A charge-insensitive single-atom spin-orbit qubit in silicon

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High fidelity entanglement of an on-chip array of spin qubits poses many challenges. Spin-orbit coupling (SOC) can ease some of these challenges by enabling long-ranged entanglement via electric dipole-dipole interactions, microwave photons, or phonons. However, SOC exposes conventional spin qubits to decoherence from electrical noise. Here we propose an acceptor-based spin-orbit qubit in silicon offering long-range entanglement at a sweet spot where the qubit is protected from electrical noise. The qubit relies on quadrupolar SOC with the interface and gate potentials. As required for surface codes, 105 electrically mediated single-qubit and 104 dipole-dipole mediated two-qubit gates are possible in the predicted spin lifetime. Moreover, circuit quantum electrodynamics with single spins is feasible, including dispersive readout, cavity-mediated entanglement, and spin-photon entanglement. An industrially relevant silicon-based platform is employed.

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In recent years, the coherence and control fidelity of solid-state qubits has dramatically improved[1, 5] and spin qubits[6, 8] with highly desirable properties have been demonstrated.[9] However, many obstacles remain to efficiently entangle a large array of spin qubits on a chip. For example, exchange is inherently vulnerable to decoherence from electrical fluctuations[11, 13], coupling spin to charge noise. Minimizing decoherence and improving control in the face of noise is the key issue for large-scale quantum computing, because it ultimately determines if the error-correction resources can be managed for a large qubit array.[14] Moreover, exchange-based entanglement is inherently short-ranged, making fabrication challenging for gates in quantum dot arrays[6], and placing strict demands on Si:P donor placement.[7]

Here we propose a single-acceptor spin-orbit qubit where the unique properties of hole spins give a host of desirable attributes. First, spin-orbit coupling (SOC) enables long-ranged entanglement via microwave photons or electric dipole-dipole interactions[15, 25], of interest for hybrid quantum systems[26, 30], improving error correction[31], and reducing fabrication demands compared with exchange coupled schemes. Second, and most remarkably, we find a sweet spot where coherence is insensitive to electrical noise and electric dipole spin resonance[32, 34] (EDSR) is maximized. Consequently, coherence and gate timings are protected from electrical noise at the Hamiltonian level, and one- and two-qubit gate times are optimized. In comparison, electric field noise dephases conventional spin-orbit qubits[32–34] and acceptor charge qubits.[23, 37] The coherence of our spin-orbit qubit benefits from reduced hyperfine coupling of holes[36] and 28Si enrichment[39], and has much longer phonon relaxation times than acceptor charge qubits.[23, 37] Finally, the acceptors naturally confine single holes that can be manipulated in silicon nanoelectronic devices.[10]

The exceptional properties of the qubit derive from the quadrupolar SOC[11, 44] contained in the spin-3/2 Luttinger Hamiltonian[45] and in the interaction with the inversion asymmetric interface potential, not studied previously for acceptors. This SOC is unusually strong for acceptors because it acts directly on the low-energy spin manifold, contrasting its indirect role in hole quantum dots.[19, 20, 46–49] The SOC must be considered non-perturbatively to obtain the sweet spot, and the interface strongly enhances EDSR relative to a bulk acceptor. We find 0.2 ns one-qubit gate times, charge-noise immunity, and long phonon relaxation times at the sweet spot, allowing for > 10^5 operations in the coherence time. Two-qubit entanglement based on spin-dependent electric dipole-dipole interactions[15, 17] is feasible with √SWAP times of 2 ns, and 10^4 operations in the coherence time. EDSR also enables circuit quantum electrodynamics[26–30] (cQED) with single-spin dispersive readout, and long distance spin-spin entanglement with √SWAP times of 200 ns. Resonant spin-photon coupling with g_e = 5 MHz is also feasible.

Qubit Concept. The qubit is a hole spin bound to a single Si:B dopant[40, 50, 51], implanted[52] or placed by scanning tunneling microscopy[53, 54] near an interface, in a strained silicon-on-insulator (SOI) substrate (Fig. 1A). The key quadrupolar interactions, associated with interface inversion asymmetry and products {J_i, J_j} = (J_i J_j + J_j J_i) of spin-3/2 matrices where i(j) = x, y, z, originate from strong SOC in the valence band, and have no analog in the conduction band.[11, 44] This SOC acts on the 4×4 ground state manifold |Ψ_m, ω⟩, i.e., the m_J = ±1/2 and m_J = ±3/2 Kramers doublets composed mostly of |J = ±1/2, m_J⟩ Bloch states.[55] For Si:B they are well isolated by ∼ 20 meV from orbital excited states and 46 meV from the valence band edge[56] (Fig. 1B).
The key quadrupolar interactions include the acceptor hole spin-mixing that is linear in electric fields, $H_{\text{electric}} = 2p\sqrt{3}(E_z_J_x, J_y) + c.p.$, associated with $\Gamma_3$ symmetry in the central cell. Here, $p = 0.26$ D is known for Si:B [55] (1 D = 0.021 e-nm). An electric field $E_z$ further breaks the envelope function parity by mixing excited states outside the $\{|\Psi_m\rangle\}$ manifold [55]. Projected into the $\{|\Psi_m\rangle\}$ subspace, this interaction is governed by $H_E = b(J^2_z - \frac{1}{2})E_z^2 + (2d/\sqrt{3})(J_y, J_z)E_yE_z + (J_x, J_z)E_xE_z$, where $b$ and $d$ split and mix the doublets, respectively. We verified that this holds for triangular interface wells, using (i) a Schrieffer-Wolff transformation [55] [60] with higher excited states in the spherical spin-3/2 basis [61], and (ii) numerical, non-perturbative Luttinger-Kohn (LK) calculations with explicit ion and interface well potentials [62, 63]. We find that the level-repulsion gap $\Delta$ is temporarily neglected.

Including magnetic fields, strain $\Delta \propto \epsilon$, and the interface well, but not in-plane electric fields, we find an operating point Hamiltonian,

$$H_{\text{op}} = \begin{pmatrix}
\Delta(E_z) & -i\varepsilon Z & i\frac{\sqrt{3}}{2}\varepsilon Z & -ipE_z \\
-i\varepsilon Z & \Delta(E_z) & \frac{i\sqrt{3}}{2}\varepsilon Z & 0 \\
-i\frac{\sqrt{3}}{2}\varepsilon Z & -ipE_z & 0 & 0 \\
i\frac{\sqrt{3}}{2}\varepsilon Z & 0 & 0 & 0
\end{pmatrix}$$

where $\varepsilon = g_A\mu_B B$ is the Bohr magneton, $g_A = 1.07$ is the Landé g-factor for Si:B [55], and $\Delta(E_z) = \Delta_W(E_z) - \Delta_x$ is the splitting between the light and heavy holes. The cubic g-factor [55] $g_x \ll g_A$ is temporarily neglected.

