Monetary Policy Uncertainty and Firm Dynamics*

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Abstract

This paper uses a FAVAR model with external instruments to show that monetary policy uncertainty shocks are recessionary and are associated with an increase in firms' exit and a decrease in entry. At the same time, the stock price drops and the equity premium rises. Total factor productivity is increasing in the medium run. To explain this result, we build a medium scale DSGE model featuring firm heterogeneity and endogenous firm entry and exit. These features are crucial in matching the empirical responses. Versions of the model with constant firms or exogenous firms' exit are unable to re-produce the FAVAR response of firms' entry and exit and suggest a much smaller effect of this shock on real activity.

Key words: Monetary uncertainty, Firm dynamics, FAVAR, DSGE.

JEL codes: C5, E1, E5, E52

"*[...] we must explain much better to the general public what we are doing and why, and we must talk to people that we do not normally reach.*" Lagarde (2020)

1 Introduction

Global events like the sluggish recovery after the Great Recession, sovereign debt crisis in Europe, Brexit, and finally Covid pandemic, all contributed to considerably raise macroeconomic uncertainty in most of the developed countries. The recent experience has shown that sharp and timely interventions of policymaker might be crucial in distress times. Announcements of policy strategies that aim to contrast the crisis and foster recovery have often helped to reassure financial markets and significantly reduce uncertainty. Instead, delayed and unclear responses by policymakers might fuel uncertainty and, likely, curb further the economy. Although the literature agrees on the recessionary effects of uncertainty shocks, less clear is the impact of heightened uncertainty about the action of policymakers. Some previous studies find that policy uncertainty influences capital flows, the business cycle, and the speed of economic recovery (Mumtaz and Zanetti (2013), Fernández-Villaverde et al. (2015), Mumtaz and Surico (2018), Bloom et al. (2018), Caggiano et al. (2020)).

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However, Born and Pfeifer (2014) claim that policy risk is unlikely to play a major role in business cycle fluctuations, with their DSGE model suggesting that policy uncertainty shocks are small and their impact is not sufficiently amplified. In this paper, we revisit the question and consider the role of firm dynamics in propagating the impact of monetary policy uncertainty shocks. We ask how important are firms' entry and exit decisions for the transmission of monetary uncertainty shocks.

We refer to monetary policy uncertainty as to the perceived uncertainty that economic agents have around the future possible realizations of monetary policy. When monetary policy is uncertain, households and firms are unsure about the value of interest rates and inflation. For the productive sector, this uncertainty has also implications for the decisions of participating in the market. Firms that become more unsure about whether the discounted future cash flows will cover the cost of entry might decide to not enter the market. Firms that become more unsure whether the discounted future cash flows will guarantee the break-even and thereby, production might decide to exit from the market. Overall, the increased uncertainty about monetary policy might imply a lower entry and higher exit of firms, which ultimately, affect economic activity.

We investigate the transmission channel of firms dynamics for monetary policy uncertainty shocks in a FAVAR model, where the monetary volatility shock is identified using an external instrument (a là Husted et al. (2019)). We show that the shock is recessionary. Moreover, firms' births decrease and firms' deaths increase.[1] The evidence is robust both at the aggregate level, namely for establishments' births and deaths in the total private sector, and at the industry level. In response to higher monetary policy uncertainty, the stock price decreases, while the equity premium surges. The utilization-adjusted TFP series reacts positively, at least in the medium-long run.

To better understand the relative importance of uncertainty and level shocks for monetary policy in affecting firms' entry and exit decisions and economic activity, we also estimate our FAVAR model to study the effects of an unexpected tightening of the monetary policy stance. As an instrument to identify the monetary policy level shock, we use the surprises in the federal funds futures around FOMC annoualsoncements, as computed by Gertler and Karadi (2015a). We show that the two shocks are qualitatively similar, yet the monetary policy uncertainty shock has stronger effects on firm dynamics and eventually on economic activity. Further, the monetary policy level shock implies a stronger negative reaction at impact for total factor productivity, but none rebounds thereafter. In contrast to the monetary policy uncertainty shock, total factor productivity does not overshoot the long-run level but remains negatively affected as the monetary policy tightening transmits to the economy.

We rationalize the empirical evidence on monetary policy uncertainty shocks in the second part of the paper. We consider a medium-scale New Keynesian model extended by adding firm heterogeneity and endogenous firm entry and exit. In the intermediate sector, firms are heterogeneous in terms of their specific productivity. Similar to Rossi (2019), firms decide to produce as long as their specific productivity is above a cut-off level, which is determined by the level of productivity that makes the present discounted value of the stream of profits equal to the firms' liquidation value. The advantage of this framework is that firms' exit and average productivity evolve endogenously, bringing about endogenous TFP variations. During a recession, firms with specific productivity below an endogenous threshold exit the market, so that the average productivity and the TFP increase. The opposite occurs in an expansionary period. As in the seminal contribution by Bilbiie et al. (2012), firms enter the market up to the point where the expected discounted value of the future profits equals the sunk cost of entry. The investment in new firms is financed by households through the accumulation of shares in a portfolio of firms. This implies that the stock price fluctuates endogenously in response to shocks. Further, in the model, the equity premium is

[1] Notice that in the paper we use entry and exit and birth and death as synonymous.

strongly related to firm dynamics and, among others, to firms' defaulting probability, which is endogenously determined. The higher the probability of firm default the higher the equity premium. Under this framework, we study the transmission of a monetary uncertainty shock implemented as an innovation to the time-varying volatility of the monetary policy shock. The main results of the theoretical model can be summarized as follows. First, as in the empirical evidence, the shock is recessionary, implying a prolonged fall in output, consumption, and investment in physical capital, i.e. the intensive margin of investment. Inflation and the policy rate decrease as well. Also, the stock price falls followed by a drop in investment in new firms, i.e. the extensive margin of investment. The number of exiting firms increases further amplifying the negative response of output. The equity premium surges mainly driven by the increased firms' defaulting probability. The recession improves resource allocation by driving out less productive producers and increasing the TFP.

To disentangle the role of the two margins of firm dynamics, namely the entry and exit, the baseline model is compared against two alternative specifications: a model with constant firms and a model with endogenous entry, but a constant defaulting probability. We show that our baseline model outperforms the two alternative specifications being more in line with the empirical evidence provided by the FAVAR model. By construction, the model with constant firms cannot replicate the dynamics of firms and implies a lower reduction of output, consumption, investment in physical capital, and a muted response of the TFP and the stock price. The model with endogenous entry, but a constant defaulting probability, shows a declining firm exit and a negative and almost muted equity premium, which is at odds with the dynamics of the FAVAR. Also, the fall in output is lower and the propagation of the shock is weaker than in our baseline model. Overall, we argue that both firm dynamics and firm heterogeneity are crucial in the theoretical framework to replicate the qualitative results found in the FAVAR analysis, particularly for dynamics of the equity premium and TFP.

This paper relates to two main strands of literature. It contemporaneously relates to the literature studying the macroeconomic effects of policy uncertainty shocks, and the literature investigating the role of firm dynamics for the business cycle analysis. After Bloom (2009), many papers discuss the macroeconomic impact of uncertainty shocks.[2] Among others, several contributions focused the consequences of policy-related uncertainty shocks over the business cycle, e.g. Fernández-Villaverde et al. (2015), Born and Pfeifer (2014), Mumtaz and Surico (2018). Some of these contributions drew on the availability of measures of policy uncertainty, e.g. Baker et al. (2016), Husted et al. (2019), Istrefi and Mouabbi (2018), to evaluate the impact of these shocks to the economy. Overall, the literature highlights the relevance of uncertainty shocks in explaining a large share of the fluctuations in the business cycle, and the contractionary effects on the main real variables, namely output, employment, consumption, and investment. For investment, however, most of the papers limited the analysis to the impact of higher uncertainty on the intensive margin of investment, namely the decisions about new investments of firms already participating in the market. Surprisingly, the effects of uncertainty shocks on the extensive margin of investment concerning the firms' decisions about participating in the market have been largely ignored in the literature. This paper highlights the importance of considering both margins of investment to monetary policy uncertainty shocks. To our knowledge, only Brand et al. (2019) has already studied in a macroeconomic model the effects of second-moment shocks on firm creation and destruction. Brand et al. (2019) build up and estimate a theoretical model with search and monitoring costs in the credit market to study how the higher dispersion in firm productivity affects macro-financial

[2] For instance, Fernández-Villaverde et al. (2011), Gilchrist et al. (2013), Caggiano et al. (2014), Christiano et al. (2014), Bachmann and Bayer (2014), Leduc and Liu (2016), Caldara et al. (2016), Basu and Bundick (2017), Bloom et al. (2018). Mumtaz and Theodoridis (2019).

aggregates and firm dynamics. We differ from their contribution along at least three dimensions. First, they provide an alternative way to formalize firm dynamics based on search frictions between entrepreneurs and banks. Second, while we focus on the effects of monetary policy uncertainty shocks, they consider uncertainty in firms' idiosyncratic productivity. Third, they do not provide evidence on firm dynamics at the industry level.

The impact of firm dynamics on business cycle fluctuations has been extensively studied in papers investigating the effects of first moment shocks, that is level shocks. The seminal paper by Bilbiie et al. (2012) in the DSGE literature shows that endogenous entry generates a new and potentially important endogenous propagation mechanism for real business cycle models. Among others, Jaimovich and Floetotto (2008), Lewis and Poilly (2012), Etro and Colciago (2010), Clementi and Palazzo (2016), Lewis and Stevens (2015) provide evidence that the number of producers varies over the business cycle and that firms dynamics may play an important role in explaining business cycle statistics. Bilbiie et al. (2014) consider a DSGE model with monopolistic competition and sticky prices and find that deviations from long-run stability of product prices are optimal in the presence of endogenous producer entry and product variety, whereas price stability would be optimal in the absence of entry. Hamano and Zanetti (2014) and Casares et al. (2018) introduce endogenous firms exit in a DSGE model, but consider different timing and exiting schemes. While Hamano and Zanetti (2014) study the effects of a negative technology shock in a simple RBC model, Casares et al. (2018) consider a medium-scale model and estimates the effects of a set of level shocks on business cycle dynamics. Differently from our framework, in their paper firms exit at the end of the production period, implying that the average productivity remains exogenous and constant even in the short run. This prevents the TFP from varying along the business cycle. Closer to our theoretical framework is Rossi (2019), who however considers a simple small-scale New Keynesian model with endogenous entry and exit interacting with banking frictions to study the effects of first-moment shocks to the aggregate productivity level.

Our paper, therefore, makes two clear contributions. First, it extends the literature on policy uncertainty shocks by considering the role of firm dynamics from an empirical and theoretical perspective. To the best of our knowledge, the role of firm dynamics in propagating a monetary policy uncertainty shock has not been investigated in the existing literature. We show that this feature is a crucial component in amplifying the effect of this shock in DSGE models. Second, from an econometric perspective, the paper proposes a FAVAR model that allows for mixed-frequency and missing data, allowing us to utilize series on aggregate and industry-specific firms' entry and exit which are available at a lower frequency and contain missing observations.

The remainder of the paper is organized as follows. Section 2 introduces the FAVAR model and provides empirical evidence. Specifically, Section 2.4 presents the dynamics after the monetary policy uncertainty shock, while Section 2.5 discusses some robustness checks and Section 2.6 compares its transmission to the monetary policy level shock. Section 3 spells out the DSGE model economy. Section 4 comments on the simulation of the monetary policy uncertainty shock in the theoretical model. Section 5 finally concludes.

2 Empirical analysis

We use a factor augmented VAR (FAVAR) to estimate the response to monetary policy uncertainty shocks for the US economy over the period 1985:m1 to 2016:m6. Relative to a small-scale VAR, the FAVAR offers three key advantages. First, it allows the inclusion of data on sector-specific entry and exit, thus capturing the relationship between sectors. Second, the FAVAR can easily handle mixed frequencies and missing data allowing us to use monthly data on variables related to monetary policy uncertainty together with industry-specific data that is only available at a quarterly frequency. Finally, the use of a large data set makes it less likely that the model suffers from information insufficiency (see Forni and Gambetti (2014)).

The observation equation of the FAVAR model is defined as

$$\begin{pmatrix} Z_t \\ \tilde{X}_t \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & \Lambda \end{pmatrix} \begin{pmatrix} Z_t \\ F_t \end{pmatrix} + \begin{pmatrix} 0 \\ v_t \end{pmatrix} \tag{1}$$

where Z_t is the monetary policy uncertainty index built by Husted et al. (2019). $\tilde{X}_t$ is a $M \times 1$ vector of variables that includes aggregate measures of macroeconomic and financial conditions provided by FRED-MD database (McCracken and Ng (2016)). $\tilde{X}_t$ also contains aggregate and sector-specific measures of firms' entry and exit provided by Bureau Labor Statistics-BED database. Details of the data used are in the Technical Appendix. F_t denotes a $K \times 1$ vector of unobserved factors while Λ is a $M \times K$ matrix of factor loadings. Finally, v_t is a $M \times 1$ vector that holds the idiosyncratic components. We assume that each row of v_t follows an $AR(P)$ process:

$$v_{it} = \sum_{p=1}^{P} \rho_{ip} v_{it-p} + e_{it}, \tag{2}$$

$$e_{it} \tilde{} N(0, r_i), R = diag\left([r_1, r_2, .., r_M]\right) \tag{3}$$

where $i = 1, 2, .., M$.

Collecting the factors in the $N \times 1$ vector $Y_t = \begin{pmatrix} Z_t \\ F_t \end{pmatrix}$, the transition equation can be described as:

$$Y_t = BX_t + u_t, \tag{4}$$

$$u_t \tilde{} N(0, \Sigma) \tag{5}$$

where $X_t = [Y'_{t-1}, .., Y'_{t-P}, 1]'$ is $(NP + 1) \times 1$ vector of regressors in each equation and B denotes the $N \times (NP + 1)$ matrix of coefficients $B = [B_1, ..., B_P, c]$. The covariance matrix of the reduced form residuals u_t is given by Σ. Note that the structural shocks are defined as $\varepsilon_t = A_0^{-1} u_t$, where $\varepsilon_t \sim N(0, 1)$ and $A_0 A'_0 = \Sigma$.

2.1 Temporal aggregation and missing data

The data on firms' entry and exit is only available at a quarterly frequency and also contains missing observations at the beginning of the sample period. The data on the total factor productivity from Fernald (2014) is available at a quarterly frequency as well. Measures of stock market return and stock price are taken from Caldara et al. (2016) and contain missing observations for the last year of the sample period. For all these series (x_t), the observation equation is defined as:

$$\hat{x}_{jt} = \delta_j F_t + v_{jt} \tag{6}$$

where $\hat{x}_{jt}$ denotes unobserved monthly growth rates of the jth series in x_t and δ_j are the associated factor loadings. Over years where quarterly observations are available, we assume the following relationship between quarterly and monthly growth rates:

$$x_{jt}^{Q} = \sum_{j=0}^{2} \hat{x}_{jt} \tag{7}$$

In other words, the quarterly growth rates are assumed to be the sum of the unobserved monthly growth rates in that quarter. In detail, we treat $\hat{x}_{jt}$ as additional unobserved states and add a step in our MCMC algorithm to draw from their conditional posterior distribution.

2.2 Identification

We are interested in identifying the monetary policy uncertainty shock, that we denote ε_t^{MPU} and order first in the vector ε_t for convenience. We employ an external instrument approach to identify the structural shock of interest as in Stock (2008) and Mertens and Ravn (2013). Following Husted et al. (2019), our instrument is constructed by orthogonalizing the monetary policy volatility on FOMC meeting days to observed monetary policy surprises. In detail, we take as an instrument the residual from the regression of the daily conditional volatility of 1-month ahead options on 1-year interest rate swaps taken by Carlston and Ochoa (2016), over monetary policy surprises on FOMC meeting days.[3] We consider the same three measures of monetary policy surprises of Rogers et al. (2018), which cover three components: target rate, forward guidance, and asset purchase.[4] The estimation is carried out using data on FOMC meeting days from October 2008 to December 2015, when all monetary policy surprises are available. The residual from that regression, m_t, can be interpreted as the measure of monetary policy volatility on FOMC meeting days that is unexplained by the change in the monetary policy itself. We take this daily measure as our instrument to identify the monetary policy uncertainty shock.[5] The instrument is available for a shorter period than the rest of the data. This is similar to other papers identifying structural shocks using high frequency data as Gertler and Karadi (2015b) and Husted et al. (2019). While we use the full dataset spanning from 1985:m1 to 2016:m6 to estimate the FAVAR model in reduced form, we take the reduced form residuals and the instrument for the period in which the latter is available to identify the shock.

We assume that the instrument satisfies the *relevance* and *exogeneity* conditions:

$$E\left(m_t, \varepsilon_t^{MPU}\right) = \alpha, \alpha \neq 0 \tag{8}$$

$$E\left(m_t, \varepsilon_t^{-}\right) = 0 \tag{9}$$

That is, the instrument is assumed to be correlated with the monetary policy uncertainty shock ε_t^{MPU} and uncorrelated with the remaining shocks ε_t^{-}. The instrument is incorporated into the FAVAR model via the following equation

$$m_t = b\varepsilon_t^{MPU} + \sigma\hat{v}_t, \quad \hat{v}_t \tilde{\ } N(0,1) \tag{10}$$

[3] See Bauer et al. (2019) for a review of the literature on market-based measures of monetary policy uncertainty.

[4] We thank Marcelo Ochoa and John Rogers for sharing the data on respectively, the swaptions volatility and the three measures of monetary policy surprises.

[5] One possible concern of using daily series as an instrument for high frequency identification is that more economic announcements might be issued on the days of the observations. In this case, the information contained in the instrument could be distorted by economic releases that do not relate to the structural shock to be identified. The related literature on high frequency identification of monetary policy shocks (Gurkaynak et al. (2005)) indicate the employment report releases issued at FOMC meeting days as one of the economic announcements that could imply a daily response of financial markets and, therefore, of the instrument that does not depend on FOMC decisions. However, it is worth noting that over the sample period we consider for the instrument only on one day, i.e. on 12^{th} December 2012, FOMC meeting coincided with the release of an employment report.

2.3 Estimation and specification

The FAVAR model is estimated using Bayesian methods. Following Bruns (2021) and Miescu and Mumtaz (2019), we extend the algorithm proposed by Caldara and Herbst (2019) for proxy VARs. The priors and the Gibbs sampling algorithm are described in detail in the Technical Appendix. Caldara and Herbst (2019) highlight that the prior for b and σ^2 are critical as they influence the reliability of the instrument. As in Mertens and Ravn (2013), we define the reliability statistic as the squared correlation between m_t and ε_t^{MPU}, or $\rho^2 = b^2/\left(b^2 + \sigma^2\right)$. In the benchmark specification of the FAVAR, we set the priors for b and σ^2 implying that $\rho \approx 0.6$. In the sensitivity analysis, we check the robustness of the empirical findings by setting priors that reflect the belief that the instrument is less relevant.

We fix the number of factors to 6. Following Bernanke et al. (2005), in the robustness analysis, we test that main results are similar when the number of factors is higher. In order to keep the number of unobserved states at a manageable level, the lag lengths in equation (4) and (2) are fixed at 6 and 1, respectively. The algorithm is run for 100,000 iterations with a burn-in of 75,000 iterations. Every fifth remaining draw is used to approximate the posterior distributions. The Technical Appendix presents evidence that is consistent with convergence.

2.4 Empirical results

2.4.1 Monetary policy uncertainty shocks

Figure 1. shows the impulse responses of selected macroeconomic and financial aggregate variables together with the responses of total private establishments' birth and deaths to the monetary policy uncertainty shock. Figures 2-3 show the responses of establishments' birth and deaths at the industry-level. The dynamic responses on the y-axis of the figures are in percent changes but in percentage points for the interest rates, inflation, unemployment, the measure of net entry of firms, and the equity premium. On the x-axis, there are reported the months after the shocks. Figure 1.reports dynamics responses up to the 2-year horizon.

