on evaluating the effect of debt relief on debt levels, the incidence of default, and welfare. Although many countries that have qualified for the HIPC Initiative have levels of output per capita below those in emerging markets, a few of the countries in the initiative are quite similar to emerging markets.[32] Honduras, Bolivia, and Guyana, for example, which are part of the HIPC initiative, have output per capita ranging from \$5,000 to \$9,000 PPP U.S. dollars, levels similar to those of Nigeria, El Salvador, and the Philippines, which are emerging markets that are not part of the initiative. We interpret, therefore, this experiment as evaluating the effects of debt relief for these borderline countries.

The third column of Table 7 shows the results of the experiment in which we lower κ while keeping all other parameters at their benchmark values. Lowering the recovery factor increases haircuts from default episodes but does not substantially change the length or the heterogeneity of default episodes. The properties of partial default are similar to those in the baseline, but the levels of debt and the volatility of spreads are lower with higher debt relief. With higher haircuts, borrowing becomes more restricted because creditors adjust the bond price schedule to avoid losses. These tighter schedules result in lower levels of debt. In terms of the sovereign's welfare, higher debt relief increases welfare in high debt states but decreases welfare in low debt states. Debt relief also leads to lower welfare on average, both overall and across states with positive partial default or zero partial default. The tight debt schedules are costly and explain these welfare losses. Debt relief also lowers ex-post values for lenders because of the higher haircuts.[33]

These results illustrate that higher debt relief lowers debt sustainability levels because creditors respond with tighter bond price schedules, leaving equilibrium mean spreads and partial default unchanged. In practice, the HIPC Initiative has had mixed results in terms of alleviating the debt burden of countries, as argued in Arslanalp and Henry (2006). Our counterfactual illustrates that general equilibrium forces respond to these policies and offset the potential benefits.

[32] Most of the HIPC countries—for example, Burundi, Niger, Mozambique, Sierra Leone, Madagascar, Togo, Guinea Bissau, and Burkina Faso—have output per capita of less than \$2,000 PPP U.S. dollars in 2018.

[33] These welfare implications could also be interpreted as resulting from a one-time switch from the baseline to each of the counterfactuals, but any normative prescription would depend on the Pareto weights for the sovereign and its lenders. For example, lenders lose 18% of their value when switching from the baseline to an economy with higher debt relief when debt is 64% and productivity z_L.

5.3 No-Dilution Covenant

The final counterfactual analyzes the impact of no-dilution covenants on default episodes. Chatterjee and Eyigungor (2015) and Hatchondo, Martinez, and Sosa-Padilla (2016) have shown no-dilution covenants to be powerful instruments in reducing the incidence of default, in some calibrated examples by about 80%. We analyze them in our model of rich default episodes.

We adopt the specification of Hatchondo, Martinez, and Sosa-Padilla (2016). The covenant specifies that the sovereign has to pay $\mathcal{C}$ to the holder of previously issued long-term bonds to compensate for diluting the value of the debt. The payment is the difference between the counterfactual bond price that would have been observed without the extra borrowing or accumulated defaulted coupons, $q(\delta a, d, z)$, and the observed bond price $q(a', d, z)$: $\mathcal{C}(a, a', d, z) = \max\{q(\delta a, 0, z) - q(a', d, z), 0\}$. The partial default decision applies equally to the coupon payment and the covenant payment, and the value of lenders is modified to include the covenant payment.[34]

The last column of Table 7 presents the results of the experiment. No-dilution covenants have only moderate effects on default episodes. The covenants reduce the length of episodes to six years but do not substantially change the haircuts or the maturity extensions from default episodes, nor do they change the time series properties of partial default. The frequency, mean, and standard deviation of partial default in this experiment are very similar to those in the baseline. This result contrasts with the previous literature, which argues that default frequencies decrease very significantly with no-dilution covenants. Debt levels and spread volatilities, however, are reduced with no-dilution covenants, consistent with this literature. In terms of welfare, no-dilution covenants tend to increase welfare when debt is low and when the economy does not partially default, but they tend to reduce welfare when debt is high and the economy is partially defaulting.[35] Overall, welfare is a bit lower with no-dilution covenants. The reason for the reduction in welfare is that no-dilution payments tend to be especially high during default episodes with high debt, because these are the cases in which dilution is most severe.

Another way to summarize the welfare implications from these counterfactuals is by means of Figure 5, in which we plot as a function of debt the consumption equivalent welfare, relative to the baseline, for the low output shock z_L when $y = z_L$. As discussed above, the welfare rankings of

[34] The sovereign's budget constraint is $c = y - (1-d)\ a(1 + \delta\mathcal{C}(a, a', d, z)) + q(a', d, z)\ b$, and the value of lenders is $H(a, y, z) = (1-d)\,(1 + \delta\mathcal{C}(a, a', d, z)) + 1/R\ \ [\delta + (R-\delta)\ \kappa\ d] \sum_{z'} \pi(z', z)\ H\,(a', z'\Psi(d, z'), z')\,$, where $a' = a'(a, y, z)$ and $d = d(a, y, z)$.

[35] The ex-post value for lenders also tends to be non-monotonic; relative to that of the baseline model, it is higher with no-dilution covenants when debt is low and lower when debt is high, in excess of 60% of output.

these counterfactuals depend on how indebted the economy is. As found in previous literature, for lower levels of debt, pari passu clauses and no-dilution covenants are preferred because they effectively improve bond price schedules. For high levels of debt, however, those clauses worsen the welfare of the economy. These higher debt level states are relevant in our model because, unlike in previous literature with full default, the economy in our model remains with high debt during default episodes and therefore experiences in equilibrium these welfare costs.