Inspecting $H_{\text{op}}$, $E_z$ mixes $\{|\Psi_{\pm1/2}\rangle\}$ and $\{|\Psi_{\pm3/2}\rangle\}$ and these states have an avoided crossing when the interface well splitting compensates strain, i.e., $\Delta(E_z) = 0$. In Fig. 1A we show that for appropriate strains $\Delta_x > \Delta_y$, the anti-crossing can be obtained at $E_z \sim 15$ MV/m for $z_0 \sim 5$ nm acceptor depths.

The field $E_z$ at such an anti-crossing is large enough that the level-repulsion gap $\Delta_{\text{gap}} = 2pE_z^2$ exceeds the Zeeman interactions, i.e., $\varepsilon Z/\Delta_{\text{gap}} \sim 0.1$. This unusual aspect of our hole spin-orbit qubit c.f. other proposals [11, 20] follows from the tunability of the spin-3/2 levels with strain and confinement, giving rise to the anti-crossing, and the strength of quadrupolar SOC [55] relative to typical spin qubit Larmor frequencies. We treat the quadrupolar SOC term $pE_z$ by a rotation that maps $pE_z$ exactly to the diagonal, to a basis $\{|\pm\rangle\}$ leaving Zeeman terms $\varepsilon Z$ off-diagonal. We find $\{|\pm\rangle\} = a_L \{|\pm 1\rangle\} \pm i a_H \{|\mp 3/2\rangle\}$, a low-energy Kramers pair with energy $\varepsilon_L = \frac{1}{2}(\Delta - \sqrt{\Delta^2 + 4E_z^2p^2})$, and an excited Kramers pair $|\pm 3/2\rangle = a_L \{|\mp 3/2\rangle\} \mp i a_H \{|\pm 1/2\rangle\}$ with energy $\varepsilon_u = \frac{1}{2}(\Delta + \sqrt{\Delta^2 + 4E_z^2p^2})$. Here, $a_L = \varepsilon_L/\sqrt{E_z^2p^2 + \varepsilon_u^2}$ and $a_H = \sqrt{1 - a_L^2} = E_zp/\sqrt{E_z^2p^2 + \varepsilon_u^2}$. In the basis $\{|\pm\rangle\}$ Eq. 1 becomes

$$H_{\text{op}} = \begin{pmatrix}
\varepsilon_L & \frac{1}{2}\lambda Z_1 & \frac{1}{2}\lambda Z_0 & 0 \\
\frac{1}{2}\lambda Z_1 & \varepsilon_L & 0 & \frac{1}{2}\lambda Z_0 \\
\frac{1}{2}\lambda Z_0 & 0 & \varepsilon_u & \frac{1}{2}\lambda Z_0 \\
0 & \frac{1}{2}\lambda Z_0 & \frac{1}{2}\lambda Z_0 & \varepsilon_u
\end{pmatrix}$$

Here, the Zeeman terms $\lambda Z_1$ depend explicitly on $E_z$ due to the gate-induced mixing of $|\pm 1/2\rangle$ and $|\pm 3/2\rangle$. We find $\lambda Z_1 = 2\varepsilon_z(\sqrt{3}a_{\Sigma}a_H - i\lambda Z_1^0)$, $\lambda Z_2 = 2\varepsilon_z(\sqrt{3}a_{\Sigma}a_H - i\lambda Z_2^0)$, and $\lambda Z_0 = 2\varepsilon_z(-a_{\Sigma}a_H + i\lambda Z_0^0 - i\lambda Z_2^0)$. We perform a final rotation that exactly maps $|\pm 1/2\rangle$ and $|\pm 3/2\rangle$ to the diagonal, leaving $\lambda Z_0$ off-diagonal, defining a basis $\{|\pm\rangle\}$ with $|\pm\rangle = \frac{\sqrt{3}}{2} \{|\pm 1/2\rangle\} \pm i \{|\pm 3/2\rangle\}$ (see Supplemental Material [61]). To zeroth order in $\lambda Z_0/\varepsilon_L$, the splitting of the Kramers pair qubit states $|\pm\rangle$ is $\hbar\omega = |\lambda Z_1|$. When mixed by the gate electric field, the spin 1/2 and spin 3/2 states with different Zeeman terms define a qubit $|\pm\rangle$ where $\hbar\omega$ is maximized (independent of electric fluctuations) and $E_z$ to first order when $|\pm\rangle = \frac{\sqrt{3}}{2} |\Psi_{\pm 1/2}\rangle \pm i(-\frac{1}{2}) |\Psi_{\mp 3/2}\rangle$ (see Supplemental Material [61]).
As we will subsequently show, the qubit is also insensitive to in-plane electric noise $\delta E_{x,y}$, while a similar analysis yields another sweet spot at $E_z = 0$.

Energy levels $\varepsilon_{\pm} = \varepsilon_0 \pm \frac{1}{2} |\lambda_{Z_l}|$ for the qubit are shown alongside excited levels $\varepsilon_{e\pm} = \varepsilon_u \pm \frac{1}{2} |\lambda_{Z_u}|$ for $z_0 = 4.6$ nm (6.9 nm) in Fig. 2A (Fig. 2B). Here, blue (red) hue denotes the amplitude of $a_{\lambda L}$ ($a_{\lambda H}$). The qubit frequency is shown in Fig. 2C and Fig. 2D for approximate (black) and exact (green) solutions to $H_{sp}$, alongside the numerics (squares). The maxima in $\hbar \omega$ in Fig. 2C (Fig. 2D) defines the sweet spot at $E_z = 17$ MV/m (14.8 MV/m), for $|a_{\lambda L}|^2 = 3/4$, as expected. We note that the approximate solution (Fig. 2C.D, black lines) captures the essential behaviour of the analytic model (Fig. 2C.D, green lines). Corrections to Zeeman interactions from interface inversion asymmetry and cubic Landé $g$-factor, although included in the numerics (squares), have been neglected in the analytic model (green). Note that the interface prevents ionization; although $E_z \sim 15$ MV/m is much smaller than silicon’s breakdown field, it well exceeds the ionization field of Si:B.[62]

**In-plane electric fields: EDSR and noise immunity.** We express interactions with in-plane electric fields in the basis $\{|-,+\}, \{e-,+\}$, yielding

$$
\hat{H} = \begin{pmatrix}
\varepsilon_0 & \frac{\hbar \omega}{2} & 0 & \alpha E_1 + \lambda_{Z_1} & \alpha E_2 + \lambda_{Z_2} \\
0 & \varepsilon_0 + \frac{\hbar \omega}{2} & \alpha E_2 + \lambda_{Z_2} & \alpha E_1 + \lambda_{Z_1} & 0 \\
0 & 0 & \alpha E_1 + \lambda_{Z_1} & \alpha E_2 + \lambda_{Z_2} & 0 \\
\alpha E_1 + \lambda_{Z_1} & \alpha E_2 + \lambda_{Z_2} & 0 & \varepsilon_u - \frac{\lambda_{Z_1}}{2} & 0 \\
\alpha E_2 + \lambda_{Z_2} & 0 & \alpha E_1 + \lambda_{Z_1} & 0 & \varepsilon_u + \frac{\lambda_{Z_1}}{2}
\end{pmatrix}.
$$

Here, $|+\rangle$ and $|-\rangle$ are our Kramers pair qubit states, $\lambda_{Z_1} \propto \lambda_{Z_0}$ and $\lambda_{Z_2} \propto \lambda_{Z_0}$ are Zeeman terms, and $E_{1,2}$ are interaction terms with in-plane electric fields, where $E_1 = i(\sin \theta + \eta \cos \theta)E_x + i(\cos \theta + \eta \sin \theta)E_y$, $E_2 = (\cos \theta + \eta \sin \theta)E_x + (\sin \theta - \eta \cos \theta)E_y$, $\theta = \theta_u - \theta_l$, $\lambda_{Z_i} = |E_{Z_i}| \exp(i\theta_i)$, and $\eta = p/\alpha$.

