DIGITALLY MOVING AN ELECTRIC GUITAR PICKUP

Zulfadhli Mohamad*, Simon Dixon, Christopher Harte,
Centre for Digital Music, Queen Mary University of London, London, United Kingdom
Zulfadhli Mohamad is supported by the Malaysian government Centre for Digital Music, Queen Mary University of London, London, United Kingdom
z.b.mohamad@qmul.ac.uk
s.e.dixon@qmul.ac.uk
chris@melodient.com

ABSTRACT
This paper describes a technique to transform the sound of an arbitrarily selected magnetic pickup into another pickup selection on the same electric guitar. This is a first step towards replicating an arbitrary electric guitar timbre in an audio recording using the signal from another guitar as input. We record 1458 individual notes from the pickups of a single guitar, varying the string, fret, plucking position, and dynamics of the tones in order to create a controlled dataset for training and testing our approach. Given an input signal and a target signal, a least squares estimator is used to obtain the coefficients of a finite impulse response (FIR) filter to match the desired magnetic pickup position. We use spectral difference to measure the error of the emulation, and test the effects of independent variables fret, dynamics, plucking position and repetition on the accuracy. A small reduction in accuracy was observed for different repetitions; moderate errors arose when the playing style (plucking position and dynamics) were varied; and there were large differences between output and target when the training and test data comprised different notes (fret positions). We explain results in terms of the acoustics of the vibrating strings.

1. INTRODUCTION
The electric guitar revolutionised Western popular music and was for several decades the most important instrument in most pop, rock and blues music. Although it may no longer hold such unique supremacy, the electric guitar remains an essential element of many of the styles it helped to define. Perhaps more so than with other instruments, many famous guitar players are recognisable by their distinctive electric guitar tone, and guitar enthusiasts are keen to know the “secrets” behind the unique sound of their favourite artist. In order to replicate the sound of their favourite guitar player, they often purchase the same model of guitar and other musical equipment used, and then adjust each of their parameters manually until similar tone is achieved. In recent years, digital replication of electric guitar, guitar amplifiers and guitar effects has grown rapidly in the research community and the music industry.

Several decades of literature exist that deal with synthesising the sound of plucked string instruments. Research can be divided into physical modelling, which involves solving the wave equation describing the vibrating string [1][2], and more abstract models which attempt only to imitate the resulting sound, such as the Karplus-Strong model [3][4]. These models have been extended to account for the characteristics of the electric guitar, based on waveguide theory [5][6][7].

The magnetic pickup contributes heavily to the timbre of an electric guitar. Thus, accurately modelling the magnetic pickup of an electric guitar is essential. In [5], the Karplus-Strong model is extended by introducing a pickup position model that emulates the comb-filtering effect of the position of the magnetic pickup along the string, whereby harmonics with nodes at or near the pickup position are suppressed. A parametric synthesis model of a Fender Stratocaster electric guitar is described in [6] that alters its level and timbre depending on the distance of the pickup and also includes the inharmonic behaviour of the pickup. A more detailed model for a magnetic pickup in [7] includes the width of the pickup, its nonlinearity and circuit response adding to the tonal colouration of the electric guitar.

As mentioned in [6], the playing technique of a musician also alters the timbre of an electric guitar. Plucking with fingers or a plectrum are the two common styles of exciting an electric guitar. The size, shape and material of the plucking device affect the tone of the guitar [8] and the plucking point along the string also contributes to the timbre. A plucking point close to the bridge will produce a brighter sound, while plucking near the fingerboard will produce a warmer sound [9][10]. This is caused by the low amplitude of harmonics that have a node at or near the plucking point. Varying how strongly a string is plucked will also affect the level of higher harmonics [6].

Outside academia, commercial products including guitar synthesisers are available that are able to emulate the sound of most popular electric guitars on the market by modifying the sound of a standard guitar [11][12]. Each string is detected individually by a hexaphonic pickup and processed by the magnetic pickup model of the selected electric guitar sound.

