RADIATION CHARACTERISTICS OF
CORNER REFLECTOR ANTENNAS

A Thesis presented for the
degree of Doctor of Philosophy
in the Faculty of Engineering of
the University of London

by

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January, 1998
ACKNOWLEDGEMENTS

The author would like to express his most sincere thanks to:

- Professor A.D. Olver for his supervision and continued guidance throughout the project.

- The Queen Mary & Westfield College for the financial support which enabled me to undertake the research.

- Professor P.J.B. Clarricoats for his active interest and his advice.

- Professor H. Khakzar who opened my horizon beyond the boarders of Germany.

- Dr. M. Rayner for his help and hours of fruitful discussions.

- Mr. J. Dupuy and Mr. M. Johnson for their expert help and assistance with the experimental programme.

- The other members of the Microwave Research group for their help.

- Ms M. Hedderich for her never ending patience and support.

- My former teachers Mr. Moßbeck, Mr. Härle, Dr. Freytag, Prof. Martin, Prof. Herter and Prof. Höfer who did so much more than they were obliged to and made all the difference.
ABSTRACT

This thesis presents a study of the radiation characteristics of corner reflector antennas. The influence of the design parameters on the radiation characteristics are assessed using an analytical method and the Finite Difference Time Domain (FDTD) method.

The FDTD method for corner reflector antennas which are electrically small to medium sized antennas is developed in detail. The important subject of the Absorbing Boundary Conditions (ABCs) is studied including a study of Mur ABC and Perfectly Matched Layers. It is shown that both methods reduce the reflections from the boundaries sufficient so that the far-field radiation pattern can be computed accurately.

An analytical solution to compute the far-field radiation pattern for infinite corner reflector antennas is derived and used to understand the radiation mechanisms. Based on those results, the FDTD method is used to conduct a parametric study on finite sized corner reflector antennas. Experimental antennas have been built and measured in order to verify the computational predictions. Very good agreement is reported.

The novel idea of a variable beam-width corner reflector antenna is developed and practical designs of such an antenna are presented. The principle is to design the corner reflector antenna such that the beam-width of the antenna can be precisely modified. Data on the gain and beam-width are presented. This has been done both by computational and by an experimental model.

The influence on the performance of the corner reflector antenna when substituting the solid reflector plates by rods has been investigated. The computational predictions have been verified by measurements of an experimental antenna. Very good agreement has been achieved.
The possibility of modifying the shape of the corner reflector antenna is investigated. It is shown that a modified corner reflector antenna with less depth produces the same far-field pattern as a standard corner reflector antenna. It is also shown that the performance of small aperture size corner reflector antennas is superior to a cylindrical parabolic reflector antenna.
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XVIII
1. INTRODUCTION

1.1. Introduction

Reflector antennas can be classified according to the geometric form of the surface. Surfaces of revolution include paraboloids, ellipsoids, hyperboloids, and spheres generated from conic sections. In addition, special shaped reflector surfaces are applied to synthesis contoured antenna patterns. Paraboloids of revolution are most frequently used. These reflector antennas are mainly employed as high gain collimating antennas. Reflectors of arbitrary shape or array feeds can be used to shape the beam. Since the reflector size is large in terms of wavelengths it is possible to employ geometric optical approximations for the design.

If only a moderate gain is needed the electrical size of the antenna can be small. Reflector antennas are normally feed by directive sources such as horn antennas. For small reflectors the blockage due to the feed becomes large and deteriorates the performance of the antenna. In this case the corner reflector antenna provides a possible solution.

The corner reflector antenna was invented 60 years ago by J. D. Kraus. The antenna has a low to medium gain and is used particularly at UHF and low microwave frequencies. It is usually linearly polarised. A picture of a corner reflector antenna is shown in fig. 1.1. The antenna consists of a source, normally a half wave dipole, located in front of a reflector which is constructed of two planar conductor plates which intersect at an angle \( \psi \), called the apex angle.

The H-plane radiation pattern of the corner reflector antenna is shaped by adjusting the apex angle and the position of the feed. If shaping of the E-plane is required the corner reflector antenna has to be arranged in vertical arrays. The E-plane radiation pattern is then mainly determined by the excitation coefficients of the feeds.
A corner reflector antenna without a feed can be used as a passive reflector or target for radar waves. In this application the intersection angle is always 90°. Reflectors with this angle have the property that an incident wave is reflected back to its source, the reflector acting as a retroreflector.

![Fig. 1.1: Picture of a corner reflector antenna](image)

Although an analytical solution for the infinite size corner reflector antenna was developed in 1954 no analytical solution for the finite size corner reflector antenna has been derived until today. The knowledge of the influence of a finite size reflector on the radiation characteristic of the corner reflector antenna is very limited. This lack of information on a widely used antenna was the motivation for the study presented in this thesis.

The finite extent of the reflector can be accounted for by using the Geometric Theory of diffraction (GTD), Method of Moments (MM), or Finite Difference Time Domain (FDTD) methods. The Geometric Theory of diffraction is a ray based approach which computes fields by summing up the energy that is incident at any point in space. The computation of the fields in the shadow region is allowed by accounting for the
diffraction effects. The Method of Moments uses integral equations to compute the induced currents flowing on the surface of an antenna geometry, from which the radiation can be computed. The Finite Difference Time Domain method calculates the time domain fields around an antenna geometry which can be used to compute the desired characteristics.

Prior to the introduction of the Finite Difference Time Domain method the Method of Moments was probably the most widely adopted method for the analysis of electrically small antennas. The introduction of the Finite Difference Time Domain method has brought about a significant change in the subject area of electromagnetic antenna simulation as some features of the method make it superior to its more mature companions.

An efficient modelling tool allows extensive parametric studies. To reduce the computing effort of such a parametric study a hybrid approach is possible. The analytical solution for infinite size corner reflector antennas can be used to identify which designs yield a desired radiation pattern. The advantage is that the computational effort for the analytical solution is smaller than for the simulation of a finite size corner reflector antenna. The influence of the finite size of the reflector can be investigated by using the computational more intensive simulation method. This investigation can be reduced to those designs which showed a promising radiation pattern for the infinite size reflector. Analysing the results of the parametric study helps to understand how the corner reflector antenna works and how a compact and efficient corner reflector antenna can be designed.

A number of modifications to the Kraus corner reflector antenna are possible. The results and the understanding derived from a parametric study can be employed to judge which options are sensible approaches for a compact and easily build design of a high gain corner reflector antenna.
One option to reduce wind resistance and costs of antennas is to replace solid reflectors by rods. The rod diameter and the rod separation have to be chosen such that the radiation characteristic of the antenna can be sustained. A further reduction of costs and weight of the antenna can be achieved if the rods are non-uniformly distributed.

Another option is to modify the actual shape of the reflector plates. By changing the shape of the reflector plates it has to be considered that a shape which is easy to build usually consists of straight plates. In this context it is certainly of interest how the performance of a corner reflector antenna compares to other reflector shapes such as the cylindrical paraboloid.

A further option is the source of the corner reflector antenna. The usually employed dipole can be replaced in order to enhance the performance of the antenna. The functioning of the antenna can be improved in two different ways by changing the source. Either the gain of the antenna or the bandwidth can be increased. If possible both criteria can be improved at the same time.

The following section gives an outline of the contents of the thesis with a brief description of the work presented in each of the chapters.

1.2. Plan of thesis

In chapter 2, an overview of the historical development and the typical use of the corner reflector antenna is given. A detailed literature survey shows what was done in the field of corner reflector antenna up to now. The possibility of an analytical solution is investigated and it is shown that it is feasible to compute the far-field of a corner reflector antenna if the plates are infinite in size.

Chapter 3 describes the development of the FDTD method from the time dependent Maxwell's equations for use in antenna radiation problems. The problem of absorbing boundary conditions is addressed and two methods are described. The Mur ABC and,
in more detail, the Berenger PML. The development of a 2-D FDTD method is described in detail. The FDTD predictions are verified experimentally by building two antennas and measuring them in a Compact Antenna Test Range. A comparison to other simulation techniques, Method of Moments and Geometrical Theory of diffraction, is also given.

Chapter 4 presents a detailed parametric study of solid corner reflector antennas. Details of how the corner reflector has to be modelled are discussed and the influence of the ABC is established. The FDTD method was used to study the effects of varying parameters such as apex angle, plate length and width, and the distance of the dipole to the apex. From the results of the parametric study the important design criteria for the corner reflector antenna are identified. The quantities of H-plane directivity and aperture efficiency are described and there importance to judge the performance of an antenna is highlighted. A ray tracing model is used to explain the results.

A novel idea of a variable beam-width corner reflector antenna is developed. A design for a practical antenna is given and the far-field and input impedance predictions are compared with measured results.

Chapter 5 is concerned with the replacement of the solid plates by rods. The difficulties of simulating a round object with a FDTD code based on a Cartesian coordinate system are investigated. A comparison between measured data, 2-D-, and 3-D FDTD prediction convey interesting information. The influence of the rod separation on the band-width is investigated. A parametric study shows the impact of rod spacing on H-plane directivity and front-to-back ratio.

In chapter 6 the shape of the reflector plates are varied in order to improve the performance of the antenna. The different shapes are parabolic, box, and modified corner reflector.

