Aspects of Brane World-Volume Dynamics in String Theory

Gianni Tallarita †

A thesis submitted for the degree of Doctor of Philosophy

† *Centre for Research in String Theory, School of Physics*
Queen Mary University of London
Mile End Road, London E1 4NS, UK
g.tallarita@qmul.ac.uk

To my Parents

Abstract

This thesis investigates the non-abelian dynamics of D-Brane systems in String Theory, specifically focussing on the fate of the open string Tachyon. Starting from the action of two coincident non-BPS D9-branes, we investigate kink configurations of the $U(2)$ matrix tachyon field, considering both symmetrised (Str) and conventional (Tr) prescriptions for the trace over gauge indices of the non-BPS action. Non-abelian tachyon condensation in the theory with Tr prescription, and the resulting fluctuations about the kink profile, are shown to give rise to a theory of two coincident BPS D8-branes.

Next we investigate magnetic monopole solutions of the non-abelian Dirac-Born-Infeld (DBI) action describing two coincident non-BPS D9-branes in flat space. These monopole configurations are singular in the first instance and require regularization. We discuss a suitable non-abelian ansatz which describes a point-like magnetic monopole and show it solves the equations of motion to leading order in the regularization parameter. Fluctuations are studied and shown to describe a codimension three BPS D6-brane, a formula is derived for its tension.

Finally, we investigate the dynamics of a pair of coincident D5 branes in the background of k NS5 branes. We extend Kutasov's original proposal to the non-abelian case of multiple D-Branes and find that the duality still holds provided one promotes the radial direction to a matrix valued field associated with a non-abelian geometric tachyon and a particular parametrization for the transverse scalar fields is chosen. Analytic and numerical solutions for the pair's equations of motion are found in certain simplified cases in which the $U(2)$ symmetry is broken to $U(1) \otimes U(1)$. For certain range of parameters these solutions describe periodic motion of the centre of mass of the pair bouncing off a finite sized throat whose minimum size is limited by the D5 branes separation.

Acknowledgements

I would like to thank my supervisor Steve Thomas for sharing his knowledge with me and introducing me to the world of research. I'm also grateful to David Berman for his support and encouragement.

I would like to acknowledge all the people that made the last three years easier. First of all fellow students at Queen Mary, special thanks go to Vincenzo Calo' for teaching me how to write a paper, Andrew Low for being a fantastic friend and to Will Black and Ilya Bachmatov for bearing my singing and whistling in the office with almost no complaints. My friends Stefano Orani (Teo), Dionigi Benincasa (Dizzi) and Gregorio Benincasa (il Principe) for living with me in the best and worst of my conditions and Oliviero Ursino (Buddinsh) for honest and loyal support. Special thanks to Michel Buck and Gregor Kuglitsch for countless hours of german entertainment and access to the sauna. Thank you to Nicky and Ash Gupta for taking care of me for nothing in return.

I'm grateful to Francesca Dell'Aria whose patience and understanding has been of undeniable support in moments of need. I thank her for introducing me to a wonderful world I knew nothing about.

I'm mostly grateful to my family, who have always believed in me. Without any of them none of this would have been possible, I hope that with this I can partly repay the sacrifices they have made.

Declaration

Except where specifically acknowledged in the text the work in this thesis is the original work of the author. Much of the research presented in this thesis has appeared in the following publications by the author: [1–3] together with Steven Thomas and Vincenzo Calo'.

Additionally, during the course of the preparation of this thesis the author has published several other articles [4–6] the results of which are not included in this thesis.

Contents

CHAPTER 1
INTRODUCTION

1 The Birth of a New Theory

One can safely say that the two greatest achievements of human thought within the area of theoretical physics are the invention of Quantum Mechanics (QM) and the complete formulation of the General theory of Relativity (GR). The first is the completely counter-intuitive notion that the universe at extremely small scales is intrinsically probabilistic, there is no real sense in trying to explore the universe in its most intimate form with the hope of extracting certain answers. Though ideologically and philosophically difficult to digest quantum mechanics in its most mathematical form has led to some of the greatest theoretical models of previously puzzling experimental observations and, through many more of these, has become an undeniable truth about the nature of the universe we live in. Thanks to an ad-hoc quantisation of phase space we have gained a complete understanding of the energy levels of the harmonic oscillator, thanks to the wave-particle interpretation of light we have defeated the double-slit experiment, thanks to the quantisation of the orbital radius we have unveiled the true beauty of the atom, and many many more.

However, it is only during the field theory revolution of theoretical physics that Quantum Mechanics has shown its true unpredictable power. Quantum Field Theory, which as the name suggests involves the quantisation of fields, forms the backbone of all modern theoretical understanding of particle interactions. By merging quantum field theory with the mathematical framework describing the symmetries of our universe theoretical physicists have unified three fundamental forces into one model, creating what is currently the most successful theoretical model of the universe: the Standard Model. The Standard Model describes the interactions of particles through electric, weak and strong forces, it is the Bible of any particle physicist and is the crowning achievement of complex research originating from Quantum Mechanics. Its theoretical foundations awarded five years of Nobel prizes [1] alone and uncountable further results were obtained in areas of research which, by the essential nature of the theory, the Standard Model embraces and reinforces. It is based on the fundamental notion that oscillations in quantum fields generate particles and that these fields form representations of the symmetry group

$$G_{SM} = U(1) \otimes SU(2) \otimes SU(3) \tag{1.1}$$

where $U(1) \otimes SU(2)$ is the symmetry group of the Electro-Weak forces and $SU(3)$ that of Quantum ChromoDynamics (QCD) describing the interactions of quarks and the strong force. Forces are mediated by interchange of virtual particles, called gauge bosons which are massless for long range forces (such as the photon for the electromagnetic force) or massive for short range forces (the case of gauge bosons in representations of a non-abelian symmetry group is not so simple, in this case the indefinite propagation of such massless bosons is restricted by self-interactions). To account for the observed mass of the Weak force gauge bosons the model relies on spontaneous symmetry breaking and predicts the existence of a yet unobserved (but long sought after) Higgs boson, a further Nobel prize to be awarded upon its confirmation. Until the Higgs boson is observed, one is allowed to call this a "weakness" of the Standard Model, together with the numerous free parameters which are adjusted according to experimental observations rather than determined from the theory itself (such as the strength of the coupling constants and masses of particles).

The Standard Model is an incredibly successful theory of three fundamental forces, however four are believed to exist in nature. The fourth is the force of gravity whose description has followed a very different path, diametrically opposite to that originating through the studies of Quantum Mechanics. Gravity is the domain of Einstein whose revolutionary thought and lifetime of research led to a completely different understanding of this force. Einstein related the force of gravity to the geometry of space-time itself, postulating that it is the latter that in its full mathematical description encapsulates the behaviour of gravity and

[1] These were in chronological order: 1969 - Gell-Mann, 1979- Glashow, Weinberg, Salam, 1999- t'Hooft, Veltman, 2004- Gross, Wilczek, Politzer, 2008- Kobayashi, Maskawa, Nambu

that the two descriptions are interchangeable, i.e. gravity produces geometry as a geometry is a form of gravity. What then is the source of this geometry? What “makes” gravity? Well, it is nothing but matter, mass or energy. This is the core structure of Einstein’s equations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu} \tag{1.2}$$

where $R_{\mu\nu}$ is the Ricci tensor formed from the metric $g_{\mu\nu}$ which encapsulates the geometry of space-time and thus describes gravity, Λ is the cosmological constant, G is Newton’s gravitational constant and $T_{\mu\nu}$ is the Energy-Momentum tensor describing the matter content of space-time. Hence, matter creates geometry and geometry is gravity and objects that move and exist in the geometry formed from this matter experience its form of gravity. As a straight line describes the path of least length between two points on a sheet of paper, objects in a gravitational field follow geodesics of the geometry they live in. The gravitational field, and thus the geometry of space-time is something which is dynamic, produced by sources of energy and constantly evolving to adjust for changes in its matter content. Light itself is just a traveller through the geometry and suffers from the same laws as any (massless) observer, it bends and follows geodesics of the gravitational field. Since the observation of the bending of light due to massive objects GR has been confirmed in numerous experiments (Eddington’s famous solar eclipse experiment is an example) and, on a par with QM for the very small, is the best model for how gravity behaves in our universe. It comprises the more classical motion (in its Newtonian approximation) of a ball thrown on earth up to motion of planets and the dynamics and geometrical nature of the universe as a whole. The theory has given birth to the study of Cosmology and it forms the essence of modern theories of the origin of the universe (such as inflation).

Therefore the modern theoretical physicist is armed with two weapons with which to fight the constant struggle to determine the real nature of the universe. In his right hand he holds Quantum Mechanics, with which he can study the very small structures of atoms and particle interactions, on the other he has General Relativity which gives him the power to master the geometrical structure of the universe. A natural question arises: can he put his hands together to form one big weapon, a Grand Unified Theory (GUT) with which he can answer any question scaling from the extremely small to the extremely large, which flows smoothly between QM and GR, a quantum theory of gravity? The answer is not fully known, attempts at unifying QM with GR in a brute force way by quantising the metric $g_{\mu\nu}$ seem to fail from any angle the problem is approached, the theory of the canonically quantised graviton is non-renormalizable. The theories work extremely well by themselves but fail to work together. A new, radical approach to the problem is needed.

2 String Theory

String Theory is a possible answer to the problem. It is a candidate for a consistent theory of quantum gravity and a general unifying theory of all the forces [2]. The basic idea at its core, and from which its name derives, is to abandon the concept of zero-dimensional point particles and assume that the most basic object in nature is a one-dimensional string of length of the order of the Planck length $l_p \approx 1.6 \times 10^{-35} m$. Then as per the frequency of oscillation of a guitar string taut between two end points creates different notes, different modes of oscillation of the fundamental string form different particles, one of which is the graviton. Thus, string theory is a theory of strings whose oscillation modes are quantised to form all ordinary QFT particles plus the graviton. In this sense string theory generates its own geometry (it is a source for the graviton) and is a quantised theory of gravity. Both open strings and closed strings (those that form a closed loop) can exist and are essential to complete the theory. This section is devoted to introducing and summarising the main mathematical ideas and formulations for consistent string theories and the main results and key concepts these theories give birth to. Many of the results and concepts investigated below are general to string theory and not directly in contact with the work presented in this thesis, they are however important in gaining a more general understanding of the completeness of the theory before delving in the more intricate details of the particular topic investigated. When results are reached which have a more important connection with material in the thesis these are pointed out and dealt with in more detail. The

[2] there exist other attempts to create such a theory, e.g. Loop Quantum Gravity, Causal sets, Causal Dynamical Triangulations.

notation used and overall treatment of the ideas presented follow [7], for details on calculations of the presented material the reader is referred to this source.

A natural point to start off with is writing a suitable action describing the dynamics of the string. The first step to consider in order to achieve this is the presence of the extra dimension imposed by dropping the notion of a point particle. For a free particle the action is derived by minimising its world-line, the trajectory the particle sweeps moving through space-time. For a string embedded in space-time, its motion will sweep out a world-sheet rather than a line, and this is the object which needs to be minimised. Thus if σ and τ are the coordinates of the world-sheet and $X^\mu(\tau,\sigma)$ describes its embedding, we can write an action for its area in Minkowski space as

$$S_{NG} = -T \int d\sigma d\tau \sqrt{(\dot{X}\cdot X')^2 - \dot{X}^2 X'^2} \tag{1.3}$$

or

$$S_P = -\frac{1}{2}T \int d^2\sigma \sqrt{-h} h^{\alpha\beta}\partial_\alpha X \partial_\beta X \tag{1.4}$$

(with $\dot{} = \frac{\partial}{\partial\tau}$ and $' = \frac{\partial}{\partial\sigma}$), which are the Nambu-Goto and Polyakov actions for the bosonic string of tension T with world sheet metric $h^{\alpha\beta}$. The two actions are equivalent in the sense that they yield the same equations of motion. The resulting equations for the string embedding obey the wave equation with general solution given by closed string or open string mode expansions composed of sums of equal numbers ($N_L = N_R$) of right-movers X^μ_R and left-movers X^μ_L with the appropriate boundary conditions. The condition $N_L = N_R$ is not imposed by hand, it is a requirement for the theory to be consistent. It is derived by requiring that the modes of the quantised stress-energy tensor (which obey the Virasoro algebra) describe physical (positive norm) states. For the closed string,

$$X^\mu_R = \frac{1}{2}x^\mu + \frac{1}{2}l_s^2 p^\mu(\tau-\sigma) + \frac{i}{2}l_s \sum_{n\neq 0}\frac{1}{n}\alpha^\mu_n e^{-2in(\tau-\sigma)} \tag{1.5}$$

$$X^\mu_L = \frac{1}{2}x^\mu + \frac{1}{2}l_s^2 p^\mu(\tau+\sigma) + \frac{i}{2}l_s \sum_{n\neq 0}\frac{1}{n}\tilde{\alpha}^\mu_n e^{-2in(\tau+\sigma)} \tag{1.6}$$

where x^μ is a centre of mass position and p^μ is the total string momentum describing the free motion of the string centre of mass. A similar expansion exists for the open string solution and the quantisation procedure follows that of the closed string very closely, we will not present this here. The parameter l_s known as the string length scale, is related to the string tension by $T = \frac{1}{2\pi\alpha'}$ where α' is the open string Regge slope parameter $\alpha' = \frac{1}{2}l_s^2$. The terms in the sum $(\alpha_m^\mu, \tilde{\alpha_m^\mu})$ represent the string excitation modes. Here is the key idea of string theory: it is these modes of oscillation which are quantised. By imposing canonical commutation relations on the excitation modes

$$[a_m^\mu, a_n^{\dagger\nu}] = [\tilde{a}_m^\mu, \tilde{a}_n^{\dagger\nu}] = \eta^{\mu\nu}\delta_{m,n}, \qquad [\alpha_m^\mu, \tilde{\alpha}_n^\nu] = 0 \tag{1.7}$$

with $a_m^\mu = \frac{1}{\sqrt{m}}\alpha_m^\mu$ and $a_m^{\dagger\mu} = \frac{1}{\sqrt{m}}\alpha_{-m}^\mu$ one obtains the familiar algebra of quantum mechanical harmonic oscillators with the addition of negative norm states $[a_m^0, a_m^{\dagger 0}] = -1$. These states are unphysical and can be successfully removed from the theory to obtain a consistent spectrum of physical states of the quantised bosonic string, however one must pay the price for this: it can only be achieved in $D = 26$ space-time dimensions. This is a new ingredient of any quantum theory, the emergence of new space-time dimensions above the four that we live in. Clearly, a consistent theory of low energy physics has to make touch with the well-known four dimensional quantum field theory of the Standard Model, this is a far from trivial task and one which string theorist are yet to complete. Furthermore, even though a state may be physical in the sense of having positive norm, it may be the case that the particle it describes has negative mass, i.e. it is Tachyonic. Tachyons are particles which travel faster than light, they are naturally emerging from spectra of string theory and denote an instability of the system. Due to their unphysical nature they cannot be part of a unified theory of gravity plus the Standard Model, they must be removed manually. From the open string sector we obtain

- a Tachyonic state with negative mass^2 $\alpha' M^2 = -1$
- a massless vector boson in the vector representation of $SO(24)$ and
- a symmetric traceless second rank tensor representation of $SO(25)$, i.e. a single massive spin-two state.

Note the appearance of a tachyonic state. From the closed string sector the spectrum consists of

- a Tachyonic ground state
- a trace term singlet of $SO(24)$, a massless scalar called the dilaton $\Phi(X)$,
- a symmetric traceless representation of $SO(24)$, a massless spin-two particle: the graviton $g_{\mu\nu}(X)$, and
- an antisymmetric second rank tensor representation of $SO(24)$, which is a massless two-form gauge field $B_{\mu\nu}(X)$.

Hence, the graviton appears naturally in the spectrum of the closed bosonic string upon quantisation of its oscillation modes. This is, in its simplest form, the major achievement of string theory. It is a theory which sources the graviton in a quantised context: it is a quantum theory of gravity. We can consistently couple the graviton to the string world sheet by allowing a term in the action of the form

$$S_g = \frac{1}{4\pi\alpha'} \int_M \sqrt{h} h^{\alpha\beta} g_{\mu\nu}(X) \partial_\alpha X^\mu \partial_\beta X^\nu d^2 z \tag{1.8}$$

with $z = e^{2(\tau - i\sigma)}$ the conformal coordinates on the string world sheet M[3]. Note the resemblance to the Polyakov action 1.4 which considered only the flat space case of $g_{\mu\nu} = \eta_{\mu\nu}$. We have previously mentioned that Tachyonic states represent unphysical particles and need to be removed, however what are the roles of the new massless scalar and antisymmetric tensor particles? Firstly consider the antisymmetric second rank bosonic gauge field $B_{\mu\nu}(X)$. String theory contains many antisymmetric forms of distinct dimensions. In the more common case of one dimension (the usual gauge field A_μ of electrodynamics) one naturally couples the form to the world line of a charged particle

$$S_A = q \int A_\mu \left(\frac{dx^\mu}{d\tau} \right) d\tau \tag{1.9}$$

where q is the charge of the particle and τ is its world-line parameter, hence

[3]the change of coordinates $\sigma, \tau \rightarrow z$ is done here purely for notational convenience

when the rank of the form is increased it makes sense to consider generalised couplings of the form

$$S_B = \frac{1}{4\pi\alpha'} \int_M \epsilon^{\alpha\beta} B_{\mu\nu}(X) \partial_\alpha X^\mu \partial_\beta X^\nu d^2 z, \tag{1.10}$$

with $\epsilon^{\alpha\beta}$ the rank-two totally antisymmetric tensor. As particles which couple to the Maxwell field are charged, so are strings which couple to two-forms, and therefore these describe in general charged couplings of the string world-sheet. The dilaton plays a fundamentally new and important role in string theory, its appearance in the string action is in a term of the form

$$S_\Phi = \frac{1}{4\pi} \int_M \sqrt{h} \Phi(X) R_2(h) d^2 z, \tag{1.11}$$

where h is the world-sheet metric and $R_2(h)$ is the corresponding Ricci scalar. Consider the case where the dilaton is a constant, then S_Φ is simply the topological invariant quantity

$$\chi(M) = \frac{1}{4\pi} \int_M \sqrt{h} R_2(h) d^2 z, \tag{1.12}$$

which is the Euler characteristic of M, it is a quantity derived from topological features of the world-sheet. String scattering amplitudes are closely related to zero-dimensional particle scatterings, however one must make sure to replace the world-lines of particles with the world-sheets of strings, as one did for the construction of their action. Therefore to calculate string scatterings one needs to sum over all surfaces spanned by interacting open and closed strings. Consider the partition function for scattering amplitudes of strings

$$Z = \int Dh \int DX^\mu ... e^{-S[h,X,...]} \tag{1.13}$$

where $\int Dh$ means the sum over all Riemann surfaces (M, h) and S is the overall string action containing the factors from the two-form, the graviton and the dilaton shown above. Then if the dilaton is a constant $\Phi = \phi_0$ this contributes an overall factor of $e^{-\phi_0 \chi(M)}$ to the partition function, i.e. it can be interpreted as a string coupling $g_s = e^{-\phi_0}$. This is a beautiful result, string theory sources it's own coupling strength by making it the vacuum expectation value (VEV) of the dilaton field, which fully illustrates its role. Note that in α' dimensions the dilaton is dimensionless and thus appears at next order (1 loop) in a coupling expansion, this is not to be expected from a usual coupling. This is indeed a more general result of string theory as a whole, all dimensionless parameters in string theory can be derived from VEVs of scalar fields, the theory has no free parameters (except l_s). Indeed this seems to be an improvement over the well-known Standard Model, which is abundant in free parameters, and is a promising sign of a healthy theory.

Therefore the bosonic string hints at a complete theory of quantum gravity with no free parameters in D=26 space-time dimensions (except l_s). However, it is unsatisfactory if it is required to describe nature for two main reasons: firstly, it lacks a clear understanding of how to eliminate the extra dimensions, secondly and of equal importance the spectrum contains no fermions. Fermions account for all the leptons and quarks present in the Standard model and must arise naturally in any theory which endows itself the task of merging it with gravity. The incorporation of fermions in string theory has been coined the "first string theory revolution" and has led to the invention of the Superstring[4]. Superstring theory is the union of string theory with supersymmetry, a symmetry which relates fermions to bosons. The theory of supersymmetry has been extensively studied in the literature (for a review see [8, 9]). Its major claim is that the universe at higher energies possesses an extra symmetry which interchanges a boson with a fermion. Hence, for every boson and fermion we see in nature there should exist a high energy supersymmetric partner, the sfermion and the bosino. Up to the present day, none of these supersymmetric partners have been observed in particle accelerators.

There are two distinct procedures to include supersymmetry in string theory, the Ramond-Neveu-Schwarz (RNS) [10] formalism which is supersymmetric on the string world-sheet, or the Green-Schwarz (GS) [11, 12] formalism which is supersymmetric on background Minkowski space-time. The two formulations are equivalent in the sense of obtaining the same spectrum for the superstring, however the RNS formalism appears mathematically simpler to present. The theory can

[4]The notion of the superstring was present before the "first string theory revolution" which in fact originates from the miraculous E_8 gauge group anomaly cancellations, but it is during this same period that its true potential was appreciated

be most elegantly expressed on superspace, an extension of ordinary space-time to include anti-commuting Grassmanian numbers. Fields defined on superspace are called superfields. In the RNS formalism one extends the string world-sheet and defines a string super-world-sheet with coordinates $(\sigma^\alpha, \theta_A)$ where $\sigma^\alpha = (\tau, \sigma)$ are the usual string world-sheet coordinates and $\theta_A = (\theta_-, \theta_+)$ are anticommuting Grassmann coordinates, i.e.

$$\{\theta_A, \theta_B\} = 0. \tag{1.14}$$

The Supercharges which generate supersymmetry transformations of the super-world-sheet coordinates are

$$Q_A = \frac{\partial}{\partial \bar{\theta}_A} - (\rho^\alpha \theta)_A \partial_\alpha, \tag{1.15}$$

where ρ^α are two-dimensional Dirac matrices obeying the Dirac algebra $\{\rho^\alpha, \rho^\beta\} = 2\eta^{\alpha\beta}$ and $\bar{\theta} = i\rho^0\theta$. It is possible then to introduce general superfields $Y^\mu(\sigma^\alpha, \theta)$ on the super-world-sheet by considering a general expansion in powers of θ (in Dirac notation),

$$Y^\mu(\sigma^\alpha, \theta) = X^\mu(\sigma^\alpha) + \bar{\theta}\psi^\mu(\sigma^\alpha) + \frac{1}{2}\bar{\theta}\theta B^\mu(\sigma^\alpha), \tag{1.16}$$

where $\psi^\mu(\sigma^\alpha) = (\psi^\mu_-, \psi^\mu_+)$ is a Dirac spinor obeying (classically) $\{\psi^\mu, \psi^\nu\} = 0$ and $B^\mu(\sigma^\alpha)$ is an auxiliary field added to the theory to ensure the supersymmetry transformations close off-shell. Then by the action of the supercharges on the superfield $\delta Y^\mu = [\bar{\epsilon}Q, Y^\mu]$ one obtains the supersymmetry transformations

$$\delta X^\mu = \bar{\epsilon}\psi^\mu \tag{1.17}$$

$$\delta \psi^\mu = \rho^\alpha \partial_\alpha X^\mu \epsilon + B^\mu \epsilon \tag{1.18}$$

$$\delta B^\mu = \bar{\epsilon}\rho^\alpha \partial_\alpha \psi^\mu, \tag{1.19}$$

which interchange fermions and bosons. From these a supersymmetry invariant string-world-sheet action can be derived

$$S = \frac{i}{4\pi} \int d^2\sigma d^2\theta \bar{D} Y^\mu D Y_\mu, \tag{1.20}$$

where $D_A = \frac{\partial}{\partial \theta_A} + (\rho^\alpha \theta)_A \partial_\alpha$ is the supercovariant derivative and a very similar procedure follows that of the bosonic string to produce the full spectrum of the superstring. There are however new ingredients, by ensuring that the variation of the action vanish one finds that there are distinct possibilities on the boundary conditions for the fermions. In the open string sector in light-cone gauge one sets $\psi_+^\mu\big|_{\sigma=0} = \psi_-^\mu\big|_{\sigma=0} = 0$ and is faced with the option of choosing the relative sign at the other end of the string. One choice are Ramond boundary conditions, where one sets

$$\psi_+^\mu\big|_{\sigma=\pi} = \psi_-^\mu\big|_{\sigma=\pi} \tag{1.21}$$

in which case the most general mode expansion for the fermionic field satisfying its equations of motion takes the form

$$\psi_-^\mu(\sigma,\tau) = \frac{1}{\sqrt{2}} \sum_{(n\in\mathbb{Z})} d_n^\mu e^{-in(\tau-\sigma)} \tag{1.22}$$

$$\psi_+^\mu(\sigma,\tau) = \frac{1}{\sqrt{2}} \sum_{(n\in\mathbb{Z})} d_n^\mu e^{-in(\tau+\sigma)} \tag{1.23}$$

where d_n^μ are the modal expansion coefficients similar to the α_n^μ for the bosonic string. A Majorana condition on the fermions requires that these satisfy $d_{-n}^\mu = d_n^{\mu\dagger}$. If instead we pick the Neveu-Schwarz boundary conditions

$$\psi_+^\mu\big|_{\sigma=\pi} = -\psi_-^\mu\big|_{\sigma=\pi} \tag{1.24}$$

then the modal expansion takes the form

$$\psi_-^\mu(\sigma,\tau) = \frac{1}{\sqrt{2}} \sum_{(n\in\mathbb{Z}+\frac{1}{2})} b_n^\mu e^{-in(\tau-\sigma)} \tag{1.25}$$

$$\psi_+^\mu(\sigma,\tau) = \frac{1}{\sqrt{2}} \sum_{(n\in\mathbb{Z}+\frac{1}{2})} b_n^\mu e^{-in(\tau+\sigma)} \tag{1.26}$$

where the major difference in the two choices is the modes being integers or half (odd) integers. In the closed string sector both choices also exist: imposing the boundary conditions $\psi_\pm(\sigma) = \pm\psi_\pm(\sigma+\pi)$ one can write similar modal expansions over integers or half (odd) integers. As per the bosonic string, one quantises the modes of oscillation of the superstring, but crucially one needs to impose anti-commutator relations on the fermionic modes, hence

$$\{b^\mu_r, b^\nu_s\} = \eta^{\mu\nu}\delta_{r+s,0} \tag{1.27}$$

$$\{d^\mu_r, d^\nu_s\} = \eta^{\mu\nu}\delta_{r+s,0} \tag{1.28}$$

which also give rise to negative norm states. Their elimination follows a very similar procedure to that of the bosonic string, however with the superstring one finds that the negative norm states can only be consistently eliminated in $D = 10$ dimensions. Even though this is still far from the goal of making contact with a four-dimensional theory it is a vast improvement over the $D = 26$ dimensions imposed by the bosonic string alone: supersymmetry has dramatically reduced the number of extra dimensions the theory imposes. Note that the fermionic modes furnish a representation of the Dirac algebra $\{\Gamma^\mu, \Gamma^\nu\} = 2\eta^{\mu\nu}$ which means that there exists degenerate ground states such that $d^\mu_0|a\rangle = \frac{1}{\sqrt{2}}\Gamma^\mu_{ab}|b\rangle$. Therefore there exists a natural relation between the fermionic modes and the Gamma matrices of the Dirac algebra $\Gamma^\mu = \sqrt{2}d^\mu_0$ which is crucial for chirality operations, as mentioned later in this section.

The spectrum of open superstring theory, or type I superstring theory, has $\mathcal{N} = 1$ Supersymmetry, for the open string in the NS sector it consists of

- a Tachyonic scalar ground state
- a massless $SO(8)$ vector + tower of massive states

whilst from the Ramond sector we obtain

- a 32-component spinor ground state
- a tower of excited states representing space-time spinors.

The 32 component ground state spinor can be further decomposed by applying Majorana and Weyl conditions, which leads to two possible ground states representing the different ten-dimensional chiralities. The Tachyonic ground state can be consistently removed by taking a GSO projection (see [7] for details), this also makes the spectrum manifestly supersymmetric. This projection is not a choice, it is required to obtain modular invariance without which there are anomalies in global world-sheet diffeomorphisms which would render the theory inconsistent. From the open string type I theory one forms the closed string, type II, spectrum by coupling left-movers and right-movers. This gives a choice between four possible sectors: the R-R, R-NS, NS-R and NS-NS sectors, plus the choice of chirality of the ground state. For type IIB string theory, one couples the left and right moving sectors of the Ramond sector with the same chirality whilst for type IIA string theory the chiralities are opposite. This leads to a total of sixty four states in each of the four massless closed string sectors

- NS-NS: IIA=IIB, one scalar (dilaton), an antisymmetric two-form gauge field and a symmetric traceless two-tensor (the graviton)
- NS-R and R-NS , a spin $\frac{3}{2}$ gravitino and a spin $\frac{1}{2}$ fermion (dilatino). The gravitinos differ in chirality in type IIA whilst have equal chiralities in type IIB.
- R-R, type IIA: one-form vector gauge field and a three form gauge field. IIB: a scalar gauge field (0-form), a two-form gauge field and a four-form gauge field.

Note the appearance of the fermions, in particular the gravitino and dilatino which are superpartners to the graviton and dilaton. Hence the spectrum can be consistently freed of negative norm states and Tachyonic states by reducing the space-time dimension to $D = 10$ and more importantly it now contains both bosons and fermions. Furthermore, one can obtain phenomenologically more appealing string theories by mixing superstring sectors with bosonic string theory sectors. Heterotic string theories are obtained in this way by combining the left moving degrees of freedom of bosonic string theory with the right moving degrees of freedom of superstring theory. These theories are excellent candidates for models of grand unification based on the $SO(32)$ and $E_8 \otimes E_8$ gauge groups.

