Abstract

This paper connects the rise in polarization to changes in media technology, which allow for higher precision in the advertising strategy of candidates. Candidates place ads in media outlets whose audiences differ in ideology and size. We develop the concept of media centrality, which takes into account differences in the media network and in ideology. Media centrality leads to a simple characterization of both the optimal targeting strategy and policy. We show that generally a more fragmented media landscape leads to an increase in polarization.

Keywords: Targeting, Networks, Voting

JEL Classification: D85, D72, D83
1 Introduction

Election campaigns optimize their advertising strategies utilizing new technologies that improved the targeting of voters. This is documented by the following quote:

Facebook advertising is highly targeted and tailored to the recipient, battle-tested for effectiveness, yet invisible to anyone but the end user. There are no spending limits on digital ads…

The Guardian, May 3rd 2017

Candidates have the means to learn about voter’s characteristics and ideology and can narrowly target voters according to these characteristics without the information being transmitted to other members of the electorate.\footnote{Providing information is an important goal of advertising, see Freedman et al. (2004), who among others, shows that campaigns are crucial in informing voters.} This practice has been commonly referred to as micro-targeting. At the same time, polarization in the U.S., that is the difference between Democratic and Republican policy positions, has been increasing (Poole and Rosenthal (2000)).

We connect the rise in polarization to changes in media technology, which allow for higher precision in the advertising strategy of candidates. Whereas it is empirically well-established that media has an impact on political outcomes, we lack a theory of how media structure affects policy.\footnote{For empirical evidence that media matters, see for example DellaVigna and Kaplan (2007), Gentzkow et al. (2011), Strömberg (2004) among several others.} We therefore make two contributions: First, we develop a model of targeting, which takes into account that voters differ in their media and policy preferences. Candidates target through media outlets and tailor their policy to the voters they target taking into account voters’ preferences. Targeted voters know the policy platform, whereas non-targeted voters cast their vote based on a prior belief. We characterize the optimal targeting strategy and introduce the concept of media centrality. This measure of centrality takes into account not only the viewership patterns of voters, that is the structure of the media network, but also their policy preferences. Second, we relate media structure to polarization. We allow for advertising to target more precisely, in line with recent developments in the media landscape. Candidates now tailor their policy to a more narrow subset of voters, which induces more extreme policies and thus, higher polarization.

In our model, two candidates compete to win an election. Both candidates maximize their chance of winning by simultaneously selecting a policy platform they commit to as well as an advertising, or targeting, strategy to convince voters. We distinguish between attached and unattached voters, capturing the seminal idea of Hinich and Ordeshook (1969). Attached voters support one of the candidates and parties, but will never switch to the other and always
know what the policy is. Unattached voters can be swayed by either candidate. Candidates therefore focus on targeting unattached voters through media outlets. They advertise their platform in certain outlets and voters that follow these outlets receive information about a candidate’s platform. A media network describes which voters access a given outlet. To make ideas precise, consider the example depicted in Figure 1. Some voters are connected exclusively to some media outlets (voters one and three) and others follow several outlets (voter two). These outlets can be thought of as following a facebook page or watching a TV show. The question that arises is what policies do the candidates choose. Do they advertise

![Figure 1: 3 Voters & 2 Outlets](image)

their policy platform in outlet one, two, or both?

Depending on the advertising strategy, unattached voters differ in their knowledge about the candidate’s policy. If the ad has been broadcast to them they know the platform. Otherwise, the unattached voters stick to a prior belief about the candidate’s platform. In this respect, we mirror Baron (1994) and Grossman and Helpman (1996) in their assumption of uninformed voters who cannot infer the platform if not directly targeted. Voters learning policy positions only if directly targeted has also been established with rational voters (Schultz (2007), Coate (2004), Callander and Wilkie (2007)) and is in line with ample empirical evidence which shows that voters are misinformed and make systematic mistakes (Delli Carpini and Keeter (1997), Caplan (2011)).

Unattached voters do not only differ in their beliefs, but also their policy preferences. Both factors affect the voting decision. Voters choose their preferred candidate according to a standard probabilistic voting model, based on their policy preferences and beliefs about the candidates’ platforms. Unattached voters are more likely to vote for a candidate whose platform they believe to be closer to their own preferences, while attached voters are more likely to vote if the policy is closer to their preference.

Candidates select in which media outlets to advertise depending on the voter’s preferences. Candidates are interested in targeting unattached voters whose bliss points are very different

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3Based on this evidence, models with behavioral voters have been introduced by Espinosa and Pouzo (2016), Spiegler (2013), Levy and Razin (2015), Bisin et al. (2015) and Ortoleva and Snowberg (2015).
from their prior about the candidate’s policy choice, taking into account the policy preferences of their attached voters. To see this, note that it is never optimal to cater to voters believing that the party implements their preferred policy. By targeting them, and thus selecting a policy that matches their position, the probability of these voters to vote for the candidate remains the same as if the party did not disclose to them. However, selecting voters whose bliss point is very different from their prior yields an increase in the probability of voting for the candidate. By selecting these voters and a policy that coincides with their preferred policy, parties can increase the number of their supporters. The choice of such a policy does not alienate unattached voters whose bliss point is close to the prior as they do not learn about the true policy and therefore stick to the prior, based on which they cast the vote. Advertising thus allows candidates to increase the number of supportive unattached voters, without loosing the unattached voters already in favor of the party.

Additionally, we allow for media outlets to differ in how many voters they reach. We show that candidates are more likely to target an outlet if it is has a large audience. We then take these considerations together: both the number of voters reached as well as their bliss points matter for the optimal target set.

Based on this we propose a new measure of centrality, which we refer to as *media centrality*. Media centrality captures the trade off between voters’ preferred policies and the number of voters targeted in a very simple and tractable way. This is, to the best of our knowledge, the first measure that explicitly highlights the trade off between an agent’s characteristics, here their bliss points, and their position in the network. While our focus here is on policy advertising, the concept of media centrality can be used more broadly for any advertising or targeting problem. Media centrality generates the optimal targeting strategy and the policy platform that maximizes the probability of winning.

Note however that the optimal policy hinges on the prior beliefs that unattached voters attribute to the candidate’s platforms. In order to rule out unrealistic priors, we extend our model to a dynamic setting. In particular, we allow for the prior belief to change in each period, which leads candidates to set different policies in each election. We further assume that the beliefs of voters regarding the candidate’s platform adjust adaptively. If voters do not receive any information about a candidate’s platform, they assume that the current policy corresponds to last period’s policy. Then, a candidate caters to the voters whose preferred policy has the greatest distance from the new prior (controlling for the size of the audience). This leads to policy cycles, a result that matches the electoral cycles in polarization we establish empirically. Our measure of polarization takes into account these
cycles, that is we calculate the difference in policy platforms across a cycle.\footnote{A model with adaptive expectations outperforms a setting with rational agents that face uncertainty about the media network. We show that the optimal targeting strategy with rational voters and uncertainty yields polarization, but fails to generate electoral cycles. Therefore, our assumption of adaptive beliefs is not only a simplifying one, but matches observed patterns of polarization. For further details, see Section 3.4.}

Given our definition of polarization across an electoral cycle, we can now connect recent developments in media networks to the policy implemented, and thus polarization. We focus on the increase in the number of media outlets. This has made it easier for voters to find programs or websites that match their interests. As demographic characteristics predict viewership as well as voting behavior, it has become feasible to target a certain set of voters specifically, reducing spillovers.\footnote{Voters, although not directly targeted, might be able to observe a campaign ad because a friend posts a link on Twitter or Facebook, or a blog reports about it. In this sense, voters are connected to a higher number of media outlets than they have been previously, that is they have a higher number of links to various outlets. We show in Section 3.4 that this development can also lead to an increase in polarization.} Therefore, the current media structure allows for a precise sorting of voters according to their policy preferences. Candidates offer a policy which reflects the preferences of a more homogeneous subset of voters. Thus if candidates target the voters with more extreme preferences, they choose a more extreme policy, whereas if they target more moderate voters, they choose a more moderate position. Generally, the effect on the more extreme policies outweigh those on the moderate policies, leading overall to an increase in polarization. Thus, an increased fragmentation of the media network leads to a rise in polarization, while mass media is associated with lower levels of polarization implying that our explanation cannot only account for the recent increase in polarization but also for the polarization patterns since the 1870s. In this respect, our work provides a theoretical foundation of the empirical findings of Gentzkow et al. (2014) and Campante and Hojman (2013).

To demonstrate that micro-targeting leading to increased polarization cannot be easily falsified, we further proxy media fragmentation by the increase in internet penetration and measure its effect on polarization among politicians. The internet has made it easier to target specific voters and to provide them with ads that match their preferences and interests. We use data from the Federal Communication Commission on internet adoption rates and connect it to polarization of U.S. congressmen. We indeed find that districts with higher levels of internet penetration elect congressmen that are relatively more polarized. Together with other work on media and polarization, this finding provides some support of our theoretical prediction.

\textbf{Related Literature} \hspace{1em} Our paper contributes to various strands of literature which we discuss in turn.
Targeting & Advertising in Networks There is a wide range of papers that look at targeting in networks.\(^6\) The key difference is that we allow not only for heterogeneity in network positions, but also for differences in preferences among agents. Most importantly, we derive a tractable measure of centrality that balances network characteristics and agent’s preferences, which can be applied in a variety of settings. This allows us to connect the network structure to polarization. The impact of networks on polarization is also analyzed by Galeotti and Mattozzi (2011). There, parties can only choose one of two policies. They do not allow for arbitrary ideological bias among voters and the policy does not emerge endogenously. Therefore, they do not obtain varying degrees of polarization, but rather a fixed level of polarization versus no polarization at all, which does not help to understand the increase in polarization in recent years. Political networks are also at the heart of the analysis in Murphy and Shleifer (2004). Their setting, unlike ours, does not allow for overlap between different social communities, which drives opinion divergence among voters and leads to polarization.\(^7\)

Polarization & Media Our work adds to the numerous explanations of the increase in polarization. It is well-established that there has been an increase in elite or party polarization, see McCarty et al. (2006). Whether there has also been an increase in polarization in the electorate has been disputed.\(^8\) Even if there has been an increase in polarization among voters, this rise has been moderate compared to the polarization among politicians. Our model provides a novel explanation for this increasing gap between the voters’ preferences and the politicians policy choices - the heightened fragmentation of media outlets.\(^9\) Other explanations of polarization are policy-motivation, entry deterrence, incomplete information among voters or candidates, differential candidate valence, politicians catering to their own electorate and increased campaign spending.\(^10\) Unlike our mechanism, these explanations fail

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\(^6\)Early work (Galeotti and Goyal (2009), Dziubiński and Goyal (2013), Goyal and Vigier (2015, 2014)) analyzes the targeting strategy of a monopolist, whereas Goyal et al. (2014) show how two competitors target a network of homogeneous consumers. This is similar to Bimpikis et al. (2013) who study the trade-off between an agent’s characteristics and his centrality, but different to our work are unable to give a measure that combines the two explicitly. Groll and Prummer (2015) also allow for networks with heterogeneous nodes. Their set up does not lend itself to the analysis of arbitrary networks, though. Battaglini and Patacchini (2016) analyze how to influence politicians. A different perspective has been taken by Fainmesser and Galeotti (2015, 2016) who consider the improved knowledge of firms about the network due to better information technologies.

\(^7\)Work that deals explicitly with microtargeting, without networks, is Schipper and Woo (2014) and Hoffmann et al. (2013, 2014).

\(^8\)Abramowitz and Saunders (2008), Abramowitz and Stone (2006) argue that there has been an increase and Fiorina and Abrams (2008), Fiorina et al. (2008), Fiorina et al. (2005), Fiorina and Levendusky (2006) state that polarization has remained the same.

\(^9\)For an overview on polarization, its causes and consequences, see Layman et al. (2006).

to account for the sharp rise in polarization in recent years (the exception being a distinct and complementary approach by Herrera et al. (2008)) and more importantly, for observed polarization patterns since the 1870s.

There is an extensive literature on media and politics, surveyed by Gentzkow and Shapiro (2008), Prat and Strömberg (2013), and Sobbrio (2014). In line with our basic idea, Gul and Pesendorfer (2011) connect increased media competition to higher polarization. Their setting focuses on the impact of media competition on media slant, which then leads to polarization, whereas we emphasize the importance of media structure. Prat (2017) analyzes the impact of media on voters’ decisions. In his model, agents have a bounded capacity to absorb news, similar to our setting, but are additionally ideologically homogeneous, whereas we allow for ideological diversity. Empirically, we relate to the literature on media and policy outcomes.\(^\text{11}\) Media can affect voter’s beliefs and polarize them, see e.g. the effect of Fox News (DellaVigna and Kaplan (2007), Clinton and Enamorado (2014)), which may lead politicians to respond by adjusting policies. In contrast to Fox News, the internet did not seem to lead to an increase in extreme positions among voters, see Boxell et al. (2017).\(^\text{12}\) Our model can reconcile these findings as it highlights that changes in media structure itself can lead to an adjustment in policies, without an increase in voters’ polarization. Gentzkow et al. (2014) document an instance where a more fragmented media landscape, measured by newspaper competition, leads to more ideologically diverse sources of news, while Campante and Hojman (2013) relate mass media and polarization directly, for details see Section 4.2. We add to this work by relating internet penetration and polarization.

**Targeting Swing vs Core Voters**

Last, this paper adds to the question of whether core voters are targeted (Cox and McCubbins (1986)) or whether it is more beneficial to target swing voters (Lindbeck and Weibull (1987)). Theoretical work connecting the aforementioned papers is Dixit and Londregan (1995, 1996).\(^\text{13}\) Our paper implies that both strategies can be advantageous – each in a different election. If candidates are perceived to cater to their core voters they have an incentive to choose a policy that reflects swing voter’s preferences. On the other hand, if candidates are regarded to be more in line with swing voters, they should

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\(^\text{11}\) This is in contrast to the literature of media and voter turnout, see Gentzkow and Shapiro (2008), Prat and Strömberg (2013), and Sobbrio (2014).

\(^\text{12}\) Similarly, Falck et al. (2014) report that broadband does not affect vote choices in Germany. Gerber et al. (2009) also do not find an effect of providing a newspaper to voters.