For all physical modelling systems, parameters of the copied electric guitar must be known accurately in order to model its sound. Many influential guitar players used electric guitars and amplifiers that are now considered vintage items; prohibitively expensive and difficult to obtain today. Certain instruments may have been discontinued by the manufacturer and modern examples may not produce a sufficiently similar sound to the older models. The lack of availability of such instruments makes it difficult to measure the physical properties of the guitar, thus making it challenging to model the instrument digitally. In our research, we are exploring the concept of replicating the electric guitar sound from an audio recording without having prior knowledge of the physical parameters of the desired electric guitar sound. In particular, in this study we analyse recordings of an electric guitar made using different pickup positions, and we compute filters to trans-
form the sound recorded by one pickup into the sound of another pickup. We test the effects of the following variables on the quality of learnt filters: plucking position, dynamics, fret position and random variation in human plucking. Although the problem of inverting a notch filter may appear ill-posed, we show that good results can be obtained in practice.

Section 2 describes the sound samples used in this study and the limitations of replicating a desired sound. Section 3 presents an overview of mathematical foundations for determining the optimal FIR coefficients to estimate the desired sound. Furthermore, Section 4 explains the calculation of the accuracy of the estimated signal. The transformation of the sound of one pickup into that of another pickup at a different position is explained and the optimal FIR coefficients to estimate the desired sound. Furthermore, inverting a notch filter may appear ill-posed, we show that good results can be obtained in practice.

Section 6 describes the sound samples used in this study and the limitations of replicating a desired sound. Section 5 presents an overview of mathematical foundations for determining the optimal FIR coefficients to estimate the desired sound. Furthermore, Section 4 explains the calculation of the accuracy of the estimated signal. The transformation of the sound of one pickup into that of another pickup at a different position is explained and the optimum number of coefficients is described in Section 5. In Section 6 the robustness of the filters when applied to an input signal with different repetitions, plucking positions, plucking dynamics and fret positions is measured. Lastly, the conclusions are presented in Section 7.

2. AN ELECTRIC GUITAR SOUND ANALYSIS DATA SET

![Modified Squier Stratocaster diagram](image)

Figure 1: The modified Squier Stratocaster diagram. Three 1/8″ output jacks allow us to tap separate signals from each magnetic pickup simultaneously. The three plucking positions are directly above each pickup.

The purpose of this research is to investigate whether it is possible to transform the sound produced by an arbitrary electric guitar to a desired guitar sound in an audio recording using optimisation techniques. The initial experiments in this study simplify the purpose by attempting to transform the sound of a pickup position into another on the same electric guitar. This is an important step because the magnetic pickup has a large impact on the timbre of an electric guitar.

Since the playing style of a guitarist affects the colouration of the electric guitar tone, it is preferable that the input signal is played at the same plucking point and plucking dynamic as the target signal. In other words, our aim is to account for differences due to the instrument or its settings (which we assume are fixed) from those due to playing technique. Thus we also assume that the timing and pitch of notes in the input and target signals coincide, and in particular that the input signal is played at the same fret position and on the same string as the target. This work utilises a modified guitar that allows us to tap the signals from each individual pickup simultaneously. This means that each signal will be played at exactly the same plucking point, dynamic, pitch and timing.

The electric guitar that is used in this study is a Squier Stratocaster with three stock magnetic pickups rewired so that each pickup can be simultaneously recorded (see Figure 1), in order to isolate differences due to the pickup from those due to time alignment or playing style. The pickup positions are situated at 158.75 mm (neck pickup), 101.6 mm (middle pickup) and 38.1 mm - 50.8 mm (slanted bridge pickup) from the bridge. The scale length of the guitar is 648 mm. The strings used are nickel wound strings with gauges .010, .013, .017, .026, .036 and .046.

The sound samples used in this paper consist of each string being plucked using a plectrum at three different fret positions, three plucking positions and three plucking dynamics, with each combination being repeated three times. The plucking dynamics are forte (loud), mezzo-forte (moderately loud) and piano (soft); the three different plucking positions are when the electric guitar is plucked near the neck (158.75 mm from the bridge), at a central playing position (101.6 mm from the bridge) and near the bridge (45 mm from the bridge); the fret positions are played at open string, fifth fret and twelfth fret; and lastly, all of the combinations are played on each string and repeated for three times. This leads to a total of $6 \times 3 \times 3 \times 3 = 486$ different variations that are recorded from each of the three pickup positions. Thus, 1458 sound samples are available to be analysed. The duration of the audio samples ranges from 3 to 28 seconds depending on the decay rate for each strings. It is planned to make this dataset publicly available for research purposes.

