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Boundary vibration control of a floating wind turbine system with mooring lines[☆]

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ABSTRACT

In this paper, we investigate dynamic modeling, active boundary control design, and stability analysis for a coupled floating wind turbine (FWT) system, which is connected with two flexible mooring lines. It is a coupled beam-strings structure, and we design two boundary controllers to restrain the vibrations of this flexible system caused by external disturbances based on the coupled partial differential equations and ordinary differential equations (PDEs–ODEs) model. Meanwhile, significant performance of designed boundary controllers and system's stability are theoretically analyzed, and a set of simulation results are provided to show efficacy of the proposed approach.

1. Introduction

The offshore area has abundant wind energy resource, especially in deep water area (> 100 m), and floating platforms are widely used in energy exploitation in offshore engineering. In offshore area, floating wind turbine (FWT) system, Fig. 1, is a large wind energy conversion equipment including wind generator tower, floating platform, and mooring lines. It is suitable for a harsh deep-sea environment where the bottom-mounted tower is infeasible (Lamas & Fernandez, 2011). However, the complex marine environment has a great impact on FWT system, and turbine tower and mooring lines will generate wave-induced oscillation, which may cause irreversible damage and premature fatigue. Thus, for the conversion efficiency and safety concern, structural vibrations of FWT system need to be studied and addressed.

In recent years, many works focus on vibration control design and dynamic analysis for the FWT systems. In previous works, many researchers use passive control methods to address the vibration reduction of the system, In general, passive control is a control without external energy, and the control force is generated passively by vibration deformation of the control device along the structure or the movement of the device itself. Common passive control includes tuned mass damper (De Domenico & Ricciardi, 2018; Stewart & Lackner, 2013; Tong, Zhao, & Zhao, 2017), tuned liquid damper (Fu, Jiang, & Wu, 2019; Zhang, Basu, & Nielsen, 2019), friction damper, etc. In Si, Karimi, and Gao (2013), the OC3-Hywind FWT system is discussed with a passive structural controller. Another vibration is active control which is to use external energy to exert the control force and the control device can use external energy to tune its own parameters for adjusting control force. Therefore, active control may have a better performance than passive control. In Bakka, Karimi, and Christiansen (2014), the authors analyze dynamic behavior of an offshore wind turbine system by considering linear varying parameter and constrained information, and establish a dynamic model. In Bakka and Karimi (2013), an output-feedback control method for an offshore FWT system with constraint is proposed to reduce wave-oscillations. In Bakka and Karimi (2012), many kinds of linear models are introduced to represent a wind turbine system. In addition, a robust output feedback controller integration using H_∞ control with pole setting constraints is investigated for a wind turbine system. In Li and Gao (2016), the authors propose a novel H_∞ control method based on a linear model and apply it to an FWT system. These works make great achievements in the modeling and control of the FWT system. However, they analyze dynamics of the FWT and address the control problem based on the linear or finite-dimensional model because of regarding the wind turbine tower as a

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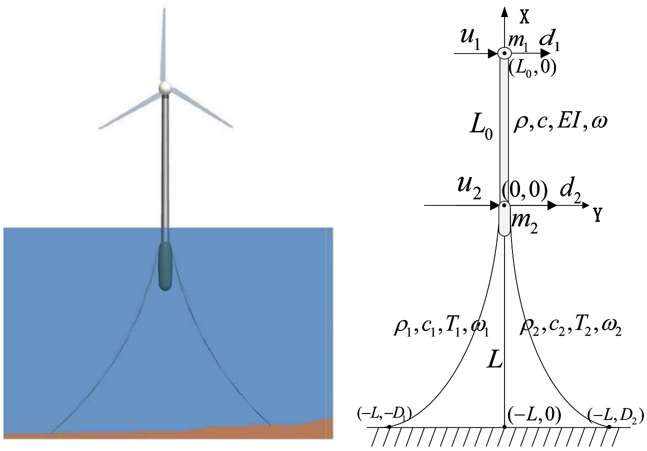

Fig. 1. The floating wind turbine system.

rigid body system. From the structure of FWT, it is a coupled beam-strings system which is characterized with infinite modes (Guo & Jin, 2015; Ren, Wang, & Krstic, 2013; Wu & Wang, 2014; Wu, Wang, & Li, 2012), namely, using PDEs to describe the system's dynamics. In this work, we consider the FWT system as a flexible coupled structure and address the vibration suppression problem of the turbine tower and mooring lines.

For vibration reduction of flexible structures, many vibration control methods are developed (Bhikkaji, Moheimani, & Petersen, 2012; Chen & Chang, 2013; Do, 2017; He, He, Shi & Sun, 2017; He, He & Sun, 2017; Jin & Guo, 2015; Liu, He, Zhao, Ahn, & Li, 2020; Pota & Alberts, 1995), e.g., adaptive control method (Chao, Chenguang, & Zhaopeng, 2020; Chen, Shao, & Jiang, 2017; Dai, He, Wang, & Yuan, 2018; He, He, Liu, Wang, Li, & Wang, 2018; Li, Chen, Fu & Sun, 2016; Li et al., 2017; Li, Wang, Du & Boulkroune, 2016; Ren, Chen, & Liu, 2020; Xie, Sun, Wen, Hei, & Qian, 2019), optimal control method (Sun & Xia, 2009; Wu, Sun, & Chen, 2018) and sliding mode control method (Wang, Liu, Ren, & Chen, 2015), force control method (Endo, Sasaki, & Matsuno, 2017; Endo, Sasaki, Matsuno, & Jia, 2016), etc., are also investigated. Compared with rigid structure (Li, Chen, Wang, Zhang, & Wang, 2019; Ming, Yanlu, Huifang, & Junzhi, 2020), flexible structures has the advantages of light weight and strong flexibility. In Moheimani, Pota, and Petersen (1999), a novel spatial balanced model is established for a flexible pinned–pinned beam, and vibration control is proposed based on the system model. In Bhikkaji et al. (2012), the vibration control problem of a flexible beam with collocated sensor/actuator pairs is investigated. Notice that some flexible systems mentioned above are discretized as a truncated model, namely considering finite modes of the system. Furthermore, the control design for flexible systems is conducted based on the discretized model. However, these unmodeled modes, including high-frequency modes, may have a great impact on the system performance, and it is valuable to consider the influences of these modes.