\(^\text{13}\) While we focus on targeting with policies, essentially convincing voters of a candidate, there is a growing literature that analyzes vote versus turnout buying (Nichter (2008)). The optimality of either approach has been discussed in Gans-Morse et al. (2014).
select a platform that mirrors their base supporters’ political ideologies. Stokes (2005) and Diaz-Cayeros et al. (2016) consider this question in a multiperiod setting with the former reaching the conclusion that swing voters are targeted, while the latter draws the opposite conclusion, both finding empirical support for their theory. This pattern emerges generally in empirical studies; namely, both theories find relatively equal support for targeting swing voters (Wright (1974), Stein and Bickers (1994), Bickers and Stein (1996), Denemark (2000), Case (2001), Dahlberg and Johansson (2002), Herron and Theodos (2004)) versus core supporters (Levitt and Jr. (1995), Balla et al. (2002), Calvo and Murillo (2004), Bickers and Stein (2000), Finan and Schechter (2012), Stokes et al. (2013)). Our theory can reconcile these different outcomes.

The remainder of this paper is structured as follows. We first present our model of targeting in Section 2. We derive and characterize media centrality in Section 3.1. Section 3.2 defines polarization. The impact of media fragmentation on polarization is analyzed in Section 3.3. Section 4 provides empirical support for our predictions. Section 5 concludes. All proofs are collected in the Appendix.

2 Model

We introduce a model of probabilistic voting on a network. Two candidates, A and B, aim to win an election while catering to their party. The candidates choose a policy platform and decide how to advertise it in a media network. The targeted audience observes the policy platform chosen by the politician. The non-targeted unattached voters vote based on their degenerate prior beliefs, while attached voters always know the policy and decide whether to vote or not.

**Voters** Voters are modelled as in Lindbeck and Weibull (1987). Thus, the utility of a voter with bliss point \( x \) and partisan preferences \( \theta \), who believes that candidate \( c \) will implement policy \( y \) amounts to

\[
U(c, y|x) = \begin{cases} 
    u(y|x) & \text{if } c = A \\
    u(y|x) + \theta & \text{if } c = B
\end{cases}
\]

Policy preferences as well as beliefs are contained in the unit interval \([0, 1]\). We further assume, as is common in the literature, that the ideological preferences are uniformly, independently and identically distributed across voters on \([-1, 1]\) – so that \( \theta \sim U[-1, 1] \). To simplify later discussions, we assume, as is standard, a quadratic loss function for the baseline policy.
preferences of voters – so that \( u(y|x) = -(y - x)^2 \).\(^{14}\) The quadratic loss function implies that \( u(y|x) \) is single-peaked, concave and symmetric in \( y \). Thus, a voter with bliss point \( x \) would prefer voting for politician \( A \) when facing platforms \( y_A \) and \( y_B \) if and only if \( u(y_A|x) - u(y_B|x) > \theta \). This specification makes the problem symmetric for party \( B \). The probability that a voter with bliss point \( x \) votes for politician \( A \) when facing platforms \( y_A \) and \( y_B \) then amounts to

\[
P_A(y_A, y_B|x) = \frac{1}{2} [1 + u(y_A|x) - u(y_B|x)],
\]

while the probability of voting for \( B \) is given by \( P_B(y_A, y_B|x) = 1 - P_A(y_A, y_B|x) \).\(^{15}\)

Additionally, we introduce attached voters, loyal to one party. They obtain utility \( u(y_c|x) \) if they choose to support their party and utility \( \overline{u} + \theta \) otherwise. We impose the same assumptions on \( \theta \) as previously and obtain a probability of supporting candidate \( c \),

\[
P_c(y_c|x) = \frac{1}{2} [1 + u(y_c|x) - \overline{u}].
\]

**Outlets and the Media Network** Let \( K \) denote the set of unattached voters, with \( k \) the number of unattached voters. Candidates may not know the exact policy preference of each voter. Therefore, we assume that each unattached voter’s policy preference is distributed according to a cumulative distribution \( G_i(x) \), which admits a density function \( g_i(x) \) with support on \([0, 1]\). We denote its expectation by \( E_i(X) \).

Similarly, the number of attached voters is denoted by \( l \) and is the same for both parties. The bliss points of attached voters have a cumulative distribution \( G_c(x) \), which admits a density function \( g_c(x) \) with support on \([0, 1]\). The attached voters of parties \( A \) and \( B \) have expected bliss points \( E_A(X) \) and \( E_B(X) \), respectively, with \( E_A(X) < E_B(X) \). We assume that every unattached voter \( i \) is more moderate than the attached voters, \( E_A(X) \leq E_i(X) \leq E_B(X) \).\(^{16}\)

There are \( m \) possible media outlets where politicians can advertise the platform they select. Let \( M \) denote the set of media outlets. The media network \( \{K, M, N\} \) is a bipartite network that describes which voters have access to a given outlet. Formally, \( N \subseteq K \times M \) and voter \( i \) observes outlet \( j \) if and only if \( ij \in N \). Denote the neighborhood of media outlet \( j \)

\(^{14}\)Our results hold for any quadratic loss function. If we consider a utility function with \( u(y|x) = \alpha_0 - \alpha_1(y - x)^2 \) and adjust the uniform distribution appropriately to \( \theta \sim U[-\alpha_1, \alpha_1] \), then the candidates’ optimization problem is unchanged.

\(^{15}\)Kamada and Kojima (2013) show that costly voting is equivalent to probabilistic voting, which implies that our results are unchanged if we were to consider a model in which voters abstain due to a non-negative cost of voting.

\(^{16}\)Attached voters are modeled to be more extreme and better informed, in line with evidence first provided by Palfrey and Poole (1987).
by $K(j) = \{i \in K \mid ij \in N\}$. Thus, $K(j)$ consists of all voters that are connected to a given outlet $j$. Without loss of generality, assume that every voter observes at least one outlet – or formally: $K = \bigcup_{j \in M} K(j)$.

**Candidates: Targeting and Policy Setting** Candidates simultaneously select the platform they are going to implement and the media outlets through which to advertise it, so as to maximize their expected vote share.\(^{17}\) Candidates can target an arbitrary number of outlets.

A strategy for a candidate $c \in \{A, B\}$ consist of a pair $\{x_c, T_c\}$. We denote the policy of candidate $c$ by $x_c \in [0, 1]$ and the target subset of media outlets in which the candidate chooses to advertise by $T_c \subseteq M$. If an outlet is targeted by candidate $c$, $c$ commits to a platform $x_c$. If a voter is connected to any outlet targeted by $c$, they know that the candidate will set platform $x_c$. However, if a voter is not targeted, unattached voters stick to their prior belief about the candidates’ policy which is denoted by $\pi_c$ and consists of a single policy in $[0, 1]$.\(^{18}\)

For any subset of media outlets $T \subseteq M$, define its coverage $K(T)$ as the set of voters that are linked to at least one outlet in $T$. Formally, this set is defined as $K(T) = \bigcup_{j \in T} K(j)$. Denote its cardinality by $k(T)$, and define unattached voter $i$’s posterior belief as

$$y_i^c(T) = \begin{cases} x_c & \text{if } i \in K(T) \\ \pi_c & \text{if } i \notin K(T) \end{cases}$$

Unlike unattached voters, attached voters always know about the platforms parties commit to. The problem faced by candidate $c \in \{A, B\}$ is then given by

$$\max_{x_c, T_c} \left( \sum_{i \in K} \int_0^1 P_c(y_i^A(T_A), y_i^B(T_B)|x)dG_i(x) + l \int_0^1 P_c(x_c|x)dG_c(x) \right), \quad (3)$$

where the expected vote share is the sum of the expected vote share among unattached and attached voters. An equilibrium is thus defined by a strategy profile $\{x_c, T_c\}_{c \in \{A, B\}}$ which solves (3) for every candidate $c \in \{A, B\}$.

\(^{17}\)Maximizing expected vote share, expected plurality (the vote share relative to that of the other party) and maximizing the probability of winning are equivalent in our setting. For a general discussion of the relation between these concepts, see Banks and Duggan (2005).

\(^{18}\)Alternatively, we can interpret the policy space as that of a policy dimension. As we have a probabilistic voting model, it is straightforward to extend our framework to a multidimensional policy space, where the analysis would then be carried out for each policy dimension separately.
3 Media Centrality, Fragmentation & Polarization

3.1 Media Centrality

We develop the concept of media centrality, which allows us to derive the optimal targeting strategy as well as the equilibrium policy. Based on this policy, we define polarization and then discuss the effect of media fragmentation on implemented policies. For ease of exposition, we set \( l = 0 \), that is we assume there are only unattached voters. Given the voting probabilities in expression (1), the maximization problem of candidate \( c \) will have the same solution as the following problem which abstracts from the competitor’s strategy:

\[
\max_{x_c, T_c} \int_0^1 u(x_c|x) \sum_{i \in K(T_c)} dG_i(x) + \int_0^1 u(\pi_c|x) \sum_{i \in K \setminus K(T_c)} dG_i(x). \tag{4}
\]

The first part of expression (4) maximizes the utility of the voters that are contained in the target set, whereas the second part refers to the voters not contained in the target set, which we denote by \( K \setminus K(T_c) \). The problem that candidates face is therefore to determine which outlets to target depending on their audience, as well as to choose the optimal policy for the outlets that are targeted.

We first show that the optimal policy coincides with the average of the bliss points of the voters in the targeted outlets. We denote this expected bliss point by \( E(X|T_c) \).\(^{19}\) We then establish that the optimal targeting strategy prescribes the selection of the target set with the highest media centrality \( W_c(T) \). Formally, we define media centrality as

\[
W_c(T) = k(T) [E(X|T) - \pi_c]^2, \tag{5}
\]

where \( k(T) \) denotes the number of voters in the target set. Media centrality takes into account the preferences of the voters, \( E(X|T) \), as well as the size of the audience, \( k(T) \). It can therefore be seen as a weighted network centrality, where \( k(T) \) captures the degree while the squared distance between average bliss point and prior reflects the ideology of voters. It highlights that the most valuable targets are media outlets with a large coverage, which have viewers whose ideal policies are not aligned with the prior beliefs about the politician’s platform. This is summarized in Proposition 1.

Proposition 1. In every equilibrium \( \{x_c, T_c\} \), for every candidate \( c \in \{A, B\} \):

(1) The policy set amounts to the average bliss point in the target’s coverage, \( x_c = E(X|T_c) \).
(2) The outlets targeted, \( T_c \), maximize media centrality \( W_c(T) \).

\(^{19}\) A formal definition of \( E(X|T_c) \) is provided in the Appendix.
Candidates select a policy platform that caters to a subset of voters whose bliss point has the greatest distance from the prior. Unless all voters have similar expected bliss points, candidates never choose to target all outlets and all voters, even though there is no cost to targeting. Candidates can essentially segment voters according to their bliss points (subject to the constraint of the media network). They choose a policy that matches the preferences of the voters they target. This induces these voters to cast the vote in their favor, as the candidate increases the utility of the targeted voter. At the same time, the voters who are not targeted do not learn about the platform and vote based on their belief. As the non-targeted voters have a preferred policy close to their prior, this still leads them to vote for the candidate with a high probability. While the actual utility of the non-targeted voters decreases (the actual policy moves away from their preferences), their perceived utility is higher, inducing them to still vote for a candidate who does not target them.

Candidates can improve their expected vote share, above what they would obtain were they to disclose in all outlets with a policy that is the average of all voters. But it is not straightforward to show how many voters will be targeted. On the one hand adding voters to the target set allows candidates to take their policy preferences into account. This increases their probability of voting on the candidate’s behalf. But voters with a bliss point that is further from the prior than that of the newly added voters are now less likely to vote for the candidate as the policy has moved away from their bliss point. The optimal target set balances this trade off. We provide in what follows a brief informal characterization of the target set, which highlights the simplicity in maximizing media centrality. The formal discussion is presented in the Appendix.

**Remark 1 (Characterization Target Set).** Consider some non-empty target set $T$.

1. If the target set $T$ is such that the expected bliss point of its audience is to the left (right) of the prior, then media centrality decreases if an outlet, whose audience has an expected bliss point to the right (left) of the prior, is added to the target set.
2. Including an outlet whose audience has an expected bliss point closer to the expected bliss point of the audience in target set $T$ relative to the prior yields a higher media centrality.
3. If media centrality increases when including outlets with an audience whose policy preferences are further away from the expected bliss point of the audience in $T$, then it further increases if outlets with an audience closer to the expected bliss point in $T$ are included.

When adding an outlet and its audience to a target set, we need to keep track of whether members of the audience are already included in a given target set. Therefore, including an outlet in the target set implies that only voters that have not yet been targeted are relevant
for the analysis, they are the new audience. We write in what follows $T' - T$ if we refer to the voters contained in target set $T'$, but not in $T$. Thus, we denote the expected bliss points of these voters by $E(X|T' - T)$ and their cardinality by $k(T' - T)$.

Then, it cannot be optimal for outlets with voters whose bliss points are on different sides of the priors to be included in the same target set. The policy associated with the target set will lie to one side of the prior, implying that one outlet and its audience will have policy preferences closer to the prior. Therefore, if these voters voted based on the prior, they would be more likely to vote for a given candidate and therefore they should not be targeted.

Second, it is optimal to include voters in a target set, which has a bliss point closer to their policy preference than the prior. In this case adding voters to the target set unambiguously increases media centrality. This is not necessarily the case if the bliss point of the voters following a given outlet is closer to the prior, but on the same side of the prior as the target set’s optimal policy. Two forces drive the politician in opposite directions – on the one hand more outlets guarantee more coverage, on the other hand they dilute the effect of disclosure as the new policy is closer to the prior.

In the latter case, we can only draw one further conclusion: if it is optimal to include an outlet with a bliss point closer to the prior than the bliss point of another outlet, then it is also optimal to include the other outlet.

Overall, the targeting patterns described in Remark 1 make it straightforward to determine which outlets will be targeted and which ones are not. This analysis does not only apply to politicians trying to influence voters through a media network, but is more widely applicable to influence setting, such as when lobbyists target politicians. Further, the set of results allows us to analyze how target sets are affected if the underlying media networks change.

