Sustainable Energy Technologies and Assessments

Thermo-economic and entropy generation analyses of magnetic natural convective flow in a nanoliquid-filled annular enclosure fitted with fins

--Manuscript Draft--

Manuscript Number:	
Article Type:	VSI:Solar thermal efficiency
Keywords:	Thermo-Economic Analysis; Fins; Magnetic natural convection of nanofluids; FEM; Entropy generation
Corresponding Author:	A. Sattar Dogonchi Babol Noshirvani University of Technology Babol, Iran, Islamic Republic of
First Author:	Tahar Tayebi
Order of Authors:	Tahar Tayebi
	A. Sattar Dogonchi
	Nader Karimi
	Ali J. Chamkha
Abstract:	A numerical analysis on the thermo-natural convection as well as entropy generation of Al2O3-H2O nanofluid enclosed by two circular cylinders in the presence of magnetic fields was performed. The internal hot cylinder is fitted with rectangular fins of different lengths. Irreversibilities related to the thermo-effects, friction effects, magnetic effects were considered. FEM was selected as the solving method. Results described the impact of active parameters on the thermo-natural convective flow and heat transfer behaviour as well as entropy generation characteristics. In addition, a basic economic analysis has been proposed to consider the cost of nanofluids in comparison to their contribution in enhancing the heat transfer rate.
Suggested Reviewers:	Khalil Ur Rehman, PhD University of Waterloo kurrehman@uwaterloo.ca
	Ishak Hashim, PhD The National University of Malaysia ishak_h@ukm.edu.my
	Kai-Long Hsiao, PhD Taiwan Shoufu University hsiao.kailong@msa.hinet.net
	W.A. Khan, PhD Beijing Institute of Technology waqarazeem@bit.edu.cn
	Abdelraheem M. Aly, PhD King Khalid University ababdallah@kku.edu.sa
	M. Waqas, PhD National University of Technology muhammadwaqas@nutech.edu.pk
	Mohammad Reza Safaei, PhD Florida International University msafaei@fiu.edu
Opposed Reviewers:	

COVER LETTER FOR SUBMISSION OF MANUSCRIPT

Subject: **Submission of manuscripts**

Dear Editor,

With this letter, I have attached a copy of manuscript entitled “**Thermo-economic and entropy generation analyses of magnetic natural convective flow in a nanoliquid-filled annular enclosure fitted with fins**”. It is declared that the work described has not been published previously, that it is not under consideration for publication elsewhere, that its publication is approved by all authors and that, if accepted, it will not be published elsewhere in the same form, in English or in any other language, without the written consent of the Publisher.

Suggested reviewers:

1- Khalil Ur Rehman (kurrehman@uwaterloo.ca)

Department of Applied Mathematics, University of Waterloo, Waterloo, Ontario N2L3G1, Canada

2- Ishak Hashim (ishak_h@ukm.edu.my)

School of Mathematical Sciences, The National University of Malaysia, Bangi, Malaysia

3- Kai-Long Hsiao (hsiao.kailong@msa.hinet.net)

Department of the Digital Recreation and Game Design, Taiwan Shoufu University, 168, Nansh Li, Madou District, Tainan City, Taiwan, Republic of China

4- W.A. Khan (waqarazeem@bit.edu.cn)

School of Mathematics and Statistics, Beijing Institute of Technology, Beijing, China

5- Abdelraheem M. Aly (ababdallah@kku.edu.sa)

Department of Mathematics, King Khalid University, Saudi Arabia

6- M. Waqas (muhammadwaqas@nutech.edu.pk)

NUTECH School of Applied Sciences and Humanities, National University of Technology, Islamabad, 44000, Pakistan

7- Mohammad Reza Safaei (msafaei@fiu.edu)

Department of Civil and Environmental Engineering, Florida International University, Florida, USA

With best wishes and highest personal regards.

Sincerely yours,

A.S. Dogonchi

Thermo-economic and entropy generation analyses of magnetic natural convective flow in a nanoliquid-filled annular enclosure fitted with fins

Tahar Tayebi[1,2], A. Sattar Dogonchi[3*], Nader Karimi[4], Ali J. Chamkha[5,6]

[1]Department of Mechanical Engineering, Faculty of Sciences and Technology, Mohamed El Bachir El Ibrahimi University, Bordj Bou Arreridj, El-Anasser, 34030, Algeria.
[2]Energy Physics Laboratory, Department of Physics, Faculty of Exact Sciences, Frères Mentouri Constantine1 University, 25000 Constantine, Algeria,
[3]Independent researcher in Mechanical Engineering
[4]School of Engineering and Materials Science, Queen Mary University of London, London, United Kingdom
[4]Faculty of Engineering, Kuwait College of Science and Technology, Doha District, Kuwait
[5]Center of Excellence in Desalination Technology, King Abdulaziz University, P.O. Box 80200, Jeddah 21589, Saudi Arabia

***Corresponding author(s):** A.S. Dogonchi (sattar.dogonchi@yahoo.com)

Abstract:

A numerical analysis on the thermo-natural convection as well as entropy generation of Al_2O_3-H_2O nanofluid enclosed by two circular cylinders in the presence of magnetic fields was performed. The internal hot cylinder is fitted with rectangular fins of different lengths. Irreversibilities related to the thermo-effects, friction effects, magnetic effects were considered. FEM was selected as the solving method. Results described the impact of active parameters on the thermo-natural convective flow and heat transfer behaviour as well as entropy generation characteristics. In addition, a basic economic analysis has been proposed to consider the cost of nanofluids in comparison to their contribution in enhancing the heat transfer rate.

Keywords: Thermo-Economic Analysis, Nanofluids; Fins, Magnetic natural convection; FEM; Entropy generation.

1. Introduction

In several thermal devices and equipment, thermo-natural convection is one of the vital heat transfer mechanisms. Many external and internal factors affect thermo-natural convection. Therefore, to ameliorate the performance of these devices, various variables need to be explored for optimizing heat transfer and reducing energy consumption. A key disadvantage of the thermal efficiency of thermal devices is considered to be the low thermal conductivity of common heat transfer liquids like water, ethylene glycol, oil, etc. A revolutionary technique for optimizing thermal transmission using nanomaterials suspended in a base liquid, referred as nanofluid, has been largely examined in recent years with respect to heat transfer improvement. Also, the presence of a magnetic field in the thermo-natural convection process, commonly called Magneto-hydrodynamics (MHD), is also a very influential external factor. The study of thermo-natural convective flows of nanofluids in the presence of magnetic field has generated considerable interest which resulted in a large number of studies due to its use in a number of technological applications especially since nanofluids have the characteristics of magnetic properties and fluid simultaneously [1–10]. Besides, the perusal of natural convection based on the entropy generation inspection can accurately evaluate the performance of thermal systems [11–20]. Kashaniet al. [21] perused the entropy generation and thermo-natural convection of nanoliquid inside an enclosure with diverse patterns of vertical walls that are considered waved. Their investigations indicate that there is a reduction in the entropy generation by a growth in nanoparticles into base water, especially at higher Rayleigh numbers. Cho et al. [22] discussed the thermal efficiency and entropy production of thermo-natural convection in a nanoliquid-loaded U-shaped domain. The findings revealed that the overall heat transmission improves and the total irreversibilities decrease as the concentration in volume of nanoparticles increases. While the overall heat transmission and the total irreversibilities are both boosted as the Rayleigh values increase. In the presence of magnetic and heat sink/source effects in a square porous domain loaded with Alumina-water nanofluid, Rashad et al. [23] perused the thermo-natural convective process and entropy production. They conveyed that by raising the amount of Alumina nanomaterials and enhancing the Hartmann number, the Nusselt goes down. In comparison, the preferred location and length of the heat source/sink are identified respectively at 0.7 and 0.2. Mahmoudi et al. [24] simulated the thermo-free convection of alumina-water nanofluid-filled square domain. The domain is affected by an external magnetic field and regular generation/absorption of heat energy. Kefayati [25] performed a CFD analysis, based on the second law of thermodynamics, to evaluate the thermal performance of laminar thermo-natural convective flow of a non-Newtonian nanoliquid in a square domain by considering an external horizontal magnetic field. Seyyedi et al. [26] performed an entropy generation-based analysis in magnetic thermo-convective flow of nanoliquid-filled L-shaped enclosure. They implemented for the first time, an economic evaluation to estimate the enclosure's thermal efficiency with consideration cost of used nanoparticles. The entropy generation features and heat transfer caused by the thermo-natural convection and of the Cu-Al_2O_3/water hybrid nanoliquid enclosed by square cavities containing various conductive elements with different geometries are further explored by Tayebi and Chamkha [27-28]. In these studies, they identified that the flow, heat transmission rate and entropy production in the hybrid nanoliquid is parameters dependent. Selimefendigil and Öztop [29] have considered a centered conductive curved solid wall and magnetic field to evaluate the thermo-natural convective flow and entropy generation in an oblique cavity which was loaded by a nanofluid. Tayebi and Öztop [30] conducted a numerical study utilizing Al_2O_3 -Cu/water hybrid nanofluid as heat transfer agent to determine entropy generation for the thermo-natural convective flow between two confocal elliptical cylinders. Hashemi-Tilehnoee et al. [31] examined via ANSYS Fluent, effects of magnetic field on thermo-natural convection and entropy production of Al_2O_3-water nanoliquid circulating within an incinerator-shaped porous cavity in the presence of a wavy solid wall. Thermo-effects, fluid friction, magnetic effects, and porous medium irreversibilities are considered.