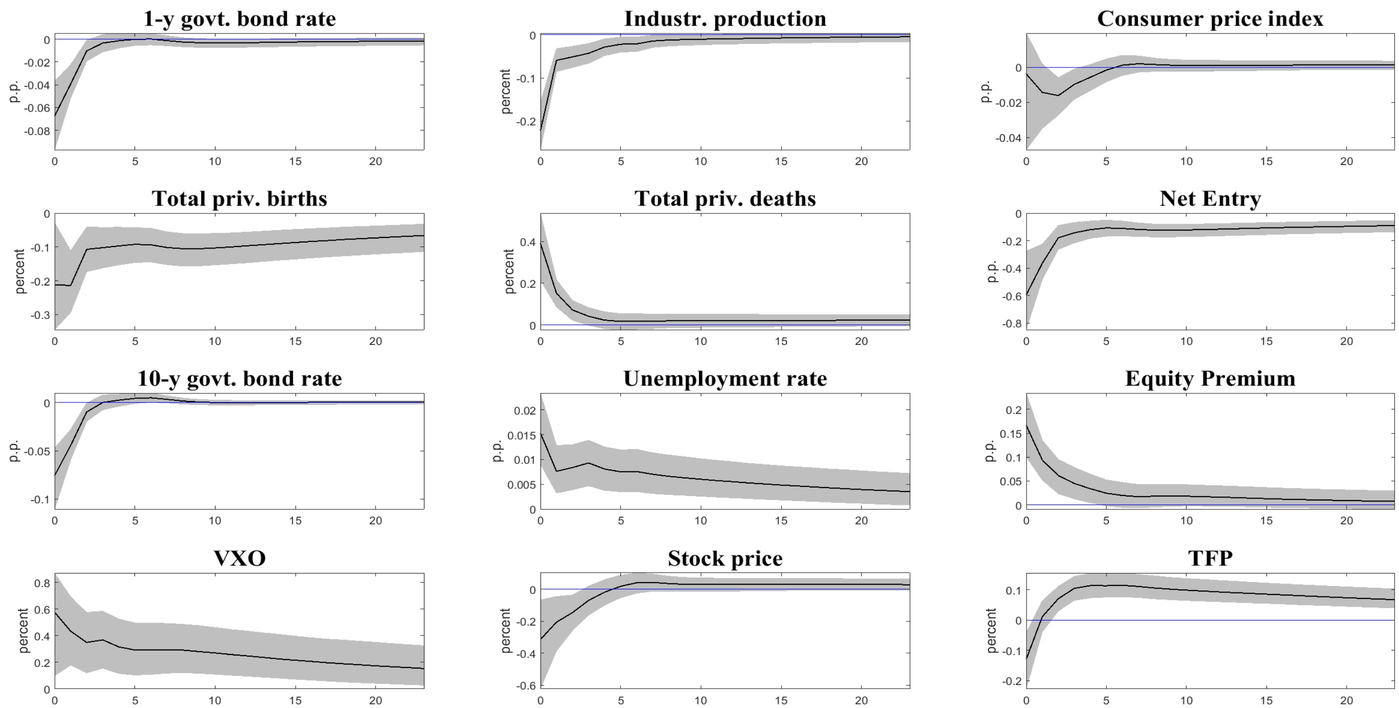

Figure 1: Impulse responses of a set of macroeconomic and financial variables to a one standard deviation monetary policy uncertainty shock. The solid line is for the median response. The shaded area represents the 68% error band. Annualized impact for interest rates, inflation, equity premium.

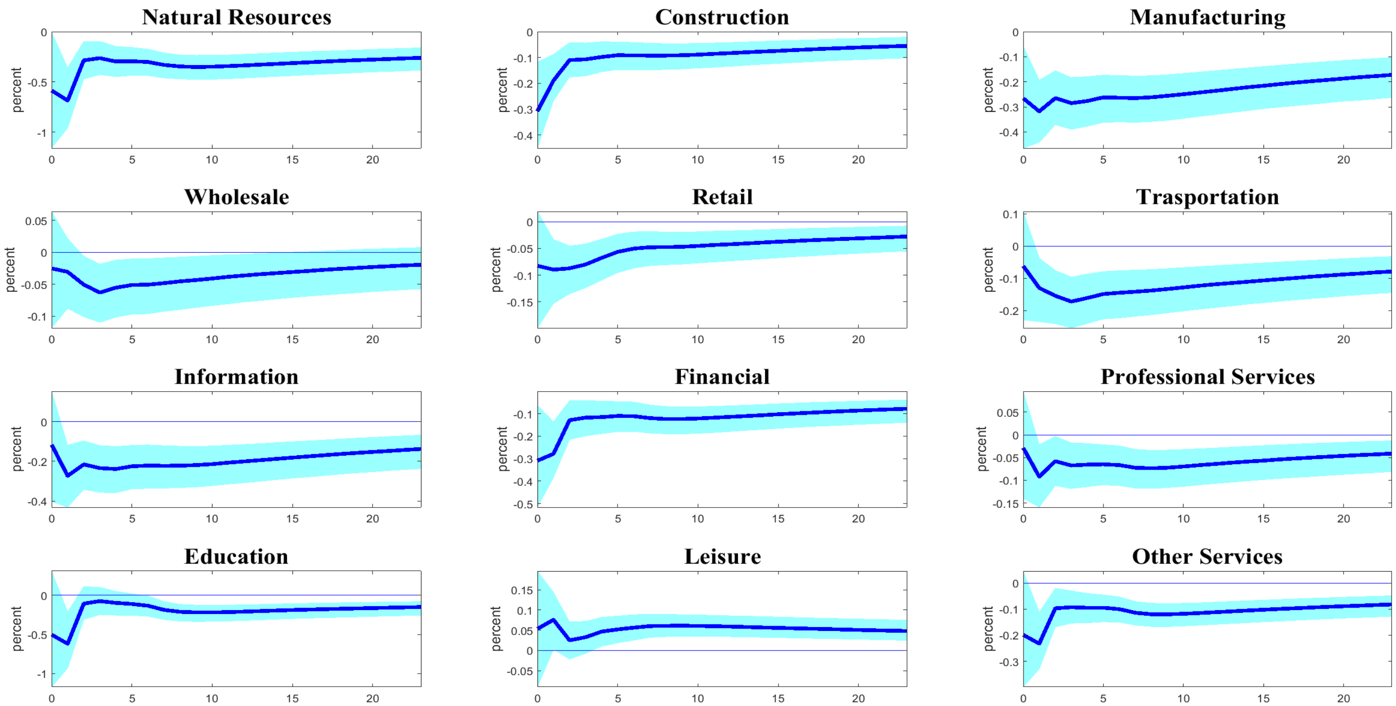

Figure 2: Impulse responses of establishments' entry at the industry level. The solid line is for the median response. The shaded area represents the 68% error band.

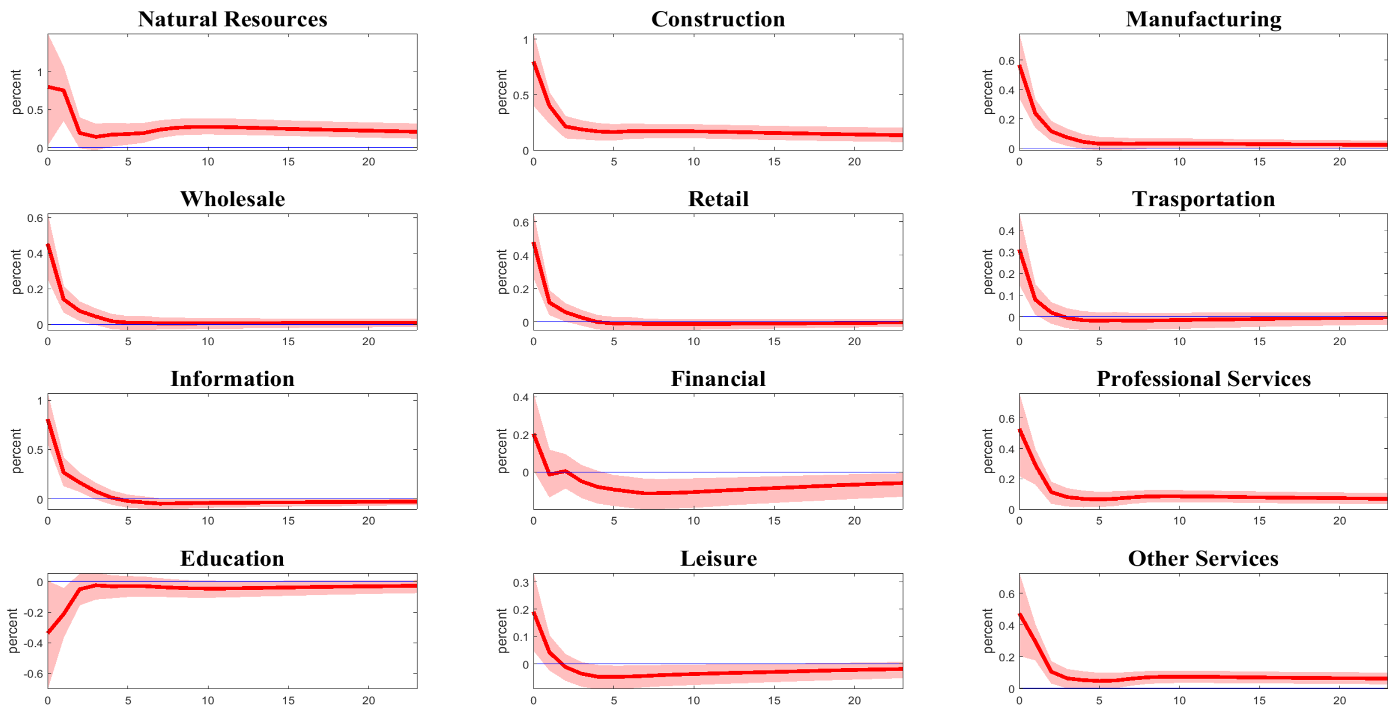

Figure 3: Impulse responses of establishments' exit at the industry level. The solid line is for the median response. The shaded area represents the 68% error band.

We study a one standard deviation shock that increases the news-based monetary policy index by Husted et al. (2019) by about 20%.[6] Meanwhile, the VXO index rises by 0.6% at the impact. This suggests that while the identified shock triggers an overwhelming surge in the measure of uncertainty about monetary policy, the impact for the less specific measure of volatility VXO is sensibly weaker. Industrial production reduces by around 0.2%.[7] The economic activity further declines in the subsequent periods. The recession lasts for at least 6 months after the shock and the rise in unemployment is persistent. The unemployment rate lays above the long-run level for more than two years after the shock. Following the contraction in the economic activity, consumer price inflation declines but its response is milder. The stock price reduces as well. For both consumer and stock prices, the transmission of the shock is faster than for the other indicators of the economic activity and it is absorbed after few months. It is worth stressing that the macroeconomic effects of the monetary policy uncertainty shock resemble a negative demand shock. In response to the joint decline in output and inflation, the monetary policy becomes accommodative to foster the recovery. Both short- and long-run interest rates fall in response to the shock. As a consequence of the recession, also the equity premium rises as expected. We measure the response of the equity premium as the difference between the percentage change in the value-weighted total stock market return and that in the federal fund rate. Different from the rest of the variables depicted in Figure 1, the response of the total factor productivity changes sign at the short to the medium horizon. The variation in the TFP is negative at the impact of the shock before turning positive from the third month ahead. Importantly, the overshooting of the response in the TFP is persistent and systematically different from zero even at the 2-year horizon. While the overall effect of the shock is recessionary, it brings about an eventually improvement in TFP.

The monetary policy uncertainty shock has clear implications for firms' participation in the market. Establishments' births and deaths of the total private sector move in opposite directions. Our measure of firms' entry reduces in response to the shock whilst the measure of firms' exit increases. At the median, the surge in establishments' death is almost twice as large as the drop in establishments' births. Interestingly, the transmission of the shock is asymmetric. While births of new establishments are still decreasing at the 2-year horizon, deaths remain positive only for few months after the shock. The impact on the net entry of firms, measured by the difference between the percentage change in establishments' entry and exit, is therefore negative and long-lasting. Figure 2 and 3 report the responses of, respectively, establishments' births and deaths at the industry-level. The response of industry-level data is similar to the aggregate response: the impact is negative for measures of firms' entry and positive for firms' exit. As for aggregates, the transmission is more persistent in establishments' births than in establishments' deaths. The size of the impact at the industry-level is, however, mixed. Among the good-producing industries, the response is larger on establishments' births and deaths in Construction and Manufacturing, while it is weaker in Natural Resources and Mining. Within the nine industries of the service-providing composite sector,[8] establishments' births drop more in Financial activities and Education and Health Services. Construction and Manufacturing are the goods-producing sectors that show a stronger reaction to establishments' deaths. In particular, the impact of deaths in Construction more than twince as large as the aggregate. Information and Professional Services are the service-

[6]The magnitude is consistent with other uncertainty shocks estimated in the literature. For instance, Basu and Bundick (2017) estimate in a small VAR a one standard deviation uncertainty shock bringing about an increase in the VXO index of 15%.

[7]Remarkably, the drop in the real activity is close to the estimated impact Fernández-Villaverde et al. (2015) find for the GDP to a policy uncertainty shock.

[8]That is Wholesale, Retail, Transportation, Information, Financial, Professional Services, Education, Leisure, Other Services.

providing sectors that report the highest peaks in establishments' deaths.

In summary, the estimates suggest four main conclusions. First, the shock is both recessionary and deflationary. All the responses of macroeconomic and financial variables we considered indicate that the economy is severely hit by such an innovation. In particular, the transmission of the monetary policy uncertainty shock is equivalent to that of a negative demand shock. Second, productivity in the economy is not affected negatively. Total factor productivity recovers immediately after the shock and improves further in the medium horizon. Third, the entry and exit of firms respond to the shock in the opposite directions. While the monetary policy shock reduces births of new establishments for several periods, establishments' deaths rise at the impact but the effect is short-lived. Taking jointly the two flows, the net entry declines and results as procyclical to output. Fourth, the empirical evidence on firm dynamics is robust both at the aggregate and industry-level.

To better understand the role of firm dynamics in shaping the behavior of economic activity after the monetary policy uncertainty shock, in the next Section we run counterfactual exercises that switch off the effects on firms' entry and exit.

2.4.2 Contribution of firms' entry and exit

The evidence provided by Section (2.4.1) indicates that the measures of firms' entry and exit are significantly affected by the monetary policy uncertainty shock. What is the role of firm dynamics in propagating this shock? To investigate the issue, we carry out a counterfactual exercise that switches off the transmission of the shock to firms' entry and exit. To be precise, the counterfactual analysis is developed by solving for shocks in the transition equation of the FAVAR to impose the restrictions that the response of establishments' birth and death in the private sector, respectively, equals zero over the entire horizon, while the counterfactual response of the monetary policy uncertainty index should equal the actual estimate. These conditions reproduce the counterfactual scenario where monetary policy uncertainty does not affect firms' entry and exit at the aggregate level. The panels in Figure 4 confirm the relevance of the firm dynamics channel in the transmission of monetary uncertainty shock. We consider a monetary policy uncertainty shock that decreases the 1-year government bond rate by 1%. Noteworthy, compare to the benchmark, the impact of the shock is smaller. For industrial production, unemployment rate, and equity premium, the responses are not differently from zero, especially under the counterfactual assumption that the shock does not affect births. Furthermore, consumer and stock prices increase initially increase meaning that uncertainty shocks would work as negative supply shocks. Lastly, the overshooting of the total factor productivity is ruled out in the counterfactual scenarios.

2.5 Robustness

To validate that the transmission of the monetary policy shock does not hinge upon the specification of the FAVAR, we perform some robustness checks. A detailed description of the sensitivity analysis and its results is left in the Technical Appendix. Here, we summarize the findings.

We develop the sensitivity analysis of the benchmark FAVAR along two main lines. First, we assume a different number of factors in the model. We estimate the FAVAR as the benchmark but using, respectively, five, seven, eight factors. Notably, we do not find evidence that responses in the FAVAR to the monetary policy uncertainty shock are driven by the number of factors. The dynamics of the variables we investigate in Section 2.4 is firmly robust across the different specifications we estimated.

Second, we modify the prior concerning the variance of the error term in the instrument equation

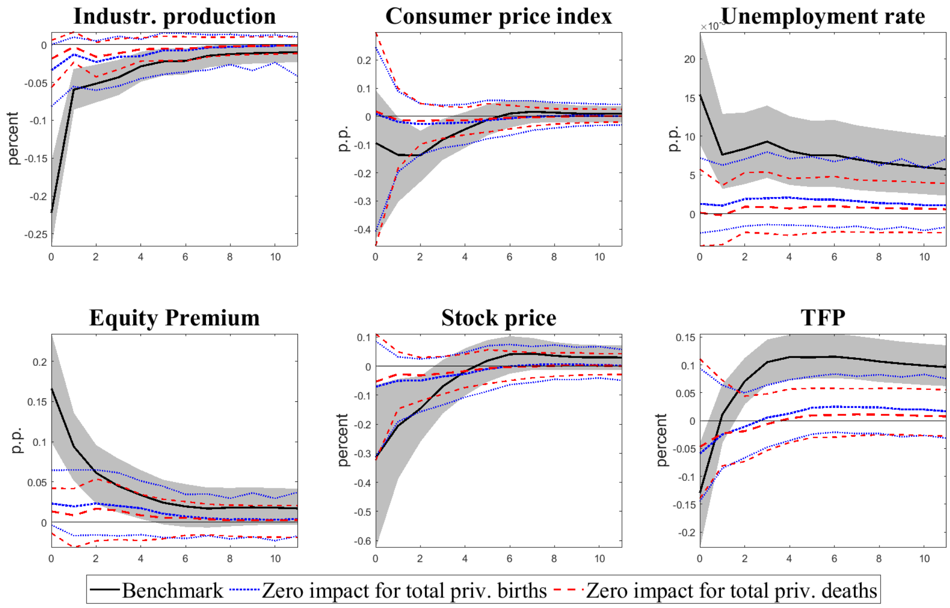

Figure 4: Normalized impulse responses to the monetary policy uncertainty shock. The solid line is for the median response. The shaded area represents the 68% error band. Industrial production, stock price, TFP in percent. Unemployment in percentage points. Inflation and equity premium in annualized percentage points.

(10). As pointed by Caldara and Herbst (2019), that prior is critical for the reliability of the instrument. We test the findings of the benchmark FAVAR with a flatter prior, which reflects a weaker belief in the reliability of the instrument. We find that changing the priors barely affects the results. Though the responses are less precisely estimated as expected, the sign and magnitude remain consistent with the benchmark FAVAR.

While the sensitivity analysis of this Section provides robustness to the effects of the monetary policy uncertainty shock, in the next Section we re-estimate the FAVAR to compare its transmission to that of the monetary policy shock.

2.6 Monetary policy innovations: uncertainty versus level

It is worth mentioning that previous literature studying the transmission of policy-related shocks to firm dynamics has focused on first-moment shocks. For instance, Lewis and Poilly (2012) study the transmission of the monetary policy shock on entry of firms, while Lewis and Winkler (2017) study the propagation of the government spending shock on the same variable. Lewis and Stevens (2015) consider jointly monetary and government spending when estimating a DSGE model that encompasses firms' entry. In all these works, the empirical counterpart of firms' entry is measured by the index of net business formation from the BEA's Survey of Current Business, that is by the difference between entry and exit. In our analysis, instead, we employ measures of entry and exit separately and back out the response of net entry as the difference between the impulse responses of the flows of births and deaths. To compare the transmission of the second-moment shocks with that of corresponding first-moment shocks, we re-estimate our benchmark FAVAR model to identify a monetary policy level shock. Consistent with the strategy we followed for the uncertainty shock, we rely on an external instrument for the identification. In particular, we employ the surprises in the fourth federal funds futures (FF4) around FOMC announcements, as computed by Gertler and Karadi (2015a). The instrument is available from the 1990:m1 to 2016:m6. Similar to the identification of the uncertainty shock, we use a shorter sample to identify the impact of monetary policy surprises than to estimate the lag coefficients. The estimation procedure and specification are the same as described for the benchmark FAVAR (see Section 2.3). Following Gertler and Karadi (2015a), we take the 1-year government bond rate as our policy indicator. We study a tightening monetary shock that raises the policy indicator by 1%. Figure 5 shows the impulse responses of industrial production, consumer price inflation, total factor productivity, firms' entry, firms' exit, and net entry to the monetary policy shock (in blue). For comparability, Figure 5 also reproduces the responses of the same variables to the monetary policy uncertainty shock that is normalized to reduce the policy indicator by 1% (in red). Unsurprisingly, both shocks imply a contraction in economic activity and inflation. While the size of the response in inflation is similar across the shocks, the impact of the uncertainty shock on industrial production is significantly larger. At the peak of the recession, the industrial production falls less than 0.5% in the case of the monetary policy shock, while it shrinks by about 3% in response to the monetary policy uncertainty shock. Also, the transmission of the shocks to total factor productivity is not the same. At the impact, the drop is almost identical but thereafter the pattern is different. In the case of the monetary policy shock, total factor productivity does not overshoot the long-run level but remains negatively affected as the innovation transmits to the economy. The response of net entry of firms is declining in response to both shocks. As expected, shocks that negatively affect the economy dampen the net creation of new firms in the market. Notice that, the result is consistent with the empirical findings of the previous literature on the effects of monetary policy shocks on net business formation. However, including the series of establishments' births and deaths allows us to discriminate between the responses of the two flows. The evidence of Figure 5 suggests that only firms' exit behaves similarly in response to the two shocks. The response of the firm's entry is remarkably different. Our measure of firms' entry does not plummet in response to the monetary policy shock as it does to the monetary policy uncertainty shock. In the former case, the response is overall imprecisely estimated indicating that the establishment's births are hardly affected by the monetary policy level shock. Such findings are somehow confirmed by looking at the relative contributions of the two shocks to the variability of the variables of interest. Figure 6 reports on the forecast error variance decomposition of industrial production, total factor productivity, firms' entry, and exit. In detail, the panels depict the relative contribution up to 20 months of the monetary policy level shock (in blue) and the monetary policy uncertainty shock (in red). Taking the median values, the contribution of the uncertainty shock is significantly higher than that of the level shock for the economic activity, firms' entry, and exit. On impact, the uncertainty shock explains almost 20% of the variability of the industrial production and establishment's deaths, while around 5% of the variability of establishments' births. However, the contribution of the uncertainty shock to firms' entry more than double after few months. About the level shock, its contribution at the impact is lower than 5% for industrial production and establishments' births, and slightly above 5% for establishments' deaths. The contribution of the level shock is higher for total factor productivity at the impact -more than 20%- but reduces thereafter. In contrast, the contribution of the uncertainty shock for total factor productivity is rising over time-it increases up to 20% at the 20-month horizon.