So far our analysis did not take into account attached voters as their presence does not affect derivation of the optimal target set. Note that attached voters can be treated as voters that are connected to all outlets. Therefore, in the presence of attached voters, candidates weigh the preferences of attached and unattached voters and choose the following policy

$$x_c = \frac{k(T_c)E(X|T_c) + lE_c(X)}{k(T_c) + l}.$$  \hspace{1cm} (6)

If attached voters are sufficiently important, that is their number $l$ is high enough, then the platform selected by candidate $B$ lies to the right of $A$’s policy independently of the priors. Party $A$ becomes left-wing, party $B$ right-wing.
Lemma 1. There exists a threshold $\bar{l}$, such that for any $l > \bar{l}$ and any prior $\pi_A, \pi_B$ the policy platform of candidate $A$ always lies below that of candidate $B$, $x_A \leq x_B$.

We provide an explicit characterization of the threshold $\bar{l}$ in the Appendix.

Proposition 1 suggests that it is optimal to target the unattached voters whose preferences have the greatest distance from the prior. This effect is moderated in the presence of attached voters. If a candidate is perceived to be extreme, then he chooses a policy that caters to moderate voters, whereas candidates who are perceived to be moderate have an incentive to cater to more extreme voters. This prediction is in line with advertising strategies of political parties, for an overview see Cox (2009), who highlights that empirically candidates target both moderate swing voters as well as more extreme core supporters. That these differences in targeting may indeed be driven by the mechanism suggested here, is supported by suggestive anecdotes. First, during presidential elections it seems common that candidates choose positions that are at odds with the perception voters have. In the last presidential election, Hilary Clinton, who was widely perceived as a moderate Democrat, adjusted her policies catering to the left supporters. Similarly, the moderate Republican Mitt Romney, increasingly moved to the right in his presidential campaign. These patterns are exactly as suggested by the model presented here and thus we provide a novel explanation for them, namely varied perceptions of candidate’s policy stance, which induce different targeting strategies.

3.2 Polarization

So far, policies depend on the prior belief, which can take any value. We rule out unrealistic priors by allowing for priors to change adaptively, that is today’s prior equals last period’s policy. We denote the prior about candidate $c$ in period $t$ by $\pi^t_c$ and we index the policy in each period with $t$. We impose the following assumption.

Assumption 1. Adaptive Learning: $\pi^{t+1}_c = x^t_c$.

This assumption is in line with a well-established literature on retrospective voting that shows that voters base their decision on who to vote for on past policies, see the seminal book by Fiorina (1981) and the work building on it.

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21 https://www.theguardian.com/commentisfree/2012/aug/12/michael-cohen-republicans-embrace-extreme-right

22 An observation that may at first be at odds with this mechanism is that presidential candidates focus their efforts on a few, traditionally moderate swing states. However, within these states there is a significant variation in terms of ideology among voters and the question is who is being targeted within these states. For this the evidence is again mixed.
We further assume that candidates maximize expected vote share for one period. Additionally, we restrict attention to the non-trivial case of \( m \geq 2 \), that is the number of media outlets is at least two.

It is straightforward to see that if priors change, policies are also affected (independently of whether the prior changes adaptively) and generally, each prior leads to a different policy. Media centrality is affected by a change of prior and this leads to a new optimal target set and policy. Note that this does not imply that a given outlet cannot be targeted again.

With adaptive priors, we can additionally show that policy platforms of each party alternate between exactly two policies in the long run. We refer to these to policies for each party as a cycle. The two policies are denoted by by \( x_{cL} \) and \( x_{cR} \) with \( x_{cL} < x_{cR} \). However, the electoral cycles are not necessarily unique. There can be two cycles with distinct target sets and distinct policies. We denote the policies in the first cycle as \( x_{cL} \) and \( x_{cR} \), those in the second one by \( x'_{cL} \) and \( x'_{cR} \). There is cycling between \( x_{cL} \) and \( x_{cR} \) if one of the two platforms is reached and cycling between \( x'_{cL} \) and \( x'_{cR} \) if either \( x'_{cL} \) and \( x'_{cR} \) is set at some point.

**Proposition 2.** A candidate’s electoral cycle has the following properties

1. There exists a \( \bar{t} \) such that for all \( t \geq \bar{t} \), a candidate’s platform alternates between exactly two policies.
2. For any two distinct cycles, it must hold that \( x_{cL} < x'_{cL} < x_{cR} < x'_{cR} \).

Our proof is solely based on how the optimal policies change over time. We can show that eventually each policy in the cycle either moves to the left or the right, that is each policy chosen lies weakly to the left or the right of the one two periods before, formally \( x_{c}^{t} \leq x_{c}^{t+2} \) and \( x_{c}^{t+1} \leq x_{c}^{t+3} \) or \( x_{c}^{t} \geq x_{c}^{t+2} \) and \( x_{c}^{t+1} \geq x_{c}^{t+3} \) for large enough \( t \). As the policy space is bounded, this implies that in the long run, there is cycling between exactly two alternatives. However, this cycle does not necessarily have to be unique for each party. It can be the case that there are two cycles which are shifts of each other, that is one cycle contains policies that lie to the left of the policies of the other cycle. We will show that these cycles emerge empirically, in line with our theoretical predictions. While adaptive expectations may seem restrictive, they match the data well, see Section 4.1.

As we have established that in the long run, there are exactly two policies for each candidate, we can now define polarization over a cycle.

**Definition 1 (Polarization).** Polarization is the average distance between parties’ platforms for an electoral cycle, \( \Delta_P = \frac{1}{2} (x_{BL} + x_{BR} - x_{AL} - x_{AR}) \).

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[23] This captures the fact that a candidate cares first and foremost about winning a given election.
3.3 Media & Polarization

Given our definition of polarization, we consider the effect of media fragmentation. We first define media fragmentation, before analyzing its effect on implemented policies.

**Media Fragmentation**  We aim to capture recent developments in information technologies that have broadened the supply of entertainment and analyze their effect on the policy set.

Figure 2: Media Fragmentation

In particular, the way in which voters are connected to media outlets has changed. Given the increase in the number of media outlets it has become easier for everyone to find programs that match their interests. As demographic characteristics predict viewership as well as voting behavior, this has made it possible to target a certain set of voters specifically. In order to analyze the effect of this development, we define the concept of media fragmentation. Intuitively, the notion of fragmentation is captured in Figure 2. In this example we take voter 3, and assign them to a newly created outlet 3, severing the ties to any other outlet. We create a new outlet $j_i$ and denote the new set of outlets by $M' = M \cup j_i$. Similarly, we denote by $K'(\cdot)$ and $N'(\cdot)$ the new operators. We then formalize our definition of media fragmentation.

**Definition 2 (Media Fragmentation).** Consider a voter $i$ who is connected to outlet $j$, $i \in K(j)$. Let $k(j) > 1$, that is outlet $j$ has more than one person in its audience. A media network $M'$ is more fragmented than $M$ if there is a newly created outlet $j_i$ with only voter $i$ in its audience and $i$ no longer follows outlet $j$, formally $ij_i \in N'$ and $K'(j_i) \cap K'(j) = \emptyset \ \forall j \in M'$.

We obtain an increasingly fragmented network by repeatedly choosing voters and assigning them to a single outlet.\(^{24}\)

\(^{24}\)The concept of Media Fragmentation is related to that of a partition. To see this consider the example. In the original media structure on the left, the only partition of voters is the set containing all voters. In the fragmented structure on the right, voters can be split up in sets \(\{1,2\}, \{3\}, \{4\}\). A fragmentation therefore leads to an increase in the number of sets that constitute a cover and splits up the cover into finer subsets.
Symmetry & Fixed Set of Targeted Voters  In order to analyze the impact of frag-
mentation, we restrict attention to symmetric media networks. This allows us to connect
the media structure to polarization, while abstracting from potential differences in the media
network of left and right leaning voters.

A media network is symmetric if for each media outlet that lies to the left of 1/2, there
exists a symmetric media outlet to the right of one half, with a symmetric audience.\(^{25}\) As we
aim to preserve the symmetry of the underlying network, we require the media fragmentation
to be symmetric, that is for each voter \(i_\) assigned to a new outlet \(j_\), there exists a voter
\(i_+\) that is assigned to \(j_+\) with \(E_{i_-(X)} = 1 - E_{i_+(X)}\). We denote the bliss points of the
new outlets by \(E(X|j_+), E(X|j_-)\); without loss of generality, \(E(X|j_+) > E(X|j_-)\). We
further assume that the preferences of attached voters are symmetric, \(E_B(X) = 1 - E_A(X)\).
Symmetry of the network and of the preferences of attached voters imply that there exists a
symmetric equilibrium with \(x_{AL} = 1 - x_{BR}\) and \(x_{AR} = 1 - x_{BL}\) (Lemma 8). Each of these
policies comes with a target set and we denote a candidates left target set by \(T_{cL}\) and their
right target set by \(T_{cR}\). Their left target set is associated with policy \(x_{cL}\), their right target
set with policy \(x_{cR}\).

![Figure 3: Symmetric Equilibrium](image)

We focus on these symmetric equilibria. Note that as policies are symmetric, target sets
can either be the same for both parties or they differ, but are still symmetric.\(^{26}\)

Last, we aim to keep the number of voters that are targeted in an electoral cycle fixed.\(^{27}\)
Media fragmentation does not increase the number of voters that are targeted, rather it makes
it possible to target voters more precisely, without spillovers. The following assumption
ensures that media fragmentation only allows for better sorting of voters according to their
preferences, without increasing the number of voters that are targeted across an electoral
cycle, that is \(k(T_{cL}) + k(T_{cR}) = k'(T'_{cL}) + k'(T'_{cR})\).

Assumption 2 (Limited Overlap: Formal Definition). For any two target sets \(T_{cL}\) and
\(T_{cR}\), with \(E(X|T_{cL}) < E(X|T_{cR})\), there does not exist an outlet \(j \in M\) such that the voters

\(^{25}\)Formally, for any outlets \(j, j' \in M\), there exist outlets \(j'', j''' \in M\) such that \(k(j - j') = k(j'' - j''')\) and
\(1 - E(X|j - j') = E(X|j'' - j''')\).

\(^{26}\)An illustration of this is provided in the Online Appendix.

\(^{27}\)What happens if the number of targeted voters increases is discussed in depth when we consider adding
links in the network.
who follow \( j \), but are not included in \( T_{cL} \) have preferences closer to \( x_{cR} \) than \( x_{cL} \), that is \( E(X|j - T_{cL}) > \frac{x_{cL} + x_{cR}}{2} \). Similarly, voters who follow \( j \), but are not included in \( T_{cR} \) do not have preferences closer to \( x_{cL} \) than \( x_{cR} \), that is \( E(X|j - T_{cR}) < \frac{x_{cL} + x_{cR}}{2} \).

Inclusion of an outlet depends on the policy preferences of voters connected to outlet \( j \). To see this, recall that any outlet with voters that have a bliss point closer to the policy than the prior should be added to the target set, see Remark 1. Here, the prior is last period’s policy. Therefore, if the bliss points of the voters in a given outlet are closer to the policy associated with the left target set, then the outlet should be added to this target set. Otherwise, the outlet should be added to the right target set.

It can be the case that there are outlets that are never added to a target set. This happens if the audience of an outlet \( j \) after accounting for overlaps, that is voters already contained in the left target set without including \( j \), is such that the remaining audience is too far right for the left target set; simultaneously, if voters connected to \( j \) not already contained in right target set have their bliss points too far left to be included in the right target set. Note that this is possible as the remaining audience of \( j \) varies due to differing overlaps with the target sets.\(^{28}\) However, as long as overlaps do not change the preferences of the audience of an outlet too much, all outlets are targeted and Assumption 2 always holds. We essentially focus on media networks in which each outlet has a sufficiently large voter base that it is only connected to it. Another way of interpreting this, is that we focus on outlets which have a sufficiently homogeneous audience. This is what micro-targeting is about: focusing on a more narrow subset of voters and conveying a policy specifically to them.

**Fragmentation & Polarization** Due to symmetry, we can analyze the problem from \( B' \)'s perspective. We denote by \( T_{BL} \) the left target set of candidate \( B \), associated with long run policy \( x_{BL} \), and by \( T_{BR} \) his right one, leading to policy \( x_{BR} \). While there can be multiple cycles and thus multiple target sets, our analysis holds for any of them.\(^{29}\)

**Lemma 2.** Let \( l > l' \). Then, the number of voters in \( B' \)'s left target set is weakly larger than the number of voters in his right target set, \( k(T_{BL}) \geq k(T_{BR}) \).

It follows immediately that \( A' \)'s right target set is weakly larger than his left target set. To see the intuition behind this result note that it is never optimal to include two symmetric outlets in \( B' \)'s right target set. By definition, two symmetric outlets have a joint expected

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\(^{28}\)An example of this is given in the Online Appendix.

\(^{29}\)To clarify, if there are any two cycles with platforms \((x_{BL}, x_{BR}), (x'_{BL}, x'_{BR})\) and respective symmetric platforms for \( A \), \((x_{AL}, x_{AR}), (x'_{AL}, x'_{AR})\) then the following results hold for both \((x_{BL}, x_{BR}, x_{AL}, x_{AR})\) and \((x'_{BL}, x'_{BR}, x'_{AL}, x'_{AR})\).
ideology of $\frac{1}{2}$. This is lower than $x_{BL}$, see Figure 3 and thus it would be better for $B$ to include the symmetric outlets in his left target set, rather than his right (see also Remark 1). Based on this, we turn to our second key result.

**Proposition 3.** Let media networks and fragmentation be symmetric and let the number of targeted voters across a cycle be constant.

(1) If both candidates select the same target sets, fragmentation weakly increases polarization.

(2) Let candidates select different target sets.
   
   (a) If $i_- \in K(T_{BL})$ and $i_+ \in K(T_{BR})$, polarization weakly increases.

   (b) Let $i_-, i_+ \in K(T_{BL})$. If $E(X|j_+)$ is sufficiently high, media fragmentation increases polarization, otherwise polarization weakly decreases.

First, note that there are two cases, namely (i) both fragmented outlets are contained in $T_{BL}$ and (ii) one fragmented outlet is contained in $T_{BL}$, the other in $T_{BR}$. We can focus on these two cases due to Lemma 2, which rules out that two symmetric outlets are contained in $B$’s right target set.

Proposition 3 shows that if both fragmented outlets are contained in different target sets, polarization unambiguously increases. The intuition behind this result is as follows.