6 Reference Model with Bargaining

The model used so far has proven helpful in addressing the observed facts about partial defaults. However, there are other models in the literature based on debt renegotiation after a full default that could, in principle, also speak to the variables we study here. In those models, following a non-payment, the sovereign does not have access to credit. However, a successful renegotiation may yield a debt reduction, and a correspondingly lower repayment, that can be interpreted as partially defaulted payments. We thus consider a renegotiation model as a reference and compare its implications with those of the partial default model.

We posit a model with the main elements that characterize state-of-the-art work in the literature. Asonuma and Joo (2020) and Benjamin and Wright (2013) have renegotiation over short-term debt, while Dvorkin et al. (2021) have long-term debt and renegotiation over both debt resizing and maturity. To make the model comparable with our partial default model, we construct a bargaining model with long-term debt and negotiation over debt reduction. We give the details of our version of the bargaining model in Appendix B. In brief, the sovereign decides whether to default on the totality of the bond payment, and if it defaults, it triggers a default episode that ends with a successful renegotiation. During the default episode, the sovereign suffers an output loss—driven by the same penalty function as in the partial-default model—and neither pays coupons nor borrows. The sovereign and the lenders make alternating offers on a new debt contract, with the sovereign proposing with a constant probability π. The proposed new debt contract has the same maturity structure as the old debt but a different coupon payment. The new contract terms may imply repayments that are less than the original coupon, and the default episode ends when a proposal is accepted. Therefore, this model can, in principle, generate episodes of varying length as well as defaults that are partial in the periods of a negotiated settlement.

To assess the quantitative performance of this renegotiation reference model, we again use a minimum distance estimator to choose parameter values. Both the set of parameters to be estimated and the set of moments to match are similar to those used for the partial default baseline, except for the fact that two parameters of the partial default model, γ and κ, are not present, and the probability of the sovereign making the offer, π, must be added. All in all, we estimate 8 parameters and target 11 moments, as discussed in Appendix B.

Table 8 compares the results from this renegotiation reference model with those from our baseline partial default model as well as with the data. We see that the moments that pertain to the debt to output, the debt service to output, the debt due to output and the spreads are reasonably well replicated by both the baseline and the renegotiation models. However, those that relate to partial default are not. Partial defaults are very rare in the renegotiation model, and their implied size is very large, twice the size of those in the data or the baseline model.

Default episodes in the renegotiation model are short and homogeneous, in contrast with the long and heterogeneous episodes in the baseline model and in the data. The renegotiation model features default episodes that typically last about two years, because there are hardly any delays in debt restructuring, and two years is the minimum effective number of periods before the sovereign can complete the renegotiation. The episodes in this case start with a full default on the coupon and end the following period with a renegotiation that tends to give a partial default during that period.[36] The model lacks meaningfully long delays, because by renegotiating and settling right away, the country and its lenders can avoid the deadweight costs of default and share a positive surplus.

This finding is consistent with the short duration of default episodes found in the renegotiation literature—for instance, the 2.4 and 5.8 quarters in Asonuma and Trebesch (2016), the 4 to 6 quarters in Asonuma and Joo (2020), or the 2.3 years in Dvorkin et al. (2021). The models in these papers feature additional elements, such as risk-averse lenders, book-value considerations for lenders, extended exclusion periods after renegotiations, and no-default costs if income is below a threshold. These elements are introduced in those papers to explore additional mechanisms for maturity extensions and haircuts but may also affect the length of default episodes.[37]

An exception in the literature regarding the size of delays is Benjamin and Wright (2013), which obtains default episodes that, while still shorter than those in our data, are substantially longer than those in the other papers. This paper exploits an intricate stochastic structure for bargaining probabilities that co-vary with the country's income, which adds a number of parameters to the

[36] In fact, out of 675,000 simulated periods, we had 9,801 default episodes with 98.99% of them lasting two years, 0.88% lasting three years, and only 0.11% lasting four years.

[37] We have also re-calibrated our renegotiation model under the restriction made in this literature that the sovereign experiences no default losses when productivity is below the threshold z^* (such that $\phi_0 = 0$). The average episode length increases slightly to 2.5 years, which, interestingly, is nearly the same as in Dvorkin et al. (2021). The fit of the model is poorer than in the reported unconstrained case. We provide further details in Appendix B.

estimation. It would be interesting to confront this model with the additional empirical moments of partial default and evaluate its ability to generate the distribution of short and long default episodes.

As also shown in Table 8, haircuts in the renegotiation model are larger than those in the data, and the maturity extensions are much shorter. The correlation between episode length and partial default is also close to zero. In sum, we find that the structure of this model is too rigid to be able to fit these data closely in a moment-matching exercise. As found in previous literature, the model does generate a debt-to-output ratio and a debt service to output like those in the data, and also delivers high but still plausible volatile sovereign spreads.

In Figure 6, we plot the dynamics of the variables of interest for the typical default episode in our baseline model and in the renegotiation reference model. The left panel shows the time path of partial default. The default episode starts in period 1 with a full default; in period 2 the country renegotiates and pays its lenders the corresponding coupon, resulting in a partial default of about one-half, and by period 3 the episode has ended. These short paths differ sharply from those in our partial default model, which gives longer and smoother episodes. The middle panel shows that debt to output is cut in half in the renegotiations, which contrasts with our baseline model, in which debt is not reduced. Finally, the right panel shows the consumption dynamics. The renegotiation model features depressed consumption during the default episode, owing to a reduced output, and a consumption boom upon completion of the renegotiation. This consumption boom upon exit arises because of a quick recovery in output and also because the country's debt is being reduced significantly, affording it a "fresh start" that allows for large borrowing. In contrast, in our baseline model consumption increases only mildly and is close to trend as the default episode ends.

In summary, we find that the renegotiation reference model, while successful in delivering haircuts comparable with those in the data, is at odds in several dimensions. Its main shortcomings are the very short default episodes it delivers and the failure to fit the moments of partial default with respect to its frequency, mean, and standard deviation. An additional discrepancy with the data is the sharp reduction in debt upon completion of the default episode, which gives rise to a consumption boom.

