Technology Entry in the Presence of Patent Thickets

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ABSTRACT

We analyze how patent thickets affect entry into patenting. A model of entry into patenting that allows for variation in technological opportunity, technological complexity and the extent of patent thickets is developed and analyzed. Using UK data we then show that patent thickets are associated with a reduction of first time patenting in a technology controlling for the level of technological complexity and opportunity. Technologies characterized by more technological complexity and opportunity attract more entry into patenting. Our evidence indicates that patent thickets raise entry costs, which leads to less entry into technologies regardless of a firm's size.

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1 Introduction

The past two decades have seen an enormous increase in patent filings worldwide (Fink, Khan, and Zhou, 2016). There are signs that the high level of patenting may be reducing innovation in certain technologies (FTC, 2003, Jaffe and Lerner, 2004, Bessen and Meurer, 2008, FTC, 2011, Schankerman and Schuett, 2016). Companies drawing on these technologies face elevated legal costs of commercializing innovative products when patents that contain overlapping claims form so-called 'patent thickets' (Shapiro, 2001). Patent thickets arise where products draw on technology protected by hundreds or even thousands of patents and these patents have fuzzy boundaries. The precision with which patent claims are formulated varies across technologies. Paradoxically claim language is quite loose in some high technology fields in which the volume of applications has been high.[1] In addition, resource constraints at patent offices have contributed to a flow of poorly delineated patents (Lei and Wright, 2017). Patents in thickets belong to many competing firms. This complicates licensing negotiations, raises the incidence of litigation, and creates incentives to add more, often weak patents to the patent system (Allison, Lemley, and Schwartz, 2015). The increased transaction costs associated with patent thickets reduce profits from commercialization of innovation, and ultimately may reduce incentives to innovate.

Empirical research on patent thickets has been largely concerned with showing that they exist and measuring their density (Ziedonis, 2004, Graevenitz, Wagner, and Harhoff, 2011). There is less evidence on the effects patent thickets have on firms' objectives. Cockburn and MacGarvie (2011) demonstrate that patenting levels affect product market entry in the software industry. This result echoes earlier findings by Lerner (1995) who showed that first-time patenting in a given technology is affected by the presence of other companies' patents in a small sample of U.S. biotech companies. Both papers use patent counts in narrow technological fields to measure thickets. In this paper we use a network measure of patent overlap by technology area as a proxy for thickets. The measure is correlated with increased patenting (Graevenitz, Wagner, and Harhoff, 2013), increased acquisition of patents by Non-Practicing Entitites (NPEs) (Fischer and Henkel, 2012) and a lower likelihood of patent opposition proceedings (Harhoff, von Graevenitz, and Wagner, 2016).

Bessen and Meurer (2013) argue that patent thickets will lead to increased litigation due to hold-up. They use the term to describe a situation where an alleged infringer faces the threat of an injunction or high licensing costs after she has sunk investment.[2] Patent thickets

[1] Allison, Lemley, and Schwartz (2015) document that for the population of patents for which litigation was initiated in 2008 and 2009 in the U.S., none or very few failed due to indefiniteness in Mechanical Engineering, Biotechnology or Chemistry, whereas this was true for nearly a third of cases in Electronics and a quarter in Software. Bessen and Meurer (2008) argue that language used to specify patent boundaries in Chemistry and Biology is more scientific than that used for software patents. Allison and Ouellette (2015) study all cases since 1982 in the U.S. decided on basis of claim indefiniteness. They find that patents from the Computer/Electronics industry failed on the basis of enablement more frequently than other industries. Enablement is the requirement that a patent must 'teach one skilled in the art to make and use' an invention (Burk and Lemley, 2008).

[2] 'High licensing costs' refers to costs that are higher than those that would have been negotiated *ex ante* in the

have remained a concern of antitrust agencies and regulators in the U.S. for over a decade (FTC, 2003, USDoJ and FTC, 2007, FTC, 2011). Reforms that address some of the factors contributing to the growth of patent thickets have recently been introduced in the U.S. (America Invents Act of 2011) and by the European Patent Office (EPO).

Another perspective is provided by authors who argue that patent thickets are a feature of rapidly developing technologies in which technological opportunities abound (Teece, 2018). Here thickets are a reflection of fast technological progress that is paired with increased technological complexity (Lewis and Mott, 2013). Increased transaction costs associated with patent thickets and the benefits of technological complexity and opportunity often coincide. There may be a trade-off between technological opportunity and growth on the one hand and increased transaction costs due to the emergence of patent thickets on the other - if the transaction costs of patenting in complex technologies are not avoidable. The challenge in assessing technologies with high levels of patenting is to develop a framework that captures the main factors that incentivize patenting and the costs and benefits thereof.

This paper focuses on entry into patenting across a wide range of industries. This is a focal outcome that has been analysed in specific industries (Lerner, 1995, Cockburn and MacGarvie, 2011). We make two contributions to this literature: first, we introduce a model to show how patent thickets, technological opportunity and complexity interact to determine levels of entry and second, we test predictions derived from the model using firm-level data on entry into patenting by firms in the United Kingdom.

We model how entry decisions are affected by technological opportunity and legal uncertainty over patent boundaries building on previous work by Graevenitz, Wagner, and Harhoff (2013). The model focuses on the interaction between firms through two channels: (i) legal costs associated with patent enforcement and (ii) incumbency advantages in R&D fixed costs. In contrast to Graevenitz, Wagner, and Harhoff (2013), we distinguish between technological complexity *per se*, which is a feature of some technologies, and patent thicket density, which arises from poor drafting of patents in a complex technology. Poor drafting increases transaction costs for firms. Specifically, transaction costs may rise due to actual hold-up, or through higher costs of licensing and greater complexity of clearing products when patent breadth is uncertain. We refer to all three as hold-up potential. Our model shows that patent thickets reduce entry into patenting.[3] The model also shows that higher complexity and opportunity are associated with increased entry into patenting, because competition for each innovation is reduced and the probability that entrants can establish themselves in a technology is increased. Where incumbency implies lower costs of R&D, incumbents patent more than entrants.

These predictions are tested empirically using data from the UK. We quantify of the importance of technological opportunity, complexity, and patent thickets on entry into patenting. To

presence of possible 'invent around' before the alleged infringer sank her investment. This possibility can arise because of either prohibitive search costs or fuzzy patent boundaries or both (Mulligan and Lee, 2012).

[3]The model generates the same comparative statics for patent application *levels* as Graevenitz, Wagner, and Harhoff (2013).

do this, separate measures of technological opportunity, technological complexity, and hold-up potential due to thickets are constructed and validated. Separating technological complexity and hold-up potential in patent thickets empirically is an important improvement over the analysis in Graevenitz, Wagner, and Harhoff (2013), who conflated complexity and hold-up potential arising from existing patent portfolios. We introduce a new measure of technological complexity that relies on U.S. patent data to mitigate endogeneity concerns and we sharpen the definition of the network measure of patent overlap as a proxy for hold-up potential.[4]

The analysis of entry in this paper confirms that greater technological opportunity and complexity *increase* entry and that hold-up potential *reduces* entry substantially. We show that these findings are robust to various assumptions underlying our empirical approach. While we cannot quantify the overall net welfare effect, our results indicate that patent thickets raise entry costs for large and small firms alike. This is true regardless of any positive effects that arise from greater technological opportunity and complexity. To the extent that more original and radical, rather than incremental ideas come from new entrants rather than incumbents (Tushman and Anderson, 1986, Henderson, 1993), reduced entry is likely to have negative long-run consequences on innovation and product market competition. In combination with earlier results by Graevenitz, Wagner, and Harhoff (2013), who point to a positive correlation between patenting levels and the presence of thickets, our results suggest that any increases in transaction costs due to thickets can potentially have important dynamic effects on innovation.

The remainder of this paper is organized as follows. Section 2 presents a model of entry into patenting in a technology area and derives several testable predictions. Section 3 describes the data, and the empirical measurement of the key concepts in the model. Section 4 discusses our results and Section 5 provides concluding remarks.

2 Theoretical Model

This section summarizes results of a model of entry into patenting.[5] We show how firms' decisions to enter into patenting depend on (i) complexity of a technology, (ii) technological opportunity and (iii) the potential for hold-up in patent thickets. The model has two stages: entry and patenting. The patenting stage generalizes analysis in Graevenitz, Wagner, and Harhoff (2013).[6] Here we focus on novel predictions derived from free entry that are then tested in Section 4. We solve the model by backward induction. Main results on entry into patenting are that greater technological opportunity and complexity increase entry, while the threat of

[4]To further validate our approach, we verify the effect of distinguishing between technological complexity and hold-up in the data used by Graevenitz, Wagner, and Harhoff (2013) for their analysis. The results, which are reported in Appendix D, are consistent with our interpretation of the patents thickets measure as a measure of hold-up potential, and of the citation network density as a measure of complexity.

[5]Details are relegated to Appendices A and B.

[6]We generalize their model to allow analysis of entry. Their main findings on patenting levels still hold. For sake of brevity we relegate analysis on levels of patenting to the appendix.

increased legal costs in patent thickets reduces entry.

In the model a technology consists of a set of opportunities, each of which consists of a number of patentable 'facets'. Opportunities within a technology share the same number of facets, while complexity of the technology is determined by the count of facets per opportunity. More opportunity within a technology attracts entrants as more avenues arise to earn a profit through application of the technology. Greater complexity of a technology also attracts entrants, because entrants are more likely to gain a share of profits flowing from opportunities. Where multiple firms hold patents on the same opportunity, licensing negotiations or litigation ensue as firms divide the profits flowing from the opportunity. We assume that holding a larger share of patents on an opportunity is beneficial for firms in terms of licensing or litigation, but less so when thickets arise from poorly delineated patents that provide increased options to litigate. This captures the costs imposed by thickets on patentees.

2.1 Notation and Assumptions

The key variables of the model are the complexity of a technology k, measured by $\mathrm{F_k}$ ($F_k \in \mathbb{R}_0^+$), the degree of technological opportunity, measured by O_k ($O_k \in \mathbb{R}_0^+$), and hold-up potential h_k. The value of all $\widetilde{F}_k$ patents granted in an opportunity is V_k. In the simplest discrete setting this is the value of the one patent (facet) that covers each technological opportunity. In complex technologies this is the value of owning rights to use all patents (facets) granted for a technological opportunity. Firms (indexed by i) choose the number of opportunities o_i to invest in and the number of facets f_i per opportunity to seek to patent.

In equilibrium only $\tilde{F}_k = (1 - (1 - \hat{f}_k/F_k)^{N_o+1})$ facets are patented,[7] where $\hat{f}_k$ is the equilibrium number of facets chosen by applicants and N_O is the number of firms that applied for patents on a specific opportunity.[8] As $\widetilde{F}_k$ may be smaller than F_k the total value of patenting in a technology is $V_k(\widetilde{F}_k) \leq V_k(F_k)$.

To simplify the modeling of simultaneous patenting of facets on multiple opportunities we assume that firms choose how many opportunities o_i and facets f_i to invest in. Which subset of facets per opportunity each firm invests in is random. The allocation of a facet among the firms seeking to patent it is also random. Then probability p_i that a facet is allocated to firm i is:[9]

$$p_i(\mathbf{f}_{i\!\!/}, F_k, N_O(O_k, \mathbf{o}_{i\!\!/}, N)) = \sum_{j=0}^{N_O} \frac{1}{j+1} \binom{N_O}{j} \prod_{l=0}^{N_O-j} \left(1 - \frac{f_l}{F_k}\right) \prod_{m=N_O-j}^{N_O} \frac{f_m}{F_k} \quad . \tag{1}$$

where $\mathbf{f}_{i\!\!/}, \mathbf{o}_{i\!\!/}$ are vectors containing the choices of the number of facets and the number of opportunities to invest in, made by all rival firms j. The expected number of patents a firm owns when it applyies for f_i facets is $\gamma_i \equiv p_i f_i$.

[7] See Appendix A.3 for more details.

[8] The properties of N_0 are summarized in Appendix A.2.

[9] See Appendix A.1.

Profits of firm i patenting technology k, π_{ik}, increase in the share of patents the firm owns per opportunity s_{ik}, where $s_{ik} \equiv p_i f_i / \tilde{F}_k$. Profits are concave in this share through $\Delta(s_{ik})$, capturing the decreasing marginal benefit of patent portfolio size in complex technologies.

In sum, the assumptions we make on the value function and portfolio size benefits are:

$$\text{(VF): } V_k(0) = 0, \ \frac{\partial V_k}{\partial \tilde{F}_k} > 0; \tag{2}$$

$$\text{(PB): } \Delta(0) = 0, \ \frac{d\Delta(s_{ik})}{ds_{ik}} > 0 \text{ and } \frac{d^2\Delta(s_{ik})}{d^2 s_{ik}} < 0\,. \tag{3}$$

The model contains three types of patenting costs:

- R&D costs per opportunity, a function of total R&D activity per opportunity: $C_o(\sum_j^{N_o} o_j)$;
- maintaining each granted patent in force: C_a;
- coordinating R&D on *different* technological opportunities $C_c(o_i)$, where $\frac{\partial C_c}{\partial o_i} > 0$.

These assumptions imply that R&D costs are fixed costs.[10] We allow for the endogenous determination of the level of R&D fixed costs, which rise as more opportunities are researched simultaneously by rival firms. This reflects competition for inputs into R&D, e.g. scientists and engineers that are in fixed supply in the short run (Goolsbee, 1998).

Where multiple firms own facets on an opportunity, their legal costs $L(\gamma_i, s_{ik}, h_k)$ depend on the absolute number of patented facets γ_i, on the share of patents per opportunity that a firm holds s_{ik}, and on the extent to which they face hold-up h_k. The first two channels capture the costs of defending a patent portfolio as the number of patents increases, while leaving scope for effects on bargaining costs that derive from the share of patents owned: The hold-up parameter captures contexts in which several firms' core technologies become extremely closely intertwined. Then each firm has to simultaneously negotiate with many others to commercialize its products, which significantly raises transaction costs.

$$\begin{aligned} \text{(LC): } L(\gamma_i, s_{ik}, h_k), \text{ where } & \frac{\partial L}{\partial \gamma_i} > 0, \frac{\partial^2 L}{\partial {\gamma_i}^2} \geq 0, \frac{\partial L}{\partial s_{ik}} \leq 0, \frac{\partial^2 L}{\partial {s_{ik}}^2} \geq 0, \\ & \frac{\partial L}{\partial h_k} > 0, \frac{\partial^2 L}{\partial \gamma_i \partial h_k} > 0, \frac{\partial^2 L}{\partial s_{ik} \partial h_k} > 0 \quad . \end{aligned} \tag{4}$$

All remaining cross partial derivatives of the legal costs function are zero.

In what follows, we use the following definitions:

$$\omega_{ik} \equiv \frac{o_i}{O_k}, \quad \phi_{ik} \equiv \frac{f_i}{F_k}, \quad \mu_k \equiv \frac{\tilde{F}_k}{V_k(\tilde{F}_k)} \frac{\partial V_k(\tilde{F}_k)}{\partial \tilde{F}_k}, \quad \xi_{ik} \equiv \frac{s_{ik}}{\Delta(s_{ik})} \frac{\partial \Delta(s_{ik})}{\partial s_{ik}}, \quad \eta_{ik} \equiv \frac{f_i}{\tilde{F}_k} \frac{\partial \tilde{F}_k}{\partial f_i} \quad . \tag{5}$$

[10]It also implies that there is no technological uncertainty. Introducing technological uncertainty into the model does not change the main comparative statics results.

Here ω_{ik} is the share of opportunities each firm chooses to pursue, ϕ_{ik} is the share of facets each firm seeks to patent per opportunity, μ_k is the elasticity of the value function with respect to the level of complexity, ξ_{ik} is the elasticity of the benefits function Δ with respect to the share of patents each firm is granted and η_{ik} is the elasticity of the number of covered facets with respect to the number of patent applications of each firm.

2.2 Patenting and Entry

Firm *i*'s profits in technology *k*, $\pi_{ik}(o_i, f_i, F_k, O_k, N_k, h_k)$, are a function of the number of opportunities o_i which the firm invests in, the number of facets per opportunity f_i the firm seeks to patent, the total number of patentable facets per opportunity F_k, the number of technological opportunities a technology offers O_k, the number of firms entering the technology N_k, and the degree of hold-up in that technology h_k.

In this section we analyze the following two-stage game *G**:

Stage 1: Firms enter until $\pi_{ik}(o_i, f_i, F_k, O_k, N_k, h_k) = 0$;[11]

Stage 2: Firms simultaneously choose the number of opportunities, o_i, to invest in and the number of facets per opportunity f_i to patent in order to maximize profits π_{ik}.