Fragmentation generally allows for a resorting of individuals according to their ideology. Before fragmentation, also extreme voters are targeted if the policy is moderate and some moderates are targeted if the policy is extreme, due to spillovers. Fragmentation allows for micro-targeting, reducing the spillovers in information. Therefore, candidates offer a policy which reflects the preferences of a more homogeneous subset of voters. Thus, if candidates target extreme voters, their policy is more extreme, if they target moderate voters, their policy is more moderate. However, the target set associated with the moderate policy is larger than the extreme target set. By symmetry, half of the outlets lie to the left of $\frac{1}{2}$ and by Remark 1 must be included in $B$’s left target set, making this target set weakly larger (Lemma 2). This implies that the influence of attached voters is relatively larger in the extreme target set, leading to a greater change in the policy associated with the extreme target set and consequently to an increase in polarization.

While this intuition generally holds, there can be instances of individuals changing target sets and polarization decreasing. Polarization can decrease if relatively moderate voters select new outlets and are included in the target set associated with the extreme policy.

If, due to the fragmentation, both moderate and extreme voters can be targeted specifically, polarization increases. This observation is in line with the "Theory of the Long Tail", an idea put forward by Anderson (2006). He argues that the number of differentiated goods
that can be offered is increasing. In particular, it has become easier to offer niche products as cost constraints disappear. This implies that for new entrants it is often more attractive to produce products that satisfy the preferences of a minority rather than to offer another mainstream product. In line with this theory, there has been an emergence of new media channels with highly specific programmes in the US. As an example, Fox news caters mainly to conservative viewers. The same pattern holds true for TV programs, for example televangelists that tend to cater to a conservative minority. Moreover, individuals’ preferences and their political attitudes are correlated. Therefore, it has not only become easier to identify and reach moderate voters, but also more extreme ones, leading to an increase in polarization as the media network becomes more fragmented.

The latter holds always if we compare the levels of polarization between a media network that is a symmetric, complete bipartite graph and any other symmetric media network. Note that Assumption 2 is automatically fulfilled in this case.

**Corollary 3.1.** Polarization increases if the media network transitions from a symmetric, complete bipartite graph, keeping the network symmetric.

The proof is omitted as it follows immediately from Proposition 3.

If we move from a media structure where all voters follow all media outlets to any other symmetric media structure, polarization strictly increases. Such a change seems to have occurred in the US, with 90% of viewers in the 80s following the Big Three broadcast networks (ABC, CBS and NBC), while this number has declined to 30% in 2005 (Hindman and Wiegand (2008)), opening the door to other, more targeted forms of information provision.

### 3.4 Discussion

So far we presented a simple model of targeting that can help understand the increase in polarization. This is based on the following assumptions, which we discuss in turn.

**Rational Voters & Uncertainty** We assumed so far that voters do not make an inference about the policy if they are not targeted and we refer to this model as our baseline model. We believe this to be a reasonable assumption, as we allow for arbitrarily many voters to be connected to all outlets. These voters always know the policy that will be implemented. This implies that the results of the baseline model hold as long as there is an arbitrarily small share of voters who fail to make an inference if they are not targeted. Our approach is in this respect similar to Baron (1994) and Grossman and Helpman (1996), who also distinguish between informed and uninformed voters and assume that only the uninformed voters can be swayed by advertising.
Moreover, we analyze a second model in which voters make an inference about the policy if they are not targeted while simultaneously introducing uncertainty about the environment. Voters face a set of states, which describe the structure of the media networks as well as the bliss points of voters. This implies that we are allowing voters not only to be uncertain about the overall structure of the media outlet, they also do not know the bliss point of media audiences. Unlike voters, candidates know the realization of the state, that is they know the exact structure and bliss points of voters in the network.

Our result, which is described in detail in the Appendix, implies that the policy patterns in our baseline model are the same as in the model with rational voters and uncertainty. The only difference is the extent of the polarization which is greater in our baseline model.

Thus, at one extreme voters can perfectly infer the policy which is the case in standard policy setting models. This leads to the mean voter theorem with probabilistic voting. At the other extreme, voters cannot infer the policy if they are not targeted as is the case in our baseline model. This implies more extreme policies in which only the preferences of a subset of voters are reflected. Last, a setting with uncertainty and rational voters leads to a policy in which the preferences of some voters are taken into account more than those of others leading to some polarization, a result in line with the findings of Schultz (2007), Callander and Wilkie (2007) and Bernhardt et al. (2008).

**Adaptive Learning** In our derivation of polarization, we rely on the notion that voters update adaptively. While this assumption is in line with a well-established literature on retrospective voting (Fiorina (1981)), we also derive in the Online Appendix results of media fragmentation on polarization, keeping the prior fixed. Our results still hold, but are less sharp, which is one more reason for why we have chosen to stick with adaptive priors. Further, our results continue to hold if only a small share of the electorate updates in an adaptive manner.

Additionally, we turned to data to see whether it is possible to see cyclical movements in polarization that stem from party platforms, as predicted by our model, and we show that this is indeed the case, see Section 4.1.

**Adding Links** We focused so far on a specific change in the media structure, media fragmentation. New technologies, however, potentially also increased spill-overs. We capture this notion by adding links to the network. We show that if a voter adds a link, then polarization can increase or decrease. While in the complete network, where every voter is connected to every outlet, polarization is at its lowest, there are non-monotonicities which can lead to higher levels of polarization. We provide examples of this and a formalization of the problem.

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30 This seems realistic as voters only have limited information about the audience of a given media outlet.
in the Appendix. The analysis is more involved compared to fragmentation as the number of voters that are targeted can no longer be kept constant. As the number of voters that are targeted across a cycle increases, polarization decreases. However, if more ideologically extreme voters connect to outlets that belong to the extreme target set, polarization can increase. Again, this implies that if voters with more distinct preferences choose to look for new outlets, this can increase polarization.

Positive Campaigning We restrict attention to positive advertising, that is candidates only disclose information about their own policies. This is a specific type of advertising, in contrast to much of the advertisement that focuses on the valence of the opponent. If negative advertising focuses on the latter, but not on the specific platforms, our analysis remains unaffected. If candidates also disclose information on the opponents’ policies, the information of unattached voters can change. If voters discount the information they receive from the opponent to some extent, then our qualitative results remain unchanged. However, even if voters do not discount the information provided by the opponent, the results still hold if we consider multiple policy dimension. If the policy is interpreted as a policy dimension (which is feasible as it is straightforward to extend a probabilistic voting model to multiple dimensions), then this would require the opponent to disclose in each of the dimensions. Otherwise, in some dimension voters would still hold incorrect priors and the average, overall policy position will still be incorrect and qualitative results are unchanged.

Commitment to Platforms Candidates reveal their platform and can commit to it. If candidates could lie, voters should not believe anything politicians promise. Additionally, it is not obvious, that lying is a good strategy. Theoretically, Callander and Wilkie (2007) have shown that lying is limited and empirically the evidence seems to be mixed (Lau et al. (2007)). As Callander and Wilkie (2007) point out, it is still an unresolved issue whether campaign promises have implications for policies. I assume throughout that it is the case, following Hotelling (1929).

Abstention The model does not allow for abstention of unattached voters. This is without loss of generality, see Kamada and Kojima (2013). They show that introducing a cost of voting does not change any of the strategic considerations.
4 Media & Polarization: Empirical Support

4.1 Polarization in US Politics

A key prediction of the paper is that media fragmentation leads to a higher level of polarization. The following empirical exercise documents that this prediction cannot be falsified with the data available. Further, the model implies political cycles, which, in fact, are present in the data.

Trend in Polarization The increase in polarization has first been shown by Poole and Rosenthal using the DW-NOMINATE data set. Poole and Rosenthal collected data on roll-call voting in the House of Representatives from 1879 to 2013. They use multidimensional scaling techniques in order to project the roll-call voting data on a one- or two-dimensional space. They assume that politicians have symmetric and single-peaked utility functions that are centered around an ideal point. Further, politicians vote probabilistically and assign a higher probability to their preferred outcome. Poole and Rosenthal perform a maximum likelihood estimation, simultaneously estimating the points of the roll-call votes and the ideal points of the politicians on the one- or two-dimensional space. They show that the one-dimensional policy space explains most of the votes and they argue that this dimension is the left-right, socio-economic dimension. Therefore, we focus in what follows on this first dimension. The optimal parameters from the maximum likelihood estimation are then placed on a scale from -1 to 1, with -1 being liberal and 1 being conservative. Ideologically similar politicians are placed close to each other, ideologically different politicians are set further apart. They then aggregate the information about individual legislators to derive the party positions in the House and Senate and calculate polarization as the difference in party means. Their measure of polarization is depicted in Figure 4, highlighting the increase of polarization in recent years.

Cyclical Component of Polarization While the long run trend in polarization is well established, our contribution is to highlight how polarization changes around its trend. We therefore de-trend polarization using an HP filter. The cyclical component we are left with

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31 For a detailed description of the data see Keith Poole’s website voteview.com, their findings are summarized in Poole and Rosenthal (2000).

32 We use an HP filter with a smoothing parameter of 6.25. The parameter of 6.25 was suggested for yearly data by Ravn and Uhlig (2002). To the best of our knowledge there does not exist a benchmark smoothing parameter for biannual data. We therefore report the results for a smoothing parameter of 6.25 for our main results. We have used a variety of smoothing parameters, both larger and smaller that have not yielded qualitatively different results as can be seen in the Online Appendix. One reason for sticking to 6.25 are also the problems with too small of a smoothing parameter as pointed out by Harvey and Trimbur (2008). They show that choosing the parameter too small results in standard errors that are too small and that trends can wrongly absorb cycles.
Poole & Rosenthal, DW-Nominate, voteview.com explains roughly 2% of the trend and is depicted in Figure 5. We estimate various ARMA specifications and select the best model based on the information criteria (AIC and BIC). The results for polarization, given in Table 1, show that there is a significant positive correlation with the first lag and a clear negative correlation with the second lag. This establishes the existence of a political cycle in which the level of polarization is similar for four years only to change afterwards. In particular, if polarization is high for a given congress, then polarization is also high for the following one and low, relative to the trend, in the Congress thereafter.

In the Online Appendix, we show that these cycles in polarization are due the cyclical component in the Democratic and Republican platforms, that is party platforms diverge and converge again, relative to the trend. We further conduct a variety of robustness checks, see the Online Appendix. We control for presidential elections, incumbency, we only consider first time members of Congress to ensure that results are not driven by the smoothing parameters.
Table 1: Cyclical Components of Polarization

<table>
<thead>
<tr>
<th></th>
<th>Senate</th>
<th>House of Representatives</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st Lag</td>
<td>0.840***</td>
<td>1.414***</td>
</tr>
<tr>
<td></td>
<td>(0.132)</td>
<td>(0.0927)</td>
</tr>
<tr>
<td>2nd Lag</td>
<td>-0.412***</td>
<td>-0.636***</td>
</tr>
<tr>
<td></td>
<td>(0.116)</td>
<td>(0.104)</td>
</tr>
<tr>
<td>Observations</td>
<td>68</td>
<td>68</td>
</tr>
</tbody>
</table>

* p < 0.05, ** p < 0.01, *** p < 0.001, standard errors in parentheses

Note: Cyclical Components in polarization from Poole and Rosenthal's DW-NOMINATE scores of first dimension obtained by using an HP-filter with smoothing parameter 6.25. Processes selected according to BIC and AIC were an ARMA (2,1) for the Senate and an ARMA (2,2) for the House of Representatives.

used in DW NOMINATE to adjust positions of member of Congress over time. Additionally, we correlate political platforms and polarization with GDP growth. In all of our specifications the cycles emerge highlighting their robustness.

4.2 Polarization, Newspapers, Radio, and TV

Figure 4 highlights that polarization was high at the turn of the century, decreased thereafter reaching a low in the 1940s, only to increase and surge in recent times. This paper proposes media fragmentation as a potential explanation of this pattern of polarization, which is empirically supported by Gentzkow et al. (2014). They establish that there was significant competition among ideologically diverse newspapers at the turn of the century. Additionally, individuals preferred newspapers in line with their own ideological convictions. This allows candidates to transmit a specific message to a narrow subset of readers, which our model predicts to lead to a higher level of political polarization, in line with the evidence. From the 1930s onwards the radio became more popular, followed by the introduction of TV. Both radio and TV at this time are mass media, in the sense that very few programmes are broadcast to everyone. Our model would then predict a decrease in polarization as the media network corresponds to an environment in which every voter is connected to the same few outlets. Our prediction is confirmed by Campante and Hojman (2013), who show that higher prevalence of radio and TV is related to a decrease in polarization in the 1940s and 1950s. We argue that after this time period, the increase in polarization may be attributed to the fact that TV became increasingly more fragmented with the introduction of many new TV channels and programmes as well as the advent of the internet.

33In this spirit, Gerbner et al. (1980) argue that TV creates a common narrative.
34Relatedly, the internet can create "echo chambers" (Sunstein (2009), Bishop (2009)), which make it easier to identify voters’ views and ideologies.
4.3 Internet & Polarization

The internet and the emergence of micro-targeting has made it particularly simple to target specific voters. We therefore proceed to analyse the effect of internet penetration on the level of polarization on the Members of the US Congress. If individuals spend more time online, then candidates can adjust their advertising strategy to the observed browsing history and provide the ad this person is interested in. We test our hypothesis using data on internet penetration collected by the Federal Communications Commission (FCC) for each county in the U.S. In order to match the counties to U.S. congressional districts, we use Census data. To measure polarization we use the DW-NOMINATE scores.

**Data Description** We combine the DW-NOMINATE data with information about internet penetration. The FCC collects data for each county on three measures of internet penetration.\(^{35}\) The data comprises of information about how many households in a given county have a fixed high-speed connection over 200 kbps and how many households have a speed of 3 Mbps downstream and 768 per kbps upstream (broadband). Additionally, the data provides information on the overall number of internet providers in a given county. In previous studies the number of internet providers has been the best measure of internet penetration.\(^{36}\) However, the available data from the FCC allows a more direct measure of actual internet connection and speed and therefore improves on previous studies. All three measures of internet penetration have been collected bi-annually since December 2008. We use the data until 2013 as we have all three measures of internet penetration only for these years.

