NON-LINEARITIES IN THE THERMOACOUSTIC RESPONSE OF A PREMIXED SWIRL BURNER

A Thesis Submitted to the University of London for the Degree of Doctor of Philosophy

by

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Declaration

The work presented in this thesis was conducted in Queen Mary University of London, Department of Engineering and Material Science.

I certify that this thesis, and the research to which it refers, are the product of my own work, and that any ideas or quotations from the work of other people, published or otherwise, are fully acknowledged in accordance with the standard referencing practices of the discipline. No part of this dissertation has been already, or being concurrently submitted for any other degree, diploma or qualification.

S. M. Reza Hosseini
December 2, 2008
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Abstract

Lean premixed combustion remains one of the simplest and most effective methods of reducing NOx emissions in industrial gas turbines. Lean premixed flames are however prone to an undesirable side effect known as combustion instability, reducing lifetime or in severe cases causing irreversible damage to the turbine. Previous studies on this subject mostly concentrated on the prediction and control of linear instabilities, whereas the current study pays particular attention to the non-linear response. In this work, scaled axial and radial swirl burners were used under atmospheric conditions to investigate the characteristics of the Flame Transfer Function (FTF) between the heat release from methane/air flames and the imposed velocity fluctuations. The velocity fluctuations imposed upon the air flow of the burners encompassed frequencies of 40 to 200 Hz, each with stepwise increase of velocity amplitude, until blow-off occurred. The work was carried out with non-intrusive, phase-locked optical diagnostic techniques, such as Particle Image Velocimetry (PIV) for flow field visualisation and an Intensified Charged Couple Device (ICCD) for analysis of the OH* chemiluminescent intensity distribution of the flame.

It is concluded that there are two dominant mechanisms responsible for the non-linear response of the flame for both swirler geometries at low (below 140 Hz) and high (above 140 Hz) frequencies of excitation. At low frequencies the flame response is governed by equivalence ratio fluctuations due to the ‘stiff’ fuel system and volumetric fluctuations of the input air caused by the forcing. At high frequencies the flame response is governed by the flow features such as vortex roll-up, stretching the flame over the high speed annular jet, and in some cases, causing some flame extinction.
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<td>4.99</td>
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<tr>
<td>4.101</td>
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Chapter 1

Introduction
1.1 Motivation

The dependence of almost all aspects of modern life on electrical power is a well established fact. In the UK for example, the main consumers of electricity in 2006 were industry (34%) and households (34%) as shown in figure 1.1, [61]. The largest consuming industries were chemicals, paper and food, accounting for 40% of industrial consumption.

There are various types of electrical power station. These include: conventional steam stations, nuclear stations, gas turbines, combined cycle gas turbine (CCGT), natural flow hydro-electric stations and pumped storage hydro-electric stations, [61]. The operating principal in gas turbines is based on the rotation of a bladed wheel connected to a shaft, the bladed wheel is driven by pressurised combustion gases from fuel burned (usually natural gas or oil gas) in combustion chambers. The shaft drives a generator and produces electricity. The combined cycle gas turbine incorporates the steam and gas turbines to increase the efficiency of the plant. The waste heat from the gas turbine is used to generate steam with the aid of boilers. The high pressure steam drives a conventional steam turbine which is connected to an electrical generator.

In 2006, 98% of UK electricity supply was home produced and 2% was from imports net of exports. Figure 1.2 shows the electricity supplied by each fuel type. The total supply of electricity decreased by 0.5% in 2006 due to high electricity prices, mild weather and adoption of energy efficient measures. Gas, nuclear and coal are still the dominant fuel types for electricity production. The production power capacity of combined cycle gas turbines has also significantly increased in 2006 compared to that of 1993 as illustrated in figure 1.3, [61].

Electrical power generation is also a major contributor to emissions and accounted for 32% of the UK’s carbon dioxide emissions in 2006. The latest emission estimates are presented in table 1.1, where it is evident that gas fuel produces the least amount of emissions compared to other fossil fuels.

In recent years the use of a lean premixed turbulent flames in industrial gas turbines has
1.1. Motivation

<table>
<thead>
<tr>
<th>Fuel</th>
<th>Emissions (tonnes of carbon per GWh electricity supplied)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coal</td>
<td>239</td>
</tr>
<tr>
<td>Oil</td>
<td>161</td>
</tr>
<tr>
<td>Gas</td>
<td>101</td>
</tr>
<tr>
<td>All fossil fuels</td>
<td>172</td>
</tr>
<tr>
<td>All fuels (including nuclear and renewables)</td>
<td>131</td>
</tr>
</tbody>
</table>

**Table 1.1:** Estimated carbon dioxide emissions from power stations in 2006, [61].

become a more viable option in an effort to further reduce NO\textsubscript{x} emissions, satisfying new environmental legislation. Such method provides lower combustion temperatures which, through the Zeldovich thermal NO\textsubscript{x} mechanism, results in suppression of thermal NO\textsubscript{x} formation, [84]. The other advantage of operating near the lean limit is that of a more homogeneous temperature distribution at the turbine inlet, thereby reducing the thermal wear, increasing combustion intensity, shortening flame length and yielding better fuel burnout. However, operation near the lean blow-off limit causes sensitivity to small perturbations in fuel concentration, flow velocity, temperature and pressure fluctuations. This mode of operation has led to self-excited combustion-induced vibrations known as humming. High amplitude vibrations cause flame blow-off or flash-back, reduced part life, and in severe cases can lead to complete system failure. The root cause of the instabilities is usually investigated under the heading of thermoacoustic. The meaning of the term thermoacoustic is self evident as a combination of thermal (heat) effects and sound, [70–72]. Thermoacoustics describes the relationship between pressure and temperature oscillations for a sound wave in gas. An example is the oscillating heat flow to and from small channels as the sound waves travels through them.

The cross section schematic of an industrial gas turbine is shown in figure 1.4, [82], where of particular interest is the burner responsible for maintaining combustion and the accompanying combustion chamber (combustor) which encloses the combustion and guides the reacted mixture towards the expansion section of the turbine. The vibrations occur as a result of closed-loop interactions between the combustion and the acoustic processes, [30]. The combustor walls reflect the acoustic disturbances that occur as a result of heat-release variations.
Acoustic losses then reduce the magnitude of the pressure disturbances; however the pressure disturbances still manage to play a role in the system by enhancing or reducing the heat release rate and hence closing the feedback loop. The timing of the feedback is another essential factor; the instability could simply grow in amplitude and cause irreversible damage or it could grow to a limit cycle depending on the timing. Dilution holes are usually drilled into combustion chambers to damp low amplitude oscillations. A balance however needs to be maintained, large number of holes reduce the thermodynamic efficiency of the gas turbine through entrance of cold flow into the combustion chamber and reduction of outlet temperature, [14]. Modern gas turbines have already achieved a considerable reduction in greenhouse gases compared to their old counterparts, mainly through improvements in efficiency, [57].

1.2 Flame Stabilisation

There are many different methods of flame stabilisation; in the case of the industrial gas turbine the flame stabilisation should avoid undesirable effects like flash-back, lift-off and blow-off over the operating range of the gas turbine. The stabilisation should also allow for the anchoring of the flame at the desired location. There are several methods of flame stabilisation. They include:

- Bluff-body flame holders
- Swirl or jet-induced recirculation flows
- Rapid increase in flow area creating recirculating separated flow

The swirl or jet-induced flow is of particular importance and is the method of stabilisation in most modern gas turbines. The swirl number $S$ is defined as [4]:

$$S = \frac{G\phi}{G_x R} \tag{1.1}$$
where $G_\phi$ is the angular momentum thrust calculated by equation 1.2 and $G_x$ is the axial momentum thrust obtained by equation 1.3:

$$ G_\phi = \int_0^R 2\pi u w \rho r dr $$ (1.2)

$$ G_x = \int_0^R 2\pi u^2 \rho r dr $$ (1.3)

where $u$ is the axial velocity, $w$ is the circumferential velocity, $r$ is the radius, $\rho$ is the density of the medium passing through the swirler and $R$ is the burner exit radius. Swirl burners increase the stability of the flame by inducing recirculation zones. These zones transfer the hot products from the top of the flame to the base of flame, increasing the flame’s resistance to unwanted phenomena such as instability or blow-off but not eliminating it.

### 1.3 Turbulent Premixed Flame

In general there are two types of flame: premixed and diffusion. The premixed flame is characterised by mixed fuel and oxidiser before chemical reaction. The diffusion flame however is characterised by reactants diffusing into one another during the chemical reaction. Practically it is impossible to mix the reactants perfectly in a premixed flame. So it is appropriate to say that in a premixed flame the reactants are mixed to a much higher extent when compared to a diffusion flame.

Turbulent flow is associated with continuous mixing of fluid elements and this is an important difference between laminar and turbulent premixed flames. Due to continuous mixing of fluid elements in turbulent combustion, the transport of momentum, energy and species is much more rapid than in laminar flow.

The propagation velocity in turbulent combustion is dependent on the characteristic of the flow and mixture properties, whereas in laminar flow it is only dependent on mixture properties (i.e. thermal and chemical properties of the mixture). There are a variety of
1.3. Turbulent Premixed Flame

definitions in the literature regarding the burning velocity. The burning velocity may be defined as a displacement speed, that is the flow velocity normal to the flame zone at the cold boundary of the flame, or a consumption speed that is related to the rate of the consumption of reactants, [80]. The turbulent flame consumption speed can be defined as:

\[ S_t = \frac{\dot{m}}{A\rho_u} \]  

(1.4)

where \( \dot{m} \) is the reactant flow rate, \( \rho_u \) is the unburned gas density and \( A \) is the defined flame area. Turbulent flames are divided into four groups on the basis of length scales. The laminar flame thickness is defined as the thickness of reaction zone controlled by molecular, not turbulent, transport of heat and mass, and it is paramount in flame characterisation. The Borghi diagram shown in figure 1.5 provides a useful classification of turbulent combustion regimes based on non-dimensional numbers such as the Karlovitz (\( Ka \)) and Damkohler (\( Da \)) numbers. The Damkohler number is the ratio of the characteristic flow time to the characteristic chemical time. The Karlovitz number is defined as the ratio of the flame reaction to Kolmogorov time scales.

The wrinkled and corrugated flamelets regimes are characterised by reactions that occur in thin-sheets that retain their laminar structure. The distributed-reaction regime occurs when the laminar flame thickness is larger than the Kolmogorov microscale but smaller than the Taylor macroscale. In this regime all the length scales are within the reaction zone. And finally the well-stirred reactor denotes the limit in which mixing occurs homogeneously over a distributed volume. Figure 1.5 illustrates the role that the Karlovitz, Damkohler and Reynolds number have in separating the different zones. The combustion occurs as wrinkled-flamelets for (RMS velocity / flame speed) ratio of less than unity, and the combustion regime for higher (RMS velocity / flame speed) ratios is determined by the Karlovitz and Damkohler numbers. The wrinkled and corrugated flamelets regimes exist up to a Karlovitz number of one, the distributed-reaction regime exists between the Karlovitz and Damkohler number of one, and finally the well-stirred reactor exists for Damkohler number larger than unity.
1.4 Chemiluminescence Light Emissions

Chemiluminescence is defined as the radiation emitted from electronically excited molecules as they return to lower state energy levels. The wavelength and complexity of the radiation is dependent on the particular molecule, transition it undergoes and the complexity of the molecule. Simple molecules such as OH*, CH* or C2* exhibit a simple spectrum with major peaks at 307, 431 and 513 nm respectively. The complex molecule CO2* however exhibits a continuous radiation spectrum, [21].

The relationship between signal strength of the OH*, CH*, CO2* and C2* chemiluminescence sensor with respect to equivalence ratio, flow rate and fuel type was investigated as early as 1958, [11]. The work concentrated on a Bunsen burner and proved the linear dependence of the chemiluminescence on fuel flow rate in the laminar flow regime. The slope of the linear relationship was however dependent on equivalence ratio.

The next stage in the progress of chemiluminescence technique was the development of correlation between the root mean square of a pressure (measured at some distance from the flame by a microphone) and the time derivative of the chemiluminescence signal strength, [29, 69]. The direct proportionality between the chemiluminescence and heat release rate deduced by Hurle et al. [29] and Price et al. [69] became the basis of the instability analysis in lean premixed gas turbines in pioneering works of Cho et al. [10], Paschereit et al. [65] and Bloxsidge et al. [7].

In hydrocarbon flames OH in its excited state (OH*) forms primarily from reaction between CH and O2 (equation 1.5):

\[ \text{CH} + \text{O}_2 \rightarrow \text{CO} + \text{OH}^* \]  (1.5)

where the concentration of the CH is proportional to the OH*, [21]. As the OH* decays through transition from \(^2\Sigma\) to \(^2\Pi\) energy levels, it emits UV radiation at wavelengths of 306.36, 306.76 and 309.04 nm in about 1 µs. For a given equivalence ratio and fuel type the intensity of radiation provides an indication of the combustion heat release rate. The OH*
chemiluminescence emission increases with equivalence ratio independently of the heat release rate.

The proportionality of the chemiluminescence to heat release however breaks down at regions of high local strain and curvature or near extinction, [64]. The OH* chemiluminescence is also less sensitive to equivalence ratio compared to CH* chemiluminescence and therefore a better indicator of heat release rate, [22]. The figure 1.6 depicts the OH* chemiluminescence of the flame, obtained by Laser Induced Pre-dissociation Fluorescence (LIPF) for different areas of the Borghi diagram presented in figure 1.5.

1.5 Jump Equation

One-dimensional acoustic wave theory is usually used when dealing with the effects of the acoustic wave on the industrial gas turbine burner. This is because of the burner’s geometric configuration. The elongated and tube-like shape of the burner means that three-dimensional acoustic waves essentially have the effect of one-dimensional waves in the direction of the elongation. The one-dimensional acoustic theory also adequately describes the acoustic conditions upstream and downstream of a turbulent premixed flame, as confirmed experimentally by Lawn [44].

The mass continuity, momentum and energy equations are used when deriving the required acoustic and thermoacoustic equations. The partial differential form of the continuity equation 1.6, the momentum equation 1.7 and the energy equation 1.8 for inviscid flow in the x-direction in the absence of body forces are:

\[
\frac{D\rho}{Dt} + \rho \frac{\partial u}{\partial x} = 0 \quad (1.6)
\]

\[
\rho \frac{D u}{Dt} = -\frac{\partial p}{\partial x} \quad (1.7)
\]

\[
\rho \frac{D h}{Dt} = \dot{Q} + \dot{W} = \rho T \frac{Ds}{Dt} + \frac{Dp}{Dt} \quad (1.8)
\]
where \( D/Dt \) is the total rate of change with respect to time, \( \partial/\partial x \) is the rate of change with respect to x-direction, \( h \) is the specific total enthalpy, \( s \) is the specific entropy, \( T \) is the temperature and \( p \) is the pressure. Also \( Q \) is the heat added to the system and \( W \) is the work done on the system per unit volume.

By assuming that the reactants and products behave as perfect gases, the fluctuations are rapid enough to be adiabatic, the Mach number is much smaller than unity and flame is acoustically compact, which means that the flame is thin in comparison with the wavelength, equation 1.9 was derived as explained in appendix B, [46].

\[
(u_2' - u_1') A = \frac{(\gamma - 1)}{\bar{\rho} c^2} \int q' dA dx
\]

(1.9)

where \( u_1' \) and \( u_2' \) is the fluctuating velocity before and after the flame respectively, over-bars denote the mean values and primes, the perturbations. Equation 1.9 expresses the well-known jump phenomenon in the volumetric flux which is proportional to the total fluctuating heat release \( h' \).

\[
h' = \int Q' dx = \int q' dA dx
\]

(1.10)

where \( Q' \) is the fluctuating heat-release per unit length and \( q' \), per unit volume. Equation 1.9 states that the perturbations in the equivalence ratio or through a turbulent flow field cause fluctuations in the heat release, which result in fluctuations in the velocity.

The specific acoustic impedance is defined as the ratio of the acoustic pressure to the associated particle speed in a medium. A normalised form of the acoustic impedance may be expressed:

\[
Z = \frac{p'}{\bar{\rho} u' c}
\]

(1.11)

where \( p' \) is the acoustic pressure, \( u' \) is the acoustic velocity, \( c \) is the speed of sound and \( \bar{\rho} \) is the mean density. By using equation 1.11 and the approximations used for equation 1.9,
equation 1.12 was derived:

\[
\frac{h'}{p'_1} \propto \frac{u'_2}{p'_1} - \frac{u'_1}{p'_1} \propto \frac{1}{Z_2 \rho_2 c_2} - \frac{1}{Z_1 \rho_1 c_1} \propto \frac{c_2}{Z_2} - \frac{c_1}{Z_1}
\] (1.12)

Equation 1.12 defines the proportionality between the total fluctuating heat release, the velocity jump and the impedances before and after the flame if the ratio of the specific heat, \(\gamma\), is constant. The proportionality between the jump in the volumetric flux across the flame and the instantaneous heat release from the whole flame obtained by the chemiluminescence emission at 307 nm was also confirmed experimentally by Lawn [44].

### 1.6 Thermoacoustics

In simple terms, the Rayleigh criterion relates the effect of pressure fluctuations to heat release fluctuations. Unstable acoustic modes are amplified if the added acoustic energy is greater than the losses. The Rayleigh parameter is:

\[
R(t) = \int_V p'q'dV
\] (1.13)

where the integral is over the entire combustion domain, \(p'\) and \(q'\) are the fluctuating pressure and heat release rate. Positive values indicate net energy addition and instability growth and negative values indicate net energy loss. Consider a control volume \(V\) with surface \(S\) with combustion occurring inside. By using equation B.15 from appendix B and the linearised form of the mass conservation and momentum equations for the three directions, equation 1.14 was obtained.

\[
\frac{\partial}{\partial t} \int_V \left( \frac{1}{2} \bar{\rho} u'^2 + \frac{1}{2} \frac{\rho'^2}{\rho_0^2} \right) dV =
\]

\[
\frac{\gamma - 1}{\rho_0 c_0^2} \int_V p'q'dV - \int_S \left( p'u'_j - u'_i \tau_{ij} \right) dS_j - \int_V \frac{\partial u_i}{\partial x_j} \tau_{ij} dV
\] (1.14)

Equation 1.14 is the three-dimensional form of the equation 1.9 for Mach numbers above unity.

The term on the left hand side of the equation 1.14 is the rate of change of the sum of kinetic
1.7 Flame Transfer Function

and potential energies within the volume $V$. The first term on the right hand side describes the exchange of the energy between the combustion and the acoustic waves. According to Rayleigh’s criterion, when $p'$ and $q'$ are in phase the acoustic energy tends to increase. The second term on the right hand side is a surface term and accounts for the loss of energy across the bounding surface $S$. The final term on the right hand side is the rate of viscous dissipation. Therefore according to this equation disturbances grow if their net energy gain from the combustion is greater than the sum of their energy losses across the boundary and due to dissipation. Therefore the acoustic mode grows in amplitude if:

$$\gamma - \frac{1}{\bar{\rho}c^2} \int_V p'q'dV > \int_S (p'u_j - u_i\tau_{ij})dS_j - \int_V \tau_{ij} \frac{\partial u_i}{\partial x_j}dV$$

(1.15)

The combustor will exhibit thermoacoustic instability if equation 1.15 is satisfied, so that the linear waves increase in amplitude until non-linear effects limit them.

1.7 Flame Transfer Function

The flame transfer function (FTF) relates the amplitude and phase of the total fluctuating heat release to the input acoustic velocity of the premixture at the burner exit. The FTF is a function of equivalence ratio, fuel consumption (burner load), frequency and amplitude of the self-excitation or forced-excitation. Burner design factors such as technique of flame stabilisation and fuel delivery methods also influence the FTF. In this investigation two methods were used to calculate the transfer function, the first method is known as the non-normalised FTF, see equation 1.16:

$$\frac{Q'}{u'_{in}} = \psi e^{i\theta} = \psi e^{-i2\pi f\tau_{eff}}$$

(1.16)

where $f$ is the frequency, $\tau_{eff}$ is the effective time delay, $\psi$ is the non-normalised FTF magnitude and $\theta$ is the FTF phase. The second method calculates the normalised flame transfer
1.8. Specific Objectives

The main aim of this work is to investigate the mechanisms involved in the limit cycle behaviour of a lean premixed methane flame stabilised by two burner geometries under self-excitation. The investigations were carried out for two equivalence ratios. Forced-excitation is the main experimental approach in the current work, since it provides a method of controlling the applied frequencies and amplitudes on the flame. Forced-excitation is a more practical way of studying the flame response and also provides a larger range of conditions for investigation of instabilities.

The main diagnostic methods are non-intrusive optical techniques such as phase-locked Intensified Charged Couple Device (ICCD) imaging to obtain OH$^*$ chemiluminescence for assessing the flame structure, phase-locked Particle Image Velocimetry (PIV) for analysis of flow features, hot wire in conjunction with photodiode measurement for determination of FTF (through measurement of velocity fluctuations and OH$^*$ chemiluminescence emission of the entire flame) and finally two microphone method for confirming the velocity fluctuations and determining acoustic properties of the burners.

\[ \frac{Q'_n}{u'_n/u_{in}} = \Psi e^{-i\Theta} = \Psi e^{-i2\pi f_{eff}} \] (1.17)

where \( \Theta \) is the FTF phase and \( \Psi \) is the normalised FTF magnitude, which is proportional to the non-normalised FTF magnitude (\( \psi \)) in equation 1.16 for fixed burner conditions. Also for fixed burner conditions the FTF phases (\( \theta \) and \( \Theta \)) should be the same.
### 1.9 Figures

**Figure 1.1:** Electricity demand by sector in 2006 for UK, [61].

**Figure 1.2:** Fuel used in electricity generation for 2001 and 2006, [61].
Figure 1.3: Generating capacity of major power producers, 1993-2006. 1) Gas turbines, oil engines and renewables other than hydro, 2) Natural flow and pumped storage and 3) Includes gas fired stations that are not CCGTs, [61].

Figure 1.4: The cross section schematic of an industrial gas turbine. Part number 30 is the combustion chamber, number 33 is the exit of the combustion chamber and number 50 is the burner, [82].
1.9. Figures

Chapter 1. Introduction

Figure 1.5: The Borghi diagram, [53].

Figure 1.6: Flame shapes occurring under different $Ka$ and $Da$ numbers. A) Wrinkled flamelets, B) Corrugated flamelets, C) Distributed or thin reaction zones and D) Well-stirred reactor, [81].
Chapter 2

Literature Review
2.1 Mechanisms Of Instability

Mechanisms of instability involved in lean premixed swirl stabilised flame excitation are summarised in figure 2.1, [49]. The instability can result from one dominant mechanism, although more than one mechanism could be present. There is also the possibility of instability occurring as a result of interaction between different mechanisms. The dominant pathways are:

- Kinematic perturbations due to acoustic velocity fluctuations at the flame front, which gives rise to area perturbations and hence heat release rate fluctuations (pathway 1).

- Acoustic velocity variations causing perturbations in the turbulent burning velocity and hence fluctuations in the fuel consumption rate. The acoustic velocity fluctuations may also give rise to large-scale vortex structures which in turn affect the fuel consumption rate. This mode can be broken down further into:
  - Local shearing causing the instantaneous perturbations (pathway 2a).
  - Fluctuations which originate from vortex shedding (i.e. coherent vortices and shear flow) and are associated with a time delay in which they are convected from the origin (pathway 2b).

- Equivalence ratio fluctuations which could result from perturbations in air or fuel or both (pathway 3a and 3b).

Other mechanisms that have an effect on the instability are included below:

- Inner and outer recirculation zone unsteadiness

- Swirl fluctuations

Some instability mechanisms such as entropy waves encountered in a combustor with choked exit [67] or effects of background noise on the instability [56] become important at high Mach
numbers and are not shown in figure 2.1. The pilot flame also play an important role in flame stabilisation [38], the details of which are outside the scope of this study.

The instabilities cause an increase in the amplitude of self-excited oscillations. The unstable linear mode grows in amplitude until non-linear effects dominate and hence a limit cycle is achieved. The frequency of the limit cycle could be obtained from linear stability models; however these models do not predict the amplitude of the limit cycle, [85].

The mechanisms of instability will be considered in more detail in the following section. Table 2.2 in section 2.6 summarises the most important and relevant literature on the mechanisms of instability, concentrating mainly on the experimental studies, and detailing experimental conditions and techniques.

2.1.1 Inner And Outer Recirculation Zone Unsteadiness

The structure of the swirl is dependent on the swirl number, [60]. A desired effect of high swirl number is the development of an Inner Recirculation Zone (IRZ) and an Outer Recirculation Zone (ORZ). The classical shape of a confined swirl stabilised burner is shown in figure 2.2 where the regions of central or IRZ and side or ORZ are clearly indicated and may dominate the flame stability, depending on the swirl number and fuel equivalence ratio. In a swirling flow the vortex aligned with the axis of the swirler breaks down in a spiral form and rotates around the axis of the swirler, resulting in a phenomenon known as a Precessing Vortex Core or PVC. The effect of the PVC as a mechanism of instability and sometimes even its presence during combustion is debated, [73, 78]. The advantage of IRZ on the other hand as a stabilising mechanism is well known. The IRZ has the task of bringing or recirculating products and partially reacted premixture from the top of the combustion zone to the flame front and hence igniting the incoming fuel / oxidant streams. This results in a high rate of heat release, and a compact and stable flame, [60].
Hedman et al. [23] used OH*-PLIF (Planar Laser-Induced Fluorescence), axial and tangen-
tional Laser Doppler Velocimetry (LDV) and instantaneous gas temperature measurements
with a Coherent Anti-Stokes Raman Spectrometer (CARS) to investigate the flame behaviour
of a premixed swirl stabilised natural gas burner. PLIF was employed since it is a non-intrusive
optical technique of measuring trace amounts of short lived combustion species such as OH*
or CH* with a great temporal resolution. High OH*-PLIF intensities are considered to be an
indication of reacting fuel/air mixtures or post-combustion gases. However CH*-PLIF is a
measure of the heat release rate and in high concentrations is related to the fuel consumption
layer of the reaction zone. The experiments were conducted for two swirl numbers of 0.47
and 1.29 and two equivalence ratios of 0.8 and 0.65. Although phase-locked measurements
were not performed for unstable modes, a statistical analysis was carried out to assess the
flame behaviour. Hedman et al. [23] concluded that the location of the flame front, variation
of flame structure and the presence/strength of the recirculation zones where all tied to the
swirl intensity and premixture equivalence ratio. A bi-stable mode was observed for the low
swirl, low equivalence ratio case, in which the flame oscillated between an anchored mode and
a lifted mode. Hedman et al. [23] suggested that this was due to operation close to the lean
limit of flammability causing the flame to be susceptible to instabilities. The recirculation
zones were defined from the LDV measurements by identifying zero axial and radial velocity
contours. The study of the height and distance between each recirculation zone revealed that
for the low swirl case the flow rates of the ORZ were much greater and extended much higher
into the combustion chamber compared to the high swirl case. The IRZ of the high swirl
case showed higher flow rates compared to the low swirl case, resulting in a more efficient
transfer of the hot gases from the combusting zone down to the ignition zone. The CARS
data illustrated the highest levels of temperature in the shear layer between the IRZ and ORZ.

Weigand et al. [89] investigated a swirl/diffusion stabilised lean flame with the aid of
LDV, OH*-PLIF, CH*-PLIF and CARS technique and used them to measure major species
concentration, temperature and mixture fraction. Employed conditions are shown in table 2.1.
Flame B exhibited the highest amplitude of self-excitation, flame C was second and flame A was the last. Flame C also demonstrated sudden lift-off and flash-back, since it was operated near the lean extinction limit. Phase-locked measurements demonstrated the existence of a strong IRZ and ORZ for all flames. High turbulence intensity was observed in the shear layer between the inflow and the IRZ, causing intense mixing of cold fresh gas and hot burned gases arriving from the IRZ for all flames. The IRZ was not a stable structure and streamline plots of the flow showed a considerable variation in its positions. However, coherent structures such as vortex rings were not observed in the measured data for all flames. Variation of equivalence ratio resulted in different thermal expansions, which mainly influence the axial velocity. The combustion also increased the axial acceleration and hence reduced the swirl number. Phase-locked measurements revealed that the IRZ size varied mainly in the axial direction whereas ORZ size oscillated in the radial direction.

The OH*-PLIF and CH*-PLIF measurements revealed that broad areas of the flame showed OH* emissions with strongly wrinkled contours and some isolated islands of unburned mixture. The IRZ above the nozzle showed the highest OH* concentration and therefore temperature, irrespective of flame type. The dominant area of heat release was in the shear layer for flame A only. In flame B due to periodic pulsations this was not the case and the flame surface area in A was larger than in B or C. The local equivalence ratio at which reaction occurred varied from the global equivalence ratio, noting that local area where $\phi = 1$ was generally not achieved.

Mixture fraction, temperature and species mole fractions deduced that the nozzle design yielded fast mixing, as demonstrated by small values of mixture fraction just above the burner nozzle. The IRZ showed the highest values of mixture fraction compared to the ORZ. This
resulted in elevated local temperatures compared to the global temperature and therefore assisted flame stabilisation. The IRZ was subjected to significant turbulent fluctuations; this suggested that the vortex was not stable in time. Periodic movement of the recirculation zones correlated with the phase-locked temperature measurements. As the ORZ reached its maximum expansion and the IRZ penetrated the burner nozzle, the flow temperature increased due to the mixing of recirculating exhaust gas and fresh gas. The data demonstrated that heat conduction to the burner plate resulted in a reduction in temperature of ORZ. The mixture fraction analysis indicated that additional mixing of the fuel, air and exhaust gases took place as a result of periodic oscillations in the flow field. The work of Weigand et al. [89] proved that the IRZ and ORZ contribute significantly to the mixing of the fuel, air and exhaust gases and hence increase the reaction progress.

Meier et al. [63] investigated the thermo-chemical states of the same flames examined by Weigand et al. [89], see table 2.1. The finite-rate chemistry affects the operation of the combustion through local flame extinction, ignition delay and changes in the emissions of the pollutants. For the flames under consideration in this study the ignition delay times are usually of the same order as the turbulent mixing and convective transport. The correlation between the temperature and mixture fraction suggested that the flame was partially premixed and the IRZ had the highest temperatures. Meier et al. [63] concluded that in the IRZ the gas mixtures were mostly completely reacted and relatively fuel rich. The thermo-chemical state of the IRZ was also close to that expected at adiabatic equilibrium. Gas mixture temperature in the outer flame regions were reduced due to heat conduction to the burner plate. Partially reacted and un-reacted samples in the vicinity of the shear layer as a result of local flame extinction or ignition delay caused the reduction of ORZ temperatures with respect to the IRZ. In these regions O2 and CH4 co-existed. Flame C exhibited random fluctuations of flame stabilisation position due to low global flame temperatures and mostly fuel lean mixture before reaching the stabilisation region.

Huang and Yang [28] implemented Large Eddy Simulation (LES) to investigate the tran-
sition from a stable to unstable flame when the inlet temperature exceeded a threshold value. The code modelled a gas turbine combustor with \( S = 0.76 \) and a bluff body stabiliser at a pressure of 4.63 atm. The code demonstrated the presence of the IRZ as a result of swirl and ORZs, which the authors attributed to the sudden expansion of the combustion chamber. The frequency of the vortex shedding from the bluff body also caused small amplitude fluctuations in the pressure and velocity fields. The effects of increasing the inlet temperature were that for a fixed mass flow rate, the flow velocity increased and the flame moved downstream. The flame speed increased and the flame propagated upstream and finally small local flow velocity caused the flame to become prone to flash-back.

Huang and Yang [28] argued that the combined effect of the above parameters determined the final flame structure. Therefore as the critical inlet temperature was passed, the flame abandoned its original anchoring position at the IRZ and flashed back to the ORZ. This resulted in a compact flame shape stabilised by both the IRZ and ORZ. The flame then oscillated in its new anchored position resulting in unsteady heat release and causing pressure oscillations to reach a new limit cycle. Stable operation could only be resumed if the inlet temperature was reduced below the critical temperature. Huang and Yang [28] concluded that equivalence ratio and inlet temperature are two important parameters regarding the combustion stability.