Chapter 7 describes the conclusion drawn from the work and proposes possible extensions for the future.
2. THE CORNER REFLECTOR ANTENNA: HISTORY AND ANALYSIS

2.1. Introduction

The corner reflector antenna was invented in 1938 by J. D. Kraus [1, 2]. At that time he was working on the radiation from a dipole parallel to and closely spaced from a flat reflecting sheet. His basic idea was that by folding the sheet the number of images predicted by Image Theory would increase and with it the directivity of the antenna. A patent for the corner reflector antenna was granted in 1942 [3]. Image Theory and its application to the problem of the corner reflector antenna are presented in section 2.4.1.

The first corner reflector antenna made by Kraus operated at a frequency of 60MHz and consisted of two flat plates intersecting at an angle of 90° in which case he called the corner reflector antenna a square-corner reflector. Due to the physical size of the corner reflector antenna at this low frequency the plates were built with wire screens. Today the physical size of the corner reflector antennas is much smaller because higher frequencies are generally used.

The early analysis of the corner reflector antenna was restricted to infinite size plates and apex angles which are submultiple of π (180°, 90°, 60°, ...). It was not until the 1950's that an analytical formula for the far-field of the corner reflector antenna with arbitrary apex angles was developed individually via two different approaches by Wait [4] and Klopfenstein [5]. A discussion of the results and the implications for the design of a corner reflector antenna are given in 2.4.2. A good understanding of Bessel functions is necessary to understand the expression derived in the above references and therefore background information is given in 2.4.2.1.

But there was still the restriction of infinite size reflector plates. At that time it was not possible to establish the influence of the finite size of the reflector plates other
than through measurements. This cumbersome task was first undertaken by Harris [6] who performed a comprehensive set of measurements. The results were also reproduced by Jasik [7]. Five years later Cottony and Wilson [8] who performed a comprehensive set of measurements focused on the gain of finite size corner reflector antennas. They discovered that there are limits to the size of the plates beyond which the gain does not increase and in some cases even decreases if the width of the plates is increased. Two years later, in a second and very comprehensive study Cottony and Wilson [9] looked at the radiation pattern of a finite size corner reflector antenna. They varied the length from 0.5λ to 5λ and the width of the plates from 1λ to 10λ. The apex angle was varied such that the maximum gain was achieved.

The corner reflector antenna found its way into antenna books. The most comprehensive treatment is in Moullin 'Radio Aerials' [10] which was published 1949. It includes an extensive discussion of radiation patterns, radiation resistance, and gain for infinite sized reflector plates. Additionally, some results of measurements are presented and compared with theory. A comparison between the corner reflector antenna and parabolic reflectors was conducted and measurements were included to highlight the similarity between the two shapes. The theory presented in this book was the basis of [4].


Publications [1-11] are the fundamental papers and books on the subject of the corner reflector antenna. All further publications only deal with certain details or improved computational methods. A comprehensive discussion of the papers is given in section 2.2.

Until today there is no analytical solution for the radiation characteristic of the corner reflector antenna with finite size plates. Simulation techniques that take the effects of finite size plates into account are presented in chapter 3. Knowledge of the
performance of finite size antennas is essential to meet modern communication systems specifications. The difference between infinite sized and finite size plates is assessed in chapter 4.

2.2. Definition and application

The basic construction of the corner reflector antenna can be seen in fig. 2.1. Literature also refers to it as the dihedral corner reflector antenna to distinguish it from other related geometries which are used in the field of antenna and radar technique.

The corner reflector antenna consists of two flat conducting plates which meet at an angle to form a corner. The point where the plates meet is called the 'apex'. The angle where they meet is called 'apex angle' \( \psi \). The length \( l \) and the width \( w \) of the plate describe the size of the plate. The antenna is usually fed by a dipole which is \( \rho_0 \) distant from the apex. These parameters define the geometry of the corner reflector antenna.
Due to the simple design the antenna it is easily built with a low budget. This is one reason why the corner reflector antenna is employed in a wide range of applications where a medium gain antenna is needed.

Applications suggested by Kraus in his early papers are point-to-point communication and radio links. He also stated that because of the broad-band behaviour the antenna would be suited for TV reception. A look onto the roofs confirms his assumption. The Square-Corner-Yagi-Uda antenna is one of the standard receiving antennas for television programmes. Later it became obvious that a modified version of the corner reflector, the trihedral corner reflector, could be employed in order to calibrate radar systems or be part of radar applications.

More recently the corner reflector antenna has been also used as a base station antenna for mobile communication systems. The concept of cellular networks and frequency reuse in different cells demands an exact knowledge of the antenna's performance.

As already mentioned there are other related geometries which finds applications in radar and remote sensing.

First of all there is the 'three dimensional corner reflector antenna' which was introduced by Inagaki [12] and used as an antenna. The additional plate at the bottom of the reflector (fig. 2.2) increases the gain. The disadvantage is that the boresight direction is no longer at $\theta=90^\circ$ but depends on several design parameters and is in the range of $30^\circ-50^\circ$.

For radar calibration and applications trihedral and quadrihedral corner reflector are used, see fig 2.3. Those reflectors consist of either three or four plates that are mounted in such a way that they have a plane aperture. Due to their regular shape the back-scatter cross-section can be analytically determined. The radar cross section (RCS) can be made high and stays that way over a wide range of angles of incoming
waves. Consequently they are used as a calibration target for radars in guiding and navigation systems.

![Three dimensional corner reflector antenna](image)

**Fig. 2.2: Three dimensional corner reflector antenna**

![Trihedral CR and Quadrihedral CR](image)

**Fig. 2.3: Trihedral and quadrihedral corner reflector**

These geometries find usage in applications such as car-borne radar to assist the driver of a vehicle and runway identification systems.

### 2.3. Literature survey

The fundamental publications have already been described in section 2.1. Kraus, who is the inventor of the corner reflector antenna, deals in his first papers [1, 2] with the
analysis of infinite size corner reflector antenna. He compares measured data for radiation pattern with results from Image Theory and derives expressions for the radiation impedance and for the gain. All of the expressions are only valid for the corner reflector antenna with an apex angle which is a submultiple of $\pi$. To indicate the practical use of the corner reflector antenna he presents a portable corner reflector antenna which can be folded for easier transport. Kraus proposes ideas for changes of construction to achieve radiation pattern tailored for radio broadcasting. The restriction to apex angles which are submultiple of $\pi$ were independently overcome by Wait [4] and Klopfenstein [5]. Wait's derivation is based on the wave equation in cylindrical form and Klopfenstein employs the dyadic Green's function for the perfectly conducting wedge to get a solution for arbitrary apex angles. The results are identical in both cases and consist of a single infinite series composed of Bessel functions of the first kind which generally have a fractional order.

The influence of the finite size of the reflector plates was assessed in three experimental studies by Harris [6] and Wilson and Cottony [8, 9]. The data of Harris is based on measurements conducted with an antenna where the plates were $w=4\lambda$ and $l=6\lambda$. He changed the dipole to apex distance from 0.2$\lambda$ to 2.0$\lambda$ and the apex angle from 60° to 270°. Harris also investigated the tilt of the main beam while the dipole is off-set from the centreline. The influence of replacing the solid plates by rods was measured for a corner reflector antenna when the size of the plates was finite but large. The results are presented in paper [6] as polar plots with a linear scale that makes the utilisation difficult.

Wilson and Cottony also varied the size of the plates. In their first study on the gain of finite size corner reflector antenna [8] they changed the width and length of the plates from 0.4$\lambda$ to 5$\lambda$ in order to highlight the influence of the finite size. The apex angle was varied from 20° to 180°. In their conclusion they argued that the ideal position of the dipole at a given apex angle depends on the length and, to a less extent, on the width of the plates. It became clear that increasing the length of the plates also
increased the gain. However, by increasing the width of the plates in some cases the gain of the antenna slightly reduced.

Their next investigation concentrated on the radiation patterns of finite size corner reflector antenna [9]. The measurements include plate dimensions of 0.5\( \lambda \)-5\( \lambda \) for the length and 1\( \lambda \)-10\( \lambda \) for the width of the plates. The apex angle had been adjusted to achieve the highest gain. The influence of the widths and lengths of the plates on the beam-width and the front-to-back ratio is summarised in a series of 56 curves. In contrast to Harris [6] the measured results are presented in x-y diagrams with a logarithmic scale that enables a much better interpretation of the results.

The idea of adding chokes on the corner reflector antenna to increase the front-to-back ratio was investigated and it was shown that the idea had no desirable influence on the patterns.

The most comprehensive book on the subject of corner reflector antenna is by Moullin [10]. His discussion on the gain and radiation pattern of the infinite corner reflector antennas is well organised. In order to back up his theoretical assumptions Moullin offers measured results whenever the opportunity arises. His attempt to generalise the formula for the radiation pattern to arbitrary apex angles was shown by Wait [4] to be incorrect. In a comparison between the corner reflector antenna and a parabolic reflector antenna he emphasised their similarity.

Other books that deal with the corner reflector antenna in some detail are Jasik [7] and Wolff [11]. In the later book the use of Image Theory for computing the radiation pattern, radiation resistance, and gain of infinite sized corner reflector antenna are discussed.

In the following paragraphs a number of publications are reviewed which deal with aspects of the corner reflector used as reflector for an antenna or as a reflector in radar and remote sensing applications.
Several ideas to improve the performance of the corner reflector antenna have been proposed over the years. The front-to-back ratio was increased by Hirasawa [13] by loading the corner reflector antenna reactively.

