Essays in Asset Pricing

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Statement of originality

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Abstract

My thesis is divided in three chapters. In the first I extend the application of Bandi and Tamoni (2014)'s time series decomposition to other asset classes, such as fixed income, credit and credit derivatives, and other models, such as the Fama and French three factor model. I document a significant increase in R squared from using the decomposition across all the asset classes and models. I also exploit the time-domain properties of the decomposition to compute time-varying betas and analyse the determinants of risk across time.

In the second chapter I present some stylized facts about political risk, as proxied by Baker, Bloom and Davis' (2016) Economic Policy Uncertainty index. The log differential is stationary and exhibits a leptokurtic distribution, with heavy tails on both sides. The index also exhibits a time varying conditional variance, as highlighted by an ARMA(1,1)-GARCH(1,1) model. I also investigate the persistent nature of the index with various Heterogeneous Autoregression (HAR) specifications: the basic version, augmented with factors, and taking into consideration asymmetric effects. I finally investigate the out of sample conditional predictive ability of these models.

In the third chapter, I introduce a model that allows separating political risk from other risks, by extracting two uncorrelated factors in a two factor Heston setting. I employ the Economic Policy Uncertainty index by Baker, Bloom and Davis (2016) as a noisy proxy of political risk. The model allows to price options and recover the impact of political risk on the whole $\mathbb{P}$-distribution of asset returns. I find that political risk has a sizeable effect on the distribution of the S&P 500 index returns and that it impacts all moments. I also find that political risk has a positive and statistically significant impact on the S&P 500 index Equity Risk Premium.

Dedication

Ai miei genitori, Anna e Paolo

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Chapter 1

Business cycle risk in equities, fixed income and credit markets.

1.1 Introduction.

An ongoing challenge for empirical asset pricing is to explain the cross section of expected asset returns using the exposures to a parsimonious set of risk factors. The failures of the Consumption CAPM has led researchers to search for new risk factors. Rather than adding factors, recent research has showed that decomposing a risk factor in high and low frequency components leads to better pricing ability and a richer structure in the prices of risk.

In this chapter I extend the application of Bandi & Tamoni (2017)'s time series decomposition to other asset classes, such as fixed income, credit and credit derivatives, and to other models, such as the Fama and French three factor model.

First, I document the well known poor pricing ability of the standard Consumption CAPM for equity, fixed income and credit markets. I find results in line with the literature[1]. Second, I show that, by decomposing consumption in frequency components, the adjusted R^2 increases sizeably: from 19% to 63% for equities, from zero to 98% for US treasuries, from 87% to 93% for corporate credit and from 38% to 58% for credit derivatives. In a Fama and French setting, I show that the decomposition increases the quarterly alpha along the size

[1] See for example Mankiw & Shapiro (1986)

(HML) and value (SMB) dimensions, from 0.99% to 1.66%, and from 0.22% to 1.51% respectively. I finally exploit the time-domain properties of Bandi an Tamoni's decomposition to estimate time-varying Betas. I find that the 8 quarter cycle is the most important driver of risk exposure across time. This is consistent with Bandi & Tamoni (2017)'s result, where they find a priced factor of 2-3 years. This result is at odds with Fourier-based methods such as Dew-Becker & Giglio (2013) and Bansal & Yaron (2004), where the priced factor is the very low frequency one (more than 10 years).

Bandi & Tamoni (2017) have showed that one of the causes for the failure of standard CCAPM is that it restricts the price of risk to be the same for all frequencies of the consumption growth process. Relaxing this assumption and estimating frequency dependent risk prices leads to a sizeable improvement in the ability of consumption betas to explain the cross section of asset returns. Moreover, some specific frequencies are more relevant than others to explain cross sectional asset returns. For example, business cycles from 2 to 8 years seem to have a very strong explanatory power, whereas higher frequency shocks to consumption do not seem particularly relevant for asset pricing purposes. I am also able to document the strong pricing ability of the 2 years cycle. The technique used to compute frequency specific betas - previously developed by Ortu *et al.* (2013) and Bandi *et al.* (2017) - relies on a rich decomposition of consumption in components that exhibit heterogeneity in persistence and pricing ability. The separation into J details provides more granularity in the analysis of fluctuations with different cycles than in the case with traditional two component filters, such as Beveridge & Nelson (1981). This technique conveniently delivers an additive decomposition of the covariance between consumption and returns along a discrete set of relevant scales. This additive decomposition is then key in estimating specific betas for consumption fluctuations occurring at specific scales. Investors then weigh different layers of the consumption process differently for the purpose of asset pricing. This reasoning resonates with a large literature on the use of consumption shocks to price asset returns, for example Campbell & Cochrane (1999), Bansal & Yaron (2004) and Hansen *et al.* (2008) among others. The economic reasoning behind the use of business cycle asset pricing models is that economic activity evolves according to cycles, and therefore some frequencies may have better pricing abilities than others. Intuitively, investors care more for what happens over the business cycle than what happens at very short, or very long, frequencies.

Merton (1973) was the first to introduce dynamic asset pricing restrictions in a continuous time setting. The main intuition behind Merton's intertemporal capital asset pricing model (ICAPM) was to understand that the investment opportunity set varies across time. Returns are not only a function of their covariance with the market portfolio but also of their covariance with the set of variables z_t that can induce changes in the investment opportunity set. Breeden (1979) then showed that the set of variables z_t can be reduced to consumption growth alone, reconciling intertemporal portfolio choice with the classical Consumption CAPM of Rubinstein (1976) and Breeden & Litzenberger (1978). The consumption capital asset pricing model (CCAPM) then expresses expected returns as functions of their covariance with consumption growth. The beauty of the CCAPM is that it reconciles dynamic asset pricing with the basic economic intuition of investment decisions: agents invest in securities to smooth consumption. Unfortunately, the classic CCAPM does not provide an adequate fit to the cross section of asset returns[2].

The work is organized as follows. In Section 2 I investigate dynamic asset allocation in an environment driven by business cycle consumption and show how this is different from Breeden (1978)'s CCAPM. In Section 3 I impose some equilibrium conditions and derive some testable implications for equilibrium asset prices. To provide further evidence that the approach pioneered by Ortu *et al.* (2013) is a successful one, in Section 4 I investigate other asset pricing specifications, including the famous Fama and French three and five factor models.

1.2 The Business Cycle CCAPM.

1.2.1 Expanding the state-space

In Merton's ICAPM, the demand for financial assets depends on the covariance between those asssets and a set of state variables X_t that drive the investment opportunity set. In 1979, Breeden showed that the collection of all the possible variables that may shift the investment opportunity set can collapse to aggregate consumption only[3]. This new asset

[2]The aggregate consumption process is too smooth to generate enough co-variation with returns to suitably account for several empirical puzzles, for example Mehra & Prescott (1985)'s equity risk premium puzzle or Weil (1989)'s risk free interest rate puzzle. This has left researchers struggling to reconcile the appealing features of the CCAPM with its empirical failure.

[3]The reasoning proceeds from observing that Merton's first order condition evaluated at the optimal consumption reads:

$$T(\mu_S - r_f) = \Sigma_{SW} c_W + \Sigma_{SX} c_X$$

with $T = -U_c/U_{cc}$,$J_W = U_c$, $J_{WX} = U_{cc} c_X$ and $J_{WW} = U_{cc} c_W$. Breeden then showed that, since optimal consumption $c(W, X, t)$ is a function of wealth and state variables, by applying Ito one gets:

$$\Sigma_{SC} = \Sigma_{SW} c_W + \Sigma_{SX} c_X$$

pricing relation was called Consumption CAPM (CCAPM).

Even if Merton's ICAPM provides a good fit to the data, Breeden's critique demolished the theoretical justification for adding variables in X_t beyond aggregate consumption. In this chapter, I show that we can still expand the state space to include time-scale decompositions of aggregate consumption.

The time-scale decomposition is the one proposed by Ortu *et al.* (2013) and Bandi *et al.* (2017). Each component of the decomposition exhibits different levels of persistence and enables to disentagle short run from long run consumption dynamics. This decomposition is also able to provide a more granular analysis that in the case with traditional two component filters, such as Beveridge & Nelson (1981). For a complete description of the decomposition used, see Appendix A.

Following the Wiener-Khinchin theorem, we can define the power spectral density of returns $r(t)$ as the Fourier transform of its autocorrelation function $R_r(t,\tau)$:

$$S_r(t,f) = F_{tf}\left\{R_r(t,\tau)\right\} = \int_{-\infty}^{+\infty} \mathbb{E}\left[r\left(t+\frac{\tau}{2}\right) r\left(t-\frac{\tau}{2}\right)\right] e^{-2\pi f \tau} d\tau$$

If the time series is stationary, the power spectral density depends only on the frequency: $S_r(f)$. If the time series is not stationary, then the autocorrelation function will vary across time and the power spectral density will be localized in time and frequency. This is called a time varying spectrum or an evolutive spectrum. If we drop the dependence on time and invert the formula, the full time series can be reconstructed from $S_r(t,f)$, net of a constant, as:

$$r(t) = \int_{-\infty}^{+\infty} S_r(t,f) e^{2\pi f t} df$$

As such, we can express the covariance between returns and consumption as:

$$\mathbb{E}\left[r(t)c(t)\right] = \int_{-\infty}^{+\infty} \mathbb{E}\left[S_r(t,f)S_c(t,f)\right] e^{2\pi f t} df$$

The term $\mathbb{E}\left[S_r(t,f)S_c(t,f)\right]$ is the co-spectrum of the two time series. Therefore, the covariance is the Fourier transform of the co-spectrum, computed across a continuum of frequencies. In this chapter, frequencies are discretized in scales so that the integral can be computed numerically $j_{\{1,2,\dots,J\}} = \left\{(-\infty, f_1], [f_1, f_2], \dots, [f_{J-1}, f_J], [f_J, +\infty)\right\}$.

and by direct substitution he finds that the covariance of securities with consumption alone is relevant for asset pricing purposes.

The four most important properties of this time-scale decomposition are the following. The first is additivity. This means that the sum of the time-scale components yields the original series. In other words, let c_t be aggregate consumption and let c_t^j be the $j-$th component of the decomposition. The first $J-1$ components are usually called *details* while the last $J-$th components is usually called *smooth*. It always holds that $c_t = \sum_{j=0}^{J} c_t^j$. Also note that details have mean equal to zero while the smooth component has a mean equal to the mean of the original data. The second property is orthogonality, which ensures that the set of detail components is orthogonal. This means that $\mathbb{E}[c_t^{j_1} c_t^{j_2}] = 0 \quad \forall \quad j_1 \neq j_2 \neq J$. The third property is variance isometry, which ensures that the sum of the variances of each component is equal to the variance of the original series: $\mathbb{E}[c_t^2] = \sum_{j=0}^{J} \mathbb{E}[c_t^{j\,2}]$. The fourth property is covariance isometry, which ensures that the sum of the covariances of each component is equal to the covariance of the original series: $\mathbb{E}[c_t x_t] = \sum_{j=0}^{J} \mathbb{E}[c_t^j x_t]$.

The strategy is now to stack these J components in a vector X_t of state variables. Each element of X_t will now be associated with fluctuations of consumption at a different time-scale and thus convey different information on the time variation of the investment opportunity set. For instance:

$$\underbrace{d\begin{pmatrix} c_t^1 \\ c_t^2 \\ \vdots \\ c_t^J \end{pmatrix}}_{dX_t} = \underbrace{\begin{pmatrix} 0 \\ 0 \\ \vdots \\ \mu(X_t, t) \end{pmatrix}}_{\mu(X_t,t)} dt + \underbrace{\begin{pmatrix} \sigma^1(X_t,t) & 0 & \cdots & 0 \\ 0 & \sigma^2(X_t,t) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma^J(X_t,t) \end{pmatrix} \begin{pmatrix} d\mathcal{W}_t^1 \\ d\mathcal{W}_t^2 \\ \vdots \\ d\mathcal{W}_t^J \end{pmatrix}}_{\sigma(X_t,t)d\mathcal{W}_{X,t}}$$

where $\mu(X_t, t)$ is the drift of the state variable, $\sigma^j(X_t, t)$ is the volatility of the j-th component and $\mathcal{W}_t^j$ is a standard Wiener process. The set of uncorrelated Wiener processes $\left\{\mathcal{W}_t^1, ..., \mathcal{W}_t^J\right\}$ is what drives the independent sources of risk at each scale. One can then set up a dynamic optimization similar to Merton's one and recover formula (1). Each component of the intertemporal hedging demand will reflect the extent to which the optimal investor is trying to hedge short and long run fluctuations in the investment opportunity set. This approach then allows to disentangle the inter-temporal hedging effect of short run from long run risks.

1.2.2 Equilibrium

If we further assume – as in Merton (1979) – that financial assets are in zero net supply, we can then equate asset demand and asset supply to recover some testable general equilibrium condition.

The Consumption CAPM

Following Breeden (1979), the only state variable that matters for asset pricing is aggregate consumption. This can easily be relaxed by allowing other measures of consumption to matter[4]. Here, I stick with a more traditional setting and use consumption growth on services and non durable goods. Please see the data section for more details.

The main claim of the Consumption CAPM is that an asset's excess return is higher the higher its beta with respect to consumption. This restriction on asset returns can be tested by estimating two linear regressions. This has been done before various times in the literature, see for example Breeden (1979). At a first stage, I estimate each security's beta from the following time series regression:

$$r^e_{i,t} = \alpha_i + \beta_i c_t + \epsilon_{i,t}$$

where $r^e_{i,t}$ are excess returns, c_t is consumption growth and $\epsilon_{i,t}$ is the error term.

At a second stage, I estimate the price of risk from the following cross sectional regression:

$$\bar{r}^e_i = \lambda_0 + \lambda_1 \beta_i + \varepsilon_i$$

For the CCAPM to hold in the data, the following null hypothesis should fail to be rejected: (1) $\alpha_i = 0$, (2) $\lambda_0 = 0$ and (3) $\lambda_1 \neq 0$.

Business cycle Consumption CAPM

[4]For example Ait-Sahalia et al. (2004) find significant pricing ability for luxury goods consumption, Yogo (2006) uses durable consumption only, Malloy et al. (2009) use stockholder consumption to price size and value portfolios. Da and Yun (2010) , Savov (2011) and Chen and Lu (2012) employ various proxies of aggregate consumption to price the cross section of US and international stock returns, respectively electricity consumption, municipal solid waste growth and carbon dioxide emissions.

Bandi & Tamoni (2017) propose a different specification for the expected return of each security. In their model, each investor prices different layers of the consumption process differently. This gives rise to a linear restriction with various betas, one for each layer of the consumption process. To understand why this is the case, it is worth recalling the basic features of their model. They work with scale-time decompositions for real consumption growth. Write: $c_t = \sum_j^J c_t^j$. The details $\left\{c_t^j\right\}$ may be thought as uncorrelated, linear autoregressive processes with a scale specific autoregressive parameter ρ_j and scale specific shocks defined over the dilated time $t - s^j$ of each individual scale. Because j is measured in terms of quarters, each detail is associated with periodic fluctuations between 2^{j-1} and 2^j quarters. The chosen lowest frequency component $J = 5$ strikes a compromise between identifiability and richness of the model. Figure 1.2 shows the adjusted R^2 as I increase the scales. After an initial drop at scales $J = 2$, the adjusted R^2 keeps increasing up to scales $J = 5$, after which it flat-lines, showing adding scales beyond the fifth brings little improvement to the model, while adding estimation issues. I therefore fix the number of scales to five. Bandi & Tamoni (2017) show that a large percentage of the pricing ability is associated with real consumption cycle periodicities of 2 to 8 years. Through business cycle fluctuations in the consumption components, the authors are able to generate directly business cycle fluctuations in the stochastic discount factor, a feature which was previously discussed by Alvarez & Jermann (2005) and Parker & Julliard (2005) as being empirically warranted and theoretically meaningful. For a more detailed discussion of the decomposition, we address the interested reader to the Appendix.

This decomposition allows to interpret the excess return of a security as a linear combination of scale specific betas and specific lambdas. Each lambda quantifies the exposure to a certain business cycle layer of the consumption process. Again, also this model can be estimated by estimating two regressions. At a first stage, I estimate multiple betas.

$$r_{i,t}^e = \alpha_i + \beta_i^1 c_t^1 + \beta_i^2 c_t^2 + ... + \beta_i^J c_t^J + \epsilon_{i,t}$$

where $r_{i,t}^e$ are excess returns, c_t^j is the j-th layer of the consumption process and $\epsilon_{i,t}$ is the error term. At a second stage, I estimate the price of risk by running the following cross sectional regression:

$$\bar{r}_i^e = \lambda_0 + \lambda_1 \beta_i^1 + \lambda_2 \beta_i^2 + ... + \lambda_J \beta_i^J + \varepsilon_i$$

In this way, I obtain a number of prices of risk equal to the number of consumption layers that matter for asset pricing. Also this version of the consumption CCAPM can be tested on the data. In particular, for this model to hold, the following null hypothesis should fail to be rejected: (1) $\alpha_i = 0$, (2) $\lambda_0 = 0$ and (3) $\lambda_1 \neq 0, \lambda_2 \neq 0, ..., \lambda_J \neq 0$.

1.3 Empirical Section

1.3.1 Data

The state variable of choice is consumption growth. United States consumption growth is measured by taking log differences of National Income and Products Accounts (NIPA) aggregate non durable and services consumption, divided by total US population. Consumption data is taken from the Bureau of Economic Analysis whereas population data is taken from the Bureau of Labor Statistics. The risk free rate of return is measured by taking the current yield on 3-months US Treasury bills, as published by the Federal Reserve Economic Data (FRED). The market portfolio is the total return on the CRSP US value weighted index, as downloaded from the Wharton Research Data Services database. Inflation is computed by taking the first difference in the log series of the US Consumer Price Index, as published by FRED. Real excess returns are computed by subtracting the risk free rate and inflation from each portfolio return. Real consumption growth is computed by subtracting inflation from nominal consumption growth. The sample of macro variables runs from February 1959 to August 2014. All data are monthly observations, aggregated to a quarterly level by direct multiplication of $(1 + r_i)$. The sample contains 222 quarterly observations.

US Equities are proxied by twenty-five Fama and French portfolios sorted by Size and Book-to-Market ratios. Data on returns for these portfolios is obtained from Professor Ken Franch's website at Dartmouth College. For fixed income markets, I collected from FRED quarterly total return data for nine Bank of America-Merrill Lynch US corporate bond portfolios, spanning various maturities and credit ratings: 1-3 years, 10-15 years and 15+ years, AAA, AA, A, BBB, High Yield and a "Corporate Master" index that includes all data. The data is available from Q1 1989 to Q4 2013. I also recovered quarterly total return data for seventeen CRSP Fama Maturity portfolios. This dataset spans US Treasuries with maturities from zero to 120 months, at intervals of 6 and 12 months. For more details, please see Fama (1984). This dataset is available from Q1 1952 to Q4 2013. I also recovered quarterly total return data for seventeen CRSP Fama Maturity portfolios. This dataset spans US Treasuries with maturities from zero to 120 months, at intervals of 6 and 12 months. For more details, please see Fama (1984). This dataset is available from Q1 1952 to Q4 2013. I have finally collected on-the-run spreads for twenty two credit default swap indices, of which twelve can be classified as macro credit indices (eg. iTraxx Europe, Crossover, HiVol, etc.) and ten as sector specific indices (eg. Auto, Industrial, etc.). This data is available only for a short time period: macro credit indices data run from Q3 2004 until Q2 2014 while sector specific indices data ends in Q4 2011, when Markit decided to terminate the coverage of such indices. Due to several non-linearity effects, I first had to transform credit spreads into total returns. The details are available in Appendix B.

1.3.2 Empirical performance of the Consumption CAPM

To provide further evidence that the approach pioneered by Bandi and Tamoni (2014) is successful, I have estimated standard and business cycle versions of the Consumption CAPM on US equities, fixed income and credit. The number of cycles chosen is $J = 5$, corresponding to cycles of 2, 4, 8, 16 and 32 quarters. Since the data spans a time frame of about 50 years, my chosen lowest frequency component ($J = 5$, in the notation used above) strikes a compromise between identifiability (higher frequency details are easier to identify) and richness of the overall decomposition (the larger the number of details, the richer the decomposition). Figure 1.2 shows the adjusted R^2 as I increase the scales. After an initial drop at scales $J = 2$, the adjusted R^2 keeps increasing up to scales $J = 5$, after which it flat-lines, showing adding scales beyond the fifth brings little improvement to the model, while adding estimation issues.

Equities

US Equities are proxied by twenty-five Fama and French portfolios sorted by Size and Book-to-Market ratios. First, I test the standard Consumption CAPM by estimating time series regressions of nominal excess returns on nominal consumption growth. The results are in line with the literature and are reported in Table 1.1. I then decompose the consumption process in five layers and estimated the Business Cycle Consumption CAPM. I again ran time series regressions on nominal excess returns using the five layers of the consumption process as explanatory variables. I then ran second stage regressions with all five λs.

The time series regressions on consumption provide little support for the CCAPM. Adjusted R^2 range from zero to 2.29%, betas are not statistically significant and alphas are statistically not different from zero. Overall it seems that the consumption CAPM is not able to capture the salient cross sectional features of size and book-to-market ratio. The results with the business cycle CCAPM are more encouraging. Time series adjusted R^2s are sizeably higher, reaching almost 10%. Equity returns respond strongly to the two years consumption risk component. Also the one year consumption process seems to hold some explicative power now. Alphas are all statistically different from zero, showing that consumption alone cannot fully account for average equity returns. This is confirmed in Table 1.12 where a Gibbons-Ross-Shanken (GRS) test rejects the null of all alphas being jointly equal to zero.

Table 1.11 and Figure 1.3 show the estimates of the second stage regression, showing the prices of risk. The difference between aggregate consumption and the decomposed version is very big. Aggregate consumption has a cross-sectional adjusted R^2 of 19%, but by using the betas from the decomposed process, this sends the cross sectional adjusted R^2 to 63%. Table 1.13 confirms these improvements, with a lower AIC for the business cycle CCAPM and a statistically significant likelihood ratio test at the 5% level.

Fixed Income

The consumption CAPM applies in theory to any asset. In practice most tests have focused on equities. There is a growing literature on consumption based models in the fixed income space. Wachter (2006) proposes an equilibrium asset pricing model that depends on consumption as the only state variable. Lettau & Ludvigson (2001) resurrect the consumption CAPM showing that a special factor constructed from a cointegrating relationship involving consumption, wealth and income has very strong pricing abilities. In this chapter I estimate a standard and business cycle Consumption CAPM for each of the two datasets. The results are reported in Table 1.2 and Table 1.3.

We can first analyse the performance of the standard CCAPM on Bank of America-Merrill Lynch bond portfolios. First, all alphas are significantly different from zero at the 10% level, with the noticeable exception of the high yield portfolio. Adjusted R^2 are all quite low and range from 0.13% to 3.71%. First, we can notice that betas are negative, meaning that exposure to consumption risk reduces returns. I then estimate a Business Cycle CCAPM with the same dataset. These time series regressions produce adjusted R^2 of a higher order of magnitude, ranging from 8.49% to 17.70%. The statistical significance of alphas vanish for many portfolios, but remains significant for 1-3Y, 10-15Y, BBB and, noticeably, the high yield porfolio. The significance in the alphas of BBB and high yield is theoretically appealing, because I expect these portfolios to carry significant default risk premia, not captured by simple consumption risk. The significance of alphas for 1-3Y and 10-15Y is harder to justify theoretically. The significance of alphas is confirmed by a GRS test, reported in Table 1.13. By analysing now betas we can spot a number of interesting facts. The only statistically significant cycles of consumption are between four and eight quarters. The first and most striking difference is that the four quarter cycle carries negative coefficients while the eight quarters cycle carries a positive coefficient. This means that exposure to one year consumption growth risk reduces the risk of fixed income securities. In other words, one year consumption growth covaries negatively with returns, providing high hedging capacity. On the contrary, fixed income securities provide little hedging capacity for two years consumption growth, and thus coefficients for this layer are high and positive. Considering the cross section of betas, we further identify interesting patterns. First, $\hat{\beta}_2$ runs in the opposite direction of $\hat{\beta}_3$, meaning that the stronger the positive impact of one year consumption, the stronger the negative impact of two years consumption. Moreover, the patterns in betas follow a monotonic pattern along the risk dimension. For example, as credit quality deteriorates, the sensitivity of returns to one year consumption becomes more negative and the sensitivity to two years consumption becomes more positive. The riskiest bond portfolios in the sample are the High Yield one, carrying a $\hat{\beta}_2$ of -3.02 and a $\hat{\beta}_3$ of +11.18, and the 15+ years one, carrying a $\hat{\beta}_2$ of -4.18 and a $\hat{\beta}_3$ of +6.87.

Estimating a CCAPM on Fama bond porfolios provides more insight in the structure of risk along the maturity dimension. These securities are all US Treasuries, so I expect betas to be lower than in the previous dataset. The simple CCAPM yields significant alphas and an interesting monotonic structure in the betas. All betas are negative again, showing that US Treasuries provide a good hedge against consumption risk. In the Business cycle CCAPM, alphas remain statistically significant, with the exception of short maturity securities. This means that consumption risk alone fails to explain alphas beyond the 18-24 months period, showing that other risk factors start to kick in after 24 months, most noticeably default, interest rate and liquidity risks. If we analyse betas we can further conclude that only consumption cycles of two quarters drives treasury returns; all other scales lose statistical significance. This shows that US treasuries provide a good hedge only against very short term consumption shocks. A noteworthy exception is represented by the 48-120 months maturity range risk which seems to be also driven by the 32 quarters - or eight years - consumption cycles.

Table 1.11 reports the cross sectional regressions from the fixed income universe. The aggregate CCAPM had almost no cross-sectional explanatory power for the BAML portfolios (adjusted R^2 = 0%) and a large one for US Treasuries. By using the decomposition, I can improve on both, documenting an adjusted R^2 of 97% for the first dataset, and 93% for the second. Table 1.13 reports the AIC improvements for both datasets. Akaike Information Criterion is lower for the business cycle CCAPM in both cases, showing a better model fit than the regular CCAPM. In the case of Bank of America-Merrill Lynch bond portfolios, a likelihood ratio test also confirms that the business cycle CCAPM is better than the regular CCAPM with a 10% significance.

Credit

To confirm what the previous analysis seemed to suggest, default risk is harder to price with consumption alone. A simple CCAPM captures some simple stylized facts of default risk: large unexplained alphas and negative betas. A GRS test confirms alphas are jointly different from zero. The business cycle CCAPM sheds further light on what seems to drive credit risk. Alphas still remain largely unexplained, reaching as much as 5.18% per quarter in the case of Crossover 10Y. The only statistically significant cycle is the the 32 quarters or 8 years cycle consumption risk, which carries a beta of around -2.3. This means that credit risk reacts significantly to long run changes in consumption risk or, equivalently, that credit instruments provide a hedge to long run changes in consumption alone. The pattern is consistent in the cross section and it fails to show a term structure between the five and ten years contracts. Instead alphas exhibit an explicit term structure, showing once again that what drives the term structure is not consumption risk. As previously hinted, the Crossover index provides an interesting deviation from this pattern. Crossover collects the 75 most liquid sub investment grade credit default swap contracts, making it the riskiest credit index in the sample. The fact that it carries a so high default risk explains why alphas are so high. By looking at betas, $\hat{\beta}_5$ is not significant while $\hat{\beta}_2$ and $\hat{\beta}_3$ are large and significant. Default risk is therefore somehow positively correlated with one and two years consumption growth. Not surprisingly, high risk securities are pro-cyclical and do not provide a good hedge against one and two years consumption shocks.

Table 1.5 sheds light on how consumption risk affects various industries. The general pattern of high unexplained alphas and large negative $\hat{\beta}_6$s is confirmed across all industries. Ten years Industrial credit is the most negatively correlated with eight years consumption risk. A small exception is represented by automotive credit, which is influenced by shorter term consumption shocks as well. Automotive credit spreads are also the highest in the sample, so again it seems like I am picking up a high default risk effect here.

Table 1.11 reports the cross sectional regressions for both credit derivatives samples using the betas from the standard CCAPM and the Business Cycle CCAPM. The aggregate model produces adjusted R^2s of 38% and 11%, while the business cycle CCAPM is able to achieve a substantially better fit, with adjusted R^2 of 57% and 94%. These improvements are confirmed in Table 1.14 by noticing the lower Akaike Information Criterion, even if the Likelihood ratio test is not significant at any meaningful level.

1.4 Arbitrage pricing across the business cycle

In this section I will explore further applications of the decomposition. The objective is to explore which frequencies, or scales, drive the size and value strategies in equities, or which scales matter in bond pricing. The main goal is to show the flexibility of the decomposition in being applied to various linear models and managing to shed additional light.

Size and value effects

Here I analyse the business cycle implications of the standard Fama & French (1993) factor regressions. I try to investigate further what drives the failure in the CAPM to explain these effects and how Fama and French's factors help solve the problem. It has been suggested by Petkova (2006) that Fama and French factors proxy for innovations in state variables that drive the business cycle. If so, then I would expect portfolios sorted by size or value to covary strongly - i.e. to have significant betas - with certain scales in the decomposition of consumption growth or the market index. This would mean that Fama and French factors indeed proxy for ”surprises” that are correlated with the business cycle.

I first regress ten portfolios sorted by value (HML) and ten portfolios sorted by size (SMB) on aggregate consumption and, subsequently, on a five scales decomposition of aggregate consumption. Estimates are reported in Table 1.14 and Table 1.15. At an aggregate level, my results show aggregate consumption does not covary significantly with any of the two factors. Alphas are large and positive and the adjusted R^2s are very low. When I decompose the aggregate consumption process, I find that betas with the third scale are positive and highly significant for both value and size. This suggests that these factors covary strongly with the 8 quarters cyclical consumption component. This provides further evidence that Fama and French factors are indeed connected to a business cycle risk explanation.

Adjusted R^2s remain small when using consumption as the explanatory variable. To try and address this issue, I have reported in Table 1.16 and Table 1.17 the results of the same regression, but replacing consumption with the market factor. At an aggregate level, I confirm the results by Fama & French (1993): the market factor cannot price value and size portfolios and generates large alphas. I then look at the regression for the business cycle CAPM. The betas with respect to the 8 quarters component are again positive and highly significant, confirming a business cycle story behing the Fama and French factors. Adjusted R^2s are higher than with consumption, but do not show an improvement with respect to the aggregate level.

Bond returns

This subsection explores further applications of the pricing ability of the decomposition by analysing additional factors. Litterman & Scheinkman (1991) and Litterman & Scheinkman (1994) have suggested a number of factors in their seminal papers, such as inflation, and the usual three yield curve factors: level, slope and curvature.

I also regress bond total returns on a number of factors: changes in inflation, consumption growth, and interest rate curve level, slope and curvature. I find that inflation and slope do in fact have a significant impact on bond total returns. I then decomposed these factors into three layers and performed the regressions again. Bond factor returns respond positively to short term changes in the level of interest rates and to the long term changes in the slope of the interest rate curve; they respond negatively to short and medium term changes in inflation and to short run consumption shocks.

A Dynamic Business Cycle Consumption CAPM

To investigate further the properties of the consumption CAPM, I have proceeded to estimate a Dynamic Conditional Correlation (DCC) model in aggregate and to each scale for the S&P 500. This is possible because I am able to exploit a particular feature of the scale decomposition: it preserves the time domain information. Contrary to a Fourier series representation, I am able to recover the time domain information scale by scale and can exploit the variation at each scale independently to estimate the DCC. The result is that most of the dynamic evolution of the time varying correlation between aggregate consumption and the stock index is driven by the third layer, that is a frequency of 8 quarters. This confirms and reinforces the previous result of a significant 8 quarter component.

Having reconstructed a time series of time-varying betas, I then proceed to characterize the risk exposure to common risk factor, to try to asses what is driving risk at each scale. I therefore regress aggregate and scale specific betas against the spread between AAA and BBB rated bond yields (DEF), US industrial production (INDP), US short term treasury rates (SHORT), the difference between long and short term US treasury yields (TERM), the VIX, inflation and Lettau and Ludvigson (2004) *cay* factor.

These factors have been chosen because they convey information about the future real economic activity (Estrella & Hardouvelis (1991)) as well as about the future investment opportunity set (Petkova (2006)). The inflation rate and the short term rate can be considered a proxy for monetary conditions, while industrial production and *cay* are both related to the business cycle conditions in the country. The VIX and the credit spread proxy for the aggregate level of risk.

I find that the 8 quarter beta has a high negative exposure to short term interest rates and a high positive exposure to the *cay* factor. This is in line with the previous result of Bandi & Tamoni (2017) where they also find an important 2 years cycle. In the aggregate regression, both SHORT and *cay* loadings are very small, proving that decomposing risk exposures in scales helps uncover previously hidden relationships. This result is consistent with Lettau and Ludvigson (2004)'s finding that scaling the Consumption CAPM with the *cay* factor improves the model fit. This result suggests than the optimal scaling factors may be different at various frequencies. For example, the VIX does not have a significant loading in the aggregate regression, but it becomes significant for the low frequency components $\beta_{4,t}$ and $\beta_{5,t}$.

1.5 Conclusions

Separating factors into short and long run dynamics allows a richer decomposition of risk exposures. Within a CCAPM framework, the two-year component of consumption has a stronger pricing ability than aggregate consumption alone. This methodology also allows to obtain a better fit from a wide range of asset pricing factor models, with sizeable improvements in adjusted R^2s.

