Various soliton molecules in fiber systems

LEI LI,[1] HAITAO HUANG,[1] LEI SU,[2] DEYUAN SHEN,[1] DINGYUAN TANG,[1]
MARIUSZ KLIMCZAK,[3] LUMING ZHAO, [1,2,*]

[1]Jiangsu Key Laboratory of Advanced Laser Materials and Devices, Jiangsu Collaborative Innovation Center of Advanced Laser Technology and Emerging Industry, School of Physics and Electronic Engineering, Jiangsu Normal University, Xuzhou, Jiangsu 221116 China
[2] School of Engineering and Materials Science, Queen Mary University of London, London, UK
[3] Institute of Electronic Materials Technology in Warsaw, Poland
**lmzhao@ieee.org*

Abstract: Generation and propagation of various soliton molecules (SMs) in fiber system are reviewed. SMs can survive either in fibers or fiber lasers. Dispersion-managed (DM) fiber link is the only platform for SMs demonstration while various fiber lasers can support different SMs. The fundamental unit of SMs can be conventional solitons generated in anomalous dispersion regime, stretched pulses in DM fiber lasers, parabolic pulses or gain-guided solitons in normal dispersion regime. SMs with typically close soliton separation are presented. In addition, we demonstrate a new kind of SMs with nanosecond soliton separation. The narrow spectral filtering is required for the generation of SMs with such long distance interaction.

I. Introduction

Solitons refer to pulses propagating without distortion in media. It is the product of balanced diffraction and nonlinearity in space, as well as the consequence of equivalent influence between the dispersive effect and the nonlinear effect in time, which is also known as a “spatial-temporal soliton”. In fiber optics, solitons generally known as ultrashort pulses generates in fiber systems: fibers or fiber lasers, which support “temporal solitons” only. Solitons are natural binary information units where “1” stands for soliton and “0” for no soliton. Therefore, one of the most promising applications of solitons is telecommunications and lots of theoretical work and experimental demonstrations in lab have been done long time ago. For example, soliton propagation beyond 4000 km was demonstrated in 1988 [1]. However, so far there have been no practical applications of solitons in telecommunications. The wavelength-division multiplexing (WDM) technique in telecommunications is due to exhausting its capacity because of the Shannon limit [2]. This is a major challenge, because current telecommunications is based on optical fibers and on binary encoding.

Although solitons are lagged behind WDM in practical applications in telecommunications, theoretical work to improve the capability of information transmission systems based on solitons is under development. One of them is the coding with an ‘‘alphabet’’ of more than two letters [3], where a soliton represents a letter. Stable bound states of two solitons or multiple solitons are needed for the realization of the coding scheme. M. Stratmann et al. first demonstrated a bound state of temporal solitons in optical fibers [3]. The structure exists only in a dispersion-managed (DM) fiber link and it is actually a pair of bright solitons bound by a dark soliton. The two bright solitons are in antiphase. It is found that the separation between the two solitons will be maintained in an equilibrium distance, which is reminiscent of the equilibrium separation of the two constituents of a diatomic molecule. Therefore, the structure is also called a soliton molecule (SM). Further study revealed that coding with solitons and SMs allows encoding two bits of data per clock period [4, 5]. A three-pulse SM is demonstrated, which completes an alphabet of four different symbols: no soliton, single soliton, two-pulse SM, and a three-pulse SM. In addition, as SMs have a certain phase structure and can survive in two orthogonal states of polarization in a fiber link, all presently developing schemes of phase modulation and/or polarization multiplexing can be potentially combined with the SM encoding approach, of course, with the sacrifice of increased complexity. Abdelâali Boudjemâa et al. studied the stability of N-soliton molecules in DM optical fibers [6]. They found that N-soliton molecules with N=4 can survive in the fiber link, but the binding energy per soliton is saturated at N⩾7 under specific parameter sets. Consequently, multipulse SMs with soliton number larger than 3 make the further scaling up of fiber's data-carrying capacity possible [7].

Fiber lasers are a very different system compared with fibers, where in general the former is a dissipative system while the latter could be considered as a Hamilton system. However, if the loss in a fiber system is significant or any amplifier(s) is used in a fiber system, then fibers could also be represented by a dissipative system. Ultrashort pulses can be generated in fiber lasers considering the balance between the dispersion and the nonlinearity, apart from the gain and loss balance, as well as other boundary conditions. In a stable state, in a fixed position in the fiber laser, the generated ultrashort pulse maintains its profile, hence, the ultrashort pulses generated in fiber lasers can be considered as solitons. Theoretically ultrashort pulse generation in fiber lasers can be described by the Ginzburg-Landau equation and its variants, which under certain hypothesis can be simplified to the nonlinear Schrödinger equation. Therefore, solitons generated in fiber lasers have similar properties compared with solitons generated in fibers. Equation (1) shows an example of coupled Ginzburg-landau equations [8], where the dispersion effect, nonlinear effect, gain and gain dispersion effect are included, where *u* and *v* are the complex optical envelopes in orthogonal polarization mode along optical fiber. k'' is the second order dispersion coefficient, k''' is the third order dispersion coefficient, and γ represents the nonlinearity of the fiber. g is the saturable gain coefficient of the gain fiber and $\boldsymbol{\Omega}_g$ is the gain bandwidth.

$$\begin{cases} \dfrac{\partial u}{\partial z} = i\beta u - \delta \dfrac{\partial u}{\partial t} - \dfrac{ik''}{2}\dfrac{\partial^2 u}{\partial t^2} + \dfrac{ik'''}{6}\dfrac{\partial^3 u}{\partial t^3} + i\gamma(|u|^2 + \dfrac{2}{3}|v|^2)u + \dfrac{i\gamma}{3}v^2 u^* + \dfrac{g}{2}u + \dfrac{g}{2\Omega_g{}^2}\dfrac{\partial^2 u}{\partial t^2} \\ \dfrac{\partial v}{\partial z} = -i\beta v + \delta \dfrac{\partial v}{\partial t} - \dfrac{ik''}{2}\dfrac{\partial^2 v}{\partial t^2} + \dfrac{ik'''}{6}\dfrac{\partial^3 v}{\partial t^3} + i\gamma(|v|^2 + \dfrac{2}{3}|u|^2)v + \dfrac{i\gamma}{3}u^2 v^* + \dfrac{g}{2}v + \dfrac{g}{2\Omega_g{}^2}\dfrac{\partial^2 v}{\partial t^2} \end{cases} \qquad (1)$$

