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Nonextensive Statistical Mechanics, Superstatistics and Beyond: Theory and Applications in Astrophysical and Other Complex Systems

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1 Introduction

After more than 140 years of impressive success there is no reasonable doubt that the Boltzmann-Gibbs (BG) entropy is the correct one to be used for a wide and important class of physical systems. The latter are basically those systems whose (nonlinear) dynamics is strongly chaotic, such as classical systems with positive maximal Lyapunov exponent, which are mixing and ergodic. However, a plethora of other physical complex systems exists for which such simplifying dynamical hypotheses are violated. Typical examples are those for which the maximal Lyapunov exponent vanishes, leading to sub-exponential sensitivity to the initial conditions, which can of course occur in a variety of mathematical ways. Corresponding anomalies are found also in a variety of quantum systems.

In order to statistically describe the dynamics of such systems, various generalised forms of statistical mechanics have been proposed such as those using the nonadditive entropies S_q (where q is a real number which, for $q = 1$, recovers the BG entropy), kappa distributions (also known as q-Gaussians, where kappa is simply related to q), superstatistical approaches, among various others. In the last decades, these new generalised statistical mechanical formalisms have found a large variety of very successful applications, many of them beyond the realm of physics. This special issue aims to cover some of the most recent analytical, experimental, observational and computational aspects and examples where these new extended formalisms have found fruitful applications. In this special volume we include some of those contributions which were

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presented at the XVI international course of the School of Complexity held in July 2-8 2019 at the Ettore Majorana Foundation and Center for Scientific Culture in Erice (Italy).

In particular, Metzler discusses superstatistics and non-Gaussian diffusion [1]; Zhang et al. [2] investigate a class of network diffusion equation with large power-nonlinearity connected to q-statistics; Carati et al. [3] discuss via q-distributions a realistic ionic-crystal model and, within the FPU model, how the system reaches equilibrium; Wilk et al. [4] study the level of nonextensivity of the quark-gluon system described by lattice QCD; Souza et al. investigate the ground entangled state of the one-dimensional spin-1/2 Ising ferromagnet at its transverse-field critical point [5]; Suyari et al. study the advantages of the q-logarithm representation over the q-exponential one [6]; Korbel et al. [7] discuss within the information geometry framework the scaling expansions of non-exponentially growing configuration spaces; Wedemann and Plastino [8] study a nonlinear Fokker Planck equation and its stationary-state solution, proving an H-theorem obeyed by a free-energy functional that involves the generalized entropy S_q; Yoon uses the equivalence of kappa distributions and q-Gaussian distributions to focus on the non-equilibrium statistical mechanical applications to the formation of non-Maxwellian electron distribution in space [9]. Interesting applications of q-statistics for predicting ruptures and earthquakes are then discussed in the paper by Greco et al [10] et al. and in the paper by Skordas et al. [11] respectively; Singh and Roy [12] discuss an internetwork synchronisation technique for complex dynamical networks of different kinds.

We hope that these studies will stimulate further discussions opening new interesting research directions. At the ending of the 19th century, it was unnecessary to qualify *Mechanics* with expressions such as *Newtonian Mechanics* or *Classical Mechanics*, currently used nowadays. Indeed, only one such theory existed. With the emergence in theoretical physics of Einstein's special and general relativities and of quantum mechanics, the situation drastically changed as we all know. The same happened with the (additive) Boltzmann-Gibbs entropy (or entropic functional): it was just called *entropy* during nearly one century and a half. The set of works included in the present special issue, with their direct connections to nonadditive entropic functionals, neatly illustrate the present need, in contemporary theoretical physics, to specifically qualify as *BG entropy* and *BG statistical mechanics*, the concept and associated thermostatistical theory that emerged at the end of the 19th century and which is today taught and used world-wide in the courses of physics, chemistry, computer science, mathematics, and elsewhere.

Finally, we warmly thank all the contributors for sharing their studies, and express our gratitude to the editorial members of EPJST for inviting this special issue [1].

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