We solve the game by backward induction and derive local comparative statics results for the symmetric extremal equilibria of the second stage game. For the subsequent analysis it is important to note that all equilibria of this second stage game are symmetric. In case that the second stage game has multiple equilibria we focus on the properties of the extremal equilibria when providing comparative statics results (Milgrom and Roberts, 1994, Amir and Lambson, 2000, Vives, 2005).

At stage two of the game each firm maximizes the following objective function:

$$\pi_{ik}(o_i, f_i) = o_i\left(V_k(\tilde{F}_k)\Delta(s_{ik}) - L(\gamma_i, s_{ik}, h_k) - C_o(\sum_j^{N_o} o_j) - f_i p_i C_a\right) - C_c(o_i) \quad . \quad (6)$$

This expression shows that per opportunity *k*, the firm derives profits from its share s_{ik} of patented facets, while facing legal costs *L* to appropriate those profits, as well as costs of R&D C_0, costs of maintaining its patent portfolio C_a, and coordination costs across opportunities C_c.

This objective function generalizes that analyzed by Graevenitz, Wagner, and Harhoff (2013). They assume that the value of patenting increases linearly in the share of patents the firm owns per opportunity ($\Delta(s_{ik}) = s_{ik}$) and do not allow for a direct effect of hold-up (h_k) on legal costs. Under free entry the model in Graevenitz, Wagner, and Harhoff (2013) does not have a solution, while the model developed here does. Generalizing the objective function also has direct

[11] N_k is the superset of all firms applying for patents within all opportunities of technology k. We treat N_k and the N_O as a continuous variables to simplify analysis of the model.

implications for an empirical test of the theory: separate measures of complexity and hold-up are required. The measures we employ in our empirical analysis are discussed in Section 3.

2.3 Simultaneous Entry with Multiple Facets

In Appendix B we show that the results derived by Graevenitz, Wagner, and Harhoff (2013) for patenting hold in our generalized model. This section summarizes new results on entry.

Comparative statics of entry

In Appendix B.4 we show that there is a free entry equilibrium. In this equilibrium the following propositions hold:

Proposition 1
Under free entry greater complexity of a technology increases entry.

Complexity has countervailing effects: first, it increases profits, because it is less likely that duplicative R&D arises making each opportunity more valuable; this clearly increases incentives to enter. Next, given the level of patent applications $\hat{f}_k$, complexity reduces the probability that each facet is patented, which reduces profits and entry incentives. Finally, complexity reduces competition for each facet, which increases the probability of patenting and increases innovation incentives. It is shown that the positive effects outweigh the negative effects.

First, consider how equilibrium profits are affected by the complexity of the technology F_k, the degree of technological opportunity O_k, and the potential for hold-up h_k:[12]

$$\frac{\partial \hat{\pi}_k(\hat{o}_k,\hat{f}_k)}{\partial F_k} = \hat{o}_k \frac{\hat{s}_k}{F_k}\left(\left(\hat{\varepsilon}_{\tilde{F}_k,F_k} - \hat{\varepsilon}_{p_k,F_k}\hat{\eta}_k\right) \overbrace{\left[V_k\left(\hat{\tilde{F}}_k\right)\frac{\triangle(\hat{s}_k)}{\hat{s}_k}\left(\hat{\mu}_k - \hat{\xi}_k\right) + \frac{\partial L}{\partial \hat{s}_k}\right]}^{\Lambda} \right) > 0 \quad (7)$$

$$\frac{\partial \hat{\pi}_k(\hat{o}_k,\hat{f}_k)}{\partial O_k} = \hat{o}_k \frac{\partial \hat{N}_O}{\partial O_k}\frac{\hat{s}_k}{\hat{N}_O}\left(\left(\hat{\varepsilon}_{\tilde{F}_k,N_O} - \hat{\varepsilon}_{p_k,N_O}\hat{\eta}_k\right)\Lambda - \frac{\partial C_o}{\partial \hat{N}_O \hat{o}}\frac{\hat{N}_O\hat{o}}{\hat{s}_k}\right) > 0 \quad (8)$$

$$\frac{\partial \hat{\pi}_k(\hat{o}_k,\hat{f}_k)}{\partial h_k} = -\hat{o}_k\frac{\partial L}{\partial h_k} < 0 \quad (9)$$

Proposition 3 follows from the Implicit Function theorem once we know the sign of the derivative of profits with respect to F_k. Under free entry firms' profits decrease with entry:

$$\frac{\partial N_k}{\partial F_k} = -\frac{\frac{\partial \hat{\pi}_k}{\partial F_k}}{\frac{\partial \hat{\pi}_k}{\partial N_k}} \quad (10)$$

[12]Equilibrium values of the firms' choices are denoted by a hat (^) and we drop firm specific subscripts, e.g. $\hat{\phi}_k$. We define $\Lambda \equiv \left[V_k\left(\hat{\tilde{F}}_k\right)\frac{\triangle(\hat{s}_k)}{\hat{s}_k}\left(\hat{\mu}_k - \hat{\xi}_k\right) + \frac{\partial L}{\partial \hat{s}_k}\right]$ to simplify expressions.

and the sign of the effect of complexity F_k on entry depends on the sign of the effect of complexity on profits.

Equation (7) shows that the effect of complexity on profits depends on the difference between the elasticities $\hat{\varepsilon}_{\tilde{F}_k,F_k}$ and $\hat{\varepsilon}_{p_k,F_k}\hat{\eta}_k$, which are derived in Appendices A.1 and A.3. Specifically, $\hat{\varepsilon}_{p_k,F_k}$ is shown to be:

$$\hat{\varepsilon}_{p_k,F_k} = \hat{N}_O^2 \frac{\hat{\phi}_k - \frac{1}{2}\left(1 + \frac{1}{\hat{N}_O}\right)}{1 - \hat{\phi}_k} \tag{11}$$

This elasticity is negative for $\hat{\phi}_k < \frac{1}{2}$, which is also a precondition for supermodularity of game G*. We find that both terms in brackets in Equation (7) are positive, when game G* is supermodular. This implies that greater complexity raises profits and this induces entry.[13]

Proposition 2

Under free entry greater technological opportunity increases entry.

For any given number of entrants an increase in technological opportunity reduces competition between firms for patents. This increases firms' expected profits and increases entry.

Continuing from the proof of Proposition 1 above, by the Implicit Function theorem the sign of the derivative of profits with respect to technological opportunity determines the effect of technological opportunity on entry:

$$\frac{\partial N_k}{\partial O_k} = -\frac{\frac{\partial \hat{\pi}_k}{\partial O_k}}{\frac{\partial \hat{\pi}_k}{\partial N_k}} \tag{12}$$

An increase in technological opportunity increases profits and entry. In Appendix B.4 we show that the term in brackets in Equation (8) is negative under free entry and that $\frac{\partial N_O}{\partial O_k} < 0$. Profits increase as technological opportunity increases, as entry per opportunity falls.

Proposition 3

Under free entry the potential for hold-up reduces entry.

An increase in the potential for hold-up raises firms' expected legal costs. This reduces expected profits and lowers potential for entry. To derive this prediction, note that by the Implicit Function theorem the sign of the derivative of profits with respect to the level of hold-up in a technology area determines the effect of hold-up on entry:

$$\frac{\partial N_k}{\partial h_k} = -\frac{\frac{\partial \hat{\pi}_k}{\partial h_k}}{\frac{\partial \hat{\pi}_k}{\partial N_k}} \tag{13}$$

[13] When $\hat{\phi}_k \geq \frac{1}{2}$ game G* is no longer supermodular. This situation corresponds to the case where one firm has more than half the patents in a particular technology opportunity within a technology area. Thus our results may not hold when a specific opportunity is highly concentrated. In general this will not be the case, especially at our level of empirical analysis, but it would be interesting to explore this possibility in future work.

Hence, Equation (9) shows that the effect of hold-up on entry derives from the increased legal costs that the possibility of hold-up imposes on affected firms.

2.4 Entry and Incumbency

In our model firms' decisions on entry are simultaneous, which is motivated by a focus on first-order effects as in the literature on excess entry (Mankiw and Whinston, 1986, Suzumura and Kiyono, 1987). Our purpose is to make predictions across a wide range of patenting industries, which we can do without recourse to data on product market outcomes. While this means that we cannot analyze dynamic evolution of patent thickets or sequential entry, we can allow for asymmetries between firms.

In Appendix B.6 we extend the model to asymmetric equilibria, in which some firms (incumbents), face lower costs ($C_O - \Psi$, where $\Psi > 0$) of entering opportunities. This way we model observable heterogeneity in the experience of doing R&D in a technology area. The main results derived above are robust to this variant of the model. In addition, we show that more experienced incumbents enter more opportunities, crowding out new entrants.

2.5 Predictions of the Model

Here we summarize the predictions of the model that we test empirically:[14]

Prediction 1: *The probability of entry increases in technological opportunity.*

Greater technological opportunity reduces competition for facets per opportunity, which raises expected profits and thereby attracts entry.

Prediction 2: *The probability of entry increases in complexity of a technology.*

Greater complexity has countervailing effects: it reduces competition per facet as well as duplicative R&D, attracting entry. It also increases the likelihood that some of a technology remains unpatented, reducing its overall value and entry. Our model shows that overall complexity increases entry.

Prediction 3: *The probability of entry falls in the potential for hold-up.*

Hold-up potential increases expected costs of entry, thereby reducing it.

Prediction 4: *More experienced incumbents are more likely to enter technological opportunities new to them.*

We show that incumbency advantage raises the number of opportunities that incumbents enter. This implies that they also enter new opportunities, which they have not previously been active in. This expansion of activity by incumbents crowds out entry by new firms.

[14] Note that Graevenitz, Wagner, and Harhoff (2013) test predictions from a more restrictive version of the model on the *level* of patent applications using data from the EPO. We replicate their analysis in Appendix D using additonal variables suggested by the generalized model we present here.

3 Data and Empirical Model

Our empirical model is a hazard rate model of firm entry into patenting in a technology area as a function of the technological opportunity, technological complexity, and hold-up potential that characterize a technology area. Additional firm level covariates include the age, size, and prior patenting history, and the concentration of their 4-digit industry. The models we estimate are stratified at both the firm and industry level. That is, the unit of observation for each entry hazard is a firm-technology area, but the hazard shapes and levels are allowed to vary either by firm or by the industry containing the firm. This approach recognizes that patenting propensities vary across firms and industries for reasons that may not be technological (e.g., strategic reasons, or reasons arising from the historical development of the sector).

We use firm-level data for the entire population of UK firms registered with Companies House and data on patenting at the EPO and at the UK Intellectual Property Office (UKIPO). The firm data come from the data held at Companies House provided by Bureau van Dijk in their FAME database. The patent data were linked to firm register data by matching applicant names in patent documents and firm names in firm registers (see Appendix C for details).

Economic studies of entry are frequently hampered by the problem of identifying the correct set of potential entrants (Bresnahan and Reiss, 1991, Berry, 1992). In our case this problem is slightly mitigated by the fact that one set of potential entrants into patenting in a specific technology area consists of those firms that currently patent in other technology areas. We complement this group of firms with a set of comparable firms from the population of UK firms that had not patented previously.

To construct the sample we deleted all firms from the data for which we have no size measure because of missing data on assets. We select previously non-patenting firms from the population of all UK firms in two steps: 1) we delete all firms in industrial sectors with little patenting (amounting to less than 2% of all patenting); and 2) we choose a sample of non-patenting firms that matches our sample of patenting firms by industry, size class, and age class. This approach results in an endogenous (choice-based) sample at the firm level. The focus of our work is on industry and technology area level effects rather than firm-level effects. Therefore we do not expect this sampling approach to introduce systematic biases into the estimates we report. We provide a number of robustness checks, including aggregate instrumental variable regressions.

All estimates are based on data weighted by the probability that a firm is in our sample.[15] The sample that results from our selection criteria is a set of firms with non-missing assets in manufacturing, oil and gas extraction and quarrying, construction, utilities, trade, and selected business services including financial services that includes all (approximately 11,000) firms applying for a patent at the EPO or UKIPO during the 2001-2009 period and another 11,000

[15]To check this, we estimated the model with and without weights based on our sampling methodology and find little difference in the results.

firms that did not apply for a patent.

The definition of technology areas that we use is based on the 2008 version of the ISI-OST-INPI technology classification, denoted TF34 classes (Schmoch, 2008). The list is shown in Table C-1 in the appendix, along with the number of EPO and UKIPO patents that were matched to UK firms with priority dates between 2002 and 2009. A comparison of the frequency distribution of patenting across technology areas from the two patent offices shows that firms are more likely to apply for patents in Chemicals at the EPO, while Electrical and Mechanical Engineering predominate in the UK patent data (see the bottom panel in Table C-1).

We treat entry into each technology area as a separate decision made by firms. More than half of firms we observe patent in more than one area and 10% patent in more than four. From the 22,000 firms observed, each of which can potentially enter into each one of the 34 technology areas, we obtain about 550,000 observations at risk.

We cluster the standard errors by firm, so our models are effectively firm random effects models for entry into 34 technology areas. Allowing firm choices to vary by technology area is sensible under the assumption that firms' patenting strategies are contingent upon technology and industry level factors and are not homogeneous across technology areas.[16]

There are some technology-industry combinations that do not occur, e.g. audio-visual technology and the paper industry, telecommunications technology and the pharmaceutical industry. In order to reduce the size of the sample, we drop all technology-industry combinations for which Lybbert and Zolas (2014) find no patenting in their data and for which there was no patenting by any UK firm from the relevant industry in the corresponding technology category. This removes about 30% of observations from the data. We provide a robustness check for this procedure in Table E-2 in the Appendix.

3.1 Variables

Dependent Variable - Entry The dependent variable is a dichotomous variable taking the value one if a firm has entered a technology area k at time t and otherwise the value zero. Entry into a technology area is measured by the first time a firm applies for a patent that is classified in that technology area, dated by the priority year of the patent.

Technological opportunity Our first prediction from the theoretical model is that there will be more entry in technology areas with greater technological opportunity.

Opportunity to generate inventions can arise from the recombination of conventional knowledge, or it can arise from a mixture of conventional and atypical knowledge (Uzzi et al., 2013). We use two measures of opportunity, the first to capture opportunity arising from conventional knowledge and the second, to capture opportunity arising from the introduction of atypical knowledge:

[16]We confirmed the validity of this assumption through interviews with leading UK patent attorneys.

1. Opportunity for recombination of conventional knowledge is measured through the logarithm of the aggregate EPO patent applications in the technology sector in a given year.

2. Opportunity from the introduction of atypical knowledge is measured through the past 5-year growth rate in the non-patent (scientific publication) references cited in patents in a technology class at the EPO.[17]

Given the difficulty of measuring technological opportunity we note that the growth rate in non-patent references is a better predictor of entry than the level of non-patent references, which has been used previously to measure technological opportunity. Presumably the growth rate is a better predictor because it captures new or expanded technological opportunity coming from recent scientific work.

The first measure of opportunity is quite broad and may be correlated with other influences on entry. Our model of patenting predicts that aggregate patenting and entry are functions of technological opportunity, complexity and patent thickets. We control for the effect of complexity, patent thickets and science derived opportunity, so that the coefficient on aggregate patenting will reflect primarily variation in the remaining, conventional knowledge dimension of opportunity.

Technological complexity The second prediction of the theoretical model is that technological complexity increases entry, other things equal. Technology is complex when there are many ways to combine inventions in a particular field to obtain novel applications of these inventions. The opposite, a discrete technology is characterized by a series of fairly isolated inventions that do not connect to each other. To construct a measure of complexity, we use the concept of network density applied to all citations among patents that issued in the particular technology area during the decade prior to the date of potential entry. We use citations at the U.S. patent office, because these are richer (averaging 7 cites per patent during this period versus 3 for the EPO) and to minimize correlation with the thickets measure, which is based on EPO data.[18]

The network density measure is computed as follows: in any year t, there are N_{kt} patents that have been applied for in technology area k between years $t\text{-}10$ and t. Each of these patents can cite any of the patents that were applied for earlier, which implies that the maximum number of citations within the technology area is given by $N_{kt}(N_{kt}\text{-}1)/2$. We count the actual number of citations made and normalize them by this quantity, scaling the measure by one million for visibility, given its small size.