Boundary control is a control method which considers the dynamics of entire modes and has been applied to various infinite-dimensional systems widely, e.g., heat equations (Huang, Xu, Li, Xu, & Yu, 2013; Wang, Ren, & Krstic, 2012), moving string system (He, Ge, & Huang, 2015; Nguyen & Hong, 2010, 2012), etc. For the wind turbine tower system, there exist some literature to address the vibration reduction problem of the tower based on boundary control strategy. A nonlinear vibration isolator is proposed to reduce the structural impact of turbine under seismal loading and wind in Van der Woude (2011), which is a novelty work used in the wind turbine. In He and Ge (2015), a vibration controller is investigated for a nonuniform wind turbine tower with random wind loads. However, these works consider the dynamics of a single flexible structure and focus on the vibration suppression of wind turbine tower. In this work, we consider the vibration control of FWT system, which is a coupled beam-strings system. The main challenges are to consider the strong coupling between turbine tower and mooring lines, and to develop boundary controllers to reduce vibrations of FWT system. For mooring systems, motions of the mooring lines are usually regarded as an external force acted to floating platforms in the previous works. In Chen, Ge, How, and Choo (2013), Tee and Ge (2006), the authors study dynamics of mooring lines by ignoring the connection of the mooring lines and the ship. Nevertheless, the coupling relationship has a great influence on the performance of mooring system. Notice that we will consider the strong coupling between mooring lines, floating platform, and flexible turbine tower, and establish a nonlinear dynamic model. Furthermore, we will propose two boundary controllers simultaneously to reduce the vibration of flexible turbine tower and mooring lines and to compensate for the performance degradation.

In brief, main works of this paper are summarized as

(i) A coupled beam-strings model is used to illustrate the dynamical behavior of FWT system, and the coupled relationship among turbine tower, floating platform, and mooring lines is considered in dynamic analysis. A coupled hybrid PDE-ODEs model is established to describe FWT system by applying Hamilton's principle.
(ii) Two boundary controllers are designed to ensure the stability of the system simultaneously based on nonlinear coupled model under unknown environment loads. Theoretical analysis and simulation results are presented to demonstrate the effectiveness of proposed controllers.

The rest of this paper are summarized as follows. In Section 2, we establish the system model described by PDEs using Hamilton's principle, to derive the governing equations (described by PDEs) and boundary conditions (described by ODEs) of FWT system. In Section 3, the control objectives and uniform boundedness of the closed-loop system are derived by Lyapunov's direct method. In Section 4, simulation results demonstrate the efficacy of the proposed approach. In Section 5, we give conclusions of this paper and future works.

2. System modeling and preliminaries

As shown in Fig. 1, FWT system is described by a coupled beam-strings structure, where, the turbine tower is regarded as a flexible beam with a lumped-mass-like payload, and mooring lines are regarded as two coupled strings (Zhang, Yang, Nie, & He, 2015). The control force $u_1(t)$ is mounted on the top of tower and the control force $u_2(t)$ is mounted on the floating platform. L_0 and L are the tower's and mooring lines' length, m_1 and m_2 are the mass of top of tower and floating platform, respectively. The mass per unit length of the tower is ρ. D_1 and D_2 are original locations of mooring lines. ρ_1 and ρ_2 are the mass per unit length of mooring lines, respectively. c, c_1 and c_2 are the damping coefficients of flexible tower and mooring lines. ω_1 represents the displacement of left line, and ω_2 represents the displacement of right line. ω represents the displacement of wind turbine tower. Moreover, EI represents the bending stiffness of beam-like tower, and tensions of two mooring lines are expressed as T_1 and T_2, respectively. $X-Y$ coordinate system is a plane coordinate system, where, X is used to determine the position and Y is the displacement of mooring lines and flexible wind tower. Meanwhile, floating platform is fixed on the sea level by mooring lines, its position will not follow the FWT system.

Remark 1. For conciseness, the notations $w'' = \partial^2 w/\partial x^2, \ddot{w} = \partial^2 w/\partial t^2, w' = \partial w/\partial x, \dot{w} = \partial w/\partial t, w = w(x,t), w_{L_0} = w(L_0,t)$ and $w_0 = w(0,t)$ are used.

2.1. Dynamical model of floating wind turbine vibrations

The FWT system's kinetic energy $E_k(t)$ can be obtained as

$$E_k(t) = \frac{\rho_1}{2}\int_{-L}^{0}\left(\frac{\partial\left[w_1+D_1\right]}{\partial t}\right)^2 dx + \frac{\rho_2}{2}\int_{-L}^{0}\left(\frac{\partial\left[w_2-D_2\right]}{\partial t}\right)^2 dx + \frac{\rho}{2}\int_{0}^{L_0}\left(\frac{\partial w}{\partial t}\right)^2 dx + \frac{m_1}{2}\left(\frac{\partial w(L_0,t)}{\partial t}\right)^2 + \frac{m_2}{2}\left(\frac{\partial w(0,t)}{\partial t}\right)^2, \quad (1)$$

and the potential energy $E_p(t)$ can be written as

$$E_p(t) = \frac{EI}{2}\int_{0}^{L_0}\left(\frac{\partial^2 w}{\partial x^2}\right)^2 dx + \frac{T_1}{2}\int_{-L}^{0}\left(\frac{\partial\left[w_1+D_1\right]}{\partial x}\right)^2 dx + \frac{T_2}{2}\int_{-L}^{0}\left(\frac{\partial\left[w_2-D_2\right]}{\partial x}\right)^2 dx, \quad (2)$$

where, t and x denote the time variable and the position variable, respectively.

Remark 2. From Fig. 1, because the strong coupling of the two flexible structures, we have $w = w_1 = w_2$ at $x = 0$ and $\partial w/\partial t = \partial w_1/\partial t = \partial w_1/\partial t$ at $x = 0$.

The virtual work done by the model is

$$\delta W_c(t) = -c\int_{0}^{L}\frac{\partial w}{\partial t}\delta w dx - c_1\int_{0}^{L}\frac{\partial\left[w_1+D_1\right]}{\partial t}\delta w_1 dx - c_2\int_{0}^{L}\frac{\partial\left[w_2-D_2\right]}{\partial t}\delta w_2 dx. \quad (3)$$

where the FWT system damping coefficients are represented as c, c_1 and c_2.

The virtual work done by external disturbances $d_1(t)$ and $d_2(t)$ is obtained by

$$\delta W_d(t) = d_1(t)\delta w(L_0,t) + d_2(t)\delta w(0,t). \quad (4)$$

Remark 3. The lumped mass at the top boundary of the tower suffers from the boundary disturbance $d_1(t)$. $d_1(t)$ is the synthesis of all disturbances to the components at the tip boundary of tower. $d_2(t)$ is the wave disturbance mainly about waves with different height, speed etc., for example, the wave with significant wave height and the theoretical wave height (Ainsworth & Juneau, 2006; Holthuijsen, 2007) that can result in large excitation forces. All these excitation forces acting on the FWT system as external disturbances can cause vibrations of tower.