The qubit Hamiltonian $H_{qbt} = \hbar \omega \sigma_z + DE_{xy} \sigma_x$, where $\hbar \omega$ is the qubit frequency (Fig. 2C.D) and $D$ is the EDSR matrix element (Fig. 2E.F), is obtained by projecting the off-diagonal elements of $H$ to first order in $E_{X,Y}$ using a Schrieffer-Wolff transformation.[59,60] Notably, qubit coherence is protected from in-plane electric noise since $\hbar \omega$ contains no terms to first order in $E_{X,Y}$. EDSR drive comes from the transverse coupling $DE_{xy} \sigma_x$ in $H_{qbt}$. We obtain $D = \alpha(\lambda_{Z_0}[(\sin \theta - \eta) - (\sin \theta - \eta)])$, where $E_{xy} = E_{xy} \{\cos \theta || y \sin \theta\}$. Interestingly, the small splitting $\varepsilon_u - \varepsilon_l$ is essential for spin mixing at the sweet spot also causes strong EDSR, since $D \propto (\varepsilon_u - \varepsilon_l)^{-1}$. Note that the EDSR term is dominated by the in-plane inversion asymmetry quadrupolar SOC parameter $\alpha \propto 25$ (Fig. 2F), since it is $100\times$ larger than the bare $T_d$ SOC parameter $p$.

Importantly, $D$ can be maximized at the sweet spot by choosing the angle $\theta_o$ relative to $B||y$ (see Fig. 2E.F). This yields fast gate times, but it also makes $D$, and therefore all timings based on EDSR, insensitive to fluctuations in electric field, protecting gate fidelity from noise at the Hamiltonian level. Since $\eta = p/\alpha \sim 0.01$ and $\theta_o = \pi/4$ at the sweet spot, $D$ is maximized with respect to $\theta || B$ at $\theta || = -\pi/4 \pm \pi/2$. As shown for $z_0 = 4.6$ nm (6.9 nm) in Fig. 2E (Fig. 2F) $D$ is maximized with...
respect to $E_z$ for the same choice $\theta_\parallel$. This result can be easily obtained analytically, and holds for the analytic (green) and numerical (blue squares) solutions.

Qubit Operation. The one-qubit and two-qubit gates employ EDSR-mediated interactions at the sweet spot, where coherence is protected from noise, and their times $\tau$ are minimized and also insensitive to electrical noise. EDSR driven $\pi$ rotations require $\tau_1 = \hbar/(2DE_{AC}) = 1$ ns (0.2 ns) for the $z_0 = 4.6$ nm (6.9 nm) deep acceptor, assuming a modest in-plane microwave field $E_{AC} = 500$ V/cm. A $\pi/2$ (0) phase shift realizes a $\sigma_x$ ($\sigma_z$) gate, and $\sigma_y$ gates can be decomposed into a sequence of $\sigma_x$ and $\sigma_y$ gates. Readout can be accomplished by energy-dependent[55] or spin-dependent[36] tunneling, or dispersive readout in cQED.[20] Initialization can be achieved by projective readout followed by spin rotation.

Two-qubit entanglement can be achieved via long-ranged Coulomb interactions, owing to spin-dependent electric dipole-dipole interactions.[15][17] Their strength is given by $J_{dd} = (\mathbf{v}_1 \cdot \mathbf{v}_2 R^2 - 3(\mathbf{v}_1 \cdot \mathbf{R})(\mathbf{v}_2 \cdot \mathbf{R}))/4\pi R^6$, where $\mathbf{R}$ is the inter-qubit displacement and $\mathbf{v}_i$ is a spin-dependent charge dipole of qubit $i$, which has the same magnitude as the EDSR matrix element. For a 20 nm distance with negligible tunnel coupling, we obtain a $\sqrt{\text{SWAP}}$ time of $\tau_{dd} = \hbar/4J_{dd} \approx 2$ ns with $J_{dd} \approx D^2/4\pi R^3$. The $10^2$ times enhancement of EDSR from the interface reduces $\tau_{dd}$ by $10^4$ relative to acceptors in bulk silicon, and $10^5$ relative to bare magnetic dipole-dipole coupling. Entanglement by Heisenberg exchange is also possible and exchange is hydrogenic when $\Delta$ exceeds $J_c$.[51] We note that the advantage that holes do not have valley degrees of freedom[70] which may complicate Heisenberg exchange for electrons in Si.[71]

Circuit QED. Coplanar superconducting microwave cavities could be used to implement cQED including two-qubit gates, dispersive single-spin readout, and strong Jaynes-Cummings coupling on resonance with the cavity.[20][27][29] We assume a coplanar waveguide resonator operating at $B = 0.5$ T ($f = 15$ GHz) and a vacuum electric field $E_0 \approx 50$ V/m. This can be obtained using a tapered resonator gap, or a superconducting nanowire resonator.[72] At the sweet spot for $z_0 = 4.6$ nm (6.9 nm), the vacuum Rabi coupling is $g_c = cDE_0 = 2$ neV (10 neV).

For cavity mediated non-demolition readout and qubit coupling, we detune the qubit from the cavity by $\Delta = 4g_c$.[22] Here, the spin state shifts the cavity resonance by $\Delta f = g_c^2/\Delta = 0.25$ MHz (1.25 MHz) for $z_0 = 4.6$ nm (6.9 nm). The two-qubit $\sqrt{\text{SWAP}}$ time is $\tau_{dd} = \hbar/4J_e$, determined by the effective spin-spin interaction.[22] $J_e = g_c^2/\Delta = 2.5$ MHz. Operating at zero detuning, spin/photons Rabi oscillations require $\hbar \pi g_c = 1$ µs (200 ns). Assuming $Q = 10^5$ at $B_0 = 0.5$ T in state-of-the-art superconducting cavities,[72][73] $g_c\kappa = 6.7$ (33) Rabi cycles can be obtained for $z_0 = 4.6$ nm (6.9 nm), where $\kappa = f/Q$ is the cavity loss rate.

