Black-hole entropy from supergravity superstrata states

Iosif Bena,[a] **Masaki Shigemori**[b,c] **and Nicholas P. Warner**[a,d]

[a]*Institut de Physique Théorique,*
CEA Saclay, F-91191 Gif sur Yvette, France

[b]*Yukawa Institute for Theoretical Physics, Kyoto University,*
Kitashirakawa Oiwakecho, Sakyo-ku, Kyoto 606-8502 Japan

[c]*Hakubi Center, Kyoto University,*
Yoshida-Ushinomiya-cho, Sakyo-ku, Kyoto 606-8501, Japan

[d]*Department of Physics and Astronomy,*
University of Southern California, Los Angeles, CA 90089, U.S.A.

E-mail: iosif.bena@cea.fr, shige@yukawa.kyoto-u.ac.jp, warner@usc.edu

ABSTRACT: There are, by now, several arguments that superstrata, which represent D1-D5-P bound states that depend upon arbitrary functions of *two variables* and that preserve four supersymmetries, exist in string theory, and that their gravitational back-reaction results in smooth horizonless solutions. In this paper we examine the shape and density modes of the superstratum and give strong evidence that the back-reacted supergravity solution allows for fluctuation modes whose quantization reproduces the entropy growth of black holes as a function of the charges. In particular, we argue that the shape modes of the superstratum that lie purely within the non-compact space-time directions account for at least $1/\sqrt{6}$ of the entropy of the D1-D5-P black hole and propose a way in which the rest of the entropy could be captured by superstratum fluctuations. We complete the picture by conjecturing a relationship between bound states of multiple superstrata and momentum excitations of different twisted sectors of the dual CFT.

Contents

1   Introduction

1.1   Black-hole microstate structure

The prototypical example of a string theory black hole whose entropy can be accounted for microscopically is the D1-D5-P black hole. If one considers the various ways in which a combination of N_1 D1 and N_5 D5 branes can carry N_P units of momentum (in the regime of parameters where the back-reaction of these branes is not important and the physical picture of the momentum-carrying excitations is clear), one finds that the corresponding entropy is given by $2\pi\sqrt{N_1 N_5 N_P}$, which exactly matches the Bekenstein-Hawking entropy of the black hole that these branes form in the regime of parameters where their back-reaction is important. Since the original work of [1, 2], such entropy-matching calculations

have been extended to many other families of supersymmetric, or merely extremal black holes, and even near-extremal black holes. The matching of the entropies has proven remarkably successful.

In 1996, the perturbative counting of black-hole microstates at vanishing string coupling in [2] represented the first real progress on the microstate problem in many years. However, this work opened up a whole new set of questions. In particular, it remained to understand how *one* particular black-hole microstate manifests itself in the finite-coupling regime in which the classical black-hole solution exists and has a large horizon area. For a long time it had been thought that all the microstates at weak coupling develop a horizon and are indistinguishable from the classical black-hole solution (except perhaps in a Planck-size region around the singularity) [3, 4]. This intuition was challenged by the construction of several families of fully back-reacted solutions that have the same charges and mass as the black hole, but differ from the classical black-hole solution at the scale of the horizon and, in particular, are smooth and horizonless [5, 6]. Such solutions are called "microstate geometries," because, via the AdS/CFT correspondence, one can map them onto states of the dual CFT. However, despite having many properties indicating that they belong to the typical sector of the black-hole microstates, these solutions have an entropy that is parametrically lower than the black-hole entropy [7], which is presumably related to the fact that these solutions have a lot of symmetry.

If one is to try to reproduce the black hole entropy from supergravity one should therefore find solutions with less symmetry, and the first step in this direction was the construction of three-charge solutions that contain an wiggly supertube [8]. These solutions are parametrized by an arbitrary continuous function and hence can have an infinite number of continuous parameters [9]. The entropy of these solutions grows with the charges as $N^{5/4}$ [9], which is more than all other known supergravity solutions, but is still less than the black hole entropy growth, $N^{3/2}$. In [10] we have furthermore argued that if one relaxes one more symmetry one can construct smooth horizonless *superstratum* solutions that depend on arbitrary continuous functions of *two* variables, and it is the purpose of this paper to argue that the perturbative semi-classical quantization of superstrata yields a black-hole-like entropy growth, and that in the fully back-reacted regime all the three-charge black-hole entropy might be reproduced by space-time fluctuation modes of the superstrata.

In parallel with our efforts, there have also been several relatively-recent developments that support this general approach. First amongst these is Mathur's tightening [11–13] of Hawking's result to show that information can only be recovered if there are $\mathcal{O}(1)$ corrections to the semi-classical physics outside black holes. That is, in order to solve the information problem, we need to make some $\mathcal{O}(1)$ changes at the horizon scale. This discussion can be taken to a new level by asking whether these changes result in a firewall for an incoming observer, as argued by [14–23, 23, 24] or rather whether the quantum super-position of these states can result in a smooth infall experience for macroscopic infalling observers [16, 25–27]. However, finding a mechanism that can support such $\mathcal{O}(1)$ changes in the structure at the horizon scale is notoriously difficult — essentially because the horizon is null, any massive object must fall in, while any massless wave packet will dilute to nothing after several horizon-crossing times. The *only* time-independent way to support such

a structure within supergravity is to place magnetic fluxes on topologically non-trivial cycles [28, 29], and this is precisely the mechanism that underpins all the known BPS [5, 6, 30] and near-extremal [18, 31] microstate geometries. Furthermore, as we have argued in [32], this mechanism extrapolates well beyond the regime of validity of supergravity, and can manifest itself either via brane polarization [33] or via non-Abelian effects.

As explained in [32], there are two separate issues that one must address in order to understand the microstate structure of black holes and the effect that this structure has at the horizon scale. The first is how one can make changes at the horizon scale and we now know [28] that the geometric transition discovered in five dimensions [5, 6] provides the *only* way to replace the horizon with horizonless time-independent structure thereby making the $\mathcal{O}(1)$ corrections. Such geometric transitions will therefore be an essential part of any string-based resolution of black-holes. The microstate structure itself, whatever its ultimate form, can then be supported by the "canvas" provided the geometric transition to large microstate geometries.

The second issue is to determine the extent to which this microstate structure can be captured by semi-classical geometries. This paper will advance the latter goal by arguing that there is indeed a class of microstate geometries, called superstrata, that can achieve the second goal at least with sufficient fidelity to obtain the correct charge-dependence of the BPS black-hole entropy.

1.2 Superstrata

The *superstratum* is a smooth, horizonless soliton (a microstate geometry) that is $\frac{1}{8}$-BPS (preserving 4 supersymmetries), depends on several arbitrary functions of two variables and has the same charges as the D1-D5-P black hole. The existence of this object was conjectured in [10] (building on earlier work in [34]) by arguing that a certain combination of branes, Kaluza-Klein monopoles (KKM's) and momentum preserves the same supersymmetries as the D1-D5-P black hole irrespective of its orientation, and hence one can glue these branes into a supersymmetric configuration that depends on functions of two variables. Furthermore, since the superstratum locally resembles a D1-D5 supertube with a KKM dipole charge, the fully back-reacted superstratum solution should be smooth and hence be a microstate geometry. Even though there is not yet an explicit construction of a generic fully back-reacted superstratum, one can find further evidence for their existence by analyzing string emission from the D1-D5-P system [35–37], or by constructing supergravity solutions that depend of two different functions of two different variables [38], which could be thought of as limits of the more general superstratum solution.

There are several ways by which one might realize the construction of a superstratum. The first way is via a double supertube transition [10, 34, 39]: one combines the D1 branes with some momentum to give a D1-P supertube (D1's with traveling waves on them) and, at the same time, one combines some D5 branes with some momentum to obtain a D5-P supertube (D5's with traveling waves on them). One must do this in such a manner that the D1-profile lies entirely within the D5-profile. Next one "executes" a second supertube transition by locally puffing out the D1-D5 system using a Kaluza-Klein monopole and the result is a D1-D5-P bound state. Since supertube transitions give the configuration

an arbitrary profile and the second transition can, in principle, be done independently and locally on each D1-D5 segment, it seems plausible [10] that two supertube transitions could give rise to a smooth superstratum solution that can be parametrized by functions of two variables.

The second way to think of a superstratum is to begin with a D1-D5 supertube with KKM dipole charge (parametrized by several arbitrary functions of one variable) and start adding momentum to it. Again, for each original configuration, given by the Lunin-Mathur geometry [40–42] one expects to be able to add a general wave profile along the common D1-D5 direction, and hence to obtain a configuration that depends on functions of two variables. Thus, every mode of the original D1-D5 supertube will act as a momentum carrier, and therefore the number of carriers over which one can distribute a given momentum is the number of modes of the D1-D5 supertube. This suggests that such excitations should describe a moduli space of D1-D5 supertubes, and each such modulus should be able to carry momentum.

A third perspective on superstrata comes from the fact that they describe bubbled microstate geometries. Indeed, the single, circular, unexcited superstratum is identical to a D1-D5 supertube geometry and this geometry, in the near-tube limit, is, up to orbifolding, the maximally-symmetric geometry *global $AdS_3 \times S^3$* [40]. More generally, multiple superstrata are expected to describe geometries with topological 3-cycles held up by cohomological fluxes. Changing the shapes of the superstrata corresponds to changing the shapes of these cycles and letting these shape changes depend upon the compact circle in AdS_3. On a single superstratum, the modes transform under the isometries $\mathrm{SL}(2,\mathbb{R})_L \times \mathrm{SL}(2,\mathbb{R})_R \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$. If the structure is to carry momentum then supersymmetry requires that this momentum be either purely left-moving or purely right-moving and so BPS fluctuations can only excite half the modes. As we will discuss in section 4, within the D1-D5 CFT, the left-moving excitations in the space-time directions are correlated with fermionic excitations that only carry $\mathrm{SU}(2)_L$ quantum numbers.[1] It is this that places restrictions on the BPS modes and thus upon the perturbative shape fluctuations. This perturbative approach to superstrata has been developed in [35, 36] and very simple, restricted classes of fully back-reacted solutions were described in [38].

1.3 Representing black hole microstates with superstrata

The problem with the quantization of the superstratum is that we do not know its action and so we cannot start from first principles and quantize. On the other hand we do know the perturbative description of the D1-D5-P microstates that give the black-hole entropy and we know the field theory dual of the $AdS_3 \times S^3$ solution corresponding to the unexcited superstratum. From these observations we can "reverse engineer" precisely which states of the superstratum will be visible within supergravity. Our ultimate goal is to argue that the modes of the D1-D5-P system will, in supergravity, give rise to geometric modes whose

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[1]This observation also has interesting implications for future work: near-BPS and non-BPS solutions have long been obtained by exciting both left-moving and right-moving momentum [43–46] and so we expect generic shape fluctuations to be a natural way to access such non-BPS solutions.

semi-classical quantization will reproduce the exact black-hole entropy:

$$S = 2\pi\sqrt{N_1 N_5 N_P}\,. \tag{1.1}$$

We will, however, start far more conservatively with what we believe can be substantiated with a high level of confidence, namely, that the semi-classical quantization of the space-time shape modes of a *single* superstratum can lead to an entropy count of, at least,

$$S = 2\pi\sqrt{\frac{1}{6}N_1 N_5 N_P}\,. \tag{1.2}$$

This differs from (1.1) by a factor of $\frac{1}{\sqrt{6}}$ because, as we will discuss, the perturbative space-time shape modes of a single superstratum must involve only one sixth of the complete set of perturbative BPS modes. More precisely, these BPS space-time shape modes describe a sector of the CFT with central charge $c = N_1 N_5$ corresponding to half of the bosonized fermions in the D1-D5 CFT. The remaining part of the CFT, with central charge $c = 5N_1 N_5$, arises from the other half of the bosonized fermions and the original bosonic excitations of the D1-D5 CFT. These states correspond to corrections to the internal metric and fields on the T^4 upon which the D5 branes are compactified. We will examine the extent to which this "other five-sixths" of the BPS states will be visible within supergravity and argue that in the fully back-reacted regime the modes that contain internal torus fluctuations will have an energy gap that is parametrically larger than that of the typical black hole microstates. We suggest that these internal torus modes will be "pushed on the Coulomb branch" and will become visible as transverse supergravity modes of the superstratum solution.

The important point here is that, whatever the ultimate status of the internal T^4 excitations, the arguments based upon group theory and perturbation theory allow us to assert with considerable confidence that the shape modes of a single superstratum can, at least, recover the correct entropy growth $S \sim \sqrt{N_1 N_5 N_P}$ as a function of $N_1 N_5 N_P$.

It is also possible to estimate the entropy of superstrata by starting from the original argument [10] that they can be constructed as momentum-carrying fluctuations of the D1-D5 supertube. This construction appears to allow *all* the shape modes of the supertube to be promoted to momentum carriers.[2] We will argue in section 5 that the dimension of the moduli space of these shape modes is $4N_1 N_5$, which would imply that the entropy of a superstratum will come from distributing N_P units of momentum over $4N_1 N_5$ bosonic carriers and their fermionic superpartners, and this would reproduce exactly the black-hole entropy (1.1). This construction appears to be at odds with the perturbative analysis that gives the entropy (1.2). It is possible that the $4N_1 N_5$ shape modes are not independent and unobstructed moduli. It is also quite possible, as we will also discuss in section 5, that the extra shape modes that go beyond the perturbative analysis of section 4 will only emerge in the fully back-reacted superstratum solution. We therefore hope that an complete and explicit superstratum solution will clarify whether the space-time modes of the superstrata will reproduce all the black-hole entropy or only $\frac{1}{\sqrt{6}}$ of it.

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[2]This also agrees with the physics of certain explicit solutions that can be thought of as singular limits of the superstratum solution [47, 48].

In formulating the entropy-counting arguments above we have taken it as given that adding momentum charge to a BPS system of branes will always lead to transverse shape modes once the supergravity back-reaction is included. We will also assume the converse: semi-classical quantization of such supergravity shape modes will recover a full description of the Hilbert space of the original perturbative momentum modes. This is certainly true of the F1-P system, since this is simply the quantization of the fundamental string [49] and it is also true of momentum modes on many systems of branes. We do not believe that there is much danger in assuming that this is a universal result.[3]

There are two frequently-expressed concerns about any program, as the one advanced here, that involves obtaining the black-hole entropy by counting supergravity solutions. The first is that classical supergravity modes only correspond to coherent quantum states and that the states that contribute to the entropy cannot be geometric. The second is that it is possible that the fluctuations that contribute primarily to the entropy may have very small scales, and hence the corresponding solutions will have structure below the Planck scale and will not be therefore correctly described by supergravity.