We match the counties to congressional districts according to their population weights using 2010 Census data and the weights given therein.\(^{37}\) The data set contains information about the congressional districts. As we only have information about internet penetration for a limited number of years, we restrict attention to the 110th to the 113th Congress, which comprises of years 2009-2015. For these years of congress, we use the DW-NOMINATE score for each member of the House of Representatives. The descriptive statics of internet penetration, polarization and controls are depicted in Table 2.

Whether households in a county have internet access, low speed or broadband, is measured on a scale from 1 to 5 by the FCC. The FCC assigns a county a value of one if less than 200 out of 1000 households have internet access, two if less than 400 out of 1000 households have internet access etc. To simplify the interpretation of the coefficients, we redefine the ordinal scales to percentages, with one being replaced by 20%, two by 40% etc. As we are

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\(^{36}\) For a discussion of this, see Larcinese and Miner (2012) and Kolko (2010).

\(^{37}\) The data is available at [http://mcdc.missouri.edu/websas/geocorr12.html](http://mcdc.missouri.edu/websas/geocorr12.html).
Table 2: Summary Statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min.</th>
<th>Max.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dist. Mean. Pol.</td>
<td>0.519</td>
<td>0.162</td>
<td>0.016</td>
<td>1.208</td>
</tr>
<tr>
<td>Mean Polarization</td>
<td>0.156</td>
<td>0.06</td>
<td>0.084</td>
<td>0.216</td>
</tr>
<tr>
<td>Internet (200MB)</td>
<td>3.84</td>
<td>0.577</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>Broadband</td>
<td>3.094</td>
<td>0.684</td>
<td>1.473</td>
<td>5</td>
</tr>
<tr>
<td># Internet Providers</td>
<td>26.343</td>
<td>11.003</td>
<td>7.166</td>
<td>62.5</td>
</tr>
<tr>
<td>Internet Percentage (200MB)</td>
<td>0.768</td>
<td>0.115</td>
<td>0.4</td>
<td>1</td>
</tr>
<tr>
<td>Broadband Percentage</td>
<td>0.619</td>
<td>0.136</td>
<td>0.295</td>
<td>1</td>
</tr>
<tr>
<td>Percent White</td>
<td>72.522</td>
<td>14.874</td>
<td>20.8</td>
<td>96.867</td>
</tr>
<tr>
<td>Median Age</td>
<td>37.399</td>
<td>2.975</td>
<td>27.179</td>
<td>50.776</td>
</tr>
<tr>
<td>Population Density per sqm/1000</td>
<td>2.114</td>
<td>6.275</td>
<td>0.012</td>
<td>68.834</td>
</tr>
<tr>
<td>Owner Occupied</td>
<td>65.247</td>
<td>8.954</td>
<td>18.5</td>
<td>83.922</td>
</tr>
<tr>
<td>Population</td>
<td>7.127</td>
<td>3.132</td>
<td>0.001</td>
<td>74.465</td>
</tr>
<tr>
<td>Observations</td>
<td>1739</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Dist. Mean. Pol. is the absolute value of the difference between mean of polarization among members of the House of Representatives and the individual level of polarization. Mean Polarization is the average level of polarization. Internet (200 MB) and Broadband give the share of users of low speed versus Broadband internet, respectively, in a given congressional district on a scale from 1-5: 1 implies that up to 20% of individuals have this type of internet access, 2 means up to 40% etc. As we aggregate the counties to congressional districts, numbers do not have to be integers. # Internet Providers is a measure of the number of internet providers in a congressional district. % White is the share of white members in the constituency. Owner Occupied is the share of individuals who live in their own house in each constituency. Population gives the number of individuals in a congressional district. Pop. Dens. per sqm/1000 provides a measure of the population density per sqm, divided by 1000 in each district.

averaging across time over internet penetration rates, we get an average percentage of internet penetration.

We calculate the mean of polarization for each congress and consider the deviation from this mean. We document what this looks like for the 113th Congress, see Figure 6. Figure 6a shows the individual levels of polarization in the House of Representative. It can be seen that the Democrats lie clearly to the left of zero and the Republicans are to the right. Further, Republicans are more extreme compared to Democrats. The black line depicts the mean of polarization. Subtracting the individual levels of polarization from the mean and taking absolute values, gives Figure 6b. This is our dependent variable in the regression.

**Estimation** We have information about internet penetration and polarization for four consecutive congresses. To exploit the panel structure of the available data, we estimate a fixed effects model with time-state fixed effects as well as congressional district level fixed effects. We further cluster standard errors at the congressional district level. Due to our fixed effect estimation, we only use the variation within each congressional district over time. Additionally, we include time dummies to make sure our results are not driven by a time trend. Although there is no redistricting over time, the allocation of some counties to congressional
Note: Figure 6a shows the polarization in the House of Representatives for the 113th Congress. We distinguish between Democrats and Republicans. The measure used is the first dimension of the DW NOMINATE score. The black line represents the mean of polarization, which is positive. In Figure 6b we depict the absolute value of the difference between the individual level of polarization and its mean.

district changes. Counties can be split among congressional districts. As an example, suppose 70% of a county is included in congressional district one and 30% is in congressional district two for the 111th Congress. Then it can be the case that 50% are included in district one and 50% in district two for the 112th congress. This can have an impact on the voter composition in the congressional districts, which is why we include various controls.

Results The results of our estimation are given in Table 3. Columns (2) through (4) show the effect of an increase of internet penetration on polarization. We add controls in columns (5)-(7) and focus on these regressions. A one unit increase in households with low speed internet penetration is correlated with an increase in polarization by .122. In terms of standard deviations, this implies that an increase of one standard deviation of low speed internet is associated with an increase in polarization of .087 standard deviations. The effect of broadband is larger, with an increase of one standard deviation in broadband leading to an increase in polarization of .16 standard deviations, relative to the mean. Similarly, an increase in the number of internet providers has a significantly positive impact on polarization.

To summarize, we find a significantly positive effect of internet penetration on polarization.\textsuperscript{38}

Endogeneity Concerns & Alternative Explanations Our regressions highlight that a higher internet penetration in a given congressional district is associated with a higher level of polarization among legislators. While we use panel data techniques and include

\textsuperscript{38} We run various specifications with additional controls, which are available upon request. Throughout all of our specifications our qualitative outcomes remain unchanged.
Table 3: Internet Penetration and Polarization

<table>
<thead>
<tr>
<th></th>
<th>Distance Mean Polarization</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Internet % (200MB)</td>
</tr>
<tr>
<td></td>
<td>0.122*</td>
</tr>
<tr>
<td></td>
<td>(0.0671)</td>
</tr>
<tr>
<td></td>
<td>(0.0706)</td>
</tr>
<tr>
<td></td>
<td>0.0544</td>
</tr>
<tr>
<td></td>
<td>(0.0658)</td>
</tr>
<tr>
<td></td>
<td>-0.0576</td>
</tr>
<tr>
<td></td>
<td>(0.0706)</td>
</tr>
<tr>
<td></td>
<td>0.122*</td>
</tr>
<tr>
<td></td>
<td>(0.0671)</td>
</tr>
<tr>
<td></td>
<td>0.195***</td>
</tr>
<tr>
<td></td>
<td>(0.0646)</td>
</tr>
</tbody>
</table>

Standard errors in parentheses, * \( p < 0.10 \), ** \( p < 0.05 \), *** \( p < 0.01 \)

Notes: Dependent variable is always the absolute value of the difference between mean of polarization among members of the House of Representatives and the individual level of polarization. All regressions include congressional district level and time-state fixed effects as well as clustered standard errors at the congressional district level. Internet % (200 MB) and Broadband % give the percentage of users of low speed versus Broadband internet, respectively, in a given congressional district. # Internet Providers is a measure of the number of internet providers in a congressional district. The Congress dummies measure time trend, we include Republican to measure whether the Member of Congress in the district is Republican or Democrat. % White is the share of white members in the constituency, Owner Occupied is the share of individuals who live in their own house in each constituency. Population gives the number of individuals in a congressional district. Pop. Dens. per sqm/1000 provides a measure of the population density per sqm, divided by 1000 in each district.
various controls, we cannot rule out all confounding factors. One potential issue could be that higher internet penetration is related to higher polarization among voters, which then triggers the higher polarization among legislators. The effect of internet use on polarization among voters has been analyzed by Boxell et al. (2017), who show that greater internet use is not associated with faster growth in political polarization among voters. Thus, the effect found here does not seem to be due to increased polarization among voters. Moreover, a higher internet penetration could potentially lead to individuals becoming more informed about policy matters. According to our model, better information about policies generally decreases polarization. However, this effect did not emerge, implying that media structure per se is a relevant predictor of polarization. Thus, our evidence, taken together with the evidence provided by Gentzkow et al. (2014) and Campante and Hojman (2013) provides support that media structure affects polarization among politicians. A second explanation for the pattern of polarization brought forward by McCarty et al. (2006) is the level of income inequality in the US. This income inequality has lead to a small share of very wealthy donors which increased the influx of money in politics in recent years. But for the money in politics to matter, it also has to affect votes and we argue that one channel might be targeting. Thus, our model does not contradict the explanation of income inequality in polarization, but rather provides a channel of why this income inequality may matter.

5 Conclusion

We connect the rise in polarization to changes in media technology, which allow for higher precision in the advertising strategy of candidates. In order to do so, we develop a framework to analyze how media structure affects policy. In our setting, voters differ in their media and policy preferences. We provide a novel and tractable measure of network centrality, media centrality, that takes not only the network structure, but also the ideology of voters into account and is a weighted degree centrality. We then ask how the fragmentation of the media network influences the level of polarization. Fragmentation allows candidates to target policies narrowly to a subset of voters. This induces an increase in polarization if both moderate and extreme voters can be targeted more precisely. We then take our prediction to the data, which indicates that internet penetration and polarization are positively correlated, lending support that voters of all political leanings can be targeted more precisely.

In our setting, voters do not make an inference about the policy if they are not targeted.

39 Similarly, Gentzkow and Shapiro (2011) show that there is no additional segregation among individuals online which further seems to indicate that the internet has not systematically affected voters’ attitudes and beliefs.
We show that our qualitative results carry over to a setting with rational voters and uncertainty about the state. Moreover, we restrict attention to symmetric media structures, which may serve as a benchmark for future work that considers observed media structures.
References


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Appendix

Media Centrality

**Proof Proposition 1:** We first establish that for any target set, both parties disclose the average bliss point in the target’s coverage. Fix $T_c$ the target set of politician $c$, and recall that his policy decision needs to solve

$$\max_{x_c} \left[ \int_0^1 u(x_c|x) \sum_{i \in K(T_c)} g_i(x) dx + \int_0^1 u(\pi_c|x) \sum_{i \in K \setminus K(T_c)} g_i(x) dx \right]$$

Our functional form assumption and requirement on voting behaviour implicit in the model imply that the problem is equivalent to solving

$$\max_{x_c} \int_0^1 \left[ -(x_c - x)^2 \right] \sum_{i \in K(T_c)} g_i(x) dx$$

Necessary conditions for an interior optimum then require that

$$\int_0^1 (x_c - x) \sum_{i \in K(T_c)} g_i(x) dx = 0$$

By our functional form assumptions, the condition is also sufficient for an optimum, and fully characterizes the equilibrium policy for any given target decision. Simple manipulations then establish that

$$x_c = \left[ \int_0^1 x \sum_{i \in K(T_c)} g_i(x) dx \right] / \left[ \int_0^1 \sum_{i \in K(T_c)} g_i(x) dx \right] = \frac{1}{k(T_c)} \sum_{i \in K(T_c)} \int_0^1 x g_i(x) dx = E(X|T_c)$$

which establishes the first part of the proposition.

Begin by calculating the payoff of any possible targeting strategy. Consider a target set $T$, and observe that, at the optimal policy, the preference of candidate $c$ are a positive affine transformation of

$$\int_0^1 u(E(X|T)|x) \sum_{i \in K(T)} g_i(x) dx + \int_0^1 u(\pi_c|x) \sum_{i \in K \setminus K(T)} g_i(x) dx =$$

$$\int_0^1 \left[ -(E(X|T) - x)^2 \right] \sum_{i \in K(T)} g_i(x) dx + \int_0^1 \left[ -(\pi_c - x)^2 \right] \sum_{i \in K \setminus K(T)} g_i(x) dx$$
If so target set \( T \) is better than target set \( S \) if and only if
\[
\int_0^1 (E(X|T) - x)^2 \sum_{i \in K(T)} g_i(x) \, dx + \int_0^1 (\pi_c - x)^2 \sum_{i \in K \setminus K(T)} g_i(x) \, dx < \int_0^1 (E(X|S) - x)^2 \sum_{i \in K(S)} g_i(x) \, dx + \int_0^1 (\pi_c - x)^2 \sum_{i \in K \setminus K(S)} g_i(x) \, dx
\]

However, the definition of variance and simple manipulations establish that
\[
\int_0^1 (E(X|T) - x)^2 \sum_{i \in K(T)} g_i(x) \, dx = k(T) \int_0^1 (E(X|T) - x)^2 \, dG(x|T) = k(T) (E(X^2|T) - E(X|T)^2)
\]

We write in what follows \( T' - T \) if we refer to the voters contained in target set \( T' \), but not in \( T \). Thus, we denote the expected bliss points of these voters by \( E(X|T' - T) \). Formally, the distribution of ideal points in \( K(T) \) that do not belong to the coverage \( K(S) \) of media outlets \( S \subseteq M \) is defined as \( G(x|T - S) = \frac{1}{|K(T) \setminus K(S)|} \sum_{i \in K(T) \setminus K(S)} g_i(x) \). We denote the expectation of \( G(x|T - S) \) by \( E(X|T - S) \). For convenience, we also let \( k(T - S) \) denote \( |K(T) \setminus K(S)| \). Then, it follows that
\[
\int_0^1 (\pi_c - x)^2 \sum_{i \in K \setminus K(T)} g_i(x) \, dx = \int_0^1 (\pi_c^2 - 2\pi_c x + x^2) \sum_{i \in K \setminus K(T)} g_i(x) \, dx
\]
\[
= k(M - T) \left( \pi_c^2 - 2\pi_c E(X|M - T) + E(X^2|M - T) \right)
\]

Using the three previous observations then implies that
\[
k(T) (E(X^2|T) - E(X|T)^2) + k(M - T) \left( \pi_c^2 - 2\pi_c E(X|M - T) + E(X^2|M - T) \right)
\]
\[
< k(S) (E(X^2|S) - E(X|S)^2) + k(M - S) \left( \pi_c^2 - 2\pi_c E(X|M - S) + E(X^2|M - S) \right)
\]

(7)

Now, observe that for any \( T \), we have that
\[
k(T)E(X^2|T) + k(M - T)E(X^2|M - T) = kE(X^2|M),
\]
as we are averaging over all voters in \( K \). Further,
\[
k(M - S)E(X|M - S) - k(M - T)E(X|M - T) = \sum_{i \in K \setminus K(S)} E_i(X) - \sum_{i \in K \setminus K(T)} E_i(X)
\]
\[
= \sum_{i \in K(T)} E_i(X) - \sum_{i \in K(S)} E_i(X)
\]
\[
= k(T)E(X|T) - k(S)E(X|S),
\]
where the second equality folds by adding and subtracting $\sum_{i \in K} E_i(X)$. Based on the last two simplifications, inequality (7) becomes

\[ k(M - T) \left( \pi_c^2 - 2\pi_c E(X|M - T) \right) - k(T) E(X|T)^2 < k(M - S) \left( \pi_c^2 - 2\pi_c E(X|M - S) \right) - k(S) E(X|S)^2 \]

\[ \Leftrightarrow \quad k(S) \left( E(X|S)^2 + \pi_c^2 \right) - k(T) \left( E(X|T)^2 + \pi_c^2 \right) < -2\pi_c (k(T) E(X|T) - k(S) E(X|S)) \]

\[ \Leftrightarrow \quad k(S) (E(X|S) - \pi_c)^2 < k(T) (E(X|T) - \pi_c)^2 \]

As the same logic applies to every target set it follows that, it is optimal to choose that target set that maximizes $k(T) (E(X|T) - \pi_c)^2$.