Superstring theory is therefore even more appealing then its bosonic cousin as a consistent quantum description of gravity. It has reduced the number of required space-time dimensions and has included a complete spectrum of fermions. However, it relies on supersymmetry, which remains an unobserved feature of nature, the dimensions are still greater than the four we observe and it is not unique, one has the theory of the open string, both IIA and IIB theories of the closed string and two classes of heterotic theories. How do we then make sense of the multitude of possible theories? Which one is the one we should use to create a theory of everything (TOE)? It was only during the "second string revolution" that these questions were answered. It turns out that all superstring theories are connected by dualities. A duality is a fundamental concept of theoretical physics, it states that if two systems are dual then they possess the same physics, and there exists a precise map which takes you from one description to the other. This happens in superstring theory as well, there are maps (or dualities) which link superstring theories together. T-duality maps a superstring theory to another superstring theory in a T-dual geometry. The simplest example of this is provided by the closed bosonic string compactified on a circle. The space-time geometry is $\mathcal{M} = \mathcal{R}^{24,1} \times S^1$, where the circle S^1 has radius R. This endows "modified" periodic boundary conditions to the embedding coordinates on the $\mathcal{S}^1$

$$X^{25}(\sigma + \pi, \tau) = X^{25}(\sigma, \tau) + 2\pi RW, \tag{1.29}$$

where W is the winding number, the number of times the string wraps the closed circle. Following usual Kaluza-Klein theory, the momentum along the compact dimension will be quantized, i.e.

$$p^{25} = \frac{K}{R} \tag{1.30}$$

where K is an integer called the Kaluza-Klein excitation number describing the momentum mode excitation level. The equation for the mass of the modes of the spectrum of the compactified bosonic string is

$$\alpha' M^2 = \alpha' \left[\left(\frac{K}{R}\right)^2 + \left(\frac{WR}{\alpha'}\right)^2 \right] + 2N_L + 2N_R - 4, \tag{1.31}$$

where N_L and N_R are the number of left-movers and right-movers respectively ($N_L \neq N_R$ due to the compactification). This equation is invariant under the map $\left(R \to \frac{\alpha'}{R}, W \to K\right)$, which describes the T-duality of the bosonic string. It indicates that the bosonic string compactified on a circle is dual to a bosonic string compactified on a circle of inverse radius, with winding modes and Kaluza-Klein modes interchanged. It is quite a counter-intuitive geometrical result which can only be understood by realising that the radius in question is small (of the order of the string scale) and that ordinary geometrical concepts break down at these scales, as they should in a theory of quantum gravity. T-duality is a consequence of the string being a one-dimensional object, indeed if the above analysis was repeated with a point-particle we would find that its winding mode $W = 0$ and therefore no duality exists. The presence of the duality is in fact independent of the size of R. The interchange between winding number and Kaluza-Klein excitation number is indicative of a map between the spectra of dual open string theories, from it it can be shown that T-duality maps (in the compact direction) $X_R \to -X_R$ and $X_L \to X_L$. This is an essential fact that allows us to extend T-duality to the closed sector of superstrings (as these are built from combinations of left and right moving sectors of open strings). Imagine compactifying ten-dimensional superstring theory on a circle. The bosonic coordinates map in the same way as bosonic string theory under T-duality and world-sheet supersymmetry ensures that the fermionic coordinates map in the same way as the bosonic ones, $\psi_L \to \psi_L$ and $\psi_R \to -\psi_R$. The latter relationship means that the chirality of the ground states is reversed under T-duality (recall the relation between the fermionic zero modes and the Dirac Gamma matrices previously mentioned), and since this is the only thing distinguishing between the IIA and IIB superstring theories we can deduce that IIA theory compactified on a circle of radius R is T-dual to IIB theory on a circle of radius $\frac{\alpha'}{R}$. Furthermore, T-duality maps the IIA and IIB coupling constants

$$g_s^{IIB} = \frac{\alpha'}{R} g_s^{IIA} \tag{1.32}$$

hence a perturbative expansion in g_s^{IIB} corresponds to a perturbative expansion in g_s^{IIA} and T-duality holds order by order. Hence the two theories are directly linked by a duality, i.e. they possess the same physical content. Analogously to the map between a compactifying radius and its inverse, S-duality maps a string theory with coupling constant g_s to a different string theory with coupling $\frac{1}{g_s}$. When one is strongly coupled the other is weakly coupled. S-duality can be understood as an extension of the common electric-magnetic duality of Maxwell's equations, together with the Dirac quantisation condition. A simple example is $\mathcal{N} = 4$ super Yang-Mills under the electric-magnetic duality (i.e. switching electric and magnetic parameters) combined with the coupling transformation $g_{YM} \rightarrow \frac{4\pi}{g_{YM}}$. S and T dualities together form an intricate network which links string theories, the results of dualities on each theory presented so far are listed in the table below.[5]

Theory	T-duality	S-duality
Heterotic Type I $SO(32)$	Heterotic Type I $E_8 \otimes E_8$	Type I open string
Heterotic Type I $E_8 \otimes E_8$	Heterotic Type I $SO(32)$	M-theory at $g_s \rightarrow \infty$
Type IIA superstring , R, g_s	Type IIB on $\frac{\alpha'}{R}$, g_s	M-theory $g_s \rightarrow \infty$
Type IIB superstring, R, g_s	Type IIA on $\frac{\alpha'}{R}$, g_s	Type IIB on R, $\frac{1}{g_s}$
Type I open string	Type I' open string	Heterotic Type I $SO(32)$

The last of these dualities, those regarding the open string, deserve special attention. The map $(X_R \rightarrow -X_R, X_L \rightarrow X_L)$ of bosonic right moving and left moving coordinates translates to a transformations on the boundary conditions of the open string. T-duality maps open strings with Neumann boundary conditions into open strings with Dirichlet boundary conditions, on a dualised geometry. Therefore it switches strings with momentum to strings with winding in the compactified direction. It is as if the dual open string is stuck to a hyperplane of the dual geometry. In effect, the dual string has end points which are bound on a membrane. Membranes where open strings end are called Dirichlet Branes (D-Branes) and are an essential and incredibly important ingredient of string theory. The key point here is that, whereas intuitively one might think that these objects are simply mathematical hyperplanes of a specified geometry, they are in fact physical objects per se. They also exist in closed string theories such as IIB and IIA superstring theory where they couple to gauge forms by providing a similar hypersurface coupling as that of the string-world-sheet. Hence a Dp brane, or a brane which spreads in p dimensions can couple to a $p+1$ form through its world

[5]For the case of Heterotic $E_8 \otimes E_8$ and Type IIA superstring theory M-theory at strong coupling is not precisely an S-dual theory as we shall see below. However we present it here as so to illustrate the general picture of the intricate web of dualities that exist throughout string theory. Also, the T-dual to type I open string theory, type I' theory, is simply an orientifold projection of type IIA theory compactified on a dual circle of radius $\frac{\alpha'}{R}$

volume

$$S_{int} = \frac{q_p}{(p+1)!}\int A_{\mu_1....\mu_{p+1}}\frac{\partial x^{\mu_1}}{\partial \sigma_0}.......\frac{\partial x^{\mu_{p+1}}}{\partial \sigma_p}d^{p+1}\sigma \tag{1.33}$$

where q_p denotes the charge carried by the Dp brane, μ_p indicates the gauge form indices and σ_p are the coordinates on the brane world-volume. These branes are thus electrically and magnetically (through standard Hodge duality) charged and carry flux. They also have a definite tension or energy, they are all in all physically independent real objects appearing in string theory. In type IIA theory the spectrum of the R-R sector carries p-forms with odd dimensions, hence the theory contains Dp branes with even dimensions to which these forms can couple. Similarly type IIB theory contains p-forms of even dimensions and Dp branes of odd dimension. D-branes which are charged under a p-form coupling are stable, those that don't are unstable and give rise to an open string spectrum which carries a Tachyon. Hence, in both type II superstring theories any dimension Dp brane exist, but only those that couple to $(p-1)$ forms are stable. The unstable D-branes are thought to decay via emission of closed string radiation. The fate of Tachyons on unstable D-branes, and the investigation of their dynamical description, forms the core of this thesis.

With the idea that Dp branes describe physical hypersurfaces where strings can end one is naturally led to the generalisation to multiple membranes. Indeed a string can have both end points on one-brane or one end point ending on one brane and the other on another brane. In this way there would be a theory of one string stretching between two D-Branes. It is in general a good idea to assign a label to the end point of a string, this will provide it with additional degrees of freedom. Chan-Paton factors associate N degrees of freedom to string end-points. By letting one end-point carry a fundamental representation of the N degrees of freedom and the other the anti-fundamental representation one can describe a gauge theory with $U(N)$ symmetry. This result is most easily understood in string scattering calculations where the Chan-Paton factors appear explicitly in amplitudes contributing identically to standard $U(N)$ gauge indices. In terms of strings on D-branes, when there are n non-coincident D-branes strings stretching between them describe a $U(1)^n$ gauge theory. When instead these branes coincide, the symmetry is enhanced to $U(n)$. Thus we see that by assigning Chan-Paton factors to end points of strings we can easily include gauge symmetries, both abelian and non-abelian, in superstring theory. This is a crucial step in making contact with the non-trivial gauge symmetry of the Standard Model, equation 1.1. Given the fact that D-Branes are physical objects in their own right, we are faced with the challenge of finding a suitable action that describes their dynamics and that includes their gauge symmetry. The D-Brane action arises as a p-dimensional generalisation of an attempt by Born and Infeld to eliminate the infinite self-energy of the charged point particle in classical Maxwell theory. This resulted in the abelian bosonic Dirac-Born-Infeld action for the Dp-brane which is central to this thesis and has its own section (2) dedicated to it.

In summary, the theory of superstrings and D-Branes is a consistent theory of quantum gravity in $D = 10$ dimensions which includes both the graviton and fermions in its spectrum. Its major flaws are that it relies on supersymmetry which hasn't been observed yet, and that there still is no direct contact to the four dimensions we observe. The first of these problems, and possibly the simplest to conceptualise, is being dealt with through a constant struggle to find supersymmetric partners of known particles at increasingly high energies in modern day particle accelerators . There is hope that with the Large Hadron Collider (LHC) up and running we might observe a supersymmetric partner below the energies of 14 TeV. This does not mean that if the particle is not observed below this energy scale then supersymmetry is ruled out. The second however is one which must be dealt with far from the laboratory and only through the complex process of theoretical modelling. There are two main lines of thought with what to do with extra dimensions: the first asserts that the extra dimensions are simply too small to be observed at our scales of energy, i.e. they only really make a noteable impact at scales close to the string scale, the second is that the extra dimensions are simply unaccessible to us, we live on a hypersurface of a higher-dimensional world in which we cannot detect the extra dimensions (which may indeed be very large) at least at the energies accessible to present technology. The first of these is the idea of Kaluza-Klein Compactification, the second is generally called Brane-World Scenario (BWS).

In Kaluza-Klein compactification, one compactifies the ten-dimensional space-time of superstring theory on a product manifold of the form

$$\mathcal{M} = \mathcal{M}_4 \otimes \mathcal{C}_6 \tag{1.34}$$

where $\mathcal{M}_4$ is Minkowski space and $\mathcal{C}_6$ is a six-dimensional suitable chosen manifold which gives promising phenomenological theories in the compactification. The idea is that the scale of the extra six dimensions of the compactified manifold are of the order of the string scale and thus by far too small to be detectable in present day experiments. The obvious choices are compactifications which lead to a theory with the Standard Model gauge group in four dimensions and which break supersymmetry down to $\mathcal{N} = 1$ as this is the only supersymmetry consistent with chirality of the fermions. Therefore the compactifying manifold has to be chosen very carefully, if for example one should try compactifying on the simple six torus then no supersymmetry would be broken and this would make no contact with phenomenological models. It turns out that the most promising compactifying manifolds are of Calabi-Yau type. These are special kinds of n-folds which are Kahler and have $SU(n)$ holonomy. Being complex, it is three-folds (which have six real dimensions) which are used for compactifications from ten dimensions. Much progress has been achieved in compactifying the heterotic superstring on Calabi-Yau three folds, the problem is that the precise number of such three folds is unknown, and indeed whether this is a finite number is also not known although some are known to be related to each other by Mirror Symmetry. Also, compactifications result in the presence of massless scalar fields, or moduli fields, which should not be there as we know from experimental observations (for example the dilaton or the radial modulus). Models have been constructed in which all moduli fields are stabilised by flux compactifications, in which higher dimensional fluxes are compactified to provide potentials for the moduli fields, but a complete consistent reduction to the $D = 4$ Standard Model by a Kaluza-Klein reduction is still to be achieved.

The alternative to the Kaluza-Klein method of taking very small compactified dimensions is the Brane-World Scenario. In this model the world we live in is believed to reside on a three dimensional brane, which is itself embedded in the full ten dimensional space-time. The fields of the standard model live on the brane and only gravity is allowed to permeate the full space-time. Within the context of string theory this idea seems to have a natural explanation as open strings, which generate the fields we see, would be stuck by their ends to the three brane whilst only closed strings, which describe gravity through the graviton, can travel outside the brane in the full space-time. In fact, by making the outside space-time warped one can find suitable models which solve the hierarchy problem of the Standard Model, that is to explain the large differences in energy scales between standard model sectors (for example why the quark masses are so much lighter than the Planck scale, the scale at which quantum gravity effects become important). This space introduces a warp factor which warps all scales on the visible sector brane compared to the Planck scale (see [13, 14] for a review on the subject). Brane world scenarios also play an important role in models of supersymmetry breaking where the idea of us living on an independent brane is combined with a hidden sector brane, somewhere along an extra dimension of the full space-time, in which supersymmetry breaking occurs and can be mediated to us by an appropriate mechanism (see [15] for a review on the subject). Both the Kaluza-Klein compactification method and the Brane-World Scenario are promising models for making contact between string theory and the real world. On one side the major challenge is to determine exactly how many Calabi-Yau three fold compactifications exist, and hence which one is the more promising one for phenomenological models and how to consistently eliminate moduli fields from the resulting theory. On the contrary, Brane-World Scenarios provide excellent candidates for resolving hierarchy issues but lack a complete understanding of their high energy completion, there is no direct low energy string theory solution which results in acceptable models of Brane-worlds, not to mention that branes have never been directly observed.

Being theories at the order of the string scale, and thus at extremely high energies, there is a more direct method of establishing some connection between more accessible energy scales and superstring theories. If superstring theory describes a theory of quantum gravity at the order of the Planck scale, what is its low energy limit? What do we obtain if we reduce the energy scale of superstring theories manually? The answer to these questions has led to one of the greatest theoretical discoveries of string theory and one which has important implications for a concrete theory of everything. The low energy action of type IIA superstring theory is ten dimensional type IIA supergravity, its bosonic part consists of

$$S = S_{NS} + S_R + S_{CS} \tag{1.35}$$

where S_{NS} involves fields from the NS sector and S_R and S_{CS} involve R-R sector fields. More specifically,

$$S_{NS} = \frac{1}{2\kappa^2} \int d^{10}x \sqrt{-g} e^{-2\Phi} \left(R + 4\partial_\mu \Phi \partial^\mu \Phi - \frac{1}{2}|H_3|^2 \right) \tag{1.36}$$

with $2\kappa^2 = \frac{1}{2\pi}(2\pi l_s)^8$, Φ being the usual scalar field of superstring theory and H_3 is the three-form field strength,

$$S_R^{IIA} = -\frac{1}{4\kappa^2} \int d^{10}x \sqrt{-g} \left(|F_2|^2 + |\tilde{F}_4|^2 \right) \tag{1.37}$$

where F_2 and F_4 are two-form and four-form field strengths with

$$\tilde{F}_4 = dA_3 + A_1 \wedge H_3 \tag{1.38}$$

and

$$S_{CS}^{IIA} = -\frac{1}{4\kappa^2} \int B_2 \wedge F_4 \wedge F_4 \tag{1.39}$$

with B_2 the R-R two-form. Similarly the low energy bosonic action of type IIB superstring theory is type IIB supergravity, with equal S_{NS} but different $R - R$ sector actions, which results from the difference in the R-R sector superstring fields. The bosonic part of $D = 11$ supergravity is

$$S = \frac{1}{2\kappa_{11}^2} \int d^{11}x \sqrt{-G} \left(R - \frac{1}{2}|F_4|^2 \right) - \frac{1}{6} \int A_3 \wedge F_4 \wedge F_4 \tag{1.40}$$

with κ_{11} the eleven dimensional gravitational coupling constant, and by compactifying this on a circle and integrating out the modes in the compact dimension one obtains the IIA supergravity action quoted above (1.37). Hence it seems that the low energy limit of type IIA superstring theory corresponds to the compactification of eleven dimensional supergravity, a natural question arises: since $D = 10$ supergravity is the low energy limit of $D = 10$ superstring theory, is this $D = 11$

supergravity also a low-energy limit of some $D = 11$ theory? And fundamentally, since all the superstring theories are related to each other via dualities, is this high energy theory the high-energy limit of all superstring theories? The answer is yes, to both questions. The eleven dimensional supergravity action, from which one can achieve type IIA supergravity and by dualities all other low energy limits of superstring theores, is the low energy limit of M-theory. M-theory is a non-perturbative version of string theory that contains no strings, its name was introduced by E. Witten but its true significance is unknown ("magical", "mysterious", "mother" and "membrane" are all candidates) . Precisely for this reason, its lack of a perturbative expansion, it is not well understood but evidence for its existence is overwhelming. This includes the interpretation of $D0$ branes in type IIA superstring theory as the first Kaluza-Klein excitation states of the eleven dimensional massless supergravity multiplet compactified on a circle. The mass of the $D0$ branes is given by $(l_s g_s)^{-1}$ and the mass of the compactified eleven dimensional supergraviton is

$$(M_N)^2 = \left(\frac{N}{R_{11}}\right)^2 \tag{1.41}$$

where N is an integer which describes a tower of states in the compactification. Hence for the first state $N = 1$ we require that

$$R_{11} = l_s g_s. \tag{1.42}$$

Therefore the radius of compactification is proportional to the string coupling constant and hence the decompactification limit $R_{11} \to \infty$ corresponds to the strong coupling limit of type IIA superstring theory. This decompactification limit should lead us directly to M-theory, as $D = 11$ supergravity is its low energy limit. Hence the strong-coupling limit of type IIA superstring theory is nothing but M-theory. Recall however that the fundamental objects of M-theory are not strings, so indeed one can extend this correspondence further and postulate that the fundamental string in type IIA superstring theory is simply derived from a two-dimensional brane in M-theory wrapped on a circle. Similarly the $D4$ brane would be a five dimensional membrane wrapped on a circle (the $D6$ brane doesn't follow this identification, being the magnetic dual of the $D0$ brane it is interpreted as a Kaluza-Klein monopole in eleven dimensions). The dynamics of these $M2$

and $M5$ branes, which are the fundamental objects of M-theory, has been captured in terms of trialgebras by Bagger and Lambert [16–18]. Further evidence for the existence of this high-energy theory comes from dualities regarding the heterotic string, where it is eventually shown that M-theory compactified on $\frac{S^1}{\mathbb{Z}_2}$ is dual to $E_8 \otimes E_8$ heterotic string theory in ten dimensions. From these and many more we can say there is compelling theoretical evidence that all superstring theories are daughters to one fundamental high-energy theory in eleven dimensions. The study of M-theory is complex due to its non-perturbative nature and to the present day little is known about it.

Through the implementation of this web of dualities string theorists have linked the five superstring theories together first, then all of these to a larger parent theory called M-theory. This is a major achievement and indeed gives us hope that if string theory is the way the universe works then we have truly made gigantic steps towards its full understanding. However the story doesn't end here. There is a further duality arising from string theory which encapsulates the beautiful essence of its dual gauge/gravity description. One knows that higher dimensional supergravity theories, such as those derived in the low energy limits of superstring theories possess gravitational solutions describing extended black-hole like objects called p-branes. For theories such as superstring theory, where we have seen that extended objects arise naturally as hypersurfaces on which string end points can end, there is a natural interpretation in identifying the two. Polchinski [19] showed that indeed p-branes and $D-$branes can be interpreted as the same objects, and that therefore the latter arise naturally as gravitational solutions of superstring theory. In this sense the D-branes are sources of geometry, and thus of gravity. Now since gauge fields live on branes through string excitations, does there exist a way to link the gauge theory derived from strings to the gravitational geometry sourced by the branes the strings end on? Originally, the answer to this question was derived by considering stacks of N parallel $D3$ branes. In general the link works as follows: the near-horizon geometry of a collection of coincident D-branes in the limit that this number is large is dual to the world-volume gauge theory of the corresponding branes. For the specific case of a stack of N $D3$ branes, the near horizon geometry corresponds to five-dimensional anti-de-Sitter space times a five-sphere

$$\mathcal{M} = AdS_5 \otimes S_5, \tag{1.43}$$

and the low energy world-volume gauge theory is simply $\mathcal{N} = 4$ Super-Yang-Mills in four dimensions, which is conformal. The duality works as follows: the integer N which is the number of stacked $D3$ branes corresponds to the rank of the gauge group, the coupling constants from the string side and Yang-Mills side are matched through the relation $g_{YM}^2 = 4\pi g_s$, the radius of the AdS_5 is related to the t'Hooft parameter $\lambda = g_{YM}^2 N$ of the gauge theory by $R = \lambda^{\frac{1}{4}} l_s$ and the generators of the Killing isomotries of the geometry correspond to the generators of the superconformal group in four dimensions. Boundary values of bulk fields act as sources for conformal operators in the dual field theory through the relation [20]

$$\left\langle e^{\int \phi_0 \mathcal{O}} \right\rangle = e^{(-S_{os}[\phi_0])} \tag{1.44}$$

where ϕ_0 is the boundary value of the bulk field ϕ and S_{os} denotes the bulk action evaluated on-shell. This duality, proposed by Maldacena [21], was named the AdS/CFT correspondence and its implications are gigantic. It relates a strongly coupled gauge theory to a theory of pure gravity. Through this correspondence calculations in strong coupling non-perturbative regimes of field theory can be performed by the dual gravitational picture, this is the essence of the developments in understanding models of QCD [22] (AdS/QCD) in which strong coupling calculations are essential and in condensed matter theory [23–25] (AdS/CMT) in which models of holographic superconductivity can be constructed by describing a condensate in the field theory as a black hole developing a hairy profile in the gravitational picture. These are just two of a vast literature of applications of the correspondence, for a review see [26] . Whilst the duality remains a conjecture and has not been fully proven it is widely accepted as true in the theoretical community, with increasing number of calculations in the gauge and gravity side confirming it.

In conclusion, already in its bosonic formulation by considering quantisation of the open and closed string modes of oscillation string theory produced an ideologically revolutionary and mathematically elegant theory of quantum gravity. The graviton, the particle which mediates the force of gravity, appears naturally in the spectrum of the closed bosonic string. However so do Tachyons, particles which are unphysical and must be removed from the theory in order to make contact with more realistic models of the universe. Furthermore, the theory is only free of negative norm unphysical states in $D = 26$ dimensions, far greater than the four we observe and contains no fermions. At this stage string theory seems only a plausible idea but lacks the real mathematical modelling to describe anything which is phenomenologically promising. When string theory is fused with supersymmetry the idea takes a significant step further towards its goal. Superstring theories consistently include fermions, the graviton and reduce the required number of dimensions to $D = 10$. But there are five of them, and which one should be used to best describe nature? In fact, all superstring theories are part of a joined picture through dualities. These include T-duality, which is a perturbative duality between string theory compactified on a circle of radius R to a similar string theory on $\frac{\alpha'}{R}$, and S-duality which relates weak coupling to strong coupling theories via $g_s \to \frac{1}{g_s}$. We observed that progress can be made in attempting to eliminate the extra number of dimensions by Kaluza-Klein compactification and that the most phenomenologically promising manifolds on which to compactify are of the Calabi-Yau form. Through T-duality, which interchanges Neumann with Dirichlet boundary conditions for strings, higher dimensional physical objects called D-Branes were discovered and it was understood that these were simply the familiar p-brane solutions in supergravity known time before. Hence D-branes are sources of geometry and therefore gravity and through their couplings to gauge fields are also charged. Through them we constructed models which could indeed describe more common features of the universe and resolve hierarchy issues, the Brane-World Scenarios. We observed that the low energy effective supergravity action of type IIA superstring theory can be interpreted as coming from a higher dimensional $D = 11$ theory compactified on a circle. Through dualities we therefore postulated that the high energy limit of this theory is unique and common to all string theories and that it describes their high energy unification into one parent theory, called M-theory: the best candidate for a real Theory of Everything. Finally we mentioned that there exists strong evidence for a further duality which relates a gauge theory at strong coupling with a purely gravitational picture. The AdS/CFT correspondence holds within it the potential for achieving the unthinkable, it gives hope to string theorists to test results directly in a laboratory.

Of all the areas touched on by string theory, of which the above are only some, this thesis investigates aspects of non-abelian D-Brane world-volume dynamics specifically focussing on the fate of Tachyons on unstable D-Branes. In sections 2 and 3 we provide an introduction to the specifics of the subject, with a detailed derivation of the non-abelian DBI action of coincident D-Branes and an overview of previous research on Tachyons and their abelian dynamics. We will present here Sen's conjectures [27] and illustrate the general method by which solitonic solutions for the abelian Tachyon profile generate world-volume theories of co-dimension branes. We also give a detailed presentation of Kutasov's work [28] on unstable D-Brane dynamics in the proximity of $NS5$ branes and introduce the symmetry between this setup and that of the unstable Tachyon. Section 4 is devoted to extending above investigations to their corresponding non-abelian theories. The kink Tachyon solution is investigated in the case of multiple coincident branes and next we demonstrate similar procedures for the case of a monopole solution. Later in the section we investigate the idea of a geometrical Tachyon interpretation in a multiple brane non-abelian set up. We will see here that there are new non-trivial features arising from the non-abelianity of the system. Finally, in section 5 we provide a summary of the conclusions obtained and point out areas of further work.

CHAPTER 2

THE EFFECTIVE TACHYON DBI ACTION

1 The non-abelian DBI action of coincident Branes

In this section we derive the non-abelian action for N coincident Dp-branes by T-dualising the non-abelian action for coincident $D9$-Branes. The end result is a crucial starting point for further analysis in this thesis. The analysis follows [29] to which the reader is referred to for complete details.

The theory of type IIA and type IIB superstrings contains many fields of varying rank. These include the string frame metric $G_{\mu\nu}$ (note the change in notation from the previous chapter $g_{\mu\nu} = G_{\mu\nu}$), the dilaton ϕ and the anti-symmetric two-form $B_{\mu\nu}$. As explained in chapter 1, these fields couple to higher dimensional world volumes and this leads to a natural construction of appropriate actions for D-Branes. The general argument for the analysis that will lead to the end result is the following: we will start with the action generated by considering $D9$-branes which are space-filling and thus have no transverse scalar fields and T-dualise in directions along the world volume of the D-brane. This will give co-dimension one $Dp \to D(p-1)$ branes which will allow us to write the full action of $D(p < 9)$-branes including the transverse scalar fields. To this purpose we recall the action of T-duality on the background fields

$$\tilde{G}_{yy} = \frac{1}{G_{yy}}, \qquad e^{2\tilde{\phi}} = \frac{e^{2\phi}}{G_{yy}} \tag{2.1}$$

$$\tilde{G}_{\mu\nu} = G_{\mu\nu} - \frac{G_{\mu y}G_{\nu y} - B_{\mu y}B_{\nu y}}{G_{yy}}, \qquad \tilde{G}_{\mu y} = \frac{B_{\mu y}}{G_{yy}} \tag{2.2}$$

$$\tilde{B}_{\mu\nu} = B_{\mu\nu} - \frac{B_{\mu y}G_{\nu y} - G_{\mu y}B_{\nu y}}{G_{yy}}, \qquad \tilde{B}_{\mu y} = \frac{G_{\mu y}}{G_{yy}} \tag{2.3}$$

where y denotes the coordinate with respect to which T-duality is applied whilst μ, ν denote the rest of the coordinate directions. To make contact with the previously introduced geometric concept of T-dualizing, if y is made periodic on a circle, i.e. $y = y + 2\pi R$ then after T-duality the new radius becomes $\tilde{R} = \frac{\alpha'}{R}$ and the coupling constant shifts by $\tilde{g}_s = \frac{g_s l_s}{R}$. If we define a ten-dimensional matrix

$$E_{\mu\nu} = G_{\mu\nu} + B_{\mu\nu} \tag{2.4}$$

then we can define the action of T-duality on multiple world-volume directions $i, j = p+1, ..., 9$ on it by

$$\tilde{E}_{ab} = E_{ab} - E_{ai}E^{ij}E_{jb}, \quad \tilde{E}_{aj} = E_{ak}E^{kj}, \quad \tilde{E}_{ij} = E^{ij}, \tag{2.5}$$

with $a, b = 0, 1, ..., p$ denoting the remaining coordinate directions and E^{ij} being the inverse of E_{ij}. Similarly the transformation of the dilaton takes the simple form

$$e^{2\tilde{\phi}} = e^{2\phi} det\left(E^{ij}\right). \tag{2.6}$$

The starting point is the non-abelian $D9$-Brane action with $U(N)$ symmetry [29]

$$S_{DBI} = -T_9 \int d^{10}\sigma Tr\left(e^{-\Phi}\sqrt{-det\left(G_{ab} + B_{ab} + \lambda F_{ab}\right)}\right) \tag{2.7}$$

with $T_9 = \frac{1}{g_s(2\pi)^9(\alpha')^5}$, $\lambda = 2\pi\alpha'$ and there is no need to include a pull-back due to the absence of transverse scalar fields. Note that one can consistently include the fermionic degrees of freedom in the DBI action and make it manifestly supersymmetric, for simplicity we will not include the full fermionic DBI action here. The DBI action makes one crucial assumption: that the fields are "slowly varying", i.e. it ignores higher derivatives of fields appearing in the action, this approximation will be assumed throughout this thesis. As mentioned in chapter 1 the world-volume theory of N coincident branes admits non-abelian $U(N)$ gauge symmetry, captured by the usual non-abelian definitions for gauge fields, field strengths and covariant derivatives

$$A_a = A_a^n T_n \tag{2.8}$$

$$F_{ab} = \partial_a A_b - \partial_b A_a + i[A_a, A_b] \tag{2.9}$$

$$D_a \Phi^i = \partial_a \Phi^i + i[A_a, \Phi^i], \tag{2.10}$$

with T_n being $N \times N$ matrices satisfying $Tr(T_n T_m) = N\delta_{nm}$. Let us first illustrate why one cannot simply perform the non-abelianisation of the DBI action manually, that is promoting fields to matrices, derivatives to covariant derivates, field strengths to include commutators and taking an overall Trace. When we T-dualise on a direction along the D-Brane world volume $y = x^p$ then the resulting theory will be that of a $D(p-1)$-Brane with y being an extra transverse direction. Therefore the role of the pth gauge field must switch to that of a transverse scalar field

$$A_p \to \Phi_p \tag{2.11}$$

which means that the corresponding p-component of the field strength becomes

$$F_{ap} \to D_a \Phi_p \tag{2.12}$$

Therefore in the non-abelian case where F_{ab} has an extra commutator term the above relations show that T-duality transforms

$$D_p \Phi^i \to i[\Phi^p, \Phi^i] \tag{2.13}$$

and therefore this generates new world-volume interactions between the scalars that would simply be missed by a conventional ad-hoc abelian to non-abelian promotion of the fields. The complete process of promoting the DBI action to the non-abelian case has to be performed by T-duality step by step. We proceed to T-dualise on $9-p$ coordinates $x^i = p+1, ..., 9$ therefore if $D = det(E_{ab} + \lambda F_{ab})$ then this becomes

$$\tilde{D} = det \begin{pmatrix} E_{ab} - E_{ai}E^{ij}E_{jb} + \lambda F_{ab} & E_{ak}E^{kj} + \lambda D_a \Phi^j \\ -E^{ik}E_{kb} - \lambda D_b \Phi^i & E^{ij} + i\lambda[\Phi^i, \Phi^j] \end{pmatrix} \tag{2.14}$$

and by defining the following matrix

$$Q_i^j = \delta_j^i + i\lambda[\Phi^i, \Phi^k]E_{kj} \tag{2.15}$$

and using the fact that upon addition to a matrix of multiples of columns and rows of the same matrix the determinant remains invariant [1] one can show that the above determinant reduces to

$$\tilde{D} = det\left(P\left[E_{ab} + E_{ai}\left(Q^{-1} - \delta\right)^{ijE_{jb}}\right] + \lambda F_{ab}\right) det(E^{ij})det(Q_j^i). \tag{2.16}$$

where $P[G_{ab}] = G_{\mu\nu}\frac{\partial x^\mu}{\partial \sigma^a}\frac{\partial x^\nu}{\partial \sigma^b}$ is the usual pull-back to the brane world volume. Using the transformation of the dilaton

$$e^{-\phi} \to \frac{e^{-\phi}}{\sqrt{E^{ij}}} \tag{2.17}$$

the second factor in the determinant cancels from the overall resulting action and integrating out the compactified directions we obtain the change in the tension

$$T_9 \to \prod_{i=p+1}^{9}(2\pi R_i) \tag{2.18}$$

which combined with the transformations for the radii and couplings

$$R_i \to \frac{l_s^2}{R_i} \qquad g_s \to g_s\frac{l_s^{9-p}}{\prod_{i=p+1}^{9} R_i} \tag{2.19}$$

gives an overall pre-factor which is exactly $T_p = \frac{1}{g_s(2\pi)^p(\alpha')^{\frac{(p+1)}{2}}}$. Hence combining these results together we obtain the T-Dual action for N coincident Dp-Branes

$$S_{DBI} = -T_p \int d^{p+1}\sigma Tr\left(e^{-\phi}\sqrt{-det\left(P[E_{\alpha\beta} + E_{\alpha i}(Q^{-1} - \delta)^{ij}E_{j\beta}] + \lambda F_{\alpha\beta}\right) detQ_j^i}\right). \tag{2.20}$$

The second determinant contains the intrinsic non-abelian nature of the action and provides the scalar potential for the theory, as can be seen from the interaction

[1] this is easily verified in the case of abelian entries for the matrix, but in the non-abelian case this relation is not trivial and requires special care

terms arising from its expansion in the flat space limit ($G_{\mu\nu} = \eta_{\mu\nu}$ and $B_{\mu\nu} = 0$)

$$\sqrt{detQ^i_j} = 1 - \frac{\lambda^2}{4}[\Phi^i, \Phi^j][\Phi^i, \Phi^j] + ... \tag{2.21}$$

The Chern-Simons action involving the R-R couplings to the $D9$-brane is

$$S_{CSD9} = \mu_9 \int Tr\left(Ce^{B+\lambda F}\right)_{10}. \tag{2.22}$$

and its promotion to the non-abelian case follows the same procedure of T-dualisation, the end result is quoted below for completeness but is not investigated further in this thesis. The full result for the complete Chern-Simons coupling action is

$$S_{CS} = \mu_p \int Tr\left(P[e^{i\lambda i_\Phi i_\Phi} C]e^{B+\lambda F}\right) \tag{2.23}$$

where i_Φ denotes the interior product by Φ^i regarded as a vector in the transverse space. [2]

2 Effective Tachyon DBI action

As previously stated, the distinguishing feature between stable (BPS) branes and unstable (non-BPS) branes is the presence of a negative mass Tachyonic mode in the spectrum of open strings on a non-BPS brane. Also, BPS branes are charged under RR $(p+1)$ form gauge fields of string theory, whilst non-BPS branes are not. One would really like to investigate the dynamics of the Tachyonic mode, however this is not an easy task. Since the mass of the Tachyonic mode is of the same order of magnitude as that of the other heavy modes of the string, one cannot simply work with a low energy effective action which results by integrating out other heavy modes of the string. Nevertheless it is convenient to state the results

[2] Therefore, as an example, this operator acting on the two form C_2 is simply

$$i_\Phi i_\Phi C_2 = \Phi^j \Phi^i C^2_{ij} = \frac{1}{2} C^2_{ij}[\Phi^i, \Phi^j]. \tag{2.24}$$

of the analysis in terms of an effective action $S_{eff}(T, ...)$ obtained by integrating out all positive mass fields. Here ... stands for the massless Bosonic fields. This is the approach we will use here.