The first concern might equally be raised as an objection to considering the vibrational motion of a diatomic molecule to be that of a spring. Obviously this is a dramatic classical simplification of a complex quantum system and the real motions of a diatomic molecule are intrinsically quantum phenomena. However, approximating the chemical bond by a classical harmonic oscillator and semi-classically quantizing this oscillator gives an excellent description of the quantum states and the vibrational spectrum because the "spring" isolates the essential physical degrees of freedom that govern the system. It is in this spirit that we believe that microstate geometries and their semi-classical quantization will describe sufficiently many microstates of black holes and give a valuable description of their thermodynamics: while the quantum mechanical states of a black hole are manifestly not geometric, and only very few of them have classical descriptions, the important insight coming from microstate geometries is that this allows us to identify the *degrees of freedom at strong coupling* that need to be quantized in order to capture the essential underlying physics of the black-hole microstates.

The second concern is more serious in that the entropy might be coming primarily from a sector in which the supergravity approximation is failing. There are two reasonable ways around this issue. First, we know that exactly the same issue arises in other instances of adding momentum modes to branes, as with the fundamental string, and yet there is no problem with the semi-classical quantization of states. The reason why there is no difficulty is precisely because such states are based upon well-understood systems of objects that make sense in string theory. Thus the easiest answer to the second concern is that we may ultimately have to broaden the scope of the semi-classical quantization and go beyond

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[3]Strictly speaking, this must hold for the momentum added to the unique ground state of the system and does not apply to the momentum carried by the ground state itself. We are always concerned with the former. For example, a straight supertube [8] carries a fixed amount of angular (longitudinal) momentum coming from the crossing of electric and magnetic worldvolume fluxes. However, any change in the momentum on top of that leads to transverse fluctuation of the supertube shape and of the back-reacted supergravity solution [50–52].

smooth microstate geometries, whose scales, by definition, lie comfortably above the Planck length, and include *microstate solutions.* The latter are defined [32] to be horizonless, physical limits of smooth geometries that have the same mass, charge and angular momentum as a given black hole, but can have singularities that either correspond to fundamental ($\frac{1}{2}$-BPS) D-brane sources or can be patch-wise dualized into a smooth solution.

It is also possible that smooth microstate geometries will resolve these issues without needing to introduce stringy singularities. Indeed, one important realization in the study of microstate geometries was that if one wants to construct a solution that has the same charges as a five-dimensional three-charge black hole with a macroscopically-large horizon area, one must use *scaling solutions* [53–55]. In these solutions the size of the bubbles appears to shrink to zero size from the perspective of the metric of the auxiliary four-dimensional base-space that is used to construct the solutions, but, in fact, the bubbles remain finite once the supergravity back-reaction is taken into account. In the scaling limit, these bubbles descend down a very long AdS throat that resembles, more and more, that of the corresponding black hole. Hence, it is possible that adding a third charge to what appear to be very stringy two-charge microstates will expand the physical length scales and result in smooth fluctuating solutions at the bottom of a very long throat.

1.4 The present approach

Returning to our main goal, we wish to describe the detailed structure of the semi-classical superstratum in terms of the D1-D5 CFT. We therefore begin in section 2 by reviewing the D1-D5 CFT and in section 3 we describe the two-charge ($\frac{1}{4}$-BPS) states of the D1-D5 system and how they correspond to supertube profiles. In section 4 we add momentum to the system and relate the three-charge ($\frac{1}{8}$-BPS) states to profiles of the superstratum. We initially adopt a rather conservative approach by focussing on the details of the microstate structure that we are confident can be reproduced by quantizing the supergravity modes. In particular, we focus on the space-time shape modes of the superstratum and how they can be matched to perturbative modes of a particular sector of the D1-D5 CFT. This allows us to reproduce the correct charge growth of the black-hole entropy, albeit with a smaller overall coefficient. In section 5 we adopt a less conservative view of the possible modes that a superstratum can have, which is closer to the original arguments for the existence of superstrata [10] and to the physics of certain singular limits of superstratum solutions [47, 48]. This allows us to use a counting argument similar to that of Maldacena, Strominger and Witten [56] to reproduce exactly the entropy of the three-charge black hole, and to obtain the correct overall coefficient as well. We then discuss several ways in which the liberal and conservative approaches to superstrata can be reconciled, and in particular we suggest in section 6 that bound states of multiple superstrata may be a key ingredient in relating all the states of the CFT to bulk supergravity solutions. Section 7 contains our concluding remarks.

2 The D1-D5 CFT and the "visible" sector

The easiest way to quantize the two-charge system is in the F1-P frame where the states are simply those of the perturbative string. However, for the superstratum, we are going to need

the detailed description in the D1-D5 duality frame where there are N_5 D5 branes wrapped on $T^4 \times S^1$ and N_1 D1 branes wrapped on the common S^1. Let R be the radius of the S^1 and v the corresponding coordinate. For fixed v, the moduli space of the configurations is the same as that of N_1 D0 branes inside N_5 D4 branes and so it may be identified with the moduli space of N_1 instanton sector of $\mathrm{SU}(N_5)$ Yang-Mills. The dimension of this moduli space is $4N_1N_5$. These moduli can be made into functions of v and thus, in the perturbative regime, one has a CFT with $4N_1N_5$ bosons on this S^1. However, the D1-D5 system has 8 supersymmetries, which extend the CFT to an $\mathcal{N} = (4,4)$ SCFT. There are thus $8N_1N_5$ free fermions that split into $4N_1N_5$ left-movers and $4N_1N_5$ right-movers.[4]

To be more precise, the underlying field theory is the $\mathcal{N} = (4,4)$ superconformal sigma model whose target space is the orbifold, $(T^4)^N/S_N$, where $N \equiv N_1N_5$ and S_N is the permutation group on N elements.[5] There are thus $4N$ free bosons and $4N$ free fermions. Following [36, 59] the bosons will be labeled, $X_{(r)}^{\dot{A}A}(z,\bar{z})$, where $r = 1, \ldots, N$, is the copy index of the T^4 and $A, \dot{A} = 1, 2$ are spinorial indices for the $\mathrm{SO}(4)_I = \mathrm{SU}(2)_1 \times \mathrm{SU}(2)_2$ of the tangent space of T^4. The left-moving and right-moving fermions, $\psi_{(r)}^{\alpha\dot{A}}(z)$ and $\tilde{\psi}_{(r)}^{\dot{\alpha}\dot{A}}(\bar{z})$ with $\alpha, \dot{\alpha} = \pm$, transform as doublets of fixed helicity on the T^4 and as doublets of different helicities under the $\mathcal{R}$-symmetry, $\mathrm{SO}(4)_{\mathcal{R}} = \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$. Note that the fermions transforming in the $(\mathbf{2}, \mathbf{1})$ and $(\mathbf{1}, \mathbf{2})$ of the $\mathcal{R}$-symmetry are left-moving and right-moving, respectively. The T^4 is, of course, the compactification manifold of the D5's and, as usual in theories on D-branes, the $\mathcal{R}$-symmetry is generated by rotations in the (non-compact) spatial directions transverse to all the branes, that is, in the space-time directions.

In the fully back-reacted D1-D5 geometry, the near-brane limit is *global* $AdS_3 \times S^3 \times T^4$ and the symmetry outside the T^4 is $\mathrm{SL}(2,\mathbb{R})_L \times \mathrm{SU}(2)_L \times \mathrm{SL}(2,\mathbb{R})_R \times \mathrm{SU}(2)_R$. These symmetries correspond to the left-moving and right-moving (finite) conformal invariance and $\mathcal{R}$-symmetry via the holographic duality.

By construction, the excitations of the bosons, $X_{(r)}^{\dot{A}A}$, only involve motions in the compactified (T^4) directions, whereas the fermionic excitations carry polarizations ($\mathcal{R}$-charge) that *are* visible within the six-dimensional space-time. To understand what portion of the fermion Hilbert space is visible from the space-time, it is convenient to bosonize the fermions by defining the currents

$$J_{(r)}^{\alpha\beta}(z) \;\equiv\; \frac{1}{2}\psi_{(r)}^{\alpha\dot{A}}(z)\,\epsilon_{\dot{A}\dot{B}}\,\psi_{(r)}^{\beta\dot{B}}(z)\,, \qquad\qquad \tilde{J}_{(r)}^{\dot{\alpha}\dot{\beta}}(\bar{z}) \;\equiv\; \frac{1}{2}\tilde{\psi}_{(r)}^{\dot{\alpha}\dot{A}}(\bar{z})\,\epsilon_{\dot{A}\dot{B}}\,\tilde{\psi}_{(r)}^{\dot{\beta}\dot{B}}(\bar{z})\,, \qquad (2.1)$$

$$K_{(r)}^{\dot{A}\dot{B}}(z) \;\equiv\; \frac{1}{2}\psi_{(r)}^{\alpha\dot{A}}(z)\,\epsilon_{\alpha\beta}\,\psi_{(r)}^{\beta\dot{B}}(z)\,, \qquad\qquad \tilde{K}_{(r)}^{\dot{A}\dot{B}}(\bar{z}) \;\equiv\; \frac{1}{2}\tilde{\psi}_{(r)}^{\dot{\alpha}\dot{A}}(\bar{z})\,\epsilon_{\dot{\alpha}\dot{\beta}}\,\tilde{\psi}_{(r)}^{\dot{\alpha}\dot{A}}(\bar{z})\,. \qquad (2.2)$$

For each value of r, the currents $J_{(r)}^{\alpha\beta}$ and $\tilde{J}_{(r)}^{\dot{\alpha}\dot{\beta}}$ generate a level 1, $\mathrm{SU}(2) \times \mathrm{SU}(2)$ current algebra. Each such algebra may be viewed as being generated by a single boson.

If one sums over r, the currents

$$J^{\alpha\beta}(z) \;\equiv\; \sum_{r=1}^{N} J_{(r)}^{\alpha\beta}(z)\,, \qquad \tilde{J}^{\dot{\alpha}\dot{\beta}}(\bar{z}) \;\equiv\; \sum_{r=1}^{N} \tilde{J}_{(r)}^{\dot{\alpha}\dot{\beta}}(\bar{z})\,, \qquad\qquad (2.3)$$

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[4]For more details on the D1-D5 CFT, see, for example, [36, 57–59].

[5]This is the description of the CFT at the free orbifold point.

generate the level N, $\mathrm{SU}(2)_R \times \mathrm{SU}(2)_L$ current algebra of the $\mathcal{R}$-symmetry. Because of the pseudo-reality of the fermions [36, 59], the standard angular momentum operators, $J^\pm$ and J^3, are given in terms of the $J^{\alpha\beta}$ by:

$$
\begin{aligned}
J_L^3 &= J^{12} = J^{21}, & J_L^+ &= J^{11}, & J_L^- &= J^{22}; \\
J_R^3 &= \tilde{J}^{12} = \tilde{J}^{21}, & J_R^+ &= \tilde{J}^{11}, & J_R^- &= \tilde{J}^{22}.
\end{aligned}
\tag{2.4}
$$

For each value of r, the currents $K_{(r)}^{\dot{A}\dot{B}}$ and $\tilde{K}_{(r)}^{\dot{A}\dot{B}}$ also generate level 1, $\mathrm{SU}(2)_1$ current algebras but now purely on the T^4.[6] The important point is that the $J_{(r)}^{\alpha\beta}(z)$ and $K_{(r)}^{\dot{A}\dot{B}}(z)$ are completely "orthogonal" sets of operators that commute with one another[7] and similarly for $\tilde{J}_{(r)}^{\dot{\alpha}\dot{\beta}}$ and $\tilde{K}_{(r)}^{\dot{A}\dot{B}}$. Thus the N $\mathrm{SU}(2)$ current algebras generated by the $J_{(r)}^{\alpha\beta}$ and $\tilde{J}_{(r)}^{\dot{\alpha}\dot{\beta}}$ involve excitations that are purely visible from the space-time with no component of this chiral algebra creating an excitation on the torus. Conversely, the $K_{(r)}^{\dot{A}\dot{B}}$ and $\tilde{K}_{(r)}^{\dot{A}\dot{B}}$ represent the chiral algebras that are visible only from the T^4 and invisible from the space-time. Thus the perturbative excitations that are visible from the six-dimensional space-time form Hilbert spaces, $\mathcal{H}_{\mathrm{st}}$, that can be characterized by the representations of, and excitations created by, the conformal field theory:

$$
(\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R)^N / S_N,
\tag{2.5}
$$

where the $J_{(r)}$ and $\tilde{J}_{(r)}$ generate these level 1 current algebras. This theory has central charge $c = N = N_1 N_5$. Similarly, the CFT that lies purely on the internal directions has $c = 5N = 5N_1 N_5$ and is generated by the bosons, $X^{A\dot{A}}$, and the currents $K_{(r)}$ and $\tilde{K}_{(r)}$. We will denote the internal Hilbert spaces by $\mathcal{H}_{\mathrm{int}}$ and think of the states of the D1-D5 theory as being decomposed into a sums of the products of the form

$$
\mathcal{H} = \mathcal{H}_{\mathrm{st}} \otimes \mathcal{H}_{\mathrm{int}}.
\tag{2.6}
$$

The back-reaction of the fermionic and bosonic modes of the D1-D5 CFT will result in shape and charge-density modes of the corresponding supergravity solution. Conversely, we will argue, in the next section, that the semi-classical quantization of the corresponding families of BPS microstate geometries will lead to the states of the D1-D5 CFT. Indeed this is precisely what holographic field theory on $AdS_3 \times S^3$ suggests. Moreover, because of the split into $c = N = N_1 N_5$ and $c = 5N = 5N_1 N_5$ sectors detailed above, we expect that the supergravity modes in the space-time directions alone will be enough to see a $c = N_1 N_5$ sector of the CFT while the remaining $c = 5N_1 N_5$ sector will be visible from semi-classical quantization of internal modes of the D1-D5 system.

We now substantiate this view by revisiting the geometry and semi-classical structure of the two-charge system and argue how this will be modified via the addition of the third charge via momentum modes.

