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Feedback Control of a Spacecraft Electro-dynamic Thruster

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Abstract:	In this paper, the possibility of feedback servo control of a spacecraft thruster's specific impulse by a boundary feedback system is theoretically considered. The motivation to introduce feedback control is two-fold. The first is to stabilize any inherent plasma instabilities and the second is regulate the output specific impulse of the thruster. Two cases are considered: Electro-thermal thrusters and Electro-dynamic thrusters. The chamber inlet temperature in the case of the electro-thermal thruster or the boundary electric field potential responsible for generation of the velocity of the plasma ions in the case of an electrodynamic thruster, are controlled by feedback so as to regulate the thruster's specific impulse. By introducing typical disturbances in the plasma ionization voltage, it is shown using a two-dimensional fluid model and a suitable boundary feedback law, where the chamber inlet temperature or applied boundary potential is proportional to the error in the specific impulse and the desired specific impulse, that the specific impulse of the thruster may be regulated and held constant. The robustness of the control system is numerically tested, by a two dimensional simulation model using McCormacks's method. The Navier-Stockes equations, including the magneto-hydrodynamic variables were discretized and simulated, using the explicit MacCormack method for a typical nozzle domain. The numerical results for the open and closed-loop velocity fields were obtained and the specific impulse was computed from these fields. It was thus shown that not only the stability of the plasma is realised but also that the specific impulse is regulated as desired.
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Response to Reviewers:	

To
The Editor
ActaAstronautica

From
Dr Ranjan Vepa
Corresponding Author

Dear Sir
Following the review, the paper has revised as per the reviewer's recommendations. The authors reply to the feviewers comments is also attached.
Please note also that the work has not been submitted previously to the Journal (in part or in whole), that it has not been published previously (except as an abstract, part of published lecture, or academic thesis), and is not under consideration for publication elsewhere.
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Many thanks for your attention,
ranjan
11 December, 2019

Authors Response to reviewers

Review of the paper entitled "Feedback Control of a Spacecraft Electro-dynamic Thruster"

Journal: Acta Astronautica

Date: 21-02-2020

First, I would like to thank the reviewers for their efforts.

The following general corrections were made:

The following sentences were corrected:

i) While the electro-thermal thruster could be excited by a resistance coil and an electric arc or a microwave radio frequency wave source that excites the plasma by merely heating it, the electro-dynamic thruster includes an additional electro-magnetic source field, which acts as a booster further driving the plasma at the source.

ii) The Navier-Stokes equations,

iii) Rather, practically for the single input single output system under consideration, the synthesis of an output feedback law is not straight forward but could also be validated by simulating the MHD Navier-Stokes equations.

In response to the reviewers comment:

Reviewer #1: The paper is well improved, all except one of the previous suggestions were well accomplished and the content represents a novel approach on the electric space propulsion, useful for publication.

One specific clarification is still needed to clear it for publishing.

1. The second formula in the second row below formula (2.3) shows that, for resting fluid (U=0) the energy becomes equal to pressure and are expressed in the same units. This should be better explained by the author as it represents an unusual approach. The reference to the IU system, as it was entered after the first review, seems not well applicable to the case.

Authors Reply: The following was included:

The following has now been added: (section 2)

"The principal assumption on which this model is based is that the electron has negligible inertia and that its overall behaviour is very fast in comparison with the motions of the plasma flow. The electron momentum dynamics is modelled as an Ohm's law. "

After equation (2.1) the following was added:

"The most general momentum and energy equations are written in terms of the magnetic total pressure,……."

Later in the following text, the sentence below was included:

“Note that in the absence of an electromagnetic field, equations (2.2) and 2.3) reduce to the general conservative form of the hydrodynamic flow equations for the momentum and the energy. They can also be written in other alternate forms as indicated in [14, 15, 16]. “

Additionally it may be noted that, when the velocity field is zero (for a resting fluid), the pressure energy relation reduces to the adiabatic equation of state of an ideal gas: $p = (\gamma - 1)\rho\varepsilon = (\gamma - 1)e$. In the preceding equation, ε is the internal energy of the flow per unit volume of matter, e is the specific energy (per unit mass), γ is the adiabatic exponent. This has now been succinctly clarified in the paper.

The following references were included:

[14] O. Toropina, G. Bisnovatyi-Kogan, S. Moiseenko, MHD Simulation of Laboratory Jets, ASTRONUM 2017, IOP Conf. Series: Journal of Physics: Conf. Series 1031 (2018) 012022 .

[15] D. B. Araya, F. H. Ebersohn, S. E. Anderson, S. S. Girimaji, Magneto-Gas Kinetic Method for Nonideal Magnetohydrodynamics Flows: Verification Protocol and Plasma Jet Simulations, Journal of Fluids Engineering, Vol. 137(8), August 2015, pp. 081302-1 to 081302-1.

[16] F. H. Ebersohn, B. W. Longmier, J.P. Sheehan, J. V. Shebalin, S. S. Girimaji, Preliminary Magnetohydrodynamic Simulations of Magnetic Nozzles, IEPC-2013-334, Presented at the 33rd International Electric Propulsion Conference, The George Washington University, Washington, D.C., USA, October 6-10, 2013.

Highlights

Continuous plasma thruster control problem explicitly defined

Thruster dynamic modelling based finite volume formulation

Output feedback control law based on numerical model

Control law design based on spacecraft thruster simulation

Regulation of specific impulse is feasible by Electric Thrusters

Feedback Control of a Spacecraft Electro-dynamic Thruster

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ABSTRACT

In this paper, the possibility of feedback servo control of a spacecraft thruster's specific impulse by a boundary feedback system is theoretically considered. The motivation to introduce feedback control is two-fold. The first is to stabilize any inherent plasma instabilities and the second is regulate the output specific impulse of the thruster. Two cases are considered: Electro-thermal thrusters and Electro-dynamic thrusters. The chamber inlet temperature in the case of the electro-thermal thruster or the boundary electric field potential responsible for generation of the velocity of the plasma ions in the case of an electrodynamic thruster, are controlled by feedback so as to regulate the thruster's specific impulse. By introducing typical disturbances in the plasma ionization voltage, it is shown using a two-dimensional fluid model and a suitable boundary feedback law, where the chamber inlet temperature or applied boundary potential is proportional to the error in the specific impulse and the desired specific impulse, that the specific impulse of the thruster may be regulated and held constant. The robustness of the control system is numerically tested, by a two

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dimensional simulation model using McCormacks's method. The Navier-Stockes equations, including the magneto-hydrodynamic variables were discretized and simulated, using the explicit MacCormack method for a typical nozzle domain. The numerical results for the open and closed-loop velocity fields were obtained and the specific impulse was computed from these fields. It was thus shown that not only the stability of the plasma is realized but also that the specific impulse is regulated as desired.