2.1.2 Coherent Vortices And Flow Structures

One way of investigating the complex interactions of acoustic fields and lean turbulent premixed swirl stabilised flames is to simplify the problem to that of a lean laminar premixed flame and acoustic fields as suggested by Durox et al. [20]. In this instance Durox et al. [20] examined an inverted conical flame, stabilised on a central bluff body without flame confinement. Under the self-excitation mode flame roll-up was visible and a sudden flame surface area reduction was also observed, resulting in a rapid decrease in heat release and hence an intense sound pressure wave. The gain of the FTF illustrated that the flame behaved like an
amplifier for a finite range of frequencies at the lowest imposed acoustic velocity. The flame image showed that in the frequency range of interest the roll-up at the flame extremity led to the creation of large flame surface and hence rapid combustion of fresh gases. Escalation of the imposed acoustic velocity increased the strength of the roll-up at the flame extremities and flame wrinkling. For very high imposed acoustic velocities, the flame ceased to behave like an amplifier and showed a clear non-linear effect. The phase of the FTF remained independent of the imposed acoustic velocity but increased linearly with frequency indicating that the instability mechanism involves a convective time delay. Durox et al. [20] also observed that the shapes of the flames were affected by roll-up due to the interaction of vortex kernels on the shear layer which separates the reactive jet from surrounding air. Durox et al. [20] finally concluded that the vortex structures were the main cause of the flame wrinkling between the shear layer of the reacting jet and surrounding medium. The wrinkling therefore induced annihilation of neighbouring reactive elements and strong rolling up of the flame, resulting in a rapid flame surface area modulation.

Schildmacher and Koch [77] conducted extensive experiments into non-reacting and reacting flow from a burner with a concentric annular swirler design. Non-reacting experiments and simulations revealed instabilities which originated from the vortex shedding in the shear layer at the burner mouth triggered by the precessing vortex core. For the case of self-excited combustion instabilities with high oscillation sound level, the LDV images revealed flame stabilisation in the regions of high velocity gradients (i.e. the shear layer). Vortex shedding affects the instability of the flame through the modulations of the shear layer. Schildmacher and Koch [77] attributed the high axial and tangential velocity components of the reacting LDV image to the presence of harmonic frequencies in the combustion chamber. Also, the local swirl at the reaction zone suffered sudden interruption due to the presence of coherent structures, which influenced the flame through strain rates and local flame quenching. Schildmacher and Koch [77] also compared the case of instability with high oscillation sound level to that of low oscillation sound level and transition from no sound level to very low sound
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level. Thus they showed the disappearance of the coherent flow structures when moving from non-reacting case to reacting case even at very low sound level. Schildmacher and Koch [77] concluded that, under self-excitation reacting conditions, the resonant frequencies are dependent on the acoustic eigen-frequencies of the combustion system and independent of the burner air flow.

Kulsheimer and Buchner [40] presented the different feedback mechanisms involved in combustion driven oscillations in highly turbulent premixed combustion systems. The feedback mechanisms are similar to those of Lawn and Polilke [49]. However Kulsheimer and Buchner [40] also accounted for the effects of mixing units, burner geometry and type. The burner was not enclosed similar to that of Durox et al. [20]; hence the combustion chamber enclosure did not play a role in the experiments. The work pays special attention to the ring-vortices, which formed under a particular combination of amplitude and frequency of pressure fluctuations. As the formed vortices roll-up, the premixture is brought together with entrained ambient medium, due to the absence of a combustion chamber. This causes a change in the mixing patterns and reaction fields of the flame. The vortex therefore causes a fluctuation in heat release rate and pressure which could generate unstable behaviour if the Rayleigh criterion was fulfilled. If the criterion was satisfied then amplification of the flow rate could present itself as larger ring vortex generation and therefore close the feedback loop. Three swirl numbers were investigated and the required pulsation level for vortex formation was obtained under cold flow conditions and presented against the dimensionless Strouhal number $St$:

$$St = f_{puls} \cdot \frac{d_{eq}}{u_{b,x}}$$ (2.1)

where $d_{eq}$ is the equivalent diameter obtained from the free flow area perpendicular to the main flow direction and $u_{b,x}$ represents the mean volumetric velocity in the axial direction at the burner exit. For the reacting flows the magnitude of the flame transfer function decreased as the imposed frequencies increased. This was attributed to the large-scale coherent structures. Also as the pulsation level increased, coherent structures formed at lower frequencies,
as observed in cold flow conditions. Increasing the amount of swirl caused a reduction in flame transfer function because entrainment of ambient medium increases with induced swirl, resulting in a reduction of global equivalence ratio and thus flame transfer function. Kulsheimer and Buchner [40] also observed reaction in the inner recirculation zone, with moderate increases of pulsation frequency causing an increase in amplitude response of the flame transfer function. This behaviour was also observed in the enclosed flame. The flame transfer function of the enclosed and open flames showed significant differences. It was concluded that the inner recirculation zone played the major role in affecting the FTF when the flame was enclosed.

Lohrmann et al. [59] conducted further experiments on the rig used by Kulsheimer and Buchner [40] and also enclosed the flame with a quartz chamber. Lohrmann et al. [59] stated that two major mechanisms are responsible for pressure oscillations in the combustion chamber. The first one was that of mass flow oscillation of time-constant, homogenous mixture due to the quasi-steady flame geometry or coherent vortex ring structures when the flow acceleration at the burner nozzle exceeded a critical value. This resulted in a time-dependent global heat release rate and variations in pressure oscillations which led to unstable modes if the Rayleigh criterion was fulfilled. In the second mechanism the oscillation of equivalence ratio, due to the acoustic properties of air and fuel supply, led to instability if the Rayleigh criterion was fulfilled.

Lohrmann et al. [59] further investigated two premixtures. The perfect mixture was time independent and homogenous and it was periodically modulated with the aid of a stepping motor and the practical premixture in which the fuel was injected into the modulated air flow. For the perfect premixture, as long as the imposed excitation frequency was below a critical value, the flame behaviour was close to that of quasi-steady flame, irrespective of the imposed velocity amplitude. Once the imposed frequency passed the critical value the formation of ring vortices became apparent. The formation of ring vortices decreased the FTF magnitude due to fuel lean combustion conditions. The FTF also showed a dependency on the amplitude of the imposed frequency, so that the FTF of higher amplitudes of acoustic
velocity decreased faster with increasing frequency compared to lower amplitudes of acoustic velocity. This dependency was related to the size of the ring vortex, so that at high acoustic velocities the ring vortex structure became larger and hence more mixture was enclosed in the vortex. This further increased the local equivalence ratio compared to the global and caused a steeper slope in FTF magnitude. The ideal idle-time delay model with a constant time delay provided a very good approximation for the FTF phase of the flame, irrespective of imposed frequency and amplitude of acoustic velocity.

Considerable differences were noted between the phases of the perfect premixture FTF and that of a practical premixture FTF. The difference in phases were independent of the imposed acoustic velocity. For forcing frequencies below the critical value the heat-release fluctuations were attributed to the equivalence ratio modulations irrespective of the acoustic velocity, and the ideal idle-time delay model described the FTF phase adequately. For forcing frequencies above the critical value, the ideal idle-time delay model diverged from the experimental FTF phase data due to presence of vortices.

The study performed by Cabot et al. [9] was carried out on swirl ($S = 0.7$) and bluff-body stabilised lean premixed turbulent flames. This work primarily concentrated on the flame behaviour as the equivalence ratio changed from near rich conditions ($\phi = 0.9$) to the limit of lean extinction ($\phi = 0.57$). Cabot et al. [9] applied phase-locked CH* with respect to pressure oscillations occurring in the self-excited mode, noting that the self-excitation occurred only at $\phi = 0.63$. A novel method of analysing the self-excitation mode was also suggested: the method involved splitting the Abel transformed phase averaged CH* image into different regions based on the sign of the axial and radial velocity components obtained from LDV. The authors demonstrated from the LDV results that periodic vortex shedding was not present and from microphone data that the fluctuations in the fuel line were not sufficient to explain the self-excitation instability. The image however did show the effect of the flame shear layer upon the instability: under the lean conditions, some of the unburned mixture in the shear layer moved into the corner recirculation zone. These gases could then react and stretch the
shear layer even further. The resultant unburned gases then moved to the top of the flame and modified the flame structure. This cycle then repeated and the instability continued.

Huang et al. [26] performed an experimental study on a low swirl stabilised lean premixed burner in order to investigate the thermoacoustic coupling with the aid of OH$^*$-PILF. The forced-excitation method was also applied here, although the loudspeakers were positioned upstream of the flame. Due to the low swirl number the flame stabilisation was attributed to the flow divergence rather than any recirculation zones. Huang et al. [26] also observed the effect of vortices and suggested that the frequency of vortex roll-up can lock onto the frequency of the sound field, provided that the amplitude of the acoustic field is large enough with a frequency close to the natural frequency of vortex shedding. Also it was demonstrated that the OH$^*$ intensity variation is caused by the distribution of the flame surface. Huang et al. [26] concluded that the acoustic perturbation caused vortex generation which affected the shear layer at which flame is stabilised. Under the vortex roll-up the reactants are pushed ahead of the flame resulting in a high Flame Surface Density (FSD$^1$), but close to the high FSD, a low FSD is achieved since the flame is strained by the vortices. The variation in FSD ultimately leads to variation in heat release. Huang et al. [26] also concluded that due to close correlation of FSD and OH$^*$, it is possible to use OH$^*$ fluctuation for assessing the flame behaviour.

Stohr and Meier [83] experiments used phase-locked PIV to investigate self-excited oscillations of the same burner configuration presented by Weigand et al. [89]. The main aim of this work was to investigate the coherent structures, since the interaction of the chemistry and acoustics with these structures is still a challenge. The average field of the PIV images revealed a strong IRZ and ORZ separated by a shear layer with high velocity gradients. The instantaneous images however showed the presence of a precessing 3D helical vortex which formed a zig-zag of ordered vortices. The position of these vortices varied from image to image; hence their presence was not apparent in the averaged image. It was also hypothesised that

$^1$Quantifies the flame surface area modulations.
because of their position on the inside and outside of the shear layer, their effect on mixing of air and fuel is important and contributes to the stabilisation of the flame. The phase-locked measurements did not show the helical vortices either, resulting in a conclusion that they are not related to the acoustic emission of the flame, since the phase-locked measurement were referenced to the acoustic pressure of the combustion chamber. Stohr and Meier [83] also demonstrated variations in the global swirl strength from the phase-locked PIV data.

Ji and Gore [31] argue the importance of instantaneous PIV data over the mean flow measurements. Their experimental investigation was carried out on a burner with a swirl number of 2.4 and a Reynolds number of 72000, under non-reacting and reacting conditions. They observed multiple smaller recirculation zones in the instantaneous PIV images for both reacting and non-reacting conditions, which resulted in a single large recirculation zone in the mean images. The turbulence kinetic energy levels in the flame were approximately three times those in the non-reacting flows, and the swirling flame vortex length scales were smaller than those in the non-reacting swirling flow. Unfortunately the instantaneous images were taken at randomly selected instances, since the work concentrated on analysing the flow field of the flame instead of combustion instability. The authors also did not investigate the phase-locked ensemble-average for determining the flame behaviour. They concluded that combustion enhances the mean and instantaneous velocity and vorticity, based on the two-dimensional PIV measurements. The combustion reduced the longitudinal extent and increased the radial extent of the mean recirculation zone.

Stone and Menon [84] investigated the effect of swirl and equivalence ratio variations using LES. The investigation focused on two phenomena. The first was the behaviour of the system as the swirl number was increased while the equivalence ratio was kept constant. The second was the reverse, analysing a burner with a constant swirl number but different equivalence ratios. In the first part of this study a difference in flame structure was observed. The flame with a low swirl ($S = 0.56$) was long and ‘pointed’, as in the case of a jet flame. The flame with high swirl number ($S = 1.12$) however presented itself with a compact and flathead
structure, ending just upstream of the recirculation zone. The shedding of coherent large-scale vortices was apparent in both cases, although their strength decreased with increasing swirl. The recirculating zone only occurred when the swirl number was above a critical value. The presence of the coherent vortices was attributed to the acoustic forcing of combustor resonance. Stone and Menon [84] demonstrate that the coherent vortices (appearing as a ring) drag the flame along as they convect downstream and ultimately collapse. The flame then propagates toward the sudden expansion, resulting in a longitudinal pulsation cycle. In the second part of the study, Stone and Menon [84] verified the presence of instability as equivalence ratio was decreased. By combining the two they concluded that at low equivalence ratio the flame speed reduces and the flame becomes more susceptible to the dragging effect of the coherent vortices.

Roux et al. [73] investigated the global behaviour of the burner and combustion chamber using a combination of LES, acoustic analysis and experimental data obtained from LDV. In the LES approach the authors specifically avoided enforcement of an exit boundary condition on the exhaust of the combustion chamber by including sections of the surrounding atmospheric air outside the combustion chamber in the computational domain. This ensured that the acoustic waves reaching the exhaust were transmitted or reflected in a similar way to those in experiments. The cold flow analysis of the combustor revealed the presence of a low amplitude $3/4$ wave mode and a strong hydrodynamic mode due to a PVC. However, the addition of combustion to the calculation damped the PVC observed in cold flow conditions and therefore the pressure fluctuations were mainly caused by acoustic modes. The authors further argue that the PVC is important, and only present in some swirled combustors, as observed by Selle et al. [78]. The eigenmodes of the combustion chamber were also calculated using the LES temperature field and Helmholtz solver to yield low amplification $1/4$ and high amplification $3/4$ modes. Roux et al. [73] therefore concluded that under cold flow conditions the PVC and the longitudinal $3/4$ mode coexist, but under the combustion regime the PVC disappears, and the pressure fluctuations lock onto the $3/4$ mode and change the frequency...
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Selle et al. [78] carried out LES studies of a radial industrial-scale burner manufactured by TURBOMECA for reacting (with preheated air in regimes which do not exhibit large-scale combustion instabilities) and non-reacting conditions, and compared the results to experimental data. Under non-reacting flow a PVC appeared, as observed by Roux et al. [73] for a different swirl geometry. Selle et al. [78] concluded that in the reacting conditions the PVC disappeared and the pressure oscillations corresponded to the transverse-longitudinal mode of the chamber only.

2.1.3 Kinematic Perturbations

Lawn et al. [50] investigated the response of a turbulent flame front to an acoustic wave at normal incidence. A novel flame stabilisation method was used to investigate the response of CH$_4$ / air and CH$_4$ / H$_2$ / air premixtures. Phase-locked PIV and OH$^*$-PLIF were employed to measure fluctuations in flow velocity and flame height as the flow was excited with loudspeakers. The addition of small amounts of hydrogen to the fuel allowed leaner mixtures to be investigated because the resultant flames are less susceptible to thermoacoustic excitation since the mixtures are more resistant to flame stretch. The acoustic velocity fluctuations affect the wrinkling of the flame front through the angle of the incident acoustic wave and turbulent burning velocity. Lawn et al. [50] concluded that the response of the burning velocity decreases as the excitation frequency increases and addition of hydrogen had no clear effect on the flame response.

Laverdan and Thevenin [42] investigated the interaction between acoustic waves and turbulence in an H$_2$ / O$_2$ / N$_2$ flame by simulating the interaction using Direct Numerical Simulation (DNS). The 2-dimensional compressible flow was modelled by taking into account realistic transport properties and chemical kinetics. The simulation illustrated that prior to arrival at the flame front the acoustic wave had a flat profile and during interaction with
the reaction zone the acoustic wave became wrinkled. The wave and flame interaction also produced weak amplitude heat release fluctuations.

Savarianandam and Lawn [75] measured the turbulent burning velocity for ethylene / air and methane / air. The flames were stabilised in a conical diffuser and in the region of ‘weakly wrinkled’ of the Borghi diagram. Savarianandam and Lawn [75] concluded that for this region the flame wrinkling was influenced by turbulence but strongly enhanced by Darrieus-Landau instability.

2.1.4 Equivalence Ratio Fluctuations

Janus et al. [30] dismissed the effect of vortex shedding and concentrated mainly on the fluctuation in air and fuel flow rates. Although the premixture was supplied via choked orifices it was argued that acoustic compressions and rarefactions in the combustion chamber could reduce or increase the speed at which the mixture reaches the flame (also known as the transport time), hence leading to a fluctuation in the heat release rate. They investigated the effect of the inlet air temperature, humidity and fuel composition on the combustion oscillations. It was concluded that changing the inlet air temperature by approximately 13% affected the stability of the flame through changes in reaction rates and thus transport times. The study presented inconclusive results for the humidity variations and the authors reported that variations in the fuel composition on the stability were not as significant as the inlet temperature variations. On the whole, ambient conditions were important if the combustion was close to instability. The above phenomena could therefore make the system unstable through small changes in transport times.

Lawn [47] investigated the thermoacoustic response of a premixed swirl burner with the aid of OH$^*$ chemiluminescence emission and pressure measurement. A photodiode was used to measure fluctuation in OH$^*$ chemiluminescence and two microphones were placed upstream and downstream of the burner to measure pressure fluctuation in the inlet and the combustion
chamber. The effect of varying equivalence ratios, the presence of a pilot flame, and withdrawal of the axial swirler into the inlet pipe, to create an annulus of un-swirled premixture, was investigated. The impedance blockage upstream of the premixture supply was varied and the effect of changing from propane to methane was also examined. The photodiode measurements were implemented in three distinct methods, by using different optical arrangements. Lawn [47] presented an analytical acoustic analysis based upon measurements acquired via the two microphone technique implemented in this investigation. Lawn [47] proposed a Gaussian fit to describe the axial distribution of the heat release in terms of four parameters. These were the position and spread of the axial distribution of the heat release fluctuations, and the origin and effective velocity of convection of disturbances. The author also analysed the Rayleigh integral of the measured emission and demonstrated consistency between unstable frequencies and positive Rayleigh integral values. The most important conclusion was the recognition of two types of instability: a lower frequency mode, which was attributed to equivalence ratio fluctuations associated with the acoustic fluctuations in the air supply, and a higher frequency mode, which was associated with the periodic generation of large scale vortical structures in the shear layer.

Schildmacher et al. [76] carried out experiments on a concentric annular swirler operated under lean conditions of $\phi = 0.83$ and 0.71. This study implemented LDV, PLIF and numerical acoustic eigenmode analysis in order to investigate the flame and flow field behaviour. Although the combustion chamber in this work was choked and open to the atmosphere, the air volume flow was identical to that of the burner in the real gas turbine and the incoming air was also preheated to the appropriate temperature of 637 K. The numerical analysis revealed all possible acoustic eigenmode frequencies but did not indicate the excitation frequency. One calculated eigenmode was close to that of an experimentally determined unstable frequency and close examination of the pressure fluctuation phase revealed a 180° shift between the pre-chamber and combustion chamber phase. The pressure fluctuation magnitude was also higher in the pre-chamber than in the combustion chamber. The experimentally determined
unstable frequency presented itself again in the calculation of the plenum Helmholtz frequency, as shown in equation 2.2.

\[ f_H = \frac{c}{2\pi} \sqrt{\frac{A}{LV}} \]  

(2.2)

where \( A \) is the area of the burner, \( L \) its corrected length and \( V \) the volume of the plenum (the corrected length is defined as \( L = l + r \), where \( l \) is the actual length and \( r \) is the radius of the choked exit). The eigenmode and Helmholtz analysis resulted in a conjecture that the unstable frequency is the Helmholtz resonator frequency. It is dependent on the square root of pre-chamber temperature and it is independent of the equivalence ratio (and temperature) in the combustion chamber.

Cold flow experiments carried out by Schildmacher et al. [76] confirmed the existence of a PVC, which by coincidence had a frequency close to that of thermoacoustic instability obtained in the presence of combustion. The pressure measurements in the combustion chamber also revealed the presence of one dominant frequency, which excluded the existence of PVC and quarter-wave mode. Schildmacher et al. [76] therefore argued that the combustion oscillations were due to the phase lag between the pre-chamber and the combustion chamber which caused alternating patterns of fuel rich and fuel lean pockets within the flame zone. The equivalence ratio of 0.83 demonstrated high oscillation amplitudes, and phase-locked LDV and microphone data showed an interaction of pressure fluctuations and velocity field within the flame. For the high oscillation amplitudes: heat release oscillations reduce in the presence of a strongly disturbed flame, local density increases within the flame zone, axial velocity decreases within the flame zone causing a reduction in axial momentum flux, swirl number within the flame region increases and mixing in the shear layer deteriorates.

Low oscillation amplitudes were observed at an equivalence ratio of 0.71. In this case the phase lag across the burner was small and the velocity fluctuations had only a minor impact on the flame. The flame shape and position also remained invariant, since the flame was sufficiently stabilised by the swirl.
2.2 Non-Linearity Effects

A very important investigation into the non-linearity of self-excited flames was carried out by Dowling [16]. Although the author was mainly interested in the combustion oscillations of after-burners and aero-engines, the methodology and theoretical outcomes may be directly compared with the problem of non-linear thermoacoustic instabilities in industrial gas turbines. This analytical and experimental study focused on a confined bluff-body stabilised flame. Dowling [16] assumed that the acoustic waves remain linear due to small amplitude pressure fluctuations and the main cause of the non-linearity is in the heat release rate. Dowling [16] demonstrated that the mode shape and frequency of the non-linear oscillations was predicted reasonably well with linear theory, whereas non-linear theory predicted the amplitude and spectra of the limit cycle oscillations. The non-linear theory developed by Dowling [16] modified the low Mach number linear treatment by Bloxsidge et al. [7]. Stating that the relationship between the heat release rate and velocity at the flame holder must saturate as flow reversal takes place, Dowling [16] proposed that the ‘describing function analysis’ could be used to estimate the limit cycle amplitudes from the linear theory. In the ‘describing function analysis’ a complex transfer function is defined between the output and input which is dependent on both the frequency and amplitude imposed on the system under investigation. Dowling [16] implemented active control in order to reduce the self-excited oscillations of the ducted flame and it was demonstrated that any linear controller with appropriate gain and phase is capable of stabilising both the non-linear limit cycles and linear flow disturbances.

Lieuwen [55] explained the linear and non-linear processes involved in the unstable combustor with the aid of figure 2.3. In figure 2.3 the relationship between the magnitude of the driving $H(A)$ or damping $D(A)$ process determines the disturbance amplitude. At low amplitudes $A$, $H(A)$ and $D(A)$ processes remain linear whereas at high amplitudes of $A$, non-linear conditions take effect. Knowledge of non-linear characteristics of $H(A)$ and $D(A)$ are therefore crucial for prediction of limit cycle amplitude of self-excited oscillations $A_{LC}$. Lieuwen [55] argued that, although understanding of the linear acoustic characteristics of the
combustor $\varepsilon_d$ and linear response of the flame to the flow and mixture disturbances $\varepsilon_H$ has improved, the combustor's non-linear dynamics are not well developed. Therefore the fundamental source of non-linearity and its dependence upon combustor geometry and disturbance parameters (flame length or frequency) is still a challenge. The author therefore investigated the non-linear dynamics of laminar premixed flame to harmonic velocity disturbances in order to determine the effect of these disturbance parameters. The response of the flame to harmonically oscillating velocity disturbances was investigated by solving a constant flame speed front tracking equation. The work focused upon two flame geometries: a conical flame and an inverted conical flame stabilised on a bluff body. It was shown that FTF non-linearities became significant at given velocity amplitudes depending on the Strouhal number based on flame length, the ratio of the flame length to width and the flame shape in the absence of perturbations.

Lieuwen [55] demonstrated that non-linearity was enhanced by increasing the Strouhal number and suppressed at large ratios of flame length to width. The conical flames demonstrated more linear behaviour at comparable disturbance amplitudes than the inverted conical flames. The phase exhibited little amplitude dependence. The authors did not investigate the effect of turbulent flame wrinkling upon the saturation of the flame response.

The work of Lieuwen and Neumeier [58] on the subject of non-linearity was performed on a flame stabilised by a conical body with an equivalence ratio of 0.85, and a mean pressure of 1.3 atm. The results suggested a non-linear relationship between pressure and heat-release oscillations, as opposed to non-linear gas dynamic processes. Therefore, the non-linear relationship between the pressure and heat-release oscillations characterised by $\text{CH}^*$ photomultiplier tube measurement manifested itself as saturation in the amplitude of the self-excited oscillation. The investigation also revealed some non-linear link between the natural combustor mode and the frequencies, which were imposed with the aid of an actuator. The authors referred to the latter non-linear behaviour as ‘frequency locking’ of natural mode oscillations. The frequency locking caused a reduction in the amplitude of the natural mode as the amplitude
Lieuwen [54] analysed the statistical characteristics of pressure oscillations in a premixed combustor, described previously by Lieuwen and Neumeier [58], to investigate the random features encountered in the combustion oscillations which cannot be characterised within a deterministic frame-work. An example of this random feature is that of cycle-to-cycle variations in the amplitude and phase of limit cycle oscillations. Statistical characteristics are a much better way of describing the instabilities, rather than single deterministic quantities, due to the noisy environment of the combustor. Lieuwen [54] obtained good agreement between the probability density function (PDF) of the measured pressure for self-excited stable, linear unstable and non-linear unstable oscillations, and the analytically derived PDF of these conditions. This agreement supported previous studies [16, 52, 66] which suggested that a single oscillator that interacts non-linearly with itself can be used to describe the unstable gas turbine combustor dynamics. To further complete the work Lieuwen [54] carried out experiments on the effect of background noise induced by actuators on the self-excited combustion oscillations. The effect of background noise predicted by the analytical study is twofold:

1. Given enough cycles, after the initial phase drift of the instability, the phase reaches a uniform probability of having all values.

2. The background noise also reduces the amplitude of the mean instability due to non-linear interactions between the coherent self-excited oscillations and the random oscillations. A similar behaviour was described by frequency-locked, which is a known non-linear phenomena, and observed by Lieuwen and Neumeier [58].

Lieuwen [54] concluded that lack of repeatability of measured data from cycle to cycle is due to the stochastic nature of the problem and active control systems must account for this phenomena for a successful control of the oscillations.

Dowling [17] developed a kinematic flame model for the geometry studied in Dowling [16]. The author calculated the time evolution of the disturbances by noting the coupling between
the kinematic flame model and acoustic waves generated by unsteady combustion. As in the previous work Dowling [17] restated that the self-excited oscillations occurred above a critical equivalence ratio, and heat release rate was the main cause of non-linearity.

Stow and Dowling [85] presented a low order model for prediction of limit cycles in lean premixed combustors, since the available linear stability models can only predict the frequency of limit cycles, not the amplitude. The model relied on the assumption that the non-linearity in the combustion was due to flow perturbations. Therefore the main cause of the non-linearity was attributed to fuel / air fluctuations and convection of these fluctuations to the flame front as a result of velocity perturbations. Stow and Dowling [85] also assumed linear acoustic processes and occurrence of reverse flow due to small Mach number. The authors conclude that an improved non-linear flame model is required for better prediction.

Balachandran et al. [3] investigated the response of a lean premixed flame stabilised by bluff-body and swirler for three cases of no swirl, moderate swirl and high swirl. The applied experimental methods included photomultiplier tube measurements (PMT) of OH* and CH*, hotwire, two microphone method and phase-locked OH*-PLIF and OH*. The flame transfer function was obtained from the Fast Fourier Transform (FFT) of the PMT with respect to hotwire data. The phase-locked OH*-PLIF was used to determine the FSD for comparison with the phase-locked OH*. For case of no swirl, 160 Hz and acoustic velocity of 65%, the phase-locked OH* and FSD produced very comparable results. With the introduction of the moderate swirl, the phase-locked FSD exhibited a phase lag compare to the phase-locked OH* but the magnitude of the two remained the same. The phase-locked FSD images also depicted shear layer rollup as the main mechanism of instability. The shear layer consisted of a rotating vortex pair, the vortex continues to grow in size as it is convected downstream and disappears as a new vortex is formed at the base of the flame. Balachandran et al. [3] showed that saturation in amplitude could be avoided by increasing the swirl number and the phase variation remained independent of amplitude. The FTF, obtained from FSD and the FTF calculated by OH*, predicted the trends very accurately for experiments with no or low
2.2. Non-Linearity Effects

The difference between the FSD and OH$^*$ of the low swirl cases and the occurrence of the saturation, together with the formation and rolling of vortices, suggested flame surface modulation through flame annihilation events. Balachandran et al. [3] concluded that high swirl number flames have a more linear response compared to low swirl number.

Peraccchio and Proscia [66] analysed data obtained by Cohen et al. [13] on a single nozzle test rig (SNR) to derive a parametric model based on the coupling of the linear acoustics and non-linear heat release. The model incorporated the analytical expressions of acoustics, heat release, flame surface area dynamics and mixture strength. Each expression required a number of parameters which were selected on the basis of previous and similar work on the SNR. Once these values were chosen they were fixed for a given power setting. The time delay however remained dependent on equivalence ratio to account for the convection delay. The authors then linearised the parametric model in order to study the linear stability and determine the limit cycle amplitudes. The study of the linear system demonstrated the susceptibility of the model to instability as the variations in the parametric values took place. The linearised model also illustrated the effect of the parametric values on the limit cycle amplitude and frequency. The model successfully predicted the onset of the instability and the resulting limit cycle and demonstrated a gain in the instability amplitude of pressure and heat release, and a reduction in limit cycle frequency, as the mean fuel air ratio was decreased. Peracchio and Proscia [66] recognised the influence of the instabilities upon the velocity fluctuations that are responsible for variations in the flame front location / shape, and upon the local equivalence ratio of the premixed flow that enters the flame front. The magnitude of the acoustic velocity fluctuations were shown to be also greater than the pressure fluctuations due to the low Mach numbers involved in the gas turbine combustor, and hence the velocity fluctuations are the main cause of unsteadiness in the heat release rate. Peracchio and Proscia [66] also demonstrate that the limit cycle can exist even if there is no saturation between the heat release and velocity.

Matveev and Culick [62] investigated the effect of vortices on the flame stabilisation of premixed dump combustors with the aid of a reduced-order model that described the vor-
Vortex shedding, chamber acoustics and combustion process interactions. The advantage of the reduced-model is in its simple and inexpensive nature compared to the time consuming and expensive CFD or experimental analysis of the instabilities, which are usually only applicable to particular geometries. The authors attributed the instability to the coupling between the acoustic field and the periodic heat release which was due to the combustion in the shed vortices containing pockets of unmixed cold reactants and hot products. The effect of the vortices dominated even further in the form of pulse-like combustion if impingement between vortex and combustion chamber walls occurred. Under vortex shedding the combustion instability was affected by chamber acoustics, hydrodynamics (vortex shedding) and the reactant supply system.

The vortices were usually shed from the shear layer and transferred to the flame front after some induction time (time delay), which depended mainly on the moment of the vortex, but also on the characteristic combustion times. The resulting unsteady heat release excited the acoustic modes of the chamber which in turn influenced the frequency and intensity of the vortex shedding process, hence completing the feedback loop and destabilising the flame. Fluid mechanics (vortex shedding) and combustion were the main sources of non-linearity in the model, whereas the acoustics were treated linearly. Matveev and Culick [62] obtained good agreement between the frequencies and amplitudes of the reduced-model to that of experimental data obtained previously.

Bellows et al. [6] investigated the non-linearity of the flame response to forced acoustic oscillations using two-dimensional phase-locked OH*-PLIF imaging, in a lean premixed combustor utilising a swirler as a stabilisation method at a Reynolds number of 21000. The phase-locked OH*-PLIF images were computed several cycles apart and therefore not truly consecutive due to the 10 Hz operation frequency of the Nd:YAG laser. Therefore, the instantaneous images can not be used to infer global flow features, such as tracking the evolution of the flow element from image to image. The authors incorporated the two-microphone method for determination of velocity fluctuations and report a similar result obtained from the PMT-
2.2. Non-Linearity Effects

CH* and PMT-OH*. The linear and non-linear behaviour of the combustor could be broken down as:

- **Linear processes** occur at low amplitudes of oscillation by influencing the balance between the driving and damping processes and for a given frequency determine the growth rate of the disturbances.

- **Non-linear effects** dominate at high amplitudes by influencing the finite amplitude dynamic of the flame.