7 Conclusion

We have developed a theory of sovereign partial default and confronted it with 50 years of emerging market data. In our model, partial default is a flexible way to intertemporally transfer resources in addition to standard borrowing. Partial default amplifies debt crises because defaulted payments accumulate and new borrowing occurs at increasingly high interest rates; both of these factors increase future indebtedness. By combining it with an accounting framework, we show that our model is successful in replicating the data. In the model and in the data, partial default is positive about one-third of the time, has a mean of one-third, yet it has a large variance. Importantly, as in the data, default episodes in the model are long, feature hump-shaped patterns for debt and partial default, and do not reduce indebtedness. In our work, defaulted debt accumulates at a given rate; we leave for future work bringing renegotiation elements into our partial default theory to provide an endogenous determination of this rate.

Our work challenges the common view of defaults as periods of impasse, with no repayments nor borrowings, and that default episodes lead to a reduction in debt. In addressing the evidence, we have presented a tractable macro approach that analyzes the totality of the sovereign's debt obligations, paid and unpaid, and borrowings. An important next step will be understanding the micro details of these choices. In recent work, Schleg, Trebesch, and Wright (2019) show evidence of systematic patterns of repayment depending on the creditor type, whether sovereign, private bond and loans, trade credit, or multilateral. Chatterjee and Eyigungor (2015) argue in turn that the maturity of the bonds plays important roles for seniority. We think that further study of the details behind partial defaults and new borrowings during episodes is important for the literature of sovereign debt.

A Default Episodes in Russia and Argentina

This appendix discusses some of the institutional and empirical background of two default episodes: Russia from 1992 to 2012, and Argentina from 2001 to 2019. The details of these two episodes are different, yet they share some properties. Defaults are partial and lengthy, and they extend many credit market instruments. Many partial defaults go through several restructuring rounds and involve haircuts to creditors. During these defaults the sovereigns of these countries borrow both in the form of new loans, unrelated to the partial default, and as part of the restructuring agreement. These features resemble those in our theoretical model.

A.1 Russian Episode

According to our accounting framework, Russia experienced one default episode from 1992 to 2012—these are the years that we find that Russia has positive partial default. This period was volatile for Russian external debt. It included the Russian crisis of 1998 and multiple restructurings with official lenders, banks, bond holders, and trade creditors. Our reading of this episode is that the culprit of the debt problem was a failure to bring fiscal deficits under control and a rapid expansion of debt. But the details of the debt crisis are complicated and involve many actors. In a fascinating article, Santos (2003) documents in rich detail the different aspects of the debt crisis. Next, we draw upon the findings in that paper to summarize the main points of interest.

Partial Defaults and Restructurings. During this period, Russia restructured most of its debt through many separate agreements. Russia held debt from private creditors, including Eurobonds, MinFin bonds, and London Club loans; official creditors, including Paris Club loans, COMECON loans (intergovernmental obligations to eight countries), and others; and multilateral creditors, including the IMF. Santos (2003) documents that in 1996, for example, 38% of the debt was with private creditors, 52% was with official creditors, and 11% was with multilateral creditors.

Starting in 1991, Russian economic conditions were declining as the country transitioned toward a market economy. Fiscal policy was expansionary, as the government wanted to play a more active role while having arbitrary tax enforcement. The result was that the bulk of the interest payments on external debt was partially defaulted on, and the partial default resulted in an accumulation of arrears or restructuring agreements.

Russia received multiple Paris Club reschedulings, including those in 1993, 1994, 1995, 1996, and 1999. After years in arrears with the London Club, Russia also rescheduled credits with the London

Club in 1997 and 2000. In the 2000 program, the loans were exchanged for new Eurobonds. Russia also rescheduled with COMECON, mainly with Germany and the Czech Republic, in 1994 and again in 2002. Russia also owed obligations to uninsured suppliers in the form of trade credit, which were restructured in 1994 and then again in 2001, on terms similar to those of the London Club deal. Finally, during 1999, Russia restructured their external IANs, PRINs, and MinFin II bonds. As documented by Sturzenegger and Zettelmeyer (2008), these bonds were exchanged with new Eurobond bonds and a cash payment, and resulted in haircuts for creditors ranging from 40% to 60%.

Borrowing during the Episode. The Russian government had substantial external borrowings during the default episode. Using data from Standard & Poor's and J.P. Morgan, Santos (2003) documents external borrowing for the Russian government, which reached $12 billion in 1998 (see Figure 7.1 in that paper). These borrowings took the form of new bonds and loans, and also arose from the restructuring arrangements after periods with arrears and partial defaults.

An example of these borrowings is the November 1996 issue of a sizable Eurobond of $1 billion. Santos (2003) explicitly acknowledges this issue was placed "despite the fact that Russia was, at the time, in arrears to the London Club." Russia, therefore, placed a bond in international financial markets while being in partial default. This stark example could be seen as a violation of a pari passu clause, as recently interpreted in New York courts for the case of Argentina; see Olivares-Caminal (2013). More work is needed to measure the extent to which sovereigns have issued private bonds in international financial markets while in arrears with other international private creditors. The many restructured arrangements, however, also generated new financial obligations for Russia, which directly contributed to an increase in debt of about 10% of output for Russia during the episode.

A.2 Argentinian Episode

According to our measurements, Argentina experienced two default episodes. We will discuss here the default episode that started in 2001 and continued until 2019. This default episode started with the well-documented crisis of 2001, with defaults on many bonds and loans, rounds of restructurings in 2005 and 2010, and a complex holdout problem. It also included 19 years of partial default with the Paris Club and some of the Brady bonds, among others. Early in the episode, Argentina borrowed from international financial institutions, sovereigns, and domestic bonds, and it borrowed large numbers of international bonds later in the episode. We draw much of this narrative from Sturzenegger and Zettelmeyer (2008), Hornbeck (2004), and the 18-K Form for the Republic of Argentina reported by the Securities and Exchange Commission (SEC (2020)).