Our empirical analysis indicates that monetary policy uncertainty shocks explain a sizeable share of the business cycle. Importaintly, they explain more of the variability of output and firm dynamics than of monetary policy level shocks. We rationalize this evidence in the rest of the paper by proposing a DSGE model augmented with heterogenous productivity at the firm level

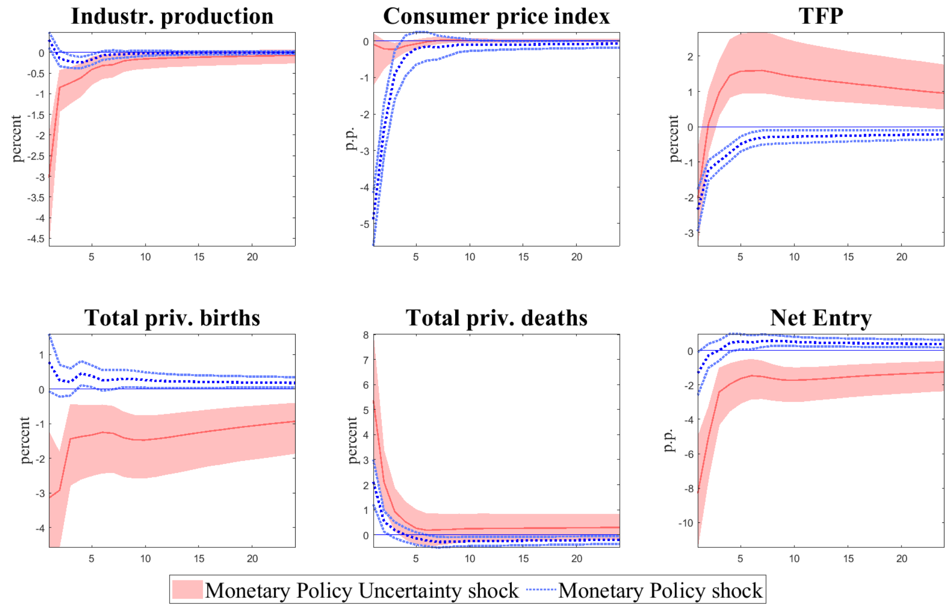

Figure 5: Normalized impulse responses to the monetary policy uncertainty shock (in red) and to the monetary policy level shock (in blue). The solid line is for the median response. The shaded area represents the 68% error band. Industrial production, TFP, entry, exit are in percent. Net entry in percentage points. Inflation in annualized percentage points.

and endogenous entry and exit.

3 Theoretical Model

In this Section, we summarize the theoretical framework of the baseline model considered all along the paper (labeled as *Baseline* henceforth). The Baseline model is a modified version of a standard medium-scale model. The main ingredients of the medium-scale model and its microfoundations are well known in the literature (Christiano et al. (2005), Smets and Wouters (2007)), so the details are not discussed here. We assume sticky nominal wages and prices a là Rotemberg (1982), adjustment costs and capacity utilization for capital, external habit persistence. On top of that, we introduce firm heterogeneity and endogenous entry and exit dynamics in the intermediate sector.

We now present a brief description of the Baseline model, underlying how it differs from the standard medium-scale model and how monetary policy uncertainty shock is introduced. The full list of the equations characterizing the model is in the Technical Appendix.

The model consists of a closed economy composed of four agents: households, firms, a monetary authority, and fiscal authority. In what follows, a brief description of the behavior of the four agents.

3.1 Households

Households consume a basket of differentiated retailer-goods, C_t, and their consumption is characterized by external habits. They supply labor, L_t, to intermediate-good producing firms, they save in the form of new risk-free bonds, B_t, of physical capital, K_{t+1}, of portfolio shares of incumbent firms, x_t, and new entrants, N_t^E. The period utility of the household is defined over the

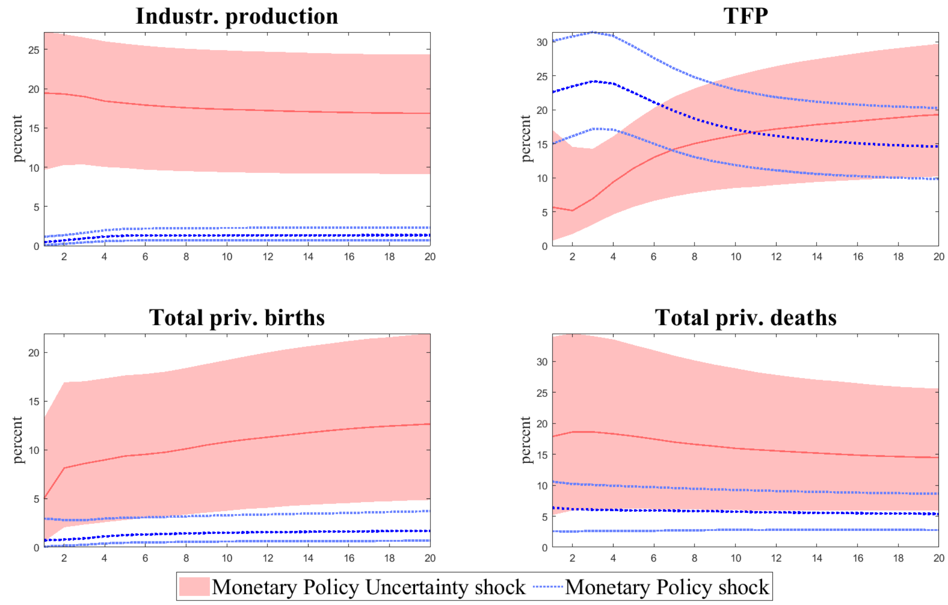

Figure 6: Forecast error variance decomposition to the monetary policy uncertainty shock (in red) and to the monetary policy level shock (in blue). The solid lines refer to the median, the shaded areas represent the 68% error bands.

Dixit-Stiglitz consumption bundle, C_t, and the labor bundle of services, L_t. It reads as follows:

$$U\left(C_t, L_t\right) \ = \frac{\left(C_t - h\overline{C}_{t-1}\right)^{1-\sigma_C}}{1-\sigma_C} \exp\left[\chi \frac{\left(\sigma_C - 1\right)\left(L_t\right)^{1+\sigma_L}}{1+\sigma_L}\right] \tag{11}$$

where h measures the degree of external habits in consumption, $\overline{C}_{t-1}$ is the last period aggregate consumption, σ_C defines the coefficient of the relative risk aversion that determines the constant intertemporal elasticity of substitution ($\frac{1}{\sigma_C}$), χ captures the relative weight assigned to labor and $\sigma_L > 0$ represents the inverse of the Frisch elasticity of the labor supply.

Households own physical capital stocks, K_t, and lease capital services, K_t^s, to firms, as in Smets and Wouters (2007). Capital services are related to the physical capital according to the following relationship:

$$K_t^s = u_t K_t \tag{12}$$

The household budget constraint is the following

$$\begin{aligned} C_t + B_t + v_t\left(\tilde{z}_t\right) x_t + I_t + FEX_t N_t^E + T_t \leq \\ w_t L_t + \left[r_t^K u_t - a(u_t)\right] K_t + \frac{1+r_{t-1}}{1+\pi_t} B_{t-1} + \\ + \left[\left(1-\eta_t\right)\left(v_t\left(\tilde{z}_t\right) + j_t\left(\tilde{z}_t\right)\right) + \eta_t l v_t\right]\left(x_{t-1} + N_{t-1}^E\right) \end{aligned} \tag{13}$$

Households enter in the period t earning the real gross income from labor, $w_t L_t$, the nominal return on bonds, $r_{t-1}B_{t-1}$, the real return of capital $\left[r_t^K u_t - a(u_t)\right] K_t$, where r_t^K is the real rental rate of capital, and $a(u_t)$ is the adjustment cost of variable capital utilization u_t. During the period t, households buy shares of incumbent firms, x_t and invest in new entrants N_t^E. In period $t+1$, with a probability $\left(1-\eta_{t+1}\right)$ measuring the survival rate of firms, households earn from firms' value and profits. Defining $\tilde{z}$ as the average level of productivity, in $t+1$ households gain from the portfolio of firms the value $v_{t+1}\left(\tilde{z}_{t+1}\right)$ and profit $j_{t+1}\left(\tilde{z}_{t+1}\right)$. With a probability η_{t+1} measuring the exit rate of firms, households earn the liquidation value lv_{t+1}. T_t is a lump-sum transfer. The households spend all the earning to consume and save. The variable FEX_t captures the cost of entry paid by households for the new startup firms, which are defined as in Casares et al. (2018), as a combination of constant and variable costs,

$$FEX_t \equiv f^E + ec_t \tag{14}$$

where f^E is the real cost of license fee paid to the fiscal authority to begin the production of a new variety, and ec_t measures congestion externalities for start-up firms:

$$ec_t = \Theta^e \left(\frac{N_t^E}{N_t}\right)^{\varsigma_e} \tag{15}$$

$\Theta^e > 0$ and $\varsigma_e > 1$.[9] Under congestion externality, entry is harder for new entrants as the greater the number of new entrants in any given period, the larger the entry costs faced by each potential entrant. As emphasized by Bergin et al. (2018) this is a common feature in the firm dynamics literature and it is analogous to familiar quadratic adjustment costs for investments in physical capital, since it serves the function of capturing the behavior of entry that responds gradually over time and not instantaneously to shocks, as observed in the data.

If a firm exits, a liquidation value is returned to households, which is a positive function of the fraction of the license fee paid at entry, f^E, and a negative function of exit congestion externalities, xc_t:

$$lv_t = (1-\tau) f^E - xc_t \tag{16}$$

where, as in Casares et al. (2018), $1-\tau$, with $0<\tau<1$, is the share of license fee returning to the households and paid by the fiscal authority once a firm exits the market, while

$$xc_t = \Theta^x \left(\frac{N_t^X}{N_t}\right)^{\varsigma_x} \tag{17}$$

with $\Theta^x > 0$ and $\varsigma_x > 1$, represents exit congestion externalities.[10]

The law of motion of the firms follows the standard one-period time-to-build assumption as

$$N_t = (1-\eta_t)\left(N_{t-1} + N_{t-1}^E\right) \ . \tag{18}$$

Hence, the stock of firms, N_t, is given by the sum of incumbent firms , $(1-\eta_t) N_{t-1}$, and surviving new entrants, $(1-\eta_t) N_{t-1}^E$. Firms' separation rate depends on an endogenous probability of defaulting, η_t, specified below. Both incumbent and new entrant firms are subject to the same endogenous exit probability. The exiting firms are thus given by

$$N_t^X = \eta_t \left(N_{t-1} + N_{t-1}^E\right) \ .$$

Households choose capital utilization and end up paying a quadratic cost for that utilization

[9] Similar assumption on entry congestion extenalities can be found in Bergin et al. (2018).

[10] As for the entry cost, it serves the function of capturing the dynamic behavior of exit over time as observed in the data. Though these costs help to capture the quantitative dynamics of entry and exit, the qualitative results of our model are not altered by the assumption of entry and exit congestion externalities.

relative to its normalized steady state value, which is equal to 1,

$$a(u_t) = \gamma_1 (u_t - 1) + \frac{\gamma_2}{2} (u_t - 1)^2 \tag{19}$$

where γ_1 and γ_2 are the parameters governing the cost of utilization of capital.

Physical capital accumulates as follows:

$$K_{t+1} = \left(1 - \delta^K - S\left(\frac{I_t}{K_t}\right)\right) K_t + I_t \tag{20}$$

where δ^K is the depreciation rate, and $S\left(\frac{I_t}{K_t}\right)$ are capital adjustment costs defined as in Hayashi (1982), as:

$$S\left(\frac{I_t}{K_t}\right) = \frac{\phi_K}{2}\left(\frac{I_t}{K_t} - \delta^K\right)^2 \tag{21}$$

The implied first-order conditions of the household problem are listed in the Technical Appendix. They are the households' labor supply, the households' investment choice, the Euler equation for consumption, for physical capital, for shares holding, and the firm entry condition.

Households supply their homogenous labor to an intermediate labor union which differentiates the labor services and sets wages subject to Rotemberg (1982) adjustment costs. As for the FOCs of the household problem, the wage New-Keynesian Phillips curve (NKPC) resulting from the union problem is reported in the Technical Appendix.

3.2 Firms

As in Rossi (2019), the supply side of the economy consists of an intermediate and a retail sector. The intermediate sector is composed of a continuum of N_t intermediate firms that compete under monopolistic competition and flexible prices to sell the intermediate goods to a continuum of measure one of retailers. Each $k \in (0, 1)$ retailer buys intermediate goods from the intermediate sector and differentiates them with a technology that transforms the intermediate goods into an aggregate industry good, $Y_t^I(k)$, solving a minimum expenditure problem. Retailers sell the differentiated industry goods to households, competing with other retailers under monopolistic competition. They face Rotemberg (1982) adjustment costs so that, due to the monopolistic competition structure, the second optimization problem gives rise to the price NKPC.

3.2.1 Intermediate Sector

Each firm in the intermediate sector produces a differentiated good under monopolistic competition and flexible prices.[11] Firms are heterogeneous in terms of their specific productivity, which is drawn from a Pareto distribution. In this context, the production function of firm ι, with $\iota \in [1, N_t]$, is

$$y_{\iota,t} = z_{\iota,t} l_{\iota,t}^{1-\alpha} \left(k_{\iota,t}^s\right)^\alpha \tag{22}$$

where $l_{\iota,t}$ and $k_{\iota,t}^s$ are respectively, the amount of labor hours and capital services employed by firm ι, while $z_{\iota,t}$ is the firm-specific productivity, which is assumed to be Pareto distributed across firms,

[11] In this model sticky prices are in the final sector and not in the intermediate good sectors, where the firm dynamism is modeled. This is for technical reasons. To satisfy the Melitz (2003) theorem of price aggregation markups should be the same across firms. Yet, the main results are not affected by the sticky price assumption, since the stickiness in the final sector transmits to the intermediate sector.

as in Ghironi and Melitz (2005). The coefficient α measures the elasticity of output with respect to capital.

This sector is characterized by endogenous firm dynamics. The timing characterizing the dynamics of firms is the following. At the beginning of the period, households invest in new firms until the entry condition is satisfied, that is until the average firms' value equals the entry costs,

$$v_t(\widetilde{z}) = FEX_t \tag{23}$$

Note that the value of the firm facing the average productivity corresponds to the stock price of the economy. The latter is so given by

$$v_t(\widetilde{z}_t) = \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \left(\left(1 - \eta_{t+1}\right) \left(v_{t+1}(\widetilde{z}_{t+1}) + j_{t+1}(\widetilde{z}_{t+1})\right) + \eta_{t+1} l v_{t+1} \right) \right], \tag{24}$$

with λ_t as the marginal utility of consumption at time t, and $j_t(\widetilde{z}_t)$ as the current profits of the average firm.

Then, incumbent and last-period entrant firms draw their firm specific productivity from a Pareto distribution. The cumulative distribution function (CDF) of the Pareto implied for productivity $z_{\iota,t}$ is $G(z_{\iota,t}) = 1 - \left(\frac{z_{\min}}{z_{\iota,t}}\right)^{\xi}$, where $z_{\min}$ and ξ are scaling parameters of the Pareto distribution.[12] After drawing the idiosyncratic level of productivity, firms observe the aggregate shock and decide whether to produce or exit the market. Using this timing assumption, the decision of last-period entrants to exit the market is identical to the decision of incumbent firms. In particular, both new entrants and incumbent firms decide to produce as long as their specific productivity $z_{\iota,t}$ is above a cutoff level $\overline{z}_t$. The latter is the level of productivity that makes the sum of current and discounted future profits equal to the liquidation value, lv_t. Separated firms exit the market before starting the production. It follows that the average output and the average firms' productivity depend on the cut-off level of productivity in the economy, $\overline{z}_t$, which is endogenously determined through the following exit condition:

$$v_t(\bar{z}_t) = lv_t, \tag{25}$$

where the value of the firm with a productivity level that is equal to the marginal value $\overline{z}_t$ reads as

$$v_t(\bar{z}_t) = j_t(\bar{z}_t) + \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \left(1 - \eta_{t+1}\right) v_{t+1}(\bar{z}_{t+1}) \right]. \tag{26}$$

Equation (26) states that the value of the marginal firm is given by its current profit $j_t(\bar{z}_t) = y_t(\bar{z}_t) - w_t l_{\bar{z},t} - r_t^K k_{\bar{z},t}^s$, with $w_t l_{\bar{z},t}$ the cost of labor and $r_t^K k_{\bar{z},t}^s$ the cost of capital services of the marginal firm.

The exit probability, $\eta_t = 1 - \left(\frac{z_{\min}}{\bar{z}_t}\right)^{\xi}$, is endogenously determined. As in Ghironi and Melitz (2005), the lower bound productivity level, $z_{\min}$, is low enough relative to the production costs, so that $\overline{z}_t$ is above $z_{\min}$. In each period, this ensures the existence of an endogenously determined number of exiting firms. The number of firms with productivity levels between $z_{\min}$ and the cutoff level $\overline{z}_t$ are separated and exit the market without producing.

[12] They represent respectively the lower bound and the shape parameter, which indexes the dispersion of productivity draws. As ξ increases, the dispersion decreases, and firm productivity levels are increasingly concentrated towards their lower bound $z_{\min}$.

3.2.2 Retailers

The retailer problem is split into two parts. First, each $k \in (0,1)$ retailer buys a fraction of the N_t intermediate goods produced by the N_t intermediate firms at the intermediate goods prices $p_{\iota,t}$. Retailers bundle the goods into an aggregate industry good, $Y_t^I(k)$, minimizing their expenditure according to a CES technology $Y_t^I(k) = \left(\int_{N_t} y_{\iota,t}^{\frac{\theta_p - 1}{\theta_p}} d\iota\right)^{\frac{\theta_p}{\theta_p - 1}}$, with $\theta_p > 1$, as the elasticity of substitution among the intermediate goods varieties. Retailer's minimum expenditure problem implies the following demand function for the intermediate good ι:

$$y_{\iota,t} = \left(\frac{p_{\iota,t}}{P_t^I}\right)^{\theta_p} Y_t^I(k), \tag{27}$$

implying the intermediate sector price index as

$$P_t^I(k) = \left(\int_{N_t} p_{\iota,t}^{\theta_p - 1} d\iota\right)^{\frac{1}{\theta_p - 1}}.$$

Second, each k retailer competes with the others under monopolistic competition to sell its bundle, $Y_t^I(k)$, to the household at the price $P_t^R(k)$, which is a markup over the intermediate sector price index, $P_t^I(k)$. Retailers adjust prices according to the Rotemberg (1982)'s model. The retailer's optimal price decision rule implies the following standard NKPC:

$$1 = \frac{\theta_p}{\theta_p - 1}\rho_t^I - \frac{\phi_p}{\theta_p - 1}(\pi_t - 1)\pi_t + \frac{\phi_p}{2}(\pi_t - 1)^2 + \frac{\phi_p}{\theta_p - 1}E_t\left\{\Lambda_{t,t+1}(\pi_{t+1} - 1)\pi_{t+1}\frac{Y_{t+1}}{Y_t}\right\} \tag{28}$$

with ϕ_p as the adjustment price parameter, and ρ_t^I as the relative price $\frac{P_t^I(k)}{P_t}$. By symmetry among the retailers, it holds $Y^R(k) = Y_t$ and $P^R(k) = P_t$. Hence, $\pi_t = \frac{P_t}{P_{t-1}}$ is the gross inflation rate.

3.3 Monetary and Fiscal Authority

Monetary Authority

To close the model we specify an equation for the behavior of the Central Bank. We simply assume that the monetary authority sets the nominal net interest rate i_t following a standard Taylor-type rule given by

$$\log\left(\frac{1+i_t}{1+i}\right) = \phi_R \log\left(\frac{1+i_{t-1}}{1+i}\right) + (1-\phi_R)\left(\phi_\pi \log\left(\frac{\pi_t}{\pi}\right) + \phi_{dy}\log\left(\frac{y_t}{y_{t-1}}\right)\right) + \varepsilon_{R,t}, \tag{29}$$

where ϕ_π and ϕ_{dy} are the elasticities of the nominal interest rate with respect to the deviation of the inflation from their long-run target and to the growth rate of output. The parameter ϕ_R is the interest rate smoothing parameter. We model the monetary uncertainty shocks by using the stochastic volatility approach proposed by Mumtaz and Zanetti (2013) and Born and Pfeifer (2014), that is by assuming time-varying volatility of the innovation to the monetary shock. Specifically, the policy uncertainty shock enters into the economy through the monetary shock, $\varepsilon_{R,t}$, that follows an AR(1) process,

$$\varepsilon_{R,t} = \rho_R \varepsilon_{R,t-1} + e^{\sigma_{R,t}} u_{\varepsilon,t} \tag{30}$$

with

$$\sigma_{R,t} = \rho_\sigma \sigma_{R,t-1} + u_{\sigma,t} \tag{31}$$

where $u_{\varepsilon,t}$ is the Gaussian innovation to the monetary shock, i.e. the level shock, while $u_{\sigma,t}$ is the Gaussian innovation to the standard deviation, $\sigma_{R,t}$, of the monetary shock, i.e. the volatility shock. An innovation to the volatility shock thereby increases the uncertainty about the path of the level shock.[13]

Fiscal Authority

The fiscal authority runs the following balanced budget:

$$T_t = f^E N_t^E - (1-\tau) f^E N_t^X$$

where T_t are lumps-sum transfers/taxes to the households, $f^E N_t^E$ are the revenues obtained from households in form of administrative fees for opening new startups, $(1-\tau) f^E N_t^X$ is the expenditure in form of liquidation value paid to households as firms exit the market.

