

1 **Frequency variability of standing Alfvén waves excited**
2 **by fast mode resonances in the outer magnetosphere**

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3 Coupled fast mode resonances (cFMRs) in the outer magnetosphere, be-
4 tween the magnetopause and a turning point, are often invoked to explain
5 observed discrete frequency field line resonances. We quantify their frequency
6 variability, applying cFMR theory to a realistic magnetic field model and mag-
7 netospheric density profiles observed over almost half a solar cycle. Our cal-

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8 culations show cFMRs are most likely around dawn, since the plasmaspheric
9 plumes and extended plasmaspheres often found at noon and dusk can pre-
10 clude their occurrence. The relative spread (median absolute deviation di-
11 vided by the median) in eigenfrequencies is estimated to be 28%, 72% and
12 55% at dawn, noon and dusk respectively, with the latter two chiefly due to
13 density. Finally, at dawn we show that the observed bimodal density distri-
14 bution results in bimodal cFMR frequencies, whereby the secondary peaks
15 are consistent with the so-called “CMS” frequencies that have previously been
16 attributed to cFMRs.

1. Introduction

17 Ultralow frequency (ULF) waves play a number of key roles within the magnetosphere
 18 such as the transport, acceleration and loss of electrons in the radiation belts [e.g. the
 19 review of *Elkington, 2006*]. One of the earliest known ULF wave modes were field line
 20 resonances (FLRs), standing Alfvén waves on field lines fixed at their ionospheric ends
 21 [*Southwood, 1974*]. At the resonant field line, position x_r (x, y, z correspond to the radial,
 22 azimuthal and field-aligned co-ordinates respectively throughout), they satisfy

$$\left[\frac{\omega}{v_A(x_r)} \right]^2 - k_z^2 = 0 \quad (1)$$

23 for angular frequency ω , wavevector component k_z , and local Alfvén speed $v_A = B/\sqrt{\mu_0\rho}$
 24 depending on both magnetic field strength B and plasma mass density ρ . The quantised
 25 frequencies of FLRs are often estimated using WKB calculations applied to models i.e.

$$\omega_l(x_r) = \pi l \left[\int \frac{dz}{v_A} \right]^{-1} \quad (2)$$

26 where $l \in \mathbb{N}$ denotes the field-aligned mode number (FLR harmonic) and the integral
 27 is taken between the field line's footpoints. These show good agreement with observed
 28 pulsations, though further sophistications have been developed [*Singer et al., 1981; Wild*
 29 *et al., 2005; Rankin et al., 2006; Kabin et al., 2007*] which yield small but non-negligible
 30 corrections (typically $\sim 20\%$ or less).

31 Often standing Alfvén waves are excited over a range of L-shells with continuous fre-
 32 quencies [e.g. *Sarris et al., 2010*]. However, discrete sets of FLRs are also observed,
 33 predominantly in the dawn/morning sector with a secondary peak around dusk [*Baker*

34 *et al.*, 2003; *Plaschke et al.*, 2008]. *Samson et al.* [1991, 1992] suggested that a set of
35 quasi-steady FLR frequencies, namely {1.3, 1.9, 2.6–2.7, 3.2–3.4} mHz known as “CMS”
36 frequencies, occur at latitudes $\sim 70^\circ$ between midnight–mid-morning. While some statis-
37 tical studies (of a few hundred events or less) seem to support this hypothesis showing
38 distinct peaks in occurrence distributions [*Fenrich et al.*, 1995; *Chisham and Orr*, 1997;
39 *Mathie et al.*, 1999; *Kokubun*, 2013], larger studies (thousands to tens of thousands of
40 events) show little or no clear peaks [*Ziesolleck and McDiarmid*, 1995; *Baker et al.*, 2003;
41 *Plaschke et al.*, 2008]. The significance of quasi-steady frequencies of discrete FLRs is
42 thus unclear.

43 A number of potential mechanisms of exciting discrete frequencies of standing Alfvén
44 waves have been proposed including Kelvin-Helmholtz surface waves [*Chen and Hasegawa*,
45 1974; *Southwood*, 1974], direct driving by solar wind dynamic pressure oscillations
46 [*Stephenson and Walker*, 2002; *Claudepierre et al.*, 2010], and so-called cavity or waveguide
47 modes [*Kivelson. et al.*, 1984; *Kivelson and Southwood*, 1985]. The latter concern radially
48 standing fast magnetosonic waves, trapped between reflecting magnetospheric boundaries
49 and/or turning points. Many types of fast mode resonance (FMR) are known such as
50 plasmaspheric, virtual, tunnelling and trapped modes [see e.g. *Waters et al.*, 2000], but
51 here we focus only on outer-magnetospheric modes which couple to an FLR on the field
52 line where equation 1 is satisfied. These modes propagate between the magnetopause,
53 position x_{mp} , and a turning point inside the magnetosphere, position $x_t \geq x_r$, satisfying
54 (assuming cold plasma)

$$\left[\frac{\omega}{v_A(x_t)} \right]^2 - k_y^2 - k_z^2 = 0 \quad (3)$$

WKB solutions (which agree within $\sim 3\%$ with full numerical solutions [Rickard and Wright, 1995]) involve radially integrating the phase

$$\Phi(x_r) \equiv \int_{x_t}^{x_{mp}} dx \sqrt{\left[\frac{\omega_l(x_r)}{v_A(x)} \right]^2 - k_y^2 - k_z^2} \quad (4)$$

and finding eigenmodes [Samson et al., 1992, 1995]. The turning point introduces a phase shift (weakly dependent on k_y) of $\pi/2$ [Rickard and Wright, 1994]. Considering the magnetopause as perfectly reflecting (nodal boundary condition), the eigenmodes correspond to $\Phi(x_r) = \pi(n - \frac{1}{4})$ for radial mode numbers $n \in \mathbb{N}$. Applying this theory, Samson et al. [1992] fitted the parameters of an assumed analytical Alfvén speed profile to the CMS frequencies. While this resulted in a reasonable $x_{mp} \sim 15 R_E$, some have questioned the field-line lengths used and large densities ($\gtrsim 25 \text{ amu cm}^{-3}$) required [Harrold and Samson, 1992; Allan and McDiarmid, 1993]. Mann et al. [1999] later showed that the magnetopause can support anti-nodal boundary conditions, with a quarter wave mode fundamental, which might be able to produce such low frequencies. FMRs with these boundary conditions have been demonstrated in global magnetohydrodynamic simulations [Claudepierre et al., 2009].

The azimuthal wavevector component is often assumed to take the form $k_y = m/x$, where m is the azimuthal mode number [Waters et al., 2000]. m takes discrete values in (closed, axisymmetric) cavity models [Kivelson et al., 1984], whereas waveguide models consider fast waves propagating towards an open tail whereby m is continuous [Sam-

73 *son et al.*, 1992]. Models of waveguide dispersion show fairly level eigenfrequencies for
74 $|m| \lesssim 3$ and almost constant azimuthal group velocities $\partial\omega/\partial k_y$ for larger $|m|$ which vary
75 only slightly with n [*Wright*, 1994; *Rickard and Wright*, 1994, 1995], hence FMRs show
76 proportionally less dispersion for higher n . While m is a free parameter in most waveg-
77 uide models, *Mann et al.* [1999] demonstrated a possible m selection mechanism for these
78 modes.

79 Few unambiguous spacecraft observations of outer-magnetospheric FMRs had been
80 found until fairly recently, largely due to observational difficulties [*Waters et al.*, 2002;
81 *Hartinger et al.*, 2012]. The overall occurrence of FMRs is unclear: *Hartinger et al.*
82 [2013] state a detection rate of $\sim 1\%$ using strict criteria (only cavity modes, biased to-
83 wards noon) whereas *Hartinger et al.* [2014] provide evidence that FMR-like events occur
84 $\sim 37\text{--}41\%$ of the time.