In any given cohort of new patents this measure captures how intensively innovations introduced by the new patents are linked to preceding innovations. The measure contains no information about whether these links indicate overlap in the innovations claimed by patent

[17] See Graevenitz, Wagner, and Harhoff (2013) for a more extensive discussion of this variable in the literature.

[18] It is important to emphasize that citations listed on U.S. patents are largely proposed by the applicant, whilst the citations listed on EPO and UKIPO patents are inserted by the examiner. This explains why the two measures are not highly correlated.

holders or not. This additional information is contained in the EPO classification of citations and is exploited in the patent thicket measure we discuss next.

Patent thickets The third prediction of the model is that greater potential for hold-up reduces entry. We measure the potential for hold-up in patent thickets using the total triples count per technology area, as previously used by Harhoff, von Graevenitz, and Wagner (2016). The triples count is the number of fully connected triads on the set of firms' critical patent references. At time *t* a unidirectional link between two firms *A* and *B* corresponds to one or more critical references to firm *A*'s patents in the set of patents applied for by firm *B* in the years *t, t-1* and *t-2*. These critical references, so-called X- and Y-references, are obtained from examiner search reports issued by the EPO and represent prior art that calls into question novelty and/or the inventive step of the patent application under examination. Triples are then formed by groups of three firms where each firm has at least one patent that is cited as critical prior research for at least one patent held by each of the other two firms. That is, in a triple, each firm holds patents that potentially block the other firms' patents creating mutually blocking triads. This indicator captures instances in which European patent examiners have identified poorly drafted claims that indicate each firm in the triple is claiming technology already claimed by the other firms in the triple. In the instances in which the examiners identify overly expansive claims, these can be re-drafted. But examiners are unlikely to spot overlapping claims in all patent applications in a technology area due to constraints on their time and ability to search for prior art. Moreover, the re-drafting of claims flagged by examiners, which often involves adding specific language to narrow the scope of claims, is unlikely to eliminate all potential overlap between the relevant patents. Therefore a higher triples count in a technology area indicates the existence of overlapping technologies and the patents that cover them, and hence an increase in hold-up potential in this technology area.[19]

The citation data used to construct this measure is extracted from PATSTAT (October 2011 edition).[20] We normalize the count of triples by aggregate EP patenting in the same technology class and year, so that the triples variable represents the intensity with which firms potentially hold blocking patents on each other relative to aggregate patenting activity in the technology.[21]

By adding a measure of technological complexity to our model we can interpret the triples count more narrowly than Graevenitz, Wagner, and Harhoff (2013), who used it as a proxy for complexity and hold-up potential together.[22] In contrast, our model separates the effect of

[19]Note that Fischer and Henkel (2012) find that NPEs are more likely to acquire patents in fields with higher triple count, providing additional support for the notion that the measure captures patent overlap and hold-up potential.

[20]Triples data was kindly provided by Harhoff, von Graevenitz, and Wagner (2016).

[21]As a robustness check, we have also explored the use of duples, i.e. the count of mutual blocking relationships, to measure hold-up potential. Combining both measures in one regression leads to thorny problems of interpretation. Taken alone the measure has similar effects as the triples measure in this context.

[22]In Appendix D, we show that this confounded the separate effects of complexity and hold-up. Including the measures of complexity and hold-up potential proposed here in their empirical model, we find that the effects on patenting incentives predicted by our theoretical model for complexity (positive) and hold-up potential (negative)

previously existing patent thickets on entry from that of technological complexity. The triples measure is more likely to be elevated in complex technologies, but complexity alone does not lead to an elevated hold-up potential. Hence we use separate measures of complexity and hold-up potential.

Covariates It is well known that firm size and industry are important predictors of whether a firm patents at all (see Bound et al., 1984, for U.S. data). Hall et al. (2013) show this for UK patenting during the period studied here. Therefore, in all of our regressions we control for firm size, industrial sector, and year of observation. We include the logarithm of the firm's reported assets and a set of year dummies in all the regressions.[23] To control for industrial sector, we stratify by industry, which effectively means that each industry has its own hazard function, which is shifted up or down by the other regressors.

We also expect the likelihood that a firm will enter a particular technology area to depend on its prior patenting experience, as well as its age. Long-established firms are less likely to be exploring new technology areas in which to compete. Thus we include the logarithm of firm age and the logarithm of the stock of prior patents applied for in any technology by the firm, lagged one year to avoid any endogeneity concerns.[24] The variables on firm size and patent stock also allow us to test *Prediction 4* about the effect of incumbency advantage on entry.

Finally, to check that our technology entry results are not driven by concentration in the firm's industrial sector, we compute the Herfindahl-Hirschman index (HHI) for each 4-digit sector using all the firms (about 3 million) on the Companies House FAME files and include that variable in our regressions. Because broad industrial sectors are being controlled for via stratification, the HHI variable only measures variations within those sectors.

3.2 Descriptive Statistics

Our estimation sample contains about 22,000 firms and 550,000 firm-TF34 sector combinations. During the 2002-2009 period there are about 14,000 entries into patenting for the first time in a technology area by these firms. Table C-2 in the appendix shows the distribution of the number of entries per firm: 3,110 enter one class, and the rest enter more than one. Table C-3 shows the population of UK firms obtained from FAME in our industries, together with the shares in each industry that have applied for a UK or European patent during the 2001-2009 period. These shares range from over 10% in Pharmaceuticals and R&D Services to less than 0.2% in Construction, Transportation, and Financial Services. Table C-4 shows the number of entrants and their share among all patentees by technology area. It shows that there is a sub-

apply in their data.

[23]The choice of assets as a size measure reflects the fact that it is the only size variable available for the majority of the firms in the FAME dataset.

[24]We compute the past stock of patents using a declining balance formula with a 15% depreciation rate, in order to reduce the impact of very old patents.

stantial amount of entry but it also varies significantly across technologies. Finally, Table C-5 shows our different measures for technological opportunity, complexity, and patent thickets by TF34 technology class and Table C-6 shows descriptive statistics for the key technology class and firm level variables.

3.3 Empirical Model

We use hazard models to estimate the probability of entry into a technology area. The models express the probability that a firm enters into patenting in a certain area conditional on not having entered yet as a function of the firm's characteristics and the time since the firm was 'at risk,' which is the time since the founding of the firm. In some cases, our data do not go back as far as the founding date of the firm, and in these cases the data are left-censored. When we do not observe the entry of the firm into a particular technology sector by the last year (2009), the data is referred to as right-censored.

We estimate two classes of failure or survival models:[25] 1) proportional hazard, where the hazard of failure over time has the same shape for all firms, but the overall level is proportional to an index that depends on firm characteristics; and 2) accelerated failure time (AFT), where the survival rate is accelerated or decelerated by the characteristics of the firm. In the body of the paper we present results using the well-known Cox proportional hazards model stratified by industry. The effect of the stratification is that we allow firms in each of the industries to have a different distribution of the time until entry into patenting conditional on the regressors. That is, each industry has its own 'failure' time distribution, where failure is defined as entry into patenting in a technology area, but the level of this distribution is also modified by the firm's size, aggregate patenting in the technology, network density, and the triples density. To check for omitted firm specific effects, we also estimate hazard models stratified by firms, where each firm has its own failure time distribution.

Appendix Table E-1 shows exploratory regressions made using various survival models. The accelerated failure time estimates are not well identified and typically have larger coefficients with larger standard errors than the other two, but of the same sign. Unlike the Weibull model, these models allow for a baseline hazard that may first increase and then decrease, which is difficult to identify in our relatively short time period.

Our data for estimation are for the 2002-2009 period, but many firms have been at risk of patenting for many years prior to that. The oldest firm in our dataset was founded in 1856 and the average founding year was 1992. Because the EPO was only founded in 1978, we chose to use that year as the earliest date any of our firms is at risk of entering into patenting. That is, we defined the initial year as the maximum of the founding year and 1978. Table E-2 in the appendix presents estimates of our model using 1900 instead of 1978 as the earliest at risk

[25] In Appendix E, we discuss the choice of the survival models that we use for analysis, how to interpret the results, and present some robustness checks.

year and finds little difference in the estimates.[26] We conclude that the precise assumption of the initial period is innocuous. Our assumption amounts to assuming that the shape of the hazard for firms founded between 1856 and 1978 but otherwise identical is the same during the 2002-2009 period.

4 Results

4.1 Main results

Our estimates of the model for entry into patenting are shown in Table 1. All regressions control for size, age, and industry. Both size and age are strongly positively associated with entry into patenting in a new technological area. Our indicator of technological opportunity and technology class size, the log of current patent applications in the technology class, is also positively associated with entry into that class, as predicted by our model.

Column 2 of Table 1 contains the basic result from our data and estimation, which is fully consistent with the predictions of our theoretical model: greater complexity as measured by citation network density increases the probability of entry into a technology area (Prediction 2), as does technological opportunity (Prediction 1), measured both as prior patenting in the class and as growth in the relevant science literature. Controlling for both technological opportunity and complexity, firms are discouraged from entry into areas with a greater density of triple relationships among existing firms (Prediction 3). We interpret this latter result as an indicator of the discouraging effect of hold-up possibilities or the legal costs associated with negotiation of rights or defense in the case of litigation.

We were concerned that our network density (complexity) and triples density (hold-up potential) measures might be too closely related to convey separate information, but we found that the raw correlation between these two variables was -0.001. To check for the impact of potential correlation conditional on year, industry, and the other variables, in column 1 of Table 1 we included the measure of thickets without that for network density and found that although the coefficient was very slightly lower in absolute value, the result still hold.[27]

As we show in Appendix E, the estimated coefficients in the table are estimates of the elasticity of the yearly hazard rate with respect to the variable, and do not depend on the industry specific proportional hazard. A one standard deviation increase in the log of network density is associated with a 7% increase in the hazard of entry (0.112×0.59), while a one standard deviation in the log of triples density is associated with a 23% decrease in the hazard of entry (-0.150×1.56). Thus the differences across these technology areas in the willingness of firms

[26]The main difference is in the firm age coefficient. Because the models are nonlinear, this coefficient is identified even in the presence of year dummies and vintage/cohort (which is implied by the survival model formulation). However it will be highly sensitive to the assumptions about vintage due to the age-year-cohort identity.

[27]In results not shown, we also included the network density variable separately, with similar effect.

to enter them is substantial, bearing in mind that the average probability of entry is only about 1% in this sample.

There are fixed costs to patenting, and a firm may be more likely to enter into patenting in a new area if it already patents in another area. To test this idea, in the third column of Table 1, we add the logarithm of past patenting by the firm. In line with Prediction 4, firms with a greater prior patenting history are indeed more likely to enter a new technology area doubling a firm's past patents leads to an almost 100% higher hazard of entry. Accounting for differences across firms in patenting propensity also changes the sign of the non-patent references coefficient, which we are using as one of the proxies for technological opportunity in the technology sector. Apparently firms with strong patenting histories are not more likely to enter sectors with recent growth in scientific input.[28] Controlling for past patenting also weakens the triples coefficient somewhat, which is consistent with the idea that patenting strength renders a firm less vulnerable to hold-up possibilities.

Industry concentration may also affect a firm's willingness to enter new technology areas. Recall that we already control for the level of entry by two-digit industry via stratified hazard rate model estimation. In the next column, we add the Herfindahl for the firm's 4-digit industry and find that within two-digit industry, variations in four-digit concentration impact entry positively, but the effect is unrelated to any of the other variables, especially those describing the technological context. That is, entry into new technology areas is more likely in concentrated industries, but the impact of complexity, potential hold-up, and technological opportunity is the same regardless of the firms industry concentration.

In the last column we interact the log of assets with the log of patents, the log of network density, the growth of non-patent literature, and the log of triples density to see whether these effects vary by firm size. The results show that the technological opportunity effect declines slightly with firm size. The triples density effect shows a small decrease with size, suggesting that hold-up concerns affect larger firms somewhat less than smaller firms. We show this graphically in Figure 1, which overlays the coefficients as a function of firm size on the actual size distribution of our firms. From the graph one can see that the impact of aggregate patenting in a sector is higher and more variable than the impact of hold-up potential, and that both fall to zero for the largest firms. Growth in non-patent literature is positively associated with technology entry for small firms, but negatively for large firms, suggesting the role played by the smaller firms in newer technologies based on science. Large firms seem not to be as active in these areas.

[28]The negative sign of the non-patent references coefficient appears to be driven by firms in the pharmaceutical industry. When we exclude firms in the pharmaceutical industry and the relevant technology categories organic fine chemistry, biotechnology, and pharmaceuticals, the coefficient on the non-patent references is close to zero and statistically not different from zero (results not reported here).

Table 1: Hazard of entry into patenting in a TF34 Class

Variable	Cox Proportional Hazard Model				
	(1)	(2)	(3)	(4)	(5)
Log (network density)		0.112***	0.118***	0.116***	0.117***
		(0.022)	(0.021)	(0.021)	(0.021)
Log (triples density	-0.147***	-0.150***	-0.111***	-0.112***	-0.117***
in class)	(0.010)	(0.009)	(0.008)	(0.009)	(0.009)
Log (patents in class)	0.558***	0.598***	0.573***	0.573***	0.605***
	(0.027)	(0.026)	(0.024)	(0.024)	(0.025)
5-year growth of non-	0.122***	0.096***	-0.126***	-0.125***	-0.094***
patent refs in class	(0.033)	(0.034)	(0.031)	(0.031)	(0.031)
Log assets	0.288***	0.287***	0.200***	0.200***	0.676***
	(0.011)	(0.011)	(0.013)	(0.013)	(0.084)
Log firm age in years	1.203***	1.205***	1.178***	1.169***	1.203***
	(0.093)	(0.093)	(0.103)	(0.103)	(0.103)
Log (pats applied for			1.074***	1.071***	1.071***
by firm previously)			(0.038)	(0.039)	(0.038)
Herfindahl for firm's				0.442**	
4-digit industry				(0.217)	
Log (network density)					0.000
× Log assets					(0.006)
Log (triples density)					0.008***
× Log assets					(0.003)
Log (patents in class)					-0.056***
× Log assets					(0.008)
Log (average NPL refs)					-0.067***
× Log assets					(0.010)
Log likelihood	-84.40	-84.38	-77.24	-76.34	-77.20
Degrees of freedom	13	14	15	16	19
Chi-squared	2450.7	2583.5	3520.8	3408.5	3452.9
551,981 firm-TF34 observations with 14,709 entries (22,316 firms)					

Notes: The sample is matched on size class, sector, and age class. Estimates are weighted by sampling probability. Time period is 2002-2009 and minimum entry year is 1978. Sample is UK firms with nonmissing assets, all patenting firms and a matched sample of non-patenting firms. A complete set of year dummies is included in the hazard function. Method of estimation is Cox proportional hazard. Coefficients for the hazard of entry into a patenting class are shown. Estimates are stratified by industry - that is, each 2-digit industry has its own baseline hazard. Standard errors are clustered on firm. *** (**) denote significance at the 1% (5%) level. The degrees of freedom are those for the chi-squared test versus a model with hazard rate only. Source: Authors' calculations.

4.2 Firm effects

In the previous regressions we controlled for firm size, age, industry and past patenting behavior. But obviously firms can differ in other unobservable ways and it would be desirable to control for these left out variables. Because firms in our data can enter into any one of 34 technology areas, this turns out to be straightforward, as we have variability across technology as well as years to provide identification. The cost is that we can no longer identify the coefficients of the firm-level variables.

Table 2 displays the results of estimating proportional hazard models on our data stratified by firm rather than industry, with standard errors also clustered by firm. The results are similar but differ in places from those using industry stratification. Complexity of a sector has a much weaker impact but the impact of the thickets or hold-up variable is strengthened, implying that firms avoid those sectors with a high potential for hold-up.

With the exception of past non-patent literature growth, the interaction coefficients (which are identified even though the simple log assets coefficient is not) all suggest weakened impacts for larger firms. The impact of growth in the past non-patent literature used by patents in the class is negative within firm and even more negative for larger firms. Looking at the raw data in the appendices, it appears that organic fine chemistry, biotechnology, and pharmaceuticals have both the lowest first time entry rates and the highest growth in the use of non-patent literature. In these technologies, it appears that other forces beyond thickets discourage entry.

