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Highlights

- This paper analyses optimal taxation with and without commitment in an environment where all tax instruments are distortionary within the period, due to endogenous labour supply, variable capital utilisation, and non-tax-deductible depreciation.
- The novelty of the paper is allowing for consumption taxation.
- Ramsey and Markov-perfect policies are close to identical with consumption taxation, while they differ substantially with only factor income taxation.
- The value of commitment is negligible with consumption taxation and substantial without, and the value of taxing consumption is much higher without commitment.

Time-Consistent Consumption Taxation☆

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Abstract

We characterise optimal tax policies when the government has access to consumption taxation and cannot credibly commit to future policies. We consider a neoclassical economy where factor income taxation is distortionary within the period, due to endogenous labour and capital utilisation and non-tax-deductibility of depreciation. Contrary to the case where only labour and capital income are taxed, the optimal time-consistent policies with consumption taxation are remarkably similar to their Ramsey counterparts. The welfare gains from commitment are negligible, while they are substantial without consumption taxation. Further, the welfare gains from taxing consumption are much higher without commitment.

JEL classification: E62, H21.

Keywords: fiscal policy, Markov-perfect policies, consumption taxation, variable capital utilisation

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1. Introduction

Most of the literature on optimal fiscal policy rules out consumption taxation, a policy instrument used in most industrialised economies. For example, as of early 2019, the value-added tax (VAT) on standard items ranges from 17 to 27 percent in European Union countries. The literature on optimal consumption taxation includes Coleman (2000), who finds, under the assumption that the fiscal authority can commit to future policies, that replacing income taxes with consumption taxes would lead to large welfare gains in the United States. Correia (2010) extends this result to a heterogeneous-agents framework. Two recent contributions highlight the role of consumption taxation as a tool to relax a constraint of the monetary authority on the nominal interest rate, either as a result of the zero lower bound (Correia et al., 2013) or in a monetary union (Farhi et al., 2014). Our study finds a new benefit of consumption taxation: time-consistent policies and the resulting allocations are almost identical to those under commitment.

Our results are derived in a neoclassical model with endogenous labour supply and variable capital utilisation. The government has to finance spending on public goods and has access to three types of proportional taxes: capital income, labour income, and consumption taxes. A key element of our baseline environment is that all tax instruments are distortionary within the period. Given endogenous labour, both the consumption tax and the labour tax affect the consumption-leisure choice of households. Given endogenous capital utilisation and that depreciation is not tax-deductible, the capital tax distorts the capital utilisation rate. Lump-sum taxes and debt (or asset) accumulation are not available to the government. Under these assumptions there is no trivial solution to the problem of raising fiscal revenues.

Our key assumptions resulting in a distortionary capital tax within the period are motivated by the facts that in practice (i) capital utilisation is not fixed, and (ii) depreciation is deductible according to accounting formulae and is not based on the actual loss of value in capital due to usage. Hence, capital taxes introduce a wedge between the return on capital services and their cost in terms of capital depreciation.

The novelty of our paper lies in analysing time-consistent fiscal policies when the policy-maker has access to consumption taxation, in addition to factor income taxation.

The existing literature on Markov-perfect policies, starting with the seminal paper Klein et al. (2008), finds that lack of commitment alters greatly the characteristics of optimal policies and the resulting allocations when capital and/or labour income are taxed. The closest contributions to our study are Martin (2010) and Debortoli and Nunes (2010). These papers consider the same economic environment as we do, and find that without commitment factor income tax rates at the steady state are close to those in the United States.

Our results can be summarised as follows. First, if the policy-maker has access to all three types of taxes and the tax rates are unrestricted, the first-best allocation can be implemented at the steady state under commitment. Optimal taxation involves no tax on capital and taxing consumption and subsidising labour at the same rate, as long as private consumption is larger than labour income. The result comes from the fact that any constant consumption tax rate is non-distortionary with respect to the household's consumption-saving decision. Given that Ramsey policies achieve the first best, they are time-consistent. In other words, the steady states under Ramsey and Markov-perfect policy-making coincide. However, these tax policies include an unrealistically large (several hundred percent) labour subsidy.

Second, we study the case where subsidising labour is prohibited, as in Coleman (2000). In this case, the labour income tax is zero. The Ramsey (Markov) policy-maker taxes consumption at 22.3 (22.1) percent at the steady state in our baseline calibration, and sets the capital income tax to zero (0.4 percent). Looking at the transition from the status quo, optimal consumption and capital tax rates vary little over time and with the level of capital, under both Ramsey and Markov policy-making.

Third, for comparison, we also analyse the case with only factor income taxation. In this case the capital income tax is 19.8 percent at the Markov steady state. Ramsey and Markov policies and allocations are not similar.

In terms of welfare-equivalent consumption, the gains from commitment are negligible with consumption taxation (0.0003 percent), while they are substantial (2.01 percent) without. The welfare gains from taxing consumption are 2.77 (1.21) percent in the case of a Markov (Ramsey) policy-maker. This means that taxing consumption generates much larger welfare gains under discretion than under commitment. With consumption taxation the welfare gains over the existing tax system in the United States are 7.745 (7.744) percent under Ramsey (Markov) policies, while without taxing consumption the gains are 7.06 (4.92) percent. Remarkably, we find higher welfare under discretion when the policy-maker has access to consumption taxation than under commitment when the government can tax only labour and capital income.

Finally, we analyse policies over the business cycle when the economy is hit by aggregate productivity shocks. We find that with access to consumption taxation also the cyclical properties of tax rates and allocations under a Ramsey and a time-consistent policy-maker are very similar.

The intuition behind these results is the following. Firstly, at the steady state, taxing consumption causes only intratemporal distortion, while taxing capital income causes intertemporal distortion as well, i.e., goods of different time periods are taxed differently, which is to be avoided by the principles of optimal commodity taxation (Atkinson and Stiglitz, 1972). At the Markov equilibrium and in the initial periods under commitment, the policy-maker optimally taxes already installed capital as much as possible, as it is viewed as a non-distortionary source of revenue. In our environment, firstly, our balanced-budget requirement limits the initial capital levy. Secondly and most importantly, the capital income tax distorts capital utilisation. With only factor income taxation, the Markov policy-maker's desire to tax 'initial' capital still dominates, and hence there are large differences in policies and allocations between Ramsey and Markov governments.

On the contrary, taxing consumption partly taxes the initial capital stock, akin to the capital tax and as opposed to the labour tax. Further, a flat consumption tax does not distort intertemporal decisions at all, while the capital tax of all periods except the initial one does. An additional trade-off the government faces is in terms of intratemporal distortions: the capital tax impacts the capital utilisation margin, while the consumption tax distorts the consumption-leisure margin. The latter distortion turns out to be the least important quantitatively in determining optimal policy, while the intratemporal distortion caused by the capital tax plays a key role.

Our main result that with consumption taxation Ramsey and Markov-perfect policies and allocations are almost identical is robust to modifying various parameter values[1] and assuming that government spending is exogenous rather than endogenous. Our results are much weakened if the capital income tax rate is non-distortionary within the period. This happens if depreciation is tax-deductible and/or capital is fully utilised. The gains from commitment with consumption taxation increase to between 0.388 and 0.633 percent and the capital tax rate is not close to zero (10.3 to 23.6 percent) at the Markov steady state. Our results reinforce the conclusions of Zhu (1995) on the importance of the capital utilisation margin when studying optimal taxation.

The rest of the paper is structured as follows. Section 2 details the economic environment. Section 3 sets up the fiscal policy problems, both (i) the Ramsey problem and (ii) the problem of a time-consistent policy-maker. Afterwards, it characterises the equilibria and presents some analytical results. Section 4 contains our quantitative results. Section 5 scrutinises the role of our key assumptions. Section 6 concludes.

2. The model

The economy is populated by a representative household, a representative firm, and a utilitarian policy-maker. The household decides on consumption, saving, leisure, and the capital utilisation rate. The firm operates in perfectly competitive markets, maximises profits, and uses capital services and labour as production inputs. The policy-maker spends on public consumption which yields utility to households, and raises revenues via proportional taxes on labour income, capital income, and consumption. Depreciation, which depends on the capital utilisation rate, is not tax-deductible. Lump-sum taxes are not available, and the government has to balance its budget in each period. Time is discrete.

A few comments are in order about our main assumptions before describing the economic environment in mathematical terms. First, endogenous capital utilisation (see Greenwood et al., 1988, Greenwood et al., 2000, and many others) and non-tax-deductible

[1] In particular (i) the coefficient of relative risk aversion for private consumption, (ii) the Frisch elasticity of labour supply, (iii) the weight of government spending, (iv) the discount factor, and (v) how the depreciation rate depends on capital utilisation.

depreciation imply that taxing capital is distortionary within the period. Zhu (1995) was the the first to argue that variable capital utilisation should be taken into account when analysing optimal fiscal policy. Variable capital utilisation is in line with the fact that capital is not fully utilised in reality and is a standard assumption in bigger Dynamic Stochastic General Equilibrium (DSGE) models. Further, in reality depreciation which is tax-deductible does not depend on the actual capital utilisation rate, instead it is given by accounting rules. These assumptions are crucial. To highlight their role, we solve our model under alternative assumptions as well. In particular, we assume that depreciation is tax-deductible and/or capital is fully utilised, so that the capital tax is no longer distortionary within the period. See Section 5 and Appendix C.

Second, we assume that the government operates under a balanced-budget rule. We do this for two main reasons. First, we wish to compare our findings with previous studies on time-consistent fiscal policies with capital accumulation, which also impose a balanced-budget requirement (Klein and Ríos-Rull, 2003; Ortigueira, 2006; Klein et al., 2008; Azzimonti et al., 2009; Martin, 2010; Debortoli and Nunes, 2010). Second, while Debortoli and Nunes (2013) and Debortoli et al. (2017)) allow for debt in a Markov-perfect policy setting, they exclude capital. We leave the study of Markov-perfect policies in an environment with both capital and government debt to future work, and focus on the trade-offs between distortions generated by different tax instruments with and without commitment in this paper. The seminal paper of Coleman (2000) on Ramsey policies with consumption taxation does not impose a balanced-budget requirement, and imposes upper bounds on the tax rates instead. In Section 5.4 we compare are results to his, in order to highlight the role our balanced-budget requirement plays.

Finally, we consider government spending to be a choice variable of the fiscal authority, following the literature on Markov-perfect policies (e.g., Klein et al., 2008). At least part of government consumption can be adjusted in response to changes in the economy, and we are interested in studying the optimal policy mix both on the revenue and the spending side. We have repeated our analysis with exogenous government spending as a robustness check, and the results are unaltered, see Section 4.3.3.

Let a_t denote the level of aggregate productivity at time t, and let $\mathbf{a}^t = \{a_1, a_2, ..., a_t\}$

denote the history of aggregate productivity realisations. The representative household takes prices and policies as given and maximises

$$\mathbb{E}_0 \left(\sum_{t=1}^{\infty} \beta^t u \left(c \left(\mathbf{a}^t \right), \ell \left(\mathbf{a}^t \right), g \left(\mathbf{a}^t \right) \right) \right), \tag{1}$$

where $\mathbb{E}_0$ represents the rational expectations operator at time 0, $\beta \in (0, 1)$ is the discount factor, $c\left(\mathbf{a}^t\right)$ is private consumption when history $\mathbf{a}^t$ has occurred, $\ell\left(\mathbf{a}^t\right)$ represents leisure, and $g\left(\mathbf{a}^t\right)$ is public consumption; subject to the time constraint

$$h\left(\mathbf{a}^t\right) + \ell\left(\mathbf{a}^t\right) = 1, \ \forall \mathbf{a}^t, \tag{2}$$

where $h\left(\mathbf{a}^t\right)$ represents hours worked given history $\mathbf{a}^t$, and the budget constraint

$$\begin{aligned} &\left(1 + \tau^c\left(\mathbf{a}^t\right)\right) c\left(\mathbf{a}^t\right) + k\left(\mathbf{a}^t\right) = \left(1 - \tau^k\left(\mathbf{a}^t\right)\right) r\left(\mathbf{a}^t\right) v\left(\mathbf{a}^t\right) k\left(\mathbf{a}^{t-1}\right) \\ &+ \left(1 - \tau^h\left(\mathbf{a}^t\right)\right) w\left(\mathbf{a}^t\right) h\left(\mathbf{a}^t\right) + \left(1 - \delta\left(v\left(\mathbf{a}^t\right)\right)\right) k\left(\mathbf{a}^{t-1}\right), \ \forall \mathbf{a}^t, \end{aligned} \tag{3}$$

where $k\left(\mathbf{a}^{t-1}\right)$ is the level of the capital stock at the beginning of the period, $v\left(\mathbf{a}^t\right) > 0$ is the capital utilisation rate, $\delta\left(v\left(\mathbf{a}^t\right)\right)$ represents the depreciation rate of capital as a function of capital utilisation, and $\tau^c\left(\mathbf{a}^t\right)$, $\tau^h\left(\mathbf{a}^t\right)$, and $\tau^k\left(\mathbf{a}^t\right)$ denote the consumption, the labour income, and the capital income tax rate, respectively, given history $\mathbf{a}^t$. Finally, the variables $r\left(\mathbf{a}^t\right)$ and $w\left(\mathbf{a}^t\right)$ are the interest rate and the wage rate, respectively, and represent the remuneration of production factors, namely, capital services and labour. The utility function $u\left(\right)$ is assumed to be twice continuously differentiable in all three of its arguments with partial derivatives $u_c > 0$, $u_{cc} < 0$, $u_\ell > 0$, $u_{\ell\ell} < 0$, $u_g > 0$, $u_{gg} < 0$, where u_x and u_{xx} denote, respectively, the first and the second derivative of the utility function with respect to the variable x.

Combining the first-order conditions with respect to consumption and leisure when history $\mathbf{a}^t$ has occurred gives

$$\frac{u_\ell\left(\mathbf{a}^t\right)}{u_c\left(\mathbf{a}^t\right)} = \frac{1 - \tau^h\left(\mathbf{a}^t\right)}{1 + \tau^c\left(\mathbf{a}^t\right)} w\left(\mathbf{a}^t\right). \tag{4}$$

It is straightforward to derive a standard Euler equation,

$$\frac{u_c\left(\mathbf{a}^t\right)}{1+\tau^c\left(\mathbf{a}^t\right)} = \beta \mathbb{E}_t \left(\frac{u_c\left(\mathbf{a}^{t+1}\right)}{1+\tau^c\left(\mathbf{a}^{t+1}\right)} \left[1-\delta\left(v\left(\mathbf{a}^{t+1}\right)\right) + \left(1-\tau^k\left(\mathbf{a}^{t+1}\right)\right) v\left(\mathbf{a}^{t+1}\right) r\left(\mathbf{a}^{t+1}\right)\right]\right). \tag{5}$$

The first-order condition with respect to $v\left(\mathbf{a}^t\right)$ is

$$\delta_v\left(\mathbf{a}^t\right) = \left(1-\tau^k\left(\mathbf{a}^t\right)\right) r\left(\mathbf{a}^t\right). \tag{6}$$

The optimal rate of capital utilisation is where the marginal benefit from utilising more capital in terms of after-tax income equals its marginal cost in terms of higher depreciation. Equation (6) implies that capital income taxation is distortionary within the period.

Examining the household's first-order conditions, the different distortions caused by the three tax instruments become apparent. The labour income tax distorts the (intratemporal) consumption-leisure margin, (4). The current consumption tax distorts the same margin. In addition, both the current and next period's consumption tax enters into the current (forward-looking) Euler equation, (5). Finally, only next period's capital income tax distorts the current Euler equation, but the current capital income tax impacts the (intratemporal) capital utilisation margin, (6). The task of the fiscal authority is to find the optimal tax mix to raise revenue given these distortions.