Relaxation and Dephasing. We consider spin-lattice (phonon) relaxation and dephasing from a host of electrical noise sources, and compare them to gate times. Since silicon is not piezoelectric, spin-lattice relaxation occurs only via the deformation potential[74][75]. For temperatures $T < \hbar \omega_c/k_B$, the spin relaxation time derived in the Supplemental Material[64] follows $T_1^{-1} = (\hbar \omega_c)^3(C_d/20\pi\hbar h^4)|\Delta Z_{00}/\Delta|^2$, where $|\Delta Z_{00}/\Delta| = \omega_c/4E_z$ at the sweet spot, and $C_d = 4.9 \times 10^{-20}$ (eV)$^2$(s/m)$^5$. We obtain $T_1 = 20$ µs (5 µs) for $z_0 = 4.6$ nm (6.9 nm) at $B_0 = 0.5$ T that are 100 times longer c.f. bulk unstrained silicon at $B = 0.5$ T.[23][57]

Random fluctuations in qubit splitting $\hbar \omega(t)$ dephase the qubit. The dephasing rate from random telegraphic signal (RTS) in charge trap occupation is $(T_2^\ast)^{-1} = (\delta \omega)^2 T_S/2$, where $\delta \omega$ is qubit frequency shift, and $T_S$ is the average switching time. We take $T_S = 10^4$ ns as the worst case, since slower fluctuations can be suppressed by dynamical decoupling. Assuming a trap 50 nm away, we find $4E \sim 2,000$ V/m and a large window of 200,000 V/m (20,000 V/m) of gate space where $T_2^\ast > 2T_1$ at the sweet spot, for $z_0 = 4.6$ nm (6.9 nm). In comparison, the same analysis gives $T_2^\ast \sim 0.1$ ns for acceptor-based charge qubits with similar gate times. It is remarkable that in comparison, electrical noise has virtually no effect on coherence in our spin-orbit qubit, illustrating the advantages of inversion asymmetry and our spin-orbit qubit’s sweet spot. We also find that dephasing from Johnson-limited gate voltage noise, and from two-level (tunneling) systems (TLS), are $\sim 10^7$ and $\sim 10^4$ times weaker, respectively, compared with RTS.[36] There are only a few spin resonance experiments on acceptors,[58][76][51], none of which feature strain and an interface. We expect hyperfine-induced decoherence in $^{30}$Si to be weak since it has only 4.7 % of spin-bearing isotopes and hyperfine interactions are weaker for holes than electrons.[38] Meanwhile, $^{28}$Si enrichment could be used to virtually eliminate the nuclear bath.[39]

The insensitivity to Johnson noise and tunneling TLS means spin-lattice $T_1$ limits coherence for few (or slow enough) traps at Si/SiO$_2$ interfaces. For $B = 0.5$ T, $r_1 > 10^4$ single qubit gates, $r_{2dd} > 10^3$ dipole-dipole two-qubit gates, and $r_2c \approx 25$ cavity-mediated two-qubit gates can be achieved in a $T_1$ limited coherence time. Therefore while $T_1$ is short compared to donors, many gate operations can performed. Since $T_1 \propto \omega^{-5}$, choosing $B = 0.25$ T increases all ratios favourably to $r_1 > 10^5$, $r_{2dd} \approx 10^4$, and $r_2c \approx 50$. Since $T_1$ is much longer at the $E_z = 0$ sweet spot, adiabatically sweeping to $E_z = 0$ opens a pathway for a long-lived quantum memory.

Conclusions. The proposed single-acceptor spin-orbit qubit exploits the tunability of the $J = 3/2$ manifold of acceptors and the associated quadrupolar SOC arising from the ion and interface potential, providing for (i) fast one-qubit and long-ranged two-qubit gates (ii) at a sweet spot where the qubit phase and all gate timings are in-
sensitive to electrical fluctuations, (iii) avoiding entirely the need for exchange interactions, (iv) in an industrially relevant silicon platform. $10^5$ single-qubit and $10^4$ two-qubit gates could be possible in the qubit coherence time. Using cQED, dispersive single-spin readout, cavity-mediated spin-spin entanglement, and Jaynes-Cummings spin-photon entanglement are possible.

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**APPENDIX**

**INTERACTIONS WITH MAGNETIC FIELDS AND STRAIN**

Interactions of acceptor-bound holes with magnetic fields and strain are known for acceptor dopants in bulk silicon. In the $|\Psi_{m_j}\rangle$ subspace, interactions with magnetic fields $B = \hat{x}B_x + \hat{y}B_y + \hat{z}B_z$ are represented by the Hamiltonian

$$H_Z = \mu_B(g_1(J_xB_x + c.p.) + g_2(J_yB_y + c.p.)) \quad (A1)$$

Here, $J_x$ are $J = 3/2$ matrices, c.p. refers to cyclic permutations, $g_1$ and $g_2$ are the linear and cubic Landé $g$-factors, and $\mu_B$ is the Bohr magneton. Interactions with strain $\epsilon_{ij}$ are represented by the Hamiltonian

$$H_s = a'd'Tr[e] + b'((J_z^2 - \frac{3}{2}I)\epsilon_{xx} + c.p.) + (2d'/\sqrt{3})((J_xJ_y\epsilon_{xy} + c.p.), \quad (A2)$$

where $\{J_x, J_y\} = \frac{1}{2}(J_xJ_y + J_yJ_x), d', b'$ and $d'$ are Bir-Pikus deformation potentials.

**ACCEPTOR STATES IN SPHERICAL SPIN-3/2 BASIS**

In the spherical spin-3/2 basis $|L, J; F, m_F\rangle$, where $F = L + J$ is an effective total angular momentum, acceptor eigenstates take the form

$$|\Psi_{m_j}\rangle = f_0(r)|L = 0, J = \frac{3}{2}; F = \frac{3}{2}, m_F\rangle + g_0(r)|L = 2, J = \frac{3}{2}; F = \frac{3}{2}, m_F\rangle \quad (A3)$$

where $J = \frac{3}{2}$ is an effective total spin, and the spin-3/2 spin-orbit interaction has coupled states with $\Delta L = 0, \pm 2$. The $f_0(r)$ and $g_0(r)$ are radial envelope wavefunctions for envelope function spherical harmonics with $L = 0$ and $L = 2$, respectively. The $|L, J; F, m_F\rangle$ are found using the Clebsch-Gordan coefficients.

States outside the $4 \times 4$ subspace, given in ref. [61], contribute to envelope function asymmetry to realize the $b$ and $d$ terms in $H_E$ from the main text. The Schrieffer-Wolff calculation mixing in these higher excited states will be presented in a future paper.

**ANALYTIC MODEL FOR LOW ENERGY STATES**

The low-energy holes are described in the $|\Psi_{m_j}\rangle$ basis by

$$H_{op} = \begin{pmatrix}
0 & -i\sqrt{2}\epsilon_Z & -ipE_z & 0 \\
\epsilon_Z & -i\epsilon_Z & -ipE_z & 0 \\
pE_z & i\epsilon_Z & -i\epsilon_Z & 0 \\
0 & i\epsilon_Z & -i\epsilon_Z & 0
\end{pmatrix} \quad (A4)$$

for $m_j = \frac{3}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}$, where $\epsilon_Z = g_1\mu BR$, and $g_1 = 1.07$ for B:Si. The unitary transform $U_0$ that diagonalizes $H_{op}(\epsilon_Z = 0)$ is

$$U_0 = \begin{pmatrix}
-i\alpha_H & 0 & 0 & a_L \\
0 & a_L & i\alpha_H & 0 \\
a_L & 0 & 0 & -i\alpha_H \\
0 & i\alpha_H & a_L & 0
\end{pmatrix} \quad (A5)$$

in the basis $\{|\Psi_{+3/2}\rangle, |\Psi_{+1/2}\rangle, |\Psi_{-1/2}\rangle, |\Psi_{-3/2}\rangle\}$.