85 Since FLRs transfer energy to radiation belt electrons [*Mann et al.*, 2013] and the iono-
86 sphere [*Hartinger et al.*, 2015], predicting when, where and why these occur is important.
87 While direct solar wind driving may account for $\sim 32\%$ of events [*Viall et al.*, 2009], such
88 an assessment for these coupled fast mode resonances (cFMRs) has not yet been possible
89 since observational evidence or lack thereof for cFMRs has often involved searching for the
90 (still heavily disputed) CMS frequencies. However, even cFMR proponents acknowledge
91 that the variability of the magnetosphere should affect these frequencies [*Samson et al.*,
92 1992; *Walker et al.*, 1992; *Mathie et al.*, 1999]. Models of FMRs typically use either fixed
93 profiles or idealised analytical expressions whereby one parameter is varied [*Allan and*
94 *McDiarmid*, 1989; *Wright and Rickard*, 1995]. It is not clear how realistic such idealised

95 profiles are and how variable these might be, thus the potential occurrence and variability
96 in frequency/location of outer-magnetospheric cFMRs is unknown. We therefore set out
97 to quantify this variability for the first time.

2. Method

98 In this study, cFMR theory is applied to dawn, noon and dusk only. Due to the
99 disparity in timescales associated with changes in magnetospheric densities (hours to
100 days [*Khazanov*, 2011]) and magnetic fields (several minutes [*Smit*, 1968]), we treat these
101 quantities independently using observed equatorial density profiles over almost half a solar
102 cycle and a realistic magnetic field model.

103 Electric Field Instrument (EFI) [*Bonnell et al.*, 2008] and Electrostatic Analyzer (ESA)
104 [*McFadden et al.*, 2008a] measurements from the inner three Time History of Events
105 and Macroscale Interactions during Substorms (THEMIS) [*Angelopoulos*, 2008] probes
106 are used between Feb 2008 – Jun 2013, yielding 5 seasons in each sector. The median
107 magnetic local time (MLT) was calculated for all inbound and outbound magnetosphere
108 crossings (between $3 R_E$ and apogee) and only those crossings with sufficient data coverage
109 ($>75\%$) whose median MLT was within 1 h of a target sector were selected. This resulted
110 in 863 (dawn: 6 ± 1 h MLT), 809 (noon: 12 ± 1 h MLT) and 893 (dusk: 18 ± 1 h MLT)
111 crossings. Excluding magnetosheath and solar wind periods using the method of *Lee and*
112 *Angelopoulos* [2014], electron density profiles n_e were calculated from the spin-averaged
113 spacecraft potential [*McFadden et al.*, 2008b] and binned by radial distance ($0.1 R_E$ reso-
114 lution). A median filter was applied to smooth the profiles but maintain distinct features
115 e.g. the plasmopause. See supporting material for an example. At dawn and dusk since

116 the THEMIS apogees did not extend far enough, a constant extrapolation to the mag-
117 netopause was applied [c.f. *Carpenter and Anderson, 1992*]. Changing the extrapolation
118 technique affects our calculations by $\sim 10\%$, but has little effect on their relative variabil-
119 ity. To arrive at the plasma mass density, we assume fixed ion compositions in each sector
120 using the results of *Lee and Angelopoulos [2014]* yielding ρ/n_e as 6.8, 2.6 and 4.0 amu cm⁻³
121 at dawn, noon and dusk respectively. The usual power law form for the density distribu-
122 tion along the field lines was assumed, using exponent $\alpha = 2$ [c.f. *Denton et al., 2015*].
123 While these fixed parameters do vary in reality, the effect on cFMR frequency variability is
124 small compared to the density and magnetic field. Figure 1d–f displays histograms (shades
125 of blue) of the density profiles in the three sectors as a function of radial distance. These
126 are largely consistent with previous results e.g. the plasmopause can be seen typically
127 between 4–6 R_E [*O'Brien and Moldwin, 2003; Liu and Liu, 2014*], and higher densities at
128 large radial distances due to either plasmaspheric plumes [*Darrouzet et al., 2008; Walsh*
129 *et al., 2013*] or an extended plasmasphere [*Carpenter and Anderson, 1992; Tu et al., 2007*]
130 are more often observed in the noon and dusk sectors.

131 A model magnetic field is used rather than observed profiles since we require self-
132 consistent FLR frequencies and equatorial Alfvén speeds. Furthermore, the time taken
133 accumulating each density profile is much longer than the variability timescale of the
134 magnetic field. Due to the large variability in equatorial densities [*Sheeley et al., 2001;*
135 *Takahashi et al., 2010, 2014*], as a first instance we apply a fixed T96 magnetic field model
136 [*Tsyganenko, 1995, 1996*] (shown in Figure 1a–c) using the median solar wind conditions
137 taken from the OMNI database over the survey period. Combining T96 with the density

138 observations we arrive at Alfvén speed (g-i) and FLR frequency (j-l) profiles, which again
 139 are largely consistent with previous observations and models [e.g. *Waters et al.*, 2000;
 140 *Archer et al.*, 2013b].

141 The cFMR theory detailed in equations 1-4 was applied to these profiles for $l = 1-3$
 142 and $|m| = 0-10$ (0.5 spacing). While in idealised box/cylinder models the fast and Alfvén
 143 modes are decoupled for $m = 0$ [*Southwood*, 1974], this is not the case in more represen-
 144 tative geometries [*Radoski*, 1971]. We use the quantisation condition

$$\Phi(x_r) = \frac{\pi}{2} \left(n - \frac{1}{2} \right) \quad (5)$$

145 whereby odd n correspond to modes with an antinode at the magnetopause (e.g. $n = 1$ is
 146 a quarter wave mode [*Mann et al.*, 1999; *Claudepierre et al.*, 2009]) whereas even n exhibit
 147 nodes [*Samson et al.*, 1992, 1995]. Solving equation 5 yields the resonance locations and
 148 eigenfrequencies, denoted $\omega_{l,n}(m)/2\pi$. The calculations assume plasma properties vary
 149 slowly with azimuth compared to the azimuthal propagation of the FMR over a bounce
 150 period, found to be $\lesssim 10^\circ$, thus are valid in this respect [c.f. *Moore et al.*, 1987].

151 Since the focus of this study is on variability, we only require that the computed cFMR
 152 frequencies are broadly correct since any (small) systematic deviation in absolute values,
 153 due to either the WKB approximation or our choice of fixed parameters, will have no
 154 effect on the relative variability. Previous studies have indeed shown that the methods
 155 used here result in FLR frequencies in good agreement with observations [*Wild et al.*, 2005;
 156 *Archer et al.*, 2013a, b]. Throughout this paper the relative spread (or variability) refers
 157 to the ratio of median absolute deviation (a robust estimator of scale given by $\text{MAD} =$

158 Median_{*i*} ($|x_i - \text{Median}_j(x_j)|$) whereby 50% of the data lie between Median \pm MAD [*Huber*,
 159 1981]) to the median. This is shown for the density (dotted) and Alfvén speed/FLR
 160 frequency (solid) as a function of radial distance in Figure 1m–o.