The key aim of this analysis is to address the irreversibility problem in MHD nanofluid flow within a gap among two differentially heated circular cylinders. The internal hot cylinder is fitted with rectangular fins that its length could be changed in size. Nonlinear PDEs are computed through the Finite Element Technique. Physical aspects of several influential variables on fluid flow features, temperature, Bejan number, and heat transmission are examined. Irreversibilities related to the thermo-effects, friction effects, and magnetic effects are considered. In addition, an economic analysis study was suggested to consider the cost of nanofluids in comparison to their contribution in boosting the heat transfer rate.

2. Basic equations and problem explanation

Thermo-natural convection of Al_2O_3-H_2O nanoliquid within a gap among two circular cylinders is perused (**Fig. 1**). The inner hot cylinder is equipped with rectangular fins that its length could be changeable. Considering the impact of magnetic field, the equations ruled over the current problem could be mentioned as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\rho_{nf}\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu_{nf}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+\sigma_{nf}\left(B_y B_x v-B_y^2 u\right) \tag{2}$$

$$\begin{aligned}\rho_{nf}\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\mu_{nf}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+\rho_{nf}\beta_{nf}\,g\left(T-T_c\right)\\ +\sigma_{nf}\left(B_y B_x u-B_x^2 v\right)\end{aligned} \tag{3}$$

$$\left(\rho C_p\right)_{nf}\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k_{nf}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \tag{4}$$

where the features for the nanofluid as function of nanoparticles' concentration could be assumed as:

$$\frac{k_{nf}}{k_f}=1+2.944\phi+19.672\phi^2, \qquad \rho_{nf}=\left(1-\phi\right)\rho_f+\phi\rho_s,$$

$$\frac{\sigma_{nf}}{\sigma_f}=1+\frac{3\left(\frac{\sigma_s}{\sigma_f}-1\right)\phi}{\left(\frac{\sigma_s}{\sigma_f}+2\right)-\left(\frac{\sigma_s}{\sigma_f}-1\right)\phi}, \qquad \frac{\mu_{nf}}{\mu_f}=1+4.93\phi+222.4\phi^2, \tag{5}$$

$$\left(\rho\beta\right)_{nf}=\left(\rho\beta\right)_f-\phi\left(\rho\beta\right)_f+\phi\left(\rho\beta\right)_s,\ \left(\rho C_p\right)_{nf}=\left(\rho C_p\right)_f-\phi\left(\rho C_p\right)_f+\phi\left(\rho C_p\right)_s$$

Considering the following parameters,

$$X = \frac{x}{L}, \quad Y = \frac{y}{L}, \quad P = \frac{pL^2}{\rho_f \alpha_f^2}, \quad U = \frac{uL}{\alpha_f}, \quad V = \frac{vL}{\alpha_f}, \quad \theta = \frac{T - T_c}{T_h - T_c} \tag{6}$$

The equations ruled over the current work along with the boundary conditions could be stated in their non-dimensional type as:

$$\begin{aligned} U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{\rho_f}{\rho_{nf}}\frac{\partial P}{\partial X} + \frac{\mu_{nf}/\mu_f}{\rho_{nf}/\rho_f}\Pr\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right) \\ + \frac{\sigma_{nf}/\sigma_f}{\rho_{nf}/\rho_f} Ha^2 \Pr\left(B'_y B'_x V - B'^2_y U\right) \end{aligned} \tag{7}$$

$$\begin{aligned} U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{\rho_f}{\rho_{nf}}\frac{\partial P}{\partial Y} + \frac{\mu_{nf}/\mu_f}{\rho_{nf}/\rho_f}\Pr\left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right) \\ + \frac{\sigma_{nf}/\sigma_f}{\rho_{nf}/\rho_f} Ha^2 \Pr\left(B'_x B'_y U - B'^2_x V\right) + \frac{(\rho\beta)_{nf}}{\rho_{nf}\beta_f}\Pr Ra\theta \end{aligned} \tag{8}$$

$$U\frac{\partial \theta}{\partial X} + V\frac{\partial \theta}{\partial Y} = \frac{k_{nf}/k_f}{(\rho C_p)_{nf}/(\rho C_p)_f}\left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2}\right) \tag{9}$$

$\theta = 0$ on the exterior cylinder

$\theta = 1$ on the interior cylinder and fin (10)

$\Psi = 0$ on all walls

where $\Pr = \nu_f / \alpha_f$ and $Ra = g\beta_f (T_h - T_c)L^3 / \alpha_f \nu_f$.

The Nusselt numbers i.e. local and average along the exterior cylinder can be depicted as:

$$Nu_{loc.} = -\frac{k_{nf}}{k_f}\frac{\partial \theta}{\partial n}, \quad Nu_{ave.} = \frac{1}{2\pi}\int_0^{2\pi} Nu_{loc.} d\zeta \tag{11}$$

3. Entropy generation analysis

Based on [26], the local entropy generation (En$_{\text{local}}$) in its dimensionless type considering magnetic field could be declared as:

$$En_{local}=\underbrace{\frac{k_{nf}}{k_f}\left[\left(\frac{\partial\theta}{\partial Y}\right)^2+\left(\frac{\partial\theta}{\partial X}\right)^2\right]}_{En_{local,HT}}+\underbrace{\frac{\mu_{nf}}{\mu_f}\chi_f\left[2\left(\frac{\partial U}{\partial Y}\right)^2+2\left(\frac{\partial U}{\partial X}\right)^2+\left(\frac{\partial U}{\partial X}+\frac{\partial U}{\partial Y}\right)^2\right]}_{En_{local,FF}}+ \\ \underbrace{\frac{\sigma_{nf}}{\sigma_f}\chi_f Ha^2\left(UB'_y-VB'_x\right)^2}_{En_{local,MF}} \tag{12}$$

The total entropy generation (En_{total}) could be gained by:

$$En_{total}=\int_V En_{local}\,\mathrm{dV}. \tag{13}$$

The Bejan number (Be_{local}, Be_{ave}), furthermore, could be stated as follow:

$$\begin{aligned} Be_{local} &= En_{local,HT}\,/\,En_{local} \\ Be_{ave.} &= \int_A Be_{local}\,\mathrm{dA}\Big/\int_A \mathrm{dA} \end{aligned} \tag{14}$$

4. A simple economic investigation

Admittedly, adding nanoparticles to the normal fluid could boost the heat transfer rate. The cost of nanofluids, nevertheless, may well be ascended by ascending the volume fraction of nanofluid which should be considered. The total cost of nanofluid (C_T) could be expressed as [26]:

$$C_T=C_{Nanoparticle}+C_{Base-fluid}+C_{other} \tag{15}$$

where $C_{Nanoparticle}$, $C_{Base\text{-}fluid}$, and C_{other} signify the cost of whole nanoparticles (€), the cost of base fluid, and the cost of preparing nanofluid, respectively. In this work, the latter one is disregarded and $C_{nanoparticle}$ could be defines as:

$$C_{Nanoparticle}=m\times C_u \tag{16}$$

here C_u denotes the unit cost of nanoparticle (€/gr) and *m* indicates the mass of nanoparticles required for the considered cavity which could be acquired as:

$$m=\phi\times V\times\rho \tag{17}$$

where ϕ, V, and ρ indicate the volume fraction of nanoliquid, the volume of the cavity (cm^3), and the density of the nanoparticle (gr/cm^3). In addition to this, the allocated values to the C_u and $C_{Base\text{-}fluid}$ are 2.721 €/gr (Al_2O_3) and 0.07076 €/dm^3 [26], respectively, and since these values are valid for 2017 so C_u and $C_{Base\text{-}fluid}$ should be updated for the current year 2021 by considering the interest rate equals to 10%. By doing so, we have:

C_u (2021) = 2.721 €/gr×$(1+0.1)^4$=3.9838161 €/gr

$C_{Base\text{-}fluid}$ (2021) =0.07076 €/dm^3 × $(1+0.1)^4$=0.103599716 €/dm^3=0.103599716×10^{-3} €/cm^3

Table 2 portrays the total cost of nanofluid for diverse values of ϕ. Furthermore, Seyyedi et al. [26] recently introduced a criterion in order to explore the thermal performance of adding nanoparticles to the base fluid versus its costs that could be determined as follow:

$$Cr = \frac{\dfrac{Nu_{ave}(\phi) - Nu_{ave}(\phi = 0\%)}{Nu_{ave}(\phi = 0\%)}}{\dfrac{C_T(\phi) - C_T(\phi = 0\%)}{C_T(\phi = 0\%)}} \tag{18}$$

5. Numerical validation and the methodology

A numerical approach called finite element method is utilized to gain the solution of non-dimensional type of equations ruled over the current problem. In addition to this, based on figure 2 one could observe a supreme compromise among the current result and those of studies done experimentally as well as numerically.

