Period doubling eigenstates in a fiber laser mode-locked by nonlinear polarization rotation

Xiongquan Yao,[1] Lei Li,[1,*] Andrey Komarov,[2] Mariusz Klimczak,[3,4] Dingyuan Tang,[5] Deyuan Shen,[1] Lei Su,[6] and Luming Zhao[1,6,*]

[1]*Jiangsu Key Laboratory of Advanced Laser Materials and Devices, Jiangsu Collaborative Innovation Center of Advanced Laser Technology and Emerging Industry, School of Physics and Electronic Engineering, Jiangsu Normal University, Xuzhou, 221116 Jiangsu, P. R. China*
[2]*Institute of Automation and Electrometry, Russian Academy of Sciences, Academician Koptyug Prospekt 1, 630090 Novosibirsk, Russia*
[3]*Faculty of Physics, University of Warsaw, Warsaw, Poland*
[4]*Institute of Electronic Materials Technology, Wólczynska 133, 01-919 Warsaw, Poland*
[5]*School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore 639798*
[6]*School of Engineering and Materials Science, Queen Mary University of London, London E1 4NS, UK*
* sdulilei@gmail.com, lmzhao@ieee.org

Abstract: Due to the weak birefringence of single mode fibers, solitons generated in fiber lasers are indeed vector pulses and exhibit periodic parameter change including polarization evolution even when there is a polarizer inside the cavity. Period doubling eigenstates of solitons generated in a fiber laser mode-locked by the nonlinear polarization rotation, i.e., period doubling of polarization components of the soliton, are numerically explored in detail. We found that, apart from the synchronous evolution between the two polarization components, there exists asynchronous development depending on the detailed operation conditions. In addition, period doubling of one polarization component together with period-one of another polarization component can be achieved. When the period tripling window is obtained, much complexed dynamics on the two polarization components could be observed.

1. Introduction

Period doubling bifurcation in mathematics refers to a bifurcation in which a slight change of a parameter value in a dynamic system's equations results in the system switching to a new behavior with twice the period of the original system. With the doubled period, the system repeats themselves taking twice as many iterations as before. Bifurcation phenomena include period doubling, period tripling, period n (n>3), and period doubling route to chaos etc. Period doubling bifurcation is one of the universal properties of nonlinear systems. Ultrafast Fiber lasers are a paradigm of nonlinear systems. Due to the feasibility of easy controlling of laser parameters, small cross-section of light propagation, and long interaction distance between light and fibers forming the laser, fiber lasers are a perfect platform for studying dynamics in nonlinear systems.

When the dispersion effect balances the nonlinear Kerr effect during pulse propagation, a soliton can be generated in a fiber laser. Period doubling of solitons in fiber lasers was predicted two decades ago [1] and soon be demonstrated [2]. Period doubling, period tripling, and the corresponding bifurcations of solitons have been reported [3-11]. Based on the classical one-dimension quantic complex Ginzburg-Landau equation (GLE) and the varying of the nonlinear gain, period doubling and period tripling were discovered [3]. By varying spectral filtering or altering group velocity dispersion coefficient of the gain fiber, period tripling bifurcation was numerically observed [4]. The changing of nonlinear gain alone, the varying of spectral filtering alone, or the altering of group velocity dispersion coefficient of the gain fiber alone is difficult to realize in practice. Zhang et al numerically achieved period-tripling bifurcation route to chaos

as well as the classical period-doubling bifurcation route to chaos by changing the small signal gain only [5]. It simplifies the theme that is capable to experimentally demonstrate period-tripling bifurcation by changing pump level only. Period doubling and its variants are intensity threshold dependent. The appearance of period doubling or period-doubling bifurcations is subject to that the accumulated nonlinearity during pulse propagation is strong enough. Therefore, period doubling is independent of the cavity dispersion of fiber lasers. So far, period doubling has been demonstrated in fiber lasers of either anomalous dispersion [6], normal dispersion [7], or around zero dispersion [8]. Research progress on period doubling bifurcation in fiber lasers is well reviewed recently [9].