From the analysis of the D-Brane actions 2.20 above we can easily include this mode into a Tachyonic effective action. For the abelian case [30]

$$S_T = -T_p \int d^{p+1}\sigma V(T)\sqrt{-det\,(A_{\mu\nu})} \tag{2.25}$$

where

$$A_{\mu\nu} = \eta_{\mu\nu} + \lambda\partial_\mu T\partial_\nu T + \lambda\partial_\mu X^i\partial_\nu X^i + \lambda F_{\mu\nu} \tag{2.26}$$

(note that we use $g_{\mu\nu} = \eta_{\mu\nu}$ and $B_{\mu\nu} = 0$) and $V(T)$ is the Tachyon potential (which sometimes includes the tension T_p) which we will discuss later in this section and X^i denote the remaining transverse scalar fields. Here $0 \leq \mu, \nu \leq p$ and $(p+1) \leq I \leq 9$. Note how the Tachyon appears simply as an extra scalar field supplemented by a suitable potential which must generate an instability. This potential has three requirements

- it is symmetric under $T \rightarrow -T$
- it has a maximum at $T = 0$
- its minima are at $T = \pm\infty$ where it vanishes.

These conditions are easily understood, a small displacement from around the origin must grow exponentially, signifying an instability of the system. With these conditions satisfied the above action 2.25 is expected to be a good effective field theory description for the Tachyon field under the further conditions that T is large and that second and higher derivatives of T are small. Hence we should keep in mind that this is at best simply an approximation to the full Tachyon action in string theory and it admits correction terms. Nevertheless, it is an excellent starting point to describe Tachyon dynamics. Altough the exact form of Tachyon potential is still unknown, there are different proposals in the literature. For instance, the one which is consistent with S-matrix element calculation is given by [31]

$$V(T) = T_9 \, (1 + \pi\alpha' m^2 T^2 + \frac{1}{2}(\pi\alpha' m^2 T^2)^2 + O(T^6)) \tag{2.27}$$

with T_9 the tension of the D9-brane and $m^2 = -\frac{1}{2\alpha'}$ the Tachyon mass. The one obtained from boundary string field theory (BSFT) computations is [32,33]

$$V(T) = T_9 \, e^{-\pi\alpha' m^2 \, T^2} \, . \tag{2.28}$$

In particular, the potential (2.27) can be obtained from (2.28) by expanding the latter around the Tachyonic vacuum, $T = 0$. There has been a more phenomenologically agreeable string theory result proposal which takes the form [34–36]

$$V(T) = \frac{\mathcal{T}_p}{cosh\left(\frac{T}{\sqrt{2}}\right)}, \tag{2.29}$$

we will see the latter result being important in later sections of this thesis, but for now choose to work with the most general form of the potential and simply leave this as arbitrary. By making use of the pull-back notation we can extend the action to the curved space case

$$S_T = -\int d^{p+1}\sigma V(T)\sqrt{-det\left(P[g_{\mu\nu} + B_{\mu\nu}] + \lambda F_{\mu\nu} + \lambda\partial_\mu T \partial_\nu T\right)} \tag{2.30}$$

with $P[g_{\mu\nu}] = g_{ab}\partial_\mu X^a \partial_\nu X^b$ which reduces to the previous action 2.25 in the flat space limit $g_{\mu\nu} = \eta_{\mu\nu}$, $B_{\mu\nu} = 0$ and the tension T_p is included in the Tachyon potential. Its non-abelian generalisation follows closely the derivation of the coincident D-Brane action presented before. Hence we begin with the non-abelian Tachyon effective action for coincident $D9$ branes, and apply T-duality rules to derive the most general Dp-Brane Tachyon effective action. The Tachyon action of two coincident $D9$'s is

$$S = -T_9 \int d^{10}\sigma Tr\left(e^{-\phi} V(T)\sqrt{-det\left(G_{\mu\nu} + B_{\mu\nu} + \lambda F_{\mu\nu} + \lambda\left(D_\mu T D_\nu T + D_\nu T D_\mu T\right)\right)}\right) \tag{2.31}$$

where we recognise the usual fields and

$$D_\mu T = \frac{\partial T}{\partial \sigma^\mu} - i[A_\mu, T]. \tag{2.32}$$

This action has an overall $U(2)$ symmetry. Note that from this action one can arrive at an effective action for a Dp-anti-Dp-brane pair proposed in [30] by projecting it with $(-1)^{F_L}$ where F_L is the spacetime left-handed fermion number. In this case, the gauge group is $U(1) \times U(1)$ and so there are two massless gauge fields $A_\mu^{(1)}$ and $A_\mu^{(2)}$, a complex Tachyon field T and scalar fields $X^I_{(1)}$, $X^I_{(2)}$ corresponding to the transverse coordinate of individual branes. In particular, the action reads

$$S = -\int d^{p+1}x\, V(T, X^I_{(1)} - X^I_{(2)}) \left(\sqrt{-\mathrm{det}G_{(1)}} + \sqrt{-\mathrm{det}G_{(2)}}\right) \tag{2.33}$$

where

$$G_{(i)\mu\nu} = \eta_{\mu\nu} + 2\pi\alpha' F^{(i)}_{\mu\nu} + \partial_\mu X^I_{(i)} \partial_\mu X^I_{(i)} + \pi\alpha'(D_\mu T)^*(D_\nu T) + \pi\alpha'(D_\nu T)^*(D_\mu T)\,. \tag{2.34}$$

This action has the nice property of admitting a vortex solution whose world volume action is given by the DBI action of a stable D$(p-2)$-brane [30].

In [37] another form of the effective action for a coincident non-BPS $D9$-brane pair has been proposed. It is given in terms of the symmetrized trace[3] [29, 38]

$$S = -Str \int d^{10}x\, V(T) e^{-\phi} \sqrt{-\det\left(g_{\mu\nu}\mathbb{1}_2 + B_{\mu\nu}\mathbb{1}_2 + 2\pi\alpha' D_\mu T D_\nu T + 2\pi\alpha' F_{\mu\nu}\right)}. \tag{2.35}$$

Various couplings in this action are consistent with the appropriate disk level S-matrix elements in string theory. In the above action the Str prescription means specifically that one has to first symmetrize over all orderings of terms like $F_{\mu\nu}, D_\mu T$ and also individual T that appear in the potential $V(T)$, and thus that the square root factor appearing in the action has to be understood as an infinite series expansion. The Tr or Str forms of the action are thus very different when one has carried out the individual symmetrizations mentioned above. As

[3] $Str(M_1 \ldots M_n) \equiv Tr \sum_\sigma M_1 \ldots M_n$ where $\sum_\sigma$ is a sum over all permutations of matrices in $M_1 \ldots M_n$ divided by $n!$.

we discussed before, by projecting this action with $(-1)^{F_L}$ one can obtain the effective action of a D_9-anti-D_9-brane pair. However, for this action there are no known solutions corresponding to a vortex whose world volume is given by the DBI action of a stable $D7$-brane. This is symptomatic of a deeper difference between the two prescriptions. As we will see in section 4 different choices for the Trace prescription lead to significantly different results.

Even though the trace prescription may require a symmetric completion, making it completely symmetric between non-abelian expressions of the form $F_{\mu\nu}$, $D_\mu T$ and individual T's appearing in $V(T)$, the covariant derivative Tachyon terms appearing in 2.31 are written in symmetric form directly. Applying the standard T-duality transformations on $i = p+1, ..., 9$ coordinates will yield the action for coincident Dp branes. The gauge fields in these directions become

$$\tilde{A}_i = \frac{X^i}{\lambda} \tag{2.36}$$

and the Tachyon remains unchanged, $\tilde{T} = T$. Hence the covariant derivative on the Tachyon field becomes

$$\tilde{D}_i \tilde{T} = -\frac{i}{\lambda}[X^i, T] \tag{2.37}$$

using the fact that under the transformation the transformed field must be independent of the coordinates σ^i. The T-duality transformations are 2.1 with

$$\tilde{F}_{ab} = F_{ab}, \qquad \tilde{F}_{ai} = \frac{1}{\lambda} D_a X^i \tag{2.38}$$

$$\tilde{F}_{ij} = -\frac{i}{\lambda^2}[X^i, X^j], \qquad \tilde{F}_{ia} = -\frac{1}{\lambda} D_a X^i. \tag{2.39}$$

The transformations of the determinant 2.16 result in

$$\tilde{D} = det \begin{pmatrix} \tilde{E}_{ab} + \lambda F_{ab} + \lambda D_a T D_b T & \tilde{E}_{aj} + D_a X^j - i D_a T [X^j, T] \\ \tilde{E}_i b - D_b X^i - i[X^i, T] D_b T & \tilde{E}_{ij} - \frac{i}{\lambda}[X^i, X^j] - \frac{1}{\lambda}[Xi, T][X^j, T] \end{pmatrix} \tag{2.40}$$

which is the same expression as in 2.5 augmented with the Tachyon terms.

Upon manipulations of the determinant one obtains

$$\tilde{D} = det\left(P[E_{ab} + E_{ai}\left(Q^{-1} - \delta\right)^{ij} E_{jb}] + \lambda F_{ab} + T_{ab}\right) det(Q^i_j) det(E^{ij}) \tag{2.41}$$

which is exactly the result obtained before 2.16 with in addition some extra Tachyon coupling terms of the form

$$\begin{aligned} Q^i_j &= \delta^i_j - \frac{i}{\lambda}[X^i, X^k]E_{kj} - \frac{1}{\lambda}[X^i, T][X^k, T]E_{kj} \\ T_{ab} &= \lambda D_a T D_b T - D_a T[X^i, T](Q^{-1})_{ij}[X^j, T]D_b T \\ &\quad - iE_{ai}(Q^{-1})^i_j[X^j, T]D_b T - iD_a T[X^i, T](Q^{-1})^j_i E_{jb} \\ &\quad - iD_a X^i(Q^{-1})_{ij}[X^j, T]D_b T - iD_a T[X^i, T](Q^{-1})_{ij} D_b X^j \end{aligned} \tag{2.42}$$

which gives the final non-abelian Tachyon DBI action as

$$\begin{aligned} S_T = &-T_p \int d^{p+1}\sigma \\ &\times Tr\left(e^{-\phi}V(T)V'(T, X^i)\sqrt{-det\left(P[E_{ab} + E_{ai}\left(Q^{-}1 - \delta\right)^{ij} E_{jb}] + \lambda F_{ab} + T_{ab}\right)}\right) \end{aligned} \tag{2.43}$$

which we recognise as the usual action of N coincident branes with the extra Tachyon couplings. The Trace prescription is symmetric in orderings of matrices.

It has been suggested however that the above Tachyon action is inconsistent with Tachyon scattering calculations involving Tachyons carrying internal degrees of freedom, as is the case here. Namely the Pauli Matrix factors carried by the Tachyons cannot be ignored in amplitude calculations and give results which originate from a field theory which possesses Chan-Paton factors explicitly [39]. Therefore the action needs to be modified to take this into account and the Tachyon field is promoted to carry an internal Pauli matrix $T^i = T\sigma^i$ with $i = 1, 2$. Then an action consistent with the above scattering calculations takes the form

$$S_{DBI} = -\frac{T_p}{2}\int d^{p+1}\sigma STr\left(V(T^iT^i)\sqrt{1+\frac{1}{2}[T^i,T^j][T^j,T^i]}\right.$$
$$\left.\times\sqrt{-det\left(\eta_{ab}+\lambda F_{ab}+\lambda D_aT^i\left(Q^{-1}\right)^{ij}D_bT^j\right)}\right) \quad (2.44)$$

with

$$Q^{ij} = I_2\delta^{ij} - i[T^i,T^j]. \quad (2.45)$$

The presence of the extra $\mathcal{O}(T^4)$ coupling comes from the Pauli matrix terms carried by the Tachyon field. Here the trace prescription is written in terms of the symmetrized trace STr to denote the symmetrisation procedure explicitly. The potential has an expansion of the form 2.27. Note that in the case of the modified action above one needs to calculate the full form of the pre-potential term

$$\frac{1}{2}STr\left(V(T^iT^i)\sqrt{1+[T^i,T^j][T^j,T^i]}\right) = \left(1-\frac{\pi}{2}T^2+\frac{\pi^2}{24}T^4+....\right)(1+T^4+.....) \quad (2.46)$$

and hence the extra terms are not believed to change the overall sign of the potential, which is therefore still expected to vanish as $T\to\infty$. Both 2.43 and 2.44 are the main forms of the actions we will use in the rest of the paper, from these we will investigate Tachyon dynamics and, more generally, D-Brane dynamics in String Theory. [4]

[4] One should note that other forms have been suggested in the literature, one important one with direct links to results presented below is that due to Kluson [40], which we won't discuss further

CHAPTER 3
ABELIAN TACHYON DYNAMICS

1 Tachyon Solutions and Sen's Conjectures

The major question we are interested in in determining Tachyon dynamics is whether the Tachyon potential $V(T)$ has a local minimum, and if it does, then how does the theory behave around this minimum? The answer to this question has been summarised by A. Sen [27] in three conjectures

- $V(T)$ does have a pair of global minima at $T = \pm T_0$ for the non-BPS Brane. At this minimum the tension of the original D-Brane configuration is exactly canceled by the negative contribution of the potential $V(T)$. Therefore

  $$V(T_0) + T_p = 0 \tag{3.1}$$

  where
  $$T_p = \sqrt{2}(2\pi)^{-p} g_s^{-1}. \tag{3.2}$$

  Therefore the total energy density vanishes at the Tachyon minima.

- Since there is no energy density at the Tachyon minima $T = \pm T_0$ and because the non-BPS brane carries no RR charge it seems natural to conjecture that the minimum describes a vacuum without any D-Brane. Hence upon quantising the theory around this minimum we expect there to be no open string states in the perturbation theory (as open strings must end on D-Branes). This is of course not what we expected, since in conventional field theories the number of perturbative states doesn't change as we go from one extremum of the potential to another.

- Even in the absence of perturbative physical states in the minimum of the potential there are non-trivial time independent classical solutions to the equations of motion for the Tachyon derived from $S_{eff}(T,...)$. The conjecture is that these solutions describe lower dimensional D-Branes. Some examples include the following (the case of the kink solution and vortex solution on the D anti-D brane pair will be investigated in detail further along this thesis as they illustrate the general method to achieve their non-abelian extensions):

  1. The effective action admits a one-dimensional $T(x^p)$ kink solution, such that the limits $x^p \to \pm\infty$ correspond to the limits $T \to \pm T_0$, with the solution interpolating between the two minima around $x^p = 0$. Since the energy density vanishes at $T = \pm T_0$ the energy density must be concentrated around a $(p-1)$ dimensional hypersurface around $x^p = 0$. Hence the kink solution describes a BPS D-(p-1)-Brane in the same theory. This result will be shown in full later in this section.
  2. The Brane-AntiBrane system admits a similar solution with the imaginary part of the Tachyon set to zero and the real part taking the kink profile. This describes a non-BPS D$(p-1)$-Brane, and is thus non stable as compared to the above solution.
  3. The Brane-AntiBrane system $Dp - \bar{D}p$ also admits vortex solutions for the Tachyon field. Here the Tachyon is a function of two coordinates x^p and x^{p-1},

$$T = T_0 f(\rho) e^{i\theta} \tag{3.3}$$

     where

$$\rho = \sqrt{(x^{p-1})^2 + (x^p)^2}, \quad \theta = tan^{-1}\left(\frac{x^p}{x^{p-1}}\right) \tag{3.4}$$

     and the function $f(\rho)$ has the property

$$f(\infty) = 1, \quad f(0) = 0. \tag{3.5}$$

     Hence the energy density of this solution vanishes as $\rho \to \infty$ and given that the gauge fields fall off sufficiently quickly at large ρ then the net energy density is concentrated around the $\rho = 0$ region. This is a co-dimension two solitonic solution describing a BPS D$(p-2)$-Brane in the same theory. This specific conjecture will also be verified explicitly in subsequent sections of this thesis.

4. If rather than taking the case of the single brane we focus on a pair of coincident non-BPS branes then, as we saw before, the effective field theory around $T = 0$ contains a $U(2)$ gauge field and the Tachyon field contains four degrees of freedom contained in a two-by-two hermitian matrix transforming in the adjoint representation of the gauge group. From the standard theory of Chan-Paton factors we know that the (ij) component of the matrix represents the Tachyon in the open string sector beginning on the i-th D-Brane and ending on the j-th D-Brane. There is a whole family of minima of the Tachyon potential found by taking the Tachyon in the configuration

$$T = T_0 \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{3.6}$$

which represents Tachyons at their respective minima on either Brane. Then the family of minima is obtained by performing an $SU(2)$ rotation of this configuration, to obtain minima of the form $T = T_0 \hat{n}_i \sigma^i$ where σ^i are standard Pauli matrices. This breaks the $SU(2)$ part of the gauge group down to a $U(1)$. The theory thus describes a 't Hooft-Polyakov monopole solution, depending on three spatial coordinates x^i $(i = 1, 2, 3)$ given by

$$T(x^i) \approxeq T_0 \frac{\sigma^i x_i}{|x^i|}, \quad F^a_{ij}(x^i) \approxeq \epsilon^{aij} \frac{x^a}{|x^i|^3} \tag{3.7}$$

with F^a_{ij} denoting the standard gauge field strength. The energy density of the solution is concentrated around $x^i = 0$ and it describes a BPS D$(p-3)$-Brane in the same theory. This solution is fully investigated in section 4 of this thesis, an initial analysis is presented later in this section.

Therefore, by combinations of the above results, if we start with sufficient numbers of non-BPS $D9$-Branes or $D9 - \bar{D}9$-branes we can describe any lower dimensional D-Brane by giving the Tachyon profile a classical solitonic solution.

These are in effect descent relations between p-dimensional branes.

Up to this point we have discussed time independent solutions for the Tachyon field, however a vast literature on time-dependent configurations exists [35,41–58]. Indeed a natural question is what happens if one displaces the Tachyon from the maximum of the potential and lets it roll down to a minimum? Also, since D-Branes act as sources for closed string fields, a time dependent string field configuration such as the rolling Tachyon acts as a time dependent source for closed string fields, and thus produces closed string radiation. This radiation can be computed with standard scattering techniques [36, 59–61] and has led to the formulation of the open string completeness conjecture [62] which states that the complete dynamics of a D-Brane is captured by the quantum open string sector without need to consider the coupling of the system to closed strings. Indeed the closed sector provides a dual description (details for this can be found in [27]). Furthermore, rolling Tachyon solutions provide important avenues for Tachyon driven inflation models [63–73]. We will not discuss the very interesting area of time dependent cosmological Tachyon solutions further, however time dependence plays a crucial part in the geometrical Tachyon interpretation of unstable D-Brane systems as will be shown in section 2 of this chapter.

1.1 The classical Kink Solution and the $D(p-1)$-Brane

We begin by analysing the descent relation $Dp \to D(p-1)$ from a Tachyon kink profile on a non-BPS Brane [30]. Therefore we start off with the action for a non-BPS Dp brane 2.25. The energy momentum tensor associated with this action is

$$T^{\mu\nu} = -V(T)\left(A^{-1}\right)_s^{\mu\nu}\sqrt{-detA} \tag{3.8}$$

where the subscript s denotes that only the symmetric part of the matrix $A_{\mu\nu}$ is taken. When the Tachyon takes the kink profile, it depends on one coordinate

$x = x^p$ only, hence the components of the energy-momentum tensor become

$$\begin{aligned} T_{xx} &= -\frac{V(T)}{\sqrt{1+(\partial_x T)^2}} \\ T_{\alpha x} &= 0 \\ T_{\alpha\beta} &= -V(T)\sqrt{1+(\partial_x T)^2}\eta_{\alpha\beta} \end{aligned} \tag{3.9}$$

where $\alpha, \beta = 0, ..., p-1$. This results in the energy-momentum conservation equation

$$\partial_x T_{xx} = 0, \tag{3.10}$$

i.e. T_{xx} is independent of x. However, for the kink solution $T \to \pm\infty$ as $x \to \pm\infty$ and $V(T) \to 0$ in this limit, hence T_{xx} must vanish as $x \to \infty$ and since it is independent of x by 3.10 it must vanish for all x. This in turn implies that we must have either $T = \pm\infty$ or $\partial_x T = \infty$ (or both) for all x. Therefore the solution is singular, but as will be shown below has finite energy density. To work with this singularity consider a field configuration of the form

$$T(x) = f(ax) \tag{3.11}$$

where

$$f(-u) = -f(u), \quad f'(u) > 0 \quad \forall u, \quad f(\pm\infty) = \pm\infty, \tag{3.12}$$

and the constant a serves as the regularisation constant which will eventually be taken to infinity to reproduce the singularity. Note that in the $a \to \infty$ limit we recover precisely the singular behaviour of the kink solution. The $a \to \infty$ limit is a direct consequence of the nature of the DBI action. Being this an infinite expansion higher order terms will have different scaling powers with respect to a which do not remain under control. Taking a truncation of the action would yield a regular kink solution. In this regularisation limit this ansatz for the Tachyon field satisfies the Tachyon equations of motion and is thus a classical solution to the system (see [30]). From the energy-momentum equations (3.9) T_{xx} vanishes in the $a \to \infty$ limit and

$$T_{\alpha\beta} = -a\eta_{\alpha\beta}V(f(ax))f'(ax), \tag{3.13}$$

from which the integrated $T_{\alpha\beta}$ associated with the codimension one soliton is

$$T^{kink}_{\alpha\beta} = -a\eta_{\alpha\beta}\int_{-\infty}^{\infty} dxV(f(ax))f'(ax) = -\eta_{\alpha\beta}\int_{-\infty}^{\infty} dyV(y), \tag{3.14}$$

where $y = f(ax)$. Note that this depends only on the form of the potential $V(y)$ and not on the function $f(ax)$, hence the energy-density result is independent of the regularisation procedure used to derive it. Since $y = f(ax)$ the greatest contribution to the integral comes from a region of x of width $\frac{1}{a}$ around $x = 0$, which approaches a delta function as $a \to \infty$. Therefore

$$T_{\alpha\beta} = -\eta_{\alpha\beta}\delta(x)\int_{-\infty}^{\infty} dyV(y) \tag{3.15}$$

which is what we would expect from a $D(p-1)$-Brane provided we associate the integral $\int_{\infty}^{\infty} dyV(y)$ to its tension, i.e.

$$\mathcal{T}_p = V(0), \quad \mathcal{T}_{p-1} = \int_{-\infty}^{\infty} V(y)dy. \tag{3.16}$$

Hence, before analysing the world-volume theory of fluctuations around this solution we have clear indication that the resulting theory is that of a codimension $D(p-1)$ brane. Note however that we started with an action believed to be valid in the limit that derivatives of the Tachyon are small. Therefore the above result can in principle be spoilt be higher derivative Tachyon corrections. However, the agreement between the properties of the soliton and those of the $D(p-1)$-Brane suggest that these higher derivatives arrange themselves so as not to spoil this result.

Let's proceed to investigate the theory of fluctuations for the bosonic fields around the kink solution to reproduce the DBI action of the $D(p-1)$-Brane. The general method of analysis presented here is central in derivations of results shown later in the thesis. Consider a fluctuation ansatz of the form

$$\begin{aligned} T(x,\xi) &= f(a(x-t(\xi))) \\ A_x(x,\xi) &= 0 \\ A_\alpha(x,\xi) &= a_\alpha(\xi) \\ Y^I(x,\xi) &= y^I(\xi) \end{aligned} \tag{3.17}$$

where ξ^α for $0 \leq \alpha \leq (p-1)$ are coordinates tangential to the kink world-volume. Hence fields denoted by lower case letters will be our fluctuations over the upper case ones. We denote with Y^I the remaining transverse scalar fields (previously X^I) to avoid confusion with coordinates x^i. With this ansatz $A_{\mu\nu}$ defined in 2.26 (with units $\alpha' = 1$) yields

$$\begin{aligned} A_{xx} &= 1 + a^2(f')^2 \\ A_{x\alpha} &= A_{\alpha x} = -a^2(f')^2\partial_\alpha t \\ A_{\alpha\beta} &= (a^2(f')^2 - 1)\partial_\alpha t \partial_\beta t + a_{\alpha\beta} \end{aligned} \tag{3.18}$$

with $f' = f'(a(x-t(\xi)))$ and

$$a_{\alpha\beta} = \eta_{\alpha\beta} + f_{\alpha\beta} + \partial_\alpha y^I \partial_\beta y^I + \partial_\alpha t \partial_\beta t \tag{3.19}$$

where $f_{\alpha\beta} = \partial_\alpha a_\beta - \partial_\beta a_\alpha$. This determinant can be easily evaluated by adding multiples of the first row and first column to other rows and columns. Hence we define

$$\begin{aligned} \hat{A}_{\mu\beta} &= A_{\mu\beta} + A_{\mu x}\partial_\beta t, \quad \hat{A}_{\mu x} = A_{\mu x} \\ \tilde{A}_{\alpha\nu} &= \hat{A}_{\alpha\nu} + \hat{A}_{x\nu}\partial_\alpha t, \quad \tilde{A}_{x\nu} = \hat{A}_{x\nu} \end{aligned} \tag{3.20}$$

so that

$$det(A) = det(\hat{A}) = det(\tilde{A}). \tag{3.21}$$

In components the matrix $\tilde{A}_{\mu\nu}$ reads

$$\begin{aligned} &\tilde{A}_{xx} = 1 + a^2(f')^2, \quad \tilde{A}_{x\alpha} = \tilde{A}_{\alpha x} = \partial_\alpha t, \\ &\tilde{A}_{\alpha\beta} = a_{\alpha\beta} \end{aligned} \tag{3.22}$$

and hence

$$det(A) = a^2(f')^2 \left(det(a) + \mathcal{O}\left(\frac{1}{a^2}\right)\right). \tag{3.23}$$

Substituting this into the action 2.25 (with the tension T_p absorbed inside the Tachyon potential) we obtain in the $a \to \infty$ limit

$$S = -\int d^p\xi \int dx V(f) a f' \sqrt{-det(a)} \tag{3.24}$$

which upon the change of variables $y = f(a(x - t(\xi)))$ becomes

$$S = -\mathcal{T}_{p-1} \int d^p\xi \sqrt{-det(a)} \tag{3.25}$$

which is precisely the world-volume action of a BPS $D(p-1)$-Brane identifying the field t as the coordinate x^p associated with the p-the direction. But what about other fluctuation ansatze for the fields? Will these lead to different results? There is a nice argument due to Sen which shows that this is not the case. Consider the more general ansatz

$$\begin{aligned}
Y^I(x,\xi) &= y^I(\xi) + \sum_{n=1}^{\infty} f_n(x - t(\xi)) y^I_{(n)}(\xi) \\
A_x(x,\xi) &= \phi_0(\xi) + \sum_{n=1}^{\infty} f_n(x - t(\xi)) \phi_{(n)} \\
A_\alpha(x,\xi) &= a_\alpha(\xi) + \sum_{n=1}^{\infty} f_n(x - t(\xi)) a^{(n)}_\alpha(\xi) - \phi(x,\xi)\partial_\alpha t,
\end{aligned} \tag{3.26}$$

where $f_n(u)$ obey the same condition of being smooth functions which vanish at $u = 0$ and which are bounded (also including the points $u = \pm\infty$). Then it can be shown that the resulting action will be independent of $y^I_n(\xi)$, $a^n_\alpha(\xi)$ for $n \geq 1$ and $\phi_n(\xi)$ for $n \geq 0$. Hence at the Tachyon vacuum a finite deformation of the A_μ and the Y^I leaves the action unchanged, and hence all such field configurations are identified at a single point in configuration space. It is therefore postulated that all such transformations are related to each other by a local transformations, and hence all such deformations associated with $\phi(x,\xi)$,$y^I_n(\xi)$ and $a^n_\alpha(\xi)$ should be regarded as pure gauge deformations.

Therefore the soliton kink configuration for the Tachyon on a non-BPS Dp-Brane describes the effective theory of a BPS $D(p-1)$-Brane, as conjectured in [27]. In Section 4 of this thesis we will investigate non-abelian kink solutions on a pair of coincident non-BPS Branes and derive similar results on descent relations linking pairs of non-BPS to pairs of BPS D-Branes. The next section will demonstrate a very similar procedure to the above by analysing vortex solutions of the $D-\bar{D}$ brane system, this will serve as a perfect introduction to the monopole analysis presented later in the same section.

1.2 Vortex Solution on the $D-\bar{D}$ system

This section will perform a similar analysis to that of the kink solution on a non-BPS D-Brane for the case of the vortex on a $D-\bar{D}$ Brane-anti-Brane pair. Apart from elucidating a key result on the main subject of this thesis it will reinforce the mathematical trickery presented previously, providing a very natural next step in complexity of calculations which will lead nicely to the results presented in the later section on non-abelian Tachyon dynamics (see chapter 4). For this setup, the Tachyon is complex as it originates from a string stretched between the brane and the anti-brane, these having opposite orientations.