Here, $a_L = \epsilon_1/\sqrt{E_{Zp}^2 + \epsilon_1^2}$ and $\alpha_H = \sqrt{1 - a_L^2} = E_Zp/\sqrt{E_{Zp}^2 + \epsilon_1^2}$. Applying this transformation we obtain an extended qubit Hamiltonian for the spin qubit states and nearest excited spin states

$$\tilde{H}_{op} = \begin{pmatrix}
\epsilon_1 & \frac{1}{2}\lambda_{Zl} & \frac{1}{2}\lambda_{Zl} & 0 \\
\frac{1}{2}\lambda_{Zl} & \epsilon_1 & 0 & \frac{1}{2}\lambda_{Zl} \\
0 & \frac{1}{2}\lambda_{Zl} & 0 & \epsilon_u \\
0 & \frac{1}{2}\lambda_{Zl} & \frac{1}{2}\lambda_{Zl} & \epsilon_u
\end{pmatrix} \quad (A6)$$

in the basis $\{|-\rangle, |+\rangle, |u_-\rangle, |u_+\rangle\}$. Here, the effective Zeeman interactions $\lambda_{Zl}$ appear on the off-diagonal. The effective Zeeman interactions depend explicitly on the gate electric field due to the $pE_z$-induced mixing of $|\Psi_{\pm 1/2}\rangle$ and $|\Psi_{\pm 3/2}\rangle$. These off-diagonal Zeeman interactions are

$$\lambda_{Zl} = 2\epsilon_Z(3a_La_H - i\alpha_H^2), \quad (A7)$$

$$\lambda_{Zu} = 2\epsilon_Z(3a_La_H - i\alpha_H^2), \quad (A8)$$

$$\lambda_{Zo} = 2\epsilon_Z(-a_La_H + \sqrt{3}a_L^2/2 - i\sqrt{3}a_H^2/2). \quad (A9)$$

The upper and lower $2 \times 2$ blocks of $\tilde{H}_{op}$ are diagonalized by

$$U_{Z0} = 2^{-\frac{1}{2}}\begin{pmatrix}
+e^{-i\theta/2} & +e^{-i\theta/2} & 0 & 0 \\
-e^{i\theta/2} & +e^{i\theta/2} & 0 & 0 \\
0 & 0 & +e^{-i\theta_u/2} & +e^{i\theta_u/2} \\
0 & 0 & -e^{i\theta_u/2} & +e^{i\theta_u/2}
\end{pmatrix} \quad (A10)$$
Applying this transformation we obtain the following extended Hamiltonian
\[
\tilde{H}_{\text{op}} = \begin{pmatrix}
\varepsilon_l - \frac{\lambda_{2z}}{2} & 0 & \lambda_{2z1} & \lambda_{2z2} \\
0 & \varepsilon_l + \frac{\lambda_{2z}}{2} & \lambda_{2z2} & \lambda_{2z1} \\
\lambda_{2z1} & \lambda_{2z2} & \varepsilon_u - \frac{\lambda_{2z}}{2} & 0 \\
\lambda_{2z2} & \lambda_{2z1} & 0 & \varepsilon_u + \frac{\lambda_{2z}}{2}
\end{pmatrix}
\]
(A13)
in the basis \{|-\rangle, |+\rangle, |e-\rangle, |e+\rangle\}, where,
\[
\lambda_{2z1} = \frac{1}{2}\lambda_{Zo} \cos(\theta_l/2 - \theta_u/2 - \theta_o),
\]
\[
\lambda_{2z2} = \frac{1}{2}\lambda_{Zo} i \sin(\theta_l/2 - \theta_u/2 - \theta_o),
\]
\[
\theta_o = \arctan(\sqrt{3} a_L^2 - a_H^2, a_H a_L).
\]
(A14)
(A15)
(A16)

Our approximate qubit model from the main text takes \(\lambda_{Zo}/(\varepsilon_u - \varepsilon_l)\) to zeroth order to give qubit states |\text{-}\rangle and |\text{+}\rangle. In this approximation we obtain a qubit frequency
\[
\hbar \omega = |\lambda_{Z|} = 2\varepsilon_Z \sqrt{3a_L^2 a_H^2 + a_H^2}. \]
(A17)

A sweet spot occurs when the qubit frequency is insensitive to small fluctuations in electric fields. For an acceptor experiencing no static external applied field along \(x\) and \(y\) directions, this occurs for the roots of
\[
\partial \hbar \omega/\partial E_z = 2\varepsilon_z (\partial a_L/\partial E_z) (3 - 4a_L^2)/\sqrt{3 - 2a_L^2}.
\]
(A18)

One root (sweet spot) is \((a_L, a_H) = (-\sqrt{3}, \frac{1}{2})\). Substituting \(\varepsilon_l\) and \(\Delta(E_z)\), we obtain an equivalent condition
\[
\Delta_G(E_z) + \Delta_h + 2p/\sqrt{3E_z} = \Delta_e
\]
(A19)

for this sweet spot. Another root (sweet spot) occurs at the roots of
\[
\partial a_L/\partial E_z = E_z^2 p^2 (\partial \varepsilon_l/\partial E_z) (\varepsilon_l^2 + E_z^2 p^2)^{-3/2},
\]
(A20)

The sweet spot associated with the above root occurs at \(E_z = 0\). The lowest order correction to \(\hbar \omega\) due to coupling to levels \(|e\pm\rangle\) is easily obtained from 2nd-order perturbation theory,
\[
\delta \hbar \omega = -\frac{1}{4} \left( \frac{\lambda_{Zo}}{\varepsilon_l - \varepsilon_u} \right)^2 (|\lambda_{Z|} - |\lambda_{Zu}| \cos(2\theta_o - \theta_l + \theta_u)).
\]

The exact solution in (main text, Fig. 2C) shows that all higher order corrections (including \(\hbar \omega^{(2)}\)) to the approximate solution presented in the main text do not qualitatively modify the qubit frequency.