The observation of SMs in fiber lasers is actually before the discovery of SMs in fibers. SMs are first observed in fiber lasers [9]. It is soon demonstrated that the solitons are phase-locked [10, 11]. SMs with soliton number larger than 2 are obtained independently by different groups [12-14]. Phase relationship between solitons in SMs are studied [15, 16]. Bound state of SMs are reported [17]. Different from that SMs exist in DM fiber link only, SMs could be observed in fiber lasers operated in various dispersion regimes, even when the fundamental units of the SMs are intrinsically different: such as traditional solitons generated in the anomalous dispersion regime [9, 11, 17], dispersion-managed solitons [10, 13-16, 18-24], pulses generated in the normal dispersion regime [12, 25-27]. Apart from the stable SMs, vibrating SMs were observed with the temporal separation and the phase relationship oscillating with a period much larger than the cavity round-trip time [28]. Provided that the vibration cycle is stable, the vibrational motion has a direct influence on the averaged optical spectra (loss of contrast in the spectral fringes) and autocorrelation traces (reduction of the amplitude of the side peaks, with a broadening of the side peaks) which can be recorded. SMs with restless internal soliton movement were reported [29, 30], where fundamental unit is the vector soliton. Very recently, a real-time probe of the internal dynamics of femtosecond SMs was reported [31]. Using the time-stretch dispersive Fourier transform [32], the formation of stable SMs and rapid internal motions for a diverse set of bound states were uncovered. Sub-femtosecond precision relative timing jitter characterization between two solitons composing a SM is enabled by the balanced optical cross-correlation method [33]. Even for a stationary SM generated from a fiber laser, there actually exists a tiny vibration between the solitons of the SM. Lili Gui et al. reported bound solitons with various phase differences and pulse separations in a net-anomalous-dispersion erbium fiber laser mode-locked by carbon nanotubes, including bound states with phase differences close to 0, $\pm\pi/2$, and π [34].

In this paper, we summarize the SMs observed in different fiber lasers and their properties. We note that SMs included in this overview refer to soliton bunches, which can function similarly to a single-pulse soliton. In Section II we recall the theoretical demonstration of SM generation in fiber systems and especially in fiber lasers. We describe SM generation in fiber lasers operated in anomalous dispersion regime in Section III. In the following section we explain the SMs consisting of DM soliton in fiber lasers. The SMs based on parabolic pulses in a normal dispersion fiber and gain-guided solitons in large normal dispersion regime, respectively, are introduced in Section V. Finally we show a new kind of SMs with nanosecond soliton separation. A narrow spectral filtering is required for such SM generation. Section VII summarizes our main conclusion.

II. Theoretical demonstration of SM generation

Boris A. Malomed predicted that two-pulse and multipulse bound states could be stable under the frame of perturbed cubic nonlinear Schrödinger equation or the quintic Ginzburg-Landau equation [35]. By studying the interaction between slightly overlapped solitons, it was found that two-pulse and multipulse bound states with weak stability could be generated. The existence and stability of two-soliton bound states were reported in ref [36], where the interaction between weakly overlapping pulses described both in the quintic Ginzburg-Landau equation and in the driven damped nonlinear Schrödinger equation are studied. Bound states of the pulses correspond to fixed points of the dynamical system. It is found that all the fixed points in the quintic model are unstable. However, a special type of fixed point, spirals, has an extremely weak instability and may be treated as a stable soliton pair. For the driven damped model, the existence of fully stable bound states is demonstrated, provided that the amplitude of the driving field exceeds a very low threshold. N. N. Akhmediev et al. analyzed stable soliton pairs transmission in an optical fiber and found that the distance between the solitons and the phase difference between them are defined by energy and momentum balance equations, rather than by equations of standard perturbation theory [37]. The stable soliton pairs are featured with a $\pi/2$ phase difference between solitons. For multipulse bound state, the phase difference could bias from $\pi/2$.

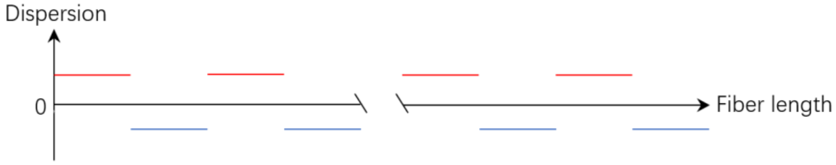

Fig. 1 Schematic of a DM fiber link.

When a pulse propagates in fibers, the anomalous group-velocity dispersion supports bright solitons while dark solitons can survive in normal dispersion fibers. By combined fiber segments with opposite dispersion, a DM fiber link as shown in Fig. 1 could be set up. By appropriate choice of the individual fiber length, the total dispersion of the fiber link can be selected to be anomalous, zero, or normal. The zero total dispersion is preferred in principle for telecommunications. Stratmann et al. numerically found that, in a DM fiber link with appropriate dispersion, there exists a stable bound state where two bright solitons are bound to each other with a dark soliton separating them in the time domain [38]. The bright solitons are in antiphase, and they sit at a certain temporal separation from each other. Diverged from the equilibrium distance will be rectified during propagation. It has also been found that the bound state is unstable in constant dispersion fibers, but only exists in DM fiber links.

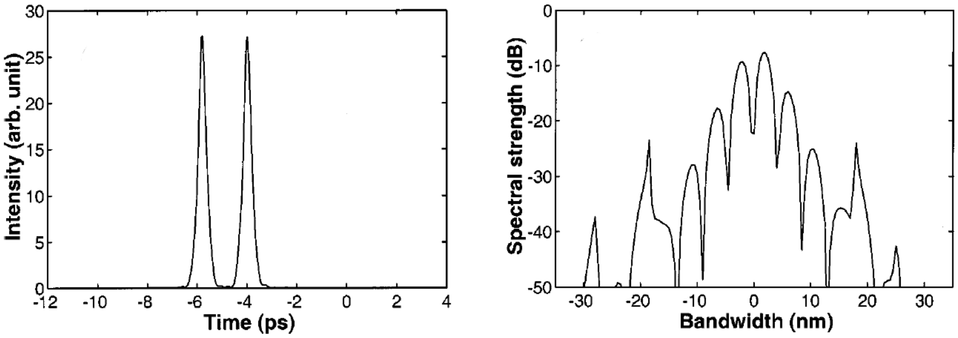

Fig. 2 Numerically obtained bound solitons (left: pulse profile; right: pulse spectrum) [11].