Partial Defaults and Restructurings. Argentina defaulted in 2001 on most of its private bonds, loans to commercial banks, and bilateral loans including with the Paris Club. As documented in Hornbeck (2004), however, before the first restructuring of 2005, about 44% of the $195 billion sovereign debt of Argentina was actually performing, and 4% was non-performing but not included in this restructuring. The performing loans included $33 billion of loans to international financial institutions, such as the IMF and World Bank, and $27 billion of the national government bonds, BODEN. As documented by Sturzenegger and Zettelmeyer (2008), this restructuring consisted of a large bond exchange program that resulted in significant haircuts for creditors of 73% on average and had an acceptance rate of 76% across holders of these bonds in 2005. In 2010, Argentina launched another bond exchange with the holdout creditors, which resolved about half of them. SEC (2020) reports that as of December 2010, Argentina had arrears of about $11 billion with these private creditors, which included both principal and accumulated interest. These debts with holdout creditors were finally resolved in 2016 with litigation, after a court in the United States applied the pari passu clauses that prevented Argentina from paying the restructured 2010 debt payments before settling with the holdout creditors. Throughout this episode Argentina maintained a partial default on loans from the Paris Club, in the amount of $6 billion in 2014, and also on a portion of the early Brady bonds, issued in 1992, which were not part of the exchanges in 2005 and 2010.

Borrowing during the Episode. During the default episode, Argentina received fresh loans from multilateral organizations, other sovereign nations, and private bonds. Datz (2009) also notes that between 2001 and 2005, Argentina issued "Bonar" bonds under Buenos Aires law, with foreign investors making up an estimated 70% of the holders. The Inter-American Development bank gave many loans to Argentina even during 2002; as of December 2019, it had $13 billion of loans with the IDB. The IMF also lent Argentina funds in 2018 through a program that was canceled in 2020. An important lender for Argentina during this period was the Chinese government. According to Horn, Reinhart, and Trebesch (2021), China granted 14 loans between 2006 and 2017 for over $10 billion. Grund (2019) reports Argentina issued many bonds in international sovereign bond markets between 2016 and 2018, which totaled $160 billion.

B Reference Model with Renegotiation

Our reference model with debt renegotiation is based on Asonuma and Joo (2020) and Benjamin and Wright (2013), whose work we extend to long-term debt and map into our accounting framework, and also on Dvorkin et al. (2021), but without their negotiation over maturity. As in our baseline model, the debt contracts are long-term perpetuities with decay rate δ.

In this environment, when the sovereign defaults, it does so on the totality of all current and future coupons associated to the existing long-term debts, cannot borrow any additional funds and, in the following period, gains a bad-standing flag and enters a process of renegotiation with its lenders. During the renegotiation process, based on the realization of an i.i.d. random variable, it is either the sovereign or the lender that makes a take-it-or-leave-it offer of a new level of debt. The sovereign regains a good standing when negotiations reach an agreement. We pose extreme value shocks to smooth the discrete decisions: the sovereign's decision of whether to default and both parties' decision of whether the offer to be made when negotiating is one that will be accepted.

In this framework, the relevant state variables are a for the debt due, $\theta \in \{0, 1\}$ for good or bad standing, z for the endowment, and $\nu \in \{s, \ell\}$ for the identity of the proposer under negotiations.

The Sovereign. Starting in a good standing state $\theta = 0$, if the sovereign does not default, it will continue in good standing next period and can borrow b and consume c, given a budget constraint equivalent to that in the baseline model. The decision problem is

$$\begin{aligned} v^{ND}(a, 0, z, \nu) &= \max_{b} \left\{ u(c) + \beta \sum_{z'} \pi(z', \nu'|z)\ v(a', 0, z', \nu') \right\}, \\ \text{s.t} \qquad & c = z - a + b\ q(b, a, 0, z, \nu), \\ & a' = \delta a + b, \end{aligned}$$

where $q(.)$ denotes the debt-pricing function, and $v(.)$ is the continuation value function. The solution gives the value function $v^{ND}(a, 0, z, \nu)$ and policy rule $b(a, 0, z, \nu)$, which implies a rule for debt due $a' = a(a, 0, z, \nu)$.

When the sovereign in good standing defaults, it does so on the totality of the coupon, loses access to credit so consumption equals output $c = z$, and carries a bad-standing flag into the next period $\theta' = 1$. The continuation debt coupon remains unchanged (without interest) the following period $a' = a$. The value associated with defaulting is therefore given by $v^D(a, 0, z, \nu) =$

$u(z) + \beta \sum_{z',\nu'} \ \pi(z', \nu'|z) \ v(a, 1, z', \nu')$.

The default outcome is determined by the values associated with the two options, $v^{ND}(a, 0, z, \nu)$ and $v^{D}(a, 0, z, \nu)$. We introduce extreme-value shocks to this decision and denote by σ_{EV} the parameter for the scale of volatility of these shocks. The solution yields an ex-ante probability of default $d(a, 0, z, \nu)$ and value function $v(a, 0, z, \nu)$.