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[6]For the full internal $\mathrm{SU}(2)$ symmetry current, we must include the contribution from the bosonic field $X^{\alpha\dot{\alpha}}$. Note that the internal rotational symmetry is, of course, broken by the compactification.

[7]One can see this most easily by viewing the indices on the fermions, $\psi_{(r)}^{\alpha\dot{A}}$, as transforming as a $(\mathbf{2},\mathbf{2})$ of $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_1$ and then the J's and K's generate these two $\mathrm{SU}(2)$'s.
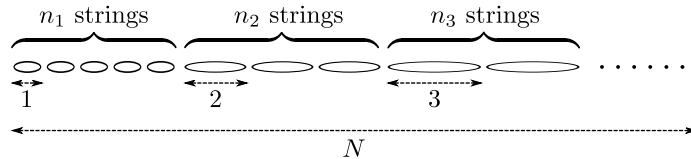
Figure 1. The "effective string" picture of the RR ground states of the D1-D5 CFT. There are n_1 strings of length 1, n_2 strings of length 2, and so on, and the total length of the system if N.

3 The two-charge states

The two-charge states of the D1-D5 system are the Ramond-Ramond (RR) ground states of the CFT and preserve half the CFT supercharges, or eight supersymmetries (note that these states are called $\frac{1}{4}$-BPS states, relative to the 32 supercharges of type IIB superstring before putting D-branes). These states have angular momenta in the range $-\frac{N}{2} \leq J_L^3, J_R^3 \leq \frac{N}{2}$. One can spectrally flow these states to the NS sector to obtain chiral primary fields and the RR ground states can viewed as being created by chiral primaries acting on the maximally-spinning RR ground state, $|\psi_0\rangle$, with $J_L^3 = J_R^3 = -\frac{N}{2}$ [60, 61]. Spectral flow takes the RR-state $|\psi_0\rangle$ to the vacuum $|1\rangle_{\text{NS}}$ of the NS sector.

The chiral primaries of the D1-D5 CFT can be obtained from the twist fields of the S_N orbifold, and these fields are labeled by the conjugacy classes of S_N. The conjugacy classes of S_N are in one-to-one correspondence with the partitions of N, which are given by collections of non-negative integers $\{n_k\}_{k \geq 1}$ satisfying

$$N = \sum_{k \geq 1} k \, n_k. \tag{3.1}$$

It is useful to imagine these as describing a collection of "effective strings." Namely, one associates the conjugacy class $\{n_k\}_{k \geq 1}$ with n_1 effective strings of length 1, n_2 effective strings of length 2, and so on. The total length of all the effective strings is N. See figure 1. The effective string of length k represents a twist field that intertwines k copies of the $c = 6$ CFT and may be viewed as taking k circles of length $2\pi R$ and twisting them into combinations of fewer but longer circles. The maximally-spinning state $|\psi_0\rangle$ is unexcited by chiral primaries and so involves no intertwining of CFTs. It thus corresponds to the partition with $n_1 = N$ and all other $n_k = 0$.

The holographic dual of the maximally-spinning state is a single, maximally-spinning, perfectly circular supertube in an $\mathbb{R}^2$ plane. In the near-supertube limit this geometry is exactly *global* $AdS_3 \times S^3$. The chiral primaries carry $\mathcal{R}$-symmetry, by definition, and also have T^4 indices. In the effective string picture, we may view the effective strings as carrying $\mathcal{R}$-symmetry and T^4 indices coming from fermion zero modes. We will focus here on the $\mathcal{R}$-charge since it is visible from six-dimensional space-time and we will suppress for now the T^4 structure.[8] The partition (3.1) is now refined according to

$$N = \sum_{k \geq 1} \sum_{\alpha, \dot\alpha = \pm} k \, n_k^{\alpha \dot\alpha}, \tag{3.2}$$

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[8]For a more detailed description of the geometries dual to effective strings that carrying T^4 indices, see [62].

where $n_k^{\alpha\dot\alpha} = 0, 1, 2, \ldots$ is the number of effective strings with length k and $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ spin $(\alpha, \dot\alpha)$. The maximally-spinning state $|\psi_0\rangle$ with $J_L^3 = J_R^3 = -\frac{N}{2}$ corresponds to the partition with $n_1^{--} = N$ and all other $n_k^{\alpha\dot\alpha} = 0$.

Introducing twist fields generates excitations in the shape and density modes and the bulk geometry dual to a generic two-charge state of the form (3.2) is the Lunin-Mathur geometry [40] which is D1-D5 supertube with KKM dipole charge and an arbitrary profile, or shape. (For a more detailed dictionary see [63].) The Lunin-Mathur geometry is completely regular [41] and parametrized by arbitrary functions of one variable, $f^i(w)$ $(i = 1, 2, 3, 4)$, describing the profile of the D1-D5 supertube in the $\mathbb{R}^4$ transverse to the D1-D5 world-volume. The SO(4) vector index i of the $f^i(w)$ in $\mathbb{R}^4$ is simply a pair of spinor indices, $(\alpha, \dot\alpha)$, of the $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ $\mathcal{R}$-symmetry. Hence we will denote these shape modes by $f^{\alpha\dot\alpha}(w)$. These functions are periodic, $f^{\alpha\dot\alpha}(w + L) = f^{\alpha\dot\alpha}(w)$, and can be expanded in Fourier series as

$$f^{\alpha\dot\alpha}(w) = \mu \sum_{\substack{k \in \mathbb{Z} \\ k \neq 0}} \frac{a_k^{\alpha\dot\alpha}}{\sqrt{|k|}} e^{2\pi i k w/L}, \qquad a_{-k}^{\alpha\dot\alpha} = (a_k^{\alpha\dot\alpha})^*, \tag{3.3}$$

where L, μ are constants.[9] The zero mode $k = 0$ has been removed by shifting the origin of the $\mathbb{R}^4$. The AdS/CFT dictionary for the two-charge states [40][10] is that the number of effective strings, specified by $n_k^{\alpha\dot\alpha}$, is identified with the magnitude of the Fourier coefficients of the profile functions, $a_k^{\alpha\dot\alpha}$, by

$$n_k^{\alpha\dot\alpha} \; \leftrightarrow \; |a_k^{\alpha\dot\alpha}|^2. \tag{3.4}$$

In the bulk viewpoint, the constraint (3.2) is nothing other than the requirement that the supertube carries N_1 units of D1-brane charge.

In this way one can substantiate the idea that semi-classical quantization of the D1-D5 profiles yields a description of the states of the D1-D5 system [40, 65]. For the two-charge system, the profiles for the typical states have curvatures of order the Planck scale and so one must appeal to the idea of *microstate solutions* [32] discussed in the Introduction, to argue that while the supergravity approximation is not strictly valid, supergravity is capturing the essential semi-classical degrees of freedom that underlie the microstate structure. On the other hand, adding the third charge to the system means that there can be *deep scaling* solutions [53–55] in which the underlying structures remain macroscopic but lie at the bottom of long AdS throats. This means that the supergravity approximation can remain valid over a large range of excitations and that the semi-classical description of smooth low-curvature geometries may be enough to account for the entropy.

This dictionary (3.4) is in complete accord with the idea that the effective strings carry $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ charges and they must represent visible microstates in the dual six-dimensional spacetime. As we argued above, the effective strings arise from twist fields that

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[9]Although we do not need their explicit expression, for completeness, they are given by $L = 2\pi g_s \alpha' N_5/R$, $\mu = \alpha'^2 g_s/(R\sqrt{V_4})$, where $(2\pi)^4 V_4$ is the volume of T^4 [64].

[10]For a precise dictionary and its subtleties, see [63].

intertwine k copies of CFT, with $k = 1, \ldots, N$. The fact that these fields carry $\mathcal{R}$-charges, i.e., space-time angular momenta, means that they have polarizations directed into the space-time and so describe fluctuations in space-time. Indeed, acting with these twist fields changes the length and spins of effective strings and, by the AdS/CFT dictionary (3.4), corresponds to changing the shape of the back-reacted D1-D5 supertube. We may look on these twist fields as providing a Landau-Ginzburg description of the shape modes of the D1-D5 system. It should be stressed that these shape modes correspond to supertube profiles in the $\mathbb{R}^4$ transverse to the D1-D5 world-volume. There will be similar shape modes in the T^4 directions but in this paper we focus on the space-time shape modes.

The correspondence between the quantization of shape modes and the states of the two-charge system is, of course, obvious in the F1-P duality frame where one is simply describing shape modes of a fundamental string. Indeed, one can go from the F1-P modes to the description of the D1-D5 modes by a suitable set of duality transformations. However, we need to work in the D1-D5 frame and see that the states in this frame are also represented by shape modes because we are now going to add a third charge to the system and it is easiest to understand what this entails if the new third charge is a momentum charge and not some other brane charge. By showing that the D1-D5 states involve shapes as a function of one variable we are now going to see that the D1-D5-P states are obtained by giving these D1-D5 shape modes an extra dependence on another direction.

4 Adding momentum: the three-charge states

4.1 Adding the momentum

As we have seen, the two-charge ($\frac{1}{4}$-BPS) states of the D1-D5 system can be mapped onto the RR ground states of the CFT on the common S^1 of the D1 and D5 branes. The three-charge ($\frac{1}{8}$-BPS) states are obtained simply if we keep the Ramond ground states in the right-moving sector, thereby preserving half of the right-moving supersymmetries, but allow any excited state, $|\chi\rangle$, in the left-moving sector, thereby breaking all the left-moving supersymmetries. (The choice of the left/right sector to break/preserve supersymmetry is purely conventional and we could have done it in the other way around.) The eigenvalue of the left-moving Virasoro generator, L_0, on a state, $|\chi\rangle$, yields the momentum, $P = L_0 - c/24$, of the corresponding $\frac{1}{8}$-BPS state. It was this construction that originally led to the perturbative counting of BPS microstates [2] and the microscopic description of the entropy (1.1). As we saw above, the $\frac{1}{4}$-BPS shape modes along the profile in the spatial $\mathbb{R}^4$ are the shapes of the D1-D5 configuration described by $f^{\alpha\dot\alpha}(w)$ (or equivalently by $a_k^{\alpha\dot\alpha}$) and these may be thought of as choices of Ramond ground states or as the states generated by acting with chiral primaries upon the maximally-spinning ground state $|\psi_0\rangle$.

Just as for fundamental strings, adding momentum to any system of branes is expected to involve excitations transverse to the branes (see footnote 3). In the fully back-reacted supergravity solution, these momentum states are reflected in a non-trivial profile that sources the solution. Conversely, the quantization of that profile yields a semi-classical description of the momentum states of the system. If we assume that these are also true in the current situation, adding momentum to the D1-D5 system means that the back-reacted

supergravity solution will now not only have a profile in the spatial $\mathbb{R}^4$, parametrized by w, but that such a profile will now also depend upon v, the coordinate along the S^1 common to the D1 and the D5 branes[11] Thus one obtains shape modes that depend upon functions of *two* variables and these functions will provide a semi-classical description of all the states of the D1-D5 system.

In particular, if we focus on the perturbative states visible within the space-time and described by $\mathcal{H}_{\text{st}}$ then these shape modes are captured by the space-time shape modes of a generic, single superstratum. We therefore expect that the two-charge profile functions, $f^{\alpha\dot\alpha}(w)$, which describe the supertube along an arbitrary curve in $\mathbb{R}^4$, will be promoted to three-charge profile functions, $f^{\alpha\dot\alpha}(w,v)$, which describe the superstratum along an arbitrary surface. Correspondingly, the one-index Fourier coefficients $a_k^{\alpha\dot\alpha}$ will be promoted to two-index ones, $a_{km}^{\alpha\dot\alpha}$.

Put differently, we can take a Landau-Ginzburg perspective in which the D1-D5 modes are created by chiral primaries and these, considered as Landau-Ginzburg fields, become momentum carriers simply through their descendant states within the left-moving Hilbert space. Thus we see how a generic perturbative BPS excitation can give rise to a double Fourier series (with coefficients $a_{km}^{\alpha\dot\alpha}$) of space-time dependent excitations of the original D1-D5 system, or unexcited superstratum.

4.2 Details of the perturbative momentum states

The connection between perturbative CFT states and the supergravity shape modes can be made very explicit. In the near-superstratum limit the geometry is simply $AdS_3 \times S^3$, which is the dual of the maximally-rotating RR ground state. The shape modes of the superstratum are simply Fourier modes of supergravity fields on the S^3 and thus correspond to representations of the $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$. While the two-charge D1-D5 shape modes carry quantum numbers of both $\mathrm{SU}(2)_L$ and $\mathrm{SU}(2)_R$, the momentum-carrying *BPS* operators that excite those states carry only the quantum numbers of $\mathrm{SU}(2)_L$ and hence adding momentum does not involve changing the D1-D5 shape modes that transform under $\mathrm{SU}(2)_R$. In particular, consider the maximally-spinning D1-D5 solution whose near-brane geometry is $AdS_3 \times S^3$. The generic D1-D5 ground states can be thought of as fluctuation modes on the S^3. In the NS sector, they are the chiral primary states and have quantum numbers under $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ given by $(\ell, m; \tilde\ell, \tilde m) = (\ell, \ell; \tilde\ell, \tilde\ell)$. Note that these D1-D5 "supertube" shape modes on the S^3 are very special, in that the quantum numbers are constrained to satisfy $\ell = m$, $\tilde\ell = \tilde m$ and, furthermore, $|\ell - \tilde\ell|$ is equal to the spin of the fields that exist in the theory. For a fixed spin field the Fourier modes are determined by one quantum number and hence correspond to one-dimensional shape modes on the S^3. In contrast, the BPS momentum carrying modes, which are of the form $(any, chiral)$ in the NS sector, allow more general excitations under $\mathrm{SU}(2)_L$, while the $\mathrm{SU}(2)_R$ quantum numbers remain unchanged. So, the generic $\frac{1}{8}$-BPS mode will have $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$

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[11]In general, the geometries dual to CFT states that are exact eigenstates of the momentum operator $P \equiv L_0 - \tilde L_0$ are v-independent, while coherent states, which are not a precise eigenstate of P, are v-dependent [36]. We are concerned with the latter because we are interested in the traveling waves on the supertube along v and their classical description is given by coherent states.

quantum numbers $(\ell, m; \tilde{\ell}, \tilde{\ell})$. Since we now have m independent of ℓ, these will generate intrinsically two-dimensional shape modes on the S^3.