KEYWORDS: Simulation, dynamic modelling of plasma thrusters, feedback control, gain margin.

1. Introduction

Spacecraft plasma thrusters offer substantially higher exhaust velocities than chemically propelled thrusters, and absorb more power to produce higher thrust densities than space-charge limited electrostatic thrusters. The fundamental thrust generation process is based on converting electrical energy into kinetic energy in a propellant chamber, by the application of electromagnetic body forces. The thrust is generated by an electromagnetic acceleration processes, which involves the coupling of compressible ionized gas dynamics and electromagnetic field theory with both particle electrodynamics and with plasma collision dynamics. A simple way to study these interactions is to model and use numerical schemes to simulate propulsive plasma flows. Such models, examples of which are given later, may also be used to study the influence of feedback control laws on the performance characteristics of the plasma thrusters which could otherwise be quite complex and cannot be modelled easily by traditional continuous dynamics models in a state space. In the operation of spacecraft thruster it is useful to regulate and maintain the specific impulse output of thruster at a constant value. When this is done the thrust itself can be linearly controlled by changing the propellant mass flow rate. It is then possible to precisely predict the thruster's outputs which in turn makes it relatively easy to optimally plan a particular mission.

The application of feedback control to spacecraft thrusters has been very limited. Although nozzle flow optimisation methods have been applied based on control theory [1] and flow control methods have been applied based on computing the adjoint flows [2] for purposes of aeroacoustic analysis, the application of feedback control theory to numerical flow models and applied to spacecraft thrusters for the purpose of controlling the specific impulse has been very limited.

For bounded plasmas, sheath instabilities can trigger instabilities in the plasma volume, leading to an anomalous diffusion of plasma particles. Also anomalous diffusion of electrons across magnetic field lines is often present. Accompanied with the anomalous diffusion are electric field fluctuations due to micro-instabilities, resistive instabilities, gradient driven instabilities perpendicular to the wall or electron driven instabilities. For ion thrusters, an instability in the resistivity, induced by an unstable plasma sheath due to strong secondary emission of electrons, leads to a fluctuation of the electric field, perpendicular to the plasma limiting wall. Moreover electrostatic turbulence can appear even without the presence of secondary emission of electrons. Plasmas, in general, can exhibit a range of instabilities. Yet only some of these are relevant to ion thrusters. The occurrence of some of these instabilities has been studied by Duras [3]. In particular one can simulate these instabilities by simulating ideal Magneto-Hydro-Dynamic equations, derived in [4], for a duct of varying area or nozzle representing the thruster with several assumptions on the length scales which must be larger than both the Larmor radius and the mean free path and frequencies which must be lower than the cyclotron frequencies. Sankaran et al [5] have proposed a numerical method for simulating propulsive plasma flows. Subramaniam and Raja [6] have considered the simulation of axisymmetric flows of plasmas with application to spacecraft thrusters. Several simulation studies based on numerical codes such as the one by Ahedo et al [7], Ramos, Merino and Ahedo [8] , Lorzel, and Mikellides [9] and Domínguez-Vázquez et al [10] have also been reported. On the experimental side, Qin et al [11, 12] have investigated, both in Hall type and Ion thrusters, unstable discharge phenomenon in which discharge voltage oscillations appear, particularly at high discharge currents or low flow rate conditions. This high amplitude of potential oscillations were likened to ionization-like instabilities. Increasing the flow rate seems to be effective in stabilizing the potential oscillations. The optimization of electric parameters on the energy distribution and thrust efficiency of an ablative pulsed plasma thruster was studied by Wu et al [13].

In this paper, the possibility of feedback servo control of a spacecraft thrusters specific impulse (I_{sp}) by a boundary feedback system is theoretically considered. Two cases are considered: Electro-thermal thrusters and Electro-dynamic thrusters. While the electro-thermal thruster could be excited by a resistance coil and an electric arc or a microwave radio frequency wave source that excites the plasma by merely heating it, the electro-dynamic thruster includes an additional electro-magnetic source field, which acts as a booster further driving the plasma at the source. The chamber inlet temperature in the case of the electro-thermal thruster or the boundary electric field potential responsible for generation of the velocity of the plasma ions in the case of an electrodynamic thruster, are controlled by feedback so as to regulate the thruster's I_{sp}. By introducing typical disturbances in the plasma ionization voltage, it is shown using a two-dimensional fluid model and a suitable boundary feedback law, where the chamber inlet temperature or applied boundary potential is proportional to the error in the I_{sp} and the desired I_{sp}, that the I_{sp} of the thruster may be regulated and held constant. In both cases it is observed that when the feedback gain is sufficiently low, the closed loop system is not stable. The feedback gains are chosen so the gain margin is at least equal to 6 dB or more. The robustness of the control system is numerically tested, by a two dimensional simulation model using McCormacks's method. The Navier-Stokes equations, including the magneto-hydrodynamic variables were discretized by the Finite Volume method, using the explicit MacCormack method for a typical nozzle domain. The MacCormack method is a two-step method (predictor-corrector) of second-order accuracy in both space and time and this method is commonly utilized in the solution of compressible fluids problems associated with spacecraft thruster models. The numerical results of velocity fields were obtained and the I_{sp} was computed from these fields.

2. Dynamic MHD Modelling of Electro thermal and Electrodynamic spacecraft thrusters

The magneto-hydrodynamic (MHD) model is a ‘single-fluid’ description of plasma with a single-temperature approximation, is used to describe plasma flow dynamics. The principal assumption on which this model is based is that the electron has negligible inertia and that its overall behaviour is very fast in comparison with the motions of the plasma flow. The electron momentum dynamics is modelled as an Ohm’s law. The resistive MHD equations as the model is quite commonly referred to, comprise of a hydrodynamic part described by the compressible Navier-Stokes equations with magnetic Lorentz force and Joule heating source terms which is coupled with Maxwell's electromagnetic equations defined by Faraday's and Ampere's laws. The entire system is closed by an equation of state and a generalized Ohm's law. A detailed derivation can be found in [4]. The first equation in the continuum model of plasma dynamics is mass continuity in the plasma flow. The second subset of equations relate to the momentum continuity in the plasma flow, characterised by the material derivative of the momentum vector and driven by the gradient of the total pressure and includes the divergence of the dissipative stress tensor. The third equation is the energy equation, where the energy generation rate of the plasma, equals to the divergence of the heat flux, Ohmic heating, momentum losses of the particles due to collisions, and the heat exchanged between the particles. The final subset of equations arise from combining the magnetic induction equation with Faraday's and Ampere's laws. The complete set of equations may be expressed as in [5, 6]. The mass conservation equation is,