In 1983 Elkamchouchi presented two ideas in his paper [14]. The first idea was to add a cylindrical reflecting surface of suitable radius at the apex of the corner reflector. This is discussed in chapter 6 of the thesis. The second proposal was based on Inagaki’s three dimensional corner reflector antenna [12]. Rather than adding a square plate at the bottom of the corner reflector antenna he suggests adding a cylindrical plate. For the latter idea he and Elrakaiby presented results using the Method of Moments (MM) in a paper which followed 13 years later [15].

Abdelazeez [16] investigated the idea of modifying the apex for different feed configurations.

Another idea of changing the geometry of the reflector to improve the performance was developed by Matthew et al [17]. They added two reflector elements as shown in fig. 2.4 and named the structure a 'triple corner-reflector antenna'. In their paper they claim an increased directivity.
Research carried out at the University of Michigan aimed to build a high gain monolithic millimetre-wave corner reflector antenna fed by a long wire antenna [18, 19]. Gain values of 17.7dB at 222GHz have been reported. This shows the possibility of using the corner reflector antenna in Monolithic Microwave Integrated Circuits (MMICs) where the antenna and the associated circuits are combined in a very compact form.

The feed of the corner reflector antenna is usually assumed to be a half-wave dipole. Several proposals have been made in order to increase the band-width of the corner reflector antenna. The sleeve dipole was used as a feed by Wong and King [20, 21, 22, 23]. A broad band source such as the log-periodic dipole array (LPDA) was investigated by Hasan [24], Stephenson and Finley [25], and Kosat et al [26]. For both attempts it is valid to say that the success was limited since the proximity of the plates reduced the band-width of the otherwise broad band sources. For the LPDA the excitation of a second resonance of the longer dipoles subsequently restricted the overall usable band-width.

Maruyama and Kagoshima [27] and Suzuki and Kagoshima [28] pursued a different approach. They proposed an antenna with the same beam-width in two bands by introducing rod-type parasitic elements close to the driven element. The reason was that two frequencies could be used at base stations for mobile communications.

The use of a triangular flat blade dipole as a feed has been investigated in a paper by Duff and Tranbargar [29] for borehole applications.

Quite a few papers deal with the computation of the characteristics of corner reflector antennas. The phase centre of the corner reflector antenna which is important for radio interferometry is investigated by Ja [30]. An extension of Kraus's approach using Image Theory to determine the far-field of the corner reflector antenna is presented by Ng and Lee [31]. They allowed for a tilted dipole as feed for corner reflector antenna with an apex angle which is a submultiple of $\pi$. In 1982 Shastry and Kumar [32]
developed a method of analysing three dimensional corner reflector antenna. Expressions for the far-field, radiation resistance, and directivity were obtained. Griesser and Balanis [33] employ Physical Optics (PO) and Physical Theory of Diffraction (PTD) to compute the back-scattered field of a trihedral corner reflector. Later the same authors use Uniform Theory of Diffraction (UTD) to establish the back-scattered fields of a dihedral corner reflector [34].

The radiation pattern of the corner reflector antenna has been computed by using the Boundary-Element Method (BEM) in a paper by Miyata and Fukai [35]. The same problem is tackled by Zhang et al [36, 37] by using GTD. Recently Lindell and Puska [38] studied the reflections of a plane wave from the corner reflector by using a reflection dyadic.

The corner reflector is of importance in radar technology. It has been used to calibrate Synthetic Aperture Radar (SAR) since the RCS of the corner reflector is well known. References 39 to 44 deal with SAR. Various techniques have been applied to determine the RCS of corner reflectors. Anderson [45, 46] studied a quadrihedral corner reflector using PO and the reduction of RCS for dihedral corner reflector for modest departures from orthogonality. The dihedral corner reflector was also investigated by Fathe et al [47]. They apply the MM to both monostatic and 90° bistatic source-receiver geometries. Park et al [48] investigated the dihedral corner reflector applying the Mode Matching Technique (MMT). The trihedral corner reflector has been investigated with an hybrid approach of PO and Method of Equivalent Currents (MEC) by Polycarpou et al [49].

The RCS of corner reflectors is relatively high since waves coming in at a range of angles are reflected into the direction they came from. The range of angles varies with the apex angle. Therefore corner reflectors finds application for runway identification (Grant et al [50]) and in airborne radar to assist drivers of vehicles in determine where the vehicle is located within the road during adverse weather conditions (Yamaguchi et al [51]).
2.4. Analytical prediction of radiation characteristics

It is possible to find analytical expressions for the far-field radiation pattern as long as infinite size reflector plates are assumed. The first approach, which is still in use today, is Image Theory. The method is restricted to apex angles which are submultiple of π. In 1954 Wait introduced a modified method [4] which is valid for arbitrary apex angles. The results were later validated by Klopfenstein [5] using a different approach. In this section the method will be referred to as Wait's method.

2.4.1. Image Theory

Image Theory is a method to analyse the performance of a source of an electromagnetic field near a perfect conducting plate of infinite size. For a single source in front of a flat plate the theory predicts an imaginary source, which is opposite in phase to the real source. It does not physically exist, but if the plate is replaced by this source, the field distribution in the half space of the source can be calculated. Since the plate is assumed to be infinite in size it is clear that there can be no currents on the back of the plate and the field behind the plate does not exist.

A good introduction to Image Theory can be found in Balanis "Antenna Theory, Analysis and Design" [52]. Shen [53] uses an expanded Image Theory for the case of a source in front of a corner of conducting walls. The flat plate is a special case of the corner reflector with an apex angle of 180°.

The basic corner reflector antenna has two infinite size plates. This can be allowed for by using Image Theory to superimpose the result of the source in front of one plate and the source in front of the other plate. For an apex angle of 90° the images are shown in fig. 2.5. The dashed line indicates that the plates have to be continued beyond the apex. From the real source number 1 image number 2 is due to the upper plate and image number 4 from the lower plate. Those two images are mirrored on the dashed extensions of the plates. Therefore this gives image number 3. For reasons of
symmetry it is clear that the Image Theory can only be applied for apex angles which are submultiple of \( \pi \).

Fig. 2.5: Image theory for an apex angle of 90°

The number of images increases when the apex angle decreases. The number of images can be computed by:

\[
\text{number of images} = 2 \frac{\pi}{\psi} - 1 \quad (2.1)
\]

For the case of \( \psi = 180° \) there is one image at the position of image number 3 in figure 2.5. According to Image Theory it is out of phase with the real source. The path difference is 2\( \rho_0 \) for \( \phi = 0° \) but changes with the cosine of \( \phi \). For \( \rho_0 = m\pi/\lambda \) with \( m=1, 2, 3, \ldots \) the interference is therefore destructive, which results in a cancellation of the field on boresight. For \( \rho_0 = m\pi/(2\lambda) \) with \( m=1, 3, 5, \ldots \) the interference is constructive and the field will be twice the value compared with the value for a dipole on its own. In-between the field varies with the sin(\( k\rho_0 \)).

The far-field on boresight for the flat plate is as follows, assuming that both sources radiate a sinusoidal field:
\[
\frac{E(t)}{E_0} = \sin(\omega t - k \rho_0) - \sin(\omega t + k \rho_0)
\]

\[
\frac{E(t)}{E_0} = 2* \cos(\frac{\omega t - k \rho_0 + \omega t + k \rho_0}{2}) \sin(\frac{\omega t - k \rho_0 - \omega t - k \rho_0}{2})
\]

\[
\frac{E(t)}{E_0} = 2\cos(\omega t)\sin(k \rho_0)
\] \hspace{1cm} (2.2)

and in the frequency domain

\[
\frac{E}{E_0} = 2\sin(k \rho_0)
\]

Similar considerations lead to simple expressions for the radiation pattern for \(\psi=90^\circ\) and \(60^\circ\).

n = 1, \(\psi = 180^\circ\): \(\frac{E}{E_0} = 2\sin(k \rho_0 \cos \phi)\)

n = 2, \(\psi = 90^\circ\): \(\frac{E}{E_0} = 2(\cos(k \rho_0 \cos \phi) - \cos(k \rho_0 \sin \phi))\) \hspace{1cm} (2.3a-2.3c)

n = 3, \(\psi = 60^\circ\): \(\frac{E}{E_0} = 4\sin(\frac{1}{2} k \rho_0 \cos \phi)\left[\cos\left(\frac{1}{2} k \rho_0 \cos \phi\right) - \cos\left(\frac{1}{2} \sqrt{3} k \rho_0 \sin \phi\right)\right]\)

Moullin [10] developed a general equation for the radiated electric field of a corner-reflector antenna based on these expressions and Image Theory:

\[
E = \frac{4k\pi I n}{c} \left[\left(-J_n(k \rho) + j Y_n(k \rho)\right) J_n(k \rho_0) \cos n\phi + \left(-J_{3n}(k \rho) + j Y_{3n}(k \rho)\right) J_{3n}(k \rho_0) \cos 3n\phi\right]
\] \hspace{1cm} (2.4)

where:

\[ k = \frac{2\pi}{\lambda}, \]

\[ c = 3 \times 10^8 \text{ m/s}, \text{ speed of light in free space} \]

\[ n = \frac{\pi}{\psi}, \]

\( \rho \) : is the distance of the observation point

\( \rho_0 \) : the distance of the source from the apex

I : is the current on the filament
If \( k\rho \) tends to infinity this becomes:

\[
\frac{E}{E_0} = 4n\left\{ e^{\frac{n\pi\rho}{2}} J_n(k\rho_0)\cos n\phi + e^{\frac{n\pi\rho}{2}} J_{3n}(k\rho_0)\cos 3n\phi + \ldots \right\} \tag{2.5}
\]

In the equations \( E_0 \) stands for the field at the given distant point due to an isolated filament carrying a current \( I \). The series in eqn. (2.4) converges quite fast when \( \rho_0 \) is within a few wavelengths of the source.

