Effects of Online Reviews and Competition on Quality and Pricing Strategies

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By providing useful product information from the perspective of other buyers, online product/service reviews can help customers better evaluate the true quality of products and judge whether a product is a good fit for them. Based on online reviews, firms may improve product quality and/or adjust selling price to compete with other firms. In this study, we investigate the effects of online reviews on product quality and pricing decisions in a duopoly market that consists of two competing firms. Specifically, we consider a stylized two-period model and study the equilibrium decisions based on a Nash game framework. In the base scenario, the selling prices are exogenously given, and the firms decide on the product quality across two periods to optimize their respective profits. We study the equilibrium decisions in the static setting (in which the product quality remains the same across the two periods) and in the dynamic setting (in which the product quality can be improved in the second period), respectively. In the extended scenario, the selling prices can be endogenously determined as well. For each scenario, we compare the equilibrium decisions under dynamic competition with those under static competition through theoretical analysis and numerical experiments, which uncover some interesting managerial insights regarding the impact of online reviews under competition.

Key words: online reviews; Nash game; quality improvement; optimal pricing

History: This paper was first submitted on June 28, 2020 and has been with the authors for 24 months for 3 revisions.

1 Introduction

In recent years, there has been a continuous shift in customer spending in favor of online shopping. This shift in customer behavior has been accelerated during the COVID-19 pandemic as more business activities are now transitioning to online platforms while physical retail stores have had to close or follow strict social distancing rules. Thus, online retail sales have achieved an astonishing

27.6% global increase to $4.28 trillion in 2020, and are expected to continue this growing trend for the foreseeable future.[1] While online shopping provides convenience and efficiency, it does not allow customers to physically touch or try out products before making a purchase. Hence, sellers usually disclose their product information in the form of text, photos, and videos online. As has been widely recognized, there is normally a significant gap between "seller's show" and "buyer's show" because a product may not fit the personalized need of a particular individual even if it has a high quality. As such, customers tend to use online reviews to acquire additional information and make informed purchasing decisions when they shop online (e.g., Chen and Xie 2008, Kuksov and Xie 2010, Kwark et al. 2014). Note that online reviews provide additional information of a product, also in the form of text, images, and even videos, but from the perspective of other customers who have bought the product. Such information is a useful supplement to the product information provided by sellers. It was reported that "90% of customers read online reviews before visiting a business" and "88% of customers trust online reviews as much as personal recommendations."[2] Deloitte (2007) reveals that 43% of customers change their opinions about which product to buy, and 9% of customers even abandon their purchase plan after reading online reviews. The above data suggest that word-of-mouth (WOM) as reflected by online reviews is essential for many businesses in a competitive e-marketplace (e.g., Kostyra et al. 2016, Liu et al. 2017).

Developing a reputable online WOM profile requires firms to devote substantial efforts and resources, which can be challenging for new market entrants. Among the strategic options available for firms to develop and improve their WOM profiles, product quality is the most important, as a majority of customers disclose quality information when posting product reviews online (Godes 2017). While some firms attempt to establish high-quality levels when they launch new products in e-marketplaces to improve online reviews and stimulate future demand, others consider reviews as a vital ingredient for customer collaboration and co-creation and incorporate them in improving the quality regularly. For example, Xiaomi, a global leading smartphone manufacturer, encouraged its customers to post product reviews on its Mi Community and social media platforms, and then used these reviews to aid in quality improvement in both software and hardware development. However, a quality-focused strategy to establish a reputable WOM profile, either premium quality offering in launching new products or gradual quality improvement incorporating customer reviews, inevitably increases cost. To balance the costs and benefits, a firm's quality strategy is often accompanied by a pricing policy. While some firms pass on the extra cost of quality improvement to customers, others improve quality internally without raising the selling prices. For example, Xiaomi provided

[1] https://www.emarketer.com/content/global-historic-first-ecommerce-china-will-account-more-than-50-of-retail-sales.

[2] https://www.e-satisfaction.com/how-important-are-customer-reviews-to-consumers/

additional cloud storage capacity expansion to its users on the Mi Community free of charge. Besides, Huawei upgraded the processor of its tablet line, MatePad 10.4-inch, from Kirin 810 to Kirin 820 in 2020, without changing the selling price of the 6GB+64GB version after the upgrade.[3]

Therefore, what is the impact of online reviews on the operational decisions, in terms of pricing, inventory, and R&D level, of firms involved in the supply chain? This research topic has attracted the interest of numerous scholars over the past several years (e.g., Wang and Li 2012, Xue et al. 2017, Caulkins et al. 2017, Jiang and Yang 2019). The majority of existing publications in the area of operations management and marketing have considered the effects of online reviews on quality and price decisions in a monopoly market (e.g., He and Chen 2018, Jiang and Yang 2019). By intuition, online reviews can be more important for firms in a competitive market because they can help improve market share, if used properly. To the best of our knowledge, Kwark et al. (2014) are among the few researchers who study the effects of online product reviews on competing firms. By considering two manufacturers selling products through the same retailer, the study explores the price competition between the two products and investigates the impact of online reviews. Note that in Kwark et al. (2014), the product quality is exogenously given. Our study is among the first to examine the impacts of online reviews on product quality decisions in the context of competition.

By considering a competitive market and the impact of online reviews on customers' purchasing behavior, this study attempts to answer the following research questions. First, how do online reviews affect the quality competition between firms selling partially substitutable products? Second, if selling prices can also be adjusted, what are the impacts of online reviews on product quality and price decisions? Finally, considering the impact of online reviews on competition, what is the profit potential if firms adjust their product quality (and selling prices), compared to some static policies under which the decisions cannot be updated?

To answer the above questions, we study a stylized game-theoretical model in which two rival firms, who sell partially substitutable products, compete on product quality (and price) over two successive periods. Despite that firms disclose some product information through product description, images, and videos, online customers cannot obtain the full product information and cannot judge the quality level and whether a product fits their needs accurately. In contrast to the first period, customers in the second period can obtain more product information from online reviews posted by customers who purchase in the first period. Starting from the choice behavior of customers, we first consider the scenario in which the selling prices are exogenous and the two firms simply compete on product quality. In a setting with static competition, the firms do not

[3] https://baijiahao.baidu.com/s?id=1680231304820144395.

improve their product quality in the second period; and in a setting with dynamic competition, they can improve product quality in the second period based on online reviews. Following a Nash-game framework, we examine the equilibrium decisions for each model, and investigate the corresponding structural properties. Furthermore, we extend our analysis to the scenario in which selling prices are endogenous, and also compare the equilibrium decisions under the static and dynamic settings. On the basis of analytical results, we conduct numerical experiments to evaluate the profit impact of dynamic competition, benchmarking on the equilibrium profits under static competition.

Our study uncovers several interesting insights. First, in the presence of online reviews, dynamically adjusting product quality in a competitive market does not necessarily guarantee an increase in firms' profitability; this finding is in sharp contrast to the existing studies for a monopoly market which show that a dynamic quality decision is at least not worse off (e.g., Zhao and Zhang 2019). The effects of dynamic quality strategy on the two competing firms' profitability are further influenced by their pricing strategy. When the prices are exogenous, although improving product quality may increase the profit of the firm with a higher price (in the case of a low cost parameter), if its price exceeds a threshold level, it always reduces the profit of the firm with a lower selling price. In contrast, when prices are endogenous variables, the dynamic quality and price strategy can have a negative or positive impact on firms' profit, depending on the market size in the second period and the R&D cost parameter. Second, the optimal quality decisions differ between the two pricing policies. When prices are exogenously given, dynamic quality competition may induce the firms to offer a higher quality level in both periods (especially when customers tend to "under-estimate" the product value) as compared to the static quality competition. Conversely, when prices are endogenously determined, dynamic competition may lead to higher or lower quality in both periods, depending on the R&D cost parameter. Third, online reviews may enable the firms to mark-up or mark-down the selling price over time, depending on the unit misfit cost, the market size, the quality difference between the two firms in the first period, and customers' private perception of product quality; this is in contrast to the finding of Jiang and Yang (2019), who show that a monopolistic firm should mark-down the selling price in the presence of online reviews. Finally, by comparing the equilibrium profits under dynamic competition with those under static competition, our numerical results show that online reviews have distinct impacts on the two firms. Specifically, when selling prices are exogenous, dynamic competition can be harmful to the firm with a cost advantage and can be beneficial to the other firm; and the magnitude of profit improvement or loss (in percentage) diminishes if online reviews can help customers judge the products' fitness even better. Interestingly, the opposite of these findings holds true for the scenario in which selling prices can be endogenously determined.

The remainder of the study is organized as follows. Section 2 provides a review of the relevant literature. Section 3 describes the base model with exogenous prices and Section 4 studies the equilibrium decisions under static and dynamic competitions, respectively. Section 5 extends the analysis to the scenario in which firms compete on both quality and price. Numerical experiments are conducted in Section 6 for additional insights. Finally, Section 7 concludes the study. All the detailed proofs of theorems and propositions are relegated to the online e-companion.

2 Literature Review

With the development of e-commerce and social media in recent years, a growing body of literature has emerged on operations management and marketing covering various aspects of online reviews and their effects on price and quality decisions. We do not attempt a comprehensive review of this literature; instead, we highlight our contributions by discussing two closely related research streams: the effects of online reviews on (i) product price and (ii) product quality.

First, as online reviews play an increasingly important role in customers' purchasing decisions, many scholars advocate the significance of incorporating online reviews in price decisions (e.g., Mayzlin 2006, Chen et al. 2011, Godes 2017). Relevant studies on price decisions start by examining the impacts of online reviews on product pricing strategies in static environments. Interested readers can refer to Chen and Xie (2005), Li and Hitt (2010), Li et al. (2011), Kwark et al. (2014), and Godes (2017) for more details.

More relevant to this work, some scholars investigate dynamic pricing in e-markets with online reviews to provide firms with strategic guidance. Among them, Li (2013) discusses the impact of online reviews on product prices by using a two-period model and finds that, in the presence of online reviews, sellers always choose to reduce product prices in the first period to improve their reputation. Yu et al. (2016) investigate how customer-generated quality information (e.g., online reviews) can affect dynamic pricing in the presence of strategic customers. They conclude that firms may reduce their initial product prices compared with the case in the absence of online reviews. Liu et al. (2017) use a two-period duopoly model to explore how the combination of online reviews and sales volume information affects a firm's pricing decisions. Their results show that firms can benefit from online reviews and sales volume information, but this is not necessarily true for customers. He and Chen (2018) examine the dynamic pricing decisions of electronic products in a market with online reviews, and find that the firm selling the higher-quality product would reduce product prices. Kuksov and Xie (2010) develop a two-period model to examine the optimal product price under the scenario that an online seller might offer unexpected frills to raise review ratings. Their results show that lowering the price, lowering the price and offering frills, or raising the price and offering frills are all beneficial to sellers. Feng et al. (2019) also show that in the presence of online reviews, adjusting product prices dynamically is an effective strategy. Thus, these studies demonstrate that online product reviews have significant effects on customer demand and firms' profits, and that dynamic pricing can improve firms' competitiveness. Although we also consider dynamic pricing decisions, our work differs from these studies in that it incorporates product quality as an endogenous variable.

Along with product pricing, many firms adopt different quality strategies to compete in a market in the presence of online reviews. Using laboratory economics experiments, Kim et al. (2019) highlight the impacts of the relational factors on the reviewer's behavior as well as the service provider's corresponding quality adjustments. Chung et al. (2020) also employ economics experiments to examine the effects of the social structure and reviewers' reputational cost on ratings and quality choices in expert review systems. There are close relationships among online reviews, product price, and quality as the price of the product can serve as a signal of product quality (Erdem et al. 2008) and as an important moderator of online reviews (Ba et al. 2020). Several scholars have investigated the effects of online reviews on quality and price decisions. Among them, Godes (2017) examines how online WOM affects quality and price decisions, taking into consideration two types of WOM (i.e., informative and persuasive). Jiang and Yang (2019) analyze the impacts of online reviews on quality and price decisions for experience goods over two selling periods, and find that price in the first period is often higher than that in the second period. Considering online reviews, Zhao and Zhang (2019) develop a dynamic model incorporating a queuing system to determine the optimal quality and price decisions for a customer-intensive service system. Unlike these papers that consider a monopoly market setting, we incorporate firm competition in examining the effects of online reviews on product quality and price decisions.

This study makes several important contributions to the literature on online reviews. First, our study complements the existing literature by examining the effects of online reviews on product price and quality decisions in a duopoly market setting. This represents a departure from studies that have assessed this topic in the monopoly market setting (e.g., Godes 2017, Jiang and Yang 2019, Zhao and Zhang 2019). As market competition has significant impacts on firms' quality and price decisions (Liu et al. 2016, 2018), it is imperative to take competition into account. Second, although competition between two upstream manufacturers is considered by Kwark et al. (2014), their focus is on price competition between substitutable products rather than on quality competition. Our research, however, prioritizes firms' quality strategy to examine the effects of online reviews on product competition. Such a setting is more in line with reality because product quality is a more prevalent strategic option than pricing for firms to develop and improve WOM profiles, as a majority of customers disclose quality information when posting product reviews online (Godes 2017). Finally, our investigation on the effects of online reviews on quality and price competition uses a two-period setting in which online reviews are unavailable in the first period, and by the end of the first period, customers who have bought a product may post online reviews that influence demand in the second period. The managerial insights derived from such a setting are particularly beneficial for new market entrants to develop appropriate quality and price strategies.

3 The Model

Consider two competing firms (1 and 2) that produce and sell imperfectly substitutable products (products 1 and 2) in an electronic market. As a convention, let $k(=1,2)$ be the index of products or firms; and without loss of generality, the unit cost for each product is normalized to 0 (Kwark et al. 2014). Faced with a common group of potential customers, the two firms compete with each other across two successive periods to optimize their respective profit. In the base scenario, the firms compete only on product quality; and in the extended scenario to be studied in Section 5 they compete on product quality as well as selling price. Due to the possible gap between "seller's show" and "buyer's show", customers can accurately evaluate the product quality and determine whether or not the product fits their needs only after they have bought and used the product. However, in contrast to the first period, in the second period customers can obtain more product information from online reviews provided by customers in the first period, and as such, better judge the product quality and fitness. Detailed sequence of events is described below.

Let p_k be the selling price of product k over the two periods. Without loss of generality, we normalize the potential market size in the first period to 1. At the beginning of the first period, firm k determines its initial product quality, denoted as u_k. The maximum value that a customer derives from a product is proportional to its quality level and depends on how the customer perceives the product. That is, customers may perceive the quality differently (Kwark et al. 2014, Hu et al. 2015). Specifically, we suppose customer i's maximum value from product k is given by $X_i u_k$, where X_i $(i=1,2,...)$ is a sequence of $i.i.d.$ random variables with mean value being $x_0 = \mathbb{E}[X_i]$. Clearly, x_0 may be greater than, equal to, or less than 1, because a customer can evaluate the product quality only from the product description, images, and videos posted online by firms. Note that before making a purchase, customer i only knows the distribution of X_i, whose actual value can be observed only when the customer has received and tried out the product.

Besides the uncertainty associated with the maximal product quality, a product may not perfectly fit the needs of a customer. Following Kwark et al. (2014), we introduce a fit attribute for each product. In particular, a typical horizontal product differentiation model is adopted to quantify the misfit cost. That is, suppose products 1 and 2 are located at the two ends of a unit-length horizontal line, and customers are distributed uniformly along the line. The misfit cost of a product to a customer is proportional to the distance between the customer and the product. Let the unit misfit cost be t. Then, the misfit costs of products 1 and 2 to a customer located at $\lambda\,(0 \le \lambda \le 1)$ are given by λt and $(1-\lambda)t$, respectively. Following Kwark et al. (2014) and Liu et al. (2017), we assume a customer cannot identify her true location accurately. Instead, she can simply observe a signal, which equals her true degree of misfit (i.e., true location) only with probability $\beta \in [0,1]$. As such, a customer who perceives her location at λ has an expected location $\beta\lambda + \frac{1-\beta}{2}$.[4] Therefore, the expected misfit cost of product 1 to a customer with perceived location λ is $(\beta\lambda + \frac{1-\beta}{2})t$, and the expected misfit cost of product 2 is $(1-\beta\lambda-\frac{1-\beta}{2})t$.

As such, in the first period, a customer's (whose perceived location is λ) expected net surplus from buying the two products, denoted as $U_k(\lambda)$, $k=1,2$, is

$$\begin{aligned} U_1(\lambda) &= x_0u_1 - [0.5(1-\beta)+\beta\lambda]t - p_1 \\ \text{and } U_2(\lambda) &= x_0u_2 - [0.5(1+\beta)-\beta\lambda]t - p_2, \end{aligned} \tag{1}$$

respectively.