The virtual work $W_u(t)$ done by the controllers $u_1(t)$ and $u_2(t)$ is denoted as

$$\delta W_u(t) = u_1(t)\delta w(L_0,t) + u_2(t)\delta w(0,t). \quad (5)$$

Then, the entire virtual work $W(t)$ done on the system can be represented as

$$\delta W(t) = \delta W_c(t) + \delta W_d(t) + \delta W_u(t). \quad (6)$$

According to the Hamilton's principle (Goldstein, 1951), governing equations of the FWT system including the turbine tower and mooring lines are obtained as

$$\rho\ddot{w} + EIw'''' + c\dot{w} = 0, \quad (x,t)\in(0,L_0)\times[0,\infty). \quad (7)$$

$$\rho_i\ddot{w}_i - T_i w_i'' + c_i\dot{w}_i = 0, i=(1,2) \text{ and } (x,t)\in(-L,0)\times[0,\infty). \quad (8)$$

with boundary conditions

$$w_0'' = w_{L_0}'' = 0, \quad (9)$$

$$m_1\ddot{w}_{L_0} - EIw_{L_0}''' = u_1(t) + d_1(t) \quad (10)$$

$$w_1(-L,t) = -D_1, \quad (11)$$

$$w_2(-L,t) = D_2, \quad (12)$$

$$m_2\ddot{w}_0 + EIw_0''' + T_1 w_1'(0,t) + T_2 w_2'(0,t) = u_2(t) + d_2(t). \quad (13)$$

$\forall t\in[0,\infty)$.

Remark 4. The governing equations (7) and (8) describe the dynamics of flexible structures including two mooring lines and a flexible tower. Meanwhile, the boundary conditions (9), (11), and (12) describe the dynamics of the boundary position of flexible structures and they satisfy the Newton's second law From the form of these equations, $\ddot{w}_0$ and $\ddot{w}_{L_0}$ are the acceleration of floating platform and top of tower. A part of boundary conditions is nonconservative force which includes that $u_2(t)$ and $u_1(t)$ are boundary control forces acting on platform and top of tower, and $d_2(t)$ and $d_1(t)$ are external boundary disturbances. EIw_{L_0}''', EIw_0''' and $T_1 w_1'(0,t)$, $T_2 w_2'(0,t)$ are strain forces of two boundary of flexible tower and tensions of two mooring lines. From Eq. (12), it is easy known that the motion of platform is related to the dynamics of mooring lines and flexible tower, and the coupling relation has an effect on the dynamic modeling of FWT system.

Remark 5. For the FWT system, the distributed disturbances have less influence on the dynamics of the flexible system comparing with the boundary disturbances. In this paper, we ignore the effect of disturbances for its less influence and simplifying the theoretical analysis.

2.2. Preliminaries

For stability analysis, we present lemmas, remark, and assumption for the subsequent development as follows.

Lemma 1 (*He, Zhang, & Ge, 2014*). *For any continuously differentiable ϕ on $[L_1, L_2]$, we have*

$$[\phi - \phi(L_1,t)]^2 \le (L_2 - L_1)\int_{L_1}^{L_2}[\phi']^2 dx. \quad (14)$$

Lemma 2 (*He et al., 2014*). *For any continuously differentiable ϕ on $[L_1, L_2]$, we have*

$$[\phi]^2 \le 2(L_2 - L_1)\int_{L_1}^{L_2}[\phi']^2 dx + 2[\phi(L_1,t)]^2. \quad (15)$$

Lemma 3 (*Poincaré Inequality Smyshlyaev and Krstic (2010)*). *For any ϕ, continuously differentiable on $[L_1, L_2]$, we have*

$$\int_{L_1}^{L_2}[\phi]^2 dx \le 2(L_2-L_1)\phi^2(L_2,t) + 4(L_2-L_1)^2\int_{L_1}^{L_2}[\phi']^2 dx, \quad (16)$$

$$\int_{L_1}^{L_2}[\phi]^2 dx \le 2(L_2-L_1)\phi^2(L_1,t) + 4(L_2-L_1)^2\int_{L_1}^{L_2}[\phi']^2 dx. \quad (17)$$

Remark 6. From Poincaré inequality (16), we further have

$$[\phi]^2 \le 4(L_2-L_1)^2\phi'^2(L_1) + 8(L_2-L_1)^3\int_{L_1}^{L_2}[\phi'']^2 dx + 2[\phi(L_1)]^2. \quad (18)$$

Assumption 1. According to the unknown external boundary disturbances $d_1(t)$ and $d_2(t)$, we assume that the two positive values $\bar{d}_1\in\mathbb{R}^+$ and $\bar{d}_2\in\mathbb{R}^+$ exist, to make $|d_1(t)|\le\bar{d}_1$ and $|d_2(t)|\le\bar{d}_2$ hold. This assumption is appropriate because the disturbances have finite energy.

3. Controller design

In this part, we design controllers $u_1(t)$ and $u_2(t)$ for vibrations suppression of the FWT system under external disturbances.

Theorem 1. *Based on the dynamic model (7) - (8) and (9)–(13), the boundary controllers are proposed as follow*

$$u_1(t) = -k_1\left[\dot{w}_{L_0} + \frac{\alpha}{\beta}w_{L_0}\right] - \text{sgn}\left[\dot{w}_{L_0} + \frac{\alpha}{\beta}w_{L_0}\right]\bar{d}_1 - \frac{\alpha}{\beta}m_1\dot{w}_{L_0}, \quad (19)$$

$$u_2(t) = -k_2\left[\dot{w}_0+\frac{\alpha}{\beta}w_0\right] - k_p w_0 - \text{sgn}\left[\dot{w}_0+\frac{\alpha}{\beta}w_0\right]\bar{d}_2 - \frac{\alpha}{\beta}m_2\dot{w}_0, \quad (20)$$

where k_1, k_2 and k_p are three positive control gains, α and β are the positive constants, and $\text{sgn}(\cdot)$ represents the signum function. The closed-loop system is uniformly bounded as the initial conditions are bounded.

Proof. Consider the Lyapunov candidate function as

$$\Theta(t) = \Theta_1(t) + \Theta_2(t) + \Theta_3(t) + \Delta(t), \quad (21)$$

where $\Theta_1(t)$, $\Theta_2(t)$, $\Theta_3(t)$ and $\Delta(t)$ are defined as

$$\begin{aligned}\Theta_1(t) =& \frac{\beta}{2}\int_0^{L_0}\rho[\dot{w}]^2dx + \frac{\beta}{2}EI\int_0^{L_0}[w'']^2dx \\ &+\frac{\beta}{2}\int_{-L}^{0}\left(\rho_1[\dot{w}_1]^2+\rho_2[\dot{w}_2]^2\right)dx \\ &+\frac{\beta}{2}\int_{-L}^{0}\left(T_1[w_1']^2+T_2[w_2']^2\right)dx, \end{aligned} \quad (22)$$

$$\Theta_2(t) = \frac{\beta}{2}m_1\left[\dot{w}_{L_0}+\frac{\alpha}{\beta}w_{L_0}\right]^2 + \frac{\beta}{2}m_2\left[\dot{w}_0+\frac{\alpha}{\beta}w_0\right]^2 + \frac{\beta}{2}k_p[w_0]^2, \quad (23)$$

$$\Theta_3(t) = \frac{\alpha}{2}\int_0^{L_0}c[w]^2dx + \frac{\alpha}{2}\int_{-L}^{0}c_1[w_1]^2dx + \frac{\alpha}{2}\int_{-L}^{0}c_2[w_2]^2dx, \quad (24)$$

$$\begin{aligned}\Delta(t) =& \alpha\int_0^{L_0}\rho\dot{w}w\,dx + \alpha\int_{-L}^{0}\left[\rho_1\dot{w}_1w_1+\rho_2\dot{w}_2w_2\right]dx \\ &-\gamma\int_{-L}^{0}x\left[\rho_1\dot{w}_1w_1'+\rho_2\dot{w}_2w_2'\right]dx,\end{aligned} \quad (25)$$

where γ is a positive weighting constant.