Tables and Figures

Table 1.1: Equities: standard and business cycle CCAPM

No.	α	β	R^2	α	β_1	β_2	β_3	β_4	β_5	R^2
1	-2.16	2.17**	2.28%	0.54	3.12**	-2.52	15.27***	-0.17	0.36	8.79%
	(1.78)	(0.95)		(2.80)	(1.38)	(2.33)	(3.93)	(4.83)	(1.74)	
2	-0.14	1.77**	2.13%	2.61	2.6**	-2.22	13.79***	0.29	-0.06	9.59%
	(1.51)	(0.81)		(2.36)	(1.17)	(1.97)	(3.32)	(4.08)	(1.46)	
3	0.33	1.51**	1.94%	2.43	2.00*	-2.03	11.52***	2.08	0.11	8.38%
	(1.35)	(0.72)		(2.13)	(1.05)	(1.77)	(2.98)	(3.67)	(1.32)	
4	1.34	1.23*	1.41%	2.81	1.42	-1.65	10.39***	0.80	0.24	6.81%
	(1.29)	(0.69)		(2.04)	(1.01)	(1.70)	(2.86)	(3.52)	(1.26)	
5	0.99	1.76**	2.29%	3.34	2.03*	-1.41	11.73***	3.35	0.19	7.49%
	(1.44)	(0.77)		(2.29)	(1.13)	(1.9)	(3.21)	(3.95)	(1.42)	
6	-0.16	1.15	0.83%	2.40	2.30*	-3.64*	12.59***	-0.91	-0.56	8.20%
	(1.58)	(0.85)		(2.48)	(1.22)	(2.07)	(3.48)	(4.28)	(1.54)	
7	0.58	1.13	1.11%	2.56	1.78*	-2.56	10.72***	1.21	-0.19	7.71%
	(1.33)	(0.71)		(2.10)	(1.03)	(1.75)	(2.94)	(3.62)	(1.30)	
8	1.57	0.85	0.78%	3.55*	1.07	-1.83	10.27***	1.00	-0.48	7.47%
	(1.20)	(0.64)		(1.89)	(0.93)	(1.57)	(2.66)	(3.27)	(1.17)	
9	1.37	1.05*	1.22%	2.52	0.89	-1.21	8.98***	1.27	0.28	5.64%
	(1.19)	(0.63)		(1.89)	(0.93)	(1.57)	(2.65)	(3.26)	(1.17)	
10	1.42	1.18*	1.25%	2.70	0.95	-0.87	8.8***	3.07	0.32	4.64%
	(1.32)	(0.7)		(2.11)	(1.04)	(1.75)	(2.96)	(3.64)	(1.31)	
11	0.52	0.74	0.42%	2.83	1.54	-3.39*	11.16***	0.12	-0.81	7.54%
	(1.43)	(0.76)		(2.24)	(1.11)	(1.87)	(3.15)	(3.87)	(1.39)	
12	1.29	0.77	0.65%	3.31*	1.25	-2.99*	10.25***	1.97	-0.58	8.79%
	(1.20)	(0.64)		(1.87)	(0.92)	(1.56)	(2.63)	(3.23)	(1.16)	
13	1.32	0.71	0.65%	3.33*	0.76	-1.04	8.73***	0.99	-0.63	6.11%
	(1.10)	(0.59)		(1.75)	(0.86)	(1.45)	(2.45)	(3.02)	(1.08)	
14	1.71	0.71	0.62%	3.41*	0.67	-1.36	9.41***	1.32	-0.43	6.85%
	(1.12)	(0.60)		(1.76)	(0.87)	(1.47)	(2.48)	(3.04)	(1.09)	
15	1.63	1.03	1.13%	3.62*	1.01	-0.33	8.46***	2.11	-0.30	4.89%
	(1.21)	(0.65)		(1.94)	(0.96)	(1.61)	(2.72)	(3.34)	(1.20)	
16	1.40	0.31	0.09%	3.87*	1.06	-3.11*	9.88***	0.31	-1.33	7.26%
	(1.30)	(0.70)		(2.05)	(1.01)	(1.7)	(2.87)	(3.53)	(1.27)	
17	1.14	0.48	0.28%	3.65**	1.08	-2.49*	9.23***	1.15	-1.20	8.22%
	(1.13)	(0.60)		(1.76)	(0.87)	(1.46)	(2.47)	(3.03)	(1.09)	
18	1.43	0.54	0.40%	2.67	0.45	-2.45*	8.52***	2.97	-0.29	7.29%
	(1.08)	(0.58)		(1.69)	(0.83)	(1.41)	(2.37)	(2.92)	(1.05)	
19	2.14**	0.33	0.15%	3.44**	-0.13	-1.42	8.42***	2.92	-0.55	6.38%
	(1.07)	(0.57)		(1.69)	(0.83)	(1.40)	(2.37)	(2.91)	(1.05)	
20	0.69	1.28**	1.73%	2.65	1.33	-1.42	9.16***	4.92	-0.02	7.04%
	(1.21)	(0.65)		(1.92)	(0.95)	(1.59)	(2.69)	(3.31)	(1.19)	
21	1.81*	-0.25	0.09%	3.72**	0.07	-3.40***	8.64***	0.95	-1.53	9.91%
	(1.02)	(0.55)		(1.58)	(0.78)	(1.32)	(2.22)	(2.73)	(0.98)	
22	1.91**	-0.23	0.10%	3.04**	-0.19	-3.36***	7.47***	1.39	-0.99	9.36%
	(0.93)	(0.50)		(1.45)	(0.71)	(1.21)	(2.03)	(2.50)	(0.90)	
23	1.49*	0.05	0.01%	3.03**	0.22	-2.73**	6.25***	2.96	-0.97	8.12%
	(0.87)	(0.47)		(1.36)	(0.67)	(1.13)	(1.91)	(2.35)	(0.85)	
24	1.25	0.28	0.15%	2.18	-0.11	-1.93	6.98***	3.33	-0.35	7.26%
	(0.90)	(0.48)		(1.41)	(0.70)	(1.17)	(1.98)	(2.44)	(0.88)	
25	1.36	0.39	0.23%	2.95*	0.32	-2.09	6.9***	3.56	-0.66	5.92%
	(1.03)	(0.55)		(1.62)	(0.80)	(1.35)	(2.28)	(2.8)	(1.01)	
26	-1.72***	-0.49***	12.36%	0.49***	0.00	0.02	0.71**	0.04	-1.97***	57.33%
	(0.16)	(0.09)		(0.19)	(0.09)	(0.16)	(0.26)	(0.32)	(0.12)	

Table 1.1 reports results from a regression analysis of 25 Fama and French portfolios sorted by Size and Book-to-Market ratios. The 26th portfolio is the CRSP US value weighted return. The regressor is real aggregate consumption growth. On the left, real aggregate consumption is used as a single regressor, while on the right it is separated in five orthogonal scales and each scale is used as a regressor. The table reports coefficients and standard errors in parenthesis. Significance is reported at the 10, 5 and 1% level with one, two and three asterisks respectively. The sample size is equal to 222 quarters.

Table 1.2: Fixed Income 1 (by portfolio type): standard and business cycle CCAPM.

Portfolio	α	β	R^2	α	β_1	β_2	β_3	β_4	β_5	R^2
1-3 Y	1.03**	-0.41*	3.71%	0.78*	-0.10	-1.56***	2.57***	-0.25	-0.16	17.30%
	(0.25)	(0.21)		(0.43)	(0.31)	(0.44)	(0.88)	(0.56)	(0.41)	
10-15 Y	2.24**	-1.00*	3.30%	2.37**	-0.70	-3.33***	6.06***	0.11	-1.10	14.03%
	(0.64)	(0.54)		(1.14)	(0.81)	(1.18)	(2.33)	(1.48)	(1.08)	
15+ Y	2.12**	-0.87	1.45%	2.29	-0.38	-4.18***	6.87**	0.51	-1.02	11.24%
	(0.85)	(0.71)		(1.52)	(1.08)	(1.56)	(3.09)	(1.96)	(1.43)	
AAA	1.42**	-0.60	2.38%	0.75	-0.78	-1.84**	2.84*	0.25	0.08	8.49%
	(0.45)	(0.38)		(0.83)	(0.59)	(0.85)	(1.69)	(1.07)	(0.78)	
AA	1.56**	-0.68*	2.93%	0.95	-0.69	-2.25***	3.62**	0.05	-0.06	11.27%
	(0.46)	(0.39)		(0.83)	(0.59)	(0.85)	(1.69)	(1.08)	(0.78)	
A	1.53**	-0.60	1.60%	1.19	-0.44	-2.93***	5.07**	0.51	-0.25	12.87%
	(0.56)	(0.47)		(1.00)	(0.71)	(1.02)	(2.03)	(1.29)	(0.94)	
BBB	1.88**	-0.80	2.46%	2.21**	-0.22	-3.53***	6.52***	0.03	-1.11	17.70%
	(0.60)	(0.50)		(1.04)	(0.74)	(1.06)	(2.11)	(1.34)	(0.98)	
High Yield	0.99	0.29	0.13%	3.28**	1.58	-3.02*	11.18***	-0.18	-1.95	15.41%
	(0.93)	(0.78)		(1.61)	(1.14)	(1.66)	(3.28)	(2.09)	(1.52)	
Corporate	1.70**	-0.71	2.30%	1.58	-0.42	-2.99***	5.31***	0.19	-0.57	14.53%
	(0.55)	(0.46)		(0.96)	(0.68)	(0.99)	(1.96)	(1.25)	(0.91)	

Table 1.2 reports results from a regression analysis of 9 Bank of America Merrill Lynch bond total returns indices. The regressor is real aggregate consumption growth. On the left, real aggregate consumption is used as a single regressor, while on the right it is separated in five orthogonal scales and each scale is used as a regressor. The table reports coefficients and standard errors in parenthesis. Significance is reported at the 10, 5 and 1% level with one, two and three asterisks respectively. The sample size is equal to 102 quarters.

Table 1.3: Fixed Income 2 (by maturity): standard and business cycle CCAPM.

From	To	α	β	R^2	α	β_1	β_2	β_3	β_4	β_5	R^2
0	6	0.12**	-0.03	0.76%	0.06	-0.05	-0.01	-0.03	-0.06	0.02	1.44%
		(0.04)	(0.02)		(0.06)	(0.03)	(0.05)	(0.08)	(0.1)	(0.04)	
6	12	0.29**	-0.10**	1.99%	0.23	-0.15**	-0.06	0.07	-0.11	-0.05	2.63%
		(0.08)	(0.05)		(0.14)	(0.07)	(0.11)	(0.19)	(0.24)	(0.09)	
12	18	0.45**	-0.15**	2.06%	0.32	-0.23**	-0.12	0.12	-0.14	-0.06	2.81%
		(0.13)	(0.07)		(0.21)	(0.1)	(0.17)	(0.29)	(0.36)	(0.13)	
18	24	0.57**	-0.21**	2.70%	0.50*	-0.28**	-0.19	0.09	-0.19	-0.16	2.99%
		(0.16)	(0.09)		(0.26)	(0.13)	(0.22)	(0.37)	(0.45)	(0.16)	
24	30	0.65**	-0.24**	2.17%	0.65**	-0.28*	-0.27	0.22	-0.11	-0.24	2.52%
		(0.2)	(0.11)		(0.33)	(0.16)	(0.27)	(0.46)	(0.56)	(0.2)	
30	36	0.78**	-0.30**	2.63%	0.79**	-0.34*	-0.33	0.21	-0.06	-0.3	2.91%
		(0.23)	(0.12)		(0.37)	(0.18)	(0.31)	(0.52)	(0.64)	(0.23)	
36	42	0.89**	-0.35**	3.03%	0.96**	-0.38*	-0.4	0.15	-0.12	-0.4	3.30%
		(0.25)	(0.13)		(0.41)	(0.2)	(0.34)	(0.57)	(0.7)	(0.25)	
42	48	0.95**	-0.39**	3.16%	1.04**	-0.41*	-0.49	0.11	-0.09	-0.45	3.50%
		(0.27)	(0.15)		(0.45)	(0.22)	(0.37)	(0.62)	(0.77)	(0.28)	
48	54	1.09**	-0.46**	3.86%	1.19**	-0.46**	-0.54	-0.13	-0.07	-0.53*	3.95%
		(0.29)	(0.16)		(0.48)	(0.23)	(0.39)	(0.66)	(0.81)	(0.29)	
54	60	1.07**	-0.50**	3.55%	1.26**	-0.54**	-0.66	0.52	0.12	-0.63*	4.65%
		(0.33)	(0.17)		(0.53)	(0.26)	(0.44)	(0.74)	(0.91)	(0.33)	
60	120	1.31**	-0.55**	3.28%	1.48**	-0.53*	-0.88*	0	0.4	-0.67*	4.06%
		(0.38)	(0.20)		(0.62)	(0.3)	(0.51)	(0.86)	(1.05)	(0.38)	
0	12	0.21**	-0.06*	1.60%	0.15	-0.10**	-0.04	0.03	-0.09	-0.02	2.15%
		(0.06)	(0.03)		(0.10)	(0.05)	(0.08)	(0.14)	(0.17)	(0.06)	
12	24	0.51**	-0.18**	2.45%	0.41*	-0.26**	-0.16	0.11	-0.15	-0.11	2.94%
		(0.14)	(0.08)		(0.24)	(0.11)	(0.19)	(0.33)	(0.40)	(0.15)	
24	36	0.75**	-0.28**	2.62%	0.71**	-0.33*	-0.32	0.15	-0.17	-0.25	2.84%
		(0.21)	(0.11)		(0.35)	(0.17)	(0.29)	(0.49)	(0.60)	(0.22)	
36	48	0.96**	-0.38**	3.16%	0.99**	-0.39*	-0.50	0.08	-0.13	-0.40	3.38%
		(0.26)	(0.14)		(0.43)	(0.21)	(0.36)	(0.60)	(0.74)	(0.27)	
48	60	1.14**	-0.49**	3.91%	1.21**	-0.52**	-0.63	0.07	0.00	-0.54*	4.28%
		(0.31)	(0.16)		(0.5)	(0.25)	(0.41)	(0.70)	(0.86)	(0.31)	
60	120	1.31**	-0.55**	3.28%	1.48**	-0.53*	-0.88*	0.00	0.40	-0.67*	4.06%
		(0.38)	(0.20)		(0.62)	(0.30)	(0.51)	(0.86)	(1.05)	(0.38)	

Table 1.3 reports results from a regression analysis of 16 Fama bond portfolios total returns. The regressor is real aggregate consumption growth. On the left, real aggregate consumption is used as a single regressor, while on the right it is separated in five orthogonal scales and each scale is used as a regressor. The table reports coefficients and standard errors in parenthesis. Significance is reported at the 10, 5 and 1% level with one, two and three asterisks respectively. The sample size is equal to 220 quarters.

Table 1.4: Credit Swaps (Macro): standard and business cycle CCAPM.

Index	α	β	R^2	α	β_1	β_2	β_3	β_4	β_5	R^2
iTraxx Europe	0.19	-0.27**	9.98%	1.49***	0.10	-0.14	-0.22	-0.21	-2.22***	52.07%
	(0.12)	(0.12)		(0.23)	(0.21)	(0.24)	(0.35)	(0.21)	(0.34)	
CDX (United States)	0.23**	-0.31**	15.32%	1.45***	0.10	-0.17	-0.23	-0.30	-2.16***	56.12%
	(0.11)	(0.11)		(0.21)	(0.19)	(0.22)	(0.32)	(0.19)	(0.31)	
Japan	0.48**	-0.29*	8.74%	1.9***	0.09	-0.07	0.25	-0.32	-2.42***	48.22%
	(0.14)	(0.15)		(0.28)	(0.25)	(0.29)	(0.43)	(0.26)	(0.41)	
Australia	0.39**	-0.35**	14.50%	1.78***	0.13	-0.26	-0.07	-0.40*	-2.45***	55.47%
	(0.13)	(0.13)		(0.25)	(0.22)	(0.26)	(0.38)	(0.23)	(0.36)	
HiVol 5Y	0.56**	-0.28*	7.78%	1.99***	0.30	-0.20	0.04	-0.36	-2.43***	48.10%
	(0.14)	(0.15)		(0.29)	(0.25)	(0.30)	(0.43)	(0.26)	(0.41)	
HiVol 10Y	1.23**	-0.24	5.57%	2.66***	0.29	-0.14	0.04	-0.37	-2.39***	43.21%
	(0.15)	(0.16)		(0.31)	(0.27)	(0.32)	(0.47)	(0.28)	(0.45)	
Crossover 5Y	1.68**	0.95	5.25%	2.86*	0.27	2.65*	5.21**	0.25	-0.71	16.90%
	(0.60)	(0.63)		(1.51)	(1.33)	(1.58)	(2.29)	(1.38)	(2.19)	
Crossover 10Y	4.60**	1.06	5.43%	5.18***	0.06	2.96*	5.24**	0.11	0.32	14.64%
	(0.66)	(0.69)		(1.68)	(1.48)	(1.77)	(2.56)	(1.54)	(2.44)	
Fin Senior 5Y	0.27*	-0.23	6.16%	1.73***	0.06	-0.09	-0.16	-0.13	-2.42***	48.01%
	(0.14)	(0.14)		(0.27)	(0.24)	(0.29)	(0.41)	(0.25)	(0.4)	
Fin Senior 10Y	0.69**	-0.22	4.81%	2.16***	0.02	-0.05	-0.12	-0.17	-2.42***	42.82%
	(0.15)	(0.15)		(0.30)	(0.27)	(0.32)	(0.46)	(0.28)	(0.44)	
Fin Subordinated 5Y	0.82**	-0.20	2.07%	2.22***	-0.17	0.10	0.37	-0.35	-2.29***	23.76%
	(0.20)	(0.21)		(0.48)	(0.42)	(0.5)	(0.72)	(0.43)	(0.69)	
Fin Subordinated 10Y	2.26**	-0.12	0.42%	3.32***	-0.44	0.31	0.92	-0.60	-1.70*	12.02%
	(0.28)	(0.29)		(0.70)	(0.61)	(0.73)	(1.06)	(0.64)	(1.01)	

Table 1.4 reports results from a regression analysis of 12 Markit credit indices total returns. The regressor is real aggregate consumption growth. On the left, real aggregate consumption is used as a single regressor, while on the right it is separated in five orthogonal scales and each scale is used as a regressor. The table reports coefficients and standard errors in parenthesis. Significance is reported at the 10, 5 and 1% level with one, two and three asterisks respectively. The sample size is equal to 40 quarters.

Table 1.5: Credit Swaps (Industrial): standard and business cycle CCAPM.

Index	α	β	R^2	α	β_1	β_2	β_3	β_4	β_5	R^2
AUTO 5Y	0.43	0.32	2.82%	1.85**	0.62	1.44*	2.66**	0.49	-1.76*	32.48%
	(0.33)	(0.34)		(0.73)	(0.67)	(0.79)	(1.10)	(0.68)	(1.04)	
AUTO 10Y	2.08**	0.38	7.62%	3.22***	0.34	1.88***	1.17*	0.85*	-1.36**	46.64%
	(0.23)	(0.24)		(0.47)	(0.43)	(0.50)	(0.70)	(0.44)	(0.67)	
IND 5Y	0.31*	-0.29*	9.38%	1.57***	0.37	-0.16	0.46	-0.20	-2.12***	44.23%
	(0.16)	(0.16)		(0.33)	(0.3)	(0.35)	(0.49)	(0.31)	(0.47)	
IND 10Y	0.63**	-0.36**	18.60%	1.95***	0.27	-0.30	-0.14	-0.08	-2.30***	58.43%
	(0.13)	(0.14)		(0.26)	(0.23)	(0.27)	(0.38)	(0.24)	(0.36)	
CONS 5Y	0.13	-0.38**	24.60%	1.23***	0.17	-0.27	-0.14	-0.30	-1.98***	54.78%
	(0.12)	(0.12)		(0.24)	(0.22)	(0.26)	(0.36)	(0.22)	(0.34)	
CONS 10Y	0.55**	-0.34**	18.69%	1.75***	0.18	-0.16	-0.32	-0.06	-2.11***	55.71%
	(0.12)	(0.13)		(0.24)	(0.22)	(0.26)	(0.36)	(0.23)	(0.35)	
ENRG 5Y	0.03	-0.33**	19.93%	1.17***	0.17	-0.24	-0.16	-0.10	-2.00***	56.01%
	(0.12)	(0.12)		(0.23)	(0.21)	(0.25)	(0.35)	(0.22)	(0.33)	
ENRG 10Y	0.29**	-0.31**	15.91%	1.54***	0.18	-0.18	-0.35	0.11	-2.17***	59.47%
	(0.13)	(0.13)		(0.23)	(0.21)	(0.25)	(0.35)	(0.22)	(0.33)	
TMT 5Y	0.38**	-0.27**	13.51%	1.57***	0.16	-0.12	-0.20	0.15	-2.04***	58.78%
	(0.12)	(0.12)		(0.22)	(0.20)	(0.24)	(0.33)	(0.21)	(0.31)	
TMT 10Y	0.07	-0.30**	18.29%	1.14***	0.17	-0.20	-0.22	-0.07	-1.89***	54.33%
	(0.11)	(0.12)		(0.23)	(0.20)	(0.24)	(0.34)	(0.21)	(0.32)	

Table 1.5 reports results from a regression analysis of 10 Markit credit indices total returns. The regressor is real aggregate consumption growth. On the left, real aggregate consumption is used as a single regressor, while on the right it is separated in five orthogonal scales and each scale is used as a regressor. The table reports coefficients and standard errors in parenthesis. Significance is reported at the 10, 5 and 1% level with one, two and three asterisks respectively. The sample size is equal to 40 quarters.

Table 1.6: Fama and French regressions

								MKT						SMB					HML			
No	α	β_{MKT}	β_{SMB}	β_{HML}	R^2	α	β_1	β_2	β_3	β_4	β_5	β_1	β_2	β_3	β_4	β_5	β_1	β_2	β_3	β_4	β_5	R^2
1	-1.17**	1.06**	1.53**	-0.32**	91.58%	-0.84	0.97***	1.04***	1.31***	0.79**	0.56	1.58***	1.55***	1.54***	1.31***	0.87**	-0.31***	-0.38***	-0.28	-0.51**	0.42	89.32%
	(0.33)	(0.04)	(0.06)	(0.06)		(1.38)	(0.08)	(0.09)	(0.11)	(0.33)	(0.47)	(0.11)	(0.19)	(0.2)	(0.35)	(0.37)	(0.11)	(0.13)	(0.17)	(0.21)	(0.76)	
2	0.08	0.98**	1.35**	0.07	94.21%	0.13	0.99***	0.95***	1.07***	0.81***	0.76**	1.37***	1.26***	1.43***	1.7***	0.43	0.11	-0.09	0.19	0.03	0.82	91.76%
	(0.23)	(0.03)	(0.05)	(0.04)		(1.03)	(0.06)	(0.06)	(0.08)	(0.25)	(0.35)	(0.08)	(0.14)	(0.15)	(0.26)	(0.28)	(0.08)	(0.1)	(0.13)	(0.16)	(0.56)	
3	0.06	0.91**	1.21**	0.32**	94.58%	-0.69	0.85***	0.92***	1***	1.18***	0.96***	1.28***	1.11***	1.19***	1.56***	0.31	0.32***	0.25***	0.31***	0.44***	1.43***	92.01%
	(0.20)	(0.03)	(0.04)	(0.04)		(0.90)	(0.06)	(0.06)	(0.07)	(0.22)	(0.31)	(0.07)	(0.12)	(0.13)	(0.23)	(0.24)	(0.07)	(0.09)	(0.11)	(0.14)	(0.50)	
4	0.56**	0.88**	1.16**	0.46**	94.95%	-0.06	0.8***	0.93***	0.94***	1.07***	0.84***	1.24***	1.09***	1.06***	1.66***	0.05	0.47***	0.38***	0.44***	0.46***	1.68***	92.69%
	(0.18)	(0.02)	(0.04)	(0.03)		(0.82)	(0.05)	(0.05)	(0.07)	(0.2)	(0.28)	(0.06)	(0.11)	(0.12)	(0.21)	(0.22)	(0.06)	(0.08)	(0.1)	(0.13)	(0.45)	
5	0.37*	1.03**	1.24**	0.78**	95.31%	0.78	0.93***	1.04***	1.2***	1.4***	0.77**	1.36***	1.21***	1.10***	1.41***	0.46*	0.82***	0.71***	0.76***	0.64***	1.19**	92.91%
	(0.2)	(0.03)	(0.04)	(0.04)		(0.91)	(0.06)	(0.06)	(0.07)	(0.22)	(0.31)	(0.07)	(0.13)	(0.13)	(0.23)	(0.25)	(0.07)	(0.09)	(0.12)	(0.14)	(0.5)	
6	-0.35	1.09**	1.08**	-0.39**	95.29%	1.65*	1.09***	1.09***	1.16***	1.07***	0.33	1.14***	1.02***	0.95***	1.31***	0.78***	-0.47***	-0.30***	-0.26**	-0.55***	-1.03**	93.64%
	(0.22)	(0.03)	(0.04)	(0.04)		(0.94)	(0.06)	(0.06)	(0.08)	(0.23)	(0.32)	(0.07)	(0.13)	(0.14)	(0.24)	(0.25)	(0.07)	(0.09)	(0.12)	(0.14)	(0.52)	
7	0.01	0.99**	0.98**	0.11**	94.74%	0.50	1.01***	0.95***	1.02***	1.07***	0.70***	1.06***	0.91***	0.80***	1.14***	0.37*	0.01	0.22***	0.13	0.25**	0.39	93.78%
	(0.19)	(0.03)	(0.04)	(0.03)		(0.78)	(0.05)	(0.05)	(0.06)	(0.19)	(0.27)	(0.06)	(0.11)	(0.11)	(0.2)	(0.21)	(0.06)	(0.07)	(0.10)	(0.12)	(0.43)	
8	0.44**	0.96**	0.81**	0.36**	94.87%	1.13	0.98***	0.97***	0.96***	0.84***	0.64**	0.78***	0.85***	0.72***	1.23***	-0.04	0.28***	0.41***	0.28***	0.55***	0.64	93.30%
	(0.17)	(0.02)	(0.03)	(0.03)		(0.73)	(0.05)	(0.05)	(0.06)	(0.18)	(0.25)	(0.06)	(0.10)	(0.11)	(0.18)	(0.20)	(0.06)	(0.07)	(0.09)	(0.11)	(0.4)	
9	0.29	0.98**	0.76**	0.59**	94.10%	-0.16	1.04***	0.94***	0.94***	1.15***	0.97***	0.74***	0.84***	0.57***	1.15***	-0.03	0.60***	0.59***	0.47***	0.72***	1.45***	91.69%
	(0.18)	(0.02)	(0.04)	(0.03)		(0.81)	(0.05)	(0.05)	(0.06)	(0.2)	(0.28)	(0.06)	(0.11)	(0.12)	(0.2)	(0.22)	(0.06)	(0.08)	(0.1)	(0.12)	(0.44)	
10	0.01	1.07**	0.89**	0.85**	95.46%	0.93	1.08***	1.08***	1.06***	1.13***	0.67**	0.95***	0.93***	0.61***	1.07***	0.21	0.82***	0.96***	0.73***	0.87***	0.95**	93.94%
	(0.18)	(0.02)	(0.03)	(0.03)		(0.76)	(0.05)	(0.05)	(0.06)	(0.19)	(0.26)	(0.06)	(0.11)	(0.11)	(0.19)	(0.21)	(0.06)	(0.07)	(0.1)	(0.12)	(0.42)	
11	0.05	1.06**	0.75**	-0.46**	95.46%	0.71	1.02***	1.05***	1.1***	1.29***	0.78***	0.82***	0.81***	0.62***	0.83***	0.31	-0.58***	-0.36***	-0.35***	-0.57***	-0.45	94.35%
	(0.19)	(0.02)	(0.04)	(0.03)		(0.8)	(0.05)	(0.05)	(0.06)	(0.19)	(0.27)	(0.06)	(0.11)	(0.11)	(0.2)	(0.22)	(0.06)	(0.08)	(0.1)	(0.12)	(0.44)	
12	0.36*	1.01**	0.63**	0.13**	93.40%	1.12	0.98***	1.06***	1***	1.41***	0.69**	0.63***	0.61***	0.5***	0.96***	0.18	0.1	0.07	0.07	0.37***	0.11	91.49%
	(0.19)	(0.03)	(0.04)	(0.04)		(0.82)	(0.05)	(0.05)	(0.07)	(0.2)	(0.28)	(0.06)	(0.11)	(0.12)	(0.21)	(0.22)	(0.06)	(0.08)	(0.1)	(0.13)	(0.45)	
13	0.10	0.97**	0.54**	0.41**	91.80%	1.12	1.04***	0.96***	0.91***	0.91***	0.65**	0.48***	0.58***	0.39***	0.81***	0.35	0.36***	0.5***	0.29***	0.6***	0.04	90.37%
	(0.20)	(0.03)	(0.04)	(0.04)		(0.81)	(0.05)	(0.05)	(0.06)	(0.2)	(0.28)	(0.06)	(0.11)	(0.12)	(0.2)	(0.22)	(0.06)	(0.08)	(0.1)	(0.12)	(0.44)	
14	0.18	1.03**	0.46**	0.65**	91.90%	1.79**	1.03***	1.05***	1.13***	0.88***	0.45*	0.39***	0.48***	0.24**	0.96***	-0.03	0.58***	0.73***	0.59***	0.72***	0.23	91.17%
	(0.2)	(0.03)	(0.04)	(0.04)		(0.78)	(0.05)	(0.05)	(0.06)	(0.19)	(0.27)	(0.06)	(0.11)	(0.11)	(0.2)	(0.21)	(0.06)	(0.07)	(0.1)	(0.12)	(0.43)	
15	0.30	1.00**	0.69**	0.8**	90.03%	1.37	1.05***	1.08***	0.96***	0.71***	0.64**	0.66***	0.46***	0.78***	0.79***	0.12	0.74***	0.92***	0.81***	0.84***	0.65	88.78%
	(0.24)	(0.03)	(0.05)	(0.04)		(0.96)	(0.06)	(0.06)	(0.08)	(0.23)	(0.33)	(0.07)	(0.13)	(0.14)	(0.24)	(0.26)	(0.07)	(0.09)	(0.12)	(0.15)	(0.53)	
16	0.53**	1.05**	0.42**	-0.46**	94.15%	1.14	1.09***	1.09***	1.01***	1.14***	0.69**	0.34***	0.50***	0.38***	0.51***	-0.13	-0.44***	-0.43***	-0.62***	-0.49***	-0.22	93.47%
	(0.20)	(0.03)	(0.04)	(0.04)		(0.78)	(0.05)	(0.05)	(0.06)	(0.19)	(0.27)	(0.06)	(0.11)	(0.11)	(0.20)	(0.21)	(0.06)	(0.07)	(0.10)	(0.12)	(0.43)	
17	-0.15	1.03**	0.35**	0.18**	89.81%	0.08	1.05***	1.03***	1.05***	1.22***	0.93***	0.34***	0.33***	0.16	0.65***	0.1	0.09	0.23***	0.15	0.37***	0.26	89.04%
	(0.23)	(0.03)	(0.04)	(0.04)		(0.87)	(0.05)	(0.05)	(0.07)	(0.21)	(0.30)	(0.07)	(0.12)	(0.13)	(0.22)	(0.24)	(0.07)	(0.08)	(0.11)	(0.13)	(0.48)	
18	-0.01	1.05**	0.25**	0.44**	90.48%	0.44	1.02***	1.07***	1.1***	1.44***	0.65**	0.22***	0.26**	0.13	0.67***	-0.21	0.37***	0.39***	0.37***	0.67***	0.82*	88.88%
	(0.21)	(0.03)	(0.04)	(0.04)		(0.84)	(0.05)	(0.05)	(0.07)	(0.20)	(0.29)	(0.07)	(0.12)	(0.12)	(0.21)	(0.23)	(0.07)	(0.08)	(0.11)	(0.13)	(0.46)	
19	0.23	1.03**	0.29**	0.57**	90.50%	1.46*	1.01***	1.05***	1.13***	1.21***	0.39	0.25***	0.36***	0.16	0.57***	-0.34	0.51***	0.55***	0.57***	0.68***	0.66	89.99%
	(0.21)	(0.03)	(0.04)	(0.04)		(0.79)	(0.05)	(0.05)	(0.06)	(0.19)	(0.27)	(0.06)	(0.11)	(0.11)	(0.20)	(0.21)	(0.06)	(0.08)	(0.10)	(0.12)	(0.44)	
20	-0.25	1.13**	0.39**	0.78**	88.90%	0.57	1.15***	1.03***	1.3***	1.27***	0.68**	0.42***	0.50***	0.20	0.37	-0.59**	0.72***	0.85***	0.83***	0.83***	1.21**	88.18%
	(0.26)	(0.03)	(0.05)	(0.05)		(0.98)	(0.06)	(0.06)	(0.08)	(0.24)	(0.34)	(0.08)	(0.14)	(0.14)	(0.25)	(0.26)	(0.08)	(0.09)	(0.12)	(0.15)	(0.54)	
21	0.46**	1.00**	-0.23**	-0.34**	94.98%	1.06	0.98***	1.03***	0.99***	0.94***	0.74***	-0.18***	-0.28***	-0.30***	-0.16	-0.83***	-0.37***	-0.28***	-0.34***	-0.33***	-0.19	92.58%
	(0.14)	(0.02)	(0.03)	(0.03)		(0.65)	(0.04)	(0.04)	(0.05)	(0.16)	(0.22)	(0.05)	(0.09)	(0.09)	(0.16)	(0.18)	(0.05)	(0.06)	(0.08)	(0.10)	(0.36)	
22	0.07	0.98**	-0.17**	0.10**	89.83%	0.04	0.99***	0.95***	1.08***	0.94***	0.92***	-0.15**	-0.17	-0.42***	0.01	-0.41**	0.07	0.06	0.01	0.23**	0.37	87.84%
	(0.19)	(0.02)	(0.04)	(0.03)		(0.76)	(0.05)	(0.05)	(0.06)	(0.19)	(0.26)	(0.06)	(0.1)	(0.11)	(0.19)	(0.21)	(0.06)	(0.07)	(0.10)	(0.12)	(0.42)	
23	-0.02	0.93**	-0.21**	0.28**	84.19%	-0.18	0.91***	0.93***	0.98***	1.16***	0.96***	-0.23***	-0.29**	-0.25**	0.03	-0.39	0.25***	0.26***	0.22*	0.49***	0.50	81.44%
	(0.22)	(0.03)	(0.04)	(0.04)		(0.88)	(0.05)	(0.05)	(0.07)	(0.21)	(0.30)	(0.07)	(0.12)	(0.13)	(0.22)	(0.24)	(0.07)	(0.08)	(0.11)	(0.13)	(0.48)	
24	-0.39**	0.98**	-0.14**	0.58**	90.94%	0.07	0.95***	0.97***	1.05***	1.08***	0.76***	-0.08	-0.16	-0.33***	-0.02	-0.54***	0.50***	0.59***	0.69***	0.63***	0.71*	88.30%
	(0.17)	(0.02)	(0.03)	(0.03)		(0.72)	(0.04)	(0.04)	(0.06)	(0.18)	(0.25)	(0.06)	(0.1)	(0.1)	(0.18)	(0.19)	(0.06)	(0.07)	(0.09)	(0.11)	(0.40)	
25	-0.34	1.06**	-0.13**	0.67**	82.72%	0.58	1.10***	0.98***	1.11***	0.69***	0.89**	-0.13	-0.07	-0.15	-0.05	-0.54*	0.63***	0.66***	0.87***	0.64***	0.34	80.29%
	(0.27)	(0.03)	(0.05)	(0.05)		(1.07)	(0.07)	(0.07)	(0.09)	(0.26)	(0.36)	(0.08)	(0.15)	(0.15)	(0.27)	(0.29)	(0.08)	(0.1)	(0.13)	(0.16)	(0.59)	
26	-2.48***	0.02	0.01	-0.01	1.43%	0.27	0.01	0.01	0.03	0.21***	-0.63***	0.00	0.03	0.02	0.06	0.20**	0.01	0.00	-0.04	0.04	-1.74***	36.47%
	(0.11)	(0.014)	(0.021)	(0.02)		(0.33)	(0.02)	(0.02)	(0.03)	(0.08)	(0.11)	(0.03)	(0.04)	(0.05)	(0.08)	(0.09)	(0.03)	(0.03)	(0.04)	(0.05)	(0.18)	

Table 1.6 reports results from a regression analysis of 25 Fama and French porfolios. The 26th portfolio is the CRSP US value weighted index. The regressors are the typical Fama and French (1993) factors downloaded by French's online data library. The table reports coefficients and standard errors in parenthesis. Significance is reported at the 10, 5 and 1% level with one, two and three asterisks respectively. The sample size is equal to 40 quarters.