Figure 1: Firm size and the effects of technological opportunity, complexity, and patent thickets

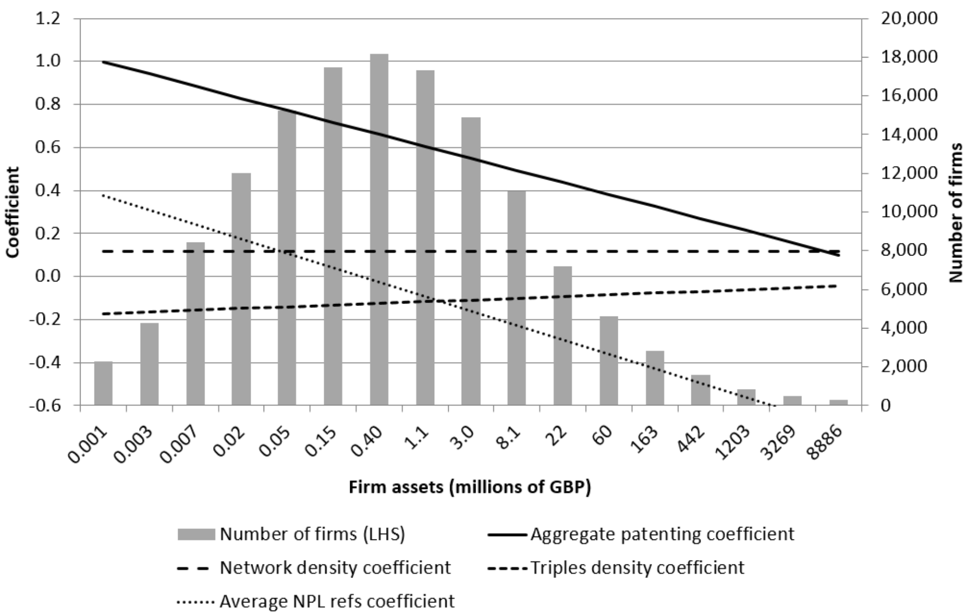

Table 2: Hazard of entry into patenting in a TF34 Class – Firm Effects

Variable	Cox Proportional Hazard Model			
	(1)	(2)	(3)	(4)
Log (network density)	0.000		0.036**	0.041**
	(0.017)		(0.017)	(0.017)
Log (triples density in class)		-0.206***	-0.207***	-0.212***
		(0.008)	(0.008)	(0.008)
Log (patents in class)	0.361***	0.735***	0.752***	0.782***
	(0.018)	(0.022)	(0.023)	(0.024)
5-year growth of non-patent refs in class	-0.574***	-0.636***	-0.644***	-0.634***
	(0.026)	(0.026)	(0.026)	(0.026)
Log (network density) × Log assets				-0.022***
				(0.006)
Log (triples density) × Log assets				0.011***
				(0.002)
Log (patents in class) × Log assets				-0.071***
				(0.008)
Log (average NPL refs) × Log assets				-0.025***
				(0.009)
Log likelihood	43.45	43.87	43.87	43.92
Degrees of freedom	3	3	4	8
Chi-squared	964.9	1468.3	1478.5	1565.2
551,981 firm-TF34 observations with 14,709 entries (22,316 firms)				

Notes: The sample is matched on size class, sector, and age class. Estimates are weighted by sampling probability. Time period is 2002-2009 and minimum entry year is 1978. Sample is UK firms with nonmissing assets, all patenting firms and a matched sample of non-patenting firms. Method of estimation is Cox proportional hazard. Coefficients for the hazard of entry into a patenting class are shown. Estimates are stratified by firm - that is, each firm has its own baseline hazard. Standard errors are clustered on firm. *** (**) denote significance at the 1% (5%) level. The degrees of freedom are those for the chi-squared test versus a model with hazard rate only. Source: Authors' calculations.

4.3 Robustness

One concern we may have with the relationship between entry and the triples variable is simultaneity. That is, technology areas with lots of entry may also be prone to a higher triples density, just because of the entries. To address this possibility, we use the aggregate form of our entry regression. For each year we regress the log of the number of first time entries in each technology-industry sector combination on the characteristics of the technology class together with industry and year dummies. As instruments for the triples density, we use the median examination lag in the technology for patents applied for 5 and 6 years prior to the current year, which is long enough so that most of them will have been granted, rejected, or withdrawn. The idea is that classes with long examination lags may also be those where it is more difficult to assess patentability, leading to the hold-up potential captured by the triples proxy variable. We find that the instrumental variables regression easily passes the specification tests for under-, weak and over-identification, justifying our choice of instruments.

Table 3: Aggregate regressions for entry into patenting classes 2001-2009

Variable	Log number of first time entries by a firm into class by sector					
	OLS			IV†		
	Coef.	s.e.‡		Coef.	s.e.‡	
	(1)			(2)		
Log (US network density)	0.034	0.030		0.054	0.031	*
Log (triples density)	-0.099	0.011	***	-0.214	0.033	***
Log (patent apps in class)	0.318	0.033	***	0.484	0.058	***
Past 5 year growth in NPL refs	-0.303	0.032	***	-0.297	0.035	***
Log (number firms in class)	0.719	0.017	***	0.664	0.020	***
Average 4-digit HHI in sector	-0.179	0.148		-0.198	0.146	
R-squared	0.625			0.600		
Standard error	0.581			0.598		
9 years×34 tech classes×25 sectors = 7,650 observations						

Notes: ‡ Standard errors are clustered on tech class-industrial sector (which allows free correlation over time). *** (*) denote significance at the 1% (10%) level. † Instruments are lag 5 and 6 median exam duration for patents in the class. Tests for under-identification and weak identification pass easily. Hansen J-stat for over-identification has a p-value of 0.826. Log of triples density is treated as endogenous in the IV estimates. Source: Authors' calculations.

Table 3 shows the results, both ordinary least squares and instrumental variables.[29] We include all the technology area variables, a count of the number of firms in the tech class-industry sector-year cell, and the average HHI for the industry of those firms. Note that we do not expect results to be identical when comparing the aggregate regressions to individual firm-level hazard rate estimations, as the functional forms and aggregation level of the models differ. However,

[29] We also estimated this model by LIML and GMM, with almost no change in the resulting coefficients (not shown).

the results are similar in sign to those in column 4 of Table 1, with the exception of the HHI coefficient, which is insignificant. For our purposes, interest centers on the coefficient of triples density. The least squares estimate of the elasticity is negative and implies a 15% reduction in entry per year when the triples density increases by one standard deviation. Instrumenting this variable doubles its coefficient, which suggests that our hazard rate estimates may be an underestimate of the true impact of potential hold-up on entry.

Table E-2 in the appendix explores some variations of the sample used for estimation in Table 1. Column 1 of Table E-2 is the same as column 3 of Table 1 for comparison. The first change (column 2) was to add back all the technology-industry combinations where Lybbert and Zolas (2014) find no patenting in their data and where there was no entry by any UK firm from the relevant industry into that technology category. These observations are about 20% of the sample. The impact of network density on entry is considerably weaker, but the impacts of triples density and the technology class size are considerably stronger. The growth in non-patent references in the class is again negative, contrary to our prediction. This may be because the sector-class combinations added were weighted towards chemicals and pharmaceuticals, where non-patent references are much more important, and where we have already seen that entry is low.

Next we explored the differences across firm size, first removing all the firms with assets greater than 12.5 million pounds and then keeping only the firms with more than one billion pounds in assets.[30] The former restriction removed only 2% of the 20,000 firms, while the latter left only 273 firms. Column 3 of Table E-2 shows that the results for the SMEs do not change a great deal, although they are somewhat stronger, and the growth in non-patent literature is no longer significant. The coefficients for the giant firms appear different, but they have very large standard errors. So our results do not appear to be dominated by a particular size class of firms.

In column 5, we removed the telecommunications technology sector from the estimation, because it is such a large triples outlier. Once again, there was little change to the estimates. The last column of Table E-2 shows the results of defining the minimum entry year as 1900. With the exception of firm age, the coefficients are nearly identical to those in column 1 of the table. Age is nearly collinear with firm entry dates so changes in that coefficient are to be expected when we redefine the entry year.

5 Conclusion

Patent thickets arise for a multitude of reasons; they are mainly driven by an increase in the number of patent filings and concomitant reductions in patent quality (that is, the extent to

[30] 12.5 million pounds is a cutoff based on the definition of Small and Medium-sized Enterprises (SMEs) as firms with fewer than 250 employees. We do not have employment for all our firms, so we assume that assets are approximately 50 thousand pounds per employee in order to compute this measure. For small firms only, this yields an assets cutoff of 2.5 million pounds.

which the patent satisfies the requirements of patentability) as well as increased technological complexity and interdependence of technological components. The theoretical analysis of patent thickets (Shapiro, 2001) and the qualitative evidence provided by the FTC in a number of reports (FTC, 2003, 2011) suggest that thickets can impose significant costs on some firms. The subsequent literature has focused on the measurement of thickets (e.g. Ziedonis, 2004, Graevenitz, Wagner, and Harhoff, 2011) and has linked thickets to changes in firms' intellectual property strategies in a number of dimensions. There is still a lack of evidence on the effect of patent thickets as well as their welfare implications at the aggregate level.

The empirical analysis of the effects of patent thickets must contend with two challenges: first, patent thickets have to be measured and secondly, effects of thickets must be separated from effects of other factors that are correlated with the growth of thickets, in particular technological complexity.

This paper confronts both challenges. We show that our empirical measure for the density of thickets captures effects of patent thickets predicted by theory. We separate the impact of patent thickets on entry from effects of technological opportunity and complexity and show that thickets reduce entry into patenting. Controlling for technological opportunity and complexity is important because both are correlated with entry into patenting and the presence of thickets. It is also worth emphasizing that our measure of thickets is purged of effects that are driven by patenting trends in particular technologies. That is, our results are not due to the level of invention and technological progress within a technology field.

Our results demonstrate that patent thickets significantly reduce entry into those technology areas in which growing complexity and growing opportunity increase the underlying demand for patent protection. These are the technology areas, which are associated most with productivity growth in the knowledge economy. However, the welfare consequences of our finding are not so clear. Reduced entry into new technology areas could be welfare-enhancing: Entry into a market may be excessive if entry creates negative externalities for active firms, for instance due to business stealing (Mankiw and Whinston, 1986, Suzumura and Kiyono, 1987). This is likely to be true of patenting too. Furthermore, Arora, Ceccagnoli, and Cohen (2008) show that the patent premium does not cover the costs of patenting for the average patent (except for pharmaceuticals). These and related facts might lead one to conclude that lower entry into patenting is likely to increase welfare and that thickets raise welfare by reducing entry.

In contrast, reduced entry into patenting in new technology areas may also be welfare-reducing, for at least two reasons. First, there is the obvious argument that the benefits from more innovation may exceed any business stealing costs (as has been shown empirically in the past by others, e.g., Bloom, Schankerman, and Van Reenen, 2013), so that some desirable innovation may be deterred by high entry costs. Even if this were not true, there is no reason to believe that firms that do not enter into patenting due to thickets are those we wish to deter. Given the incumbency advantage, it is likely that the failure to enter into patenting in these areas reflects less innovation by those who bring the most original ideas, that is, by those who are inventing 'outside the box.'

The view that firms generally identify and preempt the emergence of patent thickets through private contractual arrangements sounds optimistic in this light (Barnett, 2018). While firms have the ability to privately contract around blocking patents, transaction costs associated with contracts of this nature may be sufficiently important to deter some firms, specifically smaller ones, from doing so. Our evidence also casts doubt on the suggestion that in response to a thicket, firms will simply resort to unlicensed use of patented technology (Teece, 2018). This is much more likely to be a response adopted by large corporations with strong patent portfolios, as is apparent in the many patent cases brought by smartphone vendors after 2011 (Paik and Zhu, 2016). The key question for public policy in this context is whether or not to employ more resources to change incentives for patentees to submit clearly delineated patent claims and to strengthen the examination of patents such that patent notice is strengthened. Menell (2019) discusses a range of approaches that could be taken in this regard. Our analysis suggests that these measures might primarily benefit smaller patent applicants.

Supplementary material

Supplementary material is available on the OUP website. These are the data and replication files and the online appendix. Some of the data used in this paper are available from Bureau van Dijk's FAME database.

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ONLINE APPENDIX

Technology Entry in the Presence of Patent Thickets

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A Results from Prior Work

This appendix summarizes a number of results derived by Graevenitz, Wagner, and Harhoff (2013) as well as some additional results that are useful for understanding our theoretical results.

A.1 The Probability of Patenting a Facet

The probability p_i that firm i obtains a patent on a facet is:

$$p_i(\mathbf{f}_{\not{i}}, F, N_O(O, \mathbf{o}_{\not{i}}, N)) = \sum_{j=0}^{N_O} \frac{1}{j+1} \binom{N_O}{j} \prod_{l=0}^{N_O-j} \left(1 - \frac{f_l}{F_k}\right) \prod_{m=N_O-j}^{N_O} \frac{f_m}{F_k} \quad . \tag{A.1}$$

where $\mathbf{f}_{\not{i}}, \mathbf{o}_{\not{i}}$ are vectors containing the choices of the number of facets and the number of opportunities to invest in, made by all rival firms j.

For the comparative statics of entry stage it is useful to know that the elasticity of p_i with respect to F_k is negative if $\hat{\phi}_k < \frac{1}{2}$:

$$\frac{\partial p_i}{\partial F_k} = \sum_{i=0}^{N_O} \frac{1}{i+1} \binom{N_O}{i} (1 - \hat{\phi}_k)^{N_O - i} \hat{\phi}_k^i (-1) \left(\frac{N_O}{F_k} - \frac{N_O - i}{F_k - \hat{f}_k}\right) \tag{A.2}$$

Then the elasticity ϵ_{p_i,F_k} is:

$$\epsilon_{p_i,F_k} = N_O^2 \frac{\left(\hat{\phi}_k - \frac{1}{2}(1 + \frac{1}{N_O})\right)}{1 - \hat{\phi}_k} \tag{A.3}$$

A.2 The Expected Number of Rival Investors

The expected number of rival firms N_O that undertake R&D on the same technology opportunity as firm i can be expressed as a sum of products:

$$N_O = \sum_{j=0}^{N_k} j \binom{N_k}{j} \prod_{l=0}^{N_k-j} (1 - \omega_{lk}) \prod_{m=N_k-i}^{N_k} \omega_{mk}. \tag{A.4}$$

In the second stage equilibrium N_O can be rewritten as:

$$N_O = \sum_{j=0}^{N} j \binom{N}{j} (1-\hat{\omega}_k)^{(N-j)} \hat{\omega}_k^j. \tag{A.5}$$

Incumbency Advantage

In the case in which there are incumbents and entrants the expected number of rival firms N_O has to rewritten slightly. To do this define:

$$\omega_{ik}^I \equiv o_i^I / O_k \qquad \omega_{ik}^E \equiv o_i^E / O_k \tag{A.6}$$

We assume that in a previous period N^p firms entered and of these a fraction λ are still active. Then the expected number of rival firms $\tilde{N}_O$ that undertake R&D on the same technology opportunity as firm i is:

$$\tilde{N}_O = \sum_{j=0}^{\lambda N^P} j \binom{\lambda N^P}{j} (1-\hat{\omega}_k^I)^{(N^P-j)} (\hat{\omega}_k^I)^j + \sum_{j=0}^{N} j \binom{N}{j} (1-\hat{\omega}_k^E)^{(N-j)} (\hat{\omega}_k^E)^j. \tag{A.7}$$

A.3 The Expected Number of Facets Covered

In the second stage equilibrium the expected number of facets covered through the joint efforts of all firms investing in a technological opportunity is:

$$\tilde{F}_k = F_k \Big[1 - (1-\hat{\phi}_k)^{(N_O+1)} \Big] \tag{A.8}$$

The derivative of this expression with respect to F_k is positive:

$$\frac{\partial \tilde{F}_k}{\partial F_k} = 1 - \left(1-\hat{\phi}_k\right)^{N_O} \left(1 + \hat{\phi}_k N_O\right) \geq 0 \ . \tag{A.9}$$

The elasticities of $\tilde{F}_k$ with respect to f_k and F are:

$$\hat{\eta}_k = \frac{\hat{\phi}_k (1-\hat{\phi}_k)^{N_O}}{1-(1-\hat{\phi}_k)^{(N_O+1)}} \tag{A.10}$$

$$\hat{\epsilon}_{\tilde{F}_k, F_k} = \frac{1-(1-\hat{\phi}_k)^{(N_O)}(1+\hat{\phi}_j N_O)}{1-(1-\hat{\phi}_k)^{(N_O+1)}}. \tag{A.11}$$

which shows that $1 \geq \epsilon_{\tilde{F}_k F_k} \geq 0$ as the denominator in the fraction is always greater than the numerator. It is useful to observe that the upper bound of the elasticity $\hat{\eta}_k$ is decreasing in N_O.