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{U}) = 0 \ . \tag{2.1}$$

The most general momentum and energy equations are written in terms of the magnetic total pressure,

$$\frac{\partial (\rho \mathbf{U})}{\partial t} + \nabla \cdot \left(\rho \mathbf{U}\mathbf{U} - \mathbf{B}\mathbf{B}/\mu_0\right) = -\nabla\left(p_{tot}\right) + \nabla \cdot \overline{\overline{\tau}} \ , \tag{2.2}$$

$$\frac{\partial e_{tot}}{\partial t} + \nabla \cdot \left\{ \mathbf{U} \nabla \cdot \left(e + p_{tot}\right) - \left(\mathbf{U} \cdot \mathbf{B}/\mu_0\right)\mathbf{B} \right\} = \nabla \cdot \left(k_{th} \nabla T\right) + \nabla \cdot \left(\mathbf{U} \cdot \overline{\overline{\tau}}\right) - \nabla \cdot \left(\mathbf{J} \times \mathbf{B}/\mu_0 \sigma\right) \tag{2.3}$$

where $p_{tot} = p + \mathbf{B} \cdot \mathbf{B}/\mu_0$ is the isotropic thermodynamic total pressure, $e_{tot} = e + \mathbf{B} \cdot \mathbf{B}/2\mu_0$, is the total specific energy, $e = \rho \mathbf{U} \cdot \mathbf{U}/2 + p/(\gamma - 1)$, is the specific energy due to the hydrodynamic flow and pressure alone, $\overline{\overline{\tau}}$ is the dissipative stress tensor, T is the temperature, k_{th} is the thermal conductivity, $\mathbf{B}$ is the magnetic field, $\mathbf{J}$ is the current density, μ_0 is the permeability of free space and σ is the conductivity. Note that in the absence of an electromagnetic field, equations (2.2) and 2.3) reduce to the general conservative form of the hydrodynamic flow equations for the momentum and the energy. They can also be written in other alternate forms as indicated in [14, 15, 16]. The pressure and energy are both defined in consistent SI units. The total pressure and total energy include the contribution of the magnetic field. The momentum equation contains the electromagnetic body force per unit volume, $\mathbf{J}\times\mathbf{B}$, written as the divergence of the Maxwell stress tensor. The magnetic field evolution may be expressed as,

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla\times\mathbf{E} = \nabla\cdot\overleftrightarrow{\mathcal{E}}_{res} - \nabla\left(\mathbf{UB}-\mathbf{BU}\right),\ \nabla\times\mathbf{B} = \mu_0\mathbf{J}, \tag{2.4}$$

where $\overleftrightarrow{\mathcal{E}}_{res}$ is the resistive diffusion tensor, $\mathbf{J}$ is related to the electric and magnetic fields, $\mathbf{E}$ and $\mathbf{B}$, by the generalised Ohm's law,

$$\left(\eta\mathbf{I} - \frac{1}{n_e q}\mathbf{B}\times\right)\mathbf{J} = \mathbf{E} + \mathbf{U}\times\mathbf{B} + \frac{1}{n_e q}\left(n_e k T_e\right). \tag{2.5}$$

In equation 2.5, T_e is the electron temperature, n_e is the electron number density, k is the Boltzmann constant, q is the electron charge and η is the resistivity. Finally the resistive diffusion tensor $\overleftrightarrow{\mathcal{E}}_{res}$ may be expressed as, $\nabla\cdot\overleftrightarrow{\mathcal{E}}_{res} = -\nabla\times\left(\bar{\bar{\eta}}\cdot\frac{\nabla\times\mathbf{B}}{\mu_0}\right)$, where $\bar{\bar{\eta}}$ is the full resistivity tensor, which includes the dispersive Hall effect.

3. Numerical Simulation of Plasma Flows

Since one is interested in the synthesis of control laws that would allow one to effectively regulate the I_{sp} output, one does not have too many choices as to the methodologies that can be adopted for the synthesis of the control laws. Model predictive control is effectively one of the only options that could be used. However in the first instance one is dealing with a single input single output system. The synthesis of optimal feedback control laws, although feasible by solving the MHD Navier-Stokes equations given above along with their co-state equations, is not expected to yield a practical control law. Rather, practically for the single input single output system under consideration, the synthesis of an output feedback law is not straight forward but could also be validated by simulating the MHD Navier-Stokes equations. There are key issues that must be considered, particularly the stability of the closed loop system that cannot be taken for granted and must be evaluated carefully in the context of the assumptions and methods used for simulating the closed loop MHD Navier-Stokes equations. There are indeed a number of methods available for the numerical simulation of the plasma flows [17]. In this work, we adopted the rather simple approach of [18] with suitable modifications for introducing the closed loop output feedback control law. The methodology of optimal control which requires the solution of the adjoint equations with the associated adjoint boundary conditions was deliberately avoided as it involves full state feedback. One of the earliest numerical methods developed for solving the Navier-Stokes equations in two or in three dimensions is MacCormack's method. The MacCormack method has some important features and is largely applied to solve flow problems described by the Navier Stokes equations. The McCormack scheme is generally known to predict the solution of the Navier Stokes equations without generating any oscillations.

4. MacCormack’s method for the Plasma Navier Stokes Equations

The MacCormack is essentially a variation and simplification of the two step Lax-Wendroff method, and yet more efficient in solving the Navier Stokes equations. The original MacCormack method is an explicit two step method that calculates a

"Predictor solution" and followed by a "Corrector solution". The final solution is generally defined as the arithmetic mean of the predictor and corrector solutions. The predictor step uses a forward difference operator while the corrector step uses a backward difference operator. The method uses this scheme to ensure a second-order of accuracy in both the time and space domains.