6. Results and discussion

In the following section, we investigate thermal, flow, entropy generation and heat transfer features of nanofluids filled-annulus of a double-cylinder with rectangular fins mounted to the surface of the inner cylinder. The simulation results which represented in terms of isotherms lines, stream function isolines, U- and V-velocities components isolines and $Be_{loc.}$

isolines, and profiles of Nusselt and Bejan numbers, were evaluated for disparate values of AR, Ha, and Ra.

The thermal and flow features variations with respect to Hartmann and Rayleigh numbers are portrayed in **Fig. 3**. Despite the rise in buoyancy effects (Ra) and magnetic forces (Ha), temperatures and stream function isolines preserve the property of symmetry concerning the middle vertical line of the system, where two symmetrical vortices with two cores are formed that rotates at the same speed and in opposite directions. With the escalation of Ra and the decline in Ha, the intensity of the thermo-convective flow heightens. This is explicated by the magnitudes of stream function which are found to be ascended with Rayleigh number and descended with Hartmann number. With regard to the thermal behavior, with increasing Ra, the isotherms were gradually deformed towards the surface of the outer cylinder and found to be denser at the outer edges of the rectangular fins reflecting a higher local heat transfer rate near those zones. The distributions of the velocity components inside the annulus in **Fig. 4** reflect the flow directions and high-speed regions within the system. Positive and negative values confirm that the nanofluid flow is bi-cellular as seen by the streamlines. It was observed that the maximum values of vertical velocity (V) were reported near the surface of the cold cylinder downward and upwards near the inner cylinder. The maximum horizontal velocities, to the right or the left direction, are found at the top and bottom of the annulus with symmetry with respect to the midpoint of the annulus. The values of the velocity components increase under the action of the buoyancy forces and decrease under the action of the magnetic forces. This is explained by the fact that the magnetic force acts in the opposite direction to the buoyancy force.

The comparison of isotherms and streamlines maps for varying Rayleigh and Hartmann numbers are shown in **Fig. 5**. It is observed that the thermo-convective flow intensity reduces by augmenting the size of the rectangular fins on the inner surface since the fins act to obstructing the circulation of the nanofluid inside the annulus. Besides, As AR reduces, we observe that the isotherms lines approach the inner hot surface, which increases local heat transfer and makes the distribution of heat transfer more uniform on the inner surface. It can be concluded from **Fig. 6**, that an increase in fin size on the hot surface mainly affects the vertical velocity decrease as compared to the horizontal velocity.

Fig. 7 portrays isolines maps of local Bejan numbers for disparate values of AR, Ha, and Ra. The local Bejan numbers are generated from the ratio of local irreversible heat transfer to total local irreversibility owing to heat transfer, fluid friction, and magnetic effects. It can be seen that the maximum values of local Bejan numbers are found to be in regions with high-temperature gradients and low-velocity intensity and vice versa. At the lower Rayleigh number, the Bejan number in the almost whole system is close to 1, which means that the irreversibility related to the thermal effects is predominant. Since Rayleigh acts to irritate the intensity of the thermo-convection and augments the velocity gradients within the system, a decrease in the overall Bejan number is thus noticed at high Rayleigh values suggesting that irreversibility thermal effects are no longer the key contributors to the total irreversibility. At a given Rayleigh, with increasing Hartmann the Bejan number decreases especially in zones with high-velocity gradients. This is due to the magnetic effects irreversibilities that are added to the total production of entropy. Moreover, the overall production of entropy is more in the system in which its inner surface is fitted with large fins. This is due to the nanofluid friction irreversibilities that are apparent particularly near the fins' surfaces.

Fig. 8 shows the distribution diagram of the local heat transfer rate, which is estimated by local Nusselt numbers (Nu_{loc}), along the exterior cold cylinder for disparate values of AR, Ha, and Ra. As can be observed, at lower Ra, the Nusselt profiles are in peaks fashion where each peak corresponds to the highest heat transfer rate as there is a maximum temperature gradient on the surface of the outer cylinder at these areas. These areas where the maximum rate of heat transfer has occurred are located against the prominent fins on the inner cylinder surface. The distribution of Nusselt numbers tends to be more uniform over the outer cylinder surface as AR decreases. Moreover, as the buoyancy effects increases, the heat transfer rate becomes more apparent in the top portion of the outer surface. The effect of magnetic forces on reducing heat transfer is also becoming more noticeable. We present the mean Nusselt number as a function of the Rayleigh number for various AR values and two separate Ha values in **Fig. 9**. We can see that Nusselt is an ascending function of Ra for a given AR value. For $Ra<10^4$, the Nusselt number is less sensitive to the variation of Ra. This sensitivity increases with the increase in the Ra number. Hartmann's influence on the Nusselt number soars with growing Rayleigh. The highest average heat transfer rate was for the higher values of the fins protruding from the inner cylinder surface. This is attributed to the enhancement in the heat exchange surface by an increasing AR. The profiles of average Bejan numbers, in **Fig. 9**, indicate that the contribution of the irreversibilities due to the thermo-effects to the overall irreversibility inside the system is more prominent for lower Ra. Also, the magnetic field tends to augment these thermo-effects irreversibilities contribution.

To assess the cost of using the nanofluid to improve heat transfer in the system, **Fig. 10** shows the variation of the ratio of the heat transfer enhancement by using nanofluid versus the increment in the cost of the mixture according to the solid volume fraction (0.01, 0.02, 0.03, 0.04) for Ha= 0 and Ha=40 at $Ra= 10^5$ where convection is the predominant heat transfer mechanism. From this figure, it is predicted that while the value of *Cr* decreases with increasing the nanoparticles volume fraction in the absence of the magnetic field (Ha=0), it tends to increases as ϕ increases for the case of Ha=40. This means that improving the thermal transfer enhancement in the case of the presence of magnetic forces is more costly, and the reason is that the magnetic field plays an opposite role in improving the heat transfer rate within the cavity.

7. Conclusion

MHD-Thermo-natural convection and entropy generation analysis in a gap between two circular cylinders in which the internal hot cylinder is equipped with rectangular fins were evaluated numerically. The results described the impact of active parameters on the behaviour of thermo-natural convective flow and heat transfer, as well as the features of entropy generation in the annulus. In addition, a basic economic analysis has been proposed to consider the cost of nanofluids compared to their contribution to enhancing the rate of heat transfer. The key results of the numerical analysis can be briefed as follows:

- The magnitude of thermo-natural convective motion is found to be increased with the Rayleigh number and decreased with the Hartmann number and the size of the rectangular fins.
- As the size of the fins reduces, the local heat transfer distribution tends to be more uniform on the inner surface.
- A rise in the size of the fins on the hot surface mainly contributes to the decrease of the vertical velocity relative to the horizontal velocity.
- For low Rayleigh numbers, the irreversibility related to the thermo-effects is predominant within the annulus. By augmenting Rayleigh values, the irreversibility due to the thermo-effects is no longer the key contributor to the overall entropy production.
- The magnetic forces help to increase the thermo-effects irreversibility contribution to the overall irreversibilities.
- It is found that the thermal transfer enhancement in the case of the presence of magnetic forces is more costly in terms of nanofluids, and the reason is that the magnetic field plays an opposite role in improving the heat transfer rate within the cavity.