Table 1.7: Fixed income total returns unrestricted factor model.

Maturity (months)	α	$\beta_{d(INFL)}$	β_{CONS}	β_{LVL}	β_{SLP}	β_{CRV}	R^2
0-6	-0.03	-0.13**	-0.03	0.01	0.04	0.07	15.63%
	(0.07)	(0.03)	(0.02)	(0.01)	(0.03)	(0.06)	
6-12	-0.03	-0.3**	-0.05	0.01	0.13	0.11	15.41%
	(0.17)	(0.07)	(0.04)	(0.02)	(0.08)	(0.15)	
12-18	-0.08	-0.46**	-0.07	0.01	0.2	0.16	15.82%
	(0.26)	(0.1)	(0.07)	(0.03)	(0.12)	(0.23)	
18-24	-0.04	-0.57**	-0.1	0	0.27*	0.18	15.91%
	(0.33)	(0.13)	(0.08)	(0.04)	(0.15)	(0.29)	
24-30	-0.04	-0.7**	-0.09	0.00	0.34*	0.14	15.06%
	(0.41)	(0.16)	(0.1)	(0.05)	(0.19)	(0.36)	
30-36	-0.06	-0.8**	-0.11	0.00	0.43*	0.10	15.76%
	(0.47)	(0.18)	(0.12)	(0.06)	(0.22)	(0.41)	
36-42	0	-0.89**	-0.11	-0.01	0.47*	0.07	15.78%
	(0.51)	(0.2)	(0.13)	(0.06)	(0.24)	(0.45)	
42-48	0.12	-1.04**	-0.14	-0.03	0.48*	0.05	15.90%
	(0.56)	(0.22)	(0.14)	(0.07)	(0.26)	(0.49)	
48-54	0.17	-1.13**	-0.15	-0.04	0.55*	-0.03	16.91%
	(0.6)	(0.24)	(0.15)	(0.07)	(0.28)	(0.52)	
54-60	-0.15	-1.26**	-0.15	-0.01	0.73**	-0.31	16.33%
	(0.67)	(0.27)	(0.17)	(0.08)	(0.31)	(0.59)	
60-120	0.52	-1.52**	-0.16	-0.08	0.53	-0.01	15.47%
	(0.78)	(0.31)	(0.19)	(0.1)	(0.36)	(0.68)	
0-12	-0.03	-0.22**	-0.04	0.01	0.09	0.09	15.21%
	(0.13)	(0.05)	(0.03)	(0.02)	(0.06)	(0.11)	
12-24	-0.05	-0.52**	-0.08	0.01	0.23*	0.17	15.95%
	(0.29)	(0.12)	(0.07)	(0.04)	(0.14)	(0.26)	
24-36	0.06	-0.77**	-0.12	-0.01	0.34	0.2	15.45%
	(0.44)	(0.17)	(0.11)	(0.06)	(0.2)	(0.38)	
36-48	0.16	-0.97**	-0.14	-0.03	0.44*	0.13	15.83%
	(0.54)	(0.21)	(0.14)	(0.07)	(0.25)	(0.47)	
48-60	0.26	-1.2**	-0.18	-0.05	0.53*	0.03	16.28%
	(0.63)	(0.25)	(0.16)	(0.08)	(0.29)	(0.55)	

Table 1.7 reports results from a regression analysis of 16 Fama bond portfolios total returns, sorted by average maturity. The regressors are some common macro factors used for fixed income returns, namely changes in inflation rates, aggregate real consumption growth, and the level, slope and curvature of the US zero rates curve. The table reports coefficients and standard errors in parenthesis. Significance is reported at the 10, 5 and 1% level with one, two and three asterisks respectively. The sample size is equal to 220 quarters.

Table 1.8: Fixed income total returns restricted factor model.

Maturity (months)	α	$\beta_{d(INFL)}$	β_{CONS}	β_{LVL}	β_{SLP}	β_{CRV}	R^2
0-6	0.02	-0.12**			0.06**		12.81%
	(0.03)	(0.03)			(0.02)		
6-12	-0.01	-0.28**			0.16**		14.46%
	(0.06)	(0.07)			(0.04)		
12-18	-0.02	-0.43**			0.24**		14.79%
	(0.10)	(0.10)			(0.06)		
18-24	-0.09	-0.53**			0.33**		15.21%
	(0.12)	(0.12)			(0.08)		
24-30	-0.11	-0.66**			0.40**		14.73%
	(0.15)	(0.15)			(0.1)		
30-36	-0.15	-0.75**			0.47**		15.42%
	(0.17)	(0.18)			(0.11)		
36-42	-0.16	-0.84**			0.51**		15.49%
	(0.19)	(0.19)			(0.12)		
42-48	-0.17	-0.97**			0.53**		15.45%
	(0.21)	(0.21)			(0.13)		
48-54	-0.20	-1.05**			0.59**		16.30%
	(0.22)	(0.22)			(0.14)		
54-60	-0.32	-1.17**			0.63**		15.67%
	(0.25)	(0.25)			(0.16)		
60-120	-0.14	-1.43**			0.61**		14.78%
	(0.28)	(0.29)			(0.18)		
0-12	0.01	-0.20**			0.11**		13.91%
	(0.05)	(0.05)			(0.03)		
12-24	-0.05	-0.48**			0.29**		15.13%
	(0.11)	(0.11)			(0.07)		
24-36	-0.10	-0.72**			0.42**		14.94%
	(0.16)	(0.17)			(0.10)		
36-48	-0.14	-0.90**			0.52**		15.38%
	(0.20)	(0.20)			(0.13)		
48-60	-0.20	-1.10**			0.59**		15.56%
	(0.23)	(0.24)			(0.15)		

Table 1.8 reports results from a regression analysis of 16 Fama bond portfolios total returns, sorted by average maturity. The regressors are some common macro factors used for fixed income returns, namely changes in inflation rates, aggregate real consumption growth, and the level, slope and curvature of the US zero rates curve, where the coefficients on the statistically insignificant factors are restricted to zero. The table reports coefficients and standard errors in parenthesis. Significance is reported at the 10, 5 and 1% level with one, two and three asterisks respectively. The sample size is equal to 220 quarters.

Table 1.9: Fixed income total returns unrestricted scale factor model.

Model	α	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8	β_9	β_{10}	β_{11}	β_{12}	β_{13}	β_{14}	β_{15}	R^2
0-6	-0.07	-0.12**	-0.16*	-0.09	-0.06**	0	0.02	0.29**	-0.01	0.01	0.33**	-0.15	0.04	-0.1	0.08	0.11	23.96%
	(0.09)	(0.03)	(0.08)	(0.24)	(0.03)	(0.05)	(0.05)	(0.08)	(0.08)	(0.01)	(0.14)	(0.15)	(0.04)	(0.18)	(0.21)	(0.08)	
6-12	-0.16	-0.27**	-0.47**	0.26	-0.18**	-0.02	0.05	0.65**	0.02	0	0.78**	-0.47	0.12	-0.51	0.26	0.34*	26.62%
	(0.2)	(0.07)	(0.19)	(0.56)	(0.06)	(0.11)	(0.11)	(0.18)	(0.19)	(0.03)	(0.32)	(0.35)	(0.09)	(0.41)	(0.48)	(0.19)	
12-18	-0.34	-0.4**	-0.78**	0.38	-0.29**	-0.05	0.1	0.93**	0.06	0	0.98**	-0.85	0.22	-0.63	0.55	0.49*	28.47%
	(0.29)	(0.10)	(0.28)	(0.83)	(0.10)	(0.16)	(0.17)	(0.27)	(0.28)	(0.04)	(0.47)	(0.51)	(0.13)	(0.62)	(0.71)	(0.28)	
18-24	-0.42	-0.48**	-1**	0.79	-0.36**	-0.09	0.11	1.16**	0.06	0.00	1.24**	-1.06	0.32*	-0.79	0.84	0.56	28.13%
	(0.37)	(0.12)	(0.36)	(1.06)	(0.12)	(0.20)	(0.21)	(0.34)	(0.35)	(0.05)	(0.6)	(0.65)	(0.17)	(0.78)	(0.91)	(0.35)	
24-30	-0.49	-0.61**	-1.22**	1.49	-0.37**	-0.15	0.08	1.33**	0.1	0.00	1.56**	-1.28	0.41*	-0.98	1.08	0.63	25.41%
	(0.47)	(0.16)	(0.46)	(1.34)	(0.15)	(0.26)	(0.27)	(0.43)	(0.45)	(0.06)	(0.76)	(0.83)	(0.21)	(1.00)	(1.15)	(0.45)	
30-36	-0.63	-0.71**	-1.29**	1.32	-0.45**	-0.17	0.13	1.49**	0.01	0.00	1.71*	-1.72*	0.53**	-1.11	1.75	0.58	26.40%
	(0.53)	(0.18)	(0.52)	(1.52)	(0.17)	(0.29)	(0.3)	(0.48)	(0.51)	(0.07)	(0.85)	(0.94)	(0.24)	(1.12)	(1.3)	(0.51)	
36-42	-0.65	-0.79**	-1.50**	2.15	-0.50**	-0.22	0.08	1.50**	-0.02	0.01	1.50	-1.84*	0.60**	-1.13	2.03	0.65	26.32%
	(0.59)	(0.20)	(0.57)	(1.67)	(0.19)	(0.32)	(0.33)	(0.53)	(0.56)	(0.08)	(0.94)	(1.03)	(0.26)	(1.24)	(1.43)	(0.56)	
42-48	-0.74	-0.91**	-1.6**	2.6	-0.56**	-0.33	0.1	1.38**	-0.07	0.01	1.28	-2.43**	0.68**	-0.74	2.47	0.59	26.22%
	(0.64)	(0.22)	(0.62)	(1.83)	(0.21)	(0.35)	(0.37)	(0.58)	(0.61)	(0.08)	(1.03)	(1.13)	(0.29)	(1.36)	(1.57)	(0.61)	
48-54	-0.68	-1.02**	-1.55**	2.63	-0.57**	-0.33	0.10	1.38**	-0.2	0.00	1.45	-2.31*	0.72**	-1.26	2.69	0.64	25.62%
	(0.69)	(0.23)	(0.67)	(1.96)	(0.23)	(0.37)	(0.39)	(0.62)	(0.65)	(0.09)	(1.10)	(1.21)	(0.31)	(1.45)	(1.68)	(0.66)	
54-60	-0.92	-1.1**	-1.81**	2.52	-0.71**	-0.46	0.01	1.47**	-0.03	0.04	1.41	-1.77	0.86**	-0.92	1.81	0.38	23.54%
	(0.78)	(0.26)	(0.76)	(2.23)	(0.26)	(0.42)	(0.45)	(0.71)	(0.75)	(0.1)	(1.26)	(1.38)	(0.35)	(1.65)	(1.91)	(0.75)	
60-120	-0.77	-1.32**	-2.2**	3.28	-0.77**	-0.64	0.22	1.44*	-0.11	-0.03	0.66	-2.59	0.83**	-0.14	2.64	0.71	23.83%
	(0.90)	(0.30)	(0.87)	(2.56)	(0.29)	(0.49)	(0.51)	(0.82)	(0.86)	(0.12)	(1.44)	(1.58)	(0.40)	(1.90)	(2.19)	(0.86)	
0-12	-0.12	-0.19**	-0.32**	0.10	-0.12**	-0.01	0.03	0.49**	0.01	0.00	0.58**	-0.3	0.08	-0.31	0.13	0.23*	25.63%
	(0.14)	(0.05)	(0.14)	(0.41)	(0.05)	(0.08)	(0.08)	(0.13)	(0.14)	(0.02)	(0.23)	(0.25)	(0.06)	(0.30)	(0.35)	(0.14)	
12-24	-0.37	-0.44**	-0.89**	0.59	-0.33**	-0.07	0.10	1.04**	0.06	0.00	1.12**	-0.97	0.26*	-0.73	0.71	0.53*	28.45%
	(0.33)	(0.11)	(0.32)	(0.94)	(0.11)	(0.18)	(0.19)	(0.30)	(0.31)	(0.04)	(0.53)	(0.58)	(0.15)	(0.70)	(0.80)	(0.32)	
24-36	-0.48	-0.66**	-1.31**	1.24	-0.44**	-0.18	0.12	1.37**	0.11	-0.01	1.45*	-1.46	0.42*	-0.85	1.32	0.67	26.25%
	(0.50)	(0.17)	(0.49)	(1.43)	(0.16)	(0.27)	(0.29)	(0.46)	(0.48)	(0.06)	(0.81)	(0.88)	(0.23)	(1.06)	(1.22)	(0.48)	
36-48	-0.61	-0.85**	-1.49**	2.05	-0.53**	-0.32	0.14	1.36**	0.01	-0.01	1.15	-1.77	0.60**	-0.61	1.80	0.68	25.24%
	(0.62)	(0.21)	(0.6)	(1.77)	(0.2)	(0.34)	(0.35)	(0.56)	(0.59)	(0.08)	(1.00)	(1.09)	(0.28)	(1.31)	(1.51)	(0.59)	
48-60	-0.73	-1.03**	-1.75**	2.35	-0.67**	-0.41	0.16	1.29*	-0.15	-0.01	0.89	-2.25*	0.76**	-0.52	2.56	0.58	25.06%
	(0.73)	(0.25)	(0.71)	(2.08)	(0.24)	(0.4)	(0.42)	(0.66)	(0.70)	(0.09)	(1.17)	(1.28)	(0.33)	(1.54)	(1.78)	(0.70)	

Table 1.9 reports results from a regression analysis of 16 Fama bond portfolios total returns. The regressors are some common macro factors used for fixed income returns, namely changes in inflation rates, aggregate real consumption growth, and the level, slope and curvature of the US zero rates curve, but are decomposed in three relevant scales. The table reports coefficients and standard errors in parenthesis. Significance is reported at the 10, 5 and 1% level with one, two and three asterisks respectively. The sample size is equal to 220 quarters.

Table 1.10: Fixed income total returns restricted scale factor model.

Model	α	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8	β_9	β_{10}	β_{11}	β_{12}	β_{13}	β_{14}	β_{15}	R^2
0-6	0.01	-0.11**	-0.17*		-0.06*			0.15**					0.07**				15.50%
	(0.03)	(0.03)	(0.08)		(0.03)			(0.06)					(0.02)				
6-12	-0.05	-0.25**	-0.49**		-0.18**			0.32**					0.2**				20.20%
	(0.06)	(0.07)	(0.19)		(0.07)			(0.13)					(0.04)				
12-18	-0.1	-0.38**	-0.82**		-0.29**			0.51**					0.32**				22.10%
	(0.1)	(0.1)	(0.28)		(0.1)			(0.19)					(0.07)				
18-24	-0.18	-0.46**	-1.06**		-0.37**			0.63**					0.43**				22.80%
	(0.12)	(0.13)	(0.36)		(0.12)			(0.25)					(0.08)				
24-30	-0.23	-0.58**	-1.3**		-0.38**			0.67**					0.51**				20.40%
	(0.15)	(0.16)	(0.45)		(0.16)			(0.31)					(0.1)				
30-36	-0.29*	-0.68**	-1.4**		-0.46**			0.76**					0.61**				21.60%
	(0.17)	(0.18)	(0.51)		(0.18)			(0.35)					(0.12)				
36-42	-0.32*	-0.76**	-1.62**		-0.51**			0.87**					0.67**				22.30%
	(0.19)	(0.2)	(0.56)		(0.19)			(0.38)					(0.13)				
42-48	-0.35*	-0.89**	-1.8**		-0.56**			0.83*					0.71**				21.90%
	(0.21)	(0.22)	(0.62)		(0.21)			(0.42)					(0.14)				
48-54	-0.39*	-0.99**	-1.75**		-0.59**			0.78*					0.79**				22.10%
	(0.22)	(0.24)	(0.66)		(0.23)			(0.45)					(0.15)				
54-60	-0.51**	-1.07**	-2.03**		-0.72**			0.87*					0.83**				21.10%
	(0.25)	(0.27)	(0.74)		(0.26)			(0.51)					(0.17)				
60-120	-0.4	-1.31**	-2.52**		-0.76**			1.13*					0.88**				20.90%
	(0.29)	(0.31)	(0.85)		(0.3)			(0.58)					(0.2)				
0-12	-0.02	-0.18**	-0.34**		-0.12**			0.24**					0.14**				18.70%
	(0.05)	(0.05)	(0.14)		(0.05)			(0.1)					(0.03)				
12-24	-0.14	-0.42**	-0.94**		-0.33**			0.56**					0.37**				22.70%
	(0.11)	(0.11)	(0.32)		(0.11)			(0.22)					(0.07)				
24-36	-0.23	-0.63**	-1.41**		-0.45**			0.74**					0.56**				21.50%
	(0.16)	(0.17)	(0.48)		(0.17)			(0.33)					(0.11)				
36-48	-0.3	-0.83**	-1.66**		-0.53**			0.86**					0.69**				21.80%
	(0.2)	(0.21)	(0.59)		(0.2)			(0.41)					(0.14)				
48-60	-0.41*	-1.02**	-1.97**		-0.67**			0.91*					0.81**				22.20%
	(0.23)	(0.25)	(0.69)		(0.24)			(0.47)					(0.16)				

Table 1.10 reports results from a regression analysis of 16 Fama bond portfolios total returns. The regressors are some common macro factors used for fixed income returns, namely changes in inflation rates, aggregate real consumption growth, and the level, slope and curvature of the US zero rates curve, where the coefficients on the statistically insignificant factors are restricted to zero. The table reports coefficients and standard errors in parenthesis. Significance is reported at the 10, 5 and 1% level with one, two and three asterisks respectively. The sample size is equal to 220 quarters.

Table 1.11: Market prices of risk

Asset class	λ_0	λ_1	R^2	λ_0	λ_1	λ_2	λ_3	λ_4	λ_5	R^2
Equity	1.88***	0.48**	19.70%	2.98**	0.02	0.54***	0.05	0.04	0.12	63.61%
	(0.20)	(0.19)		(1.16)	(0.33)	(0.16)	(0.15)	(0.1)	(0.32)	
Fixed income 1	1.00***	-0.01	0.00%	0.46*	-0.28***	0.07	0.09*	0.14	-0.23	97.96%
	(0.15)	(0.22)		(0.22)	(0.09)	(0.08)	(0.05)	(0.2)	(0.19)	
Fixed income 2	0.11***	-0.63***	87.00%	0.06*	-0.24**	-0.79**	-0.07	-0.34**	0.43	93.80%
	(0.02)	(0.06)		(0.03)	(0.32)	(0.3)	(0.08)	(0.14)	(0.27)	
Credit Default Swap 1	-0.28***	0.05**	38.00%	-0.26**	0.02	0.03	-0.01	0.03	0.03	57.84%
	(0.02)	(0.018)		(0.1)	(0.02)	(0.03)	(0.01)	(0.04)	(0.05)	
Credit Default Swap 2	-0.41***	-0.03	11.50%	-1.07***	-0.05***	0.04	0.01***	-0.10	-0.33**	94.59%
	(0.06)	(0.03)		(0.22)	(0.01)	(0.03)	(0.00)	(0.07)	(0.12)	

Table 1.11 reports results from a cross sectional OLS regression of expected returns over betas. The table reports coefficients and standard errors in parenthesis. Significance is reported at the 10, 5 and 1% level with one, two and three asterisks respectively.

The linear cross-sectional regression is: $\bar{r}_i^e = \lambda_0 + \lambda_1 \beta_i + \varepsilon_i$

Table 1.12: Gibbons Ross Shanken test

Asset class	χ^2 statistic		P value	
	Standard CCAPM	Business cycle CCAPM	Standard CCAPM	Business cycle CCAPM
Equity	3.10	12.20	3.14×10^{-6}	0.00
Fixed Income 1	7.20	28.00	8.06×10^{-8}	0.00
Fixed Income 2	1.70	11.50	0.04	0.00
Credit Default Swap 1	210.30	791.30	0.00	0.00
Credit Default Swap 2	134.70	1134.10	1.88×10^{-15}	0.00

Table 1.12 reports results from a Gibbons Ross Shanken test on the alphas of each regression model. The GRS test rejects the null of all alphas being jointly equal to zero for both models.

Table 1.13: Akaike Information Criterion and Likelihood Ratio test

Asset class	AIC		LR test
	Standard CCAPM	Business cycle CCAPM	
Equity	219.60	196.71	10.98**
Fixed Income 1	147.13	91.29	7.83*
Fixed Income 2	208.89	191.47	1.00
Credit Default Swap 1	106.20	80.51	2.22
Credit Default Swap 2	119.59	82.59	2.71

Table 1.13 reports the Akaike Information Criterion and the Likelihood Ratio test statistic for both models. The lower AIC signals the better model. Significance at 10%, 5% and 1% is reported with one, two and three stars respectively.

Table 1.14: HML regressions versus real consumption growth

	α	β	R^2	α	β_1	β_2	β_3	β_4	β_5	R^2
Low	2.15**	0.51	0.39%	2.40*	0.10	-2.15	6.63***	0.46	0.06	7.81%
	(0.73)	(0.54)		(1.39)	(0.72)	(1.3)	(1.65)	(1.68)	(2.25)	
2	2.40***	0.61	0.70%	2.57*	0.39	-2.03*	5.36***	0.51	0.31	6.80%
	(0.65)	(0.49)		(1.26)	(0.65)	(1.17)	(1.50)	(1.52)	(2.03)	
3	2.45***	0.64	0.83%	3.22**	0.54	-1.83	4.99***	0.43	-0.76	6.50%
	(0.63)	(0.47)		(1.21)	(0.63)	(1.13)	(1.44)	(1.47)	(1.96)	
4	2.60***	0.42	0.33%	3.24**	0.27	-2.56**	5.29***	0.07	-0.74	7.48%
	(0.65)	(0.48)		(1.24)	(0.64)	(1.16)	(1.48)	(1.50)	(2.00)	
5	2.47***	0.66	1.02%	2.85**	0.49	-1.70	4.30***	0.50	-0.02	5.78%
	(0.58)	(0.43)		(1.13)	(0.59)	(1.05)	(1.34)	(1.36)	(1.82)	
6	2.61***	0.72	1.15%	2.57**	0.38	-2.19**	4.74***	1.41	0.80	7.39%
	(0.6)	(0.45)		(1.15)	(0.6)	(1.07)	(1.37)	(1.39)	(1.86)	
7	2.74***	0.73	1.12%	2.8**	0.24	-1.38	5.05***	0.68	0.61	6.25%
	(0.61)	(0.46)		(1.19)	(0.62)	(1.1)	(1.41)	(1.43)	(1.92)	
8	2.87***	0.80*	1.27%	2.71**	0.44	-1.93*	5.30***	1.23	1.08	7.40%
	(0.63)	(0.47)		(1.22)	(0.63)	(1.13)	(1.45)	(1.47)	(1.96)	
9	3.04***	0.87*	1.39%	3.38**	0.78	-2.35*	5.05***	1.22	0.27	7.12%
	(0.66)	(0.49)		(1.27)	(0.66)	(1.19)	(1.51)	(1.54)	(2.06)	
High	3.14***	1.42**	2.57%	4.06**	1.15	-2.02	6.31***	2.58	-0.25	7.56%
	(0.79)	(0.59)		(1.53)	(0.79)	(1.42)	(1.81)	(1.84)	(2.46)	
High - Low	0.99	0.91		1.66	1.05	0.13	-0.32	2.12	-0.31	

Table 1.14 reports results from a regression analysis of 10 portfolios sorted by Value. The regressor is aggregate consumption growth. The table reports coefficients and standard errors in parenthesis. Significance is reported at the 10, 5 and 1% level with one, two and three asterisks respectively. The sample size is equal to 220 quarters.

Table 1.15: SMB regressions versus real consumption growth

	α	β	R^2	α	β_1	β_2	β_3	β_4	β_5	R^2
Small	2.46**	1.77**	2.82%	3.73**	1.66*	-1.42	8.33***	0.75	-0.53	7.86%
	(0.93)	(0.70)		(1.81)	(0.94)	(1.68)	(2.15)	(2.18)	(2.92)	
2	2.38***	1.68**	2.84%	3.54**	1.67*	-1.74	7.76***	0.84	-0.42	8.06%
	(0.88)	(0.66)		(1.71)	(0.89)	(1.59)	(2.03)	(2.06)	(2.76)	
3	2.80***	1.31**	1.91%	3.85**	1.33	-1.93	6.93***	0.70	-0.61	7.11%
	(0.84)	(0.63)		(1.63)	(0.85)	(1.52)	(1.94)	(1.97)	(2.63)	
4	2.62***	1.24**	1.86%	3.64**	1.09	-1.59	7.01***	0.54	-0.61	7.17%
	(0.81)	(0.61)		(1.57)	(0.81)	(1.46)	(1.86)	(1.89)	(2.53)	
5	2.73****	1.19**	1.84%	3.86**	1.11	-1.72	6.28***	0.96	-0.86	6.71%
	(0.78)	(0.58)		(1.52)	(0.79)	(1.41)	(1.80)	(1.83)	(2.45)	
6	2.66****	1.09*	1.76%	3.89**	0.93	-1.45	6.43***	0.78	-1.15	7.47%
	(0.73)	(0.55)		(1.41)	(0.73)	(1.31)	(1.68)	(1.70)	(2.28)	
7	2.71***	0.96*	1.38%	3.43**	0.76	-1.66	6.19***	0.59	-0.34	6.82%
	(0.73)	(0.55)		(1.42)	(0.73)	(1.32)	(1.68)	(1.71)	(2.29)	
8	2.66***	0.82	1.09%	3.17**	0.66	-1.68	5.11***	0.70	-0.11	5.41%
	(0.70)	(0.52)		(1.36)	(0.71)	(1.27)	(1.62)	(1.64)	(2.2)	
9	2.68***	0.48	0.45%	2.92**	0.25	-2.16*	4.97***	0.24	0.04	6.28%
	(0.64)	(0.48)		(1.24)	(0.65)	(1.16)	(1.48)	(1.50)	(2.01)	
Big	2.24***	0.50	0.61%	2.21*	0.16	-2.23**	4.97***	0.85	0.56	8.06%
	(0.58)	(0.43)		(1.11)	(0.57)	(1.03)	(1.31)	(1.34)	(1.79)	
Small-Big	0.22	1.27		1.51	1.49	0.81	3.36	-0.10	-1.09	

Table 1.15 reports results from a regression analysis of 10 portfolios sorted by Size. The regressor is aggregate consumption growth. The table reports coefficients and standard errors in parenthesis. Significance is reported at the 10, 5 and 1% level with one, two and three asterisks respectively. The sample size is equal to 220 quarters.

Table 1.16: HML regressions versus MKT

	α	β	R^2	α	β_1	β_2	β_3	β_4	β_5	R^2
Low	0.79**	1.09**	87%	0.8*	1.12**	1.08**	1.03**	1.21**	1.11**	85.00%
	(0.24)	(0.03)		(0.44)	(0.05)	(0.05)	(0.07)	(0.23)	(0.24)	
2	1.22**	1.00**	92%	1.34**	1.09**	0.96**	0.97**	0.72**	0.95**	90.00%
	(0.17)	(0.02)		(0.31)	(0.03)	(0.04)	(0.05)	(0.16)	(0.17)	
3	1.35**	0.96**	91%	1.59**	1.00**	0.96**	0.94**	0.83**	0.82**	89.00%
	(0.18)	(0.02)		(0.32)	(0.04)	(0.04)	(0.05)	(0.17)	(0.18)	
4	1.37**	0.96**	86%	1.74**	0.99**	0.95**	1.02**	0.64**	0.74**	85.00%
	(0.22)	(0.03)		(0.39)	(0.04)	(0.05)	(0.06)	(0.20)	(0.21)	
5	1.55**	0.85**	82%	1.82**	0.85**	0.84**	0.90**	0.72**	0.68**	80.00%
	(0.23)	(0.03)		(0.40)	(0.04)	(0.05)	(0.07)	(0.21)	(0.22)	
6	1.67**	0.89**	85%	2.06**	0.83**	0.95**	0.91**	0.81**	0.64**	82.00%
	(0.22)	(0.03)		(0.39)	(0.04)	(0.05)	(0.06)	(0.20)	(0.22)	
7	1.81**	0.88**	79%	2.34**	0.86**	0.89**	0.93**	0.76**	0.54**	76.00%
	(0.26)	(0.03)		(0.46)	(0.05)	(0.06)	(0.07)	(0.24)	(0.25)	
8	1.98**	0.88**	74%	3.12**	0.85**	0.97**	0.92**	0.44*	0.12	75.00%
	(0.30)	(0.03)		(0.49)	(0.05)	(0.06)	(0.08)	(0.25)	(0.27)	
9	2.12**	0.93**	76%	2.85**	0.97**	0.96**	0.90**	0.30	0.45	76.00%
	(0.3)	(0.03)		(0.50)	(0.05)	(0.06)	(0.08)	(0.26)	(0.28)	
High	2.36**	1.03**	66%	3.07**	0.96**	1.08**	1.24**	0.42	0.57	65.00%
	(0.43)	(0.05)		(0.72)	(0.08)	(0.09)	(0.12)	(0.37)	(0.40)	
High - Low	1.57	-0.05		2.27	-0.16	0.00	0.21	-0.79	-0.53	

Table 1.16 reports results from a regression analysis of 10 portfolios sorted by Value. The regressor is S&P 500 total returns. The table reports coefficients and standard errors in parenthesis. Significance is reported at the 10, 5 and 1% level with one, two and three asterisks respectively. The sample size is equal to 220 quarters.

Table 1.17: SMB regression versus MKT

	α	β	R^2	α	β_1	β_2	β_3	β_4	β_5	R^2
Small	1.57**	1.23**	68.00%	3.59**	1.24**	1.23**	1.46**	0.93**	-0.12	66.00%
	(0.51)	(0.06)		(0.83)	(0.09)	(0.10)	(0.13)	(0.43)	(0.46)	
2	1.41**	1.25**	77.00%	2.99**	1.35**	1.21**	1.34**	0.85**	0.21	76.00%
	(0.40)	(0.05)		(0.66)	(0.07)	(0.08)	(0.11)	(0.34)	(0.36)	
3	1.65**	1.23**	82.00%	2.95**	1.34**	1.21**	1.23**	0.79**	0.38	82.00%
	(0.33)	(0.04)		(0.55)	(0.06)	(0.07)	(0.09)	(0.28)	(0.30)	
4	1.50**	1.20**	84.00%	2.67**	1.32**	1.16**	1.16**	0.88**	0.43	83.00%
	(0.31)	(0.04)		(0.51)	(0.05)	(0.06)	(0.08)	(0.26)	(0.28)	
5	1.60**	1.18**	87.00%	2.55**	1.26**	1.17**	1.17**	0.97**	0.56**	87.00%
	(0.26)	(0.03)		(0.44)	(0.05)	(0.05)	(0.07)	(0.23)	(0.24)	
6	1.57**	1.11**	88.00%	2.17**	1.17**	1.12**	1.10**	0.96**	0.73**	89.00%
	(0.23)	(0.03)		(0.39)	(0.04)	(0.05)	(0.06)	(0.20)	(0.22)	
7	1.53**	1.13**	90.00%	2.41**	1.16**	1.14**	1.15**	1.01**	0.56**	92.00%
	(0.20)	(0.02)		(0.35)	(0.04)	(0.04)	(0.06)	(0.18)	(0.19)	
8	1.47**	1.08**	92.00%	2.23**	1.13**	1.10**	1.07**	1.06**	0.59**	93.00%
	(0.18)	(0.02)		(0.30)	(0.03)	(0.04)	(0.05)	(0.16)	(0.17)	
9	1.42**	1.01**	93.00%	1.82**	1.00**	1.06**	0.98**	0.85**	0.76**	95.00%
	(0.14)	(0.02)		(0.27)	(0.03)	(0.03)	(0.04)	(0.14)	(0.15)	
Big	1.15**	0.90**	91.00%	0.97**	0.89**	0.90**	0.90**	0.90**	1.05**	95.00%
	(0.12)	(0.01)		(0.27)	(0.03)	(0.03)	(0.04)	(0.14)	(0.15)	
Small-Big	0.42	0.33		2.61	0.35	0.33	0.56	0.03	-1.17	

Table 1.17 reports results from a regression analysis of 10 portfolios sorted by Size. The regressor is S&P 500 total returns. The table reports coefficients and standard errors in parenthesis. Significance is reported at the 10, 5 and 1% level with one, two and three asterisks respectively. The sample size is equal to 220 quarters.