Formal mathematical methods to predict the maximum time step $\Delta t_{\max}$ for obtaining a stable solution can only be applied to relatively simple model equations. However, although the MHD Navier Stokes equations are far too complex for a formal analysis of stability, the findings from the simpler analysis provides useful guidance for determining a suitable time step when solving these equations. Because of the complexity of the MHD Navier-Stokes equations, it is not possible to obtain a closed-form stability expression for the MacCormack scheme applied to these equations. The main weakness of explicit schemes is that the largest time step that can be used is limited. Instability can arise in the computed solution when the time step Δt is too large, making the response to diverge. For this reason, the time step is chosen so as to meet the Courant-Friednchs-Levy (CFL) condition [19] when the flow is inviscid. (The condition is modified when the flow is viscous.) Physically, the CFL condition represents the fact that the time-step must be less than or equal to the time it takes for the fastest wave, in the computed response, to move from one grid point to the next. The Courant-Friedrichs-Lewy condition, for choosing the time step, for a given spatial grid spacing was first formulated by Courant, Friedrichs and Lewy [20]. Lax [21] provided a generic numerical interpretation of the CFL conditions, deriving it from purely numerical considerations by considering a first order wave equation and showing that the CFL condition was essential to arrest the temporal exponential growth of the numerical solution.

As a result of this condition, the MacCormack method skirts the instability problem in generating the solution for the steady flow case by adopting a time step reduction strategy. Thus it tends to continuously reduce the time step when it encounters an incipient instability. Knowing this fact, is useful in detecting an incipient instability in the numerical time dependent response solution. When the continuous reduction of the time step leads to an increasing response magnitude, without a significant increase in the time, it is an indicator of instability. In this paper, such an approach is adopted to recognize instability. Once instability was recognised at a particular low gain, a gain margin of 6dB was introduced into the closed loop. The two dimensional flow model in Cartesian coordinates was adopted to ensure that the flow dynamics was relatively simple so the feedback control method could be validated.

5. Applying the Boundary Feedback Control

The feedback inputs are effected through the boundary conditions. For this reason the boundary conditions are briefly discussed. Boundary conditions need to be applied at the top wall, along the line of symmetry, at the input to the flow domain and at the output of the flow domain. On the input side one could have a heat source as in an electro-thermal thruster or an applied electric field, as in a plasma dynamic thruster, which in turn influences the velocity field. Since the flow is assumed to be made up of a single fluid, the velocity of the entire input flow is empirically estimated from the flow velocity of the ions in the flow. Here it is assumed that that momentum of the ionised particles is evenly distributed across the whole cross section of the flow. The applied magnetic field on the boundary is assumed to be fixed and is not controlled.

A conservative formulation of the governing equations is used as it ensures that the variables are all conserved. The formulation is used as it facilitates the application of boundary conditions, since the fluxes are the quantities that only need to be specified at the boundaries. All the variables to be computed are defined at the vertices of the cells, as they will then coincide with the boundary, and can then be specified at the boundaries as boundary conditions. Since the magnetic field evolution equations are coupled to the Navier-Stokes and Maxwell's equations, boundary conditions need to be specified for each of these component state variables in order to proceed with the simulations. The Navier-Stokes equations represent the fluid dynamic evolution behaviour of the plasma. They are set up by specifying the pressure, temperature and velocity of the electro-thermal plasma at the inlet boundary as well as by specifying the boundary conditions at the walls and enforcing the conditions of symmetry. The far-field boundary conditions are implemented as outflow type constraints so as to allow for the velocity transients to be smoothly convected out of the simulation domain with minimal spurious components. A zero normal gradient boundary condition is also implemented along the axis boundary. The ambient pressure is assume to be a very low pressure and the expansion of thermal plasma into the low pressure background is assumed to mimic free expansion into vacuum. For the magnetic field evolution, the magnetic field along the wall is assumed to be prescribed.

In an electro-thermal thruster, such as a high temperature resisto-jet, the temperature can be changed directly by controlling the heating. In other electro thermal thrusters such as an arc jet or a radio frequency or micro-wave heated thruster, although the relationship is not direct, the nonlinear relationship between the current input to the thruster and the temperature could be established, to transform the control law in terms of the current input.

Alternately, consider the two heat sources, neglecting the contribution from the environment: plasma fluxes and RF heating. The plasma heating is due to charged particle fluxes and excited neutral radiation and the RF heating is due to eddy currents in the thruster chamber. The total power input or rate of heat flow is thus balanced by the net temperature gradient representing the spatial flow of heat. Thus since the temperature change is directly proportional to the power input, the feedback law can be used to directly control the power input at the boundary.

Stating the control law in terms of the current or power input to the thruster, would depend on the design of the thruster. For this reason the control law is stated in terms of the change in the temperature required at the boundary in the case of an electro thermal thruster or in terms of the increase in the electric field at the inlet, as in the case of electro-dynamic thruster.

The velocity of the electro-thermal plasma at the inlet boundary is updated every time step, using the linear feedback law to define either the increase in inlet temperature, as in the case of an electro-thermal thruster or the increase in the electric field at the inlet, as in the case of electro-dynamic thruster. In the case of an electro-thermal thruster, the relationship between temperature and velocity is not linear. However it may be linearized about an equilibrium flow condition. And the control law is then based on the perturbations about the equilibrium flow. The instability prediction simply refers to the initiation of instability over a parametric domain and the active controller simply inhibits the instability. Thus the use of linearized models for the purpose of control law synthesis is completely justified. In both cases the feedback input is defined in terms of the error between the desired and actual I_{sp}

multiplied by a gain. In the examples considered in this paper the magnetic field on the boundary is held fixed.

6. Typical Simulation Examples

Two classes of simulation examples are considered. The first is a class of electro-thermal thrusters where the inlet temperature is control while in the second class the inlet field is controlled. In [18] the simulation of a two-dimensional nozzle flow of plasma based on [5], without any feedback control, was considered and a MATLAB code was presented for it. This code was modified by the author to include suitable modifications to the boundary conditions and for introducing closed loop output, boundary, feedback control as discussed in the preceding section, at each time update step. The parameters of the thrusters considered are listed in Table 1.

Table 1: Typical parameter and initial state values for simulation

Parameter	Value
Initial inlet temperature, T_c	$4000°$
Atmospheric Pressure, P_{atm}	101325Pa
Inlet pressure, P_c	$25 \times P_{atm}$
Specific heat (pressure) C_p	$20.785\, J / mole \times K$
Molecular weight (Ar), M_w	$500\,\mathrm{kgm}^2$
Depth of throat, h_{th}	$0.008m$
Depth of exhaust, h_{exh}	$48 \times h_{th}$
Diameter of cathode tip	$h_{th}/4$
Current supplied to solenoid	100A
E_{i0}	$440V$

In figure 1 is illustrated the nozzle shape beyond the throat which is designed by the method of characteristics. Also shown are the distribution of the Mach number and the pressure along the nozzle.