Table 1: Thermo-physical specifications of H_2O and Al_2O_3

	$C_p\left(J/kg\ K\right)$	$\rho\left(kg/m^3\right)$	$k\left(W/m\ K\right)$
Al_2O_3	765	3970	40
H_2O	4179	997.1	0.613

Table 2: C_T for diverse values of ϕ

ϕ	$C_{Nanoparticle}$ (€)	$C_{Base\text{-}fluid}$ (€)	C_T (€)
0.00	0	0.065	0.065
0.01	99.235	0.065	99.300
0.02	198.471	0.065	198.536
0.03	297.706	0.065	297.771
0.04	396.942	0.065	397.007

Table 3: Impact of ϕ on $Nu_{ave.}$

Ra	ϕ	$Nu_{ave.}$
10^3	0.01	3.245291
	0.04	3.615988
10^4	0.01	3.253477
	0.04	3.620284
10^5	0.01	3.778663
	0.04	3.954019

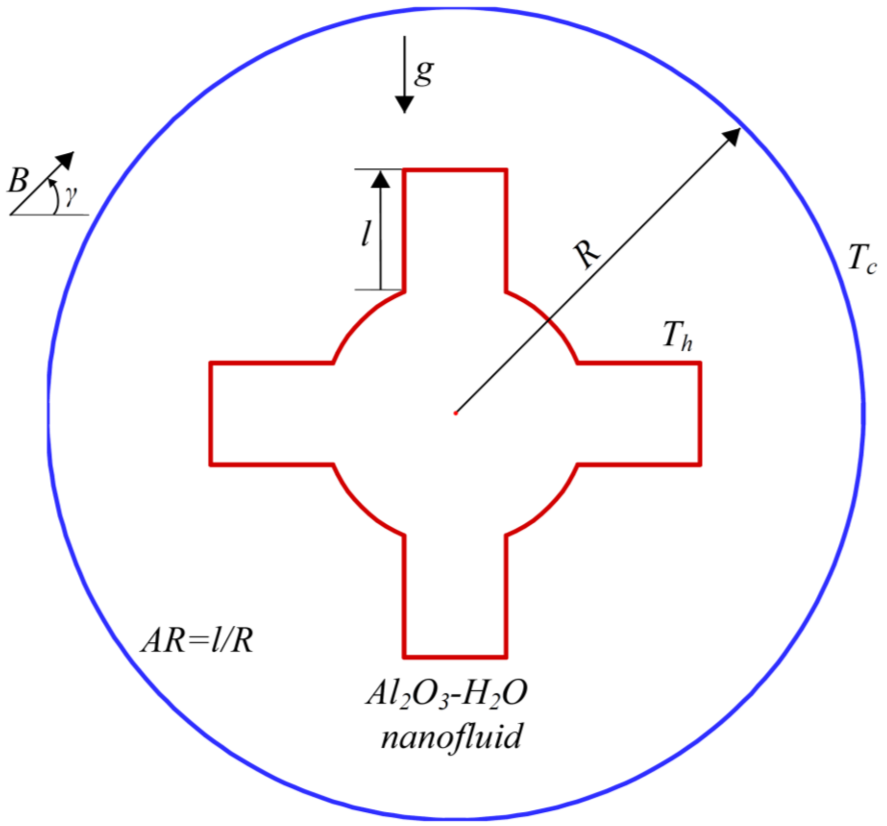

Fig. 1. Geometry of current work

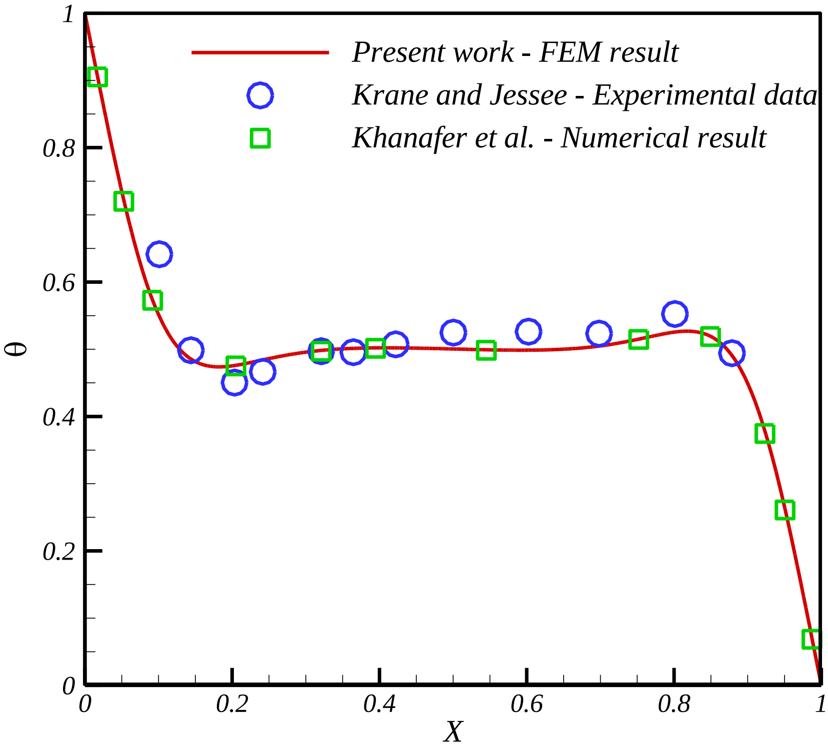

Fig. 2. Validation of current outcome with experimental data [32] and numerical result [33]

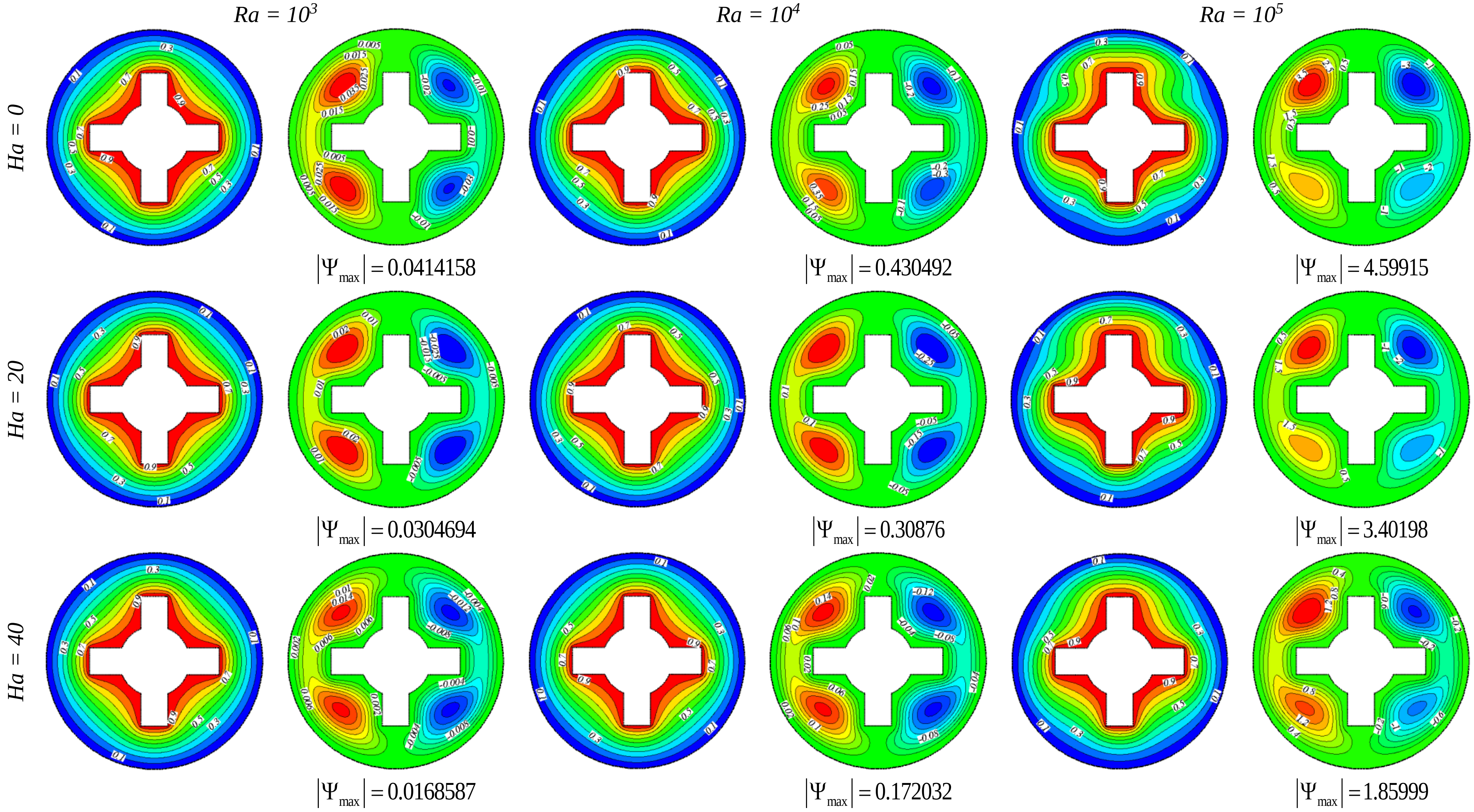

Fig. 3. Isotherms and streamlines for disparate amounts of Ha as well as Ra when AR=0.15