Different kinds of saturable absorbers (SAs) can be used in fiber lasers for generating ultrashort pulses. They are mainly classified into two types: real SAs such as semiconductor saturable absorber [12], carbon nanotube [13], graphene and other 2-D materials [14]; artificial SAs such as the nonlinear polarization rotation (NPR) technique [6], the nonlinear optical loop mirror [15] and its variants [16]. Except the NPR technique where there is generally a polarizer in the cavity, all the other SA can support vector soliton generation. Due to the bias from the theoretically symmetrical cross-section of fibers and the random operation conditions, fibers have at least weak birefringence. Consequently, solitons generated in a fiber laser are vector solitons even when the fiber laser includes polarization sensitive components, for example, a polarizer. Previously, solitons, generated in fiber lasers mode-locked by using the NPR technique [6-8], were considered as scalar solitons. The NPR technique functions as an artificial SA but the polarization of the pulse in each circulation inside the laser will be recovered to the state of the polarizer. Consequently, the period doubling phenomena were analyzed based on single polarization even the fiber laser was described by coupled GLEs. In another word, the period doubling phenomena reported are actually based on the solitons composed of two polarization components. It is a composite state instead of the eigenstates. Wu et al have demonstrated both numerically and experimentally that vector and scalar solitons can co-exist within the laser cavity in a passively mode-locked fiber laser by using the NPR technique [10]. The mode-locked pulse evolves as a vector soliton in the strong birefringent segment and is transformed into a scalar soliton after the polarizer in the laser cavity. M. Almanee et al investigated the polarization and nonlinear evolution of amplified soliton pulses in twisted fibers. Depending on the input polarization of the paired pulse input, the output ellipticity evolves toward a constant value, approaching plus or minus 45 deg. With a single pulse input the output ellipticity linearly tracks with the input ellipticity and exhibits strong fluctuations [17]. Period doubling of vector solitons has been reported a decade ago [11], where there is no polarization dependent component in the fiber laser and the mode locking mechanism is also polarization-independent. The two components of the period-doubled vector soliton each can exhibit period doubling. The pulse intensity variation between the two orthogonal polarization components could be anti-phase, which results in weak period doubling (small pulse intensity shuttle) of the vector soliton. Cui et al recently reported on the observation of a cross-phase-modulation-forced frequency-oscillating soliton whose wavelength exhibits redshift and blueshift periodically like dancing in a mode-locked fiber laser under moderate birefringence [18]. The fiber laser was mode-locked by using a polarization independent saturable absorber and the evolution of polarization components of a vector soliton was investigated. Hence, it is interesting and necessary to comprehend the detailed period doubling performance of polarization components of a soliton generated from a fiber laser mode-locked by using the NPR technique. In this paper, we numerically studied period doubling eigenstates in a fiber laser mode-locked by using the NPR technique. When period doubling of a soliton is achieved, apart from the synchronous evolution between the two polarization components of the soliton, we observed asynchronous development depending on the detailed operation conditions. In addition, period doubling of one polarization component together with period-one of another polarization component could be obtained. Much complexed dynamics on the two polarization components could be observed when the period tripling window is achieved.
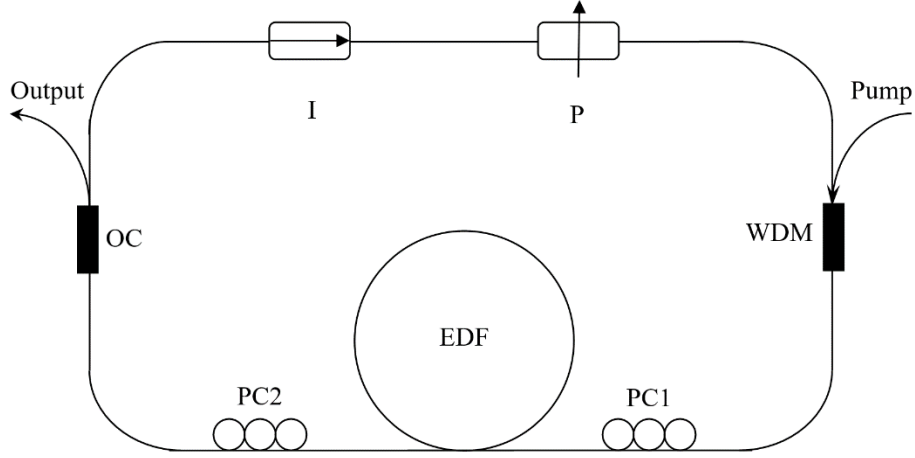
Fig. 1 Schematic of the fiber laser. WDM: wavelength division multiplexer; PC1/PC2: polarization controller; EDF: Erbium-doped fiber; OC: output coupler; I: isolator; P: polarizer.