**ELECTRIC DIPOLE SPIN RESONANCE**

The total interaction with in-plane electric fields \(E_x\) and \(E_y\) described by \(H_E\) and \(H_{E,\text{ion}}\) is
\[
H_{E||} = \alpha(E_z) \begin{pmatrix} 0 & E_- & 0 & 0 \\ E_+ & 0 & 0 & 0 \\ 0 & 0 & 0 & -E_- \\ 0 & 0 & -E_+ & 0 \end{pmatrix} + p \begin{pmatrix} 0 & -iE_+ & 0 & 0 \\ iE_- & 0 & 0 & 0 \\ 0 & 0 & 0 & +iE_+ \\ 0 & 0 & -iE_- & 0 \end{pmatrix}, \quad (A22)
\]
in the basis \(\{|\Psi_{+2/3}\rangle, |\Psi_{+1/2}\rangle, |\Psi_{-1/2}\rangle, |\Psi_{-3/2}\rangle\}\), where \(E_+ = E_x + iE_y\) and \(E_- = E_x - iE_y\). The first matrix is the coupling due to the broken inversion symmetry of the interface and gate field, along the \(z\) direction, while the second matrix describes the interaction due to the \(T_2\) symmetry of the local field of the ion. Rotated into the qubit basis using \(U_{i0} = U_{Zo} U_0\), we obtain
\[
\tilde{H} = \begin{pmatrix} \varepsilon_l - \frac{\lambda_{2z}}{2} & 0 & \alpha E_1 + Z_1 & \alpha E_2 + Z_2 \\ 0 & \varepsilon_l + \frac{\lambda_{2z}}{2} & \alpha E_2 + Z_2 & \alpha E_1 + Z_1 \\ \alpha E_1^* + Z_1^* & \alpha E_2^* + Z_2^* & \varepsilon_u - \frac{\lambda_{2z}}{2} & 0 \\ \alpha E_2^* + Z_2^* & \alpha E_1^* + Z_1^* & 0 & \varepsilon_u + \frac{\lambda_{2z}}{2} \end{pmatrix}, \quad (A23)
\]
in the basis \(\{|\Psi_{-2/3}\rangle, |\Psi_{-1/2}\rangle, |\Psi_{1/2}\rangle, |\Psi_{2/3}\rangle\}\), where, \(E_1 = i(\sin \theta + \eta \cos \theta) E_x + i(\cos \theta + \eta \sin \theta) E_y\), \(E_2 = (-\cos \theta + \eta \sin \theta) E_x + (\sin \theta - \eta \cos \theta) E_y\), \(\theta = \theta_u - \theta_l\), and \(\eta = p/\alpha\).

EDSR coupling \(D E_{||}\sigma_x\) in the qubit subspace arises from oscillating in-plane electric fields \(E_{||} = E_{||}(x \cos \theta || + y \sin \theta)\). We obtain the effective EDSR interactions \(D E_{||}\sigma_x\) in the qubit basis using a Schrieffer-Wolff transformation. Expanding in \(\eta = |\lambda_{Z|}/(\varepsilon_l - \varepsilon_u)\) and \(\eta_u = |\lambda_{Zu}|/(\varepsilon_l - \varepsilon_u)\) and grouping terms according to powers in the electric fields \(E_1\) and \(E_2\), we obtain
\[
D = 2|\lambda_{Zu}| Re(\lambda_{Z1}^* \lambda_{Z2}) / (\varepsilon_l - \varepsilon_u)^2 + \alpha Re(E_{||}^2 \lambda_{Z1} + E_{||}^2 \lambda_{Z2}) / (\varepsilon_l - \varepsilon_u) + \alpha^2 2|\lambda_{Zu}| Re(E_{||}^2 \lambda_{Z1}) / (\varepsilon_l - \varepsilon_u)^2. \quad (A24)
\]
The term proportional to \(E_{||}^2 \lambda_{Z1} + E_{||}^2 \lambda_{Z2}\) is linear in the electric field and defines the electric dipole spin resonance term. Substituting \(\lambda_{Z1}, \lambda_{Z2}, E_1\) and \(E_2\) we obtain the EDSR matrix element
\[
D = \alpha |\lambda_{Zo}| \cos(\theta_o - \theta_||) + \eta \sin(\theta_o - \theta_||)). \quad (A25)
\]
**SPIN-DEPENDENT DIPOLE-DIPOLE INTERACTION**

Because of the spin-orbit interaction, the electric dipole moment in each acceptor couples to spin. As a result, two qubits interacting only via mutual Coulomb repulsion experience a spin-dependent interaction resembling a magnetic dipole-dipole interaction. Here we determine this interaction in the coupled-qubit subspace $|−−⟩$, $|--⟩$, $|+−⟩$, $|++⟩$. For the total Hamiltonian we have $H_{\Sigma} = H_{\text{op}}^{i} + H_{\text{op}}^{2} + V_{12}^{1}$, where $H_{\text{op}}^{i}$ is the single-acceptor Hamiltonian for qubit $i = 1, 2$, and $V_{12}(r_{1} - r_{2}) = e^{2}/4\pi\epsilon|r_{1} - r_{2}|$ is the electrostatic interaction between the qubits.

We work in the tensor $16 \times 16$ tensor product subspace of the two qubits $|mn⟩ = |m^{1}\rangle \otimes |n^{2}\rangle$ where $m \in \{-+, +-, u-, u+\}$ and $n \in \{-+, +-, u-, u+\}$, explicitly ignoring anti-symmetrization, i.e., assuming spatial overlaps are negligible. Without the Coulomb interaction, the Hamiltonian is

$$\langle mn|H_{\text{op}}^{i} + H_{\text{op}}^{2}|m'n'\rangle = \langle m|H_{\text{op}}^{i}|m'\rangle \delta_{mm'} + \langle n|H_{\text{op}}^{2}|n'\rangle \delta_{nn'},$$

(A26)

where $H_{\text{op}}^{i}$ is given by Equation (A13). Meanwhile, in the direct product subspace the two-qubit Coulomb interaction is

$$\langle mn|V_{12}^{12}|m'n'\rangle = \int dr_{1}^{3} dr_{2}^{3} e^{2}\Psi_{m}^{i}(r_{1})\Psi_{n}^{i}(r_{2})\Psi_{m'}^{i}(r_{1})\Psi_{n'}^{i}(r_{2}) 4\pi\epsilon|r_{1} - r_{2}|,$$

(A27)

When the separation $R_{12}$ between the acceptors is large compared to dipole moments $\langle \delta r_{i} \rangle$ of the system, we may use the multi-pole expansion of the Coulomb interaction in Equation (A27). The lowest-order non-zero term is

$$\langle mn|V_{12}^{12}|m'n'\rangle = \frac{e^{2}}{4\pi\epsilon R_{12}^{3}} \left[ R_{12}^{2} \langle \delta r_{1} | m' n' \rangle, \langle \delta r_{2} | m'n' \rangle - 3 \langle \delta r_{1} | m'n' \rangle \cdot \langle \delta r_{2} | m'n' \rangle \right]$$

(A28)

where

$$\langle \delta r_{i} | m'n' \rangle = \int dr_{i}^{3} (r_{i} - R_{i})\Psi_{m}^{i}(r_{i})\Psi_{n}^{i}(r_{i}).$$

(A29)

$R_{i}$ is the position of qubit ion $i$, and $R_{12} = R_{1} - R_{2}$. The Coulomb interaction is now a product of single-hole dipole matrix elements known in the basis $\{−+, +, u−, u+\}$ from Equation (A23). The total Hamiltonian is

$$\langle mn|H_{\Sigma}^{i} |m'n'\rangle = \langle m|H_{\text{op}}^{i}|m'\rangle \delta_{mm'} + \langle n|H_{\text{op}}^{2}|n'\rangle \delta_{nn'} + \langle mn|V_{12}^{12}|m'n'\rangle,$$