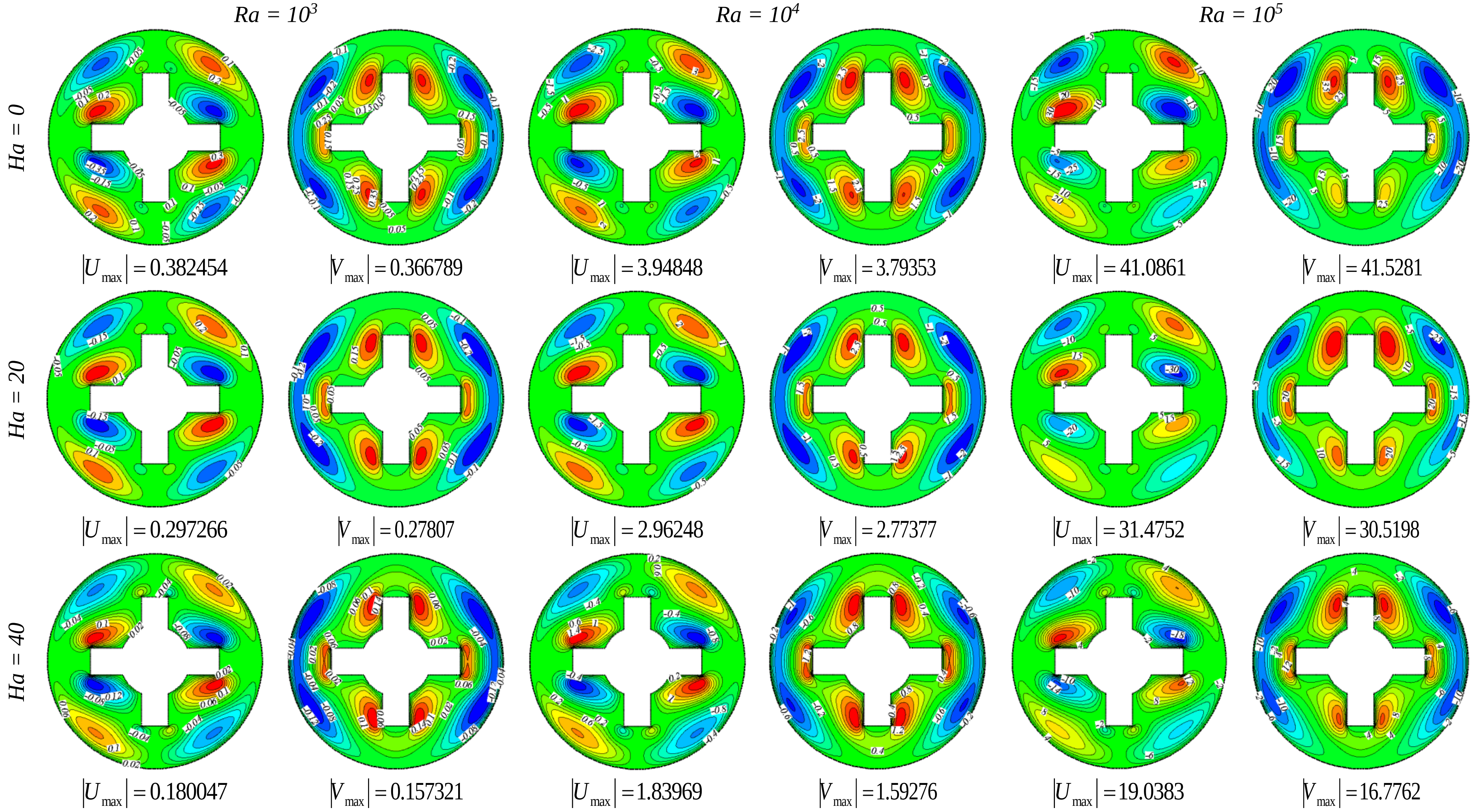

Fig. 4. Velocities (U,V) for disparate amounts of Ha as well as Ra when AR=0.15

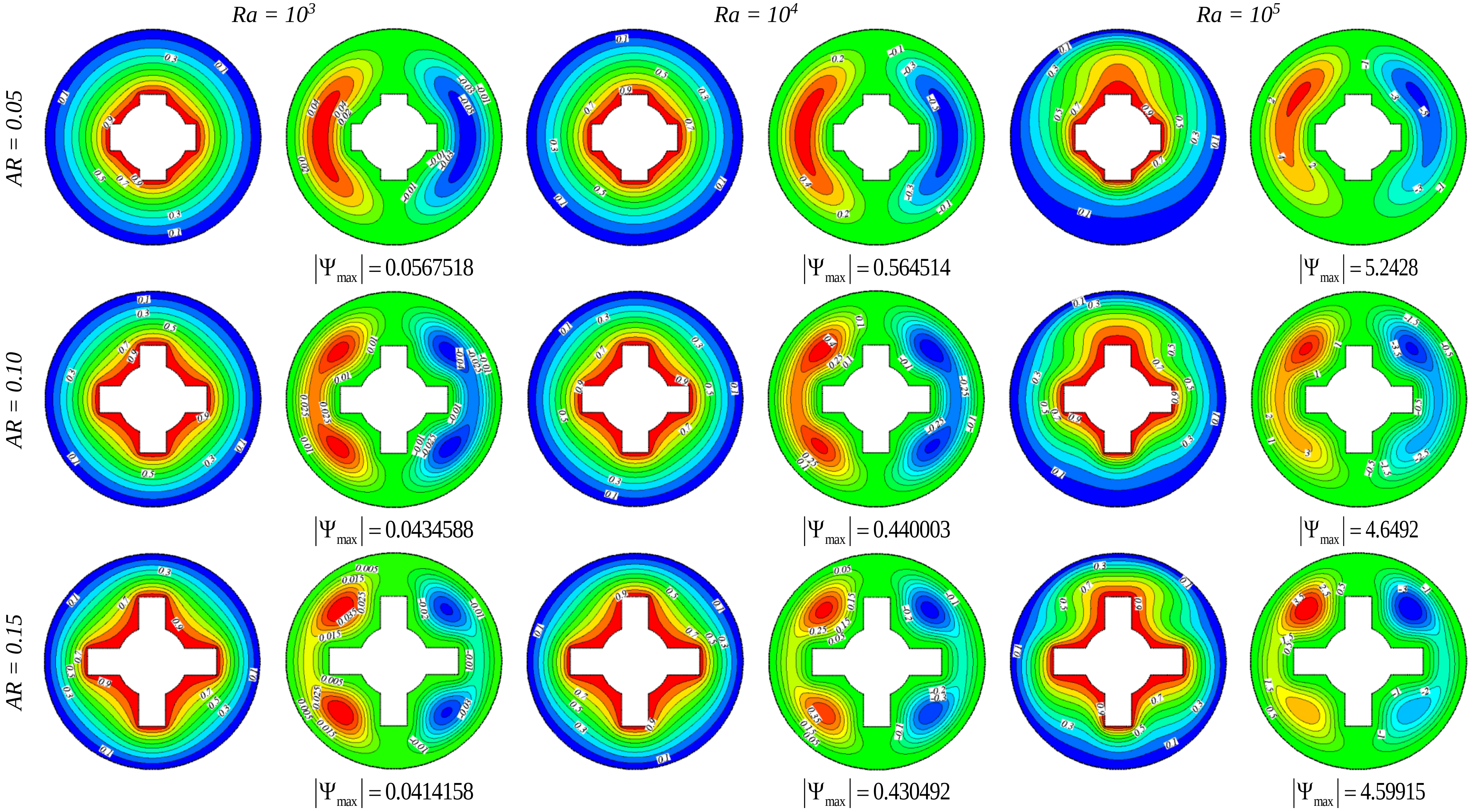

Fig. 5. Isotherms and streamlines for disparate amounts of AR at various Ra

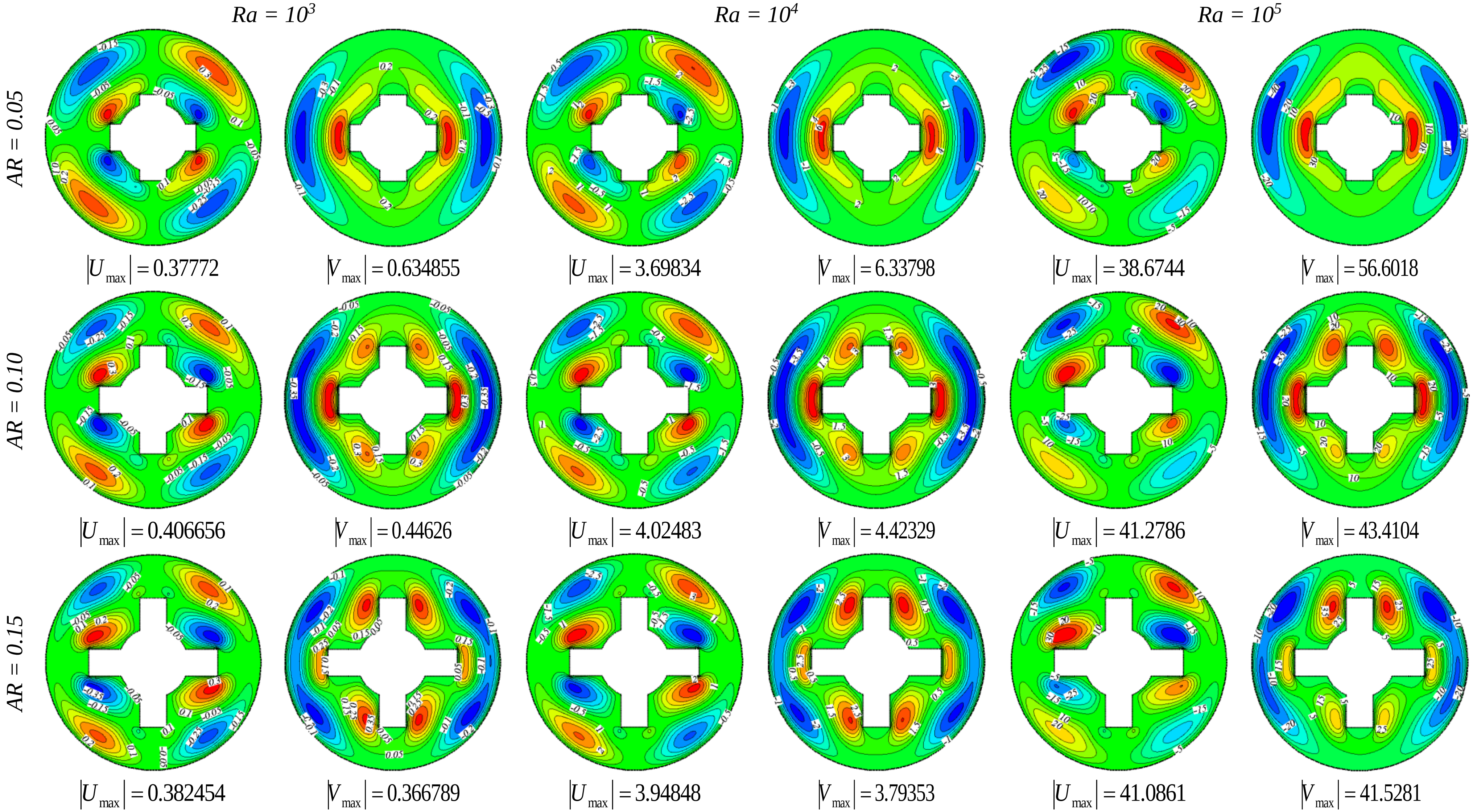

Fig. 6. Velocities (U,V) for disparate values of AR at various Ra

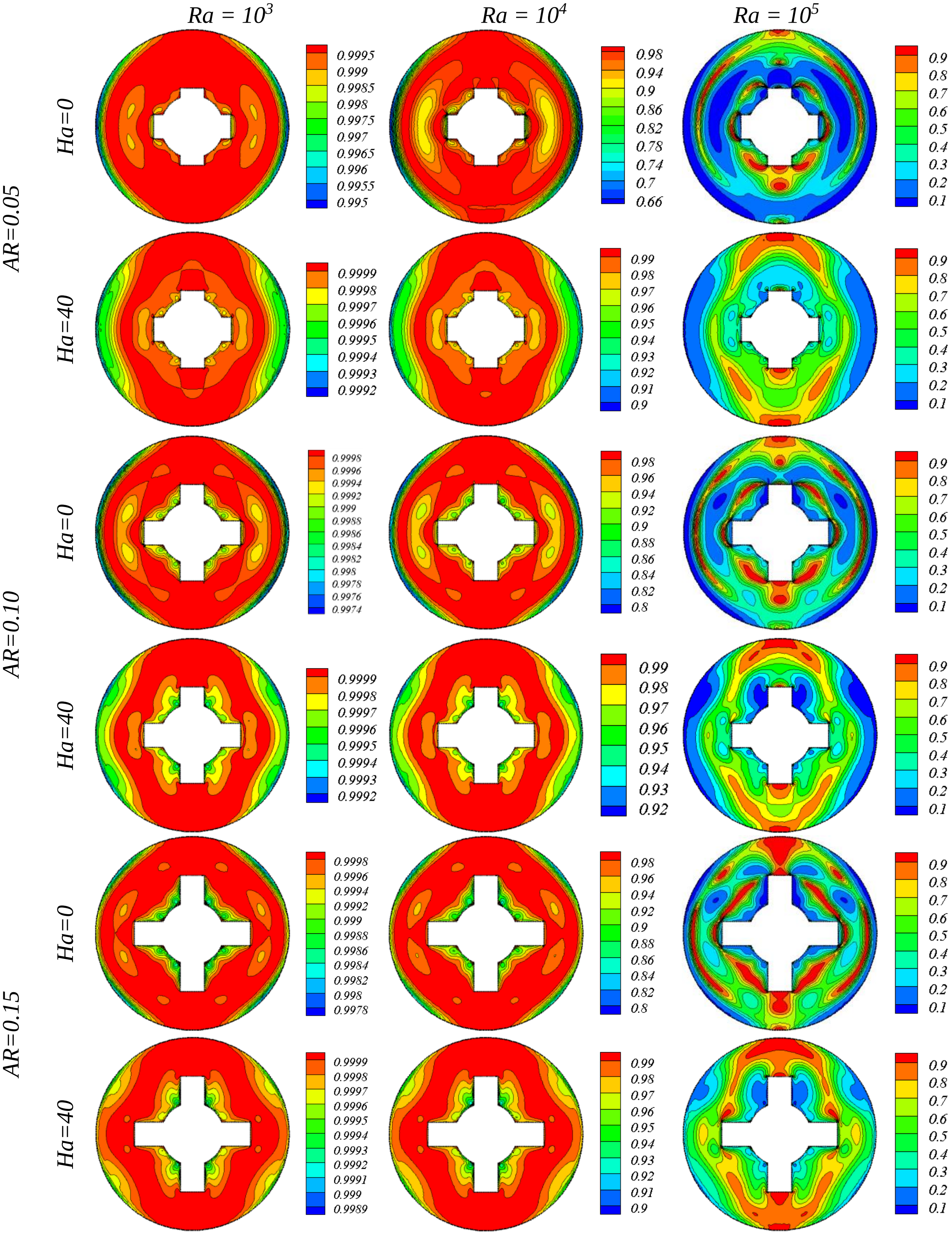