3. Occurrence

161 We investigate the possible occurrence of cFMRs (assuming a suitable driver is present
 162 at all times) by plotting the fraction of profiles which supported them i.e. a solution to
 163 equation 5 existed. This is shown in Figures 2a–c (as a function of n and l for $m = 0$) and
 164 3a–c (as a function of n and m for $l = 1$). It is clear that cFMRs should predominantly
 165 occur in the morning sector (e.g. 89% of profiles supported the fundamental mode), being
 166 less likely at dusk (65%) and noon (27%). This is in agreement with the occurrence
 167 statistics of discrete FLRs [*Baker et al.*, 2003; *Plaschke et al.*, 2008], though of course
 168 there are numerous other mechanisms of FLR excitation.

169 In Figure 1d–l, we plot the median (lines) and interquartile ranges (error bars) for those
 170 profiles which did (yellow) and did not (red) support a fundamental cFMR. These reveal,
 171 in all sectors though most notably at noon, that cFMR are not supported when the density
 172 rises immediately earthward of the magnetopause. In the cFMRs under consideration, fast
 173 magnetosonic waves only propagate in regions where $v_A(x) < v_A(x_r)$ [*Waters et al.*, 2000].
 174 Indeed, the profiles which do not support cFMR show decreases in the Alfvén speed
 175 with distance from the magnetopause due to the density rising faster than the magnetic
 176 field. The size of the cavity is restricted to the vicinity of the magnetopause under these
 177 circumstances, making cFMRs impossible. Such density rises may be due to an extended
 178 plasmasphere, often observed around noon [*Tu et al.*, 2007; *Archer et al.*, 2013a], or the

179 plasmaspheric plume in the afternoon sector [*Darrouzet et al.*, 2008; *Walsh et al.*, 2013],
 180 thereby explaining the possible occurrence of cFMR with local time.

181 Figures 2a–c and 3a–c show clear trends in possible cFMR occurrence with the mode
 182 numbers, being more likely as l increases but less likely as both $|m|$ and n increase. Again,
 183 these can be understood in terms of the theory. For a cFMR to be possible, the radial
 184 phase integral (equation 4) must become sufficiently large within the outer-magnetospheric
 185 cavity (between the magnetopause and plasmopause) such that a radial eigenmode can
 186 form (equation 5). Smaller radial mode numbers n require smaller phase integrals, hence
 187 are more likely. Increasing the field-aligned mode number l increases the integrand in the
 188 phase integral, thereby making a radial eigenmode more likely. Finally, the azimuthal
 189 mode number m decreases the integrand serving to push the resonance point Earthward
 190 compared to $m = 0$. Since the FLR frequency profiles usually exhibit a peak ahead of
 191 the plasmopause, this introduces a maximum possible $|m|$ for which cFMRs are possible,
 192 which can be seen when looking at specific examples (not shown).

4. Frequencies

4.1. Density

193 Here we assess the variability in cFMR frequencies due to density alone. Figure 2 shows
 194 the frequencies (d–f) and resonance locations (g–i) as box plots for $m = 0$, where horizontal
 195 lines display medians across the profiles, boxes indicate interquartile ranges and whiskers
 196 show 95% of the data. The eigenfrequencies are broadly within the expected ranges both
 197 theoretically [*Mann et al.*, 1999; *Claudepierre et al.*, 2009] and observationally [*Baker*
 198 *et al.*, 2003; *Plaschke et al.*, 2008; *Hartinger et al.*, 2013], being typically of the order of

199 a few mHz at dawn/dusk and tens of mHz around noon (due to the smaller cavity size
 200 and larger Alfvén speeds). As expected, cFMR frequencies increase with both l and n
 201 forming an anharmonic series i.e. they are not integer multiples of the fundamental being
 202 proportionally more tightly spaced [c.f. *Samson et al.*, 1992]. The resonance locations are
 203 at radial distances $\sim 4\text{--}10 R_E$ corresponding to magnetic latitudes of $\sim 60\text{--}75^\circ$, within
 204 the range of observed discrete FLRs on the ground [*Plaschke et al.*, 2008]. These move
 205 towards the magnetopause as l increases, because l increases the phase integrand thus the
 206 radial quantisation condition is satisfied earlier; and Earthward for increasing n , due to
 207 the larger phase integral required.

208 While an indication of variability is apparent via the size of the boxes and whiskers
 209 in Figure 2, we quantify the relative spreads over all profiles in the frequency (red) and
 210 resonance location (blue) for each mode number, shown in panels j–l. It is clear that the
 211 variability in resonance location is fairly small in all sectors: $6 \pm 2\%$ (dawn), $14 \pm 3\%$
 212 (noon), $8 \pm 1\%$ (dusk); hence our calculations suggest that the excited FLRs should recur
 213 at similar distances/latitudes. Our calculated frequencies, however, display much greater
 214 variability, particularly in the noon ($67 \pm 8\%$) and dusk ($49 \pm 2\%$) sectors compared
 215 to dawn which exhibits only $18 \pm 1\%$. The level of variability is reflective of the relative
 216 spreads in both Alfvén speed and FLR frequency in the outer magnetosphere, as displayed
 217 in Figure 1m–o (solid yellow lines for profiles which support cFMR).

218 Figure 3 indicates how the frequencies and resonance locations are altered as a function
 219 of $|m|$ i.e. dispersion. Frequencies and cavity sizes are plotted as the ratio to $m = 0$
 220 results, highlighting changes due to $|m|$ alone by removing the inherent variability at

221 $m = 0$. As previously noted, increasing $|m|$ pushes the resonance location Earthward
 222 (g-i), which serves to increase the cFMR frequency (d-f). The qualitative form of the
 223 dispersion and its proportional decrease with n are similar to previous analytical models
 224 [*Wright, 1994; Rickard and Wright, 1994, 1995*]. Interestingly, there is little spread in the
 225 frequency ratios across the profiles ($< 10\%$ at noon and $< 5\%$ at dawn/dusk) indicating
 226 that the proportional dispersion is systematic. While m is a free parameter in our cFMR
 227 model, *Mann et al. [1999]* demonstrated an m selection method. Given the systematic
 228 nature of the dispersion, we therefore do not add a contribution to the overall cFMR
 229 frequency variability due to the possible range of m .

4.2. Magnetic Field

230 So far we have considered cFMR variability due to the density only, however, changes in
 231 the magnetic field may also be important. Since the solar wind dynamic pressure P_{dyn} is
 232 the most significant source of magnetic field variability, we repeated our calculations over
 233 all density profiles changing this input into T96 by plus/minus one median absolute deviation
 234 (calculated over the survey period). This self consistently changes the magnetopause
 235 location, magnetic field lines and field strengths.

236 Changing P_{dyn} has a similarly sized effect on cFMR frequencies in all three sectors
 237 whereby enhanced P_{dyn} results in higher frequencies, due to a now smaller cavity and
 238 higher Alfvén speeds, with the opposite true when decreasing it. This variability due to
 239 the magnetic field is $21 \pm 1\%$ (dawn), $24 \pm 5\%$ (noon) and $21 \pm 2\%$ (dusk). Therefore, at
 240 dawn the spread in frequency due to changes in the magnetic field is comparable to that
 241 of the density, whereas at noon and dusk these effects are small.

242 Since we treat densities and magnetic fields independently, we combine these sources of
 243 variability to arrive at the overall relative spread in cFMR frequencies. These are found
 244 to be 28% (dawn), 72% (noon) and 55% (dusk). For comparison, the relative spread in
 245 eigenfrequencies of the proposed eigenmode of the subsolar magnetopause is 25% [*Archer*
 246 *and Plaschke, 2015*] i.e. similar to the cFMR frequency variability in the dawn sector.