We assume that the representative firm's technology is of the standard Cobb-Douglas form in capital services $v\left(\mathbf{a}^t\right) k\left(\mathbf{a}^{t-1}\right)$ and hours $h\left(\mathbf{a}^t\right)$, i.e.,

$$y\left(\mathbf{a}^t\right) = f\left(v\left(\mathbf{a}^t\right) k\left(\mathbf{a}^{t-1}\right), h\left(\mathbf{a}^t\right), a_t\right) = a_t \left(v\left(\mathbf{a}^t\right) k\left(\mathbf{a}^{t-1}\right)\right)^{\gamma} h\left(\mathbf{a}^t\right)^{1-\gamma}, \ \forall \mathbf{a}^t, \tag{7}$$

where $\gamma \in [0,1]$ represents the capital-services elasticity of output. Denoting by f_x the derivative of the production function with respect to the variable x, optimal behaviour in perfect competition implies

$$r\left(\mathbf{a}^t\right) = f_{vk}\left(\mathbf{a}^t\right) = \gamma a_t \left(\frac{h\left(\mathbf{a}^t\right)}{v\left(\mathbf{a}^t\right) k\left(\mathbf{a}^{t-1}\right)}\right)^{1-\gamma}, \ \forall \mathbf{a}^t, \tag{8}$$

$$w\left(\mathbf{a}^t\right) = f_h\left(\mathbf{a}^t\right) = (1-\gamma) a_t \left(\frac{v\left(\mathbf{a}^t\right) k\left(\mathbf{a}^{t-1}\right)}{h\left(\mathbf{a}^t\right)}\right)^{\gamma}, \ \forall \mathbf{a}^t, \tag{9}$$

i.e., factor prices equal their marginal products.

The resource constraint in this economy is

$$c\left(\mathbf{a}^t\right) + g\left(\mathbf{a}^t\right) + k\left(\mathbf{a}^t\right) = y\left(\mathbf{a}^t\right) + \left(1 - \delta\left(v\left(\mathbf{a}^t\right)\right)\right) k\left(\mathbf{a}^{t-1}\right),\ \forall \mathbf{a}^t, \tag{10}$$

where the initial level of capital $k\left(\mathbf{a}^0\right)$ is given. Finally, the government's budget constraint is

$$g\left(\mathbf{a}^t\right) = \tau^k\left(\mathbf{a}^t\right) r\left(\mathbf{a}^t\right) v\left(\mathbf{a}^t\right) k\left(\mathbf{a}^{t-1}\right) + \tau^h\left(\mathbf{a}^t\right) w\left(\mathbf{a}^t\right) h\left(\mathbf{a}^t\right) + \tau^c\left(\mathbf{a}^t\right) c\left(\mathbf{a}^t\right),\ \forall \mathbf{a}^t. \tag{11}$$

The benchmark first-best equilibrium in our environment can be defined as follows.

Definition 1 (First best). *The first-best equilibrium consists of allocations* $\left\{g\left(\mathbf{a}^t\right), c\left(\mathbf{a}^t\right), \ell\left(\mathbf{a}^t\right), h\left(\mathbf{a}^t\right), k\left(\mathbf{a}^t\right), v\left(\mathbf{a}^t\right), y\left(\mathbf{a}^t\right)\right\}_{t=1}^{\infty}$ *that maximise (1) subject to the household's time constraint,* (2)*, the production function, (7), and the market clearing condition, (10),* $\forall \mathbf{a}^t$, $k\left(\mathbf{a}^0\right)$ *and the productivity process given.*

The characterisation of the first best is presented in Appendix A.1.

We can define competitive equilibria as follows.

Definition 2 (Competitive equilibrium). *A competitive equilibrium consists of government policies,* $\left\{\tau^h\left(\mathbf{a}^t\right), \tau^k\left(\mathbf{a}^t\right), \tau^c\left(\mathbf{a}^t\right), g\left(\mathbf{a}^t\right)\right\}_{t=1}^{\infty}$*, prices,* $\left\{w\left(\mathbf{a}^t\right), r\left(\mathbf{a}^t\right)\right\}_{t=1}^{\infty}$*, and private sector allocations,* $\left\{c\left(\mathbf{a}^t\right), \ell\left(\mathbf{a}^t\right), h\left(\mathbf{a}^t\right), k\left(\mathbf{a}^t\right), v\left(\mathbf{a}^t\right), y\left(\mathbf{a}^t\right)\right\}_{t=1}^{\infty}$*, satisfying,* $\forall \mathbf{a}^t$,

(i) private sector optimisation taking government policies and prices as given, that is,

- *the household's time constraint,* (2)*, budget constraint,* (3)*, and optimality conditions,* (4)*,* (5)*, and* (6)*,*
- *the production function, (7), and the firm's optimality conditions,* (8) *and* (9)*;*

(ii) market clearing, (10), and

(iii) the government's budget constraint, (11),

$k\left(\mathbf{a}^0\right)$ *and the productivity process given.*

3. The policy problems

Both with and without commitment, the policy-maker maximises the household's lifetime utility over competitive equilibria. We assume, following most of the literature, that the policy-maker moves first in each period. We use a version of the primal approach, i.e., we write the policy problems in terms of allocations and substitute for prices and tax rates. We also substitute for output to simplify. However, we keep the consumption tax rate as a decision variable along with the allocations. This will be useful when constraining the tax rates.

First, one can eliminate three variables, output $y\left(\mathbf{a}^t\right)$ and prices $w\left(\mathbf{a}^t\right)$ and $r\left(\mathbf{a}^t\right)$, and three equations, (7), (8), and (9), in the definition of competitive equilibria, Definition 2. Second, the government's budget constraint and the resource constraint jointly imply that the household's budget constraint, (3), holds. Then six conditions are left which characterise competitive equilibria. Third, one can use the household's consumption-leisure optimality condition and the government's budget constraint to express the labour and capital income tax rates. The details of these derivations are in Appendix A.2. Then there remain four constraints: (2) and, $\forall \mathbf{a}^t$,

$$c\left(\mathbf{a}^t\right)+g\left(\mathbf{a}^t\right)+k\left(\mathbf{a}^t\right)=a_t\left(v\left(\mathbf{a}^t\right)k\left(\mathbf{a}^{t-1}\right)\right)^{\gamma}h\left(\mathbf{a}^t\right)^{1-\gamma}+\left(1-\delta\left(v\left(\mathbf{a}^t\right)\right)\right)k\left(\mathbf{a}^{t-1}\right), \tag{12}$$

$$\begin{aligned}\frac{u_c\left(\mathbf{a}^t\right)}{1+\tau^c\left(\mathbf{a}^t\right)}=\beta\mathbb{E}_t\Bigg(\frac{u_c\left(\mathbf{a}^{t+1}\right)}{1+\tau^c\left(\mathbf{a}^{t+1}\right)}\Bigg[&1-\delta\left(v\left(\mathbf{a}^{t+1}\right)\right)+a_{t+1}v\left(\mathbf{a}^{t+1}\right)\left(\frac{h\left(\mathbf{a}^{t+1}\right)}{v\left(\mathbf{a}^{t+1}\right)k\left(\mathbf{a}^t\right)}\right)^{1-\gamma}\\&-\frac{g\left(\mathbf{a}^{t+1}\right)-\tau^c\left(\mathbf{a}^{t+1}\right)c\left(\mathbf{a}^{t+1}\right)}{k\left(\mathbf{a}^t\right)}\Bigg]-u_\ell\left(\mathbf{a}^{t+1}\right)\frac{h\left(\mathbf{a}^{t+1}\right)}{k\left(\mathbf{a}^t\right)}\Bigg),\end{aligned} \tag{13}$$

$$\begin{aligned}\delta_v\left(\mathbf{a}^t\right)=a_t\left(\frac{h\left(\mathbf{a}^t\right)}{v\left(\mathbf{a}^t\right)k\left(\mathbf{a}^{t-1}\right)}\right)^{1-\gamma}&-\frac{g\left(\mathbf{a}^t\right)-\tau^c\left(\mathbf{a}^t\right)c\left(\mathbf{a}^t\right)}{v\left(\mathbf{a}^t\right)k\left(\mathbf{a}^{t-1}\right)}\\&-\frac{u_\ell\left(\mathbf{a}^t\right)}{u_c\left(\mathbf{a}^t\right)}\left(1+\tau^c\left(\mathbf{a}^t\right)\right)\frac{h\left(\mathbf{a}^t\right)}{v\left(\mathbf{a}^t\right)k\left(\mathbf{a}^{t-1}\right)}.\end{aligned} \tag{14}$$

So far we have not imposed any restrictions on the tax rates. We are also interested in the case where the labour tax has to be non-negative, as in Coleman (2000) and Correia (2010), given that in reality a labour subsidy is not observed at the aggregate level. Further, a (large) subsidy would likely lead to misreporting of hours, and verification of hours is likely to be prohibitively costly. To impose the restriction $\tau^h\left(\mathbf{a}^t\right) \geq 0$, we impose

$$\frac{u_\ell\left(\mathbf{a}^t\right)}{u_c\left(\mathbf{a}^t\right)} \leq \frac{1}{1+\tau^c\left(\mathbf{a}^t\right)}\left(1-\gamma\right) a_t \left(\frac{v\left(\mathbf{a}^t\right) k\left(\mathbf{a}^{t-1}\right)}{h\left(\mathbf{a}^t\right)}\right)^{\gamma}. \tag{15}$$

Below we write the policy problems in a general form including the constraint (15). We will, however, also study the case where (15) is ignored and the case without consumption taxation, i.e., $\tau^c\left(\mathbf{a}^t\right) = 0, \forall \mathbf{a}^t$, to compare our results with the existing literature.

3.1. The Ramsey policy-maker's problem

The Ramsey policy-maker maximises (1) choosing consumption tax rates $\left\{\tau^c\left(\mathbf{a}^t\right)\right\}_{t=0}^{\infty}$ and allocations $\left\{g\left(\mathbf{a}^t\right), c\left(\mathbf{a}^t\right), \ell\left(\mathbf{a}^t\right), h\left(\mathbf{a}^t\right), k\left(\mathbf{a}^t\right), v\left(\mathbf{a}^t\right)\right\}_{t=0}^{\infty}$, subject to (2), (12), (13), (14), and (15), $k\left(\mathbf{a}^0\right)$ and the productivity process given. We assign the Lagrange multipliers $\lambda_1\left(\mathbf{a}^t\right), ..., \lambda_5\left(\mathbf{a}^t\right)$ to the five constraints, respectively.

Note that we have assumed from the beginning that the time constraint, (2), and the resource constraint, (12), will bind. Furthermore, note that future decision variables enter into the household's current Euler equation, (13). This latter feature implies that the Ramsey problem is not recursive using only the natural state variables, a and k. Following Marcet and Marimon (1998/2017), the Lagrange multiplier on the Euler equation, $\lambda_3\left(\mathbf{a}^t\right)$, with $\lambda_3\left(\mathbf{a}^0\right) = 0$, can be introduced as a co-state variable to write a Bellman equation. The Ramsey problem can then be solved numerically by standard policy function iteration. The value function and the policy functions are time-invariant on the extended state space, the current value of which is denoted (a, k, λ_3). Next period's productivity a' is given exogenously. The policy-maker chooses the functions $\mathcal{K}'()$ and $\Lambda_3'()$, as well the policy functions for the control variables, i.e., $\mathcal{T}^c()$, $\mathcal{C}()$, $\mathscr{L}()$, $\mathcal{H}()$, $\mathcal{G}()$, $\mathcal{V}()$, $\Lambda_1'()$, $\Lambda_2'()$, $\Lambda_4'()$, and $\Lambda_5'()$, and the value function $\mathcal{W}()$.

Let unindexed variables denote the values of the policy functions for the control variables at this state, i.e., $c = \mathcal{C}\left(a, k, \lambda_3\right)$, and so on; and $k' = \mathcal{K}'\left(a, k, \lambda_3\right)$, $\lambda_3' = \Lambda_3'\left(a, k, \lambda_3\right)$, and $W = \mathcal{W}\left(a, k, \lambda_3\right)$. Let us collect terms corresponding to intra-temporal constraints in

Z, i.e.,

$$Z \equiv -\lambda_1 \left(\ell + h - 1\right) - \lambda_2 \left[c + g + k' - a\left(vk\right)^{\gamma} h^{1-\gamma} - \left(1 - \delta\left(v\right)\right) k\right]$$
$$- \lambda_4 \left[a\left(\frac{h}{vk}\right)^{1-\gamma} - \frac{g - \tau^c c}{vk} - \frac{u_\ell}{u_c}\left(1 + \tau^c\right)\frac{h}{vk} - \delta_v\right] - \lambda_5 \left[\frac{u_\ell}{u_c} - \frac{1}{1+\tau^c} a\left(1-\gamma\right)\left(\frac{vk}{h}\right)^{\gamma}\right].$$

Then we can write

$$W^{Ramsey} = \max_{\{\tau^c, c, \ell, h, g, k', v\}} \min_{\{\lambda_1, \lambda_2, \lambda_3', \lambda_4, \lambda_5\}} u\left(c, \ell, g\right) + \beta \sum_{a'} \Pr\left(a' \mid a\right) \mathcal{W}\left(a', k', \lambda_3'\right)$$
$$+ \lambda_3' \frac{u_c}{1+\tau^c} - \lambda_3 \left\{\frac{u_c}{1+\tau^c}\left[1 - \delta\left(v\right) + av\left(\frac{h}{vk}\right)^{1-\gamma} - \frac{g - \tau^c c}{k}\right] - u_\ell \frac{h}{k}\right\} + Z$$

$\lambda_5 \geq 0$, with complementary slackness conditions. Appendix A.3 presents the first-order conditions of the Ramsey policy-maker's problem.

3.2. *The time-consistent policy-maker's problem*

To characterise optimal time-consistent policies, it is convenient to assume that there is an infinite sequence of separate policy-makers, one for each period. The optimal policy problem therefore resembles a dynamic game between the private sector and all successive governments. The current policy-maker seeks to maximise social welfare from today onwards, anticipating how future policies depend on current policies via the inherited state variables. It also takes into account the optimising behaviour of the private sector. Note that, as under Ramsey policy-making, the fiscal authority moves first in every period, and commits within the period.

Without commitment, strategies for government spending and tax rates depend only on the current natural state of the economy, (a, k). We restrict our attention to stationary Markov-perfect equilibria of the policy game, following the literature (Klein et al., 2008). In a stationary Markov-perfect equilibrium, all governments employ the same policy rules. Hence, the rules must satisfy a fixed-point property: if the current policy-maker anticipates that all future governments will follow the policy rules $\{\mathcal{T}^c(a,k), \mathcal{C}(a,k), \mathscr{L}(a,k), \mathcal{H}(a,k), \mathcal{G}(a,k), \mathcal{V}(a,k), \mathcal{K}'(a,k)\}$, and similar rules for the Lagrange multipliers, then it finds it optimal to follow the same rules.

Let $\mathcal{U}_c() = u_c(\mathcal{C}(), \mathscr{L}(), \mathcal{G}())$, and similarly for $\mathcal{U}_\ell()$. Then we have

$$W^{Markov} = \max_{\{\tau^c,c,\ell,h,g,k',v\}} \min_{\{\lambda_1,\lambda_2,\lambda_3,\lambda_4,\lambda_5\}} u(c,\ell,g) + \beta \mathbb{E} \mathcal{W}(a',k')$$
$$-\lambda_3 \left\{ -\frac{u_c}{1+\tau^c} + \beta \mathbb{E} \left(\frac{\mathcal{U}_c(a',k')}{1+\mathcal{T}^c(a',k')} \left[1 - \delta(\mathcal{V}(a',k')) + a' \frac{\mathcal{H}(a',k')^{1-\gamma} \mathcal{V}(a',k')^{\gamma} k'^{\gamma}}{k'} \right. \right. \right.$$
$$\left. \left. \left. - \frac{\mathcal{G}(a',k') - \mathcal{T}^c(a',k') \mathcal{C}(a',k')}{k'} \right] - \mathcal{U}_\ell(a',k') \frac{\mathcal{H}(a',k')}{k'} \right) \right\} + Z,$$

$\lambda_5 \geq 0$, with complementary slackness condition. Appendix A.4 presents the first-order conditions of the time-consistent policy-maker's problem.

3.3. Analytical results

We present analytical results for the steady state (i) in the case where tax rates are unrestricted and (ii) excluding a labour subsidy. In both cases we consider an economy without productivity shocks, i.e., we set $a = a' = 1$. We assume that private consumption is larger than labour income, as in the data. We also consider cases where depreciation is tax-deductible and/or capital is fully utilised, i.e., $v = v' = 1$. Importantly, under these assumption, the capital income tax is no longer distortionary within the period. The details of the model with tax-deductible depreciation are in Appendix C. Our analytical results characterise steady states.[2]

Result 1. *Assume that the government has access to labour income, capital income, and consumption taxation, and all tax rates are unrestricted. Then the Ramsey steady state with $\tau^c > 0$, $\tau^c = -\tau^h$, and $\tau^k = 0$ corresponds to the first best, and hence it is time-consistent. This holds in our baseline model, when capital is fully utilised, and when depreciation is tax-deductible.*

Proof. In Appendix B. □

The government has only three instruments but faces four constraints. The first best can nonetheless be achieved, because setting the capital tax to zero leaves both the Euler equation and capital utilisation undistorted.