2. Fiber laser setup and theoretical model

Numerical simulations were carried out based on the fiber laser shown in Fig. 1. A polarizer and two polarization controllers (PCs) were used to employ the NPR technique for achieving mode locking. An isolator was used to guarantee the unidirectional operation. Roundtrip model was taken to describe pulse evolution in the laser [19]. In general, an arbitrary small pulse is used as the initial input and it circulates in the laser cavity until a steady pulse evolution is established under specific operation conditions. When the pulse goes through a component, the Jones matrix of the component is applied to the pulse. The pulse propagation in fibers is described by the coupled GLEs as shown in equation (1), where the dispersion effect, nonlinear effect, gain and gain dispersion effect are included:

$$
\begin{cases}
\dfrac{\partial u}{\partial z} = i\beta u - \delta \dfrac{\partial u}{\partial t} - \dfrac{ik''}{2}\dfrac{\partial^2 u}{\partial t^2} + \dfrac{ik'''}{6}\dfrac{\partial^3 u}{\partial t^3} + i\gamma(|u|^2 + \dfrac{2}{3}|v|^2)u + \dfrac{i\gamma}{3}v^2 u^* + \dfrac{g}{2}u + \dfrac{g}{2\Omega_g^{\,2}}\dfrac{\partial^2 u}{\partial t^2} \\[3mm]
\dfrac{\partial v}{\partial z} = -i\beta v + \delta \dfrac{\partial v}{\partial t} - \dfrac{ik''}{2}\dfrac{\partial^2 v}{\partial t^2} + \dfrac{ik'''}{6}\dfrac{\partial^3 v}{\partial t^3} + i\gamma(|v|^2 + \dfrac{2}{3}|u|^2)v + \dfrac{i\gamma}{3}u^2 v^* + \dfrac{g}{2}v + \dfrac{g}{2\Omega_g^{\,2}}\dfrac{\partial^2 v}{\partial t^2}
\end{cases}
\tag{1}
$$

where u and v are the complex optical envelopes in orthogonal polarization mode along fibers; u^* and v^* are the conjugates of u and v; $\beta = \pi\Delta n/\lambda$ is the wave-number difference between the two polarization modes of the fiber, where Δn is the difference between the effective indices of the two modes, and λ is the wavelength; $\delta = \beta\lambda/2\pi c$ is the inverse of group velocity, where c is the light speed; k'' is the second order dispersion coefficient, k''' is the third order dispersion coefficient, and γ represents the nonlinearity of the fiber. g is the saturable gain coefficient of the gain fiber and Ω_g is the gain bandwidth. For undoped fibers g = 0; for EDF, the gain saturation is:

$$
g = G\exp\left[-\frac{\int\left(|u|^2 + |v|^2\right)dt}{P_{sat}}\right]
\tag{2}
$$

To study period doubling eigenstates, the following parameter set is selected as shown in Table 1.

Table 1. Parameters used in the simulations [19]

γ	3 W^{-1}km^{-1}	k''_{EDF}	-23 ps^2/km
k''_{SMF}	-23 ps^2/km	k'''	-0.13 ps^3/km
L_b	6 m	Ω_g	50 nm
θ	0.152π	φ	$\theta+\pi/2$
P_{sat}	250 pJ	G	varying (km^{-1})
Φ_{PC}	varying between π to 2π (mode locking regime)		

where L_b is the birefringence beat length, and the laser cavity length L = 4 m, which is composed of 1 m SMF, 1 m EDF, and 2 m SMF in sequence, and the output position is at the end of the first segment of SMF, and 10% output is applied. Φ_{PC} corresponds to the linear cavity phase delay, it can be understood as the following: the cavity transmission can be described by [19]:

$$T=\sin^2\theta\sin^2\varphi+\cos^2\theta\cos^2\varphi+\frac{1}{2}\sin 2\theta\sin 2\varphi\cos\left[\Phi_l+\Phi_{nl}\right] \qquad (3)$$

where Φ_l is the linear phase delay and Φ_{nl} is the nonlinear phase delay. In the simulation, the linear phase delay includes two parts, the cavity linear phase delay bias and the linear phase shift caused by the fiber birefringence:

$$\Phi_l=\Phi_{PC}+2\pi\left(1-\frac{\Delta\lambda}{\lambda_c}\right)\frac{L}{L_b} \qquad (4)$$

The definition of other parameters can be retrieved from Ref. [19]. The general cavity length of a fiber laser can be from less than 1-m [20] to tens of meters. And a 0.335-m EDF is enough for supporting pulse generation. Therefore, 1 m of EDF and 3 m for the other passive components are reasonable for a fiber laser.