When the sovereign is in bad credit standing $\theta = 1$, income is $z\Psi(z)$, with $\Psi(z) \leq 1$, and the budget constraint depends on whether the renegotiation is successful. If it is not successful, consumption remains equal to income, $c = z\Psi(z)$, debt due next period remains $a' = a$, and the credit standing next period stays bad $\theta' = 1$. If the renegotiation is successful, then the sovereign and its lenders agree on a new debt contract with a value of debt due a' in the following period and corresponding current coupon a'/δ, so consumption is $c = z\Psi(z) - a'/\delta$. The new debt due is given by the policy function $a'(a, 1, z, \nu)$, and failure to achieve an agreement, equivalent to the probability of default, is given by the policy rule $d(a, 1, z, \nu)$. These functions are equilibrium objects to be determined under bargaining. A successful renegotiation switches the credit standing to good next period $\theta' = 0$. While in bad standing, the value to the sovereign is thus

$$\begin{aligned} v(a, 1, z, \nu) \quad &= \quad [1 - d(a, 1, z, \nu)] \ \left[u(c) + \beta \sum_{z',\nu'} \ \pi(z', \nu'|z) \ v(a', 0, z', \nu') \right] \\ &\quad + \ d(a, 1, z, \nu) \ \left[u(z\Psi(z)) + \beta \sum_{z',\nu'} \ \pi(z', \nu'|z) \ v(a, 1, z', \nu') \right], \end{aligned}$$

subject to $c = z\Psi(z) - a'_0/\delta$ and $a' = a'(a, 1, z, \nu)$.

Lenders and Bond Prices. Given the value of one unit of debt to the lenders, $H(a, z, \theta, \nu)$, a no-arbitrage condition implies a price function for debt that is traded after either the default or the renegotiation decision is made:

$$q(b, a, \theta', z, \nu) = \frac{1}{R} \sum_{z',\nu'} \ \pi(z', \nu'|z) \ H[a'(\theta'), \theta', z', \nu'],$$

where $a'(\theta') = \begin{cases} \delta a + b, & \text{if} \ \ \theta' = 0, \\ a, & \text{if} \ \ \theta' = 1. \end{cases}$

The lender's continuation value $H(.)$ depends on the equilibrium policy rules $d(.)$ and $a'(.)$ for good and bad standing. These values are:

$$
\begin{aligned}
H(a,0,z,\nu) &= 1-d(a,0,z,\nu)+ [1-d(a,0,z,\nu)] \ \frac{1}{R}\,\delta \sum_{z',\nu'} \pi(z',\nu'|z)\ H(a',0,z',\nu') \\
&+ \quad d(a,0,z,\nu) \ \ \frac{1}{R}\sum_{z',\nu'} \pi(z',\nu'|z)\ H(a,1,z',\nu'), \\
H(a,1,z,\nu) &= [1-d(a,1,z,\nu)]\ \frac{a'/\delta}{a} + [1-d(a,1,z,\nu)]\ \frac{1}{R}\ \frac{a'}{a}\sum\ \ \pi(z',\nu'|z)\ H(a',0,z',\nu') \\
&+ \quad d(a,1,z,\nu)\ \ \frac{1}{R}\ \ \sum \pi(z',\nu'|z)\ H(a,1,z',\nu'),
\end{aligned}
$$

where $a' = a'(a,j,z,\nu)$, for $j \in \{0,1\}$.

The Renegotiation. In the renegotiation stage, $\theta = 1$, the sovereign or the syndicate of lenders makes take-it-or-leave-it offers to resume borrowing in subsequent periods. When the state is $\nu = \ell$, lenders choose whether to make an acceptable offer. When making the offer, they maximize their value, which equals the payment during the renegotiation a'/δ plus the value to the lender of the future debt due a', subject to affording the sovereign a non-negative surplus, which is equivalent to a no-default (ND) outcome:

$$
w^{B,ND}(a,z) = \max_{a'} \left\{ a'/\delta + a'\frac{1}{R}\sum \pi(z',\nu'|z)H(a',0,z',\nu') \right\}
$$

$$
\text{s.t.}\quad u[z\Psi(z) - a'/\delta] + \beta\sum\ \pi(z',\nu'|z)\ v(a',0,z',\nu') \geq u[z\Psi(z)] + \beta\sum\ \pi(z',\nu'|z)\ v(a,1,z',\nu'),
$$

which has solution $a'(a,1,z,\ell)$. If lenders choose not to make an acceptable offer, they cause default (D), and they obtain the value of continuing negotiations over the current debt due a,

$$
w^{B,D}(a,z) \ \ = \ \ 0 + a\frac{1}{R}\sum_{z',\nu'} \pi(z',\nu'|z,\ell)\ H(a,1,z',\nu').
$$

In the maximization between the two options, we assume values are subject to extreme-value shocks with volatility parameter σ^{ℓ}_{EV}. The solution gives the probability of failed negotiation, $d(a,1,z,\nu=\ell)$.

When the state is $\nu = s$, the sovereign chooses whether to make the acceptable offer that maximizes their value, subject to giving lenders a non-negative surplus, or

$$\begin{aligned} v^{B,ND}(a,z) &= \max_{a'} \{u[z\Psi(z) - a'/\delta] + \beta \sum_{z',\nu'} \pi(z',\nu'|z,s)\ v(a',0,z',\nu')\} \\ \text{s.t.} \qquad & a'/\delta + \frac{1}{R} \sum_{z',\nu'} \pi(z',\nu'|z,s)\ a'\ H(a',0,z',\nu') \geq \frac{1}{R} \sum_{z',\nu'} \pi(z',\nu'|z,s)\ a\ H(a,1,z',\nu'), \end{aligned}$$

which gives $a'(a,1,z,s)$ as the solution. If the sovereign chooses not to make an acceptable offer, they obtain the value of continuing in the defaulted state $\theta' = 1$,

$$v^{B,D}(a,z) \;\; = \;\; u[z\Psi(z)] + \beta \sum_{z',\nu'} \pi(z',\nu'|z,s)\ v(a,1,z',\nu').$$

The maximization between the two options is subject to extreme-value shocks of the same scale as those affecting the default choice under good standing σ_{EV}. The solution gives the probability of failed negotiation $d(a,1,z,s)$.