D. Y. Tang et al. [11] demonstrated in a fiber laser both experimentally and numerically that two solitons binding together tightly with fixed pulse separation can function as a fundamental unit as that of a single soliton, as shown in Fig. 2. Considering that the pulse propagation in fibers is governed by the coupled Ginzburg-Landau equations, stable bound solitons with different pulse separations could be obtained under appropriate parameter settings, independent of the initial conditions. Based on the same fiber laser, D. Y. Tang et al. further studied the soliton interaction [39], identifying three types: a global type of soliton interaction caused by the existence of unstable continuous wave components, a local type of soliton interaction mediated through the radiative dispersive waves, and the direct soliton interaction. The last one results in the generation of bound solitons with close temporal separation. Based on the same theoretical model but moved to the DM fiber laser, we numerically reproduced the experimentally observed bound states of DM solitons in a DM fiber laser with near zero group-velocity dispersion [19]. As shown in Fig. 3, the modulated spectrum has a Gaussian contour without Kelly sidebands [40]. The phase difference between the two pulse peaks is about 12.5π. In addition, bound states of SMs are numerically demonstrated as shown in Fig. 4. Depending on the operation condition, different SM separations could be achieved. Very recently, the role of third-order dispersion on two- and multi-soliton bound states, both stationary and oscillatory one, is studied in a fiber laser near the zero-dispersion point, based on the complex Ginzburg–Landau equation with the cubic-quintic nonlinearity term [41]. Several specific families of robust bound states of solitons were obtained. The stationary bound states are featured with constant soliton separation while the dynamical bound states are characterized with oscillating soliton separation. Further investigation using a different fiber laser with large normal dispersion [26] suggested that bound state of gain-guided solitons could be generated due to the direct soliton-soliton interaction, as shown in Fig. 5, where the pulse profiles should be overlapped somewhere during evolution in the cavity.

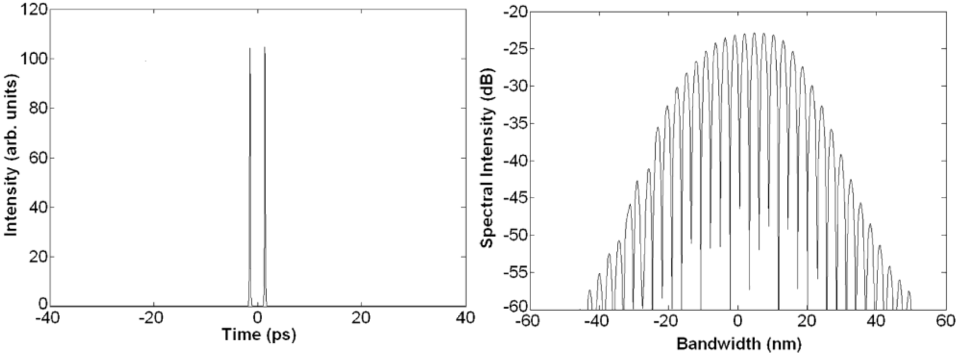

Fig. 3 Numerically obtained bound solitons in a DM fiber laser (left: pulse profile; right: pulse spectrum) [19].

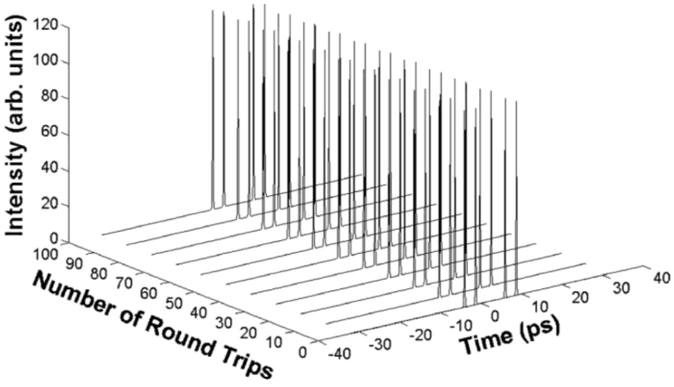

Fig. 4 Numerically obtained bound state of DM SMs [19].

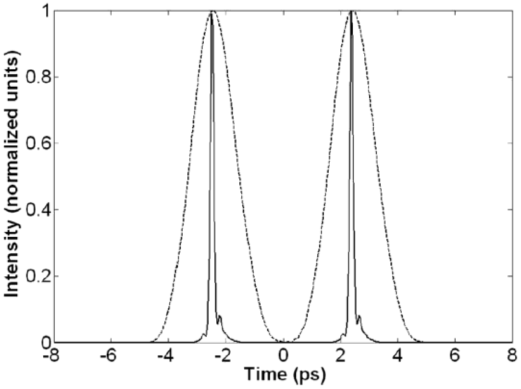

Fig. 5 Numerically obtained bound state of two gain-guided solitons at the cavity positions with the minimum and maximum pulse width [26].

Soliton bunches with 3 solitons were numerically constructed to explain the missing spectral modulation in an experimental observation of passive harmonic mode locking of bunches of single-pulse solitons or twin-pulse solitons. It is found that the large number of solitons in a bunch, which blurs the extra spectral modulation, may cause the missing spectral modulation; on the other hand, it may result from the specific phase relationship between the solitons in the bunch [20]. By taking into account the Lyot filter effect due to the strong birefringence, we demonstrated numerically the ultra-high-repetition-rate bound-solitons, which is due to the joint effect of the dissipative four-wave-mixing effect [42] and the modulation instability process [43, 44] occurring in the laser. As shown in Fig. 6, a stable SM pulse train with repetition rate of ~125 GHz could be achieved by increasing the pump power after a stable single-soliton pulse train is obtained.

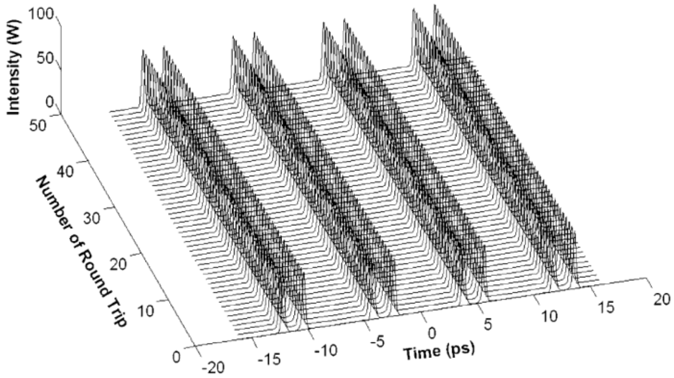

Fig. 6 Numerically obtained ultra-high-repetition-rate bound soliton train [21].