3. Simulation results

By properly choosing the linear cavity phase delay bias, which corresponds in the experiment to appropriately selecting the orientations of the polarization controllers, soliton operation can be always obtained in our simulations. As the value of gain increases, the period-doubling state can be achieved. But with different linear cavity phase delay bias, each polarization component of the soliton can show individual behavior of period doubling.

When the linear cavity phase delay bias is set at 1.35π, a stable soliton sequence can be obtained when the gain coefficient is set at $G = 700$, as shown in Fig. 2(a). Increasing the value of G and keeping all the other parameters fixed, the soliton repetition period in the cavity is doubled at $G = 800$ (Fig. 2(c)). Figure 2(b) and Fig. 2(d) shows the corresponding optical spectrum of the solitons to Fig. 2(a) and Fig. 2(c), where the optical spectrum of 10 roundtrips are overlapped. Therefore, the spectra shown in Fig. 2(d) have two different curves corresponding to period doubling while the spectra shown in Fig. 2(b) have only one curve.

To study the detailed period doubling performance of the two orthogonal polarization components, the gain coefficient is set at $G = 820$ to make the cavity operated in a period-doubling state and change the linear cavity phase delay bias to understand the pulse intensity variation between horizontal component and vertical component, as shown in Fig. 3. When $\Phi_{PC} = 1.35\pi$, the pulse train of horizontal component and vertical component is shown in Fig. 3(a) and Fig. 3(b). The evolution of horizontal component and vertical component are synchronous. When the linear cavity phase delay bias changes to $\Phi_{PC} = 1.42\pi$, the horizontal component becomes period-one state while the vertical component is period-doubling state, as shown in Fig. 3(c)/(d). Changing Φ_{PC} to 1.46π, the horizontal component and vertical component are all in period-two states but they are asynchronous, as shown in Fig. 3(e)/(f).
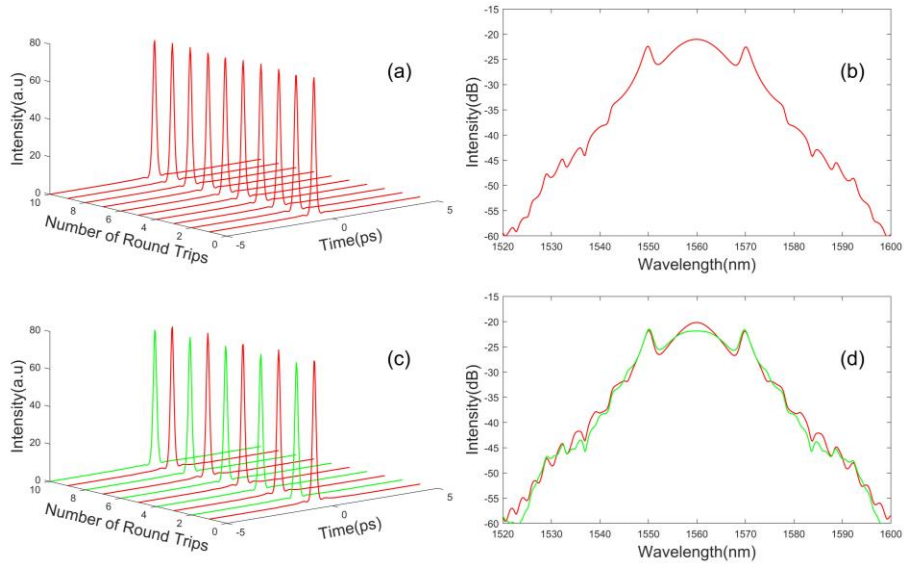
Fig. 2 Soliton pulse evolution and the corresponding optical spectra numerically calculated under different pump strength. The linear cavity phase delay bias is set as $\Phi_{PC} = 1.35\pi$. (a)/(b) Period-one soliton state, $G = 700$; (c)/(d) Period-two soliton state, $G = 800$
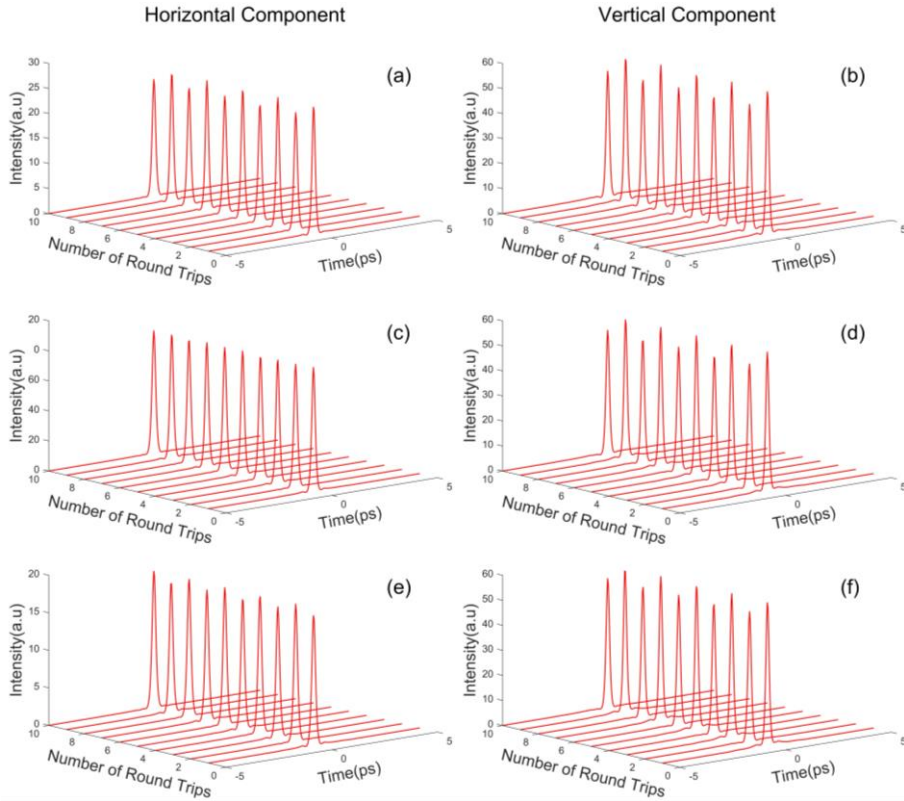