Bellows et al. [6] analysed two frequencies of forced-excitation for a range of imposed velocity amplitudes. For the frequency of 310 Hz the FTF remained linear up to imposed velocity fluctuations of 95% of mean before saturation, whereas for 410 Hz the saturation occurred at 20%, but after initial saturation the FTF showed a second linear gain before a second region of saturation at 60%. The phase of the FTF also demonstrated amplitude dependence. For the case of 310 Hz at the highest flow velocity fluctuation, the phase-locked OH*-PLIF images revealed flame rollup, forcing the flame into the IRZ causing a rapid flame area reduction and flame response saturation. In the case of 410 Hz, the low velocity amplitude demonstrated similar behaviour to that of low amplitude 310 Hz case. At high velocity amplitude variation, the flame anchoring point was cycle dependent resulting in flame area modulation and FTF saturation. Bellows et al. [6] essentially confirmed the experimental results of the non-swirling combustor obtained by Balachandran et al. [2]. The destructive effect of such vortices upon the flame surface area was also recognized by Balachandran et al. [2]. Bellows et al. [6] concluded that the non-linear flame response could be explained by two mechanisms:

1. **Vortex rollup** at large disturbance amplitudes causing a non-proportional relationship between the flame surface density and disturbance amplitude. The vortex rollup is therefore important because of instability initiation and instability saturation.

2. The dependence of the flame attachment point on the phase of the instability cycle. The
flame lift-off during part of the cycle played an important role in the flame surface area modulation.

2.3 Control Of Combustion Instability

Dowling [18] analysed the different methods available for control of combustion oscillations. Although the Rayleigh criterion explains the mathematical requirement for reduction of amplitude of combustion oscillations, the desired effect is much more difficult to achieve in practice. Methods of control of the combustion oscillations are divided into two groups of passive and active control by either increasing the energy loss at the boundaries by increasing $\int_S (p'u_j - u_jn_{ij})dS_j$ or by decreasing the source term $\int_V \tau_{ij} \frac{\partial u_i}{\partial x_j} dV$ in equation 1.15. Introduction of various time delays between the fuel distribution point and flame position is an example of a passive method that reduces velocity perturbations through modification of the source term. Use of Helmholtz resonators provides another method for reduction of combustion oscillations where the lessening in the kinetic energy is caused by a reduction of acoustic energy as it travelled through the neck of the resonator. Passive methods are however ineffective and sometimes not practical at low-instability frequencies, especially in the case of Helmholtz resonators where very large resonators are required, [18]. Active methods consist mainly of adaptive control, for example use of a loudspeaker to modify the feedback acoustic waves, with the aim of achieving stable combustion. Use of a loudspeaker is a thoroughly tested method, although it is difficult to apply at large scales. Another method of active control is that of modulating the fuel supply to stabilise the flame, as demonstrated by Langhorne et al. [41] with a premixed turbulent duct flame. The procedure presented by Langhorne et al. [41] required monitoring of the pressure perturbations in the combustion chamber. The amplified and time delayed perturbations were then used to modulate the fuel injectors, hence achieving after-burner operating conditions which were impossible to attain without control. The application of active control to the land based gas turbines is more complicated to that of
Langhorne et al. [41], since more sophisticated algorithms are required to ensure that stable combustion occurs throughout the operating range of the turbine. In the case of gas turbines, the main concern is the stability of the feedback control, since any instability of the feedback control enhances the uncontrolled combustion oscillations with damaging effect on the gas turbine.

Lepers et al. [51] carried out experimental and computational analysis of a full scale annular combustor in order to extend the operating range of the gas turbine by minimising the thermoacoustic instabilities with the aid of Helmholtz resonators. This is essentially a passive approach which introduces a damping effect on the combustor. The investigation focused on the interaction of multiple resonators attached to the full scale gas turbine combustors rather than analysis of individual resonator performance. Lepers et al. [51] showed that analysis of the turbine behaviour at low pressure was comparable to the conditions as the gas turbine was operated at high pressure. The computational analysis presented in this study employed one-dimensional acoustic transfer elements to obtain the acoustic transfer characteristics of a complete gas turbine combustor. The acoustic transfer of a given element such as ducts, diffusers, annular ducts and area discontinuities defines the relationship between in the inlet and outlet acoustic velocities and pressures. There are however some inaccuracies in calculated instability frequencies due to treatment of the three-dimensional acoustic waves as one-dimensional. The one-dimensional treatment of the gas turbine also limits the upper frequency which could be predicted by this method to that of equation 2.3 due to limit of plane wave propagation:

$$f_{\text{lim}} = \frac{c}{2W} \sqrt{1 - \frac{M}{2}}$$  \hspace{1cm} (2.3)

where $W$ is the maximum dimension of the respective combustor component in the transverse direction to that of the assumed one-dimensional sound wave propagation. Lepers et al. [51] concluded that the Helmholtz resonators may be used to increase the operating range of gas turbines and illustrated the effectiveness of acoustic transfer elements for predicting the instability frequencies, as long as the criteria in equation 2.3 is satisfied. Lepers et al.
[51] also observed that unstable combustion conditions produced a hysteresis cycle of pressure amplitudes with respect to equivalence ratio. The authors note that hysteresis phenomena stems from non-linear stability theory, however the authors did not investigate this effect further.

Stow and Dowling [86] presented an analytical study of the thermoacoustically self-excited oscillations of a lean premixed combustor in an annular geometry, using an acoustic network similar to that of Lepers et al. [51] to simulate appropriate elements of the turbine combustor. Stow and Dowling [86] suggested an arrangement for Helmholtz resonators to be added to the gas turbine in order to absorb acoustic energy and damp out self-excited oscillations. Stow and Dowling [86] concluded that the resonators should be placed evenly along half of the circumferential wave length in order to obtain the best damping effect. Good damping was also achieved if the resonators were rotated by multiplies of half a wavelength.

2.4 Summary

The work presented in this chapter cited the most important studies on the subject of linear and non-linear thermoacoustic instabilities in industrial gas turbines. Some inconsistency was noted between different authors and the suggested phenomena behind the encountered instability mechanisms. Janus et al. [30] for example reports an absence of vortex shedding whereas Stone and Menon [84] attributes the instability to interplay of $\phi$ and coherent structures. The wide range of instability mechanisms available and the different design parameters in each burner type complicates the problem even further. The difference in the combustion chamber design could start from something seemingly insignificant, such as the length of the combustor, to more significant variations, such as swirler design. It is conceivable to think that the overall design of the burner is as significant as other explained phenomena. A certain burner design could become unstable according to one, two or even a combination of processes that are explained in this chapter.
The number of studies that concentrate on non-linear flame response is limited and even fewer studies have tried to systematically investigate the effect of equivalence ratio and swirler geometry. Also the majority of the studies investigate only few frequencies and velocity fluctuations whereas a systematic investigation is much more preferable in regards to understanding the instability mechanisms that lead to non-linear flame response. The current study aims to fulfil these requirements.
### 2.5 Summary Table

<table>
<thead>
<tr>
<th>Ref</th>
<th>Authors</th>
<th>Stabilisation</th>
<th>Operating data</th>
<th>Excitation</th>
<th>Non-linear</th>
<th>Frequency</th>
<th>Diagnostic</th>
</tr>
</thead>
<tbody>
<tr>
<td>[58]</td>
<td>Lieuwen &amp; Neumeier (2002)</td>
<td>Conical flame holder</td>
<td>$\phi = 0.85$ &amp; $P = 1.3$ atm</td>
<td>Forced (actuator) &amp; self</td>
<td>Yes</td>
<td>157, 190, 235 &amp; 167 Hz</td>
<td>PMT-CH* &amp; Mic</td>
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<tr>
<td>[9]</td>
<td>Cabot et al. (2003)</td>
<td>Swirler &amp; bluff-body $S = 0.7$</td>
<td>$\phi = 0.9$ to $0.57$ &amp; $P = 1$ atm</td>
<td>Self</td>
<td>No</td>
<td>16 Hz</td>
<td>LDV, PL-CH* &amp; PL-Mic</td>
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<td>[84]</td>
<td>Stone &amp; Menon (2002)</td>
<td>$S = 0.56, 0.84$ &amp; 1.12</td>
<td>$\phi = 0.9, 0.72, 1.12$ &amp; $P = 1$ atm</td>
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<td>LES</td>
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<td>[77]</td>
<td>Schildmacher &amp; Koch (2005)</td>
<td>Concentric annular swirlers</td>
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<td>254 Hz at $\phi = 0.83$</td>
<td>PL-LDV &amp; Mic</td>
</tr>
<tr>
<td>[40]</td>
<td>Kulsheimer &amp; Buchner (2002)</td>
<td>Swirler $S = 0.21, 0.45$ &amp; 0.79</td>
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<td>PMT-OH*, Ht &amp; PL-OH*</td>
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<td>[59]</td>
<td>Lohrmann et al. (2003)</td>
<td>Swirler &amp; pilot</td>
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<td>Variable</td>
<td>PMT-OH* &amp; Ht</td>
</tr>
<tr>
<td>Ref</td>
<td>Authors</td>
<td>Stabilisation</td>
<td>Operating data</td>
<td>Excitation</td>
<td>Non-linear</td>
<td>Frequency</td>
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<tr>
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<td>Lawn (2000)</td>
<td>Axial swirler, unswirled premixture &amp; pilot</td>
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<td>204, 182, 172, 159 &amp; 156 Hz</td>
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<td>[26]</td>
<td>Huang et al. (2006)</td>
<td>Swirler</td>
<td>$\phi = 0.5, 0.6, 0.75 &amp; P = 1$ atm</td>
<td>Forced (LS)</td>
<td>No</td>
<td>13, 37, 55, 65, 85, 116, 130 &amp; 208 Hz</td>
<td>OH*-PLIF</td>
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<td>[89]</td>
<td>Weigand et al. (2006)</td>
<td>Swirling CH$_4$ / air diffusion</td>
<td>$S = 0.9, 0.55$ &amp; $P = 1$ atm</td>
<td>Self</td>
<td>No</td>
<td>380 &amp; 290 Hz</td>
<td>LDV, Mic, OH*-PLIF, CH*-PLIF &amp; Raman</td>
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<td>[20]</td>
<td>Durox et al. (2005)</td>
<td>Laminar flame</td>
<td>$\phi = 0.92$ &amp; $P = 1$ atm</td>
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<td>Mic, LDV, PIV &amp; PMT-CH*</td>
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<td>Hedman et al. (2005)</td>
<td>Swirler</td>
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<tr>
<td>[28]</td>
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<td>2-PMT-OH* &amp; Mic</td>
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<tr>
<td>[51]</td>
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<td>Full scale annular gas turbine &amp; pilot</td>
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<td>Yes</td>
<td>206 Hz</td>
<td>Th &amp; Mic</td>
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<td>[54]</td>
<td>Lieuwen (2003)</td>
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<td>Mic</td>
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<tr>
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<td>Lieuwen (2005)</td>
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<td>Yes</td>
<td>N/A</td>
<td>Analytical study</td>
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<td>[76]</td>
<td>Schildmacher et al. (2006)</td>
<td>Concentric annular swirlers</td>
<td>$\phi = 0.83$, 0.71 &amp; $P = 1$ atm</td>
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<td>No</td>
<td>254 Hz</td>
<td>PL-LDV, OH$^*$.PLIF, Ht, eigenmode analysis &amp; Mic</td>
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<tr>
<td>[42]</td>
<td>Laverdant &amp; Thevenin (2003)</td>
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<td>N/A</td>
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<tr>
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<td>Bluff-body</td>
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<td>[17]</td>
<td>Dowling (1999)</td>
<td>Bluff-body</td>
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<td>N/A</td>
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<tr>
<td>[73]</td>
<td>Roux et al. (2005)</td>
<td>Radial swirler &amp; bluff-body</td>
<td>( \phi = 0.75 )</td>
<td>Self</td>
<td>No</td>
<td>300 &amp; 570 Hz</td>
<td>LES, Mic &amp; LDV</td>
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<tr>
<td>[78]</td>
<td>Selle et al. (2004)</td>
<td>Inner &amp; outer swirler</td>
<td>( \phi = 0.5 )</td>
<td>Self</td>
<td>No</td>
<td>N/A</td>
<td>LES, Th &amp; LDV</td>
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<tr>
<td>[66]</td>
<td>Peracchio &amp; Proscia (1999)</td>
<td>Swirler</td>
<td>( \phi = 0.45 \text{ to } \phi = 0.56 )</td>
<td>Self</td>
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<td>N/A</td>
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<tr>
<td>[86]</td>
<td>Stow &amp; Dowling (2003)</td>
<td>Annular geometry (no burners)</td>
<td>N/A</td>
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<td>No</td>
<td>N/A</td>
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<tr>
<td>[85]</td>
<td>Stow &amp; Doelning (2004)</td>
<td>Swirler</td>
<td>( \phi = 0.7 \text{ &amp; } 0.75 )</td>
<td>Self</td>
<td>Yes</td>
<td>222, 239 &amp; 269 Hz</td>
<td>Analytical study</td>
</tr>
<tr>
<td>[47]</td>
<td>Lawn (2000)</td>
<td>Axial swirler, unswirled premixture &amp; pilot</td>
<td>( \phi \text{ variable} ) ( \text{C}_3\text{H}_8, \text{CH}_4 ) &amp; ( P = 1 \text{ atm} )</td>
<td>Self</td>
<td>No</td>
<td>Variable</td>
<td>PMT-OH*, Mic &amp; Analytical study</td>
</tr>
<tr>
<td>[31]</td>
<td>Ji &amp; Gore (2002)</td>
<td>Swirler ( S = 2.4 )</td>
<td>( \phi = 0.94 ) &amp; ( P = 1 \text{ atm} )</td>
<td>N/A</td>
<td>No</td>
<td>N/A</td>
<td>PIV</td>
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<td>[6]</td>
<td>Bellows et al. (2006)</td>
<td>Swirler</td>
<td>( \phi = 0.8 ) &amp; ( P = 1 \text{ atm} )</td>
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<td>Yes</td>
<td>130 &amp; 410 Hz</td>
<td>PL-OH*-PLIF, 2-Mic, Ht, PMT-CH* &amp; PMT-OH*</td>
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<td>Authors</td>
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<td>Excitation</td>
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<td>[73]</td>
<td>Lawn &amp; Savarianandam (2006)</td>
<td>Conical</td>
<td>C&lt;sub&gt;2&lt;/sub&gt;H&lt;sub&gt;4&lt;/sub&gt;, CH&lt;sub&gt;4&lt;/sub&gt; &amp; P = 1 atm</td>
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<td>No</td>
<td>N/A</td>
<td>PMT-OH* &amp; Ht</td>
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</table>

Table 2.2: Summary of the most important and relevant literature on the mechanisms of instability in lean premixed combustors. \( \phi \) stands for equivalence ratio, photomultiplier tube is abbreviated by PMT, measurement of the pressure is shown with Mic, Ht stands for hotwire anemometry and PL stands for phase-locked measurements. The PLIF stands for Planar Laser Induced Fluorescence, PIV is for Particle Image Velocimetry, LDV is for Laser Doppler Velocimetry, TH is for thermocouple and Raman denotes the CARS technique. The Large Eddy Simulated is abbreviated by LES and Direct Numerical Simulation by DNS.
Figure 2.1: Overview of acoustic interactions in the flame, [49].
2.6. Figures

Chapter 2. Literature Review

Figure 2.2: Flow patterns in confined, premixed, swirl-stabilised combustor, [23]. Central and side recirculation zones are also known as inner and outer recirculation zone respectively.

Figure 2.3: Qualitative description of the dependence of acoustic driving, $H(A)$, and damping, $D(A)$, processes upon amplitude, $A$, [55].
Chapter 3

Experimental Methods
3.1 Experimental Arrangement

3.1.1 Burner Geometry

Two burner geometries were used in the current study. The first burner geometry consisted of a sixteen bladed axial swirler as depicted in figure 3.1. The swirler is a one twelfth linear scale model of the Siemens V94.2 burner and has been used in previous studies by [25, 44-49]. The axial burner configuration is illustrated in figure 3.2 and the relevant dimensions are shown in figure 3.3. The conical converging section was separated from the connection flange by a ceramic ring, to avoid the conduction of heat from the flame through the conical body to the connection flange. This was important to prevent damage to the sensors located in the connection flange section. The fuel was delivered to the fuel chamber via a long tube with an internal diameter of 1 mm and was distributed just upstream of the swirler through a tree of 8 distribution pipes, each having 5 holes of approximately 0.5 mm in diameter. The top of the swirler also had a fuel distribution hole with diameter of 1.5 mm, which supplied pure fuel for the pilot flame. This hole could be reduced in size or completely blocked off by using appropriate plugs. The pilot flame could therefore be adjusted between 0%, 5% and 10% of the total fuel supply, as estimated from the area, [25]. No pilot flames were investigated in this research. The fuel supply system was stiff (e.g. fuel flow was close to invariant) since the pressure in the fuel distribution plenum remained in-phase with acoustic fluctuations outside it up to frequency of 200 Hz, [47]. The axial swirler was positioned in the conical converging section so that fuel and air premixture passed through the swirler vanes before entering the combustion chamber, unlike some of the pervious studies [48, 49], where the swirler was withdrawn from the conical converging section to reduce the resultant swirl number. The swirl number of the axial swirler was approximately 0.5 as estimated from the angle of the vanes. The combustion chamber was fabricated from quartz to give good UV transmission. The quartz tube had a diameter of 51 mm and was two diameters in length to avoid resonant conditions at the frequencies of interest.
The second burner arrangement in-cooperated the radial swirler as the method of flame stabilisation illustrated in figure 3.4, it is a one quarter linear scale model of a Siemens SGT-100/200 burner. The radial burner configuration is illustrated in figure 3.5, while the schematic in figure 3.6 shows the relevant dimensions. It consisted of 12 triangular vanes, each with one fuel injection port with a diameter of 0.1 mm. The small diameter of the fuel injection ports ensured that the fuel flow into the combustion chamber was choked; hence the fluctuations in the combustion chamber did not affect the fuel supply. The fuel delivery line was the same as the one used in-conjunction with the axial swirler. The air entered through the vanes of the swirler, achieving a swirl number of approximately 0.8 as estimated from the angle of the vanes.

The air was provided from the laboratory compressed air system through a pipe with diameter of 5 mm and the fuel was from a compressed BOC methane cylinder. The volume flow rate of fuel and air were controlled by appropriate rotameters, each having an accuracy of ±1.25%. The pressure elevations in the delivery lines were recorded and subsequent corrections were applied to the rotameters. Hence the volume flow rate $\dot{Q}_{co}$ corrected to the ambient conditions of the test section, was determined for air and fuel flow using the equation 3.1.

$$\dot{Q}_{co} = \frac{p_{rot}}{p_{amb}} \sqrt{\frac{T_{rot}}{T_{ref}} \frac{T_{amb}}{T_{rot}}} \dot{Q}_{unco}$$

(3.1)

where $p$ denotes pressure, $T$ denotes temperature. The subscript $amb$ stands for ambient conditions, $ref$ denotes the reference calibration conditions and $rot$ is for the rotameter conditions. The $\dot{Q}_{co}$ was kept constant at 71 l/min for each burner throughout the course of this work.

The flame stabilised by the axial swirler remained within the combustion chamber at all times even in the presence of the forced-excitation and took the shape of an inverted cone with half angle of 34 degrees at equivalence ratio of 0.56 when no external or self-excitation was present, as shown in figure 3.7. The velocity at exit was 4 m/s, yielding a Reynolds number
3.1. Experimental Arrangement

of 5067. The equivalence ratio for quasi-steady blow-off limit of the axial swirler was 0.33 at air flow rate of 71 l/min.

The flame stabilised by the radial swirler resided in the combustion chamber even in the presence of the external excitation. The radial flame is shown in figure 3.8. The appearance of the flame differed from the axial swirler case in terms of flame root, which was much broader and almost hollow in appearance. The flame root extended all the way to the face of the swirler and remained attached to it throughout the steady state, forced or self-excitations experiments. The flame root which was 31 mm long remained hidden from the view. The exit velocity was 2 m/s, yielding a Reynolds number of 3467. The equivalence ratio for quasi-steady blow-off limit of the radial swirler was 0.31 at air flow rate of 71 l/min.

3.1.2 Forced-Excitation Geometry

The forced-excitation geometry is depicted in figure 3.9. The constricted air inlet created high acoustic impedance and blocked disturbances from the air supply system, ensuring an almost constant flow. Two commercial 10 cm dual core Pioneer loudspeakers each with maximum power of 100 W, nominal power of 20 W, frequency response of 0.01 to 10 kHz and 4 Ω impedance were mounted flush to the sides of an MDF cube (‘excitation box’) with dimensions of 186 × 151 × 181 mm. The loudspeakers were not enclosed on the outside. Enclosing the loudspeakers with an incorrect volume hindered the movement of the loudspeaker cone and hence reduced the imposed velocity fluctuation on the air flow, as determined with measurement of resultant acoustic velocities for a variety of enclosure volumes. The loudspeakers were driven by a LDS PA25E power amplifier with a gain setting of 1.0 for the axial swirler and 1.5 for the radial swirler. The equivalence ratios and frequencies examined are shown in table 3.1.
3.1. Experimental Arrangement

Chapter 3. Experimental Methods

<table>
<thead>
<tr>
<th>Excitation method</th>
<th>Forced</th>
<th>Radial</th>
</tr>
</thead>
<tbody>
<tr>
<td>Swirl geometry</td>
<td>Axial</td>
<td>Radial</td>
</tr>
<tr>
<td>Equivalence ratio / no units</td>
<td>0.56 0.48 0.56 0.91</td>
<td>0.56 0.58 0.58 0.91</td>
</tr>
<tr>
<td>Combustion chamber / mm</td>
<td>100 100 100 100</td>
<td>100 100 100 100</td>
</tr>
<tr>
<td>Frequency / Hz</td>
<td>⋆ ⋆ 144 200</td>
<td>⋆ ⋆ 120 144</td>
</tr>
</tbody>
</table>

Table 3.1: Examined equivalence ratios for axial and radial burner geometries under forced-excitation. Symbol ⋆ includes frequencies of 40 to 200 Hz in steps of 20 Hz. The velocity fluctuations for each frequency were increased stepwise until flame blow-off occurred irrespective of burner type and equivalence ratio.

3.1.3 Self-Excitation Geometry

In order to remove the impact of the loudspeakers on the volume of air in the forced-excitation geometry and hence on the self-excited flame, the upstream box illustrated in figure 3.9 was replaced with the one depicted in figure 3.10 for both burner geometries in order to give a similar setup to that used by Lawn [46]. The quartz combustion chamber was also replaced with the geometry shown in figure 3.11. The self-excitation combustion chamber consisted of a 100 mm long quartz tube with inner diameter of 51 mm sealed to a metal tube with the same inner diameter. The length of the tube and the equivalence ratio were varied to obtain a high amplitude self-excited flame without blow-off. A microphone connected to the combustion chamber acquired the pressure fluctuations and spectral analysis was used for calculation of the self-excitation frequency and amplitude. The total length of the combustion chamber, the resultant dominant self-excited frequency for each burner type is shown in table 3.2.

<table>
<thead>
<tr>
<th>Excitation method</th>
<th>Self</th>
<th>Axial</th>
<th>Radial</th>
</tr>
</thead>
<tbody>
<tr>
<td>Swirl geometry</td>
<td>Self</td>
<td>Axial</td>
<td>Radial</td>
</tr>
<tr>
<td>Equivalence ratio / no units</td>
<td>0.56 0.91</td>
<td>0.58 0.91</td>
<td></td>
</tr>
<tr>
<td>Combustion chamber / mm</td>
<td>1055 1055</td>
<td>1055 1055</td>
<td></td>
</tr>
<tr>
<td>Frequency / Hz</td>
<td>144 200</td>
<td>120 144</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.2: Examined equivalence ratios for axial and radial burner geometries and the resulting self-excited frequency.

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Early investigations with the axial swirler revealed very strong self-excitation amplitudes resulting in flame blow-off or continuous blow-off or re-igniting similar to that of forced-excitation under very large amplitudes of velocity fluctuations. The large amplitude self-excited oscillations persisted as the equivalence ratio was increased from 0.56 onwards irrespective of the tube length used. A mesh plate with a large loss coefficient of 1643 at the flow rate of interest was therefore placed between the geometries illustrated in figure 3.10 and the axial burner geometry depicted in figure 3.2 in order to damp the high amplitude oscillations and to obtain a humming flame.

The flame of the radial swirler was however less prone to self-excitation and hence no violent blow-off occurred. The high impedance mesh was therefore not used since it hindered the development of humming flame.

3.2 Data Acquisition

3.2.1 Acoustic Velocity

The constant-temperature anemometer was used to determine fluctuations in the flow velocity. This is done by a probe consisting of a short length of a fine metal wire that measures the flow velocity by the heat convected away by the flow. It is therefore important to ensure that the ambient temperature at hotwire location remains constant. This was done by prohibiting convection of heat from the burner top to the hotwire location with the aid of a ceramic ring shown in figures 3.2 and 3.5 for axial and radial swirl geometries respectively. The probe was placed 65 mm (see figure 3.2) and 75 mm (see figure 3.5) upstream of the burner plate for the axial and radial swirler respectively to avoid direct radiation from the hot zone of the flame.
3.2. Data Acquisition

In constant-temperature mode of operation a Dantec type 55P11 probe, consisting of a tungsten wire with a diameter of 5 µm and a length of 1.2 mm, was kept at a pre-selected excess temperature by means of the Wheatstone balancing bridge in-cooperated in the 55M10 Dantec control module. The excess temperature is determined by setting the hot sensor resistance on the 55M10 module. This resistance is set to give an overheat ratio \(a\) and it is defined by equation 3.2:

\[
R = R_0 (1 + a)
\]

where \(R\) is the sensor resistance when hot and \(R_0\) is the sensor resistance at ambient temperature. An overheat ratio of 0.6 was chosen for the present studies. The sensitivity of the wire increases with the overheat ratio, but at a cost of reducing probe life. The frequency response of the bridge was obtained by subjecting the hotwire to the highest possible flow occurring during the measurements and imposing a square wave signal onto the balancing bridge. Under these conditions the bridge was forced into optimum alignment by adjusting the gain, high frequency filter and cable compensation values. The optimum alignment is defined as the instance where the imposed square wave produces an oscilloscope pattern showing the shortest possible impulse response without superimposed oscillation. The frequency response of the hotwire \(f_c\) was calculated to be 5 kHz using equation 3.3, [32].

\[
f_c = \frac{1}{1.3\Delta t}
\]

where \(\Delta t\) is the time taken for the signal to return to 3% of its maximum value with an undershoot of 15%. The hotwire calibration took place inside the connection flange. The hotwire tip was perpendicular to the flow and placed halfway between the wall of the connection flange and the fuel delivery line. The mean velocity across the connection flange was calculated using mass flow continuity assuming a fully developed flow, taking into account the relevant pressure drops occurring in the air delivery lines from the rotameters, as described
3.2. Data Acquisition

by equation 3.1. The calibration curve is shown in figure 3.12. A third-order polynomial was fitted to the data with an uncertainty of approximately ±5% in the gradient of the hotwire calibration curve.

**Loudspeaker Calibration**

The hotwire was also used to calibrate the loudspeakers. Due to the non-linear response of the loudspeakers, a sinusoid of given amplitude but different frequency results in a dissimilar imposed velocity fluctuation on the air flow. As illustrated in figure 3.13 for the axial swirler with an equivalence ratio of 0.56 at 200 Hz, a sine wave with a peak to peak amplitude of 9 V was required to obtain a velocity fluctuation of 30%, whereas at 80 Hz, this acoustic velocity fluctuation is obtained with a sine wave with a peak to peak amplitude of 3.5 V. The loudspeaker calibration was repeated for axial and radial burners separately; since the process was dependent on the output impedance of the burners. The axial flame response to the imposed velocity fluctuations irrespective of the equivalence ratio was divided into three distinct regions of:

- Fluctuating flame with no blow-off, lift-off or re-ignition
- Continuous blow-off and re-ignition of the flame
- Sudden extinction of the flame in form of blow-off

Figure 3.13 illustrates the velocity fluctuation up to the point of blow-off and re-ignition for axial swirl burner, with equivalence ratio of 0.56. All the forced-excitation experiments for the axial swirler were carried out up to the point of blow-off and re-ignition. The radial swirler only demonstrated sudden extinction and no region of blow-off and re-ignition could be defined.
3.2.2 Acoustic Pressure

A B&K microphone type 2615 with sensitivity of 9.5816 mV/Pa was used for the purpose of the acoustic pressure measurements upstream and downstream of the radial or axial burner outlets. The two-microphone method was also implemented to confirm the velocity fluctuations and aid with the acoustical analysis of the burner rigs using acoustic network methodology explained in appendix C. The two-microphone technique was similar to that described by Seybert and Ross [79] and Balachandran [1], the main difference in the current implementation of the two-microphone method is that of referencing the microphone to the sine wave of excitation rather than each other since only a single B&K microphone was available.

Velocity Fluctuation Measurements

Application of the two-microphone method for calculation of velocity fluctuations required a mesh plate (not shown) to be placed between the axial burner configuration depicted in figure 3.2 and the excitation geometry illustrated in figure 3.9, bearing in mind that combustion was not present and the flow was maintained at 71 l/min. The microphone tapping of the axial burner and excitation geometry is located 60 and 150 mm upstream of the axial burner plate respectively.

The acoustic velocity was calculated by rearranging the first part of the transfer matrix in equation C.1 (see appendix C), as shown in equation 3.4.

\[ u'_{in} = \frac{p'_out - p'_in}{\rho c (K_{mesh} M + ikl_{effmesh})} \]  

(3.4)

where \(u'_{in}\) is the fluctuating velocity upstream of mesh plate, \(K_{mesh}\) is the loss coefficient of the mesh, \(l_{effmesh}\) is the effective length and \(p'_in\) and \(p'_out\) are the fluctuating pressures upstream and downstream of the mesh plate respectively.

The effective length in this case was the distance between the location of the microphone
tappings, it was constant and equal to 90 mm. The loss coefficient across the mesh plate was calculated using equation 3.5.

\[ K_{\text{mesh}} = \frac{\Delta P}{0.5\rho \bar{u}^2} \]  

where $\Delta P$ is the steady pressure drop across the mesh plate, measured with Bexhill micromanometer type FC012 for a range of volumetric flow rates. The mean velocity in the connection flange is denoted by $\bar{u}$ and was calculated from the overall mass flow rate. The ambient air density is represented by $\rho$. The loss coefficient demonstrated a linear reduction as the volumetric flow rate increased. The loss coefficient at the flow rate of interest (71 l/min) was constant and equal to 9.

By realising that the signals from the microphone were referenced to the excitation signal and rewriting equation 3.4 to account for this, equation 3.6 was obtained.

\[
\left| \frac{u_{\text{in}}'}{S} \right| e^{i(\theta_{\text{in}} - \theta_S)} = \frac{\left| p_{\text{out}}'/S \right| e^{i(\theta_{\text{out}} - \theta_S)} - \left| p_{\text{in}}'/S \right| e^{i(\theta_{\text{in}} - \theta_S)}}{\rho c (K_{\text{mesh}} M + ikl_{\text{effmesh}})}
\]  

(3.6)

where $\left| \frac{X}{S} \right|$ and $\theta_X - \theta_S$ represent the frequency response magnitude and phase of $X$ referenced to $S$. The symbol $X$ could be $p_{\text{out}}$, $p_{\text{in}}$, or $u_{\text{in}}$, representing the condition where the signal from the sensor designated here as $X$ is referenced to the sine wave $S$ that drives the loudspeakers.

The magnitude and phase of the recorded velocity fluctuations were then compared to those obtained using the hotwire method. Figure 3.14 and 3.15 illustrates the magnitude and phase of the acoustic velocity fluctuation from the hotwire and the two-microphone method as function of imposed loudspeaker amplitude and frequency for the axial burner configuration without combustion.

The two-microphone method overestimates the magnitude of acoustic velocity when compared to the hotwire anemometry and the error in calculation increases to 8% as the imposed velocity fluctuation increases (see top graph of figure 3.14 for 80 Hz), however as the imposed frequency increases the error reduces to 2% irrespective of the acoustic velocity magnitude (see bottom graph, figure 3.14 for 200 Hz). The error between the phases of the two methods
remained constant and equal to 20% irrespective of frequency or amplitude of the acoustic velocity as illustrated in figure 3.15. This error in the magnitude and phase is the result of variations in $K$ measured under the steady and fluctuating flow fields, [39].