(A30)

The combined effect of the Coulomb and Zeeman interactions can be projected into the coupled-qubit subspace $|−−⟩$, $|--⟩$, $|+−⟩$, $|++⟩$ using a Schrieffer-Wolff transformation. Working out the effective interaction, to second order in off-diagonal terms and zeroth order in off-diagonal terms and zeroth order in Fermi’s golden rule, $\hbar\omega_{qs} = \hbar\nu_{qs}$ can be determined from Fermi’s golden rule,

$$\frac{1}{T_{n \rightarrow n'}} = \frac{2\pi}{\hbar} \sum_{i, j, s, q} | \langle n', n_{q} + 1|H_{eis}|n, n_{q} \rangle |^{2} \times \delta(E_{n} - E_{n'} - \hbar\omega_{qs}).$$

(A39)

**PHONON-INDUCED SPIN RELAXATION**

The relaxation from $|n'\rangle$ to $|n\rangle$ via emission of a phonon with energy $\hbar\omega_{qs} = \hbar\nu_{qs}$ can be determined from Fermi’s golden rule,
where $s = \ell, t_1, t_2$ are the phonon polarizations, $q_s$ is the phonon wavevector, $\sum_{i,j} H_{\ell_{ij}} = \sum_{i,j} D_{ij}\epsilon_{\ell_{ij}}$ is the electron-phonon interaction, and $n_q$ is the phonon population. The deformation potential matrices $D_{ij}$ are determined from the Bir-Pikus Hamiltonian

$$\sum D_{ij}\epsilon_{\ell_{ij}} = a'(\varepsilon_{xxs} + \text{c.p.}) + b'[(J_z^2 - \frac{5}{12}I)\varepsilon_{xxs} + \text{c.p.}] + (2d'/\sqrt{3})[(J_x, J_y)\varepsilon_{xy} + \text{c.p.}]$$  \hspace{1cm} (A40)

where $a', b'$, and $d'$ are Bir-Pikus deformation potentials \cite{BirPikus} and the strain $\epsilon_{\ell_{ij}} = \frac{1}{2}(\partial\delta R_s/\partial r_J + \partial\delta R_s/\partial r_s)$ of the phonon polarization $s$ is determined by the displacement \cite{BirPikus}

$$\delta R_s = (-i)\sqrt{\frac{h}{2N\rho\omega_{qs}}} \hat{e}_{qs}(a^\dagger_{qs} + a_{qs})\exp(iq_s\cdot r),$$ \hspace{1cm} (A41)

where $\hat{e}_{qs}$ is the normalized phonon polarization vector \cite{BirPikus}. $N$ is the number of unit cells, $V$ is the unit cell volume, $N V = L^3$ is the crystal volume, $\rho$ is the mass density, and $\sigma_{qs}$ creates (destroys) a phonon of wavevector $q_s$ and polarization $s$. For $\langle n'|D_{ij}\exp(iq\cdot r)|n\rangle$ we use the dipole approximation $\langle n'|D_{ij}(1 + iq\cdot r + \ldots)\rangle \approx \langle n'|D_{ij}|n\rangle$, which is appropriate since $qa \sim 10^{-2}$ where $q$ is the phonon wavevector and $a \sim 1$ nm is the Bohr radius. At low temperatures $T \ll h\omega/k \approx 0.7$ K we obtain

$$\frac{1}{T_1} = \frac{(h\omega)^3}{20h^2\pi \rho} \left[ \sum_i |\langle n'|D_{ii}|n\rangle|^2 \left( \frac{2}{v^2_{f}} + \frac{4}{3v^2_{f}} \right) \right. + \left. \sum_{i\neq j} |\langle n'|D_{ij}|n\rangle|^2 \left( \frac{2}{3v^2_{f}} + \frac{1}{v^2_{f}} \right) \right]$$ \hspace{1cm} (A42)

where $v_f$ and $v_l$ are the longitudinal and transverse sound velocities. For our qubit, the relaxation from $|+\rangle$ to $|-\rangle$ can be evaluated using a Schrieffer-Wolff transformation of the non-diagonal elements of $\hat{H} = \hat{H}_{\text{op}} + \hat{H}_{\text{eij}},$ where

$$\hat{H}_{\text{eij}} = U_{10}^\dagger(\sum_{i,j} H_{\ell_{ij}}\epsilon_{\ell_{ij}})U_{10}. \quad \text{We determine the coupling to lowest order in } \hat{H}_{Z_0} \text{ and } \hat{H}_{\text{eij}}, \text{ while treating }$$

$$\hat{H}_{E,\text{ion}}, \hat{H}_d \text{ and } \hat{H}_E \text{ exactly within the } 4 \times 4 \text{ subspace).} \quad \text{We obtain}

$$\frac{1}{T_1} = \frac{(h\omega)^3}{20h^2\pi \rho} \left( \frac{h\omega}{4pE_z} \right)^2 \left[ \frac{9}{32} b^2 \left( \frac{2}{v^2_{f}} + \frac{4}{3v^2_{f}} \right) + \frac{5}{16} d'^2 \left( \frac{2}{3v^2_{f}} + \frac{1}{v^2_{f}} \right) \right]$$ \hspace{1cm} (A43)

at the sweet spot. For comparison, we also evaluated the phonon-mediated transition rate from the first excited level $|-\frac{1}{2}\rangle$ to the ground state $|-\frac{3}{2}\rangle$ in bulk unstrained silicon, which is allowed even in zero magnetic field, since $|-\frac{1}{2}\rangle$ and $|-\frac{3}{2}\rangle$ are not time-reversal symmetric. For $B_0$ along [001] directions we obtain

$$\frac{1}{T_1} = \frac{(h\omega)^3}{20\rho\pi h^4} \left( \frac{1}{v^2_{f}} + \frac{2}{3v^2_{f}} \right) 2d'^2,$$ \hspace{1cm} (A44)

in agreement with the low-temperature result in ref. \cite{BirPikus}.

### DEPHASING FROM ELECTRIC FIELD FLUCTUATIONS

Dephasing of a spin-orbit qubit occurs due to random fluctuations $h\delta\omega(t)$ in the qubit level energy splitting. In this section we outline expected dephasing rates associated with realistic parameters for the qubit.

#### Charge noise from trap charging/discharging

First, we estimate the dephasing due to a single fluctuating charge trap (random telegraph signal), assuming that a charge trap can only fluctuate when it is in tunneling proximity to carrier reservoir or gate. The dephasing rate is given by $(T_2^\ast)^{-1} = (\delta\omega)^2\tau_{S}/2$, where $\delta\omega$ is the qubit energy shift when the trap is charged, and $\tau_{S}$ is the switching time \cite{Kane}. Together with its image in the gate, a dipole $\ell K_d$ is created, resulting in a dipole potential $V_d = \ell K_d/\pi r^2 \approx 2\pi e^3 c^2 h_{\omega}z^3/\sqrt{2}$ when the defect is a distance $R = x\hat{x} + z\hat{z}$ away from the acceptor qubit. The effective field for the dipole potential is $\vec{E} = -\nabla V_d = x\hat{x} + z\hat{z}$.

