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Corrections and Remarks

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Abstract

This document contains corrections to errors discovered to-date in, and also some remarks upon, both Samin Ishtiaq's thesis, *A Relevant Analysis of Natural Deduction* [Ish99] and also the JLC [IP98] and CSL [IP99] papers, by Ishtiaq and Pym, that follow from it.

1 Introduction

This document contains corrections to errors discovered to-date in both Samin Ishtiaq's thesis, *A Relevant Analysis of Natural Deduction* [Ish99] and also the JLC [IP98] and CSL [IP99] papers, by Ishtiaq and Pym, that follow from it. The postscript version of the thesis, available <http://www.dcs.qmw.ac.uk/~si>, is the source of the hard-bound copies submitted to the libraries of the University of London (Senate House) and Queen Mary and Westfield College. A copy of this document has been deposited with the Librarians at both of these institutions.

2 Ishtiaq's Ph.D. thesis [Ish99]

The corrections are listed by chapter title.

2.1 Introduction

On Page 12, Line 10, replace “fragment” by “variant”. (Section 2.3 below explains the reason for this replacement in full detail.)

2.2 The $\lambda\Lambda$ -calculus and the RLF logical framework

1. Definition 8 on Page 29 of the thesis defines the κ function. It should be emphasized that the function relies on the formation of a set, rather than a multiset, of variables. It might be that alternative definitions for variable sharing are possible.

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2.3 The propositions-as-types correspondence

There are some mis-statements in the description of the logic and the correspondence, which are corrected below. The main import of these corrections is that the propositions-as-types correspondence of the $\lambda\Lambda$ -calculus holds with a structural variant, with Dereliction, of a fragment of **BI**. In fact, the need for dereliction should have been clear from the context. The basic idea of the correspondence, to consider “,” as linear extension and “;” as additive extension, still seems an appropriate one for dependent type theory. We are grateful to Peter O’Hearn for asking a question which led to some of these corrections.

Items 1-3 below refer to Section 3.3 on Page 69 of the thesis.

1. In the logic we must explicitly take the rule of Dereliction:

$$\frac{(X, Y)\Gamma, \Delta \vdash_{\Sigma, \Xi} \phi}{(X; !Y)\Gamma; !\Delta \vdash_{\Sigma, \Xi} \phi} \text{Dereliction}$$

where $!Y$ is Y with each each $x : A$ replaced by $x!A$, and each $,$ replaced by $;$. The reason for taking Dereliction is that the type theory admits Dereliction and, in particular, allows the conversion of $A \multimap B$ to $A \rightarrow B$. The corresponding logic must mimic this. Also, the rule not only changes the context combining operator from “,” to “;” but also changes the variable’s “tag” from multiplicative to additive. This is in accordance with predicate **BI**’s treatment of variables.

2. We must restrict the original presentation of the logic so that the multiplicative unit rule

$$\frac{(X)\Gamma \vdash_{\Sigma, \Xi} \phi}{(X)I, \Gamma \vdash_{\Sigma, \Xi} \phi}$$

is not allowed. There is a crucial use of this rule in the “linear variable used twice” example of **BI**, where it is used to convert a additive variable into a multiplicative one. Such a proof cannot be mimicked in the type theory as there is only one unit (but cf. Proposition 61).

Note that the corresponding use of the additive unit rule, which converts a multiplicative variable into an additive one, is fine as the type theory admits Dereliction.

3. The Contraction rule will need to be taken for multiplicatively combined variables which are related by the *Axiom* relation.
4. In Definition 37 on Page 73 of the thesis there is an obvious mis-statement (note that the preceding section on the restricted logic mentions the reliance on Dereliction) in the translation of the additive bunch. The correct clause is:

$$\mathcal{T}(\Gamma; \Delta) = !\mathcal{T}(\Gamma), x_1! \delta_1, \dots, x_n! \delta_n$$

The translation must also equate any two variables related by *Axiom*. That is,

$$\mathcal{T}(\Gamma(x:A, y:A) \vdash \phi) = \mathcal{T}(\Gamma(x:A)[x/y] \vdash \phi[x/y])$$

5. There is a type-formation condition missing in the statement of Theorem 41 (Soundness) on Page 76 of the thesis. The correct statement should be: If $(X)\Gamma \vdash_{\Sigma, \Xi} \Pi : \phi$, where $(X)\Gamma = (Y; Z)\Delta; \Xi$ and $(Z)\Xi \vdash_{\Sigma, \Xi} \Pi : \phi$, then if $\mathcal{T}((Y)\Delta) \vdash_{\Sigma} \mathcal{T}(\phi) : \text{Type}$, then $\mathcal{T}((X)\Gamma) \vdash_{\Sigma} \mathcal{T}(M) : \mathcal{T}(\phi)$. The side-condition says that if there is any non-relevant context around, then it had better be used in the type formation. This condition is a natural one given the relevant type formation that the type theory enforces. (Note that completeness builds in these conditions by requiring the relevant well-formedness conditions due to the type theory.)