Fig. 3 The detailed period doubling eigenstates of period doubling solitons under different linear cavity phase delay bias. The gain coefficient is fixed at $G = 820$. (a)/(b) synchronous when $\Phi_{PC} = 1.35\pi$; (c)/(d) the horizontal component is period one while the vertical component is period doubling when $\Phi_{PC} = 1.42\pi$; (e)/(f) asynchronous when $\Phi_{PC} = 1.46\pi$
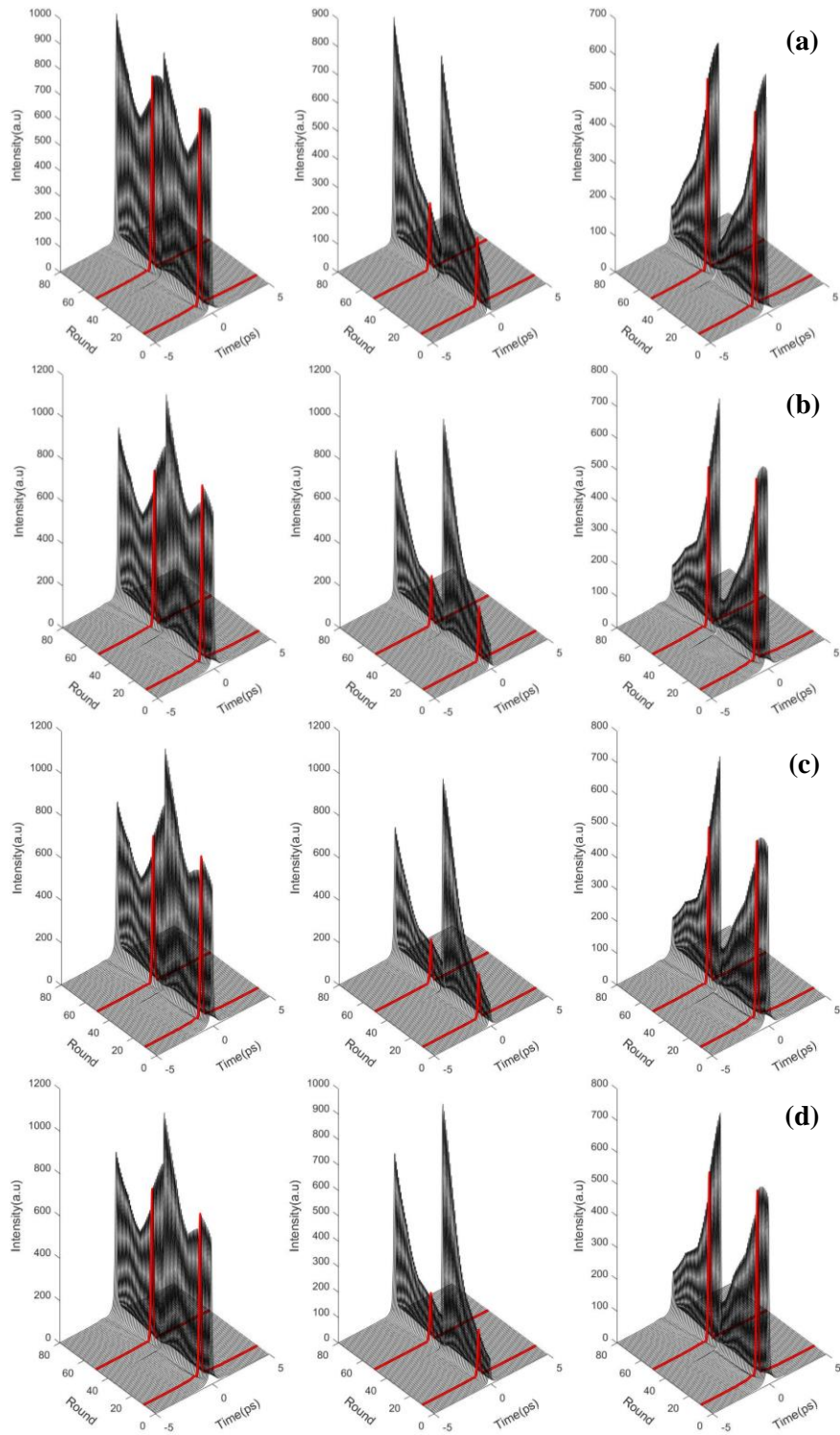
Fig. 4 Evolution of the soliton (left), the horizontal component of the soliton (middle), and the vertical component of the soliton (right) in the cavity for two adjacent roundtrips. (a) The state of period one; (b) the state of synchronous period doubling evolution; (c) the state that the horizontal component is period one while the vertical component is period doubling; (d) the state of asynchronous period doubling evolution.