3. OPTIMISATION TECHNIQUE

In this study, we aimed to transform the sound of any pickup selection into the sound of another pickup on the same guitar using an FIR filter. As an example, we are taking the sound of the neck pickup as an input and transforming its sound into the bridge pickup sound. This is achieved by convolving the input signal (neck pickup sound), $x(n)$ with an FIR filter, $h(n)$ to estimate the target signal (bridge pickup), $y(n)$.

$$y(n) = x(n) * h(n)$$

(1)

As the filter $h(n)$ is unknown, an optimisation technique is required to estimate the FIR coefficients accurately. In this paper, the coefficients of the FIR filter are obtained by using the least squares method. A least squares estimator has been used to estimate the coefficients of a filter to reverse engineer a target mix. The linear combination to estimate the desired response is given by:

$$\hat{y}(n) = \sum_{k=1}^{M} h(k) x(n - k)$$

(2)
where $h(k)$ is the filter coefficient vector for the FIR filter, $M$ is the filter order and $\hat{g}(n)$ is the estimated target response. This can also be expressed in matrix notation as:

$$\hat{y} = X\hat{h}$$

(3)

where the matrix $X$ is composed of $M$ shifted versions of $x(n)$. The estimation error and its matrix notation are given by:

$$e(n) = y(n) - \hat{y}(n) = y - X\hat{h}$$

(4)

and the set of optimal coefficients are computed by minimising the sum of squared errors:

$$\hat{h} = \arg \min_{\hat{h}} \| y - X\hat{h} \|$$

(5)

By solving the least-squares normal equations the optimal coefficients are given by:

$$\hat{h} = (X^T X)^{-1} X^T y$$

(6)

where $X^T X$ is the time-average correlation matrix, $\hat{R}$, the elements of which can be calculated by:

$$\hat{r}_{ij} = \frac{\hat{R}_{ij}}{\| R \|_2} = \sum_{n=N_i}^{N_f} x(n + 1 - i)x^*(n + 1 - j) \quad 1 \leq i, j \leq M$$

(7)

leading to:

$$\hat{r}_{i+1,j+1} = \hat{r}_{ij} + x(N_i - i)x^*(N_i - j) - x(N_f + 1 - i)x^*(N_f + 1 - j) \quad 1 \leq i, j < M$$

(8)

where $N_i$ and $N_f$ are the range of computing the process of the filtering operation. Here, we set $N_i = 0$ and $N_f = N - 1$ which is the pre-windowing method that is extensively used in least squares adaptive filtering [15].

4. TIMBRAL SIMILARITY MEASUREMENT

Once the set of optimal coefficients for the FIR filter and estimated response are obtained, the similarity between the estimated signal and target signal is measured. A more meaningful way of measuring the similarity of both signals is by measuring the distance between two sounds in a time-frequency representation. The short-time Fourier transform (STFT) is commonly used for time-frequency analysis. There are several papers that propose a genetic optimisation approach to find optimal parameters for frequency modulation matching synthesis and a method of measuring the similarity between two sounds [16, 17, 18]. The waveforms that are being measured are divided into short segments and a discrete Fourier transform is calculated for each segment. We set the length of the frame to be 1024 samples with an overlap of 512 samples, where the sampling rate of the audio signals is 44100 Hz. The raw distance (or error), $D_R(\hat{Y}, Y)$, is calculated as follows:

$$D_R(\hat{Y}, Y) = \frac{1}{T} \sum_{t=1}^{T} \sum_{f=1}^{F} |\hat{Y}_t[f] - Y_t[f]|^2$$