By the end of the first period, customers who have bought product k may post online reviews (with texts, photos, and videos) that disclose the product quality and fitness. Although the existing studies find that online reviews are biased in reporting objective and true product quality feedback (e.g., Kim et al. 2019, Chung et al. 2020), for simplicity, we suppose the online review provides complete information on the product quality in the first period. Note that the model can be readily extended to the scenario in which online reviews only provide partial information regarding product quality. For example, under the assumption that customers are risk-neutral and only care about the expected net surplus from buying a product, if the disclosed quality of product k is $u_k+\epsilon$ or $u_k\epsilon$ where random variable ϵ follows a given distribution, then incorporation of ϵ does not alter the major findings at all.

Based on the feedback from customers, the firms may improve their product quality in the second period. Let v_k represent the extent of quality improvement of product k; as such, the quality level of product k in the second period becomes u_k+v_k. Note that for sustainability consideration, neither firm is allowed to decrease its product quality (i.e., we must have $v_k \ge 0$). Let ρ be the market size of potential customers in the second period. Note that $\rho = 0$ reduces to the scenario in which the firms have only a single selling period. Like period 1, a customer can choose to buy either product 1 or 2. However, in the presence of online reviews (and true product quality in the first period), customers in the second period have a different expected utility, as illustrated below.

[4] A detailed derivation of the conditional expectation of misfit cost can be found in Appendix A.1 of Kwark et al. (2014).

(i) For the quality attribute, a customer in the second period can assess the maximal utility according to two sources of information: common assessment as revealed by online reviews and the customer's private assessment. The common assessment is unbiased such that quality level u_k in the first period can be accurately assessed. However, the perceived quality in the second period of a particular customer is still given by $X_i(u_k+v_k)$. Following Bates and Granger (1969) and Kwark et al. (2014), we assume that the overall perceived quality in the second period is a weighted summation of the private assessment [i.e., $X_i(u_k+v_k)$] and common assessment (i.e., u_k), with $\gamma \in [0,1]$ and $(1-\gamma)$ being the weights, respectively. As such, a customer has a maximum utility of $\gamma X_i(u_k+v_k)+(1-\gamma)u_k$ towards product k.

(ii) For the misfit attribute, customers are more certain about whether or not the product is a good "fit" for them in the presence of online reviews (Archak et al. 2011, Kwark et al. 2014). Therefore, a customer observes a new signal, which equals her true degree of misfit with an updated probability, denoted as $\hat{\beta}$. Owing to the additional information provided by online reviews, we must have $\hat{\beta} > \beta$, because otherwise the online reviews are indeed misleading. As such, the value of $\hat{\beta}-\beta$ reflects the information efficiency of online reviews on disclosing the fitness of the product. Like period 1, it is not difficult to show that a customer who perceives her location at λ has an expected location $\hat{\beta}\lambda + \frac{1-\hat{\beta}}{2}$.

Considering the above two-aspect impacts, a customer's (whose perceived location is λ) expected net surplus from buying product k in the second period, denoted as $V_k(\lambda)$, is

$$\begin{aligned} V_1(\lambda) &= \gamma x_0(u_1+v_1)+(1-\gamma)u_1-[0.5(1-\hat{\beta})+\hat{\beta}\lambda]t-p_1 \\ \text{and } V_2(\lambda) &= \gamma x_0(u_2+v_2)+(1-\gamma)u_2-[0.5(1+\hat{\beta})-\hat{\beta}\lambda]t-p_2, \end{aligned} \tag{2}$$

respectively.

Note that a high product quality is usually associated with a high R&D cost (Kim and Chhajed 2002). In particular, the cost is normally increasing and convex in the product quality improvement. For simplicity and following the literature (e.g., Chakraborty et al. 2019, Xie et al. 2011), we assume that the R&D cost (for both firms) as a function of the quality improvement level q takes the following non-linear form:

$$C(q) = \alpha_k q^2,\ k = 1,2, \tag{3}$$

where parameter α_k represents the extent of cost efficiency. Finally, we assume both firms are risk-neutral; they make decisions on product quality over two periods, with the objective of maximizing their respective expected profits.

4 Equilibrium Analysis

In this section we study the equilibrium decisions between the two firms. Following Kwark et al. (2014), we do not allow customers to leave empty-handed; that is, any customer buys either product 1 or 2 in both periods (note that there is no competition between the firms if a portion of customers can choose to buy nothing). First, by the expected utility function (2), we write the demand for product k in the second period as

$$d_k^{(2)}(v_1, v_2) = \frac{\rho}{2\hat{\beta}t}\Big[\hat{\beta}t + (\gamma x_0 + 1 - \gamma)(u_k - u_{3-k}) + \gamma x_0(v_k - v_{3-k}) - p_k + p_{3-k}\Big].$$

Consequently, firm k's profit in the second period is

$$\Pi_k^{(2)}(v_1, v_2) = p_k d_k^{(2)}(v_1, v_2) - \alpha_k v_k^2. \tag{4}$$

Second, by the expected utility function (1), we write the demand for product k in the first period as

$$d_k^{(1)}(u_1, u_2) = \frac{1}{2} + \frac{x_0(u_k - u_{3-k}) - p_k + p_{3-k}}{2\beta t}.$$

Consequently, firm k's profit in the first period is

$$\Pi_k^{(1)}(u_1, u_2) = p_k d_k^{(1)}(u_1, u_2) - \alpha_k u_k^2. \tag{5}$$

The two firms compete with each other, across two periods, with the objective of maximizing their respective total profit $\Pi_k^{(1)}(u_1, u_2) + \Pi_k^{(2)}(v_1, v_2)$.

4.1 Static Competition as a Benchmark

As a benchmark, we first examine the static competition between the two firms when they do not improve product quality in the second period. This corresponds to the special case with $v_1 = v_2 = 0$. Each firm only needs to determine the quality level in the first period, with the objective of maximizing his total profit

$$\Pi_k(u_1, u_2) = \Pi_k^{(1)}(u_1, u_2) + \Pi_k^{(2)}(0,0) = p_k\Big[d_k^{(1)}(u_1, u_2) + d_k^{(2)}(0,0)\Big] - \alpha_k u_k^2. \tag{6}$$

We present the equilibrium decisions in the following theorem.

THEOREM 1. *When both firms are not allowed to improve their product quality in the second period, there exist two critical values, denoted as $\alpha_1^{(1)}$ and $\alpha_2^{(1)}$ (whose formula is given in Table EC.1), such that the equilibrium quality decision of firm k (denoted as $\tilde{u}_k$) is the following:*

(i) when the R&D cost of either firm is low (i.e., $\alpha_1 \le \alpha_1^{(1)}$ or $\alpha_2 \le \alpha_2^{(1)}$), we have

$$\tilde{u}_k = \frac{[\hat{\beta}x_0 + (\gamma x_0 + 1 - \gamma)\rho\beta]p_k}{4\alpha_k\beta\hat{\beta}t}, \ k = 1, 2; \tag{7}$$

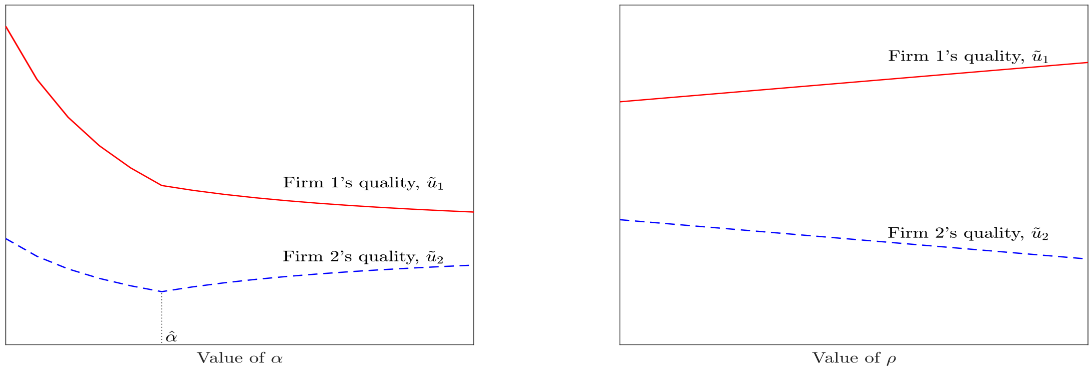
(ii) when the R&D costs of both firms are high (i.e., $\alpha_1 > \alpha_1^{(1)}$ and $\alpha_2 > \alpha_2^{(1)}$), the equilibrium is not unique. In particular, there exist two points, $A(u_1^{(2)}, u_2^{(1)})$ and $B(u_1^{(1)}, u_2^{(2)})$, such that any point on line AB is an equilibrium.

Accordingly, firm k's profit under static competition is denoted as $\tilde{\Pi}_k$. As we can see from the proof of Theorem 1, the best response curves show that a higher quality from the competitor induces a firm to decrease its quality. Given the constraint that a uniform product quality should be adopted across two periods, each firm chooses a compromised decision to account for the different choice behavior of customers arriving in different periods. As Theorem 1(ii) shows, the equilibrium may not be unique when the R&D cost parameters of the two firms are both high. When there are multiple equilibriums, we suppose the firms will choose the midpoint of line AB as an equilibrium. That is, when $\alpha_1 > \alpha_1^{(1)}$ and $\alpha_2 > \alpha_2^{(1)}$, the equilibrium quality of firm k is

$$\tilde{u}_k = \frac{t + p_1 + p_2}{2\hat{x}_0} + \frac{[\hat{\beta}x_0 + (\gamma x_0 + 1 - \gamma)\rho\beta](\alpha_{3-k}p_k - \alpha_k p_{3-k})}{8\alpha_1\alpha_2\beta\hat{\beta}t},\ k = 1, 2. \tag{8}$$

Equations (7) and (8) imply an interesting pattern on the optimal quality decision with respect to the cost parameter. When the two firms are asymmetrical in cost parameter (i.e., $\alpha_1 \neq \alpha_2$), firm k's quality decision $\tilde{u}_k$ is always decreasing in α_k; this is consistent with one's expectation. However, as Figure 1(a) shows, when the two firms are identical in cost parameter (i.e., $\alpha = \alpha_1 = \alpha_2$), the equilibrium quality $\tilde{u}_k$ is increasing in α if a firm charges a lower price than its competitor when α is above a threshold value $\hat{\alpha}$ (note that in the example $p_1 > p_2$). This implies the firm with a price advantage (i.e., with a lower selling price) has an incentive to choose an even higher product quality when the R&D cost is high. This is because a high R&D cost mitigates the quality competition between the two firms. As such, the firm with a price advantage may be incentivized by the reduced competition intensity to improve its quality in order to attract customers. Thus, the higher cost parameter may narrow the quality difference between the two firms, which is beneficial to customers of the firm with a price advantage.

As one may expect, when the future market size is large (i.e., ρ is high), firms tend to provide high-quality products in the first period to build good WOM that can help boost demand in late periods (Chen and Xie 2008). This intuition is validated by the observation that $\tilde{u}_1$ and $\tilde{u}_2$ are both increasing in ρ when the R&D cost parameter of either firm is low [see Theorem 1(i)]. However, as Equation (8) shows, $\tilde{u}_k$ may be decreasing in ρ when the R&D cost parameters of both firms are high. Specifically, when the future market size is large, a firm with a higher price or a lower cost has an incentive to increase his product quality and thus improve profit. Consequently, the competing firm with a lower price or a higher cost may have to lower her product quality, as shown in Figure 1(b) (note that in the example we assume $\frac{p_1}{\alpha_1} > \frac{p_2}{\alpha_2}$). This seems to be against one's intuition and it is mainly contributed by competition.



(a) Cost Parameter α (with $p_1 > p_2$) (b) Market Size ρ (with $\alpha_1 > \alpha_1^{(1)}$ and $\alpha_2 > \alpha_2^{(1)}$)

Figure 1 The Equilibrium Product Quality under Static Competition

Intuitively, a higher quality, which can improve a firm's online reputation, will be provided when customers pay great attention to online reviews. However, as Equations (7) and (8) show, a lower value of γ (which corresponds to customers paying more attention to online reviews) does not necessarily imply a higher quality decision $\tilde{u}_k$. Specifically, when the R&D cost parameter α_k of either firm is low, $\tilde{u}_k$ is decreasing in γ if and only if $x_0 < 1$. Whereas when the R&D cost parameters of both firms are high, a lower value of γ may lead to a lower quality level decision (a lower $\tilde{u}_k$) even if $x_0 > 1$. This will be validated only when the firm has a lower cost or a higher selling price than its competitor. That is, even when customers place more weight on online reviews (i.e., γ is low), the firms may produce a lower quality of product, which can lead to less favorable online reviews and worse firm reputation, and consequently hurt the customers.

Note that x_0 reflects the customers' private assessment towards product quality. Theorem 1 shows that when the R&D cost parameter of either firm is low, the equilibrium quality $\tilde{u}_k$ is increasing in x_0, implying that customers' higher perceived value from product quality can encourage the firm to increase his quality level. However, if the R&D cost parameters of both firms are high, the equilibrium quality level decision $\tilde{u}_k$ is decreasing in x_0 when the firm k has a lower selling price or a higher cost (i.e., $\frac{p_k}{\alpha_k} < \frac{p_{3-k}}{\alpha_{3-k}}$) than its competitor, but may not be monotonic in x_0 otherwise.

Finally, a higher value of $\hat{\beta}$ may not induce a higher product quality. Equation (7) shows that $\tilde{u}_k$ is decreasing in $\hat{\beta}$ when the R&D cost parameter of either firm is low. Note that $\hat{\beta}$ reflects the value of online reviews on the misfit attribute in the second period. This implies that online reviews that increase customers' certainty about the perceived product fit may force the firm to lower his product quality level. In other words, under competition when the firm has a low cost parameter, product evaluation of other customers (who have bought the product) on the product fit in online reviews always induces the firm to decrease his product quality. In contrast, as shown in Equation (8), the relationship of $\tilde{u}_k$ with respect to $\hat{\beta}$ also depends on the difference in ratio of price to R&D cost parameter between the competing firms (as measured by $\Delta := \frac{p_1}{\alpha_1} - \frac{p_2}{\alpha_2}$) when the R&D cost parameters of both firms are high. If $\frac{p_k}{\alpha_k} \geq \frac{p_{3-k}}{\alpha_{3-k}}$, then firm k should adopt an even lower quality level when $\hat{\beta}$ is higher. However, for the firm of a lower selling price or a higher cost than its competitor, there is an incentive to provide a higher product quality to attract more demand if $\hat{\beta}$ is higher. That is, online reviews can induce firm k with a higher cost or a lower selling price (i.e., when $\frac{p_k}{\alpha_k} < \frac{p_{3-k}}{\alpha_{3-k}}$) to improve his product quality. The reason is that with a higher $\hat{\beta}$, the impact of a high quality on customer demand is relatively insignificant. In this case, the firm with a higher selling price or a lower cost than its competitor tends to invest less in improving the quality to save the R&D costs. That is, even the future selling period has a larger market size, the firms may produce a lower quality which can lead to worse online reviews and consequently have a negative effect on demand. This implies that product evaluation of other customers on the product fit from online reviews (corresponding to a higher value of $\hat{\beta}$) does not necessarily imply a higher quality decision for both firms. Such a finding stands in contrast to that of previous studies showing that customer reviews can motivate firms to provide high-quality products (e.g., Sun and Xu 2018, Jiang and Yang 2019).

4.2 Equilibrium under Dynamic Competition

In this subsection, we study the equilibrium decisions under the dynamic scenario in which the two firms are allowed to improve their product quality in the second period. Given the decisions in the first period, firm k determines v_k to optimize his profit $\Pi_k^{(2)}(v_1, v_2)$ in the second period:

$$\Pi_k^{(2)}(v_1, v_2) = \frac{\rho}{2\hat{\beta}t}\Big[\hat{\beta}t + (\gamma x_0 + 1 - \gamma)(u_k - u_{3-k}) + \gamma x_0(v_k - v_{3-k}) - p_k + p_{3-k}\Big]p_k - \alpha_k v_k^2.$$

The equilibrium decision in the second period is presented in the following theorem.