Estimation. The eight parameters, $\Theta^R = \{\phi_0, \phi_1, z^*, \beta, \pi_s, \delta, \rho, \sigma_\eta\}$, are estimated using the same 11 moments from the data as in the estimation of the partial default model. Table A1 displays the parameters obtained. On their part, the size of the extreme-value shocks on the binary choices is set to facilitate convergence and bears practically no effect on the outcomes. Further details, including an alternative strategy, are in the online Appendix.

C Data and Code Availability

We have created a repository with the files for replication of our empirical findings and quantitative model results at the Harvard Dataverse; it can be accessed through Arellano, Mateos-Planas, and Ríos-Rull (2022). The repository contains folders with the files needed to compute the benchmark partial default model as well as the reference model with renegotiation. It also contains the files used to calculate the empirical patterns we document in the data. We also include files with detailed instructions on how to produce all the results of the paper.

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Table 1: Partial Default and Default Episodes in Percentages

Partial Default			
Frequency			36
Mean $\mid$ partial default >0			38
Standard deviation $\mid$ partial default >0			22
Default Episodes			
Episode length (years)			9
Fraction of short episodes ($\leq$ 2 years)			36
Haircut (%)			36
Maturity extension (years)			6
Default Episodes Dynamics	Partial Default	Debt	Output
Before episode	0	32	0
Beginning of episode	22	34	-2
Middle of episode	33	40	-5
After episode	0	33	-3

Note: Partial default is given by the definition in equation (4). The standard deviation of partial default is the average time series statistic across the 37 emerging countries, conditioning on positive partial default. The episode length is the average across the 70 default episodes in the data. The dynamics of the default episode are averages across episodes of the variables of interest. "Before" is the period before the start of the episode; "beginning" is the first period of the episode; "middle" is the midpoint of the episode; "end" is the period when partial default returns to zero. Debt is reported relative to output; output is logged and linearly detrended and reported relative to the level before the episode. Estimates for haircuts and maturity extensions are from Cruces and Trebesch (2013), Meyer, Reinhart, and Trebesch (2018), and Fang, Schumacher, and Trebesch (2016).

Table 2: Partial Default Bins: Spreads, Debt, and Output

		Partial default > 0		
Means (%)	No default	Small $(0-25\%)$	Medium $(25-75\%)$	Large $(25-75\%)$
Partial default	0	3	28	91
Spreads	4	6	8	18
Debt to output	24	33	43	60
Output	1	-1	-2	-4

Notes: The statistics are means of the variables in the first column after partitioning the panel data set across four bins based on partial default. The no-default bin has all the observations with zero partial default; the small bin has the observations in the first quartile of positive partial default; the medium bin has the observations in the second and third quartiles; the large bin has the observations in the top quartile. Output is logged and linearly detrended. See Section 2.2 for more details on the data construction.

Table 3: Estimated Parameter Values

Default costs	$\phi_0 = 0.0476$	$\phi_1 = 0.12$
	$\gamma = 2.0$	$z^* = 0.938 \times \bar{z}$
Discount factor	$\beta = 0.954$	
Decay parameter annual	$\delta = 0.88$	
Debt recovery factor	$\kappa = 0.70$	
Shock process	$\rho = 0.875$	$\sigma_\eta = 0.052$

Table 4: Moments in Model and Data

	Data	Model
Target Moments		
Partial default (in %)		
frequency	36	37
mean | partial default >0	38	39
st. dev. | partial default >0	22	19
Debt to output (in %)		
mean	32	32
st. dev.	18	25
Debt service to ouput (in %)		
mean	3.6	3.5
st. dev.	2.1	2.2
Debt due to output mean	4.9	5
Spread st. dev.	4.1	3.7
Output		
persistence	0.89	0.88
st. dev. (in %)	10	12
Other Moments in Panel		
Defaulted coupons to output (in %)		
mean | partial default >0	5.2	4.0
st. dev. | partial default >0	6.4	3.6
Spreads		
mean	5.3	1.6
correlation with output	-17	-38
correlation with debt	24	56
Consumption st. dev. (relative to output)	1.0	0.91

Notes: Debt service, defaulted coupons, debt due, and partial default are defined using our accounting framework as in equations (1) through (4), in the data and the model. In the model, debt and spreads are constructed from equations (6) and (8), while for the data, we use the measures from total external government debt relative to output from IDS and the EMBI+ spread. Output and consumption are logged and linearly detrended. The model statistics are computed from time series simulated data. The standard deviations and correlations from emerging market data are means across countries of the statistics using country time series data. See Section 2.2 for more details on merging the panel data with our accounting framework.

Table 5: Distribution: Partial Default, Spreads, Debt, and Output

	Partial default bins			
Means (%)	No default	Small	Medium	Large
		Data		
Partial default	0	3	28	91
Debt to output	24	33	43	60
Spreads	4	6	8	18
Output	1	-1	-2	-4
		Model		
Partial default	0	20	35	66
Debt to output	18	33	55	82
Spreads	1	1	2	8
Output	6	-11	-10	-18

Notes: The model statistics are computed from time series simulated data partitioned into four bins according to partial default percentiles. The no-default bin has all the observations with zero partial default; the small bin has the observations in the first quartile of the positive partial default; the medium bin has the observations in the second and third quartiles; the large bin has the observations in the top quartile. Output is log and linearly detrended. See notes in Tables 2 and 4 for more details.

Table 6: Default Episodes Properties

	Data	Model
Properties of Episodes		
Mean episode length (years)	9	8
Percentage of short episodes (≤ 2)	36	42
Coefficient of variation for episode length	1.1	1.5
Haircut (%)	36	37
Maturity extension	6	7
Correlation (length, partial default)	26	75
Default Episode Dynamics		
Partial Default		
Before	0	0
Beginning	22	21
Middle	33	28
End	0	0
Output		
Before	0	0
Beginning	-2	-7
Middle	-5	-9
End	-3	3
Debt		
Before	32	32
Beginning	34	35
Middle	40	44
End	33	42

Notes: The model statistics are computed from simulated data during default episodes. See notes in Tables 1 and 4 for definitions of default episode length and its dynamics, as well as partial default, debt, and output. Haircuts and maturity extensions in the model are constructed from equations (9) and (7).