Fig. 7. Be_{loc} for for disparate values of AR, Ha, and Ra

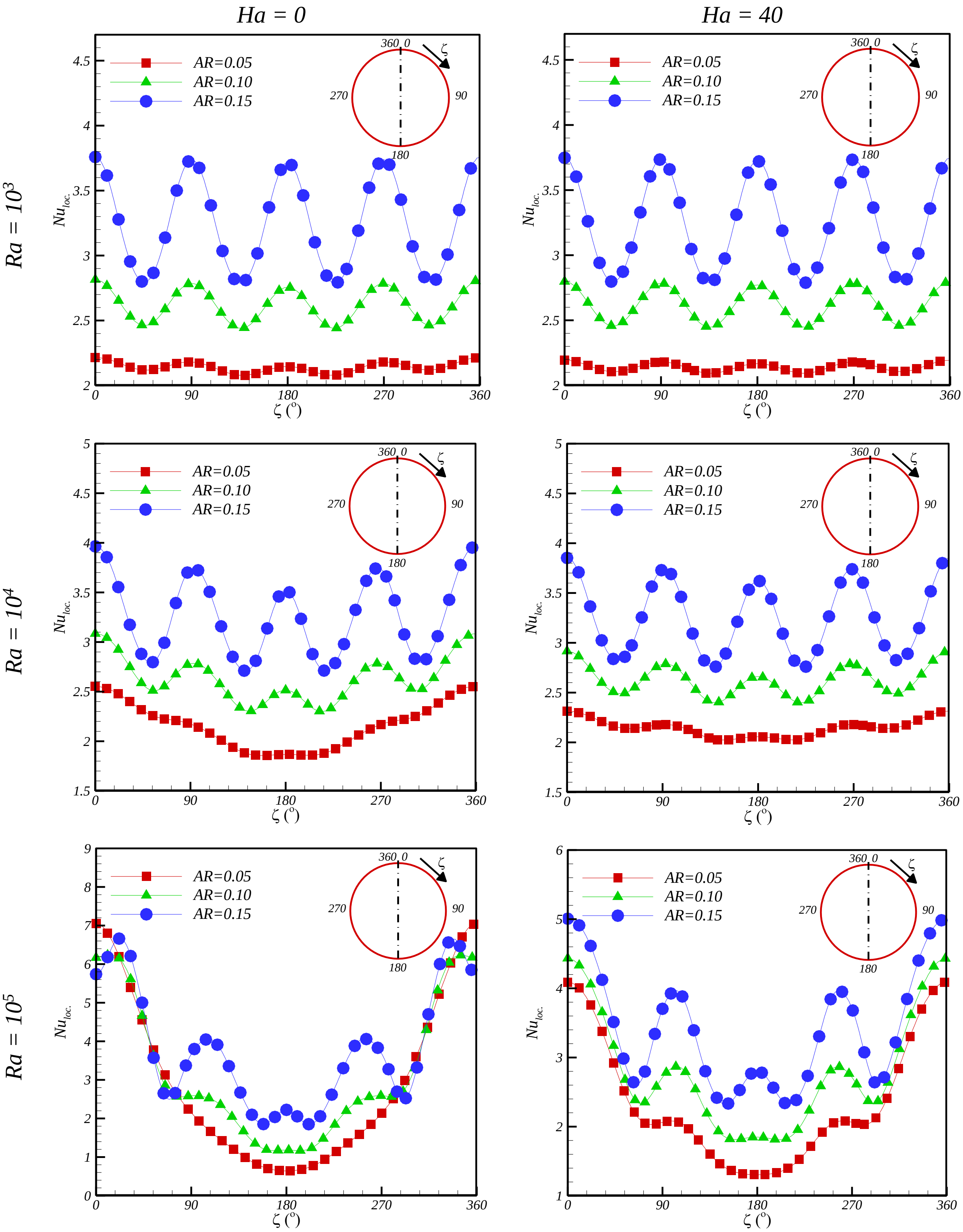

Fig. 8. Nu_{loc} for disparate values of AR, Ha, and Ra

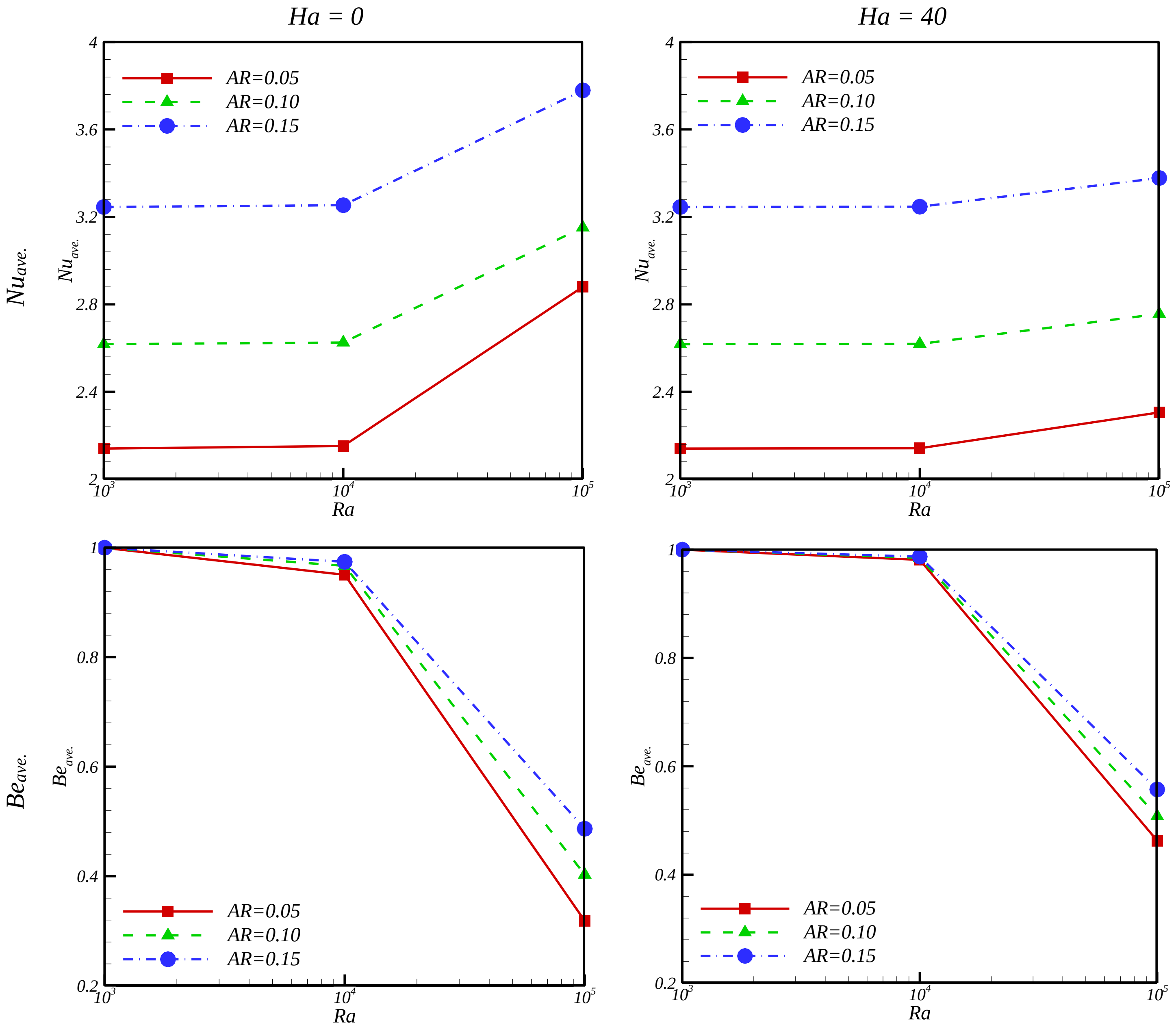

Fig. 9. Nu_{ave} and Be_{ave} for for disparate values of AR, Ha, and Ra

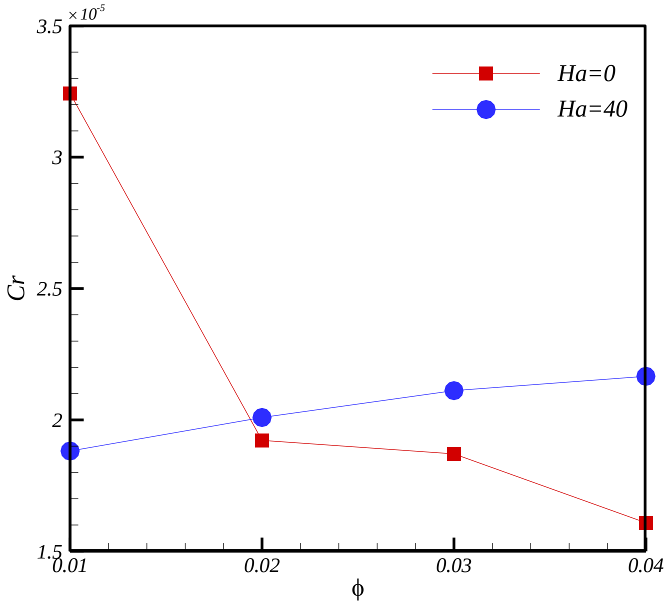

Fig. 10. *Cr* for for disparate values of ϕ and *Ha* at $Ra=10^5$

References

[1] Sheikholeslami, M., Gorji-Bandpy, M., & Ganji, D. D. (2014). Lattice Boltzmann method for MHD natural convection heat transfer using nanofluid. *Powder Technology*, *254*, 82-93.

[2] Al-Zamily, A. M. J. (2014). Effect of magnetic field on natural convection in a nanofluid-filled semi-circular enclosure with heat flux source. *Computers & Fluids*, *103*, 71-85.