THEOREM 2. *Under dynamic quality competition, for any given quality levels (u_1, u_2) in the first period, the equilibrium quality improvement decision of firm k in the second period, denoted as v_k^*, is the following:*

(i) When the initial quality level is high (i.e., $u_1 + u_2 \geq \frac{t+p_1+p_2}{\gamma x_0 + 1 - \gamma}$), we have

$$v_k^* = \frac{\rho\gamma x_0 p_k}{4\alpha_k\hat{\beta}t},\ k = 1, 2. \tag{9}$$

(ii) When the initial quality level is low (i.e., $u_1 + u_2 < \frac{t+p_1+p_2}{\gamma x_0 + 1 - \gamma}$), the equilibrium depends on the magnitude of cost efficiency. Specifically, there exist two critical values, $\alpha_1^{(2)}$ and $\alpha_2^{(2)}$, such that: (a) if $\alpha_1 \leq \alpha_1^{(2)}$ or $\alpha_2 \leq \alpha_2^{(2)}$, the equilibrium is identical to that given in Equation (9). (b) otherwise, the equilibrium is not unique. Like before, the midpoint equilibrium is given by

$$v_k^* = \frac{t + p_1 + p_2 - (\gamma x_0 + 1 - \gamma)(u_1 + u_2)}{2\gamma x_0} + \frac{\rho\gamma x_0(\alpha_{3-k}p_k - \alpha_k p_{3-k})}{8\alpha_1\alpha_2\hat{\beta}t},\ k = 1, 2. \tag{10}$$

As the proof of Theorem 2 shows, each firm will lower the degree of quality improvement if the competitor produces a higher quality, either premium initial quality offering or significantly greater quality improvement in the second period; this is consistent with the static competition scenario (Section 4.1). Under equilibrium, Theorem 2 shows that the quality improvement is independent of u_1 and u_2 when the quality in the first period is already high or the R&D cost parameter of either firm is low, and the quality improvement is decreasing in u_1 and u_2 when the initial product quality level is low and the R&D cost parameters of both firms are high. Generally, the equilibrium quality improvement is non-increasing in the quality level in the first period. It is not difficult to show that $v_1^* - v_2^*$ is proportional to $\Delta := \frac{p_1}{\alpha_1} - \frac{p_2}{\alpha_2}$, implying that a higher ratio of selling price to R&D cost parameter is always associated with a higher degree of quality improvement in the second period. It is reasonable as a higher ratio of selling price to R&D cost parameter indicates greater contribution to firm's profit from quality improvement.

When customers are more certain about the fitness of a product in the presence of online reviews, a natural instinct is that the firms may be less inclined to improve their product quality. Therefore, it is interesting to know the impact of parameter $\hat{\beta}$ on the equilibrium decisions. This intuition is validated by the observation in Theorem 2(i). However, Theorem 2(ii) shows that the conjecture holds only for the firm that has a higher ratio of selling price to R&D cost parameter than its competitor (i.e., the firm has a higher selling price or a lower cost). For the firm of a lower selling price or a higher cost, his quality improvement is increasing in $\hat{\beta}$ when the initial quality levels are low and when the R&D cost parameters of both firms are high.

Moreover, Theorem 2 shows that when the quality in the first period is high or the R&D cost parameter of either firm is low, a lower value of γ can induce the firm to decrease his product quality improvement v_k^* in the second period. That is, when customers concern online reviews more significantly (i.e., when γ is low), firms will choose to invest less in improving their quality level; this differs from the static competition setting (refer to Theorem 1), under which a lower value of γ may lead to a higher quality level if customers tend to "under-estimate" the maximal utility (i.e., $x_0 < 1$) and the R&D cost parameter α_k of either firm is low. In contrast, when the initial product quality is low and the R&D cost parameters of both firms are high, a lower value of γ does not necessarily imply a lower degree of quality improvement v_k^*. Specifically, when customers tend to "over-estimate" the maximal utility (i.e., $x_0 \geq 1$), the equilibrium quality improvement decision v_k^* is decreasing in γ if the firm has a lower ratio of selling price to the R&D cost parameter (a lower price or a higher cost) than its competitor, but may not be monotonic in γ otherwise. In contrast, if customers tend to "under-estimate" the maximal utility (i.e., $x_0 < 1$), a lower value of γ may induce the firm to increase or reduce his quality improvement level. When $u_1 + u_2 < t + p_1 + p_2$, the equilibrium quality improvement decision v_k^* is decreasing in γ if the firm has a lower ratio of selling price to the R&D cost parameter than its competitor, but may not be monotonic in γ otherwise; this is consistent with the case where customers tend to "over-estimate" the maximal utility. And if $u_1+u_2>t+p_1+p_2$, a higher value of γ can induce the firm of a higher price or a lower cost parameter to increase his quality improvement level but may not be monotonic otherwise.

Substituting Theorem 2 into $\Pi_k^{(2)}(v_1,v_2)$, we arrive at firm k's optimal profit $\Pi_k^{(2*)}$ under equilibrium decisions in the second period

$$\Pi_k^{(2*)}=\frac{p_k\rho}{2\hat{\beta}t}\left[\hat{\beta}t+(\gamma x_0+1-\gamma)(u_k-u_{3-k})+\frac{\rho(\gamma x_0)^2(\alpha_{3-k}p_k-\alpha_k p_{3-k})}{4\alpha_1\alpha_2\hat{\beta}t}-p_k+p_{3-k}\right]-\alpha_k(v_k^*)^2. \quad (11)$$

Anticipating the equilibrium decisions in the second period, the two firms compete in the first period to maximize their respective profits

$$\Pi_k(u_1,u_2)=\left(\frac{1}{2}+\frac{x_0(u_k-u_{3-k})-p_k+p_{3-k}}{2\beta t}\right)p_k-\alpha_k u_k^2+\Pi_k^{(2*)}. \quad (12)$$

The equilibrium quality decisions in the first period are presented in the following theorem.

Theorem 3. *When the firms can improve their product quality in the second period, the equilibrium quality decision of firm k in the first period, denoted as u_k^*, is the following:*

(i) When $x_0\ge 1$ and the R&D costs of both firms are high (i.e., $\alpha_1>\alpha_1^{(2)}$ and $\alpha_2>\alpha_2^{(2)}$), we have the following: If $\alpha_1\le\alpha_1^{(3)}$ or $\alpha_2\le\alpha_2^{(3)}$, the equilibrium product quality is

$$\begin{aligned}u_k^* =\ &\frac{1}{4\alpha_1\alpha_2\beta\hat{\beta}t[(2\gamma x_0)^2+2(\gamma x_0+1-\gamma)^2]}\Big[4\alpha_1\alpha_2\beta\hat{\beta}t(\gamma x_0+1-\gamma)(t+p_1+p_2)+\Big(\hat{\beta}x_0+\frac{3}{2}\rho\beta(\gamma x_0+1-\gamma)\Big)\\ &\times\Big(4(\gamma x_0)^2\alpha_{3-k}p_k+(\gamma x_0+1-\gamma)^2(\alpha_{3-k}p_k-\alpha_k p_{3-k})\Big)-(\gamma x_0)^2\rho\beta(\gamma x_0+1-\gamma)(\alpha_{3-k}p_k+\alpha_k p_{3-k})\Big].\end{aligned} \quad (13)$$

If $\alpha_1>\alpha_1^{(3)}$ and $\alpha_2>\alpha_2^{(3)}$, the equilibrium is not unique. Like before, the midpoint equilibrium is

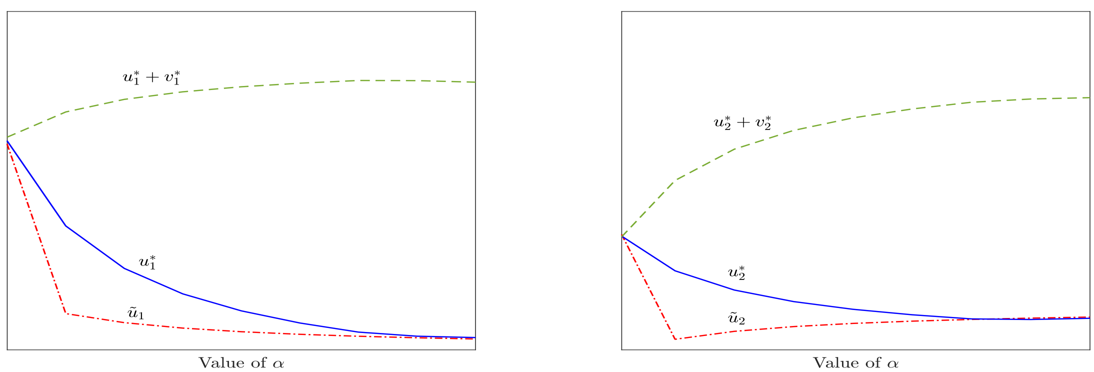
$$u_k^*=\frac{t+p_1+p_2}{2x_0}+\frac{(\alpha_{3-k}p_k-\alpha_k p_{3-k})}{8\alpha_1\alpha_2\beta\hat{\beta}t}\Big(\hat{\beta}x_0+\frac{3}{2}\rho\beta(\gamma x_0+1-\gamma)\Big). \quad (14)$$

(ii) When $x_0\ge 1$ and the R&D cost of either firm is low (i.e., $\alpha_1\le\alpha_1^{(2)}$ or $\alpha_2\le\alpha_2^{(2)}$), the equilibrium quality takes a similar form as those under the static competition scenario (refer to Theorem 1), except that $\hat{x}_0$ is replaced by x_0.

(iii) When $x_0\le 1$, the equilibrium quality decision is identical to that under the static competition scenario (refer to Theorem 1).

Note that the critical values, $\alpha_i^{(j)}$, $i=1,2$ and $j=2,3$, are defined in Table EC.1. One may expect that each firm chooses a compromised quality decision in static competition, such that the static equilibrium decision falls between the corresponding decisions in the first and second periods under dynamic competition (i.e., $u_k^*\le\tilde{u}_k\le u_k^*+v_k^*$). Interestingly, Theorem 3 shows that the opportunity of product quality improvement may only influence the quality decision in the second period, but not the first period, as compared to the static competition scenario (refer to

Theorem 1). Specifically, in part (iii) of Theorem 3, despite that both firms will choose to improve their product quality in the second period, the initial quality decision in the first period remains the same as that under the static competition scenario (i.e., $u_k^* = \tilde{u}_k < u_k^* + v_k^*$). This result is obtained because of the assumption that the product quality should be sufficiently high such that no customer leaves empty-handed, consistent with Kwark et al. (2014). In this case, the quality improvement of firm k is given by Equation (9). That is, the quality improvement in the second period is independent of the initial quality. Besides, as Theorem 3(i) shows, when customers tend to over-estimate the product utility (i.e., $x_0 \geq 1$) and the R&D cost parameters of both firms are high (i.e., $\alpha_1 > \alpha_1^{(2)}$ and $\alpha_2 > \alpha_2^{(2)}$), it is possible that dynamic competition leads to a higher product quality in both periods; as the example shown in Figure 2. Note that in Figure 2 we have $\alpha = \alpha_1 = \alpha_2$ and $p_1 > p_2$. In contrast, when customers tend to over-estimate the product utility and the R&D cost parameter of either firm is low [see part (ii) of Theorem 3], it is not difficult to derive that $\tilde{u}_k \geq u_k^* + v_k^* > u_k^*$ when $\alpha_k \geq \alpha_k^{(4)}$. That is, dynamic competition may lead to lower product quality in both periods when cost parameter α_k exceeds a threshold value.



(a) Firm 1's Quality (b) Firm 2's Quality

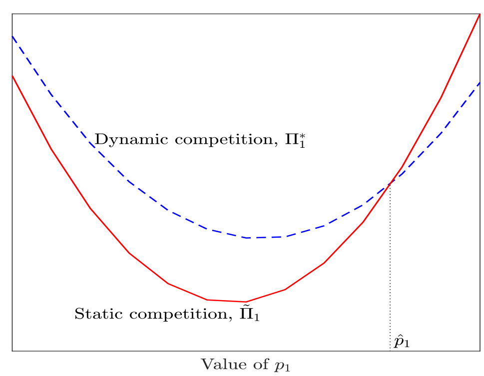
Figure 2 Equilibrium Quality Decisions: Dynamic vs. Static Competition

When $\tilde{u}_k \leq u_k^* < u_k^* + v_k^*$, dynamic competition is beneficial to customers because they can enjoy a higher product quality at the same price. In particular, when customers tend to under-estimate the product value [Theorem 3(iii)], customers are even better off if parameter $\hat{\beta}$ is low (note that v_k^* in Equation (9) is decreasing in $\hat{\beta}$). In contrast, when $\tilde{u}_k \geq u_k^* + v_k^* > u_k^*$, dynamic competition is harmful to customers. In the following, we analyze the impact of dynamic competition on the firms' total profit Π_k^*, as compared to that under static competition. For ease of presentation, let

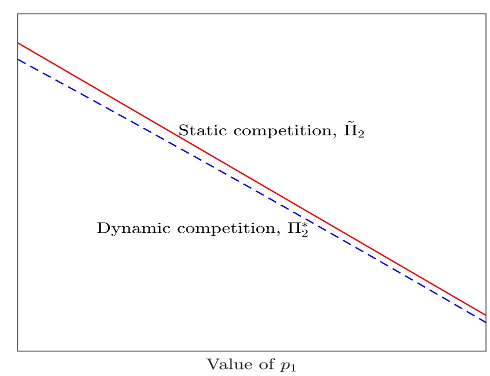
$$\Delta\Pi_k := \Pi_k^* - \tilde{\Pi}_k$$

be firm k's profit improvement from dynamic competition, benchmarking on static competition.

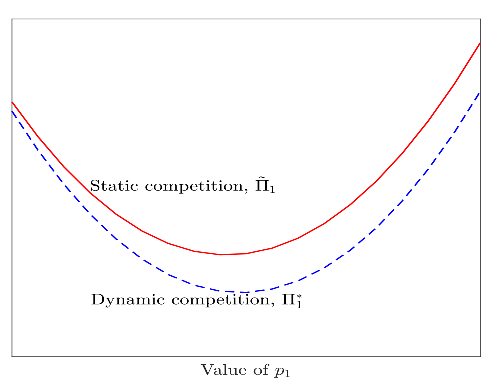
First, in part (i) and part (ii) of Theorem 3, though it is difficult (albeit still possible) to provide a detailed analytical comparison between Π_k^* and $\tilde{\Pi}_k$, Figure 3 shows that both firms may hurt from dynamic competition. As the figure suggests, when the R&D cost parameters of both firms are high [part (i)], fixing firm 2's selling price p_2, firm 1's profit under dynamic competition is even lower than that under static competition when p_1 is above a threshold value $\hat{p}_1$; firm 2, on the other hand, is always worse off from dynamic competition. In contrast, when the R&D cost parameter of either firm is low [part (ii)], dynamic competition leads to the reduced profit of firm 2 if p_1 is above a threshold value $\tilde{p}_1$, whereas it is always harmful to firm 1. This result is in sharp contrast to that under a monopolistic setting, because the monopoly should always benefit from making dynamic decisions (Zhao and Zhang 2019).



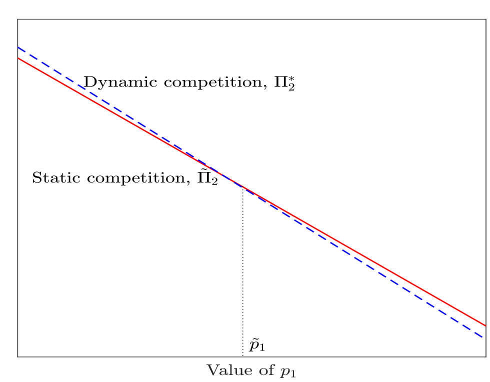
(a) Firm 1's Profit (with $\alpha_1 > \alpha_1^{(2)}$ and $\alpha_2 > \alpha_2^{(2)}$)



(b) Firm 2's Profit (with $\alpha_1 > \alpha_1^{(2)}$ and $\alpha_2 > \alpha_2^{(2)}$)



(c) Firm 1's Profit (with $\alpha_1 \le \alpha_1^{(2)}$ or $\alpha_2 \le \alpha_2^{(2)}$)



(d) Firm 2's Profit (with $\alpha_1 \le \alpha_1^{(2)}$ or $\alpha_2 \le \alpha_2^{(2)}$)

Figure 3 Equilibrium Profits: Dynamic vs. Static Competition

Second, in part (iii) of Theorem 3, we can show that the difference in profit (i.e., $\Delta\Pi_k$) simply comes from the difference in profit in the second period. Consequently, we have the following proposition.

PROPOSITION 1. *When customers tend to under-estimate the product utility [see part (iii) of Theorem 3], $\Delta\Pi_k \geq 0$ if and only if $\frac{\alpha_k}{p_k} \leq \frac{\alpha_{3-k}}{2p_{3-k}}$.*

Proposition 1 also shows that by allowing quality improvement in the second period, dynamic competition may not necessarily improve the firms' profit. In particular, dynamic competition is always harmful to the firm with a lower ratio of selling price to the R&D cost parameter than its competitor, and is beneficial to the firm with a higher ratio of selling price to the R&D cost parameter if and only if $\frac{\alpha_k}{p_k} \leq \frac{\alpha_{3-k}}{2p_{3-k}}$ (or $p_k \geq \frac{2\alpha_k p_{3-k}}{\alpha_{3-k}}$). That is, only when a firm has a significantly higher ratio of selling price to the R&D cost parameter relative to the competitor, can he benefit from dynamic competition. Therefore, product quality improvement may lead to a lose-lose situation for the two competing firms. Of course, this result is due to the existence of competition as well.

5 The Scenario with Endogenous Prices

In this section, we extend the analysis to the scenario in which the selling prices are endogenously determined. Like Section 4, we first study the static competition under which the firms cannot update their quality and price decisions in the second period, followed by dynamic competition under which the product quality and selling price decisions can be adjusted jointly.

5.1 Static Quality and Price Competition

By static competition, the product quality and price remain the same across two periods for each firm. Similar to the scenario with exogenous prices (refer to Theorem 1), the equilibrium (which is determined by the best response curves) can locate on the boundary conditions or interior points of a firm's feasible domain, depending on a set of complicated conditions. To focus on more interesting insights, in the following we simply present the interior equilibrium between the two firms.

THEOREM 4. *When neither firm adjusts product quality or price in the second period, the interior equilibrium decision of firm k, denoted as $(\tilde{u}_k, \tilde{p}_k)$, is the following:*

$$\tilde{u}_k = \frac{(1+\rho)\Big(\hat{\beta}x_0 + (\gamma x_0 + 1 - \gamma)\rho\beta\Big)\Big[6(\hat{\beta}+\rho\beta)\alpha_{3-k}\beta\hat{\beta}t - \Big(\hat{\beta}x_0 + (\gamma x_0 + 1 - \gamma)\rho\beta\Big)^2\Big]}{2(\hat{\beta}+\rho\beta)\Big[12(\hat{\beta}+\rho\beta)\alpha_1\alpha_2\beta\hat{\beta}t - (\alpha_1+\alpha_2)\Big(\hat{\beta}x_0 + (\gamma x_0 + 1 - \gamma)\rho\beta\Big)^2\Big]},$$

$$\tilde{p}_k = \frac{2(1+\rho)\beta\hat{\beta}t\Big[6(\hat{\beta}+\rho\beta)\alpha_1\alpha_2\beta\hat{\beta}t - \alpha_k\Big(\hat{\beta}x_0 + (\gamma x_0 + 1 - \gamma)\rho\beta\Big)^2\Big]}{(\hat{\beta}+\rho\beta)\Big[12(\hat{\beta}+\rho\beta)\alpha_1\alpha_2\beta\hat{\beta}t - (\alpha_1+\alpha_2)\Big(\hat{\beta}x_0 + (\gamma x_0 + 1 - \gamma)\rho\beta\Big)^2\Big]}. \tag{15}$$

As shown from the proof of Theorem 4, the best response curves indicate that the quality and price decisions of the competitor have an opposite impact on a firm's decision. That is, a higher price from the competitor induces a firm to increase its quality and price, but a higher quality from the competitor induces a firm to decrease its quality and price. As Equation (15) shows, both the equilibrium quality $\tilde{u}_k$ and the equilibrium price $\tilde{p}_k$ may not be monotonic in ρ and $\hat{\beta}$. However, as a special case, if the two firms are symmetric in R&D costs (i.e., $\alpha_1 = \alpha_2$), the equilibrium reduces to

$$\tilde{u}_k = \frac{1+\rho}{4\alpha(\hat{\beta}+\rho\beta)}\Big(\hat{\beta}x_0 + \rho\beta(\gamma x_0 + 1 - \gamma)\Big);\ \tilde{p}_k = \frac{1+\rho}{\hat{\beta}+\rho\beta}\beta\hat{\beta}t.$$

In this special case, $\tilde{u}_k$ and $\tilde{p}_k$ are both increasing in ρ (because $\hat{\beta} \geq \beta$), implying that a high product quality and a high price should be adopted if the future period has a significantly larger market size. Moreover, the equilibrium price $\tilde{p}_k$ is increasing in $\hat{\beta}$, implying that online reviews enable firms to raise their selling prices. However, a higher value of $\hat{\beta}$ does not necessarily imply a higher quality decision $\tilde{u}_k$. Specifically, we can show that $\tilde{u}_k$ is increasing in $\hat{\beta}$ if and only if $x_0 > 1$. That is, when customers tend to "over-estimate" the product value, online reviews can induce the firm to improve his product quality. In contrast, when customers tend to "under-estimate" the product value (i.e, $x_0 < 1$), online reviews can induce the firm to decrease his product quality. Interestingly, online reviews have no impact on the equilibrium product quality level at all when $x_0 = 1$. This is in contrast to the static competition with an exogenous price (refer to Theorem 1).

Moreover, Equation (15) shows that $\tilde{u}_k$ and $\tilde{p}_k$ are both decreasing in α_k when the R&D costs are heterogeneous. In contrast, when the two firms have an identical R&D cost parameter (i.e., $\alpha = \alpha_1 = \alpha_2$), the R&D cost has no effect on the equilibrium price $\tilde{p}_k$. This seems to be against one's intuition because a higher α can still induce the firm to decrease the equilibrium quality decision $\tilde{u}_k$. Therefore, under the static quality and price competition, a higher identical R&D cost is harmful to customers because they pay the same price for a lower-quality product.

5.2 Dynamic Quality and Price Competition

In this section, we study the dynamic competition under which the firms can improve their quality level (v_k) and adjust their selling price (r_k) in the second period. Like Section 5.1, we simply present the interior equilibriums. Given the decisions in the first period, the equilibrium decisions in the second period are presented in the following theorem.

THEOREM 5. *Under dynamic competition, firm k's interior equilibrium decisions in the second period are as follows:*

$$v_k^* = \frac{\rho\gamma x_0\Big[6\alpha_{3-k}\hat{\beta}t - \rho(\gamma x_0)^2 + 2\alpha_{3-k}(\gamma x_0 + 1 - \gamma)(u_k - u_{3-k})\Big]}{24\alpha_1\alpha_2\hat{\beta}t - 2(\alpha_1+\alpha_2)\rho(\gamma x_0)^2},$$

$$r_k^* = \frac{2\alpha_k \hat{\beta} t \left[6\alpha_{3-k}\hat{\beta} t - \rho(\gamma x_0)^2\right] + 4\alpha_1\alpha_2\hat{\beta} t(\gamma x_0 + 1 - \gamma)(u_k - u_{3-k})}{12\alpha_1\alpha_2\hat{\beta} t - (\alpha_1+\alpha_2)\rho(\gamma x_0)^2}. \tag{16}$$

Theorem 5 uncovers some interesting observations that are different from the static scenario. First, the equilibrium quality and price are not necessarily increasing in the market size of the second period (ρ). For the equilibrium product price, when the other firm's cost parameter α_{3-k} is below $\frac{2\alpha_1\alpha_2}{\alpha_1+\alpha_2}$, if the firm offers a lower product quality in the first period (i.e., $u_k < u_{3-k}$), r_k^* is decreasing in ρ; however, if the firm offers a higher product quality in the first period (i.e., $u_k \geq u_{3-k}$), then only when the firm's initial quality advantage $u_k - u_{3-k}$ is above $\Delta u_k^{(1)}$, should his selling price in the second period be increasing in ρ. In contrast, when the other firm's cost parameter α_{3-k} exceeds $\frac{2\alpha_1\alpha_2}{\alpha_1+\alpha_2}$, if the firm offers a higher product quality in the first period (i.e., $u_k > u_{3-k}$), r_k^* is increasing in ρ; otherwise only when the other firm's initial quality has a slight advantage, i.e., $u_{3-k} - u_k < \Delta u_k^{(2)}$, should his second-period selling price be increasing in ρ. For the equilibrium product quality improvement, when the firm has a high cost parameter, i.e., $\alpha_k > \check{\alpha}_k$, and offers a high product quality in the first period, i.e., $u_k - u_{3-k} > \Delta u_k^{(3)}$, the equilibrium quality improvement level v_k^* is increasing in ρ, implying that the firm will invest more in improving the product quality when the future market size is larger.

Second, a higher value of $\hat{\beta}$ may not induce a higher selling price or a higher product quality either. Specifically, when the difference in the product quality between the two firms in the first period $u_k - u_{3-k}$ is below $\frac{(\alpha_k - \alpha_{3-k})\rho(\gamma x_0)^2}{4\alpha_1\alpha_2(\gamma x_0 + 1 - \gamma)}$, the equilibrium product quality improvement v_k^* is increasing in $\hat{\beta}$, implying that a small difference in the initial quality level will encourage the firm to offer a higher level of quality improvement in late periods if $\hat{\beta}$ is higher. Similarly, for the equilibrium product price in the second period r_k^*, a higher price should be set with a higher $\hat{\beta}$ when the difference in the initial quality level is small, i.e., $u_k - u_{3-k} < \Delta u_k^{(4)}$.

In particular, when the two firms have an identical cost parameter (i.e., $\alpha = \alpha_1 = \alpha_2$), the equilibrium decisions in the second period reduces to

$$v_k^* = \frac{\rho\gamma x_0}{4\alpha} + \frac{\rho\gamma x_0(\gamma x_0 + 1 - \gamma)(u_k - u_{3-k})}{2\Big(6\alpha\hat{\beta} t - \rho(\gamma x_0)^2\Big)}; \; p_k^* = \hat{\beta} t + \frac{2\alpha\hat{\beta} t(\gamma x_0 + 1 - \gamma)(u_k - u_{3-k})}{6\alpha\hat{\beta} t - \rho(\gamma x_o)^2}.$$

In the symmetric setting, if the other firm offers a higher product quality in the first period (i.e., $u_{3-k} > u_k$), then r_k^* is decreasing in ρ, implying that a lower selling price should be offered when the market size in the second period becomes larger. In fact, only when firm k offers a higher product quality in the first period, should his selling price in the second period be increasing in ρ. For the optimal level of quality improvement, v_k^* is increasing in ρ if $u_k \geq u_{3-k}$, but may not be monotonic in ρ otherwise. Moreover, the relationship of (v_k^*, r_k^*) with respect to parameter $\hat{\beta}$ also depends on the quality gap in the first period. If firm k adopts a higher initial quality level, then he should adopt an even lower level of quality improvement in the second period if $\hat{\beta}$ is higher. With a higher $\hat{\beta}$, the impact of quality improvement on customer demand becomes less significant. In this case, the firm with initial quality advantage tends to invest less in quality improvement.

Considering the equilibrium decisions, we write firm k's optimal profit in the second period as

$$\Pi_k^{(2*)} = \frac{\alpha_k\rho\Big(6\alpha_{3-k}\hat{\beta}t - \rho(\gamma x_0)^2 + 2\alpha_{3-k}(\gamma x_0 + 1 - \gamma)(u_k - u_{3-k})\Big)^2\Big(8\alpha_k\hat{\beta}t - \rho(\gamma x_0)^2\Big)}{4\Big(12\alpha_1\alpha_2\hat{\beta}t - (\alpha_1+\alpha_2)\rho(\gamma x_0)^2\Big)^2}. \tag{17}$$

Anticipating the equilibrium decisions in the second period, the two firms compete in the first period to maximize their respective profits

$$\Pi_k(u_k, p_k, u_{3-k}, p_{3-k}) = \Big(\frac{1}{2} + \frac{x_0(u_k - u_{3-k}) - p_k + p_{3-k}}{2\beta t}\Big)p_k - \alpha_k u_k^2 + \Pi_k^{(2*)}, \tag{18}$$

where $\Pi_k^{(2*)}$ is given by Equation (17). Note that $\Pi_k^{(2*)}$ is independent of (p_1, p_2), implying that for each firm, determining the first-period price only needs to consider the profits in the first period. In contrast, determining the first-period quality should take into account its impact on profits in both periods. We present the equilibrium quality and price decisions in the first period in the following theorem.

THEOREM 6. *Under the dynamic competition scenario, the interior equilibrium product quality and selling price of firm k in the first period are*

$$\begin{aligned}
u_k^* = &\frac{1}{4\alpha_1\alpha_2K_1^2 - 2\alpha_kK_1K_2 - 2\alpha_{3-k}K_1K_3}\Big\{\alpha_1\alpha_2\rho(\gamma x_0 + 1 - \gamma)\Big[-K_3\Big(8\alpha_{3-k}\hat{\beta}t - \rho(\gamma x_0)^2\Big)\Big(6\alpha_k\hat{\beta}t - \rho(\gamma x_0)^2\Big) \\
&+(2\alpha_{3-k}K_1 - K_2)\Big(8\alpha_k\hat{\beta}t - \rho(\gamma x_0)^2\Big)\Big(6\alpha_{3-k}\hat{\beta}t - \rho(\gamma x_0)^2\Big)\Big] + \frac{x_0K_1}{2}(2\alpha_{3-k}K_1 - K_2 - K_3)\Big\}, \\
p_k^* = &\beta t + \frac{x_0}{3\Big(4\alpha_1\alpha_2K_1 - 2\alpha_kK_2 - 2\alpha_{3-k}K_3\Big)}\Big\{2\alpha_1\alpha_2\rho(\gamma x_0 + 1 - \gamma)\Big[\alpha_{3-k}\Big(8\alpha_k\hat{\beta}t - \rho(\gamma x_0)^2\Big)\Big(6\alpha_{3-k}\hat{\beta}t - \rho(\gamma x_0)^2\Big) \\
&-\alpha_k\Big(8\alpha_{3-k}\hat{\beta}t - \rho(\gamma x_0)^2\Big)\Big(6\alpha_k\hat{\beta}t - \rho(\gamma x_0)^2\Big)\Big] + (\alpha_{3-k} - \alpha_k)x_0K_1\Big\}.
\end{aligned} \tag{19}$$

By intuition, when the market size in the second period is large (i.e., when ρ is high), firms tend to provide high-quality products in the first period. As Theorem 6 shows, a larger future market size may not induce a higher product quality in the first period. However, in the symmetric setting with $\alpha = \alpha_1 = \alpha_2$, the equilibrium product quality and price in the first period are

$$u_1^* = u_2^* = \frac{x_0}{4\alpha} + \frac{\rho(\gamma x_0 + 1 - \gamma)\Big(8\alpha\hat{\beta}t - \rho(\gamma x_0)^2\Big)}{8\alpha\Big(6\alpha\hat{\beta}t - \rho(\gamma x_0)^2\Big)};\ p_1^* = p_2^* = \beta t. \tag{20}$$

Equation (20) shows that u_1^* and u_2^* are both increasing in ρ. On the side of misfit attribute, u_1^* and u_2^* are decreasing in $\hat{\beta}$, implying that the firms should decrease their product quality if online reviews can accurately reveal the product misfit level. This is due to the competition between the two firms and the assumption that no customer leaves empty-handed. This is in sharp contrast to the static competition scenario, where the optimal product quality is increasing in $\hat{\beta}$ if customers tend to "over-estimate" the maximal utility (i.e., $\mathbb{E}[X_i]>1$) (recall Theorem 4). Besides, as Equation (20) shows, when the two firms have an identical cost parameter, the equilibrium product price in the first period is only determined by the "fit probability" β and the unit misfit cost t. This implies that the market size in the second period and online reviews have no effect on the optimal selling price in the first period (i.e., p_1^* and p_2^* are independent of parameters ρ and $\hat{\beta}$). This differs from the static competition scenario in which product price is increasing in the market size ρ and parameter $\hat{\beta}$ (recall Theorem 4). Therefore, customers can benefit from a larger market size in the second period ρ; whereas a higher value of $\hat{\beta}$ is harmful to them. Moreover, Equation (20) shows that a lower value of γ may induce the firm to increase or decrease his quality level, depending on the customers' private quality assessment x_0, the unit misfit cost t and the market size in the second period ρ; this is different to the static competition scenario that the optimal product quality is decreasing in γ if $x_0<1$.

By substituting Theorem 6 into Theorem 5, we can show that the quality advantage in the first period does not necessarily imply a higher level of quality improvement and selling price in the second period. In particular, in the symmetric setting with $\alpha=\alpha_1=\alpha_2$, the equilibrium decisions in the second period reduce to

$$v_1^*=v_2^*=\frac{\rho\gamma x_0}{4\alpha} \text{ and } r_1^*=r_2^*=\hat{\beta}t.$$

That is, a larger market size motivates the firms to pursue a higher quality level in the second period. Combining with Equation (20), both the equilibrium initial product quality u_k^* and quality improvement v_k^* in the two periods are increasing in the market size ρ; implying that a high product quality should be offered if the future period has a significantly larger market size. In terms of the equilibrium prices in the two periods, r_k^* and p_k^*, $k=1,2$, we arrive at the following proposition.