(9)

where $\hat{Y}_t[f]$ is the magnitude spectrum of the estimated response at time frame $t$ and frequency bin $f$, $Y_t[f]$ is the magnitude spectrum of the target response at frame $t$ and bin $f$, $T$ is the number of frames in the STFT and $F$ is the total number of frequency bins in each frame. Ideally, the sound is considered to be more similar to the target response when the distance is closer to zero. Due to the variations in plucking dynamics between different trials, a normalisation is required to compensate for differences in loudness. The raw distance, $D_R$, is divided by the average energy of the target signal and the estimated signal, giving the normalised distance $D(\hat{Y}, Y)$:

$$D(\hat{Y}, Y) = \frac{2D_R(\hat{Y}, Y)}{\sum_{k=0}^{K} |y(k)|^2 + \sum_{l=0}^{L} |\hat{y}(l)|^2}$$

(10)

where $K$ is the sample length of the target signal and $L$ is the sample length of the estimated signal.

5. RESULTS: AN EXAMPLE

5.1. Learning the Filter

Table 1: Results for transforming one pickup position to another for a tone played on an open G string ($f_0 = 196$ Hz). The normalised distance between estimated signal $\hat{Y}$ and target signal $Y$ is shown in the fourth column. For comparison, the normalised distance between input $X$ and target signal is given in the third column showing a large reduction in distance after transformation.

<table>
<thead>
<tr>
<th>Input Signal ($X$)</th>
<th>Target Signal ($Y$)</th>
<th>$D(X, Y)$</th>
<th>$D(\hat{Y}, Y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>neck pickup</td>
<td>bridge pickup</td>
<td>0.802</td>
<td>0.123</td>
</tr>
<tr>
<td>bridge pickup</td>
<td>neck pickup</td>
<td>0.802</td>
<td>0.011</td>
</tr>
<tr>
<td>neck pickup</td>
<td>mid pickup</td>
<td>0.434</td>
<td>0.085</td>
</tr>
<tr>
<td>mid pickup</td>
<td>neck pickup</td>
<td>0.434</td>
<td>0.007</td>
</tr>
<tr>
<td>bridge pickup</td>
<td>mid pickup</td>
<td>0.234</td>
<td>0.007</td>
</tr>
<tr>
<td>mid pickup</td>
<td>bridge pickup</td>
<td>0.234</td>
<td>0.009</td>
</tr>
</tbody>
</table>

In this section we demonstrate the use of an FIR filter to transform the sound from the neck pickup into the sound of the bridge pickup for the open G string (3rd string, $f_0 = 196$ Hz). The electric guitar is plucked directly above the middle pickup and played forte. The filter order is set to 1024, with coefficients obtained using the least squares method described in Section 4. The estimated signal produced by convolving the input signal and the filter impulse response.

Figure 2 shows the magnitude spectra of the neck pickup signal, bridge pickup signal and estimated bridge pickup signal for a tone played on the open G string. The spectral envelope curves drawn on Figures 2(a) and 2(b) illustrate the comb filtering effect due to the different pickup positions. The neck pickup, situated approximately $\frac{1}{4}$ of the way along the string, lowers the amplitude of every 4th harmonic, while the bridge pickup, at about $\frac{3}{4}$ of the string length, lowers the amplitude of every 14th harmonic. As shown in Figure 2(c), the amplitude of every 4th harmonic is increased and every 14th harmonic is decreased relative to the input (Figure 2(a)), which indicates that the magnitude spectrum of the estimated signal matches that of the target signal. The accuracy of the estimated signal is calculated as the normalised distance $D(\hat{Y}, Y)$ between target ($Y$) and estimated ($\hat{Y}$) signals using Equation 10. For this example, the distance is 0.123, compared
Figure 2: Magnitude spectra for a guitar tone played on the open 3rd string \((f_0 = 196\, \text{Hz})\), calculated from (a) the neck pickup signal, (b) the bridge pickup signal and (c) the estimated bridge pickup signal. The spectral envelopes are drawn for illustrative purposes to show the comb filtering effect of the pickup position.

The filter coefficients can also be determined for other pairs of input and target signals. Table 1 shows the distances calculated for transforming between pickup positions for one tone. In all cases the filter is able to simulate the effect of moving the pickup position, reducing the spectral difference by 80% to 99% for the various cases. The transformation of the neck pickup sound appears to be more difficult than the other cases; we discuss reasons for this in subsection 5.3. In Section 6 we investigate the factors influencing the generalisation of these results.