To see this note that the elasticity can be expressed as:

$$\hat{\eta}_k = \frac{(1-\hat{\phi}_k)^{N_O}}{(N_o+1)\left(1-\hat{\phi}_k\frac{N_o}{2!}+\hat{\phi}_k^2\frac{N_o(N_O-1)}{3!}\dots\right)}\,. \tag{A.12}$$

This shows that the upper bound of the elasticity decreases in N_O: $\lim_{\hat{\phi}_k\to 0}\eta_k = 1/(N_O+1) \leq 1$. Here we use the binomial expansion of $(1-\hat{\phi}_k)^{N_o+1}$. The expression also shows that the lower bound of $\eta_{k|\hat{\phi}_k=1}$ is zero.

B Results

This appendix contains results on patenting for Stage 2 of game G^* and derivations of all propositions.

B.1 Stage 2: Comparative statics of patenting

The second stage of game G^* is smooth supermodular, as shown in Section B.2.

Proposition 4

The second stage patenting game, defined in particular by assumptions (VF, (2)), (PB, (3)) and (LC, (4)) is smooth supermodular if $\hat{\mu}_k > \hat{\xi}_k$ and if ownership of the technology is expected to be fragmented.

This result generalizes Proposition 1 in Graevenitz, Wagner, and Harhoff (2013).[31]

Further, we show that:

Proposition 5

The potential for hold-up in complex technologies reduces patenting incentives.

In Appendix B.3 we show that the expected legal costs of hold-up reduce the number of opportunities that firms invest in. In addition, firms with larger portfolios are more exposed to hold-up and benefit less from the share of patents they have patented per opportunity. Both effects combine to reduce the number of facets each firm applies for.

B.2 Supermodularity of the Second Stage Game

This section sets out the main results needed to show that the second stage of game G* is supermodular.

Consider the first order conditions that determine the equilibrium number of facets $\hat{f}$ and technological opportunities $\hat{o}$:

[31] Here it is no longer the case that the value function has to be increasing in the number of patented facets for supermodularity of the patenting game. We relegate further discussion of this result to Appendix B.2.

$$\frac{\partial \pi_{ik}}{\partial o_i} = V_k(\tilde{F}_k)\Delta(s_{ik}) - L(\gamma_i, s_{ik}, h_k) - C_o(\Sigma_{j=1}^{N_o} o_j) - \gamma_i C_a - \frac{\partial C_c}{\partial o_i} = 0 \quad , \tag{B.1}$$

$$\frac{\partial \pi_{ik}}{\partial f_i} = \frac{o_i p_i}{\tilde{F}_k}\left(\left[V_k(\tilde{F}_k)\mu_k\eta_{ik}\frac{\Delta(s_{ik})}{s_{ik}} - \tilde{F}_k\left(\frac{\partial L}{\partial \gamma_i} + C_a\right)\right] + \left[V_k(\tilde{F}_k)\frac{d\Delta}{ds_{ik}} - \frac{\partial L}{\partial s_{ik}}\right](1-\eta_{ik})\right) = 0 \quad . \tag{B.2}$$

Now, consider the cross-partial derivatives which must be positive, if the second stage game is supermodular. First, we derive the cross partial derivative with respect to firms' own actions:

$$\frac{\partial^2 \pi_{ik}}{\partial o_i \partial f_i} = \frac{p_i}{\tilde{F}_k}\left(\left[V_k(\tilde{F}_k)\mu_k\eta_{ik}\frac{\Delta(s_{ik})}{s_{ik}} - \tilde{F}_k\left(\frac{\partial L}{\partial \gamma_i} + C_a\right)\right] + \left[V_k(\tilde{F}_k)\frac{d\Delta}{ds_{ik}} - \frac{\partial L}{\partial s_{ik}}\right](1-\eta_{ik})\right) = 0 \quad . \tag{B.3}$$

This expression corresponds to the first order condition (B.2) for the optimal number of facets.

Now consider effects of rivals' actions on firms' own actions:

$$\frac{\partial^2 \pi_{ik}}{\partial o_i \partial o_m} = \frac{\partial \tilde{F}_k}{\partial o_m}\frac{s_{ik}}{\tilde{F}_k}\left[V_k(\tilde{F}_k)\frac{\Delta}{s_{ik}}(\mu_k - \xi_{ik}) + \frac{\partial L}{\partial s_{ik}}\right] + \frac{\partial p_i}{\partial o_m}\frac{f_i}{\tilde{F}_k}\left[\left(V_k(\tilde{F}_k)\frac{d\Delta}{ds_{ik}} - \frac{\partial L}{\partial s_{ik}}\right) - \tilde{F}_k\left(\frac{\partial L}{\partial \gamma_i} + C_a\right)\right] \tag{B.4}$$

$$- \frac{\partial C_o}{\partial \Sigma_{j=1}^{N_o} o_j},$$

$$\frac{\partial^2 \pi_{ik}}{\partial o_i \partial f_m} = \frac{\partial \tilde{F}_k}{\partial f_m}\frac{s_{ik}}{\tilde{F}_k}\left[V_k(\tilde{F}_k)\frac{\Delta}{s_{ik}}(\mu_k - \xi_{ik}) + \frac{\partial L}{\partial s_{ik}}\right] + \frac{\partial p_i}{\partial f_m}\frac{f_i}{\tilde{F}_k}\left[\left(V_k(\tilde{F}_k)\frac{d\Delta}{ds_{ik}} - \frac{\partial L}{\partial s_{ik}}\right) - \tilde{F}_k\left(\frac{\partial L}{\partial \gamma_i} + C_a\right)\right], \tag{B.5}$$

$$\begin{aligned}\frac{\partial^2 \pi_{ik}}{\partial f_i \partial o_m} &= \frac{\partial \tilde{F}_k}{\partial o_m}\left[\frac{\partial V_k}{\partial \tilde{F}_k} + \frac{\partial^2 V_k}{\partial \tilde{F}_k^2}\tilde{F}_k\eta_{ik} - \frac{\partial L}{\partial \gamma_i} - C_a + \frac{\partial^2 L}{\partial s_{ik}^2}\frac{s_{ik}}{\tilde{F}_k}(1-\eta_{ik})\right] \\ &+ \frac{\partial \eta_{ik}}{\partial o_m}\left(V_k(\tilde{F}_k)\frac{\Delta}{s_{ik}}(\mu_k - \xi_{ik}) + \frac{\partial L}{\partial s_{ik}}\right) - \frac{\partial p_i}{\partial o_m}\left[\frac{\partial^2 L}{\partial \gamma_i^2}f_i + \frac{\partial^2 L}{\partial s_{ik}^2}\frac{f_i}{\tilde{F}_k}(1-\eta_{ik})\right],\end{aligned} \tag{B.6}$$

$$\begin{aligned}\frac{\partial^2 \pi_{ik}}{\partial f_i \partial f_m} &= \frac{\partial \tilde{F}_k}{\partial f_m}\left[\frac{\partial V_k}{\partial \tilde{F}_k}\frac{\Delta}{s_{ik}}\left(\xi_{ik}(1-\epsilon_{\tilde{F}_k,f}) + \epsilon_{\tilde{F}_k,f}\right) + \frac{\partial^2 V_k}{\partial \tilde{F}_k^2}\tilde{F}_k\eta_{ik}\frac{\Delta}{s_{ik}} - \frac{\partial L}{\partial \gamma_i} - C_a + \frac{\partial^2 L}{\partial s_{ik}^2}\frac{s_{ik}}{\tilde{F}_k}(1-\eta_{ik})\right] \\ &+ \frac{\partial \eta_{ik}}{\partial f_m}\left(V_k(\tilde{F}_k)\frac{\Delta}{s_{ik}}(\mu_k - \xi_{ik}) + \frac{\partial L}{\partial s_{ik}}\right) - \frac{\partial p_i}{\partial f_m}\left[\frac{\partial^2 L}{\partial \gamma_i^2}f_i + \frac{\partial^2 L}{\partial s_{ik}^2}\frac{f_i}{\tilde{F}_k}(1-\eta_{ik})\right] \quad .\end{aligned} \tag{B.7}$$

The second stage game is supermodular, if the equations (B.4)-(B.7) are non-negative. The following results show that the conditions discussed in Section 2 must hold simultaneously if game G* is supermodular.

Using the first order condition (B.2), which will hold for any interior equilibrium, it can be shown that:

$$\left[\left(V_k(\tilde{F}_k)\frac{d\Delta}{ds_{ik}} - \frac{\partial L}{\partial s_{ik}}\right) - \tilde{F}_k\left(\frac{\partial L}{\partial \gamma_i} + C_a\right)\right] = -\eta_{ik}\left(V_k(\tilde{F}_k)\frac{\Delta}{s_{ik}}(\mu_k - \xi_{ik}) + \frac{\partial L}{\partial s_{ik}}\right). \tag{B.8}$$

If $\left(V_k(\tilde{F}_k)\frac{\Delta}{s_{ik}}(\mu_k - \xi_{ik}) + \frac{\partial L}{\partial s_{ik}}\right) > 0$, then the second term in the cross-partial derivatives (B.4) and (B.5) is the product of two negative expressions, and then Equation (B.5) is positive. Equation (B.4) can be rewritten as follows:

$$\frac{\partial^2 \pi_{ik}}{\partial o_i \partial o_m} = \frac{\partial N_O}{\partial o_m}\frac{s_{ik}}{N_O}\left[\epsilon_{\tilde{F}_k,N_O} - \epsilon_{p_k,N_O}\eta_{ik}\right]\left[V_k(\tilde{F}_k)\frac{\Delta}{s_{ik}}(\mu_k - \xi_{ik}) + \frac{\partial L}{\partial s_{ik}}\right] - \frac{\partial C_o}{\partial \Sigma_{j=1}^{N_o} o_j}. \tag{B.9}$$

Graevenitz, Wagner, and Harhoff (2013) show that as rival firms choose more opportunities to invest in the number of firms seeking to patent within each opportunity increases. It can also be shown that the first term in square brackets in Equation (B.9) is positive: $\epsilon_{\tilde{F}_k,N_O} > 0, \epsilon_{p_i,N_O} < 0, \eta_{ik} \geq 0$. Therefore, this cross-partial introduces a restriction on the costs of doing R&D per opportunity: these should not increase too steeply with respect to each rival firm's choice of total opportunities to invest in. As more firms become active in a technology the marginal effect of an additional entrant will diminish, so that this requirement is not very strong in a crowded technology area. We return to this requirement further below when considering free entry.

Turning to equations (B.6) and (B.7) we can show that:

$$\frac{\partial \eta_{ik}}{\partial o_m} = \frac{\partial^2 \tilde{F}_k}{\partial f_i \partial o_m}\frac{f_i}{\tilde{F}_k} - \frac{\partial \tilde{F}_k}{\partial f_i}\frac{\partial \tilde{F}_k}{\partial o_m}\frac{f_i}{\tilde{F}_k{}^2} = -\tilde{F}_k{}^{-1}\frac{\partial \tilde{F}_k}{\partial o_m}\left(\frac{\phi_k}{1-\phi_k} + \eta_{ik}\right) \tag{B.10}$$

$$\frac{\partial \eta_{ik}}{\partial f_m} = \frac{\partial^2 \tilde{F}_k}{\partial f_i \partial f_m}\frac{f_i}{\tilde{F}_k} - \frac{\partial \tilde{F}_k}{\partial f_i}\frac{\partial \tilde{F}_k}{\partial f_m}\frac{f_i}{\tilde{F}_k{}^2} = -\tilde{F}_k{}^{-1}\frac{\partial \tilde{F}_k}{\partial f_m}\left(\frac{\phi_k}{1-\phi_k} + \eta_{ik}\right) \tag{B.11}$$

This result allows us to rewrite equations (B.6) and (B.7) as follows:

$$\frac{\partial^2 \pi_k}{\partial f_i \partial o_m} = \frac{1}{\tilde{F}_k}\frac{\partial \tilde{F}_k}{\partial o_m}\left[\left(V\frac{\Delta}{s_{ik}}(\mu_k - \xi_{ik}) + \frac{\partial L}{\partial s_{ik}}\right)\left(1 - 2\eta_{ik} - \frac{\phi}{1-\phi}\right) + \frac{\partial^2 V}{\partial \tilde{F}_k{}^2}\tilde{F}_k\eta_{ik} + \frac{\partial^2 L}{\partial s_{ik}{}^2}\frac{s_{ik}}{\tilde{F}_k}(1-\eta_{ik})\right]$$
$$- \frac{\partial p_i}{\partial o_m}\left[\frac{\partial^2 L}{\partial \gamma_i{}^2}f_i + \frac{\partial^2 L}{\partial s_{ik}{}^2}\frac{f_i}{\tilde{F}_k}(1-\eta_{ik})\right], \tag{B.12}$$

$$\frac{\partial^2 \pi_k}{\partial f_i \partial f_m} = \frac{1}{\tilde{F}_k}\frac{\partial \tilde{F}_k}{\partial f_m}\left[\left(V\frac{\Delta}{s_{ik}}(\mu_k - \xi_{ik}) + \frac{\partial L}{\partial s_{ik}}\right)\left(1 - 2\eta_{ik} - \frac{\phi}{1-\phi}\right) + \frac{\partial^2 V}{\partial \tilde{F}_k{}^2}\tilde{F}_k\eta_{ik} + \frac{\partial^2 L}{\partial s_{ik}{}^2}\frac{s_{ik}}{\tilde{F}_k}(1-\eta_{ik})\right]$$
$$- \frac{\partial p_i}{\partial f_m}\left[\frac{\partial^2 L}{\partial \gamma_i{}^2}f_i + \frac{\partial^2 L}{\partial s_{ik}{}^2}\frac{f_i}{\tilde{F}_k}(1-\eta_{ik})\right]. \tag{B.13}$$

Given assumptions (VF) and (LC) shown in the main text, these two equations will be positive if $\left(V\frac{\Delta}{s_{ik}}(\mu_k - \xi_{ik}) + \frac{\partial L}{\partial s_{ik}}\right) > 0$ and $\left(1 - 2\eta_{ik} - \frac{\phi}{1-\phi}\right) > 0$. We analyze each condition in more detail next:

1. Given our assumptions on the legal cost function (LC, eq. 4) the condition $\left(V\frac{\Delta(\hat{s}_k)}{\hat{s}_k}(\hat{\mu}_k - \hat{\xi}_k + \frac{\partial L}{\partial \hat{s}_k}\right) > 0 \Leftrightarrow V\frac{\Delta(\hat{s}_k)}{\hat{s}_k}(\hat{\mu}_k - \hat{\xi}_k > -\frac{\partial L}{\partial \hat{s}_k}$ implies that $\hat{\mu}_k > \hat{\xi}_k$. The elasticity of the value function with respect to additional covered patents must exceed the elasticity of the portfolio benefits function with respect to the share of patents held by the firm. This condition is less restrictive than the assumption in Graevenitz, Wagner, and Harhoff (2013) that $\hat{\mu}_k > 1$, since we are assuming that $\hat{\xi}_k < 1$.

2. $(1 - 2\hat{\eta}_k) - \frac{\hat{\phi}_k}{1-\hat{\phi}_k} > 0 \Leftrightarrow (1 - 2\hat{\phi}) > (1 - \hat{\phi})^{(\hat{N}_O+1)}$. This holds for any $\hat{\phi}_k < \frac{1}{2}$ and $\hat{N}_o$ sufficiently large. These restrictions imply a setting in which the ownership of patents belonging to each opportunity is fragmented amongst many firms. It is more likely to arise if the technology is highly complex, otherwise the condition that $\hat{\phi}_k < \frac{1}{2}$ is less likely to hold.

In Appendix B.5 we derive the conditions under which the equilibrium of game G^* is unique. If there is a unique solution to the optimization problem of the firm at which profits are maximized, then this requires that $\partial^2\pi_k/\partial\hat{f}^2 < 0$. The restrictions that i) $\hat{\mu}_k < 1$ and ii) the share of overall profits which the firm obtains is decreasing at the margin in the share of patents the firm holds ($\partial^2\Delta/\partial\hat{s}_k^2 < 0$) ensure that there is always such a unique interior solution.

In this game G^* the comparative statics of patenting are the same as in the main model analyzed in Graevenitz, Wagner, and Harhoff (2013). Specifically we can show that the following effects hold in this game:

$$\frac{\partial^2\pi}{\partial o_i \partial F_k} > 0, \frac{\partial^2\pi}{\partial f_i \partial F_k} > 0, \frac{\partial^2\pi}{\partial o_i \partial O_k} < 0, \frac{\partial^2\pi}{\partial f_i \partial O_k} < 0 \qquad \text{(B.14)}$$

This implies that complexity of the technology increases firms' patent applications while increased technological opportunity reduces firms' patenting applications.