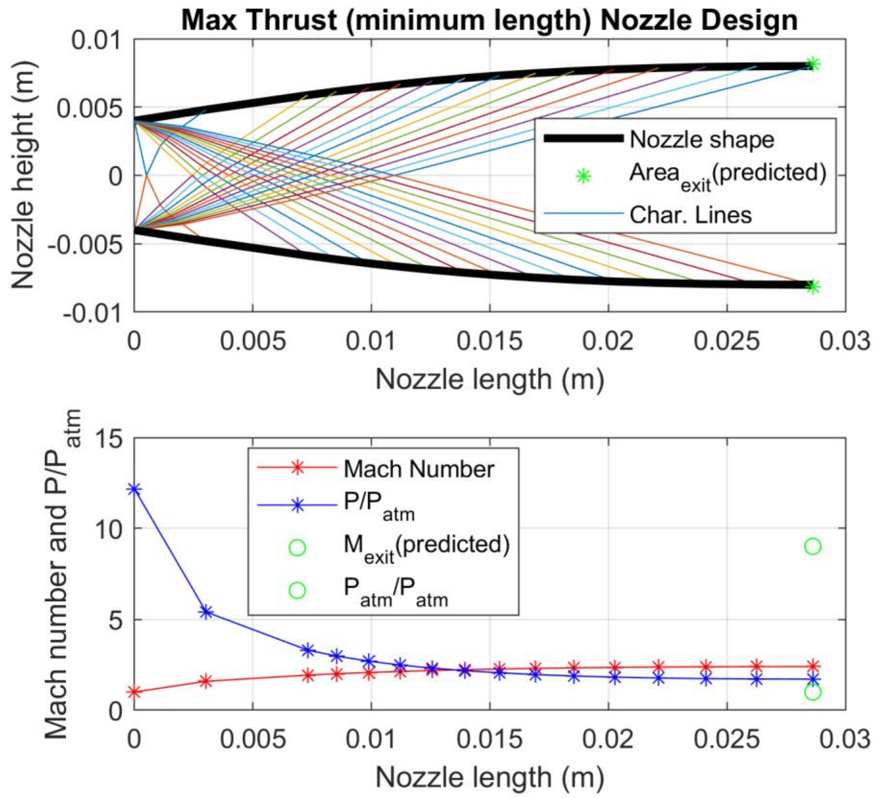

Fig. 1 a) The nozzle shape beyond the throat (upper) and b) the distribution of the Mach number and the pressure (lower).

All other parameters required are defined as and when they are required. Typically these are the $I_{sp,desired}$ which would depend on the nature of the thruster, and the control gains.

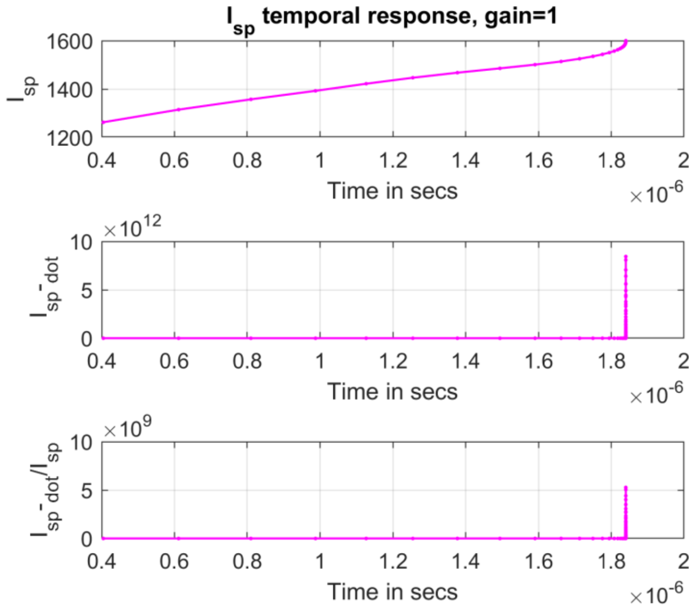

Fig. 2 The temporal growth of the I_{sp}, dI_{sp}/dt and the ratio of dI_{sp}/dt and I_{sp} with $K_{fT} = 1$.

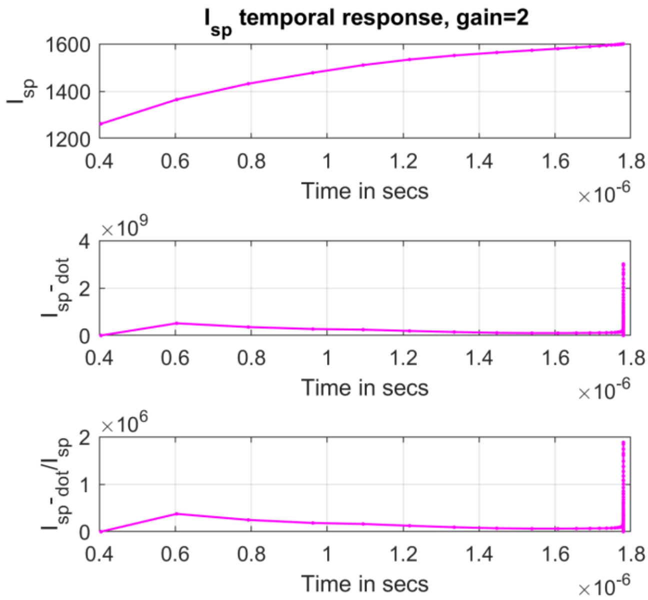

Fig. 3 The temporal growth of I_{sp} and related variables with $K_{fT} = 2$.

The next step in the simulation is to establish the critical value of the control gain. If it is assumed that the temporal growth of the specific impulse is modelled as $I_{sp} = I_{sp0} + I_{sp1} \exp(\lambda t)$ then $\dot{I}_{sp} = \lambda I_{sp1} \exp(\lambda t)$, so the ratio of the dominant unsteady component of $\dot{I}_{sp}$ and I_{sp} is λ which must have a negative real part for stability. In the case of instability, when the most unstable exponential terms dominate, $\dot{I}_{sp} / I_{sp} \to \lambda$ as $t \to \infty$, with a positive real part. Thus by examining the plots of both I_{sp} and $\dot{I}_{sp}$ as well as the ratio of latter to the former, one can make conclusive assessments of instability. In figure 2, are shown the temporal growth of the I_{sp}, dI_{sp}/dt and the ratio of dI_{sp}/dt and I_{sp}, in the case of the electro-thermal thruster with,

$$\Delta T_{in} = K_{fT} \left(I_{sp,\ desired} - I_{sp} \right). \tag{6.1}$$

In accordance with optimal control theory, the gain computed for linear-quadratic regulator has minimum, absolute, gain margin of 2. So the strategy adopted in fixing the gain magnitude is to first find the gain value at it which the thruster is just neutrally stable and then doubling this gain value to establish the value of the gain K_{fT}. The gain is set to be equal to unity and the $I_{sp,\ desired} = 1600$. For reasons discussed earlier this case was deemed to be unstable. The corresponding situation with the gain increased by a factor of 2 is shown in figure 2. Although the evolution of the I_{sp} appears smooth, it is still deemed to be unstable. The gain is increased to 2.5 step by step and in this case, shown in figure 4, the response is neutrally or marginally stable.