[2]Here we are assuming (i) convergence of the allocations to an interior steady state and (ii) convergence of the Lagrangian multipliers. As highlighted by Reinhorn (2014) and Straub and Werning (2018), these assumptions are not innocuous in optimal taxation problems. However, our assumptions exclude cases with diverging multipliers, see Straub and Werning (2018). Note also that when solving the model numerically, we do not assume convergence of multipliers.

We now turn to the case where a labour subsidy is excluded.

Result 2. *When $\tau^h \geq 0$ is imposed, the Ramsey policy-maker taxes only consumption at the steady state. This holds in our baseline model, when capital is fully utilised, and when depreciation is tax-deductible.*

Proof. In Appendix B. □

Coleman (2000) proves similar results in a framework without a balanced-budget requirement (and with capital fully utilised and depreciation tax-deductible). In that case and allowing for a labour subsidy, as in Result 1, constant taxes can be set over the transition period as well, hence the Ramsey planner can implement the first best in all periods.

4. Quantitative analysis

We now turn to numerical methods and solve a calibrated version of our economy, to assess quantitatively the optimal fiscal policy mix and welfare with and without consumption taxation and with and without commitment.

4.1. Calibration

We consider the model period to be a year and specify the utility function as

$$u(c, \ell, g) = \log(c) - \alpha_\ell \frac{(1-\ell)^{1+1/\varphi}}{1+1/\varphi} + \alpha_g \log(g), \tag{16}$$

where φ is the (constant) Frisch elasticity of labour supply, while α_ℓ and α_g are the weights of leisure and public goods relative to private consumption, respectively. Given an intertemporal elasticity of substitution equal to 1, we set the Frisch elasticity of labour supply, φ, equal to 3, as in Trabandt and Uhlig (2011), but we also check the robustness of our results to a wide range of values of φ, 0.4 to 5, see below. We assume that the depreciation rate is an increasing and convex function of capital utilisation, following Greenwood et al. (1988) and many others. That is, $\delta(v) = \eta v^\chi$, with $\eta > 0$ and $\chi > 1$. Finally, we assume that aggregate productivity follows an AR(1) process with persistence parameter ρ and standard deviation of the shock σ_a.

To pin down β, γ, α_ℓ, η, and χ, we use the private sector's first-order conditions and the resource constraint at steady state to match average macroeconomic ratios from

United States data for the period 1996-2010. We take average capacity utilisation for all industries from the Federal Reserve Board, and we compute all other macroeconomic ratios using data provided by Trabandt and Uhlig (2012).[3] The private sector takes as given the effective tax rates. We use the effective tax rates computed by Trabandt and Uhlig (2012) for each year to find average tax rates of $\tau^h = 0.221$, $\tau^c = 0.045$, and $\tau^k_\delta = 0.410$, where the lower index δ means that this capital tax rate is with depreciation allowance.[4] We target the average labour income share (60.9 percent), private consumption over GDP (69.6 percent), public consumption over GDP (15.5 percent), capital over GDP (2.349), and the fraction of time worked for the working age population (24.9 percent). To calibrate α_g, we assume that g found in the data is optimally chosen in the sense that $u_c = u_g$. Finally, we calibrate the AR(1) coefficients of the technological progress so that the unconditional persistence and the standard deviation of total output in our economy with fixed tax rates match that of de-trended US GDP for the period 1996-2010.

The calibrated parameter values are presented in Table 1.

Table 1. Calibrated parameters

Par	Value	Description
φ	3	Frisch elasticity
β	0.943	Discount factor
α_ℓ	4.154	Weight of leisure
α_g	0.223	Weight of public goods
γ	0.391	Capital elasticity
η	0.102	Depreciation parameters, $\delta(v) = \eta v^\chi$
χ	1.956	
ρ	0.619	Technology shock autoregressive parameter
σ_a	0.020	Technology shock standard deviation

Note that we have not taken into account the household's and the government's budget constraint. In reality tax revenues are raised not only to finance public consumption, but also in order to redistribute resources from richer to poorer households. At the status-quo steady state, tax revenues are higher by 11.7 percent of GDP than public consumption. In order to satisfy the budget constraints, one can imagine that the government gives a lump-sum transfer of 11.7 percent of GDP to the representative household.

[3]https://sites.google.com/site/mathiastrabandt/home/downloads/LafferNberDataMatlabCode.zip

[4]We convert it to a capital tax rate without depreciation allowance, in line with our model, taking revenue from capital income taxation as given. That is, $\tau^k_\delta (rv - \delta(v)) = \tau^k rv = \tau^k \gamma \frac{y}{vk}$. This gives $\tau^k = 0.253$.

Viewed through the lens of a representative-agent model, this is a source of inefficiency and will imply additional welfare gains for all optimal tax reforms.

4.2. Solution method

First, we solve the Ramsey problem using policy function iteration. This consists of the following steps. We discretise the state variables $k \in \left[\underline{k}, \overline{k}\right]$ and $\lambda_3 \in \left[\underline{\lambda}_3, \overline{\lambda}_3\right]$. In the stochastic case, we approximate the estimated AR(1) process by a 3-state Markov chain following Galindev and Lkhagvasuren (2010) and Kopecky and Suen (2010). Once we have found the endogenous collocation nodes, we guess the policy functions at each grid point. At each iteration we solve the system of non-linear equations at each grid point, and we approximate globally the policy functions of next period using cubic splines.

Second, using the solution to the Ramsey problem by policy function iteration as initial guess, we solve it again parameterising the policy functions using cubic splines. Then, we solve the time-consistent policy-maker's problem in the same way. The derivatives of next period's policy functions with respect to the endogenous state k' are computed using the Compecon Matlab package by Fackler and Miranda (2004). In this solution algorithm we iterate until the parameters of the policy functions converge to high accuracy. The resulting policy functions are well behaved. We use this algorithms to simulate the dynamic paths of tax policies and allocations.[5]

4.3. Results

In this section we first present the results for our baseline model, where the capital income tax is distortionary within the period, as well as results under alternative assumptions. In particular, ee describe the steady state (Section 4.3.1) and the transition (Section 4.3.2), and we discuss our results with productivity shocks in Appendix D. Section 4.3.3 summarises our robustness checks on parameter values, and Appendix E presents the results in details.

[5]We have also verified that alternative numerical methods give identical rates and allocations up to many decimals: (i) we have used Chebyshev polynomials instead of cubic splines, and (ii) we have solved the time-consistent policy-maker's problem by policy function iteration as well, using the Ramsey solution as initial guess.

4.3.1. Steady state

Table 2 shows the allocations and the tax rates at steady state for five policy models. Our first numerical result (displayed in the first column of Table 2) is that in the case of unrestricted tax rates, while the policy-maker can implement the efficient allocation (see Result 1), the tax rates are unrealistic, with a consumption tax of 324.5 percent and a labour income tax of -324.5 percent.

Table 2. Tax rates and allocations at steady state

	unrestricted	$\tau^h \geq 0$		$\tau^c = 0$	
Variable		Ramsey	Markov	Ramsey	Markov
Consumption tax rate	3.245	0.223	0.221	0.000	0.000
Labour income tax rate	-3.245	0.000	0.000	0.240	0.065
Capital income tax rate	0.000	0.000	0.004	0.000	0.198
Capital	1.801	1.548	1.539	1.467	1.106
Hours worked	0.320	0.275	0.277	0.261	0.283
Income	0.572	0.492	0.491	0.466	0.439
Consumption-income ratio	0.654	0.654	0.652	0.654	0.723
Public spending-income ratio	0.146	0.146	0.146	0.146	0.117
Welfare-eq. consumption loss	0.000	0.059	0.061	0.087	0.184

The second and main result is that if the government is prohibited from subsidising labour, the consumption and capital tax rates implied by Ramsey and Markov policies are remarkably similar. The Ramsey policy-maker (second column of Table 2) taxes consumption at 22.3 percent at the steady state and sets the labour and capital income taxes to zero.[6] The time-consistent policy-maker (third column) finances government spending mainly from taxing consumption as well, taxing it at 22.1 percent, and sets the capital income tax to 0.4 percent and the labour income tax to zero. Once a labour subsidy is ruled out, it is inefficient to tax both labour and consumption, as both taxes distort the same margin, the consumption-leisure decision of the household. The policy-maker uses the consumption tax, because it is less distortionary.

Taxing consumption is less distortionary than taxing labour for the following reasons. First, note that, at the steady state, neither tax distorts intertemporal decisions,

[6]Note that the fact that the Ramsey policy-maker sets the consumption tax rate equal to α_g when $\tau^h \geq 0$, is a consequence of logarithmic sub-utilities for both private and government consumption. For the same reason, the logarithmic sub-utilities for both private and government consumption implies that the consumption-income ratio and the public spending-income ratio are the same in the Ramsey and the first-best steady state, see Motta and Rossi (2018).

only intratemporal ones. The consumption-leisure margin is distorted by the tax wedge $\xi \equiv \frac{1-\tau^h}{1+\tau^c}$, with $\xi \leq 1$ as long as tax rates are non-negative. It is easy to see that for any tax rate $\tilde{\tau} > 0$, ξ is closer to 1 when consumption rather than labour income is taxed. That is, the same consumption tax distorts the consumption-leisure margin less than the labour income tax. Note also that the difference in ξ increases with the tax rate $\tilde{\tau}$. Relatedly, a 100 percent labour tax would imply that the economy shuts down, or, the labour tax Laffer curve peaks below 100 percent, while the economy would still function with a consumption tax of 100 percent.[7] In addition, as long as private consumption is larger than labour income as a share of GDP, as in the data and in our calibrated model, then raising a given amount of fiscal revenues requires a lower consumption tax rate than labour tax rate, which magnifies the difference between the two instruments when it comes to distorting the household's consumption-leisure choice.

As a direct consequence of the similarities in the tax rates in the two policy scenarios when $\tau^h \geq 0$ is imposed, Ramsey and Markov steady states are remarkably similar in terms of allocations and welfare. Capital and income are almost as high at the Markov as at the Ramsey steady state. Likewise, the consumption- and public spending-income ratios change very little as a result of the change in commitment. Close-to-identical allocations imply that welfare changes little with commitment: the steady-state welfare-equivalent consumption losses amount to 6.1 percent in the case of time-consistent policy and to 5.9 percent under Ramsey compared to the first best. To summarise, with consumption taxation the key feature of our steady-state results is that Ramsey and Markov policies, allocations, and welfare are very similar.

The third result is that without consumption taxation Ramsey and Markov steady states are not similar. The Ramsey policy-maker (fourth column of Table 2) taxes only labour income at the steady state (the Chamley-Judd result), while the time-consistent policy-maker (fifth column) sets the labour income tax to 6.5 percent and the capital income tax to 19.8 percent. However, the tax rates strongly depend on the Frisch elasticity of labour supply ($\varphi = 3$ in our baseline calibration) in this case, while not in the other policy scenarios, see Section 4.3.3. Such distinct tax rates under the two commit-

[7] See Trabandt and Uhlig (2011) for more details on the labour and consumption tax Laffer curves.

ment scenarios imply that allocations and welfare differ significantly at the Ramsey and Markov steady states. Compared with the Ramsey allocations, Markov policies imply significantly lower long-run capital and income, a higher consumption-income ratio, and a lower public spending-income ratio, which are all due to more distortions caused by taxation.[8] Hours worked are higher under discretion than under commitment, because of the lower labour income tax. The steady-state welfare-equivalent consumption losses amount to 18.4 percent in the case of time-consistent policy and to 8.7 percent under Ramsey. Interestingly we find that welfare is higher without commitment but with access to consumption taxation than with commitment but taxing only labour and capital income.

4.3.2. Dynamics

In this section, we study whether our steady-state results on the usefulness of consumption taxation in terms of mitigating the commitment problem of the policy-maker and improving welfare hold once we take into account transitional dynamics. Further, we aim to shed light on the key trade-offs faced by the policy-makers during the transition. In addition, we quantify for different taxation and commitment scenarios (i) the welfare gains compared to the status quo, (ii) the gains from commitment, and (iii) the gains from taxing consumption.

In order to do this, we perform the following policy exercise. We assume that initially the economy is at the status-quo steady state, described in Section 4.1. At time 1, a new policy-maker enters into office. It can be either a Markov or a Ramsey policy-maker, and with either access to consumption taxation or no access. Figures 1 and 2 show the dynamics of tax rates and allocations without and with access to consumption taxation, respectively, for both Ramsey and Markov policy-makers.

The key result displayed in Figures 1 and 2 is that with access to consumption taxation the whole dynamic paths of taxes and allocations hardly differ with and without commitment, while they differ substantially when consumption is not taxed, as in Martin (2010) and Debortoli and Nunes (2010). These results extend our second and third steady-state results.

[8] The result that the public spending-income ratio is lower under Markov policy was first noted by Klein et al. (2008) in an environment where only labour income taxes are available.

Figure 1. Ramsey and Markov policies without consumption taxation starting from the status quo

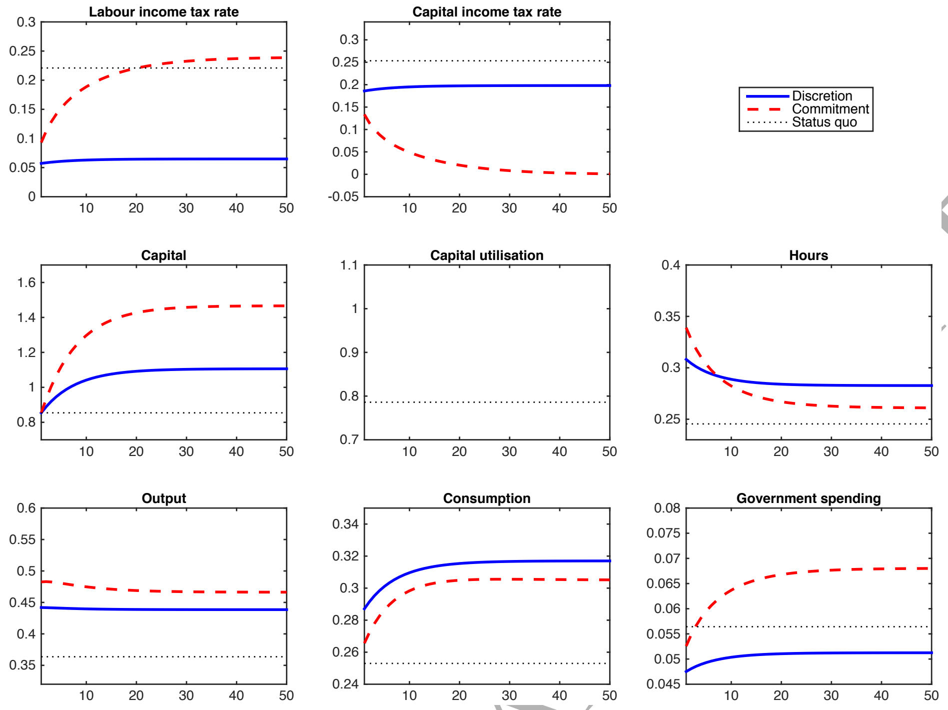
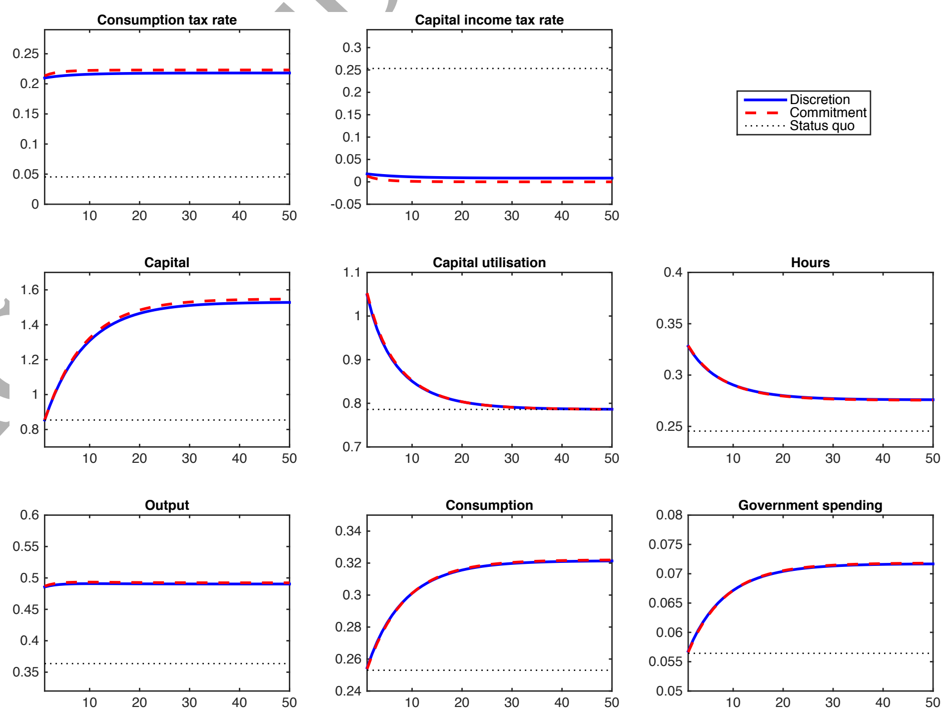


Figure 2. Ramsey and Markov policies with consumption taxation and no labour subsidy starting from the status quo



The intuition behind our results is the following. First, consider the case without consumption taxation. A high capital tax rate in the first few periods is partly levied on the initial capital stock, which is a non-distortionary way to raise revenue. Our balanced-budget assumption and the intratemporal distortion caused by the capital tax limit the initial capital levy in our environment, but it is still the key driver of policies in the first few years. The Ramsey planner takes into account that the capital income tax in each (but the first) period distorts the marginal rate of substitution between consumptions in different time periods, as is well known. The desire of the policy-maker to avoid this distortion is what drives long-run tax policies under commitment. In our environment, the government balances the above considerations with the distortions created by the capital income tax on the capital utilisation rate and by the labour income tax on the consumption-leisure margin. Variable capital utilisation modifies the calculation of the Ramsey government only marginally compared to a standard setting with capital fully utilised. The policy-maker raises the capital income tax less in the initial period and makes a greater use of the labour income tax.