The starting point will be the Tachyon effective action on the brane-anti-brane pair

$$S=-\int d^{p+1}xV\left(T,Y_1^I-Y_2^I\right)\left(\sqrt{-detA_1}+\sqrt{-detA_2}\right) \tag{3.27}$$

where

$$A^i_{\mu\nu}=\eta_{\mu\nu}+F^i_{\mu\nu}+\partial_\mu Y^I_i\partial_\nu Y^I_i+\frac{1}{2}(D_\mu T)^*(D_\nu T)+\frac{1}{2}(D_\nu T)^*(D_\mu T) \tag{3.28}$$

and

$$F^i_{\mu\nu}=\partial_\mu A^i_\nu-\partial_\nu A^i_\mu,\quad D_\mu T=\left(\partial_\mu-iA^1_\mu+iA^2_\mu\right)T. \tag{3.29}$$

For small T the potential behaves as

$$V\left(T, Y_1^I - Y_2^I\right) = \mathcal{T}_p \left[1 + \frac{1}{2}\left(\sum_I \left(\frac{Y_1^I - Y_2^I}{2\pi}\right)^2 - \frac{1}{2}\right)|T|^2 + \mathcal{O}(|T|^4)\right] \tag{3.30}$$

with $\mathcal{T}_p$ denoting the tension of the individual Dp-branes. The analysis for the vortex solution proceeds in similar fashion to that of the kink, hence the energy-momentum tensor is given by

$$T^{\mu\nu} = -V\left(T, Y_1^I - Y_2^I\right)\left[\sqrt{-detA_1}\left(A_1^{-1}\right)_S^{\mu\nu} + \sqrt{-detA_2}\left(A_2^{-1}\right)_S^{\mu\nu}\right] \tag{3.31}$$

with the subscript S denoting symmetrisation, and we will use a vortex ansatz of the form

$$T(r,\theta) = \bar{f}(r)e^{i\theta}, \quad A_\theta^1 = -A_\theta^2 = \frac{1}{2}\bar{g}(r) \tag{3.32}$$

where r and θ denote the polar coordinates in the (x^{p-1}, x^p) plane and $\bar{f}(r)$ and $\bar{g}(r)$ are real functions of r which satisfy

$$\bar{f}(0) = 0, \quad \bar{f}(\infty) = \infty, \quad \bar{g}(0) = 0, \quad \bar{g}'(0) = 0 \tag{3.33}$$

and all other fields vanish. This background yields

$$-detA_1 = -detA_2 = \left[\left(1 + (\bar{f}')^2\right)\left(r^2 + \bar{f}^2(1-\bar{g})^2\right) + \frac{1}{4}(\bar{g}')^2\right] \tag{3.34}$$

with $'$ denoting differentiation with respect to r, and non-zero components of the energy-momentum tensor given by

$$\begin{aligned}
T_{\alpha\beta} &= -2\eta_{\alpha\beta}V(T)\sqrt{\left(1 + (\bar{f}')^2\right)\left(r^2 + \bar{f}^2(1-\bar{g})^2\right) + \frac{1}{4}(\bar{g}')^2} \\
T_{rr} &= -2V(T)\left(r^2 + \bar{f}^2(1-\bar{g})^2\right) / \sqrt{\left(1 + (\bar{f}')^2\right)\left(r^2 + \bar{f}^2(1-\bar{g})^2\right) + \frac{1}{4}(\bar{g}')^2} \\
T_{\theta\theta} &= -2V(T)\left(1 + (\bar{f}')^2\right) / \sqrt{\left(1 + (\bar{f}')^2\right)\left(r^2 + \bar{f}^2(1-\bar{g})^2\right) + \frac{1}{4}(\bar{g}')^2}
\end{aligned} \tag{3.35}$$

with $V(T) = V(T, 0)$.

As per the kink solution, the energy conservation equation

$$\partial^\mu T_{\mu r} = \partial_r T_{rr} = 0 \tag{3.36}$$

means that T_{rr} must be a constant, and since $V(T)$ vanishes exponentially at large T we see that T_{rr} vanishes there also unless $\bar{g}(r)$ is singular. However it can be shown [30] that $\bar{g}$ varies monotonically between zero and one. This is achieved by a careful consideration of how the function $\bar{g}(r)$ must behave in order to minimise the integrated energy momentum tensor and thus be a solution of the equations of motion. Indeed if the function exceeds the value 1 in some range, then we can replace it by another function equal to the original when this is less than 1 and equal to 1 when this is greater, which thus minimises $\int drd\theta T_{00}$ further, showing that the original $\bar{g}(r)$ cannot be a solution of the equations of motion if it exceeds 1. The same argument applies to the lower bound and the fact that the function cannot have any local maxima. Hence T_{rr} vanishes at infinity and by the conservation equation, must vanish everywhere. This can either be achieved if $V(T) = 0$ or the denominator is infinite, which requires $\bar{f}'$ and/or $\bar{g}'$ to be singular. Now $V(T)$ is certainly not zero close to $r = 0$ so, in parallel to the kink ansatz, we look for function profiles of the form

$$\bar{f}(r) = f(ar), \quad \bar{g}(r) = g(ar) \tag{3.37}$$

and take the $a \to \infty$ limit to reproduce the singularity. In this limit we obtain

$$\begin{aligned}
-detA_1 = -detA_2 &\approxeq a^2(f'(ar))^2\left[r^2 + f(ar)^2(1-g(ar))^2 + \frac{1}{4}\left(\frac{g'(ar)}{f'(ar)}\right)^2\right] \\
T_{\alpha\beta} &\approxeq -2\eta_{\alpha\beta}V(f(ar))af'(ar)\sqrt{r^2 + f(ar)^2(1-g(ar))^2 + \frac{1}{4}\left(\frac{g'(ar)}{f'(ar)}\right)^2} \\
T_{rr} &\approxeq -2V(f(ar))\frac{r^2 + f(ar)^2(1-g(ar))^2}{af'(ar)\sqrt{r^2 + f(ar)^2(1-g(ar))^2 + \frac{1}{4}\left(\frac{g'(ar)}{f'(ar)}\right)^2}}
\end{aligned} \tag{3.38}$$

note how indeed T_{rr} vanishes in the $a \to \infty$ limit as argued above. However the integral over (r, θ) of $T_{\alpha\beta}$ gives the $(p-2+1)$ dimensional energy-momentum tensor $T^{vortex}_{\alpha\beta}$ on the vortex. Substituting the relations

$$y = f(ar), \quad \hat{r}(y) = a^{-1} f^{-1}(y), \quad \hat{g}(y) = g(ar) = g(a\hat{r}(y)) \tag{3.39}$$

this reads

$$T^{vortex}_{\alpha\beta} = -4\pi\eta_{\alpha\beta} \int_0^\infty dy V(y) \sqrt{\hat{r}(y)^2 + y^2(1-\hat{g}(y))^2 + \frac{1}{4}\hat{g}'(y)^2} \tag{3.40}$$

which further simplifies to

$$T^{vortex}_{\alpha\beta} = -4\pi\eta_{\alpha\beta} \int_0^\infty dy V(y) \sqrt{y^2(1-\hat{g}(y))^2 + \frac{1}{4}\hat{g}'(y)^2} \tag{3.41}$$

in the $a \to \infty$ limit as $\hat{r}(y)$ vanishes when this is taken. Hence in similar fashion to the kink solution the overall energy-momentum vortex tensor is independent of the choice of $f(y)$. However, it does depend on the shape of $g(y)$, which is in turn determined by its equation of motion

$$\frac{1}{4}\partial_y \left[V(y) \frac{\hat{g}'(y)}{\sqrt{y^2(1-\hat{g}(y))^2 + \frac{1}{4}\hat{g}'(y)^2}} \right] + V(y) \frac{y^2[1-\hat{g}(y)]}{\sqrt{y^2(1-\hat{g}(y))^2 + \frac{1}{4}\hat{g}'(y)^2}} = 0 \tag{3.42}$$

Hence it is the potential $V(y)$ which fully determines $T^{vortex}_{\alpha\beta}$. Most of the contribution to the energy-momentum tensor is concentrated in a region in r space of width $\frac{1}{a}$ around the origin (this was also the case for the kink solution), in the limit that $a \to 0$ we have

$$T^{vortex}_{\alpha\beta} = -4\pi\eta_{\alpha\beta}\delta(x^{p-1})\delta(x^p) \int_0^\infty dy V(y) \sqrt{y^2(1-\hat{g}(y))^2 + \frac{1}{4}\hat{g}'(y)^2} \tag{3.43}$$

which can be interpreted as an energy-momentum tensor localised on a $(p-2)$-dimensional surface. This agrees with the identification of the vortex solution as a $D-(p-2)$-Brane, where we recognise the tension of the brane as

$$\mathcal{T}_{p-2} = 4\pi \int_0^\infty dy V(y) \sqrt{y^2(1-\hat{g}(y))^2 + \frac{1}{4}\hat{g}'(y)^2}. \tag{3.44}$$

This shows that at least at the level of energy-momentum the Tachyon vortex solution on a Brane-anti-Brane corresponds to a BPS $D(p-2)$-Brane in the same theory. Note that if we take a $BSFT$ potential profile of the form $V(y) = V_0 e^{-\beta y^2}$ then 3.42 admits the analytic solution

$$g(y) = 1 - e^{-\frac{1}{\beta} y^2} \tag{3.45}$$

which gives the following tensions for the Dp-branes

$$\begin{aligned} \mathcal{T}_{p-1} &= V_0 \sqrt{\frac{\pi}{\beta}} \\ \mathcal{T}_{p-2} &= 2\pi \frac{V_0}{\sqrt{1+\beta^2}} \end{aligned} \tag{3.46}$$

in units where $2\pi\alpha' = 1$ (the first result is derived from the kink relation for the D$(p-1)$ tension 3.16). By making the choice $V_0 = \sqrt{2}\mathcal{T}_p$ and $\beta = 1$ we obtain

$$\begin{aligned} \mathcal{T}_{p-1} &= \sqrt{2\pi}\mathcal{T}_p \\ \mathcal{T}_{p-2} &= 2\pi \mathcal{T}_p \end{aligned} \tag{3.47}$$

which reproduce the correct descent relations.

As we did before for the kink, we now proceed to demonstrate this descent relation in full by analysing the world-volume theory of fluctuations around the vortex background. The coordinates transverse to the brane world-volume of the vortex are denoted by x^i with $(p-1) \leq i \leq p$ and those tangential to it as ξ^α for $0 \leq \alpha \leq (p-2)$ and re-express the vortex ansatz in Cartesian coordinates

$$A_i^1 = -A_2^i = \bar{h}_i(x), \quad T(x) = \bar{f}(x), \tag{3.48}$$

where

$$\bar{h}_{p-1}(x) = -\frac{x^p}{2r^2}\bar{g}(r), \quad \bar{h}_p(x) = \frac{x^{p-1}}{2r^2}\bar{g}(r), \quad \bar{f}(x) = \bar{f}(r), \tag{3.49}$$

and $r = |x|$, $x = (x^{p-1}, x^p)$. We take the following ansatz for the fluctuating fields

$$\begin{aligned} & A_i^1(x,\xi)=\bar{h}_i[x-t(\xi)], \quad A_i^2(x,\xi)=-\bar{h}_i[x-t(\xi)], \\ & A_\alpha^1(x,\xi)=-\bar{h}_i[x-t(\xi)]\partial_\alpha t^i+a_\alpha(\xi), \\ & A_\alpha^2(x,\xi)=-\bar{h}_i[x-t(\xi)]\partial_\alpha t^i+a_\alpha(\xi), \\ & Y_1^I(x,\xi)=T_2^I(x,\xi)=y^I(\xi), \quad T(x,\xi)=\bar{f}[x-t(\xi)], \end{aligned} \tag{3.50}$$

with lower case letters denoting the fluctuation fields on the vortex world-volume. Following calculations identical to that of the kink, evaluating the components of the matrices $A_{1,2}$ and adding rows and columns of the matrix to the same matrix in the following way

$$\begin{aligned} \hat{A}_{(s)\alpha\nu} &= A_{(s)\alpha\nu}+A_{(s)i\nu}\partial_\alpha t^i, \quad \hat{A}_{(s)i\nu}=A_{(s)i\nu} \\ \tilde{A}_{(s)\mu\beta} &= \hat{A}_{(s)\mu\beta}+\hat{A}_{(s)\mu j}\partial_\beta t^j, \quad \tilde{A}_{(s)\mu j}=\hat{A}_{(s)\mu j} \end{aligned} \tag{3.51}$$

for $0 \leq \mu, \nu \leq p$, one can show that the resulting action after taking the determinant $det(\tilde{A}_s)$ becomes

$$\begin{aligned} S=&-2\int d^{p-1}\xi \int dr d\theta V(f(ar))af'(ar) \\ &\times\sqrt{r^2+f(ar)^2[1-g(ar)]^2+\frac{1}{4}\left(\frac{g'(ar)}{f'(ar)}\right)^2} \\ &\times\sqrt{-deta_{\alpha\beta}} \end{aligned} \tag{3.52}$$

where

$$a_{\alpha\beta}=\eta_{\alpha\beta}+f_{\alpha\beta}+\partial_\alpha y^I\partial_\beta y^I+\partial_\alpha t^i\partial_\beta t^i \tag{3.53}$$

and $r=|x-t(\xi)|$, $\theta=tan^{-1}[(x^{p-1}-t^{p-1}(\xi))/(x^p-t^p(\xi))]$. Performing the integrals over r and θ this yields

$$S=-\mathcal{T}_{p-2}\int d^{p-1}\xi\sqrt{-deta_{\alpha\beta}} \tag{3.54}$$

which is the world-volume action on a BPS $D(p-2)$ brane, with t^i and y^I interpreted as coordinates transverse to the brane, and a_α as the gauge field on the brane world-volume, as was the goal to show. Hence we have shown explicitly the descent relations working for the abelian cases of the kink on a single non-BPS brane and the vortex on the Brane-anti-Brane system. The answer to the natural question of what happens when one considers multiple branes and thus a non-abelian promotion of such theories is explored in chapter 4 of this thesis, which focusses on non-abelian kinks on multiple non-BPS brane systems and the non-abelian monopole solution leading to a co-dimension $D(p-3)$-Brane.

As shown in this section, one particularly interesting aspect of Tachyon dynamics that is captured by the various effective descriptions is the existence of solitonic configurations of the Tachyon field [74], including singular Tachyon kink profiles [30, 75–77] which describe codimension one BPS branes as well as more exotic objects such as vortex solutions in Brane-anti-Brane systems.

More generally, Tachyon condensation has long been an interesting aspect of D-brane physics (for a comprehensive review see [27]). Study of the dynamics of open string Tachyons has provided a fertile arena for studying various aspects of non-perturbative string theory. A growing body of research has developed in open string field theory (for a review see [78] or [79, 80] for more recent works) boundary string field theory, (BSFT) [32, 33, 81–85] and various effective actions around the Tachyon vacuum [37, 86–89] to demonstrate Sen's results [90–94] concerning the fate of the open string vacuum in the presence of Tachyons. In related developments, it was also shown that D-brane charges take values in appropriate K-theory groups of space-time. A major result is that all lower-dimensional D-branes can be considered in a unifying manner as non-trivial excitations on the appropriate configuration of higher-dimensional branes. In type IIB superstring theory, it was demonstrated by Witten in [95] that all branes can be built from sufficiently many D9-anti-D9 pairs. In type IIA superstring theory, Horava described how to construct BPS D$(p-2k-1)$-branes as bound states of unstable Dp-branes [74].

In the next section we aim to introduce the geometrical interpretation of the Tachyon in the abelian setup and, as per the case of Tachyon descent relations presented above, leave its non-abelian extension to a later section of the thesis.

2 $NS5$ Branes and the Geometrical Tachyon

In previous chapters we have introduced the concept of S-duality as the transformation that links weak to strongly coupled theories through inversion of the coupling constant $g_s \to \tilde{g}_s = \frac{1}{g_s}$. In string theory it is the dilaton that plays the part of the string coupling and, as we have seen, this appears in terms of an exponential in D-Brane actions. Hence the S-duality inversion of the coupling can be recast in terms of the dilaton by $g_s = e^{\phi} \to \tilde{g}_s = e^{\tilde{\phi}}$ where $\tilde{\phi} = -\phi$. This in turn implies a re-definition of the string length, i.e. $\alpha' = \tilde{g}_s^{-1}\tilde{\alpha}'$ from which we can deduce the action of S-duality on D-Branes by analysing the resultant dual forms for the tension. Hence for example the D-String (or the $D1$-brane) tension is mapped to

$$\mathcal{T}_1 = \frac{1}{2\pi\alpha' g_s} \to \frac{1}{2\pi\tilde{\alpha}'} = \mathcal{T}_1^F \tag{3.55}$$

which is the tension of the fundamental string in the dual string theory. Similarly the $D3$-Brane tension is mapped to

$$\mathcal{T}_3 = \frac{1}{(2\pi)^3\alpha'^2 g_s} \to \frac{1}{(2\pi)^3\tilde{\alpha}'^2\tilde{g}_s} = \tilde{\mathcal{T}}_3 \tag{3.56}$$

which is the tension of a $D3$-Brane in the dual theory. The case of the $D5$ brane in type IIB superstring theory is however more interesting, consider the map of the $D5$-brane tension

$$\mathcal{T}_5 = \frac{1}{(2\pi)^5\alpha'^3 g_s} \to \frac{1}{(2\pi)^5\tilde{\alpha}'^3\tilde{g}_s^2} = \mathcal{T}_5^F \tag{3.57}$$

the dual tension is not the tension of a $D5$-Brane. In fact, since under the duality the R-R two form is mapped to the NS-NS two form the latter brane is magnetically charged under the latter. This type of brane is called an $NS5$ brane and it is a soliton solution in the NS-NS sector of the theory [96].

Kutasov [97, 98] has presented intriguing links between systems of unstable D-branes and the DBI effective action of open string Tachyon modes of non BPS D-branes. The former can be considered for example as a probe BPS D-brane moving in a background geometry which breaks all remaining supersymmetry. An example of such a geometric background is that due to k coincident NS5 branes [99]. It then emerges that one can associate the radial motion of the probe brane in this background with that of the open string Tachyon of non BPS D-branes. Such an association gives the former a geometrical interpretation, hence the notion of 'geometric Tachyons', as we will shortly show.

We consider a stack of k $NS5$ branes in type II string theory, then although D-Branes might be BPS their motion in the vicinity of the stack is unstable since the NS branes and D-Branes preserve different halves of the now completely broken supersymmetry. This is indeed not surprising as the gravitational picture formed by the stack is that of an infinite throat along which the string coupling grows without bound, which denotes a physical instability.

Let us proceed to study the abelian system in a bit more detail (see [97]), this will serve as a good illustration for the later section in which the non-abelian generalisation is presented. The stack of k parallel $NS5$ branes is extended in $(x^1, x^2, ...x^5)$ directions and localised in $(x^6, ...x^9)$ directions. We are interested here in the dynamics of a BPS Dp-brane in the vicinity of the $NS5$'s, we take this brane to be parallel to the fivebranes and pointlike in directions transverse to it. We label the world-volume of the brane by x^μ where $\mu = 0, 1, ...p$ with $p \leq 5$. The background fields around k parallel NS5 branes are

$$ds^2 = G_{AB}dx^A dx^B = \eta_{\mu\nu}dx^\mu dx^\nu + \delta_{mn}H(x^m)dx^m dx^n \tag{3.58}$$

$$e^{2(\Phi-\Phi_0)} = H(x^m)$$

$$H_{mnp} = -\epsilon^q_{mnp}\partial_q\Phi \tag{3.59}$$

where the index $A = (\mu, m)$ with m denoting the transversal directions. The function $H(x^n)$ is the harmonic function describing k five-branes, H_{mnp} is the field strength of the Kalb-Ramond B-field and Φ is, as usual, the dilaton field. For coincident NS5 branes the harmonic function $H(x^n)$ reduces to

$$H = 1 + \frac{kl_s^2}{r^2} \tag{3.60}$$

where $r = |\vec{x}|$ is the radial coordinate away from the five-branes in the transverse R^4 labeled by $(x^6, \cdots, x^9)$ and $l_s = \sqrt{\alpha'}$ is the string length. The dynamics of transverse scalar fields on the D-Brane $(X^6(x_\mu)......, X^9(x_\mu))$ is governed by the

DBI action

$$S_p = -\mathcal{T}_p \int d^{p+1}\sigma e^{-(\phi-\phi_0)} \sqrt{-det\left(G_{\mu\nu} + B_{\mu\nu}\right)} \tag{3.61}$$

where the standard pull-back 2.16 on the fields inside the determinant is implied. As mentioned in chapter 2 this action is expected to be reliable for the case of small string coupling, i.e. $e^{\phi} << 1$, which in this case is not a trivial constraint as we will soon see. We consider the special case where all the fivebranes are placed at $x^m = 0$ and we restrict to purely radial fluctuations of the Dp-Brane in the transverse $\mathbb{R}^4$ labelled by x^m. For this case there is only one excited field on the brane: $R(x^\mu) = \sqrt{X^m X_m(x^\mu)}$, and the B field vanishes. With this simplification the induced metric takes the form

$$G_{\mu\nu} = \eta_{\mu\nu} + H(R)\partial_\mu R \partial_\nu R \tag{3.62}$$

and the DBI action becomes

$$S_p = -\mathcal{T}_p \int d^{p+1}x \frac{1}{\sqrt{H}} \sqrt{1 + H(R)\partial_\mu R \partial^\mu R}. \tag{3.63}$$

This is similar to the Tachyon DBI action 2.25 with the field strength $F_{\mu\nu}$ and the remaining transverse scalar fields X^I vanishing and in fact one can map one into the other using the relation

$$\frac{dT}{dR} = \sqrt{H(R)} = \sqrt{1 + \frac{kl_s^2}{R^2}} \tag{3.64}$$

where the Tachyon potential becomes

$$V(T) = \frac{\mathcal{T}_p}{\sqrt{H(R(T))}}. \tag{3.65}$$

This map has an analytic solution given by

$$T(R) = \sqrt{kl_s^2 + R^2} + \frac{1}{2}\sqrt{kl_s} ln \frac{\sqrt{kl_s^2 + R^2} - \sqrt{k}l_s}{\sqrt{kl_s^2 + R^2} + \sqrt{k}l_s} \tag{3.66}$$

up to an additive constant. In the limit of close proximity $R \to 0$ and being far away from the brane stack $R \to \infty$ the solutions become

$$\begin{aligned} T(R \to 0) &\approxeq \sqrt{k} l_s ln \frac{R}{\sqrt{k} l_s} \\ T(R \to \infty) &\approxeq R \end{aligned} \tag{3.67}$$

which give limiting behaviours for the Tachyon potential as

$$\begin{aligned} \frac{1}{\mathcal{T}_p} V(T \to -\infty) &\approxeq exp\left(\frac{T}{\sqrt{k} l_s}\right) \\ \frac{1}{\mathcal{T}_p} V(T \to \infty) &\approxeq 1 - \frac{k l_s^2}{2T^2}, \end{aligned} \tag{3.68}$$

and therefore for large positive T we recognise the long range gravitational attraction between the D-Brane and the fivebranes. In the opposite case, where $T \to -\infty$ the potential vanishes exponentially. Note that even in proximity of the fivebranes there is no perturbative string Tachyon, as indeed no fundamental string can stretch between the D-Branes and the $NS5$ branes. Hence in this case the Tachyon field acquires a geometrical meaning as the radial distance between the D-Brane and the fivebranes.

This system has interesting solutions to the equations of motion for the radial mode

$$\dot{R}^2 = \frac{1}{H} - \frac{\mathcal{T}_p^2}{E^2 H^2} \tag{3.69}$$

where E is the conserved energy of the system, for the case of vanishing angular momentum and in the region $R << \sqrt{k} l_s$ one can solve this exactly and obtain

$$\frac{1}{R} = \frac{\mathcal{T}_p}{E\sqrt{k} l_s} cosh \frac{t}{\sqrt{k} l_s} \tag{3.70}$$

where $t = 0$ is (by choice) the time at which the D-Branes reach maximal distance from the fivebrane stack. Substituting this into 3.58 we can determine the behaviour of the dilaton (and thus of the string coupling) as a function of time

$$e^{\Phi} = \frac{g_s \mathcal{T}_p}{E} cosh \frac{t}{\sqrt{k} l_s} \tag{3.71}$$

and thus we see that the above solution is only reliable when $g_s\mathcal{T}_p << E << \mathcal{T}_p$ where we are safely in the $e^\Phi << 1$ region in which quantum effects are not important. Whilst the solution remains valid, it describes oscillatory motion[1] of the Dp-Branes which go from one side of the fivebranes to the other, passing through them twice in each cycle. Under the geometrical Tachyon mapping ($R \to T$) this dynamics has interesting analogues in rolling Tachyon solutions. Indeed it appears that as the D-Brane approaches the fivebranes it behaves as a pressureless fluid, which is very similar to late time behaviour of unstable D-Branes. Also, due to the open-closed completeness conjecture presented above (see discussion above section 1.1 of this chapter) we can interpret the D-Brane as shedding energy into modes which live on the $NS5$ branes as it approaches them.

Kutasov's original model was further investigated and extended in [76, 100–112]. Cosmological applications of geometrical Tachyons were considered in [76, 113–118] following Sen's original rolling Tachyons ideas in [45]. A subsection of chapter 4 is dedicated to extending this study to the case of parallel Dp-Branes approaching the stack of $NS5$'s, we will show that new non trivial dynamics is seen and investigate further the meaning of the geometrical Tachyon map in a non-abelian context.

[1] the oscillation is in proper time τ, i.e. with regards to an observer on the Branes, where $-dt^2 = G_{00}d\tau^2$

CHAPTER 4
NON-ABELIAN TACHYON DYNAMICS

1 Non-Abelian Tachyon Kink

In chapter 3 [30], the world-volume theory of the singular kink soliton solution (suitably regularised) where a single real Tachyon field 'condenses' on a single non-BPS D-brane in a flat background was investigated using the effective Dirac-Born-Infeld (DBI) framework. Remarkably, it was shown that the effective theory of fluctuations about the Tachyon kink profile, that depends only on a single spatial world-volume coordinate, are precisely those of a co-dimension one BPS brane. Furthermore, it was also shown that in brane-antibrane systems, in which a single complex Tachyon field is present, vortex solutions to the equations of motion exist, that naturally depend on two spatial worldvolume coordinates. Analysis of the fluctuations in this case show that they describe a co-dimension two BPS D-brane. Monopole solutions in certain truncations of Tachyon models have also been found and initial investigations suggest that the corresponding effective theory of fluctuations about this background correspond to co-dimension three BPS D-branes [119] (see the next section).

In this section we wish to investigate the process of Tachyon condensation starting from the effective description of two coincident non-BPS D9-branes as proposed by Garousi in [86] (see chapter 2, equation 2.31). As we mentioned previously, this theory describes a non-abelian version of the DBI action in which the Tachyon field transforms in the adjoint representation of the $U(2)$ gauge symmetry of the coincident non-BPS D9-brane world volume action. Crucially, in the original construction of this action and its generalisation to coincident non-BPS Dp-branes, a standard trace prescription (which we denote as Tr) was taken over the gauge indices. Another prescription, motivated by string scattering calculations (at least to low orders in α' [29,38]) is to take the symmetrized trace (which we denote by Str) over gauge indices. In both cases the expression being traced over is the same but the Str prescription results in significantly more complicated terms in the action compared to Tr (see the discussion in chapter 2, below equation 2.31).

The effective theory of coincident non-BPS D9-branes is the simplest example of a multiple non-BPS brane action since there are no matrix valued coordinate fields present perpendicular to the branes. We shall show that singular Tachyon profiles exist which can be regularised in a way that preserves the $U(2)$ symmetry. We will see that studying the most general fluctuations about this profile yields precisely the non-abelian DBI action of two coincident D8-branes. The only caveat is that our proof relies on assuming the standard Tr as opposed to the Str prescription for tracing over gauge indices in the DBI action of both the non-abelian non-BPS D9-brane action and the non-abelian D8-brane action. Whilst it is possible that Tachyon condensation in the non-BPS action using Str could lead to the Str form of the action for two coincident D8-branes [29,38], the exact mechanism for this to happen seems beyond a straightforward extension of the method Sen used in the case of a single non-BPS brane [30]. In this sense the Str prescription presents a challenge for non-abelian Tachyon condensation and deserves further investigation.

As a simple check of the non-abelian Tachyon condensation we also consider the case of non-abelian Tachyon kinks where the $U(2)$ symmetry is spontaneously broken to $U(1) \otimes U(1)$. The resulting effective theory of fluctuations is shown to lead to the sum of two DBI actions of separate BPS D8-branes, as expected.

The structure of the following sections is as follows. First, we study regularised kink profiles in the matrix valued Tachyon field that preserve the $U(2)$ symmetry and derive the effective world volume theory of its fluctuations. Here we also discuss the issues of Tr vs Str prescriptions and why the latter seems problematic as far as Tachyon condensation is concerned. Finally we extend these results to kink profiles that spontaneously break $U(2) \to U(1) \otimes U(1)$.

We begin our calculations by working with the action 2.31, we retain the Trace prescription throughout the analysis. To simplify our calculations we set $B_{\mu\nu} = 0$,

$g_{\mu\nu} = \eta_{\mu\nu} = (-1, 1, \ldots, 1)$ and take a constant dilaton ϕ consistent with the flat background. We also set the gauge fields to zero. The latter will be reintroduced when we consider fluctuations around the kink solution.

1.1 Energy-momentum tensor and equations of motion

In this section we shall compute the energy-momentum tensor and the equations of motion associated with the actions (2.31) and (2.35). In particular the energy-momentum tensor associated with the action (2.31) is given by

$$T_{\mu\nu} = -Tr\, V(T)\, \sqrt{-detG}\, G^{-1}_{\{\mu\nu\}} \tag{4.1}$$

where curly brackets denote symmetrisation and we defined

$$G_{\mu\nu} \equiv \eta_{\mu\nu} + B_{\mu\nu} + \pi\alpha'(D_\mu T D_\nu T + D_\nu T D_\mu T) + 2\pi\alpha' F_{\mu\nu}\,. \tag{4.2}$$

A similar expression holds for the symmetrized trace form of the action but with Tr replaced by Str.

Following Sen [30], we show that the kink solution consistent with the energy-momentum conservation and the equations of motion (e.o.m) is given by

$$T(x) = f(a\frac{x}{\sqrt{\alpha'}})\mathbb{1}_2 = f(a\frac{x}{\sqrt{\alpha'}}\,\mathbb{1}_2) \tag{4.3}$$

with gauge fields set to zero, $x \equiv x^9$ a direction longitudinal to the system and a an arbitrary dimensionless constant that we should take to infinity at the end. The function $f(u)$ can be any real function with the property that $f(u \to \pm\infty) \to \pm\infty$ and $f'(u) > 0$, $\forall u$. As a matter of fact, eq. (4.3) is a way of regularizing the Tachyon singular solution (as per the abelian case) which comes from the energy-momentum conservation condition $\partial_x T_{xx} = 0$: the latter implies that

$$T_{xx} = -Tr\, \frac{V(T)}{\sqrt{1 + 2\pi\alpha' \partial_x T \partial_x T}} \tag{4.4}$$

must be independent of x. Therefore, since for $x \to \infty$ we have that $T_{xx} \to 0$

then[1] $T_{xx} = 0, \forall\, x$. We conclude that T is singular, namely

$$T = \pm\infty \quad \text{and/or} \quad \partial_x T = \pm\infty \quad \forall\, x \tag{4.5}$$

and this singularity is regularized by taking the constant a in (4.3) to infinity. However, one can also show that this kink solution has finite energy density regardless of the way of regularizing the singularity.

Let's compute now the equation of motion for the Tachyon (keeping the gauge fields non-zero), in particular, varying eq. (4.3) w.r.t. T we obtain:

$$\pi\alpha' D_\rho \left(V(T)\sqrt{-detG}\, (G^{-1})^{\mu\nu} (D_\nu T \delta^\rho_\mu + D_\mu T \delta^\rho_\nu) \right) - \frac{\partial V(T)}{\partial T}\sqrt{-detG} = 0 \tag{4.6}$$

where we use the properties of the trace to permute all the various sources of δT factors that arise in the variation of the action. When one uses the symmetrized trace form of the action (2.35) the equations of motion for T are:

$$\Sigma_\sigma \left[\pi\alpha' D_\rho \left(V(T)\sqrt{-detG}\, (G^{-1})^{\mu\nu} (D_\nu T \delta^\rho_\mu + D_\mu T \delta^\rho_\nu) \right) - \frac{\partial V(T)}{\partial T}\sqrt{-detG} \right] = 0 \tag{4.7}$$

where $\sum_\sigma$ accounts for all symmetrical permutations of the matrices inside the squared brackets in the previous expression.