Consider a positive function $\kappa(t)$ as

$$\begin{aligned}\kappa(t) =& \int_0^{L_0}\left\{[\dot{w}]^2+[w]^2+[w'']^2\right\}dx + \int_{-L}^{0}\left\{[\dot{w}_1]^2+[w_1]^2+[w_1']^2\right. \\ &\left.+[\dot{w}_2]^2+[w_2]^2+[w_2']^2\right\}dx.\end{aligned} \quad (26)$$

From the expression of $\Theta_1(t)$ and $\Theta_3(t)$, we have

$$\mu_1\kappa(t) \le \Theta_1(t)+\Theta_3(t) \le \mu_2\kappa(t), \quad (27)$$

where

$$\mu_1 = \min\left(\frac{\beta\rho}{2},\frac{\beta EI}{2},\frac{\alpha c}{2},\frac{\beta\rho_1}{2},\frac{\beta T_1}{2},\frac{\alpha c_1}{2},\frac{\beta\rho_2}{2},\frac{\beta T_2}{2},\frac{\alpha c_2}{2}\right), \quad (28)$$

$$\mu_2 = \max\left(\frac{\beta\rho}{2},\frac{\beta EI}{2},\frac{\alpha c}{2},\frac{\beta\rho_1}{2},\frac{\beta T_1}{2},\frac{\alpha c_1}{2},\frac{\beta\rho_2}{2},\frac{\beta T_2}{2},\frac{\alpha c_2}{2}\right). \quad (29)$$

From the expression of $\Delta(t)$, we know that $\Delta(t)$ is bounded as $|\Delta(t)| \le \mu_3\kappa(t)$, where $\mu_3 = \max\left(\alpha\rho, \alpha\rho_1+\gamma L\rho_1, \alpha\rho_2+\gamma L\rho_2\right)$.

Considering α, β and γ satisfying $\mu_1-\mu_3>0$, we have

$$0 < \mu_4\left[\Theta_1(t)+\Theta_3(t)\right] \le \Theta_1(t)+\Theta_3(t)+\Delta(t) \le \mu_5\left[\Theta_1(t)+\Theta_3(t)\right], \quad (30)$$

where $\mu_4 = \frac{\mu_1-\mu_3}{\mu_2}$, and $\mu_5 = \frac{\mu_2+\mu_3}{\mu_1}$. Then the Lyapunov candidate function is bounded as

$$\lambda_1\left[\Theta_1(t)+\Theta_2(t)+\Theta_3(t)\right] \le \Theta(t) \le \lambda_2\left[\Theta_1(t)+\Theta_2(t)+\Theta_3(t)\right], \quad (31)$$

where $\lambda_1 = \min\left(\mu_4, 1\right)$, $\lambda_2 = \max\left(\mu_5, 1\right)$.

Lemma 4. *With above assumptions and defines, the time derivative of Lyapunov candidate function (21) is bounded with*

$$\dot{\Theta}(t) \le -\lambda\Theta(t) + \varepsilon, \quad (32)$$

where λ and ε are two positive constants.

Proof. Differentiation $\Theta(t)$ leads to

$$\dot{\Theta}(t) = \dot{\Theta}_1(t)+\dot{\Theta}_2(t)+\dot{\Theta}_3(t)+\dot{\Delta}(t), \quad (33)$$

From Remark 2 and boundary condition (10), using Lamma 3, and substituting the proposed control (19) and (20), and let $\gamma L-\alpha \ge 0$, we further have

$$\begin{aligned}\dot{\Theta}(t) \le& -\beta k_1\left[\dot{w}_{L_0}+\frac{\alpha}{\beta}w_{L_0}\right]^2 - \beta k_2\left[\dot{w}_0+\frac{\alpha}{\beta}w_0\right]^2 \\ &-\left(\alpha k_p-\sigma_1-\sigma_2-\sigma_3\right)[w_0]^2 \\ &-\left(\beta c-\alpha\rho\right)\int_0^{L_0}[\dot{w}]^2dx - \left(\alpha EI-8\sigma_1L_0^3\right)\int_0^{L_0}[w'']^2dx \\ &-\left(\beta c_1-\alpha\rho_1-\frac{\gamma c_1L}{2}-\frac{\gamma}{2}\rho_1\right)\int_{-L}^{0}[\dot{w}_1]^2dx \\ &-\left(\beta c_2-\alpha\rho_2-\frac{\gamma c_2L}{2}-\frac{\gamma}{2}\rho_2\right)\int_{-L}^{0}[\dot{w}_2]^2dx \\ &-\left(\alpha T_1-\frac{\gamma}{2}T_1-\frac{\gamma c_1L}{2}-2\sigma_2L\right)\int_{-L}^{0}[w_1']^2dx \\ &-\left(\alpha T_2-\frac{\gamma}{2}T_2-\frac{\gamma c_2L}{2}-2\sigma_3L\right)\int_{-L}^{0}[w_2']^2dx - \frac{\sigma_1}{2L_0}\int_0^{L_0}w^2dx \\ &-\frac{\sigma_2}{2L}\int_{-L}^{0}w_1^2dx - \frac{\sigma_3}{2L}\int_{-L}^{0}w_2^2dx + \frac{\alpha T_1}{2}D_1^2 + \frac{\alpha T_2}{2}D_2^2.\end{aligned} \quad (34)$$

where σ_1, σ_2 and σ_3 are positive constants. Then, we have

$$\dot{\Theta}(t) \le -\lambda_3\left[\Theta_1(t)+\Theta_2(t)+\Theta_3(t)\right] + \varepsilon, \quad (35)$$