As shown in Fig. 3, the details of the period doubling eigenstates are sensitive to the linear cavity phase delay bias. Generally, changing the linear cavity phase delay bias plays two roles: i) modifying pulse parameters by varying the transmission function; ii) changing the phase relationship between the two coupled components, which affects the coherent energy exchanging between two components [21]. The latter one can significantly affect the evolution of the two components. Consequently, synchronous or asynchronous period doubling of the two coupled components could be obtained.

When the NPR technique is used for achieving mode locking in fiber laser, the generated ultrashort pulse is generally considered as a scalar soliton as the pulse polarization is periodically normalized to the polarization direction of the polarizer. However, due to the existence of fiber weak birefringence, the soliton obtained in the fiber laser indeed consist of two orthogonal polarization components, although the soliton is polarization-normalized every time when it passes through the polarizer. Consequently, the period doubling eigenstates are not based on the soliton anymore but should be based on the polarization components of the soliton. Simulation results shown in Fig. 3 clearly suggests that the period doubling of the polarization component could be different from that of the soliton. When the soliton exhibits period doubling, depending on the detailed operation conditions the polarization component could show synchronous period doubling, asynchronous period doubling, or even period one, respectively.

To better understand the detailed dynamics, the evolution of the two orthogonal components of the soliton and that of the soliton inside the cavity were shown in Fig. 4. Two adjacent roundtrips inside cavity were shown, respectively. Figure 4(a) corresponds to the state of period one as shown in Fig. 2(a); Fig. 4(b) corresponds to the state of synchronous period doubling evolution of the two orthogonal components, as shown in Fig. 3(a); Fig. 4(c) corresponds to the state that the horizontal component is period one while the vertical component is period doubling, as shown in Fig. 3(c); Fig. 4(d) corresponds to the state of asynchronous period doubling evolution of the two orthogonal components as shown in Fig. 3(e). The red lines are at the position just before the output. We note that the output ratio is 10% and the evolution shown in Fig. 4 is inside the cavity.
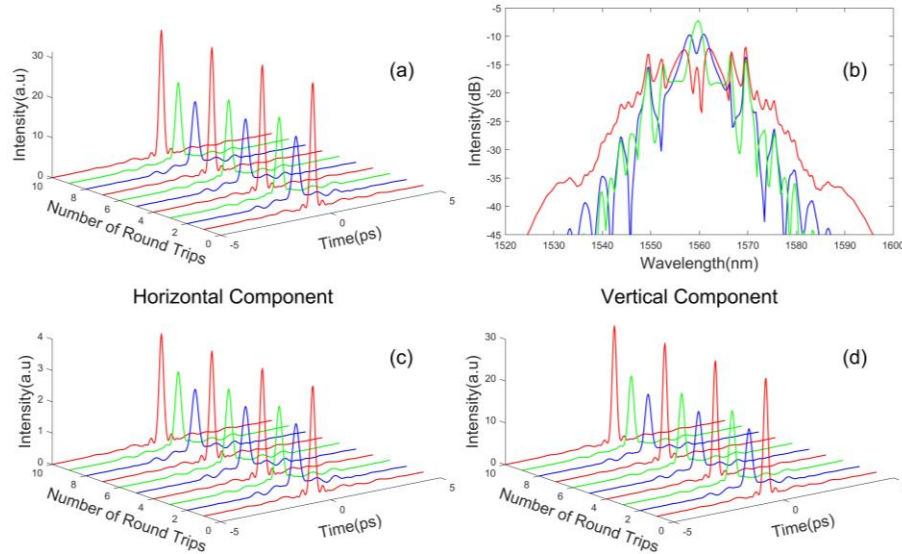

Fig. 5 Period tripling of soliton and the corresponding period tripling of individual polarization component. (a)/(b) Period tripling of soliton and the corresponding optical spectra; (c)/(d) Period tripling of soliton of individual polarization component

Numerically we also observed period tripling of soliton and the corresponding period tripling of individual polarization component. So far only the synchronous behavior was observed as shown in Fig. 5. We believe that more complexed behaviors apart from the synchronous one exist depending on the detailed operation conditions.

We checked the dependency on the initial conditions for the occurrence of period doubling. Plenty of numerical simulations suggest that the occurrence of period doubling based on the soliton is generally independent on the initial conditions. However, detailed period doubling based on the polarization component of a soliton depends on the initial conditions. All the period doubling or period tripling we obtained is stable. It is found that when the Φ_{pc} is set between 1.1π to 1.8π, the stable soliton sequence including multiple solitons could be obtained. Stable period bifurcation is easy to be obtained when the Φ_{pc} is set between 1.35π to 1.55π. Noise-like pulses may be obtained when the Φ_{pc} is larger than 1.5π. We note that so far we only exemplified that there exists period doubling eigenstates. However, as the period doubling eigenstates are based on two coupled components, it is difficult to explore the bifurcation diagrams [5] based on either polarization component.

4. Conclusions

Appearance of period doubling and its variants is one of intrinsic properties of nonlinear systems. Period doubling eigenstates are discussed in a fiber laser mode-locked by using the NPR technique. Numerical simulations suggest that each polarization component of the soliton could show independent behavior of period doubling while there exists interaction between the two orthogonal polarization components and the polarization was normalized every time when the soliton periodically passes the polarizer. Depending on the detailed parameters, either synchronous evolution or asynchronous development between the two polarization components of the generated soliton under period doubling appearance can be achieved. Specifically, period doubling of one polarization component while the other polarization component maintaining period-one can as-well be achieved. The discovery above great enriches our understanding on soliton dynamics in fiber lasers and pave the way to be closer to the real dynamics of nonlinear systems.

Funding

Disclosures

The authors declare that there are no conflicts of interest related to this article.

References

1. Sucha G, Bolton S R, Weiss S, and Chemla D S, "Nonlinear dynamics of additive pulse mode-locked lasers: period doubling and chaos," Quantum Electronics & Laser Science Conference, JTuC6, 1993.
2. Tamura K, Doerr C R, Haus H A, and Ippen E P, "Soliton fiber ring laser stabilization and tuning with a broad   intracavity filter," IEEE Photonics Technology Letters **6**(6), 697–699 (1994).
3. N. Akhmediev, J. M. Soto-Crespo, and G. Town, "Pulsating solitons, chaotic solitons, period doubling, and pulse coexistence in mode-locked lasers: Complex Ginzburg-Landau equation approach," Phys. Rev. E **63**(5), 645–651 (2001).

4. J. M. Soto-Crespo, M. Grapinet, P. Grelu, and N. Akhmediev, "Bifurcations and multiple-period soliton pulsations in a passively mode-locked fiber laser," Phys. Rev. E **70**(6), 691–738(2004).

5. Qian Zhang, Lei Li, Dingyuan Tang, and Luming Zhao*, "Period-timing Bifurcations in a Dispersion-managed Fiber Laser with Zero Group Velocity Dispersion," IEEE Photonics Journal **8**, 1504608 (2016).

6. L. M. Zhao, D. Y. Tang, F. Lin, and B. Zhao, "Observation of period-doubling bifurcations in a femtosecond fiber soliton laser with disperse management cavity," Opt. Exp. **12**(19), 4573–4578(2004).

7. L. M. Zhao, D. Y. Tang, X.Wu, and H. Zhang, "Period-doubling of gain-guided solitons in fiber lasers of large net normal dispersion," Opt. Commun. **281**(13), 3557–3560(2008).

8. L. M. Zhao, D. Y. Tang, T. H. Cheng, H. Y. Tam, C. Lu, and S. C. Wen, "Period-doubling of dispersion-managed solitons in an Erbium-doped fiber laser at around zero dispersion," Opt. Commun. **278**(2), 428–433(2007).

9. Zhao Luming*, Shu Chaojie, Wang Yufei, Li Lei, "Research progress of period doubling bifurcation in ultrafast fiber lasers (invited)," Infrared and Laser Engineering **47**(8), 803002-0803002(2018).

10. Zhichao Wu, Deming Liu, Songnian Fu*, Lei Li, Ming Tang, and Luming Zhao, "Scalar-vector soliton fiber laser mode-locked by nonlinear polarization rotation," Optics Exp. **24**(16), 18764-18771 (2016).

11. L. M. Zhao, D. Y. Tang, H. Zhang, X Wu, C. Lu, and H. Y. Tam, "Period-doubling of vector solitons in a ring fiber laser," Optics Commun. **281**(21), 5614-5617 (2008).

12. N. N. Akhmediev, A. Ankiewicz, M. J. Lederer, and B. Luther-Davies, "Ultrashort pulses generated by mode-locked lasers with either a slow or a fast saturable-absorber response," Opt. Lett. **23**, 280-282 (1998)

13. Sze Y. Set, Hiroshi Yaguchi, Yuichi Tanaka, and Mark Jablonski, "Laser Mode Locking Using a Saturable Absorber Incorporating Carbon Nanotubes," J. Lightwave Technol. 22, 51- (2004)

14. H. Zhang, D. Y. Tang, **L. M. Zhao**, Q. L. Bao, and K. P. Loh, "Large energy mode locking of an erbium-doped fiber laser with atomic layer graphene," Opt. Express 17, 17630-17635 (2009).

15. N. J. Doran and David Wood, "Nonlinear-optical loop mirror," Opt. Lett. 13, 56-58 (1988)

16. M. E. Fermann, F. Haberl, M. Hofer, and H. Hochreiter, "Nonlinear amplifying loop mirror," Opt. Lett. 15, 752-754 (1990)

17. M. Almanee, J. W. Haus, I. Armas-Rivera, G. Beltran-Perez, B. Ibarra-Escamilla, "Polarization evolution of vector wave amplitudes in twisted fibers pumped by single and paired pulses," Optics Exp. **41**(21), 4927-4930 (2016).

18. Y. Cui, Y. Zhang, Y. Xiang, Y. Song, and X. Liu, "XPM-forced frequency-oscillating soliton in mode-locked fiber laser," https://arxiv.org/abs/1910.09225.

19. L. M. Zhao, C. Lu, H. Y. Tam, P. K. A. Wai, and D. Y. Tang, "Gain dispersion for dissipative soliton generation in all-normal-dispersion fiber lasers," App. Opt. **48**(27), 5131-5137 (2009).B. C. Collings, S. T. Cundiff, N. N. Akhmediev, J. M. Soto-Crespo, K. Bergman, and W. H. Knox, "Polarization-locked temporal vector solitons in a fiber laser: experiment," J. Opt. Soc. Am. B 17, 354-365 (2000).

20. L. M. Zhao, C. Lu, H. Y. Tam, P. K. A. Wai, and D. Y. Tang, "High fundamental repetition rate fiber lasers operated in strong normal dispersion regime," IEEE Photonics Technology Letters, 21, 724-726 (2009).

21. B. C. Collings, S. T. Cundiff, N. N. Akhmediev, J. M. Soto-Crespo, K. Bergman, and W. H. Knox, "Polarization-locked temporal vector solitons in a fiber laser: experiment," J. Opt. Soc. Am. B 17, 354-365 (2000).