4.3. Dawn

247 Given that our calculated cFMRs around dawn can potentially occur most often and
 248 exhibit the least amount of variability in both frequency and resonance location, this
 249 sector warrants further investigation. Figure 4(top) shows the relationship between the
 250 cFMR frequencies for the first three radial eigenmodes ($l = 1, m = -1$) with the reciprocal
 251 square root of the outer-magnetospheric density (at apogee). As one might expect, the
 252 cFMR frequencies are found to highly correlate to this quantity and thus the Alfvén
 253 speed. The density distribution, shown as both a histogram and kernel density estimate
 254 (KDE) [*Bowman and Azzalini, 1997*] at the top left, is found to be bimodal. KDEs of the
 255 cFMR frequencies (same mode numbers as above) are displayed in bold in the bottom
 256 panel revealing similarly bimodal distributions. While the main population corresponds
 257 to densities $\sim 0.4 \text{ cm}^{-3}$ and have frequencies $\gtrsim 3$ mHz, the secondary population have
 258 larger densities $\sim 3 \text{ cm}^{-3}$ and thus lower frequencies. Curiously, the resulting secondary
 259 peaks for the $n = 1-3$ cFMR frequencies are similar (within the absolute errors of our
 260 calculations) to the first three CMS frequencies, indicated by the grey areas. We find
 261 that these secondary peaks in frequency are rather insensitive to the choice of m (lighter
 262 colours show KDEs for $-2 \leq m \leq 0$), unlike the higher frequency primary peak. Finally,

263 the resonance locations of these cFMRs (not shown) typically correspond to latitudes
264 $\sim 70^\circ$, in agreement with the original *Samson et al.* [1991, 1992] observations.

265 It had been questioned whether cFMR theory could explain such low frequencies [*Har-*
266 *rold and Samson, 1992; Allan and McDiarmid, 1993*], due to the field-line lengths and
267 large densities used by *Samson et al.* [1992]. By allowing for antinodal magnetopause
268 boundary conditions, *Mann et al.* [1999] postulated that mHz FMR eigenfrequencies may
269 be possible. We have shown that these low frequencies may indeed be explained by
270 cFMRs for a small population of observed density profiles applied to a realistic magnetic
271 field model. However, we do not preclude the possibility that other forms of FMR [e.g.
272 *Harrold and Samson, 1992; Waters et al., 2000*] might also explain similar frequency dis-
273 crete FLRs or that they may be excited via other mechanisms e.g. directly by solar wind
274 pressure oscillations [*Viall et al., 2009*].

5. Conclusions

275 Due to observational challenges and conflicting results, it has been unclear how often
276 standing Alfvén waves are excited by coupled fast mode resonances (cFMRs) in the outer
277 magnetosphere (between the magnetopause and a turning point) and what their range
278 of frequencies are. Through the use of a realistic magnetic field model and observed
279 magnetospheric density profiles over almost half a solar cycle, we have quantified their
280 possible occurrence and variability in frequency and resonance location for the first time.
281 We find that cFMRs are supported most often in the dawn sector compared to dusk and
282 noon, since the large densities associated with the plasmaspheric plume or an extended
283 plasmasphere in these sectors can preclude cFMR occurrence. This possible occurrence in

our calculations is consistent with the occurrence of observed discrete field line resonances (FLRs) on the ground [Baker et al., 2003; Plaschke et al., 2008], though numerous other mechanisms for their excitation also exist. The computed eigenfrequencies are within the range of previously observed [Baker et al., 2003; Plaschke et al., 2008; Hartinger et al., 2013] and theoretical results [Mann et al., 1999; Claudepierre et al., 2009], at typically a few mHz around dawn/dusk and tens of mHz at noon. The variability, however, is found to be much larger in the noon and dusk sectors, chiefly due to the density, whereas magnetic field changes have a comparable contribution around dawn. Overall the relative spread (ratio of median absolute deviation to the median) is estimated to be 28%, 72% and 55% at dawn, noon and dusk respectively. Finally, the observed bimodal distribution in outer-magnetospheric density at dawn results in bimodal cFMR frequency distributions, whereby the secondary population have the low “CMS” frequencies often attributed to FMRs [Samson et al., 1992] that have been called into question by some [Harrold and Samson, 1992; Allan and McDiarmid, 1993].

Future work should validate the calculated frequencies and resonance locations against observations both in space and on the ground, taking particular care in unambiguously identifying the ULF mode and driver where possible. Furthermore, by parameterising the collated density profiles in this study it should be possible to ascertain the dependence of cFMR occurrence and frequencies on e.g. the plasmopause position or radial density exponent [Allan and McDiarmid, 1989; Wright and Rickard, 1995] and with solar wind and magnetospheric conditions e.g. P_{dyn} or Kp . This would allow the prediction of FMR

305 frequencies and the discrete standing Alfvén waves they excite, of interest to e.g. the
306 radiation belt community [*Elkington*, 2006].

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311 <http://omniweb.gsfc.nasa.gov>.

References

- 312 Allan, W., and D. R. McDiarmid, Magnetospheric cavity modes and field-line reso-
313 nances: the effect of radial mass density variation, *Planet. Space Sci.*, *37*, 407–418,
314 doi:10.1016/0032-0633(89)90122-0, 1989.
- 315 Allan, W., and D. R. McDiarmid, Frequency ratios and resonance positions for magneto-
316 spheric cavity/waveguide modes, *Ann. Geophys.*, *11*, 916–924, 1993.
- 317 Angelopoulos, V., The THEMIS mission, *Space Sci. Rev.*, *141*, 5–34, doi:10.1007/s11214-
318 008-9336-1, 2008.
- 319 Archer, M. O., and F. Plaschke, What frequencies of standing surface waves
320 can the subsolar magnetopause support?, *J. Geophys Res.*, *120*, 3632–3646, doi:
321 10.1002/2014JA020545, 2015.
- 322 Archer, M. O., T. S. Horbury, J. P. Eastwood, J. M. Weygand, and T. K. Yeoman,
323 Magnetospheric response to magnetosheath pressure pulses: A low pass filter effect, *J.*
324 *Geophys. Res.*, *118*, 5454–5466, doi:10.1002/jgra.50519, 2013a.

- 325 Archer, M. O., M. D. Hartinger, and T. S. Horbury, Magnetospheric “magic” frequen-
326 cies as magnetopause surface eigenmodes, *Geophys. Res. Lett.*, *40*, 5003–5008, doi:
327 10.1002/grl.50979, 2013b.
- 328 Baker, G. J., E. F. Donovan, and B. J. Jackel, A comprehensive survey of auro-
329 ral latitude Pc5 pulsation characteristics, *J. Geophys. Res.*, *108*, SMP 11–1, doi:
330 10.1029/2002JA009801, 2003.
- 331 Bonnell, J. W., F. S. Mozer, G. T. Delory, A. J. Hull, R. E. Ergun, C. M. Cully, V. An-
332 gelopoulos, and P. R. Harvey, The electric field instrument (EFI) for THEMIS, *Space*
333 *Sci. Rev.*, *141*, 303–341, doi:10.1007/s11214-008-9469-2, 2008.
- 334 Bowman, A. W., and A. Azzalini, *Applied Smoothing Techniques for Data Analysis*, Ox-
335 ford University Press, 1997.
- 336 Carpenter, D. L., and R. R. Anderson, An ISEE/whistler model of equatorial electron
337 density in the magnetosphere, *J. Geophys Res.*, *97*, 1097–1108, doi:10.1029/91JA01548,
338 1992.
- 339 Chen, L., and A. Hasegawa, A theory of long-period magnetic pulsations: 1. steady
340 state excitation of field line resonance, *J. Geophys Res.*, *79*, 1024–1032, doi:
341 10.1029/JA079i007p01024, 1974.
- 342 Chisham, G., and D. Orr, A statistical study of the local time asymmetry of Pc5 ULF
343 wave characteristics observed at midlatitudes by SAMNET, *J. Geophys. Res.*, *102*,
344 24,339–24,350, doi:10.1029/97JA01801, 1997.
- 345 Claudepierre, S. G., M. Wiltberger, S. R. Elkington, W. Lotko, and M. K. Hudson, Mag-
346 netospheric cavity modes driven by solar wind dynamic pressure fluctuations, *Geophys.*