The abovementioned SMs in fiber lasers are all based on scalar solitons as there are polarization dependent components in the fiber lasers. A vector soliton in fibers refers to a soliton having multiple polarized components propagates as a unit [45, 46]. Recently we numerically reproduced bound states of vector dissipative solitons (VDSs) in an erbium-doped fiber laser mode locked with a semiconductor saturable absorber mirror and operated in the normal dispersion regime [22]. The fundamental unit of the bound states could be either coherently coupled VDSs or incoherently coupled VDSs. The soliton separation is fixed and invariant to operation condition changes as long as the VDSs survived. In addition, bound states of dark solitons were analyzed in the quintic Ginzburg-Landau equation [47]. In particular, the bound states exist in a wide range of parameters and are highly stable. Based on the complex Ginzburg-landau equation, bound states of one-, two-, and three-dimensional solitons [48] as well as cavity SMs [49] were demonstrated.

III SMs of traditional solitons in fiber lasers

Traditional solitons are generated in fiber lasers of anomalous dispersion. Due the balanced interaction between the dispersion effect and the nonlinear Kerr effect imposed on pulses during propagation, pulses can propagate without distortion. One of the intrinsic features of traditional solitons is the appearance of Kelly sidebands [40]. Similar spectral sidebands were predicted by BorisA. Malomed independently [50].

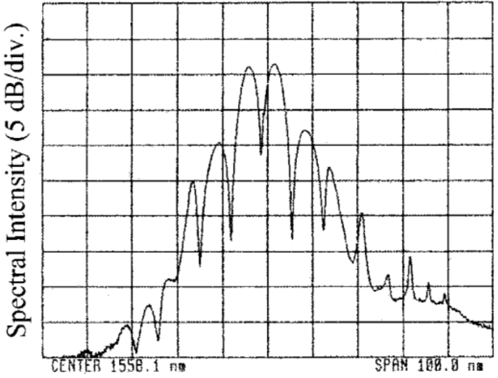

Fig. 7 Typical spectrum of a bound state of solitons [9].

D. Y. Tang et al. used an in-house fiber laser with anomalous dispersion and confirmed experimentally the existence of stable bound states of solitons with discrete, fixed soliton separations [9]. Figure 7 shows a typical spectrum of a bound state of solitons observed. The spectral modulation has almost symmetric structure with a dip in the center, which indicates that the phase difference between the bound solitons is roughly π as predicted [35]. Further experimental observation [11] suggests that the bound state of solitons can have different soliton separations. Similar to single-pulse soliton, the bound state of solitons can function as a unit and form another bound state or a multiply pulsing state. The interaction between the bound-soliton pairs clearly exhibits the particle-like nature of the soliton interaction. Three bound soliton pairs with exactly same soliton separation were observed as shown in Fig. 8 [9]. The newly appearance/disappearance of the fundamental unit with pump power is the bound-soliton pair, not the individual soliton more. Numerical simulation reproduced all the experimental details. Detailed exploration on soliton interaction [39] suggests the long-distance interaction between solitons, which explains the existence of bound solitons with a soliton separation that is larger than 5 times of the soliton pulse width.

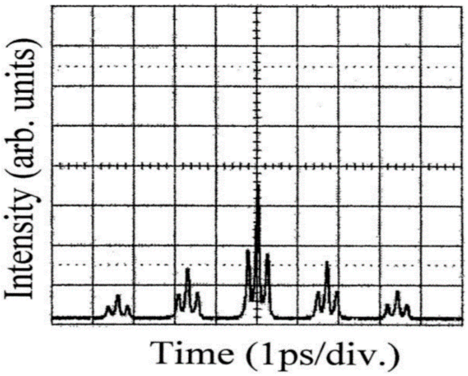

Fig. 8 A typical autocorrelation trace showing the state of 3 bound-soliton pairs [9].

Ⅳ SMs of DM solitons in fiber lasers

Grelu et al. reported phase-locked SMs in a DM fiber laser [10]. Differently from what D. Y. Tang et al. observed and reported in [9], the observed stable pulse pairs were with a ±π/2 phase difference.

Apart from the twin-pulse SMs, we present experimental evidence of multipulse bound solitons with fixed pulse separations [14]. SMs consisting of 3 solitons and 4 solitons are observed in a DM fiber laser. In addition, the SMs indeed function as a unit to form the further bound states of SMs. Example of bound states of twin-pulse and three-pulse SMs are shown in Fig. 9.

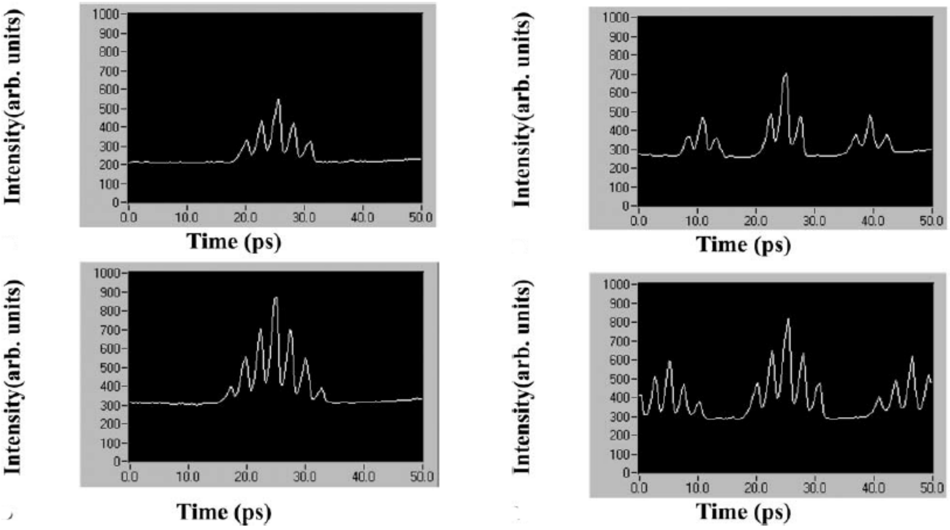

Fig. 9 Autocorrelation trace of various SMs (top-left: three-pulse SM; bottom-left: 4-pulse SM) and the bound state of SMs (top-right: bound state of twin-pulse SMs; bottom-right: bound state of three-pulse SMs) [14].