The change $\delta\hat{\omega}$ of the Larmor frequency is readily calculated from the extended qubit Hamiltonian (Eq. \ref{eq:A23}), where $\delta E_{x,y}$ couples off-diagonally through the $aE_{x,y} + \lambda_{x\lambda_{x}}$ terms, and $\delta E_z$ on-diagonally via the explicit dependence of $\Delta(E_z) \text{ and } |\lambda_{x\lambda_{x}}(E_z)|$ on $E_z$. Equivalently, the $2 \times 2$ qubit model can be used, and it is easy to show that

$$\delta\omega \approx \omega(E_z + \delta E_z) - \omega(E_z) + \frac{2D^2\delta E^2_{x,y}}{\hbar^2\omega(E_z)}.$$ \hspace{1cm} (A45)

For the estimate of dephasing in the main text we assumed $x = 50 \text{ nm}$, and $z = 5 \text{ nm}$ giving $\delta E_x \approx 600 \text{ V/m}$ and $\delta E_z \approx 2000 \text{ V/m}$.

### Gate electric field noise

Gate electric field noise is modeled as a white Johnson voltage noise $v_n(t)$ process applied across the gates producing a randomly fluctuating field of order $E(t) \approx v_n(t)/d_g$ where $d_g \approx 20 \text{ nm}$ is the shortest envisioned distance between gates. The dephasing rate is given by $(T_2^\ast)^{-1} = S(\omega = 0)$ where $S(\omega)$ is the power spectral density of the random process $\delta\omega(t)$. Further approximation to $\delta\omega$ from above gives

$$\delta\omega(t) \approx \left( \frac{\partial\omega(E_z)}{\partial E_z} \right) \frac{v_n(t)}{d_g} + \frac{2D^2}{\hbar^2\omega} \left( \frac{v_n(t)}{d_g} \right)^2.$$ \hspace{1cm} (A46)

We find that the first term is much larger for our qubit and dominates the power spectral density of $\delta\omega(t)$,

$$S(\omega) \approx \left( \frac{\partial\omega(E_z)}{\partial E_z} \right)^2 \frac{1}{d_g^2} S_{\nu\nu}(\omega).$$ \hspace{1cm} (A47)
The quantity $\partial \omega(E_z)/\partial E_z$ is determined from our analytic model. Substituting $S_{e\nu}(\omega) = 4k_B T R$ as the white noise power spectrum, $T = 1$ K and $R = 50$ ohms, we find that the dephasing rate $(T_d^*)^{-1}$ due to intrinsic Johnson voltage noise on the gate is $10^7$ times smaller than the dephasing rate from charge noise.

**NUMERICAL KOHN LUTTINGER CALCULATIONS**

All theory predictions in the main text were compared with a numerical solution of the acceptor Hamiltonian including the full spatial dependence of $H(k)$. The Hamiltonian $H = H_{LK} + H_{E,ion}$, where $H_{LK} = H(k) + H_e + H_{ion}(r)$, was computed in the $6 \times 6$ representation of valence band Bloch states $|J,m_j\rangle$,[45], $|\psi_{m_j}\rangle = \sum_{J,m_j} F_{J,m_j}(r) |J,m_j\rangle$, where $F_{J,m_j}(r)$ are envelope functions. The first step is to numerically diagonalize $H_{LK}$ using $H(k) = \Psi$ in the basis $|J,m\rangle$.

$$P + Q - \sqrt{2} R(0, \sqrt{2} S') P - Q 0 - \sqrt{2} R(0, \sqrt{2} S') R^t 0 0 0 P + D_{SO}$$

in the basis $|\frac{3}{2}, \frac{3}{2}\rangle, |\frac{3}{2}, \frac{1}{2}\rangle, |\frac{3}{2}, -\frac{1}{2}\rangle, |\frac{3}{2}, -\frac{3}{2}\rangle, |\frac{1}{2}, \frac{1}{2}\rangle, |\frac{1}{2}, -\frac{1}{2}\rangle$, with:

$$P = \frac{1}{2m_0}\gamma_1(k_z^2 + k_y^2 + k_z^2),$$

$$Q = \frac{1}{2m_0}\gamma_2(k_z^2 + k_y^2 - 2k_z^2),$$

$$R = \frac{1}{2m_0}\sqrt{3(-\gamma_2(k_z^2 - k_y^2) + 2\gamma_3 k_z k_y)},$$

$$S = \frac{1}{2m_0}2\sqrt{3}\gamma_3(k_x - ik_y)k_z,$$

$$H_{ion}(r) = -e^2/4\pi\epsilon_r, H_e = -eE_z z, H_{id} = U_0 \Theta(-z)$$

The states $|\Psi_{LK}\rangle$ experience inversion asymmetry due to explicit presence of the interface and gate field. We have verified that inversion asymmetry-induced mixing of excited states into the $4 \times 4$ subspace does not modify the symmetry of $H_E$ from the form in the main text, for different acceptor depths, by evaluating $H_{E} = \langle \Psi_{m_j,LK} | e\delta E \cdot r | \Psi_{m_j,LK} \rangle$ using eigenstates $|\Psi_{LK}\rangle$ calculated non-perturbatively, that is, with applied potentials including the ion, interface, and gate electric field. This was checked for all static fields $E_z$ in the main text, and for fields $\delta E$ for all three orthogonal directions. Values for $\Delta_{id} + \Delta(E_z)$ were obtained from eigen-energies, while values of $\alpha$ were obtained from the off-diagonal elements coupling the LH and HH blocks.

**Larmor frequency and EDSR: numerical results**

The LH-HH mixing caused by $p$ in $H_{E,ion}$ changes the Larmor frequency $\omega$ of our spin qubit $|\pm\rangle$ dramatically compared with $|\Psi_{LK}\rangle$. Numerical calculations of $\omega$ (Fig. 2C,D, blue squares) are in agreement with the approximate (Fig. 2C,D, black lines) and
alytic solutions (Fig. 2C,D, green lines) to the 4 × 4 model. In particular, our numerical calculations show that mixing of states outside the 4 × 4 subspace, and the inclusion of cubic Lande g-factor \( g_2 \) with values obtained in experiments, does not qualitatively change the behaviour of the qubit splitting. Therefore, the main electric-field dependence on electric fields is dominated by the LH-HH mixing inside the 4 × 4 subspace. The numerical results for EDSR (Fig. 2E,F, orange squares) were obtained by transforming our numerical representations of \( \hat{H}_E \) and \( \hat{H}_{E,\text{ion}} \) into the basis of \( \hat{H}_L \) using \( \langle +|e\cdot r|−\rangle = U^\dagger (\hat{H}_E + \hat{H}_{E,\text{ion}}) U' \), where \( U' \) is the unitary matrix that diagonalizes \( \hat{H}_{L,K} + \hat{H}_{E,\text{ion}} \) in the basis \( |\Psi_{L,K}\rangle \).