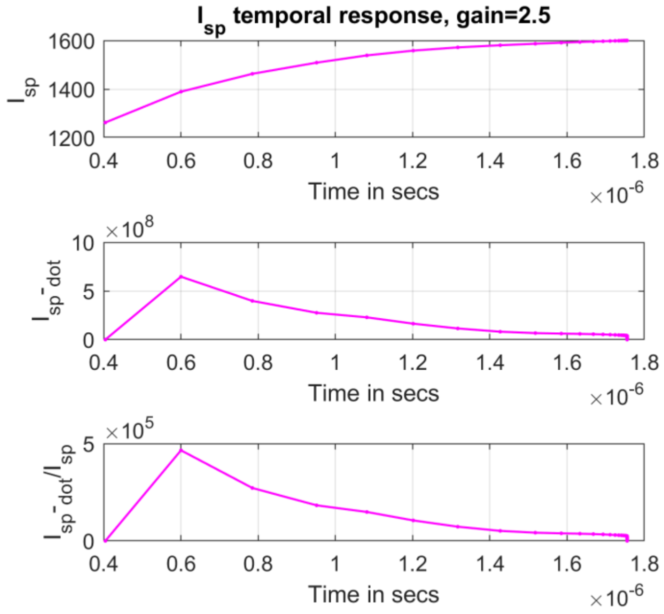

Fig. 4 The temporal growth of I_{sp} and related variables with $K_{fT} = 2.5$.

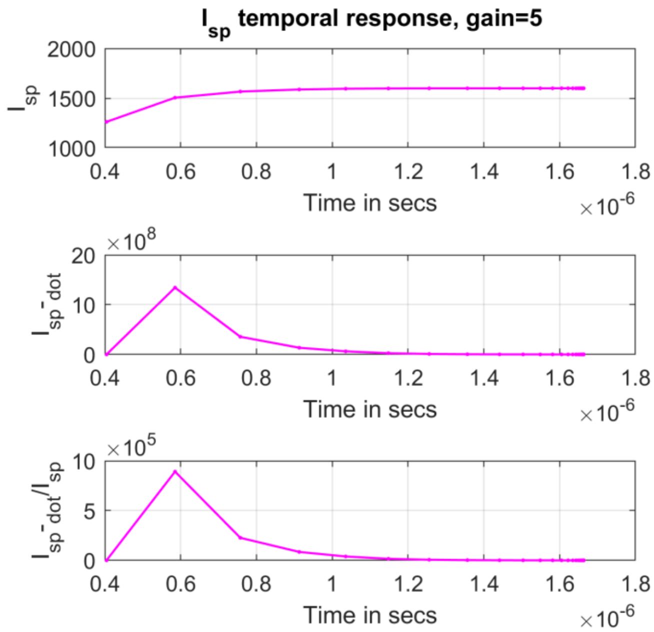

Fig. 5 The temporal growth of I_{sp} and related variables with $K_{fT} = 5$.

At this stage the absolute gain is doubled to ensure a gain margin of 2 which is equivalent to gain margin in dB of $20\log 2 \approx 6.04$ or 6 dB. The resulting responses of both the I_{sp} characteristics as well as the thruster inlet temperature, T_{in} and the feedback gain K_{fT} are shown in figures 5 and 6.

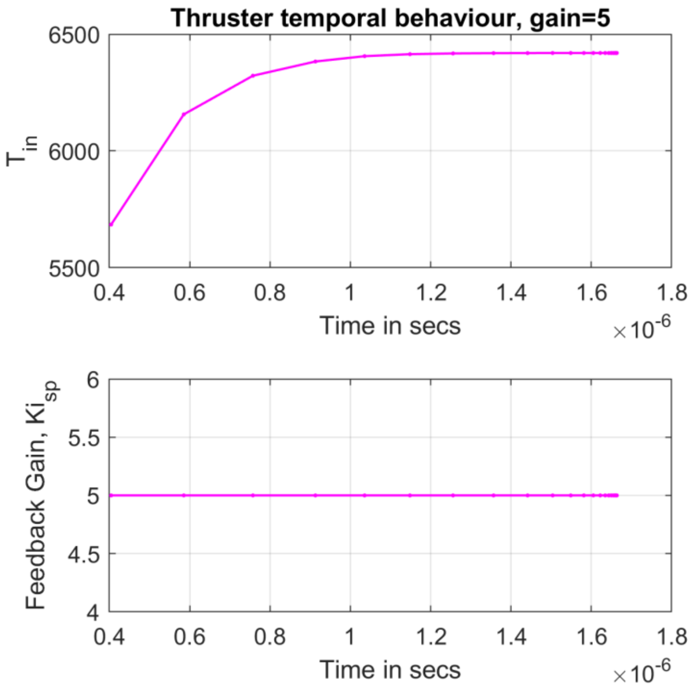

Fig. 6 The thruster inlet temperature, T_{in} and the feedback gain K_{fT} in fig. 5.

It is seen from figure 4 that the ratio of dI_{sp}/dt and I_{sp}, only approaches zero in steady state. In figure 5 the same ratio is zero in steady state. Moreover a further doubling of the gain to $K_{fT} = 10$, maintains the stability of the response. For this reason the optimum gain was assumed to be $K_{fT} = 5$.