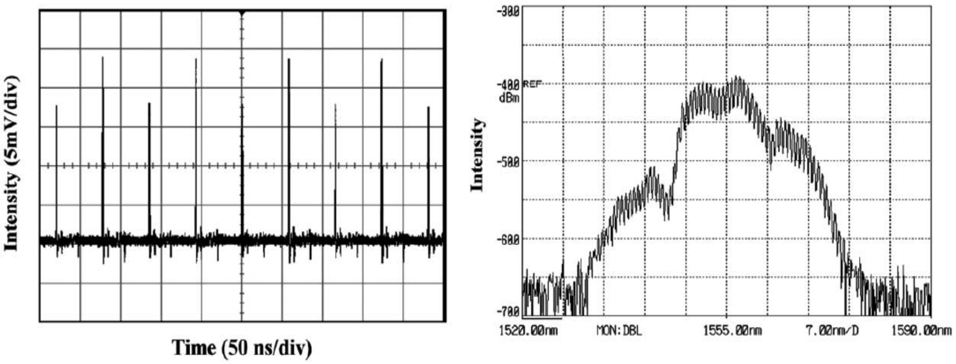

Fig. 10 Oscilloscope trace and corresponding spectrum of a period-doubling pulse train based on SMs [18].

SMs of DM solitons can exhibit other properties inherent to single-pulse solitons in DM fiber lasers. We experimentally observed period-doubled, period-quadrupled, and chaotic states of SMs in a DM fiber laser [18]. The generated bound state of solitons, or SMs, indeed function as an entity to exhibit complicated nonlinear dynamics, such as the period-doubling route to chaos. In figure 10, we show for example a period-doubling pulse train based on SMs [18]. Limited by the resolution of the measurement system, the detailed intensity variations of each of the solitons under period doubling bifurcations could not be resolved. There are three possible ways for a two-pulse bound soliton exhibiting a period-doubling intensity pattern. The two solitons simultaneously experience the period-doubling in phase, one soliton experiences the period-doubling while the other one remains stable, or two solitons simultaneously experience the period-doubling out-of-phase, but the total intensity shows the period-doubling.

Ⅴ SMs based on pulses generated in normal dispersion fiber lasers

Bright solitons can be generated in fiber lasers with normal dispersion. By carefully designing a fiber laser, Ilday et al. theoretically and experimentally demonstrated self-similar evolution of parabolic pulses in most of the fiber laser [51]. Pulse breathing happens only once even it is indeed a DM fiber laser. Self-similar propagation of intense pulses will be terminated if any bandwidth limitation is imposed [52]. Ortaç et al. reported the observation of self-similar propagation of bound state pulses in an ytterbium-doped double-clad fiber laser [25]. In addition, a triplet of parabolic pulses with different time separations was observed. All the pulses in the bound states can be extra-cavity compressed to ~100 fs.

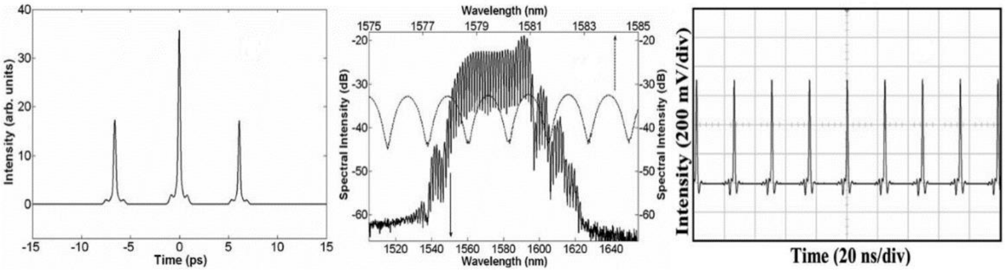

Fig. 11 A typical bound state of gain-guided solitons: left: autocorrelation trace; middle: optical spectrum; right: pulse train [26].

Dissipative solitons (DSs) could be generated in fiber lasers with large normal dispersion, where spectral filtering is required [53]. In erbium-doped fiber lasers, as the gain fiber itself can provide gain bandwidth limitation equivalent to spectral filtering, no practical spectral filtering component is required. Consequently, the generated DSs are called as "gain-guided solitons" [54, 55]. For ytterbium-doped fiber lasers, as the gain fiber has broad bandwidth, a real spectral filter with narrow bandwidth is generally required [56]. The typical feature of a DS is the steep spectral edge(s) [54-56]. We observed bound states of gain-guided solitons in a

DM fiber laser with larger normal dispersion [26]. Figure 11 shows a typical bound state of gain-guided solitons. The ratio between the pulse separation and the pulse width is 34.4, far larger than the setting criterion of direct soliton-soliton interaction [39]. Numerical simulation demonstrated that the bound state is still the consequence of direct soliton-soliton interaction as the pulse is substantially broadened at certain position in the cavity due to the pulse breath. Therefore, the ratio between the pulse separation and the pulse width could be less than 5 during evolution in cavity, which is small enough for the appearance of direct soliton-soliton interaction.

Ⅵ SMs with nanosecond soliton separation

Depending on the soliton separation, different soliton interactions are dominant. Consequently, different soliton dynamics are expected. The soliton interactions can be classified into three types [39]. The bound states of close separated solitons stem from direct soliton-soliton interaction due to overlapping of the pulse tails. To facilitate the classification, we proposed a judgement of 5 times of the pulse duration for identifying the direct soliton-soliton interaction [39]. For tightly bound states [9-11, 15-16, 19, 25, 26] with the ratio between the soliton separation and soliton duration is less than 5, the direct soliton-soliton interaction is particularly strong. With that, the bound state itself can function as a unit to form further complicated dynamics, intrinsic to a single-pulse soliton, for example such as a bound state of a bound state of two solitons [16]. For loosely bound states [18, 20, 25] with the ratio larger than 5, the bound state can still function as a unit, for example, exhibiting period doubling and forming harmonic mode locking. However, the internal structure of the loosely bound states can change with the operation conditions, for example, the soliton separation or even the soliton number will change, which is similar to a SM with different molecule energy or a new SM generation.