3.4 Aggregation and Market Clearings

The economy aggregate output is implied by the following

$$Y_t = N_t^{\frac{1}{\theta_p - 1}} \widetilde{z}_t (L_t)^{1-\alpha} (K_t^s)^\alpha \tag{32}$$

while the resource constraint of the economy is given by,

$$Y_t = C_t + I_t + a(u) K_t + N_t^E ec_t + N_t^X xc_t + PAC_t + WAC_t \tag{33}$$

where

$$PAC_t = \frac{\phi_p}{2} (\pi_t - 1)^2 Y_t \tag{34}$$

and

$$WAC_t = \frac{\phi_w}{2} \left(\frac{w_t}{w_{t-1}} \pi_t - 1 \right)^2 Y_t \tag{35}$$

are respectively the price and wage adjustment costs.

3.5 Equity premia and firm dynamics

In this Section, we focus on the two key equations affecting firms' decisions on participating in the market, namely the entry (23) and the exit condition (25). The entry condition states that households will invest in new firms up to the point in which the average firm value equals the entry cost they pay to enter. The exit condition defines the value of the marginal firm at time t, which corresponds to the one that equals firm liquidation value. Worth stressing, both the value of average firm and marginal firm are in turn defined by the intertemporal equations (24) and (26). The latter iteratively link the present equity value of firms to the expected future realizations. Working around these equations, we can rewrite them in terms of the returns on equity for the average firm, κ_t, and marginal firm, $\bar{\kappa}_t$. Thus, equation (24) can be written as

$$1 = E_t \left[M_{t+1} \bar{\eta}_{t+1} \kappa_{t+1} \right], \tag{36}$$

[13] We also tested our model specifying the stochastic processes in levels as in Basu and Bundick (2017), where the volatility $\sigma_{R,t}$ does not impact the average value of level shock. However, the transmission of the uncertainty shock remains fully consistent with the benchmark.

where we split the overall stochastic discount factor in two components: the discount factor that would prevail in an economy without firms' exit, i.e. $M_{t+1} \equiv \beta E_t\left(\frac{\lambda_{t+1}}{\lambda_t}\right)$, and the survival probability $\bar{\eta}_{t+1} \equiv \left(1-\eta_{t+1}\right)$. The equity return of the average firm is given by $\kappa_{t+1} \equiv \frac{v_{t+1}+j_{t+1}}{v_t} + \frac{1-\bar{\eta}_{t+1}}{\bar{\eta}_{t+1}}\frac{lv_{t+1}}{v_t}$, which includes the liquidation value gained by households whenever a firm exits from the market. Similar for the marginal firm, equation (26) can be rewritten as

$$1-\frac{\bar{j}_t}{\bar{v}_t} = E_t\left[M_{t+1}\bar{\eta}_{t+1}\bar{\kappa}_{t+1}\right], \tag{37}$$

where $\bar{v}_t$ and $\bar{j}_t$ represent marginal firm's value and profits at time t,, respectively. The equity return for the marginal firm is defined as $\bar{\kappa}_{t+1} \equiv \frac{\bar{v}_{t+1}}{\bar{v}_t}$.

With equations (36) and (37) in hand, we can study the effect of the monetary policy uncertainty shock on the equity premium. To do that, we take the same approach used by Bianchi et al. (2018), who identify distinct risk propagation channels for uncertainty shocks by implementing a risk-adjusted log-linearization of the equations of their model.[14] The technique offers the advantage of developing a linear approximation of the model which takes into account the risk-adjustment in the expectational equations.[15] In our case, the log-linearization of expectational equations as (36) and (37) involves second-order terms that capture the risk-adjustment. The methodology allows us to get an analytical decomposition of the equity premium that accounts for the effects of the monetary policy uncertainty. The Technical Appendix contains more details about the risk-adjusted log-linearization of equations (36) and (37). Here, we discuss the decomposition of the equity premium and illustrate how differently its components are affected by increased monetary policy uncertainty. We define two measures of equity premium as, respectively, the spread between the expected return on equity of average and marginal firm and the risk-free rate, $R_{f,t}$, that is the return on a theoretical risk-free asset that bought at time t pays one unit of the consumption good in every state of the economy at time $t+1$.

From the log-linearization with risk adjustment of equation (36), we obtained the following expression that we label as the *average* equity premium,

$$E_t\hat{\kappa}_{t+1} - \hat{R}_{f,t} = -E_t\hat{\bar{\eta}}_{t+1} + E_t\left[\begin{array}{c} -\frac{1}{2}Var_t\hat{M}_{t+1} - \frac{1}{4}Var_t\hat{\bar{\eta}}_{t+1} - \frac{1}{2}Var_t\hat{\kappa}_{t+1} \\ -Cov_t\left(\hat{M}_{t+1},\hat{\bar{\eta}}_{t+1}\right) - Cov_t\left(\hat{M}_{t+1}+\hat{\bar{\eta}}_{t+1},\hat{\kappa}_{t+1}\right) \end{array}\right]. \tag{38}$$

Variables with hat are in log-deviation from the deterministic steady state, e.g. $\hat{x}_t \equiv \log(\frac{x_t}{x})$. Notice that, the average equity premium depends negatively on the expected future survival probability for firms. A higher probability of surviving reflects a lower probability of default that makes investing in equity less risky. Hence, it will command a lower compensation on equity, everything else equal. The term in brackets in equation (38) is the risk-adjustment component. Consistent with the description provided by Bianchi et al. (2018) about the channels for which the uncertainty shocks affect the macroeconomic variables, we denote that term as the *entry risk premium.* Higher variance terms have all a negative impact on the equity return. More interestingly, the effect of the covariance terms depends on the underlying relationships between the variables. If the expected survival probability is low when the marginal utility of wealth is high, then investing in firms is risky and does not hedge households in recession. This will command a higher entry risk premium to

[14] The same methodology has been carried out in other contributions as Jermann (1998), Lettau (2003), Kaltenbrunner and Lochstoer (2010), and Malkhozov (2014).

[15] This is allowed because once the model is log-linearized, the variables in levels are log-normal distributed conditionally to the exogenous shocks.

compensate for the inverse relationship. Similarly, if the return on equity is low when the marginal utility of wealth adjusted by the survival probability is high, then investing in equity is risky and commands a higher entry risk premium.

We obtain the *marginal* equity premium by log-linearizing with risk adjustment equation (37),

$$
\begin{aligned}
E_t\widehat{\hat{\kappa}}_{t+1} - \hat{R}_{f,t} \quad = \quad & \frac{\bar{\bar{j}}}{\bar{v}-\bar{\bar{j}}}\left(\widehat{\bar{v}}_t - \widehat{\bar{j}}_t\right) - E_t\widehat{\bar{\eta}}_{t+1} \\
& + E_t\left[\begin{array}{c} -\frac{1}{2}Var_t\hat{M}_{t+1} - \frac{1}{4}Var_t\widehat{\bar{\eta}}_{t+1} - \frac{1}{2}Var_t\widehat{\bar{\kappa}}_{t+1} \\ -Cov_t\left(\hat{M}_{t+1}, \widehat{\bar{\eta}}_{t+1}\right) - Cov_t\left(\hat{M}_{t+1} + \widehat{\bar{\eta}}_{t+1}, \widehat{\bar{\kappa}}_{t+1}\right) \end{array}\right].
\end{aligned} \tag{39}
$$

The marginal equity premium is proportional to the current value of the marginal firm net of profits up to the constant. $\frac{\bar{\bar{j}}}{\bar{v}-\bar{\bar{j}}}$. The marginal equity premium is, moreover, affected by the expected future survival probability and the risk-adjustment component in brackets, which we label as the *exit risk premium.* Entry and exit risk premium share thus the same expression but the variance and covariance terms concerning the equity return refer, respectively, to the average and marginal firm.

4 Model Dynamics

This Section shows the implied model dynamics in response to an unexpected increase in monetary uncertainty. First, we illustrate the calibration strategy. Second, we compute the impulse response function (IRFS) of the main macroeconomic variables of our *Baseline* model to a positive monetary uncertainty shocks. For better understanding the role played by firms dynamics the IRFs of Baseline is compared with two alternative models: i) a model charaterized by endogenous entry and an exogenous firms' exit probability, η, modeled as in Bilbiie et al. (2012), which we label as *Exo Exit*; ii) a standard medium-scale model without firms dynamics, which we label as *No Firms*. Finally, to test the robustness of the results we consider alternative calibrations for three key parameters of the Baseline model , i.e. the elasticity of substitution in the goods market, the degree of rigidity in price adjustment, and the persistence of the monetary level shock.

4.1 Calibration

For sake of comparison with the FAVAR, the calibration of the DSGE model is set at a monthly frequency. We set parameters for the model spelled out in Section 3, i.e. the *Baseline* model. We keep fixed the same calibration for Exo Exit and No Firms.

First, we calibrate the parameters of the exogenous processes. We study one-standard-deviation shock to the volatility of the monetary shock, $\sigma_{R,t}$, that follows an AR(1) process with the IID Gaussian term $u_{\sigma,t} \sim N(0,1)$. We assume an AR(1) process for the monetary shock $\varepsilon_{R,t}$ as well, with innovation the IID Gaussian term $u_{\varepsilon,t} \sim N\left(0, 0.003^2\right)$. As a result, the volatility shock we simulate more than doubles the uncertainty around the monetary policy shock at the impact.[16] To set the persistence of the shock, we calibrate the autoregressive coefficients of both first and second-order shock processes. For the autoregressive coefficient ρ_σ, we proceed as follows. First, we back out the model-implied VXO similarly to Basu and Bundick (2017).[17] Then, we set ρ_σ

[16] From equation (30), a one standard deviation shock to $\sigma_{R,t}$ impacts the level shock by around 2.7.

[17] We construct a model counterpart to the VXO index to link the dynamics of the DSGE with the one in the FAVAR. We define the equity return and the expected conditional volatility of the equity return, i.e. the model-implied VXO index, as in Basu and Bundick (2017), (see equations 9 and 10 in the paper). Also, we test our results by using a different definition of the equity return that is closer to our set-up of endogenous firm entry and exit. The

to match the persistence of the VXO in the FAVAR to the monetary policy uncertainty shock.[18] This requires us to set the persistence of the volatility shock, ρ_σ, to 0.85. We fix the persistence of the level shock, ρ_R, to 0.5 in the benchmark calibration. We, however, check the robustness of the findings of the Baseline with different values of ρ_R.

The next set of parameters we calibrate concerns household preferences. The discount factor, β, is set at 0.9967, corresponding to an annualized real interest rate of about 4%. The coefficient of the relative risk aversion, σ_C, is set to 1.5, while the elasticity of labor supply, σ_L, to 5. The habits persistence parameter is set to 0.6. All values of the parameters in the utility function lay within admissible intervals of estimates in the literature (Smets and Wouters (2007), Christiano et al. (2005)). The capital-income share α is set to 0.33, whereas the depreciation rate of the physical capital, δ_k, is set to 0.0067, which is equivalent to around 2% every quarter. The parameter measuring the elasticity of the capital utilization adjustment cost function, γ_2, is set to 0.54 as in Smets and Wouters (2007), while the capital adjustment costs parameter, ϕ_K, is set to 5. The output in the steady state is normalized to 1.

The steady state value of the exit probability η is set to match the U.S. quarterly establishments' death ratio, which is at around 3% for the period considered in the FAVAR analysis. The parameter of the elasticity of substitution among intermediate goods, θ_p, is set equal to 4.3, corresponding to a steady state price markup of around 30%. Though this value is in line with the literature on firm dynamics (Ghironi and Melitz (2005), Bilbiie et al. (2012)), we test the robustness of the results at a different level of price markup. We set the markup in the labor market as the benchmark for the good market, so that the elasticity of substitution among labor types θ_w is fixed to 4.3. The shape parameter of the Pareto distribution ξ is set equal to 6.51 to satisfy the steady state value of the exit rate. This value also guarantees that the condition for well-behaved average productivity, i.e. $\xi > \theta_p - 1$, is satisfied. The lower bound of productivity distribution, $z_{\min}$, is equal to 1. The variable components of entry and exit costs, ec and xc, are set, respectively, to 1.6% and 1.2% of the GDP in the steady state. The elasticities of entry and exit congestion externalities, ς_e and ς_x, are set to 2 and 1. Both the variable components of sunk costs and the congestion externalities are set slightly higher for entry than for exit, which is consistent with the estimates in Casares et al. (2018). Once ec, xc, ς_e, ς_x are calibrated, the remaining constant component of the entry cost, f^E, and the parameters Θ^e and Θ^x are endogenously determined.[19] The share of the fixed entry cost of the exiting firms rebated to the households is fixed to 25% so that the parameter τ is set to 0.75.

Parameters describing the price and wage setting are calibrated as follows. We set the Rotemberg parameter of price adjustment cost ϕ_p equal to 80 so that the slope of the linearized NKPC corresponds to that in a Calvo staggered price-setting model with nearly half a year of price contract duration. Analogously, we follow Born and Pfeifer (2020) in mapping the Rotemberg parameter of nominal wage adjustment cost ϕ_w into the corresponding Calvo wage contract duration. We set ϕ_w equal to 160, which corresponds to a wage contract duration of slightly more than 6 months. Our calibration for nominal frictions is compatible with the estimates in the literature (Smets and

alternative defines the return of firm equity as $R^E_{t+1} = \frac{(1-\eta_{t+1})(v_{t+1}(\tilde{z}_{t+1})+j_{t+1}(\tilde{z}_{t+1}))+\eta_{t+1} lv_{t+1}}{v_t}$, namely it allows for the probability of exiting and the liquidation value to affect the equity return. Our simulations, however, indicate that the two measures of equity return produce almost identical responses for the expected conditional volatility.

[18] Our evidence suggests that, at the median, the effect of the uncertainty shock on the VXO index falls gradually to about 75 and 45% of its impact value after 1 and 12 months, respectively. This evidence suggests that the persistence for the VXO index would be 0.75 if we take the observation at 1 month, or about 0.94 if we take the observation at 12 months ($0.4480^{\frac{1}{12}} = 0.94$). We take the average value of these observations to broadly match the persistence of the VXO in the FAVAR at a 1-month horizon.

[19] Though entry and exit adjustment costs help to capture the quantitative dynamics of entry and exit, we tested that the qualitative results of our model are not altered by the assumption of entry and exit congestion externalities.

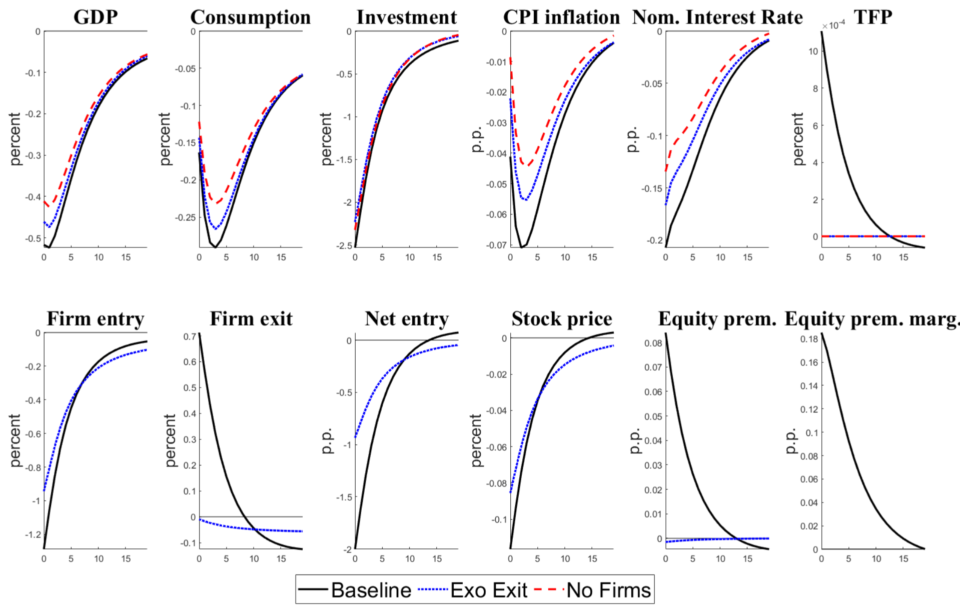

Figure 7: DSGE impulse responses to a one standard deviation monetary policy uncertainty shock. Baseline model versus the model with exogenous exit probability (Exo Exit) and the model without entry and exit (No Firms). Annualized impact for interest rates, inflation, equity premium.

Wouters (2007), Christiano et al. (2005)), and along with that, preserves a higher degree of rigidity for wages. As robustness, we compare the performance of the Baseline model by changing the parameter of price adjustment.

Finally, we set the coefficients in the Taylor rule as $\phi_R = 0.75$, $\phi_\pi = 2.5$, $\phi_{dy} = 0.05$. Being roughly in the range of the values estimated for the U.S. economy.[20], the calibration guarantees the uniqueness of the equilibrium, and specifically for the feedback to output growth, it implies a response of the policy rate of 0.15 to deviations in the output on a quarterly basis.

4.2 IRFs to monetary policy uncertainty shocks

This Sub-Section comments on the transmission of the monetary policy volatility shock in the DSGE model. To examine the dynamic effects of this second-order shock, we solve the model using third-order approximations to the equilibrium conditions around the steady state. We follow the procedure suggested by Fernández-Villaverde et al. (2015) to compute the impulse responses in deviation from the stochastic steady state.

We carried out the impulse response analysis as follows. First, we show the impulse responses of the Baseline model and compare them against those of Exo Exit and No Firms models. Second, we test the robustness of the benchmark calibration testing the Baseline over different values of i) the elasticity of substitution in the goods market, ii) the degree of price stickiness, and iii) the persistence of the monetary policy shock.

Figure 7 shows the responses to the monetary policy uncertainty shock of Baseline, Exo Exit,

[20] See for example Smets and Wouters (2007). We check, however, that the findings are not qualitatively altered by the choice of the coefficients in Taylor rule for neither the Baseline model nor the other specifications.

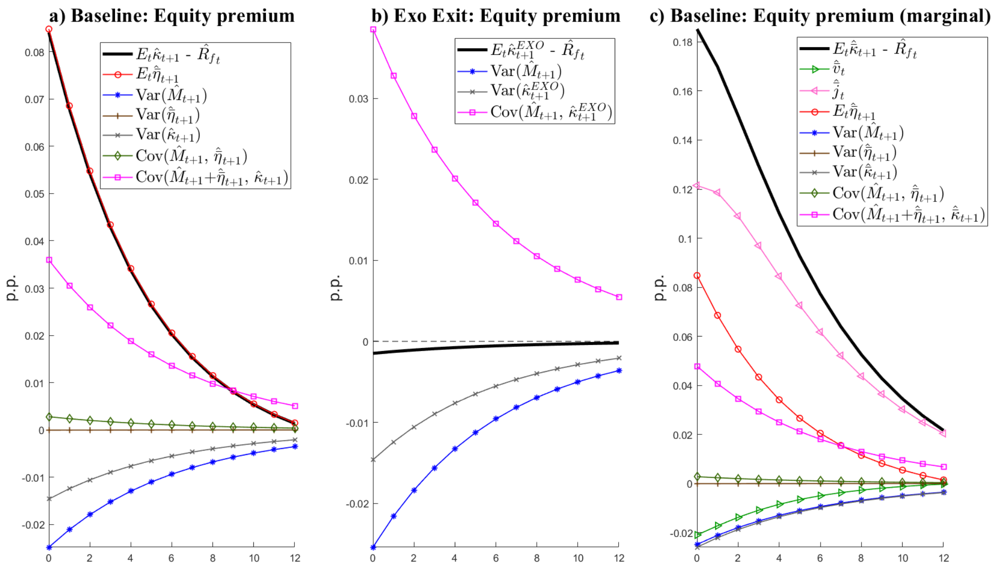

Figure 8: DSGE impulse responses of equity premium and its components to the monetary policy uncertainty shock.

and No Firms model[21]. The comparison allows us to investigate the relevance of firm dynamics and firm heterogeneity in explaining the propagation of the shock. In Exo Exit, the heterogeneity in firm productivity does not play a role because, as in Bilbiie et al. (2012) among others, the probability of defaulting is constant and does not depend on the idiosyncratic level of productivity each firm draws at the beginning of the period. In No Firms, only the intensive margin of the investment, namely the one in physical capital, is allowed to respond, while the extensive margin, namely the investment in new firms, is neglected.