The case of an electrodynamic thruster is now considered. Initially the inlet plasma ion velocity is assumed to be input and the control law is assumed to be of the form,

$$\Delta v_{in} = K_{fv}\left(I_{sp,\,desired} - I_{sp}\right), \tag{6.2}$$

where the $I_{sp,\ desired} = 7600$. It follows that the inlet plasma ion velocity and electric field are respectively given by, $v_{in} = v_{i0} + \Delta v_{in}$, and,

$$E_i = \frac{1}{2}\frac{m_i}{q}v_{in}^2 = \frac{1}{2}\frac{m_i}{q}\left(v_{i0} + \Delta v_{in}\right)^2 \simeq \frac{1}{2}\frac{m_i}{q}v_{i0}^2 + \frac{m_i}{q}v_{i0}\Delta v_{in}. \qquad (6.3)$$

Thus the electric field may be expressed as an equivalent linear control law,

$$E_i \equiv E_{i0} + \frac{m_i}{q}v_{i0}K_{fv}\left(I_{sp,\ desired} - I_{sp}\right) = E_{i0} + K_{fE}\left(I_{sp,\ desired} - I_{sp}\right). \qquad (6.4)$$

However in this case the control gain, K_{fE} is not constant. Initially K_{fv} was assumed to be equal to 0.05. In figure 7, are shown the temporal growth of the I_{sp}, dI_{sp}/dt and the ratio of dI_{sp}/dt and I_{sp}, in the case of the electro-dynamic thruster. By the same process as in the case of the electro-thermal thruster, the marginally stable gain was established to be equal to 0.4. The corresponding I_{sp} characteristics and the variations of v_{in}, E_i and K_{fE} are shown in figures 8 and 9. The gain is then doubled and the characteristics for this case are shown in figures 10 and 11.

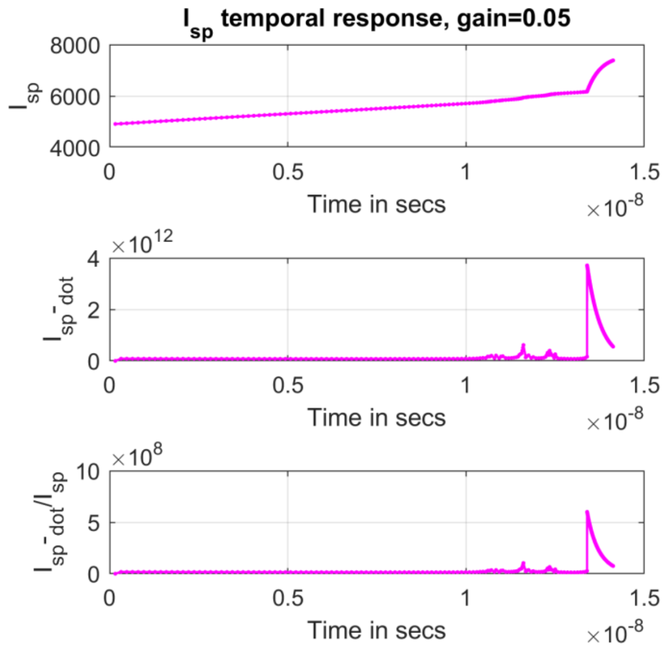

Fig. 7 The temporal growth of I_{sp} and related variables with $K_{fv} = 0.05$.

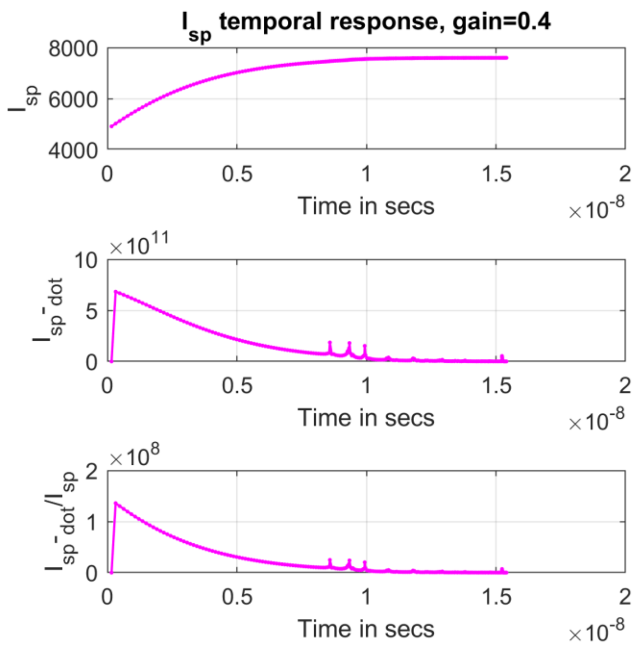

Fig. 8 The temporal growth of I_{sp} and related variables with $K_{fv} = 0.4$.

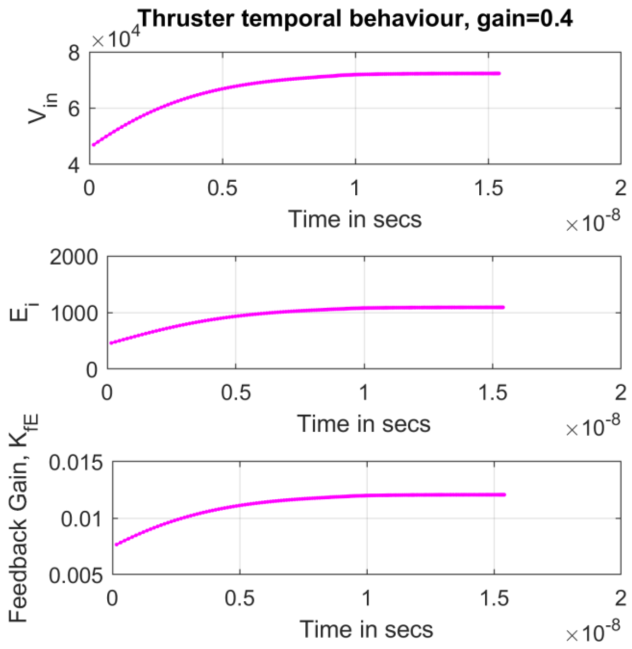

Fig. 9 The variations of v_{in}, E_i and K_{fE} corresponding to figure 8.

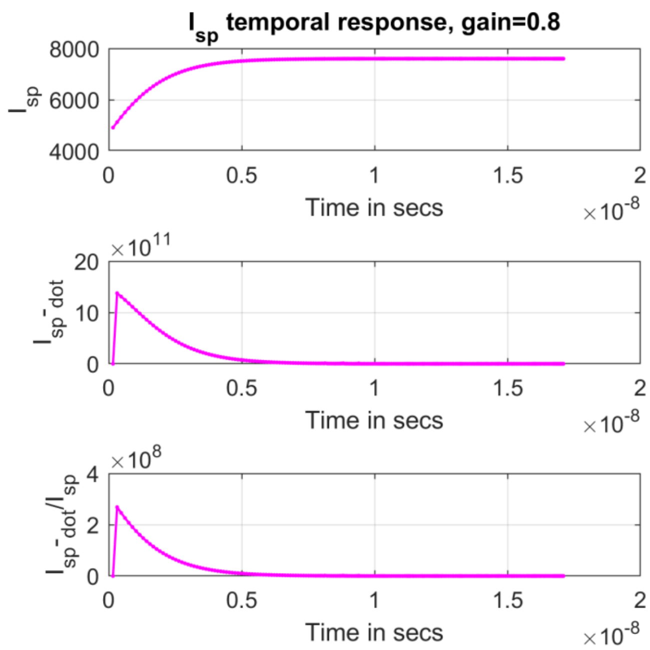

Fig. 10 The temporal growth of I_{sp} and related variables with $K_{fv} = 0.8$.