So far, all the above-mentioned studies show SMs with small internal soliton separations. In this section, we report on the observation of another type of SMs in an all-normal-dispersion fiber laser. Different from the general SMs, the soliton separation is beyond nanosecond. SMs with the same soliton separation but with different soliton numbers are observed. We suspect that the narrow spectral filtering is the reason for the generation of this new kind of SMs.

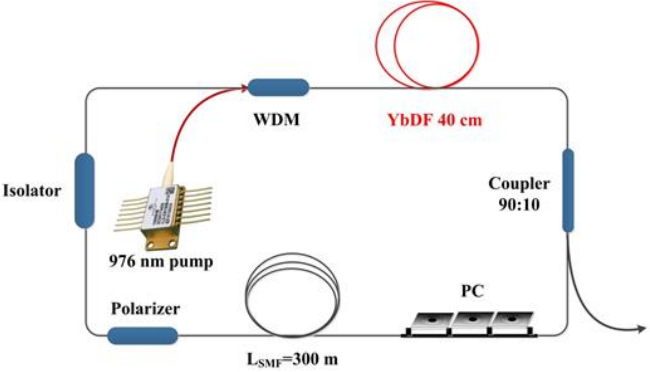

Fig. 12 Schematic of the laser setup. WDM: wavelength division multiplexer; YbDF: ytterbium-doped fiber; PC: polarization controller.

The fiber laser is shown in Fig. 12 with a ring cavity of about 317 m. The gain is provided by a 40 cm long ytterbium-doped fiber (YbDF), co-pumped by a 976 nm laser diode. All the other fiber inside the cavity is 1060XP single mode fiber. The nonlinear polarization rotation technique is used for achieving mode locking with the help of a fiber-type polarizer and a polarization controller (PC). An output coupler provides the output ratio of 10%. A polarization-independent isolator ensures unidirectional operation.

When the pump power is increased to a specific threshold, multiple solitons are automatically obtained if the PC is appropriately set. Different from the general multiple soliton states, those include multiple solitons with randomly irregular spacing, harmonic mode locking, SMs with close soliton separation, and the generated multiple solitons are always assembled together as a bunch with a large soliton separation. As shown in an example in Fig. 13(a), the bunch is comprised of 11 solitons with same soliton separation of about 7 ns. Figure 13(b), (c), and (d) show, respectively, the oscilloscope trace of the SM pulse train, the corresponding optical spectrum, and the RF spectrum. The period of the fiber laser from Fig. 13(b) is about 1.56 μs, while the fundamental repetition frequency from Fig. 13(d) is about 639 kHz. Both values agree with the cavity length. The steep spectral edges clearly suggest that the generated pulse is a DS. The edge-to-edge bandwidth is about 0.95 nm while the 3-dB bandwidth is about 0.6 nm.

The pulse shaping is caused by the invisible birefringence filter resulting from the polarizer and the cavity birefringence [57]. Assuming the average fiber birefringence is about 1 m, the effective bandwidth of the birefringence filter would be about 1.67 nm, which qualitatively matches the 0.95 nm edge-to-edge bandwidth of the generated DS. We note that for the specific PC setting, when we change the pump power, a new DS will abruptly appear/disappear with the same soliton separation instead of destroying the bunch. The structure is stable for pump powers from 220 mW to 700 mW. Pump powers outside of this range do not support mode locking. Due to the large normal dispersion, the generated DS is a heavily chirped pulse. The calculated transform-limited pulse duration is about 2.7 ps if a Gaussian pulse profile is assumed.

The narrow spectral filtering not only supports generation of DS but also excludes the closing binding of DSs. The nominal 1.67-nm bandwidth cannot support pulse separation shorter than 2 ps. Moreover, an all-normal dispersion cavity can only support heavily chirped pulses, consequently DSs with long pulse separation are expected. We currently are not sure about the factor which determines the ~7 ns pulse separation. We found that the determined pulse separation depends on the PC setting. Slightly changing the orientation of the PC would change the pulse separation, but would still maintain the SM feature – increasing/decreasing pump power only changes the soliton number with fixed soliton separation.

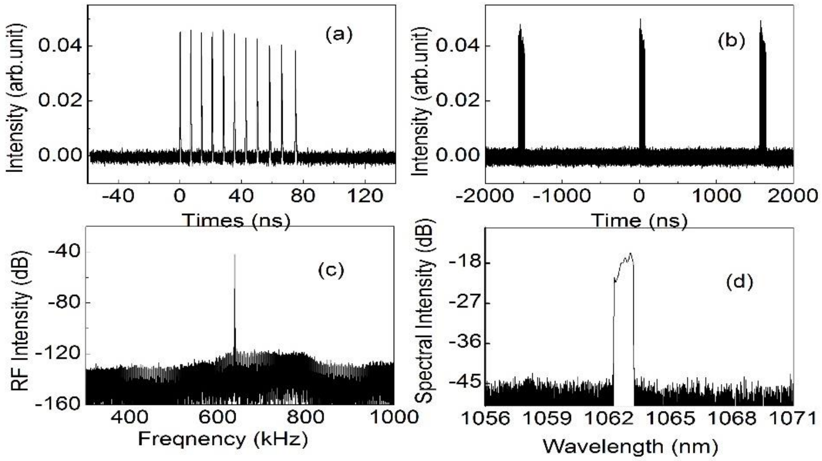

Fig. 13 (a) Temporal profile of a SM of 11 solitons with equal soliton separation; (b) Oscilloscope trace of pulse train of the SM; (c) Optical spectrum of the SM; (d) RF spectrum of the pulse train.

Ⅶ Conclusions

As it is well-known, two or several atoms can form a molecule under certain conditions. A molecule is similar to an atom in the sense that both of them could be the fundamental unit of a more complicated ensemble, analogically to a material. A single-pulse soliton can function as a fundamental unit to form all kinds of states: bound states, harmonic states etc. SMs are special soliton compounds which can also function as a fundamental unit. We have summarized various SMs in fiber lasers comprised of different fundamental pulses: the traditional solitons generated in anomalous dispersion fiber lasers; DMs; parabolic pulses and gain-guided solitons in normal dispersion fiber lasers. All the SMs observed can form further structure or exhibit properties inherent to a single-pulse soliton. A new kind of SMs with nanosecond pulse separation is reported. The narrow spectral filter is required for such SM generation.

SMs exist for all common mode-locking regimes in fiber lasers. They have plenty of features that are useful for both the fundamental research and real-life applications, with a disruptive potential similar to the one which the soliton has brought to ultrafast sciences. An indispensable prerequisite for this potential to realize is the knowledge and understanding of the complex dynamics relevant to formation, existence and dissolving of SMs. We intended this work to provide an overview on the current state of art, and thus to facilitate identification of attractive starting points for future discovery in the field of bound states of solitons.