Table 7: Counterfactuals

	Baseline	Pari Passu	Debt Relief	No-Dilution
Default Episodes				
Mean episode length (years)	8	2	8	6
Percentage of short episodes (≤ 2)	42	89	42	41
Coefficient of variation for episode length	1.5	0.8	1.4	1.2
Haircut (%)	37	32	46	36
Maturity extension	7	5	7	7
Correlation (length, partial default)	75	33	76	67
Time series in (%)				
Partial default				
frequency	37	11	35	31
mean	39	44	36	31
st. dev.	19	15	16	10
Debt to output mean	32	27	23	26
Spread st. dev.	3.7	1.1	2.3	0.9
Welfare rel. baseline (% CE)				
No debt, z_L	–	0.12	-0.11	0.07
Debt 64%, z_L	–	-0.07	0.05	-0.20
Partial default$=0$, average	–	0.05	-0.05	0.02
Partial default>0, average	–	-0.01	-0.02	-0.05
Overall Average	–	0.03	-0.04	-0.01

See notes in Tables 4, 5, and 6. Welfare is consumption equivalence measures relative to the baseline model. The average welfare measures are computed using the limiting distribution of the baseline model.

Table 8: Moments in Data and Baseline and Renegotiation Models

	Data	Baseline	Renegotiation
Target Moments (in %)			
Partial default			
frequency	36	37	3
mean	38	39	76
st. dev.	22	19	26
Debt to output			
mean	32	32	30
st. dev.	18	25	14
Debt service to output			
mean	3.6	3.5	4.4
st. dev.	2.1	2.2	1.6
Debt due to output mean	4.9	5.0	4.7
Spread st. dev.	4.1	3.7	6.0
Properties of Episodes			
Mean episode length (years)	9	8	2.01
Percentage of short episodes (≤ 2)	35	42	98.9
Coefficient of variation for episode length	1.1	1.5	0.07
Haircut (%)	36	37	54
Maturity extension	6	7	1
Correlation (length, partial default)	.26	.75	.13

See notes in Tables 4, 5, and 6. The moments targeted are the same for both models. The parameters estimated are modified accordingly. Parameters κ and γ of the baseline are absent in the renegotiation model, while π, the probability that the debt reducing offer is made by the sovereign, is absent in the baseline model.

Table A1: Renegotiation Model - Estimated Parameters

Default costs	$\phi_0 = 0.0995$	$\phi_1 = 0.0979$	$z^* = 0.970 \times \bar{z}$
Discount factor	$\beta = 0.937$		
Decay parameter annual	$\delta = 0.877$		
Sovereign's offer-making probability	$\pi_s = 0.479$		
Shock process	$\rho = 0.865$	$\sigma_\eta = 0.051$	

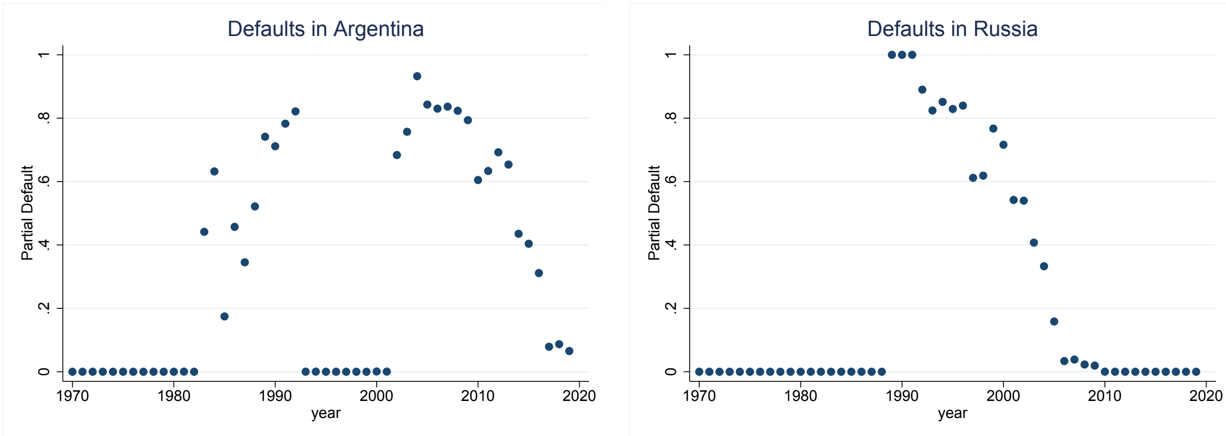
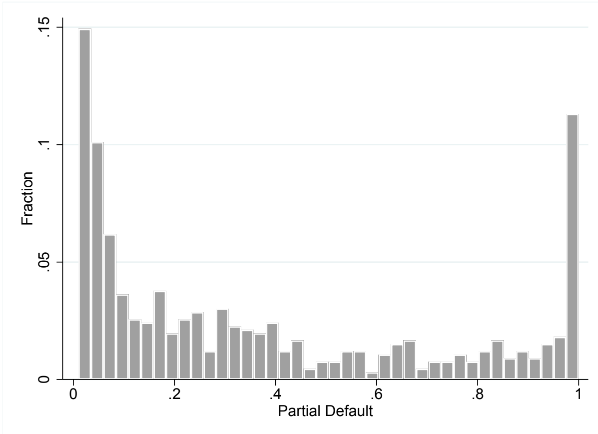
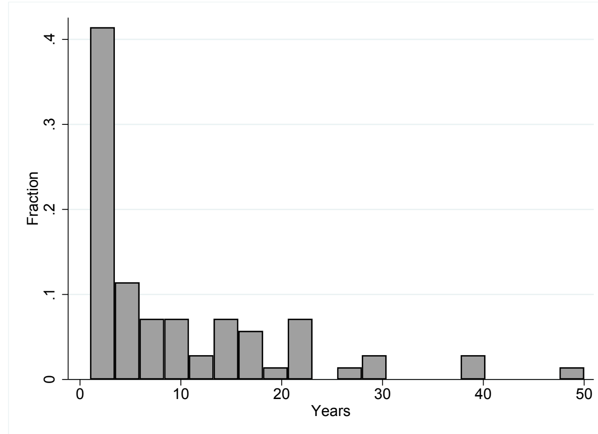