**Burner Acoustical Properties**

The two-microphone method together with hotwire anemometry was also applied to the axial and radial configurations in order to obtain the acoustic properties of the burners such as the loss coefficient and the effective length.

The hotwire was placed at location designated as ‘hotwire tapping’ for the burner geometry under investigation. In the first set of measurements a B&K microphone was placed at location ‘microphone tapping 1’ and the data from hotwire and microphone were collected. In second set of experiments the microphone was placed at ‘microphone tapping 2’ to record the pressure fluctuations in the combustion chamber (see figures 3.2 and 3.5).

Equation 3.6 was then rearranged to the form of equation 3.7 for the calculation of unknown parameters ($K_{burner}$ and $l_{effburner}$) for the flow rate of 71 l/min and equivalence ratios mentioned in the forced-excitation section of table 3.1 up to the blow-off re-ignition acoustic velocity for axial swirler and up to blow-off acoustic velocity for radial swirler.

$$K_{burner}M + ik_{effburner} = \frac{1}{\rho c} \left[ \frac{P_{out}}{S} e^{i(\theta_{out} - \theta_S)} - \frac{P_{in}}{S} e^{i(\theta_{in} - \theta_S)} \right] \left( \frac{u''_{in}}{S} e^{i(\theta_{uin} - \theta_S)} \right)$$

(3.7)

The graphs of $K_{burner}$ and $l_{effburner}$ for 80 Hz as functions of acoustic velocity and swirler type are shown in figure 3.16 and figure 3.17. The average effective length for the axial swirler was 0.17 m irrespective of the applied acoustic velocity, frequency or equivalence ratio with maximum error of 10%. The average loss coefficient of the axial swirler remained at approximately 160 irrespective of the acoustic velocity, frequency or equivalence ratio with maximum error of 15%. The average loss coefficient predicted by the two-microphone was similar to the one predicted by equation 3.5, since the total flow through the swirler was kept
constant. The average effective length for the radial swirler was 0.20 m irrespective of the applied acoustic velocity, frequency or equivalence ratio with maximum error of 5% and the averaged loss coefficient predicted by the two-microphone was 290 with maximum error of 10% similar to the one predicted by equation 3.5.

3.2.3 Photodiode Sensor

The photodiode sensor and the hotwire were used for calculation of the non-normalised flame transfer function magnitude and phase (designated here as the photodiode FTF magnitude and phase, see equation 1.16) over a wide range of imposed fluctuations and frequencies at a high sampling frequency. The photodiode sensor is similar to the one used in previous studies by Lawn [45]. However, a new amplifier was incorporated to improve the signal-to-noise ratio of the sensor. The RS-564-021 UV enhanced photodiode, has a responsivity of 0.15 Amp/Watt of incident power at 307 nm, a polar sensitivity of $18^\circ$ from the normal of approximately 90% and an area of $2.4 \text{ mm}^2$. The sensor was placed behind an Ealing Electro Optics 35-7939 filter, centred on 307.1 ± 2 nm with a bandwidth of 10 nm. The photodiode was fixed at the focal point of a 51 mm diameter quartz lens (45 mm aperture when mounted) in a telescopic arrangement. The distance from the photodiode to the burner centre was set to 200 mm so that all the flame emission was concentrated on the photodiode. The signal from the photodiode was taken to represent the fluctuating heat release.

3.2.4 Free-running Data Acquisition

Laboratory Virtual Instrumentation Engineering Workbench (LabView) is used mainly for data acquisition by controlling the analogue to digital data acquisition (DAQ) card. LabView is also capable of signal generation, signal conditioning, statistic analysis, etc, through a wide variety of supplied program libraries.

A National Instrument multipurpose (M series, NI PCI 6251) DAQ card together with
LabView 7.1.1 software was used to record data from a variety of sensors and generate analogue and digital signals. The card has 16 analogue inputs, 24 digital input/outputs and 2 analogue outputs. The connection between the BNC cables and the DAQ was managed by a BNC-2110 noise rejection shielded box and SHC68-68-EPM shielded cable manufactured by National Instruments.

In the free-running mode, the master program generated a sine wave of a known frequency and amplitude as specified in table 3.1. The program applied the hotwire and microphone calibration to the raw data before online spectrum analysis. The complete set of raw data, power spectra, frequency response magnitude and frequency response phase of the photodiode with respect to reference sensor (hotwire) were recorded for each frequency and amplitude combination experiment. It was ensured that the LabView spectrum analyser utilised the same techniques used by the Hewlett-Packard 35670A spectrum analyser. Therefore comparison of the data obtained in the current work with the past experiments was valid. To this end, the resultant output of the two analysers was further examined by feeding a sine wave of known frequency and amplitude to both systems and comparing the relevant power spectra. The error observed between the two systems was less than ±1%.

This program was also capable of re-analysing the recorded raw data to obtain coherence or to re-calculate the magnitude or phase of the frequency response sensors with respect to each other. The unit of the power spectra in this work is in terms of root-mean-square (r.m.s) unless otherwise stated. It is common practice to express power spectra as Power Spectral Density (PSD). The power spectral density is obtained by dividing the mean square of the signal in the frequency band by the bandwidth, which was set to 3.77 Hz. In the current study all data with coherence of 0.7 or less was ignored.

The sampling frequency was set to 2048 Hz. According to the Nyquist criterion the chosen sampling frequency should be at least twice as large as the maximum expected value in order to minimise distortion when analogue to digital conversion is applied. In total 20 batches of raw data were obtained for each spectrum analysis.
3.3 Imaging Systems

Two non-intrusive imaging techniques were applied to further aid the investigation of the mechanisms responsible for flame instability. These methods were OH* chemiluminescence imaging with the aid of an phase-locked Intensified Charge Couple Device (ICCD) and flow field visualisation using phase-locked Particle Image Velocimetry (PIV). The connection schematic of the ICCD, PIV and sensors are shown in figure 3.18.

3.3.1 ICCD System

The ICCD system was controlled via DaVis 7.0.11 software provided by the manufacturer. The schematic of the ICCD is depicted in figure 3.19. A LaVision IRO together with a LaVision low speed (5 frames per second with 1024 × 1024 pixel area) CCD camera were used as a means of intensifying and recording the light emissions from the flame. A narrow band filter mounted on the intensifier centred at 305 ± 2.5 nm reduced emissions to the range usually associated with the OH* chemiluminescence at 307 nm. The IRO is an electro-optical device placed in front of the CCD digital array, it consisted of relay-optics, image intensifier and ‘external control unit’ for setting variable exposure time and intensifier gain respectively. The principal of the ICCD operation is as follows: the light is focused on the photo-cathode using the objective lens. The photo-cathode converts light into electrons which pass through micro-channel plate (MCP) and are amplified in numbers. The electrons are then converted back to light by the phosphor surface and are focused on the CCD camera using the coupling optics. The exposure time of the intensifier (also known as ‘gating’) is controlled through the ‘image intensifier high voltage pulser’ by the external control unit (see figures 3.19I and 3.19J respectively). The external control unit offered an extremely variable exposure time for the intensifier gating in the range of nanoseconds, whereas the exposure time of the CCD camera, controlled by the DaVis software, was in the range of milliseconds. The gain in the number of electrons is controlled through ‘image intensifier high voltage MCP’ by the external control
3.3. Imaging Systems

unit (see figures 3.19H and 3.19J respectively). The intensifier gain settings ranged from 1 to 10, which corresponded linearly to 0 to 900 volts applied to the MCP. Depending on the amount of flame emission a high setting could damage the CCD camera and a low setting could result in an almost blank image. The setting of 8.00 was chosen in the current study in order to get the best detailed image without damaging the CCD camera. The intrinsic delay of the LaVision system is 90 ns according to the manufacturer.

### 3.3.2 PIV System

The schematic of a PIV system\(^1\) is shown in figure 3.20. Two consecutive laser pulses illuminated the region of interest in a seeded flow and the PIV-CCD camera captured the illuminated particles from each pulse in two separate frames. In the post processing stage, each frame was divided into subsections known as interrogation area. The common particle displacement and hence the velocity in the interrogation area is calculated through pixel by pixel cross-correlation of the interrogation areas from two frames. The complete analysis of the interrogation areas yields the velocity vector map over the whole target area.

The supplied Insight 3G 8.0.3 software was used to control the entire PIV system. The PIV system consisted of a Big Sky twin pulse Nd:Yag laser, with a pulse energy of 120 mJ and pulse duration of 10 ns. A light guiding arm finished with a -15 cylindrical lens and a 500 mm spherical lens converted the beam into a 1 mm thick laser sheet and was positioned to direct the sheet across the centre of the burner. The laser flash lamp responsible for generation of the laser pulse operated at frequency of 7.5 Hz. The camera used in this investigation had a resolution of four million pixels, arranged in a \(2048 \times 2048\) sensor array, where each pixel was 7.4 \(\mu\text{m}^2\). For the two-dimensional PIV measurements, one camera was positioned in order to achieve a 50 by 60 mm field of view. The light sheet and the camera were perpendicular with respect to each other throughout the course of this investigation. For the purpose of the PIV measurements, the air was seeded from two solid-particle units (type 3400a fluidised bed

\(^1\)This section is heavily based upon the work of Dr C. Gardner.
generators), each using aluminium oxide particles with average diameter of \( d_p \approx 3 \mu m \). This resulted in an orange coloured flame instead of light blue.

The PIV camera was equipped with a narrow band filter centred at 532 nm with 50\% transmission, allowing acquisition of the laser light only and prohibiting the capture of the ambient and flame light emissions. The filter distorted the particle shapes from circular to elliptical. The distortion was contributed to the coating direction of the filter, the elliptical particles changed direction as the filter mounted on the camera was rotated. For the best accuracy, the PIV post processing required circular particles that were 4 pixels wide and long. To reduce the effect of particle distortion the filter was rotated so majority of the particles in the area of interest were 4 pixels long and wide. The particles outside the area of interest however remained non-circular.

The strongly swirling flow requires careful control of the laser sheet profile, timing between the laser pulses and seeding density. This is due to strong out-of-plane motion of particles causing a reduction in the number of valid particle-image pairs. Keane and Adrian [33] suggested that the number of particle image-pairs should remain between 10 to 20 and the particles lost due to the out-of-plane motion should not exceed 25\% for optimum cross-correlation results. The relatively short time between the two laser pulses (25 \( \mu s \)) resulted in approximately 10 particles per interrogation windows and the thick laser light sheet compensated for out-of-plane motion. The fidelity of the seeding particles following the gas flow is evaluated from the relaxation time \( \tau_s \) as shown in equation 3.8.

\[
\tau_s = \frac{d_p^2 \rho_p}{18 \mu_f}
\] (3.8)

where \( d_p \) is the diameter of the seeding particles, \( \rho_p \) is the density of the particles and \( \mu_f \) is dynamic viscosity of fluid. For the current aluminium oxide particles with \( \rho_p = 3970 \text{ kg/m}^3 \) and \( d_p = 10 \mu m \), the relation times are approximately 107 and 32 \( \mu s \) in the reactants and products respectively.
Deposition of the seeding particles upon the inner surface of the combustion chamber occurred very quickly, reducing visibility of the seeding particles and compromising the image pair acquisition. To overcome this issue periodic cleaning of the combustion chamber after every set of 25 images was required. The images were also affected by reflections of the laser light sheet from inside the quartz chamber. The position of laser reflections shifted as the quartz chamber was removed for cleaning after each set of measurements. These reflections produced random erroneous results when PIV post-processing algorithms were applied. Also the high intensity reflections could damage the PIV cameras, so care was taken to minimise the risk to hardware. The reflections were minimised by directing the laser light sheet across the burner from above and selecting an appropriate region of interest for PIV post processing by cutting out high intensity reflections manifested on the image by blooming. Some reflections were still present and are apparent in the PIV data. Laser light incident onto the burner head also resulted in significant reflections from the burner top, which were minimised by using high temperature black paint and by moving the burner top below the field of view of the PIV-CCD camera.

3.3.3 Phase-Locked, Force-Excited Data Acquisition

The ICCD and PIV techniques could be applied in two different ways, either instantaneous image acquisition or phase-locked image acquisition. The phase-locked image acquisition has received much attention in this field of research [1, 5, 83], since the ensemble averaged images at given phases reveal the dominant structures as less important structures disappear in the processes. If the ICCD and PIV phase-locked acquisitions are carried out simultaneously, then the post processed OH* chemiluminescence emission and flow field structures can be superimposed for better understanding of instability mechanisms.

Prior to start of experiments the ICCD and PIV hardware and software were configured for external triggering. The phase-locking took place at intervals of 45°, starting at zero degrees of the excitation sine wave fed to the loudspeakers. The sine wave was used as the reference signal
for phase-locking, ensuring that a repeatable signal was used. In the phase-locked mode the LabView master program recorded sensors signals at the designated phases and triggered the ICCD. The triggers were in the form of standard TTL (Transistor-Transistor Logic with low state of 0 and high state of 5 volts) but with a rise time of 10 μs, irrespective of the frequency, amplitude and phase of excitation. This rise time corresponds to an error of less than one degree for all frequencies that were examined in this study. The ICCD hardware, unlike the PIV hardware, was only capable of accepting a maximum of five triggers per second, although the LabView software generated more triggers per second. This property of the ICCD was used when simultaneous phase-locked ICCD-PIV external triggering was required. The PIV was triggered by the ICCD, hence ensuring that simultaneous phase-locked ICCD-PIV image acquisition with an equal number of images from both systems could be obtained. The ICCD could therefore be operated as a stand-alone device or used together with PIV or any other system that is capable of using external triggering. The phase of the velocity fluctuation with respect to sine wave was also calculated, so that the in the presentation of the results, the phase-locked ICCD or phase-locked simultaneous ICCD-PIV could be referenced to the velocity fluctuation phase instead of imposed sine wave phase.

The timing sequence with the external triggering for the ICCD at phase lag of 90 degrees is shown in figure 3.21A, B, C and D. The intensifier and the CCD do not react to the LabView triggers until the CCD frame is activated with the duration known as frame exposure (T1). Simultaneously, with CCD frame activation, the intensifier is activated, and remains active for a period which is equal to the frame exposure (figure 3.21C). Finally LabView triggers cause the activated intensifier digital-shutter to open, projecting an image on the CCD (figure 3.21D). These images combine on the CCD until the frame exposure is finished, the frame is then transferred to the computer. It is therefore possible to combine multiple intensifier exposures within each frame of the CCD. The number of intensifier exposures per CCD frame exposure remained the same for different frequencies of loudspeaker excitation. In the current studies only one intensifier exposure was captured by each frame exposure. The time for each
intensifier exposure (T2) was chosen to correspond to five degrees at the excitation frequency. The number of the CCD frame exposures was 100 throughout this study.

The timing sequence of the PIV is depicted in figure 3.21A, B, C, E and F. The PIV is triggered from the frame activation of the ICCD to ensure a simultaneous acquisition (from figures 3.21C to 3.21E). There existed an intrinsic time delay between the reception of the trigger from the ICCD and activation of the PIV system. This intrinsic time delay (T3), if uncorrected, could lead to a phase-locking sequence that is not in synchronisation with the ICCD. This problem was rectified by adding an extra time delay (T4) so that the PIV started its acquisition sequence at the appropriate phase of the excitation cycle. Beside the added time delay which was frequency dependent, the rest of the PIV timing sequences remained the same. The first and second exposure time of the PIV cameras were different and remained constant throughout the experiments. The number of images obtained was kept constant at 100 per phase of any frequency and amplitude combination, bearing in mind that each PIV image consists of two frames.

At imposed frequency of 100 and 140 Hz the ICCD system failed to trigger the PIV due to mismatch between the acquisition frequencies of the two systems. The simultaneous ICCD-PIV acquisition therefore only covered imposed frequencies of 60, 80, 120, 144, 160, 180 and 200 Hz, each with imposed velocity fluctuation of 10%, 20% and 30% for the axial swirler only at $\phi = 0.56$. Earlier photodiode FTF magnitudes revealed that these fluctuations are related to linear, transition to from linear to non-linear and non-linear flame response stabilised by the axial swirler. The frequency of 144 Hz was specifically chosen in order to correspond to the self-excitation frequency of the axial swirler as presented in table 3.2. A complete set of phase-locked ICCD OH* chemiluminescence acquisition for all frequencies of interest except 140 Hz, with stepwise increase of velocity fluctuation until blow-off were carried out to complement the data obtained from the photodiode for both the radial and axial swirl geometries. Note that at 140 Hz the phase-locked acquisition failed due to triggering issues between the ICCD and LabView systems. The full range of investigated frequencies and amplitudes are shown
3.3. Imaging Systems

Table 3.3: The frequencies, equivalence ratios and velocity fluctuations investigated under forced-excitation with phase-locked ICCD or simultaneous phase-locked ICCD-PIV. The symbol △ stands for frequencies of 60 to 200 Hz in steps of 20 Hz except 140 Hz,▽ stands for frequencies of 80 to 200 Hz in steps of 20 Hz except 140 Hz, ⃝ stands for frequencies of 60, 80, 120, 144, 160, 180 and 200 Hz. For the ⊗ the velocity fluctuation was increased stepwise until flame blow-off but ⊖ indicates imposed fluctuations of 10%, 20% and 30% only.

The combustion chamber length for all conditions mentioned in table 3.3 was kept constant at 100 mm. Phase-locked forced-excited ICCD investigation at $\phi = 0.91$ for the radial swirler did not yield presentable results due to very high flame OH$^*$ chemiluminescence causing saturation in the ICCD image.

3.3.4 Phase-Locked, Self-Excited Data Acquisition

The master LabView program was also used for phase-locked ICCD data acquisition under self-excitation. In the first stage, the microphone signal of the combustion chamber was used to determine the dominant frequency of the self-excitation, illustrated in table 3.2. A Barr-Stroud narrow band filter type EF3, centred at the dominant frequency of the combustion chamber was used to remove the additional noise and harmonic frequencies. The signal was then amplified by a factor of 10 using a power amplifier and fed to the LabView program, which was configured to initiate the ICCD at the rising edge of the acquired microphone signal, corresponding to phase of zero degrees with respect to the pressure oscillation in the combustion chamber. The program also provided the capability of delaying the initiation of the ICCD, providing a method of phase-locking. It was ensured that the dominant frequency
was repeatable and reached constant value after few minutes of operation. This was due to
the warming up of the self-excitation combustion chamber shown in figure 3.11. The full range
of the experimental conditions are shown in table 3.4

<table>
<thead>
<tr>
<th>Apparatus</th>
<th>ICCD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Swirl geometry</td>
<td></td>
</tr>
<tr>
<td>Axial</td>
<td>Radial</td>
</tr>
<tr>
<td>Equivalence ratio / no units</td>
<td>0.56 0.91</td>
</tr>
<tr>
<td>Frequency / Hz</td>
<td>144   200</td>
</tr>
</tbody>
</table>

Table 3.4: The frequencies and equivalence ratios investigated under self-excitation with phase-
locked ICCD. The combustion chamber length was kept constant at 1055 mm.

3.4 Data Processing

3.4.1 FTF Calculation

As explained in section 3.2.3, the photodiode FTF magnitude and phase were calculated from
the photodiode and hotwire measurements of flame emission as it was forced with a range of
frequencies and amplitudes. A uniform acoustic velocity profile was assumed, even though
a mean velocity profile certainly existed in the connection flange at the hotwire location.
Therefore the percentage calculated was preserved as the flow accelerated through the burner.
After allowing 2 seconds of settling time the hotwire and photodiode signals were recorded
and, after applying the calibration to the hotwire data, analysed using spectrum analyser.
Typical spectra of the hotwire and photodiode are shown in figure 3.22 for axial swirler at
\( \phi = 0.56 \) excited at 80 Hz with acoustic velocity of 30%. As explained in equation 1.16, section
1.7, the photodiode FTF consisted of a magnitude \( \psi \) and phase \( \theta \), which is equivalent to the
frequency response magnitude and phase of the photodiode with respect to the hotwire at the
imposed frequency of excitation.
3.4.2 ICCD Image Processing

A number of image processing techniques were applied to the phase-locked instantaneous OH\textsuperscript{*} chemiluminescence images. As explained in section 3.3.1, there were 100 instantaneous images for each phase and 9 phases for each frequency and amplitude combination.

Figure 3.23 demonstrates the flowchart detailing pathways taken for phase-locked ICCD data acquisition and post processing. A number of MatLab programs were written to enable batch processing of the exported image. Unfortunately due to a mismatch between the excitation and acquisition frequency, especially at 160 and 180 Hz, some images were blank. Therefore only 50 viable instantaneous images could be selected from the data set, an example of an instantaneous image is shown in figure 3.24. An ensemble-average image from the combined instantaneous images was calculated using batch processing programs written specifically for the task. The background emission was then subtracted form the whole image, the background emission is associated with noise of the CCD camera. The background emission varied irrespective of frequency, amplitude or phase of excitation. The ensemble-averaged image (with the background correction) was first normalised with respect to the appropriate intensifier exposure time (figure 3.25), before the vertical profile of the image was calculated (figure 3.26) and used to obtain the height (H) at which the maximum intensity occurred, as a function of phase. The horizontal intensity profile across the image was also calculated at heights of 5, 10 (figure 3.27), 15, 20 and 25 mm above the burner lip.

The image-averaged OH\textsuperscript{*} chemiluminescence intensity of each ensemble-averaged and normalised ICCD image (e.g. the average of the OH\textsuperscript{*} chemiluminescence image in figure 3.25) was calculated and plotted against the phase of the excitation sine wave. The least-squares-method was used to fit a sine wave to this plot as depicted in figure 3.28, resulting in a mean, amplitude and phase-shift. The mean and amplitude corresponded to $\bar{Q}$ and $Q'$ of equation 1.17 in section 1.7. The $\bar{u}_{in}$ is the mean velocity calculated from the mean flow through the burner and $u'_{in}$ is the measured amplitude of velocity fluctuation. The ratio $Q'/\bar{Q}$ is known as the normalised global heat release and $(Q'/\bar{Q})/(u'_{in}/\bar{u}_{in})$ defines the normalised
FTF magnitude $\Psi$ (designated here as ICCD FTF magnitude). The phase-shift of the sine fit corresponds to FTF phase $\Theta$ (designated here as ICCD FTF phase). If the image-averaged OH$^*$ chemiluminescence intensity follows a sinusoidal pattern with respect to the imposed phase of excitation, then the sine fit results in accurate prediction of the mean, amplitude and phase-shift. Note that the formulation in equation 1.16 and 1.17 implies that the fuel is premixed and there are no fluctuations in the stoichiometry ($\bar{Q}$ remains constant). The ICCD FTF magnitude $\Psi$ is proportional to the photodiode FTF magnitude $\psi$ in equation 1.16 for fixed burner conditions.

The ICCD OH$^*$ chemiluminescence images must be considered as the projection of the line-of-sight images of a three-dimensional flame onto a two-dimensional CCD camera. The line-of-sight, ensemble-averaged and normalised ICCD OH$^*$ chemiluminescence image was therefore de-convoluted by the Abel transform to yield heat release from the diametral plane, [45]. The main requirement in the Abel transform calculation is that the flame is axisymmetric, which was satisfied in the case of the flames examined in this research. Before the de-convolution was carried out, the normalised flame image was divided into two sections at the centre around the y-axis; the two sides of the flame were then averaged and the Abel transform was implemented. The discretized Abel transform is shown in equation 3.9, [45],

$$i(r_n, z) \Delta y \cdot \Delta z \cdot \Delta w = \frac{(I_n - I_{n-1})}{\pi r_n(r_n - r_{n-1})} \quad \text{for} \quad r_n > 0 \quad (3.9)$$

where

$$I_{g_{m,n}} = \frac{y_m g(y_m, x)}{\sqrt{y_m^2 - r_n^2}} \quad (3.10)$$

and

$$I_n = \sum_{m=0}^{n-1} \frac{1}{2} (I_{g_{m,n}} + I_{g_{m-1,n}}) (y_{m-1} - y_m) \quad (3.11)$$

A graphical implementation of the process is presented in figure 3.29. The intensity of a given row of the image matrix $x$ and column indicated by $r_n$ was represented by $g(y_m, x)$. With this in mind one can easily calculate $I_{g_{m,n}}$, $I_{g_{m-1,n}}$, $I_n$ and $I_{n-1}$. The discretization process
was implemented on each row until $y_m = r_n$. The value of $i(r_n, z)$ is the Abel transform since $\Delta y \cdot \Delta z \cdot \Delta w$ is independent of $r$. A reverse Abel transform was also implemented to recover the averaged images and assess the numerical errors. The comparison between the averaged images obtained from the two procedures yielded an error of less than 1%.

### 3.4.3 PIV Image Processing

The Insight 3G software was used to capture and calculate the ensemble-averaged of the 100 frames per phase of each frequency and amplitude combinations\(^2\). The post processing steps are, [87]:

1. Performing image conditioning: where a signal to noise threshold is specified to filter out bad vectors, in the current study a signal to noise ratio of 1.5 was defined and vectors smaller than this threshold were filtered.

2. Generating grids: this process breaks down the image into smaller spots or interrogation areas in preparation for the next stage of analysis and enables capturing of particles that move out of the interrogation area. The result of this process is a list of the grid points, defining the pixels that make up interrogation area A (in frame A) and interrogation area B (in frame B). The ‘recursive Nyquist grid’ reducing from 64 pixels square to 32 pixels was chosen from the available grid generation algorithms, for its increased accuracy and higher spatial resolution compared to ‘Nyquist grid’, faster processing time compared to ‘deformation grid’ and less dependent on user specified grid parameters as in the case of ‘rectangular grid’ algorithm.

3. Masking spots: the implantation of the ‘FFT correlator’ in the next stage of post processing required the use of ‘zero pad mask’ algorithm in this section. This masking algorithm removed the large displacement aliasing errors and increased the signal to noise ratio. The disadvantage of the zero pad mask is that of increased processing

\(^2\)This section is heavily based upon the work of Dr C. Gardner.
times. Other available mask engines were ‘deformation mask’, ‘no mask’ and ‘Gaussian mask’. The deformation mask was not used since it required the deformation grid as its prerequisite, no mask introduced aliasing errors and Gaussian mask was not compatible with the implemented FFT correlator in the next stage.

4. Performing correlation: the FFT correlator was implemented to calculate the common particle displacement and hence the velocity in each pair of interrogation areas resulting in a vector map of recorded frames. The FFT cross-correlation is computationally less intensive compared to other available correlations (e.g. Hart and Direct).

5. Locating peaks: ‘Gaussian peak’ was used in order to locate the correlation map peak with sub-pixel accuracy, [90].

The resultant vector maps (.vec) were converted into MatLab files (.mat) and the spurious large vectors were filtered. The main MatLab tasks included:

- Integration of the velocity profile 2.3 mm above the burner head for calculation of volumetric fluctuations.
- Calculation of equivalence ratio fluctuation from volumetric fluctuations by considering a ‘stiff’ fuel system.
- Stream and strain plots superimposed with Abel de-convoluted OH* chemiluminescence images.
3.5 Figures

**Figure 3.1:** Schematic of the axial swirler and the fuel chamber. A) Fuel chamber, B) Axial swirler and C) Fuel distribution tree.

**Figure 3.2:** Schematic of the axial burner geometry. The quartz combustion chamber was placed in the recess and not shown here.
3.5. Figures Chapter 3. Experimental Methods

Figure 3.3: Dimensions of the axial swirler burner geometry. The combustion chamber is not shown. All values are in millimeters.

Figure 3.4: Schematic of the radial swirler and the fuel chamber. A) Radial swirler and B) Fuel chamber.
3.5. Figures

Figure 3.5: Schematic of the radial burner geometry. The quartz combustion chamber was placed in the recess and not shown here.

Figure 3.6: Dimensions of the radial burner geometry. The combustion chamber is not shown. All values are in millimeters.
3.5. Figures

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Figure 3.7: Image of the axial swirler flame at steady state, $\phi = 0.56$, captured by commercial digital camera.

Figure 3.8: Image of the radial swirler flame at steady state, $\phi = 0.56$, captured by commercial digital camera. The visible radial flame emission was weaker than the axial flame in figure 3.7 due to hollow structure of the radial flame.
Figure 3.9: Schematic of the forced-excitation geometry placed upstream of the burner configuration illustrated in figure 3.2 or 3.5. The tube is 205 mm long and the plenum had dimensions of $186 \times 151 \times 181$ mm. The diameter of air inlet is 5 mm.
Figure 3.10: Schematic of the self-excitation geometry placed upstream of the burner configuration illustrated in figure 3.2 or 3.5. The length of the entire tube (assuming the centre line of the bend) is 425 mm. The diameter of air inlet is 5 mm. The combustion chamber which is placed downstream of the burners is depicted in 3.11.
Figure 3.11: Schematic of the self-excitation combustion chamber placed downstream of the swirl geometries depicted in figures 3.2 or 3.5. The quartz tube is sandwiched in a metallic frame.
3.5. Figures

**Figure 3.12:** Hotwire calibration data fitted with a third order polynomial.

**Figure 3.13:** Loudspeaker calibration curve, axial swirler, $\phi = 0.56$. The data for 40 and 60 Hz are not shown for the purpose of clarity.
3.5. Figures

Figure 3.14: Comparison of acoustic velocity fluctuation magnitude obtained from two-microphone method and hotwire anemometry for 80 Hz (top graph) and 200 Hz (lower graph) for the axial swirl burner without combustion.

Figure 3.15: Comparison of acoustic velocity fluctuation phase obtained from two-microphone method and hotwire anemometry for 80 Hz (top graph) and 200 Hz (lower graph) for the axial swirl burner without combustion.
3.5. Figures

Figure 3.16: Graph of loss coefficient \( K \) against acoustic velocity fluctuation deduced for the axial and radial burner geometries for 80 Hz only.

Figure 3.17: Graph of effective length \( l_{\text{eff}} \) against acoustic velocity fluctuation deduced for the axial and radial burner geometries for 80 Hz only.
Figure 3.18: Schematic of the data acquisition system connections. ‘Signal to devices’ is shown with blue, and ‘signal from devices’ is shown with red. The hotwire and microphone were placed at U and P respectively. The LabView system is the master controller of all experiments.
Figure 3.19: The schematic of the ICCD system, [88]. A) Objective lens, B) Image intensifier, C) Photo-cathode, D) MCP channel, E) Phosphor plate, F) Coupling optics, G) CCD camera, H) Image intensifier high voltage MCP, I) Image intensifier high voltage pulser, J) External control unit, K) CCD trigger and L) Image intensifier trigger.
Figure 3.20: The schematic of the PIV data acquisition and post processing, [87].
Figure 3.21: Timing sequence (not to scale) of the phase-locked ICCD (A, B, C, and D) [88] and simultaneous phase-locked ICCD-PIV (A, B, C, D, E and F) for phase lag of 90° with respect to the sine wave of excitation. T1) Frame exposure, T2) Intensifier exposure, T3) PIV intrinsic delay, T4) Added delay for the PIV, T5) First PIV-CCD exposure and T6) Second PIV-CCD exposure.
Figure 3.22: Power spectra of the hotwire (top graph) and photodiode (lower graph), axial swirler, $\phi = 0.56$, $f_i = 80$ Hz, $U_{rms} = 30\%$. Notice the strong harmonic presence in the photodiode spectrum.
Figure 3.23: Applied routines for phase-locked ICCD data acquisition and analysis for forced-excitation of X Hz, acoustic velocity of Y% and phase lag of Z degrees. The above routines were thus repeated for all the phase lags associated with each forced-excitation frequency and amplitude combination.
Figure 3.24: Phase-locked instantaneous ICCD image, axial swirler, $\phi = 0.56$, $f_i = 60$ Hz, $U_{rms} = 30\%$, phase of $-4^\circ$. The colour bar indicates the intensity in arbitrary units.

Figure 3.25: Phase-locked ensemble-averaged (with background correction) and normalised (with respect to the intensifier exposure time) ICCD image, axial swirler, $\phi = 0.56$, $f_i = 60$ Hz, $U_{rms} = 30\%$, phase of $-4^\circ$. The colour bar indicates the intensity in arbitrary units.
3.5. Figures

Figure 3.26: Vertical OH$^*$ chemiluminescence intensity profile of the phase-locked ICCD image shown in figure 3.25, axial swirler, $\phi = 0.56$, $f_i = 60$ Hz, $U_{rms} = 30\%$, phase of $-4^\circ$. The H stands for the height at which the vertical distribution of the emission reached a maximum.