B.3 Effect of Hold-up on Patenting

Here we show that Proposition 2 holds. Consider the following cross-partial derivatives for the effects of higher legal costs L due to hold-up:

$$\frac{\partial^2\pi_k}{\partial\hat{o}\partial h_k} = -\frac{\partial L(\hat{\gamma}_k, \hat{s}_k, h_k)}{\partial h_k} < 0 \quad , \qquad \text{(B.15)}$$

$$\frac{\partial^2\pi_k}{\partial\hat{f}\partial h_k} = -\frac{\hat{o}\hat{p}_k}{\tilde{F}_k}\left(\tilde{F}_k\left(\frac{\partial^2 L}{\partial\hat{\gamma}_k\partial h_k}\right) + \frac{\partial^2 L}{\partial\hat{s}_k\partial h_k}(1 - \hat{\eta}_k)\right) < 0 \quad . \qquad \text{(B.16)}$$

The first of these conditions shows that the expected legal costs of hold-up reduce the number of opportunities a firm invests in, in equilibrium. The second condition shows that firms with larger portfolios will be more exposed to hold up and will benefit less from the share of patents they have patented per opportunity. Both of these effects reduce the number of facets each firm applies for.

B.4 Free Entry Equilibirum

Lemma 1

There is a free entry equilibrium at which the marginal entrant can just break even, if R&D fixed costs per opportunity (C_o) increase in the number of entrants.

In a free entry equilibrium it must be the case that the following conditions hold:

$$\pi_k(\hat{o}_k, \hat{f}_k, \hat{N}_k) > 0 \qquad \wedge \qquad \pi_k(\hat{o}_k, \hat{f}_k, \hat{N}_k + 1) < 0\ . \tag{B.17}$$

The effect of entry on profits at the first stage of game G^* can be shown to be:

$$\begin{aligned}\frac{\partial \pi(\hat{o}_k, \hat{f}_k)}{\partial \hat{N}_k} = \hat{o}\frac{\partial \hat{N}_O}{\partial \hat{N}_k}\Bigg(&\frac{\hat{s}_k}{\hat{\tilde{F}}_k}\frac{\partial \hat{\tilde{F}}_k}{\partial \hat{N}_O}\left[\hat{V}_k(\hat{\tilde{F}}_k)\hat{\mu}\frac{\Delta(\hat{s}_k)}{\hat{s}_k} - \left(\hat{V}(\hat{\tilde{F}}_k)\frac{\partial \Delta}{\partial \hat{s}_k} - \frac{\partial L}{\partial \hat{s}_k}\right)\right] \\ &+ \frac{\partial \hat{p}_k}{\partial \hat{N}_O}\frac{\hat{f}}{\hat{\tilde{F}}_k}\left[\left(\hat{V}(\hat{\tilde{F}}_k)\frac{d\Delta}{d\hat{s}_k} - \frac{\partial L}{\partial \hat{s}_k}\right) - \hat{\tilde{F}}_k\left(\frac{\partial L}{\partial \hat{\gamma}_k} + C_a\right)\right] - \frac{\partial C_o}{\partial \hat{N}_o \hat{o}}\hat{o}\Bigg)\ .\end{aligned} \tag{B.18}$$

This expression is very similar to the first cross-partial (B.9). Simplifying further as above we obtain:

$$\frac{\partial \pi(\hat{o}, \hat{f})}{\partial \hat{N}_k} = \hat{o}\frac{\partial \hat{N}_O}{\partial \hat{N}_k}\left(\frac{\hat{s}_k}{\hat{N}_O}\left[\hat{\epsilon}_{\tilde{F}_k, N_o} - \hat{\epsilon}_{p_i, N_o}\hat{\eta}_k\right]\left[\hat{V}\frac{\Delta(\hat{s}_k)}{\hat{s}_k}\left(\hat{\mu}_k - \hat{\xi}_k\right) + \frac{\partial L}{\partial \hat{s}_k}\right] - \frac{\partial C_o}{\partial \hat{N}_o \hat{o}}\hat{o}\right)\ . \tag{B.19}$$

This expression must be negative in a free entry equilibrium, otherwise profits would increase with additional entrants. Note that there are two main differences with respect to Equation (B.9):

i) the positive term is multiplied with $\frac{\partial \hat{N}_O}{\partial o_m}$ in Equation (B.9) and

ii) the cost function is multiplied with $\hat{o}$ here.

Graevenitz, Wagner, and Harhoff (2013) show that:

$$\frac{\partial \hat{N}_O}{\partial w_m} = N_k \omega_l^{N-1} + \sum_{i-0}^{N-1}\binom{N-1}{i}(1-\omega_l)^{(N-1-i)}\omega_l^i > 0 \quad . \tag{B.20}$$

Note that:

$$\frac{\partial \hat{N}_O}{\partial o_m} = \frac{\partial \hat{N}_O}{\partial \omega_m} \frac{1}{O_k} \quad . \tag{B.21}$$

It can be demonstrated that for arbitrary values of ω the derivative of N_O with respect to o_m is significantly larger than 1. This implies that there is always a region of parameter values for F_k, O_k, h_k, C_a in which the supermodular game G* is in equilibrium and N_k, N_O are determined by the free entry condition.

B.5 Uniqueness of the second stage equilibrium

We show above that stage 2 of game G* is supermodular. This implies that there exists at least one equilibrium of the stage game. An alternative way of deriving existence of the second stage equilibrium for game G* is to analyze the conditions under which the Hessian of second derivatives of the profit function H_π is negative semidefinite. This matrix consists of four derivatives of which only one leads to additional restrictions on the model.

It is easy to see that $\frac{\partial^2 \pi}{\partial {o_i}^2} < 0$ due to the coordination costs $C_c(o_i)$ and the restrictions we impose with assumption (FVC). The two cross-partial derivatives are both zero in equilibrium - refer to equation B.3.

Therefore, the only expression that remains to analyze is $\frac{\partial^2 \pi}{\partial {f_i}^2}$.

$$\frac{\partial^2 \pi}{\partial {f_i}^2} = \frac{o_i p_i}{\tilde{F}_k}\left[\frac{\partial^2 V}{\partial {\tilde{F}_k}^2}\left(\frac{\partial \tilde{F}_k}{\partial f_i}\right)^2 \frac{\Delta \tilde{F}_k}{p_i} + 2\frac{\partial V}{\partial \tilde{F}_k}\frac{\partial \tilde{F}_k}{\partial f_i}\frac{d\Delta}{ds_{ik}}(1-\eta_{ik}) + V\frac{d^2\Delta}{s_{ik}^2}\frac{p_i}{\tilde{F}_k}(1-\eta_{ik})^2 \right.$$
$$\left. - \frac{\partial^2 L}{\partial {\gamma_i}^2}p_i^2 - \frac{\partial^2 L}{\partial {s_{ik}}^2}\left(\frac{\partial s_{ik}}{\partial f_i}\right)^2 - 2\left(V\frac{d\Delta}{ds_{ik}} - \frac{\partial L}{\partial s_{ik}}\right)\frac{(1-\eta_{ik})\,\eta_{ik}}{f_i}\right] \quad .$$

This can be further simplified:

$$\frac{\partial^2 \pi}{\partial {f_i}^2} = \frac{o_i p_i}{\tilde{F}_k}\left[\frac{\partial^2 V}{\partial {\tilde{F}_k}^2}\left(\frac{\partial \tilde{F}_k}{\partial f_i}\right)^2 \frac{\Delta \tilde{F}_k}{p_i} + V\frac{d^2\Delta}{s_{ik}^2}\frac{p_i}{\tilde{F}_k}(1-\eta_{ik})^2 - \frac{\partial^2 L}{\partial {\gamma_i}^2}p_i^2 \right.$$
$$\left. - \frac{\partial^2 L}{\partial {s_{ik}}^2}\frac{p_i}{\tilde{F}_k}(1-\eta_{ik})^2 - 2\left(V\frac{\Delta}{s_{ik}}\xi_{ik}(1-\mu_k) - \frac{\partial L}{\partial s_{ik}}\right)\frac{(1-\eta_{ik})\,\eta_{ik}}{f_i}\right] \quad . \tag{B.22}$$

If we impose the restriction that the second derivative of the value function is negative and that the elasticity of the value function, $\mu_k < 1$, then the first and the last terms in the above expression are negative. The sign of the second term in the expression depends on $sign\{\frac{\partial^2 \Delta}{\partial {s_{ik}}^2}\}$, which we will assume is negative. The third and fourth terms in the above expression are negative given the conditions imposed on the legal cost function above.

B.6 Entry and Incumbency

In this section we analyze a game in which incumbents have lower costs of entry and demonstrate that our main predictions are robust.

We assume that a fraction λ ($0 < \lambda < 1$) of the previously active N^P firms remain as incumbents. The firms enter until the marginal profit from entry is reduced to zero.

Objective Functions

First, consider the objective functions of incumbents and entrants and the patenting game they are involved in. We analyze this game and show when it is supermodular.

Given symmetry of technological opportunities (Assumption S) the expected value of patenting for entrant and incumbent firm's in a technology area k is:

$$\pi^I_{ik}(o^I_i, f^I_i) = o^I_i \left(V(\tilde{F}_k)\Delta(s^I_{ik}) - L(\gamma^I_{ik}, s^I_{ik}) - \left(C_o(\sum_{j=1}^{N^P\lambda-1+N^E} o_j) - \Psi \right) - f^I_i p_k C_a \right) - C_c(o^I_i) \quad . \tag{B.23}$$

$$\pi^E_{ik}(o^E_i, f^E_i) = o^E_i \left(V(\tilde{F}_k)\Delta(s^E_{ik}) - L(\gamma^E_{ik}, s^E_{ik}) - C_o(\sum_{j=1}^{N^P\lambda+N^E-1} o_j) - f^E_i p_k C_a \right) - C_c(o^E_i) \quad . \tag{B.24}$$

Define a game G^E in which:

- There are $N^P\lambda$ incumbent firms and the number of entrants, N^E, is determined by free entry.
- Entrants and incumbents simultaneously choose the number of technological opportunities $o^I_i, o^E_i \in [0, O^n]$ and the number of facets applied for per opportunity $f^I_i, f^E_i \in [0, F^n]$. Firms' strategy sets S_n are elements of R^4.
- Firms' payoff functions π_{ik}, defined at (B.23,B.24), are twice continuously differentiable and depend only on rivals' aggregate strategies.
- Assumptions (VF) and (LC) describe how the expected value and the expected cost of patenting depend on the number of facets owned per opportunity.

Firms' payoffs depend on their rivals' aggregate strategies because the probability of obtaining a patent on a given facet is a function of all rivals' patent applications. Note that the game is symmetric within the two groups of firms as it is exchangeable in permutations of the players. This implies that symmetric equilibria exist, if the game can be shown to be supermodular (Vives, 2005).[32]

[32]Note also that only symmetric equilibria exist as the strategy spaces of players are completely ordered.

First order conditions for game G^E:

$$\frac{\partial \pi_{ik}^I}{\partial o_i^I} = V\Delta(s_{ik}) - L(\gamma_i, s_{ik}) - \left(C_o(\sum_{j=1}^{N^P\lambda-1+N^E} o_j) - \Psi \right) - \gamma_i C_a - \frac{\partial C_c}{\partial o_i^I} = 0 \quad , \tag{B.25}$$

$$\frac{\partial \pi_{ik}^I}{\partial f_i^I} = \frac{o_i^I p_i}{\tilde{F}_k} \left(\left[V\mu\epsilon_{\tilde{F}_k f_i} \frac{\Delta(s_{ik})}{s_{ik}} - \tilde{F}_k \left(\frac{\partial L}{\partial \gamma_i} + C_a \right) \right] + \left[V\frac{d\Delta(s_{ik})}{ds_{ik}} - \frac{\partial L}{\partial s_{ik}} \right] (1-\eta_{ik}) \right) = 0 \quad , \tag{B.26}$$

$$\frac{\partial \pi_{ik}^E}{\partial o_i^E} = V\Delta(s_{ik}) - L(\gamma_i, s_{ik}) - C_o(\sum_{j=1}^{N^P\lambda+N^E-1} o_j) - \gamma_i C_a - \frac{\partial C_c}{\partial o_i^E} = 0 \quad , \tag{B.27}$$

$$\frac{\partial \pi_{ik}^E}{\partial f_i^E} = \frac{o_i^E p_i}{\tilde{F}_k} \left(\left[V\mu\epsilon_{\tilde{F}_k f_i} \frac{\Delta(s_{ik})}{s_{ik}} - \tilde{F}_k \left(\frac{\partial L}{\partial \gamma_i} + C_a \right) \right] + \left[V\frac{d\Delta(s_{ik})}{ds_{ik}} - \frac{\partial L}{\partial s_{ik}} \right] (1-\eta_{ik}) \right) = 0 \quad . \tag{B.28}$$

Proposition 6
In game G^E the equilibrium number of facets chosen by incumbents and entrants is the same: $\hat{f}^I = \hat{f}^E$.

We show in Appendix A.2 that in the game with incumbents the number of rivals per opportunity $\tilde{N}_O$ becomes a function of both $\hat{o}^I, \hat{o}^E$. The first order conditions determining $\hat{f}^I, \hat{f}^E$ both depend on the total number of entrants per technological opportunity $\tilde{N}_O$ and so both on $\hat{o}^I, \hat{o}^E$. This is the only way in which rivals' choices of the number of opportunities to pursue enter these first order conditions[33]. Therefore the two conditions are identical and Proposition 6 holds.

Proposition 7
The second stage of game G^E is smooth supermodular under the same conditions as game G^*. Comparative statics results for game G^* also apply to game G^E.

The first order conditions characterizing the game with incumbents and entrants are identical to those for the game without incumbents as long as $\Psi = 0$. As this variable is a constant it does not enter into the second order conditions which we analyze to establish supermodularity and which underpin the comparative statics predictions in Propositions 3-5.

Proposition 8
In the second stage of game G^E incumbents enter more technological opportunities, if they have a cost advantage in undertaking R&D ($\Psi > 0$).

The first order conditions determining the equilibrium number of opportunities chosen by incumbents and entrants are identical if firms R&D fixed costs per opportunity are the same ($\Psi = 0$). Therefore $\hat{o}^I_{|\Psi=0} = \hat{o}^E$. As the cost advantage of incumbents in undertaking R&D

[33]Clearly the factors outside the brackets in equations (B.26), (B.28) also depend on these variables, but these do not affect the equilibrium values of f_i^E, f_j^I.

grows this increases the number of opportunities chosen by incumbents:

$$\frac{\partial^2 \pi_{ik}^I}{\partial o^I \partial \Psi} = 1 > 0 \quad . \tag{B.29}$$

Proposition 9
In the second stage of game G^E the number of entrants decreases as the cost advantage of incumbents increases.

Due to the supermodularity of the second stage game, increases in incumbents' choices of the number of opportunities to invest in will raise the number of opportunities entrants invest in as well as the numbers of facets entrants and incumbents seek to patent in equilibrium. The increases in $\hat{o}^I$ and $\hat{o}^E$ will raise the fixed costs of entry into new opportunities, C_o, which then reduces entry.

C Data

Our analysis relies on an updated version of the Oxford-Firm-Level-Database, which combines information on patents (UK and EPO) with firm-level information obtained from Bureau van Dijks Financial Analysis Made Easy (FAME) database (for more details see Helmers, Rogers, and Schautschick (2011) from which the data description in this section draws).

The database consists of two components: a firm-level data set and intellectual property (IP) data. The firm-level data is the FAME database that covers the entire population of registered UK firms. The original version of the database, which formed the basis for the update carried out by the UKIPO, relied on two versions of the FAME database: FAME October 2005 and March 2009. The main motivation for using two different versions of FAME is that FAME keeps details of inactive firms (see below) for a period of four years. If only the 2009 version of FAME were used, IP could not be allocated to any firm that has exited the market before 2005, which would bias the matching results. FAME is available since 2000, which defines the earliest year for which the integrated data set can be constructed consistently. The update undertaken by the UKIPO used the April 2011 version of FAME. However, since there are significant reporting delays by companies, even using the FAME 2011 version means that the latest year for which firm-level data can be used reliably is 2009.