Table 1.18: Determinants of S&P 500 risk exposure

	α	DEF	INDP	SHORT	TERM	VIX	cay	INFL	R^2
β_t	0.22*	0.48***	0.09*	-0.04	-0.05**	-0.02	0.03*	0.17***	44.94%
	(0.12)	(0.04)	(0.04)	(0.08)	(0.03)	(0.04)	(0.02)	(0.04)	
$\beta_{1,t}$	-0.75**	0.19**	0.02	-0.16*	0.04	0.07	-0.01	0.18***	19.42%
	(0.13)	(0.05)	(0.05)	(0.09)	(0.03)	(0.05)	(0.02)	(0.05)	
$\beta_{2,t}$	-4.97***	0.85**	-0.22	-0.58*	0.30**	-0.53**	-0.08	0.36**	21.75%
	(0.46)	(0.16)	(0.17)	(0.32)	(0.11)	(0.17)	(0.06)	(0.18)	
$\beta_{3,t}$	14.32***	0.61	0.03	-4.79**	0.88	-1.15	2.94***	0.99	29.26%
	(2.56)	(0.92)	(0.97)	(1.81)	(0.61)	(0.94)	(0.36)	(0.98)	
$\beta_{4,t}$	0.11	0.03	-0.10*	0.11	-0.03	0.12**	-0.04	0.01	9.39%
	(0.15)	(0.05)	(0.06)	(0.11)	(0.04)	(0.06)	(0.02)	(0.06)	
$\beta_{5,t}$	1.75**	0.25*	0.13	-0.87**	0.08	-0.94**	-0.05	0.11	32.59%
	(0.34)	(0.12)	(0.13)	(0.24)	(0.08)	(0.13)	(0.05)	(0.13)	

Table 1.18 reports the OLS regression coefficient estimates of the time varying Betas on a set of structural variables.

Consumption growth decomposition.

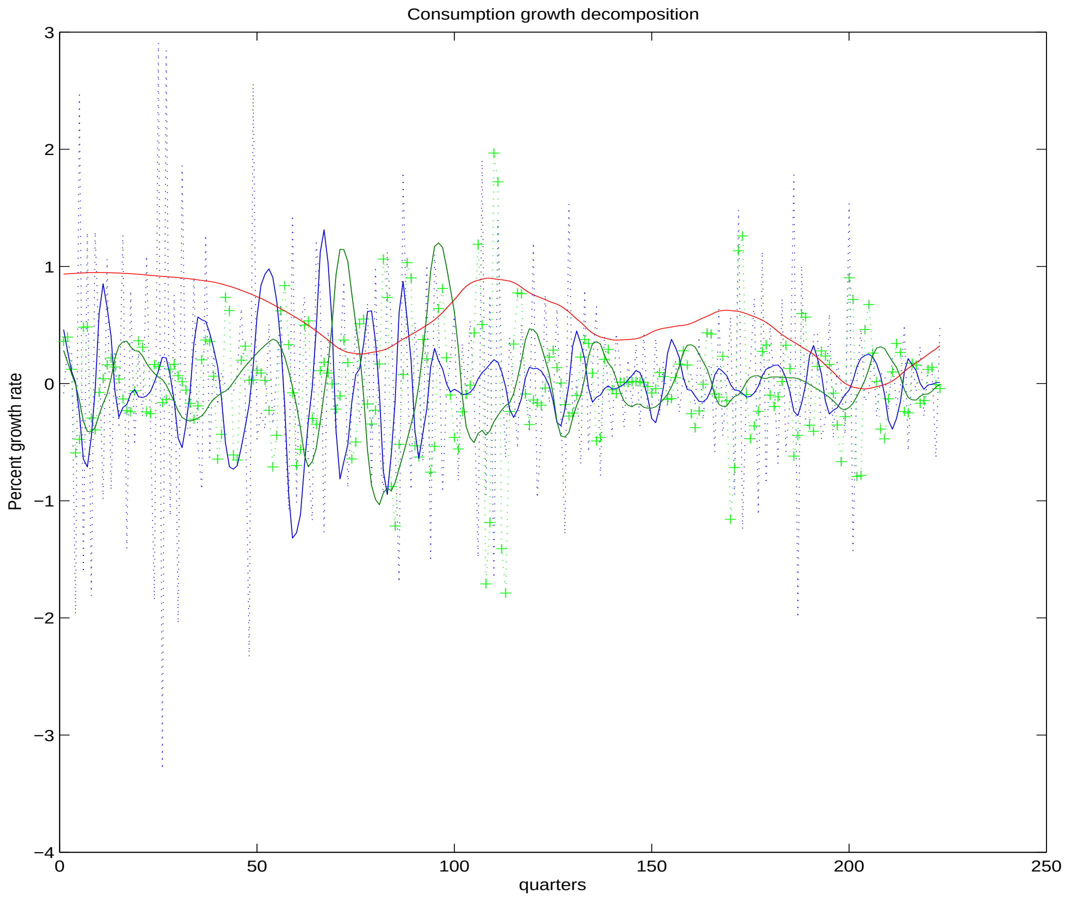


Figure 1.1: The figure shows the time series of the consumption growth process decomposed in the $J = 5$ scales. The decomposition is performed by using a Daubechies wavelet. The data is quarterly observations from February 1959 to August 2014. The sample contains 222 quarterly observations.

Adjusted R^2 across scales.

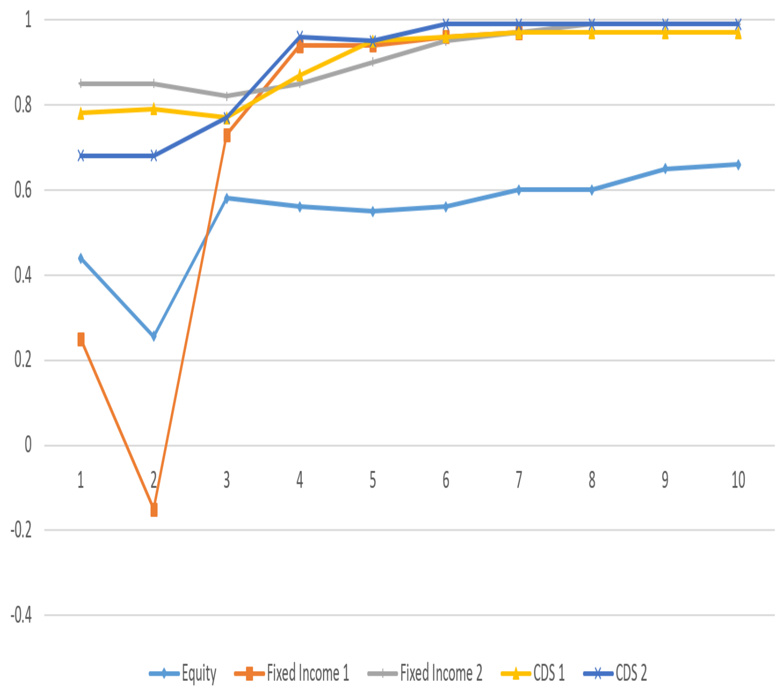
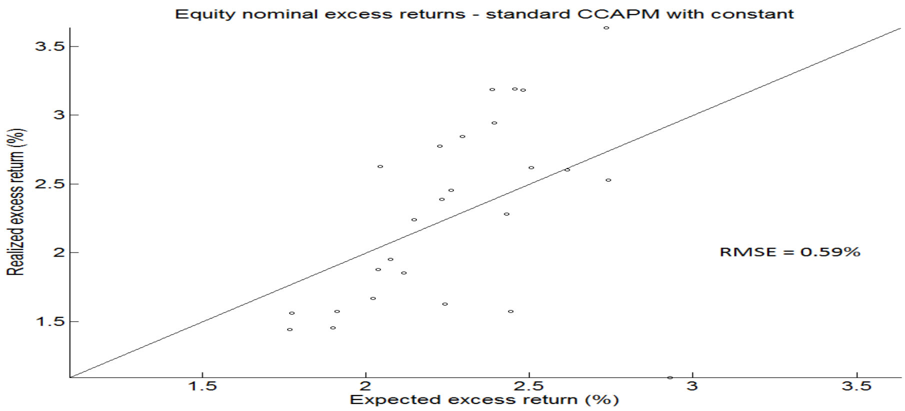


Figure 1.2: The figure shows the behaviour of the cross-sectional adjusted R^2 for various scales. After scale $J = 5$, the adjusted R^2 flat-lines for all asset classes. The adjusted R^2 shows the explained cross sectional variation from the following regression:

$$\bar{r}_i^e = \lambda_0 + \lambda_1 \beta_i^1 + \lambda_2 \beta_i^2 + ... + \lambda_J \beta_i^J + \epsilon_i$$

where J is recursively increased from $J = 1$, corresponding to the normal consumption CAPM, up to $J = 10$, a business cycle consumption CAPM with ten frequency components. The figure shows that after the fifth scale, there is not much additional explanatory power gained from adding further scales.

Second stage regression: 25 Fama and French porfolios.



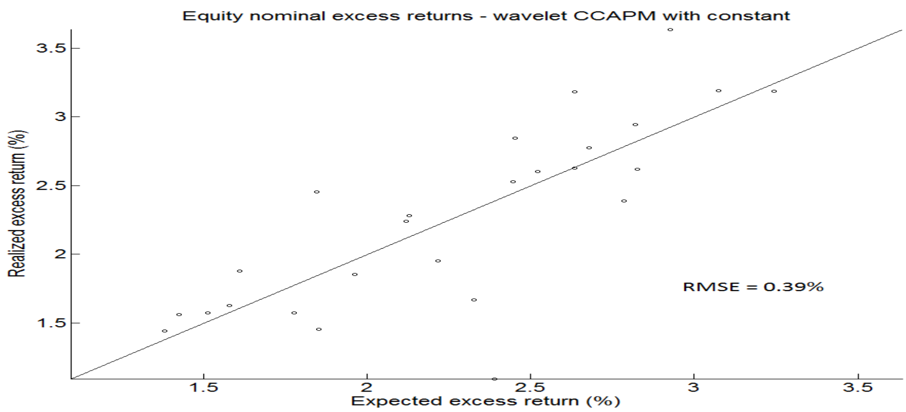


Figure 1.3: The figure presents the results from the second stage cross sectional regression for the 25 Fama and French portfolios. The top panel presents the result using aggregate consumption only, while the lower panel presents the result using the decomposed consumption. I report model implied expected returns vs realized excess returns. I also report a 45 degree line and the Relative Mean Square Error.

The first regression is an OLS estimate of: $\bar{r}_i^e = \lambda_0 + \lambda_1 \beta_i + \epsilon_i$
while the second regression is an OLS estimate of: $\bar{r}_i^e = \lambda_0 + \lambda_1 \beta_i^1 + \lambda_2 \beta_i^2 + ... + \lambda_J \beta_i^J + \epsilon_i$

Second stage regression: 9 BAML fixed income portfolios.

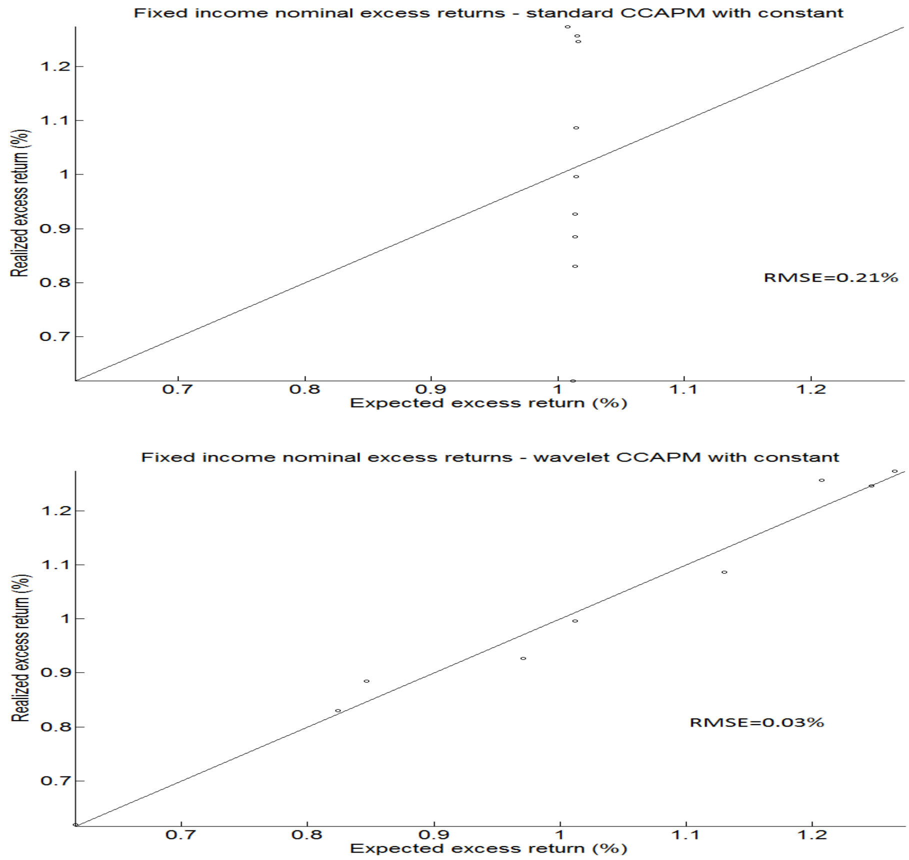


Figure 1.4: The figure presents the results from the second stage cross sectional regression for the 9 BAML fixed income portfolios. The top panel presents the result using aggregate consumption only, while the lower panel presents the result using the decomposed consumption. I report model implied expected returns vs realized excess returns. I also report a 45 degree line and the Root Mean Square Error.

The first regression is an OLS estimate of: $\bar{r}_i^e = \lambda_0 + \lambda_1 \beta_i + \epsilon_i$
while the second regression is an OLS estimate of: $\bar{r}_i^e = \lambda_0 + \lambda_1 \beta_i^1 + \lambda_2 \beta_i^2 + ... + \lambda_J \beta_i^J + \epsilon_i$

Second stage regression: 17 CRSP Fama Maturity portfolios.

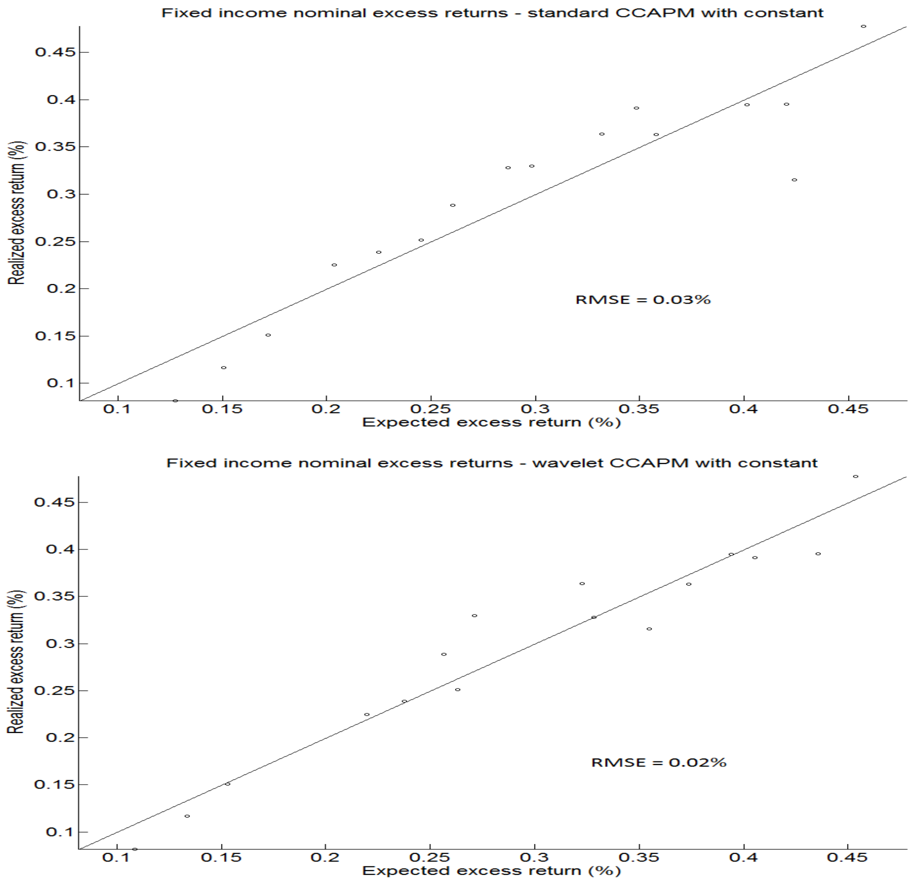


Figure 1.5: The figure presents the results from the second stage cross sectional regression for the 17 CRSP Fama Maturity portfolios. The top panel presents the result using aggregate consumption only, while the lower panel presents the result using the decomposed consumption. I report model implied expected returns vs realized excess returns. I also report a 45 degree line and the Root Mean Square Error.

The first regression is an OLS estimate of: $\bar{r}_i^e = \lambda_0 + \lambda_1 \beta_i + \epsilon_i$
while the second regression is an OLS estimate of: $\bar{r}_i^e = \lambda_0 + \lambda_1 \beta_i^1 + \lambda_2 \beta_i^2 + ... + \lambda_J \beta_i^J + \epsilon_i$

Second stage regression: 12 macro credit indices.

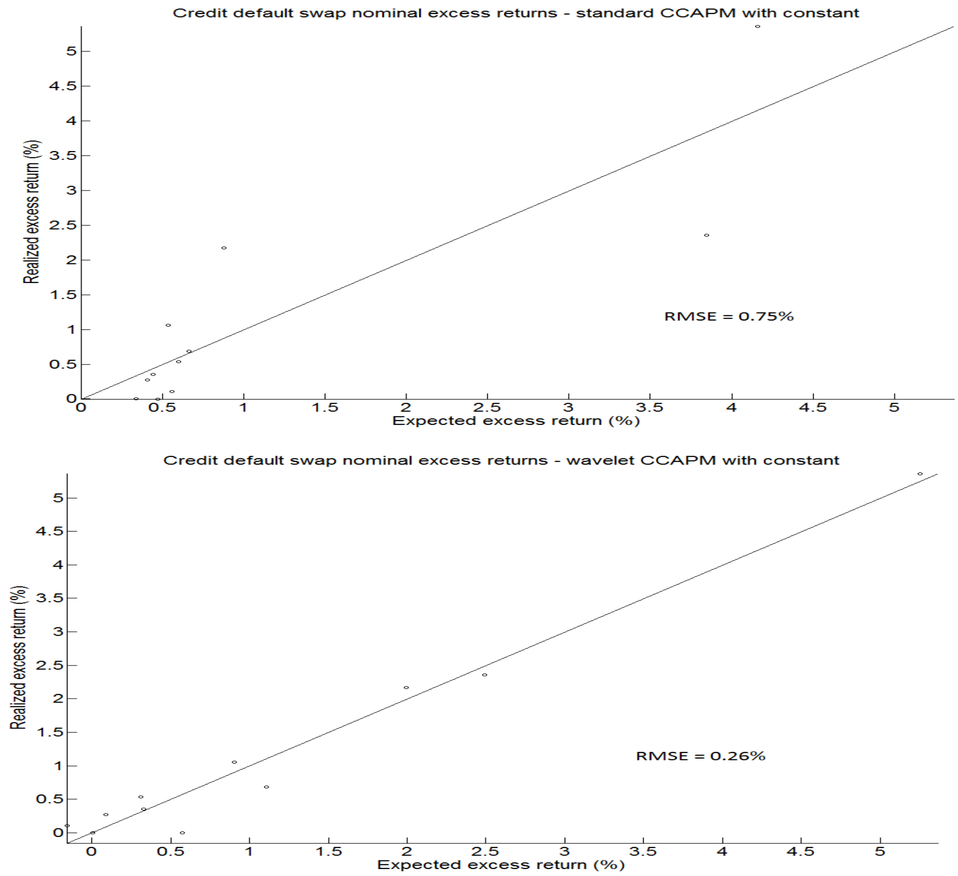


Figure 1.6: The figure presents the results from the second stage cross sectional regression for the 12 macro credit indices. The top panel presents the result using aggregate consumption only, while the lower panel presents the result using the decomposed consumption. I report model implied expected returns vs realized excess returns. I also report a 45 degree line and the Root Mean Square Error.

The first regression is an OLS estimate of: $\bar{r}_i^e = \lambda_0 + \lambda_1 \beta_i + \epsilon_i$
while the second regression is an OLS estimate of: $\bar{r}_i^e = \lambda_0 + \lambda_1 \beta_i^1 + \lambda_2 \beta_i^2 + ... + \lambda_J \beta_i^J + \epsilon_i$

Second stage regression: 10 industry specific credit indices.

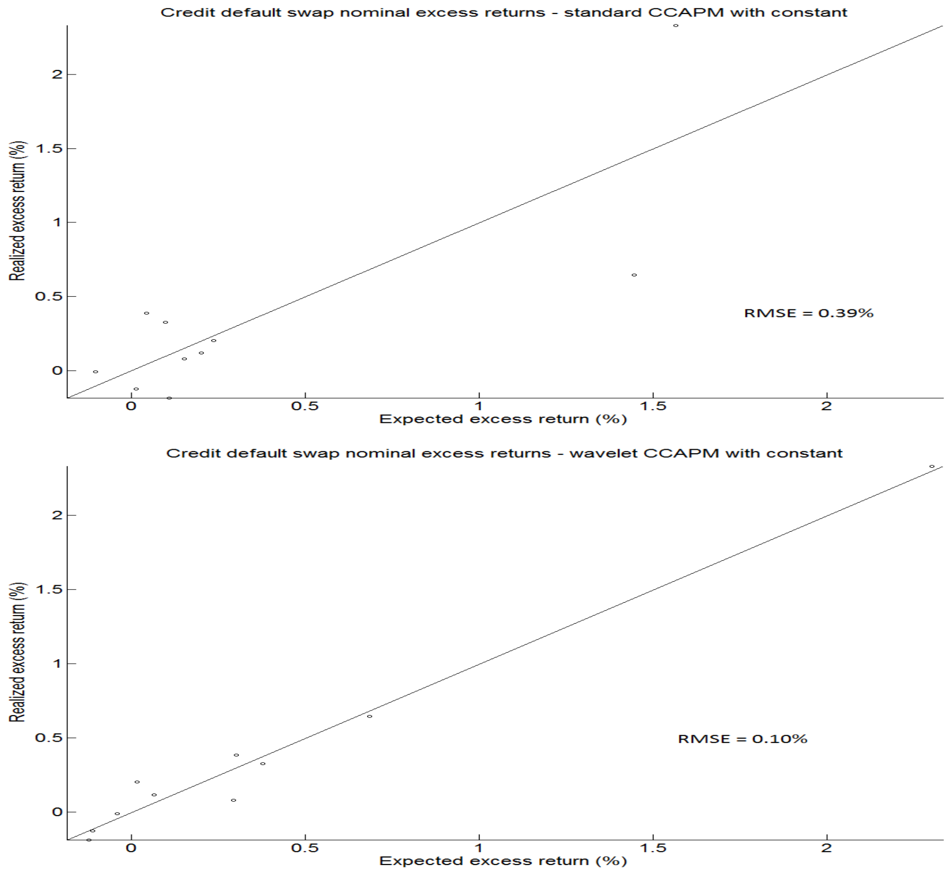


Figure 1.7: The figure presents the results from the second stage cross sectional regression for the 10 industry specific credit indices. The top panel presents the result using aggregate consumption only, while the lower panel presents the result using the decomposed consumption. I report model implied expected returns vs realized excess returns. I also report a 45 degree line and the Root Mean Square Error.

The first regression is an OLS estimate of: $\bar{r}_i^e = \lambda_0 + \lambda_1 \beta_i + \epsilon_i$
while the second regression is an OLS estimate of: $\bar{r}_i^e = \lambda_0 + \lambda_1 \beta_i^1 + \lambda_2 \beta_i^2 + ... + \lambda_J \beta_i^J + \epsilon_i$

Conditional volatilities and correlations.

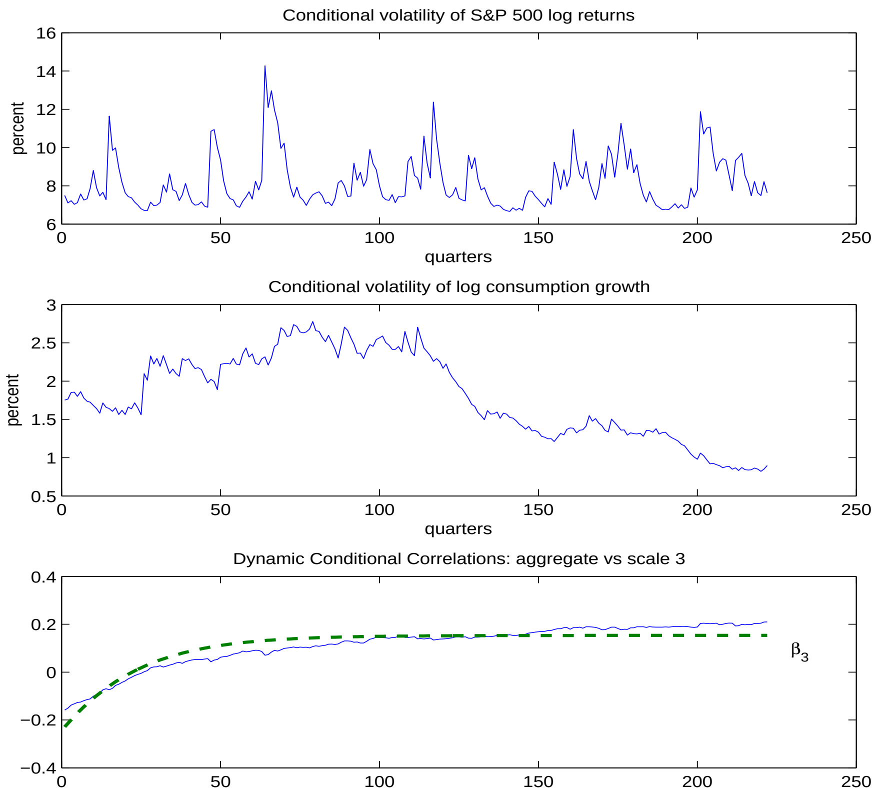


Figure 1.8: The figure shows the path of the conditional volatility of S&P 500 returns (top panel), of consumption growth (middle panel) and the dynamic conditional correlation (bottom panel) between S&P 500 returns and aggregate consumption growth (solid) and between S&P 500 returns and the third scale of consumption growth (dashed). The bottom panel shows that almost all the correlation between S&P 500 returns and consumption growth occurs at scale 3, that is at a one year frequency.

Time-varying betas.

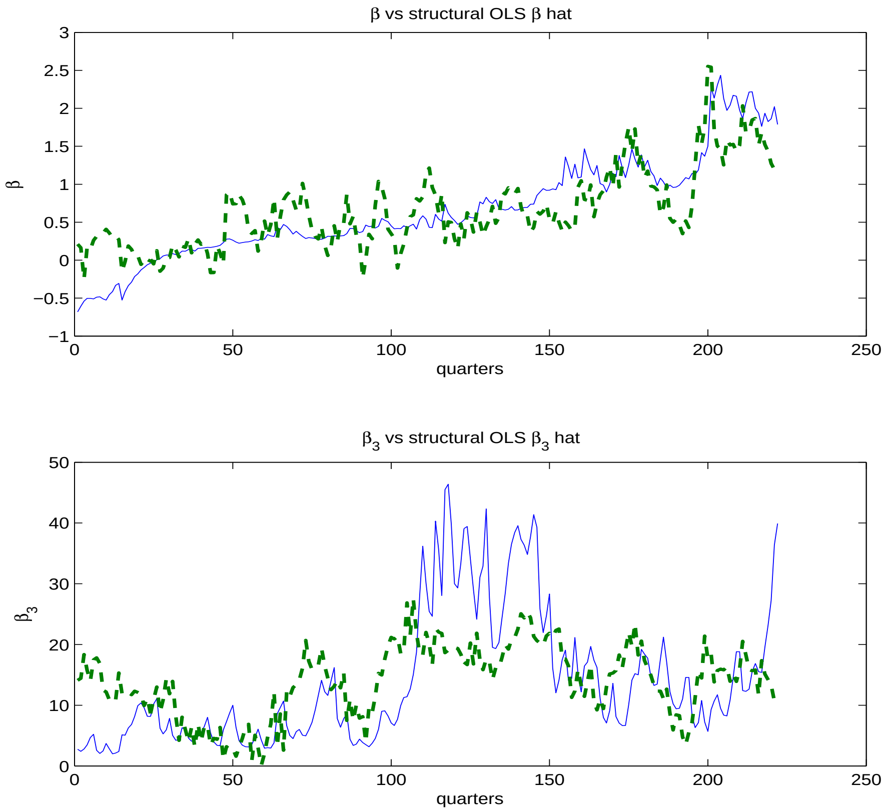


Figure 1.9: The figure shows time varying betas from a Dynamic Conditional Correlation (DCC) model. The top panel shows the beta with respect to aggregate consumption growth from a DCC model (solid), and the estimated beta from a set of structural factors (dashed). The lower panel shows the beta with respect to the third scale of consumption growth from a DCC model (solid), and the estimated beta from a set of structural factors (dashed).

The estimated betas are generated by fitting via OLS the following linear regression:

$$\beta_{i,t} = \phi_0 + \phi_1 DEF_t + \phi_2 INDP_t + \phi_3 SHORT_t + \phi_4 TERM_t + \phi_5 VIX_t + \phi_6 cay_t + \phi_7 INFL_t + u_t$$

with $i = 1, .., J$ or aggregate. The results of this regression are reported in Table 1.18.

Chapter 2

The empirical properties of political risk.

2.1 Introduction.

Since the European debt crisis of 2010, political risk has returned at the forefront of finance. Part of the challenge is to understand what is meant by "political risk". Baker *et al.* (2016) introduce a measure of economic policy uncertainty risk, based on the news coverage of certain keywords, historically associated with government crises. This has spawned a literature that tries to address the impacts of policy uncertainty risk on various economic measures, such as GDP growth, credit spreads and stock market returns.

The goal of this chapter is to take one step back and describe the main statistical properties of the Economic Policy Uncertainty index as well as the information it conveys. It is also important to understand to what extent it conveys new information with respect to more traditional measures of risk, such as the VIX index.

This is important because most of the literature that has used the EPU index until now has not investigated its statistical properties thoroughly. For example, OLS estimates are biased if the data exhibits heteroskedasticity. In general, the statistical properties unveiled here must be taken into consideration when modelling policy uncertainty risk. These properties will be useful in the next Chapter, where I present a model of political risk that takes the EPU index as an input.

The main statistical properties of the Economic Policy Uncertainty (EPU) index uncovered in this chapter are the following. First, the EPU index is not stationary, while the first differentials of the logarithm are. Second, the index exhibits a leptokurtic behaviour, with fat tails and a high peak around the mean. Third, the index has a time varying conditional variance. Heteroskedasticity only partially explains the leptokurtic behaviour. Fourth, EPU has a strong persistent nature, that can be exploited to produce forecasts of future political risk levels. And finally, the EPU index Granger-causes other factors, most importantly the S&P 500, oil, credit spreads, fed funds deviation and the VIX index.

Some of these properties may come from the way the index is constructed. The sum of three separate indices at various frequencies has been shown to mechanically produce heteroskedasticity. Unfortunately the sub indices are not available so it is impossible to decompose this effect in its constituents.

The rest of the chapter is organised as follows. In Section 2 I review the literature on political risk, in Section 3 I conduct a preliminary investigation into the statistical properties of the EPU index, in Section 4 I present a number of HAR models and their estimation. In Sections 5 I draw some conclusions.

2.2 Literature review.

Baker et al.'s index is relatively new, but it has already been used in a number of studies on political risk.

Manzo (2013) finds that a 10 percent increase in political uncertainty leads to a 3% increase in both default risk and credit risk after a month. A European regional-level analysis reveals heterogeneity in the response of sovereign risk to variations in political uncertainty. I am able to document a Granger causality between EPU and corporate credit spreads in the US, even if my analysis clearly lacks the cross sectional dimension of Manzo.

Brogaard & Detzel (2015) find that EPU positively forecasts log excess market returns. A one-standard deviation increase in EPU is associated with a 1.5% increase in the forecasted

3-month abnormal returns (6.1% annualized). Furthermore, innovations in EPU earn a significant negative risk premium in the Fama French 25 size and momentum portfolios. Among the Fama French 25 portfolios formed on size and momentum returns, the portfolio with the greatest EPU beta underperforms the portfolio with the lowest EPU beta by 5.53% per annum, controlling for exposure to the Carhart four factors as well as implied and realized volatility. These findings suggest that EPU is an economically important risk factor for equity markets.

Wisniewski & Lambe (2015) estimate a VAR on EPU and changes in CDS indices (iTraxx and CDX). They find that policy uncertainty Granger-causes corporate credit spreads. Using a structural dynamic factor model, Scheffel (2015) finds that a shock to EPU has pervasive effects on the US economy and that more globally integrated markets exhibit significantly more pronounced responses. Sum (2012) analyses the response of stock market returns to a change in EPU: with a VAR methodology the response is positive while with pooled OLS it is negative.

Baker et al.'s index is not the only proxy for political risk. There is a wider literature on policy uncertainty that goes beyond the use of the EPU index and that tries to quantify policy uncertainty in different terms, such as taxation risk or uncertainty over the government's regulatory stance.

Manelaa & Moreira (2017) construct a news based measure of uncertainty by scraping *Wall Street Journal* headlines and linking them to the VIX. They call their measure the "News VIX" or NVIX. This measure is able to forecast excess returns and is able to capture the time variation of risk aversion. It performs exceptionally well in capturing the time variation in disaster risk, with a particular focus 20th Century war risk.

Sialm (2006) analyzes the effect of stochastic taxes on asset prices, and finds that investors require a premium to compensate for the risk introduced by tax changes. Tax uncertainty also features in Croce *et al.* (2012), who explore its asset pricing implications in a production economy with recursive preferences. Croce *et al.* (2012) examines the effects of fiscal uncertainty on long-term growth when agents facing model uncertainty care about the worst-case scenario. Finally, Ulrich (2013) studies the policy problem of a government that cares about welfare as well as public spending. He analyzes the bond market implications of Knightian uncertainty about the effectiveness of government policies.

Erb *et al.* (1996) find a weak relation between political risk, measured by the International Country Risk Guide, and future stock returns. Pantzalis *et al.* (2000) and Li & Born (2006) find abnormally high stock market returns in the weeks preceding major elections, especially for elections characterized by high degrees of uncertainty. This evidence is consistent with a positive relation between the equity premium and political uncertainty. Brogaard & Detzel (2015) find a positive relation between the equity risk premium and their search-based measure of economic policy uncertainty in an international setting. Santa-Clara & Valkanov (2003) relate the equity risk premium to political cycles. Pastor & Veronesi (2013) set up an economy where agents learn in a bayesian fashion about future government policies and examines its effects on average implied volatilities. Policy heterogeneity plays a large role in generating sufficient excess volatility. Belo *et al.* (2013) link the cross-section of stock returns to firms' exposures to the government sector. Bittlingmayer (1998) , Voth (2002), and Boutchkova *et al.* (2011) find a positive relation between political uncertainty and stock volatility in a variety of settings.