For apex angles which are submultiple of \( \pi \) it can be shown that eqn. (2.5) reduces to (2.3a-2.3c) using standard transforms for Bessel series:

\[
sin(k\rho_0 \cos \phi) = 2\sum_{n=0}^{\infty} (-1)^n J_{2n+1}(k\rho_0)\cos((2n+1)\phi),
\]

\[
\cos(k\rho_0 \cos \phi) = J_0(k\rho_0) + 2\sum_{n=1}^{\infty} (-1)^n J_{2n}(k\rho_0)\cos(2n\phi), \tag{2.6}
\]

\[
\cos(k\rho_0 \sin \phi) = J_0(k\rho_0) + 2\sum_{n=1}^{\infty} J_{2n} \cos(2n)\).
\]

As Image Theory is a well established method it is used to verify results which are obtained by using the Wait's Method that is outlined in the next section.

### 2.4.2. Wait's Method

Moullin assumed that equation (2.5) was valid for arbitrary apex angles. He wrote in [10] page 129 'Consideration, however, appears to show that (3.13) must be the general solution for all values of \( n \), integral and fractional.'

If the expression is valid for arbitrary angles it can be tested by applying the equation for two corner reflector antennas with apex angles which are only 1° apart. A small change like that should not influence the radiation pattern by much. The radiation pattern of corner reflector antennas with apex angle of 90° and 91° and a dipole separation of 1\( \lambda \) compute using eqn. (2.5) is shown in fig. 2.6.
The pattern changes more than it should with a variation of the apex angle from 90° to 91°. Obviously Moullin's consideration is not valid. A fact which Wait acknowledges.

Wait states in his paper [4] that Moullin was '...rather sceptical as to whether the results expressed as a series of Bessel functions can be generalised to corner reflectors of an arbitrary value between 0 and 2π.'

Wait shows in the same paper that Moullin's theory can be extended to the continuous range of apex angles from 0° to 180° by admitting multi-valued solutions of the wave equation in cylindrical co-ordinates.

This leads to:

\[
\frac{E}{E_0} = \frac{jωμe^{-jkρ}}{\psi ρ} \sum_{n=1}^\infty e^{-jnπ^2 i 2ψ} \sin(nπφ_0 / ψ)sin(nπφ / ψ)J_{nπ/ψ}(kρ_0 \sin \theta) \tag{2.7}
\]
where:

\( \phi_0 \): is the angle at which the dipole is placed (normaly \( \psi / 2 \))

\( \phi \): azimuth angle of observation

\( \theta \): elevation angle of observation

\( J_{n\pi/\psi} \): is the Bessel function of order \( n\pi / \psi \)

Wait's equation (2.7) has been computed for various angles. The comparison of a sub-multiple value of \( \pi \) from the apex angle with a slightly different value (i.e. 60° and 61°) and a dipole separation of 0.5\( \lambda \) should validate the functionality for cases where the order of the Bessel function becomes fractional. The results in fig. 2.7 show only a minor change in the radiation pattern between the case of 60° and 61° apex angle. This validates Wait's equation. It will be used later in the thesis as a comparison for other methods.

![Fig. 2.7: Pattern for variation of 1°](image)

The broadening of the radiation pattern derives from the physics of the problem. The waves guidance is weaker for \( \psi=61° \) since the plates are further apart. Therefore, the wave will expand more which leads to a broader beam.
To further validate Wait's method a comparison can be made between Image Theory and Wait's method for an apex angle which is a submultiple of $\pi$. The author carried out this comparison for $180^\circ$, $90^\circ$, $60^\circ$, $45^\circ$ and $30^\circ$. The case of an apex angle of $60^\circ$ and a dipole separation of $0.5\lambda$ is presented as a representative example. The arrangement was chosen because it modelled an antenna that was built and measured.

The radiation pattern is shown in fig. 2.8. The agreement is excellent. The angle range is restricted to the apex angle since at the reflector plates the field has to be zero due to the method used. The same agreement can be observed of the other apex angles. Wait's method will be used to compare measurement results and predictions of modelling methods to determine the impact of the finite size of the reflector plates.

To interpret (2.7) it is necessary to investigate the behaviour of the Bessel functions. Based on these insights it is possible to relate the existence of side-lobes and the change of the main beam to the apex angle and the position of the dipole in respect to the apex.
2.4.2.1. Some properties of Bessel functions

Since the boundary-value problem of a corner reflector antenna is best described in a cylindrical co-ordinate system the solution of the vector wave equation can be transformed into the Bessel differential equation [54]. The solution of the corner reflector antenna problem consists of an infinite series of Bessel functions of the first kind which in general are of fractional order. All the equations in this section are quoted from the standard book on Bessel functions by Watson [55].

There are a few relevant features of $J_n(z)$ provided that $n$ exceeds unity, which is the case for all apex angles less than 180°. The function is then characterised by a very gradual rise to its first maximum followed by a steep descent, through zero, to its first negative maximum. Thereafter the function oscillates with an amplitude tending to vary as $z^{-0.5}$ with a period that rapidly tends to the value $2\pi$.

Furthermore, for all values of $n$, $J_n(z)$ and $J'_n(z)$ are positive and increasing for all values of $z$ less than $n$, hence the first maximum must occur at a value of $z$ which is greater than $n$.

The first zero occurs for a value $j_n$ of $z$ such that

$$j_n < \sqrt[3]{\frac{4}{3} (n + 1)(n + 5)} \quad (2.8)$$

If $j'_n$ denotes the value which makes $J'_n(z)$ zero for the first time, then it can be shown that

$$j'_n < \sqrt{n(n + 3)} \quad (2.9)$$

Also the first maximum becomes relatively sharper as $n$ increases.

2.4.2.2. Interpretation of Wait's equation

Based on the properties in the previous section it is possible to draw the following conclusions.
For \( k \rho_0 = n \) we have \( 2 \pi \rho_0 / \lambda = \pi / \psi \), thus the circumferential width, \( cw \), across a V-shaped reflector is then equal to \( \lambda / 2 \).

\[
\begin{align*}
\text{cw} &= \frac{2 \pi \varphi_0}{2n} = \frac{\pi \varphi_0}{n} \\
\end{align*}
\]

The circumferential width is the arc between the plates at the distance of the dipole and therefore:

\[
\varphi_0 = \frac{n \lambda}{2 \pi}
\]

\[
\text{cw} = \frac{\lambda}{2}
\]

\(cw\): circumference of corner - reflector at \( \varphi_0 \)

Hence the first maximum of the forward field cannot occur until this circumferential width exceeds \( \lambda / 2 \). This will occur either by increasing the apex angle or increasing the dipole distance. The limiting case of this general law is the well-known 'cut-off' property of a rectangular wave guide (when \( \psi = 0 \) and \( n = \infty \)) where the output is zero unless the width of the guide exceeds \( 0.5 \lambda \).

The number of subsidiary maximum must increase with \( n \). It is hard to say which maximum will be the largest peak. When \( n \) is very large and \( k \rho_0 \) approaches \( 3n \) the curve described the forward field for a movement of dipole further away of the apex may well be described as \( J_n(k \rho_0) \) climbing up the comparatively broad hump of \( J_{3n}(k \rho_0) \). Fig. 2.9 gives an example of relevant Bessel functions for an apex angle of 60°.

Moullin showed that a polar diagram cannot have any appreciable side lobes until the arcual width across the V-shaped reflector, at the position of the dipole, is verging at \( 3/2 \lambda \). The result cannot be anything but a simple sinusoid. At this point one should remember that the far-field is computed within the angle described by the plates. This statement is therefore no contradiction to the assumption that for a smaller aperture we would expect more side-lobes assuming a homogenous illumination. Note that
since the reflector plates are infinite in size the aperture itself must also be infinite in size.

In the interval between $k\rho O=3n$ and $5n$ the pattern will consist of a sine curve and a third harmonic, and thus is the sum of two components. When the beam is to be sharpened by the third harmonic it must be disposed such that $J_{3n}$ and $J_n$ are both to be positive or both negative if $n$ is even. But $J_{3n}$ is necessarily greater than 0 until $k\rho O$ slightly exceeds $3n$; whether or not $J_n$ is positive in this region depends on $n$.

For $n=2, 3, 6, 7$ \hspace{1cm} $J_n(3n)$ is $<0$

$n=4, 5, 8$ \hspace{1cm} $J_n(3n)$ is $>0$

Accordingly if $\psi=60^\circ, 45^\circ, 25.6^\circ, \text{or } 22.5^\circ$ (i.e. $n=2, 3, 6, 7$) the main beam tends to sharpen when the arc across the V-shaped reflector at the antenna is approaching $3/2\lambda$, but when $\psi=90^\circ, 36^\circ, 30^\circ$ (i.e. $n=4, 5, 8$) it will tend to become more blunt. For the $60^\circ$ case the Bessel functions which have to be taken into account are shown in fig. 2.9.