Proposition 2. *Under dynamic competition, comparing the equilibrium selling prices across the two periods, we have the following observations.*

(i) When $\rho<\hat{\rho}$ and $x_0\le 1$, the firms will mark-up their prices over time (i.e., $r_k^*\ge p_k^*$) if $u_k-u_{3-k}\ge \Delta u_k^{(5)}$; otherwise a lower selling price should be set in the second period (i.e., $r_k^*<p_k^*$);

(ii) When $\rho<\hat{\rho}$ and $x_0>1$, the firms will mark-up their prices over time (i.e., $r_k^*\ge p_k^*$) if $u_k-u_{3-k}\ge \Delta u_k^{(5)}$ and $t<\hat{t}$, or $u_k-u_{3-k}<\Delta u_k^{(5)}$ and $t\ge\hat{t}$; otherwise a lower selling price will be set in the second period (i.e., $r_k^*<p_k^*$);

(iii) When $\rho>\hat{\rho}$ and $x_0\le 1$, the firms will mark-up their prices over time (i.e., $r_k^*\ge p_k^*$) if $u_k-u_{3-k}\le \Delta u_k^{(5)}$; otherwise a lower selling price should be set in the second period (i.e., $r_k^*<p_k^*$);

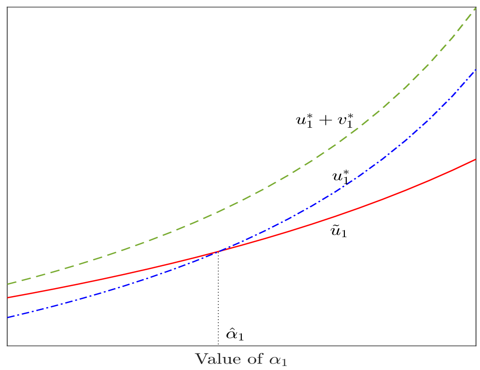
(iv) When $\rho > \hat{\rho}$ and $x_0 > 1$, the firms will mark-up their prices over time (i.e., $r_k^* \geq p_k^*$) if $u_k - u_{3-k} \leq \Delta u_k^{(5)}$ and $t < \hat{t}$, or $u_k - u_{3-k} > \Delta u_k^{(5)}$ and $t \geq \hat{t}$; otherwise a lower selling price should be set in the second period (i.e., $r_k^* < p_k^*$).

Proposition 2 compares the optimal selling prices across the two periods. By intuition, the firms will charge a higher selling price in the second period as compared to the first period because of the improved product quality. Interestingly, Proposition 2 shows that in the presence of online reviews, a higher product quality may induce a firm to charge a lower selling price in the second period. Specifically, online reviews enable the firms to mark-up or mark-down the price depending on the unit misfit cost t, the market size ρ, the quality difference in the first period, and customers' private perception of product quality x_0. This result differs from that of Jiang and Yang (2019), who indicate that a monopolistic firm should always mark-down the price in the second period in the presence of online reviews. Therefore, in contrast to the monopoly setting, competition induces firms to adopt a different pricing strategy. In particular, when the two firms have an identical cost parameter (i.e., $\alpha_1 = \alpha_2$), the equilibrium price in the second period, r_k^*, $k = 1, 2$, takes a similar form as the equilibrium price in the first period p_k^*, except that β is replaced by $\hat{\beta}$. Recall that $\hat{\beta} \geq \beta$, therefore the firms should always mark-up their prices in the second period (i.e., $r_k^* \geq p_k^*$). In this special case, p_k^* departs more significantly from r_k^* when the "fit probability" $\hat{\beta}$ is high, because a higher value of $\hat{\beta}$ induces a firm to raise the second-period selling price r_k^*. This seems to be against one's intuition because a higher value of $\hat{\beta}$ can induce the firm to reduce the quality improvement level in the second period (recall Theorem 5). However, such a result is consistent with the findings of Kostami and Rajagopalan (2014) and Chen and Jiang (2021), and is supported by the industrial practices that businesses often raise their product prices after developing a good review profile.

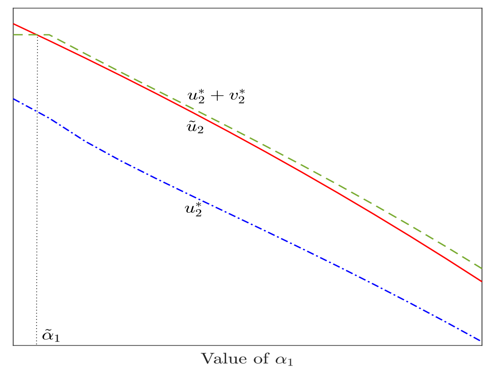
5.3 Dynamic vs. Static Competition

Recall that in the base model with exogenous prices, dynamic quality competition can induce the firms to offer a higher (or lower) product quality level (i.e., $\tilde{u}_k \leq u_k^* < u_k^* + v_k^*$ or $\tilde{u}_k > u_k^* + v_k^* > u_k^*$) as compared to the static competition scenario (see Theorem 3 and Figure 2). Similar to the base model, under the scenario with endogenous prices, as compared to static competition, it is possible that dynamic competition leads to a higher or lower product quality in both periods, depending on the R&D cost parameter. For example, when $\alpha_1 > \hat{\alpha}_1$, a higher product quality (i.e., $\tilde{u}_1 < u_1^* < u_1^* + v_1^*$) should be offered by firm 1 in both periods under the dynamic competition scenario, as shown in Figure 4(a); and for firm 2, when $\alpha_1 < \tilde{\alpha}_1$, a lower product quality (i.e., $\tilde{u}_2 > u_2^* + v_2^* > u_2^*$) is offered under the dynamic competition scenario, as shown in Figure 4(b).

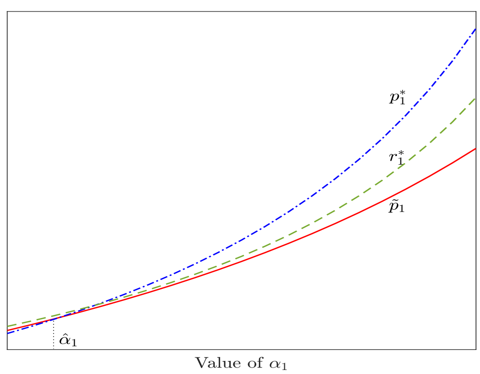
Moreover, Figure 4 shows that a higher R&D cost can induce a firm to offer a higher product quality level and selling price, whereas a higher R&D cost from the competitor can induce a firm to lower its product quality and selling price; this is in contrast to the base model with exogenous prices (recall Theorem 1).



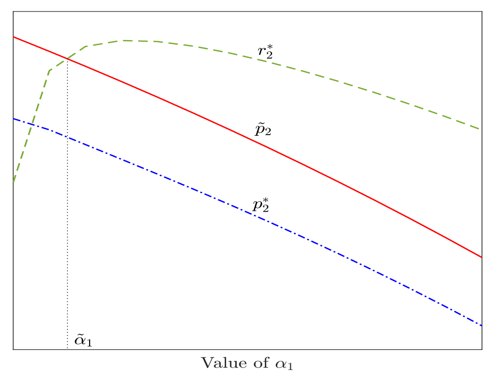
(a) Firm 1's Quality



(b) Firm 2's Quality



(c) Firm 1's Selling Prices

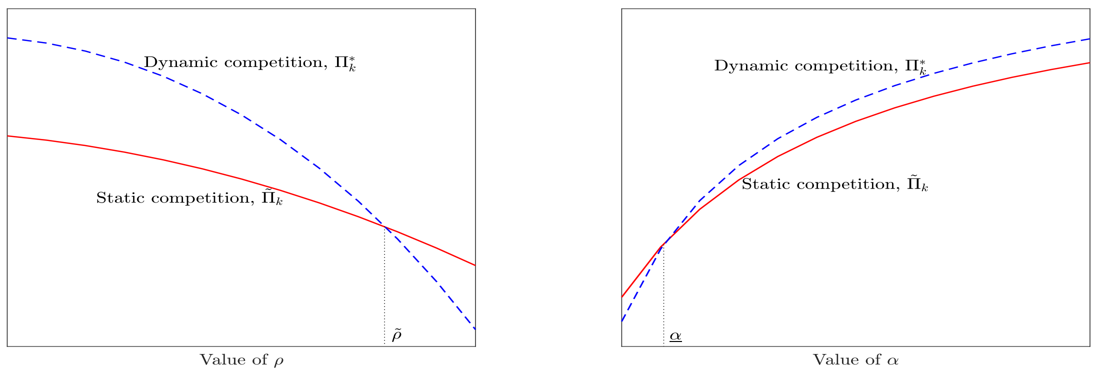


(d) Firm 2's Selling Prices

Figure 4 Equilibrium Decisions with Endogenous Prices: Dynamic v.s. Static Competition ($\alpha_1 > \alpha_2$)

For the equilibrium selling prices, similar to the equilibrium product quality, it is possible that dynamic competition leads to a higher (or lower) selling price in both periods. For example, Figure 4(c) shows that when firm 1 has a high R&D cost (i.e., $\alpha_1 > \hat{\alpha}_1$), it charges a higher selling price in both periods (i.e., $\tilde{p}_1 < p_1^*$ and $\tilde{p}_1 < r_1^*$) under the dynamic competition scenario. In particular, $\tilde{p}_1$ departs more significantly from p_1^* and r_1^* when the R&D cost parameter α_1 is high. And firm 2 adopts a lower selling price under dynamic competition when firm 1's R&D cost parameter α_1 is below $\tilde{\alpha}_1$ [see Figure 4(d)].

Next, we compare the equilibrium profits under dynamic and static competitions in order to evaluate the impact of making dynamic decisions. Evidently, Π_k^* could be lower than $\tilde{\Pi}_k$. For example, as Figure 5 shows, $\Pi_k^* < \tilde{\Pi}_k$ when $\rho > \tilde{\rho}$ or $\alpha < \underline{\alpha}$. This resembles the findings in the scenario with exogenous prices (recall Section 4.2). Therefore, it can be concluded that adjusting product quality and/or price over two periods does not necessarily improve the firms' profits (as compared to static competition).



(a) Market Size ρ (b) Cost Parameter α

Figure 5 Equilibrium Profit under Dynamic and Static Competition (with $\alpha = \alpha_1 = \alpha_2$)

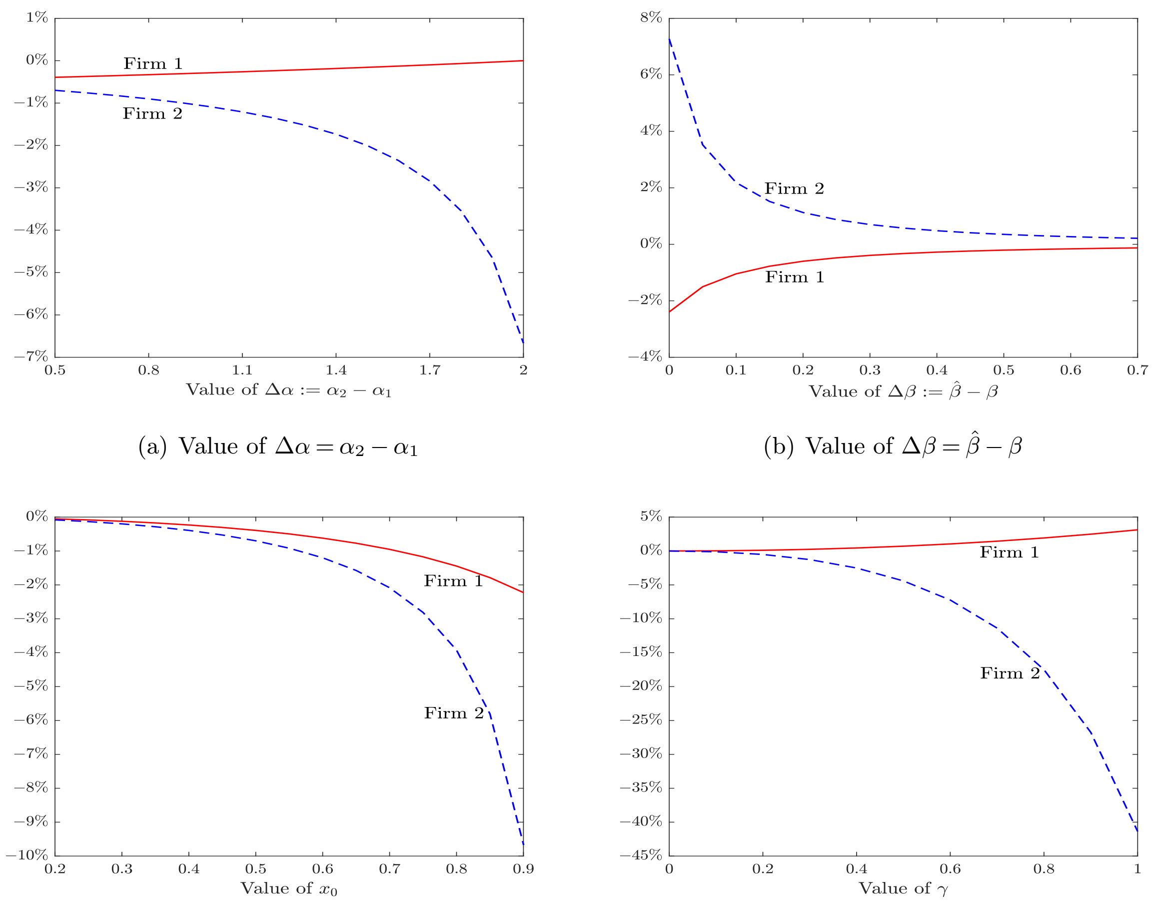
Finally, we summarize the main research findings in Table 1, which shows the similarities and differences across the two scenarios with exogenous and endogenous prices.

Table 1 Summary of Main Results

	Price	**Quality**	**Profit**
Exogenous prices	N/A	(i) Online reviews can result in a lower product quality with low α_k; (ii) Dynamic competition always leads to higher quality in both periods with $x_0 \leq 1$.	Dynamic competition can have a negative or positive effect on profits.
Endogenous prices	(i) Online reviews can push up (or down) selling price in the late period; (ii) Dynamic competition leads to higher (or lower) selling prices in both periods depending on α_k.	(i) Online reviews can lead to a lower quality level with $x_0 \leq 1$; (ii) Dynamic competition leads to higher (or lower) quality level in both periods with $x_0 \leq 1$.	Dynamic competition can have a negative or positive effect on profits.

6 Numerical Experiments

In this section, we evaluate the impact of dynamic competition, as compared to static competition, through numerical experiments. The base parameters are set as follows: $\alpha_1 = 3.5$, $\alpha_2 = 4$, $\rho = 1$, $\beta = 0.3$, $\hat{\beta} = 0.6$, $\gamma = 0.4$, $x_0 = 0.5$, $t = 1$, $p_1 = 2$, and $p_2 = 2$. In each group of experiments we alter the value of one parameter and evaluate each firm's profit improvement or loss (in percentage) from dynamic competition relative to static competition for the scenarios with exogenous prices and endogenous prices, respectively; the major results are plotted in Figures 6 and 7.



(a) Value of $\Delta\alpha = \alpha_2 - \alpha_1$

(b) Value of $\Delta\beta = \hat{\beta} - \beta$

(c) Value of x_0

(d) Value of γ

Figure 6 Profit Improvement/Loss from Dynamic Competition with Exogenous Prices

In the first group of experiments, we examine the impact of difference in R&D cost parameter between the competing firms, as measured by $\Delta\alpha := \alpha_2 - \alpha_1$. By intuition, a cost disadvantage leads to a reduced profit; this is validated by the observation from Figure 6(a), which shows that the profit of firm 2 with a higher R&D cost decreases when the two firms are allowed to improve their quality level in the second period, and a more significant cost disadvantage leads to a more significant profit loss. Interestingly, the firm with a cost advantage (i.e., firm 1) benefits from dynamic competition only when its cost advantage exceeds a threshold value. This implies that dynamic competition may lead to a lose-lose situation when the difference in R&D parameter is slight. Differing from the scenario in which prices are exogenous, dynamic competition can increase