FAME contains basic information on all firms, such as name, registered address, firm type, industry code, as well as entry and exit dates. Availability of financial information varies substantially across firms. In the UK, the smallest firms are legally required to report only very basic balance sheet information (shareholders' funds and total assets). The largest firms provide a much broader range of profit and loss information, as well as detailed balance sheet data including overseas turnover. Lack of these kinds of data for small and medium-sized firms means that our study focuses on total assets as a measure of firm size and growth.

Table C-1: Patenting by FAME firms on Patstat (priority years 2002-2009)

	Weighted by #owners & #classes*			**Sector shares**	
Technology areas	GB pats	EP pats	Total	GB pats	EP pats
Electrical machinery, apparatus, energy	1,321	1,101	2,422	6.1%	4.4%
Audio-visual technology	633	549	1,182	2.9%	2.2%
Telecommunications	1,181	1,206	2,386	5.5%	4.8%
Digital communication	590	732	1,323	2.7%	2.9%
Basic communication processes	302	146	447	1.4%	0.6%
Computer technology	1,481	1,302	2,783	6.8%	5.2%
IT methods for management	256	224	480	1.2%	0.9%
Semiconductors	269	248	518	1.2%	1.0%
Optics	392	481	873	1.8%	1.9%
Measurement	1,216	1,458	2,674	5.6%	5.8%
Analysis of biological materials	132	426	557	0.6%	1.7%
Control	592	542	1,134	2.7%	2.2%
Medical technology	996	1,561	2,558	4.6%	6.3%
Organic fine chemistry	182	1,538	1,720	0.8%	6.2%
Biotechnology	193	950	1,143	0.9%	3.8%
Pharmaceuticals	277	1,876	2,153	1.3%	7.5%
Macromolecular chemistry, polymers	114	280	394	0.5%	1.1%
Food chemistry	88	458	547	0.4%	1.8%
Basic materials chemistry	314	1,050	1,363	1.5%	4.2%
Materials, metallurgy	161	318	479	0.7%	1.3%
Surface technology, coating	287	284	571	1.3%	1.1%
Chemical engineering	507	724	1,231	2.3%	2.9%
Environmental technology	296	344	640	1.4%	1.4%
Handling	996	813	1,809	4.6%	3.3%
Machine tools	428	356	784	2.0%	1.4%
Engines, pumps, turbines	887	942	1,829	4.1%	3.8%
Textile and paper machines	235	304	539	1.1%	1.2%
Other special machines	742	623	1,365	3.4%	2.5%
Thermal processes and apparatus	410	261	671	1.9%	1.0%
Mechanical elements	1,149	854	2,002	5.3%	3.4%
Transport	1,063	930	1,993	4.9%	3.7%
Furniture, games	1,064	612	1,675	4.9%	2.5%
Other consumer goods	630	507	1,137	2.9%	2.0%
Civil engineering	2,237	960	3,196	10.3%	3.8%
Total	**21,619**	**24,959**	**46,578**		
Electrical engineering	6,032	5,508	11,540	27.9%	22.1%
Instruments	3,328	4,468	7,796	15.4%	17.9%
Chemistry	2,418	7,822	10,240	11.2%	31.3%
Mechanical engineering	5,910	5,083	10,993	27.3%	20.4%
Other Fields	3,930	2,079	6,009	18.2%	8.3%

Notes: ∗ Weighting by owners does not affect the numbers, since they all get added back into the same cell. Weighting by classes means that a patent in multiple TF34 sectors is downweighted in each of the sectors.

The patent data come from the EPO Worldwide Patent Statistical Database (PATSTAT). Data on UK and EPO patent publications by British entities were downloaded from PATSTAT version April 2011. Due to the average 18 months delay between the filing and publication date of a patent, using the April 2011 version means that the patent data are presumably only complete up to the third quarter in 2009. This effectively means that we can use the patent data only up to 2009 under the caveat that it might be somewhat incomplete for 2009. Patent data are allocated to firms by the year in which a firm applied for the patent.

Since patent records do not include any kind of registered number of a company, it is not possible to merge data sets using a unique firm identifier; instead, applicant names in the IP documents and firm names in FAME have to be matched. Both a firm's current and previous name(s) were used for matching in order to account for changes in firm names. Matching on the basis of company names required names in both data sets to be 'standardized' prior to the matching process. For more details on the matching see Helmers, Rogers, and Schautschick (2011).

Table C-2: Number of TF34 classes entered between 2002 and 2009

Number of classes	**Number of firms**	**Number of entries**
1	3,110	3,110
2	1,696	3,392
3	765	2,295
4	384	1,536
5	215	1,075
6	115	690
7	68	476
8	57	456
9	44	396
10	33	330
11	16	176
12	11	132
13	5	65
14	9	126
15 or more	26	454
Total	**6,554**	**14,709**

Table C-3: Sample population of UK firms, by industry

	Industry	Number of firms	Number of patenters	Share patenting	Number of patents
				2001-2009	
1	Basic metals	2,992	71	2.37%	659
2	Chemicals	4,140	312	7.54%	3002
3	Electrical machinery	4,242	371	8.75%	2071
4	Electronics & instruments	9,173	745	8.12%	6161
5	Fabricated metals	25,963	778	3.00%	3082
6	Food, beverage, & tobacco	10026	129	1.29%	526
7	Machinery	10,823	824	7.61%	5749
8	Mining, oil&gas	49,548	152	0.31%	1218
9	Motor vehicles	2,569	137	5.33%	1190
10	Other manufacturing	66,321	1831	2.76%	8341
11	Pharmaceuticals	1,122	132	11.76%	2180
12	Rubber & plastics	6,798	514	7.56%	2081
13	Construction	315426	563	0.18%	1638
14	Other transport	3,501	126	3.60%	1319
15	Repairs & retail trade	142,934	374	0.26%	1724
16	Telecommunications	16,305	188	1.15%	3115
17	Transportation	65,906	104	0.16%	424
18	Utilities	14,789	128	0.87%	396
19	Wholesale trade	148,511	986	0.66%	3639
20	Business services	782,561	2660	0.34%	17795
21	Computer services	199,364	1070	0.54%	4272
22	Financial services	225,434	281	0.12%	1559
23	Medicalservices	42,542	167	0.39%	902
24	Personal services	102,986	321	0.31%	1079
25	R&D services	8,693	1173	13.49%	9186
	Total	**2,262,669**	**14,137**	**0.62%**	**83,308**

Table C-4: Entry into techology area 2002-2009

Technology	Numbers			Shares		
	Total patenting in tech class by GB firms	First time patenter	Patented previously in another tech	Total entry rate	First time patenter	Patented previously in another tech
Electrical machinery, apparatus, energy	3,584	416	197	17.1%	11.6%	5.5%
Audio-visual technology	2,138	326	160	22.7%	15.2%	7.5%
Telecommunications	3,817	321	137	12.0%	8.4%	3.6%
Digital communication	2,205	209	118	14.8%	9.5%	5.4%
Basic communication processes	748	72	65	18.3%	9.6%	8.7%
Computer technology	4,181	519	165	16.4%	12.4%	3.9%
IT methods for management	923	291	128	45.4%	31.5%	13.9%
Semiconductors	898	113	105	24.3%	12.6%	11.7%
Optics	1,393	154	117	19.5%	11.1%	8.4%
Measurement	4,160	478	188	16.0%	11.5%	4.5%
Analysis of biological materials	1,125	143	95	21.2%	12.7%	8.4%
Control	1,993	367	195	28.2%	18.4%	9.8%
Medical technology	3,766	393	159	14.7%	10.4%	4.2%
Organic fine chemistry	3,495	110	77	5.4%	3.1%	2.2%
Biotechnology	2,005	119	78	9.8%	5.9%	3.9%
Pharmaceuticals	4,072	143	66	5.1%	3.5%	1.6%
Macromolecular chemistry, polymers	825	90	88	21.6%	10.9%	10.7%
Food chemistry	902	97	77	19.3%	10.8%	8.5%
Basic materials chemistry	2,480	205	118	13.0%	8.3%	4.8%
Materials, metallurgy	914	176	92	29.3%	19.3%	10.1%
Surface technology, coating	1,250	214	161	30.0%	17.1%	12.9%
Chemical engineering	2,498	345	147	19.7%	13.8%	5.9%
Environmental technology	1,184	261	111	31.4%	22.0%	9.4%
Handling	2,805	528	215	26.5%	18.8%	7.7%
Machine tools	1,237	226	163	31.4%	18.3%	13.2%
Engines, pumps, turbines	2,430	206	157	14.9%	8.5%	6.5%
Textile and paper machines	997	183	104	28.8%	18.4%	10.4%
Other special machines	2,273	396	183	25.5%	17.4%	8.1%
Thermal processes and apparatus	1,074	216	123	31.6%	20.1%	11.5%
Mechanical elements	3,155	528	231	24.1%	16.7%	7.3%
Transport	2,919	462	189	22.3%	15.8%	6.5%
Furniture, games	2,451	514	169	27.9%	21.0%	6.9%
Other consumer goods	1,916	366	188	28.9%	19.1%	9.8%
Civil engineering	4,357	763	193	21.9%	17.5%	4.4%
Total	**76,170**	**9,950**	**4,759**	**19.3%**	**13.1%**	**6.2%**

Table C-5: UKIPO and EPO patents: numbers, triples and network density 2002-2009

Technology categories	Aggregate EPO patents	Number of EPO triples‡	Triples per 1000 patents	US Citation network density†	references references	Growth in non-pat refs	Industries w/ no patents*
Electrical machinery, apparatus, energy	56,652	7751	136.8	39.3	0.300	-0.182	2
Audio-visual technology	34,753	13268	381.8	63.5	0.332	-0.300	0
Telecommunications	62,485	27049	432.9	78.5	0.886	0.355	5
Digital communication	36,837	16529	448.7	179.1	1.137	0.501	8
Basic communication processes	10,167	2289	225.1	110.7	1.022	0.012	10
Computer technology	60,755	21956	361.4	54.2	1.164	0.338	2
IT methods for management	9,436	34	3.6	141.5	0.507	0.177	7
Semiconductors	24,511	9974	406.9	94.2	0.904	-0.041	4
Optics	28,676	7767	270.9	58.0	0.672	-0.228	5
Measurement	44,304	2503	56.5	45.7	0.883	0.257	3
Analysis of biological materials	11,742	26	2.2	320.4	9.981	0.425	25
Control	17,617	308	17.5	111.9	0.328	0.007	3
Medical technology	65,903	4411	66.9	206.1	0.491	0.491	4
Organic fine chemistry	40,937	3993	97.5	33.1	3.926	0.542	8
Biotechnology	33,008	365	11.1	88.4	15.716	0.688	8
Pharmaceuticals	52,619	11222	213.3	76.9	8.171	0.603	9
Macromolecular chemistry, polymers	21,129	3722	176.2	92.2	0.957	0.262	7
Food chemistry	10,190	140	13.7	328.0	4.901	0.592	8
Basic materials chemistry	27,422	1929	70.3	84.6	1.531	0.485	5
Materials, metallurgy	16,868	405	24.0	90.4	0.821	0.020	4
Surface technology, coating	17,512	363	20.7	59.5	0.728	0.087	0
Chemical engineering	24,568	443	18.0	66.1	0.632	0.169	2
Environmental technology	12,656	858	67.8	208.7	0.396	0.094	3
Handling	30,340	252	8.3	67.2	0.049	-0.308	1
Machine tools	24,045	508	21.1	64.1	0.090	-0.496	3
Engines, pumps, turbines	32,508	6678	205.4	86.5	0.113	-0.401	2
Textile and paper machines	23,266	2640	113.5	85.2	0.256	-0.236	5
Other special machines	29,903	319	10.7	65.7	0.543	-0.168	1
Thermal processes and apparatus	15,237	335	22.0	146.1	0.096	-0.345	5
Mechanical elements	32,822	1301	39.6	57.3	0.053	-0.447	1
Transport	48,904	10929	223.5	68.2	0.067	-0.381	1
Furniture, games	19,758	206	10.4	108.3	0.050	-0.074	0
Other consumer goods	19,736	301	15.3	106.4	0.099	-0.016	1
Civil engineering	28,826	171	5.9	117.5	0.061	-0.201	0
Total	**1,026,089**	**160,945**	**156.9**	**100.3**			
Electrical engineering	295,596	98,850	334.4	60.3	0.791	0.107	
Instruments	168,242	15,015	89.2	96.3	1.270	0.191	
Chemistry	256,907	23,440	91.2	71.1	4.938	0.354	
Mechanical engineering	237,025	22,962	96.9	70.2	0.152	-0.348	
Other Fields	68,320	678	9.9	111.4			

Notes: ‡ Triples based on all EPO patenting, priority years 2002-2009 (see text for definition and further explanation). † Network density is 1,000,000 times the number of within technology citations between 1976 and the current year divided by the potential number of such citations. ∗ Number of industries with no patenting in this technology class.

Table C-6: Descriptive statistics 2002-2009

	Geom. Mean	Median	Std. dev.‡	q25	q75	Minimum	Maximum
Class variables (34 tech classes for 8 years = 272 observations)							
Network density	64.36	58.40	0.59	43.20	87.64	16.72	301.18
Triples density in class	0.047	0.054	1.56	0.013	0.209	0.001	0.533
Patents in class	3,256.13	3,367.14	0.54	2,220	4,695	995	9,515
Average non-patent refs in class	0.86	0.87	1.52	0	1	0	19
5-year growth of average non-patent refs in class	-0.08	-0.03	0.42	-0.23	0.18	-1.34	0.79
Firm variables (22,316 firms for 8 years, unbalanced = 160,211 observations)							
Assets (millions GBP)	1.18	1.39	2.73	0.21	6.08	0.001	2,053,036
Age of firm in years	15	16	0.89	8	27	1	146
Firm patent stock to date	1.27	1.00	0.72	1.00	1.02	1.00	3,214

Notes: ‡ Standard deviation of the log for all variables except the growth rate in NPL refs. The firm's patent stock is computed using a declining balance formula with 15% depreciation rate.

D Robustness Tests on Network Density and Triples

This section examines how the addition of the network density measure to the analysis undertaken in Graevenitz, Wagner, and Harhoff (2013) changes the coefficient and sign of the triples measure reported there. They examine how complexity, technological opportunity and other variables affect the number of patents firms apply for. Graevenitz, Wagner, and Harhoff (2013) do not distinguish between hold-up and complexity in their model and use the triples measure to capture complexity. The model presented in this paper separates the effects of hold-up and complexity and predicts that hold-up will reduce firms' patenting incentives, while complexity raises these. In this paper, the network density measure is introduced as a measure of technological complexity, while we argue that the triples measure captures hold-up.

The exercise undertaken in this appendix is a validation of these two measures in light of the updated model we present in this paper. The evidence provided is based on two data sets: first we report regression results obtained by adding the network density measure to the data used by Graevenitz, Wagner, and Harhoff (2013), second we report results obtained from a new dataset. This dataset covers the same period as that used by Graevenitz, Wagner, and Harhoff (2013), but it is based on the same more recent technology area classification as that used in this paper. Furthermore, it is based on the same measure of triples as that used in this paper.

The models presented below are system GMM models which include a lagged dependent variable. We demonstrate that regardless of how the data are constructed the triples measure reduces patenting while the network density measure increases patenting efforts in our data. This supports our view that the triples measure is a measure of hold-up.

In the results presented below we instrument potentially endogenous variables using lagged values. Exogeneity of the instruments is tested using difference-in Hansen tests. We instrument the lagged dependent variable and its interaction with fourth order lags. All other variables are instrumented with third order lags or higher. We include only year and area dummies in the levels equations as it is likely that the fixed effects are correlated with differences in the remaining explanatory variables.

Instrument sets are collapsed in order to reduce the number of instruments used. Throughout we rely on the Hansen test to determine whether instruments are exogenous. Where the statistic indicated that this was not the case we rejected the models. We report only those models that were not rejected by the test for which the lagged dependent variable was within the range one would expect from estimation of OLS models with the same specification.

D.1 Sample and Definition of Variables

The sample used for both tables below consists of all firms that have at least one hundred patent applications at the EPO across all technology areas between 1987 and 2002 and who have applied for patents in at least three years in the sample period in a technology area.

The two tables below include a number of variables that we do not use in this paper other than here. We briefly discuss these variables next:

Dependent variable In both tables below the dependent variable is the logarithm of the number of patents each firm has applied for in a technology area and year. To deal with missing values arising from firms not having patent applications in some years we add one to all patent counts before taking the logarithm.