Fig. 2.9: Relevant Bessel functions for $\psi=60^\circ$
The $J_n$ is opposite in sign for $\rho_0=1.5\lambda$ but for a value of about $\rho_0=1.25\lambda$ there should be an even sharper beam and therefore a higher directivity. To visualise the influence of an additional Bessel term, patterns for $\rho_0=2.5\lambda$ and $\rho_0=1.5\lambda$ are shown in fig. 2.10. Fig. 2.11 shows the same cases for $\psi=36^\circ$ (i.e. $n=5$).

For $\psi=36^\circ$ and $\rho_0=1.5\lambda$ the main beam becomes broader, as predicted. The absence of side-lobes can be explained since the $J_{3n}$ term for $n=5$ (which is $J_{15}$) is almost zero. Therefore, only the first term of the summation which consists mainly of $J_5(k\rho_0)\cos(5\phi)$ has to be taken into account. This gives zero on the plates and the peak at the half-angle. If the dipole is further away from the apex the higher order Bessel function terms come into the equation. These involve the higher frequency cosines which manifest themselves in the build-up of side-lobes as can be seen in fig. 2.11.

Fig. 2.10: Comparison of different dipole separation for $\psi=60^\circ$
Up to this point the source is a long filament and consequently there is no variation of the fields in the H-plane. If the source is a half-wave dipole the problem becomes three dimensional and thereby much more complicated. The Fourier-Bessel expansions have to be replaced by Legendre polynomials and involve powers of $\cos \theta$. This study does not include the Image Theory in three dimensions because the three dimensional case will be computed using FDTD.

2.4.3. Directivity and the peak directivity

Two important parameters for antennas are the directivity and the peak directivity. The directivity is the ratio of radiated power in the direction $(\theta_1, \phi_1)$ to the total radiated power [56]:

$$D(\theta_1, \phi_1) = \frac{4\pi \text{power radiated per unit solid angle in direction } \theta, \phi}{\text{Total power radiated}} = \frac{4\pi I(\theta_1, \phi_1)}{\int \int I(\theta, \phi) \sin(\theta) d\theta d\phi}$$  \hspace{1cm} (2.11)
where:

\[ D(\theta_1, \phi_1) : \text{Directivity in direction } (\theta_1, \phi_1) \]

\[ I : \text{Radiation Intensity in any direction } (\theta, \phi) \]

The sin(\theta) term in the denominator is called the Jacobian of the transformation [57] and results from the transformation of the co-ordinate systems:

\[
\iint_{\mathbb{R}} I(x, y)dx dy = \iint_{\mathbb{R}} I\{x(u, v), y(u, v)\}|J|dudv
\]  

(2.12)

where:

\[ |J| : \text{Jacobian of the transformation} \]

For spherical co-ordinates (2.12) becomes:

\[
\iint_{\mathbb{R}} I(x, y)dx dy = \iint_{\mathbb{R}} I\{x(\theta, \phi), y(\theta, \phi)\}\sin(\theta)d\theta d\phi
\]

(2.13)

The peak directivity is the value of the directivity in the direction of its maximum value. Based on (2.11) the peak directivity can be expressed as:

\[
D_o = \frac{4\pi I_{\text{max}}}{\iint_{0}^{\theta_{\text{max}}} I(\theta, \phi)\sin(\theta)d\theta d\phi}
\]

(2.14)

where:

\[ D_o : \text{Peak directivity} \]

\[ I_{\text{max}} : \text{Maximum Radiation Intensity} \]

Throughout the thesis, the aim is for the peak directivity to be along boresight.

Based on (2.7) and (2.11) the directivity of the corner reflector antenna excited with a vertical infinitesimal dipole can be written as:

\[
D_r(\theta_1, \phi_1) = \frac{4\pi \sin^2(\theta_1)||G(\theta_1, \phi_1)||^2}{\iint_{0}^{\theta_{\text{max}}\pi} ||G(\theta, \phi)||^2 \sin^3 \theta d\theta d\phi}
\]

(2.15)
where:

\[ G = \sum_{n=1}^{\infty} e^{-jnp\pi/2} \sin(n\pi \phi_0 / \psi) \sin(n\pi \phi / \psi) J_{n \pi / \psi}(k\rho_0 \sin \theta) \]

The integration in \( \phi \) has only to be carried out between \( \phi = 0 \) and \( \psi \) since the radiation outside the reflector is assumed to be zero due to the infinite size of the reflector plates. The expression can be simplified to:

\[ D_1(\theta_1, \phi_1) = \frac{4\pi \sin^2(\theta_1) G(\theta_1, \phi_1)^2}{\psi \sum_{n=1}^{\infty} \sin^2(n\pi \phi_0 / \psi) \int_0^{\pi/2} J^2_{n \pi / \psi}(k\rho_0 \sin \theta) \sin^3(\theta) d\theta} \]  

(2.16)

This expression represents the directivity for the corner reflector antenna when an infinite small dipole is used as a source. However, as pointed out earlier the main interest is the directivity along boresight whereby (2.16) reduces to:

\[ D_0\left(\frac{\pi}{2}, \phi_0\right) = \frac{4\pi \left| G\left(\frac{\pi}{2}, \phi_0\right) \right|^2}{\psi \sum_{n=1}^{\infty} \sin^2(n\pi \phi_0 / \psi) \int_0^{\pi/2} J^2_{n \pi / \psi}(k\rho_0 \sin \theta) \sin^3(\theta) d\theta} \]  

(2.17)

The directivity for the corner reflector antenna along boresight for a 60° apex angle is shown in fig. 2.12.

Like the radiation pattern of the antenna the directivity is also a function of the Bessel terms in the summation. It can be seen that the first null does occur at exactly the same location when \( J_3 = J_9 \), (fig. 2.9). In this case there is no radiation in the boresight direction. Thus the beam splits up and a null develops at boresight. The peak at 1.25\( \lambda \) in fig. 2.12 results from both terms in the series being opposite in sign and with high value. All subsequent maxima and minima, as \( \rho_0 \) increases, are caused by the same scenario of Bessel function terms which either add together or cancel each other.
Fig. 2.12: Directivity in boresight for a 60° corner-reflector for various $\rho_0$

Scanning all apex angles from 30° to 90° and varying the spacing of the dipole from the apex in the range of 0.5$\lambda$-2.0$\lambda$ yields a surface graph for the directivity on boresight, fig. 2.13. The upper limit of the dipole distance is chosen to get a compact antenna. If $\rho_0$ is too high the plates have to be large to guide the wave.

Fig. 2.13: Directivity for various apex angles and dipole separation, surface plot
An alternative visualisation is shown in fig. 2.14 using a contour plot of the same data.
Fig. 2.14 shows that there are areas with high directivity and some with very low directivity. Additionally, in some areas there is also very rapid change of directivity which follows from a high slope of the Bessel terms. The circles for $\psi \leq 60^\circ$ at increasing dipole separation $\rho_o$ are due to the higher order Bessel terms $J_n(k\rho_o)$ that have their first zero for higher value of $\rho_o$.

The contour graph indicates that it is crucial for the design of the corner reflector antenna that the parameters are chosen such that a high directivity is produced but not at those areas with a rapid change of directivity.

Every apex angle produces a dipole separation which brings the highest directivity for that angle. This is plotted in fig. 2.15.

![Graph showing maximum directivity for various apex angles.](image)

**Fig. 2.15: Maximum Directivity for various apex angles**

It becomes clear from the figure that the smaller the apex angle the higher the directivity. On the other hand the dipole separation has to be enlarged. For a practical system this will mean that the reflector plates have to be longer which contradicts the
aim to make a compact design for the antenna. So it appears a good compromise to have an apex angle of about $60^\circ$ and a dipole separation of about $1.25\lambda$.

2.5. Summary

An introduction into the corner reflector antenna is given. A comprehensive literature survey is presented to show the development of the analysis and applications of the corner reflector antenna. Image Theory is applied to compute the far-field pattern of the corner reflector antenna and results are compared with Wait's method.

The results of the Wait's method show clearly that certain combinations of apex angles and dipole positions are more favourable than others. Hence, it is therefore possible to narrow down the combinations which have to be investigated in order to find a compromise between high directivity and compact size of the antenna. Considering that a broad band operation might be desired and that the electrical distance of the dipole to the apex varies with frequency it is sensible to place the dipole such that the directivity is not sensitive to this parameter. Obviously the further away the dipole is placed the more the finite size of the plates have an influence. This influence is assessed in chapter 4.
3. **FINITE DIFFERENCE TIME DOMAIN (FDTD) TECHNIQUE FOR MODELLING CORNER REFLECTOR ANTENNAS**

### 3.1. Introduction

The ability to accurately predict the radiation characteristics of an antenna is very important. As a component of a system the antenna has to match the specifications to ensure the functionality of the system. There are basically two ways to establish the performance of an antenna. The first method is to build and test the antenna using suitable measurement equipment. This procedure is both very time consuming and expensive. For some problems limitations imposed by the available measurement facilities complicate matters even further.