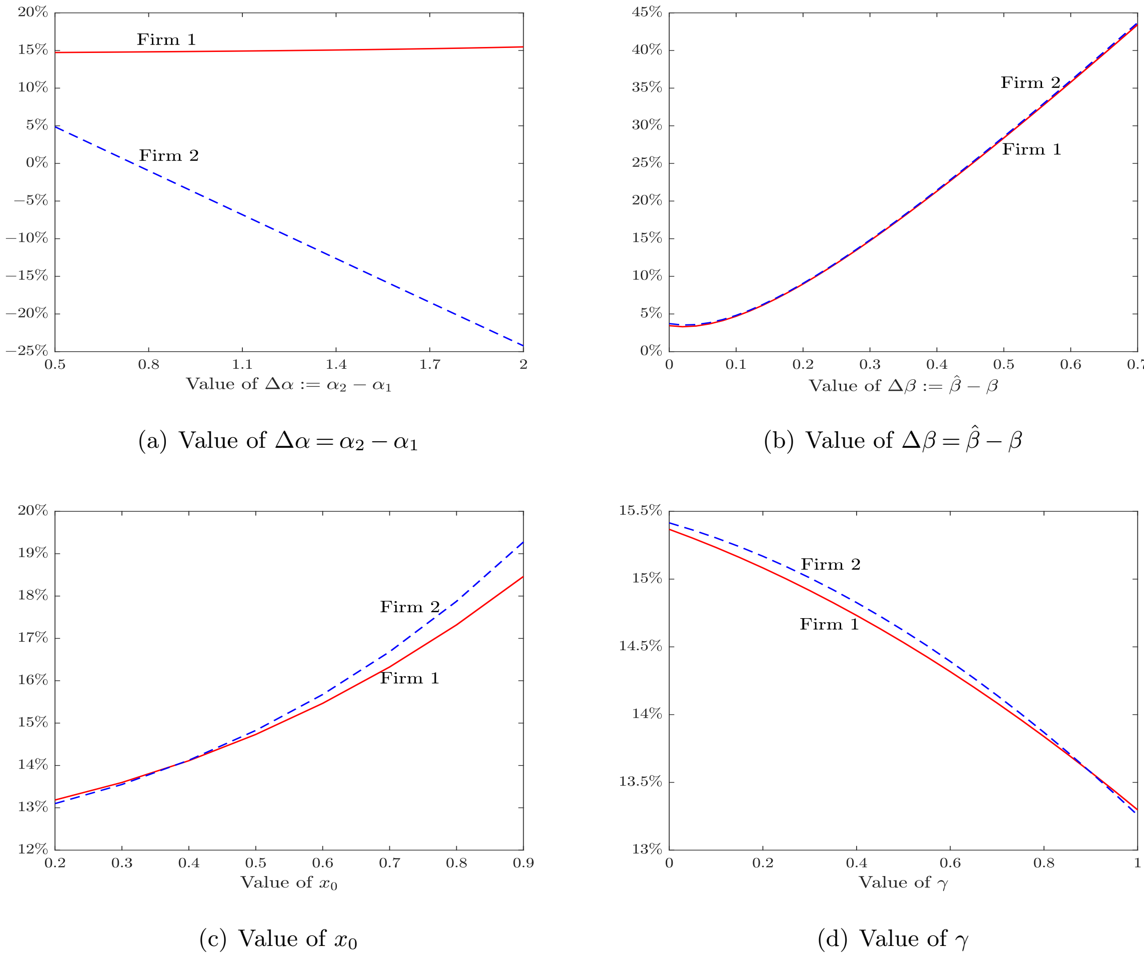


Figure 7 Profit Improvement/Loss from Dynamic Competition with Endogenous Prices

the profit of firm 1 with a cost advantage even if a small cost advantage when the two firms can endogenously set their selling prices, as shown in Figure 7(a). For firm 2 with a higher cost, dynamic competition can also result in an increased profit when his cost disadvantage is slight. This implies that incorporating endogenous selling prices can lead to increased profits for both firms when the difference in R&D cost parameter is slight. Overall, as Figure 6(a) and Figure 7(a) show, when the cost difference is slight, the dynamic quality decision alone may lead to a lose-lose situation; whereas the dynamic joint decisions on quality and price can induce a win-win situation for the two competing firms. Moreover, as compared to the firm with a lower cost parameter, the cost difference has a more significant impact on the equilibrium profit of the firm with a cost disadvantage under both competition scenarios.

In the second group of experiments, we examine the impact of the information efficiency on the product fit of online reviews, as measured by $\Delta\beta := \hat{\beta} - \beta$. As Figure 6(b) shows, firm 2 (who has a higher cost) benefits from the dynamic quality improvement decision, while higher information efficiency of online reviews (a higher value of $\Delta\beta$) may diminish such benefit. This implies that higher information efficiency regarding the fitness of products from online reviews may reduce the benefit of dynamic quality improvement. In contrast, the dynamic quality improvement strategy may hurt the profit of firm 1 who has a lower cost parameter, and the relative loss is decreasing in $\Delta\beta$. This can be primarily attributed to the fact that with higher information efficiency of online reviews (i.e., $\Delta\beta$ is high), the impact of a high quality on customer demand is relatively insignificant. However, Figure 7(b) shows that the dynamic quality and price decision can result in increased profits for both firms and that the benefits become more significant if the information efficiency of online reviews $\Delta\beta$ increases. This is because, evaluation of other customers regarding the product fit in online reviews may induce the firm to provide a lower quality level, and consequently, the quality competition lessens and both firms' profits are improved.

In the third group of experiments, we examine the impact of customers' private perception of product quality x_0. As Figure 6(c) shows, the dynamic quality improvement strategy can decrease the profitability of both firms and a higher value of customers' private perception of product quality x_0 can lead to more significantly reduced profit, especially for firm 2 with a higher cost parameter. It seems to be counter-intuitive. This is because, firms have an incentive to position even higher quality level when customers' private perception of product quality x_0 increases. However, with competition and the under-estimated ($x_0 < 1$) product value, the impact of a high quality on customer demand is relatively insignificant. In contrast, Figure 7(c) shows that the dynamic quality and price decision can increase both firms' profitability and that the profit gain is increasing in customers' private perception of product quality x_0. This is mainly due to the fact that the firms will choose to improve their profitability by using the dynamic pricing strategy that is cheaper and easier to implement to mitigate the quality competition.

Finally, Figure 6(d) and Figure 7(d) illustrate the impact of customers' weight of perceived quality from product description in the second period γ. As Figure 6(d) shows, firm 1 with a lower cost parameter can benefit from the dynamic quality improvement strategy, and this benefit is increasing in γ, whereas the dynamic quality improvement hurts firm 2 with a higher cost parameter and is more harmful if the customer weight of perceived quality from product description in the second period γ is large. That is, the dynamic quality improvement strategy can become more beneficial for the firm with a cost advantage when customers pay less attention to online reviews (which corresponds to a large value of γ) but not for competing firm 2 with a higher cost parameter. Intuitively, with customers paying more attention to online reviews (i.e., when γ is small), the firms may be incentivized by online reviews to improve the quality level to attract customers. As such, the intensified quality competition leads to the reduced profit of firm 1 with a cost advantage but the higher quality level reduces the financial loss of firm 2 with a higher cost parameter. By contrast, as shown in Figure 7(d), the joint quality and price decisions increase the profits of both firms and the benefits are more significant if customers pay more attention to online reviews (i.e., γ is small). This can be explained by the fact that, a joint quality and price strategy will neutralize the quality and price differentiation between the rival firms and, consequently, a larger customer weight on online reviews will mitigate the quality and price competition.

7 Concluding Remarks

This study explores the quality and price decisions for two rival firms selling substitutable products in the presence of online reviews following two-period game-theoretical models. We start our analysis with the benchmark scenario under which product prices are exogenous and the firms compete on product quality to optimize their respective profits. We study the equilibrium quality decisions under static and dynamic competitions, respectively. We then consider the extended scenario in which the product prices are endogenous and the firms make joint decisions on product quality and selling price over two periods. We compare the equilibrium decisions between static and dynamic competitions. Moreover, we conduct numerical experiments to evaluate the impact of key parameters on the firms' profits from dynamic competition.

The findings provide several interesting managerial insights that are different from the literature. First, we show that in a competitive market with online reviews, dynamically adjusting product quality (and price) may not always be effective in improving firms' profits; this is in sharp contrast to the existing studies for a monopoly market (e.g., Zhao and Zhang 2019). Second, the product information disclosure from online reviews may induce a firm to improve or decrease its quality level, depending on the cost parameter as well as pricing policy. This finding contradicts with previous studies, which claim that online reviews can normally motivate firms to improve product quality (e.g., Sun and Xu 2018, Jiang and Yang 2019). Third, our results show that online reviews may enable the firms to mark-up or mark-down the selling price over time, depending on the combination of parameters. This finding is different from that of Jiang and Yang (2019), who claim that the early price is often higher than that in late periods in a monopoly market. Our result highlights the importance of market competition in determining firms' price decisions (Feng et al. 2018). In particular, we show that when online review plays a more important role in impacting demands, both firms may tend to lower their product price.

This study also has some limitations, suggesting several promising future research avenues. First, our model can be extended to the scenario with more than two competing firms, selling multiple substitutable and/or complementary products, and over multiple periods. Second, to focus on the effects of online reviews and competition, this study does not consider customer returns. However, online shopping often has a high return rate due to the information asymmetry between the seller and customers (Fan and Chen 2020). High return rates lead to increased reverse logistics costs and overstocked inventories for retailers, and should be taken into consideration when making the quality and price decisions. Therefore, a potential research avenue is to incorporate customer returns into our model. Third, we do not consider the strategic choice behavior of customers. In anticipating a product with a possible better quality and a possible lower price in the second period, a portion of forward-looking customers may choose to wait in the first period (Lobel et al. 2016). Therefore, another potential research direction is to evaluate the product quality and price decisions in the presence of both strategic and myopic customers.

Acknowledgement

We are grateful to the Department Editor, the Associate Editor, and two anonymous referees for their valuable comments that have helped improve the paper significantly. This work has been partially supported by the National Key R&D Program of China under Grant No. 2020AAA0103801 and the National Natural Science Foundation of China under Grant No. 72125002.

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E-Companion for "Effects of Online Reviews and Competition on Quality and Pricing Strategies"

EC.1 A List of Critical Values

For ease of presentation, Table EC.1 provides a list of critical values used in this paper.

Table EC.1 A List of Critical Values

Notation	Formula
$\hat{x}_0$	$\min(x_0, \gamma x_0 + 1 - \gamma)$
$\alpha_1^{(1)}$	$\frac{\hat{x}_0[\hat{\beta}x_0+(\gamma x_0+1-\gamma)\rho\beta]\alpha_2 p_1}{4\alpha_2\beta\hat{\beta}t(t+p_1+p_2)-\hat{x}_0[\hat{\beta}x_0+(\gamma x_0+1-\gamma)\rho\beta]p_2}$
$\alpha_1^{(2)}$	$\frac{\rho(\gamma x_0)^2\alpha_2 p_1}{4\alpha_2\hat{\beta}t[t+p_1+p_2-(\gamma x_0+1-\gamma)(u_1+u_2)]-\rho(\gamma x_0)^2 p_2}$
$\alpha_1^{(3)}$	$\frac{4\gamma^2 x_0^3[\hat{\beta}x_0+\rho\beta(\gamma x_0+1-\gamma)]\alpha_2 p_1}{4\alpha_2\beta\hat{\beta}t(t+p_1+p_2)[4\gamma^2 x_0^4+2(\gamma x_0+1-\gamma)(1-\gamma)(1-x_0)]-4\gamma^2 x_0^3[\hat{\beta}x_0+\rho\beta(\gamma x_0+1-\gamma)]p_2}$
$\alpha_2^{(1)}$	$\frac{\hat{x}_0[\hat{\beta}x_0+(\gamma x_0+1-\gamma)\rho\beta]p_2}{4\beta\hat{\beta}t(t+p_1+p_2)}$
$\alpha_2^{(2)}$	$\frac{\rho\gamma^2 x_0^2 p_2}{4\hat{\beta}t[t+p_1+p_2-(\gamma x_0+1-\gamma)(u_1+u_2)]}$
$\alpha_2^{(3)}$	$\frac{\gamma^2 x_0^3[\hat{\beta}x_0+\rho\beta(\gamma x_0+1-\gamma)]p_2}{\beta\hat{\beta}t(t+p_1+p_2)[4\gamma^2 x_0^2+2(\gamma x_0+1-\gamma)(1-\gamma)(1-x_0)]}$
$\alpha_k^{(4)}$	$\frac{p_k\rho\gamma x_0^2(\gamma x_0+1-\gamma)}{2(t+p_1+p_2)(x_0-1)(1-\gamma)\hat{\beta}t}$
$\check{\alpha}_k$	$\frac{\alpha_{3-k}\rho^2\gamma^4 x_0^4}{24\alpha_{3-k}\hat{\beta}t(\rho\gamma^2 x_0^2-3\alpha_{3-k}\hat{\beta}t)-\rho^2\gamma^4 x_0^4}$
K_1	$[12\alpha_1\alpha_2\hat{\beta}t-(\alpha_1+\alpha_2)\rho\gamma^2 x_0^2]^2$
K_2	$2\rho\alpha_k^2\alpha_{3-k}(\gamma x_0+1-\gamma)^2[8\alpha_{3-k}\hat{\beta}t-\rho(\gamma x_0)^2]+\frac{x_0^2[12\alpha_1\alpha_2\hat{\beta}t-(\alpha_1+\alpha_2)\rho\gamma^2 x_0^2]^2}{6\beta t}$
K_3	$2\rho\alpha_k\alpha_{3-k}^2(\gamma x_0+1-\gamma)^2[8\alpha_k\hat{\beta}t-\rho(\gamma x_0)^2]+\frac{x_0^2[12\alpha_1\alpha_2\hat{\beta}t-(\alpha_1+\alpha_2)\rho\gamma^2 x_0^2]^2}{6\beta t}$
K_4	$[8\alpha_1\hat{\beta}t-\rho(\gamma x_0)^2][6\alpha_2\hat{\beta}t-\rho(\gamma x_0)^2]$
K_5	$[8\alpha_2\hat{\beta}t-\rho(\gamma x_0)^2][6\alpha_1\hat{\beta}t-\rho(\gamma x_0)^2]$
K_6	$(1+2\hat{\beta})t\sqrt{K_1}-\rho(\gamma x_0)^2[3(\alpha_1+\alpha_2)\hat{\beta}t-\rho(\gamma x_0)^2]$
$\Delta u_k^{(1)}$	$\frac{3\hat{\beta}t[2\alpha_1\alpha_2-\alpha_{3-k}(\alpha_1+\alpha_2)]}{\alpha_{3-k}(\gamma x_0+1-\gamma)(\alpha_1+\alpha_2)}$
$\Delta u_k^{(2)}$	$\frac{3\hat{\beta}t[\alpha_{3-k}(\alpha_1+\alpha_2)-2\alpha_1\alpha_2]}{\alpha_{3-k}(\gamma x_0+1-\gamma)(\alpha_1+\alpha_2)}$
$\Delta u_k^{(3)}$	$\frac{24\alpha_1\alpha_2\hat{\beta}t[\rho(\gamma x_0)^2-3\alpha_{3-k}\hat{\beta}t]-(\alpha_1+\alpha_2)\rho^2\gamma^4 x_0^4}{24\alpha_1\alpha_2\hat{\beta}t\alpha_{3-k}(\gamma x_0+1-\gamma)}$
$\Delta u_k^{(4)}$	$\frac{12\alpha_{3-k}\hat{\beta}t[6\alpha_1\alpha_2\hat{\beta}t-(\alpha_1+\alpha_2)\rho(\gamma x_0)^2]+(\alpha_1+\alpha_2)\rho^2\gamma^4 x_0^4}{2\alpha_{3-k}(\gamma x_0+1-\gamma)(\alpha_1+\alpha_2)\rho(\gamma x_0)^2}$
$\Delta u_k^{(5)}$	$-\frac{12\alpha_1\alpha_2\hat{\beta}t^2(\hat{\beta}-\beta)+\rho(\gamma x_0)^2 t[(\alpha_1+\alpha_2)\beta-2\alpha_k\hat{\beta}]}{4\alpha_1\alpha_2\hat{\beta}t(1-\gamma)(1-x_0)+\frac{1}{3}x_0(\alpha_1+\alpha_2)\rho(\gamma x_0)^2}$
$\hat{\rho}$	$\frac{12\alpha_1\alpha_2\hat{\beta}t}{(\alpha_1+\alpha_2)(\gamma x_0)^2}$
$\hat{t}$	$\frac{x_0(\alpha_1+\alpha_2)\rho(\gamma x_0)^2}{12\alpha_1\alpha_2\hat{\beta}(1-\gamma)(x_0-1)}$

EC.2 Proof of Theorems and Propositions

Proof of Theorem 1. We first investigate firm 1's optimal quality level decision in response to the other firm's decision u_2. Note that to induce all the customers to buy a product, firm 1's quality level should be higher than a critical value, i.e.,

$$u_1 \geq \max\left\{\frac{1}{x_0}, \frac{1}{\gamma x_0 + 1 - \gamma}\right\}(t + p_1 + p_2) - u_2 = \frac{t + p_1 + p_2}{\hat{x}_0} - u_2,$$

which establishes a lower bound for u_1.