Figure 3.27: Horizontal OH$^*$ chemiluminescence intensity profile of the phase-locked ICCD image shown in figure 3.25 at height of 10 mm above the burner head, axial swirler, $\phi = 0.56$, $f_i = 60$ Hz, $U_{rms} = 30\%$, phase of $-4^\circ$. 
Figure 3.28: Sine fit to the image averaged OH* chemiluminescence intensity, axial swirler, $\phi = 0.56$, $f_i = 80$ Hz, $U_{rms} = 30\%$. Each square point is the average of the ensemble-averaged (with background correction), normalised phase-locked ICCD image.

Figure 3.29: Schematic of Abel transform implementation. The $y_m$ and $y_{m-1}$ are the radial distance from the flame’s central axis, $x$ is the vertical distance from the burner head and $g(y_m, x)$ is the intensity at location $(y_m, x)$. 

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Chapter 4

Results
4.1 Forced-Excitation: Axial Swirler

As explained in section 3.1.2, table 3.1, the photodiode FTF measurements were carried out until blow-off occurred. The blow-off limits of the axial swirler with equivalence ratio of 0.56 and 0.48, as functions of imposed frequency and acoustic velocity fluctuation, are shown in figure 4.1. The required acoustic velocity fluctuation for blow-off decreased linearly as the frequency increased. The blow-off limit for the equivalence ratio of 0.56 was higher than the corresponding values with equivalence ratio of 0.48, as expected.

Figure 4.2 depicts the ICCD OH\(^\ast\) intensity measurement of a time-averaged and normalised flame (50 exposures each 999 \(\mu\)s long), stabilised by the axial swirler, for a range of equivalence ratios (0.34 to 1.03) at steady state conditions. As the equivalence ratio was increased the flame shape evolved from an inverted cone to a flat flame. The flame root location also changed. At low equivalence ratios the flame was stabilised inside the vanes of swirler (i.e. at \(\phi = 0.34\)) but as the equivalence ratios increased, the flame root moved upstream. At the highest equivalence ratio (\(\phi = 1.03\)) the flame stabilised at approximately 10 mm above the burner lip. The High Intensity Area (HIA) is a region where the flame front dominates for longer periods compared to the Low Intensity Area (LIA). The HIA therefore emit higher emissions at a wavelength of 307 nm compared to other regions of the flame, underlining a higher proportion of heat release occurring in these regions. Regions of the flame with OH\(^\ast\) chemiluminescence greater or equal to 0.6 (arbitrary units, see figure 4.2) are classified as HIA and flame regions smaller than 0.6 are designated as LIA. The variation of OH\(^\ast\) chemiluminescence intensity with respect to equivalence ratio was calculated from figure 4.2 and it is presented in figure 4.3. The OH\(^\ast\) chemiluminescence emission is a strong function of pressure and equivalence ratio, and the variation with respect to the equivalence ratio is known to be non-linear, [15, 35, 36].

The contour plot of velocity magnitude and the streamline plot for the axial swirler at \(\phi = 0.56\) with no imposed excitation is shown in figure 4.4 and figure 4.5 respectively. Figure 4.4 clearly demonstrates the shape of the shear layers on the annular jet entering the combustion...
chamber. As is to be expected the velocity in the shear layer was much higher than the outer or inner recirculation velocities and, after contact with the quartz chamber wall, the shear layer continued parallel to it downstream. The laser reflection issues as explained in section 3.3.2 were also apparent in the contour plots at axial locations of 15 and -15 mm in the form of vertical discontinuation of the shear layer velocity profile. Figure 4.5 depicts the inner and outer recirculation zones separated by the almost parallel streamlines of the shear layer. Of course the streamline plot was the result of ensemble averaging instantaneous images. The ORZ and IRZ did not actually remain stationary, even under non-excited conditions. As with the previous case shown in figure 4.4, the streamlines were not symmetrical around the burner central axis due to laser reflections at axial locations of 15 and -15 mm. The area subjected to the laser reflections is much larger at -15 mm when compared to that at 15 mm. The reflections prohibit the calculation of common particle displacement resulting in asymmetrical streamline plot, although the actual flow field is symmetrical. The symmetrical nature of the flow field was confirmed by rotating the axial swirler by angles of 45° and comparing the resulting streamline plots; the position of the laser arm remained unchanged in these experiments.

4.1.1 FTF, $\phi = 0.56$

The photodiode flame transfer function magnitude and phase, as described in sections 1.7 and 3.4.1, are presented in figures 4.6, 4.7 and 4.8. The magnitude of the photodiode FTF is presented in two graphs of low frequencies (60 to 120 Hz) and high frequencies (140 to 200 Hz). Non-linearity in the flame response is characterised by a drop in FTF magnitude whereas linearity is characterised by a constant FTF magnitude. At $\phi = 0.56$, non-linearity was observed once the imposed acoustic velocity fluctuation exceeded 20% of the mean velocity for all frequencies except 40 and 60 Hz. The latter frequencies remained linear irrespective of imposed acoustic velocity fluctuation. Unlike other frequencies the photodiode FTF magnitude at 200 Hz demonstrated an increasing gain up to $U_{rms} = 10\%$, linear response up to
$U_{rms} = 20\%$, followed by a non-linear response. The phase of the photodiode FTF remained constant for a given frequency irrespective of acoustic velocity fluctuation (see figure 4.8). The phase information is plotted in terms of phase lag by subtracting appropriate multiples of one cycle from the photodiode FTF phase. This was done to stress that the flame response should always lag the velocity fluctuations.

The normalised global heat release (e.g. $Q'/\bar{Q}$) data from the ICCD are presented in figures 4.9 and 4.10. The difference between the steady state $\bar{Q}$ and the mean of the fluctuating $Q$ was less than 4% (not shown here). The normalised global heat release was used to calculate the ICCD FTF magnitude (e.g. $(Q'/\bar{Q})/(u'_in/\bar{u}_{in})$), presented in figure 4.11, as explained in section 3.4.2 and equation 1.17. Bear in mind that the ICCD data does not include the 40 and 140 Hz cases as shown in table 3.3. The 60 Hz normalised global heat release in figure 4.9 demonstrated a linear behaviour in agreement with the photodiode FTF magnitude in figure 4.6. The rest of the frequencies shown in figures 4.9 and 4.10 saturated at $U_{rms} = 20\%$, similarly to those in figures 4.6 and 4.7. The agreement between the photodiode and ICCD FTF magnitudes (see figure 4.11) is very good except for the 200 Hz case, where the increasing gain did not occur in the ICCD FTF. The effect of the increasing gain was also not observed in the normalised global heat release (see figure 4.10).

The error in the normalised global heat release and ICCD FTF magnitude is the result of the background correction. This has an effect on the accuracy of the image-averaged OH$^*$ chemiluminescence amplitude calculated by the sine fit program. The maximum error in the normalised global heat release (and therefore the ICCD FTF magnitude) was dependent on the imposed frequency and velocity fluctuation and did not exceed 5%.

The ICCD FTF phase lags produced similar results to the photodiode FTF phase lags presented in figure 4.12. In fact the maximum discrepancy between the phases obtained from these two methods was no more than $10^\circ$.

The frequency dependence of the photodiode FTF magnitude and phase are plotted in figures 4.13 and 4.14. The photodiode FTF magnitude peaked at approximately 80 Hz. An
increase in magnitude was also noted at around 200 Hz. The photodiode FTF magnitude was also dependent on the imposed acoustic velocity fluctuation such that the highest magnitude was achieved by 10%, 15%, 20%, 25% and finally 30%. The photodiode FTF magnitudes at \( U_{rms} = 5\% \) appeared between \( U_{rms} = 10\% \) and 30%. The photodiode FTF phase was linearly dependent on frequency.

The fundamental and harmonic content of the photodiode is depicted in figure 4.15 for 80 and 200 Hz only. The fundamental curve clearly demonstrates the saturation of the signal for both frequencies once the loudspeaker amplitude of 4 volts was achieved. The loudspeaker amplitudes at which \( U_{rms} = 10\%, \ 20\% \) and 30\% occur are marked on the graph for the fundamental cases only. The harmonic content of the hotwire was less than 5\% of the fundamental irrespective of linear or non-linear photodiode response and therefore it is not shown here. The fundamental content of the hotwire demonstrated a linear behaviour with respect to loudspeaker amplitude, irrespective of the imposed frequency and amplitude. For all frequencies the photodiode harmonic content increased as the acoustic velocity fluctuation increased. At the lowest frequency (e.g. 40 Hz) the first harmonic content was 50\% of the fundamental at the highest possible \( U_{rms} \), but as the frequency increased the magnitude of the first harmonic with respect to fundamental was reduced to 15\% at the highest possible \( U_{rms} \) (see 200 Hz in figure 4.15). The slope at which the fundamental photodiode voltage saturated also varied according to the imposed excitation frequency. The 80, 100, 120, and 200 Hz showed strong saturation of the fundamental photodiode voltage by reaching a constant photodiode voltage after the \( U_{rms} = 20\% \). The 140, 160 and 180 Hz results illustrated weak saturation when compared to other frequencies after \( U_{rms} = 20\% \) mark. The weak saturation is characterised by a shallower slope when compared to the slope occurring prior to the \( U_{rms} = 20\% \). The fundamental of 40 and 60 Hz did not saturate although the harmonic content were significant.
4.1.2 FTF, $\phi = 0.48$

Equivalence ratios higher than 0.56 exhibited a very weak hum (self-excitation) in the frequency range of 40 to 200 Hz. To avoid the uncertainty introduced by this hum the equivalence ratio of 0.48 was selected. The photodiode FTF magnitude and phase for $\phi = 0.48$ are presented in figures 4.16 (for low frequencies), 4.17 (for high frequencies) and 4.18. The non-linearity for the low excitation frequencies began at $U_{\text{rms}} = 15\%$, which was lower than the $\phi = 0.56$ case. The 40 Hz case did not possess any linear regime and saturated at $U_{\text{rms}} \approx 7.5\%$. Similarly, the 140 Hz presented non-linear response irrespective of the imposed velocity fluctuations. For the cases of 160 and 180 Hz linear behaviour was observed at all excitation amplitudes. The 200 Hz case demonstrated distinct increasing gain in photodiode FTF magnitude up to $U_{\text{rms}} = 10\%$, where a linear profile was apparent until $U_{\text{rms}} = 15\%$, followed by non-linear response. The three distinct regimes of the photodiode FTF magnitude profile (increasing gain, linear and non-linear response) at 200 Hz repeated those found for the 200 Hz case of $\phi = 0.56$, but with a less pronounced increase in gain. As before the phases are shown in terms of lags, which increased with increasing frequency (see figure 4.18). The mean phase for each frequency was within $\pm 10^\circ$ when compared to its counterpart at $\phi = 0.56$.

The normalised global heat release are presented in figures 4.19 and 4.20. The difference between the steady state $\overline{Q}$ and the fluctuating $\overline{Q}$ was less than 1% (not shown here). As with the previous case the normalised global heat release was used for calculation of the ICCD FTF magnitude (figure 4.21). The agreement between the photodiode and ICCD FTF magnitude is very good except for the 200 Hz case where the increasing gain was not observed in the ICCD FTF magnitude. Such disagreement was also seen in the 200 Hz case at $\phi = 0.56$. As with the $\phi = 0.56$ case, the maximum error in the normalised global heat release (and therefore the ICCD FTF magnitude) was dependent on the imposed velocity fluctuation and frequency and did not exceed 5%.

At $\phi = 0.48$, the ICCD FTF phases (shown in figure 4.22) were consistent with photodiode FTF phases. The maximum discrepancy between the phases obtained from these two methods...
was again less than 10°.

Figures 4.23 and 4.24 illustrate the frequency dependence of the photodiode FTF magnitude and phase. The photodiode FTF magnitude measured at an acoustic velocity fluctuation of 5% was omitted due to low coherence. At frequencies of 80, 120, 160, 180 and 200 Hz, the $U_{rms} = 30\%$ was not available due to proximity to the blow-off limit. A peak in the photodiode FTF magnitude was observed at 80 Hz, while an increase in magnitude was observed at 200 Hz similar to the $\phi = 0.56$ case. The photodiode FTF magnitude decreased as the acoustic velocity fluctuation was increased from 10% to 30%. However, unlike the high equivalence ratio case, the photodiode FTF magnitudes diverged from each other at lower frequencies of excitation.

The harmonic content of the hotwire was less than 5% of the fundamental irrespective of linear or non-linear photodiode response and therefore it is not shown here. The fundamental content of the hotwire demonstrated a linear behaviour with respect to loudspeaker amplitude irrespective of the imposed frequency and amplitude, as with the $\phi = 0.56$ case. As with the previous case ($\phi = 0.56$), the harmonic content of the photodiode (not shown here) increased as the imposed velocity fluctuation increased but the ratio of the fundamental to the first harmonic content increased with increasing frequencies. The fundamental content of the photodiode demonstrated weak saturation for 120 Hz, characterised by a shallower slope compared to that occurring prior to the saturation mark at $U_{rms} = 15\%$. Strong saturation, characterised by the fundamental of the photodiode achieving a constant voltage occurred for 60, 80, 100, 200 Hz after the $U_{rms} = 15\%$ mark. The 40 Hz showed strong saturation starting at $U_{rms} \approx 7.5\%$. The fundamental of the photodiode voltage was linear for 160 and 180 Hz cases. The 140 Hz illustrated a strong saturation for the photodiode fundamental starting at up to $U_{rms} = 5\%$. 
4.1.3 Simultaneous Phase-Locked ICCD-PIV, $\phi = 0.56$

The phase-locked ICCD flame images taken on their own are not presented due to their similarity to the flame images obtained from simultaneous phase-locked ICCD-PIV displayed in this section. Note that the simultaneous phase-locked ICCD-PIV images at 100 and 140 Hz and phase-locked ICCD of 140 Hz were not taken, as explained in section 3.3.3. Therefore the Abel de-convoluted OH* chemiluminescence, calculated from the stand-alone phase-locked ICCD at the excitation frequency of 100 Hz subjected to 10% and 30% fluctuations, are shown in figures 4.25. The streamline plots superimposed with Abel de-convoluted OH* chemiluminescence of the flame, and the streamline plots superimposed with strain rates, are illustrated in figures 4.26 to 4.67 for frequencies of 60, 80, 120, 144, 160, 180 and 200 Hz, each with imposed velocity fluctuation of 10%, 20% and 30%. The vertical stripes of high or low strain rate are the result of laser reflections in the quartz combustion chamber, as explained in section 4.1 and figures 4.4 and 4.5. These stripes usually occur at a horizontal distance of approximately 10 to 15 mm from the central axis of the burner (see -184° in figure 4.31 or -213° in figure 4.37). However the stripes of high strain rate at the axial position of 25 mm are the result of flow field interaction with the combustion chamber and are not affected by laser reflections. The tensile strain rates are denoted by ‘high +ve’ and the compressive strain rates are shown by ‘low -ve’. Note that except for the 60 Hz for which flame response was linear, the $U_{rms} = 10\%$ corresponds to linear flame response, transition from linear to non-linear occurs at $U_{rms} = 20\%$, and the $U_{rms} = 30\%$ corresponds to non-linear flame response.

Low Frequency Regimes

Close inspection of the phase-locked Abel de-convoluted OH* chemiluminescence images for the low frequency regime revealed that the flame length variation and OH* chemiluminescence intensity variation increased as the imposed velocity fluctuation increased.

At $U_{rms} = 10\%$ the flame root was predominantly above the burner head at height of
approximately 3 mm as shown in figures 4.25, 4.26, 4.32 and 4.38. However the variation of the flame root became significant as the imposed velocity fluctuation was increased, with maximum heights of 6 mm from the burner head occurring at 60 Hz (−94° into the cycle, shown in figure 4.30) and decreasing to maximum of 2 mm at 120 Hz (−112° into the cycle, shown in figure 4.42). As the imposed velocity fluctuation increased, the flame root moved inside the burner irrespective of imposed frequency, for example see figures 4.26, 4.28 and 4.30. The area of the flame root that penetrated into the burner head increased as the imposed frequency was increased from 60 Hz to 120 Hz (compare flame root area in figures 4.42 and 4.30).

The variation of the OH* chemiluminescence intensity (calculated using the methodology described in section 3.4.2) against the phase of imposed velocity fluctuation is presented in figures 4.68 and 4.69 for $U_{rms} = 10\%$ and 30\% respectively for low frequency regimes. The OH* chemiluminescence intensity of 60 to 120 Hz at $U_{rms} = 10\%$ (figure 4.68) demonstrated a sinusoidal behaviour, with the 80 Hz case showing the highest OH* chemiluminescence intensity amplitude. As shown in figure 4.6 for $\phi = 0.56$, non-linear flame response was observed at $U_{rms} = 30\%$ except for the 60 Hz case. Non-sinusoidal or ‘skewed’ variation in the OH* chemiluminescence intensity is observed at 60 and 80 Hz at $U_{rms} = 30\%$ (figure 4.69), while 100 and 120 Hz follow a sinusoidal behaviour. The highest amplitude variation of OH* chemiluminescence intensity was observed at 80 Hz. As the excitation frequency increased the overall amplitude of the OH* chemiluminescence intensity decreased.

Figure 4.70 illustrates the OH* chemiluminescence intensity variation at 80 Hz as the imposed acoustic velocity fluctuation was increased from 5\% to 30\%, encompassing linear, transition from linear to non-linear and non-linear flame response. Sinusoidal variation in OH* chemiluminescence intensity is observed at low velocity fluctuation (5\% to 10\%). As the amplitude of the imposed velocity fluctuation increased, the OH* chemiluminescence intensity variation become skewed and ‘flattened’ in the later part of the cycle. This skewed OH* chemiluminescence intensity was also observed at 60 Hz for all imposed velocity fluctuations except 5\% and 10\%, where a sinusoidal variation was observed (not shown here), bearing in
mind that the 60 Hz case remained linear irrespective of imposed velocity fluctuations. The 100 and 120 Hz cases, however, exhibited sinusoidal OH* chemiluminescence intensity variation with increasing amplitude as the imposed acoustic velocity fluctuation was increased (not shown here).

At 60 Hz the flame does not enter the outer recirculation zone (ORZ), irrespective of the imposed velocity fluctuations (see figures 4.26, 4.28 and 4.30). The ORZ does not remain stationary and it consists of one recirculation zone irrespective of the phase or imposed velocity fluctuation. The flame is stabilised between the inner recirculation zone (IRZ) and ORZ, and the flame height variation increases as the imposed velocity fluctuation increases. Irrespective of the imposed velocity fluctuation the shape of the IRZ consisted of an interaction of two recirculation zones (vortices). At $U_{rms} = 10\%$ for example the interaction started from $-4^\circ$ to $-184^\circ$ and at $-229^\circ$, these two recirculation zones joined. The interaction of the two recirculation zones was dependent on the imposed velocity fluctuation. Note for example the phase of $-49^\circ$ at $U_{rms} = 10\%$, 20\% and 30\%: the lower extent of the IRZ moves further downstream as the imposed velocity fluctuation is increased. The corresponding strain rates in figures 4.27, 4.29 and 4.31 show the presence of continuous high tensile strain rates on the edge of the ORZ. The magnitude and area of the compressive strain rates increases as the imposed velocity fluctuation increases (compare $-139^\circ$ in figures 4.27, 4.29 and 4.31).

At 80 Hz, the flame remains outside the ORZ irrespective of the imposed velocity fluctuation, and the flame height variation increases with the imposed velocity fluctuation. As the velocity fluctuation increases the existence of two recirculation zones within the ORZ becomes apparent (see $-348^\circ$ in figures 4.32, 4.34 and 4.36). At $U_{rms} = 10\%$, the ORZ consists mainly of one recirculation zone, at $U_{rms} = 30\%$ however the ORZ exhibits two recirculation zones. Two recirculation zones are also apparent in the IRZ irrespective of the $U_{rms}$. The high tensile strain rates in the shear layer are continuous at $U_{rms} = 10\%$ (see figure 4.33). But as the $U_{rms}$ increases the high strain rates show areas of local reduction. The magnitude of compressive strain rate also increases substantially with the $U_{rms}$ (see $-168^\circ$ in figures 4.33, 4.35 and 4.37).
The flame maintains its conical shape to some degree and does not become flattened to the
extent observed in the 60 Hz cases. The flame at -258° in figure 4.37 was split into two sections
at a height of approximately 25 mm.

At 120 Hz case the flame does not enter the ORZ (see figures 4.38, 4.40 and 4.42). There
is more than one recirculation zone in the IRZ. The strain rates on the edge of the ORZ
for the 120 Hz case are not continuous irrespective of the imposed velocity fluctuations when
compared to the 60 or 80 Hz cases (see figures 4.39, 4.41 and 4.43). The compressive strain rate
magnitude and the flow area subjected to the compressive strain rate increase substantially
with \( U_{rms} \) (see -202° in figures 4.39, 4.41 and 4.43). In fact the 120 Hz case at \( U_{rms} = 30\% \)
exhibited the largest area of compressive strain rate compared to all other frequencies and
\( U_{rms} \) combinations. At 120 Hz case the flame shape becomes considerably different compared
to those at other frequencies as \( U_{rms} \) increases, most notably with the complete penetration
of the flame into the burner exit (see -337° in figure 4.42). The flame root at this phase is 19
mm wide, as big as the burner exit. The 80 Hz case at \( U_{rms} = 30\% \) (see -258° in figure 4.36)
was the only other frequency which possessed complete flame penetration into the burner’s
exit.

High Frequency Regimes

The flame shape excited at high frequencies was significantly different to the flame shape
excited at low frequencies. These manifested as an overall shortening of the flame length and
greater curvature of the flame front than observed in the low frequency regime, even at low
amplitudes of velocity fluctuations.

The flame root at \( U_{rms} = 10\% \) was above the burner head for almost half the cycle and
at a maximum height of approximately 2 mm, as depicted in figures 4.44, 4.50, 4.56 and 4.62.
At \( U_{rms} = 30\% \) the flame root was inside the burner for more of the cycle than for the low
frequency cases.
4.1. Forced-Excitation: Axial Swirler

Chapter 4. Results

The cyclic variation of the OH$^*$ chemiluminescence intensity for frequencies of 160 to 200 Hz also differed significantly compared to the low frequency regimes, as shown for $U_{rms} = 10\%$ and $U_{rms} = 30\%$ in figures 4.71 and 4.72 respectively. In general the amplitude of the OH$^*$ chemiluminescence intensity was reduced in comparison with the low frequency cases for both $U_{rms} = 10\%$ and $U_{rms} = 30\%$. The 160 and 180 Hz demonstrated comparable values of OH$^*$ chemiluminescence intensity amplitude variation with sinusoidal behaviour, whereas 200 Hz had the highest variation in the OH$^*$ chemiluminescence intensity amplitude with sinusoidal behaviour. The OH$^*$ chemiluminescence intensities of 160, 180 and 200 Hz demonstrated a sinusoidal behaviour with increasing amplitude as the imposed acoustic velocity fluctuation increased (not shown here).

At 144 Hz the flame area of high OH$^*$ chemiluminescence emission increased while the flame length remained almost constant through a cycle. A distinction should be made here between the variation in the high OH$^*$ chemiluminescence intensity area and the variation in the OH$^*$ chemiluminescence intensity. In the low frequency cases the variation of the total flame area is larger but the HIA is smaller than those of the high frequency cases. The compound effect is the large variation in the amplitude of the OH$^*$ chemiluminescence intensity. In the high frequency cases, the opposite is true. The flame area variation is smaller but the HIA is larger than those in the low frequency cases. Therefore the OH$^*$ chemiluminescence intensity variation in the high frequency cases is smaller than those in the low frequency cases. The IRZ continues to exhibit two recirculation zones and the area of the flame tip that penetrates the ORZ increases with $U_{rms}$ (compare -134° in figures 4.44, 4.46 and 4.48). The tensile strain rate is not continuous irrespective of $U_{rms}$. The area and magnitude of the compressive strain rate increased with $U_{rms}$ (see -224° in figures 4.45, 4.47 and 4.49).

In the 160 Hz case the HIA is reduced compared to that at 144 Hz. At 160 Hz the IRZ exhibits two recirculation zones and the tensile strain rates reveal discontinuity. The compressive strain rate and the area of compressive strain rate increase with $U_{rms}$ (see -281° figures 4.51, 4.53 and 4.55). The area of the flame tip that penetrates the ORZ also increases.
with the $U_{rms}$ (see -191° in figures 4.50, 4.52 and 4.54).

At 180 Hz the flame shows the largest HIA (consuming approximately 60% of the total flame area) compared to all other cases even at $U_{rms} = 10\%$. Note that the colour bar was adjusted so that the effect of Abel deconvolution at the burner’s central axis could be reduced. This however resulted in a white region within the flame for the 180 Hz cases; these areas therefore exhibited intensities larger than the ‘High’ mark on the colour bar. As with the previous cases the IRZ continues to exhibit two recirculation zones irrespective of $U_{rms}$ (see figures 4.56, 4.58 and 4.60), the high tensile strain rates remain in outer shear layer but are not continuous (see figures 4.57, 4.59 and 4.61) and the flame tip penetration into the ORZ increases with $U_{rms}$. The 180 Hz case is also in the only frequency where a significant portion of the flame tip penetrates into the ORZ at $U_{rms} = 30\%$ (see -198 in figure 4.60). As with the previous case the magnitude of the compressive strain rates increased as the $U_{rms}$ is increased.

The 200 Hz is the only case after 60 Hz which demonstrates a flat flame as the $U_{rms}$ is increased (see -127° in figures 4.62, 4.64 and 4.66). The HIA is reduced compared to that of the 180 Hz case and penetration of the flame tip into the ORZ increases with $U_{rms}$, similarly to the rest of the high frequency cases. The IRZ contains two recirculation zones irrespective of $U_{rms}$. The high tensile strain rates are not continuous, similarly to the rest of the high frequency cases. The area of discontinuity (low tensile strain rates) increases with the imposed frequency. For example, -37° in figure 4.67 has the largest area of discontinuity when compared to other imposed frequencies. This area also increases as $U_{rms}$ increases for any given frequency.

Figure 4.73 illustrates the vertical distance of the ORZ vortex centre from the burner face on the right hand side (RHS) and left hand side (LHS) of the burner central axis, for 160 Hz at 30%. Superimposed on this plot is the height at which the vertical distribution of the OH$^*$ chemiluminescence flame emission reached a maximum value (definition of H is shown in figure 3.26). This plot is only shown for the high frequency cases due to the importance of the flame and ORZ interaction, which only occurs in the high frequency cases. The height of the
LHS and RHS ORZ vortex centres are the same due to the symmetrical nature of the flow. Figure 4.74 depicts the ORZ vortex height on the left hand side of the burner centre and OH* chemiluminescence position for frequencies of 160 to 200 Hz for 30% fluctuation. The 144 Hz case was omitted for the purpose of clarity.

The velocity profile calculated from the PIV, 2.3 mm above the axial burner exit was rotated and integrated about the central axis to obtain the volumetric fluctuations. The volumetric fluctuations in air flow allow the magnitude of equivalence ratio fluctuations to be estimated because the fuel flow is considered to be almost invariant with the imposed frequency and velocity fluctuation. The equivalence ratio variations for 60, 120 and 200 Hz cases are shown in figures 4.75, 4.76 and 4.77 respectively. The equivalence ratio variation of the 60 Hz (figure 4.75) and high frequency cases do not dip below the steady state lean blow-off limit even at $U_{rms} = 30\%$ (figure 4.77). The equivalence ratio variation in the low frequency cases excluding the 60 Hz case, however, dip below the steady state limit as the imposed velocity fluctuation reaches 30% (see figure 4.76).

### 4.1.4 Phase-Locked ICCD, $\phi = 0.48$

A comparison of the phase-locked Abel de-convoluted OH* chemiluminescence of the axial swirler flame between the two equivalence ratios is shown in figures 4.78 and 4.79 for 80 and 200 Hz respectively, each subjected to $U_{rms} = 25\%$. The low equivalence ratio case exhibited similar variations in flame structure to the higher one, only with reduced intensity, which was to be expected since the OH* chemiluminescence emission is dependent on the equivalence ratio, as shown in figures 4.2 and 4.3. In general lowering the equivalence ratio resulted in a smaller flame size and reduced the region of high intensity compared to the high equivalence ratio cases. Also, the flame root settled at a lower level, withdrawing into the burner for the majority of cases, even at low amplitudes of oscillation. In the case of 80 Hz for example, note the disappearance of the flame tip at $-258^\circ$ when the $\phi$ is reduced from 0.56 to 0.48. The reduction in the OH* chemiluminescence of the flame tip is also apparent in the 200 Hz case.
at a phase of -262° when \( \phi \) is reduced from 0.56 to 0.48.

Comparison of the OH\(^*\) chemiluminescence intensity for \( \phi = 0.56 \) and 0.48, calculated for 80 and 200 Hz each at \( U_{rms} = 10\% \) and 25\%, is shown in figures 4.80 and 4.81 respectively. The flame exhibited non-linear response at \( U_{rms} = 25\% \), as shown in the photodiode FTF magnitude in figures 4.6 and 4.7 for \( \phi = 0.56 \), and in figures 4.16 and 4.17 for \( \phi = 0.48 \), respectively. The 80 Hz OH\(^*\) chemiluminescence variations at \( \phi = 0.56 \) and 0.48 were similar. However, the mean of the variation for \( \phi = 0.56 \) was higher than that of \( \phi = 0.48 \) as expected. For all other low frequencies (60 to 120 Hz) the intensity variation of the two equivalence ratio cases were similar irrespective of \( U_{rms} \) but with a reduced mean for the \( \phi = 0.48 \) case. The similarity encompassed the variation in OH\(^*\) chemiluminescence intensity and the phase at which the peak intensity occurred. The 60 Hz intensity variation for example, was skewed and the peak occurred at the same phase irrespective of \( \phi \) and \( U_{rms} \) (not shown here). For the 200 Hz case however the OH\(^*\) chemiluminescence variations at \( \phi = 0.56 \) and 0.48 were significantly different, although, as with the previous case, the mean of the variation for \( \phi = 0.56 \) was higher than for \( \phi = 0.48 \). This dissimilarity in the intensity variation and the phase at which peak intensity occurred between the two equivalence ratios was the characteristic difference between the two equivalence ratios of the high frequency cases (140 to 200 Hz).

4.2 Forced-Excitation: Radial Swirler

As in the case of the axial swirler, the photodiode FTF measurements were carried out until blow-off occurred. The blow-off limit of the radial swirler with equivalence ratios of 0.56 and 0.58, as a function of imposed frequency and velocity fluctuations is presented in figure 4.82. The blow-off generally increased up to imposed frequencies of 120 Hz. For the 200 Hz case, however, the blow-off limit was very low (\( U_{rms} \approx 22\% \)).

As discussed before in section 3.1.1, due to the structure of the flame, burner head configuration and the stabilising method of the radial swirler, the photodiode and the ICCD camera
did not capture any OH\textsuperscript{*} chemiluminescence emission from approximately 31 mm of the flame. Also no simultaneous phase-locked ICCD-PIV data were collected for the radial swirler.

Figure 4.83 depicts the OH\textsuperscript{*} chemiluminescence emissions of a time-averaged and normalised flame (50 exposures each 999 $\mu$s long) stabilised by the radial swirler under steady state conditions for a range of equivalence ratios ($\phi = 0.46$ to $\phi = 1.02$). Note that the colour bar was adjusted so that the intensity distribution of the flame at $\phi = 1.02$ was distinguishable. This resulted in a flame emission that appeared weak at $\phi = 0.46$. The increase in the equivalence ratio increased flame height and introduced an area of high intensity close to the burner head.