5.2. Estimating the Filter Order

The accuracy of the estimated signal was computed for various filter orders, to estimate a suitable number of coefficients for the FIR filter. Figure 3 shows that the error converges for higher order filters. We choose 1024 FIR coefficients as a reasonable compromise of accuracy and efficiency to estimate the target signal.

5.3. Comparison with Theoretical Model

The filter that was obtained in Section 5.1 is analysed and compared with a theoretical model. Transforming the sound of a neck pickup sound into bridge pickup sound is achieved by cascading an inverse neck pickup model and bridge pickup model. The theoretical model, \(H_\text{t}(z)\) is computed as follows:

\[
H_\text{t}(z) = \frac{H_\text{n}(z)}{H_\text{b}(z)} \tag{11}
\]

where \(H_\text{n}(z)\) is the neck pickup model and \(H_\text{b}(z)\) is the bridge pickup model. The inverse neck pickup model and bridge pickup model are essentially a feedback comb filter and feedforward comb filter, respectively, which include a fractional delay and a dispersive filter [6, 7, 19]. We excluded some of the details in building the theoretical model such as the nonlinearity of pickup [7, 20] and pickup response [7] which are beyond the scope of this paper. The simplified theoretical model captures the basic behaviour of the system.

Figures 4(a) and 4(b) show the frequency responses of the inverse neck pickup model and bridge pickup model respectively. Also, a comparison of the frequency responses of the theoretical model and the estimated FIR filter is shown in Figure 4(c). We observe that the estimated FIR filter tends to be closer to the the-
oretical model at or near the partial frequencies. This is because most of the energy of the input and target signals is found at the partials, so the filter optimisation is biased to give low errors at these frequencies rather than the frequencies between partials. By the same reasoning, the filter is less accurate at higher frequencies, where the signal has less energy. In addition, the nulls of the pickup model remove energy at specific frequencies. In theory, this could lead to numerical problems when inverting the model but we did not experience this problem with our data. It is either because the notches are not perfectly nulling or because the partial overtones never coincided with notch frequencies. Overall, the frequency response of the estimated FIR filter follows the curves of the theoretical model and captures the most important timbral features.

To explain the results in Table 1, we note that the inverse filter for the neck pickup has the poles most closely spaced, so more poles occur in the region where the signal has the most energy. Since we are using an FIR filter, the approximation of the poles will have some error, which is most noticeable in the case where the neck pickup is the input sound. The error in these cases is approximately one order of magnitude greater.

6. RESULTS: TESTS OF ROBUSTNESS

In guitar synthesisers, each string is processed by an individual filter to emulate the sound of an electric guitar. In Section 5 we extract a filter for a single string played at a particular plucking point and plucking dynamic. In this section we test the generality of learnt filters for each string, to assess the effect of different playing techniques such as variations in plucking dynamics and plucking positions.

In order to measure the robustness of the filter to such differences, we extract a filter for a particular input/target pair (the training pair) and test how well it performs given a different input/target pair (the testing pair). In the simplest case, the training and testing pairs are different instances (repetitions) of the same parameters (string, fret, dynamic, plucking position), but we also test for variations in one of the other parameters at a time (except for string). Thus, the variables that we analyse are repetition, plucking position, plucking dynamic and fret position. To keep the results manageable, we only report results where the input signal is from the neck pickup and target signal is from the bridge pickup.

6.1. Analysing Filter Generality

We used the guitar recordings described in Section 2. The process of analysing each variable can be explained by an example. In this case, we take the variable repetition. The steps for analysing the robustness of the filter to different repetitions are as follows:

1. Take three input/target pairs \( x_i(n) \) and \( y_i(n) \) where \( i \in \{1, 2, 3\} \) is the index of the repetition and other variables remained constant.

2. Obtain the filters \( h_i(n) \) as described in Section 5 for each repetition.

3. The input signals \( x_i(n) \) are convolved with each filter \( h_j(n) \), \( j \in \{1, 2, 3\} \) separately to obtain estimated signals \( \hat{y}_{i,j}(n) \). The distance \( D(\hat{Y}_{i,j}, Y_i) \) between the estimated signal and target signal \( y_i(n) \) is calculated.