2.3 Preliminary data analysis.

My sample of Baker *et al.* (2016)'s daily index of Economic Policy Uncertainty in the United States runs from the 2^{nd} January 1986 to the 30^{th} June 2014, totalling $n = 7083$ daily observations. Figure 2.1 shows the time series of the index. The index records the frequency of articles in 10 leading US newspapers that contain the following triple: "economic" or "economy"; "uncertain" or "uncertainty"; and one or more of "congress", "deficit", "Federal Reserve", "legislation", "regulation" or "White House". The aim of the index is to measure "policy uncertainty", as it might be an important part of total risk. For a more complete account on how the index is built or on how the raw data is collected, I redirect the reader to Baker, Bloom and Davis own work. The index is available at `http://www.policyuncertainty.com/`.

In this chapter, I assume that political risk is a stochastic process and that I observe a realization $\{\text{EPU index}_t\}_{\{t=1,\ldots,T\}}$ of this process. I will be mainly dealing with the following two variables:

$$\text{EPU levels: } X_t = \text{EPU index}_t$$

$$\text{EPU returns: } x_t = \log(\text{EPU index}_t) - \log(\text{EPU index}_{t-1})$$

Table 2.1 reports some descriptive statistics. The time series of levels oscillates between a minimum of 3.3 and a maximum of 720 and it is very noisy. The mean is around 100 and the standard deviation around 70. The data shows a positive skew of 1.81, indicating a positive tail to the right. The kurtosis is 8.62, which is higher than a normal distribution.

Table 2.2 reports the results of some stationarity tests, to see if there is a unit root in the data. The Augmented Dickey Fuller (ADF) and the Philips Perron (PP) test on the levels reject the null of a unit root with a p value of 0.1 %. The conclusion would therefore be that EPU in levels is stationary. On the other hand, the Kwiatkowski, Phillips, Schmidt and Shin (KPSS) and Leybourne-McCabe (LMC) reject the null of stationarity with p values of 1 %, leading to the conclusion that EPU in levels is not stationary. The LMC test in particular suggests that EPU follows an ARIMA(p,1,1) model. Since EPU in levels is non stationary, I will discard this dataset. I therefore proceed to test the stationarity of the log difference, and all four tests confirm stationarity, with p-values of up to 10 % for the KPSS and LMC tests. Figure 2.2 plots the autocorrelation and partial autocorrelation plots for the log differential, along with the 95 % confidence interval. Some autocorrelations remain statistically different from zero, in particular the 7-day autocorrelation. From now on, therefore, unless specified, I will be working with log differentials. Figure 2.3 shows the empirical distribution of the log differential. The empirical distribution shows some leptokurtosis, a phenomenon that could arise from time varying conditional variances.

I run a Cramer-von Mises goodness of fit test for a number of possible distribution on the log differentials. The test fails to rejects the logistic distribution and the t-Student distribution. The best fit is provided by the logistic distribution with parameters $\mu = -0.002$ and $\sigma = 0.382$. Table 2.3 presents the results of the test and Figure 2.3 shows the fit.

2.3.1 Autocorrelation.

It is well known that the autocorrelation of asset returns tend to quickly decay to zero at very small lags. The two main statistical features of volatility are positive short term autocorrelation and long term mean reversion. The positive autocorrelation means that high volatility tends to be followed by high volatility, leading to the well documented volatility clustering phenomenon. Volatility, in other words, tends to experience long, prolonged periods at low values and then bursts upwards and remains high for considerable amounts of time. This means asset managers may be tricked in thinking an asset is safer that it actually is, only to discover its true riskiness during a crisis, when liquidity is thin and disinvesting very expensive. The other feature of volatility is mean reversion, which is the tendency of volatility to hover around a long term average.

Political risk shares many features of volatility. As can be seen from Figure 2.1 political risk hovers within a bounded range. Figure 2.2 documents a negative autocorrelation of around -0.4, meaning that increases in political risk tend to be followed by decreases. The statistically significant negative autocorrelation in this case has little market implications, as political risk is not a traded asset.

2.3.2 Non linear dependence

Another dimension worth investigating is to explore the possible non-linear dependencies in the data. The autocorrelation of log differences is not the sole aspect to look at when trying to characterize dependence. In fact, it is interesting to look at dependence between non linear functions of returns as well. The first step is to model the conditional mean of x_t. Noticing that the time series exhibits some autocorrelation and partial autocorrelation, I model the returns as a ARMA(1,1), where the orders p and q is estimated via the Akaike Information Criterion. I then analyse some non-linearities in the residuals. The easiest way to do this is to start by analysing autocorrelation among powers of the residuals. This amounts at looking at the decay rates of the second, third and fourth power of residuals. These plots are reported in Figure 2.4, along with the 95 % confidence bands. Looking at the squared changes of the residuals, we can notice that the autocorrelation function is positive and decaying very slowly, in other words, very persistent. This points to the presence of volatility clustering in political risk. This means that not only political risk tends to cluster, but its own volatility tends to cluster too. This justifies introducing econometric methods of the “ARCH” class, to try to capture this volatility clustering of political risk. Note that the use of ARCH or GARCH models simply tries to capture this clustering, which remains a feature of the data and is model free. To this end, let me recall GARCH models and their estimation.

A GARCH(p,q) will look like this:

$$\epsilon_t = \sqrt{\sigma_t \nu_t}$$
$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \gamma_i \sigma_{t-i}^2 + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2$$

where ν_t is a white noise process. The model can be estimated by maximizing the log likelihood of observations:

$$l(\theta|y_0) = \sum_{t=2}^{T} \log\left(f_{Y_T|Y_{T-1}}\right) =$$
$$= -\frac{T-1}{2}\log(2\pi) - \frac{1}{2}\sum_{t=2}^{T}\log(\sigma_t^2) - \frac{1}{2}\sum_{t=2}^{T}\frac{\epsilon_t^2}{\sigma_t^2}$$

where $\theta = \left\{\omega, \{\gamma_i\}_{i=1,\ldots,p}, \{\alpha_i\}_{i=1,\ldots,q}\right\}$.

Conditional volatility models like the GARCH are tested on residuals. This means that I must specify a model for the data that will generate these residuals. I keep the previous ARMA(1,1) specification, adding GARCH(1,1) innovations. The order selection is again done by maximizing the Akaike Information Criterion (AIC).

The result of this estimation point to a time-varying conditional variance of political risk. Table 2.4 reports the results of the estimation. All coefficients are significant, except for the constant. The overall model fit is $R^2 = 64\%$. Figure 2.5 depicts the extracted conditional volatility. To check if conditional variances alone can explain the fat tail behaviour, I have checked normality via a QQ plot. In Figure 2.6, the normalized residuals still show significant heavy tails. A Jarque-Brera test rejects the null of normality at any significance (JB statistic = 766). I also perform Chen & Kuan (2003)'s normality test for GARCH innovations. The modified JB statistic is 535, so I still reject the null at any significance level.

2.3.3 Extreme correlations

When building portfolios, an asset manager always tries to diversify. The extent to which her portfolio is diversified depends on the degree of correlation between the assets in her portfolio. Unfortunately, correlation is time varying and tends to spike during recessions. This means that what used to be a well diversified portfolio might fail to deliver the benefits of diversification precisely when those are most needed, that is during market turmoil. For this reason, asset managers also look at tail correlations, or the probability that two assets will move together conditional on there being a big market movement.

For two assets, tail correlation is defined as the conditional probability of having a large negative return in one asset given a large negative return in another asset. The thresholds can be defined in terms of quantiles $q_i(\alpha)$ (usually 95 or 99%).

$$\mathbb{P}(r_i < q_i(\alpha)|r_j < q_j(\alpha)) = \frac{\mathbb{P}(r_i < q_i(\alpha) \cap r_j < q_j(\alpha))}{\mathbb{P}(r_j < q_j(\alpha))}$$

or

$$\mathbb{P}(r_j < q_j(\alpha)|r_i < q_i(\alpha)) = \frac{\mathbb{P}(r_i < q_i(\alpha) \cap r_j < q_j(\alpha))}{\mathbb{P}(r_i < q_i(\alpha))}$$

These probabilities can be estimated from a sample $\{r_i\}_{i=1,\ldots,N}$of returns as:

$$\mathbb{P}(r_i < q_i(\alpha) \cap r_j < q_j(\alpha)) = \frac{1}{N}\sum \mathbb{I}_{\{r_i<q_i(\alpha) \wedge r_j<q_j(\alpha)\}}$$
$$\mathbb{P}(r_i < q_i(\alpha)) = \frac{1}{N}\sum \mathbb{I}_{\{r_i<q_i(\alpha)\}}$$
$$\mathbb{P}(r_j < q_j(\alpha)) = \frac{1}{N}\sum \mathbb{I}_{\{r_j<q_j(\alpha)\}}$$

where $\mathbb{I}$ is an indicator function that is one if the event is verified and zero otherwise.

To estimate tail correlations, I compute the sample equivalents of the above formulas. For a pair of stationary time series r_i and r_j, and for a set of αs from 30 % to 1%, I first compute the relevant quantiles as $q_i = \mu_i \pm t_{1-\alpha}\sigma_i$ and $q_j = \mu_j \pm t_{1-\alpha}\sigma_j$. Then I compute the empirical frequencies $p_i(\alpha) = \frac{\#r_i<q_i}{N}$, $p_j(\alpha) = \frac{\#r_j<q_j}{N}$ and $p_{i,j}(\alpha) = \frac{\#r_i<q_i \wedge \#r_j<q_j}{N}$. The tail correlation between r_i and r_j is then the conditional probability $\rho_{i,j}(\alpha) = \frac{p_{i,j}(\alpha)}{p_i(\alpha)}$.

I compute the tail correlations for two scenarios: a positive return scenario, where $q_{S\&P500}(\alpha) = \mu_{S\&P500} + t_{1-\alpha}\sigma_{S\&P500}$, $q_{VIX}(\alpha) = \mu_{VIX} - t_{1-\alpha}\sigma_{VIX}$, $q_{EPU}(\alpha) = \mu_{EPU} - t_{1-\alpha}\sigma_{EPU}$, and a negative return scenario, where $q_{S\&P500}(\alpha) = \mu_{S\&P500} - t_{1-\alpha}\sigma_{S\&P500}$, $q_{VIX}(\alpha) = \mu_{VIX} + t_{1-\alpha}\sigma_{VIX}$, $q_{EPU}(\alpha) = \mu_{EPU} + t_{1-\alpha}\sigma_{EPU}$. For the positive return scenario, I compute the probability that $r_{S\&P500} > q_{S\&P500}(\alpha)$,$r_{VIX} < q_{VIX}(\alpha)$, $r_{EPU} > q_{EPU}(\alpha)$ while for the negative return scenario I compute $r_{S\&P500} < q_{S\&P500}(\alpha)$,$r_{VIX} > q_{VIX}(\alpha)$, $r_{EPU} > q_{EPU}(\alpha)$. The tail correlation is then the probability of observing an extreme co-movement, given a very positive and negative $S\&P500$ return.

Figure 2.8 depicts the tail correlation between thee pairs: S&P 500 and VIX (blue), EPU and VIX (green) and EPU and S&P 500 index (red). They all tend to slope downwards, showing that the probability that those pairs experience severe co-movements falls as I decrease α. The EPU index exhibits lower tail correlations with both the VIX index and the S&P 500 index, showing that the conditional probabilities of EPU co-moving with either the VIX or the S&P 500 are equally low. This shows that in extreme events, the EPU index conveys a different information content from both the S&P 500 and the VIX. This confirms that EPU is useful in providing new information to the market, even in stressed situations.

2.4 A HAR specification.

U. Muller & von Weizsacker (1997) introduced the idea that market participants act at different scales and this generates an additive cascade of partial volatilities. Each level of the cascade is a function of not only its past value, but also of the expected values of the other partial volatilities. Corsi (2009) shows by straightforward recursive substitutions of the partial volatilities that this additive structure for the volatility cascade leads to a simple restricted linear autoregressive model featuring volatilities realized over different time horizons. The heterogeneous nature of the model derives from the fact that, at each time scale, the partial volatility relies on different autoregressive structures. This leads to a Heterogeneous Auto Regressive (HAR) specification.

Similar to Corsi's argument, also political risk exists at various frequencies. The most obvious is the electoral cycle, which in the US tends to last four years. Also, because of the two years lag between presidential and congressional elections, there are two four year cycles. On top of this, there are state congressional elections, mayoral elections, referenda on propositions, etc. all acting at different frequencies and intensities. On top of these purely political cycles, the monetary and economic boom-bust cycles can create a fiscal cycle, and create further political complications, such as the need to raise the debt ceiling or to tackle a budget deficit. The complex overlap of all these cycles determines the heterogeneous nature of policy uncertainty and therefore renders the HAR model well suited to model this data.

The choice of the frequencies is not unique. Including many frequencies would improve model fit but would reduce the interpretability of the model. In what follows I adhere to Fernandes *et al.* (2014). Their choice of frequency starts with a common choice in the literature - 1,5 and 22 - so as to mirror daily, weekly and monthly components. They then augment the frequencies with a biweekly (10) and a quarterly (66) component.

Let $y_t = \log\left(\frac{EPU_t}{EPU_{t-1}}\right)$. Let also $y_t^h = \frac{1}{h}\sum_{s=1}^{h} y_{t-s+1}$ and define $x_t = \left[1, y_t^{h_1}, y_t^{h_2}, ..., y_t^{h_N}\right]'$ where the vector of periods is $h = 1, 5, 10, 22, 66$ days. The time series y_t then follows a HAR model if $y_t = \beta x_{t-1} + \epsilon_t$, where ϵ_t denotes a white noise process.

Following Fernandes *et al.* (2014), I consider three variations of the HAR specification. The first includes a set of additional regressors z_t such that $y_t = \beta x_{t-1} + \gamma z_t + \epsilon_t$ where $z_t = \{z_{1t}, ..., z_{Kt}\}$ is a K-dimensional vector of explanatory variables. I refer to this as the HARX specification.

Among the regressors, I include the following macrofinance variables: the h-day continuously compounded return on the S&P 500 index for $h = 1, 5, 10, 22, 66$. (S&P 500 h-day return); the h-day continuously compounded return on the one-month crude oil futures contract (oil h-day return); the first difference of the logarithm of the trade-weighted average of the foreign exchange value of the US dollar index against the Australian dollar, Canadian dollar, Swiss franc, euro, British sterling pound, Japanese yen, and Swedish kroner (USD change); the excess yield of the Moody's seasoned Baa corporate bond over the Moody's seasoned Aaa corporate bond (credit spread); the difference between the 10-Year and 3-month treasury constant maturity rates (term spread); the difference between the effective and target Federal Fund rates (FF deviation); and the h-day continuously compounded return on the VIX index.

The choice of these factors is motivated by their ubiquitousness in the literature. They all proxy for changes in the investment opportunity set as explained below. I use S&P 500 returns in order to account for possible leverage and volatility effects (see for example Fleming *et al.* (1995); Giot (2005) and Bollerslev & Zhou (2006). I include h-period returns on the S&P 500 index so as to comply with the HAR nature of the model. The remaining regressors all convey information about the market conditions in the US. Both the term spread and oil prices contain information about the future real economic activity (Estrella & Hardouvelis (1991)) as well as about the future investment opportunity set (Petkova (2006)). The credit spread can be considered a proxy for the amount of liquidity in the market, while US dollar index returns and FF deviation are both related to the macroeconomic conditions in the Country.

As a preliminary analysis, I run and Augmented Dickey Fuller test on all regressors, and reject the null of unit roots at all levels of significance. I then test Granger-causality among the macrofinance variables. The resulting F statistics from pairwise Granger causality tests are showed in Table 2.8. The EPU index does not seem to Granger-cause the other macro finance variables, but it is Granger-caused by S&P 500, VIX, BAA credit spread and term spread. As a second test, I have tested the null hypothesis of a cointegrating relationship among the structural variables. The test rejects the null at all levels of significance.

The second specification only includes lagged values: $y_t = \beta x_{t-1} + \gamma z_{t-1} + \epsilon_t$. This specification helps to uncover the predictive ability of state variables. The third variant is an HAR-type specification that controls for explanatory variables with asymmetric effects. The chief motivation is to investigate if EPU shows the same asymmetric relation with S&P 500 index returns as the VIX. In particular, the AHARX model is given by:

$$y_t = \beta x_{t-1} + \gamma_{(-)} z_t^{(-)} + \gamma_{(+)} z_t^{(+)} + \epsilon_t$$

where $z_t^{(-)} = \left\{ z_{1t}^{(-)}, ..., z_{Kt}^{(-)} \right\}$ and $z_t^{(+)} = \left\{ z_{1t}^{(+)}, ..., z_{Kt}^{(+)} \right\}$, with $z_{1t}^{(+)} = z_{kt}\mathbf{1}(z_{kt} > 0)$ and $z_{1t}^{(-)} = z_{kt}\mathbf{1}(z_{kt} < 0)$.

The models are estimated via OLS. Table 2.5 reports the estimation results. The constant is never statistically different from zero. The first column reports the results of a HAR estimation. The model exhibits a high level of persistence, at all lags considered. The coefficients of the lagged values, all statistically significant at the 1 % level, increase with the lag period h, from -0.17 to -6.87. By looking at partial R^2s, we can notice in Table 2.6 that the 1 day lagged EPU value explains about 18 % of the total variation alone. The 5 day lagged EPU explains an additional 7 %. The improvement then decays substantially to 2.64 %, 2.77 % and 1.06 % for the remaining components of the HAR. The biggest contributors to the explanatory power of the model remains EPU's own lagged values by far.

The second column of Table 2.5 reports the estimation of the HARX model, where the vector of explaining factors z_t is contemporaneous to the EPU index. The adjusted R^2 to 33.01 %, and most of this increase is driven by lagged S & P 500 returns and lagged VIX changes. The large negative and significant coefficient of the 66 day period return points to a negative relation with the S & P 500 but much more slow moving that the VIX. The 10 day oil return, the change in credit and term spreads and fed funds deviation all carry statistically significant coefficients, but with very little explanatory power, as measured by their contribution to R^2.

The third column shows the effect of lagging the set of explanatory variables one day back, so as to simulate an in sample one step ahead forecast. The R^2 falls very marginally to 32.88%. This can be a measure of predictability of the EPU index.

The fourth column displays the estimates of the Asymmetric HARX. The R^2 rises only marginally. The most interesting effect is that the 66 day period return. The effect is large and negative when the S & P 500 experiences a positive return, while the effect disappears during negative stock returns. This means that positive stock market performances tends to decrease political risk, while negative stock market performances don't seem to have a large effect. Another interesting effect is the 5-day return on the VIX, which retains a significant effect both during the upside and the downside.[1]

To investigate the predictive power of the model and to avoid over-fitting, I have estimated the above models on a rolling window of 2000 observations, and then performed a one step ahead forecast for each sample. Table 2.9 reports the mean forecast error (MFE), the standard deviation of the forecast error (SDFE), the mean squared error (MSE) and the mean absolute error (MAE) for the three models. I have also added a simple random walk model as a benchmark. All three models have a a similar MSE of 0.30, well below the value of 1.21 for the random walk. The three models perform in a similar way also in terms of MAE.

To test the predictive power of the models, I have run a Giacomini and White test on each pair of specifications, reported in Table 2.10. The loss function is the absolute forecast error.

[1]To compare these results to another time series, I have estimated another (A)HARX on the VIX, using political risk as an explanatory variable, collected in z_t. The results are reported in Table 2.7. The model performs better on the VIX, yielding a much higher R^2. While VIX had some explanatory power on EPU, the opposite doesn't seem to hold true. The VIX is also much more difficult to predict one day ahead, with an R^2 of only 3.49%.

With a p-value of zero, the test rejects the null of equal predictive ability for HAR, HARX and AHARX against the RW benchmark. It also tends to reject the null of equal predicitive ability between HARX and AHARX. The test fails to reject the null for HARX and AHARX against HAR. Judging by these results, it seems like adding the asymmetric effects significantly deteriorates the predictive power of the model. It also means that adding a set of exogenous explanatory variables does not improve the predictive performance significantly.

2.5 Conclusions.

In this chapter I have investigated the time series properties of Baker et al.'s index of political uncertainty. The log differential is stationary and exhibits a leptokurtic distribution, with heavy tails on both sides. The index also exhibits a time varying conditional variance, as highlighted by an ARMA(1,1)-GARCH(1,1) model. This effect is only partially able to explain the fat tails. Political risk also has small extreme correlations with the S & P 500, pointing to the fact that it tends to convey a different type of risk other than market risk.

The set of HAR specifications has uncovered a number of further interesting properties. The index is highly persistent and exhibits a strong heterogeneous nature, as current values are impacted by various h-period past returns. Political risk interacts interestingly with a number of macro factors, most importantly the h-period return of the S&P 500, oil, credit spreads, fed funds deviation and the VIX index. The regressive nature of EPU can be exploited to produce a one step ahead forecast of the index.

Tables and Figures

The Economic Policy Uncertainty Index.

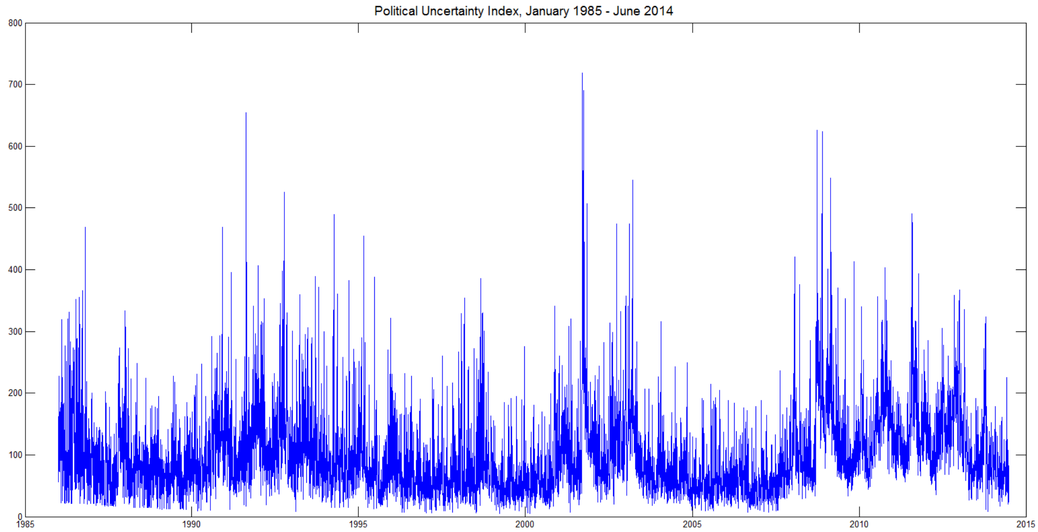


Figure 2.1: Plot of the EPU index time series, the sample period runs from from the 2[nd] January 1986 to the 30[th] June 2014, including altogether 7016 observations.

Autocorrelation

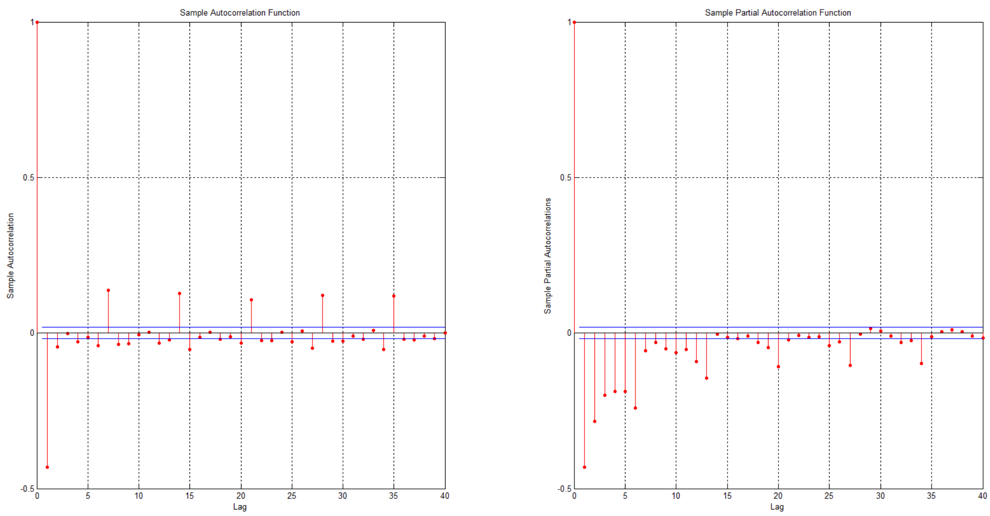


Figure 2.2: Autocorrelation and partial autocorrelation functions for x_t. The 95 % confidence bands are reported around zero.

Empirical distribution of x_t.

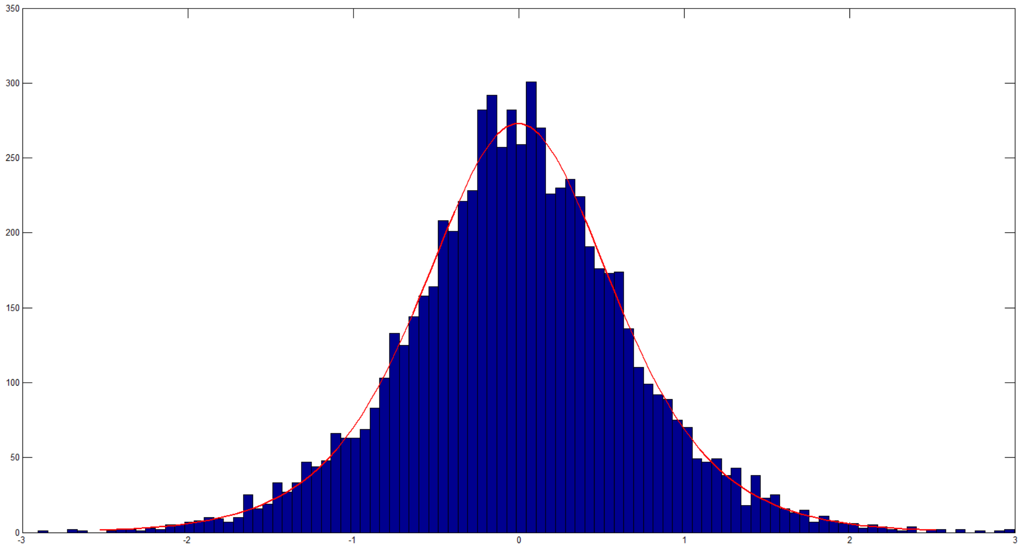


Figure 2.3: Empirical distribution of the log differential of the EPU index with a logistic distribution over-imposed. The parameters of the fitted logistic distribution are $\mu = -0.0019$ and $\sigma = 0.3826$.

Autocorrelation of residuals.

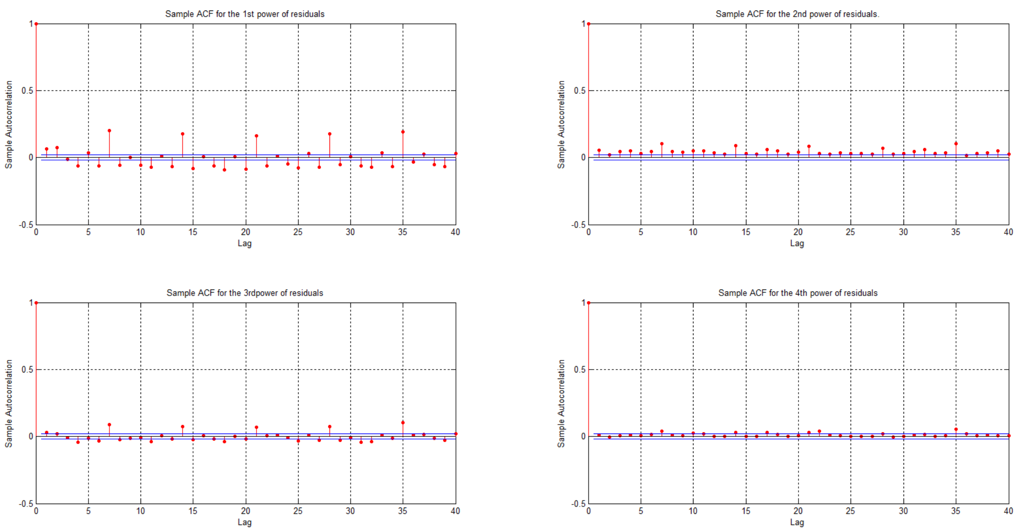


Figure 2.4: Sample autocorrelation function for various powers of an ARMA(1,1) residuals. By quadrants, clockwise, (1) first moment, (2) second moment, (3) fourth moment and (4) third moment. The residuals are computed from an ARMA(1,1) model as $\epsilon_t = x_t - \alpha - \phi_1 x_{t-1} - \psi_1 \epsilon_{t-1}$.

Conditional volatilities.

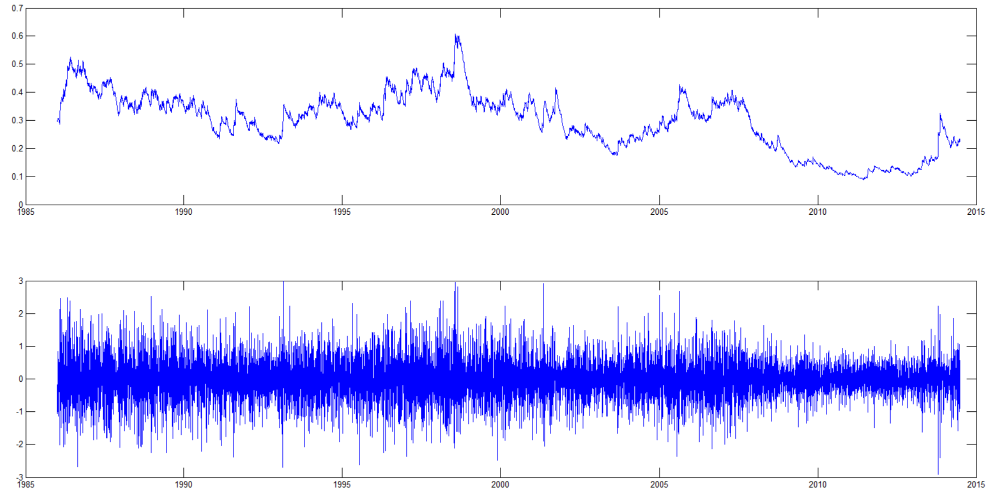


Figure 2.5: Top panel reports the conditional volatility of EPU index returns extracted from an ARMA(1,1)-GARCH(1,1) model. The bottom panel the log returns of the EPU index.

QQ plot of normalised residuals.

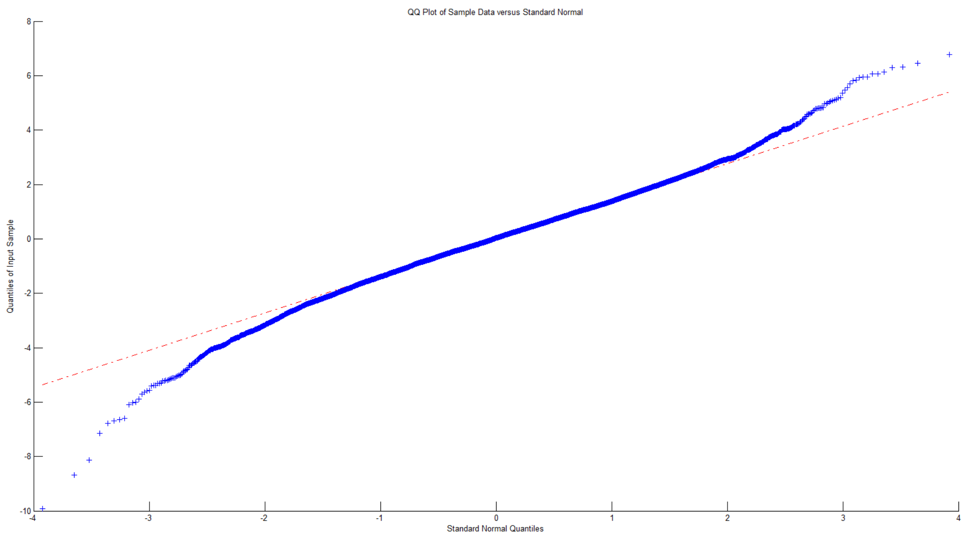


Figure 2.6: QQ plot of normalized residuals from the ARMA(1,1)-GARCH(1,1) model.

ARMA order selection.

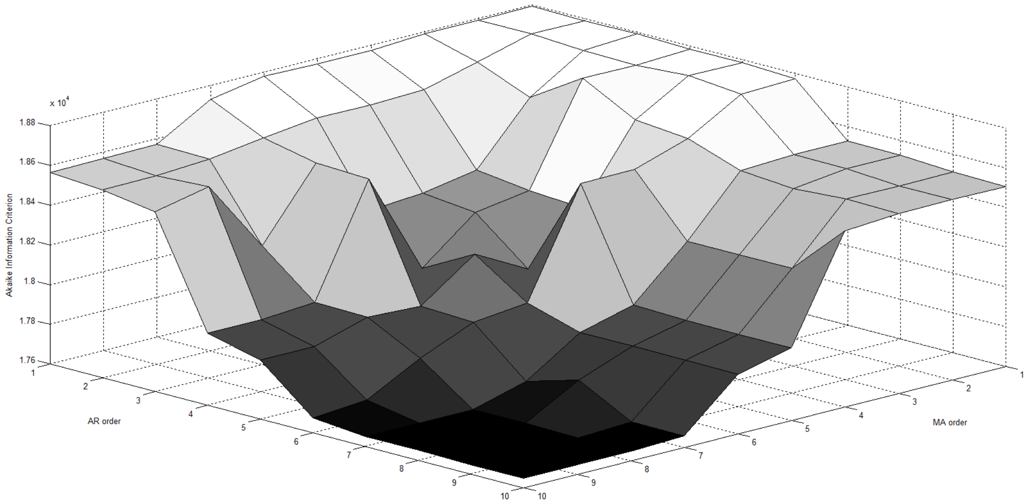


Figure 2.7: Akaike Information Criterion as a function of AR(p) and MA(q) order.

Tail correlations.