- 347 *Res. Lett.*, *36*, L13,101, doi:10.1029/2009GL039045, 2009.
- 348 Claudepierre, S. G., M. K. Hudson, W. J. G. Lotko, W. Lotko, and R. E. Denton, Solar
349 wind driving of magnetospheric ULF waves: Field line resonances driven by dynamic
350 pressure fluctuations, *J. Geophys Res.*, *115*, A11,202, doi:10.1029/2010JA015399, 2010.
- 351 Darrouzet, F., J. de Keyser, Décréau, F. El Lemdani-Mazouz, and X. Vallières, Statistical
352 analysis of plasmaspheric plumes with Cluster/WHISPER observations, *Ann. Geophys.*,
353 *26*, 2403–2417, doi:10.5194/angeo-26-2403-2008, 2008.
- 354 Denton, R. E., K. Takahashi, J. Lee, C. K. Zeitler, N. T. Wimer, L. E. Litscher, H. J.
355 Singer, and K. Min, Field line distribution of mass density at geostationary orbit, *J.*
356 *Geophys Res.*, *120*, 4409–4422, doi:10.1002/2014JA020810, 2015.
- 357 Elkington, S. R., A review of ULF interactions with radiation belt electrons, in *Magneto-*
358 *spheric ULF Waves: Synthesis and New Directions*, *Geophysical Monograph Series*, vol.
359 169, edited by K. Takahashi, P. J. Chi, R. E. Denton, and R. L. Lysak, John Wiley &
360 Sons, doi:10.1029/169GM06, 2006.
- 361 Fenrich, F. M., J. C. Samson, G. Sofko, and R. A. Greenwald, ULF high- and low-m
362 field line resonances observed with the Super Dual Auroral Radar Network, *J. Geophys.*
363 *Res.*, *100*, 21,535–21,547, doi:10.1029/95JA02024, 1995.
- 364 Harrold, B. G., and J. C. Samson, Standing ulf modes of the magnetosphere: A theory,
365 *Geophys. Res. Lett.*, *19*, 1811–1814, doi:10.1029/92GL01802, 1992.
- 366 Hartinger, M. D., V. Angelopoulos, M. B. Moldwin, Y. Nishimura, D. L. Turner, K.-
367 H. Glassmeier, M. G. Kivelson, J. Matzka, and C. Stolle, Observations of a Pc5 global
368 (cavity/waveguide) mode outside the plasmasphere by THEMIS, *J. Geophys. Res.*, *117*,

- 369 A06,202, doi:10.1029/2011JA017266, 2012.
- 370 Hartinger, M. D., V. Angelopoulos, M. B. Moldwin, K. Takahashi, and L. B. N. Clausen,
371 Statistical study of global modes outside the plasmasphere, *J. Geophys. Res.*, *118*, 804–
372 822, doi:10.1002/jgra.50140, 2013.
- 373 Hartinger, M. D., D. Welling, N. M. Viall, M. B. Moldwin, and A. Ridley, The effect
374 of magnetopause motion on fast mode resonance, *J. Geophys Res.*, *119*, 8212–8227,
375 doi:10.1002/2014JA020401, 2014.
- 376 Hartinger, M. D., M. B. Moldwin, S. Zou, J. W. Bonnell, and V. Angelopoulos, ULF wave
377 electromagnetic energy flux into the ionosphere: Joule heating implications, *J. Geophys*
378 *Res.*, *120*, 494–510, doi:10.1002/2014JA020129, 2015.
- 379 Huber, P. J., *Robust Statistics*, Wiley Series in Probability, John Wiley & Sons, 1981.
- 380 Kabin, K., R. Rankin, C. L. Waters, R. Marchand, E. F. Donovan, and J. C. Samson, Dif-
381 ferent eigenproblem models for field line resonances in cold plasma: Effect on magneto-
382 spheric density estimates, *Planet. Space Sci.*, *55*, 820–828, doi:10.1016/j.pss.2006.03.014,
383 2007.
- 384 Khazanov, G. V., *Kinetic Theory of the Inner Magnetospheric Plasma*, chap. Analysis of
385 Cold Plasma Transport, pp. 193–269, Astrophysics and Space Science Library, Springer,
386 2011.
- 387 Kivelson, M. G., and D. J. Southwood, Resonant ULF waves: a new interpretation,
388 *Geophys. Res. Lett.*, *12*, 49–52, doi:10.1029/GL012i001p00049, 1985.
- 389 Kivelson., M. G., J. Etcheto, and J. G. Trotignon, Global compressional oscillations of the
390 terrestrial magnetosphere: The evidence and a model, *J. Geophys Res.*, *89*, 9851–9856,

- 391 doi:10.1029/JA089iA11p09851, 1984.
- 392 Kokubun, S., ULF waves in the outer magnetosphere: Geotail observation 1 transverse
393 waves, *Earth Planets Space*, *65*, 411–433, doi:10.5047/eps.2012.12.013, 2013.
- 394 Lee, J. H., and V. Angelopoulos, On the presence and properties of cold ions
395 near earth's equatorial magnetosphere, *J. Geophys Res.*, *119*, 1749–1770, doi:
396 10.1002/2013JA019305, 2014.
- 397 Liu, X., and W. Liu, A new plasmplasma location model based on THEMIS observations,
398 *Sci. China Earth Sci.*, *57*, 2252–2557, doi:10.1007/s11430-014-4844-1, 2014.
- 399 Mann, I. R., A. N. Wright, K. J. Mills, and V. M. Nakariakov, Excitation of magne-
400 topheric waveguide modes by magnetosheath flows, *J. Geophys Res.*, *104*, 333–353,
401 doi:10.1029/1998JA900026, 1999.
- 402 Mann, I. R., et al., Discovery of the action of a geophysical synchrotron in the Earth's
403 Van Allen radiation belts, *Nature Commun.*, *4*, 2795, doi:10.1038/ncomms3795, 2013.
- 404 Mathie, R. A., I. R. Mann, F. W. Menk, and D. Orr, Pc5 ULF pulsations associated with
405 waveguide modes observed with the IMAGE magnetometer array, *J. Geophys. Res.*,
406 *104*, 7025–7036, doi:10.1029/1998JA900150, 1999.
- 407 McFadden, J. P., C. W. Carlson, D. Larson, M. Ludlam, R. Abiad, B. Elliott, P. Turin,
408 M. Marckwordt, and V. Angelopoulos, The THEMIS ESA plasma instrument and in-
409 flight calibration, *Space Sci. Rev.*, *141*, 277–302, doi:10.1007/s11214-008-9440-2, 2008a.
- 410 McFadden, J. P., C. W. Carlson, J. Bonnell, F. Mozer, V. Angelopoulos, K. H. Glassmeier,
411 and U. Auster, THEMIS ESA first science results and performance issues, *Space Sci.*
412 *Rev.*, *141*, 447–508, doi:10.1007/s11214-008-9433-1, 2008b.