4. Steps 1, 2 and 3 are repeated for all cases (i.e. for each combination of plucking position, plucking dynamic, fret position and string), leading to a total of 162 cases.

The same process can be used for analysing the robustness of the filters to different plucking positions, plucking dynamics and fret positions.
Table 2: Errors measured for filters applied to an input/target pair with different (a) repetition, (b) plucking position, (c) plucking dynamic, (d) fret position and (e) fret position (improved filter); where $d$ is the distance of the plucking point from the bridge. All values are averages over 162 different cases, as described in Section 6.7.

<table>
<thead>
<tr>
<th>Input signal</th>
<th>$h_1(n)$</th>
<th>$h_2(n)$</th>
<th>$h_3(n)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rep 1</td>
<td>0.186</td>
<td>0.240</td>
<td>0.260</td>
</tr>
<tr>
<td>Rep 2</td>
<td>0.216</td>
<td>0.184</td>
<td>0.238</td>
</tr>
<tr>
<td>Rep 3</td>
<td>0.217</td>
<td>0.225</td>
<td>0.187</td>
</tr>
</tbody>
</table>

(a)

<table>
<thead>
<tr>
<th>Input signal</th>
<th>$h_1(n)$</th>
<th>$h_2(n)$</th>
<th>$h_3(n)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Forte, f</td>
<td>0.195</td>
<td>0.364</td>
<td>0.535</td>
</tr>
<tr>
<td>Mezzoforte, mf</td>
<td>0.358</td>
<td>0.211</td>
<td>0.287</td>
</tr>
<tr>
<td>Piano, p</td>
<td>0.352</td>
<td>0.258</td>
<td>0.152</td>
</tr>
</tbody>
</table>

(c)

<table>
<thead>
<tr>
<th>Input Signal</th>
<th>$h_1(n)$</th>
<th>$h_2(n)$</th>
<th>$h_3(n)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Open string, 5th fret</td>
<td>0.151</td>
<td>0.421</td>
<td>0.241</td>
</tr>
<tr>
<td>5th fret</td>
<td>0.121</td>
<td>0.495</td>
<td>0.152</td>
</tr>
<tr>
<td>12th fret</td>
<td>0.644</td>
<td>0.311</td>
<td>0.333</td>
</tr>
</tbody>
</table>

(d)

<table>
<thead>
<tr>
<th>Input signal</th>
<th>$h_1(n)$</th>
<th>$h_2(n)$</th>
<th>$h_3(n)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5th fret</td>
<td>0.108</td>
<td>0.111</td>
<td>0.508</td>
</tr>
<tr>
<td>12th fret</td>
<td>0.773</td>
<td>1.949</td>
<td>0.310</td>
</tr>
</tbody>
</table>

(e)

6.2. Results

Following the steps in Section 6.1, we obtain nine distances for each of 162 cases to analyse each variable. Each of the nine distances, or errors, is then averaged over all 162 cases. Tables 2(a), 2(b), 2(c) and 2(d) show the errors for analysing the robustness of the filters when applied to an input/target pair with a different repetition, plucking position, plucking dynamic or fret position respectively. For the learnt filters $h_1(n)$, the values of the variable that we are analysing are indexed by $i$. For instance, in Table 2(b), $h_1(n)$, $h_2(n)$ and $h_3(n)$ are extracted from input/target pairs that were plucked at 158.75 mm, 101.6 mm and 45 mm from the bridge respectively. The filters are then evaluated on input/target pairs from each of the three different plucking positions. The figures in bold emphasise the cases where the training and testing pairs coincide. These can be used as reference values, to obtain the loss in accuracy due to the variable under analysis.

As shown in Table 2(c), the increase in error when the filters are applied to different repetitions ranges from 16% to 39%. We would expect the filters to be reasonably robust towards other repetitions, because the notes are being plucked at similar positions, dynamics, frets and strings. Note that we did not use a mechanical plucking device, so there are slight random variations in playing technique between the repetitions, but there should be no systematic variation. Hence all non-bold values are quite consistent, because different repetitions have similar timbre.