We now verify that the kink solution eq. (4.3) satisfy the equation of motions (4.6) in the $a \to \infty$ limit. In this case:

$$G_{\mu\nu} = \eta_{\mu\nu} + 2\pi\alpha' \partial_\mu T \partial_\nu T = \begin{pmatrix} -1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \dots & 0 & (1 + 2a^2\pi(f')^2) \end{pmatrix} \otimes \mathbb{1}_2 \tag{4.8}$$

where $'$ denotes differentiation w.r.t. the dimensionless argument of f. It follows that

$$-detG = 1 + 2a^2\pi(f')^2 \approx 2a^2\pi(f')^2 \tag{4.9}$$

[1] Recall that for a kink solution $\lim_{x\to\infty} T \to \infty$ and we assumed that the Tachyon potential is zero at infinity.

and

$$(G^{-1})^{\mu\nu} = \left[\eta^{\mu\nu} + \left(\frac{1}{1+2a^2\pi(f')^2} - 1\right)\delta_x^\mu \delta_x^\nu\right] \otimes \mathbb{1}_2 \,. \tag{4.10}$$

Substituting eqs. (4.3), (4.9) and (4.10) into eq. (4.6) one obtains

$$\begin{aligned}
& 2\pi\alpha'\partial_x\left(V(T)\sqrt{-detG}\,(G^{-1})^{xx}\partial_x T\right) - \frac{\partial V(T)}{\partial T}\sqrt{-detG} \\
= \;& 2\pi\sqrt{\alpha'}\partial_x\left(V(T)\frac{1}{\sqrt{1+2a^2\pi(f')^2}}af'\right) - \frac{\partial V(T)}{\partial T}\sqrt{1+2a^2\pi(f')^2} \\
\approx \;& \sqrt{2\pi\alpha'}\partial_x V(T) - \sqrt{2\pi}af'\frac{\partial V(T)}{\partial T} = 0
\end{aligned} \tag{4.11}$$

where in the last step we have taken the large a limit. Notice that since the solution (4.3) is such that both T and D_xT commute (indeed they are both proportional to the identity in group space), then it is equally a solution of the equations of motion derived from the Str procedure eq. (2.35) in the background in which the gauge fields are set to zero.

1.2 Study of the fluctuations

We proceed to study the fluctuations around the solution (4.3) which preserve the $U(2)$ symmetry. These fluctuations correspond just to shifts in the argument of the function $f(a\frac{x}{\sqrt{\alpha'}})$. The analysis is similar to chapter 3 [30], however, we now have two copies of the usual abelian Tachyon profile filling out the diagonal elements of the matrix Tachyon field, thus representing the two coincident D8-branes.

- $T = f(\frac{a}{\sqrt{\alpha'}}(x - t(\xi)))\mathbb{1}_2$

  As a warmup calculation we consider a fluctuation of the type

  $$T = f(\frac{a}{\sqrt{\alpha'}}(x_1 - t(\xi)))\mathbb{1}_2 \,, \tag{4.12}$$

  where ξ^α denotes all the coordinates tangential to the kink world-volume and $t(\xi)$ the field associated with the translational zero mode of the kink. Taking the group trace, Tr or Str, in the action (2.31) or (2.35) in the case where the Tachyon profile and its derivatives are proportional to the identity as in eq. (4.12), will thus give us two identical D8-brane actions[1]. Indeed, for the fluctuation (4.12),

$$-detG = 1 + 2\pi a^2 (f')^2 \left(1 + \eta^{\alpha\beta}\partial_\alpha t \partial_\beta t\right) \tag{4.13}$$

and we obtain

$$\begin{aligned} S &= -Tr \int d^9\xi \, dx \, V(f) \, \sqrt{2\pi} a f' \sqrt{1 + \eta^{\alpha\beta}\partial_\alpha t \partial_\beta t} \\ &= -2\sqrt{2\pi} a \int d^9\xi \, dx \, V(f) f' \sqrt{1 + \eta^{\alpha\beta}\partial_\alpha t \partial_\beta t} \end{aligned} \tag{4.14}$$

and by a substitution $y = f(\frac{a}{\sqrt{\alpha'}}(x - t(\xi)))$ one finds

$$S = -2\sqrt{2\pi\alpha'} \int_{-\infty}^{\infty} dy V(y) \int d^9\xi \sqrt{1 + \eta^{\alpha\beta}\partial_\alpha t \partial_\beta t} \tag{4.15}$$

which upon the identification $T_8 = \sqrt{2\pi\alpha'} \int_{-\infty}^{\infty} dy V(y)$ we recognize as the action describing two identical D8-branes (with no separation) with a single translational fluctuation mode $t(\xi)$ turned on.

- $T = f(\frac{a}{\sqrt{\alpha'}}(x\mathbb{1}_2 - t^a(\xi)\sigma_a))$

  Of course it is well known that the full DBI action for coincident BPS D8-branes should involve a nonabelian theory in which the single coordinate perpendicular to the D8-brane worldvolume is a $U(2)$ matrix-valued field and the resulting action has local $U(2)$ gauge invariance. Thus we would like to show how such an action appears by looking at the most general fluctuations around our original kink solution $T = f(\frac{a}{\sqrt{\alpha'}}x)\mathbb{1}_2$. To this end, let us keep the fluctuations in the gauge field zero for the time being and consider fluctuations of the Tachyon profile of the form:

$$T = f(\frac{a}{\sqrt{\alpha'}}(x\mathbb{1}_2 - t^a(\xi)\sigma_a)) \tag{4.16}$$

  where $\sigma^a = (\sigma^0 = \mathbb{1}_2, \sigma^i)$, σ^i being the Pauli matrices and we should regard f as a matrix-valued application expressed as an infinite power series of its argument. The above ansatz for the fluctuations is a natural non-abelian

[1]Note that in the determinant under the square root the symmetric $D_\mu T D_\nu T$ term is automatically diagonalized in the gauge indices.

generalization of the one that Sen used to describe fluctuations of regularized Tachyon kink in the abelian case [30].

If in the first instance, we make use of the quadratic approximation for the determinant:

$$detG_{\mu\nu} = \mathbb{1}_2 + 2\pi\alpha^{'}\partial_\mu T\partial^\mu T + \mathcal{O}(\alpha^{'2}) \tag{4.17}$$

the action in the large a limit becomes

$$S = -Tr\int d^{10}xV(f)\sqrt{2\pi}a\sqrt{f'^2}\sqrt{\mathbb{1}_2 + \partial_\alpha t\partial^\alpha t} \tag{4.18}$$

where t is the $U(2)$ matrix $t^a\sigma_a$.

In obtaining the above we have implicitly assumed that $\partial_\alpha f = -\frac{a}{\sqrt{\alpha'}}f'\partial_\alpha t$ while $\partial_x f = \frac{a}{\sqrt{\alpha'}} f'$ is identically the case since the dependence on x is via the unit matrix $\mathbb{1}_2$ in f. In fact, there is a subtlety associated with the former relation: since $\partial_\alpha t$ and t do not commute in general, there is an ordering issue that means that for general functions f, differentiating w.r.t. ξ^α one cannot simply use the chain rule and express the result as $-\frac{a}{\sqrt{\alpha'}}f'\partial_\alpha t$. There will be various symmetric ordering of $\partial_\alpha t$ and t that spoil this.

However there is at least one example, namely when $f(u)$ is linear in its argument (with positive coefficient so that $f' > 0$ everywhere as required) where the chain rule will hold and no ordering problems occur when differentiating.

The linear form of f has another interesting feature. If we had started with the Str form of the action, then as discussed above this implies symmetrization w.r.t. $F_{\mu\nu}, D_\mu T$ and T. For linear f we see that it follows that this Tr procedure immediately implies a similar Str procedure where we replace T with t. This is exactly what we would expect if we require that the Str procedure is the one that correctly describes coincident D8-branes with t the single transverse coordinate to the world volume.

Finally it is interesting to observe that as pointed out in [30], the linear Tachyon profile seems to play and important role in the BSFT description of Tachyon vortex solutions discussed in [32, 33].

For all these reasons the linear form of f seems to be singled out as being special. For now we will leave f in its generic form but bear in mind these issues.

The action (4.18) looks of the right form, i.e., it is a non-abelian DBI action (though with the gauge field fluctuations yet to be included). However, one faces taking the square root of the function f'^2 which is matrix valued and is thus non trivial. One has to diagonalize the matrix f first in order to take its square root and obtain a closed form expression. The terms inside the second square root part of the action are proportional to the identity and so we can diagonalize them by a $U(2)$ transformation directly [2]:

$$\begin{aligned} S &= -\sqrt{2\pi}a\,Tr\int d^{10}x\,V(f)\,\sqrt{f'^2}\sqrt{\mathbb{1}_2+\partial_\alpha t\partial^\alpha t} \\ &= -\sqrt{2\pi}a\,Tr\int d^{10}x\,U^\dagger\,V(f)U\,U^\dagger\sqrt{f'^2}U\,\sqrt{\mathbb{1}_2+\partial_\alpha t\partial^\alpha t} \\ &= -\sqrt{2\pi}a\,Tr\int d^{10}x\,V(U^\dagger fU)\,\sqrt{U^\dagger f'^2U}\,\sqrt{\mathbb{1}_2+\partial_\alpha t\partial^\alpha t}\,. \end{aligned} \tag{4.19}$$

Now,

$$\begin{aligned} U^\dagger f(\frac{a}{\sqrt{\alpha'}}(x\mathbb{1}_2+t^a(\xi)\sigma_a))U &= f(U^\dagger\frac{a}{\sqrt{\alpha'}}(x\mathbb{1}_2+t^a\sigma_a)U) \\ &= f\left(\frac{a}{\sqrt{\alpha'}}\left((x+t^0)\mathbb{1}_2+U^\dagger t^i\sigma_iU\right)\right) \\ &= f\left(\frac{a}{\sqrt{\alpha'}}\left((x+t^0)\mathbb{1}_2+\sqrt{t^at_a}\sigma_3\right)\right) \end{aligned} \tag{4.20}$$

This diagonalization then describes a matrix of the form:

$$\begin{aligned} U^\dagger f(\frac{a}{\sqrt{\alpha'}}(x\mathbb{1}_2+t^a(\xi)\sigma_a))U &= \begin{pmatrix} f\left(\frac{a}{\sqrt{\alpha'}}(x+t^0+\sqrt{t^at_a})\right) & 0 \\ 0 & f\left(\frac{a}{\sqrt{\alpha'}}(x+t^0-\sqrt{t^at_a})\right) \end{pmatrix} \\ &\equiv \mathcal{D}(f_1,f_2) \end{aligned} \tag{4.21}$$

[2]Note that whilst the results following from this action are for general f, in order to write this action in the first place one assumes that either f is linear or the DBI action arranges itself so as to be possible to factorise powers of f' in the way written in this action

where

$$\begin{aligned} f_1 &= f\left(\frac{a}{\sqrt{\alpha'}}(x+t^0+\sqrt{t^a t_a})\right), \\ f_2 &= f\left(\frac{a}{\sqrt{\alpha'}}(x+t^0-\sqrt{t^a t_a})\right). \end{aligned}$$

We also note that the matrix used to diagonalize f only depends on the variables $t^i(\xi)$ which means that $U^\dagger f' U = (U^\dagger f U)'$ and so the action (4.19) becomes

$$\begin{aligned} S &= -\sqrt{2\pi} a\, Tr \int d^{10}x\, \mathcal{D}(V(f_1), V(f_2))\, \mathcal{D}(f_1', f_2') \sqrt{\mathbb{1}_2 + \partial_\alpha t \partial^\alpha t} \\ &= -\sqrt{2\pi} a\, Tr \int d^{10}x\, \mathcal{D}(V(f_1) f_1', V(f_2) f_2')\, \sqrt{\mathbb{1}_2 + \partial_\alpha t \partial^\alpha t}\,. \end{aligned} \quad (4.22)$$

Substituting for the variables $y = f_1$ and $z = f_2$ we obtain the generalization of Sen's procedure for the non-abelian case:

$$\begin{aligned} S &= -\sqrt{2\pi\alpha'} Tr \int d^9x\, \mathcal{D}\left(\int_{-\infty}^{\infty} dy V(y)\,, \int_{-\infty}^{\infty} dz V(z)\right) \sqrt{\mathbb{1}_2 + \partial_\alpha t \partial^\alpha t} \\ &= -T_8\, Tr \int d^9x \sqrt{\mathbb{1}_2 + \partial_\alpha t \partial^\alpha t} \end{aligned} \quad (4.23)$$

which we recognize as the non-abelian DBI action for the coincident D8-branes (with gauge fields set to zero) upon identifying the tension $T_8 = \sqrt{2\pi\alpha'} \int_{-\infty}^{\infty} dy V(y)$. In order to be sure that in the $a \to \infty$ limit one really is in the vacuum of the theory we must look at the potential for the matrix form of T: the requirement that $V(f(\pm\infty)) = 0$ is enough to ensure that. Now one might also try and arrive at the Str form of the above action, by starting with the Str form of the Tachyon action for non-BPS D9-branes (2.35). The terms inside the square root part of the action are diagonal in $U(2)$ space and so one can imagine expanding out the square root factor in a power series and them symmetrizing over terms involving $\partial_\alpha T$ and T in $V(T)$. The problem one encounters then is that integrating over dx by making the change of variables as above does not look feasible due to the non-commutation between f and $\partial_\alpha t$ terms. That is, even using the cyclic properties of Tr, terms obtained through Str cannot be factorized into terms involving just powers of f times those involving $\partial_\alpha t$. Therefore, it seems that a straightforward generalization of Sen's procedure to show that non-abelian Tachyon condensation via kink solitons in coincident non-BPS brane theories gives rise to coincident Dp-branes is only possible in the Tr prescription rather than Str. It is interesting to see here a parallel to the problem of Str vs Tr prescriptions in trying to realize vortex (as opposed to kink) solutions in brane-antibrane systems obtained from coincident non-BPS D9-branes [37].

Working within the Tr prescription, let us now proceed to include the gauge field fluctuations and to go beyond the quadratic approximation of the determinant used before, to include all higher order terms. We take the following ansatz for the gauge fields [30]:

$$A_x(x,\xi) = 0\,, \qquad A_\alpha(x,\xi) = a(\xi)^a_\alpha \sigma_a\,, \tag{4.24}$$

Now let us pause briefly to comment on the action of the covariant derivative D_α on the function f appearing in the ansatz eq. (4.16) for the Tachyon kink. Just as we mentioned earlier when discussing the action of ∂_α on f, the commutator terms $[A_\alpha, f]$ cannot, in general, easily be expressed in terms of f' and $[A_\alpha, t]$ which is what we would have hoped if we are to promote the action eq. (4.23) to one that is locally $U(2)$ invariant. There are again ordering issues arising form the non-commutativity of $[A_\alpha, t]$ and t. Taking $f(u)$ linear in its argument avoids this as before. For now let us just keep f in our expressions but have in mind that it is likely to be constrained to be linear if we assume that $D_\alpha T = -\frac{a}{\sqrt{\alpha'}} f' D_\alpha t$. We can proceed with calculating the determinant of the matrix in the action using the ansatz (4.16) for the Tachyon field and (4.24) for the gauge fields. We obtain

$$G_{xx} = (1 + 2\pi a^2 f'^2) \tag{4.25}$$

$$G_{\alpha x} = -2\pi a^2 f'^2 D_\alpha t \tag{4.26}$$

$$G_{\alpha\beta} = \pi a^2 f'^2 (D_\alpha t D_\beta t + D_\beta t D_\alpha t) + a_{\alpha\beta} \tag{4.27}$$

where $a_{\alpha\beta} = \eta_{\alpha\beta} + 2\pi\alpha' F_{\alpha\beta}$. Now we can make use of Sen's trick [30] of adding rows and columns of the same matrix to simplify the computation of the determinant. In particular, we have

$$\hat{G}_{\mu\beta} = G_{\mu\beta} I_2 + \frac{1}{2} G_{\mu x} D_\beta t + \frac{1}{2} D_\beta t G_{\mu x} \tag{4.28}$$

$$\hat{G}_{\mu x} = G_{\mu x} \tag{4.29}$$

and finally:

$$\tilde{G}_{\alpha\nu} = \hat{G}_{\alpha\nu} I_2 + \hat{G}_{x\nu} D_\alpha t \tag{4.30}$$

$$\tilde{G}_{x\nu} = \hat{G}_{x\nu} \tag{4.31}$$

from which we obtain

$$\tilde{G}_{xx} = (1 + 2\pi a^2 f'^2) \mathbb{1}_2, \quad \tilde{G}_{x\alpha} = \tilde{G}_{\alpha x} = D_\alpha t(\xi)^a \sigma_a, \quad \tilde{G}_{\alpha\beta} = \tilde{a}_{\alpha\beta} \tag{4.32}$$

where

$$\tilde{a}_{\alpha\beta} = a_{\alpha\beta} + D_\alpha t^a(\xi) D_\beta t^b(\xi) \sigma_a \sigma_b \,. \tag{4.33}$$

This means that overall

$$\det(\tilde{G}_{\mu\nu}) = \det(G_{\mu\nu}) = 2\pi a^2 f'^2 det(\tilde{a}_{\alpha\beta}) + O(\frac{1}{a^2}) \,. \tag{4.34}$$

The last equation is precisely the generalization of the result Sen obtained to the case of local $U(2)$ gauge covariant quantities. Note that in the above manipulations we have taken f' to commute through expressions involving $U(2)$ matrices. For general f this would not be the case but for linear f, f' is simply proportional to the 2×2 identity matrix as noted earlier, so this is justified.

We can now substitute this result into the action to obtain

$$S = -\sqrt{2\pi} a \, Tr \int d^{10}x \, \mathcal{D}(V(f_1) f_1', V(f_2) f_2') \sqrt{-det(\tilde{a}_{\alpha\beta})} \tag{4.35}$$

which is the full non-abelian DBI action for two coincident D8-branes (using the Tr prescription) once the usual parameter substitutions are performed and the resulting integral over x identified with the D8-brane tension T_8:

$$S = -T_8 \, Tr \int d^9x \sqrt{-det(\tilde{a}_{\alpha\beta})} \,. \tag{4.36}$$

Now one should also show, as a further check, that the solutions of the equations of motion arising from the action (4.36) coincide with the solutions as derived from the original coincident non-BPS D9-brane action (2.31), upon using the non-abelian Tachyon profile given in eq. (4.16). This check was done explicitly by Sen in [30] in the case of Tachyon condensation on a single non-BPS Dp-brane. The calculation in our case would follow quite closely that of Sen, just extended to the non-abelian case relevant to two coincident D-branes.

1.3 Breaking $U(2)$ to $U(1) \otimes U(1)$

As further check on our generalized Sen ansatz eq. (4.16), we can consider modifying the argument of f so that the corresponding kink solution breaks $U(2)$ symmetry and thus should describe a pair of separated D8-branes after condensation. This amounts to allowing a vacuum expectation value to one of the $U(2)$ adjoint fields t^i. In particular, we set $t(\xi) \to t(\xi) + c\sigma_3$, where c denotes a constant v.e.v. related to the separation of the two D8-branes along their single transverse direction. In this case we expect to break the $U(2)$ invariance of the theory down to $U(1) \otimes U(1)$. The resulting action of fluctuations about this vacuum configuration should split into two abelian DBI actions, i.e., two distinct determinant terms each carrying a single $U(1)$ gauge field and perpendicular scalar fluctuation field, that describe the separate D8-branes.

We start by introducing the v.e.v. c and obtain a modification of eq. (4.33) due to this shift: in particular

$$\begin{aligned}\tilde{G}_{\alpha\beta} = \tilde{a}_{\alpha\beta} \;=\;& a_{\alpha\beta} + \partial_\alpha t \partial_\beta t - i\partial_\alpha t[A_\beta, t] - i[A_\alpha, t]\partial_\beta t - [A_\alpha, t][A_\beta, t] \\ & -ic\,\partial_\alpha t\,[A_\beta, \sigma_3] - ic[A_\alpha, \sigma_3]\partial_\beta t - c[A_\alpha, t][A_\beta, \sigma_3] - c[A_\alpha, \sigma_3][A_\beta, t] \\ & -c^2[A_\alpha, \sigma_3][A_\beta, \sigma_3] \qquad (4.37)\end{aligned}$$

where the covariant derivatives appearing in eq. (4.33) have been expanded out explicitly. To proceed we make use of a different parametrization of t that makes explicit the Goldstone modes associated with $U(2)$ symmetry breaking: we set

$$t^a \sigma_a = U^\dagger(\tilde{t}^0 \mathbb{1}_2 + \tilde{t}^3 \sigma_3)U \qquad (4.38)$$

where $U = \exp^{\frac{i}{c}(\tilde{t}^1\sigma_1+\tilde{t}^2\sigma_2)}$ and we pick a preferential gauge in which

$$\begin{aligned}(t^a\sigma_a)' &= Ut^a\sigma_a U^\dagger = \tilde{t}^0\mathbb{1}_2 + \tilde{t}^3\sigma_3 \qquad (4.39)\\ (A^a_\alpha\sigma_a)' &= U(A^a_\alpha\sigma_a)U^\dagger - (\partial_\alpha U)U^\dagger\,. \qquad (4.40)\end{aligned}$$

In this gauge, the fluctuations t are diagonal and[3]

$$\begin{aligned}\partial_\alpha t\partial_\beta t &= (\partial_\alpha t^0\partial_\beta t^0 + \partial_\alpha t^3\partial_\beta t^3)\mathbb{1}_2 + (\partial_\alpha t^0\partial_\beta t^3 + \partial_\alpha t^3\partial_\beta t^0)\sigma_3\\ \partial_\alpha t\,[A_\beta, t] &= 2it^3\partial_\alpha t^0\left(A^2_\beta\sigma_1 - A^1_\beta\sigma_2\right) - 2t^3\partial_\alpha t^3\left(A^2_\beta\sigma_2 + A^1_\beta\sigma_1\right)\\ [A_\alpha, t][A_\beta, t] &= 4(t^3)^2\left(-A^1_\alpha A^1_\beta - A^2_\alpha A^2_\beta + i\left(A^2_\alpha A^1_\beta - A^1_\alpha A^2_\beta\right)\sigma_3\right) \qquad (4.41)\end{aligned}$$

with similar expressions holding with various t^3 are replaced by the v.e.v. c. Now we redefine the gauge fields so as to absorb the v.e.v. c by setting $A^i_\alpha = \frac{1}{2c}\tilde{A}^i_\alpha$ for $i = 1, 2$. Substituting these expressions and taking the large c limit one obtains to leading order

$$\begin{aligned}\tilde{G}_{\alpha\beta} &= \eta_{\alpha\beta} + F^0_{\alpha\beta}\mathbb{1}_2 + F^3_{\alpha\beta}\sigma_3 + (\partial_\alpha t^0\partial_\beta t^0 + \partial_\alpha t^3\partial_\beta t^3)\mathbb{1}_2 + (\partial_\alpha t^0\partial_\beta t^3 + \partial_\alpha t^3\partial_\beta t^0)\sigma_3\\ &\quad \left(\partial_\alpha t^0(A^2_\beta\sigma_1 - A^1_\beta\sigma_2) + (\alpha \leftrightarrow \beta)\right) + i\left(\partial_\alpha t^3(A^1_\beta\sigma_1 + A^2_\beta\sigma_2) - (\alpha \leftrightarrow \beta)\right)\\ &\quad +(A^1_\alpha A^1_\beta + A^2_\alpha A^2_\beta)\mathbb{1}_2 - i\left(A^2_\alpha A^1_\beta - A^1_\alpha A^2_\beta\right)\sigma_3 \qquad (4.42)\end{aligned}$$

The fields $A^i_\alpha, i = 1, 2$ are non-propagating to lowest order in a $1/c$ expansion and a consistent solution of their equations of motion is to set $A^1_\alpha = A^2_\alpha = 0$. The limit of large c corresponds to considering the two coincident D8-branes as being separated by a distance that is large compared to the string length $\sqrt{\alpha'}$.

We use this and redefine the field strengths and scalar fields associated to each brane as $F^1_{\alpha\beta} = F^0_{\alpha\beta} + F^3_{\alpha\beta}$, $F^2_{\alpha\beta} = F^0_{\alpha\beta} - F^3_{\alpha\beta}$ and $\phi^1 = t^0 + t^3$, $\phi^2 = t^0 - t^3$. Then, in group space the matrix $\tilde{G}_{\alpha\beta}$ reduces to

$$\tilde{G}_{\alpha\beta} = \begin{pmatrix} \eta_{\alpha\beta} + F^1_{\alpha\beta} + \partial_\alpha\phi^1\partial_\beta\phi^1 & 0 \\ 0 & \eta_{\alpha\beta} + F^2_{\alpha\beta} + \partial_\alpha\phi^2\partial_\beta\phi^2 \end{pmatrix}$$

[3] We drop the prime sign from the gauged form of A'_α and the tilde on $\tilde{t}^0, \tilde{t}^3$.

hence,

$$\sqrt{-\det(\tilde{G}_{\alpha\beta})} = \begin{pmatrix} \sqrt{-det(\eta_{\alpha\beta} + F^1_{\alpha\beta} + \partial_\alpha\phi^1\partial_\beta\phi^1)} & 0 \\ 0 & \sqrt{-det(\eta_{\alpha\beta} + F^2_{\alpha\beta} + \partial_\alpha\phi^2\partial_\beta\phi^2)} \end{pmatrix}$$

and finally defining $\tilde{G}^1_{\alpha\beta} = \eta_{\alpha\beta} + F^1_{\alpha\beta} + \partial_\alpha\phi^1\partial_\beta\phi^1$ and $\tilde{G}^2_{\alpha\beta} = \eta_{\alpha\beta} + F^2_{\alpha\beta} + \partial_\alpha\phi^2\partial_\beta\phi^2$ we find that the action becomes

$$S = -\sqrt{2\pi}a\int d^{10}x\left(V(f_1)f_1'\sqrt{-\det(\tilde{G}^1_{\alpha\beta})} + V(f_2)f_2'\sqrt{-\det(\tilde{G}^2_{\alpha\beta})}\right) . \quad (4.43)$$

After performing the usual change of variables and using the descent relation between T_9, T_8 and V, we recognise this as being the $U(1) \otimes U(1)$ symmetric abelian DBI action for two separate D8-branes.

2 Dirac-Born-Infeld Tachyon Monopoles

2.1 Monopole Solution in the $D8$-$D6$ System

In this section we will discuss how to obtain a $Dp \to D(p-3)$ brane descent relation via a monopole solution for the tachyon profile [120]. The construction of this solution involves a t'Hooft-Polyakov monopole in $SU(2)$ Yang-Mills Theory [121, 122]. Just as in the case of kink and vortex solitonic tachyon solutions of the full DBI non-BPS actions, as previously analysed by Sen, these monopole configurations are singular in the first instance and require regularisation. We discuss a suitable non-abelian ansatz which describes a point-like magnetic monopole and show it solves the equations of motion to leading order in the regularisation parameter. Fluctuations are studied and shown to describe a co-dimension three BPS D6-brane and a formula is derived for its tension. In order to introduce the full world-volume calculation we present first an energy based method which serves to illustrate the descent relation and its main result without delving in complex calculations [120].

To obtain two parallel $D8$ branes we will start off with two non-BPS $D9$ branes and use a kink solution for the Tachyon to arrive at the $D8$'s via the method of tachyon condensation $Dp \to D(p-1)$ illustrated above. We start off with the action

$$S = \mathcal{T} STr \int d^{10}x e^{-\frac{T^2}{a}} \left(\mathbb{I} + (D_\mu T)(D^\mu T) + \frac{1}{2} F_{\mu\nu} F^{\mu\nu} \right) \tag{4.44}$$

which can be considered as the two-derivative truncated form of 2.31 with the BSFT potential 2.28 and a symmetrisation implied over T^2, $D_\mu T$ and $F_{\mu\nu}$. The equations of motion derived from this action take the form

$$\begin{aligned} \sum_\sigma \left[\left(-2D_\mu D^\mu T - \frac{2}{a} T(\mathbb{I} - D_\mu T D^\mu T + \frac{1}{2} F_{\mu\nu} F^{\mu\nu}) \right) e^{-\frac{T^2}{a}} \right] = 0 \\ \sum_\sigma \left[D_\mu \left(e^{-\frac{T^2}{a}} F^{\mu\nu} \right) + \left[T, e^{-\frac{T^2}{a}} D_\nu T \right] \right] = 0, \end{aligned} \tag{4.45}$$

where $\sum_\sigma$ denotes the symmetrisation over $U(2)$ matrices described above. These equations present a particularly interesting solution with non-vanishing gauge field:

$$\begin{aligned} T &= x_9\mathbb{I} + \Phi \\ A_i &= \epsilon_{aij}x_j\frac{W(r)}{r}\frac{1}{2}\sigma_{a-5} \end{aligned} \tag{4.46}$$

with a, i and j running from 6 to 8 and σ_{a-5} are the usual sigma matrices. The scalar field Φ is given by

$$\Phi = x_a\frac{F(r)}{r}\frac{1}{2}\sigma_{a-5}. \tag{4.47}$$

The scalar field and the gauge field take the exact form of the Prasad-Sommerfield limit of the t'Hooft-Polyakov monopole solution,

$$F(r) = \frac{C}{tanh(Cr)} - \frac{1}{r}, \quad W(r) = \frac{1}{r} - \frac{C}{sinh(Cr)} \tag{4.48}$$

and accordingly the solution satisfies the Bogomol'nyi equation $B_i + D_i\Phi = 0$ where $B_i = \epsilon_{ijk}F_{jk}$. The part proportional to the identity in the solution for T is precisely what we expect in order to construct the $D8$'s from non-BPS $D9$'s, i.e. a kink-like profile. We proceed now to show that the rest of the solution corresponds to a $D6$ brane localised between the $D8$'s. To do so, consider the energy of the system

$$E = \mathcal{T}STr\int d^9xe^{-\frac{T^2}{a}}\left((\mathbb{I} - \partial_9T)^2 + 2\partial_9T + (D_iT + B_i)^2 - 2(D_iTB_i)\right) \tag{4.49}$$

given that the solution satisfies the BPS equations $\mathbb{I} - \partial_9T = D_iT + B_i = 0$ the above provides an energy bound for the solution. Evaluating this energy for the solution above gives

$$\begin{aligned} E = \mathcal{T}STr\Bigg(2\sqrt{\pi a}\int d^8x\mathbb{I}\left[S\left(\frac{T}{\sqrt{a}}\right)\right]_{-\infty}^{\infty} \\ -2\sqrt{\pi a}\int d^5x\int dx_9\int_{r=\infty}dS_i\left[S\left(\frac{T}{\sqrt{a}}\right)B_i\right]\Bigg) \end{aligned} \tag{4.50}$$

with $S(X)$ denoting the step function. Hence the first term in this expression shows the energy of two parallel $D8$ branes as we were expecting, however the second term involves an integral over d^5x which suggests that this corresponds to a $D6$ brane, at least at the level of energy considerations. Using the gauge symmetry to diagonalise Φ such that

$$T = \begin{pmatrix} x_9 + \frac{C}{2} & 0 \\ 0 & x_9 - \frac{C}{2} \end{pmatrix} \tag{4.51}$$

(where the function $F(r)$ is expanded around $r \to \infty$), then the magnetic field also becomes diagonal

$$B_i = -\frac{1}{2}\frac{x_i}{r^3}\sigma_3 \tag{4.52}$$

and the energy evaluates to

$$E = 4\pi\sqrt{\pi a} Vol_5 \mathcal{T} \int dx_9 \left(S\left(\frac{(x_9 + C/2)}{\sqrt{a}}\right) - S\left(\frac{(x_9 - C/2)}{\sqrt{a}}\right)\right) \tag{4.53}$$

with Vol_5 the five-dimensional volume. Hence these step functions indicate that the energy-density of the $D6$ Brane is localised on the line segment $-C/2 < x_9 < C/2$ and therefore the solution corresponds to a $D6$ brane suspended between parallel $D8$ branes. Therefore this initial investigation has shown that a monopole solution for the Tachyon is related to a descent relation to a $D(p-3)$ brane, which we now proceed to investigate in full.