- 413 Moore, T. E., D. L. Gallagher, J. L. Horwitz, and R. H. Comfort, MHD wave
414 breaking in the outer plasmasphere, *Geophys. Res. Lett.*, *14*, 1007–1010, doi:
415 10.1029/GL014i010p01007, 1987.
- 416 O'Brien, T. P., and M. B. Moldwin, Empirical plasmopause models from magnetic indices,
417 *Geophys. Res. Lett.*, *30*, 1152, doi:10.1029/2002GL016007, 2003.
- 418 Plaschke, F., K. H. Glassmeier, O. D. Constantinescu, I. R. Mann, D. K. Milling,
419 U. Motschmann, and I. J. Rae, Statistical analysis of ground based magnetic field
420 measurements with the field line resonance detector, *Ann. Geophys.*, *26*, 3477–3489,
421 doi:10.5194/angeo-26-3477-2008, 2008.
- 422 Radoski, H. R., A note on the problem of hydromagnetic resonances in the magnetosphere,
423 *Planet. Space Sci.*, *19*, 1012–1013, doi:10.1016/0032-0633(71)90152-8, 1971.
- 424 Rankin, R., K. Kabin, and R. Marchand, Alfvénic field line resonances in arbitrary mag-
425 netic field topology, *Adv. Space Res.*, *38*, 1720–1729, doi:10.1016/j.asr.2005.09.034, 2006.
- 426 Rickard, G. J., and A. N. Wright, Alfvén resonance excitation and fast wave prop-
427 agation in magnetospheric waveguides, *J. Geophys. Res.*, *99*, 13,455–13,464, doi:
428 10.1029/94JA00674, 1994.
- 429 Rickard, G. J., and A. N. Wright, ULF pulsations in a magnetospheric waveguide: Com-
430 parison of real and simulated satellite data, *J. Geophys. Res.*, *100*, 3,531–3,537, doi:
431 10.1029/94JA02935, 1995.
- 432 Samson, J. C., R. A. Greenwald, J. M. Ruohoniemi, T. J. Hughes, and D. D. Wallis,
433 Magnetometer and radar observations of magnetohydrodynamic cavity modes in the
434 Earth's magnetosphere, *Can. J. Phys.*, *69*, 929–937, doi:10.1139/p91-147, 1991.

- 435 Samson, J. C., B. G. Harrold, J. M. Ruohoniemi, R. A. Greenwald, and A. D. M. Walker,
436 Field line resonances associated with MHD waveguides in the magnetosphere, *Geophys.*
437 *Res. Lett.*, *19*, 441–444, doi:10.1029/92GL00116, 1992.
- 438 Samson, J. C., C. L. Waters, F. W. Menk, and B. J. Fraser, Fine structure in the
439 spectra of low latitude field line resonances, *Geophys. Res. Lett.*, *22*, 2111–2114, doi:
440 10.1029/95GL01770, 1995.
- 441 Sarris, T. E., W. Liu, X. Li, K. Kabin, E. R. Talaat, R. Rankin, V. Angelopoulos,
442 J. Bonnell, and K. H. Glassmeier, THEMIS observations of the spatial extent and
443 pressure-pulse excitation of field line resonances, *Geophys. Res. Lett.*, *37*, L15,104, doi:
444 10.1029/2010GL044125, 2010.
- 445 Sheeley, B. W., M. B. M. B. Moldwin, H. K. Rassoul, and R. R. Anderson, An empirical
446 plasmasphere and trough density model: CRRES observations, *J. Geophys Res.*, *106*,
447 25,631–25,641, doi:10.1029/2000JA000286, 2001.
- 448 Singer, H. J., D. J. Southwood, R. J. Walker, and M. G. Kivelson, Alfvén wave resonances
449 in a realistic magnetospheric magnetic field geometry, *J. Geophys. Res.*, *86*, 4589–4596,
450 doi:10.1029/JA086iA06p04589, 1981.
- 451 Smit, G. R., Oscillatory motion of the nose region of the magnetopause, *J. Geophys. Res.*,
452 *73*, 4990–4993, doi:10.1029/JA073i015p04990, 1968.
- 453 Southwood, D. J., Some features of field line resonances in the magnetosphere, *Planet.*
454 *Space Sci.*, *22*, 483–491, doi:10.1016/0032-0633(74)90078-6, 1974.
- 455 Stephenson, J. A., and A. D. M. Walker, HF radar observations of Pc5 ULF pulsations
456 driven by the solar wind, *Geophys. Res. Lett.*, *29*, 8–1 – 8–4, doi:10.1029/2001GL014291,

- 457 2002.
- 458 Takahashi, K., R. E. Denton, and H. J. Singer, Solar cycle variation of geosynchronous
459 plasma mass density derived from the frequency of standing alfvén waves, *J. Geophys*
460 *Res.*, *115*, A07,207, doi:10.1029/2009JA015243, 2010.
- 461 Takahashi, K., R. E. Denton, M. Hirahara, K. Min, S. Ohtani, and E. Sanchez, Solar
462 cycle variation of plasma mass density in the outer magnetosphere: Magnetoseismic
463 analysis of toroidal standing Alfvén waves detected by Geotail, *J. Geophys. Res.*, *119*,
464 8338–8356, doi:10.1002/2014JA020274, 2014.
- 465 Tsyganenko, N. A., Modeling the earth's magnetospheric magnetic field confined within
466 a realistic magnetopause, *J. Geophys. Res.*, *100*, 5599–5612, doi:10.1029/94JA03193,
467 1995.
- 468 Tsyganenko, N. A., Effects of the solar wind conditions in the global magnetospheric
469 configurations as deduced from data-based field models, in *International Conference on*
470 *Substorms, Proceedings of the 3rd International Conference held in Versailles*, edited
471 by E. Rolfe and B. Kaldeich, p. 181, European Space Agency, Paris, 1996.
- 472 Tu, J., P. Song, B. W. Reinisch, and J. L. Green, Smooth electron density transition
473 from plasmasphere to the subauroral region, *J. Geophys. Res.*, *112*, A05,227, doi:
474 10.1029/2007JA012298, 2007.
- 475 Viall, N. M., L. Kepko, and H. E. Spence, Relative occurrence rates and connection of
476 discrete frequency oscillations in the solar wind density and dayside magnetosphere, *J.*
477 *Geophys. Res.*, *114*, A01,201, doi:10.1029/2008JA013334, 2009.

- 478 Walker, A. D. M., J. M. Ruohoniemi, K. B. Baker, R. A. Greenwald, and J. C. Samson,
479 Spatial and temporal behavior of ULF pulsations observed by the Goose Bay HF radar,
480 *J. Geophys. Res.*, *97*, 12,187–12,202, doi:10.1029/92JA00329, 1992.
- 481 Walsh, B. M., D. G. Sibeck, Y. Nishimura, and V. Angelopoulos, Statistical analysis
482 of the plasmaspheric plume at the magnetopause, *J. Geophys Res.*, *118*, 4844–4851,
483 doi:10.1002/jgra.50458, 2013.
- 484 Waters, C. L., B. G. Harrold, F. W. Menk, J. C. Samson, and B. J. Fraser, Field line
485 resonances and waveguide modes at low latitudes 2. a model, *J. Geophys Res.*, *105*,
486 7763–7774, doi:10.1029/1999JA900267, 2000.
- 487 Waters, C. L., K. Takahashi, D.-H. Lee, and B. J. Anderson, Detection of ultralow-
488 frequency cavity modes using spacecraft data, *J. Geophys. Res.*, *107*, 1284, doi:
489 10.1029/2001JA000224, 2002.
- 490 Wild, J. A., T. K. Yeoman, and C. L. Waters, Revised time of flight calculations for high
491 latitude geomagnetic pulsations using a realistic magnetospheric magnetic field model,
492 *J. Geophys. Res.*, *110*, A11,206, doi:10.1029/2004JA010964, 2005.
- 493 Wright, A. N., Dispersion and wave coupling in inhomogeneous MHD waveguides, *J.*
494 *Geophys. Res.*, *99*, 159–167, doi:10.1029/93JA02206, 1994.
- 495 Wright, A. N., and G. J. Rickard, ULF pulsations driven by magnetopause mo-
496 tions: Azimuthal phase characteristics, *J. Geophys. Res.*, *100*, 23,703–23,710, doi:
497 10.1029/95JA01765, 1995.
- 498 Ziesolleck, C. W. S., and D. R. McDiarmid, Statistical survey of auroral latitude Pc5
499 spectral and polarization characteristics, *J. Geophys. Res.*, *100*, 19,299–19,312, doi:

500 10.1029/95JA00434, 1995.