The panels on the first row of 7 show that, in all specifications, the monetary policy uncertainty shock generates a positive comovement in output, consumption, and investment in physical capital. The increase in monetary volatility is followed by a slump in all three variables. As emphasized by Basu and Bundick (2017), the result is standard in macroeconomic models with nominal rigidities, where the fall of the aggregate demand is sufficiently large to make both consumption and investment in capital declining in response to the heightened volatility. However, our Baseline model also embeds the extensive margin of investments, namely firm entry and exit. Because of the endogenous responses of entry and exit, the impact is larger in Baseline. A one standard deviation shock in the volatility of the monetary shock depresses the GDP by more than half of a percentage point in the Baseline model. The recession is less severe for the alternative specifications. Especially for No Firms, which neglects both the extensive margins of firms and therefore the overall contraction of output and consumption is milder. Differences in the dynamics of the three models widen for the nominal variables. Although the fall in inflation is overall weak in comparison to real variables, the impact is relatively stronger on Baseline. Unsurprisingly, given the heavier impact on output and inflation, monetary policy is more expansive in Baseline to mitigate the effects of the recession.

[21] To be consistent with the data in the FAVAR model, this Section comments on the impulse responses we obtain for DSGE models for aggregate variables that are depurated by the love of variety, $N_t^{\frac{1}{\theta_p - 1}}$.

Different from Baseline, neither No Firms nor Exo Exit imply endogenous TFP dynamics. The model response of the TFP is mute in both specifications, as long as either no firms exit the market or firms exit the market exogenously and not due to the low productivity. In both cases, firm average productivity and the aggregate TFP remain constant. In Baseline, aggregate productivity responds positively to the monetary policy uncertainty shock. The reason lies in the *selection effect* in the productive sector. As a consequence of the increased threshold $\overline{z}_t$, the less productive firms are pushed out of the market. and aggregate productivity increases after the shock. Noteworthy, in the FAVAR, the TFP shows a negative response only at the very impact of the shock. Thereafter, the TFP overshoots its long-run trend and remains persistently positive. The differences at a very short horizon can be justified by the fact that the creative destruction mechanism we emphasize in our model is only one of the possible mechanisms affecting the TFP. Another possible channel might be the dynamics of the labor market and in particular, of unemployment. During a recession, unemployment increases with lags, and thereby, total hours worked decrease with delays as well. The same occurs for the stock of capital. While output reacts immediately, sluggish adjustments in the productive factors can justify the initial reduction of the TFP. However, as soon as unemployment increases and firms with lower productivity are pushed out of the market, the TFP increases and remains positive as the shock transmits to the rest of the economy. Ultimately, the response of the aggregate productivity in Baseline is consistent with the one obtained by the FAVAR. This corroborates our claim that the DSGE specification that encompasses both heterogeneous productivity at the firm level and endogenous entry and exit is the one that fits better the empirical evidence.

The second row of 7 reports the responses of variables that are related to the extensive margin of investment. For this reason, only the cases of Baseline and Exo Exit are considered. Remarkably, the differences between the models enlarge for these variables. Focusing on the entry and exit of firms, only the Baseline model can replicate the evidence of the FAVAR that indicates that the two flows react in the opposite way to a monetary policy uncertainty shock. In both Baseline and Exo Exit, the decline in the entry is driven by the reduction in the firm value, namely the stock price in the model. As in the FAVAR, the stock price falls in both specifications but in Baseline the contraction is stronger. As equation (24) shows, the firm value in Baseline is given by the present discounted value of the stream of expected future profits. After the monetary policy uncertainty shock, firm profits decline and the minimum level of productivity, $\overline{z}_t$, which guarantees the market participation, increases. The exit probability, depending on the minimum level of productivity (see Section 3.2.1), rises as well. This has implications for entry and exit. On the entry side, the increased exit probability affects the overall stochastic discount factor at which future firm profits are discounted. This dampens the average firm value and makes the fall in entry larger than in Exo Exit, where the exit probability η, remains constant. For the same reason, the exit of firms reacts differently between Baseline and Exo Exit. In the former, firms' exit surges after the shock, while in the latter declines. In Exo Exit, the flow of exiting firms is proportional to the stock of firms that participate in the market. As the number of firms falls because of the reduced entry, the number of exiting firms reduces proportionally too. At odds with the FAVAR and the Baseline model, firms' exit is thereby procyclical to output in Exo Exit. As a consequence, although net entry reduces in both Baseline and Exo Exit, the fall in the former is twofold at the impact.

The last two panels in the second row of 7 show the responses of the average and marginal equity premium, respectively. Both premia surge in Baseline. In particular, the rise in the premium for equity return of marginal firm doubles that of the average firm. The positive response of equity premia in Baseline is consistent with the FAVAR and at odds with Exo Exit, where the equity premium is almost unaffected. It needs to be stressed that in Exo Exit there is not an exit condition as in Baseline. Hence, we can define only one measure of the equity premium, which is calculated as the average equity premium in Baseline but with the exiting probability as a constant.[22]

4.3 Firm Dynamics and Equity Premia

We now investigate the different shock transmissions in Baseline and Exo Exit further by inspecting the response of the equity premia and its relation with firm dynamics. In particular, we exploit the log-linearization with risk adjustment introduced in Section 3.5, and decompose the aggregate response of the equity premia into contributions of single components. Figure 8 shows the decomposition of responses of average equity premium in Baseline (panel a) and Exo Exit (panel b), and marginal equity premium in Baseline (panel c). Back solid lines refer to the aggregate responses. The response of the survival probability is depicted with a red dashed line. The dotted lines refer to the variance terms and the lines with triangle markers to the covariances. Looking at the average equity premium in Baseline, the propagation of the shock seems almost entirely driven by the survival probability. The increased uncertainty around monetary policy reduces the expected survival probability, which is, however, negatively associated with the equity premium, and therefore contributes to increasing the latter. A closer inspection of Figure 8, reveals that also the terms of the entry risk premium are affected by the shock but their responses compensate each other making its contribution as minor. Three components of the risk equity premium work in the opposite direction. As holding equity has become riskier after the monetary policy uncertainty shock, the covariance between the adjusted stochastic discount factor and equity premium shrinks and commands a higher equity premium. On the other side, both the variance of stochastic discount factor and of equity premium ramp up and contribute to dampening the equity premium. A similar analysis of Panel B helps to understand why the response of equity premium in Exo Exit is negligible. Different from Baseline, in Exo Exit the survival probability does not play a role in explaining the equity premium because it is constant. The dynamics of the latter is only driven by the entry risk premium, whose components react to the shock but, as for Baseline, almost offside each other. As a result, the equity premium in Exo Exit is nearly unaffected by the shock. Panel C completes the analysis showing the decomposition of the response of marginal equity premium in Baseline. Notably, the contribution of the expected survival probability and second-order components is almost identical to the case of the average equity premium. The difference of the aggregate responses lies in the contribution of the current variables of the marginal firm in equation (39): i) the firm value, $\widehat{\bar{v}}_t$, and ii) the profits, $\widehat{\bar{j}}_t$. The shock reduces both, but the drop in profits is stronger. Thus, $\frac{\bar{j}}{\bar{v}-\bar{j}}\left(\widehat{\bar{v}}_t - \widehat{\bar{j}}_t\right)$ is greater than zero, contributing to rising the marginal equity premium.[23] This also explaines the relatively higher effect of the shock on the marginal equity premium than on the average equity premium.

4.4 Robustness checks

We test the robustness of the transmission of the monetary policy uncertainty shock for the Baseline model allowing alternative calibrations for three key parameters, i.e. the elasticity of substitution in the goods market, the degree of rigidity in price adjustment, and the persistence of the monetary level shock. Figures 11-10 illustrate the responses of a bunch of variables. Note that, as described in Section 4.1, the benchmark calibration assumes $\theta_p = 4.3$, $\phi_p = 80$, $\rho_R = 0.5$.

[22] As In Exo Exit firms are severed exogenously and it does not hold an exit condition as in Baseline, we cannot define the equity premium for the marginal firm.

[23] Notice indeed that for the calibration used the constant $\frac{\bar{j}}{\bar{v}-\bar{j}}$ is greater than zero. The result is robust to alternative empirically plausible calibration.

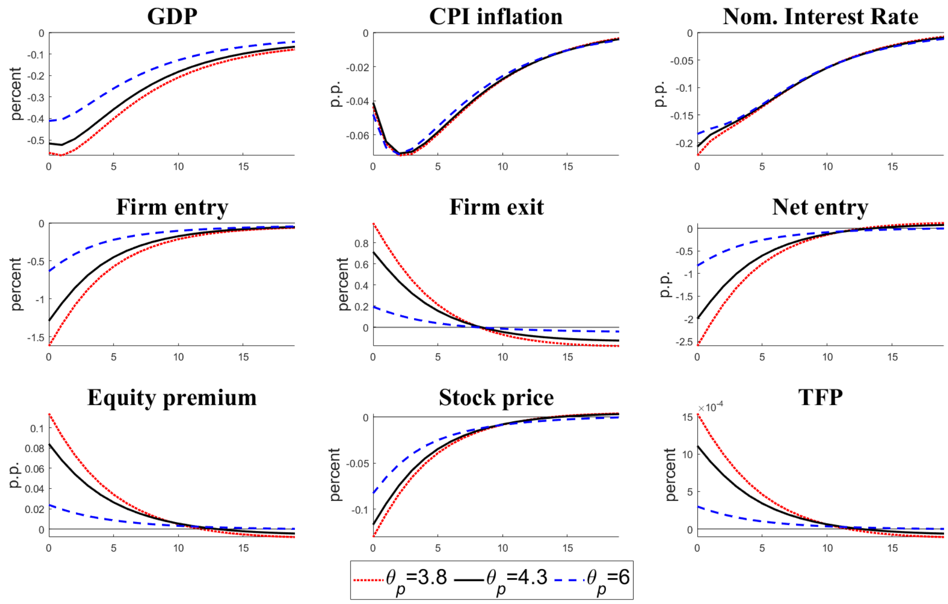

Figure 9: DSGE impulse responses under different goods elasticity of substitution. Annualized impact for interest rates, inflation, equity premium.

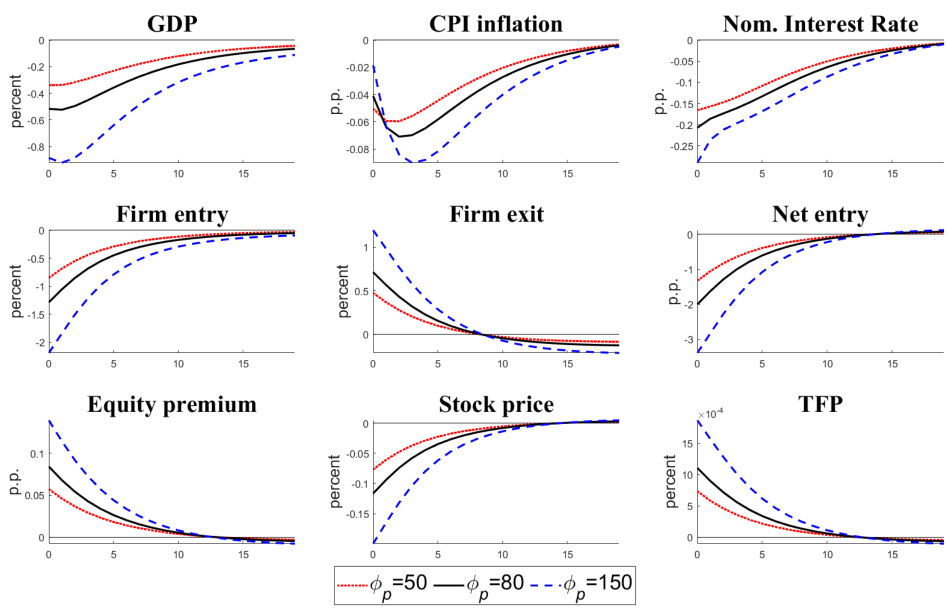

Figure 10: DSGE impulse responses under different degree of rigidity in price adjustment. Annualized impact for interest rates, inflation, equity premium.

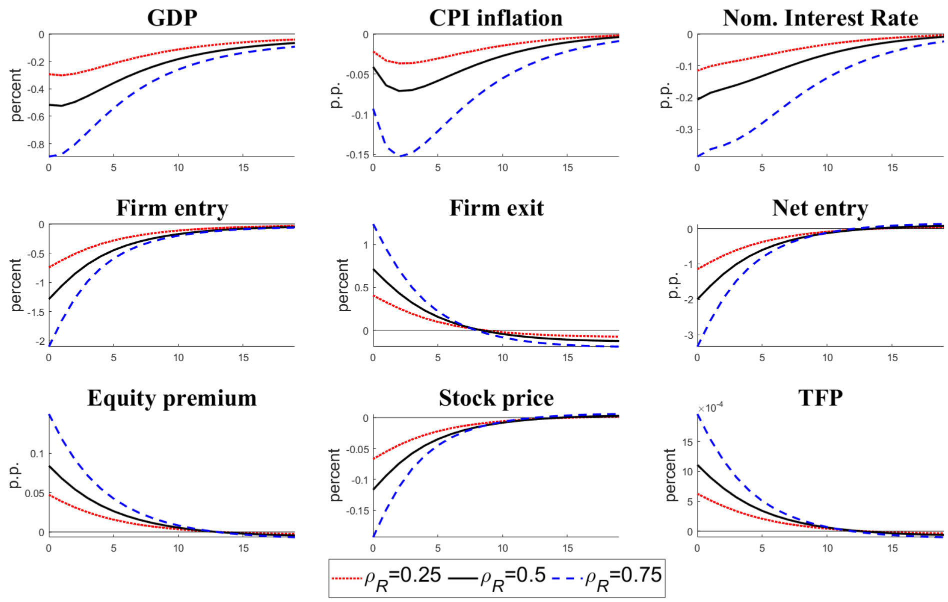

Figure 11: DSGE impulse responses under different persistence of the monetary level shock. Annualized impact for interest rates, inflation, equity premium.

Price markup Setting the elasticity of substitution among intermediate goods, θ_p, is equal to 4.3 in the benchmark calibration, it implies a steady state price markup, μ, of around 30% in the intermediate sector. Although such a level might be argued as at an upper bound according to some empirical evidence, it is even conservative if compared to the parametrization used in several DSGE models with firm dynamics. As highlighted by Bilbiie et al. (2012), in models without any fixed cost, this is a measure of both markup over marginal cost and average cost. In a model with entry costs as ours, free entry ensures that firms earn zero profits net of the entry cost. Equivalently, although the parametrization implies a high markup over the marginal cost, intermediate firms eventually price at the average cost including the entry cost. We check how the impulse responses to the monetary volatility shock are affected by the steady state price markup by setting θ_p to, respectively, 3.8 as in Bilbiie et al. (2012) and close to the estimate in Casares et al. (2018), and 6 as in Rotemberg and Woodford (1992) among others. Figure 11 shows that the dynamics of the Baseline model is not qualitatively altered by different elasticities of substitution among intermediate goods. Overall, a higher elasticity, or equivalently a lower steady state markup, reduces the impact of the shock. While the change is negligible for the nominal variables, the shrink in the responses of real variables is sizeable, especially for firm entry and exit.

Price stickiness In Section 4.2, we show the responses of the Baseline model when the price adjustment cost parameter ϕ_p is fixed to 80. This parametrization implies that in a corresponding Calvo setting the average duration of price contract is of 5.5 months, really close to the findings of Bils and Klenow (2004). Figure 9 shows the responses of the Baseline model with the same calibration, but for the price stickiness parameter ϕ_p fixed to 50 and 150. At the first glance, the dynamics of the Baseline model to the monetary policy uncertainty shock is fairly robust to different costs for firms in adjusting prices. Higher price adjustment costs make the effects of an uncertainty shock stronger and lasting for much more periods. The fall in output worsens by around one-third when the price contract duration in the corresponding Calvo pricing scheme rises from four ($\phi_p = 50$) to more than five ($\phi_p = 80$) months. The impact on firm flows and aggregate productivity is enhanced by a similar magnitude. The impact of the shock is still worse when the price contract duration is set higher, i.e. more than seven months ($\phi_p = 150$). In this case, the fall in real variables doubles the scenario with the lowest price rigidities. Also, though inflation falls more during the propagation of the shock, it contracts less at the impact.

The related literature has pointed several channels for which uncertainty shocks affect inflation. The overall impact is driven by channels that bring about opposite effects. Beyond the declining effect due to the fall of the aggregate demand, Fernández-Villaverde et al. (2015) and Born and Pfeifer (2014) for instance highlight that, when uncertainty increases, it might be convenient for firms to increase the selling prices because of the convexity of the marginal profit curve. When the uncertainty about future outcomes is increased, keeping prices high could be more profitable, which is denoted as the *inverse Oi–Hartman–Abel* effect in Born and Pfeifer (2014). Responses of inflation in Figure 9 show that, though the negative aggregate demand effect ultimately prevails across the calibrations, when it is more costly for firms to update prices, the pricing bias plays a role in minimizing the initial drop.

Monetary shock persistence Figure 10 compares the responses of the Baseline model when ρ_R is set to 0.25, 0.5, 0.75, and ϕ_p to 40. Although the monetary policy uncertainty shock is a second-moment shock, the persistence of the corresponding first-moment shock, i.e. the monetary policy shock, matters for the propagation of the former. Equation (30) clarifies the relationship between the volatility shock, $\sigma_{R,t}$, and level shock, $\varepsilon_{R,t}$. Changing the persistence of the monetary policy shock does not alter qualitatively the responses to the monetary policy uncertainty shock. However, the plots in Figure 10 indicate that the impact is magnified and lasts for more periods when the persistence of the monetary shock increases. In particular, the differences in the shock propagation are consistent with those found in Figure 9 by varying the price rigidity parameter.

5 Conclusion

In this paper, we use a FAVAR model to show that a shock that increases uncertainty around monetary policy is associated with a drop in output and inflation, declining stock prices, lower entry of new firms, and increased firms' exit and the equity premium. Further, the utilization-adjusted TFP increases persistently in the medium-run. We show that the recession triggered by the uncertainty shock is even more severe than after a tightening of the monetary policy. Importantly, we show that the contribution of entry and exit is critical to explain differences in monetary policy level and volatility shocks. To rationalize these results, we provide a medium-scale DSGE model with heterogeneous firms and endogenous firm dynamics. Unlike the standard DSGE model, the extended model can match the response for firms' entry and exit to the monetary policy uncertainty shock. Also, the dynamics of stock price and equity premium are consistent with the FAVAR. Our model suggests that the larger impact on real activity is driven by the propagation of the shock through firm dynamics. Moreover, thanks to the presence of firm heterogeneity and endogenous firm default, a monetary uncertainty shock improves resource allocation in the model by driving out less productive producers and increasing the TFP as in the FAVAR.

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Technical Appendix: Monetary Policy Uncertainty and Firm Dynamics

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1 FAVAR model

The FAVAR model is defined by the following equations

$$\begin{pmatrix} Z_t \\ \tilde{X}_t \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & \Lambda \end{pmatrix} \begin{pmatrix} Z_t \\ F_t \end{pmatrix} + \begin{pmatrix} 0 \\ v_t \end{pmatrix} \tag{1}$$

$$Y_t = BX_t + u_t \tag{2}$$

$$m_t = b\varepsilon_{1t} + \sigma\hat{v}_t \tag{3}$$

$$v_{it} = \rho_i v_{it-1} + e_{it} \tag{4}$$

where Z_t is the monetary policy uncertainty index built by Husted et al. (2019). $\tilde{X}_t$ is a $M \times 1$ vector of variables that include aggregate measures of macroeconomic and financial conditions. F_t denotes a $K \times 1$ vector of unobserved factors while Λ is a $M \times K$ matrix of factor loadings. $X_t = [Y'_{t-1}, .., Y'_{t-P}, 1]'$ is $(NP+1) \times 1$ vector of regressors in each equation and B denotes the $N \times (NP+1)$ matrix of coefficients $B = [B_1, ..., B_P, c]$. The disturbances of the model are defined as:

$$\begin{pmatrix} u_t \\ \hat{v}_t \\ e_t \end{pmatrix} \tilde{} N \left(0, \begin{pmatrix} \Sigma & 0 & 0 \\ 0 & \sigma^2 & 0 \\ 0 & 0 & R \end{pmatrix} \right) \tag{5}$$

where $e_t = [e_{1t}, e_{2t}, .., e_{Mt}]$.

The covariance matrix, Σ, of the reduced form residuals, u_t, can be written as:

$$\Sigma = (Aq)(Aq)' \tag{6}$$

where A is the lower triangular Cholesky decomposition of Σ, and q is an element of the family of orthogonal matrices of size N, satisfying $q'q = I_N$.