Monopole solutions in certain truncations of tachyon models have already been studied in [119] as shown above. In [123] the authors extended their results to include all higher derivatives using the boundary string field theory (BSFT) approach and thus argued the ansatz for the tachyon monopole introduced in [119] survives higher derivative corrections. However, in this section we wish to investigate magnetic monopole solutions arising from the full non-linear non-abelian DBI like action, i.e., without assuming an action truncated in an expansion in derivatives of the tachyon field. From our understanding of the DBI tachyon kink and vortex solutions discussed above, we expect (and find) that such monopole solutions will again be singular in the first instance and require regularisation. We find solutions that are in perfect agreement with those obtained in BSFT and so provides an independent check of the tachyon monopole ansatz first presented in [119, 123].

Our starting point will be the effective description of two coincident non-BPS D9-branes proposed in [37]. As discussed, this theory describes a non-abelian version of the DBI action in which the tachyon field transforms in the adjoint representation of the $U(2)$ gauge symmetry of the coincident non-BPS D9-brane world volume action. In the original construction of this action and its generalisation to coincident non-BPS Dp-branes, a standard trace prescription (which we denote as Tr) was taken over the gauge indices. As mentioned earlier, the other prescription, motivated by string scattering calculations (at least to low orders in α' [29, 38]) is to take the symmetrized trace (which we denote by Str) over gauge indices. In both cases the expression being traced over is the same but the Str prescription results, in general, in significantly more complicated terms in the action compared to Tr. Throughout this section we will adopt the Str procedure and we will find that its implementation in the case of a tachyon monopole profile is straightforward and leads to the correct expression for the D6-Brane tension.

The structure of the analysis is as follows. We begin in section 2.2 with a 't Hooft-Polyakov monopole like ansatz for the $U(2)$ non-abelian DBI tachyon world volume theory and show how it leads to the correct expression for the resulting D6 brane tension, realised as a co-dimension 3 solution of the equations of motion, with a suitable regularisation. In section 2.3 a study of the fluctuation spectrum about these monopoles shows them to be precisely described by a DBI action of a single BPS D6 brane in flat space, in the limit where the regularisation is removed.

Finally in Appendix A, we show how the tachyon monopole ansatz satisfies the correct Dirac quantisation of magnetic charge.

2.2 The 't Hooft-Polyakov Monopole and the DBI action

We begin by reviewing an effective DBI action for the coincident non-BPS D9-brane pair [37]. This system is unstable and it contains a tachyon in its spectrum, in particular, around the maximum of the tachyon potential, the theory contains a $U(2)$ gauge field and four tachyon states represented by a 2×2 hermitian matrix-valued scalar field transforming in the adjoint representation of the gauge group.

We therefore start with action 2.31 with a full symmetrised trace prescription, for the potential we shall only assume that a family of minima can be found by taking (up to a SU(2) rotation)

$$T = \begin{pmatrix} +\infty & 0 \\ 0 & -\infty \end{pmatrix} \tag{4.54}$$

which represent the tachyon on the first D-brane at its minimum $T_0 = +\infty$ and the tachyon on the second D-brane at its minimum $T_0 = -\infty$. We shall also assume that the potential vanishes at $T = T_0$. The monopole solution of the DBI action (2.31) corresponds in taking the tachyon and the gauge fields to depend on three worldvolume coordinates x_i, with $i = 1, 2, 3$ whereas $\alpha, \beta = 0, 4 \ldots 9$ will label the other worldvolume coordinates including time.

Apart from a $U(1)$ subgroup, the effective theory of two unstable D-branes, admits as a solution the 't Hooft-Polyakov monopole, which in the limit of zero-size core is of the form

$$\begin{aligned} T(x) &= t(r)\frac{x \cdot \sigma}{r} \, , \\ A_i(x) &= \frac{1}{2}\epsilon_{ijk}\,\frac{x_j}{r^2}\sigma_k \end{aligned} \tag{4.55}$$

where r is the radial distance from the origin in the three transverse directions [4]. In [124] it was shown that the limit of zero-size core correctly reproduces also the Ramond-Ramond couplings of a D6-brane. It is actually more convenient to work in spherical coordinates

$$x_1 = r\cos\theta \ , \quad x_2 = r\sin\theta\cos\phi \ , \quad x_3 = r\sin\theta\sin\phi \tag{4.56}$$

to make use of the spherical symmetry of the solution. In these coordinates the tachyon takes the form

$$T = t(r) x_r \cdot \sigma \tag{4.57}$$

and the gauge fields

$$A_r = 0 \ , \quad A_\theta = -\frac{1}{2\sin\theta} x_{\phi r} \cdot \sigma \ , \quad A_\phi = \frac{1}{2}\sin\theta \, x_{\theta r} \cdot \sigma \tag{4.58}$$

[4] A similar ansatz was proposed by [119] in the context of a truncated tachyon DBI action and later extended to include all higher derivatives via BSFT in [123]

where $x^i_r = \partial_r x^i$ and $x^i_{\phi r} = \partial_r \partial_\phi x^i$ and so on. The covariant derivatives of the tachyon are

$$D_r T = t'(r) x_r \cdot \sigma \ , \quad D_\theta T = D_\phi T = 0 \ , \tag{4.59}$$

the gauge field strength

$$F_{r\theta} = F_{r\phi} = 0 \ , \quad F_{\theta\phi} = -\frac{1}{2} \sin\theta x_r \cdot \sigma \ . \tag{4.60}$$

Finally, the determinant becomes:

$$-\text{det}G = (1 + \lambda D_r T D_r T) \left(r^4 \sin^2\theta + \lambda^2 F^2_{\theta\phi} \right) . \tag{4.61}$$

We now compute the energy-momentum tensor

$$T^{\mu\nu} = -\text{STr} \left(V(T) \sqrt{-\text{det}G} (G^{-1})^{\mu\nu} \right) \tag{4.62}$$

The elements with one r-component are

$$\begin{aligned} T_{rr} &= -\text{STr} \left[\frac{V(T)\sqrt{r^4 \sin^2\theta + \lambda^2 F^2_{\theta\phi}}}{\sqrt{1 + \lambda D_r T D_r T}} \right] , \\ T_{r\theta} &= T_{r\phi} = 0 \ . \end{aligned} \tag{4.63}$$

From the previous expressions it is clear that the conservation equation for the r-component reduces to $\partial_r \ T_{rr} = 0$. If we assume that the potential vanishes at infinity, then T_{rr} must vanish everywhere because of the conservation equation, hence T_{rr} should vanish for all r. However, for r close to the origin, the potential is finite and T_{rr} doesn't vanish and so at least for small r we require $t'(r)$ to blow up. This forces us to consider a regularization of the form

$$T = \hat{t}(kr) x_r \cdot \sigma \tag{4.64}$$

such that in the $k \to \infty$ limit $t'(r)$ goes to infinity while keeping $t(r)$ fixed. In particular, in the large k limit:

$$D_r T D_r T = k^2 \hat{t}'^2 (x_r \cdot \sigma)^2 \tag{4.65}$$

and the energy-momentum tensor which goes like $T_{rr} \sim 1/k$ vanishes everywhere as required. This shows that the monopole solution is indeed a solution to the conservation equation and hence a consistent solution of the system e.o.m. Let us now calculate the tension associated with the D6-brane: the energy-momentum tensor along the directions orthogonal to the monopole is

$$T_{\alpha\beta} = -\eta_{\alpha\beta}\,\mathrm{STr}\left[V(T)\sqrt{(1+\lambda D_rTD_rT)\left(r^4\sin^2\theta+\lambda^2F^2_{\theta\phi}\right)}\right] \tag{4.66}$$

which, by taking the large k-limit and by performing the following coordinate transformation,

$$y = \hat{t}(kr)\,, \quad r \equiv \hat{r}(y) = k^{-1}\hat{t}^{-1}(y)\,, \tag{4.67}$$

becomes, after integrating over the x_i world-volume coordinates

$$T^{int}_{\alpha\beta} = -\frac{1}{2}\lambda^{3/2}\eta_{\alpha\beta}\,\mathrm{STr}\left[\int dyd(-\cos\theta)d\phi\,V(T(y))(x_r\cdot\sigma)^2\right] \tag{4.68}$$

In a similar fashion to the kink and vortex calculations seen in chapter 3 [30] most of the contribution to $T_{\alpha\beta}$ comes from a small region in r space centered around $\frac{1}{k}$. We can identify the tension of the D6-brane as:

$$\mathcal{T}_6 = \frac{1}{2}\lambda^{3/2}\mathrm{STr}\int d(-\cos\theta)d\phi\,dyV(y)(x_r\cdot\sigma)^2 \tag{4.69}$$

The tension of the D6-brane is determined only by the tachyon potential. Now we try to evaluate the previous expression by choosing an explicit expression for the tachyon potential. One which gives a lot quantitative agreements with string theory results is 2.29 [5]:

$$V(T) = \frac{\sqrt{2}\mathcal{T}_9}{\cosh(\sqrt{\pi}T)} = \sqrt{2}\mathcal{T}_9\sum_{i=0}^{\infty}\frac{E_{2i}(\sqrt{\pi}y)^{2i}(x_r\cdot\sigma)^{2i}}{(2i)!} \tag{4.70}$$

where E_i is ith Euler number. We see that in order to compute the tension of the D6-brane we need to evaluate

$$\mathrm{STr}\left[(x_r\cdot\sigma)^{2m}\right] = \mathrm{Tr}\left[(x_r\cdot\sigma)^{2m}\right] = 2\ . \tag{4.71}$$

[5] For example, this potential reproduces the correct tachyon mass $\mathcal{L}\sim\mathrm{Tr}\left(D_\mu TD^\mu T - \frac{1}{2\alpha'}T^2\right)$

Therefore, the tension becomes

$$\mathcal{T}_6 = \sqrt{2}\mathcal{T}_9\lambda^{3/2}4\pi\int_0^\infty dy\,\frac{1}{\cosh(\sqrt{\pi}y)} = (2\pi\sqrt{\alpha'})^3\mathcal{T}_9\,. \tag{4.72}$$

which correctly reproduces the D-brane tension descent relation between the $\mathcal{T}_9$ and the $\mathcal{T}_6$ tension.

2.3 World-volume action and the monopole

This section is devoted to analyzing the world-volume fluctuations of the tachyon monopole background described in the previous section. We plan to show that the world-volume theory of the monopole condensed on a Dp-brane results in a D(p-3)-brane, described by an action with a U(1) gauge theory. Although our analysis involves the presence of non-abelian tachyon and gauge fields, what follows is similar to [30] because all our computations are carried out inside the STr operation, in which objects are effectively commutative. We begin by recasting the ansatz for the monopole in the following way:

$$\begin{aligned} T(\vec{x}) &= f(r)x_i\sigma_i \\ A_i(\vec{x}) &= g(r)\epsilon_{ijk}x_j\sigma_k \end{aligned} \tag{4.73}$$

where $g(r) = 1/(2r^2)$ and $f(r) = t(r)/r$. We make the following ansatz for the fluctuating fields:

$$\begin{aligned} \bar{T}(\vec{x},\xi) &= T(\vec{x}-\vec{\phi}(\xi)) = f(\hat{r})(x_i-\phi_i(\xi))\sigma_i \\ \bar{A}_i(\vec{x},\xi) &= A_i(\vec{x}-\vec{\phi}(\xi)) = g(\hat{r})\epsilon_{ijk}(x_j-\phi_j(\xi))\sigma_k \\ \bar{A}_\alpha(\vec{x},\xi) &= -\bar{A}_i(\vec{x},\xi)\partial_\alpha\phi^i + a_\alpha(\xi)\otimes\mathbb{1} \end{aligned} \tag{4.74}$$

In the previous expressions, $\phi_i(\xi)$ are scalar fluctuations which depend on the worldvolume coordinate of the D-brane and we have defined

$$\hat{r}^2 = (x_i-\phi_i(\xi))(x^i-\phi^i(\xi)) \tag{4.75}$$

Using the fact that at the end we have to take the symmetrized trace we can write $\partial_\alpha \bar{T} = -\partial_\alpha \phi^i \partial_i \bar{T}$ and $[\bar{A}_\alpha, \bar{T}] = -\partial_\alpha \phi^i [\bar{A}_i, \bar{T}]$ to obtain

$$D_\alpha \bar{T} = -D_i \bar{T}\, \partial_\alpha \phi^i \tag{4.76}$$

and similarly, using the fact that $\partial_\alpha \bar{A}_j = -\partial_\alpha \phi^i \partial_i \bar{A}_j$ and defining $f_{\alpha\beta} \equiv \partial_\alpha a_\beta - \partial_\beta a_\alpha$, we have

$$\begin{aligned}
F_{\alpha\beta} &= \bar{F}_{ij} \partial_\alpha \phi^i \partial_\beta \phi^j + f_{\alpha\beta} \mathbb{1} \\
F_{\alpha j} &= -\partial_\alpha \phi^i \bar{F}_{ij} \\
F_{i\alpha} &= -\bar{F}_{ij} \partial_\alpha \phi^j \,, \\
F_{ij} &= \partial_i \bar{A}_j - \partial_j \bar{A}_i - i[\bar{A}_i, \bar{A}_j]
\end{aligned}$$

From these we can proceed to compute the matrix elements of our determinant, by defining

$$\gamma_{ij} \equiv \lambda D_i \bar{T} D_j \bar{T} + \lambda \bar{F}_{ij} \tag{4.77}$$

we have

$$\begin{aligned}
G_{\mu\nu} &= \begin{pmatrix} G_{\alpha\beta} & G_{\alpha j} \\ G_{i\beta} & G_{ij} \end{pmatrix} = \\
& \begin{pmatrix} \eta_{\alpha\beta} + \lambda f_{\alpha\beta} + g_{ij} \partial_\alpha \phi^i \partial_\beta \phi^j & -\partial_\alpha \phi^i g_{ij} \\ -g_{ij} \partial_\beta \phi^j & \delta_{ij} + g_{ij} \end{pmatrix}
\end{aligned}$$

Next, we introduce a new matrix $\hat{G}_{\mu\nu}$ whose elements are $\hat{G}_{\alpha\nu} \equiv G_{\alpha\nu} + \partial_\alpha \phi^i G_{i\nu}$ and $\hat{G}_{i\nu} = G_{i\nu}$, namely

$$\begin{aligned}
\hat{G}_{\mu\nu} &= \begin{pmatrix} \hat{G}_{\alpha\beta} & \hat{G}_{\alpha j} \\ \hat{G}_{i\beta} & \hat{G}_{ij} \end{pmatrix} \equiv \begin{pmatrix} G_{\alpha\beta} & G_{\alpha j} \\ G_{i\beta} & G_{ij} \end{pmatrix} + \partial_\alpha \phi^i \begin{pmatrix} G_{i\beta} & G_{ij} \\ 0 & 0 \end{pmatrix} \\
&= \begin{pmatrix} \eta_{\alpha\beta} + f_{\alpha\beta} & \partial_\alpha \phi_j \\ G_{i\beta} & G_{ij} \end{pmatrix}
\end{aligned} \tag{4.78}$$

If we were considering matrices whose elements were commuting, then clearly $\det G_{\mu\nu} = \det \hat{G}_{\mu\nu}$ because in that case the determinant would be invariant under the addition of a multiple of a row(column) to another row(column). This property follows from the fact that if each element in a row(column) is a sum of two terms, the determinant equals the sum of the two corresponding determinants. In our case the entries of the matrix $G_{\mu\nu}$ are $su(2)$ algebra-valued elements and therefore it is not clear *a priori* whether in this case that result should hold. However, notice that also in our case

$$\begin{aligned}
\det\hat{G}_{\mu\nu} &\equiv \begin{vmatrix} G_{\alpha\beta} + \partial_\alpha\phi^i G_{i\beta} & G_{\alpha j} + \partial_\alpha\phi^i G_{ij} \\ G_{i\beta} & G_{ij} \end{vmatrix} \\
&= \begin{vmatrix} G_{\alpha\beta} & G_{\alpha j} \\ G_{i\beta} & G_{ij} \end{vmatrix} + \begin{vmatrix} \partial_\alpha\phi^i G_{i\beta} & \partial_\alpha\phi^i G_{ij} \\ G_{i\beta} & G_{ij} \end{vmatrix}
\end{aligned} \tag{4.79}$$

and the latter determinant is zero because $\partial_\alpha\phi^i$, being proportional to the identity in group space, commutes with all the other elements and, therefore, $\det G_{\mu\nu} = \det\hat{G}_{\mu\nu}$. Using the same arguments, we perform a final redefinition by introducing the matrix $\tilde{G}_{\mu\nu}$ whose elements are $\tilde{G}_{\mu\beta} = \hat{G}_{\mu\beta} + \hat{G}_{\mu j}\partial_\beta\phi^j$ and $\tilde{G}_{\mu j} = \hat{G}_{\mu j}$, namely

$$\begin{aligned}
\tilde{G}_{\mu\nu} &= \begin{pmatrix} \tilde{G}_{\alpha\beta} & \tilde{G}_{\alpha j} \\ \tilde{G}_{i\beta} & \tilde{G}_{ij} \end{pmatrix} \equiv \begin{pmatrix} \hat{G}_{\alpha\beta} & \hat{G}_{\alpha j} \\ \hat{G}_{i\beta} & \hat{G}_{ij} \end{pmatrix} + \begin{pmatrix} \hat{G}_{\alpha j} & 0 \\ \hat{G}_{ij} & 0 \end{pmatrix} \partial_\beta\phi^j \\
&= \begin{pmatrix} \eta_{\alpha\beta} + f_{\alpha\beta} + \partial_\alpha\phi^i\partial_\beta\phi_i & \partial_\alpha\phi_i \\ \partial_\beta\phi_i & G_{ij} \end{pmatrix}
\end{aligned} \tag{4.80}$$

Now, we take the determinant of the previous expression. Notice that the determinant of G_{ij} is given by (4.61) upon the replacement of r by $|\vec{x} - \phi(\vec{\xi})|$. This determinant has an explicit factor of k^2 which becomes dominant in the large k limit, hence, we can ignore the off-diagonal contributions in computing $\det\tilde{G}_{\mu\nu}$. We have

$$-\det\tilde{G}_{\mu\nu} \approx -\det G_{ij}\,\det\tilde{G}_{\alpha\beta} \tag{4.81}$$

So substituting this into the action gives:

$$\begin{aligned}
S &= -\lambda^{1/2}\mathrm{STr}\int d^7\xi \int dr d(-\cos\theta) d\phi\; V(\hat{t}(kr))k\,\hat{t}'(kr) \\
&\quad \times\sqrt{r^4\sin^2\theta + \lambda^2 F_{\theta\phi}}\sqrt{-\det(\tilde{G}_{\alpha\beta})}
\end{aligned} \tag{4.82}$$

Performing the coordinate transformation in (4.67) and taking the large k-limit, we find

$$\begin{aligned} S &= -\frac{1}{2}\lambda^{3/2}\mathrm{STr}\int d^7\xi \int dyd(-\cos\theta)d\phi \\ &\qquad \times V(y)(x_r\cdot\sigma)^2\sqrt{-\det\tilde{G}_{\alpha\beta}} \\ &= -\mathcal{T}_6\int d^7\xi\sqrt{-\det\tilde{G}_{\alpha\beta}} \end{aligned} \tag{4.83}$$

where

$$\tilde{G}_{\alpha\beta} = \eta_{\alpha\beta} + \lambda f_{\alpha\beta} + \partial_\alpha\phi^i\partial_\beta\phi_i \tag{4.84}$$

This we recognize as the action of a BPS D6-brane, with the correct U(1) gauge theory.

There is therefore a natural non-abelian extension of Sen's descent relations for Tachyon condensation regarding the Kink solution and the conjectured relation between a Dp and a D$(p-3)$ brane via condensation into a monopole solution is proven from the world-volume analysis shown above. In the next section, motivated by the discussion introduced in the previous chapter, we attempt to perform a similar non-abelian extension of Kutasov's geometric Tachyon interpretation. This will provide a more complete analysis of non-abelian D-brane world-volume dynamics and a more general understanding of non-abelian Tachyon systems in String Theory.

3 Non-Abelian Geometrical Tachyon

In section 2 of the previous chapter we introduced Kutasov's geometric interpretation of the Tachyon field. We showed how the dynamics of a D-Brane moving in the vicinity of a stack of NS5 branes is unstable and can be directly described by a non-BPS Tachyon DBI action under a mapping where the Tachyon field is identified with the radial mode of the D-brane dynamics.

In this section we want to investigate what happens if we consider not just a single probe D-brane but rather a coincident pair of probe D-branes moving in the background of k coincident NS5 branes. For k large, this coincident pair of branes can still be regarded as probes in the sense that one may neglect the back-reaction on the geometry to first approximation.

The analysis proceeds as follows: in the first two sections we consider different ansatze for the scalar fields which realise the map of the unstable D-brane system to one described by a non-abelian Tachyonic mode. Next we show the importance of a careful choice of definition for the harmonic function H describing the NS5 branes background and we stress the differences between the matrix and function approach. Finally we make use of a symmetry breaking ansatz to expose a simplified version of the non-abelian system and give solutions for the equations of motion of the Tachyonic field, these will be shown to reduce to the known single brane results in the abelian limit.

3.1 Multiple D-branes in the NS5-brane background

Consider a stack of k parallel NS5 branes in type II string theory, stretched in the directions $x^\mu = (t, x^1, \ldots, x^5)$, $\mu = 0, \ldots 5$, and localised in $x^m = (x^6, \ldots, x^9)$, $m = 6, \ldots 9$. Then the background is that shown in 3.58, and we will make use of it here. We are interested in the dynamics of two coincident BPS D5 branes in the background of the five-branes. We can label the world-volume coordinates of the D-branes by ξ^μ, and by using reparametrisation invariance on their world-volume we set $\xi^\mu = x^\mu$.

The low-energy dynamics of the D5-brane pair is described by a non-abelian U(2) gauge theory [29] (see also [38]). The dynamics of the open string sector lightest degrees of freedom, namely the adjoint valued scalar fields $(X^6(\xi^\mu), \ldots, X^9(\xi^\mu))$ which describe the position of the pair in the transverse directions $(x^6, \ldots, x^9)$, the non-abelian gauge field A_μ as well as the lightest degrees of freedom of the closed string sector, namely the metric G_{AB}, the dilaton ϕ and the Kalb-Ramond field B_{AB} is governed by the non-abelian DBI action 2.20

$$\begin{aligned} S &= \\ &- T_5 \int d^6x \, \mathrm{STr} \left(e^{-(\Phi-\Phi_0)} \sqrt{-det \left(P\left[E_{\mu\nu} + E_{\mu m}\left(Q^{-1} - \delta\right)^{mn} E_{n\nu}\right] + \lambda F_{\mu\nu}\right) det(Q^m_n)} \right) \end{aligned}$$

with

$$\lambda = 2\pi l_s^2, \quad E_{AB} = G_{AB} + B_{AB}, \quad \text{and} \quad Q^m{}_n = \delta^m{}_n + i\lambda[X^m, X^k]E_{kn}. \tag{4.85}$$

The field strength $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]$, P denotes the pullback to the brane world-volume and STr denotes the symmetrised trace.

3.2 Fuzzy-sphere ansatz for the bulk scalars

'Fuzzy sphere' configurations for the adjoint scalars X^m in the previous non-abelian DBI action have been considered in the past [29, 76, 125, 126] . Let us generalise for the moment and consider the case of N coincident D5-branes rather than just two and consider the following 'fuzzy sphere' ansatz for the transverse scalar fields:

$$X^i = \hat{R}(x^\mu)\alpha^i \ , \quad i = 1, 2, 3, \tag{4.86}$$

where α^i give some $N \times N$ matrix representation of the $SU(2)$ algebra

$$[\alpha_i, \alpha_j] = 2i\epsilon_{ijk}\alpha^k \ . \tag{4.87}$$

We define the physical radius of the 5-dimensional transverse space as

$$R^2(x^\mu) = \frac{\lambda^2}{N} \sum_{i=1}^{3} \mathrm{Tr}\left[X^i(x^\mu)^2\right] = \lambda^2 C \hat{R}(x^\mu)^2 \tag{4.88}$$

where C is the Casimir of the particular representation of the generators under consideration, defined by the identity

$$\sum_{i=1}^{3} \alpha^i \alpha^i = C \mathbb{1}_N \tag{4.89}$$

Now, given this ansatz the DBI action becomes

$$\begin{aligned} S &= -T_5 \int d^6x \, \mathrm{STr} \left(\frac{1}{\sqrt{H}} \sqrt{1 + \lambda^2 H \partial_a \hat{R} \partial^a \hat{R} \alpha^i \alpha^i + 4 \frac{\lambda R^2 H (1 - 2\lambda R^2 H) \alpha_i \alpha^i}{1 + 4\lambda^2 R^4 H^2 \alpha^i \alpha_i} \partial_a R \partial^a R} \right. \\ & \left. \times \sqrt{1 + 4\lambda^2 \hat{R}^4 H^2 \alpha^i \alpha^i} \right) \end{aligned} \tag{4.90}$$

with

$$H = 1 + \frac{k l_s^2}{\hat{R}^2 \alpha^i \alpha^i} \tag{4.91}$$

where it is understood that H is an $N \times N$ matrix. Note that the symmetrized trace in the action ensures that one cannot simply replace all $\alpha^i \alpha^i$ by the Casimir C, there will be ordering issues which spoil this.

This action resembles a modified DBI action in flat background of N non-BPS D5-branes proposed in [39], namely

$$\begin{aligned} S_{DBI} &= -T_5 \int d^6x \; \mathrm{STr} \left(V(T_i T_i) \sqrt{1 + \frac{1}{2} [T_i, T_j] [T_j, T_i]} \right. \\ & \left. \times \sqrt{-\det \left(\eta_{ab} + \lambda \partial_a T_i \left(Q^{-1}\right)_{ij} \partial_b T_j \right)} \right) \end{aligned} \tag{4.92}$$

where

$$Q_{ij} = \mathbb{1}_N \delta_{ij} - i [T_i, T_j] \tag{4.93}$$

with $T_1 = T\sigma_1$ and $T_2 = T\sigma_2$ and there is no sum over i, j, with the inclusion of an extra term with a $\alpha^i \alpha_i$ factor in the denominator. Now we proceed to study the two limits of large and small R, which correspond to the probe D-branes being close and far from the NS5's respectively. The results quoted in these sections are for the case where we ignore the contribution from the extra term, however bear in mind that this term is present and unless it can be ignored by taking a particular region of R space it may spoil these. It is not clear a priori whether this region exists or not and certainly deserves further attention, the difficulties arise because of the extra factors of $\alpha_i\alpha^i$ in the denominator which in order to investigate require a full expansion of the square root factor and order by order matching of the terms after performing the symmetrised trace manually.

3.3 The limit of Large radius

In this section we are looking for a limit of $\hat{R}$ space in which $H \to 1$. Hence we need $R^2 >> kl_s^2$ but, to obtain an expandable DBI action which is crucial to performing calculations involving the Symmetrised Trace we must also have $\lambda R^2 = 2\pi l_s^2 R^2 << 1$. In the approximation where we can safely ignore the second term in the extra contribution in the square root (assuming this approximation is consistent with the above limits for R) we obtain that the DBI action (4.90) becomes:

$$S = -T_5 \int d^6x STr\left(\frac{1}{\sqrt{H}}\sqrt{1+\lambda^2\partial_a R\partial^a R\alpha_m\alpha^m}\sqrt{1+4\lambda^2R^4\alpha_n\alpha^n}\right). \quad (4.94)$$

Expanding both square roots and performing the symmetrised trace manually we find that this action is dual to

$$S = -T_5 \int d^6x STr\left(V(T_iT_i)\sqrt{1+2T^4}\sqrt{1+\tilde{\lambda}\partial_a T\partial^a T\alpha_i\alpha^i}\right) \quad (4.95)$$

under the map $T^4 = 2\lambda^2R^4CN$ (where $\tilde{\lambda} = \sqrt{\frac{N}{2C}}\lambda$ and C is the Casimir of the N dimensional representation generated by the α_i) up to order λ^2R^4 which given the choice of limit means this term is small and higher order terms are progressively less important. This is simply the large[6] T expansion of (4.92), where to carry out this expansion one needs to be careful in the choice of $i = 1, 2$ as detailed in [39].

[6] large here is a slight misnomer, with the map used we still have $T^4 << 1$, which is important for the latter action to be expandable

The potential takes the form:

$$\frac{1}{T_5}V(T^2) = 1 - \frac{1}{2}\frac{2Ckl_s^2\tilde{\lambda}}{T^2} \tag{4.96}$$

which is simply the long range gravitational attraction between the D-branes and the five-branes.

3.4 The limit of Small radius

In this section we are looking for a region of R-space where

$$H \sim \frac{kl_s^2}{\hat{R}^2}, \tag{4.97}$$

which is achieved for $R^2 < kl_s^2$. However we still want to have a DBI action which is expandable, hence we also need $R^2 > \lambda^2 kl_s^2$, which is a sensible enough region provided R^2 is not too small compared to kl_s^2 originally (recalling that $\lambda = 2\pi l_s^2$). We take the approximation where the extra term can be ignored to obtain:

$$S = -T_5 \int d^6x \, \mathrm{STr}\left(\frac{\hat{R}}{\sqrt{k}l_s}\sqrt{1+\lambda^2\frac{kl_s^2}{\hat{R}^2}\partial_a\hat{R}\partial^a\hat{R}\alpha^i\alpha^i}\sqrt{1+4\lambda^2(kl_s^2)^2\alpha^i\alpha^i}\right)$$

If we set

$$T = \sqrt{k}l_s \ln\frac{\hat{R}}{\sqrt{k}l_s} \tag{4.98}$$

the previous action becomes,

$$S = -T_5 \int d^6x \, \mathrm{STr}\left(e^{\frac{T}{\sqrt{k}l_s}}\sqrt{1+4\lambda^2(kl_s^2)^2\alpha^i\alpha^i}\sqrt{1+\lambda^2\partial_aT\partial^aT\alpha_i\alpha^i}\right)$$

which is the Tachyon-DBI action with Tachyon potential which is corrected from the usual $e^{\frac{T}{\sqrt{k}l_s}}$ by terms which are derived by expanding the action, taking the symmetrised trace and matching terms order by order.

Therefore the fuzzy sphere ansatz seems to provide a valuable avenue in order to form a duality between the $NS5$ system and Garousi's Tachyon action, at least for the case in which the above mentioned approximation is valid. This is however not a trivial approximation and it's validity must be investigated further by explicit expansion of the DBI actions and order by order matching of the terms before the duality can be claimed true overall.

3.5 Commutative Ansatz

Here we shall consider a different ansatz to that in the previous section. Inspired by [97], where purely radial fluctuations of the fields on the branes give a geometrical description of a dual Tachyonic system, we re-write the non-abelian action in terms of a radial "direction" defined as $X^m X_m = R^2$, and we parametrize the scalar fields as

$$X^m = f^m(\theta, \phi, \chi)\tilde{R} \tag{4.99}$$

where f^m are angular functions with $f^m f_m = 1$ and R is an adjoint valued $U(2)$ matrix which we rewrite as a linear combination of $U(2)$ adjoint matrices α_a in the following way

$$\tilde{R} = \tilde{R}_a(\xi)\alpha^a \tag{4.100}$$

where we have also included the $U(1)$ field R_0 and defined $\alpha_0 = \mathbb{1}_2$. With this parametrization it is clear that the commutator of the scalar fields vanishes $[X^m, X^n] = 0$, in particular, $Q^m_n = \delta^m_n$. Thus in contrast to the fuzzy sphere ansatz of the previous section, one might call this a 'commutative' ansatz.