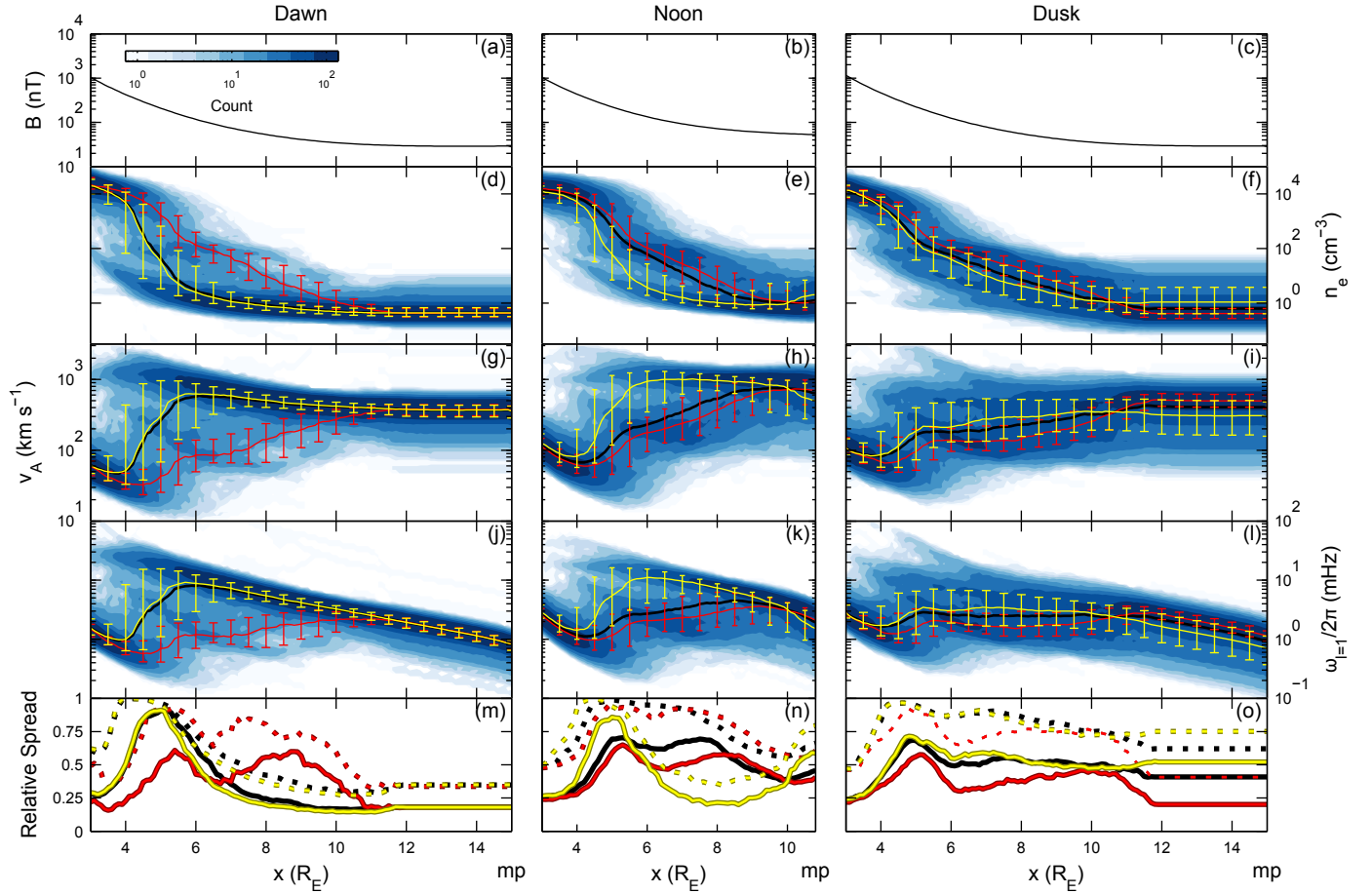


Figure 1: Profiles as a function of radial distance in the dawn (left), noon (middle) and dusk (right) sectors of equatorial (a–c) magnetic field strength, (d–f) electron number density, (g–i) Alfvén speed, (j–l) fundamental FLR frequency, (m–o) relative spreads in the density (dotted) and speed/frequency (solid). Medians (solid lines) and interquartile ranges (error bars) are shown over all profiles (black), profiles which support a fundamental cFMR (yellow), and profiles which don't (red).