<table>
<thead>
<tr>
<th>Analysis Method</th>
<th>Type of Application</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geometrical optics (GO)</td>
<td>Electrically large antennas where diffraction can be neglected</td>
</tr>
<tr>
<td>Finite Elements (FE)</td>
<td>Quasi static transmission line problems, and also used for full non-quasi static electromagnetic problems</td>
</tr>
<tr>
<td>Finite Difference Time Domain (FDTD)</td>
<td>Used for electrically small to medium sized (&lt;5λ) antennas</td>
</tr>
<tr>
<td>Method of Moments (MM)</td>
<td>Electrically small antennas and Scatterers. Particularly wire antennas, but other types can be modelled</td>
</tr>
<tr>
<td>Geometrical Theory of Diffraction (GTD)</td>
<td>Medium to large antennas where diffraction is important</td>
</tr>
<tr>
<td>Transmission Line Method</td>
<td>Problems which can be modelled as transmission lines</td>
</tr>
<tr>
<td>Transmission Line Method (TLM)</td>
<td>Can be used for to model the corner reflector antenna</td>
</tr>
<tr>
<td>Physical Optics (PO)</td>
<td>Analysis of Reflector Antennas</td>
</tr>
<tr>
<td>Spherical Wave Theory (SWT)</td>
<td>Applied to Boundary Value Solution of Scattering Problems, and other problems such as Near to Far Field Transformations</td>
</tr>
</tbody>
</table>

**Table 3.1 Electromagnetic Analysis Techniques**

The second possibility is to utilise one of many available electromagnetic analysis methods. Which method is best suited for a given problem depends on a number of modelling factors including the geometry and size of the antenna, whether dielectrics
need to be included in the model and computer resources considerations. Table 3.1 shows an overview of the most common methods with an indication of the class of problems for which the method is typically applied.

The antenna under investigation in this study is a few wavelengths in size and therefore FDTD, MM and GTD/GO are suitable. In the next section the FDTD method is presented in some detail to explain why this method is employed in the study.

3.2. The Finite Difference Time Domain (FDTD) method

This method is relatively young. It was introduced by Yee in 1966 [58] but only became popular with the availability of fast computers. The basis of the method is the solution of Maxwell's equation in the time domain. In the beginning the method was mainly used for scattering and radar cross section (RCS) problems. The technique is now applied to a wide range of problems. Some examples are:

- Medicine, Piket-May et al [59, 60]
- PCB Simulation together with SPICE, Piket-May et al [61], and Thomas et al [62]
- Fibre optics, Goorjian et al [63]
- Antennas, Rayner et al [64]

The co-ordinate system in which the FDTD is realised can be tailored to the specific properties of the antenna. Throughout this study the Cartesian co-ordinate system is used. In general the plates of the corner reflector antenna have to be approximated using Cartesian co-ordinates. The exception is the case of apex angles of 180° or 90° in which case the plates can be aligned along the principle axis. The influence of this approximation is assessed in chapter 4.

A typical configuration of a FDTD simulation is shown in fig. 3.1.
An iterative, time stepping algorithm to calculate the near-field radiated electric and magnetic fields in both space and time is used. The fields are calculated in a finite volume of space called the 'spatial domain' or 'problem space'. The spatial domain is created using a number of smaller cells on which the vector components of the electromagnetic fields are positioned. To reduce the reflections from the domain boundaries the spatial domain is terminated using absorbing boundary conditions (ABCs). Since the reflections are reduced but not totally avoided the absorbing boundary conditions are an important limitation on the FDTD method. The structure which is investigated is defined inside the spatial domain by applying the necessary initial boundary value conditions at the required vector field calculation points. The simulation starts with the switching on of the source, or sources, at time $t=0$ and the time stepping algorithm is then applied at small time intervals until steady state conditions are achieved.

In the following sections, the creation of the spatial domain, the development of the FDTD time stepping algorithm for antenna problems, and a comparison between two ABCs formulations is presented.
3.2.1. The FDTD Spatial Domain

The dimension of the simulated structure defines the size of the spatial domain. The spatial domain consists of a grid of interleaved nodal points at which either a component of the electric or a component of the magnetic vector field is calculated. In a standard Cartesian co-ordinate system, the spatial domain is formed using a number of smaller sub cells. Yee proposed a cell in which the Cartesian components of the electric and magnetic fields are arranged as illustrated in fig. 3.2.

The cell is shown in two parts for clarity. The spatial cell locates the vector electric field components along the edges of the spatial cell and the vector magnetic field components in the centres of the faces of the cell. Cells which use the field arrangement of fig. 3.2 are known as 'Yee cells'.

The dimension of the Yee cells are determined by the precise geometry of the structure and the need to satisfy the assumptions of linearity made in the FDTD equations.

As a general rule a cell size of $\lambda/10$ is regarded as the maximum for the assumption of linearity to hold true. This figure should not be considered as a strict rule but as a
guideline for FDTD modelling as there will be exceptions where a grid spacing of greater than \( \lambda/10 \) can be tolerated.

The spatial domain has to cover the whole structure and must extend beyond it so that the fields can propagate into free space. The size to which the spatial domain must extend beyond the structure should be theoretically as large as possible. The reason for this is that the numerical boundaries of the domain interact with incident waves and cause reflections which do not occur in practice. The smaller the distance between the ABCs and the structure the higher will be the reflections. Since the additional spatial domain increases both computational effort and memory requirement the distance between the ABCs and the structure has to be limited to the minimum necessary amount such that the influence of the imperfections can be tolerated. As a general rule a distance of \( \lambda/2 \) between the structure and the ABCs is most of the time sufficient.

3.2.2. Formulation of the FDTD Algorithm

The development of the FDTD algorithm in any chosen co-ordinate system begins with the Maxwell’s time dependent equations for electromagnetic radiation. For an isotropic source free medium, the time dependent Maxwell’s equations are:

\[
\nabla \times \mathbf{E} = -\mu \frac{\partial \mathbf{H}}{\partial t} - \mathbf{J}_m \tag{3.1}
\]

\[
\nabla \times \mathbf{H} = \varepsilon \frac{\partial \mathbf{E}}{\partial t} + \mathbf{J}_s \tag{3.2}
\]

where:

- \( \mu \) = Magnetic Permeability
- \( \varepsilon \) = Electric Permittivity
- \( \mathbf{H} \) = Vector Magnetic Field
- \( \mathbf{E} \) = Vector Electric Field
- \( \mathbf{J}_s \) = Electric Current Density
- \( \mathbf{J}_m \) = Fictitious Magnetic Current
The symmetry of the Maxwell's equations which is achieved by the inclusion of the fictitious magnetic current term \( J_m \) permits some mathematical functions to be simplified. The differential equations (3.1) and (3.2) must be written in the appropriate form for the co-ordinate system in which the algorithm is being developed. In case of a Cartesian co-ordinate system, equations (3.1) and (3.2) are expressed in terms of the Cartesian components of the vector electric and magnetic fields to yield the following set of linear partial differential equations.

\[
\begin{align*}
\frac{\partial H_x}{\partial t} &= \frac{1}{\mu} \left( \frac{\partial E_y}{\partial z} - \frac{\partial E_z}{\partial y} - \rho'H_x \right) \\
\frac{\partial H_y}{\partial t} &= \frac{1}{\mu} \left( \frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z} - \rho'H_y \right) \\
\frac{\partial H_z}{\partial t} &= \frac{1}{\mu} \left( \frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x} - \rho'H_z \right) \\
\frac{\partial E_x}{\partial t} &= \frac{1}{\varepsilon} \left( \frac{\partial H_y}{\partial z} - \frac{\partial H_z}{\partial y} - \sigma E_x \right) \\
\frac{\partial E_y}{\partial t} &= \frac{1}{\varepsilon} \left( \frac{\partial H_z}{\partial x} - \frac{\partial H_x}{\partial z} - \sigma E_y \right) \\
\frac{\partial E_z}{\partial t} &= \frac{1}{\varepsilon} \left( \frac{\partial H_x}{\partial y} - \frac{\partial H_y}{\partial x} - \sigma E_z \right)
\end{align*}
\]

where:

- \( E_x, E_y, E_z \) : Cartesian Electric Field Components
- \( H_x, H_y, H_z \) : Cartesian Magnetic Field Components
- \( \sigma \) : Electric Conductivity
- \( \rho' \) : Equivalent Magnetic Resistivity

In the previous set of equations, the electric current density, \( J_s \) has been replaced by the term \( \sigma E \). This allows lossy dielectric materials to be modelled. The fictitious magnetic current, \( J_m \) has been replaced by \( \rho'H \) to permit the possibility of having a magnetic loss mechanism.
For numerical solution of the above set of partial differential equations the partial derivatives in equations (3.3)-(3.8) have to expressed in terms of finite differences.