where $\varepsilon = \frac{\alpha T_1}{2}D_1^2 + \frac{\alpha T_2}{2}D_2^2$, and

$$\begin{aligned}\epsilon_1 &= \beta c-\alpha\rho > 0, \qquad \epsilon_2 = \alpha EI - 8\sigma_1L_0^3 > 0,\\ \epsilon_3 &= \beta c_1-\alpha\rho_1-\frac{\gamma c_1L}{2}-\frac{\gamma}{2}\rho_1 > 0,\\ \epsilon_4 &= \beta c_2-\alpha\rho_2-\frac{\gamma c_2L}{2}-\frac{\gamma}{2}\rho_2 > 0,\\ \epsilon_5 &= \alpha T_1-\frac{\gamma}{2}T_1-\frac{\gamma c_1L}{2}-2\sigma_2L > 0,\\ \epsilon_6 &= \alpha T_2-\frac{\gamma}{2}T_2-\frac{\gamma c_2L}{2}-2\sigma_3L > 0,\\ \epsilon_7 &= \beta k_1 > 0, \qquad \epsilon_8 = \beta k_2 > 0,\\ \epsilon_9 &= \frac{\sigma_1}{2L_0} > 0, \qquad \epsilon_{10} = \frac{\sigma_2}{2L} > 0,\\ \epsilon_{11} &= \frac{\sigma_3}{2L} > 0, \qquad \epsilon_{12} = \alpha k_p-\sigma_1-\sigma_2-\sigma_3 > 0,\\ \lambda_3 &= \min\left(\frac{2\epsilon_1}{\beta\rho},\frac{2\epsilon_2}{\beta EI},\frac{2\epsilon_3}{\beta\rho_1},\frac{2\epsilon_4}{\beta\rho_2},\frac{2\epsilon_5}{\beta T_1},\frac{2\epsilon_6}{\beta T_2},\frac{2\epsilon_7}{\beta m_1},\frac{2\epsilon_8}{\beta m_2},\right.\\ &\left.\frac{2\epsilon_9}{\alpha c},\frac{2\epsilon_{10}}{\alpha c_1},\frac{2\epsilon_{11}}{\alpha c_2},\frac{2\epsilon_{12}}{\beta k_p}\right) > 0.\end{aligned}$$

Combining Ineqs. (31) and (35), we have

$$\dot{\Theta}(t) \le -\lambda\Theta(t) + \varepsilon, \quad (36)$$

where $\lambda = \lambda_3/\lambda_2$. Subsequently, we have the following main result of this paper as follows. Integrating of Ineq. (36), we obtain

$$\Theta(t) \le \left(\Theta(0)-\frac{\varepsilon}{\lambda}\right)e^{-\lambda t} + \frac{\varepsilon}{\lambda} \le \Theta(0)e^{-\lambda t} + \frac{\varepsilon}{\lambda} \in \mathcal{L}_\infty. \quad (37)$$

Utilizing inequality (15) and the Poincaré inequality (18), we have

$$\begin{aligned}\frac{\beta}{16L_0^3}EI[w]^2 \le& \frac{\beta}{2}EI\int_0^{L_0}[w'']^2dx + \frac{\beta}{2}k_p[w(0,t)]^2 \\ &-\left(\frac{\beta}{2}k_p - \frac{\beta}{8L_0^3}EI\right)[w(0,t)]^2.\end{aligned} \quad (38)$$

We design k_p satisfying $k_p - \frac{EI}{4L_0^3} \ge 0$, and we further have

$$\begin{aligned}\frac{\beta}{16L_0^3}EI[w]^2 \le& \frac{\beta}{2}EI\int_0^{L_0}[w'']^2dx + \frac{\beta}{2}k_p[w(0,t)]^2 \\ &\le \Theta_1(t)+\Theta_2(t) \le \frac{1}{\lambda_1}\Theta(t) \in \mathcal{L}_\infty.\end{aligned} \quad (39)$$

Suitably rewritten the above inequality, uniformly bounded of w can be obtained as

$$|w| \le \sqrt{\frac{16L_0^3}{\beta\lambda_1EI}\left(\Theta(0)e^{-\lambda t}+\frac{\varepsilon}{\lambda}\right)} \le D_0, \quad (40)$$

where the constant $D_0 = \sqrt{\frac{16L_0^3}{\beta\lambda_1 EI}(\Theta(0)+\frac{\varepsilon}{\lambda})}$, $\forall x \in [0, L_0]$. In addition, as t trends to infinity, we have

$$\lim_{t\to\infty}|w| \le \sqrt{\frac{16L_0^3\varepsilon}{\beta\lambda_1\lambda EI}}, \quad \forall x \in [0, L_0]. \tag{41}$$

In a similar manner, from Inequality (14), we have

$$\frac{\beta}{2L}T_1[w_1+D_1]^2 \le \frac{\beta}{2}T_1\int_{-L}^{0}[w_1']^2dx \le \Theta_1(t) \le \frac{1}{\lambda_1}\Theta(t) \in \mathcal{L}_\infty, \tag{42}$$

$$\frac{\beta}{2L}T_2[w_2-D_2]^2 \le \frac{\beta}{2}T_2\int_{-L}^{0}[w_2']^2dx \le \Theta_1(t) \le \frac{1}{\lambda_1}\Theta(t) \in \mathcal{L}_\infty. \tag{43}$$

Then, we obtain $w_1 + D_1$ and $w_2 - D_2$ are uniformly bounded as follows

$$|w_1+D_1| \le \sqrt{\frac{2L}{\beta\lambda_1 T_1}\left(\Theta(0)e^{-\lambda t}+\frac{\varepsilon}{\lambda}\right)} \le D, \tag{44}$$

$$|w_2-D_2| \le \sqrt{\frac{2L}{\beta\lambda_1 T_2}\left(\Theta(0)e^{-\lambda t}+\frac{\varepsilon}{\lambda}\right)} \le D \tag{45}$$

where the constant $D = \sqrt{\frac{2L}{\beta\lambda_1\min(T_1,T_2)}(\Theta(0)+\frac{\varepsilon}{\lambda})}$, $\forall x \in [-L, 0]$. Moreover, as t trends to infinity, we have

$$\lim_{t\to\infty}|w_1+D_1| \le \sqrt{\frac{2L\varepsilon}{\beta\lambda_1\lambda T_1}}, \tag{46}$$

$$\lim_{t\to\infty}|w_2-D_2| \le \sqrt{\frac{2L\varepsilon}{\beta\lambda_1\lambda T_2}}, \tag{47}$$

$\forall x \in [-L, 0]$. ■

Remark 7. For this FWT system, the value of system parameters EI, T_1, T_2, L and L_0 are relative larger than positive constants α, β, γ, and σ_1 -σ_3. According to inequalities $\epsilon_1 - \epsilon_{12}$, when the system parameters EI, T_1, T_2, L, L_0, ρ, ρ_1, ρ_2, c, c_1 and c_2 are determined, α, β, γ and σ_1-σ_3 can be chosen to be small. Further, the value of control gains k_1, k_2 and k_p can be determined to make ϵ_7, ϵ_8, and ϵ_{12} hold.

Remark 8. From Eqs. (41), (46) and (47), it is easy to know that the system's displacement w, w_1 and w_2 can converge to bounded range. Therefore, we can obtain that proposed control laws (19) and (20) are effective for suppressing vibrations of the FWT system.

Remark 9. All of the required information in the two controllers can be received by a backward difference method or obtained through a series of sensors. Using the laser displacement one can get w_{L_0} and w_0, which are the tip boundary of the tower and the bottom base for the floating tower respectively (Queiroz, Dawson, Nagarkatti, & Zhang, 2000). $\dot{w}_{L_0}$ and $\dot{w}_0$ with only one time derivative, which can be obtained by applying the backward difference algorithm.