Triples count In Table D-1 below we use the triples count employed by Graevenitz, Wagner, and Harhoff (2013). They count how often firm triples arise, such that each firm in a triple holds patents that are cited as limiting one or more patent applications submitted by each of the other two firms. Their measure of triples is constructed using only the ten most frequently cited firms in each applicant's patent portfolio in any area and year. In Table D-2 we use the same triples count as in this paper, i.e. the restriction to the most frequently cited firms is removed.

Fragmentation The fragmentation measure used here is based on Ziedonis (2004). The measure is based only on critical references and captures the concentration of prior art cited in the patent portfolio of a firm in a year and area.

Large / Relative Size In Table D-1 below we use a dummy variable that is one for all firms above the median firm by size of patent portfolio in each area and year. In Table D-2 we capture relative size by measuring the size of each firm's patent portfolio by area up to a given year relative to the total number of all firms' patent applications in that area and year.

Table D-1: GMM models for patent applications - Old data

Variable	SGMM **E**	SGMM **K**	SGMM **L**	SGMM **M**
log Patentcount$_{t-1}$	0.749***	0.879***	1.020***	0.976***
	(0.093)	(0.099)	(0.113)	(0.106)
log Patentcount$_{t-1}$× Triples	−0.017***	−0.017***	−0.013***	−0.012***
	(0.003)	(0.002)	(0.002)	(0.002)
Non Patent References (NPR)	1.553***	1.863***	1.648***	1.389***
	(0.254)	(0.252)	(0.240)	(0.183)
NPR × Triples	−0.036***	−0.038***	−0.033***	−0.028***
	(0.006)	(0.005)	(0.005)	(0.004)
NPR × Triples × Large	0.007***	0.006***	0.005***	0.005***
	(0.002)	(0.001)	(0.001)	(0.001)
NPR × Large	−0.366***	−0.386***	−0.339***	−0.340***
	(0.081)	(0.062)	(0.051)	(0.043)
Fragmentation	−0.474**	−0.521**	−0.543**	−0.490***
	(0.170)	(0.182)	(0.174)	(0.129)
Fragmentation × Triples	0.006	0.009*	0.005	0.004
	(0.006)	(0.004)	(0.004)	(0.004)
Triples	0.055***	0.052***	0.046***	0.039***
	(0.010)	(0.007)	(0.007)	(0.005)
Areas	0.096***	0.084***	0.049**	0.050**
	(0.012)	(0.010)	(0.017)	(0.016)
Large	0.342**	0.476***	0.427***	0.424***
	(0.117)	(0.105)	(0.091)	(0.072)
Network Density			0.002**	0.002**
			(0.001)	(0.001)
Year dummies	YES	YES	YES	YES
Primary area dummies	YES	YES	YES	YES
Constant	−1.443***	−1.846***	−2.017***	−1.772***
	(0.319)	(0.267)	(0.270)	(0.237)
N	173448	173448	173448	173448
m1	−10.860	−10.470	−9.318	−9.634
m2	4.739	6.307	6.2	6.305
m3	.896	.509	−.071	−.302
Hansen	10.988	2.017	4.454	12.506
p-value	.052	.569	.216	.052
Degrees of freedom	5	3	3	6

* p<0.05, ** p<0.01, *** p<0.001

1. Asymptotic standard errors, asymptotically robust to heteroskedasticity are reported in parentheses

2. m1-m3 are tests for first- to third-order serial correlation in the first differenced residuals.

3. Hansen is a test of overidentifying restrictions. It is distributed as χ^2 under the null of instrument validity, with degrees of freedom reported below.

4. In all cases GMM instrument sets were collapsed and lags were limited.

D.2 Data used by Graevenitz, Wagner, and Harhoff (2013)

The results presented here are based on adding the network density measure of complexity constructed for this paper from US patent data to the data used by Graevenitz, Wagner, and Harhoff (2013). They study the determinants of the level of patent applications at the EPO.

Column E in Table D-1 above is reported by Graevenitz, Wagner, and Harhoff (2013) and is presented here as a reference point. Models K,L,M contain new results. Model K replicates model E closely, differences are likely due to updates to the code used to estimate these models. Model L is like model K with the network density measure added. In model M we adjust the set of instruments to obtain a model with a lagged dependent variable that is significantly below 1 at the 5% level at the mean of the triples variable.

These results show that adding the network density measure to the data does not change the sign or significance of the other variables reported in Table D-1. Network density itself has a positive and significant effect on patenting in models L and M. We would expect to see this, if this measure captures complexity.

Adding network density does have an important and not immediately obvious effect. The range of values of non-patent references for which an increase in the hold-up measure (triples) reduces patenting incentives is larger in model M than model E at the mean of the patent count: in model E non-patent references must lie beyond 1.22 for an increase in triples to have a negative effect on patenting incentives, while in model M non-patent references beyond 1.1 have the same effect. Similarly the range of values of the patent count for which an increase in the hold-up measure (triples) reduces patenting incentives is larger in model M than model E at the mean of non patent references. This shows that the triples measure contained in this data has a negative effect on patenting incentives in sufficiently complex technologies. We would expect to see this, if triples is a measure of hold-up.

D.3 New Data at 34 Area Level

Here we present results based on an updated dataset of patenting in Europe that is based on PATSTAT, October 2014, but covers the same range of years (1987-2002), for better comparability with the data presented in the previous section.

The main difference between the data used here and the older data used by Graevenitz, Wagner, and Harhoff (2013) is that we now rely on a more recent, slightly finer specification of the number of technology areas: the current classification contains 34 rather than 30 areas.

In addition, the triples measure we use now captures all triples and not just those affecting each firm and its ten closest technology rivals as in the earlier data. Due to the larger number of areas we now exclude slightly more patentees when applying the criterion that a firm must have at least one hundred patents in a technology area and must have at least three years of patent activity in an area to be included in the analysis.

Table D-2 demonstrates that the predicted negative effects of triples and non-patent references are present in all specifications we report. We also show that network density is either not significant or positive and significant. The model in which the measure is positive and significant is our prefered model, due to the low instrument count and the better test of overidentifying restrictions.

Table D-2: GMM models for patent applications - New data

Variable	SGMM E	SGMM K	SGMM L
log Patentcount$_{t-1}$	0.454***	0.799***	0.808***
	(0.111)	(0.080)	(0.082)
Non Patent References (NPR)	−0.014***	−0.014***	−0.014***
	(0.001)	(0.001)	(0.001)
Triples	−0.091***	−0.100***	−0.117***
	(0.016)	(0.011)	(0.012)
Network Density	−0.216	0.195	0.217*
	(0.129)	(0.101)	(0.107)
Fragmentation	0.899***	0.658***	0.804***
	(0.155)	(0.093)	(0.085)
Areas	0.020*	0.016**	−0.001
	(0.009)	(0.006)	(0.006)
Relative Size	−0.010***	0.000	−0.000
	(0.003)	(0.003)	(0.002)
Constant	−0.033	−0.208***	−0.149**
	(0.045)	(0.052)	(0.046)
N	168066	168066	168066
m1	−7.598	−12.948	−13.08
m2	3.871	9.964	9.896
m3	−1.450	.538	.055
Hansen	3.49	3.79	2.1
p-value	.48	.29	.55
Degrees of freedom	4	3	3

* $p<0.05$, ** $p<0.01$, *** $p<0.001$

1. Asymptotic standard errors, asymptotically robust to heteroskedasticity are reported in parentheses

2. m1-m3 are tests for first- to third-order serial correlation in the first differenced residuals.

3. Hansen is a test of overidentifying restrictions. It is distributed as χ^2 under the null of instrument validity, with degrees of freedom reported below.
4. In all cases GMM instrument sets were collapsed and lags were limited.

Overall, both sets of models demonstrate that an interpretation of the triples measure as a measure of hold-up only, and of the network density as a measure of complexity, is consistent with the effects we observe in the data on patent applications presented here.

E Estimating Survival Models

This appendix gives some further information about the various survival models we estimated and the robustness checks that were performed. We estimated two general classes of failure or survival models: 1) proportional hazard, where the hazard of failure over time has the same shape for all firms, but the overall level is proportional to an index that depends on firm characteristics; and 2) accelerated failure time, where the survival rate is accelerated or decelerated by the characteristics of the firm. We transform (2) to a hazard rate model for comparison with (1), using the usual identity between the probability of survival to time t and the probability of failure at t given survival to $t-1$. The first model has the following form:

$$Pr\left(i \text{ first patents in } j \text{ at } t \,|\, i \text{ has no patents in } j \ \forall s < t, X_i\right)$$
$$h(X_i, t) = h(t)\exp\left(X_i, \beta\right)$$

where i denotes a firm, j denotes a technology sector, and t denotes the time since entry into the sample. $h(t)$ is the baseline hazard, which is either a non-parametric or a parametric function of time since entry into the sample. The impact of any characteristic x on the hazard can be computed as follows:

$$\frac{\partial h(X_i,t)}{\partial x_i} = h(t)\exp\left(X_i,\beta\right)\beta \quad \text{or} \quad \frac{\partial h(X_i,t)}{\partial x_i}\frac{1}{X_i,t} = \beta \tag{E.1}$$

Thus if x is measured in logs, β measures the elasticity of the hazard rate with respect to x. Note that this quantity does not depend on the baseline hazard $h(t)$, but is the same for any t. We use two choices for $h(t)$: the semi-parametric Cox estimate and the Weibull distribution pt^{p-1}. By allowing the Cox $h(t)$ or p to vary freely across the industrial sectors, we can allow the shape of the hazard function to be different for different industries while retaining the proportionality assumption.

In order to allow even more flexibility across the different industrial sectors, we also use two accelerated failure time models, the log-normal model and the log-logistic model. These have the following basic form:

$$\text{Log-normal} : S(t) = 1 - \Phi \left\{ \frac{\ln(t) - \mu}{\sigma} \right\} \tag{E.2}$$

$$\text{Log-logistic} : S(t) = \frac{1}{1 + (\lambda t)^{\frac{1}{\gamma}})} \tag{E.3}$$

where $S(t)$ is the survival function and $\lambda_i = \exp(X_i\beta)$. We allow the parameters σ (log-normal) or γ (log-logistic) to vary freely across industries (j). That is, for these models, both the mean and the variance of the survival distribution are specific to the 2-digit industry. In the case of these two models, the elasticity of the hazard with respect to a characteristic x depends on time and on the industry-specific parameter (σ or γ), yielding a more flexible model. For example, the hazard rate for the log-logistic model is given by the following expression:

$$h(t) = \frac{-d \log S(t)}{dt} = \frac{\lambda^{\frac{1}{\gamma}} t^{\frac{1}{\gamma}-1}}{\gamma \left(1 + (\lambda t)^{\frac{1}{\gamma}})\right)} \tag{E.4}$$

From this we can derive the elasticity of the hazard rate with respect to a regressor x:[34]

$$\frac{\partial \log h_{ij}(t)}{\partial x_i} = \frac{-\beta}{\left(1 + (\lambda t)^{\frac{1}{\gamma}}\right)} \tag{E.5}$$

One implication of this model is therefore that both the hazard and the elasticity of the hazard with respect to the regressors depend on t, the time since the firm was at risk of patenting. We sample the firms during a single decade, the 2000s, but some of the firms have been in existence since the 19th century. This fact creates a bit of a problem for estimation, because there is no reason to think that the patenting environment has remained stable during that period. We explored variations in the assumed first date at risk in Tables E-1 (1978) and E-2 (1900), finding that the choice made little difference. Accordingly, we have used a minimum at risk year of 1978 for estimation in the main table in the text.

[34]We assume that x is in logarithms, as is true for our key variables, so this can be interpreted as an elasticity.

Table E-1: Hazard of entry into patenting in a TF34 Class – Comparing models

Variable	**Proportional hazard**		**AFT**	
	Cox PH	Weibull	Log logistic	Log normal
Log (network density)	0.119***	0.118***	-0.306***	-0.217***
	(0.021)	(0.021)	(0.072)	(0.036)
Log (triples density	-0.111***	-0.108***	0.534***	0.252***
in class)	(0.008)	(0.009)	(0.046)	(0.027)
Log (patents in class)	0.573***	0.579***	-2.224***	-1.030***
	(0.024)	(0.024)	(0.177)	(0.107)
5-year growth of non-	-0.122***	-0.160***	0.504***	0.205***
patent refs in class)	(0.031)	(0.030)	(0.110)	(0.049)
Log assets	0.200***	0.195***	-0.419***	-0.206***
	(0.013)	(0.013)	(0.037)	(0.023)
Log firm age in years	1.178***	-0.189*	-1.409***	0.170*
	(0.103)	(0.106)	(0.181)	(0.093)
Log (pats applied for	1.074***	1.105***	-10.433***	-6.411***
by firm previously)	(0.038)	(0.037)	(1.158)	(0.803)
Log likelihood	-77.3	-132,032.1	-154,859.4	-153,780.3
Degrees of freedom	15	39	39	39
Chi-squared	3522.6	4322.7	364.4	607.9
551,981 firm-TF34 observations with 14,709 entries (22,316 firms)				

Notes: All estimates are weighted estimates, weighted by sampling probability, and stratified by 2-digit industry. For the Cox and Weibull models, coefficients shown are elasticities of the hazard w.r.t. the variable. For the log-logistic and lognormal AFT models, estimates will be opposite in sign to the proportional hazard estimates. AFT - Accelerated Failure Time models. Time period is 2002-2009 and minimum entry year is 1978. Calendar dummies included in all estimations. Sample is UK firms with nonmissing assets, all patenting firms and a matched sample of non-patenting firms. See text for precise sample definition. *** (**) denote significance at the 1% (5%) level. The degrees of freedom are those for the Chi-squared test versus a model with no covariates.

Table E-2: Hazard of entry into patenting in a TF34 Class - Robustness

Variable	(1) **Table 1**	(2) **Include zeros**	(3) **SMEs**	(4) **Giants**	(5) **No Telecomm**	(6) **Entry 1900**
Log (network density)	0.119***	0.041**	0.124***	0.062	0.114***	0.106***
	(0.021)	(0.018)	(0.023)	(0.125)	(0.020)	(0.021)
Log (triples density	-0.111***	-0.202***	-0.134***	-0.051	-0.112***	-0.111***
in class)	(0.008)	(0.007)	(0.010)	(0.047)	(0.008)	(0.008)
Log (patents in class)	0.573***	0.739***	0.659***	0.111	0.575***	0.575***
	(0.024)	(0.023)	(0.029)	(0.127)	(0.024)	(0.025)
5-year growth of non-	-0.122***	-0.582***	-0.039	-0.323	-0.113***	-0.171***
patent refs in class)	(0.031)	(0.025)	(0.037)	(0.196)	(0.031)	(0.031)
Log assets	0.200***	0.195***	0.177***	0.517**	0.199***	0.192***
	(0.013)	(0.012)	(0.019)	(0.214)	(0.013)	(0.013)
Log firm age in years	1.178***	0.988***	0.413***	0.382	1.205***	-0.551***
	(0.103)	(0.117)	(0.138)	(0.949)	(0.102)	(0.181)
Log (lagged firm-level	1.074***	1.273***	1.119***	1.071***	1.080***	1.125***
patent stock)	(0.038)	(0.033)	(0.064)	(0.170)	(0.038)	(0.041)
Observations	551,981	702,982	484,725	8,258	536,634	551,981
Firms	22,316	22,316	19,902	273	22,316	22,316
Entries	14,709	14,709	11,000	351	14,251	14,709
Entry rate	2.66%	2.09%	1.96%	4.25%	2.66%	2.66%
Log likelihood	-77.27	-72.54	-53.31	-0.27	-74.92	-70.66
Degrees of freedom	15	15	15	15	15	15
Chi-squared	3522.6	5134.0	1968.5	233.1	3512.0	2879.5

Notes: All estimates are weighted estimates, weighted by sampling probability. Larger coefficients indicate increases in entry probability. *** (**) denote significance at the 1% (5%) level. The degrees of freedom are those for the Chi-squared test versus a model with no covariates. Sample is UK firms with nonmissing assets, all patenting firms and a matched sample of non-patenting firms. Time period is 2002-2009 and minimum entry year is 1978, with the exception of column (6). Calendar year dummies in all models. Cox proportional hazard model stratified by industry. (1) Estimates from Table 1, for comparison. (2) Observations for tech sectors of firms whose industry has no such patenting (Lybbert-Kolas) and where there is no entry by any UK firm in that industry are included. (3) SMEs: firms with assets¡12.5 million GBP. (4) Firms with assets¿100 million GPB. (5) The Telecom tech sector is removed. (6) The minimum founding year is 1900 instead of 1978.

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