A particular subset of the BPS states involve arbitrary excitations created by operators in the SU(2) current algebras, $J_{(r)}^{\alpha\beta}$, defined in (2.1). As noted above, these currents and the associated left-moving CFT in (2.5) reflect purely space-time modes and will be visible in the perturbative space-time shape modes of the superstratum.

To make this more precise, one can easily describe the complete set of two-charge supertube shape and density modes within supergravity and express the result in terms of exact supergravity solutions in six dimensions. One can also realize the action of the superconformal algebra on the geometry and, in particular, implement the action of the currents (2.3) in terms of rotations on the supergravity solutions. In this way one can, at the linearized level, generate the linearized supergravity solutions with shape modes in the $(\ell, m; \tilde{\ell}, \tilde{\ell})$ representations by starting with the D1-D5 shape modes $(\ell, \ell; \tilde{\ell}, \tilde{\ell})$ that correspond to chiral primaries in CFT. Realizing this procedure has been one of the major goals of [35–37]. The fact that BPS equations of the six-dimensional supergravity are essentially linear means that knowing the linearized solutions is almost enough to construct the fully back-reacted solutions [47]. This observation was exploited to significant effect in [37, 38]. To construct the fully back-reacted BPS fluctuations of the superstratum and show that there is indeed an intrinsically two-dimensional BPS shape modes in space-time one simply needs to take the special fluctuating modes considered in [38] and use the current algebra action, as in [37, 66], to find the generic supergravity modes and then try to compute the fully back-reacted solution using [47].

The foregoing procedure of rotating supertube fluctuation modes by the generators of the asymptotic symmetry algebra corresponds to acting by the *total* $J^{\alpha\beta} = \sum_{r=1}^{N} J_{(r)}^{\alpha\beta}$ and not by the *individual* $J_{(r)}^{\alpha\beta}$. Moreover, one really only needs the zero modes of $J^{\alpha\beta}$ to obtain the fluctuations with quantum numbers of the form $(\ell, m; \tilde{\ell}, \tilde{\ell})$. Put differently, this is equivalent to a rather trivial statement that acting on a chiral primary by the generators of the finite Lie algebras $\mathrm{SL}(2, \mathbb{R})_L \times \mathrm{SU}(2)_L$ only gives the descendant of a chiral primary but certainly does not yield generic $\frac{1}{8}$-BPS states that are descendants of the non-chiral primaries. It therefore seems, at first sight, that the procedure we have outlined only generates an extremely small subset of the general momentum-carrying states, which require all the modes of all the individual currents $J_{(r)}^{\alpha\beta}$.

However, this is not exactly what we are doing: we are not simply rotating a complete, known classical BPS state. Instead we are using rotations to generate all the individual fluctuating modes of some of the fields but discarding all of the rest of the rotated solution. We then take arbitrary linear combinations of those modes as seeds to generate new classical solutions using the linear BPS system replete with its sources that depend non-linearly on the fluctuating modes. In this way we construct the most general, fully back-reacted fluctuating supergravity solution. In the quantum theory, classical solutions can be regarded as coherent quantum states and so taking such classical linear combinations amounts to taking tensor products of the corresponding quantum states. The products of descendants of chiral primaries generically yield the descendant of non-chiral primaries [67, 68]. Therefore,

if we complete the fully back-reacted supergravity solution based on linear combinations of modes, they will represent the descendants of the non-chiral primaries.

Thus the process of feeding a general superposition of classical fluctuations into the complete BPS system will certainly generate the most general exact classical BPS states and we claim that this will also give a semi-classical description of the most general BPS quantum state. Indeed, precisely this sort of result was established in [68] where it was shown that the space of supergravity fluctuations in a finite neighborhood of the $AdS_3 \times S^3$ background precisely reproduced the elliptic genus of the CFT (ref. [68] is when the internal manifold is K3; for T^4, see [69]).

It is important to note that the result of [68, 69] was only established using a perturbative supergravity "gas" around a solution that lay outside the black-hole regime and so one may quite reasonably doubt the applicability of this result within microstate geometries that look like black holes. However, to make a microstate geometry that looks like a black hole one does not simply use small perturbations of $AdS_3 \times S^3$: one must incorporate the back-reaction of the momentum to obtain *deep, scaling* microstate geometries in which the topological cycles descend a long AdS_2 throat. We will discuss this further in the next section, but here we want to note that $AdS_3 \times S^3$ represents a good local model of individual topological bubbles and it is expected that their fluctuations will give the microstate structure only when these bubbles are located at the bottom of a *deep, scaling* throat. All we therefore need from [68, 69] is the result that the that semi-classical quantization of supergravity modes on $AdS_3 \times S^3$ captures the quantum CFT states locally. It is then expected that these states generate the correct microstate structure of a black hole when they are located deep within a scaling solution and greatly red-shifted as a result.

Before concluding this section we want to return to the other classical modes that live on the internal T^4 and whose semi-classical quantization should give rise to $\mathcal{H}_{\text{int}}$ in (2.6). Indeed, one of the points emphasized in [35, 36] is that *all* the perturbative excitations of D1-D5 system will be visible within the ten-dimensional supergravity description of the superstratum. The left-moving $c = N$ theory (2.5) whose states lie in $\mathcal{H}_{\text{st}}$ will indeed be visible within the space-time of the effective six-dimensional theory but the remaining modes, lying in $\mathcal{H}_{\text{int}}$ and described in terms of the other $c = 5N$ part of the full CFT, will be also visible as perturbative fluctuations of geometry and fluxes in the full ten-dimensional solution. Thus, even though the space-time shape modes of the superstratum will only lead to an entropy (1.2), one might hope that the internal supergravity modes should lead to the full accounting for the entropy (1.1).

However, as we will now describe, there is a subtlety in the supergravity back-reaction that suggests that only the space-time shape modes will have sufficient resolution to capture a large enough section of the Hilbert space of the D1-D5-P system.

4.3 The supergravity back-reaction and holography

One of the important features of the CFT dual of black-hole microstates is the fact that the CFT can have an energy gap as low as $E_{\text{gap}} \sim c^{-1} \sim \frac{1}{N_1 N_5}$. This can be viewed as coming from the scaling dimensions of the longest twist operators or from the longest-wavelength momentum excitations of the longest effective strings. For a long time it was a puzzle as to

how such fractionation, and the energy gap in particular, could emerge from fluctuations of smooth microstate geometries. Such a match is crucial if the semi-classical quantization of supergravity is to reproduce the perturbative states of the CFT with sufficient fidelity to obtain the entropy.

To understand the holographic description of the correct E_{gap}, one should first recall that the only way to construct microstate geometries whose charges correspond to a five-dimensional black hole with a finite horizon area is to use *deep, scaling* BPS geometries have a very long AdS throat that is smoothly capped off by bubbles, or homology cycles. The energy gap of these solutions then emerges holographically [53] by taking the longest-wavelength fluctuation of the microstate geometry and red-shifting it according to the depth of the throat. The depth of the throat is typically a free classical parameter in the microstate geometry however semi-classical quantization of such geometries sets the throat depth and thus fixes the energy gap [7, 55, 70]. It was thus one of the triumphs of the microstate geometry program that this correctly reproduced the energy gap of the dual CFT. The simplest microstate geometries, in which the holographic energy gap was first computed, can then be viewed as containing unexcited superstrata and so the semi-classical quantization of the superstratum will reproduce the correct energy levels.

Thus, in the holographic dual, modes of with energy $E_{\text{gap}} \sim \frac{1}{N_1 N_5}$ come from *space-time* fluctuations whose wavelengths are of order the diameter of throat of the BPS black hole.[12] If there is only a handful of bubbles or superstrata, then this wavelength is set by the longest wavelength fluctuation of homology cycles that spread across the throat. If there are a lot of bubbles or superstrata then this wavelength should be thought of as the longest wavelength collective mode of all the bubbles and superstrata.

This result relies upon the crucial structure of the warp factors in the metric. In the IIB formulation, the ten-dimensional metric takes the form:

$$
\begin{aligned}
ds_{10}^2 &= -2\frac{1}{\sqrt{Z_1 Z_2}}\,(dv + \beta)\big(du + k - \tfrac{1}{2}\,Z_3\,(dv + \beta)\big) \ + \ \sqrt{Z_1 Z_2}\,ds_4^2 \ + \ \sqrt{\frac{Z_1}{Z_2}}\,ds_{T^4}^2 \\
&= -\frac{1}{Z_3\sqrt{Z_1 Z_2}}\,(dt + k)^2 \ + \ \frac{Z_3}{\sqrt{Z_1 Z_2}}\,(dz + A^{(3)})^2 \ + \ \sqrt{Z_1 Z_2}\,ds_4^2 \ + \ \sqrt{\frac{Z_1}{Z_2}}\,ds_{T^4}^2 \,.
\end{aligned}
\tag{4.1}
$$

For BPS solutions, the base metric, ds_4^2, is hyper-Kähler and ambi-polar; the deep, scaling solutions come from taking limits in which a cluster of two-cycles in this base appear to scale to zero size. In the physical metric (4.1) the warp factor $(Z_1 Z_2)^{\frac{1}{2}}$ modifies this so that the cluster of cycles limits to a finite size determined by $Q_1 Q_2$ in the spatial directions of the base. In the full ten-dimensional metric, the two-cycles are lifted to three-cycles via the v fiber and their volume also involves Q_3. The important point is that the "area" of the throat scales with $Q^{3/2}$ and so, as a result of the warp factor, the longest wavelength mode that fits across the throat scales as $Q^{-1/2}$. The red-shift of the deep throat then gives an additional factor of $Q^{-3/2}$ to obtain $E_{\text{gap}} \sim Q^{-2}$ [53]. On the other hand the warp factors

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[12]This should, of course be defined as the area of the throat to some suitable power. Alternatively, for a microstate geometry where the throat is capped off, this scale can also be defined by the diameter of all the microstate structure.

in the T^4 directions are $\mathcal{O}(Q^0) = \mathcal{O}(1)$ and so the T^4 *does not* expand to the typical size of the throat. This suggests that fluctuations around the T^4 will develop the wrong energy gap, $E_{T^4 \text{ gap}} \sim Q^{-3/2}$.

Thus it seems that the supergravity fluctuations of the superstratum in the space-time directions do give rise to the correct spectrum of microstates but the supergravity fluctuations on the T^4 will lead to a rather coarse sampling of the microstate structure. It is possible that our supergravity analysis of the T^4 fluctuations is too simplistic and we will return to these issues in section 5 where we will conjecture how the T^4 modes may ultimately be accounted for in the supergravity back-reaction.

4.4   Recapitulation

To finish this rather conservative analysis based upon perturbation theory, we want to reiterate two important conclusions from our discussion. First, and most important, is that whatever the ultimate outcome is on the holography of the T^4 modes, we have provided a good match between the supergravity shape modes and the perturbative microstate structure at least for the states in $\mathcal{H}_{\text{st}}$, with central charge $c = N_1 N_5$. Thus quantizing the superstratum should, at least, reproduce (1.2) and thus obtain the correct growth in entropy with $N_1 N_5 N_P$. This is already huge progress. In particular, since these microstate geometries describe a *macroscopic* fraction of the black-hole entropy, this means that all the typical states that contribute to the black-hole entropy will have a finite transverse size. Hence the entire system will not be surrounded by a horizon and thus we will have established the fuzzball proposal for BPS black holes in string theory.

The other thing we want to stress is that we have studied the perturbative properties of a single, round superstratum and our work and conclusions so far are based upon this rather conservative but fairly detailed correspondence. In section 5 and section 6 we will argue that superstrata that have more complicated shapes, and possibly split into bound states of multiple superstrata will in fact be able to capture the full black-hole entropy.

5   Towards the full black-hole entropy

Our conservative counting of superstrata entropy in section 4 was based on the description of the maximally-spinning supertube in the dual D1-D5 CFT and on the fact that in this CFT the left-moving (supersymmetric) fermions are charged under $\text{SU}(2)_L$ but do not carry $\text{SU}(2)_R$ angular momentum, and hence only a fraction of the shape modes of the supertube will be able to carry momentum. In this section, we will be slightly bolder and discuss how the "missing" shape modes might re-emerge and account for the full entropy of the D1-D5-P black hole.

5.1   The shape modes of the superstratum

From the perspective of the original argument for the existence of the superstratum [10] and from the perspective of supergravity solutions that describe certain superstratum components, the restriction on the possible shape modes encountered in section 4.2 appears rather puzzling.

Indeed, if one constructs the superstratum by gluing together 16-supercharge plaque-ttes that preserve the D1-D5-P Killing spinors irrespective or their orientation [10], there appears to be no restriction on the possible shapes of the resulting object, and hence the general superstratum solution might be expected to be determined by four arbitrary continuous functions of two variables.

This picture is further supported by the explicit construction of supersymmetric solutions that have all the charges and dipole charges of superstrata except one (the KKM dipole moment), and depend also on four arbitrary continuous functions of two variables [48]. These solutions are dubbed *supersheets*. Recall that, as mentioned in section 1.2, the first way to get a superstratum is to use a supertube transition to "puff out" D1 branes and momentum into a D1-P supertube and D5 branes and momentum into a D5-P supertube (first stage), and then to use a second supertube transition to puff out again the resulting (boosted and rotated) D1-D5 system into a KKM dipole charge (second stage). Because supersheets do not have a KKM dipole moment, they must be describing the first stage of this bubbling process and, consequently, represent singular supergravity solutions. The solution is expected to become a smooth superstratum once the KKM dipole moment is added and it was shown in [10] that adding the KKM dipole is compatible with super-symmetry. If the circle wrapped by the KKM dipole charge is small, this will only affect the solution in the immediate vicinity of the supersheets and hence one might reasonably expect that the KKM will not upset the shape and the supersymmetry.

Based on the foregoing arguments, we are going to assume in the rest of section 5 that a suitably generic superstratum can be given four independent shape functions. However, before proceeding on this assumption, we wish to raise several issues that might lead to restrictions on the BPS shape modes and limit such modes to those described in section 4.

First, it was noted in [10] that adding a KKM monopole requires the orientation of the KKM to be properly aligned with the underlying compactification circles, a fact that also was manifest in [38] and leads, potentially, to restrictions on the orientations of the solutions. Nevertheless, it is unclear whether this condition leads to significant restrictions on the moduli space.