The structural shocks of the FAVAR model ε_t are defined as

$$\varepsilon_t = A_0^{-1} u_t, \varepsilon_t \sim \mathcal{N}(0, I_N) \tag{7}$$

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where $A_0 = Aq$. The shock of interest is the first shock ε_{1t} in the $N \times 1$ vector of shocks $\varepsilon_t = [\varepsilon_{1t}, \varepsilon_{\cdot t}]$, where $\varepsilon_{\cdot t}$ contains the remaining $N-1$ elements in ε_t. To indentify the effect of ε_{1t}, we employ an instrument m_t described by the equation (3), where $\tilde{v}_t \sim \mathcal{N}(0,1)$ and $\mathbb{E}(\hat{v}_t \varepsilon_t) = 0$.

1.1 Priors

We assume the following prior distributions:

1. VAR parameters $b = vec(B'), \Sigma$. We use a natural conjugate prior implemented via dummy observations (see Banbura et al. (2010)):

$$Y_{D,1} = \begin{pmatrix} \frac{diag(\gamma_1 \sigma_1 ... \gamma_N \sigma_N)}{\tau} \\ 0_{N \times (P-1) \times N} \\ .... \\ diag(\sigma_1 ... \sigma_N) \\ .... \\ 0_{1 \times N} \end{pmatrix}, and\ X_{D,1} = \begin{pmatrix} \frac{J_P \otimes diag(\sigma_1 ... \sigma_N)}{\tau} & 0_{NP \times 1} \\ 0_{N \times NP+1} & \\ .... & \\ 0_{1 \times NP} & I_1 \times c \end{pmatrix} \tag{8}$$

   where γ_1 to γ_N denotes the prior mean for the coefficients on the first lag, τ is the tightness of the prior on the VAR coefficients, c is the tightness of the prior on the constant terms and N is the number of endogenous variables, i.e. the columns of Y_t. In our application, the prior means are chosen as the OLS estimates of the coefficients of an AR(1) regression estimated for each endogenous variable. We use principal component estimates of the factors F_t^{PC} for this purpose. We set $\tau = 0.1$. The scaling factors σ_i are set using the standard deviation of the error terms from these preliminary AR(1) regressions. Finally we set $c = 1/10000$ in our implementation indicating a flat prior on the constant. We also introduce a prior on the sum of the lagged dependent variables by adding the following dummy observations:

$$Y_{D,2} = \frac{diag(\gamma_1 \mu_1 ... \gamma_N \mu_N)}{\lambda}, \ X_{D,2} = \begin{pmatrix} \frac{(1_{1 \times P}) \otimes diag(\gamma_1 \mu_1 ... \gamma_N \mu_N)}{\lambda} & 0_{N \times 1} \end{pmatrix} \tag{9}$$

   where μ_i denotes the sample means of the endogenous variables calculated using F_t^{PC}. The prior tightness is set as $\lambda = 10\tau$.

2. Factor loadings Λ. We obtain an initial estimate of the factors F_t using an EM algorithm (F_t^{PC}). Using this estimate we obtain an OLS estimate of the factor loadings Λ_{ols}. Denote the factor loading for the ith series in $\tilde{X}_t$ as Λ_i. The prior for Λ_i is assumed to be $\mathcal{N}(\Lambda_{i,0}, V_\Lambda)$ where V_Λ is set as a diagonal matrix with diagonal elements equal to 0.1 and $\Lambda_{i,0}$ equals Λ_{ols} for the ith series.

3. Factors F_t. The initial values for the factors are assumed to be $\mathcal{N}\left(F_{0\backslash 0}, P_{0|0}\right)$. $F_{0\backslash 0}$ is assumed to be the initial value of F_t^{PC} and $P_{0\backslash 0}$ is set equal to an identity matrix

4. Equation for idiosyncratic errors. We use a normal prior for $\rho_i : \mathcal{N}(\rho_{i0}, V_{\rho i})$. The prior for r_i is inverse Gamma: $IG(r_{i0}, T_0)$. We set $\rho_{i0} = 0$ and $V_{\rho i} = 10$. For the inverse Gamma of r_i, we set the mean equal to the variance obtained from OLS estimation of the measurement equation (1), while the variance is set to 10.

5. Instrument equation. The prior for b is normal $\mathcal{N}(b_0, V_0)$. The prior for σ^2 is inverse Gamma with mean σ_0 and standard deviation v_0. b_0 is set equal to the OLS estimate from the

regression $m_t = b_{ols}\hat{u}_t + \hat{v}_t$ where $\hat{u}_t$ are the residuals obtained by estimating a VAR using $\begin{pmatrix} Z_t \\ F_t^{PC} \end{pmatrix}$. σ_0 is set equal to the $3VAR(\hat{v}_t)$. V_0 is set to 1, while $v_0 = 10$.

1.2 Gibbs sampling algorithm

The Gibbs algorithm samples from the following conditional posterior distributions. The symbol Θ denotes all other parameters and states.

Step 1. $p\left(\tilde{B}|\Theta_{-\tilde{B}_t}, Y_{1:T}, m_{1:T}\right)$. We write the model in state-space form:

$$\begin{aligned} \begin{pmatrix} Y_t \\ m_t \end{pmatrix} &= \begin{pmatrix} I_N \otimes X_t' \\ 0 \end{pmatrix} \tilde{B}_t + \begin{pmatrix} u_t \\ m_t \end{pmatrix} \text{ observation} \\ \tilde{B}_t &= \tilde{B}_{t-1} \text{ transition} \end{aligned}$$

where $\tilde{B} = vec\,(B')$. The covariance matrix of the observation equation residuals is:

$$cov\left(\begin{array}{c} u_t \\ m_t \end{array} \middle| \Theta_{-\tilde{B}_t} \right) = \begin{pmatrix} AA' & Aq_1'b \\ bq_1A' & b^2 + \sigma^2 \end{pmatrix}$$

This system is conditionally linear and Gaussian. As m_t is observed, one can re-write the model using the conditional normal distribution. In particular, partition the covariance $cov\left(\begin{array}{c} u_t \\ m_t \end{array} \middle| \Theta_{-\tilde{B}} \right)$ as:

$$cov\left(\begin{array}{c} u_t \\ m_t \end{array} \middle| \Theta_{-\tilde{B}} \right) = \begin{pmatrix} \sigma_{u_t u_t} & \sigma_{u_t m_t} \\ \sigma_{u_t m_t}' & \sigma_{m_t m_t} \end{pmatrix} \tag{10}$$

Then

$$u_t|m_t \tilde{} \mathcal{N}\left(\mu_{u|m}, \Omega_{u|m}\right) \tag{11}$$

where

$$\begin{aligned} \mu_{u|m} &= \sigma_{u_t m_t} \left(\sigma_{m_t m_t}\right)^{-1} m_t' \\ \Omega_{u|m} &= \sigma_{u_t u_t} - \sigma_{u_t m_t} \left(\sigma_{m_t m_t}\right)^{-1} \sigma_{u_t m_t}' \end{aligned} \tag{12}$$

The model can be written as a standard VAR

$$\begin{aligned} Y_t^* &= \left(I_N \otimes X_t'\right) \tilde{B} + u_t|m_t, \\ & u_t|m_t \tilde{} \mathcal{N}\left(0, \Omega_{u|m}\right) \end{aligned}$$

where:

$$Y_t^* = Y_t - \mu_{u|m}'$$

Thus the conditional posterior for $\tilde{B}$ is normal: $\mathcal{N}\,(M, V)$ where:

$$\begin{aligned} M &= vec\left(\left(x^{*'}x^*\right)^{-1}\left(x^{*'}y^*\right)\right) \\ V &= \Omega_{u|m} \otimes \left(x^{*'}x^*\right)^{-1} \end{aligned}$$

with:

$$y^* = \begin{pmatrix} Y_t^* \\ Y_{D,1} \\ Y_{D,2} \end{pmatrix}, x^* = \begin{pmatrix} X_t \\ X_{D,1} \\ X_{D,2} \end{pmatrix}$$

Step 2. $p\left(\Sigma|\Theta_{-\Sigma_t}, Y_{1:T}, m_{1:T}\right)$. We follow Caldara and Herbst (2019) and use a Metropolis step to sample Σ.

(a) Draw a candidate Σ_{new} from the proposal $Q(.) = IW\left(u^{*\prime}u^*, T + T_D - K\right)$. The proposal density is the conditional posterior distribution of the error covariance matrix in the case of a standard Bayesian VAR where u^* denotes the residuals $\tilde{y}^* - x^*M$ with $\tilde{y}^* = \begin{pmatrix} Y_t \\ Y_{D,1} \\ Y_{D,2} \end{pmatrix}$, T_D denotes the number of dummy observations and K denotes the number of regressors in each equation.

(b) Accept the draw with probability $\alpha = \min\left[\frac{\frac{p\left(m_{1:t}, Y_{1:t}, \Sigma_{new}, \Theta_{-\Sigma}\right)}{Q(\Sigma_{new})}}{\frac{p\left(m_{1:t}, Y_{1:t}, \Sigma_{old}, \Theta_{-\Sigma}\right)}{Q(\Sigma_{old})}}, 1\right]$. Here $p\left(m_{1:t}, Y_{1:t}\right)$ denotes the joint posterior distribution.

Step 3. $p\left(q_1|\Theta_{-q_1}, Y_{1:T}, m_{1:T}\right)$. Following Caldara and Herbst (2019) we use a Metropolis step to sample q_1 :

(a) Draw a candidate from as $q_{1,new} = \frac{z}{\|z\|}$ where z is a $N \times 1$ vector from the $\mathcal{N}(0, 1)$ distribution

(b) Accept the draw with probability $\alpha = \min\left[\frac{P\left(m_{1:t}|Y_{1:t}, q_{1,new}, \Theta_{-q_1}\right)}{P\left(m_{1:t}|Y_{1:t}, q_{1,old}, \Theta_{-q_1}\right)}, 1\right]$

Step 4 $p\left(b, \sigma|\Theta_{-[b,\sigma]}, Y_{1:T}, m_{1:T}\right)$. The structural shock of interest ε_{1t} can be calculated as $\varepsilon_{1t} = Aq_1u$. Conditional on $\Theta_{-[b,\sigma]}$ equation 3 is a standard linear regression, so specifying a conditional Normal-Gamma prior delivers a Normal-Gamma posterior. Particularly, we first draw $p\left(\sigma^2|\Theta_{-[b,\sigma]}, Y_{1:T}, m_{1:T}\right)$. Assuming an inverse-Gamma prior, this conditional posterior is also inverse-Gamma. As the prior is parameterised in terms of mean σ_0 and standard deviation v_0, it is convenient to draw the precision $\frac{1}{\sigma^2}$ using Gamma distribution. Note that $\frac{1}{\sigma^2} \sim \mathcal{G}\left(\alpha, \beta\right)$ where $\alpha = \frac{\nu_1}{2}$, $\beta = \frac{2}{s_1}$. The parameters of this Gamma density are given by $\nu_1 = \nu_0 + T$ and $s_1 = s_0 + \hat{v}_t'\hat{v}_t$ where $\hat{v}_t = m_t - be_{1t}$. s_0 can be calculated as $2\sigma_0\left(1 + \frac{\sigma_0^2}{v_0^2}\right)$ while $\nu_0 = 2\left(2 + \frac{\sigma_0^2}{v_0^2}\right)$. Moreover, assuming a prior for $b|\sigma^2, \Theta_{-[b,\sigma]} \sim \mathcal{N}(\underline{b}, \underline{V}^{-1})$, the posterior is also conditional Normal $p\left(b|\Theta_{-[b,\sigma]}, \sigma, Y_{1:T}, m_{1:T}\right) \sim \mathcal{N}(\tilde{b}, \tilde{V}^{-1})$, where $\tilde{b} = \tilde{V}^{-1}\left[\sum_{t=1}^T m_t\varepsilon_{1t} + \underline{V}\underline{b}\right]$ and $\tilde{V} = \underline{V} + \frac{1}{\sigma^2}\sum_{t=1}^T \varepsilon_{1t}^2$.

Step 5 $p\left(\Lambda|\Theta_{-\Lambda}, Y_{1:T}, m_{1:T}\right)$. Given the factors F_t, the observation equation is set of M independent linear regressions with serial correlation

$$X_{it} = F_t\Lambda_i' + v_{it}$$

where Λ_i denotes the ith row of the factor loading matrix. The serial correlaton can be dealt with via a GLS transformation of the variables:

$$\tilde{X}_{it} = \tilde{F}_t\Lambda_i' + e_{it}$$

where $\tilde{X}_{it} = X_{it} - \sum_{p=1}^{P} \rho_p X_{it-p}$ and $\tilde{F}_{kt} = F_{kt} - \sum_{p=1}^{P} \rho_p F_{kt-p}$. The conditional posterior is normal $\mathcal{N}(M, V)$:

$$\begin{aligned} V &= \left(\Sigma_0^{-1} + \frac{1}{r_i} \tilde{F}_t' \tilde{F}_t \right)^{-1} \\ M &= V \left(\Sigma_0^{-1} \Lambda_{i0} + \frac{1}{r_i} \tilde{F}_t' \tilde{X}_{it} \right) \end{aligned}$$

To account for rotational indeterminancy the top $K \times K$ block of Λ is set to an identity matrix.

Step 6 $p(r_i | \Theta_{-r}, Y_{1:T}, m_{1:T})$. The conditional posterior for r_i is $IG(T_0 + T, e_{it}' e_{it} + D_0)$ where T is the sample size.

Step 7 $p(\rho | \Theta_{-r}, Y_{1:T}, m_{1:T})$. Given a draw of the factors, the AR coefficients are drawn for each i independently. The conditional posterior is normal $\mathcal{N}(m, v)$

$$\begin{aligned} v &= \left(\Sigma_{\rho 0}^{-1} + \frac{1}{r_i} x_{it}' x_{it} \right)^{-1} \\ m &= V \left(\Sigma_{\rho 0}^{-1} \rho_0 + \frac{1}{r_i} x_{it}' y_{it} \right) \end{aligned}$$

where $y_{it} = v_{it}$ and $x_{it} = [v_{it-1}, .., v_{it-P}]$

Step 8 $p(F_t | \Xi_{-F_t}, Y_{1:T}, m_{1:T})$. To draw the factors, we write the model in state-space form taking into account the covariance between m_t and u_t and the serial correlation in the idiosyncratic components. The observation equation is defined as:

$$\underbrace{\begin{pmatrix} Z_t \\ \tilde{X}_t \end{pmatrix}}_{x_t} = \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & \Lambda \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & \tilde{\Lambda}_1 \end{pmatrix}, \ldots, \begin{pmatrix} 0 & 0 \\ 0 & \tilde{\Lambda}_P \end{pmatrix}}_{H} \underbrace{\begin{pmatrix} Z_t \\ F_t \\ . \\ . \\ . \\ Z_{t-P} \\ F_{t-P} \end{pmatrix}}_{f_t} + \underbrace{\begin{pmatrix} 0 \\ v_t \end{pmatrix}}_{V_t}$$

where $\tilde{X}_t = \begin{pmatrix} X_{1t} - \sum_{p=1}^{P} \rho_{1p} X_{1t-p} \\ . \\ . \\ X_{Mt} - \sum_{p=1}^{P} \rho_{Mp} X_{Mt-p} \end{pmatrix}$. The blocks of the H matrix contain the factor loadings multiplied by the negative of the corresponding serial correlation coefficient. For example $\tilde{\Lambda}_1 = \begin{pmatrix} -\Lambda_1 \rho_{11} \\ . \\ . \\ -\Lambda_M \rho_{M1} \end{pmatrix}$ where Λ_i denotes the factor loadings for the ith variable X_{it}.

Finally, the variance of V_t is $R = diag([0, r_1, .., r_M])$. The transition equation is defined as:

$$f_t - \tilde{\mu}_{u|m} = \mu + \tilde{B} f_{t-1} + U_t$$

where $\tilde{B} = \begin{pmatrix} B_1 & \cdot & \cdot & B_P \\ I_{N(P-1)\times NP} & & & \end{pmatrix}$, $\mu = \begin{pmatrix} c \\ 0_{N(P-1)} \end{pmatrix}$, $U_t = \begin{pmatrix} u_t|m_t \\ 0_{N(P-1)} \end{pmatrix}$, $\tilde{\mu}_{u|m} = \begin{pmatrix} \mu_{u|m} \\ 0_{N(P-1)} \end{pmatrix}$. The non-zero block of $cov(U_t)$ is given by $\Omega_{u|m}$. In other words, the structure of the transition equation accounts for the relationship between the instrument and the reduced form residuals. Given this Gaussian linear state-space, the state vector can be drawn from the normal distribution using Carter and Kohn (1994)'s algorithm.

Step 9 $p(\hat{x}_t|\Theta)$. Conditional on the remaining parameters, an independent state-space model applies for each quarterly series with missing observations. The observation equation is:

$$x^Q_{jt} = \begin{pmatrix} 1 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} \hat{x}_{jt} \\ \hat{x}_{jt-1} \\ \hat{x}_{jt-2} \\ v_{jt} \end{pmatrix} \text{ if } x^Q_{jt} \neq nan$$

$$x^Q_{jt} = \tilde{u}_{jt} \text{ if } x^Q_{jt} = nan$$

where $var(\tilde{u}_{jt}) = 1e10$. With the assumption of one lag in equation 4, the transition equation is:

$$\begin{pmatrix} \hat{x}_{jt} \\ \hat{x}_{jt-1} \\ \hat{x}_{jt-2} \\ v_{jt} \end{pmatrix} = \begin{pmatrix} F_t \Lambda_i' \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & \rho_i \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \rho_i \end{pmatrix} \begin{pmatrix} \hat{x}_{jt-1} \\ \hat{x}_{jt-2} \\ \hat{x}_{jt-3} \\ v_{jt-1} \end{pmatrix} + \begin{pmatrix} e_{jt} \\ 0 \\ 0 \\ e_{jt} \end{pmatrix}$$

where $var \begin{pmatrix} e_{jt} \\ 0 \\ 0 \\ e_{jt} \end{pmatrix} = \begin{pmatrix} r_j & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & r_j \end{pmatrix}$.

1.2.1 Missing data for instrument

If the instrument has missing observations over some of the sample period and is only available for time periods $\tilde{T}$, some steps of the algorithm need to be modified to account for this:

Step 1 $p\left(\tilde{B}|\Theta_{-\tilde{B}_t}, Y_{1:T}, m_{1:T}\right)$:The VAR model is written as:

$$\begin{aligned} Y_t^* &= \left(I_N \otimes X_t'\right)\tilde{B} + u_t|m_t, u_t|m_t \tilde{} \mathcal{N}\left(0, \Omega_{u|m}\right) \text{ if } m_t \neq nan \\ Y_t &= \left(I_N \otimes X_t'\right)\tilde{B} + u_t, u_t \tilde{} \mathcal{N}\left(0, \sigma_{u_t u_t}\right) \text{ if } m_t = nan \end{aligned}$$

In other words, the VAR model is heteroscedastic with the covariance matrix changing over time. This can be handled using a GLS step to draw $\tilde{B}$ from its conditional posterior distribution. The conditional posterior distribution for $\tilde{B}$ in this heteroscedastic setting is normal with mean and variance given by:

$$m = v\left(vec\left(\sum_{t=1}^{T}\left(X_t\left(y_t\right)'\left(\Omega_t\right)^{-1}\right)\right) + \left(S_0\right)_0^{-1}\tilde{B}'\right)$$

$$v = \left(\sum_{t=1}^{T}\left(\left(\Omega_t\right)^{-1}\otimes X_t X_t'\right) + \left(S_0\right)^{-1}\right)^{-1}$$

where:

$$\begin{aligned} y_t &= Y_t^*, \Omega_t = \Omega_{u|m} \text{if } m_t \neq nan \\ y_t &= Y, \Omega_t = \sigma_{u_t u_t} \text{if } m_t = nan \end{aligned}$$

and the mean and the variance of the prior for the coefficients is denoted by $\tilde{B}_0, S_0$ respectively.

Steps 2,3,4 In the draws $p\left(\Sigma|\Theta_{-\Sigma_t}, Y_{1:T}, m_{1:\tilde{T}}\right), p\left(q_1|\Theta_{-q_1}, Y_{1:T}, m_{1:\tilde{T}}\right), p\left(b,\sigma|\Theta_{-[b,\sigma]}, Y_{1:T}, m_{1:\tilde{T}}\right)$ only the non-missing values of the instrument are used.

Step 8. $H\left(F_t|\Xi_{-F_t}, Y_{1:T}, m_{1:T}\right)$: The transition equation for the state-space model changes when the instrument is missing. In this case, the transition equation is simpler and given by

$$f_t = \mu + \tilde{B} f_{t-1} + U_t$$

where $\tilde{B} = \begin{pmatrix} B_1 & \cdot & \cdot & B_P \\ I_{N(P-1)\times NP} & & & \end{pmatrix}, \mu = \begin{pmatrix} c \\ 0_{N(P-1)} \end{pmatrix}, U_t = \begin{pmatrix} u_t \\ 0_{N(P-1)} \end{pmatrix}$. The non-zero block of $cov\left(U_t\right)$ is given by $\sigma_{u_t u_t}$. In other words, over periods where the instrument is missing the correlation between the instrument and the residuals does need to be directly modelled.