The choice of finite difference formulation is determined by the arrangement of the electric and magnetic field points in the Yee cell. The Yee field arrangement leads to the use of centred finite differences as described by Jordan [65]. Considering the general function $F(x)$, the first partial derivative of the function $F(x)$ with respect to $x$ can be expressed as

$$\frac{\partial F(x)}{\partial x} = \frac{\left(F(x + (\Delta x / 2)) - F(x - (\Delta x / 2))\right)}{\Delta x} + O((\Delta x)^2)$$

where $\Delta x$ is the range of $x$ over which the central difference approximation is applied. This approximation is second order accurate in space as indicated by the error term at the end of the expression. Therefore the equations (3.3)-(3.8) can be written as a set of six linear finite difference equations that represent the time domain Maxwell's equations for electromagnetic radiation. This set of equations forms the heart of the FDTD time stepping algorithm. Using the notation proposed by Yee,

$$A_{m}^{n}(i,j,k) = A_{m}(i\Delta x,j\Delta y,k\Delta z,n\Delta t) = A_{m}(x,y,z,t)$$

where:

$A$ : Field type ('E' Electric, 'H' Magnetic)

$m$ : Vector field component ($x,y,z$)

$\Delta t$ : Temporal increment (Time Step)

$\Delta x$ : 'x' spatial increment

$\Delta y$ : 'y' spatial increment

$\Delta z$ : 'z' spatial increment

$i,j,k$ : Spatial grid coordinates

$n$ : Time step number
the FDTD finite difference equations for the $H_x$ and $E_z$ vector field components using a Yee cell spatial domain are:

$$H_x^{n+\Delta t}(i,j+\Delta y,k+\Delta z) = \left(1 - \frac{\rho'(i,j+\Delta y,k+\Delta z)\Delta t}{2\mu(i,j+\Delta y,k+\Delta z)}\right) H_x^{n}\Delta t(i,j+\Delta y,k+\Delta z) + \left(1 + \frac{\rho'(i,j+\Delta y,k+\Delta z)\Delta t}{2\mu(i,j+\Delta y,k+\Delta z)}\right)$$

$$E_z^{n+\Delta t}(i,j+\Delta y,k+1) = E_z^{n}(i,j+\Delta y,k) + \frac{\Delta t}{\mu(i,j+\Delta y,k+\Delta z)} \left(\frac{1}{1 + \frac{\rho'(i,j+\Delta y,k+\Delta z)\Delta t}{2\mu(i,j+\Delta y,k+\Delta z)}}\right)$$

The equations for the other four vector field components are obtained directly from (3.10) and (3.11) by the necessary rearrangements. The equations are suitable for modelling perfect conductors, lossy dielectrics and lossy magnetic materials. For modelling the corner reflector antenna only perfect conductors in free space need to be modelled. In this case equations (3.10) and (3.11) can be simplified since $\sigma = \infty$, $\varepsilon = \varepsilon_0$ and $\mu = \mu_0$. 

\[3.10\] 

\[3.11\]
For modelling of perfect conductors, equations (3.10) and (3.11) simplify to:

\[ H_{x}^{n+\delta}(i, j + \frac{\delta}{2}, k + \frac{\delta}{2}) = H_{x}^{n}(i, j + \frac{\delta}{2}, k + \frac{\delta}{2}) \]

\[ + \frac{\Delta t}{\mu(i, j + \frac{\delta}{2}, k + \frac{\delta}{2})} \left[ \frac{E_{y}^{n}(i, j + \frac{\delta}{2}, k - \frac{\delta}{2}) - E_{y}^{n}(i, j + 1, k - \frac{\delta}{2})}{\Delta y} \right] \]

\[ \frac{\Delta \tau}{\varepsilon(i, j, k + \frac{\delta}{2})} \left[ \frac{H_{y}^{n+\delta}(i + \frac{\delta}{2}, j - \frac{\delta}{2}, k + \frac{\delta}{2}) - H_{y}^{n+\delta}(i + \frac{\delta}{2}, j + 1, k + \frac{\delta}{2})}{\Delta y} \right] \]

\[ \frac{\Delta \tau}{\varepsilon(i, j, k + \frac{\delta}{2})} \left[ \frac{E_{x}^{n+\delta}(i, j, k + \frac{\delta}{2}) - E_{x}^{n+\delta}(i, j + 1, k - \frac{\delta}{2})}{\Delta y} \right] \]

\[ E_{z}^{n+1}(i, j, k + \frac{\delta}{2}) = E_{z}^{n}(i, j, k + \frac{\delta}{2}) \]

\[ + \frac{\Delta t}{\varepsilon(i, j, k + \frac{\delta}{2})} \left[ \frac{H_{y}^{n+\delta}(i + \frac{\delta}{2}, j - \frac{\delta}{2}, k + \frac{\delta}{2}) - H_{y}^{n+\delta}(i + \frac{\delta}{2}, j + 1, k + \frac{\delta}{2})}{\Delta y} \right] \]

\[ \frac{\Delta \tau}{\varepsilon(i, j, k + \frac{\delta}{2})} \left[ \frac{H_{x}^{n+\delta}(i, j + \frac{\delta}{2}, k + \frac{\delta}{2}) - H_{x}^{n+\delta}(i, j - \frac{\delta}{2}, k + \frac{\delta}{2})}{\Delta y} \right] \]

(3.12)

(3.13)

3.2.3. Excitation Functions

In general there are two classes of excitation functions for an FDTD simulation. The first approach is a plane wave in which case the excitation can be determined exactly at all points in the spatial domain at each time step. This approach is employed for scattering and RCS problems. For radiation problems, one or more sources are positioned at discrete points in the spatial domain. The most commonly used type of discrete source is the 'delta gap' source. This source is also employed to model the dipoles used for corner reflector antennas.

The FDTD method allows wide band results to be obtained from a single FDTD simulation by using a suitable pulse excitation (usually a Gaussian distribution). If data at only one frequency is required, a monochromatic sinusoidal source function can be used.

The formation of the delta gap source is similar to that used in the method of moments. To model a z-directed wire antenna with a delta gap source in FDTD, the
normal component of the $E_z$ electric field at the centre of the antenna is determined according to the equation

$$E_z = \frac{V(t)}{\Delta}$$

(3.14)

where $V(t)$ is the time domain excitation function and $\Delta$ is the spatial cell dimension in the appropriate direction. This source is particularly appropriate for use in modelling array antennas since the excitations for each element in the array can be specified by exciting a single electric field component in each element with the required time domain signal.

### 3.2.4. Numerical Stability

Processing the near-field data is only possible if the field at the Yee points has stabilised. Numerical stability must be ensured if convergence is to be obtained. Stability in the FDTD algorithm is achieved by employing the ‘Courant Stability Condition’ or ‘Courant Condition’. The condition described by Taflove [66] relates the maximum permissible time step for algorithm stability to the spatial domain cell dimensions. In three dimensions, the Courant stability condition is

$$\Delta t \leq \frac{v^{-1}}{\sqrt{\left(\frac{1}{(\Delta x)^2} + \frac{1}{(\Delta y)^2} + \frac{1}{(\Delta z)^2}\right)^{-1}}}$$

(3.15)

where $\Delta t$ is the maximum permissible time step, $\Delta x$, $\Delta y$ and $\Delta z$ are the dimensions of the cells in the spatial domain and $v$ is the velocity of electromagnetic propagation. In the case of cubic cells (i.e. $\Delta x=\Delta y=\Delta z=\Delta$) the 3-D Courant condition (3.15) can be simplified:

$$\Delta t \leq \frac{\Delta}{v\sqrt{3}}$$

(3.16)

The propagation velocity $v$ is approximately $3\times10^8$ m/s as long as the wave propagates in free space. Care must be taken when dielectrics are part of the model. The
propagation velocity in dielectrics is slower and therefore a larger time step is permissible. Stability is guaranteed if the time step in (3.15) is calculated using the maximum possible velocity of propagation in the model being studied.

When variable sized cells are used in a single grid. Stability can be assured by using the maximum speed of propagation and the cells with the smallest dimensions to calculate the time step.

3.2.5. Numerical Dispersion

One inherit limitation of the FDTD method is the numerical dispersion of the simulated modes in the spatial domain. Numerical dispersion occurs as a result of variations in the phase velocity of the propagating wave modes. It is a function of both propagation direction and the discretisation of the spatial domain. Taflove [67] has considered the effects of spatial dispersion in a 3-D Cartesian spatial domain and shown that the phase velocity is maximum in the 45° direction and minimum in the directions parallel to the domain axes. This numerical variation results in an artificial anisotropy that is inherent in the FDTD algorithm.

The effects of numerical dispersion are most noticeable when pulse excitation is used. When a sinusoidal excitation is used, only one frequency component is present and the spatial discretisation is chosen such that a sufficient number of spatial samples per wavelength are allowed in the spatial domain. In the case of a pulse, there is more than one frequency component present and the choice of spatial discretisation is less obvious. Appropriate choice of the spatial discretisation is essential for the correct propagation of a pulsed waveform. Choosing a spatial discretisation which is too large will result in significant errors in the near-field data as a result of the insufficient sampling of the higher frequency components in the pulse. The errors occur as a result of the variations in propagation velocity of the spectral components of the pulse. Frequency components in the pulse for which the spatial discretisation corresponds to a sampling rate of 10 or more samples per wavelength are supported by the spatial
domain and can propagate correctly, while spectral components which have insufficient spatial sampling are distorted or rejected. The slower propagating higher frequency components which are present are seen as oscillations on the trailing edge of the waveform. These effects are illustrated in the work by Yee [58]. Taflove has shown that choosing 10 spatial samples per wavelength for the principal spectral components in the pulse results in a less than 1% variation in the phase velocities of the highest frequency components regardless of the wave propagation angle in the grid.

3.2.6. Geometry Specification

The boundary conditions for the material that forms the antenna must be modelled by inserting the geometry into the spatial domain and enforcing the field points to take discrete values. Modelling of perfect conductors is most easily accomplished by specifying that the tangential electric fields to the metallic surface, or the magnetic fields normal to the metallic surface are zero. It is only necessary to set the tangential electric fields to zero since the normal components of the magnetic fields will be forced to zero automatically by the finite difference equations.

The regular shape of the corner reflector antenna avoids mathematically more complex FDTD algorithms such as the 'contour' FDTD algorithm described by Katz et al [68]. The Cartesian co-ordinate system which is implemented in the code used enables the accurate modelling of corner reflector antenna with apex angles of 180° and 90°. In this case the plates are along the principle axes. For apex angles other than those the plates have to approximated.