4. Numerical simulations

In this part, we make numerical simulations to validate the effectiveness of proposed vibration control method for the FWT system. The dynamical behavior of the FWT system is analyzed in the set $\Omega = \{(x_0, x_1, t) : 0 \le x_0 \le L_0, 0 \le x \le L, 0 \le t \le T\}$. The temporal step size k and spacial step sizes h_0 and h as below

$$\begin{aligned} h_0 &= \Delta x_0 = \frac{L_0}{M_0}, \\ h &= \Delta x = \frac{L}{M} \\ k &= \Delta t = \frac{T}{N} \end{aligned} \tag{48}$$

where, M_0, M and N are the spacial subdivisions of the length of tower and mooring lines, and the time subdivisions, respectively. Referring for previous works (He & Ge, 2015; Lyu, Zhang, & Li, 2019; Musial, Butterfield, & Boone, 2004; Ruzzo, Saha, & Arena, 2019; Wang, Hu, & Meng, 2018), system parameters of the FWT system is given in Table 1, and disturbances are given as $d_1(t) = [10 + \sin(5\pi t)] \times 10^3$N and $d_2(t) = [2 + 3\cos(10\pi t)] \times 10^4$N.

Table 1
Parameters of the FWT system.

Parameter	Value	Parameter	Value
L_0	20 m	ρ	10 kg/m
EI	2000 Nm^2	c	8 Ns/m^2
L	100 m	ρ_1, ρ_2	1.5 kg/m
T_1, T_2	10^4 N	c_1, c_2	0.8 Ns/m
m_1	150 kg	m_2	100 kg
D_1	5 m	D_2	5 m

Initial conditions of the FWT system are given as $w(x,0) = \frac{x}{L_0}$m, $w_1(x,0) = \sqrt{\frac{D_1^2 x}{L}} - D_1$ m, $w_2(x,0) = -\sqrt{\frac{D_2^2 x}{L}} + D_2$ m, and $\dot{w}(x,0) = \dot{w}_1(x,0) = \dot{w}_2(x,0) = 0$m/s, the time duration $t_f = 25$ seconds.

In the numerical simulation, three control cases will be taken into consideration, namely, without control, with PD control, and with proposed vibration control.

Without control: In this case, there is no control input acting on the FWT system, namely, $u_1(t) = 0, u_2(t) = 0$. Fig. 2 illustrates the movements of the FWT system, where, the red thick line represents the motion of the tower, and the two blue lines describe the motion of two mooring lines. From Fig. 2, it is obvious that there exist vibrations in the flexible wind turbine tower and mooring lines under the effect of external disturbances, and the vibrations of FWT system are increasing with increasing time from 0 to 25 s. Moreover, three-dimensional Figs. 5, 8 and 11 illustrate displacements of the turbine tower and mooring lines. It is also seen that increasing structural vibrations of FWT system have a great influence on system performance, even will damage the flexible structures. Therefore, it is necessary to propose vibration control to stabilize the FWT system.

PD control: In Fig. 3, the control performance for suppressing the vibration of FWT system of PD control is shown, where, PD controllers are given as $u_1(t) = 5000\dot{w}_{L_0} + 1000w_{L_0}$, $u_2(t) = 5000\dot{w}_0 + 1000w_0$. From these figures, it can be seen that vibrations of the FWT system are reduced, and the vibration reduction for each mooring lines and tower are shown in Figs. 6, 9 and 12, respectively. According to these figures, we can see that there still exist small vibrations in turbine tower and mooring lines under PD control, which implies that PD control has a positive effect on the vibration suppression, while the control performance is not enough. Therefore, it is necessary to develop a better control scheme to reduce the vibration of FWT system. In Fig. 14, control inputs of the PD control are given. The input of controller $u_1(t)$ is in the range of -1300N to 1000N from $0s$ to $25s$, and the input of controller $u_2(t)$ is in the range of -2×10^4N to 2×10^4N from $0s$ to $25s$.

Active Boundary control: In this case, active boundary controllers $u_1(t)$ (19) and $u_2(t)$ (20) are implemented in the FWT system with a set of control parameters $k_1 = 5000$, $k_2 = 3000$, $k_p = 800$, $\alpha = 20$, $\beta = 100$ and $\gamma = 0.5$. Similar to the above cases, Fig. 4 illustrates the whole dynamics of FWT system, and from Fig. 4, we can see that vibrations of the FWT system are suppressed well. Moreover, the dynamics of each mooring lines and turbine tower are shown in Figs. 7, 10 and 13. Fig. 15 shows the input control signals of two proposed controllers: the input of controller $u_1(t)$ is in the range of -1000N to 1000N from $0s$ to $25s$, and the input of controller $u_2(t)$ is in the range of -3×10^4N to 3×10^4N from $0s$ to $25s$.

According to Figs. 3–13, it can be seen that the proposed active boundary controllers has a better control performance than PD control in suppressing the vibration of FWT system, where, vibrations of turbine tower and mooring lines have better convergence. Moreover, it is seen that the designed controllers are still able to reject the effect of the external disturbances and to ensure the FWT system stable. These simulations results are consensus with the theoretical results derived in (41), (46) and (47), and demonstrate that the proposed control laws are effective in vibration suppression for the FWT system.

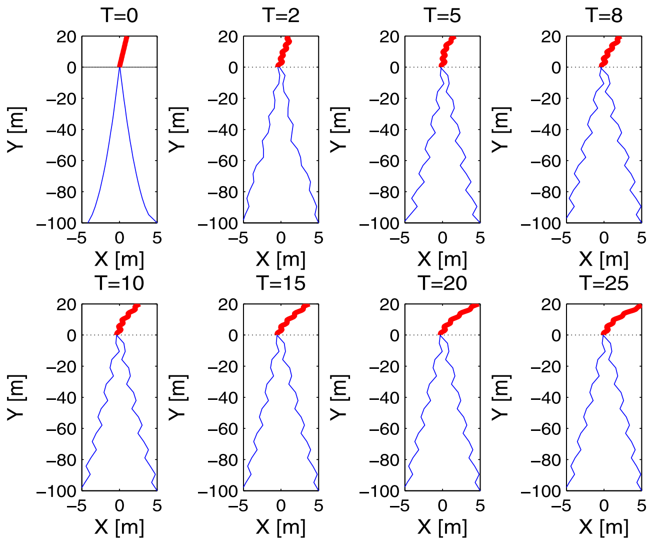

Fig. 2. Displacement of the FWT system: without control.