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References

1. L. F. Mollenauer and K. Smith, "Demonstration of soliton transmission over more than 4000 km in fiber with loss periodically compensated by Raman gain," Opt. Lett. **13**(8), 675-677 (1988).
2. C. E. Shannon, "A Mathematical Theory of Communication," Bell Syst. Tech. J. **27**(3), 379–423 (1948).
3. Stratmann M, Pagel T and Mitschke F, "Experimental observation of temporal soliton molecules," Phys. Rev. Lett. **95**(14), 143902-1-5 (2005).
4. Philipp R, Alexander H and Fedor M, "Solitons beyond binary: possibility of fibre-optic transmission of two bits per clock period," Sci. Rep. **2**(11), 866-1-4 (2012).
5. P. Rohrmann, A. Hause, and F. Mitschke, "Two-soliton and three-soliton molecules in optical fibers," Phys. Rev. A **87**, 043834-1-10 (2013).
6. Abdelâali Boudjemâa and U. Al Khawaja, "Stability of N-soliton molecules in dispersion-managed optical fibers," Phys. Rev. A **88**, 045801-1-4 (2013).
7. Abdelâali Boudjemâa, "Many-soliton bound states in dispersion-managed optical fiber: Possibility of fiber-optic transmission of three bits per clock period," Int. J. Mod. Phys. B **31**(26), 1750178-1-10 (2017).
8. D. Y. Tang, B. Zhao, D. Y. Shen, C. Lu, W. S. Man and H. Y. Tam, "Compound pulse solitons in a fiber ring laser," Phys. Rev. A. **68**, 013816-1-8 (2003).
9. D. Y. Tang, P. D. Drummond and W. S. Man, "Observation of bound states of solitons in a passively mode-locked fiber laser," Phys. Rev. A **64**, 033814-1-3 (2001).
10. Ph. Grelu, F. Belhache, F. Gutty and J.-M. Soto-Crespo, "Phase-locked soliton pairs in a stretched-pulse fiber laser," Opt. Lett. **27**(11), 966-968 (2002).
11. D. Y. Tang, B. Zhao, D. Y. Shen, C. Lu, W. S. Man and H. Y. Tam, "Bound-soliton fiber laser," Phys. Rev. A **66**, 033806-1-6 (2002).
12. B. Ortac, A. Hideur, T. Chartier, M. Brunel, P. Grelu, H. Leblond and F. Sanchez, "Generation of bound states of three ultrashort pulses with a passively mode-locked high-power Yb-doped double-clad fiber laser," IEEE Photon. Technol. Lett. **16**(5), 1274-1276 (2004).
13. P. Grelu and J.M. Soto-Crespo, "Multisoliton states and pulse fragmentation in a passively mode-locked fibre laser," J. Opt. B: Quantum Semiclass. Opt. **6**(5), 271-278 (2004).
14. D. Y. Tang, L. M. Zhao and B. Zhao, "Multipulse bound solitons with fixed pulse separations formed by direct soliton interaction," Appl. Phys. B-Lasers O 80(2), 239-242 (2005).
15. P. Grelu, F. Belhache, F. Gutty and J.M. Soto-Crespo, "Relative phase locking of pulses in a passively mode-locked fiber laser," J. Opt. Soc. Am. B **20**(5), 863-870 (2003).
16. N.H. Seong and D.Y. Kim, "Experimental observation of stable bound solitons in a figure-eight fiber laser," Opt. Lett. **27**(15), 1321-1323 (2002).