The time-consistent policy-maker differs in not internalising the impact of the current capital income tax on past consumptions, and the desire to tax the 'initial' capital stock plays a key role. The Markov policy-maker balances these considerations with only the intratemporal distortions caused by the two tax instruments. This leads to dramatic differences in policies and allocations between Ramsey and Markov governments.

On the contrary, taxing consumption partly taxes the initial capital stock as well. Further, a time-constant consumption tax does not distort intertemporal decisions, while the capital income tax does (except for period 1). Moreover, in our framework with endogenous capital utilisation, the initial capital tax distorts capital utilisation, while the consumption tax distorts the consumption-leisure margin. This last distortion turns out to be the least important quantitatively in determining optimal policy. The distortion that the capital income tax causes within the period, combined with fact that the consumption tax is partially levied on initial capital, imply that it is optimal to raise most fiscal revenue by taxing consumption, including in the initial period. Then, the time-inconsistency features of policies under commitment are negligible, and the intuition for the Markov equilibrium with consumption taxation follows from the Ramsey case. At the Markov equilibrium in every period the trade-offs are as in the initial period of the Ramsey equilibrium, hence fiscal revenue is raised mainly by taxing consumption by the Markov government as well in all periods.

Using our simulation results above, one can compute the welfare gains in terms of welfare-equivalent consumption from the different taxation and commitment scenarios compared to the existing tax system. In the case of a Ramsey policy-maker, the welfare gains are 7.745 percent and 7.06 percent with and without taxing consumption, respectively. Ceteris paribus, in the case of a Markov policy-maker, the welfare gains are 7.744 percent and 4.92 percent, respectively. Notice that the welfare gains are larger with consumption taxation and without commitment than without consumption taxation and with commitment. The fact that welfare gains in the Ramsey and Markov cases are almost identical follows from the close similarity of tax policies and allocations.

We also quantify the welfare gains from commitment both with and without consumption taxation, starting from the corresponding Markov steady state. The welfare gains from commitment are 0.0003 percent with consumption taxation and 2.01 percent taxing labour income instead. Hence, the gains from commitment are negligible with access to consumption taxation, while they are substantial without. Finally, the gains from taxing consumption without (with) commitment are 2.77 (1.21) percent, taking into account the transition from the Markov (Ramsey) steady state without consumption taxation. Hence, the welfare gains from taxing consumption are much larger under discretion than under commitment.

Table 3 summarises our welfare results including transitions.

Table 3. Welfare gains in welfare-equivalent consumption (percent)

Welfare gains...	With cons. tax	Without cons. tax	
...from commitment	0.0003	2.01	
	...compared to the existing tax system		...from taxing consumption
Ramsey	7.745	7.06	1.21
Markov	7.744	4.92	2.77

4.3.3. *Robustness*

In this section we show that our main result, that Ramsey and Markov equilibria are close to identical with consumption taxation, is robust to modifying parameter values and to assuming that government spending is exogenous rather than endogenous.

First of all, we consider a wide range of values for the Frisch elasticity of labour supply: (i) $\varphi = 0.4$, which is in line with recent micro estimates such as Domeij and Floden (2006) (see also Guner et al., 2012), (ii) $\varphi = 1$, which is often chosen in the macro literature (e.g. Christiano et al., 2005), and (iii) $\varphi = 5$ as a high value, which is sometimes chosen to better match the intertemporal variation of aggregate hours (e.g. Galí et al., 2007). We adjust α_ℓ appropriately in each case as described in Section 4.1. The steady-state results for all policy scenarios are in the first three panels of Table E.7. The change in φ only affects tax rates and allocations without consumption taxation and without commitment. In that case, as φ increases from 0.4 to 5, τ^h decreases from 17.3 to 4.2 percent and τ^k increases from 7.8 to 22.1 percent. This is because increasing the elasticity of labour supply increases the distortion caused by τ^h compared to τ^k. With $\varphi = 1$ we recover the result of the existing literature (Martin, 2010; Debortoli and Nunes, 2010) that the two income tax rates are near-equal at the Markov steady state.

It is also worth noting that under discretion as φ increases, the public spending-income ratio decreases with only factor income taxation, but hardly changes with consumption taxation. This is because the Markov planner chooses lower taxes as the distortionary effects of fiscal policy are greater. Note, however, that a higher Frisch elasticity implies larger welfare losses compared to the first best under all four policy scenarios. This is because, ceteris paribus, a higher φ implies a stronger response of hours to any given distortion of the consumption-leisure margin.

We have verified that our conclusions are robust to various other parameter changes. We have considered (i) a coefficient of relative risk aversion for private consumption equal to 2 (in this case we have recalibrated the utility weights $\alpha_\ell = 16.791$ and $\alpha_g = 0.050$ to keep hours at 0.249 and g/y at 0.155 before the reform), (ii) $\beta = 0.96$, the most commonly used value in the macro literature for yearly models, (iii) $\chi = 1.8$ and adjust $\eta = 0.098$ so that the capital utilisation rate at the steady state be the same as in the data (note that this implies a depreciation rate of 0.076 at the steady state), and (iv) $\alpha_g = 0.3$, which reduces the consumption tax base compared to the baseline. The steady-state results in all these scenarios and in panels four to seven of Table E.7.

Remarkably, under all parameterisations considered, taxing consumption is more important than being able to commit. As reported in Tables 2 and E.7, welfare is always higher under Markov policy-making and consumption taxation than under Ramsey without taxing consumption.

Finally, we assume that government spending is exogenous. The new policy problem consists of raising fiscal revenues in order to finance an exogenous and fixed level of public consumption, $\bar{g}$. The government budget constraint can now be written as

$$\bar{g} = \tau^k \left(\mathbf{a}^t\right) r \left(\mathbf{a}^t\right) v \left(\mathbf{a}^t\right) k \left(\mathbf{a}^{t-1}\right) + \tau^h \left(\mathbf{a}^t\right) w \left(\mathbf{a}^t\right) h \left(\mathbf{a}^t\right) + \tau^c \left(\mathbf{a}^t\right) c \left(\mathbf{a}^t\right), \ \forall \mathbf{a}^t.$$

We further assume that households do not value government spending, i.e.

$$\mathbb{E}_0 \left[\sum_{t=1}^{\infty} \beta^t u \left(c \left(\mathbf{a}^t\right), \ell \left(\mathbf{a}^t\right) \right) \right].$$

We calibrate $\bar{g}$ so that $\bar{g}/y$ at steady state, given the status-quo tax rates, is equal to its value in the data, 0.155. Then we keep $\bar{g}$ constant across all policy regimes. All other parameters are kept at their benchmark values as described in Section 4.1. The steady-state results of this exercise are reported in Table E.8. The close similarity of Ramsey and Markov policies and allocations when consumption is taxed optimally holds in this case as well.

5. On capital utilisation, the tax-deductibility of depreciation, and budget balance

In this section we discuss the role of three key assumptions adopted in our model, namely, (i) variable capital utilisation, (ii) non-tax-deductibility of depreciation, and (iii) the balanced-budget requirement imposed on the government.

In order to highlight the role of the first two key assumptions, we solve our model under three alternative sets of assumptions: full (instead of variable) capital utilisation, or tax-deductible (instead of non-tax-deductible) depreciation, or both. The key feature in all three alternative settings is that the capital income tax is no longer distortionary within the period. We find that Ramsey and Markov policies with consumption taxation are no longer close to identical.

The literature has studied the case where capital taxation is non-distortionary within the period without consumption taxation. In particular, Klein et al. (2008) assume that capital is fully utilised and show that, if the labour income tax rate has to be non-negative, then only capital is taxed at the Markov solution in all periods. Martin (2010) allows for a labour subsidy and shows that no Markov-perfect equilibrium exists for standard calibrations when the capital tax is non-distortionary within the period. See also Debortoli and Nunes (2010). More precisely, for any given rate of time preference, a Markov-perfect equilibrium exists only if the preference for the public good is low enough, much lower than in standard calibrations. This is because the Markov government wants to tax capital and subsidise labour, and ends up confiscating the whole capital stock, hence the economy shuts down (Martin, 2010). Therefore, below we present results with consumption taxation but not without.

5.1. Full capital utilisation

First we analyse the case where capital is fully utilised. To do this, we set $v = 1$ and $\lambda_4 = 0$ in all periods. The first two columns of Table 4 present, respectively, the Ramsey and Markov steady-state results with consumption taxation in this case. Figure 3 presents the dynamic paths of policies and allocations.

Table 4. Tax rates and allocations at steady state, alternative assumptions

	non-tax-ded. δ, $v = 1$		tax-ded. δ, variable v		tax-ded. δ, $v = 1$	
Variable	Ramsey	Markov	Ramsey	Markov	Ramsey	Markov
Consumption tax rate	0.223	0.160	0.223	0.156	0.223	0.160
Capital income tax rate	0.000	0.103	0.000	0.231	0.000	0.236
Capital	1.215	1.031	1.548	1.144	1.215	1.031
Hours worked	0.288	0.292	0.275	0.287	0.288	0.292
Income	0.505	0.478	0.492	0.473	0.505	0.478
Consumption-income ratio	0.618	0.638	0.654	0.654	0.618	0.638
Public spending-income ratio	0.138	0.142	0.146	0.146	0.138	0.142

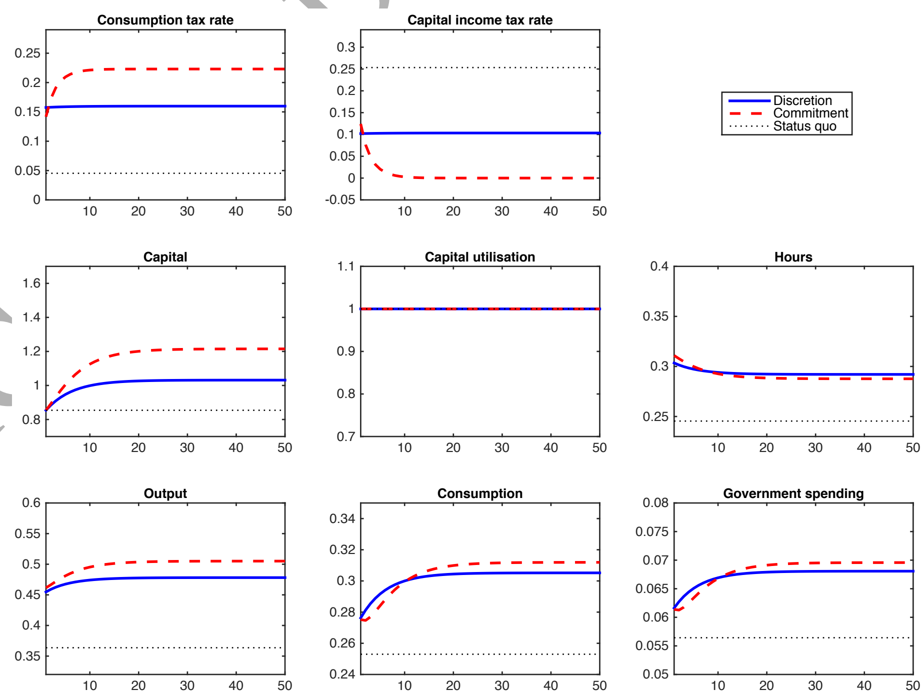
We first discuss the Ramsey equilibrium. The key difference from our baseline model is a higher capital tax in the first few years. The lack of intratemporal distortion caused by capital taxation creates an incentive for the Ramsey planner to tax capital at a higher rate initially. The government also uses consumption taxes from the beginning, due to the fact that this tax instrument is partly levied on the initial capital stock as well. Capital income taxes converge to zero, while consumption taxes increase during the transition to satisfy the government's budget constraint. The paths of allocations are similar to those in our baseline model.

Turning to the Markov equilibrium, first of all, it exists. This follows from what happens at the Ramsey equilibrium in the initial period. Consumption is taxed rather than subsidised optimally, and the capital income tax is not confiscatory, for our standard calibration. The same is true in each period when policies are time-consistent.

Contrary to our baseline model, the capital income tax rate is not close to zero at the steady state, instead it is 10.3 percent. Consumption is taxed at approximately 16 percent in all periods. As such, this tax component remains an important source of revenues even when capital is fully utilised. However, compared to its Ramsey counterpart, Markov-perfect taxation inhibits capital accumulation significantly.

Finally, we highlight the welfare gains compared to the existing tax system, and from being able to commit, see the first column of Table 5. Our baseline results are weakened

Figure 3. Ramsey and Markov policies with consumption taxation and no labour subsidy starting from the status quo, full capital utilisation



if capital is fully utilised. The gain from commitment is 0.338 percent with full capital utilisation, compared to 0.0003 at baseline.

Table 5. Welfare gains in welfare-equivalent consumption (percent), alternative assumptions

Welfare gains...	non-tax-ded. δ, $v = 1$	tax-ded. δ, variable v	tax-ded. δ, $v = 1$
...from commitment	0.338	0.633	0.338
	...compared to the existing tax system		
Ramsey	6.816	7.793	6.816
Markov	6.447	7.709	6.447

5.2. *Tax-deductible depreciation*

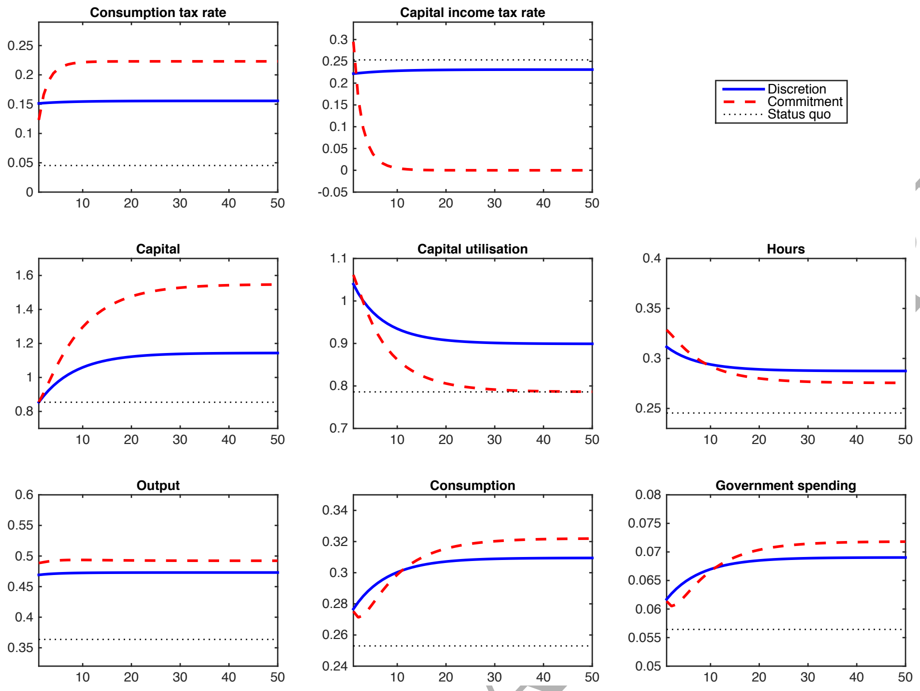
Now we consider the case where depreciation is tax-deductible and capital utilisation is endogenous. Appendix C contains the model setup and the Ramsey and Markov policy problems for this case. Tax rates and allocations at the steady state are presented in the third and fourth columns of Table 4, Figure 4 shows the dynamic paths, and welfare is reported in the second column of Table 5.