Another issue is that the shape modes outlined in [10] were based upon brane config-urations that were not fully back-reacted and the description of shape modes was based upon the local geometry of the solution. In the fully back-reacted superstratum some of the directions necessarily pinch off to make the smooth underlying topological cycles. Moreover, the directions that get pinched off are typically those upon which the shape modes depend. For a smooth solution the shape modes must therefore be required to die off as they approach these "pinch-off" points. This may well lead to restrictions on the allowed BPS modes that can be smoothly excited on a superstratum and some of these restrictions were encountered and analyzed in [38]. It remains to be seen what the full range of allowable smooth shape modes can be for a single cycle but it may be only the modes considered in section 4.

Finally, there is an interesting intermediate ground between the two extremes of four shape modes and the modes of section 4. It is possible that some of the shape modes have been suppressed by focusing on a single topological cycle and, in particular, on the

scale-invariant $AdS_3 \times S^3$ near-superstratum limit. The "missing" degrees of freedom could then emerge either as one restores the asymptotic flatness or adds more structure so as to introduce a scale. In the same vein, it may be that when one tries to make a KKM resolution of a BPS supersheet of arbitrary shape, it is possible that one may not be able to do it with a single topological bubble but that it will require several such bubbles and that the combination of the modes on such a multi-bubble solution can lead to more functions of two variables. We will pursue this idea further in section 6.

5.2 The MSW counting of black-hole entropy

As we have argued, it is possible that once the full non-perturbative superstratum is constructed, the original picture of the BPS superstratum [10] could prove correct in terms of predicting the number of shape modes. We will therefore examine what this would mean for the superstratum and in particular we will argue that that such fluctuation modes reproduce *all* the entropy of the three-charge black hole.

To see how this comes about, it is useful to recall the "second" way to get a superstratum by starting with a D1-D5 supertube with KKM dipole charge and subsequently adding momentum to it. Then the counting is very similar to the Maldacena-Strominger-Witten (MSW) counting of the entropy of four-dimensional black holes [56]: one argues that the number of momentum carriers on a superstratum is equal to the dimension of the moduli space of deformations of the D1-D5 supertubes and then derives the entropy by counting the ways of distributing the momentum amongst these moduli. At first glance the number of supertube moduli is infinite, since an arbitrary shape can be decomposed into an infinite Fourier series with arbitrary components. However, the quantization of the shapes of the supertubes reduces the range of the Fourier modes and hence renders the dimension finite. As we explained in section 2, this can be seen from the dictionary to the dual D1-D5 CFT, which restricts the length of the maximal effective string on the boundary (which corresponds to the Fourier mode of the round supertube) to $N_1 N_5$, and since there are four functions determining the embedding of the supertube in spacetime this corresponds to a moduli space dimension $4N_1 N_5$.[13]

There is another way to figure out that the dimension of the moduli space of spacetime deformations of two-charge supertubes is $4N_1 N_5$. As we explained in section 3, these supertubes can be dualized to fundamental strings carrying momentum, and the entropy of this system comes from the various ways of splitting a given amount of momentum, N_P, among different fractionated momentum carriers that carry momentum quantized in units of $1/N_1$ [1, 71]. This entropy is given by the number of possible ways of writing

$$N_1 N_P = \sum_{k \geq 1} k n_k \,, \tag{5.1}$$

much as in equation (3.1). Upon taking into account the fact that the fundamental string has eight species of bosonic momentum carriers (corresponding to its 8 transverse directions) and their fermionic partners, the number of partitions reproduces the entropy of the

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[13]More precisely, because of the constraint (3.2) imposed on the $4N_1 N_5$ Fourier modes, the moduli space dimension is $4N_1 N_5 - 1$, but this difference is negligible for the entropy counting.

two charge system. The dimension of the moduli space of these configurations is given by the number of modes carrying momentum that can be excited, and for one species alone this number is given by the maximal value of k, which is the product of its two charges: $N_1 N_P$. Hence, the dimension of the moduli space of oscillations that will become D1-D5 supertube oscillations in the *transverse* four-dimensional space is again $4N_1 N_5$.

One can also argue that the dimension of the supertube moduli space is of order $N_1 N_5$ by considering the maximally-spinning (round) supertube and counting its entropy à la Marolf and Palmer [50–52]. This supertube has angular momentum $J = N_1 N_5$, and if one tries to change its shape the angular momentum becomes smaller. One can use the Born-Infeld action describing this supertube to quantize the possible deformations of the maximally-spinning supertube and find that this entropy comes from integer partitions of $N_1 N_5 - J$. This counting therefore implies that the dimension of the moduli space of a supertube with angular momentum J is equal to $N_1 N_5 - J$ (again for each bosonic mode). Strictly speaking, this counting is only valid in the vicinity of the maximally-spinning supertube configuration (when $N_1 N_5 - J \ll N_1 N_5$), but if one extrapolates it to a supertube with zero angular momentum one finds again the dimension of the moduli space of transverse oscillations to be $4N_1 N_5$.

In the foregoing discussion, we only counted the dimension of the moduli space of the supertube fluctuations in the transverse non-compact $\mathbb{R}^4$ directions (label them 1234) and not the internal T^4 directions (label them 6789). This restriction can be justified by a supersymmetry analysis similar to the one in [10]. As mentioned above, the "first" way to get a superstratum is to first puff out D1 branes and momentum, P, into a D1-P supertube inside $\mathbb{R}^4$ and, simultaneously, puff out D5 branes and P into a D5-P supertube inside $\mathbb{R}^4$. If the resulting D1-profile lies entirely within the D5-profile, it is locally the same as the D1-D5 system which can be puffed out again into a KKM dipole charge. However, at the first stage, instead of puffing out the D1 branes and P into a curve inside $\mathbb{R}^4_{1234}$, we could have puffed them out into a curve inside T^4_{6789}. For example, D1(5) and P(5) can be puffed out into D1(6) and P(6) dipoles, where the numbers in the parentheses denote the directions along which the object is extending. Correspondingly, D5(56789) and P(5) can be puffed out into D3(789) and F1(6) dipoles (dissolved as fluxes inside the D5 worldvolume). However, it is an straightforward algebraic exercise [10] to show that these puffed-out charges cannot undergo a second supertube transition. Therefore, interestingly, the second supertube transition is kinematically (supersymmetrically) allowed only if the first transition is in the transverse $\mathbb{R}^4$ directions. This holds true even if the internal manifold is not T^4 but K3, because there is no difference between T^4 and K3 in the local geometry.

Hence, the dimension of the moduli space of bosonic fluctuations of D1-D5 supertubes in the transverse space is $4N_1 N_5$. Much as for the MSW black-hole entropy calculation, this dimension gives the number of bosonic modes that carry momentum, and one expects by supersymmetry that there should be an equal number of fermionic momentum carriers. As we explained above, there is a tension between the perturbative analysis of these modes (described in section 4) which indicates that only $N_1 N_5$ of these modes can carry momentum supersymmetrically, and the original argument for the existence of superstrata

and the solutions of [47, 48], which suggests that all the four bosonic modes, and hence all their four fermionic partners as well, can carry momentum supersymmetrically.

If there really are four bosonic modes and four fermionic counterparts then they will give a semi-classical description of momentum-carrying states with $c = 6N_1N_5$, and the entropy of the superstrata is given by the possible ways of carrying N_P units of momentum:

$$S_{\text{superstrata}} = 2\pi\sqrt{\frac{c}{6}N_P} = 2\pi\sqrt{N_1N_5N_P}\,, \qquad (5.2)$$

which reproduces exactly the Bekenstein-Hawking entropy of the three-charge black hole. Since this entropy comes entirely from spacetime modes and their fermionic partners, this entropy count also reproduces the entropy of the D1-D5-P black hole if one replaces the T^4 by $K3$.

We have thus argued that the shape modes of the superstratum have the *capacity* to describe a full set of semi-classical microstates of a black hole and while this would represent a very happy state of affairs, there are some words of caution to be made. First, as we explained at the end of section 5.1, adding a KKM monopole and pinching off circles to make topological cycles could potentially restrict the shape modes [10]. Second, we have argued that one should think of the $4N_1N_5$ spatial shape modes of the superstratum as independent "moduli" just as those of the MSW string and hence can independently be assigned momentum states. It remains unclear whether these moduli are sufficiently independent and unobstructed. Indeed, these excitations have to satisfy the constraint (3.2) and this restricts the size and degeneracies of the putative moduli space. This constraint will be modified once one adds momentum and previously indistinguishable CFT states become distinguishable. Thus the independence of, and restrictions upon, the supertube moduli remain unclear but as we have seen, it is conceivable that the complete set of shape modes can capture the complete BPS black-hole entropy.

5.3 In search of the lost 5/6$^{\text{th}}$'s

The analysis of section 4 starts from a single round supertube, corresponding to a state of the D1-D5 CFT in which the long effective string of length N_1N_5 is split into N_1N_5 effective strings each of length one, and considers adding supersymmetric (left-moving) momentum perturbatively on this object. The left-moving momentum modes are only charged under $\text{SU}(2)_L$ but not under $\text{SU}(2)_R$, which implies that only the modes that give one sixth of the central charge of all the modes that one might have hoped to promote to momentum carriers are in fact supersymmetric. Moreover, in the original discussion of the superstratum [10] it was pointed out that, while it seemed plausible that the shape modes could be excited independently in the two directions of the superstratum surface, this independence was not established rigorously. So the most conservative conclusion of the perturbative analysis of section 4 is that the space-time modes of superstrata are still given by functions of two variables, as argued in [10], but that these modes only give $\frac{1}{\sqrt{6}}$ of the entropy of the black hole.

It is important to examine the tension between the results of section 4 and the arguments of the previous subsection. Indeed, the results of section 4 indicate that 5/6 of the modes that give rise to the black-hole entropy should appear as semi-classical fluctuations

on the internal T^4 and only 1/6 of these modes are visible in space-time. This suggests that we should simply be looking at the full supergravity solution in ten dimensions and the shape modes on the T^4 in particular. On the other hand the arguments we presented above suggest that *all* the modes that carry the black hole entropy can be visible as superstratum space-time modes. We thus appear to be in danger of over-counting.

One possible solution to this tension could be that the restrictions on the supersymmetric momentum carriers coming from the perturbative analysis are valid only in the vicinity of the maximally-spinning supertube configuration in the free orbifold limit, and that far away from that point in the CFT moduli space these restrictions will be lifted.[14] Indeed, the supersheets of [48] and other singular solutions that have black hole charges and carry momentum with both $SU(2)_L$ and $SU(2)_R$ angular momentum [47] can be thought of as limits of superstrata solutions in which one has turned off the KKM dipole charge. This can be done by making the radius of the second supertube transition very small, which can be achieved by taking the number of KKM's to be very large.[15] From the perspective of the dual CFT, the number of KKM's is the length of the effective strings, and increasing this number brings one very far away from the state we considered in section 4, where there are N length-one effective strings carrying $J_R^3 = \mathcal{O}(N)$ as a whole, towards the sector where there are a few long effective strings of length $\mathcal{O}(N)$ carrying $J_R^3 = \mathcal{O}(1)$. Incidentally, this is also the sector where the black-hole entropy lives, so if the superstratum counting that gives the entropy (5.2) is correct, this entropy comes exactly from where it should come. Starting with this sector with $J_R^3 = \mathcal{O}(1)$, one has a large degree of freedom to increase/decrease J_R^3 by creating short effective strings and making them carry the desired J_R^3. However, we must note that we do have the unitarity bound $-\frac{N}{2} \leq J_R^3 \leq \frac{N}{2}$, which is still in a apparent conflict with the fact that, on the original supersheet, we could consider arbitrary $SU(2)_R$ fluctuations.

Another possible way to reconcile the two analyses above could be to consider multiple superstrata and allow different superstrata (or even different parts of one superstratum) to have different orientations so that the correlation with angular momentum might change between superstrata. It is possible for the momentum modes on one of these superstrata to be charged under $SU(2)_L$ and for the modes on the other to be charged under $SU(2)_R$. Thus, from a suitable distance, a generic collection of superstrata could appear to replicate generic space-time shape modes. Moreover, it is possible to bring two superstrata close to each other and to join them into a figure-eight configuration that looks like a deformation of a superstratum with dipole charge two. One can similarly argue that a superstratum with a very large dipole charge, of the type that is expected to describe the CFT states that give the black-hole entropy, can be deformed into configurations that contain multiple superstrata, which can in turn carry momentum modes with all angular momenta.

While these observations suggest that superstrata may have a much larger set of space-time configurations than the single, round superstratum considered in section 4, it does

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[14]Recall that the perturbation taking the CFT away from the free orbifold point is a twist operator insertion which mixes effective strings with different lengths.

[15]This can appear paradoxical, but increasing the number of KKM's decreases the radius of the KKM's and therefore reduces their influence on the geometry.

not resolve the over-counting danger associated with having both the T^4 modes and the full set of space-time shapes corresponding to states. However, one can argue that, in the regime of parameters where the black hole exists, the modes that look like internal shape modes in the perturbative analysis of section 4 will be suppressed and, in addition, it is possible that they give rise to fluctuations in the transverse space.

Indeed, our analysis of section 4.3 indicates that in the fully back-reacted supergravity regime where the classical black-hole solution exists, the modes that correspond to fluctuations in the internal directions will have the wrong mass gap and will not be therefore capable of describing the modes that give the black hole entropy. This will then suppress such semi-classical states in the total entropy. A "pessimist" would then take the view that only the perturbative space-time shapes have the correct energy gap and thus contribute to the entropy, leading to the result (1.2).

However, based upon our experience with five-dimensional microstate geometries, we know that details of "internal sectors" of the dual field theory corresponding to degrees of freedom on the compactification directions can become visible within the space-time geometry. The Coulomb-Higgs map [72, 73] is a classic example in which Higgs-branch fields create composite operators that give rise to strong effects within the space-time geometry that are more typically associated with the Coulomb branch of the field theory. Sometimes this leakage of information onto the Coulomb branch can be complete in that it yields complete information about the Higgs branch states and sometimes it can be very incomplete in that it only captures a small fraction the data about the "internal states" of the system. Thus one can take the optimistic view that the analysis of section 4.3 suppresses the shape modes from exploring the T^4, thereby protecting us from over counting, but these modes then leak into the "floppier" space-time directions for which the energy gap is *much* lower.