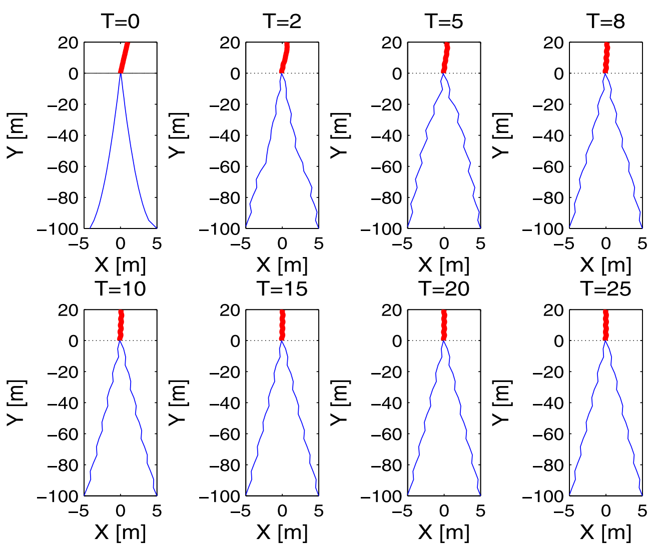

Fig. 3. Displacement of the FWT system movements: with the PD control.

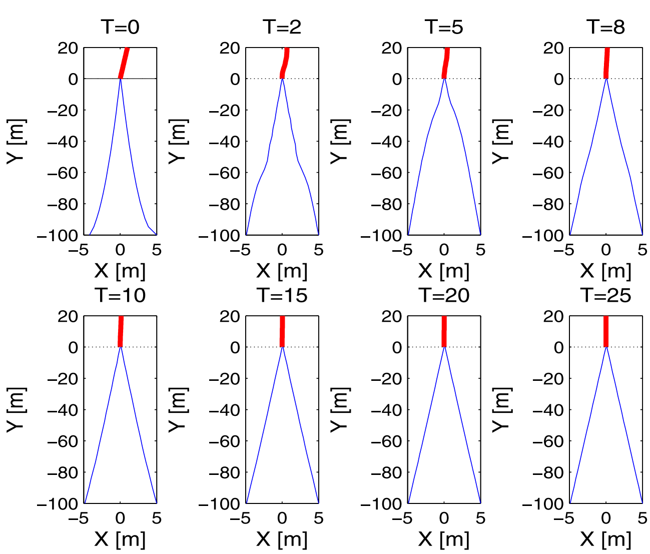

Fig. 4. Displacement of the FWT system movements: with the boundary control.

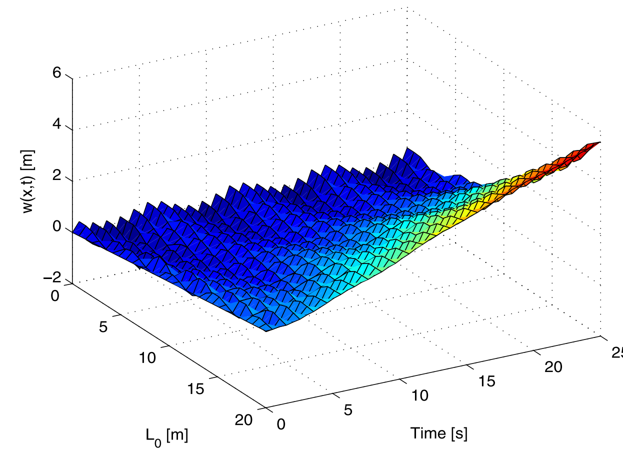

Fig. 5. $w(x,t)$, vibration of the tower without control.

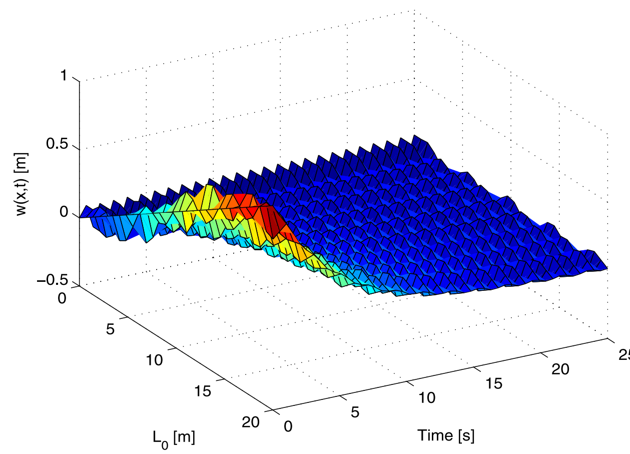

Fig. 6. $w(x,t)$, vibration of the tower with the PD control.

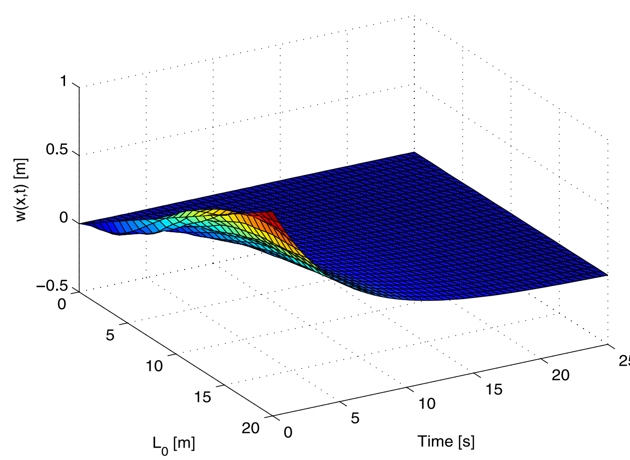

Fig. 7. $w(x,t)$, vibration of the tower with the proposed control.

5. Conclusion

In this paper, we address the stabilization problem of the FWT system, which is a beam-string coupled structure. Through proposing active vibration control method, vibrations of turbine tower and

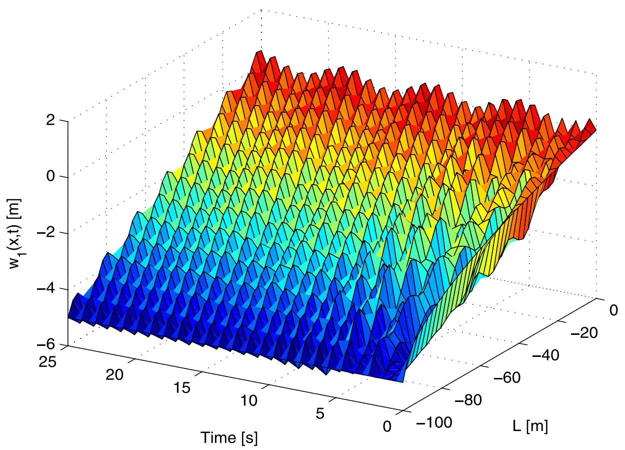

Fig. 8. $w_1(x,t)$, vibration of the left mooring line without control.

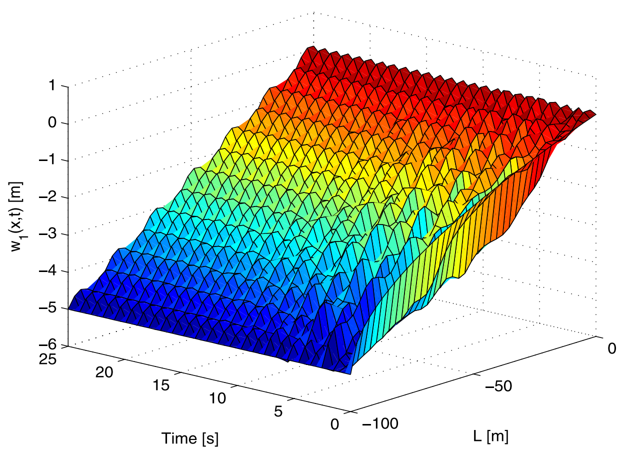

Fig. 9. $w_1(x,t)$, vibration of the left mooring line with PD control.