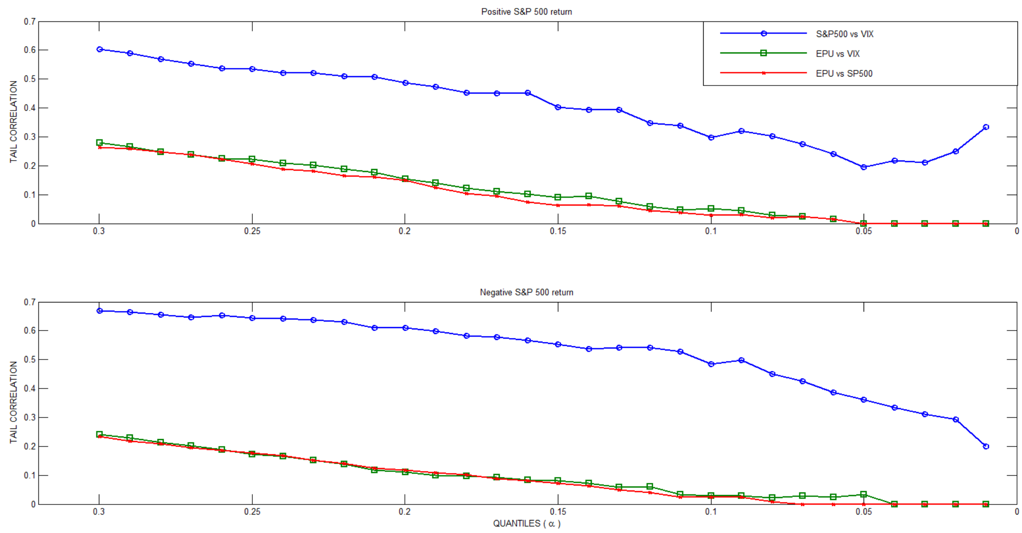


Figure 2.8: Tail correlation for three pairs: the VIX index with the S&P 500 index (blue), the EPU index with the VIX index (green) and the EPU index with the S&P 500 index (red). The figure reports various tail correlations for α from 30% to 1%. For a pair of stationary time series r_i and r_j, and for a set of αs from 30 % to 1%, I first compute the relevant quantiles as $q_i = \mu_i \pm t_{1-\alpha}\sigma_i$ and $q_j = \mu_j \pm t_{1-\alpha}\sigma_j$. The I compute the empirical frequencies $p_i(\alpha) = \frac{\#r_i<q_i}{N}$, $p_j(\alpha) = \frac{\#r_j<q_j}{N}$ and $p_{i,j}(\alpha) = \frac{\#r_i<q_i \wedge \#r_j<q_j}{N}$. The tail correlation between r_i and r_j is then the conditional probability $\rho_{i,j}(\alpha) = \frac{p_{i,j}(\alpha)}{p_i(\alpha)}$. I compute the tail correlations for two scenarios: a positive return scenario, where $q_{S\&P500} = \mu_{S\&P500} + t_{1-\alpha}\sigma_{S\&P500}$, $q_{VIX} = \mu_{VIX} - t_{1-\alpha}\sigma_{VIX}$, $q_{EPU} = \mu_{EPU} - t_{1-\alpha}\sigma_{EPU}$, and a negative return scenario, where $q_{S\&P500} = \mu_{S\&P500} - t_{1-\alpha}\sigma_{S\&P500}$, $q_{VIX} = \mu_{VIX} + t_{1-\alpha}\sigma_{VIX}$, $q_{EPU} = \mu_{EPU} + t_{1-\alpha}\sigma_{EPU}$.

Table 2.1: Empirical moments of EPU index.

	Mean	Standard dev.	Median	Skew	Kurtosis
EPU_t	101.64	69.69	83.69	1.81	8.62

Table 2.1 reports the first four empirical moments of the Economic Policy Uncertainty index.

Table 2.2: Stationarity test

Test	H_0	H_1	$\log(EPU_t)$ (p values)	Result	$\log\left(\frac{EPU_t}{EPU_{t-1}}\right)$ (p values)	Result
Agumented Dickey Fuller (ADF)	x_t is unit root	x_t is not unit root	0.1 %	Stationary	0.1 %	Stationary
Phillips-Perron (PP)	x_t is unit root	x_t is not unit root	0.1 %	Stationary	0.1 %	Stationary
Kwiatkowski, Phillips, Schmidt and Shin (KPSS)	x_t is stationary	x_t is not stationary	1 %	Not stationary	10 %	Stationary
Leybourne-McCabe (LMC)	x_t follows AR(p)	x_t follows ARIMA(p,1,1)	1 %	Not stationary	10 %	Stationary

Table 2.2 reports the p-values for four stationarity tests, performed on $\log(EPU_t)$ and its first difference.

Table 2.3: Goodness of fit test for EPU log differentials.

Distribution	Parameters			pValue	CvMstat
	mu	sigma			
Logistic	-0.0019	0.3826		0.8861	0.0483
	mu	sigma	nu		
t Location-Scale	-0.001	0.60	8.72	0.4436	0.1345
	k	sigma	mu		
Generalized Extreme Value	-0.2048	0.6914	-0.27	0.0010	5.1738
	mu	sigma			
Normal	-0.00018	0.69		0.0010	1.7567
	B				
Rayleigh	0.488			0.0010	890
	mu	sigma			
Extreme value	0.3469	0.7373		0.0010	23.43

Table 2.3 reports the result from a Cramer-Mises goodness of fit test. For each distribution tested, I report the values of the relevant parameters, the p value of the test and the test statistic.

Table 2.4: Results of ARMA(1,1)-GARCH(1,1) estimation.

ARIMA(1,1,1) Model:

Parameter	Value	Error	Statistic
Constant	-0.00012888	0.000544319	-0.36
AR1	0.167194	0.0100245	16.67
MA1	-0.92625	0.00382862	-241.93

GARCH(1,1) Conditional Variance Model:

Parameter	Value	Error	Statistic
Constant	0.000123846	5.55291e-05	2.23
GARCH1	0.990875	0.0009109	1087.80
ARCH1	0.008690	0.00088713	9.80
R^2		61.64 %	

Table 2.4 presents the results of the stimation of an ARIMA(1,1,1)-GARCH(1,1) on $X_t = \log \mathrm{EPU}_t$. This is equivalent to an ARMA(1,1) on $x_t = \log \mathrm{EPU}_t - \log \mathrm{EPU}_{t-1}$.

Table 2.5: Coefficient estimates for HARX and AHARX for EPU

Factors	Only lag	No lag	One day lag	Asymmetric	
Constant	-0.0008	0.0039	0.0055	-0.0155	
1 day lagged EPU	-0.17***	-0.16***	-0.16***	-0.16***	
5 days lagged EPU	-0.70***	-0.68***	-0.69***	-0.68***	
10 days lagged EPU	-1.09***	-1.10***	-1.10***	-1.12***	
22 days lagged EPU	-3.23***	-3.40***	-3.45***	-3.45***	
66 days lagged EPU	-6.88***	-8.14***	-8.29***	-8.16***	
		z_t	z_{t-1}	z_t^+	z_t^-
1 day S/P 500 return		0.07	-1.47*	1.07	-1.30
5 day S/P 500 return		4.91**	5.05*	5.31*	4.30
10 day S/P 500 return		-1.16	-3.33	0.52	-1.67
22 day S/P 500 return		-6.13	0.49	-9.90	-0.21
66 day S/P 500 return		-17.47**	-19.21	-35.63***	7.23
1 day oil return		0.05	0.24	0.75*	-0.55
5 day oil return		0.86	0.13	0.57	1.15
10 day oil return		-3.03**	-1.38	-2.04	-4.36**
22 day oil return		3.49*	2.12	-0.22	7.80**
66 day oil return		-1.24	-0.09	-0.70	-1.74
1 day USD index return		-1.36	-1.59	3.86*	-5.85**
change in credit spread		-0.30***	0.13	-0.52***	-0.03
change in term spread		-0.32***	0.15	0.10	-0.77***
Fed Funds deviation		0.09***	-0.01	0.11	0.02
1 day VIX index change		-0.40	-0.32**	-0.64***	-0.25
5 day VIX index change		2.39***	2.17***	1.96***	2.76***
10 day VIX index change		-0.34	-0.72	0.65	-1.25
22 day VIX index change		2.91**	4.14***	4.34**	1.26
66 day VIX index change		4.84**	3.47*	3.74	5.86*
Adjusted-R^2	31.56 %	33.01 %	32.88 %	33.29 %	

Table 2.5 reports results from three different HARX specifications estimated on the EPU. Column one "No lag" reports estimates of $y_t = \alpha + \beta' x_{t-1} + \gamma' z_t$, column two "One day lag" reports the results of $y_t = \alpha + \beta' x_{t-1} + \gamma' z_{t-1}$ and column three reports the results for the asymmetric specification AHARX $y_t = \alpha + \beta' x_{t-1} + \gamma^+ z_t^+ + \gamma^- z_t^-$. One, two and three starts correspond to significance at 10, 5 and 1%.

Table 2.6: Partial R^2

Factors	R^2	ΔR^2
1 day lagged EPU	18.11 %	+18.11 %
5 days lagged EPU	25.14 %	+7.03 %
10 days lagged EPU	27.78 %	+2.64 %
22 days lagged EPU	30.55 %	+2.77 %
66 days lagged EPU	31.61 %	+1.06 %
lagged S/P 500 returns	32.26 %	+0.65 %
lagged oil returns	32.32 %	+0.06 %
1 day USD index return	32.33 %	+0.01 %
change in credit spread	32.34 %	+0.01 %
change in term spread	32.40 %	+0.06 %
Fed Funds deviation	32.45 %	+0.05 %
lagged VIX index changes	33.05 %	+0.6 %

Table 2.6 reports the partial R^2 from sequential regressions. Each regression includes all the previous factors, so the change in R^2 shows the marginal contribution of that factor, given the inclusion of all previous factors.

Table 2.7: Coefficient estimates for HARX and AHARX for VIX

Factors	No lag	One day lag	Asymmetric	
Constant	0.0005	-0.0007	-0.0093***	
1 day lagged VIX	-0.045***	-0.0879***	-0.0435***	
5 days lagged VIX	-0.1687***	-0.1842***	-0.2038***	
10 days lagged VIX	-0.0698	-0.1323	-0.1029**	
22 days lagged VIX	-0.177**	-0.0701	-0.2288***	
66 days lagged VIX	-0.5848***	-0.59***	-0.6833***	
	z_t	z_{t-1}	z_t^+	z_t^-
1 day S/P 500 return	-4.1351***	-0.1604*	-2.9917***	-5.1198***
5 day S/P 500 return	-0.2835*	0.143	0.4393**	-1.0895***
10 day S/P 500 return	0.3567	0.0995	0.808**	0.3087
22 day S/P 500 return	-0.0327	0.843	0.2954	0.6805
66 day S/P 500 return	0.7963	1.042	0.7422	4.4592***
1 day oil return	0.0582***	0.0603**	0.071**	0.05
5 day oil return	0.0659	-0.0048	0.0788	0.0645
10 day oil return	-0.1197	-0.1926	-0.2987**	0.0497
22 day oil return	0.1023	0.3784*	-0.0438	0.2149
66 day oil return	0.1418	-0.0154	-0.3495	1.1054***
1 day USD index return	0.0205	-0.1271	0.3189*	-0.2537
change in credit spread	-0.0551***	-0.0121	-0.0323**	-0.0585***
change in term spread	0.0333***	0.0063	0.0226*	0.0229*
Fed Funds deviation	0.0041*	-0.008**	0.0052*	0.0005
1 day EPU index change	0.0014*	-0.0012	0.0005	0.0033**
5 day EPU index change	0.0001	-0.0238***	-0.002	0.0017
10 day EPU index change	-0.003	0.0046	0.0133	-0.0232*
22 day EPU index change	0.0072	0.0397*	0.0352	-0.0489*
66 day EPU index change	0.0134	0.1097*	-0.085	0.1176*
Adjusted-R^2	56.91 %	3.49 %	58.97 %	

Table 2.7 reports results from three different HARX specifications estimated on the VIX. Column one ”No lag” reports estimates of $y_t = \alpha + \beta' x_{t-1} + \gamma' z_t$, column two ”One day lag” reports the results of $y_t = \alpha + \beta' x_{t-1} + \gamma' z_{t-1}$ and column three reports the results for the asymmetric specification AHARX $y_t = \alpha + \beta' x_{t-1} + \gamma^+ z_t^+ + \gamma^- z_t^-$. One, two and three starts correspond to significance at 10, 5 and 1%.

Table 2.8: Granger causality test results

	Δ log SP500	Δ log VIX	Δ log EPU	Δ log AAA	Δ log BAA	Δ Y10-Y3	Δ log EFF	Δ log USD	Δ log OIL	Δ log TFF	Δ FF Dev
Δ log SP500	1.53	3.46	1.84	1.80	19.91*	11.34*	1.73	5.92*	3.14	2.09	2.1
Δ log VIX	0.49	0.39	2.17	2.43	0.62	1.53	1.24	1.25	3.21	0.84	0.44
Δ log EPU	18.75*	23.69*	0.81	1.68	6.16*	14.99*	1.73	2.105	0.82	2.68	1.13
Δ log AAA	12.48*	5.24*	1.269	0.06	17.51	11.87*	0.59	0.19	0.14	20.84*	0.36
Δ log BAA	23.79*	17.49*	3.13	6.95	8.32	15.19*	5.26	0.08	0.47	0.25	6.14*
Δ Y10-Y3	0.32	0.09	0.37	0.36	4.39*	0.0012	1.66	0.37	2.05	0.42	0.50
Δ log EFF	15.09*	0.84	0.94	4.54*	4.73*	15.50*	0.01	0.01	0.58	58.88*	28.81*
Δ log USD	45.62*	18.78*	1.16	1.73	0.26	56.74*	2.59	0.20	3.37	14.58*	0.89
Δ log OIL	7.53*	3.49	1.87	5.76*	2.54	13.30	1.08	3.38	8.03	15.93*	1.26
Δ log TFF	0.044	0.06	0.45	0.60	0.62	0.11	22.69*	11.35*	3.06	0.0004	0.0537
Δ FF Dev	2.44	1.06	2.62	2.01	10.16*	2.02	7.99*	0.93	8.65*	0.61	0.60

Table 2.8 reports the F statistic from pairwise Granger causality tests. Significance is reported at 5%. All the time series are stationary.

Table 2.9: Forecasting performance.

	MFE	SDFE	MSE	MAE
RW	0.0001	1.1021	1.2144	0.8431
HAR	0.0006	0.549	0.3014	0.4186
HARX	0.0015	0.5476	0.2998	0.4186
AHARX	-0.0026	0.5502	0.3027	0.4201

The sample period runs from from the 2nd January 1986 to the 30th June 2014, including altogether 7016 observations. I use a rolling window of 2000 time-series observations to estimate the different models and then perform a one day ahead out-of-sample forecasting evaluation in the remaining time series. I consider the following specifications: a random walk (RW), a heterogeneous autoregression (HAR), a heterogeneous autoregression with explanatory variables (HARX) and a heterogeneous autoregression with explanatory variables and asymmetric effects (AHARX). I gauge the forecasting performance by evaluating the mean forecast error (MFE), the standard deviation of forecast errors (SDFE), the mean squared error (MSE) and the mean absolute error (MAE).

Table 2.10: Giacomini-White tests for the absolute forecast error.

	RW	HAR	HARX
RW			
HAR	0.0001		
HARX	0.0001	0.93	
AHARX	0.0001	0.20	0.017

The sample period runs from from the 2^{nd} January 1986 to the 30^{th} June 2014, including altogether 7016 observations. I use a rolling window of 2000 time-series observations to estimate the different models and then perform a one day ahead out-of-sample forecasting evaluation in the remaining time series. I consider the following specifications: a random walk (RW), a heterogeneous autoregression (HAR), a heterogeneous autoregression with explanatory variables (HARX) and a heterogeneous autoregression with explanatory variables and asymmetric effects (AHARX). The statistics in each entry correspond to the p-value associated with the Giacomini-White test statistic for the null hypothesis that the column and row model perform equally well in terms of absolute forecast errors.

Chapter 3

The policy risk premium in equity derivatives.

3.1 Introduction.

This chapter introduces a model that allows to measure the impact of policy risk on the dynamics of the S&P 500 index using option data. I quantify the impact of policy risk on the whole $\mathbb{P}$-distribution of assets, not just on volatility, as most literature on policy risk does. I document that this type of risk is priced and has a sizeable impact on expected returns, volatilities, skewness and kurtosis. A one percent increase in the Economic Policy Uncertainty index leads to a 2.25 percentage points increase in the Equity Risk Premium.

Policy risk has often materialized with very dramatic consequences in the recent past; one example of such an event is the Greek debt crisis of 2010. In late 2009, New Democracy incumbent Greek Prime Minister Kostas Karamanlis faced an electoral defeat to PASOK's George Papandreou, who became the New Prime Minister, with 44% of the vote. The new government had to tackle a 12% budget deficit hangover from the 2008 recession and an economy that was struggling to grow. By the early months of 2010, the spread between 10 year Greek debt and the Bund reached 300 basis points. Between March and April, the Greek parliament passed a series of deficit reduction measures. These were not deemed effective by the markets and the Greek sovereign spread kept increasing, reaching 900 basis points in late April. A number of rating agencies started downgrading the sovereign debt of Greece, some to junk status. As borrowing on financial markets had become very expensive, on April 23rd Papandreou formally requested a bailout package from the EU, the ECB and the IMF. By May, international creditors had loaned the first 110 billion Euro to Greece and further deficit reduction measures were approved by the Greek parliament. Riots started to appear on the streets of Athens. Even with a comparatively small economy, the events in Greece rattled financial markets and sent the VIX index surging from 17 to 34 in just a few weeks.

These events left some researchers puzzled: how could such a small economy trigger such a global panic? How much of this crisis was due to real economic issues or the prospect of a default, and how much was due to the behaviour of bickering politicians? The problem in identifying these causes is of course that they tend to reinforce each other: poor growth could make politicians more desperate, and more desperate politicians may make policy mistakes that hamper economic growth. An interesting question is therefore how to separate the effects of policy risk from all "other" risks - economic, demographic, etc. - and how to estimate its impact on asset prices.

I address this issue by estimating a two-factor Heston model. The first factor, p_t, is policy risk. The factor is observable and is measured with error by the EPU index of Baker, Bloom and Davis (2016). I will discuss more about this index below. The second factor, v_t, is latent, constructed as uncorrelated with the first, and therefore captures all "other" risks, different from policy. My model is reduced form and allows me to price options and estimate risk parameters directly from market data. I can estimate the effect of political risk not only on asset volatilities, but on the whole distribution of asset returns under the model assumptions. The model is estimated by running an Unscented Kalman Filter on the data and by finding the parameters that maximise the likelihood of observing the actual market data, given the model specification.

This chapter has produced a number of contributions. The first is that I am able to extract the policy and non-policy risk factors from the data, a panel of option prices (more on my data later). This gives an indication of how much market risk is driven by political events and how much is left to be explained by "other" factors. I find that considering the average 3.93% variance of the S&P 500 index, 1.52% is due to policy risk while 2.41% is "other risks". The second result is that I recover the whole $\mathbb{P}$-distribution of asset prices as a function of policy risk. This allows to estimate the effects of policy risk on the whole distribution, i.e. on all the moments of the asset price. For example, Table 3.2 reports a simulation: on January 1^{st} 2009, a 3 percentage points increase in policy risk would have increased expected returns by 3.18% per quarter, from 14.76% to 17.94%, volatility by 1.97% from 19.29% to 21.26% and skewness from 0.35 to 0.36. The third result is that I can test if policy risk is priced. I find that policy risk is indeed priced, and carries a statistically significant risk premium of 2.25, while "other risks" are priced with a coefficient of 1.35. This means that a 1% increase in policy risk would lead to an increase in the ERP of 2.25% per year.

This chapter is closely related to Pastor & Veronesi (2013). While they adopt a general equilibrium approach, my model is reduced form. My model loses some of the economic intuitions of theirs, but, on the other hand, is able to recover the impact of policy risk on the whole distribution of an asset.

The rest of the chapter is organized as follows. In Section 2 I review the literature on policy risk, in Section 3 I conduct a preliminary investigation into the EPU index, in Section 4 I present my model. In Sections 5 and 6 I present my dataset and my estimation technique. I then in Section 7 present my results and draw some conclusions.

3.2 Literature review on political risk.

Policy uncertainty risk has been usually researched from two different perspectives: a macroeconomic perspective and a finance perspective.

In terms of the policy uncertainty effects on the macroeconomy, work has been done on tax uncertainty and earnings uncertainty. For example Sialm (2006) analyzes the effect of stochastic taxes on asset prices, and finds that investors require a premium to compensate for the risk introduced by tax changes. Tax uncertainty also features in Croce *et al.* (2012). They explore the asset pricing implications of tax uncertainty in a production economy with recursive preferences. They explore the long-run implications of public financing policies aimed at short-run stabilization when: (i) agents are sensitive to model uncertainty, as in Hansen and Sargent (2007), and (ii) growth is endogenous, as in Romer (1990). They find that countercyclical deficit policies promoting short-run stabilization reduce the price of model uncertainty at the cost of significantly increasing the amount of long-run risk. Ultimately these tax policies depress innovation and long-run growth and may produce welfare losses. Firm-level political risk features in Hassan *et al.* (2017). The authors build their own measure of political risk by measuring the share of their quarterly earnings conference calls they devote to discussing political risks. They find that exposure to political risk is negatively associated with investment and hiring and positively with lobbying. Macroeconomic uncertainty also features in Bali *et al.* (2016). They use Jurado *et al.* (2015)'s measure of uncertainty, which is the conditional volatility of the unforecastable component of a large number of economic indicators. They find that exposure to economic uncertainty carries a 6% risk premium.

The financial perspective has generally tried to characterize the risk exposure to policy uncertainty through the lens of beta exposures. Belo *et al.* (2013) link the cross-section of stock returns to firms' exposures to the government sector. Bittlingmayer (1998), Voth (2002), and Boutchkova *et al.* (2011) find a positive relation between policy uncertainty and stock volatility in a variety of settings.

The paper that more closely resembles mine is Pastor & Veronesi (2013). They set up an economy where agents learn in a Bayesian fashion about future government policies and examine their effects on average implied volatilities. Policy heterogeneity plays a large role in generating sufficient excess volatility. My paper improves on their contribution for various reasons. First, I am able to characterize the impact of policy uncertainty on all moments, not just volatility. This is important as an increase in volatility could be coupled with an increase in expected returns, potentially improving the investment opportunity set during periods of high policy uncertainty. Second, my model is reduced form, so I do not assume any particular underlying process for equity or for the learning process of agents. Third, my approach uses a latent factor approach, which has become very common in finance and is very easily interpretable in its intuitions.

3.3 Preliminary data analysis.

Baker, Bloom and Davis' main EPU index is compiled monthly. This is made of three components:

(a) The first component is an index of search results from 10 large newspapers. The newspapers included in their index are USA Today, the Miami Herald, the Chicago Tribune, the Washington Post, the Los Angeles Times, the Boston Globe, the San Francisco Chronicle, the Dallas Morning News, the New York Times, and the Wall Street Journal. From these papers, they construct a normalized index of the volume of news articles discussing economic policy uncertainty.

(b) The second component of their index draws on Congressional Budget Office (CBO) reports that compile lists of temporary federal tax code provisions. They create annual dollar-weighted numbers of tax code provisions scheduled to expire over the next 10 years, giving a measure of the level of uncertainty regarding the path that the federal tax code will take in the future.

(c) The third component of their policy-related uncertainty index draws on the Federal Reserve Bank of Philadelphia's Survey of Professional Forecasters. They utilize the dispersion between individual forecasters' predictions about future levels of the Consumer Price Index, Federal Expenditures, and State and Local Expenditures to construct indices of uncertainty about policy-related macroeconomic variables.

The authors also publish a daily index for the United States. It only consists of the first component as CBO reports and Professional Forecasts are not available daily. The daily index tracks closely the monthly one and the two have a 0.87 correlation coefficient. I use the daily index, as it is better suited to explain the daily variation in the option-implied forward density of the S&P 500 index.

As a preliminary analysis, I have checked that the EPU index has indeed some explanatory power on volatility by regressing it against the VIX index, which measures the 30-day ahead expected realized volatility of the S&P 500 index. The estimated coefficients are highly significant, with p-values indistinguishable from zero and an adjusted-R^2 of 7.7%. The positive slope suggests that an increase in the measure of policy uncertainty by 100 units leads to an increase in index implied volatility of about 3.6%.

$$\begin{aligned} \mathrm{VIX}_t &= 0.17288 + 0.00036987 \quad \mathrm{EPU}_t \\ \textit{t-stats:} \quad & (95.52) \qquad\quad (24.48) \end{aligned}$$

A time series plot and a look at moments show that the daily EPU index is also highly persistent and exhibits high positive skewness (see Table 3.1).

In my previous chapter, I have analyzed the EPU index in more detail and have uncovered a number of its features. The first is that the index is not stationary, while its first difference is. The second is that it exhibits a time varying conditional variance. The third is that it shows an autoregressive nature, and the fourth is that the EPU index Granger-causes the VIX index.

3.4 The model.

This model is set out to achieve three main objectives: to capture the empirical features uncovered in my previous analysis, to be simple enough to price options quickly, and to deliver some economic insight on the dynamics of risk and returns. I model policy risk using a two-factor Heston model where one of the two factors, policy risk, is observable. The other factor captures the volatility not due to policy risk and is constructed as uncorrelated with policy risk.

Let $(\Omega, \mathfrak{F}, \{\mathfrak{F}_t\}_{t\geq 0}, \mathbb{P})$ be a filtered probability space satisfying the following assumptions:

i Set Ω is the set of all possible outcomes $\{\omega_1, ..., \omega_n\}$

ii $\mathfrak{F}$ is a sigma algebra such that:

  (a) $A_k \in F \vee k$ implies $U_{k=1}^{\infty} A_k \in \mathfrak{F}$

  (b) $A \in \mathfrak{F}$ implies $A^c \in \mathfrak{F}$

  (c) $\emptyset \in \mathfrak{F}$

iii $\{\mathfrak{F}_t\}_{t\geq 0}$ is a sub-sigma algebra of $\mathfrak{F}$ with $\mathfrak{F}_s \subset \mathfrak{F}_t$ if $s < t$

iv $\mathbb{P}$ is a probability measure such that $\mathbb{P}(\Omega) = 1$, $\mathbb{P}(\emptyset) = 0$ and $\mathbb{P}(A) = 1 - \mathbb{P}(A^c) \geq 0$. A risk neutral measure $\mathbb{Q}$ is another probability measure such that $\int d\mathbb{P} = \int \frac{d\mathbb{Q}}{d\mathbb{P}} d\mathbb{P}$ where $\frac{d\mathbb{Q}}{d\mathbb{P}}$ is a Radon-Nikodym.

Let S_t be the S&P 500 index, p_t the latent policy risk factor and v_t the variable collecting ”other risks”.

The dynamics of index returns under $\mathbb{P}$ are:

$$\frac{dS_t}{S_t} = (r_f + \lambda_v v_t + \lambda_p p_t)dt + \sqrt{v_t}dW_t^{(1)} + \sigma_{EPU}\sqrt{p_t}dW_t^{(2)}$$

$$dv_t = k_v^{\mathbb{P}}(\theta_v^{\mathbb{P}} - v_t)dt + \sigma_v^{\mathbb{P}}\sqrt{v_t}dW_t^{(3)}$$

$$dp_t = k_p^{\mathbb{P}}(\theta_p^{\mathbb{P}} - p_t)dt + \sigma_p^{\mathbb{P}}\sqrt{p_t}dW_t^{(4)}$$

with the following correlations:

$$\mathbb{E}[dW_t^{(1)}dW_t^{(3)}] = \rho_{sv}dt$$

$$\mathbb{E}[dW_t^{(2)}dW_t^{(4)}] = \rho_{sp}dt$$

where $W_t^{(\cdot)}$ denotes the four main Brownian motions that drive risk in this setting. Two sources of risk appear in the index equations, and are proportional to the underlying risks. The other two sources of risk drive the factor dynamics. The two processes for $\{v_t\}_{t\geq 0}$ and $\{p_t\}_{t\geq 0}$ are instantaneously uncorrelated between them, but are correlated with the index dynamics, generating the well known leverage effect. The model allows to estimate the impact of policy risk on the S&P 500 index excess returns, quantifying the risk premium associated with policy risk. This model is able to capture the empirical features on policy risk expressed in my previous chapter. In fact, the first difference of the Ornstein Uhlenbeck process proposed is stationary, it has a time varying conditional variance equal to ${\sigma_p^{\mathbb{P}}}^2 p_t$, it exhibits an autoregressive nature controlled by parameter κ_p, and the variable p_t determines partially the value of the VIX index.

The same model under the risk neutral measure $\mathbb{Q}$ reads:

$$\frac{dS_t}{S_t} = r_f dt + \sqrt{v_t}dW_t^{(1)} + \sigma_{EPU}\sqrt{p_t}dW_t^{(2)}$$

$$dv_t = k_v^{\mathbb{Q}}(\theta_v^{\mathbb{Q}} - v_t)dt + \sigma_v^{\mathbb{Q}}\sqrt{v_t}dW_t^{(3)}$$

$$dp_t = k_p^{\mathbb{Q}}(\theta_p^{\mathbb{Q}} - p_t)dt + \sigma_p^{\mathbb{Q}}\sqrt{p_t}dW_t^{(4)}$$

A feature of the two-factor Heston model is that we can recover a closed form expression for the VIX index. In fact,

$$\text{VIX}_t^\tau = \mathbb{E}_t^{\mathbb{Q}}\left[\int_t^{t+\tau}(v_s + p_s)ds\right] = \int_t^{t+\tau}\mathbb{E}_t^{\mathbb{Q}}[v_s]ds + \int_t^{t+\tau}\mathbb{E}_t^{\mathbb{Q}}[p_s]ds$$

This fact will be used later during estimation, as it allows to understand how policy risk affects volatility.

Another feature of the Heston model is that its characteristic function is known in closed form. This allows to employ characteristics function techniques to price options fast. It also means that the forward $\mathbb{Q}$-density of the S&P 500 index is known short of a Fourier transform and that it depends on the level of p_t.

In particular, the price of a call can be computed as:

$$C(S_0, K, \tau) = S_0 e^{-q\tau} P_1 - K e^{-r\tau} P_2$$

where S_0 is the spot price of the index, K is the strike price, r is the risk free rate, q is the dividend yield and τ is the maturity. P_1 and P_2 are two probabilities that can be recovered via Fourier inversion as:

$$P_j = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty Re\left[\frac{e^{-iu\log K} f_j(u, \log S_T, v_t, p_t)}{iu}\right] du \qquad \text{for } j = 1, 2$$

where $Re(x)$ takes the real part of the complex number x and $f_j(u, \log S_T, v_t, p_t)$ is the conditional characteristic function of the model and has a general form as:

$$f_j(u, \log S_t, v_t, p_t) = e^{A_j(u) + B_{j,v}(u)v_t + B_{j,p}(u)p_t + iu\log S_t}$$

where coefficients A,B,C and D solve the corresponding Ordinary Differential Equation. The characteristic function completely defines a probability distribution. Recall, for example, that the k-th moment can be recovered from the characteristic function as the k-th derivative:

$$\mathbb{E}\left[x^k\right] = (-i)^k f_j^{(k)}(0)$$

The details of the option pricing formulas are available in Appendix C.

3.5 Data.

My data is composed of two time series for the S&P 500 index and the Economic Policy Uncertainty index, and a panel dataset for European vanilla options written on the S&P 500 index. The original data is daily, even if I then use only Wednesday prices to remove intra-week trading seasonalities. The reference period runs from January 1^{st} 1996 to December 31^{st} 2013. All the securities data was downloaded through Wharton Research Data Services. In particular, S&P 500 index data was provided by the Center for Research in Security Prices (CRSP), while option data was downloaded from Ivy DB OptionMetrics. The Economic Policy Uncertainty index, available from Baker, Bloom and Davis' website at www.policyuncertainty.com, is at a daily frequency, but I have kept Wednesday prices only for consistency.

In addition to S&P 500 index values, I use European vanilla option data on the S&P 500 index from Ivy DB. I collect weekly Wednesdays call option quotes from January 1996 to December 2013. I filter the option quotes by log moneyness from -3% to +3%. I also drop the contracts with zero trading volume. In terms of maturities, I drop the very short end of the maturity spectrum and the long end, so that my option sample has maturities from 62 to 117 days. This leaves me with 930 trading days and a total of 47,192 option contract data points across moneyness and maturities, that is an average of fifty options per day. **??** reports descriptive statistics for various characteristics of the option panel.

3.6 Estimation strategy.

My estimation problem can be viewed as a traditional dynamic system described by a state transition equation and a measurement equation:

$$x_{t+1} = g(x_t, \theta) + \epsilon_t$$
$$\tilde{y}_{t+1} = f(x_t, \theta) + u_t$$

The state variable x_t is unobserved. In my case $x_t = \{v_t, p_t\}$. What I do observe is the return of the S&P 500 index, the level of the EPU index and a vector of options on the S&P 500 index, which I collect in the vector $\tilde{y}_{t+1}$, which is my vector of market observables or measurements.

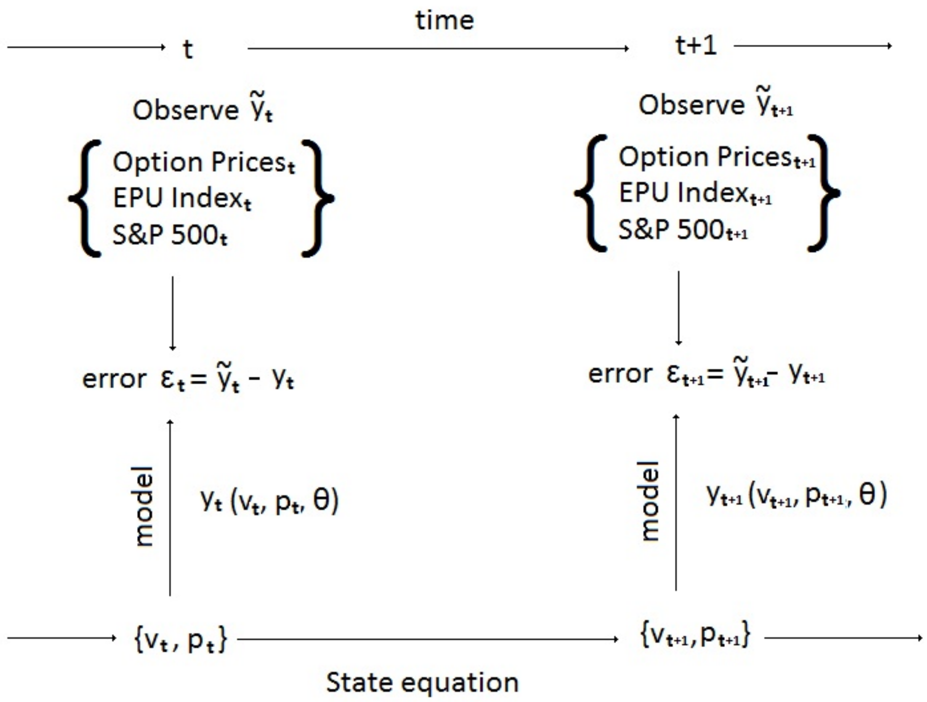
The parameters are estimated using an Unscented Kalman Filter (UKF) over the whole time series of data. Filtering means using a set of market observables, called measurements, to extract the dynamics of the unobservable state variables at each point in time. A filter can then be coupled with maximum likelihood techniques to estimate the parameter vector θ.