It is also possible that the "missing 5/6$^{\text{th}}$'s" will not be visible semi-classically within supergravity and that we can only obtain the entropy (1.2). As we have already stressed, this still represents major progress. On the other hand, we prefer to take the optimistic view that the missing 5/6$^{\text{th}}$'s should still be visible within supergravity. One might therefore hope that the internal shape modes of the single superstratum migrate to Coulomb branch and become visible as space-time shape modes. It is interesting to ask whether these modes will manifest themselves as superstratum modes, or as some other mode complicated collective modes. The first possibility would reconcile the superstratum analysis in this section with that of section 4. The second possibility would indicate there exists a space-time object more complicated than the single, isolated superstratum and such an object will account for 5/6 of the modes that give the entropy of a black hole, while the single, isolated superstratum accounts for the other 1/6. This more complicated object might be some multi-superstrata state or even something new. Either way, finding and understanding this more complicated object would clearly be a key priority.

We now make some first steps in suggesting the role of multi-superstrata states.

6 Multi-superstrata

Independent of the bulk considerations of the previous section, we will argue that the structure of the three-charge states in CFT suggests that bound states of multiple superstrata
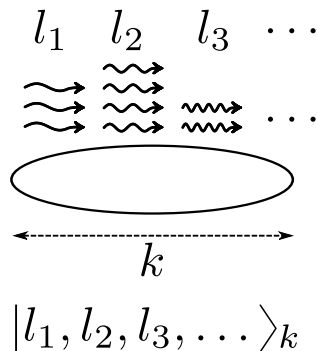
Figure 2. The excited state $|l_1, l_2, \ldots\rangle_k$ of a single effective string of length k. On the string, we have l_1 quanta carrying $\frac{1}{k}$ units of momentum, l_2 quanta carrying $\frac{2}{k}$ units of momentum, and so on. The standard projection in the orbifold procedure imposes the condition $\sum_k m l_m / k \in \mathbb{Z}$.

are the most natural candidate for the holographic duals of the CFT states. To explain this, we begin by unpacking more of the details of the states described in sections 3 and 4.

6.1 Structure of three-charge states in CFT

In the D1-D5 CFT, a two-charge BPS state, i.e. the RR ground state is made of multiple effective strings of various length. Ignoring the $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ charge, it is specified by the numbers $\{n_k\}_{k \geq 1}$ satisfying (3.1) and is of the following form:

$$\prod_{k \geq 1}(|0\rangle_k)^{n_k} = (|0\rangle_1)^{n_1}(|0\rangle_2)^{n_2}(|0\rangle_3)^{n_3} \cdots, \tag{6.1}$$

where $|0\rangle_k$ is the ground state of the $c = 6k$ CFT living on the effective string of length k. See figure 1. The bulk dual of this is a D1-D5 supertube whose profile function $f(w)$ has Fourier coefficients a_k given by

$$|a_1|^2 = n_1, \qquad |a_2|^2 = n_2, \qquad |a_3|^2 = n_3, \qquad \ldots. \tag{6.2}$$

Note that we are ignoring the $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ charge for simplicity of presentation and therefore the spin indices $\alpha, \dot{\alpha}$ on $f(w), a_k$ are also omitted.

The three-charge states are obtained by exciting momentum-carrying modes on the effective strings. In particular, on an effective string of length k lives the $\mathrm{SU}(2)_L$ current $J_L^3(z)$,[16] whose modes we denote by $J_{\frac{m}{k}}$, $m \in \mathbb{Z}$. Note that the mode numbers are in units of $\frac{1}{k}$ because the length of the string is k. We can use these modes to obtain momentum-carrying states on a single effective string as follows:

$$(J_{-\frac{1}{k}})^{l_1}(J_{-\frac{2}{k}})^{l_2} \cdots |0\rangle_k \equiv |l_1, l_2, \ldots\rangle_k, \tag{6.3}$$

with the S_N-orbifold constraint that the total momentum on the effective string is an integer, namely, $\sum_{m \geq 1} m l_m / k \in \mathbb{Z}$. See figure 2 for a pictorial description of this state. Since

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[16]Here, $J_L^3(z)$ is defined to be $J_L^3(z) = J_{L(r)}^3(z)$ with $2\pi(r-1) \leq \arg(z) < 2\pi r$ and is multi-valued, where $r = 1, \ldots, k$ is the copy index. In particular, $J_L^3(z)$ is not the sum of the individual currents, $\sum_{r=1}^k J_{L(r)}^3(z)$.

the modes $J_{-\frac{m}{k}}$ carry non-vanishing $SU(2)_L$ charge, they are visible in six-dimensional space-time. If we excite the J_L^3 modes on all the effective strings in the two-charge state (6.1), we obtain the general three-charge state that can be created by J_L^3 excitations.[17] In doing so, we must remember that effective strings of identical length k are indistinguishable if they are in the ground state but, once we excite J_L^3 modes, they become distinguishable (unless they have identical excitation numbers $\{l_1, l_2, \cdots\}$). Thus, for each k, the n_k states will be broken into distinguishable and indistinguishable effective strings.

To be concrete, let us focus on effective strings with one particular value of length k, say, $k = 3$, for a moment. If we have, e.g., seven of strings of length 3, we have the following two-charge state:

$$(|0\rangle_3)^7. \tag{6.4}$$

The seven strings are indistinguishable because they are all in the same ground state. So, this two-charge state is completely specified by a single number $n_3 = 7$. Now, three-charge states are obtained by exciting momentum modes on these strings, as in (6.3). For example, take two of them and excite the first ($m = 1$) momentum mode three times on each; namely, we have two strings, all in the state $(J_{-\frac{1}{3}})^3|0\rangle_3 = |3, 0, 0, \ldots\rangle_3$. For four of the remaining five strings, excite the $m = 1$ mode once and the $m = 2$ mode four times; namely, all four strings are in the state $(J_{-\frac{1}{3}})(J_{-\frac{2}{3}})^4|0\rangle_3 = |1, 4, 0, \ldots\rangle_3$. Finally, let the last string be in the state $(J_{-\frac{1}{3}})^6(J_{-\frac{3}{3}})^1|0\rangle_3 = |6, 0, 1, \ldots\rangle_3$. Note that the total momentum in each string is an integer. The three-charge state thus obtained is

$$(|3, 0, 0, \ldots\rangle_3)^2 \, (|1, 4, 0, \ldots\rangle_3)^4 \, (|6, 0, 1, \ldots\rangle_3)^1. \tag{6.5}$$

The $n_3 = 7$ indistinguishable strings in (6.4) have split into three distinguishable groups. If $n_3^{(i)}$ denotes the number of strings in the ith group, we have the splitting

$$n_3 = 7 = 2 + 4 + 1 = \sum_{i=1}^{3} n_3^{(i)}. \tag{6.6}$$

The $n_3^{(i)}$ strings in the ith group are all in the same excited state and indistinguishable. Let $n_{3m}^{(i)}$, $m \geq 1$ denote the momentum excitation numbers for the state of the ith group. In the present example,

$$
\begin{aligned}
1^{\text{st}} \text{ group:} \quad & (n_3^{(1)} \equiv n_{30}^{(1)}; n_{31}^{(1)}, n_{32}^{(1)}, n_{33}^{(1)}, \ldots) = (2; 3, 0, 0, \ldots), \\
2^{\text{nd}} \text{ group:} \quad & (n_3^{(2)} \equiv n_{30}^{(2)}; n_{31}^{(2)}, n_{32}^{(2)}, n_{33}^{(2)}, \ldots) = (4; 1, 4, 0, \ldots), \\
3^{\text{rd}} \text{ group:} \quad & (n_3^{(3)} \equiv n_{30}^{(3)}; n_{31}^{(3)}, n_{32}^{(3)}, n_{33}^{(3)}, \ldots) = (1; 6, 0, 1, \ldots),
\end{aligned} \tag{6.7}
$$

where we defined $n_{30}^{(i)} \equiv n_3^{(i)}$. More generally, it is clear that the general three-charge state of length-3 strings is completely specified by the numbers $\{n_{3m}^{(i)}\}_{m \geq 0, i \geq 1}$. Distinguishability between different groups with $i \neq i'$ means that $\{n_{3m}^{(i)}\}_{m \geq 1} \neq \{n_{3m}^{(i')}\}_{m \geq 1}$.

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[17]Of course, there are other momentum-carrying states that cannot be obtained by the action of J^3 but, for simplicity, we focus on the states that can be simply labeled as in (6.3).

The general three-charge state built on the general two-charge state (6.1) is obtained by multiplying excited strings with different values of k together. Namely, for each k, we index the distinguishable families of momentum excitations by (i) and let $n_{k0}^{(i)}$ denote the number of *indistinguishable* strings in each family (they are indistinguishable because they have identical excitation numbers). Therefore, the two-charge constraint (3.1) is refined to:

$$\sum_{i \geq 1} n_{k0}^{(i)} = n_k \,, \qquad \sum_{k \geq 1} \sum_{i \geq 1} n_{k0}^{(i)} = N \,. \tag{6.8}$$

Let $n_{km}^{(i)}$ ($m \geq 1$) denote the momentum excitations, as in (6.7), of the ith set of effective strings of length k:

$$|n_{k1}^{(i)}, n_{k2}^{(i)}, \cdots\rangle_k \,. \tag{6.9}$$

Distinguishability from the other strings of length k means that the momentum excitations must be different: $\{n_{km}^{(i)}\}_{m \geq 1} \neq \{n_{km}^{(i')}\}_{m \geq 1}$ if $i \neq i'$.

The three-charge states thus obtained are:

$$\prod_{k \geq 1} \prod_{i \geq 1} \left(|n_{k1}^{(i)}, n_{k2}^{(i)}, \ldots\rangle_k \right)^{n_{k0}^{(i)}} = \left(|n_{11}^{(1)}, n_{12}^{(1)}, \ldots\rangle_1 \right)^{n_{10}^{(1)}} \left(|n_{11}^{(2)}, n_{12}^{(2)}, \ldots\rangle_1 \right)^{n_{10}^{(2)}} \cdots$$
$$\times \left(|n_{21}^{(1)}, n_{22}^{(1)}, \ldots\rangle_2 \right)^{n_{20}^{(1)}} \left(|n_{21}^{(2)}, n_{22}^{(2)}, \ldots\rangle_2 \right)^{n_{20}^{(2)}} \cdots \tag{6.10}$$

where the powers represent the fact that there are $n_{k0}^{(i)}$ indistinguishable effective strings in the same state. The three-charge states (6.10) are thus specified by the non-negative integers, $\{n_{km}^{(i)}\}$. The index $k \geq 1$ is associated with the Fourier mode in the w-direction (the loop in $\mathbb{R}^4$ of the original D1-D5 system) and the index $m \geq 0$ is associated with the momentum Fourier modes in the v-direction. Note that we have identified $n_{k0}^{(i)}$ introduced above (6.8) with the $m = 0$ mode number. Thus we have sufficient data to describe the shape modes as a function of two variables, as expected of a superstratum. However, there remains an additional index (i) — this means that *the general three-charge states in the D1-D5 CFT naturally parametrize multiple functions of two variables*. What is the physical interpretation of this fact?
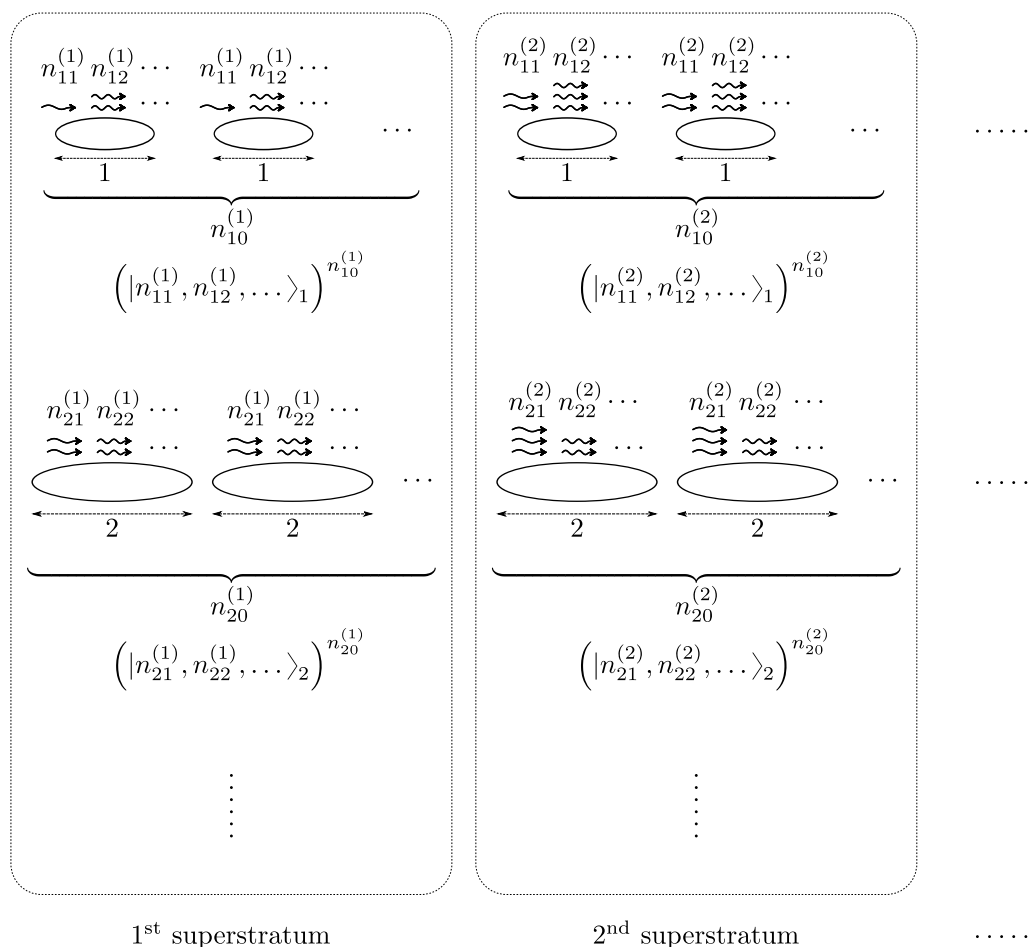
6.2 Multi-superstrata interpretation

The index (i) labels distinguishable effective strings of the same length: sets of effective strings that only became distinguishable by virtue of the momentum excitations on them. It is therefore tempting to interpret (i) as labeling the multiple superstrata into which the original D1-D5 supertube has split. The momentum excitations promote the original profile function, $f(w)$, into a function of two variables, $f(v, w)$, but we conjecture that the two-charge profile function actually gets promoted into multiple functions of two variables labeled by (i):

$$f(w) \quad \rightarrow \quad f^{(1)}(w, v), \quad f^{(2)}(w, v), \quad f^{(3)}(w, v), \quad \ldots, \tag{6.11}$$

where $f^{(i)}(w, v)$ describes the world-volume of the ith superstratum. The Fourier coefficients $a_{km}^{(i)}$ of these functions are then given by

$$|a_{km}^{(i)}|^2 = n_{km}^{(i)}. \tag{6.12}$$

Figure 3. Momentum carrying excitations on multiple effective strings and their possible multi-superstratum interpretation. For each string length k, strings on which identical momentum modes are excited are grouped together. For fixed k, the $n_{k0}^{(1)}$ strings in group 1 are all in the same state $|n_{k1}^{(1)}, n_{k2}^{(1)}, \ldots\rangle_k$ and are indistinguishable, the $n_{k0}^{(2)}$ strings in group 2 are all in the same state $|n_{k1}^{(2)}, n_{k2}^{(2)}, \ldots\rangle_k$ and are indistinguishable, and so on. The shape of the 1$^{\text{st}}$ superstratum is specified by the number of strings in group 1 for all possible values of k, namely by $\{n_{km}^{(1)}\}$. The shape of the 2$^{\text{nd}}$ superstratum is specified by $\{n_{km}^{(2)}\}$, and so on. See the text for more detail.