The action of the D5-brane pair becomes:

$$S = -T_5 \int d^6x \, \mathrm{STr}\left(\frac{1}{\sqrt{H}}\sqrt{-det\left(\eta_{\mu\nu} + HD_\mu R^a D_\nu R^b \alpha_a \alpha_b + F_{\mu\nu}\right)}\right) \tag{4.101}$$

where

$$H = 1 + \frac{kl_s^2}{X^m X_m} \tag{4.102}$$

where again it is understood that H is an $N \times N$ matrix, and

$$X^m X_m = (R_0\alpha_0)^2 + 2R_0R^i\alpha_0\alpha_i + (R_i\alpha_i)^2 \tag{4.103}$$

This action resembles that of two non-BPS D5-branes proposed in [86] in the case of vanishing transverse scalar fields:

$$S = -T_5 \int d^6x \ \mathrm{STr}\left(V(T)\sqrt{-\det\left(\eta_{\alpha\beta} + \lambda D_\mu T D_\nu T + \lambda F_{\mu\nu}\right)}\right). \qquad (4.104)$$

3.6 The limit of Small radius

In the limit in which $R_0 \sim R_1 \sim R_2 \sim R_3 \sim 0$ the action (4.101) reduces to

$$S = -T_5 \int d^6x \, \mathrm{STr} \frac{1}{\sqrt{H}} \sqrt{1 + kl_s^2 \frac{\partial_\mu R_0 \partial^\mu R_0 \alpha_0^2 + 2\partial_\mu R_0 \partial^\mu R^i \alpha_i + \partial_\mu R^i \partial^\mu R^i \alpha_i \alpha_i}{(R_0\alpha_0)^2 + 2R_0 R^i \alpha_0 \alpha_i + (R_i\alpha_i)^2}}$$

If we set

$$T = \sqrt{k} l_s \ln\left(R^m \alpha_m\right) = \ln R \qquad (4.105)$$

the above action becomes

$$S = -\int d^6x \, \mathrm{STr}\left(V(T)\sqrt{1 + \partial_\mu T \partial^\mu T}\right) \qquad (4.106)$$

and in this limit the potential is given by

$$V(T) = \frac{T_5}{\sqrt{k} l_s} e^{\frac{T}{\sqrt{k} l_s}} \qquad (4.107)$$

If we define $T = T^m \alpha_m$ then in order to obtain an explicit expression for the different components T^m of the Tachyon matrix we would expand (4.105) and match order by order each T^m components on the l.h.s. with the respective α_m component on the r.h.s.

There is an important point which must be noted here. The map 4.105 is non-linear and hence to show that the duality truly holds one needs to show that this non-linearity is consistent in the symmetrisation procedure. The result quoted above for the dual action is true only under a symmetrisation with respect to the original field R, and not the new field T, which is what it would have to be in order for it to be the Tachyon DBI action. In order to show that this action is dual even under the symmetrisation procedure one needs to expand the original action in terms of R, perform the symmetrisation and then match order by order under a linear map for T. For this case this is a hard task due to the difficult powers of $\alpha_i \alpha_j$ appearing in the expansion.

Note that this form of the map allows one to map the fully non-abelian actions, including the covariant derivatives. In particular, using eq. (4.105), under the STr we have that

$$\begin{aligned} D_\alpha R &= \partial_\alpha R + i[A_\alpha, R] & (4.108)\\ &= \sqrt{k} l_s e^{\frac{T}{\sqrt{k} l_s}} \partial_\alpha T + i\sqrt{k} l_s \, e^{\frac{T}{\sqrt{k} l_s}} [A_\alpha, T] = \sqrt{k} l_s \, e^{\frac{T}{\sqrt{k} l_s}} D_\alpha T & (4.109)\end{aligned}$$

where in the second line we used the fact that $[f(R), \sigma_a] = f'(R)[R, \sigma_a]$ for $f(R)$ a continuous power series function of a matrix $R = R^a \sigma_a$ (see B). This means that

$$\frac{1}{R^2} D_\alpha R D^\alpha R = D_\alpha T D^\alpha T + \frac{\sqrt{k} l_s}{R^2} [\exp \frac{T}{\sqrt{k} l_s}, D_\alpha T]. \tag{4.110}$$

The symmetrized trace STr in the action will ensure that the commutator vanishes everywhere, so the non-abelian map including the covariant derivatives is realised in this limit.

3.7 The limit of Large radius

In the other case, namely when $R_0 \sim R_1 \sim R_2 \sim R_3 \to \infty$ the map is realized if we set

$$T = R \tag{4.111}$$

and it is trivial to map the components of T with those of R. In this case the Tachyon potential becomes

$$V(T) = \frac{T_5}{\sqrt{1 + \frac{k l_s^2}{T^2}}} \sim T_5 \left(1 - \frac{1}{2} \frac{k l_s^2}{T^2}\right) \tag{4.112}$$

which is the long-range gravitational attraction between multiple D5-branes and the NS5 branes.

In this limit it is trivial to map the covariant derivatives in the action, one simply has $D_\alpha T = D_\alpha R$ and also note that there are now no symmetrisation issues in matching the actions.

3.8 General solution

Given the ansatz in eq. (4.99), we would like to show that one can find a general map for all values of R between the two actions (4.101) and (4.104). In [97], it was shown that for the case of a single probing D-brane (where now R is a function rather than a matrix) one could map the two systems by finding an analytical solution to the following differential equation:

$$\frac{dT}{dR} = \sqrt{H(R)} \tag{4.113}$$

and by identifying the Tachyon potential with the harmonic function H as follows:

$$V(T) = \frac{T_5}{\sqrt{H(R)}} \ . \tag{4.114}$$

In the small and large R limits the map gave useful insight into the dynamics of the probing brane and provided useful information regarding rolling Tachyonic solutions [97] and the nature of unstable D-brane systems. In the non-abelian case, the general requirement to realise the map is

$$\mathrm{STr}\left(HD_\mu RD_\nu R\right) \;\; = \;\; \mathrm{STr}\left(\lambda D_\mu TD_\nu T\right) \tag{4.115}$$

When the system is promoted to a non-abelian one such a map is still possible. However, one needs to be careful with the choice of definition of H. One possibility is that H can be thought of as a matrix, in which case $H(RR)$ depends in a general way on the matrix product of R, or we could understand H to depend on R via the non-abelian distance $H(Tr(R^2))$ so that H is a function and not a matrix. We will see in the following analysis that the choice is important. Careful string scattering calculations should reveal the true form of the Harmonic function appearing in the non-abelian DBI action and we think that once these calculations are performed the functional form of H will be obtained. However, being unaware of this result in the present literature, we decided to pursue both routes and obtain significantly differing results.

3.9 H function

Consider the case where H is chosen to be a function. We will show here that analytical solutions for $T(R^a)$ still exist and furthermore that they yield the expected single brane results of [97] in the abelian limit. We consider the simplifying case where the gauge fields are turned off.

In this case we define a physical radius as

$$R^2 = \sum_{m=1}^{3} \frac{1}{N} \mathrm{Tr} X^m X^m = \frac{1}{N} \mathrm{Tr} \tilde{R}^2 = R_0^2 + R_1^2 + R_2^2 + R_3^2 \tag{4.116}$$

With this choice one has[7]

$$H\left(X^m X^m\right) = 1 + \frac{k l_s^2}{\mathrm{Tr}\ X^m X^m} = 1 + \frac{k l_s^2}{R_0^2 + R_1^2 + R_2^2 + R_3^2} \tag{4.117}$$

In this case we can solve the full map analytically. For every value of R we need to solve

$$\partial_\mu T = \sqrt{H(R^2)} \partial_\mu R \ . \tag{4.118}$$

If we write $T = T^m \alpha_m$ then for each $m = 0, \ldots 3$ we have to solve

$$\partial_\mu T^m = \sqrt{1 + \frac{k l_s^2}{R_0^2 + R_1^2 + R_2^2 + R_3^2}} \partial_\mu R^m. \tag{4.119}$$

In the abelian case with a single D5-brane we would find the solution

$$\int \sqrt{1 + \frac{k l_s^2}{R^2}} dR = \sqrt{k l_s^2 + R^2} + \frac{1}{2} \ln \frac{\sqrt{k l_s^2 + R^2} - \sqrt{k} l_s}{\sqrt{k l_s^2 + R^2} + \sqrt{k} l_s} = T^{kut}(R) \tag{4.120}$$

where T^{kut} refers to the Tachyon field of the single probe brane case of [97]. By contrast, in the non abelian case we have to solve, for example, for the $m = 0$

[7] Notice that another ansatz which makes H a function is $H\left(X^m X^m\right) \sim \mathrm{Tr} H\left(X^m X^m\right)$. This ansatz would lead to different results from those we find below and we do not pursue this approach any further.

component

$$\int \sqrt{1+\frac{kl_s^2}{R_0^2+d^2}}dR_0 = -i\sqrt{d^2+kl_s^2}E\left(i\sinh^{-1}\left(\frac{R_0}{d}\right), \frac{d^2}{d^2+kl_s^2}\right) \quad (4.121)$$

where we define $d^2 = R_1^2 + R_2^2 + R_3^2$ for simplicity and $E(z, \omega)$ is the incomplete elliptic integral of the second kind. Although it is trivial to take the limit in which $d \to 0$ on the l.h.s., one has to take care with this limit on the r.h.s. due to divergences appearing in the argument of the elliptic integral . In order to explore the differences between the abelian and non-abelian case it is instructive to expand the explicit expression for the integrand on the l.h.s. in the limit in which $R_1^2 + R_2^2 + R_3^2 \ll R_0^2$ Then we obtain:

$$\begin{aligned} T_0 &= T^{kut}(R_0) + \frac{1}{4}\frac{d^2}{R_0^2}\left(\sqrt{kl_s^2+R_0^2} - \frac{R_0^2\sinh^{-1}\left(\frac{\sqrt{k}l_s}{R_0}\right)}{\sqrt{k}l_s}\right) \\ &+ \mathcal{O}\left(\left(\frac{d^2}{R_0^2}\right)^4\right), \end{aligned}$$

the second term here denotes the non-abelian corrections to the abelian result.

For reference we write below the full solution for all m

$$\begin{aligned} T_m &= \\ &c(R^{j\neq m}) - i\sqrt{kl_s^2+R^2-\tilde{R}_m^2}\,\mathrm{E}\left[i\,\sinh^{-1}\left[\tilde{R}^m\sqrt{\frac{1}{R^2-\tilde{R}_m^2}}\right], \frac{R^2-\tilde{R}_m^2}{kl_s^2+R^2-\tilde{R}_m^2}\right] \end{aligned}$$

for $m = 0, 1, 2, 3$. where $R^2 = R_0^2 + R_1^2 + R_2^2 + R_3^2$ and $\tilde{R}_m$ corresponds to the component of T_m one wants to solve for and $c(R^{j\neq m})$ is an integration constant.

3.10 H Matrix

In this case we would like to solve the map (4.115) where H is in general a non-diagonal matrix. The map is non-trivial (in the case where no particular limit for R is taken) unless H is diagonalised, this can be achieved by choosing an a priori diagonal ansatz for R. In this case the full $U(2)$ symmetry of the problem would be broken to $U(1)\otimes U(1)$. To illustrate this take $R = R^0\sigma_0 + R^3\sigma_3$, then

$$dT = \begin{pmatrix} \sqrt{1-\frac{kl_s^2}{(R^0+R^3)^2}} & 0 \\ 0 & \sqrt{1-\frac{kl_s^2}{(R^0-R^3)^2}} \end{pmatrix} \times \begin{pmatrix} dR^0+dR^3 & 0 \\ 0 & dR^0-dR^3 \end{pmatrix} \tag{4.122}$$

and substituting for $R_+ = R^0+R^3, R_- = R^0-R^3, T_+ = T^0+T^3$ and $T_- = T^0-T^3$ then one arrives at the map

$$dT_+ = \sqrt{1-\frac{kl_s^2}{{R_+}^2}}dR_+ \tag{4.123}$$

$$dT_- = \sqrt{1-\frac{kl_s^2}{{R_-}^2}}dR_- \tag{4.124}$$

which has as solutions two copies of the solution found in [97]. In particular, the action

$$S = -T_5\int d^4x\mathrm{STr}\frac{1}{\sqrt{H}}\left(\sqrt{1-H\partial_\alpha R\partial^\alpha R}\right) \tag{4.125}$$

becomes

$$S_T = -T_5\int d^4x\left(e^{\frac{T_+}{\sqrt{k}l_s}}\sqrt{1-\partial_\alpha T_+\partial^\alpha T_+} + e^{\frac{T_-}{\sqrt{k}l_s}}\sqrt{1-\partial_\alpha T_-\partial^\alpha T_-}\right) \tag{4.126}$$

which is the $U(1)\otimes U(1)$ symmetric double copy of the single brane case. This is to be expected from a diagonal ansatz, the D-brane probes effectively separate and have independent single probe dynamics.

3.11 Dynamics of the Coincident Brane set-up

In the case where H is regarded as a diagonal matrix we have seen the effective action is just the direct sum of two independent actions each describing the dynamics of a single probe D5-brane, which has already been investigated in [97]. Regarding H as a function of the non-abelian distance defined in eq. (4.99) produces a dynamical system where there is a non-trivial interaction between the probe branes if we choose to separate them (which breaks $U(2)\to U(1)\times U(1)$) . Such an interaction vanishes in the flat space limit, as one would expect because then the probe branes are fully BPS and no force exists between them whether separated or coincident.

We take a symmetrical parametrization of the scalar fields, and demand that they depend only on time t via

$$X^m(t) = f^m(\theta, \phi, \chi) R(t) \ . \tag{4.127}$$

Starting from the following action[8]

$$S = -T_5 \int d^6x \, \mathrm{STr} \left(\frac{1}{\sqrt{H}} \sqrt{-det\left(\eta_{\mu\nu} + H \partial_\mu R \partial_\nu R\right)} \right) \ , \tag{4.128}$$

we make a diagonal ansatz for the scalar field R

$$R = R_0 \sigma_0 + R_3 \sigma_3 \tag{4.129}$$

and finally we set

$$\begin{aligned} \phi &= R_0 + R_3 \\ \chi &= R_0 - R_3 \end{aligned} \tag{4.130}$$

One finds the action (4.128) reduces to

$$S = T_5 \int d^6x \frac{1}{\sqrt{H}} \left(\sqrt{1 - H\dot{\phi}^2} + \sqrt{1 - H\dot{\chi}^2} \right) \tag{4.131}$$

where the harmonic function H is now given by

$$H = 1 + \frac{kl_s^2}{\chi^2 + \phi^2} \tag{4.132}$$

The equation of motion that follows in the limit in which $\phi \sim \chi \ll kl_s^2$ is

$$\frac{\phi}{kl_s^2 \sqrt{\frac{\chi^2+\phi^2}{kl_s^2} - \dot{\chi}^2}} + \frac{\phi}{kl_s^2 \sqrt{\frac{\chi^2+\phi^2}{kl_s^2} - \dot{\phi}^2}} + \frac{\ddot{\phi}}{\sqrt{\frac{\chi^2+\phi^2}{kl_s^2} - \dot{\phi}^2}} - \frac{\dot{\phi}\left(\chi\dot{\chi} + \dot{\phi}\left(\phi - kl_s^2 \ddot{\phi}\right)\right)}{kl_s^2 \left(\frac{\chi^2+\phi^2}{kl_s^2} - \dot{\phi}^2\right)^{3/2}} = 0 \tag{4.133}$$

[8]We set again the gauge fields to zero manually

with an analogous one in which χ and ϕ are interchanged. Exact solutions to these equations are hard to find but one can consider the conservation of the energy which results in a simpler first order differential equation. The energy E of the system is defined as

$$E = P_\phi \dot{\phi} + P_\chi \dot{\chi} - \mathcal{L} \tag{4.134}$$

and we investigate the following ansatz[9]

$$\phi = \frac{1}{2}(R_0 + C) \tag{4.135}$$

$$\chi = \frac{1}{2}(R_0 - C) \tag{4.136}$$

where C is a constant. In the small R_0 limit the conservation of the energy gives

$$\dot{R_0}^2 = \frac{2\left(C^2 + R_0^2\right)}{kl_s^2} - \frac{4T_5^2\left(C^2 + R_0^2\right)^2}{E^2\left(kl_s^2\right)^2}. \tag{4.137}$$

By imposing reality of the solution one obtains an important inequality

$$\frac{2kl_s^2}{R_0^2 + C^2} \geq \frac{4T_5^2}{E^2} - 1, \tag{4.138}$$

we see that there are solutions at a critical energy $E_{crit} = 2T_5$ which can escape to infinity.

The energy equation (4.137) has analytical solutions for C non-zero

$$R_0 = \pm iC \text{JacobiSN}\left[\sqrt{\frac{2}{kl_s^2}}\sqrt{-1 + 2\frac{T_5^2}{E^2}C^2}t \mp i\sqrt{-1 + 2\frac{T_5^2}{E^2}C^2}c_1, \frac{2T_5^2C^2}{-E^2 + 2T_5^2C^2}\right] \tag{4.139}$$

where c_1 is an integration constant. In Figure 1 we present a plot of this solution (with given choice of signs) for certain values of the parameters E, k, l_s, c_1 and the parameter $C = 0, 0.01, 0.1$. They all correspond to the regime where the throat approximation to H is valid. The case $C = 0$ corresponds to the abelian case where $R_0(t)$ describes motion which is isomorphic to that of a single probe brane in an infinite throat, with energy less than the critical energy required to escape

[9] by a slight abuse of notation we have used the notation R_0 though it is not strictly exactly the same as the quantity occurring in eq. (4.129).

to infinity [97]. What is particularly interesting in the case where $C \neq 0$ is that the solutions appear to bounce, at least if we identify the solutions with negative values of R_0 as separated probe branes moving up the throat. This does not involve patching two distinct solutions together dis-continually, it is an issue with how to exactly interpret the negative R_0 solutions. After all only $R^2 = R_0^2 + C^2$ makes sense in this setup. Looking at Figure 4.1 we see that plotting R_0^2 would mean that the solution is "bouncing" as described above. Looking at the harmonic function H it is clear that in the case $C \neq 0$, the geometry seen by the probes is one of a finite cutoff throat, with C acting as a cutoff parameter. So the resulting centre of mass dynamics of the separated probe pair is equivalent to a single probe brane moving in a cutoff throat background.

In this interpretation, the sub-critical energy probe falls down the throat but then reflects off the boundary and back up the throat reaching a certain maximum distance, the motion being repeated forever. It's clear from the plots that the period of oscillation increases with decreasing C. This makes sense as in the limit $C \to 0$ we recover the solution found in [97] which does not oscillate (at least not in coordinate time t) but corresponds to an infalling probe brane taking infinite coordinate time to reach the throat bottom. By patching the two solutions (which differ by a minus sign) together in the regions where $R_0(t)$ is negative one finds an explicit change of sign in the velocity $\dot{R}_0(t)$ of the branes as they reach the throat cutoff, as is expected from a perfectly elastic bouncing solution. We now investigate the behaviour of the string coupling with time using the relation

$$e^{2\phi} = g_s H(Tr(RR)) \tag{4.140}$$

In Figure 4.2 we show a plot of the effective string coupling using the solution (4.139), valid in the throat approximation. The thick curve corresponds to the abelian case $C = 0$ and shows, as one expects, a rapidly increasing effective string coupling as the probe falls down the infinite throat. Thus after some time $t = t_{max}$ the solution is no longer within the perturbative string approximation. As argued in [97], the value of t_{max} depends analytically on the energy E of the probe and there is an energy 'window' $g_s T_5 \ll E \ll T_5$ for which the probe brane moves in the throat and remains within perturbation theory.

By contrast , the case where $C \neq 0$ (regular and dashed curves in Figure 2),

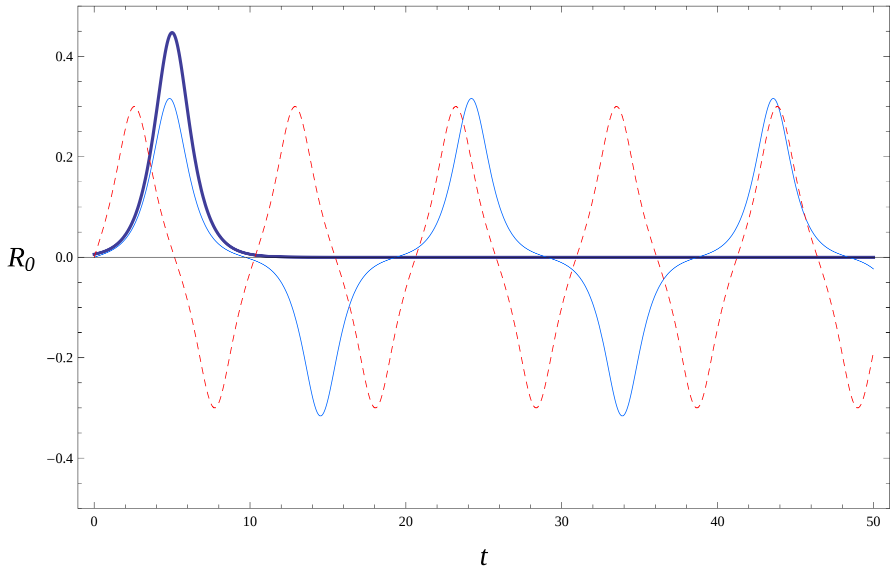

Figure 4.1: Plots of R_0 vs t. The bold curve has $C = 0$ and corresponds to the abelian case. The continuous regular curve corresponds to $C = 0.01$, and finally the dashed curve to $C = 0.1$. In all cases we have chosen $\frac{2T_5^2}{E^2} = 10$, $c_1 = 0$ and $kl_s^2 = 1$

we see the effective coupling as oscillating in time as the probes oscillate in the throat. By choosing the value of E and/or C it is possible to control the motion such that the string coupling is always in the perturbative regime and for the probes to remain in the throat region for all time.

Due to the complexity of the solution (4.139) one cannot derive a simple expression for a bound on the energy and/or C in order for the above to hold, even for small C. Instead one has to use the full expression for the JacobiSN function for $C \neq 0$ and thus we are limited to numerical plots as in Figure 2.

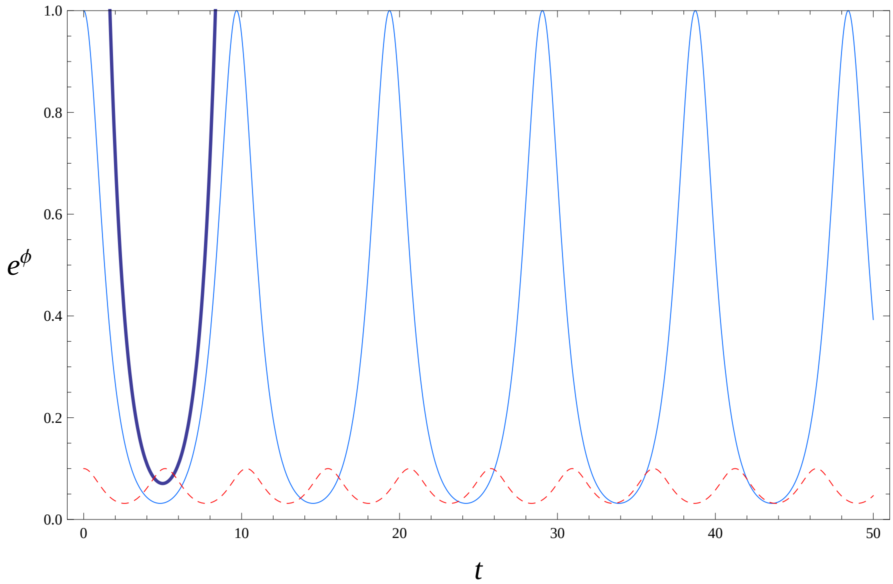

Figure 4.2: Plots of the effective string coupling e^ϕ vs t. The bold curve has $C = 0$ and corresponds to the abelian case. The continuous regular curve corresponds to $C = 0.01$, and finally the dashed curve to $C = 0.1$. In all plots we have chosen $\frac{2T_5^2}{E^2} = 10$, $c_1 = 0$ and $kl_s^2 = 1$. The value of $g_s = 0.0001$.

CHAPTER 5
CONCLUSIONS

This thesis has concentrated on non-abelian aspects of D-Brane world-volume dynamics, specifically focussing on the fate of the open string Tachyon. In chapter 4 we initially considered the generalisation of Sen's Tachyon condensation mechanism to the formation of two coincident BPS D8-branes on the world volume of Tachyon kink-like configurations of two coincident non-BPS D9-branes. We found a natural extension of Sen's regularisation of the singular Tachyon kink profile, to the case of $U(2)$ Tachyon valued field in the latter theory. What is apparent is the very different properties of the Str vs Tr prescription in taking the gauge trace in the non-abelian, non-BPS DBI action. The former leads to a series of very complicated terms that mix $D_\mu T, F_{\mu\nu}$ and more problematically individual T in the Tachyon potential $V(T)$. In particular, the latter consequence of taking Str over gauge indices makes it very difficult to see Tachyon condensation occurring in a way that is calculable and which yields the Str prescription of the action of two coincident BPS D8-branes.

Starting with the Tr prescription however, we have explicitly shown that Tachyon condensation gives rise directly to the BPS action of two coincident D8-branes. This stark contrast between the Str and Tr prescriptions, parallels similar issues found by Garousi in [37] regarding the existence (or not) of vortex solutions in Brane-anti-Brane actions derived from coincident non-BPS D9-brane actions with Tr or Str prescriptions.

Regarding further work in this area, firstly, it would be interesting to investigate non-abelian Tachyon condensation, along the lines presented in this thesis, where one starts with e.g. two coincident non-BPS Dp-branes with $p < 9$. Then one expects to find the action of two coincident D$(p-1)$ BPS branes after Tachyon condensation. The resulting action should presumably have the same structure as the one proposed by Myers [29]. Since the latter action is obtained via T-duality of the coincident D9-brane action, understanding the details of how non-abelian Tachyon condensation works in this case would allow us to see if T-duality 'commutes' with it. On the other hand, since the Myers action has a *Str* prescription, it is by no means obvious how one may realise such actions through the process of non-abelian Tachyon condensation. Secondly, there are obvious extensions of our results to the case of multiple coincident non-BPS D9-branes and Tachyon condensation leading to the action of multiple coincident BPS D8-branes. Finally, it would be interesting to show how one can inherit the correct Wess-Zumino terms for the BPS D$(p-1)$ branes from those that are part of the non-BPS action recently proposed in [127, 128].

Next, we investigated co-dimension 3 magnetic monopole solutions arising from the DBI-like action of two coincident non-BPS D9-branes. We showed the existence of singular monopoles that require regularisation in a similar fashion to the kink and vortex soliton solutions investigated by Sen in [30]. An analysis of the fluctuations shows that in the limit where the regularisation is removed, we recover the correct DBI action corresponding to a single BPS D6-brane. This extends the earlier results found by using truncated DBI like actions [119]. Our results are complementary to those presented in [123] within the BSFT framework, where the authors showed that the basic Tachyon monopole ansatz survives all higher order derivative corrections. Our results put magnetic monopoles alongside kinks and vortices as the possible products of Tachyon condensation occurring in the full non-linear, non-BPS DBI actions and which yield fluctuation spectra that are described by the full DBI action corresponding to co-dimension 1, 2 and 3 BPS branes.

These results were obtained within the framework of the non-BPS action presented in [37]. Recently, [39], a modified version of this action (based on the results of [129, 130]) has been proposed (this was important also in the discussion in chapter 4). In this modified version, the Tachyon field carries internal Pauli matrices σ_1 and σ_2 and was obtained by considering the disk level S-matrix element of one Ramond-Ramond field and three Tachyon fields. In [39] the modified action was shown to be consistent with the S-matrix element of one gauge field and four Tachyon fields. The modified action amounts to a multiplication of the Tachyon potential $V(T_i)$ in the symmetrized trace version of the non-BPS action [37] by a factor $\sqrt{1+\frac{1}{2}[T_i,T_j][T_i,T_j]}$ where $T_i = T\sigma_i,\ i = 1,2$. For large Tachyon field values it was argued in [130] that one may compute the Str by expanding $V(T_i)$ and that such modifications resulted in effectively the potential $V(T)$ being multiplied by a factor of T^4. The resulting modified potential still vanishes as $T \to \infty$, so Tachyon condensation is still expected to occur. Indeed one might argue that since the Tachyon field configurations describing kinks, vortices and as we have shown, monopoles, are 'large' almost everywhere in the regularised theory (the Tachyon field is infinite everywhere except at the maximum of $V(T)$ where it is zero, in the unregularised theory) this large T approximation is justified. Nevertheless it would be interesting to see the details of Tachyon condensation in such a modified DBI action, including an analysis of the fluctuation spectrum, and to see if they give the same results starting with the unmodified action in [37]. A first glance shows that at the very least, the formulae for the various tensions of the co-dimension 1, 2 and 3 BPS branes will change in that $V(T)$ will be replaced by $V(T)T^4$.

Note that we have only discussed Tachyon condensation in flat space. When one considers curved backgrounds there are non-vanishing Ramond-Ramond forms and thus Wess-Zumino (WZ) terms appear in both the actions of BPS and non-BPS branes. Therefore it is natural to consider the origin of such Wess-Zumino terms when BPS D-branes emerge as a result of Tachyon condensation. This has been studied some time ago in [124] in the case where a normal trace (as opposed to symmetrized trace) prescription is taken for the WZ term in the non-BPS D-brane action. More recently [128] and [127] have studied higher order derivative corrections to the WZ terms in non-BPS D-brane actions via disk amplitude S-matrix calculations. It is certainly an interesting question to consider how such corrections modify the results of [124] when one considers Tachyon condensation producing codimension 1, 2 and 3 BPS D-branes.

Finally we attempted to generalise the notion of Kutasov's geometric interpretation of the open string Tachyon, in the scenario where a D-brane is moving in a background geometry of k NS5 branes that render the system non-BPS [97]. The generalisation we investigated considered a pair of coincident probe D5-branes moving in this background instead of a single probe D5 brane discussed in [97]. The single real geometric Tachyon field that appears in the single probe case is, in the simplest scenario, related to purely transverse radial motion of the probe.

This system is abelian in that there is a $U(1)$ gauge theory on the probe brane world volume.

When we consider the case where, for example, one has as a probe two coincident D5-branes, then the situation becomes more subtle. Firstly the probe world-volume now supports non-abelian $U(2)$ gauge fields and secondly, as is well known, the coordinates transverse to this probe stack become matrix valued. This latter phenomenon raises the question of how one interprets the geometrical quantities such as the harmonic function H sourced by the NS5 branes. In one interpretation, we can define a notion of non-abelian distance in the transverse matrix geometry via the quantity $Tr(X^m X^m)$ where X^m are the matrix valued transverse coordinates. Then $H(X^m)$ can be thought of as a function via $H = H(Tr(X^m X^m))$. Another possible interpretation is that H becomes a matrix through its dependence on X^m.

Both definitions seem to give rise to well defined actions since ultimately the Lagrangians are matrix valued objects in each case and Str is taken over all free gauge indices. However as we have shown, the resulting definition of the matrix valued geometric Tachyon field and the resulting dynamics is different in the two interpretations.

As an illustration of this we saw that in the case where H is treated as a function of non-abelian distance defined above, the Tachyon map can be found exactly and in the limit where the $U(2)$ adjoint valued radial coordinate R is dominated by the terms proportional to the 2×2 identity matrix, we recovered the single probe brane Tachyon map of Kutasov.

On the other hand, a general solution for the Tachyon map in the case of H being a matrix is very complicated and its explicit form is not known. However we found that at least in the symmetry breaking case where $U(2) \to U(1) \times U(1)$ the system reduces to two non-interacting copies of single geometrical Tachyon fields. By contrast, the same $U(2)$ breaking configuration of the probe stack, in the case where H is a function and not a matrix, yields a dynamical system involving two coupled geometric Tachyon fields.