The above points leave open the general question of a corresponding type theory for substructural logics. However, some of the technology developed for the $\lambda\Lambda$ -calculus might be useful: the elimination rule for \forall_{new} , for instance, will need to deal with the context and type well-formedness.

6. In the Substitution clause of Definition 34 on Page 67, Lines 22-24 of the thesis, replace

$$\textit{Substitution} \frac{(X(x:A))\Gamma \vdash_{\Sigma, \Xi} \phi \quad (Y) \vdash_{\Sigma, \Xi} t:A}{(X(Y)[u/x'])\Gamma[t/x][t/x'] \vdash_{\Sigma, \Xi} \phi[t/x][t/x']}$$

for all x' such that $\textit{Axiom}(x, x')$, where u denotes I or 1 (here we abuse notation and write just $\textit{Axiom}(x, x')$ rather than pick out x and x' from arbitrary bunches).

by

$$\textit{Substitution} \frac{(X(x:A))\Gamma \vdash_{\Sigma, \Xi} \phi \quad (Y) \vdash_{\Sigma, \Xi} t:A}{(X(Y))\Gamma[t/x] \vdash_{\Sigma, \Xi} \phi[t/x]} \textit{Axiom}(Y, x')$$

for all x' such that $\textit{Axiom}(x, x')$ (here we abuse notation and write just $\textit{Axiom}(Y, x')$ rather than pick out ys from Y).

The correct Substitution statement makes sense according to the Cut rule [Pym00]

$$\textit{Cut} \frac{(X(X_1))\Gamma(\psi(X_1)) \vdash_{\Sigma, \Xi} \phi \quad (Y(X_2))\Delta \vdash_{\Sigma, \Xi} \psi(X_2)}{(X(Y[X_1/X_2]))[u/x']\Gamma(\Delta[X_1/X_2]) \vdash_{\Sigma, \Xi} \phi}$$

where $\textit{Axiom}(X_1, X_2)$ and for all x' in $Y(-)$ such that $\textit{Axiom}(x', Y)$, where u denotes I or 1 according to the occurrences of x' .

7. In Definitions 35 and 36 on Pages 70 and 72 of the thesis, the coherence equivalence rule, E , is missing a $X \equiv X'$ side-condition.

2.4 Kripke resource semantics

1. In Definition 51 on Page 99 of the thesis, the condition $1 \cong I$ should actually be $1 \not\cong I$, as should be clear from the context. For the term model, of course, both units have to be taken in the type theory — as, in fact, Proposition 61 on Page 116 of the thesis rightly does. Of course, the set-theoretic model has both units.

2. The proof of Proposition 61 on Page 116 of the thesis requires, to be convincing, a bit more work than that given. The intension of the Proposition is to define the monoidal operators as “an extension with A and an extension with $!A$ ” but, to be correct, these have to be monoidal combinations of arbitrary objects (contexts, in this case). The definition of the category $C(\Sigma)$ is actually as follows. Objects of the category are Δ_Γ . *i.e.*, $\vdash_\Sigma \Gamma, \Delta$ context. Extension by a type is not context combination at all, because the extending type isn’t in general an object of the category. The objects of $C(\Sigma)$ are now legitimate contexts. The monoidal operators are defined by induction, using the *join* operator:

$$\begin{aligned} \Delta \otimes \Gamma &= \text{join}(\Delta, \Gamma) \\ \Delta \times 1 (= 1 \times \Delta) &= \Delta \\ \Delta \times \Gamma &= \text{join}(!\Delta, !\Gamma) \end{aligned}$$

The requirement that the category $C(\Sigma)$ be doubly-monoidal seems, in retrospect, technically unnecessary. A simpler way to have proceeded would have been to have defined the base category as having two *extensions* rather than two *combinations*. The disadvantage of such an approach might be a scarcity of examples.

3. In the pull-back diagram on Pages 97 and 99 of the thesis, the arrow $f \times 1_A$ should be $f \times A$. (Recall, too, that \times is not a cartesian product and that p^* is inclusion.)
4. In Section 4.3.1 on Page 95 of the thesis, in the general discussion of Kripke-style models of the internal logic, the forcing clause for \rightarrow omits the “and $s \models \phi$ ” assumption.
5. There is a missing context in the definition of the *share* functor in Definition 53 on Page 104 of the thesis. The condition $\exists y: B(x) \in \Gamma'$ should be $\exists y: B(x) \in \Gamma', \Delta'$.