1.2.2 Testing the algorithm

To test the algorithm and computer code we carry out a simple Monte Carlo experiment. Artificial data is generated from the following model:

$$\begin{pmatrix} Z_t \\ \tilde{X}_t \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & \Lambda \end{pmatrix}\begin{pmatrix} Z_t \\ F_t \end{pmatrix} + \begin{pmatrix} 0 \\ v_t \end{pmatrix} \tag{13}$$

$$Y_t = BX_t + u_t \tag{14}$$

$$m_t = b\varepsilon_{1t} + \sigma\hat{v}_t \tag{15}$$

$$v_{it} = \rho_i v_{it-1} + e_{it} \tag{16}$$

where $\tilde{X}_t$ contains 50 series where 10 are subject to temporal aggregation. We assume two factors with $\Lambda\tilde{\ }\mathcal{N}(0,1)$. To calibrate B and $var\left(u_t\right)$ we use OLS estimates from a VAR model that includes GDP growth, inflation and corporate bond spread for the US. We assume that the VAR has three lags. $b = \begin{pmatrix} 1.1 \\ -1.1 \\ 2 \end{pmatrix}$ and $var\left(\hat{v}_t\right) = 0.1$. Finally, $\rho_i\tilde{\ }U(0,1)$ while $var\left(e_{it}\right)$ is set as the exponential of a draw from the standard normal distribution. We draw 340 observations, discarding the first 100. The experiment is repeated 100 times.

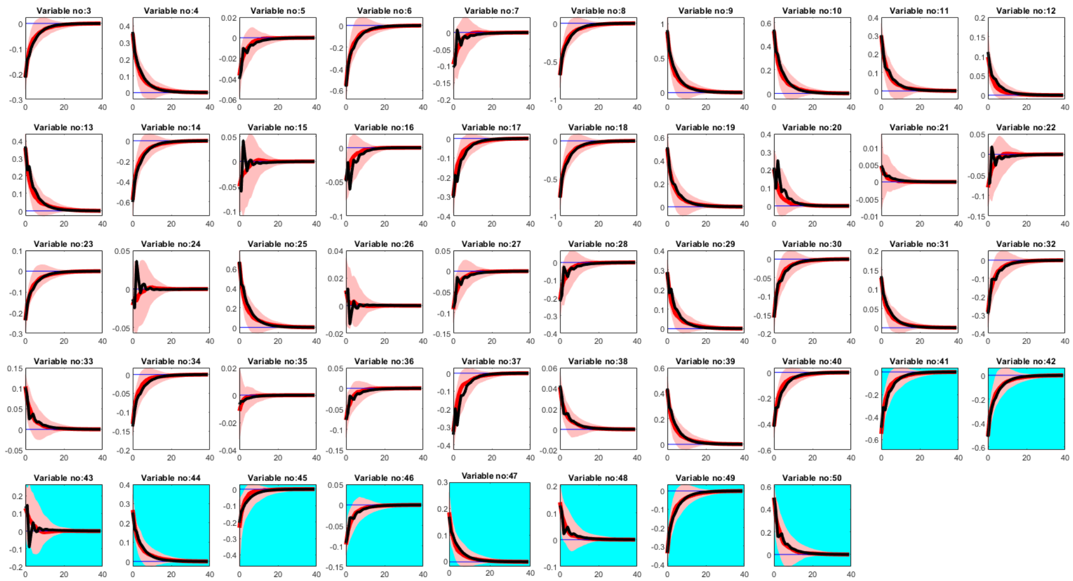

Figure 1: The black line is the true response. The red line and shaded area represent the median and 68% interval estimated using 100 Monte-Carlo replications. The plots highlighted with blue show series subject to temporal aggregation.

Figure 1 compares the true impulse response to the shock to Z_t with that obtained over the 100 Monte Carlo replications. The estimates suggest that the algorithm performs well and is able to recover the true impulse responses.

1.3 Convergence

Figure 2 shows that the inefficiency factors are fairly low. This provides evidence in favour of convergence.

1.4 Data

The list of monthly series used in the FAVAR model are shown in Table 1-3. Quarterly series follow in Table 4-. In the tables, FRED-MD refers to Federal Reserve monthly database for macroeconomic research (https://research.stlouisfed.org/wp/more/2015-012), BLS-BED refers to Business Employment Dynamics database provided by the U.S, Bureau of Labor Statistics (https://www.bls.gov/bdm/). The sample period for the monthly series spans from 1985:m1 to 2016:m6. the sample period for quarterly series on establishments' birth and death spans from 1992:q3 to 2016:q2, the sample period for quarterly growth rates of the utilization-adjusted TFP spans from 1985:q1 to 2016:q2.

	Variable	Source		Variable	Source
1)	Real Personal Income	FRED-MD	30)	All Employees: Goods-Producing Industries	FRED-MD
2)	Real personal income ex transfer receipts	FRED-MD	31)	All Employees: Mining and Logging	FRED-MD
3)	IP Index	FRED-MD	32)	All Employees: Construction	FRED-MD
4)	IP: Final Products and Nonindustrial Supplies	FRED-MD	33)	All Employees: Manufacturing	FRED-MD
5)	IP: Final Products (Market Group)	FRED-MD	34)	All Employees: Durable goods	FRED-MD
6)	IP: Consumer Goods	FRED-MD	35)	All Employees: Nondurable goods	FRED-MD
7)	IP: Durable Consumer Goods	FRED-MD	36)	All Employees: Service-Providing Industries	FRED-MD
8)	IP: Nondurable Consumer Goods	FRED-MD	37)	All Employees: Trade, Transportation & Utilities	FRED-MD
9)	IP: Business Equipment	FRED-MD	38)	All Employees: Wholesale Trade	FRED-MD
10)	IP: Materials	FRED-MD	39)	All Employees: Retail Trade	FRED-MD
11)	IP: Durable Materials	FRED-MD	40)	All Employees: Financial Activities	FRED-MD
12)	IP: Nondurable Materials	FRED-MD	41)	All Employees: Government	FRED-MD
13)	IP: Manufacturing (SIC)	FRED-MD	42)	Avg Weekly Hours : Goods-Producing	FRED-MD
14)	IP: Residential Utilities	FRED-MD	43)	Avg Weekly Overtime Hours : Manufacturing	FRED-MD
15)	IP: Fuels	FRED-MD	44)	Avg Weekly Hours : Manufacturing	FRED-MD
16)	Capacity Utilization: Manufacturing	FRED-MD	45)	Avg Hourly Earnings : Goods-Producing	FRED-MD
17)	HWI Help-Wanted Index for United States	FRED-MD	46)	Avg Hourly Earnings : Construction	FRED-MD
18)	Ratio of Help Wanted/No. Unemployed	FRED-MD	47)	Avg Hourly Earnings : Manufacturing	FRED-MD
19)	Civilian Labor Force	FRED-MD	48)	Housing Starts: Total New Privately Owned	FRED-MD
20)	Civilian Employment	FRED-MD	49)	Housing Starts, Northeast	FRED-MD
21)	Civilian Unemployment Rate	FRED-MD	50)	Housing Starts, Midwest	FRED-MD
22)	Average Duration of Unemployment (Weeks)	FRED-MD	51)	Housing Starts, South	FRED-MD
23)	Civilians Unemployed - Less Than 5 Weeks	FRED-MD	52)	Housing Starts, West	FRED-MD
24)	Civilians Unemployed for 5-14 Weeks	FRED-MD	53)	New Private Housing Permits (SAAR)	FRED-MD
25)	Civilians Unemployed - 15 Weeks & Over	FRED-MD	54)	New Private Housing Permits, Northeast (SAAR)	FRED-MD
26)	Civilians Unemployed for 15-26 Weeks	FRED-MD	55)	New Private Housing Permits, Midwest (SAAR)	FRED-MD
27)	Civilians Unemployed for 27 Weeks and Over	FRED-MD	56)	New Private Housing Permits, West (SAAR)	FRED-MD
28)	Initial Claims	FRED-MD	57)	New Private Housing Permits, South (SAAR)	FRED-MD
29)	All Employees: Total nonfarm	FRED-MD	58)	Real personal consumption expenditures	FRED-MD

Table 1: List of monthly series in the FAVAR model

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	Variable	Source		Variable	Source
59)	Retail and Food Services Sales	FRED-MD	88)	Moody's Seasoned Aaa Corporate Bond Yield	FRED-MD
60)	Real Manu. and Trade Industries Sales	FRED-MD	89)	Moody's Seasoned Baa Corporate Bond Yield	FRED-MD
61)	New Orders for Durable Goods	FRED-MD	90)	30-Year Fixed Rate Mortgage Avg in the U. S.	FRED-MD
62)	New Orders for Nondefense Capital Goods	FRED-MD	91)	3-Month Commercial Paper Minus FEDFUNDS	FRED-MD
63)	Unfilled Orders for Durable Goods	FRED-MD	92)	3-Month Treasury C Minus FEDFUNDS	FRED-MD
64)	Total Business Inventories	FRED-MD	93)	6-Month Treasury C Minus FEDFUNDS	FRED-MD
65)	Total Business: Inventories to Sales Ratio	FRED-MD	94)	1-Year Treasury C Minus FEDFUNDS	FRED-MD
66)	Consumer Sentiment Index	FRED-MD	95)	5-Year Treasury C Minus FEDFUNDS	FRED-MD
67)	M1 Money Stock	FRED-MD	96)	10-Year Treasury C Minus FEDFUNDS	FRED-MD
68)	M2 Money Stock	FRED-MD	97)	Moody's Aaa Corp. Bond Minus FEDFUNDS	FRED-MD
69)	Real M2 Money Stock	FRED-MD	98)	Moody's Baa Corp. Bond Minus FEDFUNDS	FRED-MD
70)	St. Louis Adjusted Monetary Base	FRED-MD	99)	Trade Weighted U.S. $ Inx: Major Currencies	FRED-MD
71)	Total Reserves of Depository Institutions	FRED-MD	100)	Switzerland / U.S. Foreign Exchange Rate	FRED-MD
72)	Reserves Of Depository Institutions	FRED-MD	101)	PPI: Finished Goods	FRED-MD
73)	Commercial and Industrial Loans	FRED-MD	102)	PPI: Finished Consumer Goods	FRED-MD
74)	Real Estate Loans at All Commercial Banks	FRED-MD	103)	PPI: Intermediate Materials	FRED-MD
75)	Total Nonrevolving Credit	FRED-MD	104)	PPI: Crude Materials	FRED-MD
76)	Nonrevolving cons. credit to Personal Income	FRED-MD	105)	Crude Oil, spliced WTI and Cushing	FRED-MD
77)	MZM Money Stock	FRED-MD	106)	PPI: Metals and metal products	FRED-MD
78)	Consumer Motor Vehicle Loans Outst.	FRED-MD	107)	CPI : All Items	FRED-MD
79)	Total Consumer Loans and Leases Outst.	FRED-MD	108)	CPI : Apparel	FRED-MD
80)	Securities in Bank Credit at All Comm. Banks	FRED-MD	109)	CPI : Transportation	FRED-MD
81)	Effective Federal Funds Rate	FRED-MD	110)	CPI : Medical Care	FRED-MD
82)	CP3Mx 3-Month AA Fin. Comm. Paper Rate	FRED-MD	111)	CPI : Commodities	FRED-MD
83)	TB3MS 3-Month Treasury Bill	FRED-MD	112)	CPI : Durables	FRED-MD
84)	6-Month Treasury Bill	FRED-MD	113)	CPI : Services	FRED-MD
85)	1-Year Treasury Rate	FRED-MD	114)	CPI : All Items Less Food	FRED-MD
86)	5-Year Treasury Rate	FRED-MD	115)	CPI : All items less shelter	FRED-MD
87)	10-Year Treasury Rate	FRED-MD	116)	CPI : All items less medical care	FRED-MD

Table 2: List of monthly series in the FAVAR model

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	Variable	Source		Variable	Source
117)	Personal Cons. Expend.: Chain Index	FRED-MD	126)	CP3MxMTB3MS	FRED-MD
118)	Personal Cons. Exp: Durable goods	FRED-MD	127)	Moody's Seasoned Baa Corp. Bond Yield Relative to Yield on 10-Year Treasury	FRED-MD
119)	Personal Cons. Exp: Nond. goods	FRED-MD	128)	MRTM10	FRED-MD
120)	Personal Cons. Exp: Services	FRED-MD	129)	Monetary Policy Uncertainty	Husted et al. (2019)
121)	S&P's Common Stock Price Inx: Comp.	FRED-MD	130)	Economic Policy Uncertainty	Baker et al. (2016)
122)	S&P's Common Stock Price Inx: Indust.	FRED-MD	131)	Excess Bond Premium	Gilchrist and Zakrajšek (2012)
123)	S&P's Comp. Common Stock: Div. Yield	FRED-MD	132)	Value-weighted total stock market return	Caldara et al. (2016)
124)	S&P's Comp. Common Stock: Price-Earn. Ratio	FRED-MD	133)	S&P's Goldman Sachs Commodity Index	Caldara et al. (2016)
125)	VXO	FRED-MD	134)	Utilization-adjusted TFP	Fernald (2014)

Table 3: List of monthly series in the FAVAR model

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	Variable		Source		Variable		Source
135)	Total private	Establishm. births, s.a.	BLS-BED	150)	Total private	Establishm. deaths, s.a.	BLS-BDS
136)	Goods-producing	Establishm. births, s.a	BLS-BED	151)	Goods-producing	Establishm. deaths, s.a.	BLS-BDS
137)	Natural resources	Establishm. births, s.a	BLS-BED	152)	Natural resources	Establishm. deaths, s.a.	BLS-BDS
138)	Construction	Establishm. births, s.a	BLS-BED	153)	Construction	Establishm. deaths, s.a.	BLS-BDS
139)	Manufacturing	Establishm. births, s.a	BLS-BED	154)	Manufacturing	Establishm. deaths, s.a.	BLS-BDS
140)	Service-providing	Establishm. births, s.a	BLS-BED	155)	Service-providing	Establishm. deaths, s.a.	BLS-BDS
141)	Wholesale trade	Establishm. births, s.a	BLS-BED	156)	Wholesale trade	Establishm. deaths, s.a.	BLS-BDS
142)	Retail trade	Establishm. births, s.a	BLS-BED	157)	Retail trade	Establishm. deaths, s.a.	BLS-BDS
143)	Transportation	Establishm. births, s.a	BLS-BED	158)	Transportation	Establishm. deaths, s.a.	BLS-BDS
144)	Information	Establishm. births, s.a	BLS-BED	159)	Information	Establishm. deaths, s.a.	BLS-BDS
145)	Financial activities	Establishm. births, s.a	BLS-BED	160)	Financial activities	Establishm. deaths, s.a.	BLS-BDS
146)	Professional services	Establishm. births, s.a	BLS-BED	161)	Professional services	Establishm. deaths, s.a.	BLS-BDS
147)	Education services	Establishm. births, s.a	BLS-BED	162)	Education services	Establishm. deaths, s.a.	BLS-BDS
148)	Leisure and hospitality	Establishm. births, s.a	BLS-BED	163)	Leisure and hospitality	Establishm. deaths, s.a.	BLS-BDS
149)	Other services	Establishm. births, s.a	BLS-BED	164)	Other services	Establishm. deaths, s.a.	BLS-BDS

Table 4: List of quarterly series in the FAVAR model

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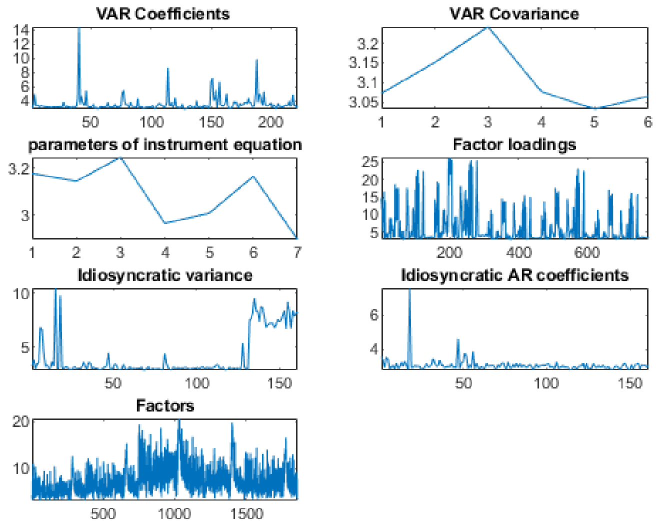

Figure 2: Inefficiency factors.

1.5 Robustness

We check the robustness of the empirical evidence obtained with the benchmark FAVAR along two lines. First, we change the number of factors in the FAVAR model. Second, we modify the prior on the reliability of the instrument. Note that in the benchmark FAVAR, we assume, respectively, six factors and the priors for b and σ^2 implying that the correlation between the instrument and the monetary policy uncertainty shock is $\rho \approx 0.6$.

Figures 3-5 illustrate the responses of the FAVAR when we estimate the same specification as the benchmark but with five, seven, eight factors, respectively. Remarkably, the transmission of the shock is not altered by setting a different number of factors. The responses are qualitatively equal to the benchmark. There are some minor variations in the size of the impact. Further, the response of inflation becomes less negative as the number of factors increases.

Figure 6 shows the responses of the FAVAR when we set a flatter prior for the variance σ^2 of the residual of the instrument equation than in benchmark. Specifically, we adjust the prior such that the correlation between m_t and ε_t^{MPU} is reduced to around 0.4. The alternative FAVAR provides responses that are very similar to those of the benchmark. However, as the prior on the reliability of the instrument is flatter, the estimated responses are less precise. The bands reported in Figure 6 are indeed wider than for the benchmark FAVAR.

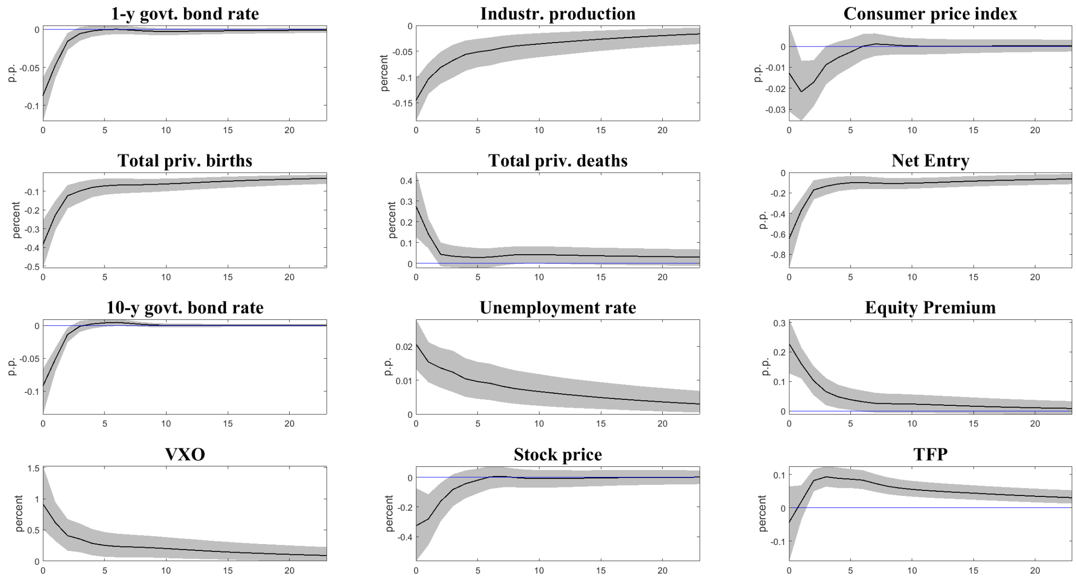

Figure 3: Impulse responses of the FAVAR with 5 factors.

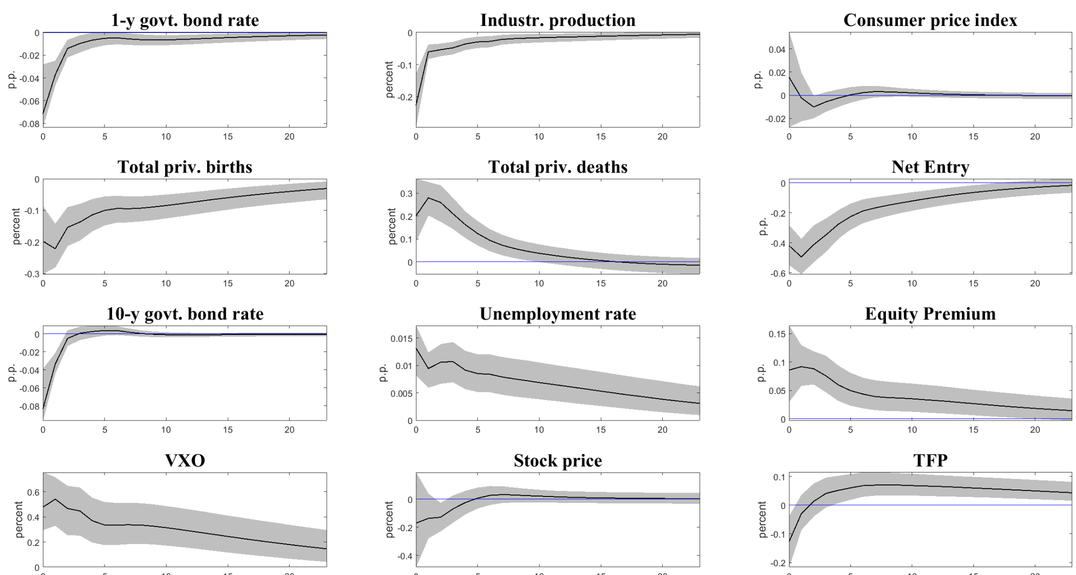

Figure 4: Impulse responses of the FAVAR with 7 factors.