By Equation (6), firm 1's profit function is

$$\Pi_1(u_1, u_2) = \Big(\frac{1}{2} + \frac{x_0(u_1 - u_2) - p_1 + p_2}{2\beta t}\Big)p_1 - \alpha_1 u_1^2 + \Big(\frac{1}{2} + \frac{(\gamma x_0 + 1 - \gamma)(u_1 - u_2) - p_1 + p_2}{2\hat{\beta} t}\Big)\rho p_1,$$

which is apparently concave in u_1. The first-order condition of $\Pi_1(u_1, u_2)$ with respect to u_1 yields the following solution:

$$u_1^{(1)} = \frac{[\hat{\beta} x_0 + (\gamma x_0 + 1 - \gamma)\rho\beta]p_1}{4\alpha_1 \beta\hat{\beta} t}.$$

Therefore, the best response curve of firm 1, written as a function of u_2, is

$$\tilde{u}_1(u_2) = \begin{cases} u_1^{(1)} & \text{if } u_2 \geq u_2^{(2)}, \\ \frac{t + p_1 + p_2}{\hat{x}_0} - u_2 & \text{otherwise}, \end{cases}$$

where

$$u_2^{(2)} = \frac{t + p_1 + p_2}{\hat{x}_0} - \frac{[\hat{\beta} x_0 + (\gamma x_0 + 1 - \gamma)\rho\beta]p_1}{4\alpha_1 \beta\hat{\beta} t}.$$

Similarly, given firm 1's quality decision u_1, firm 2's best response curve is

$$\tilde{u}_2(u_1) = \begin{cases} u_2^{(1)} & \text{if } u_1 \geq u_1^{(2)}, \\ \frac{t + p_1 + p_2}{\hat{x}_0} - u_1 & \text{otherwise}, \end{cases}$$

where

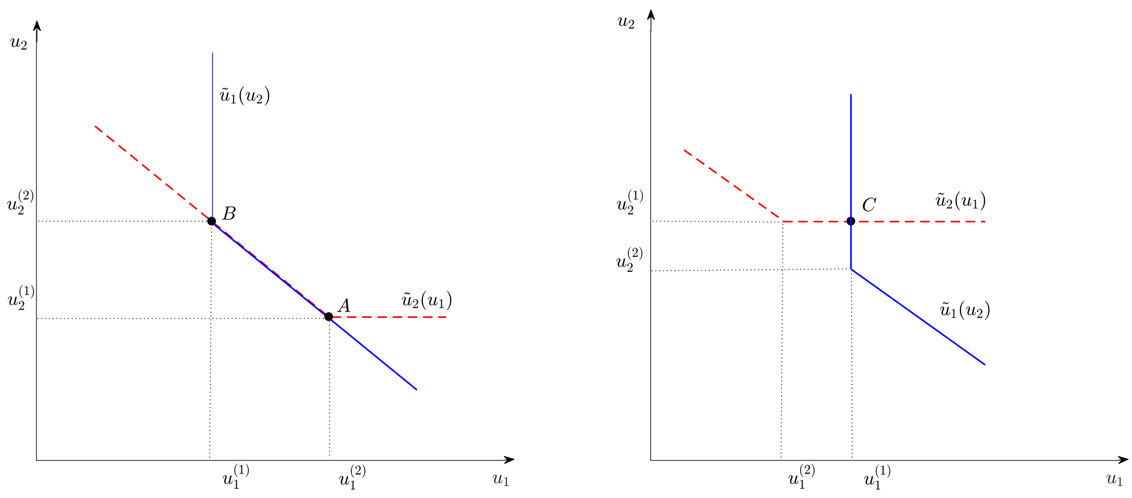
$$\begin{aligned} u_2^{(1)} &= \frac{[\hat{\beta} x_0 + (\gamma x_0 + 1 - \gamma)\rho\beta]p_2}{4\alpha_2 \beta\hat{\beta} t}, \\ u_1^{(2)} &= \frac{t + p_1 + p_2}{\hat{x}_0} - \frac{[\hat{\beta} x_0 + (\gamma x_0 + 1 - \gamma)\rho\beta]p_2}{4\alpha_2 \beta\hat{\beta} t}. \end{aligned}$$

Consider the following two cases.

- If $u_1^{(1)} < u_1^{(2)}$, i.e., $\alpha_1 > \alpha_1^{(1)}$ and $\alpha_2 > \alpha_2^{(1)}$, we can show that

$$u_2^{(2)} = \tilde{u}_2(u_1^{(1)}).$$

As such, the best response curves of firms 1 and 2 are illustrated as Figure EC.1(a). Clearly, any point located on line AB is an equilibrium.



(a) $u_1^{(1)} < u_1^{(2)}$ (b) $u_1^{(1)} \geq u_1^{(2)}$

Figure EC.1 Best Response Curves under Static Competition with Exogenous Prices

- If $u_1^{(1)} \geq u_1^{(2)}$, i.e., $\alpha_1 \leq \alpha_1^{(1)}$ or $\alpha_2 \leq \alpha_2^{(1)}$, we must have $u_2^{(1)} \geq u_2^{(2)}$. As such, the best response curves of firms 1 and 2 are illustrated as Figure EC.1(b). Clearly, point $C(u_1^{(1)}, u_2^{(1)})$ is the unique equilibrium between the firms.

This completes the proof. $\square$

Proof of Theorem 2. We first investigate firm 1's optimal quality improvement decision in response to the other firm's decision v_2 for any given initial quality level (u_1, u_2) in the first period. Note that to induce all the customers to buy a product, firm 1's quality improvement level should be higher than a critical value, i.e.,

$$v_1 \geq -v_2 + \frac{t + p_1 + p_2 - (\gamma x_0 + 1 - \gamma)(u_1 + u_2)}{\gamma x_0},$$

which establishes a lower bound for v_1.

By Equation (4), firm 1's profit function in the second period is

$$\Pi_1^{(2)}(v_1, v_2) = \Big(\frac{1}{2} + \frac{(\gamma x_0 + 1 - \gamma)(u_1 - u_2) + \gamma x_0 (v_1 - v_2) - p_1 + p_2}{2\hat{\beta} t}\Big)\rho p_1 - \alpha_1 v_1^2,$$

which is apparently concave in v_1. Therefore, the first-order condition of $\Pi_1^{(2)}(v_1, v_2)$ with respect to v_1 yields the following solution:

$$v_1^{(1)} = \frac{\rho \gamma x_0 p_1}{4\alpha_1 \hat{\beta} t}.$$

Therefore, the best response curve of firm 1 can be written as a function of v_2

$$v_1^*(v_2) = \begin{cases} v_1^{(1)} & \text{if } v_2 \geq v_2^{(2)}, \\ -v_2 + \frac{t + p_1 + p_2 - (\gamma x_0 + 1 - \gamma)(u_1 + u_2)}{\gamma x_0} & \text{otherwise,} \end{cases}$$

where

$$v_2^{(2)} = -\frac{\rho\gamma x_0 p_1}{4\alpha_1\hat{\beta}t} + \frac{t+p_1+p_2-(\gamma x_0+1-\gamma)(u_1+u_2)}{\gamma x_0}.$$

Similarly, given firm 1's quality improvement decision v_1, firm 2's best response curve is

$$v_2^*(v_1) = \begin{cases} v_2^{(1)} & \text{if } v_1 \geq v_1^{(2)}, \\ -v_1 + \frac{t+p_1+p_2-(\gamma x_0+1-\gamma)(u_1+u_2)}{\gamma x_0} & \text{otherwise,} \end{cases}$$

where

$$v_2^{(1)} = \frac{\rho\gamma x_0 p_2}{4\alpha_2\hat{\beta}t},$$
$$v_1^{(2)} = -\frac{\rho\gamma x_0 p_2}{4\alpha_2\hat{\beta}t} + \frac{t+p_1+p_2-(\gamma x_0+1-\gamma)(u_1+u_2)}{\gamma x_0}.$$

It's not difficult to show that when $u_1+u_2 \geq \frac{t+p_1+p_2}{\gamma x_0+1-\gamma}$, $(v_1^{(1)}, v_2^{(1)})$ is the only equilibrium between the firms. On the other hand, when $u_1+u_2 < \frac{t+p_1+p_2}{\gamma x_0+1-\gamma}$, we have the following two cases:

- If $v_1^{(1)} < v_1^{(2)}$, i.e., $\alpha_1 > \alpha_1^{(2)}$ and $\alpha_2 > \alpha_2^{(2)}$, we can show that

$$v_2^{(2)} = v_2^*(v_1^{(1)}).$$

As such, the best response curves of firms 1 and 2 are illustrated as Figure EC.2(a). Clearly, any point located on line AB is an equilibrium.

- If $v_1^{(1)} \geq v_1^{(2)}$, i.e., $\alpha_1 \leq \alpha_1^{(2)}$ or $\alpha_2 \leq \alpha_2^{(2)}$, we must have $v_2^{(1)} \geq v_2^{(2)}$. As such, the best response curves of firms 1 and 2 are illustrated as Figure EC.2(b). Clearly, point $C(v_1^{(1)}, v_2^{(1)})$ is the unique equilibrium between the firms.

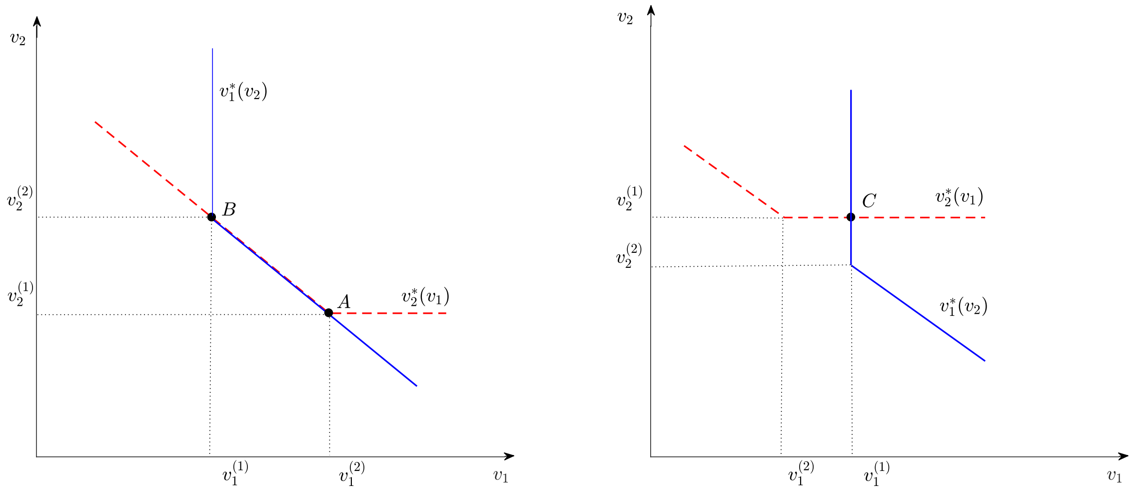
This completes the proof. □

Proof of Theorem 3. We first study the firm 1's optimal initial quality decision in response to the other firm's decision u_2. Note that to induce all the customers to buy a product in the first period, firm 1's initial quality level should be higher than a critical value, i.e.,

$$u_1 \geq \frac{t+p_1+p_2}{x_0} - u_2,$$

which establishes a lower bound for u_1. Note that the equilibrium quality improvement decision is given by Equation (9) or Equation (10). First, by substituting v_k^* in Equation (9) into Equation (12), firm 1's total profit function is

$$\begin{aligned}\Pi_1(u_1,u_2) = &\Big(\frac{1}{2} + \frac{x_0(u_1-u_2)-p_1+p_2}{2\beta t}\Big)p_1 - \alpha_1 u_1^2 - \alpha_1\Big(\frac{\rho\gamma x_0 p_1}{4\alpha_1\hat{\beta}t}\Big)^2 \\ &+ p_1\rho\Big[\frac{1}{2} + \frac{1}{2\hat{\beta}t}\Big((\gamma x_0+1-\gamma)(u_1-u_2) + \frac{\rho(\gamma x_0)^2(\alpha_2 p_1 - \alpha_1 p_2)}{4\alpha_1\alpha_2\hat{\beta}t} - p_1 + p_2\Big)\Big],\end{aligned}$$



(a) $v_1^{(1)} < v_1^{(2)}$ (b) $v_1^{(1)} \geq v_1^{(2)}$

Figure EC.2 Best Response Curves in Period 2 under Dynamic Competition

which is apparently concave in u_1. The first-order condition with respect to u_1 yields the following solution:

$$u_1^{(1)} = \frac{[\hat{\beta}x_0 + (\gamma x_0 + 1 - \gamma)\rho\beta]p_1}{4\alpha_1\beta\hat{\beta}t}.$$

Therefore, the best response curve of firm 1, written as a function of u_2, is

$$u_1^*(u_2) = \begin{cases} u_1^{(1)} & \text{if } u_2 \geq u_2^{(2)}, \\ \frac{t+p_1+p_2}{x_0} - u_2 & \text{otherwise}, \end{cases}$$

where

$$u_2^{(2)} = -\frac{[\hat{\beta}x_0 + (\gamma x_0 + 1 - \gamma)\rho\beta]p_1}{4\alpha_1\beta\hat{\beta}t} + \frac{t+p_1+p_2}{x_0}.$$

Similarly, given firm 1's quality level decision u_1, firm 2's best response curve is

$$u_2^*(u_1) = \begin{cases} u_2^{(1)} & \text{if } u_1 \geq u_1^{(2)}, \\ \frac{t+p_1+p_2}{x_0} - u_1 & \text{otherwise}, \end{cases}$$

where

$$u_2^{(1)} = \frac{[\hat{\beta}x_0 + (\gamma x_0 + 1 - \gamma)\rho\beta]p_2}{4\alpha_2\beta\hat{\beta}t},$$

$$u_1^{(2)} = -\frac{[\hat{\beta}x_0 + (\gamma x_0 + 1 - \gamma)\rho\beta]p_2}{4\alpha_2\beta\hat{\beta}t} + \frac{t+p_1+p_2}{x_0}.$$

Therefore, the best response curves take a similar form as those under static competition (refer to the proof of Theorem 1), except that $\hat{x}_0$ is replaced by x_0. Consequently, the equilibrium takes a similar form as the equilibrium in the Theorem 1.

On the other hand, by substituting v_k^* in Equation (10) into Equation (12), firm 1's total profit function is

$$\begin{aligned}\Pi_1(u_1,u_2) = & \Big(\frac{1}{2}+\frac{x_0(u_1-u_2)-p_1+p_2}{2\beta t}\Big)p_1-\alpha_1 u_1^2 \\ & +p_1\rho\Big[\frac{1}{2}+\frac{1}{2\hat{\beta}t}\Big((\gamma x_0+1-\gamma)(u_1-u_2)+\frac{\rho(\gamma x_0)^2(\alpha_2p_1-\alpha_1p_2)}{4\alpha_1\alpha_2\hat{\beta}t}-p_1+p_2\Big)\Big] \\ & -\alpha_1\Big(\frac{t+p_1+p_2-(\gamma x_0+1-\gamma)(u_1+u_2)}{2\gamma x_0}+\frac{\rho\gamma x_0(\alpha_2p_1-\alpha_1p_2)}{8\alpha_1\alpha_2\hat{\beta}t}\Big)^2,\end{aligned}$$

which is apparently concave in u_1. The first-order condition with respect to u_1 yields the following solution:

$$\begin{aligned}\bar{u}_1 = & -\frac{(\gamma x_0+1-\gamma)^2}{(2\gamma x_0)^2+(\gamma x_0+1-\gamma)^2}u_2+\frac{2(\gamma x_0)^2}{\alpha_1[(2\gamma x_0)^2+(\gamma x_0+1-\gamma)^2]}\Big[\frac{\Big(\hat{\beta}x_0+(\gamma x_0+1-\gamma)\rho\beta\Big)p_1}{2\beta\hat{\beta}t} \\ & +2\alpha_1\Big(\frac{t+p_1+p_2}{2\gamma x_0}+\frac{\rho\gamma x_0(\alpha_2p_1-\alpha_1p_2)}{8\alpha_1\alpha_2\hat{\beta}t}\Big)\frac{\gamma x_0+1-\gamma}{2\gamma x_0}\Big].\end{aligned}$$

Therefore, the best response curve of firm 1, written as a function of u_2, is

$$u_1^*(u_2)=\begin{cases}\bar{u}_1 & \text{if } u_2\geq u_2^{(2)},\\ \frac{t+p_1+p_2}{x_0}-u_2 & \text{otherwise,}\end{cases}$$

where

$$\begin{aligned}u_2^{(2)} = & \frac{(2\gamma x_0)^2+(\gamma x_0+1-\gamma)^2}{(2\gamma x_0)^2}.\frac{t+p_1+p_2}{x_0}-\frac{1}{2\alpha_1}\Big[\frac{\Big(\hat{\beta}x_0+(\gamma x_0+1-\gamma)\rho\beta\Big)p_1}{2\beta\hat{\beta}t} \\ & +2\alpha_1\Big(\frac{t+p_1+p_2}{2\gamma x_0}+\frac{\rho\gamma x_0(\alpha_2p_1-\alpha_1p_2)}{8\alpha_1\alpha_2\hat{\beta}t}\Big)\frac{\gamma x_0+1-\gamma}{2\gamma x_0}\Big].\end{aligned}$$

Similarly, given firm 1's quality level decision u_1, firm 2's best response curve is

$$u_2^*(u_1)=\begin{cases}\bar{u}_2 & \text{if } u_1\geq u_1^{(2)},\\ \frac{t+p_1+p_2}{x_0}-u_1 & \text{otherwise,}\end{cases}$$

where

$$\begin{aligned}\bar{u}_2 = & -\frac{(\gamma x_0+1-\gamma)^2}{(2\gamma x_0)^2+(\gamma x_0+1-\gamma)^2}u_1+\frac{2(\gamma x_0)^2}{\alpha_2[(2\gamma x_0)^2+(\gamma x_0+1-\gamma)^2]}\Big[\frac{\Big(\hat{\beta}x_0+(\gamma x_0+1-\gamma)\rho\beta\Big)p_2}{2\beta\hat{\beta}t} \\ & +2\alpha_2\Big(\frac{t+p_1+p_2}{2\gamma x_0}+\frac{\rho\gamma x_0(\alpha_1p_2-\alpha_2p_1)}{8\alpha_1\alpha_2\hat{\beta}t}\Big)\frac{\gamma x_0+1-\gamma}{2\gamma x_0}\Big], \\ u_1^{(2)} = & \frac{(2\gamma x_0)^2+(\gamma x_0+1-\gamma)^2}{(2\gamma x_0)^2}.\frac{t+p_1+p_2}{x_0}-\frac{1}{2\alpha_2}\Big[\frac{\Big(\hat{\beta}x_0+(\gamma x_0+1-\gamma)\rho\beta\Big)p_2}{2\beta\hat{\beta}t} \\ & +2\alpha_2\Big(\frac{t+p_1+p_2}{2\gamma x_0}+\frac{\rho\gamma x_0(\alpha_1p_2-\alpha_2p_1)}{8\alpha_1\alpha_2\hat{\beta}t}\Big)\frac{\gamma x_0+1-\gamma}{2\gamma x_0}\Big].\end{aligned}$$

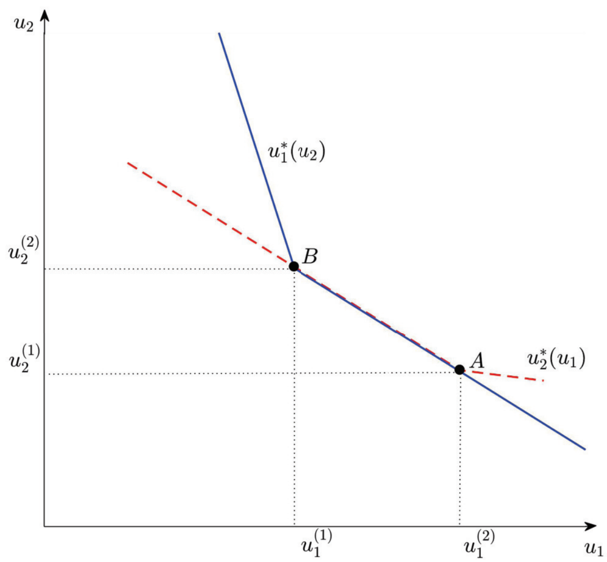
Consider the following two cases.