We can use the concept of media centrality to compare target sets. The following lemmas will be useful for this.

**Lemma 3.** Consider any two target sets $T, T' \subseteq M$ such that $T \subseteq T'$. Then $W_c(T) < W_c(T')$ if and only if

\[ k(T) \left[ E(X|T' - T) - E(X|T) \right]^2 < k(T') \left[ E(X|T' - T) - \pi_c \right]^2. \] (8)

**Proof Lemma 3:** Observe first that, since $T \subseteq T'$,

\[ W(T') = k(T') \left[ E(X|T') - \pi_c \right]^2 = k(T') \left[ \frac{k(T) E(X|T) + k(T' - T) E(X|T' - T)}{k(T')} - \pi_c \right]^2. \]

If so, $W(T) < W(T')$ is equivalent to

\[ k(T) \left[ E(X|T) - \pi_c \right]^2 < k(T') \left[ \frac{k(T) E(X|T) + k(T' - T) E(X|T' - T)}{k(T')} - \pi_c \right]^2. \]

Simple manipulations then show that this is equivalent to

\[ k(T) \left[ E(X|T' - T) - E(X|T) \right]^2 < k(T') \left[ E(X|T' - T) - \pi_c \right]^2. \]

\[ \blacksquare \]

**Lemma 4.** Let $\max\{E(X|T), E(X|T')\} < \pi_c$ and $E(X|T' - T) \geq \pi_c$. Then, $W_c(T) > W_c(T')$.

**Proof Lemma 4:** Lemma 4 highlights that outlets may be added to a given target set, $T$, only if their bliss points, $E(X|T)$ and $E(X|T' - T)$, lie to the same side of the prior. Intuitively, voters contained in target set $T'$, but not in $T$ are more likely to vote for the
If so, condition (10) must hold as well since Lemma 5. A symmetric logic applies to the converse scenario. Observe that \( E(X|T') \geq \pi_c \) implies that \( E(X|T) < \pi_c \), as

\[
E(X|T') = \frac{k(T)E(X|T) + k(T' - T)E(X|T' - T)}{k(T) + k(T' - T)} < \pi_c.
\]

Moreover, this implies that

\[
\pi_c - E(X|T) > \pi_c - E(X|T') > 0. \tag{9}
\]

For a candidate not to be willing to add outlets \( T' \setminus T \) to the target set it must be that

\[
k(T') [\pi_c - E(X|T')]^2 < k(T) [\pi_c - E(X|T)]^2.
\]

Moreover by (9) the latter condition is equivalent to

\[
k(T')^{1/2} [\pi_c - E(X|T')] < k(T)^{1/2} [\pi_c - E(X|T)].
\]

Writing explicitly the expected target set yields the following

\[
k(T')^{-1/2} \sum_{i \in K(T')} [\pi_c - E_i(X)] < k(T)^{-1/2} \sum_{i \in K(T)} [\pi_c - E_i(X)].
\]

Thus the politician does not target outlets \( T' \setminus T \) if

\[
\sum_{i \in K(T')} [\pi_c - E_i(X)] < \left( \frac{k(T')}{k(T)} \right)^{1/2} \sum_{i \in K(T)} [\pi_c - E_i(X)]. \tag{10}
\]

But, as \( \pi_c - E(X|T' - T) \leq 0 \) is equivalent to \( \sum_{i \in K(T')} [\pi_c - E_i(X)] \leq 0 \), we have that

\[
\sum_{i \in K(T')} [\pi_c - E_i(X)] \leq \sum_{i \in K(T)} [\pi_c - E_i(X)].
\]

If so, condition (10) must hold as well since \( k(T') > k(T) \).

Lemma 5. Let \( E(X|T) < \pi_c \). If either of the following conditions hold, \( W_c(T') > W_c(T) \):

1. \( E(X|T' - T) \leq [\pi_c + E(X|T)] / 2 \);
2. \( W_c(T) > W_c(T'') \) for some \( T'' \subset T \) with \( E(X|T - T'') \in (E(X|T' - T), \pi_c) \).

Proof Lemma 5: (1) Including voters whose bliss point is closer to \( E(X|T) \) than to the prior unambiguously increases media centrality. To see this, consider the case in which
\( E(X|T' - T) \leq E(X|T) \). If so,
\[
E(X|T') = \frac{k(T)E(X|T) + k(T' - T)E(X|T' - T)}{k(T')} \leq E(X|T),
\]
and therefore \( W_c(T) \leq W_c(T') \), as it is possible to both increase the number of voters targeted – as \( k(T') \geq k(T) \) – without decreasing the difference between the expected bliss point and the prior – as \( \pi_c - E(X|T') \geq \pi_c - E(X|T) \).

Next consider the case in which \( E(X|T' - T) > E(X|T) \). It is better to disclose also in outlet \( j \) if
\[
k(T') \left[ \frac{k(T)E(X|T) + k(T' - T)E(X|T' - T)}{k(T')} - \pi_c \right]^2 \geq k(T) \left[ E(X|T) - \pi_c \right]^2.
\]
This inequality by Lemma 3 and some algebra is equivalent to
\[
k(T' - T) \left( E(X|T' - T) - \pi \right)^2 \geq k(T) \left( E(X|T) - \pi_c \right) \left( E(X|T) + \pi_c - 2E(X|T' - T) \right). \quad (11)
\]
However by assumption we have that \( E(X|T) + \pi_c > 2E(X|T' - T) \) and \( E(X|T) < \pi_c \). Thus the RHS of (11) is negative, whereas the LHS of (11) is positive; and thus inequality of (11) must hold.

(2) If adding more moderate voters increases the support for a politician, then adding voters with more extreme bliss points also increases support. Suppose there is a target set \( T'' \).
Then, if it is optimal to add outlets with bliss points \( E(X|T - T'') \) such that target set \( T \) emerges, it is also optimal to add any other outlet with \( E(X|T' - T) < E(X|T - T'') \).

By part (1), including outlets \( T' \setminus T \) to the target set increases the candidate’s payoff if \( E(X|T' - T) < E(X|T) < \pi_c \). Thus, suppose that \( E(X|T' - T) > E(X|T) \). By assumption \( T \) has a higher media centrality than \( T'' \) and thus by Lemma 3 we have that
\[
k(T) \left[ \pi_c - E(X|T - T'') \right]^2 > k(T'') \left[ E(X|T'') - E(X|T - T'') \right]^2.
\]
We want to establish that
\[
k(T') \left[ \pi_c - E(X|T' - T) \right]^2 > k(T) \left[ E(X|T) - E(X|T' - T) \right]^2. \quad (12)
\]
Observe that, as \( E(X|T - T'') \in (E(X|T' - T), \pi_c) \), we have that
\[
[\pi_c - E(X|T' - T)]^2 > [\pi_c - E(X|T - T'')]^2,
\]
so that (12) holds.
Similarly, as $E(X|T' - T) > E(X|T)$, we have that

$$[E(X|T) - E(X|T - T'')]^2 > [E(X|T) - E(X|T' - T)]^2.$$ 

But, from the previous inequalities we have that

$$k(T') [\pi_c - E(X|T' - T)]^2 > k(T') [\pi_c - E(X|T - T'')]^2$$

$$> \frac{k(T')k(T'')} {k(T)} [E(X|T'') - E(X|T - T'')]^2$$

$$= \frac{k(T')k(T)} {k(T'')} [E(X|T) - E(X|T - T'')]^2$$

$$> k(T) [E(X|T) - E(X|T - T')]^2$$

$$> k(T) [E(X|T) - E(X|T' - T)]^2.$$ 

where the equality follows as $k(T'')E(X|T'') = k(T)E(X|T) - k(T - T'')E(X|T - T'')$. This completes the proof by establishing (12). \hfill \Box

**Proof Lemma 1** We define two target sets $T_{max}$ and $T_{min}$, where $T_{max} \in \arg \max_T k(T)E(X|T)$ and $T_{min} \in \arg \min_T k(T)E(X|T)$. The policy platform of candidate $c$ is given by

$$x_c = \frac{k(T_c)E(X|T_c) + lE_c(X)} {k(T_c) + l}$$

We want to ensure that

$$\frac{k(T_{max})E(X|T_{max}) + lE_A(X)} {k(T_{max}) + l} < \frac{k(T_{min})E(X|T_{min}) + lE_B(X)} {k(T_{min}) + l}$$

This can be rewritten as

$$k(T_{max})k(T_{min}) (E(X|T_{max}) - E(X|T_{min})) + l (k(T_{max})E(X|T_{max}) - k(T_{min})E(X|T_{min}))$$

$$< l^2 (E_B(X) - E_A(X)) + l (k(T_{max})E_B(X) - k(T_{min})E_A(X))$$

It is always the case that

$$l (k(T_{max})E(X|T_{max}) - k(T_{min})E(X|T_{min})) \leq l (k(T_{max})E_B(X) - k(T_{min})E_A(X))$$
as $E_A(X) \leq E_i(X) \leq E_B(X)$. Therefore, if

$$k(T_{\text{max}})k(T_{\text{min}})(E(X|T_{\text{max}}) - E(X|T_{\text{min}})) \leq l^2 (E_B(X) - E_A(X))$$

$$\Leftrightarrow \sqrt{k(T_{\text{max}})k(T_{\text{min}})} \sqrt{E(T_{\text{max}}) - E(T_{\text{min}})} \equiv \sqrt{E_B(X) - E_A(X)}$$

it must be that $x_A \leq x_B$. ■

**Polarization**

We add an additional lemma, which turns out to be helpful to see what target sets are optimal if priors change. The proof is immediate and therefore omitted.

**Lemma 6.** Let one of the following conditions hold.

\[ C.1 \quad 0 < E(X|T) < E(X|T') < \pi' < \pi < 1 \]

\[ C.2 \quad 0 < E(X|T') < E(X|T) < \pi < \pi' < 1 \]

\[ C.3 \quad 0 < E(X|T) < \pi < \pi' < E(X|T') < 1 \]

Then, if $T$ is preferred to $T'$ given prior $\pi$, it is also preferred given prior $\pi'$.

**Proof Proposition 2** (1) We want to show that a politician’s platform $x^t_c$ alternates between two policies that is for some $t \geq \bar{t}$, $x^t_c = E(X|T_L)$, $x^{t+1}_c = E(X|T_R)$ and $x^{t+2}_c = E(X|T_L)$.

Note that for a given $\pi$, there exists a generically unique target set $T$ that maximizes $W_c(T)$. To see this denote by $P(M)$ the power set of outlets $M$. The power set contains any possible target set. Every set has a maximum. This maximum is generically unique.\(^{40}\)

Denote by $T^t$ the target set in period $t$. Such a target set leads to policy $E(X|T^t)$.

Suppose first that $E(X|T^1) < E(X|T^2) < E(X|T^3)$. We are then interested in where $E(X|T^4)$ lies.

We first establish that

$$E(X|T^1) < E(X|T^4) < E(X|T^3) \quad (13)$$

To see this note that it must have been optimal to select $E(X|T^2)$ over $E(X|T^3)$ as the platform given prior $E(X|T^1)$, that is

$$k(T^2) (E(X|T^2) - E(X|T^1))^2 > k(T^3) (E(X|T^3) - E(X|T^1))^2$$

\(^{40}\)This can be established by the same logic as used in the proof of Proposition 5 in the Online Appendix.
Further, it must be the case that it is better to select platform \( E(X|T^3) \) over \( E(X|T^4) \) given prior \( E(X|T^2) \),

\[
k(T^3) \left( E(X|T^3) - E(X|T^2) \right)^2 > k(T^4) \left( E(X|T^4) - E(X|T^2) \right)^2
\]

Last, it must be the case that \( E(X|T^4) \) is preferred to \( E(X|T^2) \) for prior \( E(X|T^3) \),

\[
k(T^4) \left( E(X|T^4) - E(X|T^3) \right)^2 > k(T^2) \left( E(X|T^2) - E(X|T^3) \right)^2
\]

Taking these three equations together, it is straightforward to show that they hold if and only if \( E(X|T^1) < E(X|T^4) < E(X|T^3) \). If \( E(X|T^4) = E(X|T^2) \), that is the target sets and policies in period 2 and 4 are identical, then we have established that there is indeed cycling between two alternatives.

If, on the other hand, \( E(X|T^4) \neq E(X|T^2) \), it has to be the case that

\[
k(T^2) \left( E(X|T^2) - E(X|T^1) \right)^2 > k(T^4) \left( E(X|T^4) - E(X|T^1) \right)^2.
\]

It has to be better to choose \( E(X|T^2) \) rather than \( E(X|T^4) \) given prior \( E(X|T^1) \). This can only hold for

\[
E(X|T^1) < E(X|T^4) < E(X|T^2).
\]

We then have a sequence of policy platforms, where \( E(X|T^1) < E(X|T^4) < E(X|T^2) < E(X|T^3) \).