There are several differences compared to the case with full capital utilisation. First, capital income taxes are higher in the Markov equilibrium (23.1 percent at the steady state). This is due to the fact that with depreciation allowance, the capital income tax base is smaller, thus creating the need of a higher rate for a given level of fiscal revenues. Second, capital is significantly higher, but income is similar, given that capital is not fully utilised. Finally, the welfare gains from commitment are larger in this case (0.633 percent), even further from our baseline result.

5.3. *Full capital utilisation and tax-deductible depreciation*

Finally, we assume that capital is fully utilised and depreciation is tax-deductible. That is, we set $v = 1$ and $\lambda_4 = 0$ in the model of Appendix C. Tax rates and allocations at the steady state are presented in the fifth and sixth columns of Table 4 and welfare is reported in last column of Table 5. The results are identical to the case where depreciation is not tax-deductible and capital is fully utilised, except for the capital income tax rate at the Markov steady state (23.6 percent). The difference compared to Section 5.1 is due to the difference in tax bases, as with variable capital utilisation. We do not include a separate figure of the dynamic paths of tax rates and allocations, as they look identical to those of Figure 3, except for the level of the capital income tax under discretion.

Figure 4. Ramsey and Markov policies with consumption taxation and no labour subsidy starting from the status quo, with depreciation allowance



5.4. Budget balance

Compared to all scenarios we have considered, one difference remains between our model and that of Coleman (2000): We have assumed throughout that the government has to balance its budget, while Coleman (2000) allows for government debt under commitment. Solving our policy problems with both capital and government debt is beyond the scope of this paper, as well as of the literature on Markov-perfect policies, to our knowledge. Nonetheless, we can compare our Ramsey setting with consumption taxation, full capital utilisation, and tax-deductible depreciation to that of Coleman (2000). Then the only remaining difference concerns the government's budget.[9]

To avoid trivial solutions to the public finance problem, instead of budget balance, Coleman (2000) imposes upper bounds on the initial capital income tax rate (50 or 100 percent). Then the consumption tax becomes a very useful instrument to impose an initial capital levy, and hence the government can reduce its debt or accumulate assets. For example, with an upper bound of 100 percent on the capital income tax, Coleman (2000)

[9]Coleman (2000) assumes exogenous government spending, but this feature is not key.

finds an initial consumption tax of 117 percent. This motive is not strong in our setting with budget balance, and the consumption tax follows as increasing path, see Figure 3.

6. Concluding remarks

In this paper we have considered a representative-agent framework, hence we studied the optimal tax mix from an efficiency perspective only. An important task for future research is to analyse the distributional impact of different tax instruments with and without commitment in a model with heterogeneous households. In addition, we have imposed a balanced-budget requirement on the policy-maker. In a recent paper, Debortoli et al. (2017) allow for debt in a Markov-perfect policy setting, but they exclude capital, and the only tax instrument available to the government is a labour income tax. The study of tax policy trade-offs in the presence of debt when the government cannot commit is another interesting avenue for future investigation.

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Appendices

Appendix A. Analytical characterisations

Appendix A.1. First-best allocation

Denote by $\lambda_1\left(\mathbf{a}^t\right)$ the Lagrange multiplier on the time constraint, (2), and by $\lambda_2\left(\mathbf{a}^t\right)$ the Lagrange multiplier on the resource constraint, (10), when history $\mathbf{a}^t$ has occurred. Use (7) to replace for $y\left(\mathbf{a}^t\right)$ in (10). Then we can write the problem as

$$\max_{\{c(\mathbf{a}^t),\ell(\mathbf{a}^t),h(\mathbf{a}^t),g(\mathbf{a}^t),k(\mathbf{a}^t),v(\mathbf{a}^t)\}_{t=1}^{\infty}} \min_{\{\lambda_1(\mathbf{a}^t),\lambda_2(\mathbf{a}^t)\}_{t=1}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{u\left(c\left(\mathbf{a}^t\right), \ell\left(\mathbf{a}^t\right), \left(\mathbf{a}^t\right)\right)\right.$$
$$+\lambda_1\left(\mathbf{a}^t\right)\left(1-\ell\left(\mathbf{a}^t\right)-h\left(\mathbf{a}^t\right)\right)$$
$$\left.+\lambda_2\left(\mathbf{a}^t\right)\left[a_t\left(v\left(\mathbf{a}^t\right)k\left(\mathbf{a}^{t-1}\right)\right)^{\gamma} h\left(\mathbf{a}^t\right)^{1-\gamma} + \left(1-\delta\left(v\left(\mathbf{a}^t\right)\right)\right)k\left(\mathbf{a}^{t-1}\right) - c\left(\mathbf{a}^t\right) - g\left(\mathbf{a}^t\right) - k\left(\mathbf{a}^t\right)\right]\right\},$$

where we have used (7) to replace for $y\left(\mathbf{a}^t\right)$ in (10). The first-order conditions with respect to $c\left(\mathbf{a}^t\right), \ell\left(\mathbf{a}^t\right), h\left(\mathbf{a}^t\right), g\left(\mathbf{a}^t\right), k\left(\mathbf{a}^t\right), v\left(\mathbf{a}^t\right), \lambda_1\left(\mathbf{a}^t\right)$, and $\lambda_2\left(\mathbf{a}^t\right)$, respectively, are

$$u_c\left(\mathbf{a}^t\right) = \lambda_2\left(\mathbf{a}^t\right), \tag{A.1}$$
$$u_\ell\left(\mathbf{a}^t\right) = \lambda_1\left(\mathbf{a}^t\right), \tag{A.2}$$
$$a_t\left(1-\gamma\right)\left(\frac{v\left(\mathbf{a}^t\right)k\left(\mathbf{a}^{t-1}\right)}{h\left(\mathbf{a}^t\right)}\right)^{\gamma} = \lambda_1\left(\mathbf{a}^t\right), \tag{A.3}$$
$$u_g\left(\mathbf{a}^t\right) = \lambda_2\left(\mathbf{a}^t\right), \tag{A.4}$$
$$\lambda_2\left(\mathbf{a}^t\right) = \beta\mathbb{E}_t\left(\lambda_1\left(\mathbf{a}^{t+1}\right)\left[a_{t+1}\gamma v\left(\mathbf{a}^{t+1}\right)^{\gamma}\left(\frac{h\left(\mathbf{a}^{t+1}\right)}{k\left(\mathbf{a}^t\right)}\right)^{1-\gamma} + 1 - \delta\left(v\left(\mathbf{a}^{t+1}\right)\right)\right]\right), \tag{A.5}$$
$$\delta_u\left(\mathbf{a}^t\right) = a_t\gamma\left(\frac{h\left(\mathbf{a}^t\right)}{v\left(\mathbf{a}^t\right)k\left(\mathbf{a}^{t-1}\right)}\right)^{1-\gamma}, \tag{A.6}$$
$$\ell\left(\mathbf{a}^t\right) + h\left(\mathbf{a}^t\right) = 1, \tag{A.7}$$
$$c\left(\mathbf{a}^t\right) + g\left(\mathbf{a}^t\right) + k\left(\mathbf{a}^t\right) = a_t\left(v\left(\mathbf{a}^t\right)k\left(\mathbf{a}^{t-1}\right)\right)^{\gamma} h\left(\mathbf{a}^t\right)^{1-\gamma} + \left(1-\delta\left(v\left(\mathbf{a}^t\right)\right)\right)k\left(\mathbf{a}^{t-1}\right). \tag{A.8}$$

Straightforward combinations of (A.1)-(A.8) lead to the following equations which characterise the first-best allocation:

$$u_g\left(\mathbf{a}^t\right) = u_c\left(\mathbf{a}^t\right), \tag{A.9}$$
$$\frac{u_\ell\left(\mathbf{a}^t\right)}{u_c\left(\mathbf{a}^t\right)} = a_t\left(1-\gamma\right)\left(\frac{v\left(\mathbf{a}^t\right)k\left(\mathbf{a}^{t-1}\right)}{h\left(\mathbf{a}^t\right)}\right)^{\gamma}, \tag{A.10}$$

$$h\left(\mathbf{a}^{t}\right)+\ell\left(\mathbf{a}^{t}\right)=1, \tag{A.11}$$

$$u_{c}\left(\mathbf{a}^{t}\right)=\beta\mathbb{E}_{t}\left(u_{c}\left(\mathbf{a}^{t+1}\right)\left[1-\delta\left(v\left(\mathbf{a}^{t+1}\right)\right)+a_{t+1}\gamma v\left(\mathbf{a}^{t+1}\right)\left(\frac{h\left(\mathbf{a}^{t+1}\right)}{v\left(\mathbf{a}^{t+1}\right)k\left(\mathbf{a}^{t}\right)}\right)^{1-\gamma}\right]\right), \tag{A.12}$$

$$c\left(\mathbf{a}^{t}\right)+g\left(\mathbf{a}^{t}\right)+k\left(\mathbf{a}^{t}\right)=a_{t}\left(v\left(\mathbf{a}^{t}\right)k\left(\mathbf{a}^{t-1}\right)\right)^{\gamma}h\left(\mathbf{a}^{t}\right)^{1-\gamma}+\left(1-\delta\left(v\left(\mathbf{a}^{t}\right)\right)\right)k\left(\mathbf{a}^{t-1}\right), \tag{A.13}$$

$$\delta_{v}\left(\mathbf{a}^{t}\right)=a_{t}\gamma\left(\frac{h\left(\mathbf{a}^{t}\right)}{v\left(\mathbf{a}^{t}\right)k\left(\mathbf{a}^{t-1}\right)}\right)^{1-\gamma}, \tag{A.14}$$

$\forall\mathbf{a}^{t}$, $k\left(\mathbf{a}^{0}\right)$ and the productivity process given.

Appendix A.2. Constraints of the policy problems

The following six equations characterise competitive equilibria once three variables (output $y\left(\mathbf{a}^{t}\right)$ and prices $w\left(\mathbf{a}^{t}\right)$ and $r\left(\mathbf{a}^{t}\right)$) and four equations ((7), (8),(9), and (3)) are eliminated in Definition 2: $\forall\mathbf{a}^{t}$,

$$h\left(\mathbf{a}^{t}\right)+\ell\left(\mathbf{a}^{t}\right)=1,\ \forall\mathbf{a}^{t}, \tag{A.15}$$

$$\frac{u_{\ell}\left(\mathbf{a}^{t}\right)}{u_{c}\left(\mathbf{a}^{t}\right)}=\frac{1-\tau^{h}\left(\mathbf{a}^{t}\right)}{1+\tau^{c}\left(\mathbf{a}^{t}\right)}a_{t}\left(1-\gamma\right)\left(\frac{v\left(\mathbf{a}^{t}\right)k\left(\mathbf{a}^{t-1}\right)}{h\left(\mathbf{a}^{t}\right)}\right)^{\gamma}, \tag{A.16}$$

$$\frac{u_{c}\left(\mathbf{a}^{t}\right)}{1+\tau^{c}\left(\mathbf{a}^{t}\right)}=\beta\mathbb{E}_{t}\left(\frac{u_{c}\left(\mathbf{a}^{t+1}\right)}{1+\tau^{c}\left(\mathbf{a}^{t+1}\right)}\left[1-\delta\left(v\left(\mathbf{a}^{t+1}\right)\right)+\left(1-\tau^{k}\left(\mathbf{a}^{t+1}\right)\right)a_{t+1}\gamma v\left(\mathbf{a}^{t+1}\right)\left(\frac{h\left(\mathbf{a}^{t+1}\right)}{v\left(\mathbf{a}^{t+1}\right)k\left(\mathbf{a}^{t}\right)}\right)^{1-\gamma}\right]\right), \tag{A.17}$$

$$\delta_{v}\left(\mathbf{a}^{t}\right)=\left(1-\tau^{k}\left(\mathbf{a}^{t}\right)\right)a_{t}\gamma\left(\frac{h\left(\mathbf{a}^{t}\right)}{v\left(\mathbf{a}^{t}\right)k\left(\mathbf{a}^{t-1}\right)}\right)^{1-\gamma}, \tag{A.18}$$

$$g\left(\mathbf{a}^{t}\right)=a_{t}\left[\tau^{k}\left(\mathbf{a}^{t}\right)\gamma+\tau^{h}\left(\mathbf{a}^{t}\right)\left(1-\gamma\right)\right]\left(v\left(\mathbf{a}^{t}\right)k\left(\mathbf{a}^{t-1}\right)\right)^{\gamma}h\left(\mathbf{a}^{t}\right)^{1-\gamma}+\tau^{c}\left(\mathbf{a}^{t}\right)c\left(\mathbf{a}^{t}\right), \tag{A.19}$$

$$c\left(\mathbf{a}^{t}\right)+g\left(\mathbf{a}^{t}\right)+k\left(\mathbf{a}^{t}\right)=a_{t}\left(v\left(\mathbf{a}^{t}\right)k\left(\mathbf{a}^{t-1}\right)\right)^{\gamma}h\left(\mathbf{a}^{t}\right)^{1-\gamma}+\left(1-\delta\left(v\left(\mathbf{a}^{t}\right)\right)\right)k\left(\mathbf{a}^{t-1}\right), \tag{A.20}$$

which is (12) in the main text.

Then, we use the household's intratemporal optimality condition (A.16), and the government's budget constraint, (A.19), to express the labour and capital income tax rates when history $\mathbf{a}^t$ has occurred, respectively, as

$$\tau^h\left(\mathbf{a}^t\right) = 1 - \frac{u_\ell\left(\mathbf{a}^t\right)}{u_c\left(\mathbf{a}^t\right)} \frac{1+\tau^c\left(\mathbf{a}^t\right)}{(1-\gamma)\, a_t \left(v\left(\mathbf{a}^t\right) k\left(\mathbf{a}^{t-1}\right)\right)^\gamma h\left(\mathbf{a}^t\right)^{-\gamma}}, \tag{A.21}$$

$$\tau^k\left(\mathbf{a}^t\right) = \frac{g\left(\mathbf{a}^t\right) - \tau^c\left(\mathbf{a}^t\right) c\left(\mathbf{a}^t\right)}{\gamma a_t \left(v\left(\mathbf{a}^t\right) k\left(\mathbf{a}^{t-1}\right)\right)^\gamma h\left(\mathbf{a}^t\right)^{1-\gamma}} - \frac{1-\gamma}{\gamma}\tau^h\left(\mathbf{a}^t\right). \tag{A.22}$$

Replacing for $\left(1-\tau^k\left(\mathbf{a}^{t+1}\right)\right)$ in (A.17) using (A.22) and in turn for $\tau^h\left(\mathbf{a}^{t+1}\right)$ using (A.21), we can write the Euler equation as

$$\begin{aligned} \frac{u_c\left(\mathbf{a}^t\right)}{1+\tau^c\left(\mathbf{a}^t\right)} = \beta \mathbb{E}_t \Bigg(\frac{u_c\left(\mathbf{a}^{t+1}\right)}{1+\tau^c\left(\mathbf{a}^{t+1}\right)} \Bigg[1 - \delta\left(v\left(\mathbf{a}^{t+1}\right)\right) + a_{t+1} v\left(\mathbf{a}^{t+1}\right) \left(\frac{h\left(\mathbf{a}^{t+1}\right)}{v\left(\mathbf{a}^{t+1}\right) k\left(\mathbf{a}^t\right)}\right)^{1-\gamma} \\ - \frac{g\left(\mathbf{a}^{t+1}\right) - \tau^c\left(\mathbf{a}^{t+1}\right) c\left(\mathbf{a}^{t+1}\right)}{k\left(\mathbf{a}^t\right)} \Bigg] - u_\ell\left(\mathbf{a}^{t+1}\right) \frac{h\left(\mathbf{a}^{t+1}\right)}{k\left(\mathbf{a}^t\right)} \Bigg), \end{aligned} \tag{A.23}$$

which is (13) in the main text. Similarly, we can eliminate $\tau^k\left(\mathbf{a}^t\right)$ from (A.18) and rewrite it as

$$\begin{aligned} \delta_v\left(\mathbf{a}^t\right) = a_t \left(\frac{h\left(\mathbf{a}^t\right)}{v\left(\mathbf{a}^t\right) k\left(\mathbf{a}^{t-1}\right)}\right)^{1-\gamma} - \frac{g\left(\mathbf{a}^t\right) - \tau^c\left(\mathbf{a}^t\right) c\left(\mathbf{a}^t\right)}{v\left(\mathbf{a}^t\right) k\left(\mathbf{a}^{t-1}\right)} \\ - \frac{u_\ell\left(\mathbf{a}^t\right)}{u_c\left(\mathbf{a}^t\right)} \left(1+\tau^c\left(\mathbf{a}^t\right)\right) \frac{h\left(\mathbf{a}^t\right)}{v\left(\mathbf{a}^t\right) k\left(\mathbf{a}^{t-1}\right)}, \end{aligned} \tag{A.24}$$

which is (14) in the main text.