See figure 3 for a pictorial description of the state (6.10) and the multi-superstrata interpretation.

We hasten to note the important fact that the foregoing description of three-charge states, such as (6.10), is valid only at the free orbifold point in the moduli space of the D1-D5 CFT, whereas the actual supergravity sits at a very different point in the moduli space. Deforming the CFT away from the orbifold point corresponds to turning on twist operator perturbations (see [59] for a recent detailed account). Twist operators mix different twist sectors and therefore the picture of each individual state gets modified. However, it is the *number* of states that is important for our proposal, and it is not changed by such deformations. Namely, the deformation does not change the crucial fact that more data

than can fit on a single superstratum is needed to account for general three-charge states. Therefore, this does not invalidate our proposal that general three-charge states are represented by multiple superstrata, although the precise dictionary between the superstrata shape functions $f^{(i)}(w, v)$ and the CFT states may not be as simple as described above. For example, it is quite conceivable a state that looks like a multi-strata state in CFT corresponds to a single-stratum state in supergravity, and vice versa.[18] This is analogous to the fact that, in AdS_5/CFT_4, once interactions are turned on, the single/multi-trace operator basis of the CFT Hilbert space is different from (and a unitary transformation of) the single/multi-particle basis in of the supergravity Hilbert space.

Our multi-superstrata proposal raises several important issues. First, all the states we are discussing in (6.10) are states within *the same CFT* and *not* states in distinct CFT's. Arguing that some of these states correspond to different superstrata suggests that we are factoring the CFT into different CFT's. At a more basic level, if one accepts that the distinguishable families factor into different superstrata then why do we not accept that the same must happen in the two-charge D1-D5 system: why aren't effective strings of different lengths simply different supertubes?

The resolution of all these issues comes from remembering that multiple supertubes have no $E \times B$ interactions, and therefore can be separated at arbitrary distances. If we consider a solution that contains only two-charge supertubes placed at the bottom of a long AdS throat, these supertubes are not trapped at the bottom of the throat and can move freely out of the throat. They represent therefore unbound states dual to factorized CFT's. On the other hand, two generic superstrata will always have non-trivial $E \times B$ interactions, and hence a solution that has multiple superstrata at the bottom of a long AdS throat will represent a bound state of the CFT. Solutions with different numbers of superstrata will have different topology, and hence will belong to different sectors of this CFT.

Another important consideration is the fact that the bubbling transition to create microstate geometries with non-trivial cycles requires the three-charge system. The bubble equations [74–76], which relate the sizes of cycles to the fluxes through those cycles, degenerate for two charges or if a flux through a cycle vanishes and so the corresponding bubble collapses. Thus the possibility of separate superstrata forming a bound state in a CFT can only occur if one excites the momentum modes in the D1-D5 system and only if one excites momenta in distinct ways so that the fluxes on bubbles do not vanish. Conversely, if two superstrata have exactly the same shape and charge distribution then they will coalesce within a given AdS throat or, if they are not in an AdS throat, there will be no force between them and they can be moved arbitrarily far away from each other, which is not describable within one dual CFT [77].

It is worth noting that the "moulting phase" of the D1-D5 system [78] that appears in the three-charge situation with large angular momentum has structures rather similar to

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[18]This point is particularly clear for the three-charge states built on the two-charge state with $n_1 = N, n_{k \geq 2} = 0$ (short string sector) which we denote by $|\psi_0\rangle$. The foregoing CFT picture (the $k = 1$ version of (6.5)) says that we can build multiple-strata states on this state. On the other hand, in supergravity, the state $|\psi_0\rangle$ corresponds to a single, circular, unexcited superstratum, which is nothing but pure $AdS_3 \times S^3$. Momentum-carrying excitations on it are small deformations of the S^3, which do not seem to lead to multi-superstrata.

the ones proposed here. In [78], the following problem was studied: for given momentum charge and angular momentum $J_L = \mathcal{O}(N)$, what is the ensemble of states that has the largest entropy? In the CFT (at the orbifold point), the most entropic states were found to be made of two sectors of effective strings, reminiscent of (6.11). The first sector is made of a long string with length $\mathcal{O}(N)$, which carries all the momentum charge as well as the entropy, while the second sector consists of many ($\mathcal{O}(N)$) short strings of length one, which carry $J_L, J_R = \mathcal{O}(N)$ but no entropy. On the other hand, in supergravity, the most entropic configuration was found to be a two-center solution in an asymptotically AdS space. One center is a BMPV black hole carrying all the momentum charge and entropy, while the other center is a supertube carrying $J_L, J_R = \mathcal{O}(N)$ but no entropy.[19] (Because the BMPV black hole can be thought of as "shedding" or "moulting" a supertube, it was dubbed the "moulting phase"). The fact that the multi-sector states of the CFT correspond to a multi-center solution in supergravity can be thought of as evidence in support of our conjecture (even though these configurations are not microstates but phases with finite entropy).

Apart from the natural way in which the correspondence of distinguishable twisted sectors and bound states of multiple superstrata appears to work, one can obtain further evidence for the conjecture by re-examining the arguments of [7, 53, 55, 70] that obtain the CFT gap from the supergravity solution. We first note that the longest effective string corresponds to

$$n_N = 1, \qquad n_k = 0, \quad 1 \leq k < N, \tag{6.13}$$

and so can only involve a single superstratum, no matter how we add momentum. This sector of the theory is also the sector with $E_{\text{gap}} \sim \frac{1}{N_1 N_5}$ and was obtained holographically by considering an excitation of a bubbled geometry that has a wavelength equal to the size of the AdS throat. Such a wavelength would be the natural fundamental oscillation of a superstratum whose scale is that of the entire throat. In multiple, bound superstrata the bubbles of geometry will be smaller than the throat and the scale of an individual bubble will be roughly set by the scale of the throat divided by the some appropriate power of the number of bubbles. Thus the fundamental modes of such individual bubbles will have a shorter wavelength and a higher energy gap. Indeed, the energy gap of such a configuration should be $E_{\text{gap}} \sim \frac{p}{N_1 N_5}$, where p is the approximate number of bubbles that span the "diameter" of the throat. This, at least qualitatively, fits very nicely with the corresponding decreased lengths of the effective strings in the CFT. Obviously more work is needed to fully substantiate our conjecture but we think it is promising enough to warrant our description here.

7 Conclusions

In this paper we have argued that the BPS microstates of the D1-D5-P system will manifest themselves in the regime in which the classical black hole exists as smooth horizonless

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[19]Although the configurations in CFT and supergravity seem quite similar to each other, the entropy of the CFT states and that of the bulk two-center solution do not quite agree (the CFT entropy is always larger than the supergravity entropy), which is presumably caused by the partial lifting of states at strong coupling.

"superstratum" solutions. Despite the absence of an explicit solution describing the generic superstratum, we have been able to account for their entropy using the intuition that adding momentum modes to any system of branes will, upon back-reaction, emerge as shape modes in supergravity, and, conversely, that the semi-classical quantization of such shape modes will reconstruct the original Hilbert space of momentum states.

We first considered the construction of a superstratum in terms of fluctuations around a maximally-spinning supertube and have argued, from the dual D1-D5 CFT, that the number of supersymmetric momentum carriers of the superstratum is given by the product, $N_1 N_5$, of its D1 and D5 charges. This conservative estimate, which we believe can be substantiated with a high level of confidence, gives the entropy:

$$S = 2\pi \sqrt{\frac{1}{6} N_1 N_5 N_P} \tag{7.1}$$

and this is expected to come entirely from smooth supergravity solutions.

Then we went on to make a somewhat bolder proposal for counting the entropy of superstrata using an approach similar to that of Maldacena, Strominger and Witten [56]. Specifically, we argued that the space of transverse fluctuations of two-charge supertubes must have dimension $4N_1 N_5$. One can then view this as the moduli space of the superstratum and, much as in the original construction of superstrata [10], all these moduli could carry momentum. Assuming these moduli are independent and unobstructed, there are thus $4N_1 N_5$ bosonic modes which, when combined with their fermionic superpartners, would give an entropy:

$$S = 2\pi \sqrt{N_1 N_5 N_P} \,. \tag{7.2}$$

This exactly matches the black-hole entropy. We have also discussed the possible ways to reconcile this estimate to the more conservative estimate above, and have argued that, in the regime of parameters where the black hole exists, all the modes in the internal directions should somehow manifest themselves as fluctuations in the transverse space. We have also argued that one cannot match all the states of the CFT by counting perturbatively around a single superstratum solution, and that multiple superstrata bound states are a natural candidate for matching these states.

Modulo the explicit construction of superstratum solutions that depend on arbitrary functions, we have presented what we believe to be strong evidence that the so-called fuzzball proposal is the correct description of extremal supersymmetric black holes within string theory. Indeed, if one can obtain a *macroscopic* fraction of the black-hole entropy from horizonless supergravity solutions, this implies that all the typical states that contribute to the black-hole entropy will have a finite transverse size, and hence the entire system will not be surrounded by horizon. This in turn would imply that the correct way to think about the textbook black-hole solution is as a thermodynamic approximation of a huge number of horizonless configurations, much as a continuous fluid is a thermodynamic approximation of a huge number of molecule configurations.

The conservative and bolder views of superstrata lead to significant differences in the structure of typical black-hole microstates. If all the black-hole microstates are visible as transverse superstrata modes, then it is possible that upon full back-reaction these

modes will all give rise to low-curvature solutions that have a long black-hole-like throat and end in a smooth cap. This would imply that the modes captured by six-dimensional supergravity are enough to account for the black-hole entropy, which would establish the fuzzball proposal in its strong form.

If, however, only $1/\sqrt{6}$ of the black-hole entropy comes from transverse modes, then the typical black-hole microstates will still be horizonless, but will not be describable as smooth solutions of six-dimensional supergravity: the typical microstates will necessarily involve stringy or Kaluza-Klein modes. This would establish the "weak version" of the fuzzball proposal, which is enough for solving the information paradox, but it may not offer us a framework, at least within supergravity, for doing rigorous computations that could help establish, for example, whether an incoming observer feels a firewall or falls through the fuzzball states unharmed.

Clearly, there are two essential steps that should be done next. The first is the explicit construction of the superstratum solutions that depend on functions of two variables. This would represent major progress toward establishing the fuzzball proposal for extremal black holes. The dramatic simplification of the BPS system of equations underlying these solutions [47] means that it might be possible to construct the BPS supergravity excitations at full non-linear order. The discussion at the beginning of section 4 showed that arbitrary space-time shape modes break all the supersymmetry and that only the representations $(\ell, m; \tilde{\ell}, \tilde{\ell})$ of $SU(2)_L \times SU(2)_R$ can be excited in the $\frac{1}{8}$-BPS superstratum. This observation also underlies the analysis in [37, 38] and it will provide invaluable insight into how to address the construction of a fully back-reacted superstratum that depends upon a general function of two variables.

The second, and most difficult, step is to extend this work to non-extremal black holes. A very useful insight comes of our analysis here where we noted that certain momentum carriers that are charged under $SU(2)_R$ may break supersymmetry.[20] Hence, adding these fluctuations to a typical BPS superstratum state may allow us to move away from extremality and to argue that the supergravity structure of the black-hole microstates that we have analyzed in this paper is robust when supersymmetry is broken. This, in turn, would imply that near-extremal, and quite possibly generic, black holes are thermodynamic approximations of horizonless solutions and that the pure states of a black hole would be represented by horizonless configurations. This would solve the black-hole information paradox and allow us to address, far more rigorously, the puzzles that the information-theory analysis of black hole has revealed [14–24, 79].

Acknowledgments

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[20]A similar phenomenon happens for supertubes, and there taking into account the supersymmetry-breaking modes is crucial if one is to quantize correctly the supersymmetric modes [50].

References

[1] A. Sen, *Extremal black holes and elementary string states*, [*Mod. Phys. Lett.* **A 10** (1995) 2081](#) [[hep-th/9504147](#)] [[INSPIRE](#)].

[2] A. Strominger and C. Vafa, *Microscopic origin of the Bekenstein-Hawking entropy*, [*Phys. Lett.* **B 379** (1996) 99](#) [[hep-th/9601029](#)] [[INSPIRE](#)].

[3] G.T. Horowitz and J. Polchinski, *A correspondence principle for black holes and strings*, [*Phys. Rev.* **D 55** (1997) 6189](#) [[hep-th/9612146](#)] [[INSPIRE](#)].

[4] T. Damour and G. Veneziano, *Selfgravitating fundamental strings and black holes*, [*Nucl. Phys.* **B 568** (2000) 93](#) [[hep-th/9907030](#)] [[INSPIRE](#)].

[5] I. Bena and N.P. Warner, *Bubbling supertubes and foaming black holes*, [*Phys. Rev.* **D 74** (2006) 066001](#) [[hep-th/0505166](#)] [[INSPIRE](#)].

[6] P. Berglund, E.G. Gimon and T.S. Levi, *Supergravity microstates for BPS black holes and black rings*, [*JHEP* **06** (2006) 007](#) [[hep-th/0505167](#)] [[INSPIRE](#)].