Based on this we turn to \( E(X|T^5) \). If \( E(X|T^5) = E(X|T^3) \), then we have again established that there is cycling between two alternatives. If \( E(X|T^5) \neq E(X|T^3) \), we again need to characterize where \( E(X|T^5) \) can lie. The details of the proof can be found in the online appendix. We can show that eventually for any target set, \( E(X|T^{i+1}) < E(X|T^i) \). As the policy space is bounded, this implies that at some time \( \bar{t} \), \( E(X|T^\bar{t}) = E(X|T^{\bar{t}+2}) \). Denote by \( E_0(X|T_L) \) and \( E_0(X|T_R) \) the policies that emerge in the long run for \( \gamma = 0 \).

**Proof Proposition 2:** (2) We aim to show that \( E(X|T_L) < E(X|T_L') < E(X|T_R) < E(X|T_R') \). By Lemma 6 it cannot be the case that

\[
E(X|T_L') < E(X|T_L) < E(X|T_R) < E(X|T_R')
\]
Further, it cannot be the case that

\[ E(X|T_L) < E(X|T_R) < E(X|T'_L) < E(X|T'_R) \]

For these cycles to be optimal, the following equations have to hold

\[
\begin{align*}
    k(T_R) (E(X|T_R) - E(X|T_L))^2 &> k(T'_L) (E(X|T'_L) - E(X|T_L))^2, \\
    k(T'_L) (E(X|T'_L) - E(X|T'_R))^2 &> k(T_R) (E(X|T_R) - E(X|T'_R))^2,
\end{align*}
\]

which cannot occur.

**Media & Polarization**

Before turning to the proof of Proposition 3, we establish a sequence of Lemmas which help to simplify the proof. We first establish a general result on fragmentation in the static setting, which highlights the effect of fragmentation on the voters in the target set.

**Lemma 7.** If \( \max\{E(X|T), E'(X|T')\} < \pi, j \in T, \) and \( K(j) \) is fragmented, then \( K'(T') \subseteq K(T) \) and \( E'(X|T') \leq E(X|T) \).

**Proof Lemma 7** Fragmentation of the voters connected to outlet \( j \) can either lead to (i) no change at all (ii) an outlet dropped from the target set.

Suppose first that the voters in the target set remain the same. Then, \( k'(T') = k(T) \), \( E'(X|T') = E(X|T) \), which implies \( W'(T') = W(T) \). By Lemma 3, it then follows that \( o \notin T \) implies \( o \notin T' \) as \( k'(o - T') = k(o - T) \) and \( E'(X|o - T') = E(X|o - T) \).

If an outlet is dropped from the target set such that fewer voters remain in the target set, \( K'(T') \subset K(T) \), then by the definition of media centrality, it must hold that \( E'(X|T') < E(X|T) \) as \( k'(T') < k(T) \). We then show that it cannot be optimal for an outlet \( o \notin T \) to be added, that is it must hold that \( o \notin T' \). We denote the outlets that are dropped by \( j_i \). Given that is has not been optimal to add outlet \( o \) to target set \( T \) it must hold that

\[
k(T) (E(X|o - T) - E(X|T))^2 > (k(T) + k(o - T)) (E(X|o - T) - \pi)^2 \quad (15)
\]

It is not optimal to add outlet \( o \) to target set \( T' \), if

\[
k'(T') (E'(X|o - T') - E'(X|T'))^2 > (k'(T') + k'(o - T')) (E'(X|o - T') - \pi)^2 \quad (16)
\]
Note first that given $K'(j_i) \cap K'(o) = \emptyset$, $E(X|o - T) = E'(X|o - T')$ and $k(o - T) = k'(o-T')$. This implies that RHS of inequality (15) is larger than the RHS of inequality (16). Additionally, it has to hold that

$$k'(T') (E'(T') - \pi)^2 > k(T) (E(T) - \pi)^2$$

(17)

If $E'(X|o - T') > \pi$, then by Lemma 4, $o \notin T'$. We therefore focus on $E'(X|o - T') < \pi$. By Lemma 5 for $o \notin T$, it must be that $E(X|T) < E(X|o - T)$. This implies that

$$E(X|T') < E(X|T) < E(X|o - T) < \pi$$

Lemma 6 C.1 together with inequality (17) implies that the LHS of inequality (16) is greater than the LHS of inequality (15) which establishes that inequality (16) is fulfilled.

Next, we show that given the assumption of symmetry on outlets and partisans, we can always find a policy that is symmetric as well.

**Lemma 8 (Symmetric Policy Platforms).** If media outlets and party members are symmetric, there exists an equilibrium in which policies are symmetric, that is $x_{AL} = 1 - x_{BR}$ and $x_{AR} = 1 - x_{BL}$.

**Proof Lemma 8:** For a given target set $T_i$ there always exists a target set $T_{-i}$ by symmetry, with $E(X|T_i) = 1 - E(X|T_{-i})$ and $k(T_i) = k(T_{-i})$. Then, for any given target set the following holds

$$\frac{k(T_i)E(X|T_i) + lE_B(X)}{k(T_i) + l} = 1 - \frac{k(T_{-i})E(X|T_{-i}) + lE_A(X)}{k(T_{-i}) + l}$$

(18)

as simplifying yields

$$\frac{k(T_i)E(X|T_i) + k(T_i)(1 - E(X|T_i)) + lE_B(X) + lE_A(X)}{k(T_i) + l} = 1 \Leftrightarrow \frac{k(T_i) + l}{k(T_i) + l} = 1$$

Therefore, equation (18) holds. This implies that for each policy that is available for candidate $A$ there exists a symmetric policy for candidate $B$. Suppose that $T_{AL}$ and $T_{AR}$ are the target sets for candidate $A$. Then, there exists target sets $T_{BL}$ and $T_{BR}$ such that $x_{AL} = 1 - x_{BR}$ and $x_{AR} = 1 - x_{BL}$. Additionally, if $T_{AL}$ and $T_{AR}$ are optimal, then it also has to be the case that $T_{BL}$ and $T_{BR}$ are optimal due to the symmetry. This completes the proof.

We focus on symmetric equilibria and derive the following comparative statics result.
Lemma 9. In equilibria with symmetric policies, higher levels of polarization are associated with an increase in $\sum_{i \in K(T_{BL})} E_i(X)$ and $\sum_{i \in K(T_{BR})} E_i(X)$ and with a decrease in $k(T_{BL})$ and $k(T_{BR})$.

Proof Lemma 9 We take derivatives of

$$\Delta_P = \frac{1}{2} \left( \frac{2 \left( \sum_{i \in K(T_{BL})} E_i(X) \right) - k(T_{BL})}{k(T_{BL}) + l} + \frac{2 \left( \sum_{i \in K(T_{BR})} E_i(X) \right) - k(T_{BR})}{k(T_{BR}) + l} ight) + l \left( E_B(X) - E_A(X) \right) \left( \frac{1}{k(T_{BL}) + l} + \frac{1}{k(T_{BR}) + l} \right),$$

with respect to $\sum_{i \in K(T_{BL})} E_i(X)$, $\sum_{i \in K(T_{BR})} E_i(X)$ and $k(T_{BL})$ and $k(T_{BR})$.

1. Change in $\sum_{i \in K(T_{BL})} E_i(X)$

$$\frac{\partial \Delta_P}{\partial \sum_{i \in K(T_{BL})} E_i(X)} = \frac{2}{k(T_L) + l} > 0$$

2. Change in $\sum_{i \in K(T_{BR})} E_i(X)$

$$\frac{\partial \Delta_P}{\partial \sum_{i \in K(T_{BR})} E_i(X)} = \frac{2}{k(T_R) + l} > 0$$

3. Change in $k(T_{BL})$

$$\frac{\partial \Delta_P}{\partial k(T_{BL})} = \frac{-l - 2 \left( \sum_{i \in K(T_{BL})} E_i(X) \right) - l \left( E_B(X) - E_A(X) \right)}{(k(T_L) + l)^2} < 0$$

4. Change in $k(T_{BR})$

$$\frac{\partial \Delta_P}{\partial k(T_{BR})} = \frac{-l - 2 \left( \sum_{i \in K(T_{BR})} E_i(X) \right) - l \left( E_B(X) - E_A(X) \right)}{(k(T_R) + l)^2} < 0$$

Lemma 9 establishes that polarization increases if the voters in the target sets become more extreme and if the number of targeted voters decreases. Note that this is not only a local effect, but holds as long as we restrict attention to symmetric equilibria.

We then highlight that the target sets are of weakly different sizes, implying that the influence of attached voters differs.

Proof Lemma 2 We want to show that $k(T_{BL}) \geq k(T_{BR})$. If both candidates select the same target sets, it is clear that the target sets are of equal size. We therefore focus on the case
where candidates select different target sets that yield symmetric policies. Due to symmetry, it must hold that the number of voters to the left of \( \frac{1}{2} \) equals the number of voters to the right of \( \frac{1}{2} \). As \( l > \bar{l} \), we know that for any possible target set \( T_{BL} \), \( x_{BL} > \frac{1}{2} \). To see this note that for \( x_{BL} < \frac{1}{2} \), by symmetry it must hold that \( \frac{1}{2} < x_{AR} \). But then \( x_{AR} > x_{BL} \), which yields a contradiction as \( l > \bar{l} \). Suppose now by contradiction that the number of outlets in \( T_{BR} \) is higher than the number of outlets in \( T_{BL} \). This can only be the case if for some outlet \( j_+ \), the symmetric outlet \( j_- \) is also contained in \( T_{BR} \). Otherwise, the number of voters in each target set would be half of all voters. By symmetry, it must be that \( E(X|j_- \cup j_+) = \frac{1}{2} \). Lemma 4 establishes that it cannot be optimal to include outlets with \( E(X|j_- \cup j_+) < x_{BL} \), which yields a contradiction and completes the proof. ■

We now turn to our second key result:

**Proof Proposition 3** It can always be the case that the target sets are not affected by media fragmentation in which case polarization remains unchanged. In what follows we focus on what happens if target sets are changed due to fragmentation. We consider the problem from \( B’ \)’s perspective

**Same Target Sets** Consider first the case of candidates selecting the same target sets. We first establish that for a newly created outlet \( j_i \), \( i \in \{+, -\} \), it cannot be the case that it is contained in both \( T_{BL} \) and \( T_{BR} \). We denote by \( E(X|j_i) \) the bliss point of the voter connected to \( j_i \). We first consider the case of an outlet omitted from \( B’ \)’s left target set. Denote the bliss point of the target set with \( j_i \) omitted by \( E(X|T_{BL} - j_i) \). Then by Lemma 3 for \( j_i \) to be contained in both target sets it must be that both inequalities hold:

\[
(k(T_{BR}) + l) \left( E(X|j_i) - \frac{k(T_{BR})E(X|T_{BR}) + lE_c(X)}{k(T_{BR}) + l} \right)^2 < (k(T_{BR}) + k(j_i) + l) \left( E(X|j_i) - \frac{k(T_{BL})E(X|T_{BL}) + lE_c(X)}{k(T_{BL}) + l} \right)^2
\]

\[
(k(T_{BL} - j_i) + l) \left( E(X|j_i) - \frac{k(T_{BL} - j_i)E(X|T_{BL} - j_i) + lE_c(X)}{k(T_{BL} - j_i) + l} \right)^2 < (k(T_{BL}) + l) \left( E(X|j_i) - \frac{k(T_{BR})E(X|T_{BR}) + k(j_i)E(X|j_i) + lE_c(X)}{k(T_{BR}) + l + k(j_i)} \right)^2
\]

We can show that these equations never hold simultaneously taking into account that by Lemmas 4 and 5, it must be that

\[
\frac{k(T_{BR})E(X|T_{BR}) + k(j_i)E(X|j_i) + lE_c(X)}{k(T_{BR}) + l + k(j_i)} > E(X|j_i) > \frac{k(T_{BL})E(X|T_{BL}) + lE_c(X)}{k(T_{BL}) + l}
\]
Thus, it must be the case that $j_i$ either in $T_{BL}$ or $T_{BR}$. Due to symmetry, $i_\pm \in K(T_{BL})$, $i_+ \in K(T_{BR})$. Then, there are three cases to consider, (i) $j_-, j_+ \in T'_{BL}$, (ii) $j_-, j_+ \in T'_{BR}$, and (iii) $j_- \in T'_{BR}$, $j_+ \in T'_{BL}$. We discuss each of these cases in return.

**Case 1: $j_-, j_+ \in T'_{BL}$**

This implies that voters originally contained in $B'$ right target set now belong to his left target set. The new level of polarization is given by

$$\frac{2 \left( \sum_{i \in K(T_{BL})} E_i(X) + \sum_{i' \in K'(j_+)} E_i(X) \right) - k(T_{BL}) - k(j_+)}{k(T_{BL}) + k(j_+) + l}$$

$$+ \frac{2 \left( \sum_{i \in K(T_{BR})} E_i(X) - \sum_{i' \in K'(j_+)} E_i(X) \right) - k(T_{BR}) + k(j_+)}{k(T_{BR}) + k(j_+) + l}$$

$$+ l \left( E_B(X) - E_A(X) \right) \left( \frac{1}{k(T_{BL}) + l + k(j_+)} + \frac{1}{k(T_{BR}) + l - k(j_+)} \right)$$

It is straightforward to show that this term is larger than $\frac{2l}{k(T_{BL})+l} \left( E_B(X) - E_A(X) \right)$, the old level of polarization and thus polarization increases.

**Case 2: $j_-, j_+ \in T'_{BR}$**

This can never occur, see Lemma 2.

**Case 3: $j_- \in T'_{BR}$, $j_+ \in T'_{BL}$**

Target sets change symmetrically, as $k(j_-) = k(j_+)$ and $E(X|j_-) = 1 - E(X|j_+)$. This implies that the level of polarization is unchanged, that is polarization still equals $\frac{2l}{k(T_{BL})+l} \left( E_B(X) - E_A(X) \right)$

**Different Target Sets** We then turn to symmetric equilibria in which candidates select different target sets. Again, we can show that $j_i$ is only contained in one target set, but not in both, which follows from the same equations as before. We assume first that $i_\pm \in K(T_{BL})$, $i_+ \in K(T_{BR})$. Then, we distinguish again between the following two cases, (i) $j_-, j_+ \in T'_{BL}$ and (ii) $j_- \in T'_{BR}$, $j_+ \in T'_{BL}$. 