Note that policy risk is latent but observed with error as:

$$p_t = EPU\ index_t + \sigma_{\epsilon_p} \epsilon_{t,p}$$

where ϵ_p is a standard normally distributed error term.

The choice of the Unscented Kalman Filter is motivated by the strong non-linearities present in the observations equation. At each point in time, my measurements include options, and options are not a linear function of the underlying state variables. In some cases one can attempt to linearize the measurements, like in the Extended Kalman Filter (EKF). The EKF approximates the measurement equation to the first order of a Taylor series expansion. When the function is highly non-linear, this procedure generates large errors. Instead, the UKF does not attempt to linearize the function. The filter, developed by Wan & Merwe (2000), approximates the state distribution by using a minimal set of carefully chosen points, called sigma points. These completely capture the mean and covariance matrix of the state distribution and, when propagated forward through the transition equation, are able to capture the posterior mean and covariance matrix of the measurements accurately at the third order of a Taylor series expansion, rather than the first order as the EKF. This makes the UKF superior to the EKF, because a first derivative expansion of an option is not generally very accurate if market movements are big. As I use weekly data, the movements in the underlying index and state variables over a seven day period can be big enough to deteriorate the quality of the EKF.



The estimation procedure is structured as follows.

I first discretize the continuous time state transition dynamics under $\mathbb{P}$ using an Euler discretization as follows:

$$\begin{bmatrix} v_{t+1} \\ p_{t+1} \end{bmatrix} = \begin{bmatrix} \kappa_v \theta_v \\ \kappa_p \theta_p \end{bmatrix} \Delta t + \begin{bmatrix} 1-\kappa_v & 0 \\ 0 & 1-\kappa_p \end{bmatrix} \begin{bmatrix} v_t \\ p_t \end{bmatrix} \Delta t + Q(v_t, p_t) \begin{bmatrix} \epsilon_{t,v} \\ \epsilon_{t,p} \end{bmatrix}$$

Since I use weekly observations, Δt is set to $1/52$. In the equations above, $\epsilon_{t,i}$ is a standard normally distributed random variable. These two variables are uncorrelated among each other, but each of them is separately correlated with the stock market index. The matrix of conditional variances for the state equation $Q(v_t, p_t)$ is given by:

$$Q(v_t, p_t) = \begin{bmatrix} \sigma_v^2 \left(1 - e^{-\kappa_v \Delta t}\right)^2 \\ \sigma_p^2 \left(1 - e^{-\kappa_p \Delta t}\right)^2 \end{bmatrix} + \begin{bmatrix} \sigma_v^2 \frac{e^{-\kappa_v \Delta t} - e^{-2\kappa_v \Delta t}}{\kappa_v} \\ \sigma_p^2 \frac{e^{-\kappa_p \Delta t} - e^{-2\kappa_p \Delta t}}{\kappa_p} \end{bmatrix} \begin{bmatrix} v_t \\ p_t \end{bmatrix}$$

As previously mentioned, the measurement equation includes three components: the stock market index, the policy risk index and a vector of options, with various maturities and strike prices. For the stock market index and the vector of options the measurement is calculated by taking the log differences between model and market observations. As such, each week I observe:

$$\tilde{y}_t = \begin{bmatrix} \log(S\&P500_t) - \log(S\&P500_{t-1}) \\ EPU\ index_t \\ \left\{ \begin{matrix} \log OptionModel(v_t, p_t; K, T, \theta)_t - \log OptionMarket(K,T)_t \\ \vdots \end{matrix} \right\}_{\substack{K \in K_{min}, K_{max} \\ T \in T_{min}, T_{max}}} \end{bmatrix}$$

The first term is simply the gross weekly index return. The second measurement equation assumes policy risk is observed with some error. The third component is the option mispricing, where $OptionModel(v_t, p_t; K, T, \theta)_t$ is the price of the option as estimated by my model, given the parameters θ and the value of the state variables v_t and p_t; $OptionMarket(K, T)_t$ is a vector of market option observations, for various maturities and strike prices. Since the set of available observations changes every day, the size of my option sample is not, in general, fixed. To clear the matters further, I also report below the model-implied observation equation. Market observations should be, on average, equal to model-implied ones:

$$y_t = \begin{bmatrix} (r_f + \mu)\Delta t \\ p_t \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

where the vector of zeros is the expected error in mispricing options. The elements of y_t are the expected values of $\tilde{y}_t$. As the model should price options correctly, y_t includes a vector of zeros. Keeping the parameter μ in the model-implied observations allows to extract the average Equity Risk Premium for my sample. Note that this is replaced by $\lambda_v v_t + \lambda_p p_t$ when I estimate the prices of risk.

To estimate the model, it is necessary to make some distributional assumptions on the measurement errors. The error $\epsilon_t = \tilde{y}_t - y_t$ is assumed to follow a normal distribution with mean zero and covariance matrix:

$$R = \begin{bmatrix} v_t + \sigma^2_{EPU} p_t & 0 & \dots & \dots & 0 \\ 0 & \sigma^2_{\epsilon_p} & 0 & \dots & \vdots \\ \vdots & 0 & \sigma^2_{\epsilon_{Options_i}} & \dots & \vdots \\ \vdots & 0 & 0 & \dots & \vdots \\ 0 & \dots & \dots & \dots & \sigma^2_{\epsilon_{Options_i}} \end{bmatrix} \Delta t$$

As assuming the homoskedasticity of the errors would be too strong, I allow the variance of the errors to depend on the moneyness and the maturity of the contract. The measurement error for the option sample is therefore parametrized as:

$$\sigma^2_{\epsilon_{Options_i}} = \phi_0 + \phi_1 \mid \log \frac{K_i}{S_t} \mid + \phi_2 T_i$$

where K_i is the strike price and T_i is the time to maturity. The state variables are initialized at their sample means. At each iteration, the state variables are spread out to create "sigma points". Each sigma point is then propagated forward through the state transition equation, in order to generate a set of sigma points for $t+1$. Each of these sigma points is then passed through the measurement equation, so that for each sigma point there is a model-implied measurement. These model-implied measurements are then compared to the observed measurements. At each step, I compute the measurement error and its log-likelihood. The likelihood of options is scaled by the number of options, so that the weights of the three measurements (stock index, EPU index and options) are equal. The filter is iterated through the whole sample. The outcome is a sum of log-likelihoods, computed as a function of the model parameters.

The estimation problem can be formalized as follows:

Let $\theta = \left\{ \mu, \lambda_v, \lambda_p, \kappa_v^{\mathbb{P}}, \theta_v^{\mathbb{P}}, \sigma_v^{\mathbb{P}}, \kappa_v^{\mathbb{Q}}, \theta_v^{\mathbb{Q}}, \sigma_v^{\mathbb{Q}}, \rho_v, \kappa_p^{\mathbb{P}}, \theta_p^{\mathbb{P}}, \sigma_p^{\mathbb{P}}, \kappa_p^{\mathbb{Q}}, \theta_p^{\mathbb{Q}}, \sigma_p^{\mathbb{Q}}, \rho_p, \sigma^2_{EPU} \sigma^2_{\epsilon_p}, \phi_0, \phi_1, \phi_2 \right\}$ be the vector of parameters to be estimated.

Given an initial guess θ_0, the UKF generates a time-series of state variables $v_t(\theta_0), p_t(\theta_0)$ and another time series of model-implied measurements $y_t(v_t, p_t, \theta_0)$. As a result, at each point in time I observe the market measurements $\tilde{y}_t$ and compute an error $\epsilon_t(v_t, p_t, \theta_0)$. The problem is then to find the parameter vector than maximizes the log likelihood function of the error, or that minimizes its opposite.

$$\theta^* = \arg\min_\theta - \log L(\theta) = \sum_{t=1}^{T} \log L(\epsilon_t, \theta)$$

The likelihood function exhibits several non-linearities. Therefore, I minimize the log-likelihood numerically on a grid of parameters values. As a by product of this optimization, I also extract the optimal paths for v_t and p_t, that is, the paths that are more likely for the state variables given the observations. I then compute the asymptotic variance of the parameter estimates via the inverse of the Fisher information criterion. I numerically compute the derivatives, choosing a step equal to 1% of the parameter value.

The Equity Risk Premium is the expected excess return of the market over the risk free rate, under the physical measure $\mathbb{P}$.

$$ERP_t = \mathbb{E}_t^{\mathbb{P}} \left[\log \left(\frac{SP500_{t+1}}{SP500_t} \right) \right] - r_{f,t} = (\lambda_v v_t + \lambda_p p_t) \Delta t$$

As my model is estimated under $\mathbb{P}$, I am able to extract the Equity Risk Premium from my option sample.

3.7 Results and discussion.

Table 3.3 reports the estimation of the model parameters. The state variable p_t captures policy risk. Policy risk exhibits a long term mean of 1.65% and a volatility of 43.20%. With a correlation coefficient of -0.33, policy risk is negatively correlated with stock returns. It also exhibits strong mean reversion (26.5). A plot of log EPU differentials and p_t show that the two are almost indistinguishable. My model therefore successfully reproduces the empirical properties of EPU uncovered in my previous chapter. The state variable v_t, orthogonal to p_t, captures all "other risks", that is all non-policy risks. It has a mean of 0.14% and a volatility of 24%. Like policy risk, v_t is also negatively correlated with stock returns, by about the same order of magnitude (-0.4). v_t is also less mean reverting. The estimated average equity risk premium for the whole sample is 8%, which is in line with the literature.

We can now proceed to analyze the extracted paths of the state variables. The two factors exhibit a correlation coefficient of -0.18, which is weakly significant. Figure 3.5 shows the extracted factors. Compare this with overall market risk in Figure 3.2. As we can see, the big fall in market risk in 2009 was not driven by reduced policy risk. Despite a spike in early 2009, policy risk remained largely flat for the whole of 2009. Also, the big spike in risk in the summer of 2010 does not seem to be driven by an increase in policy risk. On the contrary, at the end of 2011, it seems that the spike in policy risk drove overall risk higher. The main focus of policy risk at the end of 2011 was the US debt ceiling crisis. Briefly, in the US only Congress has the power to authorize government indebtedness. The US Treasury keeps accumulating debt, so it has to ask periodically Congress to raise the debt ceiling. A failure to raise the debt ceiling could, in principle, generate a government shut-down and a credit event for the holders of US debt. This analysis shows that the risk posed by the policy events of late 2011 was significant.

In terms of option fit, the average error is -0.72% Black & Scholes implied volatility percentage points. Figure 3.8 shows the option mispricing as a function of moneyness and of time to maturity. The mispricings are scattered uniformly over the sample. Table 3.4 reports the average mispricing as a function of moneyness and time to maturity. The model consistently under prices almost all options. The most under priced options are the Out-of-The-Money options with a mispricing of 1.14 percentage points of implied volatility. At-The-Money options have an average mispricing of 0.8 percentage points and In-The-Money options have a 0.18 percent mispricing. Along the time-to-maturity dimension, the error rises from 0.32% to 1.25% as the maturity moves from under 0.22 to over 0.28. Figure 3.7 shows two smiles for two different maturities of two and three months, for the specific date January 6^{th} 2010. The mispricing is acceptable for In-The-Money options but deteriorates for Out-Of-The Money contracts. To put these figures into context, Table 3.5 reports the mispricings for a regular one factor model (the original Heston model). The two models are comparable because both models have only one latent factor, in the case of Heston it is the unobserved stochastic volatility, in my case v_t. My model improves on the original Heston model along all maturities and strikes.

Table 3.3 also report the estimates for the risk premia associated to policy risk. The coefficients are significant and show that both risks are priced. To investigate the effect of higher policy risk, I have simulated two $\mathbb{P}$-distributions, one with low policy risk at 3% and another with high policy risk at 6%. Figure 3.6 and Table 3.2 report the simulations. The higher policy risk has the effect of increasing the ERP from 14.76% to 17.94%, the volatility from 19.29% to 21.26% and skewness slightly from 0.35 to 0.36. The effects on the distribution are more visible in Figure 3.6: as policy risk rises, volatility rises and the distribution tilts to the right, increasing the ERP and slightly increasing skewness.

To summarise, I find that policy uncertainty risk has a positive impact on the equity risk premium and that it impacts all moments. It is also able to filter a policy uncertainty factor that has economic meaning. Finally, the two factor model is able to price options accurately.

3.8 Conclusions.

Disentangling policy risk from other risks helps to shed light on the drivers of the risk and return of asset prices. In this chapter I have introduced a two-factor model where one of the factors is observed while the other is latent. This reduced form model allows me to extract important information from option prices, such as the path of these factors and how much risk is due to each. By exploiting the reduced form nature of my model, I can also estimate the effect that policy risk has on the whole $\mathbb{P}$-distribution of returns, rather than just on volatility. An increase in policy risk affects the first three moments, pushing expected returns, volatilities and skewnesses higher. Finally I have established that policy risk drives the Equity Risk Premium up and that the effect is statistically significant.

These results convey the importance of policy uncertainty as a priced factor in the market. The economic message for asset managers is that policy uncertainty has a profound impact on return dynamics, and it may affect the long term ability to deliver stable wealth creation for investors. It may therefore be useful to try and hedge exposure to policy uncertainty away. The economic message to policy-makers is that uncertainty around their actions have statistically significant effects on markets, and therefore adopting stable and predictable policies would remove most of this source of risk.

Tables and Figures

Monthly EPU index.

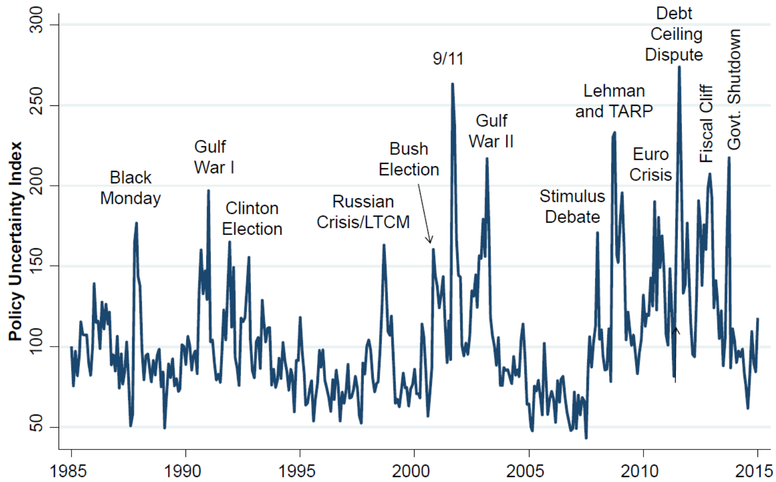


Figure 3.1: Monthly Economic Policy Uncertainty index for the US, 1985 to 2014.

EPU and VIX indices.

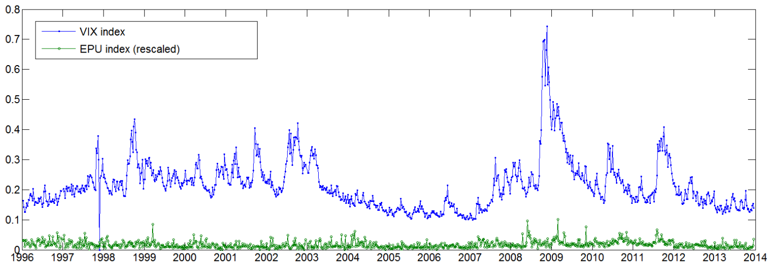


Figure 3.2: EPU index compared to 30 day VIX index, January 1996 to December 2013.

EPU and VIX indices, different axis.

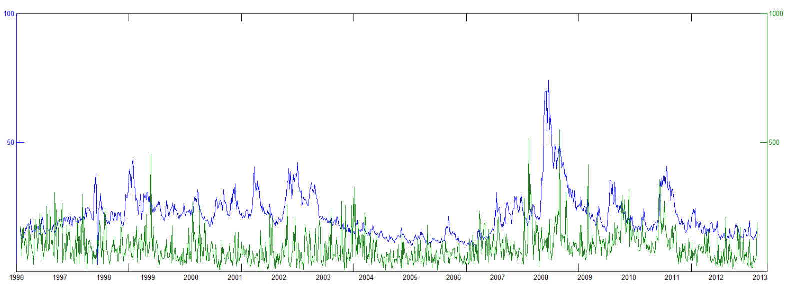


Figure 3.3: EPU index compared to 30 day VIX index, January 1996 to December 2013. The two time series are plotted on two different axes to show their correlation.

Scenario analysis.

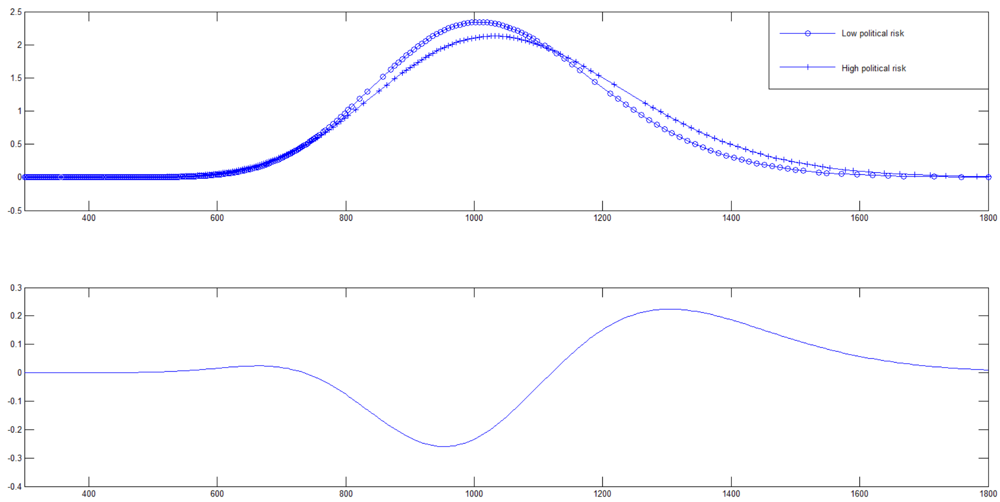


Figure 3.4: Above, model implied distribution of terminal stock price S_T for two scenarios, low and high policy risk. Below, the difference in distributions. The probability distribution is the $\mathbb{P}$-distribution generated from the model $\mathbb{P}$-parameters.

Extracted factors.

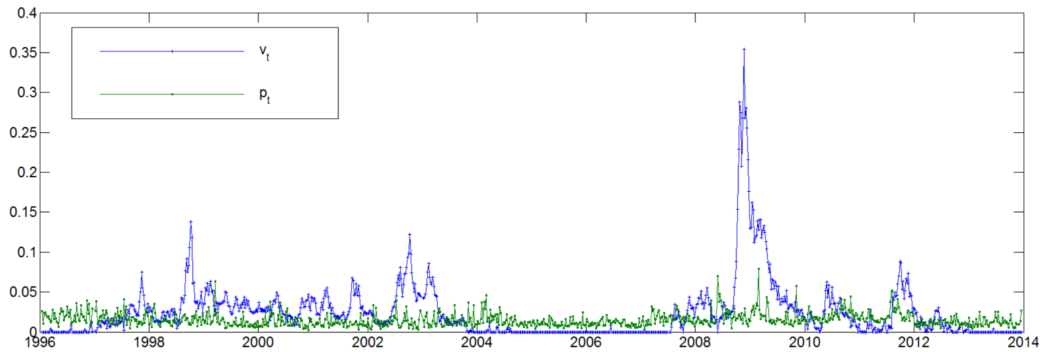


Figure 3.5: Extracted trajectories for the factors v_t (blue) and p_t (green) from the Unscented Kalman Filter.

Equity Risk Premium across time.

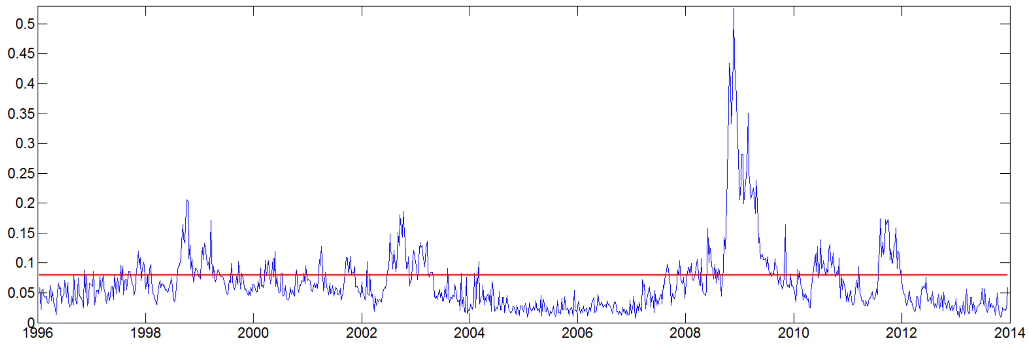


Figure 3.6: Estimated Equity Risk Premium from the $\mathbb{P}$-distribution.
This is equal to $ERP_{t,\tau} = \mathbb{E}_t^{\mathbb{P}}\left[\log\left(\frac{SP500_{t+\tau}}{SP500_t}\right)\right] - r_{f,t} = \lambda_v v_t + \lambda_p p_t$. Highlighted in red the 8 % average.

Volatility smiles: market vs model.

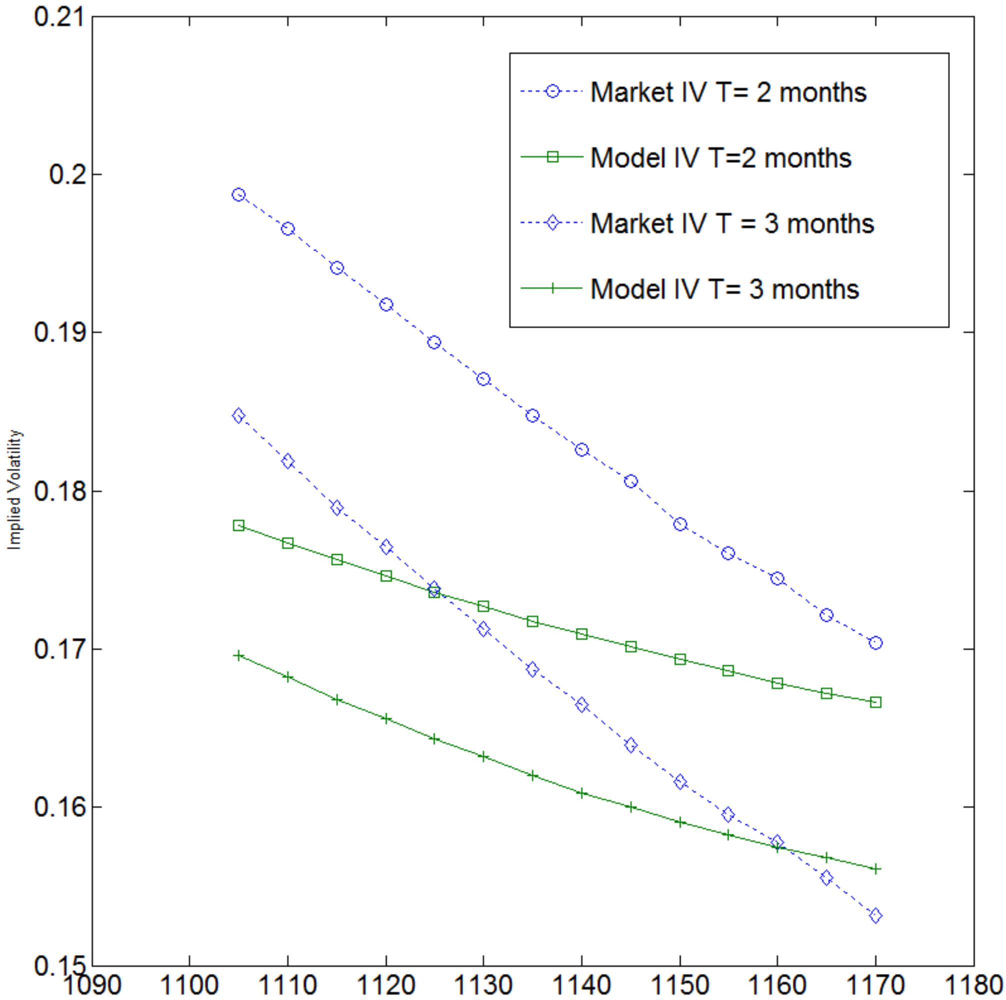


Figure 3.7: Volatility smiles for market quotes vs policy uncertainty model, for two different maturities of 2 and 3 months, on the 6 th of January 2010. The x axis shows the strike prices and the y axis shows the Black and Scholes implied volatilities.

Option mispricings across moneyness and maturities.

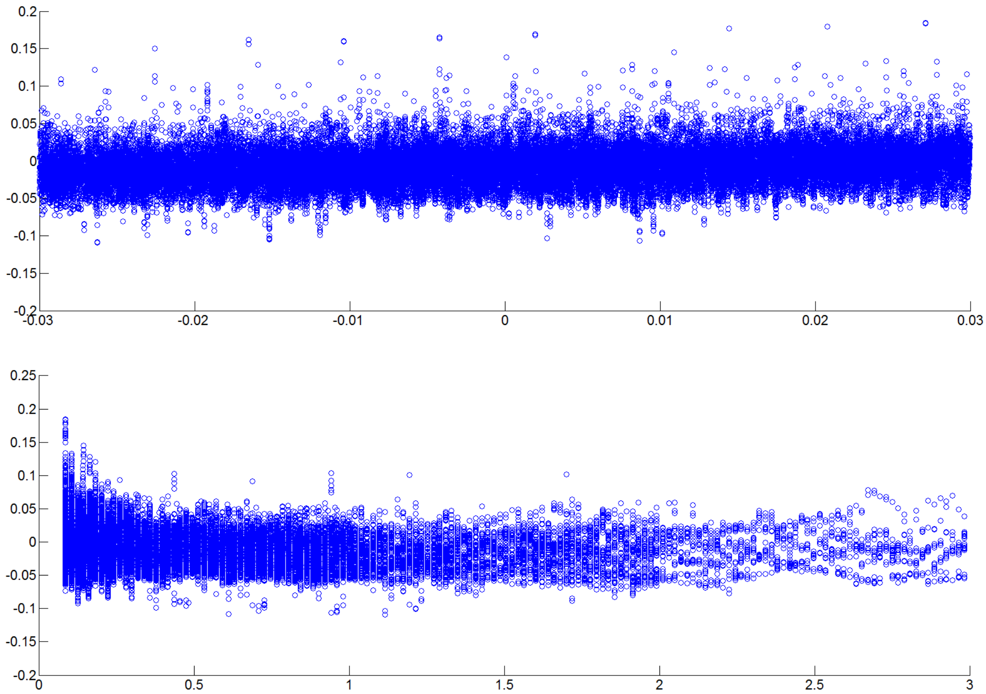


Figure 3.8: This picture plots the mispricing of the policy uncertainty model for various log moneyness levels (top panel) and time to maturities (bottom panel). At each point in time, I compute the model-implied price and compare it to the market price. The average mispricing is measured as percentage points of Black and Scholes implied volatility. Time to maturity is in years and moneyness is measured as a percentage from the forward price $k_{t,i} = \log \frac{K_i}{F(t,T_i)}$.

Monte Carlo simulation.

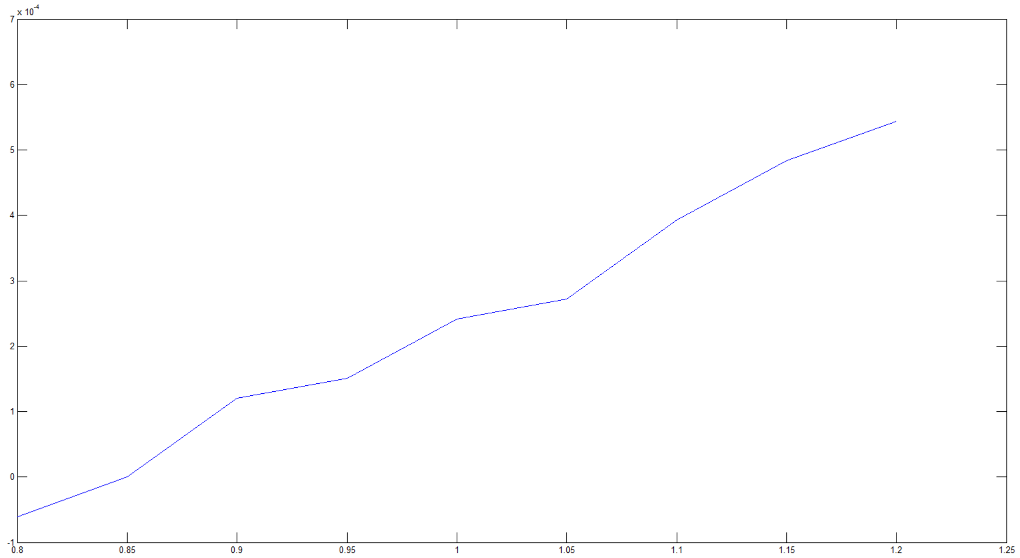


Figure 3.9: Difference in Black & Scholes implied volatilities between closed-form Heston prices and Monte Carlo prices. The difference are plotted as a function of moneyness, computed as $k_i = \log \frac{K_i}{F(T_i)}$. The scale of the error is in the order of 10^{-4}.

Table 3.1: Empirical moments of VIX and EPU indices.

	Mean (μ)	Standard deviation (σ)	Skew (s)	Kurtosis (κ)	Correlation ($\rho(\text{EPU}_t,\text{VIX}_t)$)
EPU_t	1.75%	1%	1.91	9.70	+0.28
VIX_t	21.54%	8.43%	1.92	6.76	

Table 3.1 reports the empirical moments of the VIX index and the EPU index from the 1st January 1996 to the 31st December 2013.

Table 3.2: The effect of policy risk on the moments of S_T.

Moments	Low policy risk (EPU = 3 %)	High policy risk (EPU = 6%)	Δ
μ	14.76 %	17.94 %	+3.18 %
σ	19.29 %	21.26 %	+1.97 %
s	0.35	0.36	+0.01
κ	3.2610^{-5}	2.6010^{-5}	

Table 3.2 reports the moments of the $\mathbb{P}$-distribution. The parameters used are the optimal parameters extracted from the Unscented Kalman filter. The remaining values are the values of the market the 1st of January 2009, that is S&P 500 = 906.50, VIX = 43.39%, T=0.23, $v_t = 0.2247$, $r = 0\%$ and $q = 0\%$. The moments are computed numerically on a 300 points grid, spanning [300 1800], that is -3.27σ to +4.90σ.

Table 3.3: Parameter estimates from the Unscented Kalman filter.

	Policy risk (p_t) under $\mathbb{P}$	Other risks (v_t) under $\mathbb{P}$	Policy risk (p_t) under $\mathbb{Q}$	Other risks (v_t) under $\mathbb{Q}$
Mean reversion κ	26.5	2.64	11.2	1
	[20.51 , 32.48]	*[1.29 , 3.99]*	*[9.43 12.97]*	*[0.85 , 1.15]*
Mean θ	1.65 %	0.14 %	3 %	2.80 %
	[1.51 % , 1.79 %]	*[0.14 % , 0.14 %]*	*[2.78 % , 3.22 %]*	*[2.20 % , 3.40 %]*
Volatility σ	43.20 %	24 %	116 %	19.20 %
	[36.74 % , 49.66 %]	*[10.79 % , 37.21 %]*	*[96 % , 136 %]*	*[16.73 % , 21.67 %]*
Correlation ρ	-0.33	-0.40		
	[-0.38 , -0.27]	*[-0.49 , -0.31]*		
Prices of risk λ	2.25	1.35		
	[0.57 , 3.93]	*[-0.66 , 3.36]*		
Total Equity Risk Premium	8.00%			
	[7.77 % , 8.23 %]			

Table 3.3 reports coefficient estimates from the Unscented Kalman Filter. I also report the 95 % confidence intervals, with standard errors computed by inverting the Fisher information criterion matrix. The estimates are generated by numerically minimizing the log-likelihood function of the deviations from the filtered measurements. For each parameter, I have generated a grid of 31 points and numerically found the minimum of the log-likelihood. If a minimum was not achieved, I shifted the window in order to attempt another numerical minimization. The Fisher information criterion is computed by numerically approximating the second derivative by finite difference, evaluated at the minimum.

Table 3.4: Pricing performance of the policy risk model.

	$T < 0.22$	$0.22 < T < 0.28$	$T > 0.28$	All maturities
$k < -1\%$	-0.85%	-1.26%	-1.48%	-1.14%
$-1\% < k < 1\%$	-0.36%	-0.77%	-1.37%	-0.80%
$k > 1\%$	-0.29%	-0.21%	-0.85%	-0.18%
All strikes	-0.32%	-0.70%	-1.25%	-0.72%

Table 3.4 reports the average mispricing for various time to maturities and various moneyness levels. At each point in time, I compute the model implied price and compare it to the market price. The average mispricing is measured as percentage points of Black and Scholes implied volatility. Time to maturity is in years and moneyness is measured as a percentage from the forward price $k_{t,i} = \log \frac{K_i}{F(t,T_i)}$.

Table 3.5: Pricing performance of a one-factor model.

	$T < 0.22$	$0.22 < T < 0.28$	$T > 0.28$	All maturities
$k < -1\%$	-1.22%	-1.93%	-2.97%	-2.30%
$-1\% < k < 1\%$	-0.74%	-1.42%	-2.85%	-1.23%
$k > 1\%$	-0.62%	-0.47%	-1.54%	-0.47%
All strikes	-0.60%	-1.20%	-1.93%	-0.93%

Table 3.5 reports the average mispricing for various time to maturities and various moneyness levels. At each point in time, I compute the model implied price and compare it to the market price. The average mispricing is measured as percentage points of Black and Scholes implied volatility. Time to maturity is in years and moneyness is measured as a percentage from the forward price $k_{t,i} = \log \frac{K_i}{F(t,T_i)}$.