2.5 Bibliography

There are a couple of BIBTEX errors in the bibliography listing.

1. On Page 148, Lines 8-9 of the thesis, replace the [AGM92] entry
 [AGM92] S Abramsky, D Gabbay, and TSE Maibaum, editors. Oxford Science Publications, 1992.
 by
 [AGM92] S Abramsky, D Gabbay, and TSE Maibaum, editors, Handbook of Logic in Computer Science. Oxford Science Publications, 1992.
2. On Page 151, Line 3 of the thesis, replace the [HP91] entry
 [HP91] G Huet and G Plotkin, editors. Cambridge University Press, 1991.
 by
 [HP91] G Huet and G Plotkin, editors, Logical Frameworks. Cambridge University Press, 1991.

3 JLC paper [IP98]

1. There is a minor typo in the JLC paper where “ λ_I -calculus” is written as “ λI -calculus”. This occurs in the first paragraph of Section 5.3 on Page 835 of the paper. The corresponding section, Section 2.5.3 on Page 61 of the thesis uses the correct name.
2. Definition 3.6 on Pages 820-21 of the JLC paper defines the κ function. It should be emphasized that the function relies on the formation of a set, rather than a multiset, of variables. It might be that alternative definitions for variable sharing are possible.

4 CSL paper [IP99]

1. In Section 1, towards the end of the first paragraph, we mention that the $\lambda\Lambda$ -calculus arises from a consideration of **BI**. The last sentence should end with “. . . stands in propositions-as-types correspondence with a structural variant, with Dereliction, of a fragment of **BI**, as will be clear from context”.
2. Example 3 on Page 240 incorrectly states that $x:A, x:A \vdash_{\Sigma} cxx:\text{Type}$ is an example of the concept of multiple occurrences. A correct example, albeit a more complex one, is as follows. Suppose $C, D : A \multimap \text{Type}$, then we can construct:

$$\begin{array}{c}
 \vdash_{\Sigma} d : \Lambda x:A. Cx \multimap Dx \multimap \text{Type} \quad x:A \vdash_{\Sigma} x:A \\
 \hline
 \begin{array}{ccc}
 \frac{x:A \vdash_{\Sigma} x:A}{x:A \vdash_{\Sigma} dx:Cx \multimap Dx \multimap \text{Type}} & \frac{x:A \vdash_{\Sigma} Cx:\text{Type}}{x:A, y:Cx \vdash_{\Sigma} y:Cx} & \frac{}{x:A \vdash_{\Sigma} Dx:\text{Type}} \\
 \hline
 \frac{x:A, y:Cx \vdash_{\Sigma} z:Dx \multimap \text{Type}}{x:A, y:Cx, x:A, z:Dx \vdash_{\Sigma} dxyz:\text{Type}} & & \frac{}{x:A, z:Dx \vdash_{\Sigma} z:Dx} \\
 \hline
 \end{array}
 \end{array}$$

The last two applications have a non-trivial κ action which forces one of the $x:A$ s to be shared. It can be checked that all the constants used in the proof are well-typed.

Again, we should emphasize that κ relies on the formation of a set, rather than a multiset, of variables.

The original example is not included in the thesis, so there are no corrections to be made there.

3. In Section 3.1 on Page 241 of the CSL paper, in the general discussion of Kripke-style models of the internal logic, the forcing clause for \rightarrow omits the “and $s \models \phi$ ” assumption.
4. In Definition 4 on Page 243 of the CSL paper, the condition $1 \cong I$ should actually be $1 \not\cong I$, as should be clear from the context. For the term model both units must be formally taken in the type theory — as, in fact, Proposition 61 on Page 116 of the thesis rightly does. Of course, the set-theoretic model has both units.

References

- [IP98] SS Ishtiaq and DJ Pym. A Relevant Analysis of Natural Deduction. *Journal of Logic and Computation*, 8(6):809–838, 1998.
- [IP99] S Ishtiaq and DJ Pym. Kripke Resource Models of a Dependently-Typed, Bunched λ -calculus. In M Rodríguez-Artalejo J Flum, editor, *Computer Science Logic*, volume 1683 of *Lecture Notes in Computer Science*, pages 235–249. Springer-Verlag, 1999.
- [Ish99] S Ishtiaq. *A Relevant Analysis of Natural Deduction*. PhD thesis, Queen Mary and Westfield College, University of London, 1999.
- [Pym00] DJ Pym. The Semantics and Proof Theory of the Logic of Bunched Implications, 2000. In preparation.