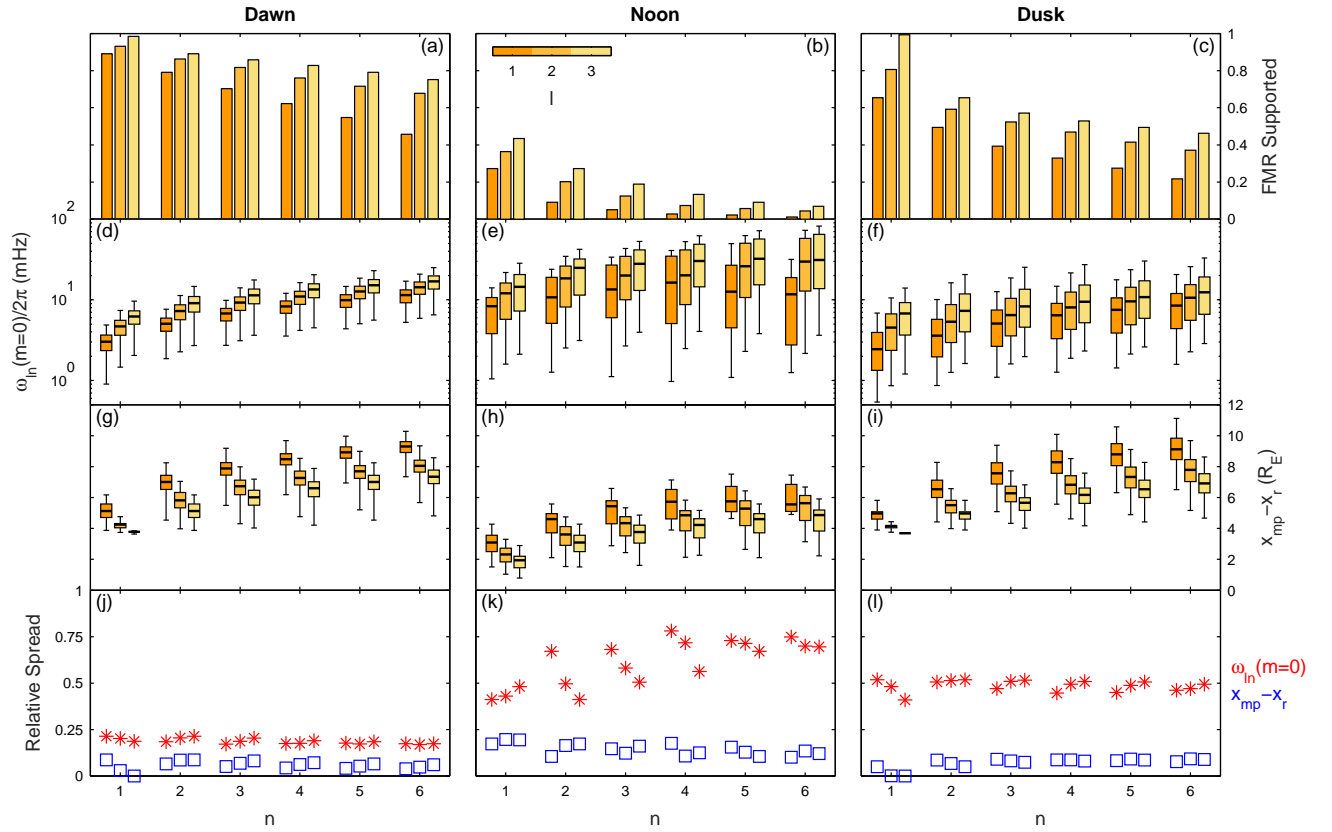


Figure 2: cFMR results as a function of n (groups) and l (colours) for $m = 0$ in the dawn (left), noon (middle) and dusk (right) sectors. (a–c) Fraction of cFMRs supported, (d–f) cFMR frequency and (g–i) cavity size as box plots with whiskers indicating 95% of the data, (j–l) relative spreads in the frequency (red) and cavity size (blue).

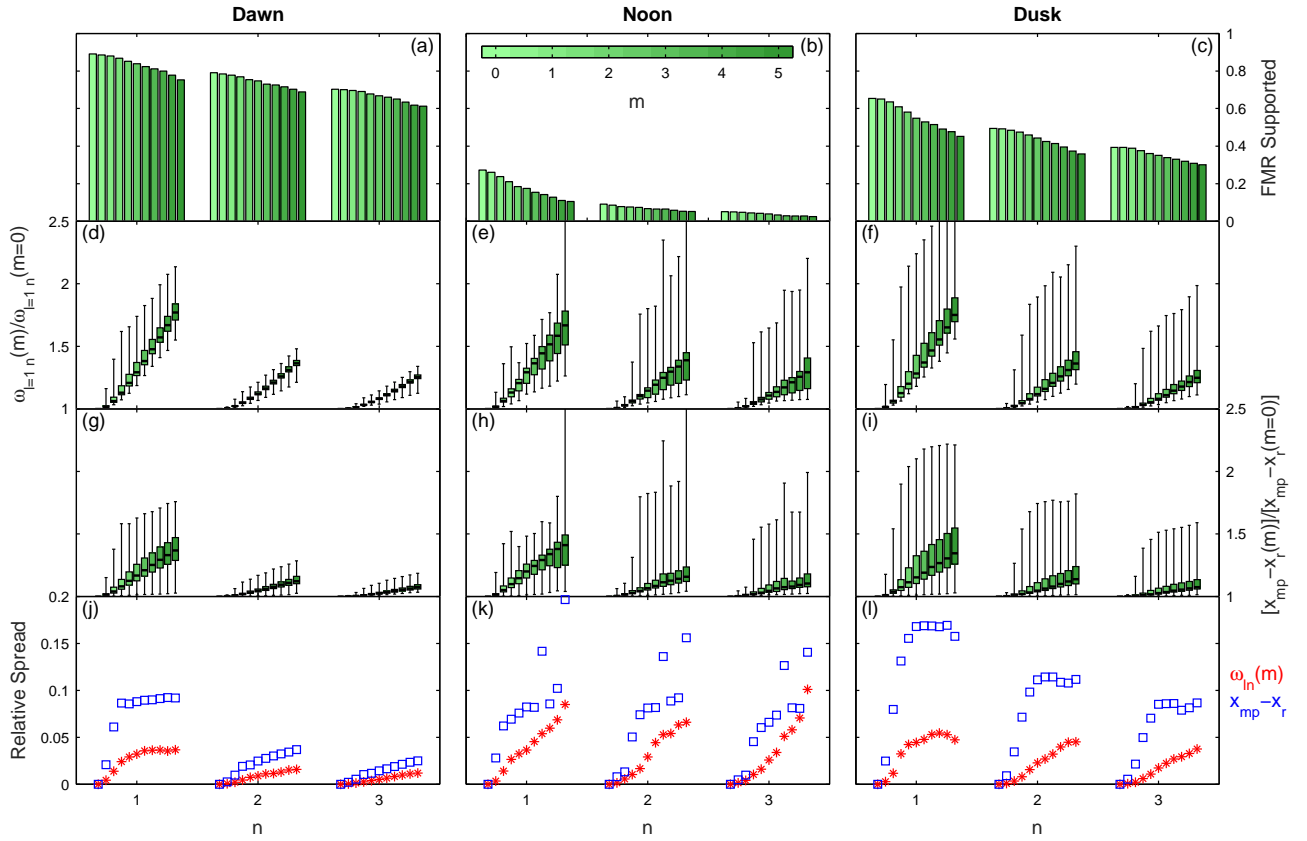


Figure 3: cFMR results as a function of n and m for $l = 1$ in a similar format to Figure 2. In panels d–l ratios to the $m = 0$ results are shown.

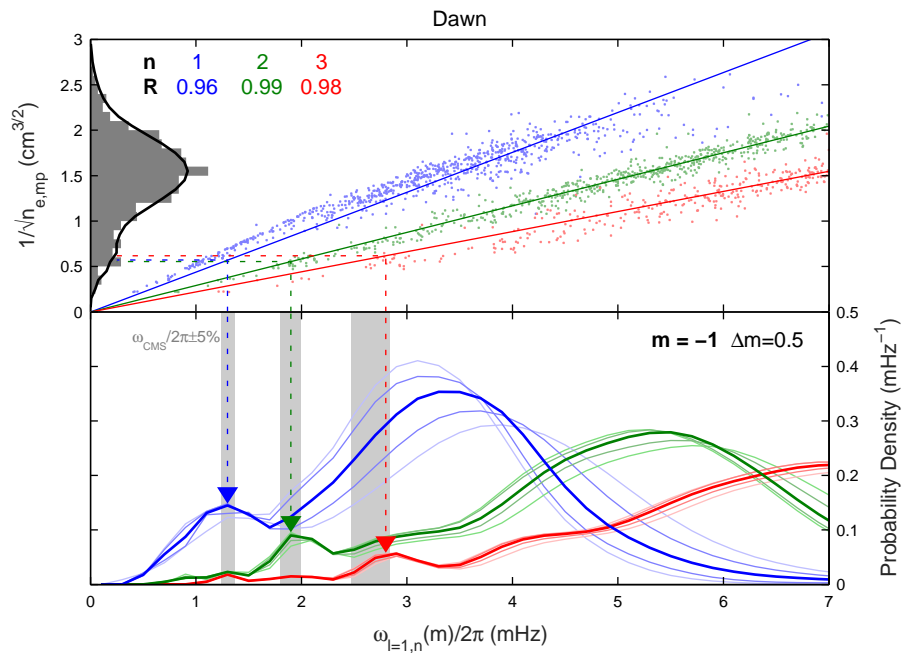


Figure 4: (Top) Relationship between cFMR frequency ($n = 1-3$ in blue, green, red) and the reciprocal square root of the outer-magnetospheric density (at apogee) at dawn. A histogram (grey) and kernel density estimate (KDE, black) of the latter is also shown. (Bottom) KDEs of the cFMR frequency distributions. Shaded areas show the CMS frequencies $\pm 5\%$.