Appendix A.3. First-order conditions of the Ramsey policy-maker's problem

We assume that the utility function is separable with respect to its three arguments, hence the second cross-derivatives are zero.

The FOCs with respect to $\tau^c, c, \ell, h, g, k', v$, and $\lambda_1, \lambda_2, \lambda_3', \lambda_4, \lambda_5$, respectively, are

$$\begin{aligned} 0 = \frac{1}{(1+\tau^c)^2} \Bigg\{ -\lambda_3' u_c + \lambda_3 u_c \left[1 - \delta(v) + av\left(\frac{h}{vk}\right)^{1-\gamma} - \frac{g - \tau^c c}{k} \right] \\ -\lambda_5 (1-\gamma)\, a \left(\frac{vk}{h}\right)^\gamma \Bigg\} - \lambda_4 \left(\frac{c}{vk} - \frac{u_\ell}{u_c}\frac{h}{vk}\right) - \lambda_3 \frac{u_c}{1+\tau^c}\frac{c}{k}, \end{aligned} \tag{A.25}$$

$$\begin{aligned}
0 &= u_c - \lambda_2 + \lambda_3' \frac{u_{cc}}{1+\tau^c} - \lambda_3 \frac{u_{cc}}{1+\tau^c}\left[1 - \delta(v) + av\left(\frac{h}{vk}\right)^{1-\gamma} - \frac{g-\tau^c c}{k}\right] \\
&\quad - \lambda_3 \frac{u_c}{1+\tau^c}\frac{\tau^c}{k} - \lambda_4\left[\frac{\tau^c}{vk} + \frac{u_\ell}{u_c^2}u_{cc}\left(1+\tau^c\right)\frac{h}{vk}\right] + \lambda_5 \frac{u_\ell}{u_c^2}u_{cc}, && \text{(A.26)}\\
0 &= u_\ell - \lambda_1 + \lambda_3 u_{\ell\ell}\frac{h}{k} + \lambda_4 \frac{u_{\ell\ell}}{u_c}\left(1+\tau^c\right)\frac{h}{vk} - \lambda_5\frac{u_{\ell\ell}}{u_c}, && \text{(A.27)}\\
0 &= -\lambda_1 + \lambda_2 a\left(1-\gamma\right)\left(\frac{vk}{h}\right)^{\gamma} - \lambda_3\left[\frac{u_c}{1+\tau^c}a\left(1-\gamma\right)\frac{v^\gamma}{h^\gamma k^{1-\gamma}} - \frac{u_\ell}{k}\right] && \text{(A.28)}\\
&\quad - \lambda_4\left[a\left(1-\gamma\right)h^{-\gamma}\left(vk\right)^{\gamma-1} - \frac{u_\ell}{u_c}\left(1+\tau^c\right)\frac{1}{vk}\right] - \lambda_5\frac{1}{1+\tau^c}a\gamma\left(1-\gamma\right)\left(vk\right)^{\gamma}h^{-\gamma-1}, \\
0 &= u_g - \lambda_2 + \lambda_3\frac{u_c}{1+\tau^c}\frac{1}{k} + \lambda_4\frac{1}{vk}, && \text{(A.29)}\\
0 &= -\lambda_2 + \beta\mathbb{E}\left(\lambda_2'\left[a'v'^{\gamma}\gamma\left(\frac{h'}{k'}\right)^{1-\gamma} + 1 - \delta\left(v'\right)\right]\right. \\
&\quad +\lambda_3'\left[\frac{u_c'}{1+\tau^{c\prime}}\left(a'v'^{\gamma}\left(1-\gamma\right)k'^{\gamma-2}h'^{1-\gamma} - \frac{g'-\tau^{c\prime}c'}{k'^2}\right) - u_\ell'\frac{h'}{k'^2}\right] && \text{(A.30)}\\
&\quad +\lambda_4'\left[a'\left(1-\gamma\right)k'^{\gamma-2}\left(\frac{h'}{v'}\right)^{1-\gamma} - \frac{g'-\tau^{c\prime}c'}{v'k'^2} - \frac{u_\ell'}{u_c'}\left(1+\tau^{c\prime}\right)\frac{h'}{v'k'^2}\right] \\
&\quad \left. +\lambda_5'\frac{1}{1+\tau^{c\prime}}a'\left(1-\gamma\right)\gamma\frac{v'^{\gamma}}{k'^{1-\gamma}h'^{\gamma}}\right), \\
0 &= \lambda_2\left(a\gamma v^{\gamma-1}k^{\gamma}h^{1-\gamma} - \delta_v k\right) + \lambda_3\frac{u_c}{1+\tau^c}\left[\delta_v - a\gamma v^{\gamma-1}\left(\frac{h}{k}\right)^{1-\gamma}\right] \\
&\quad + \lambda_4\left[a\left(1-\gamma\right)v^{\gamma-2}\left(\frac{h}{k}\right)^{1-\gamma} - \frac{g-\tau^c c}{v^2 k} - \frac{u_\ell}{u_c}\left(1+\tau^c\right)\frac{h}{v^2 k} + \delta_{vv}\right] && \text{(A.31)}\\
&\quad + \lambda_5\frac{1}{1+\tau^c}a\left(1-\gamma\right)\gamma v^{\gamma-1}\left(\frac{k}{h}\right)^{\gamma}, \\
0 &= \ell + h - 1, && \text{(A.32)}\\
0 &= c + g + k_{t+1} - a\left(vk\right)^{\gamma}h^{1-\gamma} - \left(1-\delta\left(v\right)\right)k, && \text{(A.33)}\\
0 &= -\frac{u_c}{1+\tau^c} + \beta\mathbb{E}\left(\frac{u_c'}{1+\tau^{c\prime}}\left[1-\delta\left(v'\right) + a'v'\left(\frac{h'}{v'k'}\right)^{1-\gamma} - \frac{g'-\tau^{c\prime}c'}{k'}\right] - u_\ell'\frac{h'}{k'}\right), && \text{(A.34)}\\
0 &= a\left(\frac{h}{vk}\right)^{1-\gamma} - \frac{g-\tau^c c}{vk} - \frac{u_\ell}{u_c}\left(1+\tau^c\right)\frac{h}{vk} - \delta_v, && \text{(A.35)}\\
0 &\geq \frac{u_\ell}{u_c} - \frac{1}{1+\tau^c}a\left(1-\gamma\right)\left(\frac{vk}{h}\right)^{\gamma}, && \text{(A.36)}
\end{aligned}$$

$\lambda_5 \geq 0$, with complementary slackness condition.

Appendix A.4. First-order conditions of the time-consistent policy-maker's problem

The FOCs with respect to $\tau^c, c, \ell, h, g, k', v$, respectively, are

$$0 = -\lambda_3 \frac{1}{(1+\tau^c)^2} u_c - \lambda_4 \left(\frac{c}{vk} - \frac{u_\ell}{u_c} \frac{h}{vk} \right) - \lambda_5 \frac{1}{(1+\tau^c)^2} (1-\gamma) a \left(\frac{vk}{h} \right)^\gamma , \tag{A.37}$$

$$0 = u_c - \lambda_2 + \lambda_3 \frac{u_{cc}}{1+\tau^c} - \lambda_4 \left[\frac{\tau^c}{vk} + \frac{u_\ell}{u_c^2} u_{cc} (1+\tau^c) \frac{h}{vk} \right] + \lambda_5 \frac{u_\ell}{u_c^2} u_{cc}, \tag{A.38}$$

$$0 = u_\ell - \lambda_1 + \lambda_4 \frac{u_{\ell\ell}}{u_c} (1+\tau^c) \frac{h}{vk} - \lambda_5 \frac{u_{\ell\ell}}{u_c}, \tag{A.39}$$

$$0 = -\lambda_1 + \lambda_2 (1-\gamma) a \left(\frac{vk}{h} \right)^\gamma - \lambda_4 \left[a (1-\gamma) h^{-\gamma} (vk)^{\gamma-1} - \frac{u_\ell}{u_c} (1+\tau^c) \frac{1}{vk} \right]$$
$$- \lambda_5 \frac{1}{1+\tau^c} (1-\gamma) \gamma a (vk)^\gamma h^{-\gamma-1}, \tag{A.40}$$

$$0 = u_g - \lambda_2 + \lambda_4 \frac{1}{vk}, \tag{A.41}$$

$$0 = \beta \mathbb{E} \frac{\partial \mathcal{W}(a',k')}{\partial k'} - \lambda_2 - \beta \lambda_3 \mathbb{E} \left(\frac{\partial \frac{\mathcal{U}_c(a',k')}{1+\mathcal{T}^c(a',k')}}{\partial k'} \left[1 - \delta\left(\mathcal{V}(a',k')\right) \right. \right. \tag{A.42}$$
$$\left. + a' \frac{\mathcal{H}(a',k')^{1-\gamma} \mathcal{V}(a',k')^\gamma k'^\gamma}{k'} - \frac{\mathcal{G}(a',k')}{k'} + \frac{\mathcal{T}^c(a',k') \mathcal{C}(a',k')}{k'} \right] - \frac{\partial \mathcal{U}_\ell(a',k') \frac{\mathcal{H}(a',k')}{k'}}{\partial k'}$$
$$\left. + \frac{\mathcal{U}_c(a',k')}{1+\mathcal{T}^c(a',k')} \left[- \frac{\partial \delta\left(\mathcal{V}(a',k')\right)}{\partial k'} + a' \frac{\partial \frac{\mathcal{H}(a',k')^{1-\gamma}\mathcal{V}(a',k')^\gamma k'^\gamma}{k'}}{\partial k'} - \frac{\partial \frac{\mathcal{G}(a',k')}{k'}}{\partial k'} + \frac{\partial \frac{\mathcal{T}^c(a',k')\mathcal{C}(a',k')}{k'}}{\partial k'} \right] \right)$$

$$0 = \lambda_2 \left(a\gamma v^{\gamma-1} k^\gamma h^{1-\gamma} - \delta_v k \right) + \lambda_4 \left[a (1-\gamma) v^{\gamma-2} \left(\frac{h}{k} \right)^{1-\gamma} - \frac{g - \tau^c c}{v^2 k} - \frac{u_\ell}{u_c} (1+\tau^c) \frac{h}{v^2 k} + \delta_{vv} \right]$$
$$+ \lambda_5 \frac{1}{1+\tau^c} a (1-\gamma) \gamma v^{\gamma-1} \left(\frac{k}{h} \right)^\gamma , \tag{A.43}$$

where

$$\frac{\partial \delta\left(\mathcal{V}(a',k')\right)}{\partial k'} = \delta_v \frac{\partial \mathcal{V}(a',k')}{\partial k'},$$

$$\frac{\partial \frac{\mathcal{U}_c(a',k')}{1+\mathcal{T}^c(a',k')}}{\partial k'} = \frac{\frac{\partial \mathcal{U}_c(a',k')}{\partial k'} \left(1+\mathcal{T}^c(a',k')\right) - \frac{\partial \mathcal{T}^c(a',k')}{\partial k'} \mathcal{U}_c(a',k')}{\left(1+\mathcal{T}^c(a',k')\right)^2},$$

$$\frac{\partial \frac{\mathcal{H}(a',k')^{1-\gamma}\mathcal{V}(a',k')^\gamma k'^\gamma}{k'}}{\partial k'} = \mathcal{V}(a',k')^{\gamma-1} \mathcal{H}(a',k')^{-\gamma} k'^{\gamma-2} \left[(1-\gamma) \mathcal{V}(a',k') k' \frac{\partial \mathcal{H}(a',k')}{\partial k'} \right.$$
$$\left. + \gamma \mathcal{H}(a',k') k' \frac{\partial \mathcal{V}(a',k')}{\partial k'} - (1-\gamma) \mathcal{V}(a',k') \mathcal{H}(a',k') \right],$$

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$$\frac{\partial\frac{\mathcal{G}(a',k')}{k'}}{\partial k'} = \frac{\frac{\partial\mathcal{G}(a',k')}{\partial k'}k' - \mathcal{G}(a',k')}{k'^2},$$

$$\frac{\partial\frac{\mathcal{T}^c(a',k')\mathcal{C}(a',k')}{k'}}{\partial k'} = \frac{\left[\frac{\partial\mathcal{T}^c(a',k')}{\partial k'}\mathcal{C}(a',k') + \frac{\partial\mathcal{C}(a',k')}{\partial k'}\mathcal{T}^c(a',k')\right]k' - \mathcal{T}^c(a',k')\mathcal{C}(a',k')}{k'^2},$$

$$\frac{\partial\mathcal{U}_\ell(a',k')\frac{\mathcal{H}(a',k')}{k'}}{\partial k'} = \frac{\left[\frac{\partial\mathcal{U}_\ell(a',k')}{\partial k'}\mathcal{H}(a',k') + \frac{\partial\mathcal{H}(a',k')}{\partial k'}\mathcal{U}_\ell(a',k')\right]k' - \mathcal{U}_\ell(a',k')\mathcal{H}(a',k')}{k'^2}.$$

Applying the envelope theorem gives

$$\frac{\partial\mathcal{W}(a,k)}{\partial k} = \lambda_2\left[a\gamma v^\gamma\left(\frac{h}{k}\right)^{1-\gamma} + 1 - \delta(v)\right] + \lambda_4\left[a(1-\gamma)\left(\frac{h}{v}\right)^{1-\gamma}k^{\gamma-2} - \frac{g-\tau^c c}{vk^2} - \frac{u_\ell}{u_c}(1+\tau^c)\frac{h}{vk^2}\right] + \lambda_5\frac{1}{1+\tau^c}a(1-\gamma)\gamma\frac{v^\gamma}{k^{1-\gamma}h^\gamma},$$

hence,

$$\begin{aligned}\frac{\partial\mathcal{W}(a',k')}{\partial k'} = {}& \Lambda_2(a',k')\left[a'\gamma\mathcal{V}(a',k')^\gamma\left(\frac{\mathcal{H}(a',k')}{k'}\right)^{1-\gamma} + 1 - \delta(\mathcal{V}(a',k'))\right] \\ &+ \Lambda_4(a',k')\left[a'(1-\gamma)\left(\frac{\mathcal{H}(a',k')}{\mathcal{V}(a',k')}\right)^{1-\gamma}k'^{\gamma-2}\right. \\ &\left.- \frac{\mathcal{G}(a',k') - \mathcal{T}^c(a',k')\mathcal{C}(a',k')}{\mathcal{V}(a',k')k'^2} - \frac{\mathcal{U}_\ell(a',k')}{\mathcal{U}_c(a',k')}(1+\mathcal{T}^c(a',k'))\frac{\mathcal{H}(a',k')}{\mathcal{V}(a',k')k'^2}\right] \\ &+ \Lambda_5(a',k')\frac{1}{1+\mathcal{T}^c(a',k')}a'(1-\gamma)\gamma\frac{\mathcal{V}(a',k')^\gamma}{k'^{1-\gamma}\mathcal{H}(a',k')^\gamma}.\end{aligned}$$