[7] J. de Boer, S. El-Showk, I. Messamah and D. Van den Bleeken, *A bound on the entropy of supergravity?*, [*JHEP* **02** (2010) 062](#) [[arXiv:0906.0011](#)] [[INSPIRE](#)].

[8] D. Mateos and P.K. Townsend, *Supertubes*, [*Phys. Rev. Lett.* **87** (2001) 011602](#) [[hep-th/0103030](#)] [[INSPIRE](#)].

[9] I. Bena, N. Bobev, S. Giusto, C. Ruef and N.P. Warner, *An infinite-dimensional family of black-hole microstate geometries*, [*JHEP* **03** (2011) 022](#) [*Erratum ibid.* **1104** (2011) 059] [[arXiv:1006.3497](#)] [[INSPIRE](#)].

[10] I. Bena, J. de Boer, M. Shigemori and N.P. Warner, *Double, double supertube bubble*, [*JHEP* **10** (2011) 116](#) [[arXiv:1107.2650](#)] [[INSPIRE](#)].

[11] S.D. Mathur, *The information paradox: a pedagogical introduction*, [*Class. Quant. Grav.* **26** (2009) 224001](#) [[arXiv:0909.1038](#)] [[INSPIRE](#)].

[12] S.D. Mathur, *The information paradox: conflicts and resolutions*, *Pramana* **79** (2012) 1059 [arXiv:1201.2079] [INSPIRE].

[13] S.D. Mathur, *Black holes and holography*, *J. Phys. Conf. Ser.* **405** (2012) 012005 [arXiv:1207.5431] [INSPIRE].

[14] S.L. Braunstein, S. Pirandola and K. Życzkowski, *Better late than never: information retrieval from black holes*, *Phys. Rev. Lett.* **110** (2013) 101301 [arXiv:0907.1190] [INSPIRE].

[15] A. Almheiri, D. Marolf, J. Polchinski and J. Sully, *Black holes: complementarity or firewalls?*, *JHEP* **02** (2013) 062 [arXiv:1207.3123] [INSPIRE].

[16] S.D. Mathur and D. Turton, *Comments on black holes I: the possibility of complementarity*, *JHEP* **01** (2014) 034 [arXiv:1208.2005] [INSPIRE].

[17] L. Susskind, *Singularities, firewalls and complementarity*, arXiv:1208.3445 [INSPIRE].

[18] I. Bena, A. Puhm and B. Vercnocke, *Non-extremal black hole microstates: fuzzballs of fire or fuzzballs of fuzz?*, *JHEP* **12** (2012) 014 [arXiv:1208.3468] [INSPIRE].

[19] L. Susskind, *The transfer of entanglement: the case for firewalls*, arXiv:1210.2098 [INSPIRE].

[20] S.G. Avery, B.D. Chowdhury and A. Puhm, *Unitarity and fuzzball complementarity: 'Alice fuzzes but may not even know it!'*, *JHEP* **09** (2013) 012 [arXiv:1210.6996] [INSPIRE].

[21] S.G. Avery and B.D. Chowdhury, *Firewalls in AdS/CFT*, arXiv:1302.5428 [INSPIRE].

[22] A. Almheiri, D. Marolf, J. Polchinski, D. Stanford and J. Sully, *An apologia for firewalls*, *JHEP* **09** (2013) 018 [arXiv:1304.6483] [INSPIRE].

[23] E. Verlinde and H. Verlinde, *Passing through the firewall*, arXiv:1306.0515 [INSPIRE].

[24] S.D. Mathur and D. Turton, *The flaw in the firewall argument*, *Nucl. Phys.* **B 884** (2014) 566 [arXiv:1306.5488] [INSPIRE].

[25] S.D. Mathur, *The information paradox and the infall problem*, *Class. Quant. Grav.* **28** (2011) 125010 [arXiv:1012.2101] [INSPIRE].

[26] S.D. Mathur and C.J. Plumberg, *Correlations in Hawking radiation and the infall problem*, *JHEP* **09** (2011) 093 [arXiv:1101.4899] [INSPIRE].

[27] S.D. Mathur, *Black holes and beyond*, *Annals Phys.* **327** (2012) 2760 [arXiv:1205.0776] [INSPIRE].

[28] G.W. Gibbons and N.P. Warner, *Global structure of five-dimensional fuzzballs*, *Class. Quant. Grav.* **31** (2014) 025016 [arXiv:1305.0957] [INSPIRE].

[29] P.A. Haas, *Smarr's formula in eleven-dimensional supergravity*, arXiv:1405.3708 [INSPIRE].

[30] I. Bena and N.P. Warner, *Black holes, black rings and their microstates*, *Lect. Notes Phys.* **755** (2008) 1 [hep-th/0701216] [INSPIRE].

[31] I. Bena, A. Puhm and B. Vercnocke, *Metastable supertubes and non-extremal black hole microstates*, *JHEP* **04** (2012) 100 [arXiv:1109.5180] [INSPIRE].

[32] I. Bena and N.P. Warner, *Resolving the structure of black holes: philosophizing with a hammer*, arXiv:1311.4538 [INSPIRE].

[33] R.C. Myers, *Dielectric branes*, *JHEP* **12** (1999) 022 [hep-th/9910053] [INSPIRE].

[34] J. de Boer and M. Shigemori, *Exotic branes and non-geometric backgrounds*, *Phys. Rev. Lett.* **104** (2010) 251603 [`arXiv:1004.2521`] [INSPIRE].

[35] S. Giusto and R. Russo, *Perturbative superstrata*, *Nucl. Phys.* **B 869** (2013) 164 [`arXiv:1211.1957`] [INSPIRE].

[36] S. Giusto and R. Russo, *Superdescendants of the D1D5 CFT and their dual 3-charge geometries*, *JHEP* **03** (2014) 007 [`arXiv:1311.5536`] [INSPIRE].

[37] M. Shigemori, *Perturbative 3-charge microstate geometries in six dimensions*, *JHEP* **10** (2013) 169 [`arXiv:1307.3115`] [INSPIRE].

[38] B.E. Niehoff and N.P. Warner, *Doubly-fluctuating BPS solutions in six dimensions*, *JHEP* **10** (2013) 137 [`arXiv:1303.5449`] [INSPIRE].

[39] J. de Boer and M. Shigemori, *Exotic branes in string theory*, *Phys. Rept.* **532** (2013) 65 [`arXiv:1209.6056`] [INSPIRE].

[40] O. Lunin and S.D. Mathur, *AdS/CFT duality and the black hole information paradox*, *Nucl. Phys.* **B 623** (2002) 342 [`hep-th/0109154`] [INSPIRE].

[41] O. Lunin, J.M. Maldacena and L. Maoz, *Gravity solutions for the $D1 - D5$ system with angular momentum*, `hep-th/0212210` [INSPIRE].

[42] R. Emparan, D. Mateos and P.K. Townsend, *Supergravity supertubes*, *JHEP* **07** (2001) 011 [`hep-th/0106012`] [INSPIRE].

[43] C.G. Callan and J.M. Maldacena, *D-brane approach to black hole quantum mechanics*, *Nucl. Phys.* **B 472** (1996) 591 [`hep-th/9602043`] [INSPIRE].

[44] G.T. Horowitz and A. Strominger, *Counting states of near extremal black holes*, *Phys. Rev. Lett.* **77** (1996) 2368 [`hep-th/9602051`] [INSPIRE].

[45] J.C. Breckenridge et al., *Macroscopic and microscopic entropy of near extremal spinning black holes*, *Phys. Lett.* **B 381** (1996) 423 [`hep-th/9603078`] [INSPIRE].

[46] G.T. Horowitz, J.M. Maldacena and A. Strominger, *Nonextremal black hole microstates and U duality*, *Phys. Lett.* **B 383** (1996) 151 [`hep-th/9603109`] [INSPIRE].

[47] I. Bena, S. Giusto, M. Shigemori and N.P. Warner, *Supersymmetric solutions in six dimensions: a linear structure*, *JHEP* **03** (2012) 084 [`arXiv:1110.2781`] [INSPIRE].

[48] B.E. Niehoff, O. Vasilakis and N.P. Warner, *Multi-superthreads and supersheets*, *JHEP* **04** (2013) 046 [`arXiv:1203.1348`] [INSPIRE].

[49] A. Dabholkar, J.P. Gauntlett, J.A. Harvey and D. Waldram, *Strings as solitons and black holes as strings*, *Nucl. Phys.* **B 474** (1996) 85 [`hep-th/9511053`] [INSPIRE].

[50] B.C. Palmer and D. Marolf, *Counting supertubes*, *JHEP* **06** (2004) 028 [`hep-th/0403025`] [INSPIRE].

[51] D. Bak, Y. Hyakutake and N. Ohta, *Phase moduli space of supertubes*, *Nucl. Phys.* **B 696** (2004) 251 [`hep-th/0404104`] [INSPIRE].

[52] D. Bak, Y. Hyakutake, S. Kim and N. Ohta, *A geometric look on the microstates of supertubes*, *Nucl. Phys.* **B 712** (2005) 115 [`hep-th/0407253`] [INSPIRE].

[53] I. Bena, C.-W. Wang and N.P. Warner, *Mergers and typical black hole microstates*, *JHEP* **11** (2006) 042 [`hep-th/0608217`] [INSPIRE].

[54] F. Denef and G.W. Moore, *Split states, entropy enigmas, holes and halos*, *JHEP* **11** (2011) 129 [hep-th/0702146] [INSPIRE].

[55] I. Bena, C.-W. Wang and N.P. Warner, *Plumbing the abyss: black ring microstates*, *JHEP* **07** (2008) 019 [arXiv:0706.3786] [INSPIRE].

[56] J.M. Maldacena, A. Strominger and E. Witten, *Black hole entropy in M-theory*, *JHEP* **12** (1997) 002 [hep-th/9711053] [INSPIRE].

[57] O. Aharony, S.S. Gubser, J.M. Maldacena, H. Ooguri and Y. Oz, *Large-N field theories, string theory and gravity*, *Phys. Rept.* **323** (2000) 183 [hep-th/9905111] [INSPIRE].

[58] J.R. David, G. Mandal and S.R. Wadia, *Microscopic formulation of black holes in string theory*, *Phys. Rept.* **369** (2002) 549 [hep-th/0203048] [INSPIRE].

[59] S.G. Avery, *Using the D1D5 CFT to understand black holes*, arXiv:1012.0072 [INSPIRE].

[60] W. Lerche, C. Vafa and N.P. Warner, *Chiral rings in $N = 2$ superconformal theories*, *Nucl. Phys.* **B 324** (1989) 427 [INSPIRE].

[61] W. Lerche, C. Vafa and N.P. Warner, *Addendum to 'Chiral rings in $N = 2$ superconformal theories'*, HUTP-88-A065-ADD (1988).

[62] I. Kanitscheider, K. Skenderis and M. Taylor, *Fuzzballs with internal excitations*, *JHEP* **06** (2007) 056 [arXiv:0704.0690] [INSPIRE].

[63] I. Kanitscheider, K. Skenderis and M. Taylor, *Holographic anatomy of fuzzballs*, *JHEP* **04** (2007) 023 [hep-th/0611171] [INSPIRE].

[64] S.D. Mathur, *The fuzzball proposal for black holes: an elementary review*, *Fortsch. Phys.* **53** (2005) 793 [hep-th/0502050] [INSPIRE].

[65] V.S. Rychkov, *D1-D5 black hole microstate counting from supergravity*, *JHEP* **01** (2006) 063 [hep-th/0512053] [INSPIRE].

[66] S.D. Mathur, A. Saxena and Y.K. Srivastava, *Constructing 'hair' for the three charge hole*, *Nucl. Phys.* **B 680** (2004) 415 [hep-th/0311092] [INSPIRE].

[67] J. de Boer, *Six-dimensional supergravity on $S^3 \times AdS_3$ and $2D$ conformal field theory*, *Nucl. Phys.* **B 548** (1999) 139 [hep-th/9806104] [INSPIRE].

[68] J. de Boer, *Large-N elliptic genus and AdS/CFT correspondence*, *JHEP* **05** (1999) 017 [hep-th/9812240] [INSPIRE].

[69] J.M. Maldacena, G.W. Moore and A. Strominger, *Counting BPS black holes in toroidal Type II string theory*, hep-th/9903163 [INSPIRE].

[70] J. de Boer, S. El-Showk, I. Messamah and D. Van den Bleeken, *Quantizing $N = 2$ multicenter solutions*, *JHEP* **05** (2009) 002 [arXiv:0807.4556] [INSPIRE].

[71] O. Lunin and S.D. Mathur, *Metric of the multiply wound rotating string*, *Nucl. Phys.* **B 610** (2001) 49 [hep-th/0105136] [INSPIRE].

[72] M. Berkooz and H.L. Verlinde, *Matrix theory, AdS/CFT and Higgs-Coulomb equivalence*, *JHEP* **11** (1999) 037 [hep-th/9907100] [INSPIRE].

[73] I. Bena, M. Berkooz, J. de Boer, S. El-Showk and D. Van den Bleeken, *Scaling BPS solutions and pure-Higgs states*, *JHEP* **11** (2012) 171 [arXiv:1205.5023] [INSPIRE].

[74] B. Bates and F. Denef, *Exact solutions for supersymmetric stationary black hole composites*, *JHEP* **11** (2011) 127 [hep-th/0304094] [INSPIRE].

[75] I. Bena and N.P. Warner, *One ring to rule them all . . . and in the darkness bind them?*, *Adv. Theor. Math. Phys.* **9** (2005) 667 [`hep-th/0408106`] [INSPIRE].

[76] J.P. Gauntlett and J.B. Gutowski, *General concentric black rings*, *Phys. Rev.* **D 71** (2005) 045002 [`hep-th/0408122`] [INSPIRE].

[77] N. Seiberg and E. Witten, *The D1/D5 system and singular CFT*, *JHEP* **04** (1999) 017 [`hep-th/9903224`] [INSPIRE].

[78] I. Bena, B.D. Chowdhury, J. de Boer, S. El-Showk and M. Shigemori, *Moulting black holes*, *JHEP* **03** (2012) 094 [`arXiv:1108.0411`] [INSPIRE].

[79] D. Harlow and P. Hayden, *Quantum computation vs. firewalls*, *JHEP* **06** (2013) 085 [`arXiv:1301.4504`] [INSPIRE].