**Case 1: $j_-, j_+ \in T'_{BL}$**

Here again outlets are omitted from $B'$s right target set and added to his left target set.

$$\frac{2 \left( \sum_{i \in K(T_{BL})} E_i(X) + \sum_{i' \in K'(j_+)} E_i(X) \right) - k(T_{BL}) - k(j_+)}{k(T_{BL}) + k(j_+) + l}$$

$$+ \frac{2 \left( \sum_{i \in K(T_{BR})} E_i(X) - \sum_{i' \in K'(j_+)} E_i(X) \right) - k(T_{BR}) + k(j_+)}{k(T_{BR}) - k(j_+) + l}$$

$$+ l \left( E_B(X) - E_A(X) \right) \left( \frac{1}{k(T_{BL}) + l + k(j_+)} + \frac{1}{k(T_{BR}) + l - k(j_+)} \right)$$

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Before fragmentation the level of polarization was given by

\[
\frac{2 \left( \sum_{i \in K(T_{BL})} E_i(X) \right) - k(T_{BL})}{k(T_{BL}) + l} + \frac{2 \left( \sum_{i \in K(T_{BR})} E_i(X) \right) - k(T_{BR})}{k(T_{BR}) + l} + l \left( E_B(X) - E_A(X) \right) \left( \frac{1}{k(T_{BL}) + l} + \frac{1}{k(T_{BR}) + l} \right)
\]

First, it is simple to show that

\[
l (E_B(X) - E_A(X)) \left( \frac{1}{k(T_{BL}) + l + k(j_+)} + \frac{1}{k(T_{BR}) + l - k(j_+)} \right)
\]

\[
> l (E_B(X) - E_A(X)) \left( \frac{1}{k(T_{BL}) + l + k(j_+)} + \frac{1}{k(T_{BR}) + l} \right)
\]

\[
\Leftrightarrow \frac{1}{k(T_{BL}) + l + k(j_+)} + \frac{1}{k(T_{BR}) + l - k(j_+)} > \frac{1}{k(T_{BL}) + l} + \frac{1}{k(T_{BR}) + l}
\]

as \( k(T_{BL}) > k(T_{BR}) \). Next, we focus on

\[
\frac{2 \left( \sum_{i \in K(T_{BL})} E_i(X) + \sum_{i \in K'(j_+)} E_i(X) \right) - k(T_{BL}) - k(j_+)}{k(T_{BL}) + k(j_+) + l} + \frac{2 \left( \sum_{i \in K(T_{BR})} E_i(X) - \sum_{i \in K'(j_+)} E_i(X) \right) - k(T_{BR}) + k(j_+)}{k(T_{BR}) - k(j_+) + l} + \frac{2 \left( \sum_{i \in K(T_{BL})} E_i(X) \right) - k(T_{BL})}{k(T_{BL}) + l} + \frac{2 \left( \sum_{i \in K(T_{BR})} E_i(X) \right) - k(T_{BR})}{k(T_{BR}) + l}
\]

(19)

The bliss point of the voter switching has a bliss point below or above \( \frac{1}{2} \). If the bliss point is above \( \frac{1}{2} \) then it holds trivially that polarization increases. To see this note that \( \frac{\sum_{i \in K(T_{BL})} E_i(X)}{k(T_{BL})} < \frac{1}{2} \). Thus, as \( \frac{\sum_{i \in K'(j_+)} E_i(X)}{k(j_+)} > \frac{1}{2} \) it follows that the bliss point of B’s left target set shifts to the right. Additionally, the bliss point of B’s right target set shifts to the right. By symmetry, the bliss points of A’s target set shift to the left and polarization increases. If the bliss point of the voters switching is below \( \frac{1}{2} \), then the bliss point of B’s left target set decreases, while the bliss point in B’s right target set increases. To see this note that

\[
2 \sum_{i \in K'(j_+)} E_i(X) - k(j_+) > 0 \quad \Leftrightarrow \quad \frac{\sum_{i \in K'(j_+)} E_i(X)}{k(j_+)} > \frac{1}{2}
\]

Then, expression (19) can be written as

\[
\frac{-\alpha_L - \beta}{k(T_{BL}) + k(j_+) + l} + \frac{\alpha_R + \beta}{k(T_{BR}) - k(j_+) + l} > \frac{-\alpha_L}{k(T_{BL}) + l} + \frac{\alpha_R}{k(T_{BR}) + l}
\]
where \(-\alpha_L = 2 \sum_{i \in K(T_{BL})} E_i(X) - k(T_{BL})\), \(\alpha_R = 2 \sum_{i \in K(T_{BR})} E_i(X) - k(T_{BR})\), and \(\beta = -\left(\sum_{i \in K'(j_+)} E_i(X) - k(j_+)\right)\). It is straightforward to show that this expression indeed holds and we can again establish that polarization has increased due to fragmentation.

**Case 2: \(j_- \in T'_{BR}, j_+ \in T'_{BL}\)**

Again, outlets switch between target sets. This implies that the number of voters in each set remains unchanged. It therefore suffices to show that

\[
\frac{2 \left(\sum_{i \in K(T_{BL})} E_i(X) + \sum_{i \in K'(j_+)} E_i(X) - \sum_{i \in K'(j_-)} E_i(X)\right) - k(T_{BL})}{k(T_{BL}) + l} + \frac{2 \left(\sum_{i \in K(T_{BR})} E_i(X) - \sum_{i \in K'(j_+)} E_i(X) + \sum_{i \in K'(j_-)} E_i(X)\right) - k(T_{BR})}{k(T_{BR}) + l} > \frac{2 \left(\sum_{i \in K(T_{BL})} E_i(X)\right) - k(T_{BL})}{k(T_{BL}) + l} + \frac{2 \left(\sum_{i \in K(T_{BR})} E_i(X)\right) - k(T_{BR})}{k(T_{BR}) + l}
\]

which always holds due to the symmetry and \(\frac{\sum_{i \in K'(j_-)} E_i(X)}{k(j_-)} > \frac{1}{2}\). Otherwise it can never be optimal to add \(j_-\) to \(T_{BR}\), see Lemma 4.

Next we consider the case where \(i_-, i_+ \in K(T_{BL})\). Then, outlets can be omitted from \(B'\)’s left target set and added to his right target set and polarization can increase or decrease. Polarization increases if and only if

\[
\frac{2 \left(\sum_{i \in K(T_{BL})} E_i(X) - \sum_{i \in K'(j)} E_i(X)\right) - k(T_{BL}) + k(j)}{k(T_{BL}) - k(j) + l} + \frac{2 \left(\sum_{i \in K(T_{BR})} E_i(X) + \sum_{i \in K'(j)} E_i(X)\right) - k(T_{BR}) - k(j)}{k(T_{BR}) + k(j) + l} + l (E_B(X) - E_A(X)) \left(\frac{1}{k(T_{BL}) + l - k(j_+)} + \frac{1}{k(T_{BR}) + l + k(j_+)}\right)
\]

\[
> \frac{2 \left(\sum_{i \in K(T_{BL})} E_i(X)\right) - k(T_{BL})}{k(T_{BL}) + l} + \frac{2 \left(\sum_{i \in K(T_{BR})} E_i(X)\right) - k(T_{BR})}{k(T_{BR}) + l} + l (E_B(X) - E_A(X)) \left(\frac{1}{k(T_{BL}) + l} + \frac{1}{k(T_{BR}) + l}\right)
\]

Equating the left and right hand side of the inequality and solving for \(\sum_{i \in K'(j)} E_i(X)\) shows that polarization is increasing for \(\sum_{i \in K'(j)} E_i(X)\) sufficiently large.
Rational Voters & Uncertainty

In what follows, we allow for voters to be rational and to make an inference about the policy if they are not targeted while simultaneously introducing uncertainty about the environment. Voters face a set of states, which describe the structure of the media networks as well as the bliss points of voters. This implies that we are allowing voters not only to be uncertain about the overall structure of the media outlet, they also do not know the bliss point of the different audiences. Unlike voters, candidates know the realization of the state, that is they know the exact structure and bliss points of voters in the network. The set of states is denoted by \( S = \{1, 2, \ldots, n\} \). A realization of the state is given by \( s \). The probability that a state \( s \) has been realized is \( p_s \).

In each state \( s \) a candidate selects an optimal policy \( x_s \) and a target set \( T_s \), taking into account the beliefs of the voters. If voters are targeted, they learn about the policy that is being implemented. If a voter \( i \) is not targeted, the voters use Bayesian updating to assign a posterior probability to each possible policy, which we denote by \( q_{is} \). We can show that without attached voters (\( l = 0 \)) the optimal target set minimizes

\[
\left( k(T_{\bar{s}}) + \sum_{i \in K \setminus K(T_s)} q_{is} \right) \left( E^w(X^2|T_{\bar{s}}) - E^w(X|T_{\bar{s}})^2 \right) + \int_0^1 \sum_{s \neq \bar{s}, s \in S} (x_s - x)^2 \sum_{i \in K \setminus K(T_s)} q_{is} g_i(x) dx,
\]

where \( E^w(X|T_{\bar{s}}) \) is the expected bliss point of the target set also taking into account the preferences of the non-targeted voters (weighted by their posterior). \( E^w(X^2|T_{\bar{s}}) \) the expectation of these bliss points squared. Their explicit definition is given by

\[
E^w(X|T_s) = \frac{\sum_{i \in K(T_s)} E_i(X) + \sum_{i \in K \setminus K(T_s)} q_{is} E_i(X)}{k(T_s) + \sum_{i \in K \setminus K(T_s)} q_{is}}
\]

\[
E^w(X^2|T_s) = \frac{\sum_{i \in K(T_s)} E_i(X^2) + \sum_{i \in K \setminus K(T_s)} q_{is} E_i(X^2)}{k(T_s) + \sum_{i \in K \setminus K(T_s)} q_{is}}
\]

We denote the realized state of the world by \( \bar{s} \). The two parts of equation (20) are two measures of variance. The first part of the equation measures the loss from informing voters, the second part entails the loss of votes from not disclosing. To see the difference to the model

\[\text{We assume that there are no two states with the same policy and/or same target set. It is always feasible to find conditions on the states and the network structure that occurs that guarantees this.}\]
where voters are not fully rational, note that in this case the optimal target set minimizes

\[ k(T) \left( E(X^2|T) - E(X|T)^2 \right) + \int_{0}^{1} (\pi_c - x)^2 \sum_{i \in K \setminus K(T_s)} q_i g_i(x) dx \] (21)

If voters are rational, then candidates take into account that not-disclosing reveals some information and therefore adjust their policy to also cater to some extent to non-targeted voters. However, as long as \( q_s \) lies below one, that is voters cannot perfectly infer the policy, candidates still place a greater weight on the preferences of the voters they target and the basic targeting strategy that is optimal with voters who do not make an inference if not targeted is still valid for Bayesian agents. To see this more clearly, we discuss an example in more detail in the Online Appendix.

One key assumption we have made so far is that there is no state in which voters are not targeted. Suppose instead that such a state exists with probability \( \overline{q} \) and voters think the implemented policy equals \( \pi \) in this state. We can show that the media centrality we develop is the limit case of the setting with rational voters if \( \overline{q} \to 1 \).

**Proposition 4.** Let \( \overline{q} \to 1 \). Then, the optimal targeting strategies in the baseline model and the setting with rational voters and uncertainty coincides with equations (20) and (21) being equivalent.

**Proof Proposition 4** The optimal policy for a given target set in state \( s \) is

\[ x_s = \frac{\sum_{i \in K(T_s)} E_i(X) + \sum_{i \in K \setminus K(T_s)} q_s E_i(X)}{k(T_s) + \sum_{i \in K \setminus K(T_s)} q_s} \]

As \( \overline{q} \to 1 \), \( q_s \to 0 \). Then,

\[ x_s = \frac{\sum_{i \in K(T_s)} E_i(X)}{k(T_s)} = E(X|T_s) \]

\[ E^w(X|T_s) = \frac{\sum_{i \in K(T_s)} E_i(X)}{k(T_s)} = E(X|T_s) \]

\[ E^w(X^2|T_s) = \frac{\sum_{i \in K(T_s)} E_i(X^2)}{k(T_s)} = E(X^2|T_s) \]

This implies that equation (20) simplifies to

\[ k(T_s) \left( E(X^2|T_s) - E(X|T_s)^2 \right) + \int_{0}^{1} \sum_{i \in K \setminus K(T_s)} \left( \sum_{s \neq s, s \in S} (x_s - x)^2 q_{is} + (\pi - x)^2 \overline{q} \right) g_i(x) dx \]
It is immediate that
\[
\lim_{q \to 1} \int_0^1 \sum_{i \in K \setminus K(T_s)} \left( \sum_{s \neq \tilde{s}, s \in S} (x_s - x)^2 q_{is} + (\pi - x)^2 \widetilde{q} \right) g_i(x) dx = \int_0^1 \sum_{i \in K \setminus K(T_s)} (\pi - x)^2 g_i(x) dx,
\]
which completes the proof. □

Adding Links

We provide here two examples that show that adding links can increase or decrease polarization, a more detailed discussion is provided in the Online Appendix.

**Figure 7: Polarization Decreases**

For an example when polarization decreases, consider Figure 7. Let \( E_B(X) = 1 \), \( E_R(X) = \frac{3}{4} \), by symmetry it follows that \( E_L(X) = \frac{1}{4} \). Let \( l = 2 \). The level of polarization is given by 1. Once links are added symmetrically, it is still optimal to set \( T'_{BL} = \{1\} \) and \( T'_{BR} = \{2\} \). Polarization is now given by \( \frac{4}{5} \) and has thus decreased. Consider next an example where

**Figure 8: Polarization Increases**

polarization increases. Let the network be as depicted in Figure 8. Let \( E_M(X) = \frac{1}{2} \). The other bliss points are as in the previous example. Then, \( T_L = \{1, 2\} \), \( T_R = \{3, 4\} \) and \( T'_L = \{1, 2, 3\} \), \( T'_R = \{4\} \). Polarization is initially given by \( \frac{1}{3} \), after links are added polarization is at \( \frac{7}{16} \).