Table 3.6: Pricing errors from Monte Carlo simulation.

N, number of simulations	OTM [$0.8 < k < 0.95$]	ATM [$0.95 < k < 1.05$]	ITM [$1.05 < k < 1.2$]
100.000	0.065%	0.083%	0.0123%
1.000.000	0.045%	0.072%	0.0107%
10.000.000	0.0161%	0.063%	0.0092%

Table 3.6 reports the pricing errors from the Monte Carlo simulation. Errors are computed as the percentage points in implied volatility extracted from the Monte Carlo price $C^{MC}_{T,K}$ and the closed form Heston price $C^{Heston}_{T,K}$. The errors are very close to zero for all levels of moneyness. The parameters used for the simulation are: $\kappa_v^{\mathbb{Q}} = 5$, $\theta_v^{\mathbb{Q}} = 0.25$, $\sigma_v^{\mathbb{Q}} = 0.65$, $\kappa_v^{\mathbb{Q}} = 5$, $\theta_v^{\mathbb{Q}} = 0.05$, $\sigma_v^{\mathbb{Q}} = 0.3$, $\rho_{sv} = -0.65$, $\rho_{sp} = -0.65$.

Table 3.7: Descriptive statistics for the option panel, 1996 - 2013

					Percentiles				
	Mean	Std	Min	5%	10%	50%	90%	95%	Max
S&P500 index	1246	251	598	786	904	1264	1554	1659	1810
EPU index	93.14	62.72	4.75	23.11	33.27	78.78	173.76	206.51	548.95
Implied Volatility (IV)	18.84%	6.38%	6.71%	11.06%	12.07%	17.91%	26.27%	29.29%	72.13%
Time to Maturity (T)	0.55	0.52	0.16	0.16	0.18	0.32	1.29	1.75	2.98
Log-moneyness (k)	-0.0002	0.01	-0.03	-0.02	-0.02	0.00	0.02	0.02	0.03
Market Price, $ (P)	54.01	36.43	1.38	12.26	17.32	45.32	103.41	129.75	251.76
Strike Price, $ (K)	1246	251	585	785	900	1265	1555	1670	1865

Table 3.7 reports descriptive statistics for the S&P 500 index option panel. I collect weekly call option quotes from January 1996 to December 2013. I filter the option quotes by log moneyness from -3% to +3%. I also drop the contracts with zero trading volume. I only use Wednesday prices to remove intra week trading seasonalities. In terms of maturities, I drop the very short end of the maturity spectrum and the long end, so that my option sample has maturities from 62 to 117 days. This leaves me with 930 trading days and a total of 47,192 option contract data points across moneyness and maturities, that is an average of fifty options per day.

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Appendices

Appendix A

An Orthogonal Decomposition.

The following exposition is based on Percival and Walden (2006) and it adopts a transform approach. For a time series oriented exposition, the interested reader is referred to Tamoni, Tebaldi and Ortu (2011).

Orthonormal transforms: Orthonormal transforms are of interest because they can be used to re-express a time series in such a way that we can easily reconstruct the series from its transform. In a loose sense, the 'information' in the transform is thus equivalent to the 'information' in the original series; to put it differently, the series and its transform can be considered to be two representations of the same mathematical entity.

Let $\{X_t : t = 1, ..., N\}$ represent a time series of N real-valued variables. Let $\mathbf{X}$ represent an N dimensional column vector whose t-th element is X_t for $t = 1, ..., N$. If $\mathbf{Y}$ is another such vector containing $\{Y_t : t = 1, ..., N\}$, the inner product of $\mathbf{X}$ and $\mathbf{Y}$ is given by

$$\langle \mathbf{X}\mathbf{Y} \rangle = \mathbf{X}^{\mathsf{T}}\mathbf{Y} = \sum_{t=1}^{N} X_t Y_t$$

The squared norm of $\mathbf{X}$ is given by $||\mathbf{X}||^2 = \langle \mathbf{X}\mathbf{X} \rangle = \mathbf{X}^{\mathsf{T}}\mathbf{X} = \sum_{t=1}^{N} X_t^2$. I will refer to the quantity $\xi_{\mathbf{X}} = ||\mathbf{X}||^2$ as the variance in the time series X_t.

Let $\mathcal{O}$ be an N x N real valued matrix satisfying the orthonormality property $\mathcal{O}^{\mathsf{T}}\mathcal{O} = I_N$, where I_N is the N x N identity matrix. Let $\mathcal{O}_{j,\bullet}$ be a column vector whose elements contain the j-th row vector of $\mathcal{O}$. With this notation, we can write

$$\mathcal{O} = \begin{bmatrix} \mathcal{O}_{1,\bullet}^{\mathsf{T}} \\ \mathcal{O}_{2,\bullet}^{\mathsf{T}} \\ \ldots \\ \mathcal{O}_{N,\bullet}^{\mathsf{T}} \end{bmatrix} = [\mathcal{O}_{1,\bullet}, \mathcal{O}_{2,\bullet}, \ldots, \mathcal{O}_{N,\bullet}]$$

The orthonormality property can be restated in terms of the inner product as

$$\left\langle \mathcal{O}_{j,\bullet}, \mathcal{O}_{j',\bullet} \right\rangle = \delta_{j,j'} = \begin{cases} 1 & \text{if } j = j'; \\ 0 & \text{otherwise,} \end{cases}$$

where $\delta_{j,j'}$ is the Kronecker delta function. Orthonormality also implies that the inverse $\mathcal{O}^{-1}$ of the matrix $\mathcal{O}$ is just its transpose $\mathcal{O}^{\mathsf{T}}$. Thus $\mathcal{O}\mathcal{O}^{-1} = \mathcal{O}\mathcal{O}^{\mathsf{T}} = I_N$. Hence the columns of $\mathcal{O}$ are an orthonormal set of vectors.

We can decompose a time series X_t with respect to the orthonormal matrix $\mathcal{O}$ by premultiplying $\mathbf{X}$ by $\mathcal{O}$ to obtain

$$O = \mathcal{O}\mathbf{X} = \begin{bmatrix} \mathcal{O}_{1,\bullet}^{\mathsf{T}} \\ \mathcal{O}_{2,\bullet}^{\mathsf{T}} \\ \ldots \\ \mathcal{O}_{N,\bullet}^{\mathsf{T}} \end{bmatrix} \mathbf{X} = \begin{bmatrix} \mathcal{O}_{1,\bullet}^{\mathsf{T}}\mathbf{X} \\ \mathcal{O}_{2,\bullet}^{\mathsf{T}}\mathbf{X} \\ \ldots \\ \mathcal{O}_{N,\bullet}^{\mathsf{T}}\mathbf{X} \end{bmatrix} = \begin{bmatrix} \langle \mathbf{X}, \mathcal{O}_{1,\bullet} \rangle \\ \langle \mathbf{X}, \mathcal{O}_{2,\bullet} \rangle \\ \ldots \\ \langle \mathbf{X}, \mathcal{O}_{N,\bullet} \rangle \end{bmatrix}$$

where we use the fact that $\mathcal{O}_{j,\bullet}^{\mathsf{T}}\mathbf{X} = \left\langle \mathcal{O}_{j,\bullet}, \mathbf{X} \right\rangle = \left\langle \mathbf{X}, \mathcal{O}_{j,\bullet} \right\rangle$. The N dimensional column vector O is referred to as the transform coefficients for $\mathbf{X}$ with respect to the orthonormal transform $\mathcal{O}$. The j-th transform coefficient is O_j and is given by the inner product $\left\langle \mathbf{X}, \mathcal{O}_{j,\bullet} \right\rangle$. If we premultiply both sides of the above equation by $\mathcal{O}^{\mathsf{T}}$ and recall that $\mathcal{O}^{\mathsf{T}}\mathcal{O} = I_N$, we obtain the synthesis equation

$$\mathbf{X} = \mathcal{O}^{\mathsf{T}} O = [\mathcal{O}_{1,\bullet}, \mathcal{O}_{2,\bullet}, \ldots, \mathcal{O}_{N,\bullet}] \begin{bmatrix} O_1 \\ O_2 \\ \ldots \\ O_N \end{bmatrix} = \sum_{j=1}^{N} O_j \mathcal{O}_{j,\bullet} = \sum_{j=1}^{N} \left\langle \mathbf{X}, \mathcal{O}_{j,\bullet} \right\rangle \mathcal{O}_{j,\bullet}$$

which tells us how to reconstruct $\mathbf{X}$ from its transform coefficients O. I have used the fact that $O_j = \left\langle \mathbf{X}, \mathcal{O}_{j,\bullet} \right\rangle$. The above holds for arbitrary $\mathbf{X}$, so the $\mathcal{O}_{j,\bullet}$ vectors form a basis for the finite dimensional space $\mathbb{R}^N$ of all N-dimensional real valued vectors; i.e. any real valued N-dimensional column vector can be expressed as a unique linear combination of $\mathcal{O}_{0,\bullet}, ..., \mathcal{O}_{N,\bullet}$.

An important fact about an orthonormal transform is that it preserves variance, in the sense that the variance of the transform coefficients O is equal to the variance of the original series $\mathbf{X}$, as can be seen from the following argument

$$\xi_O = ||O||^2 = O^\intercal O = (\mathcal{O}\mathbf{X})^\intercal(\mathcal{O}\mathbf{X}) = \mathbf{X}^\intercal \mathcal{O}^\intercal \mathcal{O}\mathbf{X} = \mathbf{X}^\intercal\mathbf{X} = ||\mathbf{X}||^2 = \xi_{\mathbf{X}}$$

A transform that preserves variance is sometimes called isometric. Since $\xi_{\mathbf{X}} = \sum_t X_t^2$ and $\xi_O = \sum_t O_j^2$, we can argue that the series X_t^2 describes how variance is decomposed across time, whereas the series O_j^2 describes how variance is decomposed across different transform indices; in my case, scale.

Suppose we wish to approximate $\mathbf{X}$ using a linear combination of the vectors $\mathcal{O}_{0,\bullet}, ..., \mathcal{O}_{N',\bullet}$, where $N' < N$. The projection theorem states that the best approximation $\hat{\mathbf{X}}$ to $\mathbf{X}$ that is formed using just the $N' < N$ vectors $\mathcal{O}_{0,\bullet}, ..., \mathcal{O}_{N,\bullet}$ is given by

$$\hat{\mathbf{X}} = \sum_{j=1}^{N'} O_j \mathcal{O}_{j,\bullet}$$

where 'best' is to be interpreted in a least square sense; i.e. the norm of the error vector $\mathbf{e} = \hat{\mathbf{X}} - \mathbf{X}$ is minimized.

Wavelet transform: Let $\mathcal{W}$ be an N x N real valued matrix satisfying the othonormality property $\mathcal{W}^\intercal\mathcal{W} = I_N$, where I_N is the N x N identity matrix. Let $\mathcal{W}_{j,\bullet}$ be a column vector whose elements contain the j-th row vector of $\mathcal{W}$. With this notation, we can write $\mathcal{O} = [\mathcal{W}_{1,\bullet}, \mathcal{W}_{2,\bullet}, ..., \mathcal{W}_{N,\bullet}]$.
The orthonormality property can be restated in terms of the inner product as

$$\left\langle \mathcal{W}_{j,\bullet}, \mathcal{W}_{j',\bullet} \right\rangle = \delta_{j,j'} = \begin{cases} 1 & \text{if } j = j'; \\ 0 & \text{otherwise,} \end{cases}$$

where $\delta_{j,j'}$ is the Kronecker delta function. Orthonormality also implies that the inverse $\mathcal{W}^{-1}$ of the matrix $\mathcal{W}$ is just its transpose $\mathcal{W}^\intercal$. Thus $\mathcal{W}\mathcal{W}^{-1} = \mathcal{W}\mathcal{W}^\intercal = I_N$. Hence the columns of $\mathcal{W}$ are an orthonormal set of vectors.

We can decompose a time series X_t with respect to the orthonormal matrix $\mathcal{W}$ by premultiplying $\mathbf{X}$ by $\mathcal{W}$ to obtain

$$W = \mathcal{W}\mathbf{X} = \begin{bmatrix} \mathcal{W}_{1,\bullet}^\intercal \\ \mathcal{W}_{2,\bullet}^\intercal \\ ... \\ \mathcal{W}_{N,\bullet}^\intercal \end{bmatrix} \mathbf{X} = \begin{bmatrix} \mathcal{W}_{1,\bullet}^\intercal \mathbf{X} \\ \mathcal{W}_{2,\bullet}^\intercal \mathbf{X} \\ ... \\ \mathcal{W}_{N,\bullet}^\intercal \mathbf{X} \end{bmatrix} = \begin{bmatrix} \langle \mathbf{X}, \mathcal{W}_{1,\bullet} \rangle \\ \langle \mathbf{X}, \mathcal{W}_{2,\bullet} \rangle \\ ... \\ \langle \mathbf{X}, \mathcal{W}_{N,\bullet} \rangle \end{bmatrix}$$

where we use the fact that $\mathcal{W}_{j,\bullet}^\intercal \mathbf{X} = \langle \mathcal{W}_{j,\bullet}, \mathbf{X} \rangle = \langle \mathbf{X}, \mathcal{W}_{j,\bullet} \rangle$. The N dimensional column vector W is referred to as the wavelet coefficients for $\mathbf{X}$ with respect to the wavelet transform $\mathcal{W}$. The j-th wavelet coefficient is W_j and is given by the inner product $\langle \mathbf{X}, \mathcal{W}_{j,\bullet} \rangle$. If we premultiply both sides of the above equation by $\mathcal{W}^\intercal$ and recall that $\mathcal{W}^\intercal \mathcal{W} = I_N$, we obtain the synthesis equation

$$\mathbf{X} = \mathcal{W}^\intercal W = [\mathcal{W}_{1,\bullet}, \mathcal{W}_{2,\bullet}, ..., \mathcal{W}_{N,\bullet}] \begin{bmatrix} W_1 \\ W_2 \\ ... \\ W_N \end{bmatrix} = \sum_{j=1}^{N} W_j \mathcal{W}_{j,\bullet} = \sum_{j=1}^{N} \langle \mathbf{X}, \mathcal{W}_{j,\bullet} \rangle \mathcal{W}_{j,\bullet}$$

which tells us how to reconstruct $\mathbf{X}$ from its wavelet transform coefficients W. I have used the fact that $W_j = \langle \mathbf{X}, \mathcal{W}_{j,\bullet} \rangle$. The above holds for arbitrary $\mathbf{X}$, so the $\mathcal{W}_{j,\bullet}$ vectors form a basis for the finite dimensional space $\mathbb{R}^N$ of all N-dimensional real valued vectors; i.e. any real valued N-dimensional column vector can be expressed as a unique linear combination of $\mathcal{W}_{0,\bullet}, ..., \mathcal{W}_{N,\bullet}$.

An important fact about an orthonormal transform is that it preserves variance, in the sense that the variance of the transform coefficients W is equal to the variance of the original series $\mathbf{X}$, as can be seen from the following argument

$$\xi_W = ||W||^2 = W^\intercal W = (\mathcal{W}\mathbf{X})^\intercal (\mathcal{W}\mathbf{X}) = \mathbf{X}^\intercal \mathcal{W}^\intercal \mathcal{W}\mathbf{X} = \mathbf{X}^\intercal \mathbf{X} = ||\mathbf{X}||^2 = \xi_{\mathbf{X}}$$

Since $\xi_{\mathbf{X}} = \sum_t X_t^2$ and $\xi_W = \sum_t W_j^2$, we can argue that the series X_t^2 describes how variance is decomposed across time, whereas the series W_j^2 describes how variance is decomposed across scales.

The projection theorem holds for the wavelet transform as well. Suppose we wish to approximate $\mathbf{X}$ using a linear combination of the vectors $\mathcal{W}_{0,\bullet}, ..., \mathcal{W}_{N',\bullet}$, where $N' < N$. Then, the best least square approximation $\hat{\mathbf{X}}$ to $\mathbf{X}$ is given by $\hat{\mathbf{X}} = \sum_{j=1}^{N'} W_j \mathcal{W}_{j,\bullet}$.

Let us now decompose the elements of W into $J+1$ subvectors. The first J subvectors are denoted by W_j, $j = 1, ..., J$, and the j-th subvector contains all the wavelet coefficients for scale j. The final subvector is denoted as V_J and it contains the residual coefficient W_J. We can then write

$$\mathcal{W}\mathbf{X} = \begin{bmatrix} \mathcal{W}_1 \\ \mathcal{W}_2 \\ ... \\ \mathcal{W}_J \\ \mathcal{V}_J \end{bmatrix} \mathbf{X} = \begin{bmatrix} \mathcal{W}_1\mathbf{X} \\ \mathcal{W}_2\mathbf{X} \\ ... \\ \mathcal{W}_J\mathbf{X} \\ \mathcal{V}_J\mathbf{X} \end{bmatrix} = \begin{bmatrix} W_1 \\ W_2 \\ ... \\ W_J \\ V_J \end{bmatrix} = W$$

Within each $\mathcal{W}_j$, the rows are circularly shifted versions of each other, but nevertheless are pairwise orthonormal (because of the orthonormality of $\mathcal{W}$). The wavelet coefficients in the vector W_j are associated with differences of various orders in adjacent weighted averages over a scale of $\tau_j = 2^j$, while the scaling coefficient in V_J is equal to $\sqrt{N}$ times the sample mean $\bar{X}$ of $\mathbf{X}$. We can now write the isometric condition as

$$||\mathbf{X}||^2 = ||W||^2 = \sum_{j=1}^{J} ||W_j||^2 + ||V_J||^2$$

so that $||W_j||^2$ represents the contribution to the variance of $\{X_t\}$ due to changes at scale τ_j. Because of the orthonormality of $\mathcal{W}$ and the special form of V_J, we can decompose the variance of $\mathbf{X}$ into elements associated with scales $\tau_1, ..., \tau_J$:

$$\hat{\sigma}^2_{\mathbf{X}} = \frac{1}{N}||\mathbf{X}||^2 - \bar{X}^2 = \frac{1}{N}||W||^2 = \frac{1}{N}\sum_{j=1}^{J}||W_j||^2$$

Using the same partitioning of $\mathcal{W}$ and W, we can express the synthesis of $\mathbf{X}$ ad the addition of $J+1$ vectors of length N, the first J of which J are associated with a particular scale τ_j, while the final vector S_J has all elements equal to the sample mean

$$\mathbf{X} = \mathcal{W}^{\intercal} W = [\mathcal{W}_1^{\intercal}, \mathcal{W}_2^{\intercal}, ..., \mathcal{W}_J^{\intercal}, \mathcal{V}_J^{\intercal}] \begin{bmatrix} W_1 \\ W_2 \\ ... \\ W_J \\ V_J \end{bmatrix} = \sum_{j=1}^{J} \mathcal{W}_j W_j + \mathcal{V}_J^{\intercal} V_J = \sum_{j=1}^{J} D_j + S_J$$

The collection of orthonormal coefficients $\{D_j\}$ is called "detail coefficients" while S_J is called "smooth coefficients". The knowledge of $\{D_j\}$ and S_J allows us to perfectly reconstruct the original series $\mathbf{X}$. Because $||D_j||^2 = ||W_j||^2$ for $j = 1, .., J$, the decomposition of variance on a scale by scale basis can be expressed as

$$\hat{\sigma}_{\mathbf{X}}^2 = \frac{1}{N} \sum_{j=1}^{J} ||D_j||^2$$

Clearly there are infinite choices of $\mathcal{O}$ or $\mathcal{W}$ which possess the orthonormality principle. The $\mathcal{W}_{j,\bullet}$ vectors form a basis for the finite dimensional space $\mathbb{R}^N$ of all N-dimensional real valued vectors so that any real valued N-dimensional column vector can be expressed as a unique linear combination of $\mathcal{W}_{0,\bullet}, ..., \mathcal{W}_{J,\bullet}$. Vectors $\mathcal{W}_{j,\bullet}$ are usually built from so called basis functions. Different basis functions have different properties, and the corresponding vectors $\mathcal{W}_{j,\bullet}$ inherit some or all of these peculiar properties. For example, if one uses trigonometric functions as basis functions, $\mathcal{O}_{j,\bullet}$s will form the so called Orthonormal Discrete Fouriers Transform (ODFT), an orthonormal version of the discrete Fourier transform. Trigonometric functions have only one relevant parameter, frequency. This property is then inherited by the ODFT: transform coefficients are localized in frequency only and the transform completely erases the time-domain information of the original series $\mathbf{X}$.

The Discrete Wavelet Transform (DWT) uses a special type of functions as basis functions, called wavelets. Wavelet functions have a number of interesting properties, the most important of which is that they are functions simultaneously localized in frequency *and* time. This allows the researcher to perform a richer analysis with respect to the ODFT, because the DWT is able to retain time domain information as well as information at a discrete set of frequencies, called *scales*.

A practical example: To illustrate this point, consider the matrix $\mathcal{W}$ built by chosing the

Haar[1] wavelet function as the basis function. If we choose $N = 16 = 2^4$ (so $j = 4$ scales), the resulting 16 x 16 matrix $\mathcal{W}$ will look like:

$$\mathcal{W} = \begin{bmatrix}
-\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\
-\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\
-\frac{1}{\sqrt{8}} & -\frac{1}{\sqrt{8}} & -\frac{1}{\sqrt{8}} & -\frac{1}{\sqrt{8}} & \frac{1}{\sqrt{8}} & \frac{1}{\sqrt{8}} & \frac{1}{\sqrt{8}} & \frac{1}{\sqrt{8}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{\sqrt{8}} & -\frac{1}{\sqrt{8}} & -\frac{1}{\sqrt{8}} & -\frac{1}{\sqrt{8}} & \frac{1}{\sqrt{8}} & \frac{1}{\sqrt{8}} & \frac{1}{\sqrt{8}} & \frac{1}{\sqrt{8}} \\
-\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\
\frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4}
\end{bmatrix}$$

By construction the rows of $\mathcal{W}$ have unit variance. It is also easy to see that the inner product of any two distinct rows must be zero, thus establishing that $\mathcal{W}$ is orthonormal. Then the discrete wavelet transform of $\mathbf{X}$ is simply given by $W = \mathcal{W}\mathbf{X}$.

Let me now define exactly what the notion of scale means. For any positive integer λ, let

$$\bar{X}_t(\lambda) = \frac{1}{\lambda} \sum_{l=0}^{\lambda-1} X_{t-l}$$

represent the average of contiguous data values with indices from $t - \lambda + 1$ to t. We refer to $\bar{X}_t(\lambda)$ as the sample average for scale λ over the set of times $t - \lambda + 1$ to t. Note that $\bar{X}_t(1) = X_t$ which we can regard as a 'single point average', and that $\bar{X}_N(J) = \bar{X}$, which is the sample average of all N values.

[1] The Haar wavelet function was the first wavelet to be used in 1909 by Alfred Haar during his attempt to construct an orthonormal basis for the set of all square integrable functions.

Since $W = \mathcal{W}\mathbf{X}$, considerations of the rows of $\mathcal{W}$ shows that we can write

$$W = \begin{bmatrix} W_1 \\ ... \\ W_8 \\ W_9 \\ ... \\ W_{12} \\ W_{13} \\ ... \\ W_{16} \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}}(X_2 - X_1) \\ ... \\ \frac{1}{\sqrt{2}}(X_{16} - X_{15}) \\ \frac{1}{2}(X_4 + X_3 - X_2 - X_1) \\ ... \\ \frac{1}{2}(X_{16} + X_{15} - X_{14} - X_{13}) \\ \frac{1}{\sqrt{8}}(X_8 + ... + X_5 - X_4 - ... - X_1) \\ ... \\ \frac{1}{4}(X_{16} + X_{15} + ... + X_1) \end{bmatrix}$$

Using the definition for $\bar{X}_t(\lambda)$ we can write the W_js as $W_1 = \frac{1}{\sqrt{2}}(\bar{X}_2(1) - \bar{X}_1(1)), W_8 = \frac{1}{\sqrt{2}}(\bar{X}_{16}(1) - \bar{X}_{15}(1)), W_9 = \bar{X}_4(2) - \bar{X}_2(2), W_{12} = \bar{X}_{16}(2) - \bar{X}_{14}(2), W_{13} = \sqrt{2}(\bar{X}_8(4) - \bar{X}_4(4)), W_{16} = 4\bar{X}_{16}(16)$. Note that the eight wavelet coefficients $W_1, ..., W_8$ are proportional to differences in adjacent averages of $\{X_t\}$ on a unit scale; the next coefficients $W_9, ..., W_{12}$ re proportional to differences in adjacent averages on a scale of two; W_{13} and W_{14} are proportional to differences in a scale of four; W_{15} is proportional to a difference on a scale of eight; and the final coefficient W_{16} is proportional to the average of all data. Each wavelet coefficient at each scale is also localized in time: W_1 only involves time $t = 1$ and 2, whereas W_9 involves time $t = 1, 2, 3$ and 4. In comparison, recall that the ODFT coefficients are not localized in time in any meaningful sense, which is an important distinction between 'global' transforms such as the ODFT and 'localized' transforms such as the DWT.

Appendix B

From Credit Default Swap spreads to Total Returns.

The credit default swap data consists of on-the-run credit spreads on various CDS contracts. Each CDS contract has the following parameters: a reference entity, a maturity T, a starting date t_0, a valuation date t_k, a notional contingent payment in case of default N and a recovery rate R. At inception, the spread is chosen such that the value $V(t_k, t_0, T, S_0, S_t)$ of the CDS is zero. In order to compute total returns, one has to convert on the run spreads $S(t_k, t_0, T)$ on a credit swap into total returns. To do this, I follow this process for each t:

- sell protection at the current spread $S(t_0, t_0, T)$ on a newly created CDS. The value of the position is \$0 by definition
- compute the value of the position $V(t_1, t_0, T, S(t_0, t_0, T), S(t_1, t_0, T))$. This means repricing the swap contract on the new credit curve, reducing the maturity of the contract by one quarter. This will generate a profit or a loss on the contract
- to extract returns, divide the profit or loss at the previous point by the notional N

Recall that a CDS is a swap contract made of two legs: a premium leg, corresponding to periodic payments $T_{a+1}, ..., T_b$ until default τ equal to S, and a protection leg, corresponding to only one contingent payment in case of default, equal to $(1 - R)$. In formulas:

$$\Pi_{a,b}(t) = \sum_{k=a+1}^{b} D(t, T_k)\alpha_k S \mathbf{1}_{\{\tau > T_k\}} - \sum_{k=a+1}^{b} \mathbf{1}_{\{T_{k-1} < \tau \leq T_k\}} D(t, T_k)(1 - R)$$

where $D(t, T_k)$ are discount factors and α_k are year fractions. By no arbitrage, $\mathbb{E}[\Pi_{a,b}(t)] = 0$ so the equilibrium CDS spread is

$$S = \frac{\sum_{k=a+1}^{b} \mathbb{E}[\mathbf{1}_{\{T_{k-1}<\tau\leq T_k\}}]D(t, T_k)(1-R)}{\sum_{k=a+1}^{b} D(t, T_k)\alpha_k \mathbb{E}[\mathbf{1}_{\{\tau>T_k\}}]}$$

which clearly requires the estimation of default and survival probabilities. If I assume that these probabilities do not change much during one quarter and that discount factors are constant, I can express the profit and loss on a CDS contract as:

$$\text{PnL}(t) = \Pi_{a,b}(t) - \Pi_{a,b}(t-1) =$$

$$\begin{aligned}
&= \sum_{k=a+2}^{b} D(t, T_k)\alpha_k S(t)\mathbf{1}_{\{\tau>T_k\}} - \sum_{k=a+1}^{b} D(t, T_k)\alpha_k S(t-1)\mathbf{1}_{\{\tau>T_k\}} \\
&= \sum_{k=a+2}^{b} D(t, T_k)\alpha_k S(t)\mathbf{1}_{\{\tau>T_k\}} - D(t, T_1)\alpha_1 S(t-1) - \sum_{k=a+2}^{b} D(t, T_k)\alpha_k S(t-1)\mathbf{1}_{\{\tau>T_k\}} \\
&= [S(t) - S(t-1)] \sum_{k=a+2}^{b} D(t, T_k)\alpha_k S(t)\mathbf{1}_{\{\tau>T_k\}} - D(t, T_1)\alpha_1 S(t-1)
\end{aligned}$$

For more details, the interested reader may consult Brigo and Mercurio. (2007) .

Appendix C

Derivative pricing.

In the two-factor Heston model, the price of a call can be computed as:

$$C(S_0, K, \tau) = S_0 e^{-q\tau} P_1 - K e^{-r\tau} P_2$$

where S_0 is the spot price of the index, K is the strike price, r is the risk free rate, q is the dividend yield and τ is the maturity. P_1 and P_2 are two probabilities than can be recovered via Fourier inversion as:

$$P_j = \frac{1}{2} + \frac{1}{\pi} \int_0^\infty Re \left[\frac{e^{-iu \log K} f_j(u, \log S_T, v_t, p_t)}{iu} \right] du$$

where $Re(x)$ take the real part of the complex number x and $f_j(u, \log S_T, v_t, p_t)$ is the conditional characteristic function of the model and has a general form as:

$$f_j(u, \log S_t, v_t, p_t) = e^{A_j(u) + B_{j,v}(u) v_t + B_{j,p}(u) p_t + iu \log S_t}$$

where coefficients A_j, $B_{j,v}$ and $B_{j,p}$ solve the corresponding Ordinary Differential Equation and are available in closed form as:

$$d_v = (\kappa_v - \rho_{sv} \sigma_v iu)^2 + \sigma_v^2 u(u + i)$$

$$d_p = (\kappa_p - \rho_{sp} \sigma_p iu)^2 + \sigma_p^2 u(u + i)$$

$$G_v = \frac{\kappa_v - \rho_{sv}\sigma_v ui - d_v}{\kappa_v - \rho_{sv}\sigma_v ui + d_p}$$

$$G_p = \frac{\kappa_p - \rho_{sp}\sigma_p ui - d_p}{\kappa_p - \rho_{sp}\sigma_p ui + d_v}$$

$$X_v = \frac{1 - G_v e^{-d_v\tau}}{1 - G_v}$$

$$X_p = \frac{1 - G_p e^{-d_p\tau}}{1 - G_p}$$

$$B_{1,v}(u) = \frac{(\kappa_v - \rho_{sv}\sigma_v ui - d_v)(1 - e^{-d_v\tau})}{\sigma_v^2(1 - G_v e^{-d_v\tau})}$$

$$B_{1,p}(u) = \frac{(\kappa_p - \rho_{sp}\sigma_p ui - d_p)(1 - e^{-d_p\tau})}{\sigma_p^2(1 - G_p e^{-d_p\tau})}$$

$$A_1(u) = r_f ui\tau + \kappa_v \frac{\theta_v}{\sigma_v^2}(\kappa_v - \rho_{sv}\sigma_v ui - d_v)\tau - 2\log(X_1) + \kappa_p \frac{\theta_p}{\sigma_p^2}(\kappa_p - \rho_{sp}\sigma_p ui - d_p)\tau - 2\log(X_2)$$

and where:

$$B_{1,v}(u) = B_{2,v}(u - i) - B_{2,v}(-i)$$

$$B_{1,p}(u) = B_{2,p}(u - i) - B_{2,p}(-i)$$

$$A_1(u) = A_2(u - i) - B_2(-i)$$

The $\mathbb{P}$ and $\mathbb{Q}$ distributions can be recovered by plugging the respective coefficients in the above formulas. In the case of the $\mathbb{P}$ distribution computations, the formula for coefficient $A_1(u)$ has to be slightly modified. Instead of just r_f, one has to use $r_f + \lambda_v v_t + \lambda_p p_t$.

Putting all together, the formula for $\mathbb{P}(x; S_t, v_t, p_t)$ is:

$$\mathbb{P}(x; S_t, v_t, p_t) = \frac{1}{2\pi} \int \text{Real} \left[e^{-iux} e^{A_2^{\mathbb{P}}(u) + B_{2,v}^{\mathbb{P}}(u)v_t + B_{2,p}^{\mathbb{P}}(u)p_t + iu\log S_t} \right] du$$

Appendix D

A Monte Carlo simulation.

In order to check that my option pricing and estimation methods are correct, I have conducted a Monte Carlo simulation. The goal is to compare European option prices calculated in closed form with Monte Carlo prices calculated numerically as average of terminal payoffs. The first thing is to simulate the path of volatility and of the log of stock price according to:

$$Z_{v_1} = \xi_{0,1}^{(1)}$$

$$Z_{v_2} = \xi_{0,1}^{(2)}$$

$$Z_{s_1} = \rho Z_{v_1} + \sqrt{1 - \rho_1^2 \xi_{0,1}^{(3)}}$$

$$Z_{s_2} = \rho Z_{v_2} + \sqrt{1 - \rho_2^2 \xi_{0,1}^{(4)}}$$

$$v_1(t) = v_1(t-1) + \kappa_{v_1}(\theta_{v_1} - v_1(t-1))\Delta t + \sqrt{v_1(t-1)}\sigma_{v_1} Z_{v_1}\sqrt{\Delta t}$$

$$v_2(t) = v_2(t-1) + \kappa_{v_2}(\theta_{v_2} - v_2(t-1))\Delta t + \sqrt{v_2(t-1)}\sigma_{v_2} Z_{v_2}\sqrt{\Delta t}$$

$$s(t) = s(t-1) + (r - \frac{1}{2}v_1(t-1) - \frac{1}{2}v_2(t-1))\Delta t + \sqrt{v_1(t-1)}Z_{s_1}\sqrt{\Delta t} + \sqrt{v_2(t-1)}Z_{s_2}\sqrt{\Delta t}$$

where $\xi_{0,1}^{(j)}$ is a normal random variable with zero mean and unit variance. The final Monte Carlo option price for a European call is computed as follows:

$$C_{T,K}^{MC} = e^{-rT}\frac{1}{N}\sum_{i=1}^{N}(e^{(s_j(T))} - K)^+$$

Here N is the number of simulations and T the maturity of the option.

In **??** I compare $C_{T,K}^{MC}$ with the closed form Heston price $C_{T,K}^{Heston}$ and compute pricing errors in terms of Black and Scholes implied volatilities. I report errors for various moneyness levels and for various number of simulations. I find that errors are extremely small and that the recovered implied volatility skews are almost indistinguishable from each other. The results are similar to Boyle (1977). In Figure 8 I plot the Implied Volatility errors as a function of moneyness.