According to Table 2(b), error increases by a factor of 2 to 5 when the input signal is convolved with a filter learnt from a different plucking position. In this case, the comb filter effect of plucking position creates nodes which effectively suppress information about the filter to be learnt. The filter $h_3(n)$ has a lower off-diagonal error than filters $h_1(n)$ and $h_3(n)$. Likewise input/target pair 2 has lower off-diagonal errors than the other pairs. It appears that the effect of the middle plucking point, like the position itself, is closer to the other plucking points than they are to each other.

Table 2(c) shows that the error also increases when the filter is applied to a signal with different plucking dynamics. Here the effect of changes in plucking dynamics is approximately a doubling of error, although for the filter $h_3(n)$ learnt from a quiet pluck, a much larger error is observed. The reason for this could be a lack of information for filter estimation at high frequencies, due to the signal’s energy being concentrated towards lower frequencies in the case of $p$ (piano) dynamic.

By far the largest errors are recorded in Table 2(d), when the filters are applied to different fret positions. Two reasons can be given for this result: first, the filter is learnt accurately only at the partials of the training tone; for a different testing tone, the frequency response of the filter is inaccurate. The second reason is that each different fret position results in a different comb filtering effect of the pickup. For example, the neck pickup is $\frac{3}{4}$ of the way along an open string, but $\frac{5}{4}$ of the way between the 5th fret and bridge, and $\frac{7}{4}$ of the way between the 12th fret and bridge. The comb filtering effect of the pickup is to attenuate every 4th, 3rd or 2nd partial in the respective cases.

In order to improve the results from Table 2(d), we concatenated the input (respectively target) signals played on the open string, at the fifth fret and at the twelfth fret, in order to learn a composite filter. The filter $h_1(n)$ was learnt from the open string and fifth fret signal pairs, $h_2(n)$ from the open string and twelfth fret pairs, $h_3(n)$ from the fifth fret and twelfth fret pairs and filter $h_4(n)$ from all three pairs. The same filter order of 1024 coefficients is used for the composite filters. The filters were then evaluated on input/target pairs for the three fret positions. Table 2(e) shows considerable improvement compared to the errors measured in Table 2(d). The filter with the least error is $h_4(n)$, which uses information from all input/target pairs.

Figure 5 shows the composite filter, $h_4(n)$. We can observe
that the frequency response of the composite filter is flatter than the filter in Figure 4(c). This is because the composite filter has more information from which it can learn the frequency response. In particular, the frequency response of the filter is more accurate at the partials of fifth ($f_0 = 262\text{ Hz}$) and twelfth fret ($f_0 = 392\text{ Hz}$).

6.3. Comparisons Between Variables

Table 3 summarises the results of Table 2. The second column shows the averages of the diagonal values, which is the global average error of transforming the neck pickup sound into the bridge pickup sound across all cases where the training and testing pairs coincide (thus it is independent of row). The third column shows the averages of the off-diagonal values for Tables 2(a), 2(b), 2(c) and 2(d) and the averages for the fifth column in Table 2(e). These results give insight into the relative contributions of the variables, and thus the robustness of the filters to changes in repetition, plucking position, plucking dynamic and fret position. The filters are most robust to changes in repetition, which should have no systematic difference. Changes in plucking dynamics double the error on average, and changes in plucking position triple the error. The filters are least robust to changes in fret position, which result in a 5-fold increase in error. By learning composite filters across multiple notes, a significant reduction in error appears possible.

Table 3: Summary table for comparisons between each variables.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Mean</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Diagonal</td>
<td>Off-diagonal</td>
</tr>
<tr>
<td>Replittings</td>
<td>0.186</td>
<td>0.233</td>
</tr>
<tr>
<td>Plucking positions</td>
<td>0.186</td>
<td>0.558</td>
</tr>
<tr>
<td>Plucking dynamics</td>
<td>0.186</td>
<td>0.359</td>
</tr>
<tr>
<td>Fret positions</td>
<td>0.186</td>
<td><strong>0.934</strong></td>
</tr>
</tbody>
</table>

8. REFERENCES