Figure 1: Time Series for Partial Default



(A) Partial Default Histogram (from panel data)



(B) Default Episode Length (from 70 episodes)

Figure 2: Partial Default and Default Episode Length

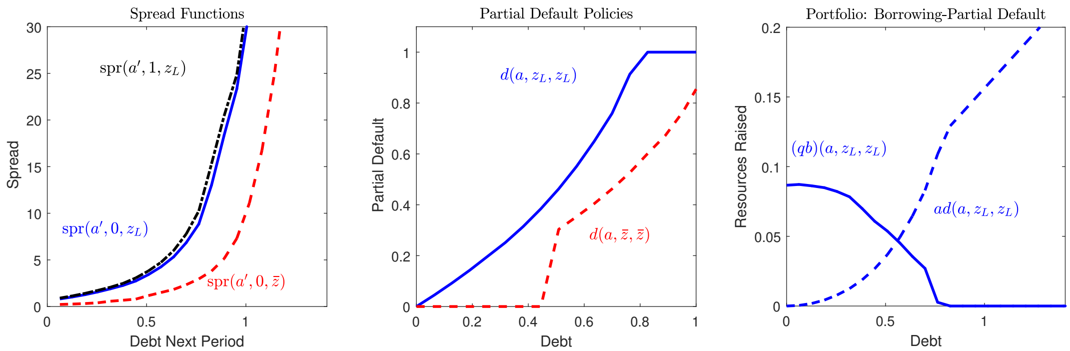


Figure 3: Spreads, Partial Default, and Portfolio

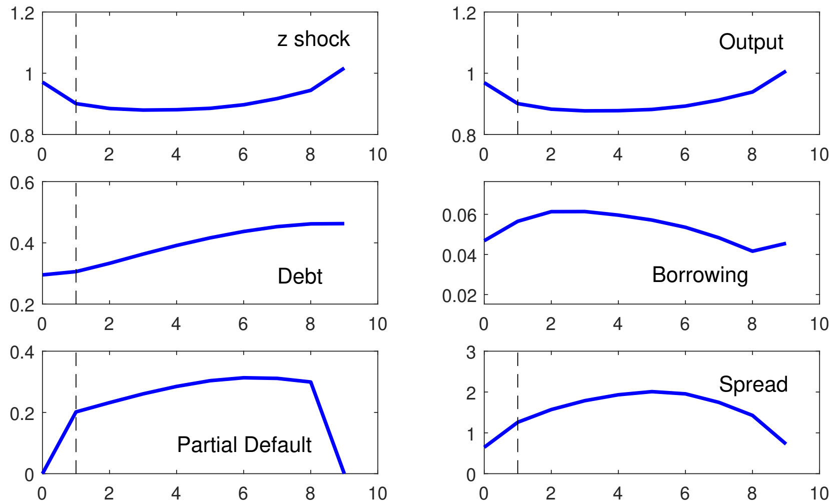


Figure 4: Dynamics for the Average Default Episode of Eight Years

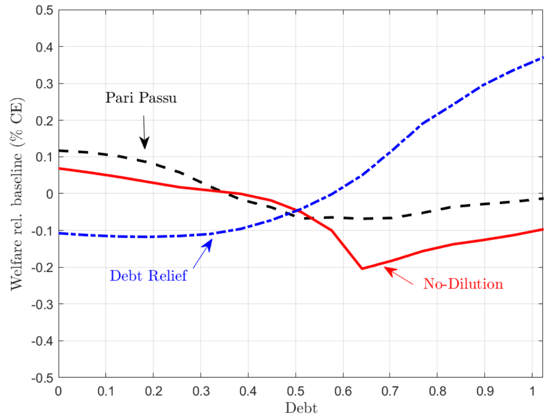


Figure 5: Relative Welfare from Counterfactuals for z_L

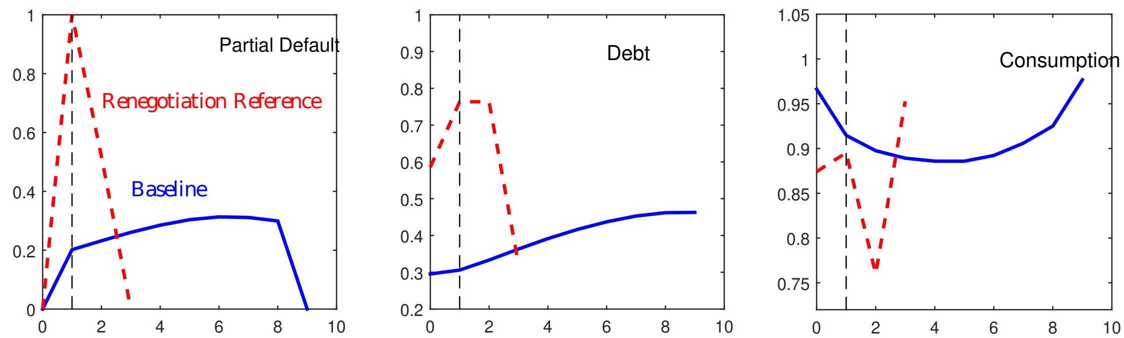


Figure 6: Default Episodes in Baseline and Reference