[3] Sheremet, M. A., Oztop, H. F., & Pop, I. (2016). MHD natural convection in an inclined wavy cavity with corner heater filled with a nanofluid. *Journal of Magnetism and Magnetic Materials*, *416*, 37-47.

[4] Dogonchi, A. S., Sheremet, M. A., Pop, I., & Ganji, D. D. (2018). MHD natural convection of Cu/H2O nanofluid in a horizontal semi-cylinder with a local triangular heater. *International Journal of Numerical Methods for Heat & Fluid Flow*. Vol. 28 No. 12, pp. 2979-2996.

[5] Alsabery, A.I., Tayebi, T., Chamkha, A.J. and Hashim, I. (2018), Effects of two-phase nanofluid model on natural convection in a square cavity in the presence of an adiabatic inner block and magnetic field, *International Journal of Numerical Methods for Heat & Fluid Flow*, Vol. 28 No. 7, pp. 1613-1647.

[6] Dogonchi, A. S., Sheremet, M. A., Ganji, D. D., & Pop, I. J. J. O. T. A. (2019). Free convection of copper–water nanofluid in a porous gap between hot rectangular cylinder and cold circular cylinder under the effect of inclined magnetic field. *Journal of Thermal Analysis and Calorimetry*, *135*(2), 1171-1184.

[7] Sheikholeslami, M., & Rokni, H. B. (2017). Numerical modeling of nanofluid natural convection in a semi annulus in existence of Lorentz force. *Computer Methods in Applied Mechanics and Engineering*, *317*, 419-430.

[8] Rashad, A. M., Chamkha, A. J., Ismael, M. A., & Salah, T. (2018). Magnetohydrodynamics natural convection in a triangular cavity filled with a Cu-Al2O3/water hybrid nanofluid with localized heating from below and internal heat generation. *Journal of Heat Transfer*, *140*(7).

[9] Dogonchi, A. S., Nayak, M. K., Karimi, N., Chamkha, A. J., & Ganji, D. D. (2020). Numerical simulation of hydrothermal features of Cu–H 2 O nanofluid natural convection within a porous annulus considering diverse configurations of heater.

Journal of Thermal Analysis and Calorimetry, 1-17. https://doi.org/10.1007/s10973-020-09419-y

[10] Tayebi, T., & Chamkha, A. J. (2020). MHD buoyancy- driven flow in a nanoliquid filled- square enclosure divided by a solid conductive wall. *Mathematical Methods in the Applied Sciences*. https://doi.org/10.1002/mma.6598

[11] A. Bejan, A study of entropy generation in fundamental convective heat transfer, J. Heat Transfer, 101 (1979), pp. 718-725

[12] A. Bejan, Second-law analysis in heat and thermal design, Adv. Heat Transfer, 15 (1982), pp. 1-58

[13] A. Bejan, Entropy Generation Minimization, CRC press, Boca Raton, NY (1996)

[14] Selimefendigil, F., Öztop, H. F., & Abu-Hamdeh, N. (2016). Natural convection and entropy generation in nanofluid filled entrapped trapezoidal cavities under the influence of magnetic field. *Entropy*, *18*(2), 43.

[15] Hussain, S., Mehmood, K., & Sagheer, M. (2016). MHD mixed convection and entropy generation of water–alumina nanofluid flow in a double lid driven cavity with discrete heating. *Journal of Magnetism and Magnetic Materials*, *419*, 140-155.

[16] Cho, C. C., Chiu, C. H., & Lai, C. Y. (2016). Natural convection and entropy generation of Al2O3–water nanofluid in an inclined wavy-wall cavity. *International Journal of Heat and Mass Transfer*, *97*, 511-520.

[17] Chamkha, A., Ismael, M., Kasaeipoor, A., & Armaghani, T. (2016). Entropy generation and natural convection of CuO-water nanofluid in C-shaped cavity under magnetic field. *Entropy*, *18*(2), 50.

[18] Sheremet, M., Pop, I., Öztop, H. F., & Abu-Hamdeh, N. (2017). Natural convection of nanofluid inside a wavy cavity with a non-uniform heating. *International Journal of Numerical Methods for Heat & Fluid Flow*.

[19] Alsabery, A. I., Mohebbi, R., Chamkha, A. J., & Hashim, I. (2019). Impacts of magnetic field and non-homogeneous nanofluid model on convective heat transfer and entropy generation in a cavity with heated trapezoidal body. *Journal of Thermal Analysis and Calorimetry*, *138*(2), 1371-1394.

[20] Alsabery, A. I., Hashim, I., Hajjar, A., Ghalambaz, M., Nadeem, S., & Saffari Pour, M. (2020). Entropy Generation and Natural Convection Flow of Hybrid Nanofluids in a Partially Divided Wavy Cavity Including Solid Blocks. *Energies*, *13*(11), 2942.

[21] Kashani, S., Ranjbar, A. A., Mastiani, M., & Mirzaei, H. (2014). Entropy generation and natural convection of nanoparticle-water mixture (nanofluid) near water density inversion in an enclosure with various patterns of vertical wavy walls. *Applied Mathematics and Computation*, *226*, 180-193.

[22] Cho, C. C., Yau, H. T., Chiu, C. H., & Chiu, K. C. (2015). Numerical investigation into natural convection and entropy generation in a nanofluid-filled U-shaped cavity. *Entropy, 17*(9), 5980-5994.

[23] Rashad, A. M., Armaghani, T., Chamkha, A. J., & Mansour, M. A. (2018). Entropy generation and MHD natural convection of a nanofluid in an inclined square porous cavity: effects of a heat sink and source size and location. *Chinese journal of physics*, *56*(1), 193-211.

[24] Mahmoudi, A., Mejri, I., Abbassi, M. A., & Omri, A. (2014). Analysis of the entropy generation in a nanofluid-filled cavity in the presence of magnetic field and uniform heat generation/absorption. *Journal of Molecular Liquids*, *198*, 63-77.

[25] Kefayati, G. R. (2016). Simulation of heat transfer and entropy generation of MHD natural convection of non-Newtonian nanofluid in an enclosure. *International Journal of Heat and Mass Transfer*, *92,* 1066-1089.

[26] S.M. Seyyedi, A.S. Dogonchi, M. Hashemi-Tilehnoee, M. Waqas, D.D. Ganji, Entropy Generation and Economic Analyses in a Nanofluid Filled L-Shaped Enclosure Subjected to an Oriented Magnetic Field, Applied Thermal Engineering (2019), doi: https://doi.org/10.1016/j.applthermaleng.2019.114789

[27] Tayebi, T., & Chamkha, A. J. (2019). Entropy generation analysis during MHD natural convection flow of hybrid nanofluid in a square cavity containing a corrugated conducting block. *International Journal of Numerical Methods for Heat & Fluid Flow*. Vol. 30 No. 3, pp. 1115-1136.

[28] Tayebi, T., & Chamkha, A. J. (2020). Entropy generation analysis due to MHD natural convection flow in a cavity occupied with hybrid nanofluid and equipped with a conducting hollow cylinder. *Journal of Thermal Analysis and Calorimetry*, *139*(3), 2165-2179.

[29] Selimefendigil, F., & Öztop, H. F. (2020). Effects of conductive curved partition and magnetic field on natural convection and entropy generation in an inclined cavity filled with nanofluid. *Physica A: Statistical Mechanics and its Applications*, *540,* 123004.

[30] Tayebi, T., & Öztop, H. F. (2020). Entropy production during natural convection of hybrid nanofluid in an annular passage between horizontal confocal elliptic cylinders. *International Journal of Mechanical Sciences*, *171*, 105378.

[31] Hashemi-Tilehnoee, M., Dogonchi, A. S., Seyyedi, S. M., Chamkha, A. J., & Ganji, D. D. (2020). Magnetohydrodynamic natural convection and entropy generation analyses inside a nanofluid-filled incinerator-shaped porous cavity with wavy heater block. *Journal of Thermal Analysis and Calorimetry*, 141, 2033–2045.

[32] R.J. Krane, J. Jessee, Some detailed field measurements for a natural convection flow in a vertical square enclosure, Proceedings of the First ASME-JSME Thermal Engineering Joint Conference 1 (1983) 323-329.

[33] K. Khanafer, K. Vafai, M. Lightstone, Buoyancy-driven heat transfer enhancement in a two-dimensional enclosure utilizing nanofluids, International Journal of Heat and Mass Transfer 46 (2003) 3639-3653.

Conflict of Interest

Declaration of interests

☒ The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

☐The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: