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Intonation in Unaccompanied Singing: Accuracy, Drift and a Model of Reference Pitch Memory

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This paper presents a study on intonation and intonation drift in unaccompanied singing and proposes a simple model of reference pitch memory that accounts for many of the effects observed. Singing experiments were conducted with 24 singers of varying ability under 3 conditions (Normal, Masked, Imagined). Over the duration of a recording, approximately 50 seconds, a median absolute intonation drift of 11 cents was observed. While smaller than the median note error (19 cents), drift was significant in 22% of recordings. Drift magnitude did not correlate with other measures of singing accuracy, singing experience or with the presence of conditions tested. Furthermore, it is shown that neither a static intonation memory model nor a memoryless interval-based intonation model can account for the accuracy and drift behaviour observed. The proposed causal model provides a better explanation as it treats the reference pitch as a changing latent variable.

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I. INTRODUCTION

Unlike other musical instruments, the vocal apparatus is common to all human beings, and in every known human culture people use it to make music (Brown, 1991, as reproduced by Pinker (2002)). There is good evidence that vocal music was practiced even in prehistoric human societies, and it might even have preceded language (Mithen, 2007). Yet science is only beginning to understand the control processes involved in human singing. This paper aims to provide some insights into intonation, a parameter that is crucial to many singing styles but has so far received little academic attention.

Intonation is defined as “accuracy of pitch in playing or singing” (Swannell, 1992), or “the act of singing or playing in tune” (Kennedy, 1980). Both of these definitions imply the existence of a reference pitch, which could be internal or external. We treat intonation as the signed pitch difference relative to the reference pitch, measured in semitones on an equal-tempered scale (see detailed discussion in Section III).

In choirs, intonation is the main reported priority in daily rehearsals (Ganschow, 2013) and the focus of guides on choral practice (e.g. Crowther, 2003). Such ensembles frequently observe a change in tuning over periods of tens of seconds or even a whole piece, a phenomenon called intonation drift or pitch drift (Seaton et al., 2013). According to Alldahl (2006) “the problem is mainly a lowering of pitch”, i.e. downward intonation drift. Seaton et al. (2013) offer a literature review on choral intonation drift, and their pilot survey on drift in choral singing corroborates Alldahl’s observation that drift mainly occurs in the downward direction. Several scientific studies suggest that one cause for the propensity to drift is the harmonic progression (Terasawa, 2004; Howard, 2007; Devaney et al., 2012) (see also Section II).

Yet harmonic effects cannot be the only cause for intonation drift, since it also occurs in solo singing: in a study on folk song analysis, Müller et al. (2010), tracking the tuning on a stanza level, report that intonation drift is common in unaccompanied solo folk singing. Intriguingly, their example shows that strong rises in tuning were observed, but no further investigations are reported. Ryyänen (2004) built adaptive tuning into a note transcription system based on the observation that “non-professional singers tend to change their tuning (typically downwards) during long melodies”. Here, intonation drift is treated as a nuisance factor. Dalla Bella et al. (2007) investigate pitch stability as one of several variables describing pitch in singing. To our knowledge no other studies on unaccompanied solo singing exist that investigate intonation drift in its own right.

Hence, the main motivation for the present study is to improve the scientific understanding of intonation drift in unaccompanied solo singing, without additional influences of harmonic consonance or ensemble interaction. Findings drawn from observations in this simpler setting are likely to play a role in explaining drift in complex ensemble situations as well. In order to understand which mechanisms may cause drift, we study three different conditions, Normal, Masked and Imagined (Section III).

The remainder of the paper is structured as follows. Section II discusses existing work related to singing intonation and musical memory. Section III describes our intonation experiments, including the three experimen-
tal conditions as well as a basic outline of the analysis setup. Section IV defines and illustrates several metrics of singing accuracy and drift. In the results section (V) we show what intrinsic and external factors influence accuracy and drift. The following section (VI) introduces a simple model of pitch reference memory which is able to account for the intonation stability and drift we observed. Section VII provides a discussion of achievements and future work, and a summary of our conclusions is found in section VIII.

II. PREVIOUS WORK

Only since the advent of precise pitch analysis in the form of the tonoscope (Seashore, 1914) has it been possible to study intonation quantitatively. Carl Seashore’s Psychology of Music (Seashore, 1967, originally published in 1938) already featured analyses of vibrato based on this technique. Since then, less burdensome methods for pitch analysis have been devised (e.g. Schroeder, 1968; Markel, 1972; de Cheveigné and Kawahara, 2002). These methods, along with computer programs like Praat (Boersma, 2002) and the advent of fast, affordable computers have made intonation analysis accessible to anyone with a microphone and a computer.

Recently, progress has been made on quantifying differences in intonation between singers. In the music informatics domain, singing tuition applications (e.g. Cano et al., 2012) have driven the development of singing assessment methods that often focus on intonation aspects (for an overview, see Molina, 2012). In the music psychology literature, the phenomenon of so-called “poor singers” has gained some interest (e.g. Berkowska and Dalla Bella, 2009; Dalla Bella and Berkowska, 2009; Dalla Bella et al., 2007; Pfordresher et al., 2010). Welch (1985) proposed a theory of singing production, with special regards to how children acquire singing skills.

Vurma and Ross (2006) investigated professional singers’ ability to sing intervals and reported average standard deviations of 22 cents in interval size, and 34 cents in absolute pitch relative to a tuning fork reference. Immediately after singing, the singers were unable to judge whether their intervals were out of tune, but after listening to a recording of their singing, their judgements were not significantly different from other expert listeners. Judgements of out of tune singing correlated with pitch errors, but errors of even 40 cents were not reliably judged out of tune by the majority of listeners.

Dalla Bella et al. (2007) compared occasional and professional singers performing a well-known melody in a free memory recall scenario. Two groups of occasional singers made errors in singing intervals of around 0.6 and 0.9 semitones on average, while professional singers’ errors were only 0.3 semitones. A correlation with tempo was also observed, and a second experiment was performed, which confirmed that errors decreased significantly when the same singers sang more slowly. In a further study, Dalla Bella and Berkowska (2009) used both free recall and repetition paradigms to characterise poor singing in terms of timing accuracy, relative pitch (interval) accuracy and absolute pitch accuracy, and found that poor singers could have deficits in any one or any combination of these attributes.

Pfordresher et al. (2010) distinguished the accuracy (mean deviation from a target pitch) and precision (consistency in repeated attempts to produce a pitch) of singers in order to classify “poor” singers. They found that the majority (56%) of singers were imprecise (standard deviation of pitch error greater than one semitone), but only 13% of singers were inaccurate (absolute value of average error greater than 1 semitone). It was also observed that errors were greater for the imitation task than for a recall task.

Most existing research on intonation is concerned with a fixed tuning system, but some authors have also studied intonation drift. Terasawa (2004), Howard (2007) and Devaney et al. (2012) investigated pitch drift in unaccompanied vocal ensembles. In such a context, physics predicts that perfect consonance conflicts with pitch stability over time. The idea goes back at least to the 16th century, when music theorist Giovanni Benedetti wrote a piece of three-part singing designed to result in various amounts of pitch drift. The evidence from the new studies for a reliably predictable effect is not entirely conclusive, partly due to small sample sizes: Devaney et al. (2012) reported only negligible effects on the original Benedetti composition, while Howard (2007) reported drifts roughly in line with predictions on specially composed new pieces. Dalla Bella et al. (2007) also measured pitch stability and found absolute deviations between repeated sequences of notes of 0.3 semitones in professional singers and 0.6 semitones in occasional singers.

III. METHOD

A. Participants

A total of 31 participants from the UK and Germany took part in the experiment. They were recruited from musicology students, office colleagues, lab members and the choir of the Wolfson College in Cambridge UK. Our aim is to study intonation of subjects who are not “poor” singers (Pfordresher and Brown, 2007). Hence, two participants were excluded because they produced a melody that matched Happy Birthday rhythmically, but

<table>
<thead>
<tr>
<th>Musical Background</th>
<th>Choir Experience</th>
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<tr>
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<td>None</td>
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<tr>
<td>Amateur</td>
<td>As a child</td>
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<tr>
<td>Semi-professional</td>
<td>No longer active</td>
</tr>
<tr>
<td>Professional</td>
<td>Still active</td>
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</tbody>
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<table>
<thead>
<tr>
<th>Singing Skill</th>
<th>Singing Experience</th>
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</thead>
<tbody>
<tr>
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<td>None</td>
</tr>
<tr>
<td>Low</td>
<td>Some</td>
</tr>
<tr>
<td>Medium</td>
<td>A lot</td>
</tr>
<tr>
<td>High</td>
<td>Professional</td>
</tr>
<tr>
<td>Very High</td>
<td>(no response)</td>
</tr>
</tbody>
</table>

TABLE I: Self-reported musical experience.
not tonally (they consistently sang a different melody). A third singer had an unstable voice from which we were unable to draw suitable pitch estimates. Also excluded were four further participants, who were detected as outliers and hence classified as “poor” singers. The outlier classification was performed using multivariate outlier detection (Filzmoser et al., 2005) on two singer-based metrics: mean absolute interval error (see Section IV.C) and ratio of intervals within a semitone of the true interval. After these exclusions, 24 subjects remained in the study. The age of the participants ranged from 13 to 62 with a median of 32.5 years (mean: 34.5). The gender ratio was imbalanced with 6 females and 18 males in the sample. The musical experience of participants was wide-spread. Fourteen singers considered themselves amateur musicians, 9 professionals or semi-professionals, and 1 reported no musical background. Thirteen participants reported “a lot” of singing experience, 9 some or no experience, one subject sings on a professional level, and one did not respond. Eleven subjects are still active in some choir, while 8 had previous choir experience, and 5 have never sung in a choir (see Table 1). Since we had a large share of male participants, baritone was the most common voice type with a total of 13 subjects, followed by soprano with 6 subjects.

B. Material

Since we chose to employ a free memory recall paradigm with a variety of subjects from two different countries, the choice fell on “Happy Birthday”, probably the single best-known and most wide-spread song in the world. Happy Birthday cannot be considered a very easy song, since it contains a variety of different intervals, some of them being large jumps (see Fig. 1). The ambitus is exactly one octave using a full major scale and comprising a total of 25 notes in 4 phrases of 6, 6, 7, and 6 notes each.

C. Procedure

Each participant sang a total of 9 renditions of “Happy Birthday”, in three recordings of three runs each. Details are given below. For a particular recording each participant was asked to sing three consecutive runs of “Happy Birthday”. The participants could choose the starting pitch at their own comfort in order to limit effects of regression to their comfort pitch. They were provided with a click track of moderate tempo (96 bpm) and instructed to wait four bars before beginning to sing. Subjects were instructed to sing the syllable “na” throughout. Subjects were recorded at a sample rate of 44100 Hz with a bit depth of 32 bit (stored to 16-bit PCM) using Audacity 2.0 running on a Windows Laptop or a MacBook Pro. A conventional headset (Logitech USB Headset 390) functioned both as microphone and headphones, through which participants were provided with the click track and the noise in the Masked condition (see below).

Three such recordings were made of each participant to test three different conditions, which differed by the way the second run of “Happy Birthday” was performed.

Normal. The participant sang three renditions of “Happy Birthday” as described above.

Masked. Pink noise at a moderate sound pressure level was applied over the headphones during the second of three renditions of “Happy Birthday”.

Imagined. The participant was asked to remain silent during the second rendition of “Happy Birthday”, while imagining to sing, and to resume singing at the start of the third rendition.

The reasoning behind these conditions was to study whether the absence of vocal strain reduces the tendency to drift (Imagined condition) and whether an impediment to auditory feedback would increase the tendency to drift (Masked condition). Note that the Imagined condition does not only remove vocal strain, but also auditory and kinesthetic feedback, as the participants can neither hear their singing nor feel singing-induced movements or the state of the vocal tract in the vicinity of the vocal folds. Anaesthetising the vocal folds has been shown to lead to a decrease in singing accuracy (Kleber et al., 2013).

The sequence of conditions was held constant (in increasing order of difficulty). In each condition, subjects sang 75 notes except in the Imagined condition with only 50 notes. Most of the German singers sang the German version of the melody which divides note 17 into two syllables at the same pitch; this extra note was disregarded in the analysis. One singer consistently missed note 19.

D. Analysis

We use as our reference tuning system equal temperament. We will see in Section IV that for the purposes of our study the assumption of equal temperament does not substantially affect our results. We also assume that pitch, a perceptual quantity, is adequately represented by its physical correlate, fundamental frequency, for harmonic sounds such as singing (Vurma and Ross, 2006).

We relate fundamental frequency $f_0$ to musical pitch $p$ as follows:

$$p = 69 + 12 \log_2 \frac{f_0}{440}. \quad (1)$$

This scale is chosen such that a difference of 1 corresponds to one semitone; for integer pitches the representation coincides with the MIDI pitch scale, with reference
pitch A4 tuned to 440 Hz \( (p = 69) \). As pitch differences are generally small, we often use the unit cent, equal to a hundredth of a semitone in equal temperament.

For example, middle C (60 on the MIDI pitch scale) has a frequency of 261.63 Hz. A note measured at 257 Hz has a pitch of 59.69, and thus an intonation difference to middle C of \(-0.31\) semitones (or \(-31\) cents).

We use the word nominal to refer to the ideal intervals or pitches with respect to a reference in equal temperament. For instance, if we consider an upward interval of a perfect fifth, then its nominal size is 7 semitones. This allows us to contrast this with the size of an observed interval, which in general differs from its nominal size.

The recorded songs were analysed using a semi-automatic pitch tracking process. The second author (kf) annotated onsets and offsets of note events by visually identifying the stable part in the estimated pitch track using Sonic Visualiser 2.0 (Cannam et al., 2010) and subsequent auditory verification. Automatically calculated onsets and offsets were adjusted manually, and the resulting annotations were fed into customised pitch tracking software (Mauch and Dixon, 2014), which is based on the YIN algorithm (de Cheveigné and Kawahara, 2002). The resulting note tracks were then analysed using R (2008). In order to obtain note-wise pitch estimates we take the median pitch estimate over the annotated duration of the note, as illustrated in Figure 2. A total of 4789 notes in 72 recordings were collected this way.

To test the reliability of the note timing annotations, 12 randomly selected blocks (of 3 runs) were also annotated manually by the other two authors and submitted to the note tracking algorithm. A comparison of onset and offset annotations reveals that these coders chose voiced/voiced boundaries and included note transitions, while kf consistently placed onsets later and offsets earlier in the sound event, capturing only the stable pitch portion of the note. A comparison of the different resulting pitch tracks showed that the median statistic is robust to such varying interpretations of note onsets and offsets, and no significant differences for the note pitch estimates were found. The average difference of the two other coders to the first coder was less than 0.2 cents, and only 1.4% of \( F_0 \)-differences were larger than 5 cents.

**IV. METRICS OF ACCURACY AND DRIFT**

In this section we introduce how we measure intonation (in terms of interval and pitch error), singer-wise performance measures and drift. We start by defining interval and pitch errors for individual notes and illustrate these using some examples from our data. Then we introduce measures of intonation accuracy and drift based on the error definitions.

**A. Interval Error**

The distance between two pitches is referred to in musical terms as an interval, corresponding in physical terms to the ratio of the constituent fundamental frequencies. For the sake of this paper, we express the interval leading to the \( i \)th pitch \( p_i \) (see Eq. (1)) as the signed distance \( \Delta p_i = p_i - p_{i-1} \) in semitones between the \( i \)th and the preceding note. The interval error of the observed interval \( \Delta p_i \) can then be written as

\[
e_i^{\text{int}} = \Delta p_i - \Delta p_i^0, \tag{2}
\]

where \( \Delta p_i^0 \) is the nominal interval in semitones using equal temperament (ET). Figure 3a shows a box plot of interval error by nominal interval. A first observation is that the two largest upward intervals of 8 semitones (minor sixth) and 12 semitones (octave) are significantly flat, i.e. smaller than expected (one sample \( t \) test \( t(186) = -6.96, t(183) = -9.09, \) both \( p < 0.0001 \)). This phenomenon is called compression and is well known in the literature (Pfordresher et al., 2010).

The prime interval, a repetition of the same pitch (0 semitone nominal interval), is systematically sharp, i.e. sung too high (one sample \( t \) test: \( t(753) = 17.96, p < 0.0001 \)) by approximately 0.29 semitones. The fact that all prime intervals occur between the first and second note of each phrase (see Figure 3b) suggests two possible explanations. Either the first note is sung flat as the vocal cords re-adjust from low tension in the rest between phrases to the higher tension required to sing the intended pitch, or the second note is sharp in preparation for an upward interval occurring after the note. This second possibility cannot explain the sharpness of note 21, which is followed by a downward interval, but we will obtain further insights by considering pitch error.

**B. Pitch error**

Defining pitch error is not as straightforward as defining interval error, because in our unaccompanied singing data we have no external reference pitch against which intonation could be measured. Instead, the tuning emerges as singers sing and may change over the course of the
FIG. 3: Box-and-whisker plot of interval errors in semitones for all recordings of all 24 singers. Values are relative to the score using equal temperament. The numbers at the top of the figure indicate sample sizes.

FIG. 4: Example of pitch error estimation, showing pitch measurements $p_i$ (empty bullets) and local tonic estimates $t_i$ (filled) using a linear fit. The stems represent the pitch error $e_i$.

As a result, no single best way of defining intonation is possible. In order to obtain a reference we will use a linear fit to the local tonic estimate, as explained below. For the measured pitch $p_i$ of the $i$th note we can find an estimate of the implied tonic pitch by subtracting from $p_i$ the nominal pitch $s_i$ relative to the estimated tonic. These nominal pitches for “Happy Birthday” are given in Figure 5b. For example, if the first note in a run is sung at $p_1 = 50.45$ (see Eq. 1), then the implied tonic is $t_1 = 50.45-(−5) = 55.45$ because the first note is 5 semitones below the tonic. This is shown in Figure 4, which also illustrates the next steps: for every run (a third of the performance) we use linear regression to fit a line to the 25 values $t_i$ with note number $i$ as independent variable, obtaining fitted values $t'_i$. (Linear regression was chosen as the simplest approach allowing for tonic changes.) We define the note error $e_i$ as the difference between the implied tonic and the fitted tonic:

$$e_i = t_i - t'_i.$$  (4)

The individual errors are represented by the stems between the linear fit and the filled markers in Figure 4.

With the ability to measure the pitch error, we can now investigate the relative effects of phrase beginnings and note jump preparation, as hypothesised in Section IV.A. A linear model predicting pitch error by the independent variables is-beginning-of-phrase and interval-to-next-note shows that both correlate significantly ($F(4667) = 254.94$, both $p < 0.0001$) with interval error. Hence, neither hypothesis can be rejected — it is likely that both influence intonation. Being at the beginning of a phrase ‘makes’ notes about 21 cents flat. Each signed semitone in the following interval leads to a sharpening of 1.3 cents (upward octave example: $12 \times 1.3 = 15.6$ cents). Together, the two variables account for 9.8% of the variance (as measured by $R^2$).

While using other reference temperaments would be possible, they do not provide substantially differing errors, which is in line with previous results by Devaney et al. (2011). In fact, in terms of mean absolute pitch error (see Section IV.C), equal temperament is a significantly better hypothesis than just intonation ($t(4774) = -14.1927$, $p < 0.0001$), but the actual difference is very small (1.3 cents). Lastly, note that interval and pitch errors indicate deviation from the mathematically defined equal temperament grid, not an aesthetic judgement.
C. Metrics of singing accuracy and precision

In order to assess singing accuracy we use two metrics: mean absolute pitch error (MAPE), defined as

$$MAPE = \frac{1}{M} \sum_{i=1}^{M} |e_i|.$$  \hspace{1cm} (5)

and mean absolute interval error (MAIE), defined as

$$MAIE = \frac{1}{M-1} \sum_{i=2}^{M} |e_i^{\text{int}}|.$$  \hspace{1cm} (6)

Both metrics are always non-negative. MAIE does not reflect any tendency to sing larger or smaller intervals, but it is in our view a natural way to indicate how closely intervals match their target (and is equivalent to interval deviation, Dalla Bella et al., 2007).

D. Metrics of pitch drift

Each of our recordings has a first and a third run of "Happy Birthday", each consisting of 25 notes. We estimate drift based on pitch differences between corresponding notes in these two runs of the song. Hence, for a particular recording we define pitch drift $D$ as the mean difference

$$D = \frac{1}{25} \sum_{i=1}^{25} p_{i+50} - p_i.$$  \hspace{1cm} (7)

The drift metric $D$ conveys information about the magnitude and direction of drift. In order to consider only the magnitude we use the metric absolute drift, i.e. $|D|$, which is equivalent to pitch stability (Dalla Bella et al., 2007), see also (Flowers and Dunne-Sousa, 1990, p. 105).

In the more general case without repeated sequences drift can be estimated as the slope of a linear model predicting the local tonic estimates $t_i$ with the note numbers 1,..,75 as the covariate. We have already used the same technique to calculate pitch error (Section IV.B). As we will see in the following section, this linear drift, denoted $D_L$, is very highly correlated with $D$, so for most of our analyses we will use only $D$ and $|D|$. From the model used to determine $D_L$ for a particular recording we also calculate the associated $p$-value, which is an indicator of the significance of the drift effect.

V. RESULTS

The metrics summarising accuracy and drift defined in Section IV allow us to analyse recordings and assess the correlations with test condition (Normal, Masked, Imagined) and participant factors such as choir experience. In order to prepare for the correlation analyses, we first present the distributions of recording-wise summary statistics themselves.

A. Distributions of accuracy and drift

We calculated the mean absolute pitch error (MAPE, see Section IV.C) for each of the 72 recordings. Figure 6a provides a histogram of the distribution of MAPE, showing that the average error magnitude is less than 0.5 semitones for all recordings, with most recordings having a MAPE of around 0.2 semitones (mean: 0.189; median: 0.187; std. dev.: 0.051). While this result shows that the singing in most recordings was systematically compatible with equal temperament, it is also clear that 0.2 semitones (20 cents) is slightly larger than the just noticeable difference, which for typical singing frequencies up to 800Hz is usually below 1%, i.e. below 17 cents (Henning, 1955). The distribution of MAIE (6b) is similar, with slightly larger magnitudes of around 26 cents (mean: 0.263; median: 0.267; std. dev.: 0.051). Turning to Table II, we observe that MAPE and MAIE are
FIG. 6: Distributions of singing accuracy metrics over all conditions and participants.

FIG. 7: Significance of drift, showing p-values (logarithmic scale) against $D_L$ for each recording. p-values below 0.01 are considered significant.
ttude, which is incompatible with an intonation model in which pitch errors propagate, as we will explore in Section VI. First, however, we investigate correlations with the singers’ self-assessment and experimental conditions.

B. Correlation with self-assessment

We investigated the relation between the quantitative intonation metrics and the singers’ self-assessment, taken from a survey they filled in. Three self-reported metrics take values from 1 to 5: singing ability (poor to very high), singing experience (none to professional) and choir experience (none to still active), and musical background (none to professional) takes values from 1 to 4. Table II shows the Spearman (i.e. rank) correlation values between all metrics, with significant correlations \(p < 0.01\) highlighted in bold print. We observe that most of the self-reported measures are inter-correlated, with the only exception of singing experience/musical background. In fact, the self-reported general level of musical background does not correlate with any of the quantitative measures either. Further study may reveal whether singing skills are indeed partially independent of general levels of musicality, as has been suggested before (Hutchins and Peretz, 2012).

However, two kinds of self-assessment ratings, singing ability and choir experience, do significantly correlate with our quantitative measures MAPE and MAIE. All of the four combinations have absolute correlations \(≥ 0.37\). While the correlation of accurate singing and choir membership is expected, the singers’ assessment of their singing ability, too, is in line with our measurements of intonation accuracy.

As we have mentioned in Section V.A, we observed little correlation between the measures of accuracy, MAPE and MAIE, and measures of drift, \(D\) and \(|D|\). In fact, the only two metrics that correlate with drift \(D\) are those that are indeed directly related: linear drift, which is a different measure of the same phenomenon, and absolute drift \(|D|\), which correlates because most of the \(D\) values are actually positive, i.e. they coincide with \(|D|\). Again, other than these direct connections, no other metrics correlate with either \(D\) or \(|D|\), in particular, none of the self-reported measures, including singing experience and choir experience.

C. Effect of experimental conditions: Normal, Masked, Imagined

To see whether the three conditions (Normal, Masked, Imagined, see Section III) have an influence on our measures of accuracy and drift, an analysis of variance was conducted. Since all four accuracy and precision variables are not normally distributed (right-skewed), a set of non-parametric Kruskal-Wallis tests was performed, but no significant differences between conditions and runs were found (MAPE: \(\chi^2(2) = 0.89, p = 0.64\); MAIE: \(\chi^2(2) = 2.43, p = 0.30\); \(D\): \(\chi^2(2) = 2.51, p = 0.28\); \(|D|\): \(\chi^2(2) = 0.42, p = 0.81\)). Even the middle run in the Masked condition did not significantly deteriorate singing intonation, in contrast with some other findings (e.g. Mürbe et al., 2002), but consistent with others who used low-level noise similar to that in our experiments (e.g. Pfndrescher and Brown, 2007). One observation during the experiments was that singers tend to sing louder in the Masked condition, compensating for the deprived auditory feedback (the so-called Lombard effect, Lombard, 1911), which is likely to have made the auditory feedback inhibition ineffective. The fact that the Imagined condition has little bearing on intonation is in line with perceptual experiments which found little difference in pitch acuity between listening and imagining conditions (Janata and Paroo, 2006). In summary, the conditions had no significant effect on the parameters we tested.

VI. A MODEL FOR INTONATION STABILITY

In this section we consider the question: how do singers stay in tune at all? While significant pitch drift was detected in many recordings, the tuning difference over three runs of “Happy Birthday” stayed remarkably small, despite large intonation errors on individual notes (see Section V.A). It appears that even amateur singers possess a mechanism that prevents them from chaotically drifting out of tune. This stabilising mechanism, we hypothesise, is mainly based on the retention of a pitch reference in short-term memory.

A. Production with Memory of a Changing Reference Pitch

A simple pitch production model can be built on the assumption that the intonation of the \(i^{th}\) note consists mainly of two components: a reference pitch \(r_i\), and the score information relative to that reference pitch. We choose to encode the melody notes in semitones relative to the tonic. (This is arbitrary; any other reference yields an equivalent model.) Assuming an additive Gaussian pitch error \(\varepsilon_i \sim N(0, \sigma_i)\), the pitch production process can then be written as

\[
p_i = r_i + s_i + \varepsilon_i, \tag{8}
\]

where \(p_i\) is the pitch of the \(i^{th}\) note, \(r_i\) is the reference pitch and \(s_i\) is the fixed score information given relative to the tonic. The error \(\varepsilon_i\) models all additional noise, e.g. from physiological effects.

Our results on pitch drift (see Section V.A) indicate that the singers’ reference pitch changes over time. We assume that the memory of the pitch reference cannot be perturbed by future events and hence model \(r_i\) as the causal process

\[
r_i = \mu r_{i-1} + (1 - \mu) (p_{i-1} - s_{i-1}), \tag{9}
\]

which depends on the previous reference pitch \(r_{i-1}\) and a point-estimate of the reference pitch \((p_{i-1} - s_{i-1})\), where \(\mu \in [0, 1]\) is a parameter relating to the memory of the previous reference pitch \(r_{i-1}\). Re-writing (9) as

\[
r_i = r_{i-1} + (1 - \mu) e_{i-1}. \tag{10}
\]
illustrates that the reference pitch is "pulled" in the direction of observed error $e_{i-1} = (p_{i-1} - s_{i-1}) - r_{i-1}$. A similar model, based on updated tuning histograms, was proposed by Ryynänen (2004) to deal with the transcription of monophonic melodies in an engineering context.

Since no reference pitch is available before the first observation, Eq. (9) is not defined for $i = 1$, i.e. we have a cold start problem. We choose the first phrase (six notes) to initialise the smoothed reference pitch estimate $r^* = \frac{1}{6} \sum t_i = \frac{1}{6} \sum (p_i - s_i)$. The first six notes in every recording are then excluded from any further analysis of this model, and the recursive update (9) is applied from $i = 7$. Figure 8 shows the local and smoothed reference pitches for an example recording under the Normal condition.

FIG. 8: Example of observed tonality estimates $t_i$ (marked as +) and the estimated reference pitch $r_i$ (filled bullets) with parameter $\mu = 0.85$.

B. Boundary models: no memory and absolute memory

The extreme cases $\mu = 0$ and $\mu = 1$ generate models with no memory of the reference pitch (in the Markovian sense) and perfect memory of the reference pitch, respectively. If $\mu = 0$, only the previous note realisation is used for reference, i.e. the reference pitch is simply $r_i = (p_{i-1} - s_{i-1})$, and hence

$$p_i = p_{i-1} + (s_i - s_{i-1}) + \varepsilon_i.$$  

That is, pitch production is based on the interval from the previous note realisation. This also means that errors from the previous note are fully passed on. Mathematical formalisation confirms that with an arbitrary starting pitch $p_0$ the pitch variance $\text{Var}[p_i - p_0] = \sum_{j=1}^i \text{Var}[\Delta p_j]$ is the sum of the interval error variances (assuming that intervals are independent). At the average observed interval variance of $\text{Var}[\Delta p_i] = 0.147$ the expected variance of two notes spaced 50 notes apart is $50 \times \text{Var}[\Delta p_i] = 7.36$.

This corresponds to a standard deviation of 2.71 semitones, which is very clearly different from the 0.28 semitones standard deviation observed in our study (see Section V.A).

The other extreme is $\mu = 1$, in which case the original reference pitch is perfectly maintained, and no information is passed on from one note to the next. In our case the reference pitch remains $r^*$ throughout the piece. Given a fixed reference pitch $r^*$, the constant reference pitch model predicts that the variance of the error $t_i - r^*$ remains constant across a recording, which is another way of saying that no drift occurs. To test this prediction, we proceed as follows: we calculate the errors $t_i - r^*$ with respect to the reference $r^*$ (based on the first phrase, as in Section VI.A) and estimate per-note variances across all recordings. We use a linear model with pitch error as a covariate in order to subtract the linear effect of pitch error variances in individual notes. The resulting pitch-error-corrected residuals show a highly significant increase of variance with notes: note number explains 31.3% of the variance ($F(67) = 30.51$, $p < 0.0001$). Over 75 notes, the standard deviation of residuals increases by 0.27 semitones. On these grounds it is very unlikely that a constant reference pitch is used, and we have to reject the boundary model for $\mu = 1$.

Hence, both boundary models are at odds with our observations: one predicts extremely volatile drifts, the other—in its assumption of perfect reference pitch memory—predicts zero drift. The question is then whether a model with an intermediate value of $\mu \in (0, 1)$ will fit the data better.

C. An intermediate memory parameter $\mu$

Having rejected the boundary models for $\mu = 0$ and $\mu = 1$ we are interested in finding whether any intermediate $\mu$ provides a more adequate model. A good model should predict the observed individual note pitches with little error.
Since $r_i$ is meant to represent $t_i = (p_i - s_i)$ up to a note-wise error, as illustrated in Figure 8, it seems plausible that, for some parameter $\mu$ the prediction error can become small. We measure the model’s mean absolute pitch error (model MAPE) with respect to this reference. Figure 9 shows the error on a grid of $\mu$ values (equidistant with hop size 0.01). The best model is achieved for $\mu = 0.85$, leading to a model MAPE of 22 cents, with errors substantially higher towards the extremes of $\mu = 0$ (27 cents) and $\mu = 1$ (29 cents). While the figure shows that the linear model prediction is better ($\text{MAPE} = 19$ cents), only the memory model is psychologically plausible because it is causal, i.e. it does not depend on future events.

We also determined the $\mu$ values that minimise the error on individual recordings and averaged them by singer to obtain singer-wise $\mu$ values. Figure 10 shows a histogram of these singer-wise estimates, which range from $\mu = 0.62$ to $\mu = 0.98$ (mean: 0.832, median: 0.850, std. dev.: 0.105).

The model behaviour in both pitch prediction and spread of drift suggests that a memory model such as the one defined by Equations (8) and (9) is reasonable for values around $\mu = 0.85$.

VII. DISCUSSION AND FUTURE WORK

New knowledge of intonation drift may have implications for practitioners of singing, especially in choirs. Our findings in Section V.A suggest that unaccompanied solo singing without a harmonic context or interaction with other musicians rarely results in significant intonation drift. The median of 11 cents drift observed is not only smaller than the mean absolute error per note, but also in the range of differences of concurrent pitches measured in choirs (10 to 15 cents, according to Ternström and Sundberg (1988)). This adds further evidence to other causes for drift, such as the interaction between temperament and intonation in polyphonic singing (Devaney and Ellis, 2008; Howard, 2007).

In terms of individual singers, the intonation memory model presented in Section VI is particularly interesting because the parameter $\mu$ can reflect the capacity of a singer to stay in tune and that—unlike interval error—is not immediately obvious when a person starts to sing. With three recordings per participant our data has allowed us to study some characteristics of individual singers, but more recordings of individual singers are necessary to refine our models and our understanding of intonation memory. For example, our model is stationary, i.e. it predicts zero long term drift. A non-zero drift term might yield a more realistic model.

For this study we chose to use “Happy Birthday” as our example tune, and while it is the most widely known song among non-professional singers, using only a single melody is an obvious limitation. More different melodies are needed to study intonation behaviour in more detail and with more claim to generality.

While we found that in our study equal temperament was as good a reference grid as just intonation, we hope that further experiments will enable us to infer more precisely the intonation intended by singers.

The analyses carried out in this paper all rely on individual notes as the fundamental musical unit. Future studies will include the temporal development of pitch within the duration of notes (e.g. glide, vibrato) and investigations on the effect of the duration itself.

VIII. CONCLUSIONS

This paper has presented a study on intonation and intonation drift in unaccompanied solo singing. The main focus of the paper was the relations between drift (going out of tune) on the one hand and measured pitch accuracy, different feedback conditions and participants’ self-assessment on the other. Our main finding is that drift is common in solo singing. However, its extent is often small (less than 0.2 semitones over 50 notes) and not correlated to pitch accuracy, interval accuracy, or musical background. Most significant drifts in our particular study are upward drifts.

No significant difference was found between the three different singing conditions Normal, Masked and Imagined, suggesting that in our study, vocal strain and auditory feedback had little impact on the singers’ capability of staying in tune.

Using our findings on solo intonation drift we motivate a causal model of reference pitch memory with a single parameter $\mu$ representing the memory strength. We show that values around $\mu = 0.85$ minimise the model mean absolute pitch error.

The fact that significant drift occurs even in unaccompanied solo singing suggests that tuning changes in more complex situations such as choir singing can partially be accounted for by drift. The small magnitude of drift observed in our study indicates that this is not inconsistent with earlier studies that highlight other causes.