### 4.2.1 FTF, $\phi = 0.56$

For $\phi = 0.56$, the photodiode FTF magnitude for the radial swirler is presented in two separate graphs, figures 4.84 and 4.85, based on the imposed excitation frequency. The 40 Hz was highly sensitive to the imposed velocity fluctuation and no region of distinct linear or non-linear response could be identified. The 60 and 120 Hz photodiode FTF magnitudes demonstrated non-linear behaviour when $U_{\text{rms}} = 30\%$ was reached. The 80 Hz demonstrated a distinct profile consisting of an increasing gain up to $U_{\text{rms}} = 30\%$, followed by linearity up to $U_{\text{rms}} = 40\%$, followed by non-linear response until blow-off. The increasing gain, linear and non-linear variation was also present in the response of the axial swirler for 200 Hz at $\phi = 0.48$ and 0.56. The 100 Hz photodiode FTF consisted only of an increasing gain up to $U_{\text{rms}} \approx 12.5\%$ and then a linear response until blow-off. For the photodiode FTF magnitude at high imposed frequencies (see figure 4.85), one could clearly see the linear behaviour at 200 Hz. The photodiode FTF magnitudes at 140 and 160 Hz on the other hand demonstrated a strong non-linear behaviour when $U_{\text{rms}} = 30\%$ was reached. The flame response at 180 Hz was linear up to $U_{\text{rms}} \approx 22\%$, followed by non-linear response until blow-off.

The phase of the photodiode FTF is shown in figure 4.86 for all frequencies of excitation.
Unlike photodiode FTF phases obtained from the axial swirler (see figure 4.8), the photodiode FTF phases of the radial swirler (see figure 4.86) for imposed frequencies of 40, 120, 140, 160 and 180 Hz were dependent on the imposed velocity fluctuations.

Normalised global heat release, ICCD FTF magnitude and phase could not be obtained for the radial swirler due to the partial view of chemiluminescence emission. The frequency dependence of the photodiode FTF magnitude and phase are illustrated in figures 4.87 and 4.88. The photodiode FTF magnitude peaked at 80 Hz, similarly to the axial swirler. However, unlike the axial swirler the rise in magnitude at 200 Hz did not occur and instead an even higher peak existed at 40 Hz. Also as the high frequencies were approached, the photodiode FTF magnitudes tended towards each other, as opposed to the lower frequencies at which they dispersed. The frequency dependence of the photodiode FTF phase decreased linearly with respect to the imposed excitation frequency for all imposed acoustic velocity fluctuations except 35% and 40%, as shown in figure 4.88.

4.2.2 FTF, $\phi = 0.58$

For $\phi = 0.58$, the photodiode FTF magnitude for the radial swirler is plotted in two separate graphs, figures 4.89 and 4.90, based on the forced-excitation frequency. As with the case of $\phi = 0.56$ the 40 Hz case was highly dependent on the imposed velocity fluctuation. However this time the 40 Hz case showed increasing gain up to $U_{rms} = 25\%$, followed by a decreasing response until flame blow-off. The 80 Hz case consisted of increasing gain up to $U_{rms} = 40\%$ and then decreasing response until flame blow-off. The 100 Hz photodiode FTF magnitudes showed three regimes of increasing gain up to $U_{rms} = 25\%$, linear up to $U_{rms} = 40\%$ and non-linear response until blow-off. The 60 Hz photodiode FTF magnitude exhibited increasing gain up to $U_{rms} = 40\%$ and non-linear response until blow-off. For 120 Hz the linearity in response continued up to $U_{rms} = 30\%$, followed by non-linearity up to flame blow-off. As with the lower equivalence ratio experiment the 140 and 160 Hz cases demonstrated a strong non-linear response, starting at $U_{rms} = 30\%$ and 5\% respectively. The 180 and 200 Hz cases
displayed linear response until flame blow-off.

The phase of the photodiode FTF is shown in figure 4.91. The phases were dependent on the imposed velocity fluctuations as with the previous case.

The frequency dependence of the photodiode FTF magnitude and phase are illustrated in figures 4.92 and 4.93 respectively. The comparison of photodiode FTF magnitudes at low and high equivalence ratio (figures 4.87 and 4.92) highlighted the absence of a response peak at 60 Hz for the $\phi = 0.56$ case. There existed only one peak in the photodiode FTF magnitude at 80 Hz, and similarly to figure 4.87, the rise at 200 Hz was not observed. As the frequency of excitation increased the photodiode FTF magnitudes tended towards each other irrespective of the imposed acoustic velocity fluctuation and at lower frequencies of excitation the photodiode FTF magnitudes diverged, similarly to the $\phi = 0.56$ case (see figure 4.87).

### 4.2.3 Phase-Locked ICCD, $\phi = 0.56$

As explained in section 4.2.1 and shown in the photodiode FTF magnitude graphs in figures 4.84 and 4.85, the radial flame response unlike that of the axial flame exhibited regimes of increasing gain, linear, decreasing or a combination of these response. And unlike that of the axial flame the onset of each regime was dependent on the imposed frequency. The phase-locked Abel de-convoluted OH$^*$ chemiluminescence images that are shown in this section (see table 4.1) are therefore for the velocity fluctuations at which linear and non-linear flame response occurs.

<table>
<thead>
<tr>
<th>Frequency / Hz</th>
<th>Linear $U_{rms} / U%$</th>
<th>Non-linear $U_{rms} / U%$</th>
<th>Figure</th>
</tr>
</thead>
<tbody>
<tr>
<td>80</td>
<td>37%</td>
<td>45%</td>
<td>4.94</td>
</tr>
<tr>
<td>100</td>
<td>15% and 30%</td>
<td>N/A</td>
<td>4.95</td>
</tr>
<tr>
<td>120</td>
<td>15%</td>
<td>33%</td>
<td>4.96</td>
</tr>
<tr>
<td>160</td>
<td>10%</td>
<td>30%</td>
<td>4.97</td>
</tr>
<tr>
<td>180</td>
<td>10%</td>
<td>30%</td>
<td>4.98</td>
</tr>
<tr>
<td>200</td>
<td>10% and 20%</td>
<td>N/A</td>
<td>4.99</td>
</tr>
</tbody>
</table>

**Table 4.1:** The frequencies, velocity fluctuations and the corresponding flame responses for the Abel plots of the radial flame, $\phi = 0.56$. 
By comparing the steady state OH* chemiluminescence images at $\phi = 0.56$ in figure 4.83 to figures 4.94 to 4.99, one can clearly see that the imposed frequency and velocity fluctuations directly affect the flame length, shape and intensity distribution.

**Low Frequency Regimes**

The 80 and 100 Hz cases demonstrated the highest OH* chemiluminescence emission, flame area variation and the largest HIA, as the imposed velocity fluctuations increased. The Abel de-convoluted images of $U_{rms} = 37\%$ and $45\%$ presented in figure 4.94 show no significant difference, even though at $U_{rms} = 37\%$ the flame response is linear and at $U_{rms} = 45\%$ non-linear flame response occurs. The flame brush is continuous and the flame tip curves away from the burner’s central axis. The flame tip is also long enough to interact with the combustion chamber as shown in -234° and -279° of the $U_{rms} = 45\%$ in figure 4.94. The 100 Hz case also exhibits flame tip interaction with the combustion chamber and the flame length variation is smaller than that of 80 Hz case (see figure 4.95). Similarly to the 80 Hz the flame tip curves away from the burner’s central axis. Note that $U_{rms} = 15\%$ and $30\%$ were in the linear flame response regime.

**High Frequency Regimes**

At 120 Hz the immediate difference from that of 80 and 100 Hz cases is the reduction in the OH* chemiluminescence emission, even though the flame is subjected to the same velocity fluctuations. The flame length at low $U_{rms}$, however, is longer than in the 80 and 100 Hz cases. For example at -7° and $U_{rms} = 15\%$ (linear flame response) in figure 4.96, the flame brush approaches the combustion chamber wall with a constant cone angle, and after the height of approximately 20 mm, it continues downstream parallel to the wall. At $U_{rms} = 33\%$ (non-linear flame response) however, the flame becomes considerably shorter and the flame tip curves both towards and away from the burner’s central axis. At $U_{rms} = 33\%$ the flame is no longer continuous and the flame brush becomes detached from the burner outlet.
4.2. Forced-Excitation: Radial Swirler

The flame structures of the 160 and 180 Hz cases are similar (see figures 4.97 and 4.98 respectively). For example the continuous flame brush and interaction with the combustion chamber wall is observed at $U_{rms} = 10\%$, where both frequencies exhibit linear flame response. The flame brush curvature towards and away from the burner’s central axis and dis-continuation of the flame brush also occurred at $U_{rms} = 30\%$, where both frequencies exhibit non-linear flame response. At 160 and 180 Hz, however, unlike that at 120 Hz, the flame brush remained attached to the burner outlet.

The photodiode FTF magnitude of the 200 Hz case was linear irrespective of imposed velocity fluctuation (see figure 4.85). The $U_{rms} = 10\%$ and $20\%$ in figure 4.99 therefore correspond to linear flame response. The flame brush is continuous and attached to the burner outlet irrespective of the imposed velocity fluctuation. The flame only curves away from the burner’s central axis for $U_{rms} = 20\%$. The flame is also attached to the burner’s exit irrespective of $U_{rms}$.

### 4.2.4 Phase-Locked ICCD, $\phi = 0.58$

Comparison of the phase-locked Abel de-convoluted OH$^*$ chemiluminescence of the radial swirler flame between the two equivalence ratios is shown in figure 4.100 and 4.101 for 80 and 200 Hz for $U_{rms} = 40\%$ and $20\%$ respectively. The $U_{rms} = 40\%$ and $20\%$ corresponded to non-linear and linear flame response for both equivalence ratios. At 80 Hz the magnitude of the intensity distribution in the $\phi = 0.58$ case was marginally higher to that of $\phi = 0.56$, as expected. The flame shape, flame front curvature and the shape and location of the HIA remained predominantly the same for both equivalence ratio cases.

At 200 Hz, the flame length and shape remained the same irrespective of equivalence ratio (compare $-41^\circ$ in figure 4.101). The flame in both cases continued to curve away from the burner’s central axis and the OH$^*$ chemiluminescence of the $\phi = 0.58$ case was marginally higher to that of $\phi = 0.56$, as expected.
4.3 Self-Excitation

As explained in section 3.3.3 the forced-excitation OH\(^*\) chemiluminescence images were phase-locked to the phase of the excitation sine wave and were presented with respect to the phase of acoustic velocity fluctuation. But in the case of self-excitation the pressure in the combustion chamber had to be used for phase-locking (see section 3.3.4). This meant that the phases of the forced-excitation and self-excitation images were not directly comparable, but still one can see the evolution of the flame structure in the two methods and assess how faithfully the forced-excitation response compared to that of self-excitation.

4.3.1 Axial Swirler

The FFT spectra of the photodiode sensor for the axial swirler at \(\phi = 0.56\) and 0.91 are shown in figure 4.102. At \(\phi = 0.91\) the dominant self-excitation frequency was 200 Hz with a weak harmonic presence. At \(\phi = 0.56\), however, the dominant peak was observed at 144 Hz. As shown, the self-excitation peak at 144 Hz was not a pure tone and it consisted of broad band frequencies and three peaks at 20, 144 and 200 Hz, noting that the peak at 20 Hz had the same amplitude as the 144 Hz case.

The self-excitation at \(\phi = 0.91\) is shown in figure 4.105, where the flame structure differed considerably at various phases. The OH\(^*\) chemiluminescence intensity is very weak in some of the phases (0\(^\circ\) to 90\(^\circ\)) unlike the forced-excitation case (figure 4.106). The flame root also remained in a lifted position throughout the phases, similar to the forced-excitation experiments results with \(\phi = 0.91\).

4.3.2 Radial Swirler

The FFT spectra of the photodiode sensor for the radial swirler at \(\phi = 0.58\) and 0.91 are shown in figure 4.107. At \(\phi = 0.91\) the dominant self-excitation frequency was 144 Hz with a
4.4 Summary

4.4.1 Axial Swirler

- The agreement between the photodiode and ICCD FTF magnitude and phase is very good across the imposed frequency, velocity fluctuations and equivalence ratios.

For the photodiode and ICCD FTF magnitude at $\phi = 0.56$ compare figures 4.6 and 4.7 to figure 4.11. For the photodiode and ICCD FTF phase at $\phi = 0.56$ compare figure 4.8 to figure 4.12.

For the photodiode and ICCD FTF magnitude at $\phi = 0.48$ compare figures 4.16 and 4.17 to figure 4.21. For the photodiode and ICCD FTF phase at $\phi = 0.48$ compare figure 4.18 to figure 4.22.

- The magnitude and phase of the FTF is dependent on the imposed frequency and velocity fluctuation.
• Non-linear flame response is a function of equivalence ratio, imposed frequency and velocity fluctuation.

• Flame response consisted of linear and non-linear regimes irrespective of \( \phi \), except for the 60 Hz case at \( \phi = 0.56 \) where the flame response was only linear, and the 200 Hz cases at \( \phi = 0.56 \) and 0.48 where an increasing gain in the flame response was observed.

• The low frequency cases have the larger variation in flame area and \( \text{OH}^* \) chemiluminescence intensity, and smaller HIA, compared to the high frequency cases, irrespective of \( \phi \). The high frequency cases have smaller variation in the flame area and \( \text{OH}^* \) chemiluminescence intensity but larger variation in HIA compared to the low frequency cases, irrespective of \( \phi \).

• For \( \phi = 0.56 \), the flame tip for the low frequency cases does not substantially penetrate the ORZ, irrespective of the imposed velocity fluctuation. For the high frequency cases, the area of the flame tip that penetrates the ORZ increases with the imposed velocity fluctuation.

• The area of high tensile strain rates between the two recirculation zones in the ORZ becomes more shredded as the imposed frequency is increased.

• The ORZ of the examined frequencies at \( \phi = 0.56 \), mostly, exhibited two recirculation zones, irrespective of imposed frequency and velocity fluctuation.

• The IRZ of the examined frequencies at \( \phi = 0.56 \) exhibited two recirculation zones, irrespective of the imposed frequency and velocity fluctuations.

• For \( \phi = 0.56 \), the area of compressive strain rate increases with the imposed velocity fluctuation, irrespective of the imposed frequency.

• The equivalence ratio variation of the \( \phi = 0.56 \) case showed the following: at 60 Hz and high frequency cases the equivalence ratio variation did not dip below the steady state lean blow-off limit, the equivalence ratio variation of the low frequency cases however
dipped below the steady state lean blow-off limit as the imposed velocity fluctuation of the 30% was approached.

4.4.2 Radial Swirler

- The photodiode FTF magnitude response of the radial swirler flame exhibited increasing gain, linear and decreasing gain or a combination of these regimes for the low frequency cases, irrespective of equivalence ratio. The onset of each regime was frequency dependent.

- The photodiode FTF phases were dependent on the imposed velocity fluctuations.

- For the high frequency cases the photodiode FTF magnitudes consisted of linear and then non-linear regimes, irrespective of the equivalence ratio.

- The \textit{OH} \textsuperscript{*} chemiluminescence emission from the root of the radial flame were not captured and hence the ICCD FTF magnitude and phase were not calculated.

- The flame length and \textit{OH} \textsuperscript{*} chemiluminescence variation for the low frequency cases is significantly larger than the high frequency cases.

- The high frequency cases exhibit distinct flame front curvature towards and away from burner’s central axis. The flame length however remained predominantly the same, the flame brush is narrow and attached to the burner exit.
Figure 4.1: Blow-off limits of the axial swirler as function of imposed acoustic velocity fluctuation and frequency. Note the inverse dependence of the blow-off amplitude with respect to frequency.
Figure 4.2: Time-averaged and normalised ICCD OH* chemiluminescence images of the axial swirler flame for increasing equivalence ratios under steady state conditions. The colour bar indicates the intensity in arbitrary units.
4.5. Figures

Figure 4.3: Mean of time-averaged and normalised OH$^+$ chemiluminescence emissions for increasing equivalence ratios under steady state conditions, axial swirler.

Figure 4.4: Contour plot of velocity magnitude under steady state conditions, $\phi = 0.56$, axial swirler. The colour bar indicates the velocity in m/s. Note the vertical stripes at axial positions of -15 and 15 mm that occur due to laser reflections.
Figure 4.5: Stream plot under steady state conditions, $\phi = 0.56$, axial swirler. The flow field is not symmetrical around the burner’s central axis due laser reflections.

Figure 4.6: Photodiode FTF magnitude for low excitation frequencies, axial swirler, $\phi = 0.56$. Non-linearity occurs at $U_{rms} = 20\%$ except for $f_i = 40$ and $60$ Hz, which remain linear.
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Figure 4.7: Photodiode FTF magnitude for high excitation frequencies, axial swirler, $\phi = 0.56$. Non-linearity occurs at $U_{rms} = 20\%$, note the increasing gain in the response of the $f_i = 200$ Hz case up to $U_{rms} = 10\%$.

Figure 4.8: Photodiode FTF phase for all excitation frequencies, axial swirler, $\phi = 0.56$. The phase lag increases with increasing frequency.
Figure 4.9: Normalised global heat release for low excitation frequencies, axial swirler, $\phi = 0.56$. Non-linearity occurs at $U_{rms} = 20\%$ except for $f_i = 60\,Hz$, which remained linear, similar to that of figure 4.6.

Figure 4.10: Normalised global heat release for high excitation frequencies, axial swirler, $\phi = 0.56$. Non-linearity occurs at $U_{rms} = 20\%$, similar to that of figure 4.7.
Figure 4.11: ICCD FTF magnitude for all excitation frequencies, axial swirler, $\phi = 0.56$. Nonlinearity in 180 and 200 Hz is not apparent due to the $y$-axis scale. The $f_i = 60$ Hz response is linear.

Figure 4.12: ICCD FTF phase for all excitation frequencies, axial swirler, $\phi = 0.56$. The agreement with the photodiode FTF phase shown in figure 4.8 is very good.
Figure 4.13: Frequency dependence of the photodiode FTF magnitude, axial swirler, $\phi = 0.56$. The acoustic velocity fluctuation is shown in terms of percentage of the mean.

Figure 4.14: Frequency dependence of the photodiode FTF phase, axial swirler, $\phi = 0.56$. The acoustic velocity fluctuation is shown in terms of percentage of the mean.
Figure 4.15: Harmonic content of the photodiode signal, axial swirler, $\phi = 0.56$. Note the saturation of the fundamental signal and the rise of the harmonic content.

Figure 4.16: Photodiode FTF magnitude for low excitation frequencies, axial swirler, $\phi = 0.48$. Non-linear response starts at $U_{rms} = 15\%$ except for 40 Hz where it starts at $U_{rms} \approx 7.5\%$. 

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Figure 4.17: Photodiode FTF magnitude for high excitation frequencies, axial swirler, $\phi = 0.48$. The 160 and 180 Hz cases are linear, the 140 Hz case is non-linear starting at $U_{rms} = 5\%$. Note the increasing gain in the 200 Hz case up to $U_{rms} = 10\%$.

Figure 4.18: Photodiode FTF phase for all excitation frequencies, axial swirler, $\phi = 0.48$. The phase lag increases with increasing frequency similar to the $\phi = 0.56$ case presented in figure 4.8.
Figure 4.19: Normalised global heat release for low excitation frequencies, axial swirler, $\phi = 0.48$. Non-linear behaviour started at $U_{rms} = 15\%$ similar to that of figure 4.16.

Figure 4.20: Normalised global heat release for high excitation frequencies, axial swirler, $\phi = 0.48$. Non-linear behaviour started at $U_{rms} = 15\%$ except for 160 and 180 Hz which remained linear, similar to that of figure 4.17.
Figure 4.21: ICCD FTF magnitude for all excitation frequencies, axial swirler, \( \phi = 0.48 \). Non-linearity in 200 Hz case is not apparent due to y-axis scale. The 160 and 180 Hz cases are linear.

Figure 4.22: ICCD FTF phase for all excitation frequencies, axial swirler, \( \phi = 0.48 \). The agreement with the photodiode FTF phase shown in figure 4.18 is very good.
Figure 4.23: Frequency dependence of the photodiode FTF magnitude, axial swirler, $\phi = 0.48$. The acoustic velocity fluctuation is shown in terms of percentage of the mean.

Figure 4.24: Frequency dependence of the photodiode FTF phase, axial swirler, $\phi = 0.48$. The acoustic velocity fluctuation is shown in terms of percentage of the mean.
Figure 4.25: Abel plot, axial swirler, $f_i = 100$ Hz, $U_{rms} = 10\%$ and $U_{rms} = 30\%$, $\phi = 0.56$. The colour bar indicates the intensity in arbitrary units.
Figure 4.26: Stream and superimposed Abel plot, $f_i = 60$ Hz, $U_{rms} = 10\%$, $\phi = 0.56$, axial swirler.
Figure 4.27: Strain and superimposed stream plot, $f_i = 60$ Hz, $U_{rms} = 10\%$, $\phi = 0.56$, axial swirler.
Figure 4.28: Stream and superimposed Abel plot, $f_i = 60$ Hz, $U_{rms} = 20\%$, $\phi = 0.56$, axial swirler.
Figure 4.29: Strain and superimposed stream plot, $f_i = 60$ Hz, $U_{rms} = 20\%$, $\phi = 0.56$, axial swirler.
Figure 4.30: Stream and superimposed Abel plot, $f_i = 60$ Hz, $U_{rms} = 30\%$, $\phi = 0.56$, axial swirler.
Figure 4.31: Strain and superimposed stream plot, $f_i = 60$ Hz, $U_{rms} = 30\%$, $\phi = 0.56$, axial swirler.
Figure 4.32: Stream and superimposed Abel plot, $f_i = 80$ Hz, $U_{rms} = 10\%$, $\phi = 0.56$, axial swirler.
Figure 4.33: Strain and superimposed stream plot, $f_i = 80$ Hz, $U_{rms} = 10\%$, $\phi = 0.56$, axial swirler.
Figure 4.34: Stream and superimposed Abel plot, $f_i = 80$ Hz, $U_{rms} = 20\%$, $\phi = 0.56$, axial swirler.
Figure 4.35: Strain and superimposed stream plot, $f_i = 80$ Hz, $U_{rms} = 20\%$, $\phi = 0.56$, axial swirler.
Figure 4.36: Stream and superimposed Abel plot, $f_s = 80$ Hz, $U_{rms} = 30\%$, $\phi = 0.56$, axial swirler.
Figure 4.37: Strain and superimposed stream plot, $f_i = 80$ Hz, $U_{rms} = 30\%$, $\phi = 0.56$, axial swirler.
Figure 4.38: Stream and superimposed Abel plot, $f_i = 120$ Hz, $U_{rms} = 10\%$, $\phi = 0.56$, axial swirler.
Figure 4.39: Strain and superimposed stream plot, $f_s = 120$ Hz, $U_{rms} = 10\%$, $\phi = 0.56$, axial swirler.
Figure 4.40: Stream and superimposed Abel plot, $f_i = 120$ Hz, $U_{rms} = 20\%$, $\phi = 0.56$, axial swirler.
Figure 4.41: Strain and superimposed stream plot, $f_i = 120$ Hz, $U_{rms} = 20\%$, $\phi = 0.56$, axial swirler.
Figure 4.42: Stream and superimposed Abel plot, $f_i = 120$ Hz, $U_{rms} = 30\%$, $\phi = 0.56$, axial swirler.
Figure 4.43: Strain and superimposed stream plot, $f_i = 120$ Hz, $U_{rms} = 30\%$, $\phi = 0.56$, axial swirler.
Figure 4.44: Stream and superimposed Abel plot, $f_i = 144$ Hz, $U_{rms} = 10\%$, $\phi = 0.56$, axial swirler.
Figure 4.45: Strain and superimposed stream plot, $f_i = 144$ Hz, $U_{rms} = 10\%$, $\phi = 0.56$, axial swirler.
Figure 4.46: Stream and superimposed Abel plot, \( f_i = 144 \) Hz, \( U_{rms} = 20\% \), \( \phi = 0.56 \), axial swirler.
Figure 4.47: Strain and superimposed stream plot, $f_i = 144$ Hz, $U_{rms} = 20\%$, $\phi = 0.56$, axial swirler.
Figure 4.48: Stream and superimposed Abel plot, $f_i = 144$ Hz, $U_{rms} = 30\%$, $\phi = 0.56$, axial swirler.
Figure 4.49: Strain and superimposed stream plot, $f = 144$ Hz, $U_{rms} = 30\%$, $\phi = 0.56$, axial swirler.
Figure 4.50: Stream and superimposed Abel plot, $f_i = 160$ Hz, $U_{rms} = 10\%$, $\phi = 0.56$, axial swirler.
Figure 4.51: Strain and superimposed stream plot, \( f_i = 160 \text{ Hz}, U_{rms} = 10\% \), \( \phi = 0.56 \), axial swirler.
Figure 4.52: Stream and superimposed Abel plot, $f_i = 160$ Hz, $U_{rms} = 20\%$, $\phi = 0.56$, axial swirler.
Figure 4.53: Strain and superimposed stream plot, \( f_i = 160 \text{ Hz}, U_{rms} = 20\%, \phi = 0.56 \), axial swirler.
Figure 4.54: Stream and superimposed Abel plot, $f_i = 160$ Hz, $U_{rms} = 30\%$, $\phi = 0.56$, axial swirler.
Figure 4.55: Strain and superimposed stream plot, $f_i = 160 \text{ Hz}$, $U_{rms} = 30\%$, $\phi = 0.56$, axial swirler.
Figure 4.56: Stream and superimposed Abel plot, \( f_i = 180 \) Hz, \( U_{rms} = 10\% \), \( \phi = 0.56 \), axial swirler.
Figure 4.57: Strain and superimposed stream plot, $f_i = 180$ Hz, $U_{rms} = 10\%$, $\phi = 0.56$, axial swirler.
Figure 4.58: Stream and superimposed Abel plot, $f_i = 180$ Hz, $U_{rms} = 20\%$, $\phi = 0.56$, axial swirler.
Figure 4.59: Strain and superimposed stream plot, $f_i = 180$ Hz, $U_{rms} = 20\%$, $\phi = 0.56$, axial swirler.
Figure 4.60: Stream and superimposed Abel plot, $f_i = 180$ Hz, $U_{rms} = 30\%$, $\phi = 0.56$, axial swirler.
Figure 4.61: Strain and superimposed stream plot, $f_i = 180$ Hz, $U_{rms} = 30\%$, $\phi = 0.56$, axial swirler.
Figure 4.62: Stream and superimposed Abel plot, $f_i = 200$ Hz, $U_{rms} = 10\%$, $\phi = 0.56$, axial swirler.
Figure 4.63: Strain and superimposed stream plot, $f_i = 200$ Hz, $U_{rms} = 10\%$, $\phi = 0.56$, axial swirler.
Figure 4.64: Stream and superimposed Abel plot, $f_i = 200$ Hz, $U_{rms} = 20\%$, $\phi = 0.56$, axial swirler.
Figure 4.65: Strain and superimposed stream plot, $f_i = 200$ Hz, $U_{rms} = 20\%$, $\phi = 0.56$, axial swirler.
Figure 4.66: Stream and superimposed Abel plot, $f_i = 200$ Hz, $U_{rms} = 30\%$, $\phi = 0.56$, axial swirler.
Figure 4.67: Strain and superimposed stream plot, $f_i = 200$ Hz, $U_{rms} = 30\%$, $\phi = 0.56$, axial swirler.
Figure 4.68: Variation of the OH$^\ast$ chemiluminescence intensity calculated from phase-locked, ensemble-averaged and normalised ICCD images of the axial swirler flame for low frequency regimes, $U_{rms} = 10\%$, $\phi = 0.56$. According to figure 4.6 the flame response is in the linear region for $U_{rms} = 10\%$.

Figure 4.69: Variation of the OH$^\ast$ chemiluminescence intensity calculated from phase-locked, ensemble-averaged and normalised ICCD images of the axial swirler flame for low frequency regimes, $U_{rms} = 30\%$, $\phi = 0.56$. According to figure 4.6 the flame response is in the non-linear region for $U_{rms} = 30\%$ except for the 60 Hz case.
Figure 4.70: Variation of the OH$^*$ chemiluminescence intensity calculated from phase-locked, ensemble-averaged and normalised ICCD images of the axial swirler flame with increasing acoustic velocity fluctuation, $f_i = 80$ Hz, $\phi = 0.56$.

Figure 4.71: Variation of the OH$^*$ chemiluminescence intensity calculated from phase-locked, ensemble-averaged and normalised ICCD images of the axial swirler flame for high frequency regimes, $U_{rms} = 10\%$, $\phi = 0.56$. According to figure 4.7 the flame response is in the linear region for $U_{rms} = 10\%$. 

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Figure 4.72: Variation of the OH\textsuperscript{*} chemiluminescence intensity calculated from phase-locked, ensemble-averaged and normalised ICCD images of the axial swirler flame for high frequency regimes, $U_{rms} = 30\%$, $\phi = 0.56$. According to figure 4.7 the flame response is in the non-linear region for $U_{rms} = 30\%$.

Figure 4.73: Height of left hand side (LHS) and right hand side (RHS) ORZ vortex centre from the burner face and the height at which vertical OH\textsuperscript{*} chemiluminescence distribution reached a maximum, axial swirler, $\phi = 0.56$, $f_i = 160$ Hz, $U_{rms} = 30\%$. Two consecutive cycles are shown in this graph. Refer to figure 3.26 for definition of H.
Figure 4.74: Height of left hand side (LHS) ORZ vortex centre from the burner face and the height at which vertical OH\(^*\) chemiluminescence distribution reached a maximum, axial swirler, \(\phi = 0.56\), \(U_{rms} = 30\%\). Two consecutive cycles are shown in this graph. Refer to figure 3.26 for definition of H.

Figure 4.75: Equivalence ratio fluctuations, axial swirler, \(\phi = 0.56\), \(f_i = 60\) Hz, \(U_{rms} = 10\%, 20\%\) and 30\%. The steady state lean blow-off limit is 0.33 and shown with the dotted line.
Figure 4.76: Equivalence ratio fluctuations, axial swirler, $\phi = 0.56$, $f_i = 120$ Hz, $U_{rms} = 10\%$, 20\% and 30\%. The steady state lean blow-off limit is 0.33 and shown with the dotted line.

Figure 4.77: Equivalence ratio fluctuations, axial swirler, $\phi = 0.56$, $f_i = 180$ Hz, $U_{rms} = 10\%$, 20\% and 30\%. The steady state lean blow-off limit is 0.33 and shown with the dotted line.
Figure 4.78: Abel plot, axial swirler, $f_i = 80$ Hz, $\phi = 0.56$ and $\phi = 0.48$, $U_{rms} = 25\%$. The colour bar indicates the intensity in arbitrary units.
Figure 4.79: Abel plot, axial swirler, $f_i = 200$ Hz, $\phi = 0.56$ and $\phi = 0.48$, $U_{rms} = 25\%$. The colour bar indicates the intensity in arbitrary units.
Figure 4.80: Variation of the OH\textsuperscript{*} chemiluminescence intensity calculated from phase-locked, ensemble-averaged and normalised ICCD images of the axial swirler flame for different equivalence ratios, $U_{rms} = 10\%$.

Figure 4.81: Variation of the OH\textsuperscript{*} chemiluminescence intensity calculated from phase-locked, ensemble-averaged and normalised ICCD images of the axial swirler flame for different equivalence ratios, $U_{rms} = 25\%$.
Figure 4.82: Blow-off limits of the radial swirler as a function of imposed acoustic velocity fluctuation and frequency. Note the scatter of the blow-off limit with respect to imposed frequency as opposed to that of axial swirler illustrated in figure 4.1.
Figure 4.83: Time-averaged and normalised ICCD OH* chemiluminescence images of the radial swirler flame for increasing equivalence ratios under steady state conditions. The colour bar indicates the intensity in arbitrary units.
Figure 4.84: Photodiode FTF magnitude for low excitation frequencies, radial swirler, $\phi = 0.56$.

Figure 4.85: Photodiode FTF magnitude for high excitation frequencies, radial swirler, $\phi = 0.56$. 

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Figure 4.86: Photodiode FTF phase for all excitation frequencies, radial swirler, $\phi = 0.56$.

Figure 4.87: Frequency dependence of the photodiode FTF magnitude, radial swirler, $\phi = 0.56$. 
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**Figure 4.88:** Frequency dependence of the photodiode FTF phase, radial swirler, $\phi = 0.56$.

**Figure 4.89:** Photodiode FTF magnitude for low excitation frequencies, radial swirler, $\phi = 0.58$. 
Figure 4.90: Photodiode FTF magnitude for high excitation frequencies, radial swirler, \( \phi = 0.58 \).

Figure 4.91: Photodiode FTF phase for all excitation frequencies, radial swirler, \( \phi = 0.58 \).
Figure 4.92: Frequency dependence of the photodiode FTF magnitude, radial swirler, $\phi = 0.58$.