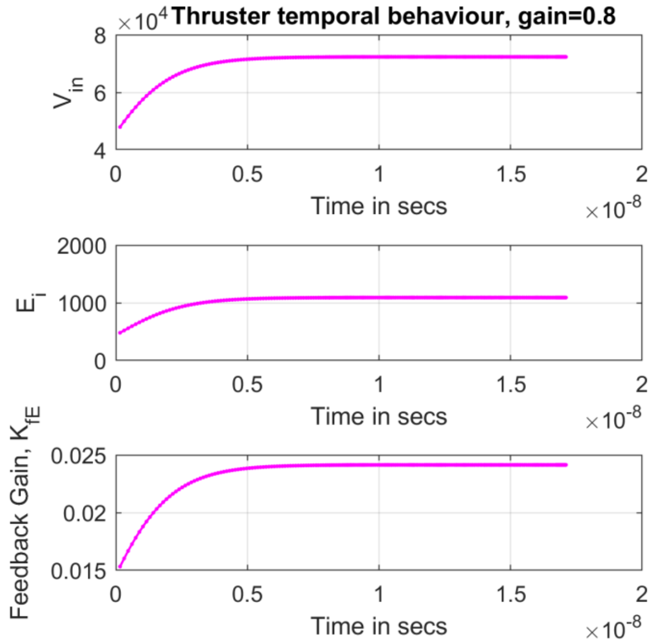

Fig. 11 The variations of v_{in}, E_i and K_{fE} corresponding to figure 10.

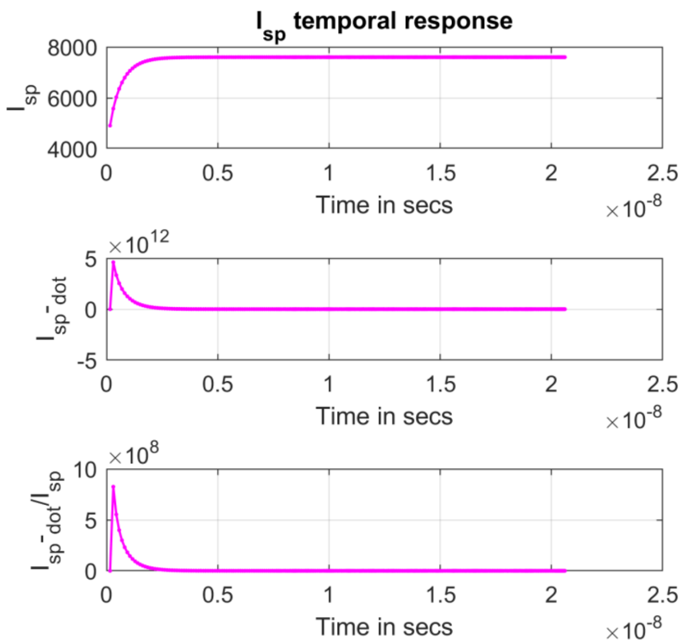

Fig. 12 The temporal growth of I_{sp} and related variables with $K_{fE} = 0.05$.

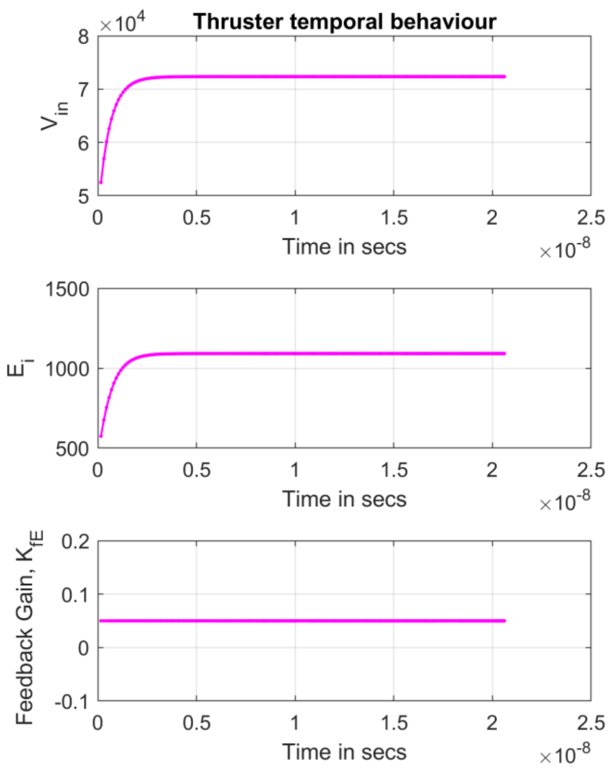

Fig. 13 The variations of v_{in}, E_i and K_{fE} corresponding to figure 12.

It is desirable that one has a constant gain controller. Based on the experience gained from the above results, constant gain controller was design and the corresponding I_{sp} characteristics and the variations of v_{in}, E_i and K_{fE} are shown in figures 12 and 13.

7. Discussion and Conclusion

In this paper, the synthesis of a feedback controller for regulating the I_{sp} of a spacecraft thruster was successfully demonstrated, using a two dimensional flow model of the plasma fluid within the nozzle domain. The methodology is based on using the MacCormack method which ensures the numerical stability of the computational solution of the MHD Navier Stokes equations by meeting the CFL condition at every time step. An essential feature of the feedback controller is the fact that it guarantees, physically stable operation of the thruster at a constant I_{sp}, thus ensuring that thrust generated can be precisely controlled by also regulating the fuel flow, using a relatively slow outer loop. The closed stability of the I_{sp} regulator, which is independent of the numerical stability of the MacCormack method, is assessed from the temporal response of the thruster and the gain margin is adequately adjusted to guarantee the desired closed loop performance. The methodology is currently being extended to axisymmetric flows which could be modelled as two-and-half dimension flows as well as to three dimension flows. Yet the two dimensional flow model allows on to easily develop the methodology for the controller synthesis without the need to consider the coupling between the numerical and physical instabilities. The methodology is also being currently extended to magnetic nozzles to facilitate the complete regulation and control of the I_{sp} and the thrust of an electrodynamic thruster.

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Declaration of Interest Statement

There are no conflicts of interest.