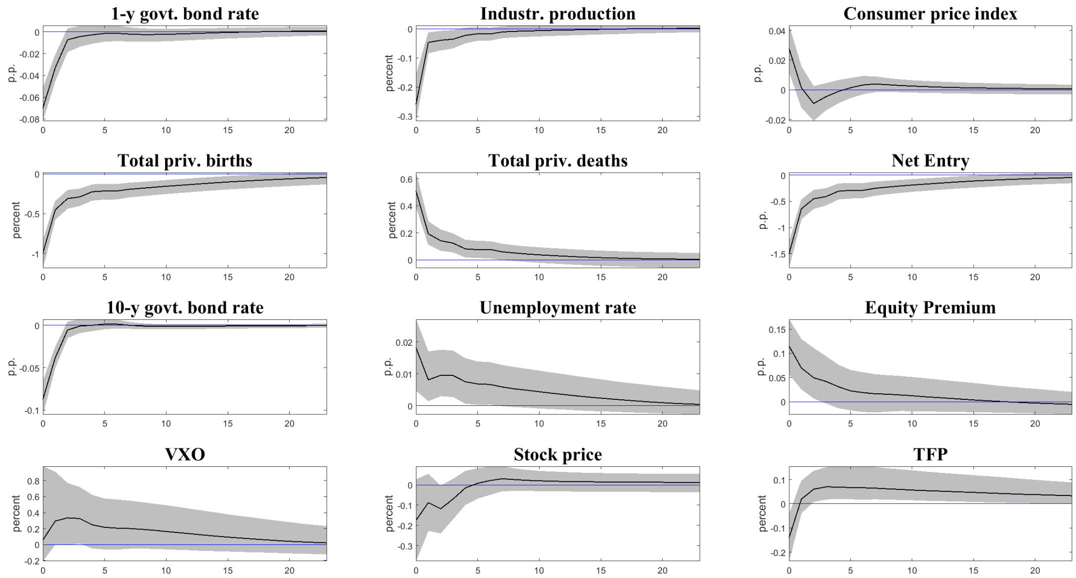

Figure 5: Impulse responses of the FAVAR with 8 factors.

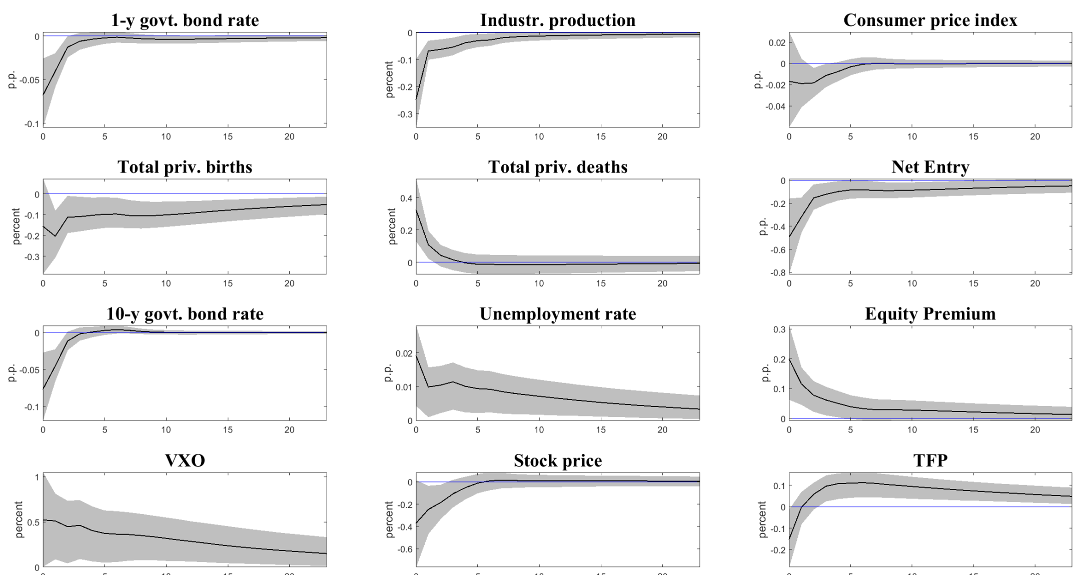

Figure 6: Impulse responses of the FAVAR with a flat prior for σ^2.

2 DSGE model

2.1 List of equations

The system of non-linear equation is summarized in Table 5-7.

	Description	Equations
1)	Marginal utility of consumption	$\lambda_t = \left(C_t - h\bar{C}_{t-1}\right)^{-\sigma_C} \exp\left(\chi \frac{(\sigma_C - 1)(L_t)^{1+\sigma_L}}{1+\sigma_L}\right),$
2)	Marginal rate of substitution	$mrs_t = \chi\left(C_t - h\bar{C}_{t-1}\right)(L_t)^{\sigma_L},$
3)	Law of motion of capital	$K_{t+1} = (1 - \delta^K - \frac{\phi_K}{2}\left(\frac{I_t}{K_t} - 1\right)^2)K_t + I_t,$
4)	Euler equation	$\lambda_t = \beta E_t\left[\lambda_{t+1}(1 + r_t)\right],$
5)	Euler equation for incumbent firm	$v_t(\widetilde{z}_t) = \beta E_t\left[\frac{\lambda_{t+1}}{\lambda_t}\left(\left(1 - \eta_{t+1}\right)\left(v_{t+1}(\widetilde{z}_{t+1}) + j_{t+1}(\widetilde{z}_{t+1})\right) + \eta_{t+1} l v_{t+1}\right)\right],$
6)	Euler equation for entrant firm	$v_t(\widetilde{z}_t) = f^E + ec_t,$
7)	Euler equation for capital	$\psi_t = \beta E_t\left[\begin{array}{c}\lambda_{t+1}\left(r^K_{t+1}u_{t+1} - a(u_{t+1})\right) + \psi_{t+1}(1 - \delta_K) \\ -\psi_{t+1}\left(\frac{\phi_K}{2}\left(\frac{I_{t+1}}{K_{t+1}} - \delta^K\right)^2 - \phi_K\left(\frac{I_{t+1}}{K_{t+1}} - \delta^K\right)\frac{I_{t+1}}{K_{t+1}}\right)\end{array}\right],$
8)	Euler equation for investments	$1 = q_t\left(1 - \phi_K\left(\frac{I_t}{K_t} - \delta^K\right)\right),$
9)	Tobin q	$q_t = \frac{\psi_t}{\lambda_t},$
10)	FOC variable capital utilization	$r^k_t = \gamma_1 + \gamma_2(u_t - 1),$
11)	Variable capital utilization adj. costs	$a(u_t) = \gamma_1(u_t - 1) + \frac{\gamma_2}{2}(u_t - 1)^2,$
12)	Law of motion of firms	$N_t = (1 - \eta_t)\left(N_{t-1} + N^E_{t-1}\right),$
13)	Wage NKPC	$1 = \frac{\theta_w}{\theta_w - 1}\frac{mrs_t}{w_t} - \frac{\phi_w}{\theta_w - 1}(\pi^w_t - 1)\pi_t\frac{Y_t}{w_{t-1}L_t} + \frac{\phi_w}{\theta_w - 1}E_t\left[\frac{\lambda_{t+1}}{\lambda_t}\left(\pi^w_{t+1} - 1\right)\pi^w_{t+1}\frac{Y_{t+1}}{w_t L_t}\right],$
14)	Wage inflation	$\pi^w_t = \frac{w_t}{w_{t-1}}\pi_t,$
15)	Wage adj. costs	$WAC_t = \frac{\phi_w}{2}(\pi^w_t - 1)^2 Y_t,$

Table 5: System of non-linear equations

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	Description	Equations
16)	Price NKPC	$1 = \frac{\theta_p}{\theta_p - 1}\rho_t^I - \frac{\phi_p}{\theta_p - 1}\left(\pi_t - 1\right)\pi_t + \frac{\phi_p}{2}\left(\pi_t - 1\right)^2 + \frac{\phi_p}{\theta_p - 1}E_t\left[\Lambda_{t,t+1}\left(\pi_{t+1} - 1\right)\pi_{t+1}\frac{Y_{t+1}}{Y_t}\right],$
17)	Love of variety equation	$\tilde{\rho}_t = N_t^{\frac{1}{\theta_p - 1}}\rho_t^I,$
18)	Stochastic discount factor	$\Lambda_{t,t+1} = \beta E_t\left[\frac{\lambda_{t+1}}{\lambda_t}\left(1 - \eta_{t+1}\right)\right],$
19)	Entry congestion externalities	$ec_t = \Theta^e\left(\frac{N_t^E}{N_t}\right)^{\varsigma_e},$
20)	Exit congestion externalities	$xc_t = \Theta^x\left(\frac{N_t^X}{N_t}\right)^{\varsigma_x},$
21)	Exiting firms	$N_t^X = \eta_t\left(N_{t-1} + N_{t-1}^E\right),$
22)	Liquidation value	$lv_t = \left(1 - \tau\right)f^E - xc_t,$
23)	Exit probability	$\eta_t = 1 - \left(\frac{z_{\min}}{\bar{z}_t}\right)^{\xi},$
24)	Average productivity	$\tilde{z}_t = \left(\frac{\xi}{\xi + 1 - \theta_p}\right)^{\frac{1}{\theta_p - 1}}\bar{z}_t,$
25)	Value of the marginal firm	$v_t\left(\bar{z}_t\right) = j_t\left(\bar{z}_t\right) + \beta E_t\left[\Lambda_{t,t+1}v_{t+1}\left(\bar{z}_{t+1}\right)\right],$
26)	Exit condition	$v_t\left(\bar{z}_t\right) = lv_t,$
27)	Profits of the marginal firm	$j_t\left(\bar{z}_t\right) = \left(\frac{\theta_p}{\theta_p - 1} - 1\right)mc_t\left(\widetilde{z}_t\right)\left(\frac{\tilde{z}_t}{\bar{z}_t}\right)^{1-\theta_p}y_t\left(\widetilde{z}_t\right),$
28)	Optimal price of the average firm	$\tilde{\rho}_t = \mu mc_t\left(\widetilde{z}_t\right),$
29)	Mark-up of the intermediate firm	$\mu = \frac{\theta_p}{\theta_p - 1},$
30)	Profits of the average firm	$j_t\left(\widetilde{z}_t\right) = N_t^{-1}\left(Y_t - w_tL_t - r_t^kK_t^s\right),$

Table 6: System of non-linear equations

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	Description	Equations
31)	Labor demand	$w_t = \tilde{z}_t mc_t(\widetilde{z}_t)(1-\alpha)\left(\frac{L_t}{K_t^s}\right)^{-\alpha},$
32)	Capital service demand	$r_t^K = \tilde{z}_t mc_t(\widetilde{z}_t)\alpha\left(\frac{L_t}{K_t^s}\right)^{1-\alpha},$
33)	Output of the average firm	$y_t(\widetilde{z}_t) = \widetilde{z}_t (L_t)^{1-\alpha}(K_t^s)^{\alpha},$
34)	Capital-capital service relation	$K_t^s = u_t K_t,$
35)	Aggregate output	$Y_t = N_t^{\frac{1}{\theta_p-1}}\widetilde{z}_t (L_t)^{1-\alpha}(K_t^s)^{\alpha},$
36)	Aggregate resource constraint	$Y_t = C_t + I_t + a(u_t)K_t + N_t^E ec_t + N_t^X xc_t + PAC_t + WAC_t,$
37)	Price adj. costs	$PAC_t = \frac{\phi_p}{2}(\pi_t - 1)^2 Y_t,$
38)	Government budget constraint	$T_t = f^E N_t^E - (1-\tau) f^E N_t^X,$
39)	Taylor rule	$\left(\frac{1+i_t}{1+i}\right) = \left(\frac{1+i_{t-1}}{1+i}\right)^{\phi_R}\left(\left(\frac{\pi_t}{\pi}\right)^{\phi_\pi}\left(\frac{y_t}{y_{t-1}}\right)^{\phi_{dy}}\right)^{(1-\phi_R)} e^{\varepsilon_{R,t}},$
40)	Fisher equation	$1 + i_t = (1 + r_t) E_t[\pi_{t+1}],$
41)	Equity premium	$E_t\hat{\kappa}_{t+1} - \hat{R}_{f,t} = \begin{pmatrix} -E_t\widehat{\widehat{\eta}}_{t+1} - \frac{1}{2}Var_t\hat{M}_{t+1} - \frac{1}{4}Var_t\widehat{\widehat{\eta}}_{t+1} - \frac{1}{2}Var_t\hat{\kappa}_{t+1} \\ -Cov_t\left(\hat{M}_{t+1}, \widehat{\widehat{\eta}}_{t+1}\right) - Cov_t\left(\hat{M}_{t+1} + \widehat{\widehat{\eta}}_{t+1}, \hat{\kappa}_{t+1}\right) \end{pmatrix},$
42)	Equity premium (marginal)	$E_t\widehat{\widehat{\kappa}}_{t+1} - \hat{R}_{f,t} = \frac{\bar{j}}{\bar{v}-\bar{j}}\left(\widehat{\widehat{v}}_t - \widehat{\widehat{j}}_t\right) + \begin{pmatrix} -E_t\widehat{\widehat{\eta}}_{t+1} - \frac{1}{2}Var_t\hat{M}_{t+1} - \frac{1}{4}Var_t\widehat{\widehat{\eta}}_{t+1} - \frac{1}{2}Var_t\widehat{\widehat{\kappa}}_{t+1} \\ -Cov_t\left(\hat{M}_{t+1}, \widehat{\widehat{\eta}}_{t+1}\right) - Cov_t\left(\hat{M}_{t+1} + \widehat{\widehat{\eta}}_{t+1}, \widehat{\widehat{\kappa}}_{t+1}\right) \end{pmatrix}$
43)	Monetary shock	$\varepsilon_{R,t} = \rho_R\varepsilon_{R,t-1} + e^{\sigma_{R,t}}u_{\varepsilon,t},$
44)	Uncertainty shock	$\sigma_{R,t} = \rho_\sigma\sigma_{R,t-1} + u_{\sigma,t},$

Table 7: System of non-linear equations

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2.2 Risk-adjusted log-linearization of the entry and exit conditions

In this section, we log-linearize with risk-adjustment the entry and exit conditions for the Baseline model. The non-linear version of the entry condition is given by

$$v_t\left(\widetilde{z}_t\right) = \beta E_t\left[\frac{\lambda_{t+1}}{\lambda_t}\left(\left(1-\eta_{t+1}\right)\left(v_{t+1}\left(\widetilde{z}_{t+1}\right)+j_{t+1}\left(\widetilde{z}_{t+1}\right)\right)+\eta_{t+1} l v_{t+1}\right)\right] \tag{17}$$

We rewrite equation (17) as

$$1 = E_t\left[M_{t+1}\bar{\eta}_{t+1}\kappa_{t+1}\right], \tag{18}$$

where we define the stochastic discount factor net of the exit probability as $M_{t+1} \equiv \beta E_t\left(\frac{\lambda_{t+1}}{\lambda_t}\right)$, the survival probability as $\bar{\eta}_{t+1} \equiv \left(1-\eta_{t+1}\right)$, the transformed equity return as $\kappa_{t+1} \equiv \frac{v_{t+1}+j_{t+1}}{v_t} + \frac{\eta_{t+1}}{1-\eta_{t+1}}\frac{lv_{t+1}}{v_t}$. Log-linearizing with risk-adjustment equation (18) yields

$$1 = M\bar{\eta}\kappa + M\bar{\eta}\kappa\begin{pmatrix} E_t\hat{M}_{t+1} + E_t\widehat{\bar{\eta}}_{t+1} + E_t\hat{\kappa}_{t+1} \\ +\frac{1}{4}Var_t\hat{M}_{t+1} + \frac{1}{4}Var_t\widehat{\bar{\eta}}_{t+1} + \frac{1}{2}Var_t\hat{\kappa}_{t+1} \\ +Cov_t\left(\hat{M}_{t+1},\widehat{\bar{\eta}}_{t+1}\right) + Cov_t\left(\widehat{M_{t+1}\bar{\eta}_{t+1}},\hat{\kappa}_{t+1}\right)\end{pmatrix}, \tag{19}$$

where M, $\bar{\eta}$, κ denote the steady state values of respectively, M_t, $\bar{\eta}_t$, κ_t, while variables with hat are in log-deviation from the deterministic steady state, e.g. $\hat{x}_t \equiv \log(\frac{x_t}{x})$. Worth noting, the log-linearization with risk-adjustment of the risk-free interest rate yields

$$\hat{R}_{f,t} = -E_t\hat{M}_{t+1} - \frac{1}{2}Var_t\hat{M}_{t+1}. \tag{20}$$

Therefore, the log-linearization with risk-adjustment of the entry condition (17) boils down to

$$E_t\hat{\kappa}_{t+1} - \hat{R}_{f,t} = -E_t\widehat{\bar{\eta}}_{t+1} + E_t\left[\begin{array}{c} -\frac{1}{2}Var_t\hat{M}_{t+1} - \frac{1}{4}Var_t\widehat{\bar{\eta}}_{t+1} - \frac{1}{2}Var_t\hat{\kappa}_{t+1} \\ -Cov_t\left(\hat{M}_{t+1},\widehat{\bar{\eta}}_{t+1}\right) - Cov_t\left(\hat{M}_{t+1}+\widehat{\bar{\eta}}_{t+1},\hat{\kappa}_{t+1}\right)\end{array}\right] \tag{21}$$

We proceed to log-linearize with risk-adjustment the exit condition,

$$v_t\left(\bar{z}_t\right) = j_t\left(\bar{z}_t\right) + \beta E_t\left[\frac{\lambda_{t+1}}{\lambda_t}\left(1-\eta_{t+1}\right)v_{t+1}\left(\bar{z}_{t+1}\right)\right]. \tag{22}$$

Equation (22) is rewritten as

$$1 - \frac{\bar{j}_t}{\bar{v}_t} = E_t\left[M_{t+1}\bar{\eta}_{t+1}\bar{\kappa}_{t+1}\right], \tag{23}$$

where we define $\bar{v}_t$, $\bar{j}_t$, and $\bar{\kappa}_{t+1} \equiv \frac{\bar{v}_{t+1}}{\bar{v}_t}$as respectively, the value, profit, equity return of the firm with the marginal value of idiosyncratic productivity. Log-linearizing with risk-adjustment equation (23) yields

$$E_t\widehat{\bar{\kappa}}_{t+1} - \hat{R}_{f,t} = \frac{\bar{j}}{\bar{v}-\bar{j}}\left(\widehat{\bar{v}}_t - \widehat{\bar{j}}_t\right) - E_t\widehat{\bar{\eta}}_{t+1} + E_t\left[\begin{array}{c} -\frac{1}{2}Var_t\hat{M}_{t+1} - \frac{1}{4}Var_t\widehat{\bar{\eta}}_{t+1} - \frac{1}{2}Var_t\widehat{\bar{\kappa}}_{t+1} \\ -Cov_t\left(\hat{M}_{t+1},\widehat{\bar{\eta}}_{t+1}\right) - Cov_t\left(\hat{M}_{t+1}+\widehat{\bar{\eta}}_{t+1},\widehat{\bar{\kappa}}_{t+1}\right)\end{array}\right]. \tag{24}$$

Finally, we calculate the equity premium for the Exo Exit model. The alternative DSGE specification consists of a model with endogenous entry as Baseline but with exogenous and constant survival probability as $\bar{\eta} \equiv (1-\eta)$. Hence, the corresponding entry condition in Exo Exit is given by

$$v_t(\widetilde{z}) = \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \left((1-\eta)\left(v_{t+1}(\widetilde{z}) + j_{t+1}(\widetilde{z}) \right) + \eta l v_{t+1} \right) \right] \tag{25}$$

where $\widetilde{z}$ indicates the average level of firm productivity, which is constant over time. Equation (17) can be rewritten as

$$\frac{1}{\bar{\eta}} = E_t \left[M_{t+1} \kappa_{t+1}^{EXO} \right], \tag{26}$$

with $\kappa_{t+1}^{EXO} \equiv \frac{v_{t+1}+j_{t+1}}{v_t} + \frac{\eta}{1-\eta}\frac{lv_{t+1}}{v_t}$ as the equity return. Log-linearizing with risk-adjustment equation (26) yields

$$E_t \hat{\kappa}_{t+1}^{EXO} - \hat{R}_{f,t} = E_t \left[-\frac{1}{2} Var_t \hat{M}_{t+1} - \frac{1}{2} Var_t \hat{\kappa}_{t+1}^{EXO} - Cov_t \left(\hat{M}_{t+1}, \hat{\kappa}_{t+1} \right) \right] \tag{27}$$

Different from Baseline, in Exo Exit does not an equivalent exit condition. Hence, we cannot define the marginal equity premium in the alternative model.

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