Plugging this condition into (A.42), we obtain

$$\begin{aligned}0 = {}& -\lambda_2 + \beta\mathbb{E}\left(\Lambda_2(a',k')\left[a'\gamma\mathcal{V}(a',k')^\gamma\left(\frac{\mathcal{H}(a',k')}{k'}\right)^{1-\gamma} + 1 - \delta(\mathcal{V}(a',k'))\right]\right. \\ &+\Lambda_4(a',k')\left[a'(1-\gamma)\left(\frac{\mathcal{H}(a',k')}{\mathcal{V}(a',k')}\right)^{1-\gamma}k'^{\gamma-2} - \frac{\mathcal{G}(a',k') - \mathcal{T}^c(a',k')\mathcal{C}(a',k')}{\mathcal{V}(a',k')k'^2}\right. \\ &\left.\left.-\frac{\mathcal{U}_\ell(a',k')}{\mathcal{U}_c(a',k')}(1+\mathcal{T}^c(a',k'))\frac{\mathcal{H}(a',k')}{\mathcal{V}(a',k')k'^2}\right] + \Lambda_5(a',k')\frac{1}{1+\mathcal{T}^c(a',k')}a'(1-\gamma)\gamma\frac{\mathcal{V}(a',k')^\gamma}{k'^{1-\gamma}\mathcal{H}(a',k')^\gamma}\right) \\ &-\beta\lambda_3\mathbb{E}\left(\frac{\partial\frac{\mathcal{U}_c(a',k')}{1+\mathcal{T}^c(a',k')}}{\partial k'}\left[1-\delta(\mathcal{V}(a',k')) + a'\frac{\mathcal{H}(a',k')^{1-\gamma}\mathcal{V}(a',k')^\gamma k'^\gamma}{k'} - \frac{\mathcal{G}(a',k')}{k'} + \frac{\mathcal{T}^c(a',k')\mathcal{C}(a',k')}{k'}\right]\right. \\ &\left.+\frac{\mathcal{U}_c(a',k')}{1+\mathcal{T}^c(a',k')}\left[-\frac{\partial\delta(\mathcal{V}(a',k'))}{\partial k'} + a'\frac{\partial\frac{\mathcal{H}(a',k')^{1-\gamma}\mathcal{V}(a',k')^\gamma k'^\gamma}{k'}}{\partial k'} - \frac{\partial\frac{\mathcal{G}(a',k')}{k'}}{\partial k'} + \frac{\partial\frac{\mathcal{T}^c(a',k')\mathcal{C}(a',k')}{k'}}{\partial k'}\right] - \frac{\partial\mathcal{U}_\ell(a',k')\frac{\mathcal{H}(a',k')}{k'}}{\partial k'}\right).\end{aligned}$$

Finally, the first-order conditions with respect to $\lambda_1, \lambda_2, \lambda_3, \lambda_4$, and λ_5, respectively, are

$$0 = \ell + h - 1, \tag{A.44}$$
$$0 = c + g + k' - a\,(vk)^{\gamma}\,h^{1-\gamma} - (1-\delta(v))\,k, \tag{A.45}$$

$$0 = -\frac{u_c}{1+\tau^c} + \beta\mathbb{E}\left(\frac{\mathcal{U}_c(a',k')}{1+\mathcal{T}^c(a',kx')}\left[1-\delta\left(\mathcal{V}(a',k')\right) + a'\frac{\mathcal{H}(a',k')^{1-\gamma}\,\mathcal{V}(a',k')^{\gamma}\,k'^{\gamma}}{k'} - \frac{\mathcal{G}(a',k') - \mathcal{T}^c(a',k')\,\mathcal{C}(a',k')}{k'}\right] - \mathcal{U}_\ell(a',k')\,\frac{\mathcal{H}(a',k')}{k'}\right), \tag{A.46}$$

$$0 = a\left(\frac{h}{vk}\right)^{1-\gamma} - \frac{g-\tau^c c}{vk} - \frac{u_\ell}{u_c}(1+\tau^c)\,\frac{h}{vk} - \delta_v, \tag{A.47}$$

$$0 \geq \frac{u_\ell}{u_c} - \frac{1}{1+\tau^c}(1-\gamma)\,a\left(\frac{vk}{h}\right)^{\gamma}, \tag{A.48}$$

$\lambda_5 \geq 0$, with complementary slackness conditions.

Appendix B. Proofs

Proof of Result 1. Consider first the case where capital is fully utilised, i.e., $v = 1$. Beside the technological constraints, i.e., the time constraint (2) and the resource constraint (12), which constrain the first best as well, the government faces only three constraints: its budget constraint plus the Euler and consumption-leisure optimality condition of households. Given a constant τ^c at the steady state, comparing (A.12) and (A.34) gives $\tau^k = 0$. Also, comparing (A.10) and (A.16) gives $\frac{1-\tau^h}{1+\tau^c} = 1$, hence $\tau^c = -\tau^h$. Finally, as long as consumption is larger than labour income, the only way to raise revenue to finance g is by setting $\tau^c > 0$. In the case where households choose the capital utilisation rate, the policy-maker has to satisfy an additional incentive constraint, (A.18), while it has no more instruments. However, given that $\tau^k = 0$, the capital utilisation margin is not distorted, hence the first-best steady state can still be implemented. In addition, it is well known that when a Ramsey equilibrium attains the first best, it is time-consistent. A similar argument can be made with tax-deductible depreciation.

Proof of Result 2. Consider our baseline framework first, i.e., endogenous capital utilisation and non-tax-deductible depreciation. By the usual Kuhn-Tucker argument, if (A.36) is not satisfied when it is ignored, we can impose it as equality, hence $\tau^h = 0$. Next, note that at the steady state combining (A.34) and (A.36) as equality gives

$$\frac{1}{\beta} = 1 - \delta(v) + \gamma v^{\gamma}\left(\frac{h}{k}\right)^{1-\gamma} - \frac{g-\tau^c c}{k}.$$

Then, using this and (A.36) as equality again, we can rewrite (A.30) as

$$0=\lambda_5\gamma\frac{u_\ell}{u_c}+\left(\lambda_2-\lambda_3\frac{u_c}{1+\tau^c}\frac{1}{k}-\lambda_4\frac{1}{vk}\right)(g-\tau^c c).$$

Now, from (A.29) $\lambda_2-\lambda_3\frac{u_c}{1+\tau^c}\frac{1}{k}-\lambda_4\frac{1}{vk}=u_g>0$, hence $g=\tau^c c$ if $\lambda_5=0$. Finally, from (A.28), this holds if $\lambda_1=\lambda_2(1-\gamma)\left(\frac{vk}{h}\right)^\gamma=\lambda_2 w$, which says that the marginal value of leisure relative to consumption is the real wage, which obviously holds. Now, with $v=1$ and $\lambda_4=0$ and/or with tax-deductible depreciation it easy to see that the above argument still holds.

Appendix C. With tax-deductible depreciation

We focus on the deterministic case (and drop aggregate productivity a from the equations to simplify), and use the notation of the recursive version of our model. We assume that the capital tax is levied on $(rv-\delta(v))k$, i.e., (actual) depreciation is tax-deductible.

Appendix C.1. Environment and policy problems

The description of the economy is modified as follows. The government's and the household's budget constraints are, respectively,

$$g=\tau^k(rv-\delta(v))k+\tau^h wh+\tau^c c,$$

$$(1+\tau^c)c+k'=\left(1-\tau^k\right)rvk+\tau^k\delta(v)k+\left(1-\tau^h\right)wh+(1-\delta(v))k.$$

The private sector's optimality conditions change as follows. The labour income tax can be expressed from the household's consumption-leisure FOC as in (A.21), as before, and we can express the capital income tax from the government's budget constraint as

$$\tau^k=\frac{g-(1-\gamma)h\left(\frac{vk}{h}\right)^\gamma+\frac{u_\ell}{u_c}h(1+\tau^c)-\tau^c c}{\left[\gamma v\left(\frac{h}{vk}\right)^{1-\gamma}-\delta(v)\right]k},$$

where we have replaced for τ^h. The household's Euler now is

$$\frac{u_c}{1+\tau^c}=\beta\mathbb{E}_t\left(\frac{u_c'}{1+\tau^{c\prime}}\left[1-\delta(v')+\left(1-\tau^{k\prime}\right)\gamma v'\left(\frac{h'}{v'k'}\right)^{1-\gamma}+\tau^{k\prime}\delta(v')\right]\right).$$

Replacing for $\tau^{k\prime}$ and rearranging give

$$\frac{u_c}{1+\tau^c}=\beta\mathbb{E}_t\left(\frac{u_c'}{1+\tau^{c\prime}}\left[1-\delta(v')+v'\left(\frac{h'}{v'k'}\right)^{1-\gamma}-\frac{g'-\tau^{c\prime}c'}{k'}\right]-u_\ell'\frac{h'}{k'}\right),$$

that is, the Euler is unchanged. Finally, the first-order condition with respect to v now is

$$\delta_v = r = \gamma \left(\frac{h}{vk} \right)^{1-\gamma},$$

hence capital income taxation is not distortionary within the period.

The recursive Lagrangian of the Ramsey policy-maker's problem is

$$\begin{aligned} W = & \max_{\{\tau^c, c, \ell, h, g, k', v\}} \min_{\{\lambda_1, \lambda_2, \lambda_3', \lambda_4, \lambda_5\}} u(c, \ell, g) + \beta \sum_{a'} \Pr(a' \mid a) \mathcal{W}(a', k', \lambda_3') \\ & - \lambda_1 (\ell + h - 1) - \lambda_2 \left[c + g + k' - a (vk)^\gamma h^{1-\gamma} - (1 - \delta(v)) k \right] \\ & + \lambda_3' \frac{u_c}{1+\tau^c} - \lambda_3 \left\{ \frac{u_c}{1+\tau^c} \left[1 - \delta(v) + av \left(\frac{h}{vk} \right)^{1-\gamma} - \frac{g - \tau^c c}{k} \right] - u_\ell \frac{h}{k} \right\} \\ & - \lambda_4 \left[\gamma \left(\frac{h}{vk} \right)^{1-\gamma} - \delta_v \right] - \lambda_5 \left[\frac{u_\ell}{u_c} - \frac{1}{1+\tau^c} a (1-\gamma) \left(\frac{vk}{h} \right)^\gamma \right], \end{aligned}$$

$\lambda_5 \geq 0$, with complementary slackness conditions.

The Markov policy-maker solves

$$\begin{aligned} W = & \max_{\{\tau^c, c, \ell, h, g, k', v\}} \min_{\{\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5\}} u(c, \ell, g) + \beta \mathbb{E} \mathcal{W}(k') \\ & - \lambda_1 (\ell + h - 1) - \lambda_2 \left[c + g + k' - a (vk)^\gamma h^{1-\gamma} - (1 - \delta(v)) k \right] \\ & - \lambda_3 \left\{ - \frac{u_c}{1+\tau^c} + \beta \mathbb{E} \left(\frac{\mathcal{U}_c(k')}{1+\mathcal{T}^c(k')} \left[1 - \delta(\mathcal{V}(k')) + a' \frac{\mathcal{H}(k')^{1-\gamma} \mathcal{V}(k')^\gamma k'^\gamma}{k'} \right. \right. \right. \\ & \left. \left. \left. - \frac{\mathcal{G}(k') - \mathcal{T}^c(k') \mathcal{C}(k')}{k'} \right] - \mathcal{U}_\ell(k') \frac{\mathcal{H}(k')}{k'} \right) \right\} \\ & - \lambda_4 \left[\gamma \left(\frac{h}{vk} \right)^{1-\gamma} - \delta_v \right] - \lambda_5 \left[\frac{u_\ell}{u_c} - \frac{1}{1+\tau^c} (1-\gamma) a \left(\frac{vk}{h} \right)^\gamma \right], \end{aligned}$$

$\lambda_5 \geq 0$, with complementary slackness conditions.

Appendix C.2. First-order conditions of the Ramsey policy-maker's problem

The FOCs with respect to $\tau^c, c, \ell, h, g, k', v$, and $\lambda_1, \lambda_2, \lambda_3', \lambda_4, \lambda_5$, respectively, are

$$\begin{aligned} 0 = & \frac{1}{(1+\tau^c)^2} \left\{ -\lambda_3' u_c + \lambda_3 u_c \left[1 - \delta(v) + v \left(\frac{h}{vk} \right)^{1-\gamma} - \frac{g - \tau^c c}{k} \right] \right. \\ & \left. - \lambda_5 (1-\gamma) \left(\frac{vk}{h} \right)^\gamma \right\} - \lambda_3 \frac{u_c}{1+\tau^c} \frac{c}{k}, \end{aligned} \tag{C.1}$$

$$
\begin{aligned}
0 &= u_c - \lambda_2 + \lambda_3' \frac{u_{cc}}{1+\tau^c} - \lambda_3 \frac{u_{cc}}{1+\tau^c}\left[1-\delta(v)+v\left(\frac{h}{vk}\right)^{1-\gamma} - \frac{g-\tau^c c}{k}\right] \\
&\quad - \lambda_3 \frac{u_c}{1+\tau^c}\frac{\tau^c}{k} + \lambda_5 \frac{u_\ell}{u_c^2} u_{cc}, && \text{(C.2)} \\
0 &= u_\ell - \lambda_1 + \lambda_3 u_{\ell\ell}\frac{h}{k} - \lambda_5 \frac{u_{\ell\ell}}{u_c}, && \text{(C.3)} \\
0 &= -\lambda_1 + \lambda_2 (1-\gamma)\left(\frac{vk}{h}\right)^{\gamma} - \lambda_3\left[\frac{u_c}{1+\tau^c}(1-\gamma)\frac{v^\gamma}{h^\gamma k^{1-\gamma}} - \frac{u_\ell}{k}\right] \\
&\quad - \lambda_4 \gamma(1-\gamma) h^{-\gamma}(vk)^{\gamma-1} - \lambda_5 \frac{1}{1+\tau^c}\gamma(1-\gamma)(vk)^\gamma h^{-\gamma-1}, && \text{(C.4)} \\
0 &= u_g - \lambda_2 + \lambda_3 \frac{u_c}{1+\tau^c}\frac{1}{k}, && \text{(C.5)} \\
0 &= -\lambda_2 + \beta \mathbb{E}\left(\lambda_2'\left[v'^\gamma \gamma \left(\frac{h'}{k'}\right)^{1-\gamma} + 1 - \delta(v')\right]\right. \\
&\quad + \lambda_3' \left\{\frac{u_c'}{1+\tau^{c\prime}}\left[v'^\gamma (1-\gamma) k'^{\gamma-2} h'^{1-\gamma} - \frac{g'-\tau^{c\prime} c'}{k'^2}\right] - u_\ell' \frac{h'}{k'^2}\right\} && \text{(C.6)} \\
&\quad \left. -\lambda_4' \gamma(\gamma-1) k'^{\gamma-2}\left(\frac{h'}{v'}\right)^{1-\gamma} + \lambda_5' \frac{1}{1+\tau^{c\prime}}(1-\gamma)\gamma \frac{v'^\gamma}{k'^{1-\gamma} h'^\gamma}\right), \\
0 &= \lambda_2\left(\gamma v^{\gamma-1} k^\gamma h^{1-\gamma} - \delta_v k\right) + \lambda_3 \frac{u_c}{1+\tau^c}\left[\delta_v - \gamma v^{\gamma-1}\left(\frac{h}{k}\right)^{1-\gamma}\right] \\
&\quad - \lambda_4\left[\gamma(\gamma-1) v^{\gamma-2}\left(\frac{h}{k}\right)^{1-\gamma} - \delta_{vv}\right] + \lambda_5 \frac{1}{1+\tau^c}(1-\gamma)\gamma v^{\gamma-1}\left(\frac{k}{h}\right)^{\gamma}, && \text{(C.7)} \\
0 &= \ell + h - 1, && \text{(C.8)} \\
0 &= c + g + k_{t+1} - (vk)^\gamma h^{1-\gamma} - (1-\delta(v))k, && \text{(C.9)} \\
0 &= -\frac{u_c}{1+\tau^c} + \beta \mathbb{E}\left(\frac{u_c'}{1+\tau^{c\prime}}\left[1-\delta(v') + v'\left(\frac{h'}{v'k'}\right)^{1-\gamma} - \frac{g'-\tau^{c\prime}c'}{k'}\right] - u_\ell' \frac{h'}{k'}\right), && \text{(C.10)} \\
0 &= \gamma\left(\frac{h}{vk}\right)^{1-\gamma} - \delta_v, && \text{(C.11)} \\
0 &\geq \frac{u_\ell}{u_c} - \frac{1}{1+\tau^c}(1-\gamma)\left(\frac{vk}{h}\right)^{\gamma}, && \text{(C.12)}
\end{aligned}
$$

$\lambda_5 \geq 0$, with complementary slackness condition.

When capital is fully utilised, we set $v = 1$ and $\lambda_4 = 0$, and ignore (C.7) and (C.11).