- If $u_1^{(1)} < u_1^{(2)}$, i.e., $\alpha_1 > \alpha_1^{(3)}$ and $\alpha_2 > \alpha_2^{(3)}$, we can show that

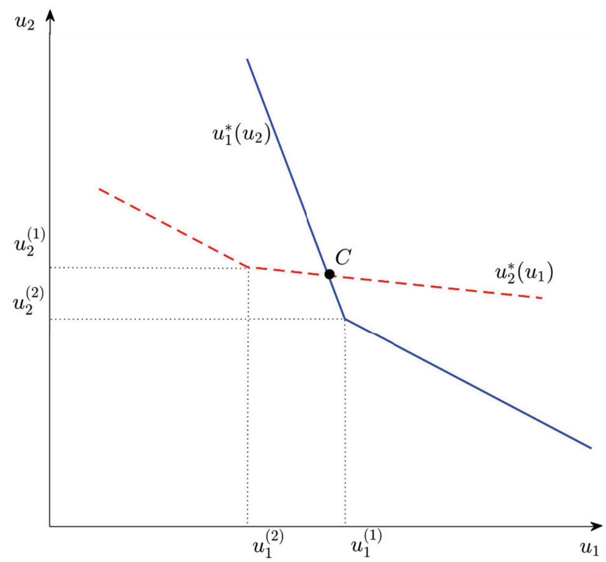
$$u_2^{(2)} = u_2^*(u_1^{(1)}).$$

As such, the best response curves of firms 1 and 2 are illustrated as Figure EC.3(a). Clearly, any point located on line AB is an equilibrium.

- If $u_1^{(1)} \geq u_1^{(2)}$, i.e., $\alpha_1 \leq \alpha_1^{(3)}$ or $\alpha_2 \leq \alpha_2^{(3)}$, we must have $u_2^{(1)} \geq u_2^{(2)}$. As such, the best response curves of firms 1 and 2 are illustrated as Figure EC.3(b). Clearly, point C is the unique equilibrium between the firms.



(a) $u_1^{(1)} < u_1^{(2)}$



(b) $u_1^{(1)} \geq u_1^{(2)}$

Figure EC.3 Best Response Curves in Period 1 under Dynamic Competition

This completes the proof. $\square$

Proof of Proposition 1. By Theorem 1, Equation (11) and Theorem 3 (iii), when customers tend to under-estimate the product value, the firm k's difference in profits between the dynamic and static competition scenarios is

$$\Delta\Pi_k = \Pi_k^* - \tilde{\Pi}_k = \frac{(\rho\gamma x_0)^2 p_k}{16\alpha_1\alpha_2(\hat{\beta}t)^2}(\alpha_{3-k}p_k - 2\alpha_k p_{3-k}).$$

Therefore: $\Delta\Pi_k \geq 0$ if and only if $\frac{\alpha_k}{p_k} \leq \frac{\alpha_{3-k}}{2p_{3-k}}$; otherwise $\Delta\Pi_k < 0$ must hold.

This completes the proof. $\square$

Proof of Theorem 4. We first investigate firm k's optimal decision in response to the other firm's decision. Firm k's profit over two periods is

$$\Pi_k(u_k, p_k, u_{3-k}, p_{3-k}) = \Big(\frac{1}{2} + \frac{x_0(u_k - u_{3-k}) - p_k + p_{3-k}}{2\beta t}\Big)p_k - \alpha_k u_k^2$$

$$+\Big(\frac{1}{2}+\frac{(\gamma x_0+1-\gamma)(u_k-u_{3-k})-p_k+p_{3-k}}{2\hat{\beta}t}\Big)\rho p_k.$$

The first-order conditions result in the following set of equations:

$$\begin{cases}\tilde{p}_k=\frac{1}{2}p_{3-k}+\frac{1}{2(\hat{\beta}+\rho\beta)}\Big[(1+\rho)\beta\hat{\beta}t+\Big(\hat{\beta}x_0+(\gamma x_0+1-\gamma)\rho\beta\Big)(u_k-u_{3-k})\Big],\\ \tilde{u}_k=\frac{\Big(\hat{\beta}x_0+(\gamma x_0+1-\gamma)\rho\beta\Big)p_k}{4\alpha_k\beta\hat{\beta}t},\end{cases}$$

solving which we arrive at the following best response curves:

$$\begin{aligned}\tilde{u}_k&=\frac{3(1+\rho)\beta\hat{\beta}t\Big(\hat{\beta}x_0+(\gamma x_0+1-\gamma)\rho\beta\Big)-\Big(\hat{\beta}x_0+(\gamma x_0+1-\gamma)\rho\beta\Big)^2u_{3-k}}{12\alpha_k\beta\hat{\beta}t(\hat{\beta}+\rho\beta)-\Big(\hat{\beta}x_0+(\gamma x_0+1-\gamma)\rho\beta\Big)^2},\\ \tilde{p}_k&=\frac{(1+\rho)\beta\hat{\beta}t}{\hat{\beta}+\rho\beta}+\frac{\Big(\hat{\beta}x_0+(\gamma x_0+1-\gamma)\rho\beta\Big)(u_k-u_{3-k})}{3(\hat{\beta}+\rho\beta)}.\end{aligned}$$

It is not difficult to show that the set of best response curves induces a unique equilibrium, which is given by Equation (15). The parameter condition of the interior equilibrium is given by Equation (EC.1).

$$t\in\begin{cases}I_1t^2-\Big(I_2+I_3x_0(1+\rho)\Big)t+x_0(1+\rho)\Big(\hat{\beta}x_0+(\gamma x_0+1-\gamma)\rho\beta\Big)^3\le 0,\\ I_1t^2-\Big(I_2+I_3(\gamma x_0+1-\gamma)(1+\rho)\Big)t+(\gamma x_0+1-\gamma)(1+\rho)\Big(\hat{\beta}x_0+(\gamma x_0+1-\gamma)\rho\beta\Big)^3\le 0\end{cases}\tag{EC.1}$$

Note that for ease of presentation, we define several critical values used in Equation (EC.1):

$$\begin{aligned}I_1&=12(\hat{\beta}+\rho\beta)\alpha_1\alpha_2\beta\hat{\beta}[\hat{\beta}+\rho\beta+2(1+\rho)\beta\hat{\beta}],\\ I_2&=(\alpha_1+\alpha_2)[\hat{\beta}x_0+(\gamma x_0+1-\gamma)\rho\beta]^2[\hat{\beta}+\rho\beta+2(1+\rho)\beta\hat{\beta}],\\ I_3&=3(\alpha_1+\alpha_2)[\hat{\beta}x_0+(\gamma x_0+1-\gamma)\rho\beta](\hat{\beta}+\rho\beta)\beta\hat{\beta}.\end{aligned}$$

This completes the proof. □

Proof of Theorem 5. To investigate the equilibrium decisions, we first study the best response of firm k for any given decision made by the other firm. Firm k's profit function in the second period is

$$\Pi_k^{(2)}(v_k,r_k,v_{3-k},r_{3-k})=\Big(\frac{1}{2}+\frac{(\gamma x_0+1-\gamma)(u_k-u_{3-k})+\gamma x_0(v_k-v_{3-k})-r_k+r_{3-k}}{2\hat{\beta}t}\Big)\rho r_k-\alpha_k v_k^2.$$

Therefore, the following first-order conditions are necessary and sufficient for optimality:

$$\begin{aligned}\frac{\partial\Pi_k^{(2)}(v_k,r_k,v_{3-k},r_{3-k})}{\partial r_k}&=\Big(\frac{1}{2}+\frac{(\gamma x_0+1-\gamma)(u_k-u_{3-k})+\gamma x_0(v_k-v_{3-k})-2r_k+r_{3-k}}{2\hat{\beta}t}\Big)\rho,\\ \frac{\partial\Pi_k^{(2)}(v_k,r_k,v_{3-k},r_{3-k})}{\partial v_k}&=\frac{\rho\gamma x_0r_k}{2\hat{\beta}t}-2\alpha_kv_k.\end{aligned}$$

Solving the above equations induces the following best response curves:

$$\begin{aligned} r_k^* &= \frac{1}{2}\Big(r_{3-k} + \hat{\beta}t + (\gamma x_0 + 1 - \gamma)(u_k - u_{3-k}) + \gamma x_0 (v_k - v_{3-k})\Big), \\ v_k^* &= \frac{\rho \gamma x_0 r_k}{4\alpha_k \hat{\beta} t}. \end{aligned}$$

Equation (16) is obtained by solving the set of best response curves. The parameter condition of the interior equilibrium is given by Equation (EC.2).

$$t \in I_5 K_1^2 + I_6 K_1 + 2\alpha_1\alpha_2\rho(\gamma x_0 + 1 - \gamma)^2 \Big(12\alpha_1\alpha_2\hat{\beta}t - (\alpha_1+\alpha_2)\rho(\gamma x_0)^2\Big)(K_2K_4 + K_3K_5) \le 0. \quad \text{(EC.2)}$$

Note that for ease of presentation, we define several critical values used in Equation (EC.2):

$$\begin{aligned} I_5 &= 4\alpha_1\alpha_2 K_6 - x_0(\gamma x_0 + 1 - \gamma)(\alpha_1+\alpha_2)\sqrt{K_1} - \rho(\gamma x_0)^2(\gamma x_0 + 1 - \gamma)x_0(\alpha_2 - \alpha_1)^2, \\ I_6 &= \sqrt{K_1}(\gamma x_0 + 1 - \gamma)[x_0(K_2 + K_3) - 2\alpha_1\alpha_2\rho(\gamma x_0 + 1 - \gamma)(\alpha_2 K_4 + \alpha_1 K_5)] \\ &\quad -(2\alpha_1 K_2 + 2\alpha_2 K_3) + 2\alpha_1\alpha_2\rho^2(\gamma x_0)^2(\gamma x_0 + 1 - \gamma)^2(\alpha_2 - \alpha_1)(\alpha_2 K_4 - \alpha_1 K_5). \end{aligned}$$

This completes the proof. $\square$

Proof of Theorem 6. We first study the optimal product quality and price decisions of firm k for any given decision made by the other firm. We write firm k's total profit function as

$$\begin{aligned} \Pi_k(u_k, p_k, u_{3-k}, p_{3-k}) &= \Big(\frac{1}{2} + \frac{x_0(u_k - u_{3-k}) - p_k + p_{3-k}}{2\beta t}\Big)p_k - \alpha_k u_k^2 \\ &\quad + \frac{\alpha_k\rho\Big(6\alpha_{3-k}\hat{\beta}t - \rho(\gamma x_0)^2 + 2\alpha_{3-k}(\gamma x_0 + 1 - \gamma)(u_k - u_{3-k})\Big)^2\Big(8\alpha_k\hat{\beta}t - \rho(\gamma x_0)^2\Big)}{4\Big(12\alpha_1\alpha_2\hat{\beta}t - (\alpha_1+\alpha_2)\rho(\gamma x_0)^2\Big)^2}. \end{aligned}$$

For any (u_{3-k}, p_{3-k}), firm k's optimal decision is jointly determined by the following first-order conditions:

$$\begin{aligned} \frac{\partial \Pi_k(u_k, p_k, u_{3-k}, p_{3-k})}{\partial u_k} &= \frac{x_0 p_k}{2\beta t} - 2\alpha_k u_k + \frac{\Big(6\alpha_{3-k}\hat{\beta}t - \rho(\gamma x_0)^2 + 2\alpha_{3-k}(\gamma x_0 + 1 - \gamma)(u_k - u_{3-k})\Big)}{\Big(12\alpha_1\alpha_2\hat{\beta}t - (\alpha_1+\alpha_2)\rho(\gamma x_0)^2\Big)^2}. \\ &\quad \alpha_1\alpha_2\rho(\gamma x_0 + 1 - \gamma)\Big(8\alpha_k\hat{\beta}t - \rho(\gamma x_0)^2\Big), \\ \frac{\partial \Pi_k(u_k, p_k, u_{3-k}, p_{3-k})}{\partial p_k} &= \frac{1}{2} + \frac{x_0(u_k - u_{3-k}) - 2p_k + p_{3-k}}{2\beta t}. \end{aligned}$$

The above equations yield the following best response curves:

$$\begin{aligned} u_k^* &= \frac{-K_3 u_{3-k} + \frac{x_0 K_1}{2} + \alpha_1\alpha_2\rho(\gamma x_0 + 1 - \gamma)\Big(8\alpha_k\hat{\beta}t - \rho(\gamma x_0)^2\Big)\Big(6\alpha_{3-k}\hat{\beta}t - \rho(\gamma x_0)^2\Big)}{2\alpha_k K_1 - K_3}, \\ p_k^* &= \frac{1}{2}\Big(p_{3-k} + \beta t + x_0(u_k - u_{3-k})\Big). \end{aligned}$$

Equation (19) is obtained by solving the set of best response curves. The parameter condition of the interior equilibrium is given by Equation (EC.3)

$$t \in \Big(4\alpha_1\alpha_2(1+2\beta)t - (\alpha_1+\alpha_2)x_0\Big)K_1^2 + I_4 K_1 + 2\alpha_1\alpha_2\rho(\gamma x_0 + 1 - \gamma)(K_2K_4 + K_3K_5) \le 0. \quad \text{(EC.3)}$$

Note that for ease of presentation, we define a critical value used in Equation (EC.3):

$$I_4 = x_0(K_2 + K_3) - 2(1+2\beta)(\alpha_1 K_2 + \alpha_2 K_3)t - 2\alpha_1\alpha_2\rho(\gamma x_0 + 1 - \gamma)(\alpha_2 K_4 + \alpha_1 K_5).$$

This completes the proof. □

Proof of Proposition 2. By Equation (16) and the proof of Theorem 6, the firm k's difference in the equilibrium selling prices between the two periods in the dynamic competition scenario with endogenous prices is

$$\begin{aligned} r_k^* - p_k^* = & \frac{1}{12\alpha_1\alpha_2\hat{\beta}t - (\alpha_1+\alpha_2)\rho(\gamma x_0)^2}\Big[12\alpha_1\alpha_2\hat{\beta}t^2(\hat{\beta}-\beta) + \rho(\gamma x_0)^2 t\Big((\alpha_1+\alpha_2)\beta - 2\alpha_k\hat{\beta}\Big) \\ & + (u_k - u_{3-k})\Big(4\alpha_1\alpha_2\hat{\beta}t(1-\gamma)(1-x_0) + \frac{1}{3}x_0\rho(\gamma x_0)^2(\alpha_1+\alpha_2)\Big)\Big]. \end{aligned}$$

Proposition 2 is obtained by solving the above equation.

This completes the proof. □