In this case we found analytic expressions for homogeneous time dependent solutions at least in the situation where we consider only diagonal degrees of freedom in the non-abelian Tachyon field, which corresponds to $U(2)$ symmetry breaking. Interestingly we found oscillating or 'bouncing' solutions in this case where the separation parameter between the two D5 probes acting as an effective cutoff on the NS5 brane infinite throat.

It would be very interesting to find (even numerically) dynamical solutions which involve the full non-abelian degrees of freedom in the $U(2)$ valued Tachyon field in the action 2.43 including non-vanishing gauge fields. Another extension is to look at different arrangements of background NS5 branes other than the point like ones considered in this thesis. For example one can also consider the k NS5 branes arranged around a ring of finite radius. This is a known supergravity solution and the corresponding metric and harmonic function are known [131,132]. This would extend to the non abelian case the results found in [106].

APPENDIX A
DIRAC QUANTIZATION OF MAGNETIC CHARGE

To evaluate the magnetic charge associated to the ansatz (4.55), we need to have a definition of the magnetic field. In a U(2) gauge theory, there is no unambiguous definition, but in a spontaneously broken theory, with unbroken group U(1), provided that the fields are close to the vacuum, a magnetic field can be defined:

$$F^{EM}_{\mu\nu} = \frac{1}{2} F^a_{\mu\nu} \hat{T}^a \tag{A.1}$$

where $\hat{T}^a$ is a unit vector that points along the direction of the 'Higgs' field (in the present case the adjoint tachyon field T^a). In particular, $\hat{T}^a = \frac{x^a}{r}$ and the physical magnetic field becomes:

$$B_i = \frac{1}{2}\epsilon_{ijk} F^{EM}_{jk} = \frac{1}{4}\epsilon_{ijk} F^a_{jk} \frac{x^a}{r}\,. \tag{A.2}$$

To find the total magnetic flux which is equal to the magnetic charge m, we have to integrate the magnetic field over S^2_∞, the 2-sphere at infinity. The magnetic charge m enclosed in some Gaussian surface Σ enclosing the magnetic charge density is given by

$$m = \int_{S^2_\infty} B_i dS_i = \lim_{r\to\infty} \frac{1}{4} \int_{S^2} \epsilon_{ijk} F^a_{jk} \frac{x^a}{r} dS_i \tag{A.3}$$

Now $dS_i = \epsilon_{ijk} dx^j \wedge dx^k$, so

$$m = \lim_{r\to\infty} \frac{1}{2} \int_{S^2} F^a_{jk} \frac{x^a}{r} dx^j \wedge dx^k \tag{A.4}$$

in polar coordinates, we can write

$$dx^j \wedge dx^k = \partial_m x^j(r,\theta,\phi) \partial_n x^k(r,\theta,\phi)\, d\xi^m \wedge d\xi^n \tag{A.5}$$

where ξ^n, $n = 1, 2$, correspond to the coordinates θ and ϕ. We have

$$\begin{aligned} m &= \lim_{r\to\infty} \frac{1}{2} \int_{S^2} F^a_{jk} \frac{x^a}{r} \partial_m x^j(r,\theta,\phi) \partial_n x^k(r,\theta,\phi) d\xi^m \wedge d\xi^n \\ &= \lim_{r\to\infty} \int_{S^2} F^a_{\theta\phi} \frac{x^a(r,\theta,\phi)}{r} d\theta d\phi \end{aligned} \tag{A.6}$$

where the S^2 has radius r. Using the definition of $x^a(r,\theta,\phi)$ and the expressions derived before for $F^a_{\theta\phi}$ we find

$$m = -\frac{1}{2} \int_{S^2_\infty} \sin\theta \, d\theta d\phi = -2\pi \tag{A.7}$$

The Dirac quantization of magnetic charge requires that

$$m = \frac{2\pi n}{e} \tag{A.8}$$

for a charge m magnetic monopole where e is the electric charge. From the definition of the covariant derivative of the tachyon field T^a it is clear that $e = -1$. So for an $n = +1$ magnetic monopole, the magnetic charge is

$$m = \frac{2\pi n}{e} = -2\pi \ . \tag{A.9}$$

APPENDIX B
PROOF OF COMMUTATION RELATION

In this appendix we wish to prove the relation

$$[f(R), \sigma_a] = f'(R)[R, \sigma_a] + \mathcal{O}([,]) \tag{B.1}$$

for $f(R)$ a continuous power series function of the matrix R and σ_a the usual Pauli Matrices, $\mathcal{O}([,])$ denotes terms which are pure commutators involving the matrices R and σ_a which will be unimportant due to the explicit symmetrisation over the Trace in the action.

Proof: Let $f(R)$ be a continuous power series function of the matrix R, then

$$f = \sum_n c_n R^n \tag{B.2}$$

for some coefficients c_n.

Hence

$$\begin{aligned}
[f(R), \sigma_a] &= \sum_n c_n [R^n, \sigma_a] \\
&= \sum_n c_n [R^{n-1} R, \sigma_a] \\
&= \sum_n c_n \left(R^{n-1}[R, \sigma_a] + [R^{n-1}, \sigma_a] R \right)
\end{aligned} \tag{B.3}$$

which becomes

$$
\begin{aligned}
&= \sum_n c_n \left(R^{n-1}[R,\sigma_a] + [R^{n-2}R,\sigma_a]R\right) \\
&= \sum_n c_n \left(R^{n-1}[R,\sigma_a] + R^{n-2}[R^n,\sigma_a]R + [R^{n-2},\sigma_a]R^2\right) \\
&= \sum_n c_n \left(R^{n-1}[R,\sigma_a] + R^{n-2}\left(R[R,\sigma_a] - [R,[R,\sigma_a]]\right) + [R^{n-2},\sigma_a]R^2\right)
\end{aligned}
\tag{B.4}
$$

but

$$R^{n-2}[R,[R,\sigma_a]] = 2[R^{n-1}, R\sigma_a] \tag{B.5}$$

and hence all such terms arising in the expansion are pure commutator terms. By induction one derives

$$= \sum_n \left(nR^{n-1}[R,\sigma_a] + \mathcal{O}([,])\right) \quad = f'(R) + \mathcal{O}([,]) \tag{B.6}$$

which is the result set out to prove.

Bibliography

[1] V. Calò, G. Tallarita, and S. Thomas, "Non Abelian Tachyon Kinks," *JHEP* **08** (2009) 094, arXiv:0904.0601 [hep-th].

[2] V. Calò, G. Tallarita, and S. Thomas, "Dirac-Born-Infeld actions and Tachyon Monopoles," *Phys.Rev.D* **81** (2010) 086007, 0908.3379. http://arxiv.org/abs/0908.3379.

[3] V. Calo, G. Tallarita, and S. Thomas, "Non Abelian Geometrical Tachyon," *JHEP* **1011** (2010) 063, 1003.6063. http://arxiv.org/abs/1003.6063.

[4] G. Tallarita, "AdS Phase Transitions at finite kappa," *JHEP* **1108** (2011) 048, 1011.3222. http://arxiv.org/abs/1011.3222.

[5] G. Tallarita and S. Thomas, "Maxwell-Chern-Simons Vortices and Holographic Superconductors," *JHEP* **1012** (2010) 090, 1007.4163. http://arxiv.org/abs/1007.4163.

[6] G. Tallarita, "Connection Constraints from Non-Abelian Supersymmetric Quantum Mechanics," 1005.3245. http://arxiv.org/abs/1005.3245.

[7] J. H. Schwarz, K. Becker, and M. Becker, *String Theory and M-Theory, a modern introduction*. Cambridge University Press, 2007.

[8] Y. Shirman, "TASI 2008 Lectures: Introduction to Supersymmetry and Supersymmetry Breaking," 0907.0039. http://arxiv.org/abs/0907.0039.

[9] M. Drees, "An Introduction to Supersymmetry," hep-ph/9611409. http://arxiv.org/abs/hep-ph/9611409.

[10] P. Ramond, "Dual theory for free fermions," *Phys. Rev. D73,2415* (1971) .

[11] A. Neveu and J. H. Schwarz, "Factorizable dual model of pions," *Nucl. Phys. B31, 86* (1971) .

[12] A. Neveu, J. H. Schwarz, and C. B. Thorn, "Reformulation of the dual pion model," *Phys. Lett. B35, 529* (1971) .

[13] C. Csaki, "TASI Lectures on Extra Dimensions and Branes," `hep-ph/0404096v1`. `http://arxiv.org/abs/hep-ph/0404096v1`.

[14] Y. Kim, C. O. Lee, I. Lee, and J. Lee, "Brane World of Warp Geometry: An Introductory Review," *J. Kor. Astron. Soc.* **37** (2004) 1–14, `hep-th/0307023`. `http://arxiv.org/abs/hep-th/0307023`.

[15] M. A. Luty, "2004 TASI Lectures on Supersymmetry Breaking," `hep-th/0509029v1`. `http://arxiv.org/abs/hep-th/0509029v1`.

[16] J. Bagger and N. Lambert, "Modeling Multiple M2's," *Phys.Rev.D* **75** (2007) 045020, `hep-th/0611108`. `http://arxiv.org/abs/hep-th/0611108`.

[17] J. Bagger and N. Lambert, "Comments on Multiple M2-branes," *JHEP* **0802** (2008) 105, `0712.3738`. `http://arxiv.org/abs/0712.3738`.

[18] J. Bagger and N. Lambert, "Three-Algebras and N=6 Chern-Simons Gauge Theories," *Phys.Rev.D* **79** (2009) 025002, `0807.0163`. `http://arxiv.org/abs/0807.0163`.

[19] J. Polchinski, "Dirichlet-Branes and Ramond-Ramond Charges," *Phys.Rev.Lett.* **75** (1995) 4724–4727, `hep-th/9510017`. `http://arxiv.org/abs/hep-th/9510017`.

[20] E. Witten, "Anti De Sitter Space And Holography," *Adv.Theor.Math.Phys.* **2** (1998) 253–291, `hep-th/9802150`. `http://arxiv.org/abs/hep-th/9802150`.

[21] J. M. Maldacena, "The Large N Limit of Superconformal Field Theories and Supergravity," *Adv.Theor.Math.Phys.* **2** (1998) 231–252, `hep-th/9711200`. `http://arxiv.org/abs/hep-th/9711200`.

[22] J. Erdmenger, N. Evans, I. Kirsch, and E. Threlfall, "Mesons in Gauge/Gravity Duals - A Review," 0711.4467v2. http://arxiv.org/abs/0711.4467v2.

[23] C. P. Herzog, "Lectures on Holographic Superfluidity and Superconductivity," 0904.1975v2. http://arxiv.org/abs/0904.1975v2.

[24] S. A. Hartnoll, "Lectures on holographic methods for condensed matter physics," 0903.3246v3. http://arxiv.org/abs/0903.3246v3.

[25] G. T. Horowitz, "Introduction to Holographic Superconductors," 1002.1722v2. http://arxiv.org/abs/1002.1722v2.

[26] O. Aharony, S. Gubser, J. Maldacena, H. Ooguri, and Y. Oz, "Large N Field Theories, String Theory and Gravity," *Phys.Rept.* **323** (2000) 183–386, hep-th/9905111. http://arxiv.org/abs/hep-th/9905111.

[27] A. Sen, "Tachyon Dynamics in Open String Theory," *Int.J.Mod.Phys.A* **20** (2005) 5513–5656, hep-th/0410103. http://arxiv.org/abs/hep-th/0410103.

[28] D. Kutasov, "D-Brane Dynamics Near NS5-Branes," hep-th/0405058. http://arxiv.org/abs/hep-th/0405058.

[29] R. C. Myers, "Dielectric-branes," *JHEP* **12** (1999) 022, hep-th/9910053.

[30] A. Sen, "Dirac-Born-Infeld Action on the Tachyon Kink and Vortex," *Phys.Rev.D* **68** (2003) 066008, hep-th/0303057. http://arxiv.org/abs/hep-th/0303057.

[31] I. Pesando, "On the effective potential of the Dp Dp-bar system in type II theories," *Mod. Phys. Lett.* **A14** (1999) 1545–1564, arXiv:hep-th/9902181.

[32] P. Kraus and F. Larsen, "Boundary string field theory of the DD-bar system," *Phys. Rev.* **D63** (2001) 106004, hep-th/0012198.

[33] T. Takayanagi, S. Terashima, and T. Uesugi, "Brane-antibrane action from boundary string field theory," *JHEP* **03** (2001) 019, hep-th/0012210.

[34] C. Kim, H. B. Kim, Y. Kim, and O.-K. Kwon, "Electromagnetic String Fluid in Rolling Tachyon," *JHEP* **0303** (2003) 008, `hep-th/0301076`. `http://arxiv.org/abs/hep-th/0301076`.

[35] F. Leblond and A. W. Peet, "SD-brane gravity fields and rolling tachyons," *JHEP* **0304** (2003) 048, `hep-th/0303035`. `http://arxiv.org/abs/hep-th/0303035`.

[36] N. Lambert, H. Liu, and J. Maldacena, "Closed strings from decaying D-branes," *JHEP* **0703** (2007) 014, `hep-th/0303139`. `http://arxiv.org/abs/hep-th/0303139`.

[37] M. R. Garousi, "On the effective action of D-brane-anti-D-brane system," *JHEP* **12** (2007) 089, `arXiv:0710.5469 [hep-th]`. `http://arxiv.org/pdf/0710.5469`.

[38] A. A. Tseytlin, "On non-abelian generalisation of the Born-Infeld action in string theory," *Nucl. Phys.* **B501** (1997) 41–52, `hep-th/9701125`.

[39] M. R. Garousi, "On modified tachyon DBI action," *Nucl. Phys.* **B817** (2009) 252–264, `arXiv:0811.4334 [hep-th]`.

[40] J. Kluson, "Proposal for non-BPS D-brane action," *Phys.Rev.* **D62** (2000) 126003, `hep-th/0004106`. `http://arxiv.org/abs/hep-th/0004106`.

[41] A. Sen, "Time Evolution in Open String Theory," *JHEP* **0210** (2002) 003, `hep-th/0207105`. `http://arxiv.org/abs/hep-th/0207105`.

[42] O. Bergman and M. Gaberdiel, "Stable non-BPS D-particles," *Phys.Lett.* **B441** (1998) 133–140, `hep-th/9806155`. `http://arxiv.org/abs/hep-th/9806155`.

[43] O. Bergman and M. R. Gaberdiel, "Non-BPS Dirichlet branes," *Class.Quant.Grav.* **17** (2000) 961–969, `hep-th/9908126`. `http://arxiv.org/abs/hep-th/9908126`.

[44] A. Sen, "Type I D-particle and its Interactions," *JHEP* **9810** (1998) 021, `hep-th/9809111`. `http://arxiv.org/abs/hep-th/9809111`.

[45] A. Sen, "Rolling tachyon," *JHEP* **04** (2002) 048, `hep-th/0203211`.

[46] A. Sen, "Tachyon Matter," *JHEP* **0207** (2002) 065, hep-th/0203265. http://arxiv.org/abs/hep-th/0203265.

[47] S. Bhowmick and S. Mukherji, "Anisotropic Power Law Inflation from Rolling Tachyons," 1105.4455. http://arxiv.org/abs/1105.4455.

[48] J. Cui, L. Zhang, J. Zhang, and X. Zhang, "New agegraphic dark energy as a rolling tachyon," *Chin.Phys.B* **19** (2010) 019802–6, 0902.0716. http://arxiv.org/abs/0902.0716.

[49] N. Jokela, M. Jarvinen, and E. Keski-Vakkuri, "N-point functions in rolling tachyon background," *Phys.Rev.D* **79** (2009) 086013, 0806.1491. http://arxiv.org/abs/0806.1491.

[50] L. Joukovskaya, "Rolling Solution for Tachyon Condensation in Open String Field Theory," 0803.3484. http://arxiv.org/abs/0803.3484.

[51] O.-K. Kwon, "Marginally Deformed Rolling Tachyon around the Tachyon Vacuum in Open String Field Theory," *Nucl.Phys.B* **804** (2008) 1–18, 0801.0573. http://arxiv.org/abs/0801.0573.

[52] T. Lee, "Applications of Thirring Model to Inhomogenous Rolling Tachyon and Dissipative Quantum Mechanics," *JHEP* **0802** (2008) 090, 0711.0512. http://arxiv.org/abs/0711.0512.

[53] J. A. Hutasoit and N. Jokela, "A Thermodynamic Interpretation of Time for Superstring Rolling Tachyons," *Phys.Rev.D* **77** (2008) 023521, 0709.1319. http://arxiv.org/abs/0709.1319.

[54] I. Ellwood, "Rolling to the tachyon vacuum in string field theory," *JHEP* **0712** (2007) 028, 0705.0013. http://arxiv.org/abs/0705.0013.

[55] T. Lee, "The Final Fate of the Rolling Tachyon," *JHEP* **0611** (2006) 056, hep-th/0606236. http://arxiv.org/abs/hep-th/0606236.

[56] N. Jokela, E. Keski-Vakkuri, and J. Majumder, "On Superstring Disk Amplitudes in a Rolling Tachyon Background," *Phys.Rev.* **D73** (2006) 046007, hep-th/0510205. http://arxiv.org/abs/hep-th/0510205.

[57] T. Lee and G. W. Semenoff, "Fermion Representation Of The Rolling Tachyon Boundary Conformal Field Theory," *JHEP* **0505** (2005) 072, hep-th/0502236. http://arxiv.org/abs/hep-th/0502236.

[58] Y. Kim and O.-K. Kwon, "Exact Rolling Tachyon in Noncommutative Field Theory," *Mod.Phys.Lett.* **A21** (2006) 421–432, hep-th/0501016. http://arxiv.org/abs/hep-th/0501016.

[59] D. Gaiotto, N. Itzhaki, and L. Rastelli, "Closed Strings as Imaginary D-branes," *Nucl.Phys.B* **688** (2004) 70–100, hep-th/0304192. http://arxiv.org/abs/hep-th/0304192.

[60] A. Sen, "Open and Closed Strings from Unstable D-branes," *Phys.Rev.D* **68** (2003) 106003, hep-th/0305011. http://arxiv.org/abs/hep-th/0305011.

[61] A. Sen, "Rolling Tachyon Boundary State, Conserved Charges and Two Dimensional String Theory," *JHEP* **0405** (2004) 076, hep-th/0402157. http://arxiv.org/abs/hep-th/0402157.

[62] A. Sen, "Open-Closed Duality at Tree Level," *Phys.Rev.Lett.* **91** (2003) 181601, hep-th/0306137. http://arxiv.org/abs/hep-th/0306137.

[63] M. Fairbairn and M. H. Tytgat, "Inflation from a Tachyon Fluid?," *Phys.Lett.* **B546** (2002) 1–7, hep-th/0204070. http://arxiv.org/abs/hep-th/0204070.

[64] A. Feinstein, "Power-Law Inflation from the Rolling Tachyon," *Phys.Rev.* **D66** (2002) 063511, hep-th/0204140. http://arxiv.org/abs/hep-th/0204140.

[65] L. Kofman and A. Linde, "Problems with Tachyon Inflation," *JHEP* **0207** (2002) 004, hep-th/0205121. http://arxiv.org/abs/hep-th/0205121.

[66] M. Sami, "Implementing Power Law Inflation with Tachyon Rolling on the Brane," *Mod.Phys.Lett.A* **18** (2003) 691, hep-th/0205146. http://arxiv.org/abs/hep-th/0205146.

[67] D. Choudhury, D. Ghoshal, D. P. Jatkar, and S. Panda, "On the Cosmological Relevance of the Tachyon," *Phys.Lett.* **B544** (2002) 231–238, hep-th/0204204. http://arxiv.org/abs/hep-th/0204204.

[68] A. Frolov, L. Kofman, and A. Starobinsky, "Prospects and Problems of Tachyon Matter Cosmology," *Phys.Lett.* **B545** (2002) 8–16, hep-th/0204187. http://arxiv.org/abs/hep-th/0204187.

[69] G. W. Gibbons, "Cosmological Evolution of the Rolling Tachyon," *Phys.Lett.B* **537** (2002) 1–4, hep-th/0204008. http://arxiv.org/abs/hep-th/0204008.

[70] S. Mukohyama, "Brane cosmology driven by the rolling tachyon," *Phys.Rev.* **D66** (2002) 024009, hep-th/0204084. http://arxiv.org/abs/hep-th/0204084.

[71] T.Padmanabhan, "Accelerated expansion of the universe driven by tachyonic matter," *Phys.Rev.D* **66** (2002) 021301, hep-th/0204150. http://arxiv.org/abs/hep-th/0204150.

[72] X. zhou Li, J. gang Hao, and D. jun Liu, "Can Quintessence Be The Rolling Tachyon?," *Chin.Phys.Lett.* **19** (2002) 1584, hep-th/0204252. http://arxiv.org/abs/hep-th/0204252.

[73] G. Shiu and I. Wasserman, "Cosmological Constraints on Tachyon Matter," *Phys.Lett.* **B541** (2002) 6–15, hep-th/0205003. http://arxiv.org/abs/hep-th/0205003.

[74] P. Horava, "Type IIA D-branes, K-theory, and matrix theory," *Adv. Theor. Math. Phys.* **2** (1999) 1373–1404, arXiv:hep-th/9812135.

[75] C. Kim, Y. Kim, O.-K. Kwon, and C. O. Lee, "Tachyon kinks on unstable Dp-branes," *JHEP* **11** (2003) 034, arXiv:hep-th/0305092.

[76] S. Thomas and J. Ward, "Geometrical Tachyon Kinks and NS5 Branes," *JHEP* **0510** (2005) 098, hep-th/0502228. http://arxiv.org/abs/hep-th/0502228.

[77] J. Kluson, "Note about tachyon kink in nontrivial background," *JHEP* **08** (2005) 032, hep-th/0506250. http://arxiv.org/pdf/hep-th/0506250.

[78] W. Taylor and B. Zwiebach, "D-branes, tachyons, and string field theory," `arXiv:hep-th/0311017`.

[79] M. Schnabl, "Analytic solution for tachyon condensation in open string field theory," *Adv. Theor. Math. Phys.* **10** (2006) 433–501, `arXiv:hep-th/0511286`.

[80] I. Ellwood and M. Schnabl, "Proof of vanishing cohomology at the tachyon vacuum," *JHEP* **02** (2007) 096, `arXiv:hep-th/0606142`.

[81] E. Witten, "On background independent open string field theory," *Phys. Rev.* **D46** (1992) 5467–5473, `hep-th/9208027`.

[82] E. Witten, "Some computations in background independent off-shell string theory," *Phys. Rev.* **D47** (1993) 3405–3410, `arXiv:hep-th/9210065`.

[83] S. L. Shatashvili, "Comment on the background independent open string theory," *Phys. Lett.* **B311** (1993) 83–86, `hep-th/9303143`.

[84] D. Kutasov, M. Marino, and G. W. Moore, "Some exact results on tachyon condensation in string field theory," *JHEP* **10** (2000) 045, `hep-th/0009148`.

[85] D. Kutasov, M. Marino, and G. W. Moore, "Remarks on tachyon condensation in superstring field theory," `hep-th/0010108`.

[86] M. R. Garousi, "Tachyon couplings on non-BPS D-branes and Dirac-Born-Infeld action," *Nucl. Phys.* **B584** (2000) 284–299, `hep-th/0003122`. `http://arxiv.org/pdf/hep-th/0003122`.

[87] M. R. Garousi, "On-shell S-matrix and tachyonic effective actions," *Nucl. Phys.* **B647** (2002) 117–130, `hep-th/0209068`.

[88] M. R. Garousi, "Slowly varying tachyon and tachyon potential," *JHEP* **05** (2003) 058, `arXiv:hep-th/0304145`.

[89] M. R. Garousi, "D-brane anti-D-brane effective action and brane interaction in open string channel," *JHEP* **01** (2005) 029, `hep-th/0411222`. `http://arxiv.org/pdf/hep-th/0411222`.

[90] A. Sen, "Stable Non-BPS Bound States of BPS D-branes," *JHEP* **9808** (1998) 010, `hep-th/9805019`. `http://arxiv.org/abs/hep-th/9805019`.

[91] A. Sen, "Tachyon Condensation on the Brane Antibrane System," *JHEP* **9808** (1998) 012, `hep-th/9805170`. `http://arxiv.org/abs/hep-th/9805170`.

[92] A. Sen, "SO(32) Spinors of Type I and Other Solitons on Brane-Antibrane Pair," *JHEP* **9809** (1998) 023, `hep-th/9808141`. `http://arxiv.org/abs/hep-th/9808141`.

[93] A. Sen, "BPS D-branes on Non-supersymmetric Cycles," *JHEP* **9812** (1998) 021, `hep-th/9812031`. `http://arxiv.org/abs/hep-th/9812031`.

[94] A. Sen, "Universality of the Tachyon Potential," *JHEP* **9912** (1999) 027, `hep-th/9911116`. `http://arxiv.org/abs/hep-th/9911116`.

[95] E. Witten, "D-branes and K-theory," *JHEP* **12** (1998) 019, `arXiv:hep-th/9810188`.

[96] *D-Branes*. Cambridge University Press, Clifford V. Johnson.

[97] D. Kutasov, "D-brane dynamics near NS5-branes," `arXiv:hep-th/0405058`.

[98] D. Kutasov, "A geometric interpretation of the open string tachyon," `arXiv:hep-th/0408073`.

[99] C. G. Callan, Jr., J. A. Harvey, and A. Strominger, "Supersymmetric string solitons," `arXiv:hep-th/9112030`.

[100] Y. Nakayama, Y. Sugawara, and H. Takayanagi, "Boundary states for the rolling D-branes in NS5 background," *JHEP* **07** (2004) 020, `arXiv:hep-th/0406173`.

[101] D. A. Sahakyan, "Comments on D-brane Dynamics Near NS5-Branes," *JHEP* **0410** (2004) 008, `hep-th/0408070`. `http://arxiv.org/abs/hep-th/0408070`.

[102] J. Kluson, "Non-BPS D-brane near NS5-branes," *JHEP* **11** (2004) 013, `arXiv:hep-th/0409298`.

[103] J. Kluson, "Non-BPS Dp-brane in the background of NS5-branes on transverse R**3 x S**1," *JHEP* **03** (2005) 032, `arXiv:hep-th/0411014`.

[104] B. Chen, M. Li, and B. Sun, "Dbrane near NS5-branes: With electromagnetic field," *JHEP* **12** (2004) 057, `arXiv:hep-th/0412022`.

[105] Y. Nakayama, K. L. Panigrahi, S.-J. Rey, and H. Takayanagi, "Rolling down the throat in NS5-brane background: The case of electrified D-brane," *JHEP* **01** (2005) 052, `arXiv:hep-th/0412038`.

[106] S. Thomas and J. Ward, "D-brane dynamics near compactified NS5-branes," *JHEP* **0506** (2005) 062, `hep-th/0501192`. `http://arxiv.org/abs/hep-th/0501192`.

[107] S. Thomas and J. Ward, "Inflation from geometrical tachyons," *Phys. Rev.* **D72** (2005) 083519, `arXiv:hep-th/0504226`.

[108] E. Papantonopoulos, I. Pappa, and V. Zamarias, "Geometrical tachyon dynamics in the background of a bulk tachyon field," *JHEP* **05** (2006) 038, `arXiv:hep-th/0601152`.

[109] A. Sen, "Geometric Tachyon to Universal Open String Tachyon," *JHEP* **0705** (2007) 035, `hep-th/0703157`. `http://arxiv.org/abs/hep-th/0703157`.

[110] J. Kluson and K. L. Panigrahi, "On the Universal Tachyon and Geometrical Tachyon," *JHEP* **06** (2007) 015, `arXiv:0704.3013 [hep-th]`.

[111] A. Das, S. Panda, and S. Roy, "Origin of the geometric tachyon," *Phys. Rev.* **D78** (2008) 061901, `arXiv:0804.2863 [hep-th]`.

[112] D. Erkal, D. Kutasov, and O. Lunin, "Brane-Antibrane Dynamics From the Tachyon DBI Action," `arXiv:0901.4368 [hep-th]`.

[113] H. Yavartanoo, "Cosmological Solution from D-brane motion in NS5-Branes background," *Int.J.Mod.Phys.* **A20** (2005) 7633–7645, `hep-th/0407079`. `http://arxiv.org/abs/hep-th/0407079`.

[114] A. Ghodsi and A. E. Mosaffa, "D-brane dynamics in RR deformation of NS5-branes background and tachyon cosmology," *Nucl. Phys.* **B714** (2005) 30–50, `arXiv:hep-th/0408015`.

[115] S. Panda, M. Sami, and S. Tsujikawa, "Inflation and dark energy arising from geometrical tachyons," *Phys. Rev.* **D73** (2006) 023515, arXiv:hep-th/0510112.

[116] B. Gumjudpai, T. Naskar, and J. Ward, "A Quintessentially Geometric Model," *JCAP* **0611** (2006) 006, hep-ph/0603210. http://arxiv.org/abs/hep-ph/0603210.

[117] K. L. Panigrahi and H. Singh, "Assisted Inflation from Geometric Tachyon," 0708.1679v3. http://arxiv.org/abs/0708.1679v3.

[118] K. L. Panigrahi and H. Singh, "Spinflation from Geometric Tachyon," *JHEP* **05** (2008) 088, arXiv:0802.4230 [hep-th].

[119] K. Hashimoto and S. Hirano, "Branes ending on branes in a tachyon model," *JHEP* **04** (2001) 003, hep-th/0102173. http://arxiv.org/pdf/hep-th/0102173.

[120] K. Hashimoto and S. Hirano, "Branes Ending On Branes In A Tachyon Model," *JHEP* **0104** (2001) 003, hep-th/0102173. http://arxiv.org/abs/hep-th/0102173.

[121] G. 't Hooft, "Magnetic monopoles in unified gauge theories," *Nucl.Phys.B* **79** no. 276, (1974) .

[122] A. Polyakov, "Particle Spectrum in quantum field theory," *JETP Lett.* **20** no. 194, (1974) .

[123] K. Hashimoto and S. Hirano, "Metamorphosis of tachyon profile in unstable D9-branes," *Phys. Rev.* **D65** (2002) 026006, hep-th/0102174.

[124] M. Billò, B. Craps, and F. Roose, "Ramond-Ramond couplings of non-BPS D-branes," *JHEP* **06** (1999) 033, arXiv:hep-th/9905157.

[125] S. Ramgoolam, B. Spence, and S. Thomas, "Resolving brane collapse with 1/N corrections in non-Abelian DBI," *Nucl.Phys.* **B703** (2004) 236–276, hep-th/0405256. http://arxiv.org/abs/hep-th/0405256.

[126] S. Thomas and J. Ward, "Fuzzy Sphere Dynamics and Non-Abelian DBI in Curved Backgrounds," *JHEP* **0610** (2006) 039, hep-th/0508085. http://arxiv.org/abs/hep-th/0508085.

[127] M. R. Garousi and E. Hatefi, "On Wess-Zumino terms of Brane-Antibrane systems," *Nucl. Phys.* **B800** (2008) 502–516, `arXiv:0710.5875 [hep-th]`.

[128] M. R. Garousi and E. Hatefi, "More on WZ action of non-BPS branes," *JHEP* **03** (2009) 008, `arXiv:0812.4216 [hep-th]`.

[129] M. R. Garousi, "Tachyon Couplings to Fermion," *JHEP* **12** (2008) 059, `arXiv:0810.2256 [hep-th]`.

[130] M. R. Garousi, "On effective actions of non-BPS D-branes and their higher derivative corrections," *Nucl. Phys.* **B809** (2009) 525–546, `arXiv:0802.2784 [hep-th]`.

[131] K. Sfetsos, "Branes for Higgs phases and exact conformal field theories," *JHEP* **9901** (1999) 015, `hep-th/9811167`. `http://arxiv.org/abs/hep-th/9811167`.

[132] K. Sfetsos, "Rotating NS5-brane solution and its exact string theoretical description," *Fortsch.Phys.* **48** (2000) 199–204, `hep-th/9903201`. `http://arxiv.org/abs/hep-th/9903201`.