Appendix C.3. First-order conditions of the time-consistent policy-maker's problem

The FOCs with respect to $\tau^c, c, \ell, h, g, k', v$, respectively, are

$$0 = -\lambda_3 \frac{1}{(1+\tau^c)^2} u_c - \lambda_5 \frac{1}{(1+\tau^c)^2} (1-\gamma) \left(\frac{vk}{h}\right)^{\gamma}, \tag{C.13}$$

$$0 = u_c - \lambda_2 + \lambda_3 \frac{u_{cc}}{1+\tau^c} + \lambda_5 \frac{u_\ell}{u_c^2} u_{cc}, \tag{C.14}$$

$$0 = u_\ell - \lambda_1 - \lambda_5 \frac{u_{\ell\ell}}{u_c}, \tag{C.15}$$

$$0 = -\lambda_1 + \lambda_2 (1-\gamma) \left(\frac{vk}{h}\right)^{\gamma} - \lambda_4 \gamma (1-\gamma) h^{-\gamma} (vk)^{\gamma-1} - \lambda_5 \frac{1}{1+\tau^c} (1-\gamma)\gamma (vk)^{\gamma} h^{-\gamma-1}, \tag{C.16}$$

$$0 = u_g - \lambda_2, \tag{C.17}$$

$$\begin{aligned}
0 = & -\lambda_2 + \beta \mathbb{E} \Bigg(\Lambda_2(k') \left[\gamma \mathcal{V}(k')^{\gamma} \left(\frac{\mathcal{H}(k')}{k'}\right)^{1-\gamma} + 1 - \delta(\mathcal{V}(k')) \right] \\
& -\Lambda_4 \gamma(\gamma-1) \left(\frac{h'}{v'}\right)^{1-\gamma} k'^{\gamma-2} + \Lambda_5(k') \frac{1}{1+\mathcal{T}^c(k')} (1-\gamma)\gamma \frac{\mathcal{V}(k')^{\gamma}}{k'^{1-\gamma}\mathcal{H}(k')^{\gamma}} \Bigg) \\
& - \beta\lambda_3 \mathbb{E} \Bigg(\frac{\partial \frac{\mathcal{U}_c(k')}{1+\mathcal{T}^c(k')}}{\partial k'} \left[1 - \delta(\mathcal{V}(k')) + \frac{\mathcal{H}(k')^{1-\gamma} \mathcal{V}(k')^{\gamma} k'^{\gamma}}{k'} - \frac{\mathcal{G}(k')}{k'} + \frac{\mathcal{T}^c(k')\mathcal{C}(k')}{k'} \right] \\
& + \frac{\mathcal{U}_c(k')}{1+\mathcal{T}^c(k')} \left[-\frac{\partial \delta(\mathcal{V}(k'))}{\partial k'} + \frac{\partial \frac{\mathcal{H}(k')^{1-\gamma}\mathcal{V}(k')^{\gamma}k'^{\gamma}}{k'}}{\partial k'} - \frac{\partial \frac{\mathcal{G}(k')}{k'}}{\partial k'} + \frac{\partial \frac{\mathcal{T}^c(k')\mathcal{C}(k')}{k'}}{\partial k'} \right] - \frac{\partial \mathcal{U}_\ell(k') \frac{\mathcal{H}(k')}{k'}}{\partial k'} \Bigg),
\end{aligned}$$

where we have applied the envelop theorem,

$$\begin{aligned}
0 = & \lambda_2 \left(\gamma v^{\gamma-1} k^{\gamma} h^{1-\gamma} - \delta_v k\right) - \lambda_4 \left[\gamma(\gamma-1) \left(\frac{h}{k}\right)^{1-\gamma} v^{\gamma-2} - \delta_{vv} \right] \\
& + \lambda_5 \frac{1}{1+\tau^c} (1-\gamma) \gamma v^{\gamma-1} \left(\frac{k}{h}\right)^{\gamma}.
\end{aligned} \tag{C.18}$$

Finally, the first-order conditions with respect to $\lambda_1, \lambda_2, \lambda_3, \lambda_4$, and λ_5, respectively, are

$$0 = \ell + h - 1, \tag{C.19}$$

$$0 = c + g + k' - (vk)^{\gamma} h^{1-\gamma} - (1 - \delta(v)) k, \tag{C.20}$$

$$\begin{aligned}
0 = & -\frac{u_c}{1+\tau^c} + \beta \mathbb{E} \Bigg(\frac{\mathcal{U}_c(k')}{1+\mathcal{T}^c(k')} \Bigg[1 - \delta(\mathcal{V}(k')) + \frac{\mathcal{H}(k')^{1-\gamma}\mathcal{V}(k')^{\gamma}k'^{\gamma}}{k'} \\
& - \frac{\mathcal{G}(k') - \mathcal{T}^c(k')\mathcal{C}(k')}{k'} \Bigg] - \mathcal{U}_\ell(k') \frac{\mathcal{H}(k')}{k'} \Bigg),
\end{aligned} \tag{C.21}$$

$$0 = \gamma \left(\frac{h}{vk} \right)^{1-\gamma} - \delta_v, \tag{C.22}$$

$$0 \geq \frac{u_\ell}{u_c} - \frac{1}{1+\tau^c} (1-\gamma) \left(\frac{vk}{h} \right)^{\gamma}, \tag{C.23}$$

$\lambda_5 \geq 0$, with complementary slackness condition.

When capital is fully utilised, we set $v = 1$ and $\lambda_4 = 0$, and ignore (C.18) and (C.22).

Appendix D. Response to aggregate productivity shocks

We now study the cyclical properties of policy instruments and allocations. We are interested in whether the close similarity between Ramsey and Markov policies when consumption is taxed optimally still holds when the economy faces aggregate productivity shocks. For each policy scenario, we simulate the model and calculate sample statistics from the simulated data.[10] The results of this exercise for our baseline calibration are reported in Table D.6. We have also solved the stochastic model with $\varphi = 1$ to check the robustness of the cyclical properties of tax rates and allocations. The main features remain unchanged. The results are available upon request.

When consumption taxes are not available and the policy-maker can credibly commit, we recover the well-known labour tax smoothing result of the Ramsey literature (Chari et al., 1994). Under discretion, the policy-maker uses both capital and labour income taxes in response to unexpected productivity changes. This is due to the fact that the Markov policy-maker is less able to smooth the effects of random productivity events intertemporally, as it is less able to use the capital income tax as shock absorber, given that it relies heavily on this instrument to raise fiscal revenue. The volatility of output and hours worked are slightly higher under commitment than under discretion, while the opposite is true for private and public consumption. Finally, both tax rates are countercyclical. These patterns are, for the most part, very similar to the ones presented in Klein and Ríos-Rull (2003) and in Debortoli and Nunes (2010), although the class of economies they look at is slightly different. In particular, Klein and Ríos-Rull (2003) study a model with full capital utilisation, exogenous government spending, and a capital income tax which is determined one or more periods in advance, while Debortoli and Nunes (2010) consider a utility function with variable Frisch elasticity of labour supply.

The differences between Ramsey and Markov policies are greatly reduced when the policy-maker can tax consumption. As in the case without consumption taxation, under commitment the coefficient of variation of capital income taxes is larger than that of the alternative tax instrument, in this case, the consumption tax. However, under Markov

[10] We proceed as follows. We assume that in the initial period the system is in its stochastic steady state. We simulate the model for 1000 periods, using the same shocks across policy scenarios, and compute sample statistics. Finally, we take the median values of the sample statistics over 101 repetitions.

Table D.6. Cyclical properties of taxes and allocations

	$\tau^h \geq 0$		$\tau^c = 0$	
	Ramsey	Markov	Ramsey	Markov
	Consumption tax		Labour income tax	
Mean	0.223	0.221	0.240	0.065
Standard deviation	0.005	0.004	0.002	0.002
Coefficient of variation	0.021	0.019	0.009	0.026
Autocorrelation	0.609	0.640	0.922	0.507
Correlation with output	0.9998	0.996	-0.700	-0.853
Capital income tax				
Mean	0.000	0.004	0.000	0.198
Standard deviation	0.008	0.007	0.009	0.003
Coefficient of variation	10.807	1.699	30.693	0.017
Autocorrelation	0.609	0.638	0.463	0.504
Correlation with output	-0.9995	-0.996	-0.801	-0.909
Public spending				
Mean	0.072	0.072	0.068	0.051
Standard deviation	0.001	0.001	0.001	0.001
Coefficient of variation	0.013	0.013	0.019	0.020
Autocorrelation	0.976	0.976	0.776	0.809
Correlation with output	0.400	0.431	0.880	0.886
Public spending-income ratio				
Mean	0.146	0.146	0.146	0.117
Standard deviation	0.004	0.004	0.004	0.002
Coefficient of variation	0.027	0.026	0.026	0.020
Autocorrelation	0.521	0.524	0.514	0.504
Correlation with output	-0.894	-0.893	-0.936	-0.885
Consumption				
Mean	0.322	0.322	0.305	0.317
Standard deviation	0.004	0.004	0.005	0.005
Coefficient of variation	0.013	0.013	0.017	0.016
Autocorrelation	0.976	0.976	0.939	0.935
Correlation with output	0.400	0.431	0.621	0.675
Hours				
Mean	0.275	0.276	0.261	0.283
Standard deviation	0.005	0.005	0.007	0.006
Coefficient of variation	0.018	0.018	0.027	0.022
Autocorrelation	0.524	0.527	0.513	0.503
Correlation with output	0.859	0.856	0.936	0.895
Output				
Mean	0.492	0.492	0.467	0.439
Standard deviation	0.015	0.015	0.020	0.016
Coefficient of variation	0.030	0.030	0.042	0.036
Autocorrelation	0.603	0.607	0.570	0.597
Welfare-eq. consumption loss	0.054	0.058	0.080	0.168

policies capital taxes still play the main role in absorbing shocks, as opposed to without consumption taxation. A new feature of tax policies with consumption taxation is that the consumption tax rate is highly procyclical. The capital income tax rate remains countercyclical, but its correlation with output increases when consumption is taxed. Consumption, public spending, hours, and output all vary less with consumption taxation than without.

Finally, we compute long-run expected welfare as a percentage increase in welfare-equivalent consumption in all periods and all states in a particular policy scenario that is necessary to make the representative household as well off as at the first best.[11] The values are very similar to those for the deterministic steady state. Therefore, the business cycle results confirm the similarities between Ramsey and Markov equilibria when the policy-maker has access to consumption taxation, as well as the welfare benefits of taxing consumption.

[11] In order to do this, for some percentage increase in consumption ε, we simulate the economy over 600 periods, compute per-period utility for the last 500 periods, and finally take the average over 501 such simulations. Then we find the ε such that the average per-period utility matches the one found for the first best from similar simulations.

Appendix E. Robustness checks

Table E.7. Tax rates and allocations at steady state with alternative parameter values

Variable	unrestricted	$\tau^h \geq 0$		$\tau^c = 0$	
		Ramsey	Markov	Ramsey	Markov
$\varphi = 0.4$					
Consumption tax rate	3.245	0.223	0.221	0.000	0.000
Labour income tax rate	-3.245	0.000	0.000	0.240	0.173
Capital income tax rate	0.000	0.000	0.003	0.000	0.078
Capital	1.553	1.466	1.459	1.436	1.273
Hours worked	0.276	0.261	0.261	0.255	0.259
Income	0.494	0.466	0.465	0.457	0.439
Consumption	0.323	0.305	0.305	0.300	0.298
Public spending-income ratio	0.146	0.146	0.146	0.146	0.136
Welfare-eq. consumption loss	0.000	0.022	0.023	0.032	0.069
$\varphi = 1$					
Consumption tax rate	3.245	0.223	0.220	0.000	0.000
Labour income tax rate	-3.245	0.000	0.000	0.240	0.125
Capital income tax rate	0.000	0.000	0.005	0.000	0.133
Capital	1.648	1.490	1.480	1.437	1.181
Hours worked	0.293	0.265	0.265	0.256	0.265
Income	0.524	0.474	0.472	0.457	0.433
Consumption	0.343	0.310	0.310	0.299	0.302
Public spending-income ratio	0.146	0.146	0.146	0.146	0.128
Welfare-eq. consumption loss	0.000	0.039	0.041	0.058	0.119
$\varphi = 5$					
Consumption tax rate	3.245	0.223	0.221	0.000	0.000
Labour income tax rate	-3.245	0.000	0.000	0.240	0.042
Capital income tax rate	0.000	0.000	0.003	0.000	0.221
Capital	1.887	1.595	1.588	1.502	1.099
Hours worked	0.336	0.284	0.284	0.267	0.295
Income	0.600	0.507	0.506	0.477	0.449
Consumption	0.392	0.332	0.332	0.312	0.329
Public spending-income ratio	0.146	0.146	0.146	0.146	0.112
Welfare-eq. consumption loss	0.000	0.066	0.067	0.098	0.208

Tax rates and allocations at steady state with alternative parameter values (continued)

Variable	unrestricted	$\tau^h \geq 0$ Ramsey	$\tau^h \geq 0$ Markov	$\tau^c = 0$ Ramsey	$\tau^c = 0$ Markov
$\sigma = 2$					
Consumption tax rate	3.245	0.234	0.224	0.000	0.000
Labour income tax rate	-3.245	0.000	0.000	0.253	0.099
Capital income tax rate	0.000	0.000	0.010	0.000	0.197
Capital	1.522	1.402	1.380	1.358	1.015
Hours worked	0.271	0.249	0.250	0.242	0.259
Income	0.484	0.446	0.443	0.432	0.402
Consumption	0.317	0.289	0.289	0.279	0.282
Public spending-income ratio	0.146	0.152	0.150	0.154	0.137
Welfare-eq. consumption loss	0.000	0.031	0.035	0.046	0.147
$\beta = 0.96$					
Consumption tax rate	3.245	0.223	0.222	0.000	0.000
Labour income tax rate	-3.245	0.000	0.000	0.240	0.059
Capital income tax rate	0.000	0.000	0.002	0.000	0.202
Capital	3.352	2.882	2.872	2.719	2.039
Hours worked	0.322	0.277	0.277	0.261	0.283
Income	0.732	0.629	0.628	0.593	0.557
Consumption	0.479	0.412	0.411	0.388	0.404
Public spending-income ratio	0.146	0.146	0.146	0.146	0.115
Welfare-eq. consumption loss	0.000	0.059	0.060	0.088	0.187
$\chi = 1.8$					
Consumption tax rate	4.628	0.223	0.223	0.000	0.000
Labour income tax rate	-4.628	0.000	0.000	0.234	0.081
Capital income tax rate	0.000	0.000	0.0004	0.000	0.172
Capital	1.669	1.435	1.434	1.366	1.071
Hours worked	0.326	0.280	0.280	0.267	0.285
Income	0.583	0.502	0.502	0.478	0.452
Consumption	0.373	0.321	0.321	0.306	0.318
Public spending-income ratio	0.143	0.143	0.143	0.143	0.117
Welfare-eq. consumption loss	0.000	0.0560	0.0562	0.081	0.157
$\alpha_g = 0.3$					
Consumption tax rate	29.558	0.300	0.297	0.000	0.000
Labour income tax rate	-29.558	0.000	0.000	0.303	0.081
Capital income tax rate	0.000	0.000	0.005	0.000	0.233
Capital	1.885	1.548	1.537	1.438	1.032
Hours worked	0.335	0.275	0.276	0.256	0.284
Income	0.599	0.492	0.491	0.457	0.428
Consumption	0.369	0.303	0.303	0.281	0.302
Public spending-income ratio	0.185	0.185	0.185	0.185	0.141
Welfare-eq. consumption loss	0.000	0.088	0.091	0.135	0.264

Table E.8. Tax rates and allocations at steady state with exogenous government spending

Variable	unrestricted	$\tau^h \geq 0$ Ramsey	$\tau^h \geq 0$ Markov	$\tau^c = 0$ Ramsey	$\tau^c = 0$ Markov
Consumption tax rate	1.179	0.167	0.165	0.000	0.000
Labour income tax rate	-1.179	0.000	0.000	0.240	0.078
Capital income tax rate	0.000	0.000	0.003	0.000	0.211
Capital	1.718	1.548	1.541	1.467	1.077
Hours worked	0.306	0.275	0.276	0.261	0.283
Income	0.546	0.492	0.491	0.466	0.434
Consumption	0.381	0.337	0.337	0.305	0.309
Public spending-income ratio	0.103	0.115	0.115	0.121	0.130
Welfare-eq. consumption loss	0.000	0.036	0.037	0.039	0.088