Figure 4.93: Frequency dependence of the photodiode FTF phase, radial swirler, $\phi = 0.58$. 

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Figure 4.94: Abel plot, radial swirler, $f_i = 80$ Hz, $U_{rms} = 37\%$ and $45\%$, $\phi = 0.56$. The colour bar indicates the intensity in arbitrary units.
Figure 4.95: Abel plot, radial swirler, $f_i = 100$ Hz, $U_{rms} = 15\%$ and $30\%$, $\phi = 0.56$. The colour bar indicates the intensity in arbitrary units.
Figure 4.96: Abel plot, radial swirler, $f_i = 120$ Hz, $U_{rms} = 15\%$ and 33\%, $\phi = 0.56$. The colour bar indicates the intensity in arbitrary units.
Figure 4.97: Abel plot, radial swirler, $f_i = 160$ Hz, $U_{rms} = 10\%$ and $30\%$, $\phi = 0.56$. The colour bar indicates the intensity in arbitrary units.
Figure 4.98: Abel plot, radial swirler, $f_i = 180$ Hz, $U_{rms} = 10\%$ and $30\%$, $\phi = 0.56$. The colour bar indicates the intensity in arbitrary units.
Figure 4.99: Abel plot, radial swirler, $f_i = 200$ Hz, $U_{rms} = 10\%$ and $20\%$, $\phi = 0.56$. The colour bar indicates the intensity in arbitrary units.
Figure 4.100: Abel plot, radial swirler, $f_i = 80$ Hz, $\phi = 0.56$ and $\phi = 0.58$, $U_{rms} = 40\%$. The colour bar indicates the intensity in arbitrary units.
Figure 4.101: Abel plot, radial swirler, $f_i = 200 \text{ Hz}$, $\phi = 0.56$ and $\phi = 0.58$, $U_{rms} = 20\%$. The colour bar indicates the intensity in arbitrary units.
Figure 4.102: Power spectra of the photodiode for $\phi = 0.91$ (top graph, dominant peak at 200 Hz) and $\phi = 0.56$ (lower graph, dominant peak at 144 Hz) for the axial swirler under self-excitation conditions.
Figure 4.103: Phase-locked, ensemble-averaged and normalised OH* chemiluminescence images under self-excitation, axial swirler, $f_i = 144$ Hz, $\phi = 0.56$. The phase is with respect to the combustion chamber pressure.
Figure 4.104: Phase-locked, ensemble-averaged and normalised \( \text{OH}^* \) chemiluminescence images under forced-excitation, axial swirler, \( f_i = 144 \text{ Hz}, \ U_{\text{rms}} = 10\% \) and \( 25\% \), \( \phi = 0.56 \). The phase is with respect to the imposed velocity fluctuation.
Figure 4.105: Phase-locked, ensemble-averaged and normalised OH\(^*\) chemiluminescence images under self-excitation, axial swirler, \(f_i = 200\) Hz, \(\phi = 0.91\). The phase is with respect to the combustion chamber pressure.
Figure 4.106: Phase-locked, ensemble-averaged and normalised OH$^*$ chemiluminescence images under forced-excitation, axial swirler, $f_i = 200$ Hz, $U_{rms} = 10\%$ and $30\%$, $\phi = 0.91$. The phase is with respect to the imposed velocity fluctuation.
Figure 4.107: Power spectra of the photodiode for $\phi = 0.91$ (top graph, dominant peak at 144 Hz) and $\phi = 0.58$ (lower graph, dominant peak at 120 Hz) for the radial swirler under self-excitation conditions.
Figure 4.108: Phase-locked, ensemble-averaged and normalised OH$^\ast$ chemiluminescence images under self-excitation, radial swirler, $f_i = 120$ Hz, $\phi = 0.58$. The phase is with respect to the combustion chamber pressure.
Figure 4.109: Phase-locked, ensemble-averaged and normalised OH$^*$ chemiluminescence images under forced-excitation, radial swirler, $f_i = 120$, $U_{rms} = 10\%$ and $30\%$, $\phi = 0.58$. The phase is with respect to the imposed velocity fluctuation.
Figure 4.110: Phase-locked, ensemble-averaged and normalised OH\textsuperscript{*} chemiluminescence images under self-excitation, radial swirler, $f_i = 144$ Hz, $\phi = 0.91$. The phase is with respect to the combustion chamber pressure.
Chapter 5

Discussion
5.1 Response Analysis

The magnitude (Ψ) and effective time delay (τ_{eff}) of the flame transfer function, as defined in section 1.7, equation 1.17, determines whether a particular mode is damped or amplified. It is believed that the acoustic processes remain linear in the low Mach number limit; therefore the instability mechanisms are limited by the non-linearity in the flame response, [16, 34, 58]. If the FTF magnitude remained constant with respect to the imposed velocity fluctuation, any unstable mode would grow until it is limited by non-linear effects. As shown in the graphs of the FTF magnitude, such behaviour does not usually occur due to the non-linearities in the flame response.

Lawn and Polifke [49] already hypothesised the existence of two mechanisms which are responsible for instability of the axial swirler used in this study. These mechanisms are:

- Fluctuations which originate from vortex shedding (i.e. coherent vortices and shear flow) and are associated with a time delay in which they are convected from the origin (pathway 2b in figure 2.1).

- Equivalence ratio fluctuations which results from perturbations in the air (pathway 3a in figure 2.1).

The model discussed by Lawn and Polifke [49] mathematically described the flame response (e.g. the gain Ψ) in terms of the above mechanisms. The model neglected the effects of acoustic pressure fluctuations on the equivalence ratio perturbations and the fluctuations in the density of the unburned premixture, because of the very small Mach numbers involved. Lawn and Polifke [49] also neglected the effects of local and global swirl fluctuations due to the fixed angle of the swirler and the short distance between the vanes and the burner mouth. The model was solved analytically and the result is shown in figure 5.1. The solution suggests that the equivalence ratio fluctuation is the main mechanism contributing to the total gain for the low frequencies (< 120 Hz). For high frequencies (> 120 Hz) however, the effect of
coherent vortices out-weighs the equivalence ratio mechanism and dominates the total flame response.

Polifke and Lawn [68] reanalysed the comprehensive flame model proposed by Lawn and Polifke [49] and modified it so that the correct limiting behaviour at low frequency was obtained. The limiting constraint requires the FTF magnitude to tend to zero as the frequency tends to zero, for any combustion system with a constant fuel mass flow rate.

The effect of these two mechanisms (equivalence ratio and vortex shedding) on non-linear flame response was also observed by Lohrmann et al. [59] for a practical (i.e. imperfect) premixture. Lohrmann et al. [59] also investigated a perfect premixture and observed that the vortex shedding started when the imposed frequency passed a critical value. However, the effect of the amplitude of velocity fluctuations on the onset of vortex shedding was not investigated. The total response due to the equivalence ratio mechanism however disappeared, giving rise to the conclusion that generating a perfect premixture is an ideal method to test the validity of flame response instability due to equivalence ratio or vortex shedding mechanisms.

The mechanisms described by Lawn and Polifke [49] are also believed to be responsible for the radial swirler instabilities; non-linear flame response however occurs only because of the ‘equivalence ratio mechanism’. Non-linear flame response involves the OH$^+$ chemiluminescence (an indication of heat release rate) ceasing to increase proportionally with the imposed velocity fluctuations, [55]. This is due to flame extinction and / or annihilation events. Flame extinction occurs when the local and / or global equivalence ratio becomes so low that the flame is incapable of continuing the chemical reaction. According to Sardi et al. [74], global extinction occurs if the amplitude of the forcing exceeds a critical limit, provided the duration is longer than a critical timescale. For lean premixed flames the time required for global extinction is longer than 60 ms for velocity fluctuations smaller than 50%, [74]. Therefore global extinction does not occur in the current studies.

The flame annihilation events usually occur when the flow field strain rates cause flame stretch to the extent of prohibiting combustion. Flame annihilation (also known as flame
5.2. Equivalence Ratio Mechanism

(Continued)

destruction) could also occur when different flame fronts are brought together with aid of vortex, as seen in the work of Balachandran [1]. In this study it is difficult to distinguish whether the non-linearity is the result of local flame extinction or annihilation events. It is also important to note that saturation between the heat release and velocity fluctuation is not a necessity for non-linear flame response, [66]. Turbulent flame wrinkling could also cause saturation of the flame response, [55]. This however occurs for flows with very high turbulent intensities, unlike the flows in the current study.

The frequencies for linear and non-linear flame response are summarised in table 5.1:

<table>
<thead>
<tr>
<th>Swirl geometry</th>
<th>Axial</th>
<th>Radial</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equivalence ratio / no units</td>
<td>0.56</td>
<td>0.56</td>
</tr>
<tr>
<td>Linear response</td>
<td>60 Hz</td>
<td>60 Hz</td>
</tr>
<tr>
<td>Non-linear response (Equivalence ratio mechanism)</td>
<td>80, 100, 120 Hz</td>
<td>80, 100, 120 Hz</td>
</tr>
<tr>
<td>Non-linear response (Vortex shedding mechanism)</td>
<td>160, 180, 200 Hz</td>
<td>160, 180, 200 Hz</td>
</tr>
</tbody>
</table>

Table 5.1: Summary of the frequencies at which linear and non-linear mechanisms occur. Note that the onset of non-linear flame response is dependent on the imposed velocity fluctuations.

5.2 Equivalence Ratio Mechanism

As explained in section 3.1.1, the axial swirler fuel system was ‘stiff’ as deduced by the in-phase relationship between the pressure in the fuel distribution plenum and acoustic fluctuations outside it, up to a frequency of 200 Hz, [47]. Therefore, alternate patterns of fuel rich and fuel lean pockets reach the flame zone with a time delay. This time delay leads to fluctuations in $\text{OH}^*$ chemiluminescence which are proportional to the heat release rate perturbations. The work of Stow and Dowling [85] shows that equivalence ratio fluctuations and convection of

\[ \text{The ‘time delay’ is 8 ms and it is calculated from the mean speed of the flow and the distance from the axial swirler fuel distribution holes to the burner mouth (figure 3.2). It is different from the ‘effective time delay’.} \]
these fluctuations to the flame front cause non-linear flame response.

The cyclic variation of the equivalence ratio (see figure 4.76 for 120 Hz) calculated from PIV 2.3 mm above the burner outlet shows that the skewed variation of the $\phi$ for the 30% case dips below the lean limit of extinction for frequencies of 80 and 120 Hz. The heat release variation shows the highest amplitude for the cases that are subjected to the equivalence ratio mechanism (see 80 and 120 Hz in figure 4.69) and supports the hypothesis of large variations in the global $\phi$. Also larger variation in flame length and size compared to 144, 160, 180 and 200 Hz cases is another indication of substantial equivalence ratio fluctuations.

In a subsidiary experiment, a perfect premixture ($\phi = 0.56$) was fed to the axial swirler in order to assess whether or not the equivalence ratio fluctuations caused non-linear flame response. The perfect premixture removed the alternate patterns of fuel rich and fuel lean pockets within the flame zone, [76]. The perfect premixture was obtained by using a mixing chamber upstream of the forced-excitation chamber. The mixing chamber was connected to the forced-excitation chamber through the high acoustic impedance inlet. This blocked the disturbances from the air supply system (see figure 3.9), ensuring a constant flow from the mixing chamber. Under these conditions the photodiode FTF magnitude did not respond at all until blow-off for the frequencies shown in the ‘equivalence ratio mechanism’ group of table 5.1. This suggests that non-linear flame response resulted from local flame extinction and / or annihilation events due to variations in the local and / or global equivalence ratio.

### 5.3 Vortex Shedding Mechanism

A vortex is defined as any circulatory motion of fluid particles around a common centre. Like momentum, vorticity is a transportable quantity and once generated it can be convected by the local velocity and diffused by the action of viscosity, [12]. The immediate difference between the frequency group affected by a ‘vortex shedding mechanism’ in table 5.1 to other frequency groups is the penetration of the flame tip into the ORZ. The vortices are shed from the lip of
the burner at the phase of the cycle when the velocity is zero but acceleration of the fluid is at a maximum, [40]. Kulsheimer and Buchner [40] deduced the onset of vortex shedding into the ORZ based on imposed velocity fluctuation and Strouhal number $St$, as shown in figure 5.2. The Strouhal number is described in section 2.1.2, equation 2.1. Therefore the vortex shedding into the ORZ in the current studies should occur when the velocity fluctuations are much larger than the limits specified in table 5.2.

<table>
<thead>
<tr>
<th>Frequency / Hz</th>
<th>60</th>
<th>80</th>
<th>100</th>
<th>120</th>
<th>160</th>
<th>180</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>$St_{axial}$</td>
<td>0.06</td>
<td>0.08</td>
<td>0.10</td>
<td>0.12</td>
<td>0.16</td>
<td>0.18</td>
<td>0.20</td>
</tr>
<tr>
<td>$U_{rms,axial}$</td>
<td>36%</td>
<td>32%</td>
<td>26%</td>
<td>25%</td>
<td>21%</td>
<td>20%</td>
<td>19%</td>
</tr>
<tr>
<td>$St_{radial}$</td>
<td>0.78</td>
<td>1.04</td>
<td>1.30</td>
<td>1.56</td>
<td>2.08</td>
<td>2.34</td>
<td>2.60</td>
</tr>
<tr>
<td>$U_{rms,radial}$</td>
<td>10%</td>
<td>10%</td>
<td>10%</td>
<td>10%</td>
<td>10%</td>
<td>10%</td>
<td>10%</td>
</tr>
</tbody>
</table>

Table 5.2: Velocity fluctuations required for the onset of vortex shedding into ORZ according to Kulsheimer and Buchner [40]. The mean velocity for the axial and radial swirlers at the exit was 4 and 2 m/s, respectively.

The limits specified in table 5.2 agree very well with the onset of vortex shedding into the ORZ in the current study. For example at 60 Hz, a velocity fluctuation much larger than 36% is required for vortex shedding into the ORZ. This does not occur at 10%, 20% and 30% as shown in figures 4.26, 4.28 and 4.30. For 200 Hz however, vortex shedding occurs for velocity fluctuations greater than 19%, as demonstrated in figures 4.64 and 4.66, for 20% and 30% respectively.

The two important shape changes of the ORZ are shown in figure 5.3 for the axial swirler. In figure 5.3A the flame tip does not penetrate the ORZ. However in figure 5.3B the two distinctive vortices create a discontinuity between them, enabling the flame to cross over the shear layer into the ORZ, where it penetrates the lower vortex. Subsequently the lower vortex entrains the flame further before extinguishing and / or annihilating the flame tip as it moves downstream, with shear rates greater than $1000 \text{s}^{-1}$ and a maximum of $2500 \text{s}^{-1}$. This agrees very well with the work of Law and Sung [43] which has shown that a tensile strain rate of $1800 \text{s}^{-1}$ is required to extinguish a turbulent premixed methane flame. It is also important to note that extinction requires that sufficiently large strain rates be imposed for sufficiently
long residence times, [19]. The extinguishing and / or annihilating events caused by the flame-vortex interaction cause a non-proportional relationship between the heat release rate and velocity fluctuations, and lead to non-linear flame response.

The flame-vortex interaction is therefore not dependent on the equivalence ratio variation and should not be affected if a perfect premixture is used. As with the previous case, a perfect premixture ($\phi = 0.56$) was used for the 160, 180 and 200 Hz cases to verify the flame-vortex interaction hypothesis. Under these conditions the photodiode FTF magnitude demonstrated non-linear flame response for the frequencies shown in the ‘vortex shedding mechanism’ group of table 5.1. The onset of non-linear flame response for a perfect premixture was within 5% (points) of the amplitude for a practical (i.e. imperfect) premixture.

The cyclic variation of the equivalence ratio for the 160, 180 and 200 Hz cases became more sinusoidal than for the lower frequencies (see figure 4.77 for 180 Hz) and did not dip below the lean limit of extinction even for the 30% velocity fluctuations. The heat release variation shows the lowest amplitude for the cases that are subjected to the shedding mechanism (see 160, 180 and 200 Hz in figure 4.72). Also the much smaller variation in flame length and size compared to 80 and 120 Hz cases suggests that the equivalence ratio fluctuation is not the critical instability mechanism.

Such interaction between the flame and the ORZ was also observed by Cabot et al. [9], where a change in $\phi$ affected the flame shape and flame stability. With the unstable $\phi$, Cabot et al. [9] observed that the flame penetrated into the ORZ, with extinction and re-ignition. Further reduction in $\phi$ caused extinction in the ORZ. The work of Cabot et al. [9] also agrees very well with the steady state flame images presented in figure 4.2. Flame penetration into the ORZ did not occur at low values of $\phi$ (i.e. $\phi < 0.41$) and therefore the flame instability did not occur. Flame-vortex interaction was also observed by Schildmacher and Koch [77] and Huang et al. [27] for a turbulent premixed flame, and by Durox et al. [20] for an unconfined laminar premixed flame.

Smaller multipoint vortical structures were also seen in the instantaneous streamline plots
in the current study (not shown) for both reacting and cold flow conditions. Ji and Gore [31] discuss the importance of these vortical structures. However the degree of their ultimate influence could not be assessed since the PIV equipment captured the images several cycles apart and therefore the streamlines were not truly consecutive, [6]. It is therefore a much better practice to concentrate on the ensemble images until such problems are solved. Stohr and Meier [83] also observed variations in the size and movement of the IRZ at different phases of the self-excitation cycle, and disregarded the helical vortices in the instantaneous images. The effect of the PVC was also disregarded since it is destroyed under reacting conditions, [73, 76, 78, 83].

5.4 Forced-Excitation: Axial Swirler

Each frequency group presented in table 5.1 is discussed in more detail in the following section, using the phase-averaged Abel de-convoluted, stream and strain plots shown in section 4.1.3.

5.4.1 60 Hz

The photodiode and ICCD FTF magnitudes demonstrated a linear behaviour for 60 Hz at $\phi = 0.56$ (see figures 4.6 and 4.11 respectively). Therefore one should examine the simultaneous phase-locked ICCD-PIV plots for an event or series of events that are distinctly different from those at other frequencies. Particularly the analysis should concentrate on the absence of the events which cause the non-linear flame response for other frequencies, i.e. equivalence ratio fluctuations which are large enough for extinction and flame-vortex interaction.

First the 60 Hz case is inspected for instability caused by equivalence ratio fluctuation. When comparison is made with 80 and 120 Hz cases, the immediate difference is the low amplitude of the mean OH* chemiluminescence variation (see figures 4.68 and 4.69). Further evidence is provided by the equivalence ratio variation in figure 4.75, which does not dip below the steady state lean blow-off limit and does not reach stoichiometric conditions even
at $U_{rms} = 30\%$, because of the sinusoidal variation.

In the second step the 60 Hz is inspected for instability caused by flame-vortex interaction. When comparison is made with 160, 180 and 200 Hz, the lack of flame penetration into the ORZ for the 60 Hz case becomes apparent. This is due to the fact that no new vortex is shed into the ORZ throughout the cycle, irrespective of the imposed velocity fluctuation in agreement with the work of Kulshheimer and Buchner [40]. This results in continuous high tensile strain rates of order of $1000 \text{s}^{-1}$ prohibiting the flame tip penetration and extinction and / or annihilation in the ORZ (see figures 4.27, 4.29 and 4.31). In two phases of $-95^\circ$ and $-139^\circ$ at 60 Hz, a very small area of the flame tip enters the ORZ, but this is insignificant compared to those that occur for frequencies of 144, 160, 180 and 200 Hz with velocity fluctuations greater than 20%. The cumulative effect of the absence of the local flame extinction and / or annihilation events, by flame-vortex interaction or equivalence ratio fluctuation, is a flame response that remains linear until blow-off at $U_{rms} \approx 42.5\%$.

The 60 Hz case does exhibit non-linear flame response at $\phi = 0.48$ (see figures 4.16 and 4.21), unlike at $\phi = 0.56$. This is however to be expected and can be explained by analysis of the equivalence ratio fluctuations of the 60 Hz with $\phi = 0.56$. It is very clear in figure 4.75 that the mean of each $\phi$ variation is 0.56 (the steady state $\phi$). Since the only change from $\phi = 0.56$ to 0.48 is that of reducing the fuel flow rate, which is insignificant compared to the air flow rate, the mean of the $\phi$ variation can be reduced from 0.56 to 0.48 as shown in figure 5.4. The $\phi$ now dips below the steady state lean blow-off limit at $U_{rms} = 30\%$.

It is hypothesised that at $\phi = 0.48$ the tensile strain rates in the flow field are strong enough to cause local flame extinction and / or annihilation, even though the flame tip does not penetrate the ORZ. The evidence for this is in the narrowing of the flame brush (not shown). The flame front is incapable of surviving the high tensile strain rates in the shear layer due to reduction in the steady state $\phi$, resulting in non-linear flame response.

It is also noted that the phase-locked OH$^*$ chemiluminescence intensity variation is slightly skewed in figure 4.69, yet the 60 Hz flame response for $\phi = 0.56$ is linear. The variation at
\( \phi = 0.48 \) also exhibited this. Therefore the skewed variation occurs irrespective of linear or non-linear flame response.

The flame shape is a function of the IRZ and ORZ interactions, which are influenced by the cyclic movement of the annular jet. The flame cannot enter the ORZ due to high tensile strain rates, as explained, and as long as the lower extent of the IRZ reaches the burner exit and the IRZ consists of one recirculation zone, the flame cannot enter the IRZ, and therefore remains in the jet between the IRZ and ORZ (see -299° in figures 4.26, 4.28 and 4.30). The inability of the flame to penetrate the IRZ is due to accumulation of the post-combustion products and long residence time of the IRZ, [23]. However as the new premixture is pushed upwards by increasing velocity, the annular jet undergoes a change of shape which manifests itself as an interaction between two recirculation zones within the IRZ (see -49° in figure 4.26). The most dominant effect of the annular jet shape change is the movement of the flame closer to the central burner axis at low velocity fluctuations (see -94° in figure 4.26), and the flattening of the flame at high velocity fluctuations due to flame tip penetration into the ORZ (see -49° in figure 4.30).

### 5.4.2 80, 100 and 120 Hz

As determined from photodiode and ICCD FTF magnitudes, the case of 80 Hz for \( \phi = 0.56 \) at \( U_{rms} = 30\% \) is in the non-linear flame response regime (see figures 4.6 and 4.11 respectively), and hence the \( U_{rms} = 30\% \) images should be compared with those at lower velocity fluctuations for determination of the cause of the non-linear flame response. Like the 60 Hz case, the flame excited at 80 Hz remains predominantly outside the ORZ, yet unlike the 60 Hz case, the 80 Hz case exhibits non-linear flame response. Again some minor penetrations into the ORZ are observed at -123° and -168° (see figure 4.34), but these are not significant and not the cause of non-linear flame response. The flame area that penetrates the ORZ is much smaller than in the 144, 160, 180 and 200 Hz cases. Note that the vertical stripes of apparent high and low strain rates at an axial position of -15 mm are the result of laser reflections.
Inspection of the phase-locked OH* chemiluminescence variation (figures 4.68 and 4.69) shows that the 80 Hz case has the highest amplitude compared to rest of the examined frequencies. This provides the first evidence that the 80 Hz is subjected to high variations of the equivalence ratio. Further evidence is provided by the equivalence ratio variation at $U_{rms} = 30\%$ (not shown here) which dips below the steady state lean blow-off limit. The equivalence ratio varies so as to exceed stoichiometric conditions and reaches $\phi = 1.2$ for $-258^\circ$. Such high variation in equivalence ratio must be responsible for local flame extinction and / or annihilation events and non-linear flame response.

The very rich premixture is the reason behind the movement of the flame root into the burner exit. This is to be expected since under lean premixed conditions, the laminar burning velocity varies rapidly with equivalence ratio. Small fluctuations in equivalence ratio can therefore lead to large fluctuations in laminar burning velocity. Turbulent lean premixed flames are also sensitive to the variations in equivalence ratio. An increase in equivalence ratio causes an increase in the turbulent burning velocity. At low air flow in the cycle the flame front therefore moves upstream as the equivalence ratio is increased.

The ORZ of the 120 Hz cases at $\phi = 0.56$ also exhibits two vortices irrespective of the imposed velocity fluctuation. The flame tip however does not cross the shear layer into the ORZ in the same way as in the 80 Hz case, yet at $U_{rms} = 30\%$ the flame response is non-linear. The equivalence ratio variation (figure 4.76) at $U_{rms} = 30\%$ dips below the steady state lean blow-off limit at $-112^\circ$ due to skewed variation. This is also the phase at which the flame root has the greatest height from the burner exit (see figure 4.42). The $\phi$ variation exceeds the stoichiometric conditions $180^\circ$ later at $-292^\circ$, reaching a value of $\phi \approx 1.1$, and causing the flame to penetrate into the burner exit. The penetration into the burner exit continues and even increases in the following phase ($-337^\circ$).

Although the stream and strain plots for 100 Hz at $\phi = 0.56$ were not available, other features such as the flame shape (figure 4.25) and phase-locked OH* chemiluminescence intensity variation (figures 4.68 and 4.69) exhibited striking similarities to those of the 80 and 120 Hz
cases and therefore place the 100 Hz case firmly in this group.

Following the analysis of $\phi = 0.56$, figure 4.76 is modified to that shown in figure 5.5 for $\phi = 0.48$. For $U_{rms} = 20\%$, the $\phi$ variation dips below the steady state lean blow-off limit. The reduction in $\phi$ also causes the narrowing of the flame brush in comparison to $\phi = 0.56$, similar to the 60 Hz case at $\phi = 0.48$. For example, compare -213° for the two equivalence ratios of the 80 Hz case in figure 4.78. The narrowing for the $\phi = 0.48$ case is the result of restriction on the flame front movement since the flame can no longer sustain the high tensile strain rates in the shear layer. This lead leads to extinction and / or annihilation events when the flame approaches the shear layer. Non-linear flame response therefore occurs at lower velocity fluctuations compared to the $\phi = 0.56$ cases.

5.4.3 144, 160, 180 and 200 Hz

As explained in section 5.3, the vortex shedding into the ORZ is the dominant mechanism for local flame extinction and / or annihilation. The onset of vortex shedding is in very good agreement with the work of Kulsheimer and Buchner [40], see table 5.2.

Visual inspection of the 144 Hz at $\phi = 0.56$ (see figures 4.44 to 4.49) shows that the flame tip area that penetrates the ORZ increases as the imposed velocity fluctuation increases (compare the -179° in figures 4.44, 4.46 and 4.48). Flame tip penetration into the ORZ is allowed due to the local reduction of the high tensile strain rates (see -179° in figures 4.45, 4.47 and 4.49). As explained in section 5.3, the flame inside the ORZ is extinguished and / or annihilated as the vortex moves upwards and a new vortex is formed in the ORZ. Flame tip extinction and / or annihilation is visually seen at -179° and -224° in figure 4.48.

Flame tip penetration into the ORZ and eventual extinction and / or annihilation as result of high tensile strain rate shredding is also visually apparent for the 160, 180 and 200 Hz cases. The flame area that penetrates the ORZ for the 144, 160, 180 and 200 Hz is at least 30% larger than the flame tip area that penetrates the ORZ for the 80 and 120 Hz cases. The
180 Hz case at 30% showed the highest flame tip area penetration into the ORZ (see -198° to -333° in figure 4.60).

The variation in equivalence ratio is a feature that occurs irrespective of the imposed frequency and velocity fluctuation. The important concept here is whether or not the equivalence ratio fluctuation is the dominant mechanism for flame instability and eventual non-linearity. The $\phi$ of the 180 Hz case at $U_{rms} = 30\%$ (figure 4.77) neither exceeds the stoichiometric condition nor dips below the steady state lean blow-off limit, similar to the 60 Hz case. The 144, 160 and 200 Hz also remain between the steady state lean blow-off limit and stoichiometric conditions (not shown). The use of the perfect premixture did not have any significant effect on the flame response or the velocity fluctuation at which the non-linear flame response for the 144, 160, 180 and 200 Hz cases started. This suggests that the flame non-linearity for the 144, 160, 180 and 200 Hz cases is not due to the equivalence ratio mechanism.

The smaller amplitude of the total OH$^*$ chemiluminescence for the 160, 180 and 200 Hz cases compared to the 80 and 120 Hz, as shown in figures 4.71 and 4.72, provides further evidence for disregarding the $\phi$ variation as a cause of non-linear flame response.

For the 160, 180 and 200 Hz with $\phi = 0.48$, the mean $\phi$ in the figure 4.77 was reduced from 0.56 to 0.48 as described in sections 5.4.1 and 5.4.2. Under these conditions the $\phi$ variations for $U_{rms} = 20\%$ and 30% dip below the steady state lean blow-off limit (see figure 5.6 for 180 Hz only). Although no stream plots are available for $\phi = 0.48$, examination of the phase-locked OH$^*$ chemiluminescence images (not shown) shows that the flame does not penetrate the ORZ. The flame brush however becomes narrow, similarly to the 60, 80, 100 and 120 Hz cases at $\phi = 0.48$. Local extinction and / or annihilation occurs due to the inability of the flame to withstand high tensile strain rates in the flow field. It is therefore believed that non-linearity occurs although it is not ‘strong’ enough to register on the FTF magnitudes.

The slope of a linear fit to the photodiode FTF phase in figure 4.14 was used for calculation of the effective time delay and convection velocity, as outlined by Hosseini and Lawn [25]. The linear fit should pass through the origin since the phase must be zero when the frequency is
5.4. Forced-Excitation: Axial Swirl

For the 160, 180 and 200 Hz cases an effective time delay of $8.5 \pm 0.3$ ms was calculated. The effective time delay agrees with the work of Komarek [37] and Hosseini and Lawn [25] which was carried out on the same apparatus. These effective time delays result in a mean convection velocity of approximately 3.4 to 3.8 m/s, consistent with the mean burner exit velocity of 4 m/s, assuming that the effect of the vortices on heat release is the same at all locations and vortex shedding occurs at the start of the cycle. For $\phi = 0.48$, figure 4.24 yielded an effective time delay of $8.8 \pm 0.3$ ms for the 160, 180 and 200 Hz cases. The change in equivalence ratio did not cause any significant change in the effective time delay.

5.4.4 FTF Frequency Response

The FTF magnitudes provide further evidence of the validity of the model proposed by Polifke and Lawn [68]. The agreement between the frequency plot of the photodiode FTF magnitude obtained from experiments and the proposed model by Lawn and Polifke [49] is very good, irrespective of the equivalence ratio (compare figures 4.13 and 4.23 to figure 5.1).

Balachandran [1] investigated the effect of harmonics on the FTF, by forced-excitation of a non-swirling premixed flame at two different frequencies. The FTF of the fundamental frequency remained unaffected until the first harmonic OH$^*$ chemiluminescence magnitude was greater than that of the fundamental. This condition was not satisfied in the current study and hence it is believed that the effect of the harmonics on the FTF was negligible.

The mechanism responsible for OH$^*$ chemiluminescence harmonics requires further investigation and it is outside the scope of current study. Its relation with non-linear flame response however is explored further. Previous studies of the flame OH$^*$ chemiluminescence FFT spectrum suggested that the presence of the harmonics is a by-product of the non-linear flame response, [58]. These harmonics in the OH$^*$ chemiluminescence FFT spectra were also seen in the current study (see figure 4.15), irrespective of equivalence ratio, swirl geometry, imposed frequency or velocity fluctuation.
The OH\textsuperscript{*} chemiluminescence harmonics (measured by photodiode) are not the result of the velocity fluctuation harmonics (measured by hotwire). This is due to the fact that the harmonic content of the velocity fluctuation was negligible compared to that of its fundamental, irrespective of the imposed frequency and loudspeaker amplitude. This essentially proves that the loudspeaker is more than capable of producing pure tones in the frequency and amplitude range of interest and hence the harmonic content of the OH\textsuperscript{*} chemiluminescence could not be attributed to the loudspeaker. The upstream excitation geometry (figure 3.9) is also not responsible for producing OH\textsuperscript{*} chemiluminescence harmonics, since there is no record of the harmonics in the hot wire FFT spectra. Examination of frequencies at which flame response remained linear irrespective of the $U_{rms}$, such as the 40 or 60 Hz cases at $\phi = 0.56$ (figure 4.6), showed that the increase in the OH\textsuperscript{*} chemiluminescence harmonic content is dependent on the magnitude of the imposed velocity fluctuation, not the linearity or non-linearity of the flame response.