17. B. Zhao, D. Y. Tang, P. Shum, X. Guo, C. Lu and H. Y. Tam, “Bound twin-pulse solitons in a fiber ring laser,” Phys. Rev. E **70**, 067602-1-4 (2004).
18. L. M. Zhao, D. Y. Tang and B. Zhao, “Period-doubling and quadrupling of bound solitons in a passively mode-locked fiber laser,” Opt. Commun. **252**(1-3), 167-172 (2005).
19. L. M. Zhao, D. Y. Tang, T. H. Cheng, H. Y. Tam and C. Lu, “Bound states of dispersion-managed solitons in a fiber laser at near zero dispersion,” Appl. Opt. **46**(21), 4768-4773 (2007).
20. L. M. Zhao, D. Y. Tang, T. H. Cheng, C. Lu, H. Y. Tam, X. Q. Fu and S. C. Wen, “Passive harmonic mode locking of soliton bunches in a fiber ring laser,” Opt. Quant. Electron. **40**(13), 1053-1064 (2008).
21. L. M. Zhao, D. Y. Tang and D. Liu, “Ultrahigh-repetition-rate bound-soliton fiber laser,” Appl. Phys. B-Lasers O **99**(3), 441-447 (2010).
22. L. M. Zhao, D. Y. Tang, H. Zhang and X. Wu, “Bound states of vector dissipative solitons,” IEEE Photon. J. **7**(6), 7802208 (2015).
23. L. Li, Q. J. Ruan, R. H. Yang, L. M. Zhao and Z. Q. Luo, “Bidirectional operation of 100 fs bound solitons in an ultra-compact mode-locked fiber laser”, Opt. Express **24**(18), 21020-21026 (2016).
24. Yiyang Luo, Jianwei Cheng, Bowen Liu, Qizhen Sun, Lei Li, Songnian Fu, Dingyuan Tang, Luming Zhao and Deming Liu, “Group-velocity-locked vector soliton molecules in fiber lasers,” Sci. Rep. **7**, 2369-1-8 (2017).
25. B. Ortaç, A. Hideur, M. Brunel, C. Chédot, J. Limpert, A. Tünnermann and F. Ö. Ilday, “Generation of parabolic bound pulses from a Yb-fiber laser,” Opt. Express **14**(13), 6075-6083 (2006).
26. L. M. Zhao, D. Y. Tang, X. Wu, D. J. Lei and S. C. Wen, “Bound states of gain-guided solitons in a passively mode-locked fiber laser,” Opt. Lett. **32**(21), 3191-3193 (2007).
27. Junsong Peng, Li Zhan, Shouyu Luo, Qishun S. Shen, “Generation of soliton molecules in a normal-dispersion fiber laser,” IEEE Photon. Technol. Lett. **25**(10), 948-951 (2013).
28. Mélanie Grapinet and Philippe Grelu, “Vibrating soliton pairs in a mode-locked laser cavity,” Opt. Lett. **31**(14), 2115-2117 (2006).
29. L. M. Zhao, D. Y. Tang, H. Zhang and X. Wu, “Bunch of restless vector solitons in a fiber laser with SESAM,” Opt. Express **17**(10), 8103-8108 (2009).
30. Regina Gumenyuk and Oleg G. Okhotnikov, “Temporal control of vector soliton bunching by slow/fast saturable absorption,” J. Opt. Soc. Am. B **29**(1), 1-7 (2012).
31. G. Herink, F. Kurtz, B. Jalali, D. R. Solli and C. Ropers, “Real-time spectral interferometry probes the internal dynamics of femtosecond soliton molecules,” Science **356**(6333), 50-54 (2017).
32. D. Solli, J. Chou and B. Jalali, “Amplified wavelength–time transformation for real-time spectroscopy,” Nat. Photonics **2**, 48–51 (2008).
33. Haosen Shi, Youjian Song, Chingyue Wang, Luming Zhao and Minglie Hu, “Observation of subfemtosecond fluctuations of the pulse separation in a soliton molecule,” Opt. Lett. **43**(7), 1623-1626 (2018).
34. Lili Gui, Xiaosheng Xiao, and Changxi Yang, “Observation of various bound solitons in a carbon-nanotube-based erbium fiber laser,” J. Opt. Soc. Am. B **30**(1), 158-164 (2013).
35. Boris A. Malomed, “Bound solitons in the nonlinear Schrödinger–Ginzburg-Landau equation,” Phys. Rev. A **44**(10), 6954-6957 (1991).
36. V. V. Afanasjev, B. A. Malomed, and P. L. Chu, “Stability of bound states of pulses in the Ginzburg-Landau equations,” Phys. Rev. A 44(10), 6954-6957 (1991).Phys. Rev. E **56**(5), 6020-6025 (1997)
37. N. N. Akhmediev, A. Ankiewicz, and J. M. Soto-Crespo, “Stable soliton pairs in optical transmission lines and fiber lasers,” J. Opt. Soc. Am. B **15**(2), 515-523 (1998).
38. M. Stratmann and F. Mitschke, Proceedings of the Conference on Quantum Electronics and Laser Science 2002 paper QThI4 (Optical Society of America, Washington, DC, 2002).
39. D. Y. Tang, B. Zhao, L. M. Zhao and H. Y. Tam, “Soliton interaction in a fiber ring laser,” Phys. Rev. E **72**, 016616-1-6 (2005).
40. S. M. J. Kelly, “Characteristic sideband instability of periodically amplified average soliton,” Electron. Lett. **28**(8), 806–807 (1992)
41. Hidetsugu Sakaguchi, Dmitry V. Skryabin, and Boris A. Malomed, “Stationary and oscillatory bound states of dissipative solitons created by third-order dispersion,” Opt. Lett. **43**(11), 2688-2691 (2018).
42. M. Quiroga-Teixeiro, C. Balslev Clausen, M. P. Sørensen, P. L. Christiansen and P. A. Andrekson, “Passive mode locking by dissipative four-wave mixing,” J. Opt. Soc. Am. B **15**(4), 1315-1321 (1998).
43. A. Hasegawa, “Generation of a train of soliton pulses by induced modulational instability in optical fibers,” Opt. Lett. **9**(7), 288-290 (1984).
44. K. Tai, A. Hasegawa and A. Tomita, “Observation of modulational instability in optical fibers,” Phys. Rev. Lett. **56**(2), 135-138 (1986).
45. C. R. Menyuk, “Stability of solitons in birefringent optical fibers. I: Equal propagation amplitudes,” Opt. Lett. **12**(8), 614-616 (1987).
46. C. R. Menyuk, “Stability of solitons in birefringent optical fibers. II. Arbitrary amplitudes,” J. Opt. Soc. Am. B **5**(2), 392-402 (1988).
47. V. V. Afanasjev, P. L. Chu, and B. A. Malomed, “Bound states of dark solitons in the quintic Ginzburg-Landau equation,” Phys. Rev. E **57**(1), 1088-1091 (1998).
48. Y. J. He, Boris A. Malomed, Dumitru Mihalache, B. Liu, H. C. Huang, H. Yang, and H. Z. Wang，“Bound states of one-, two-, and three-dimensional solitons in complex Ginzburg-Landau equations with a linear potential,” Opt. Lett. **34**(19), 1088-1091 (1998).

49. Baldeep Kaur and Soumendu Jana，“Cavity soliton molecules and all-optical push-broom effect,” J. Lightwave Technol. **36**(12), 2463-2470 (2018).
50. BorisA. Malomed, “Propagation of a soliton in a nonlinear waveguide with dissipation and pumping,” Opt. Commun. **61**(3), 192-194 (1987).
51. F. Ö. Ilday, J. Buckley, W. Clark and F. Wise, “Self-similar evolution of parabolic pulses in a laser,” Phys. Rev. Lett. **92**(21), 213902-1-4 (2004).
52. A. C. Peacock, R. J. Kruhlak, J. D. Harvey and J. M. Dudley, “Solitary pulse propagation in high gain optical fiber amplifiers with normal group velocity dispersion,” Opt. Comm. **206**(1-3), 171-177 (2002).
53. A. Chong, W. Renninger and F. W. Wise, “Properties of normal-dispersion femtosecond fiber lasers,” J. Opt. Soc. Am. B **25**(2), 140–148 (2008).
54. L. M. Zhao, D. Y. Tang and J. Wu, “Gain-guided soliton in a positive group dispersion fiber laser,” Opt. Lett. **31**(12), 1788-1790 (2006).
55. L. M. Zhao, D. Y. Tang, T. H. Cheng and C. Lu, “Gain-guided solitons in dispersion-managed fiber lasers with large net cavity dispersion,” Opt. Lett. **31**(20), 2957-2959 (2006).
56. A. Chong, J. Buckley, W. Renninger and F. W. Wise, “All-normal-dispersion femtosecond fiber laser,” Opt. Express **14**(21), 10095-10100 (2006).
57. L. M. Zhao, D. Y. Tang, X. Wu and H. Zhang, “Dissipative soliton generation in an Yb-fiber laser with an invisible intra cavity bandpass filter,” Opt. Lett. **35**(16), 2756-2758 (2010).