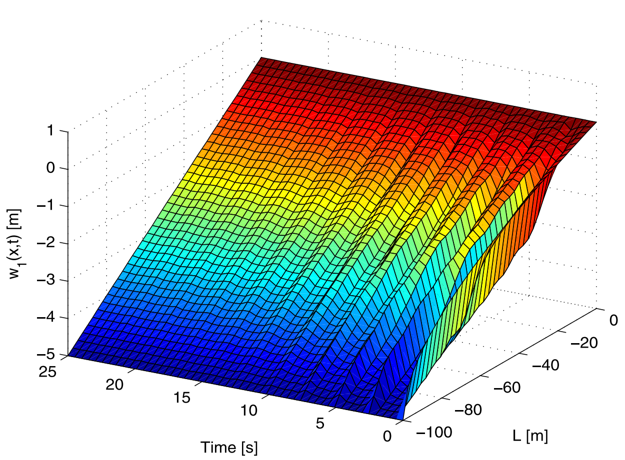

Fig. 10. $w_1(x,t)$, vibration of the left mooring line with the proposed control.

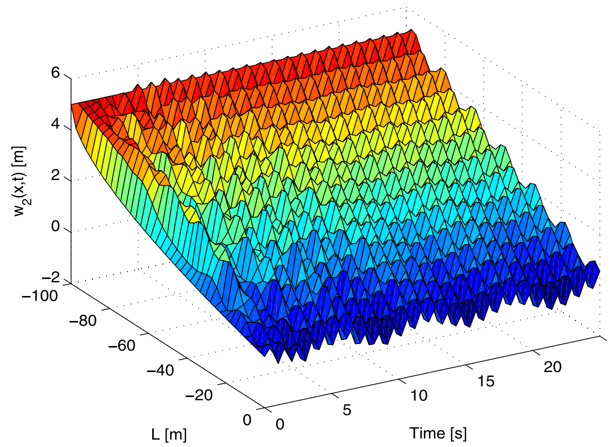

Fig. 11. $w_2(x,t)$, vibration of the right mooring line without control.

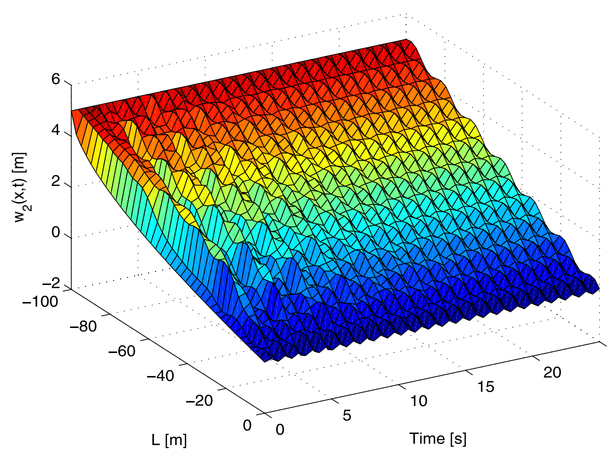

Fig. 12. $w_2(x,t)$, vibration of the right mooring line with PD control.

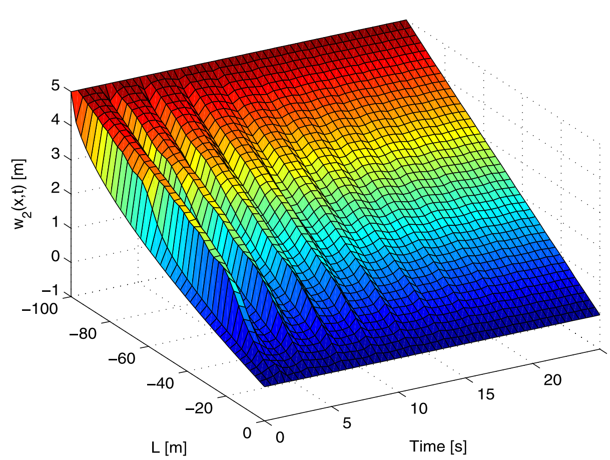

Fig. 13. $w_2(x,t)$, vibration of the right mooring line with the proposed control.

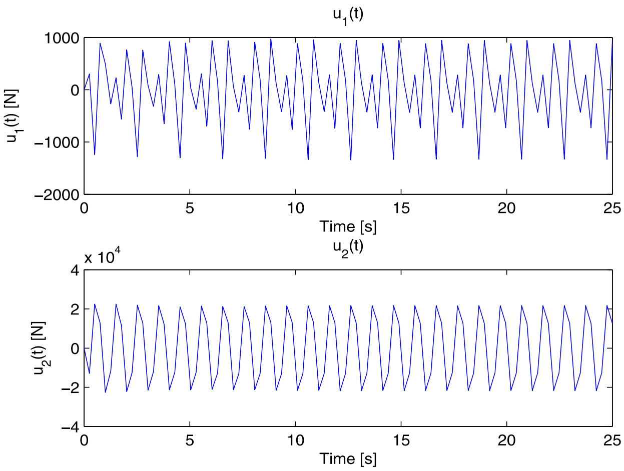

Fig. 14. PD control inputs..

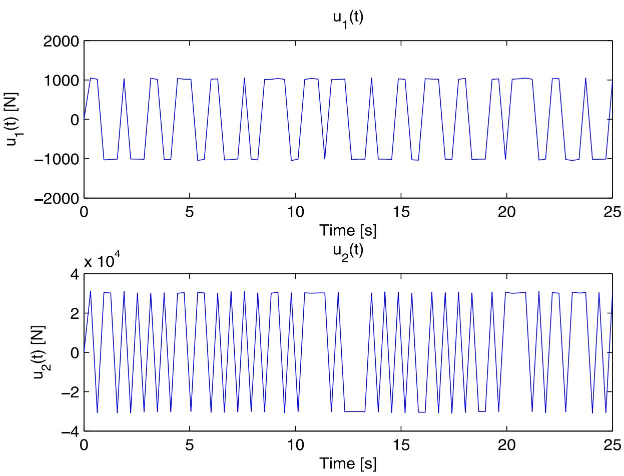

Fig. 15. Control inputs.

mooring lines can be suppressed obviously. Moreover, the stability of FWT system is also guaranteed by choosing a suitable Lyapunov function. Besides, the effectiveness of proposed controllers is validated by numerical simulation results. In future work, we will focus on the optimal control and finite-time vibration control to improve the control performances, e.g., rate of convergence, transient response, etc.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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