Previous work associated the skewed profile of the phase-locked OH\textsuperscript{*} chemiluminescence with the non-linear flame response. For example Balachandran et al. [3] attributed the non-sinusoidal (skewed cyclic) variation of the phase-locked OH\textsuperscript{*} chemiluminescence to the vortex roll-up, which induced flame shortening or annihilation events. This is however contrary to what is seen in this study. The case of 60 Hz excitation at $\phi = 0.56$ (figure 4.69) for example, showed a skewed phase-locked OH\textsuperscript{*} chemiluminescence profile, yet the photodiode FTF magnitude (figure 4.6) and the ICCD FTF magnitude (figure 4.11) both confirm that the flame response remained linear irrespective of the imposed velocity fluctuation.

The photodiode FTF magnitude of the 200 Hz for both equivalence ratios showed an increasing gain prior to linear and non-linear flame response, well below the $U_{rms} = 10\%$ (see figures 4.7 and 4.17). There are no phase-locked PIV images to examine this regime. The ICCD FTF magnitudes (see figures 4.11 and 4.21) and the normalised global heat release rates (see figures 4.10 and 4.20), however, did not exhibit increasing gains either at $\phi = 0.56$ or 0.48. It is therefore concluded that such increasing gains are not the result of a flame instability.
5.5 Forced-Excitation: Radial Swirler

Although the PIV imaging of the radial swirler was not done in the current study, reference can be made to other work in the literature with similar radial swirl burner geometry. The LES study of Roux et al. [73] revealed the streamline pattern of the flow in the combustion chamber based on the mean axial and radial velocity components (see figure 5.7). The instantaneous temperature iso-surfaces in figure 5.8 were also calculated and showed a compact flame stabilised on the face of the radial swirler, in agreement to that in the current study (compare to the time-averaged flames in figure 4.83). Note the presence of the IRZ and ORZ in the reacting flow. A CFD analysis was also done by Siemens Industrial Turbomachinery Ltd under full load gas turbine operating conditions (see figure 5.9), [8]. As shown in the CFD images, the flame is hollow and stabilised between the IRZ and ORZ.

As explained in section 4.2, OH* chemiluminescence emission from approximately 31 mm of the flame remained hidden from view throughout the experiments. The analysis therefore relies solely on the shape of the Abel de-convoluted flame and careful examination of the photodiode FTF magnitude. It is hypothesised that the mechanism responsible for non-linear flame response is that of equivalence ratio fluctuation as shown in table 5.1. The different frequencies will be discussed further in the following section.

5.5.1 80 and 100 Hz

It is important to note that, according to the work of Kulsheimer and Buchner [40], vortex shedding occurs for all the examined frequencies of the radial swirler for velocity fluctuations greater than 10% as shown in table 5.2. Like the axial swirler however the vortex shedding is not considered to be the dominant cause of non-linearity for 80 and 100 Hz cases at φ = 0.56.
In figures 4.94 and 4.95 the high variation in flame height and OH\(^{\ast}\) chemiluminescence intensity is apparent and similar to the 80, 100 and 120 Hz cases of the axial swirler (see section 5.4.2). It is therefore hypothesised that the fluctuation in equivalence ratio is also the dominant mechanism in the low frequency regimes of the radial swirler. The effect of equivalence ratio on non-linear flame response however remains debatable, since there is no global extinction and the flame is simply moving out of the view of the ICCD camera (see \(-9^\circ\) in figure 4.94). This resulted in an incorrect non-linear flame response in the photodiode FTF magnitude (figure 4.84) by causing a non-proportional relationship between the OH\(^{\ast}\) chemiluminescence and velocity fluctuation. It is therefore difficult to deduce whether the non-linear flame response is because of the local flame extinction due to the equivalence ratio fluctuation or because of the incomplete view of the flame.

As with the axial swirler case a perfect premixture (\(\phi = 0.56\)) was used in order to assess the validity of ‘equivalence ratio mechanism’. Under this condition the flame did not respond at all until blow-off, thus proving that the ‘equivalence ratio mechanism’ is responsible for flame instability and eventual non-linearity for stiff fuel systems at these low frequencies.

The second equivalence ratio (\(\phi = 0.58\)) was specifically chosen close to the first equivalence ratio (\(\phi = 0.56\)) in order to assess the effect of hidden OH\(^{\ast}\) chemiluminescence on the photodiode FTF magnitude. Comparison of the photodiode FTF magnitudes show significant differences (compare figure 4.84 and 4.89) even with such a small change in \(\phi\). Along with linear and non-linear flame response, an increasing gain is common place and was due to partial view of the flame. Therefore, as with the previous case, the FTF magnitude could not be used for assessing the flame response. The phase-locked OH\(^{\ast}\) chemiluminescence flame images for the two equivalence ratios were very similar (see figure 4.100 for 80 Hz only), but there may have been differences below the field of view.
5.5.2 120, 160, 180 and 200 Hz

For the 120, 160, 180 Hz cases a constant portion of the flame remained in the view of the ICCD camera and photodiode, therefore there was no ambiguity with regard to flame response due to incomplete view of the flame. The flame brush and length remained continuous irrespective of velocity fluctuations, and the flame tip exhibited a distinct curve towards and away from the burner’s central axis with high velocity fluctuations. The most extreme of such curvatures is that of 120 Hz (see figure 4.96 at $U_{rms} = 33\%$).

A perfect premixture ($\phi = 0.56$) showed that the flame response for the 120, 160 and 180 Hz cases was negligible until blow-off. It is therefore proved that, although the flame shape is strongly influenced by the IRZ and ORZ interaction, the eventual local flame extinction and / or annihilation and therefore non-linearity occurs because of the ‘equivalence ratio mechanism’.

The flame shapes for the two equivalence ratios remained predominantly the same (not shown here), although the OH* chemiluminescence intensity of the $\phi = 0.58$ was marginally higher than the $\phi = 0.56$ case. The $\phi = 0.58$ case also exhibited flame front curvature towards and away from burner’s central axis.

The 200 Hz cases exhibited linear flame response irrespective of $\phi$ (see figures 4.85 and 4.90). The flame front curves away from burner’s central axis into the ORZ (see -356° in figure 4.99). However, it is hypothesised that the flame tip did not penetrate the ORZ as with the 120, 160, 180 Hz cases. Under a perfect premixture, the flame did not respond, suggesting that flame instability and eventual non-linearity in the stiff fuel supply system was a result of the ‘equivalence ratio mechanism’. The OH* chemiluminescence at $\phi = 0.58$ was marginally higher than at $\phi = 0.56$ as expected, but the flame shapes were similar (see figure 4.101).

The IRZ of the radial flame is much stronger than that of the axial flame due to higher swirl number, [23]. The tensile and compressive strain rates in the flow influence the shape of the IRZ and ORZ. The IRZ is believed to be responsible for the flame curvature towards the
burner's central axis, in the same way that the ORZ in the axial flame was responsible for the flame curvature away from the burner's central axis. The schematic in figure 5.10 shows the possible interaction of the IRZ, ORZ and flame front. In figure 5.10A, the space between the two inner recirculation zones in the IRZ creates a discontinuity in high tensile strain rates, allowing the flame tip to penetrate between the IRZ and ORZ but not into the IRZ, hence extinction and/or annihilation in the IRZ does not occur. Compare figure 5.10A to -132° at $U_{rms} = 30\%$ in figure 4.97.

In figure 5.10B however the IRZ becomes continuous. There is also the possibility that different vortices in the IRZ approach each other (not shown here). In both cases the high tensile strain rates prohibit the flame tip penetration into the IRZ. The flame still manages to curve around the ORZ. The flame tip however does not penetrate the ORZ and extinction and/or annihilation in the ORZ does not occur. Compare figure 5.10B to -322° at $U_{rms} = 33\%$ in figure 4.96. The effect of any PVC was also disregarded, as in the axial swirler case, [73].

5.6 Self-Excitation

As described in section 1.8, forced-excitation has been used to investigate the self-excitation instabilities and the resultant non-linear flame response. The validity of this method and the similarities of the forced and self-excited flames at a given frequency and velocity fluctuation is shown by a number of studies, [1]. Such similarity in the flame shape and OH* chemiluminescence between the two methods of excitation was not observed in the current study. The self-excitation and the cause of dissimilarity with the forced-excitation is examined in the following sections.

5.6.1 Axial Swirler

In the case of self-excitation at $\phi = 0.56$ (see figure 4.103) the harmonic of OH* chemiluminescence observed in figure 4.102 meant that the flame was essentially subjected to 20, 144
and 200 Hz frequencies. This resulted in a flame shape that remained unchanged throughout the cycle, as shown in figure 4.103 unlike the pure tone excitation shown in figure 4.104.

In the case of $\phi = 0.91$, the flame from $0^\circ$ to $90^\circ$ in figure 4.105 is almost extinguished, a behaviour that is not observed in the forced-excited counter-part in figure 4.106. The flame extinction is due to consumption of all the available premixture from $135^\circ$ to $270^\circ$. Such drastic variation in flame shape and OH* chemiluminescence is only possible at very high velocity fluctuations.

The dissimilarity between the self and forced-excitation case is therefore due to the fact that the forced-excitation velocity fluctuation was much smaller than that of the self-excitation. It is important to note that blow-off in the self-excitation geometry (see figure 3.11) does not occur as easily as the forced-excitation geometry since the high temperatures of the self-excitation combustion chamber maintain the combustion.

### 5.6.2 Radial Swirler

The self-excitation case at $\phi = 0.58$ possessed no cyclic flame shape variation (see figure 4.108), unlike the forced-excitation case shown in figure 4.109. This was due to the low amplitude of the self-excited $U_{rms}$ and the broad band noise that is observed in the photodiode FFT spectra (see figure 4.107).

As with the case of the self-excited axial swirler at $\phi = 0.91$, the self-excited flame shape variation of the radial swirler at $\phi = 0.91$ was significant. As explained earlier the forced-excitation images are not shown here due to saturation of the ICCD camera. Therefore the flame has a higher OH* chemiluminescence intensity under forced-excitation compared to that of the self-excited case due to higher imposed velocity fluctuations. Because of this issue, comparison between the two methods of excitation was not possible.
Figure 5.1: Predicted continuations to the gain $\Psi$ of the flame response, $C_3H_8$ at $\phi = 0.58$, Lawn and Polifke [49]. Contribution from acoustic velocity and turbulence, from coherent vortices, from equivalence ratio fluctuations and the combined influence of these factors is shown in the graph.

Figure 5.2: Dynamical flow conditions characterising the formation of large-scale coherent vortices in periodically pulsed swirl flow, [40].
Figure 5.3: Schematic of the IRZ and ORZ interaction with the flame front for the axial swirler at high frequencies with vortex shedding. Flame front is shown with square dotted line, A) is the condition for flame front between the IRZ and ORZ. B) is the condition for flame front penetrating the ORZ with eventual local flame extinction and/or annihilation.
Figure 5.4: Equivalence ratio fluctuations calculated for $\phi = 0.48$ from figure 4.75, axial swirler, $f_i = 60$ Hz, $U_{rms} = 10\%, 20\%$ and 30\%. The steady state lean blow-off limit is 0.33.

Figure 5.5: Equivalence ratio fluctuations calculated for $\phi = 0.48$ from figure 4.76, axial swirler, $f_i = 120$ Hz, $U_{rms} = 10\%, 20\%$ and 30\%. The steady state lean blow-off limit is 0.33.
Figure 5.6: Equivalence ratio fluctuations calculated for $\phi = 0.48$ from figure 4.77, axial swirler, $f_i = 180$ Hz, $U_{rms} = 10\%$, 20\% and 30\%. The steady state lean blow-off limit is 0.33.

Figure 5.7: Streamline plot calculated from the LES mean axial and radial velocity components, [73].
Figure 5.8: Instantaneous temperature iso-surface at 1250 K form LES, [73].
Figure 5.9: Siemens Industrial Turbomachinery Ltd CFD analysis of the combustor under full load conditions, [8]. The colour bar shows the temperature in Kelvin.
Figure 5.10: Schematic of the hypothesised IRZ and ORZ interaction with the flame front for the radial swirler at high frequencies. The flame front is shown with square dotted line, A) is the condition for flame front curving towards burner’s central axis and B) is the condition for flame front curving away from burner’s central axis. The IRZ and ORZ are shown as closed ellipses.
Chapter 6

Conclusion
6.1 Conclusions Of This Research

The thermoacoustic instabilities of two different swirler geometries and the cause of non-linear flame response were investigated. For each swirler geometry two equivalence ratios were examined. The novel aspect of the experiments was that of using forced-excitation over a range of frequency and velocity fluctuations. Besides the traditional diagnostic techniques such as flame transfer function (FTF) measurement using hotwire and photodiode, non-intrusive optical techniques such as simultaneous phase-locked ICCD-PIV measurement and phase-locked ICCD were implemented for flow field visualisation and OH* chemiluminescence imaging of the flame. The current study extended the previous work on these swirlers and confirmed the hypotheses of ‘equivalence ratio mechanism’ and ‘flame vortex interaction’ regarding the flame instability.

Specific conclusions are:

- The velocity fluctuation for flame blow-off of the axial swirler under forced-excitation conditions was much more repeatable than for the radial swirler. However, the velocity fluctuations required for radial swirler flame blow-off were much greater than for the axial swirler flame.

- The FTF magnitude and phase of the axial swirler were calculated by two different methods (photodiode and ICCD) and generally the agreement between them was satisfactory.

- The FTF magnitude of the axial swirler generally exhibited regions of linear and then non-linear flame response as the excitation was increased. The FTF phase was mostly constant and independent of the imposed velocity fluctuations.

- The agreement between the frequency plot of the FTF magnitude obtained in this study and the model suggested by Polifke and Lawn [68] was very good for the axial swirler, irrespective of equivalence ratio. This suggests that the flame instability is the result
of equivalence ratio fluctuations in the low frequency group (80, 100 and 120 Hz) and flame-vortex interaction in the high frequency group (160, 180 and 200 Hz), as assumed in the model and proved with the aid of a perfect premixture in the current study. The instability mechanisms cause non-linear flame response through local flame extinction and / or annihilation events.

- There is a wide variety of additional evidence to support the existence of two instability mechanisms and non-linear flame response. For the axial swirler at $\phi = 0.56$ for example, in the low frequency regime the flame height and area variation were significant and larger than the high frequency cases. The flame tip did not penetrate the ORZ due to high tensile strain rates in the shear layer (approximately $1000 \, s^{-1}$). The equivalence ratio fluctuations however crossed the steady state lean blow-off limit. For the high frequency cases the flame area and height variations were smaller than the low frequency cases. The equivalence ratio remained between the steady state blow-off limit and stoichiometric but the flame tip penetrated the ORZ through discontinuities in the areas of high tensile strain rate, where flame tip extinction and / or annihilation occurred inside the ORZ, resulting in non-linear flame response. For the $\phi = 0.48$ case, however, the equivalence ratio crossed the steady state blow-off limit irrespective of imposed frequency as the imposed velocity fluctuation increased. At $\phi = 0.48$ the non-linear flame response at low frequencies (60, 80, 100 and 120 Hz) occurred due to the equivalence ratio fluctuation mechanism and at high frequencies (160, 180 and 200 Hz) the combination of flame-vortex interaction and equivalence ratio fluctuations resulted in non-linear flame response.

- The FTF magnitude should be used in conjunction with other diagnostic methods for assessing the linearity or non-linearity of the flame response. Examples have been shown (axial swirler $\phi = 0.48$ at high frequencies) where the flame response remained linear irrespective of the imposed velocity fluctuation, yet the flow field and flame image analysis exhibited local flame extinction and / or annihilation events.
• For the radial swirler the equivalence ratio fluctuation was responsible for flame instability and eventual non-linear flame response irrespective of imposed frequency, as proved with the aid of a perfect premixture.

• The radial swirler FTF magnitudes consisted mostly of increasing gain, linear and non-linear flame response for low frequencies (80 and 100 Hz). For high frequencies (120, 160, 180 Hz), however, the FTF magnitude consisted mainly of linear and non-linear regimes, as for the axial swirler.

• For the high frequency cases with the radial swirler, a flame-vortex interaction mechanism is suggested in order to explain the curvature of the flame front towards and away from burner's central axis. However a PIV analysis is required to confirm this hypothesis.

• For both swirlers, at the higher frequencies the interaction of the IRZ and ORZ has a direct influence on the flame shape through flame-vortex interaction, irrespective of equivalence ratio. The flame cannot generally penetrate the IRZ due to the existence of post-combustion products and very long residence times. Also most of the flame front cannot penetrate the ORZ due to high tensile strain rates. The flame is therefore confined to the annular jet between the IRZ and ORZ and their interaction dictates the shape of the flame, irrespective of swirl geometry.

• The onset of vortex shedding agreed very well with the work of Kulsheimer and Buchner [40], irrespective of equivalence ratio and burner swirl geometry.

• The wide range of conditions investigated produced a very complex view of flame instability and non-linear flame response. Non-sinusoidal variation of the phase-locked OH\textsuperscript{*} chemiluminescence (from phase-locked ICCD images) occurs when the first harmonic is comparable to that of the fundamental in the photodiode FFT spectrum. More importantly, in contradiction to the work available in literature, the non-sinusoidal variation of the phase-locked OH\textsuperscript{*} chemiluminescence is not the result of non-linear flame response.
• The correspondence between the self-excitation flame images and their forced-excited counter parts was very poor, but this was due to the presence of strong OH* chemiluminescence harmonics, broad band noise and the low amplitude of the self-excited velocity fluctuations. This showed the difficulties in investigating the self-excitation instabilities and proved that the forced-excitation technique is a crucial method for investigating the instability mechanisms.

6.2 Recommendations For Further Studies

• Investigation of the effect of a pilot flame on the flame structure and OH* chemiluminescence emission, bearing in mind that because of the fuel rich area around the pilot, the intensity from the ICCD camera will no longer be proportional to the heat release rate.

• Use of instantaneous phase-locked OH*-PLIF technique for analysis of flame front annihilation events.

• Redesign of the radial swirler combustion chamber so that more of the appropriate OH* chemiluminescence emission can be captured with the photodiode and ICCD equipment. The proposed arrangement will use a quartz pre-chamber instead of the current setup. However, even with a quartz pre-chamber, the view of the approximately 10 mm of flame root that sits within the radial swirler will remain hidden.

• Simultaneous phase-locked ICCD-PIV of the radial swirler for analysing the flame-vortex interaction.

• Simultaneous phase-locked ICCD-PIV for both swirl geometries under self-excitation.

• Automated control of the air and fuel flow. This will increase the accuracy of the global equivalence ratio and substantially facilitate continuous measurement without the need for periodic checking and adjusting of the air and fuel flow rates.
• Closer examination of the frequency at which the instability mechanism switches from equivalence ratio fluctuations to flame-vortex interaction.
Appendix A

List Of Publications

Some sections of this work has been published, they include:


Appendix B

Derivation Of Jump Equation

By considering equations 1.6, 1.7 and 1.8 in a region with heat input, the density \( \rho \) varies through changes in both the pressure and specific entropy. The speed of sound, \( c \) is defined by:

\[
    c^2 = \left( \frac{\partial p}{\partial \rho} \right)_s
\]  

(B.1)

because acoustic waves are isentropic by definition. From 1.8:

\[
    C_p dT = T ds + \frac{dp}{\rho} = dh = C_v dT + d \left( \frac{p}{\rho} \right) 
\]  

(B.2)

where \( C_p \) is the specific heat capacity at constant pressure and \( C_v \) is the specific heat capacity at constant volume. Assuming that the gas is perfect (i.e. \( p = \rho RT \)), the speed of sound is derived as:

\[
    \left( \frac{\partial p}{\partial \rho} \right)_s = RT + R\rho \left( \frac{\partial T}{\partial \rho} \right)_s 
\]  

(B.3)

\[
    c^2 = \left( \frac{\partial p}{\partial \rho} \right)_s = \frac{RT}{1 - R/C_P} = \gamma RT 
\]  

(B.4)

where \( \gamma \) is the ratio of specific heat capacities at constant pressure and volume. If viscous and heat conduction effects are neglected in equation 1.8, then the heat input per unit volume
Appendix B. Derivation Of Jump Equation

$q(x, t)$ is given by:

$$q(x, t) = \rho T \frac{D s}{D t} \tag{B.5}$$

Also, by assuming a perfect gas equation B.6 can be derived from equation B.2:

$$\left. \frac{\partial T}{\partial s} \right|_p = \frac{T}{C_p} \tag{B.6}$$

or

$$\left. \frac{\partial \rho}{\partial s} \right|_p = -\frac{\rho}{C_p} = -\frac{\rho T (\gamma - 1)}{c^2} \tag{B.7}$$

Using the chain rule of differentiation:

$$\frac{D \rho}{D t} = \frac{1}{c^2} \frac{D p}{D t} + \left. \frac{\partial \rho}{\partial s} \right|_p \frac{D s}{D t} \tag{B.8}$$

Substituting $\left. \frac{\partial \rho}{\partial s} \right|_p$ in equation B.8 yields:

$$\frac{D \rho}{D t} = \frac{1}{c^2} \left( \frac{D p}{D t} - (\gamma - 1) q \right) \tag{B.9}$$

Equation B.9 can be applied to a combusting gas, provided that the reactants and products behave as perfect gases. Note that each parameter is the sum of the mean and fluctuation.

By using the one-dimensional energy and mass conservation:

$$\frac{\partial \rho'}{\partial t} = -\rho \frac{\partial u'}{\partial x} - \rho' \frac{\partial u}{\partial x} - u' \frac{\partial \rho}{\partial x} - u \frac{\partial \rho'}{\partial x} \tag{B.10}$$

For $Ma \ll 1$:

$$\frac{\partial \rho'}{\partial t} = -\rho \frac{\partial u'}{\partial x} \tag{B.11}$$

Since

$$\frac{\bar{u} \frac{\partial \rho'}{\partial x}}{\frac{\partial \rho'}{\partial t}} = Ma \tag{B.12}$$
Now from B.9:

$$\frac{\partial \rho'}{\partial t} = \frac{1}{c^2} \left( \frac{\partial p'}{\partial t} - (\gamma - 1) q' \right)$$ \hspace{1cm} (B.13)

Hence using equation B.11:

$$\left( \frac{\partial u'}{\partial x} \right) = -\frac{1}{\bar{\rho}c^2} \left( \frac{\partial p'}{\partial t} - (\gamma - 1) q' \right)$$ \hspace{1cm} (B.14)

Over-bars denote the mean values and the prime, the perturbations. In the equation B.14 the small entropy production due to the frictional losses is ignored by assuming that the fluctuations are rapid enough to be adiabatic. By integrating equation B.14 across the flame zone (i.e. $dx$) for small perturbations one gets:

$$\int \left( \frac{\partial u'}{\partial x} \right) dx = -\frac{1}{\bar{\rho}c^2} \left[ \int \left( \frac{\partial p'}{\partial t} \right) dx - \int [(\gamma - 1) q'] dx \right]$$ \hspace{1cm} (B.15)

Equation B.15 is simplified even further by using the integrated form of the momentum equation. Recall the momentum equation, in the fluctuating regime and in the integrated form (for low Mach number only and for x-direction only):

$$\int \bar{\rho} \frac{\partial u'}{\partial t} dx = -\int \frac{\partial p'}{\partial x} dx = p'_1 - p'_2$$ \hspace{1cm} (B.16)

The pressure across the flame is continuous, assuming that the flame is acoustically compact, which means that the flame is thin in comparison with the wavelength, because:

$$\frac{p'}{\rho u'} \frac{\omega (k\Delta x)}{k} << 1$$ \hspace{1cm} (B.17)

By using equation B.16 the integration of the flame zone for small perturbations (i.e. equation B.15) yields:

$$\int \left( \frac{\partial u'}{\partial x} \right) dx = \frac{1}{\bar{\rho}c^2} \int [(\gamma - 1) q'] dx$$ \hspace{1cm} (B.18)
Therefore

\[(u_2' - u_1') A = \frac{(\gamma - 1)}{\rho c^2} \int q'dAdx\]  \hspace{1cm} (B.19)

Or

\[(u_2' - u_1') A = \frac{(\gamma - 1)}{\rho \gamma} h'\] \hspace{1cm} (B.20)
Appendix C

Acoustic Network

The linear acoustic network modelling is a method of predicting the self-excitation frequencies. A gas turbine with its complicated acoustic properties could be broken down into simple geometrical units, each with a known acoustic behaviour. The simplified units are then represented by four-pole elements, with the advantage that all the four-pole properties could be predicted by computational fluid dynamics (CFD) or experiments.

Another justification for using four-pole analysis is the one-dimensional nature of the acoustic wave encountered in the burner. The wave propagation is governed by the acoustic pressure and particle velocity downstream and upstream of the four-pole. The time dependence is represented by $e^{i\omega t}$, where $\omega$ is the frequency of oscillation. The relevant four-pole matrices for prediction of the self-excitation frequencies of the geometry presented in section 3.1.3 from bottom to top are as follows:

1. The transfer matrix for a simple pipe of constant diameter and no losses, bearing in mind that the Mach number in the current study is much less than one and the matrix is dependent on the pre and post-flame temperature variation through the variation in
density $\rho$ and speed of sound $c$.

$$\begin{bmatrix} p'_{\text{out}} \\ u'_{\text{out}} \end{bmatrix} = \begin{bmatrix} \cos(kx) & -i\rho c \sin(kx) \\ -i \sin(kx)/\rho c & \cos(kx) \end{bmatrix} \begin{bmatrix} p'_{\text{in}} \\ u'_{\text{in}} \end{bmatrix}$$ (C.1)

where $k$ is the wave number defined by equation C.2 and $x$ is the length of the pipe.

$$k = \frac{2\pi f}{c}$$ (C.2)

2. The transfer matrix accounting for the impedance introduced by the mesh plate, noting that cold conditions apply due to the pre-flame position of the mesh plate.

$$\begin{bmatrix} p'_{\text{out}} \\ u'_{\text{out}} \end{bmatrix} = \begin{bmatrix} 1 - (\rho c K_m M) \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} p'_{\text{in}} \\ u'_{\text{in}} \end{bmatrix}$$ (C.3)

where $K_m$ is the loss coefficient for the mesh plate.

3. Transfer matrix for a simple pipe under cold conditions similar to equation C.1.

4. For a short contraction zone with significant losses and $M << 1$, the four-pole matrix in equation C.1 is rewritten as Lawn [44]:

$$\begin{bmatrix} p'_{\text{out}} \\ u'_{\text{out}} \end{bmatrix} = \begin{bmatrix} 1 \\ -ikl_{\text{red}}/\rho c A_{\text{out}}/A_{\text{in}} \end{bmatrix} \begin{bmatrix} p'_{\text{in}} \\ u'_{\text{in}} \end{bmatrix}$$ (C.4)

where $K$ is the loss coefficient of the burner and $l_{\text{eff}}$ is the effective length. The term $KM + ikl_{\text{eff}}$ defines the burner impedance, where $M$ is the Mach number in the reference area (connection flange as depicted in figure 3.2). The reduced length $l_{\text{red}}$ and the effective length $l_{\text{eff}}$ of the burner are defined as:

$$l_{\text{red}} = \int_{x_{\text{in}}}^{x_{\text{out}}} \frac{A(x)}{A_{\text{out}}} \, dx$$ (C.5)
Appendix C. Acoustic Network

\[ l_{eff} = l_{cc,i} + l_{cc,o} \]  
(C.6)

where

\[ l_{cc} = \int_{x_{in}}^{x_{out}} \frac{A_{in}}{A(x)} \, dx \]  
(C.7)

\( A(x) \) defines the change in area of the contraction as a function of length, \( l_{cc,i} \) and \( l_{cc,o} \) are the required input and output end corrections.

5. The four-pole matrix for the flame is shown in equation C.8, [24].

\[
\begin{bmatrix}
    p_{out}' \\
    u_{out}'
\end{bmatrix} = \begin{bmatrix}
    1 & 0 \\
    0 & 1 + \left( \frac{T_{hot} - T_{cold}}{T_{cold}} \right) \Psi e^{-i\Theta}
\end{bmatrix}
\begin{bmatrix}
    p_{in}' \\
    u_{in}'
\end{bmatrix}
\]  
(C.8)

where \( T_{hot} \) is the flame temperature and \( T_{cold} \) is the ambient temperature. Symbols \( \Psi \) and \( \Theta \) stand for the normalised flame transfer function magnitudes and phase respectively, and are obtained from experiment or computational fluid dynamic simulations. Equation C.8 is also known as a generic flame model, [24].

6. The transfer matrix for a simple pipe under warm conditions, since the mean gas temperature differed from ambient due to the combustion. The mean gas temperature was calculated from \( T_{warm} = \frac{T_{hot} + T_{cold}}{2} \). Therefore equation C.1 was used but \( \rho_{cold} \) and \( c_{cold} \) were replaced with density and speed of sound at \( T_{warm} \) (i.e. \( \rho_{warm} \) and \( c_{warm} \)).

7. The end correction of the tube placed on top of the quartz combustion chamber is also important and dependent on the type of termination. The correction is dependent on \( \omega \) and calculated from:

\[
\begin{bmatrix}
    p_{out}' \\
    u_{out}'
\end{bmatrix} = \begin{bmatrix}
    1 & -\rho_{warm} c_{warm} Z_{flange} \\
    0 & 1
\end{bmatrix}
\begin{bmatrix}
    p_{in}' \\
    u_{in}'
\end{bmatrix}
\]  
(C.9)

where \( Z_{flange} \) is the specific impedance of the flanged termination. The flange impedance
Appendix C. Acoustic Network

is calculated from:

\[
Z_{\text{flange}} = \left[ 0.125 \left( \frac{\omega r}{c_{\text{warm}}} \right)^2 + i0.423 \left( \frac{\omega r}{c_{\text{warm}}} \right) \right]
\]  
\( \text{(C.10)} \)

where \( r \) is the radius of the outlet (area of the connection flange in figure 3.2) and \( \rho_{\text{warm}} \) and \( c_{\text{warm}} \) refer to the density and speed of sound at temperature \( T_{\text{warm}} \).

Using the four-pole acoustic network the self-excitation eigenvalues are found by multiplying each sectional matrix and including the initial conditions, as shown below:

\[
\begin{bmatrix}
    p'_{\text{out}} \\
    u'_{\text{out}}
\end{bmatrix} = M_{\text{exit}}(\omega) \cdot M_{\text{warm}}(\omega) \cdot M_{\text{flame}}(\omega) \cdot M_{\text{con}}(\omega) \cdot M_{\text{cold}}(\omega) \cdot M_{\text{mesh}}(\omega) \cdot M_{\text{cold}}(\omega)
\begin{bmatrix}
    p'_{\text{in}} \\
    u'_{\text{in}}
\end{bmatrix}
\]  
\( \text{(C.11)} \)

where \( M_{\text{warm}}(\omega) \) is the four-pole matrix for a simple tube with a temperature of \( T_{\text{warm}} \), and \( M_{\text{cold}}(\omega) \) is the four-pole matrix for a simple tube at ambient temperature \( (T_{\text{cold}}) \), both represented by equation C.1 provided that the appropriate density and sound speed are used. The exit condition \( M_{\text{exit}}(\omega) \) is calculated from equation C.9 and the mesh four-pole matrix \( M_{\text{mesh}}(\omega) \) is given by C.3. The \( M_{\text{flame}}(\omega) \) is the four-pole matrix of the flame represented by equation C.8 and \( M_{\text{con}}(\omega) \) is the four-pole matrix of the burner, given by equation C.4 (note that the flame four-pole matrix is dependent on \( \omega \) through the variation in ICCD FTF magnitude \( \Psi \) and phase \( \Theta \)). Since equation C.11 represents the whole system, the symbols \( p \) and \( u \) provide a relationship between the inlet pressure and velocity at the back plate depicted in figure 3.10, and the outlet pressure and velocity of the combustion chamber illustrated in figure 3.11. The inlet velocity \( (u'_{\text{in}}) \) is zero and the impedance is very high at the back plate resulting in a finite inlet pressure \( (p'_{\text{in}}) \). With this boundary condition, the equation C.11 can be solved for a range of \( \omega \) resulting in an eigenvalue solution.
Bibliography


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