3.3. Analysis of Absorbing Boundary Conditions

If FDTD is used to solve antenna problems it is usually assumed that the antenna radiates into free space. The equivalent FDTD model is an infinite big spatial domain in all three dimensions. It is obvious that memory restriction as well as the desire to
minimise the computational effort do not permit an infinite spatial domain and require that the spatial domain is kept as small as possible. The boundary conditions which are applied in the FDTD method have to allow all outward-propagating numerical waves to leave the domain almost as if the simulation was performed in an infinite spatial domain. In the process the outer boundary conditions must suppress spurious reflections of the outgoing waves to an acceptable level, permitting the FDTD solution to remain valid for all time steps, especially after the reflected waves return to the vicinity of the modelled structure. Those outer grid boundary conditions are called absorbing boundary conditions (ABCs).

ABCs cannot be directly obtained by the FDTD algorithm developed in 3.2.2 because this system employs a central spatial difference scheme that requires knowledge of the field one-half space cell to each side of the observation point. Therefore this scheme cannot be implemented at the outmost lattice planes since by definition there is no information concerning the fields at points one-half space cell outside of these planes.

Two different ABC methods are now described. In both cases the description is for the two dimensional case which allows the expressions to be simplified in order to achieve a clearer insight into the methods. The ABCs introduced by Mur [69] are described only briefly since the method is well-known and has become obsolete. The Mur ABCs are now replaced by the superior Perfect Matched Layer (PML) technique developed by Berenger [70]. A comparison between the two methods in the case of a simulation of the corner reflector antenna is given in chapter 4.

3.3.1. Mur Absorbing Boundary Conditions

The Mur ABCs are based on the one-way wave equation. That is a partial differential equation which only permits wave propagation in certain directions. In case of the FDTD method a one-way wave equation absorbs numerically impinging scattered waves. Engquist and Madja [74] derived a theory of one-way wave equations suitable for ABCs in Cartesian FDTD grids.
In the two dimensional case there are two sets of independent Maxwell's equations. One set includes $H_z$, $E_x$, and $E_y$ and is called the TE case. The other consists of $E_x$, $H_y$, and $H_z$ and is called TM case (fig. 3.3). The central difference formulation cannot be applied for field components which are tangential to the boundary. In the TM case the tangential field component is $E_z$.

The second order Mur ABC at the grid position $x=0$, left hand side of spatial domain, is given by Mur [69] as:

$$
E_{o,j}^{n+1} = -E_{h,j}^{n-1} + \frac{c\Delta t - \Delta}{c\Delta t + \Delta} (E_{o,j}^{n+1} + E_{o,j}^{n-1}) + \frac{2\Delta}{c\Delta t + \Delta} (E_{h,j}^n + E_{o,j}^n) \\
+ \frac{(c\Delta t)^2}{\Delta(c\Delta t + \Delta)} \left( E_{1,j+1}^n - 2E_{1,j}^n + E_{1,j-1}^n + E_{o,j+1}^n - 2E_{o,j}^n + E_{o,j-1}^n \right)
$$

(3.17)
where:

\[ \Delta = \Delta x = \Delta y \]

\[ \Delta t : \text{cell size} \]

\[ c : \text{speed of light} \]

\[ \Delta t : \text{time step} \]

\[ E_{0,j}^n : E \cdot \text{field at } x = 0, y = j \text{ at time step } n \]

In the corners of the spatial domain the first order Mur ABC has to be employed since the \( E_{o,j-1}^n \) field point is unknown at the bottom corner and the \( E_{o,j+1}^n \) field point at the top corner. The first order Mur ABC is given by the first three terms of (3.17).

In the three dimensional case the second order Mur ABC for a cubic lattice, \( \Delta x = \Delta y = \Delta z = \Delta \), at \( x=0 \) can be written as:

\[
W_{o,j,k}^{n+1} = -W_{1,j,k}^{n-1} + \frac{c\Delta t - \Delta}{c\Delta t + \Delta} \left( W_{1,j,k}^{n+1} + W_{o,j,k}^{n-1} \right) + \frac{2\Delta}{c\Delta t + \Delta} \left( W_{1,j,k}^n + W_{o,j,k}^n \right) + \frac{(c\Delta t)^2}{2\Delta(c\Delta t + \Delta)} \left( W_{o,j,k+1}^n - 4W_{0,j,k}^n + W_{o,j-1,k}^n + W_{1,j,k}^n - 4W_{1,j,k}^n + W_{1,j-1,k}^n + W_{0,j,k+1}^n + W_{0,j,k-1}^n + W_{1,j,k+1}^n + W_{1,j,k-1}^n \right)
\]

(3.18)

where \( W_{i,j,k}^n \) represents a Cartesian component of an E- or H-field point at \( x=i, y=j, z=k \) at time step \( n \).

The first order Mur ABC for three dimensions is the same expression as for two dimensions. Finite difference expressions for the Mur ABC at each of the other grid boundaries can be obtained by using coordinate symmetry arguments to permute the subscripts of the \( W \)'s.

The reflection at the boundaries is dependent on the angle of the incident wave.
Table 3.2: Reflection coefficient for Mur ABCs

Table 3.2 shows the approximate percentage reflection coefficient expected for the Mur boundary condition implementation for orders N=1..4 for the range of incident angles from 0° to 90°, i.e. from normal to grazing incidence.

3.3.2. Berenger Perfectly Matched Layer (PML)

Berenger [70] presented his ABC approach for two dimensions in 1994. The approach is based upon splitting electric or magnetic field components in the absorbing boundary region with the possibility of assigning losses to the individual split-field components. The net effect of this is to create a non-physical absorbing material adjacent to the outer FDTD spatial domain boundary that has a wave impedance independent of incident angle and frequency of the outgoing waves. Berenger reports effective reflection coefficients for his ABC which is 1/3000th of the second order Mur ABCs. He also reported total grid noise energies reduced to 10^-7 times the level produced by second order Mur ABCs. In [71] Katz et al confirm those claims and the method is extended to three dimensions.
In this section the principle of the method and its implementation for FDTD is described.

First consider Maxwell’s equations in two dimensions for the TE polarisation case. The electric fields lie in the x-y plane as can be seen in fig. 3.4.

![Field components for TE case](image)

Equation (3.5)-(3.7) reduce to:

\[
\begin{align*}
\varepsilon_0 \frac{\partial E_x}{\partial t} + \sigma E_x &= \frac{\partial H_z}{\partial y} \\
\varepsilon_0 \frac{\partial E_y}{\partial t} + \sigma E_y &= -\frac{\partial H_z}{\partial x} \\
\mu_0 \frac{\partial H_z}{\partial t} + \sigma^* H_z &= \frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x}
\end{align*}
\]

(3.19) (3.20) (3.21)

where:

\[\varepsilon_0 : \text{Free space permittivity}\]
\[\mu_0 : \text{Free space permeability}\]
\[\sigma : \text{Electric conductivity}\]
\[\sigma^* : \text{Magnetic loss}\]
The wave impedance of a wave in a lossless vacuum and a wave in lossy free-space medium can be matched if the relation

\[
\frac{\sigma}{\varepsilon_0} = \frac{\sigma^*}{\mu_0}
\]  

(3.22)

is satisfied. If the wave impedance is matched no reflection occurs when a plane wave propagates normally across an interface between true vacuum and lossy free-space medium.

The novel idea of Berenger is to introduce a degree of freedom in specifying loss and impedance matching by splitting \( H_z \) into two sub-components which he denotes \( H_{zx} \) and \( H_{zy} \). Equations (3.19)-(3.21) can then be written as:

\[
\begin{align*}
\varepsilon_0 \frac{\partial E_x}{\partial t} + \sigma_y E_x &= \frac{\partial (H_{zx} + H_{zy})}{\partial y} \\
\varepsilon_0 \frac{\partial E_y}{\partial t} + \sigma_x E_y &= -\frac{\partial (H_{zx} + H_{zy})}{\partial x} \\
\mu_0 \frac{\partial H_{zx}}{\partial t} + \sigma_x^* H_{zx} &= -\frac{\partial E_x}{\partial x} \\
\mu_0 \frac{\partial H_{zy}}{\partial t} + \sigma_y^* H_{zy} &= \frac{\partial E_y}{\partial y}
\end{align*}
\]  

(3.23)- (3.26)

where:

\( \sigma_x \): Electric conductivity along the x - direction
\( \sigma_y \): Electric conductivity along the y - direction
\( \sigma_x^* \): Magnetic conductivity along the x - direction
\( \sigma_y^* \): Magnetic conductivity along the y - direction

A closer look at equations (3.23)-(3.26) reveals that for certain choices of values for electric and magnetic conductivity those equations describe wave propagation in:

- vacuum if \( \sigma_x = \sigma_y = \sigma_x^* = \sigma_y^* = 0 \)
- conductive medium if \( \sigma_x = \sigma_y \) and \( \sigma_x^* = \sigma_y^* = 0 \)
• absorbing medium if $\sigma_z = \sigma_y$ and $\sigma_z^* = \sigma_y^*$

Furthermore, the equations show that a plane wave propagating along the x-axis consists of $E_y$ and $H_{zx}$. The propagation, consequently, is independent of $\sigma_y$ and $\sigma_y^*$.

Such a wave can still be absorbed even if both conductivities along the y-direction are set equal to zero. Plane waves travelling along the y-axis consist only of $E_x$ and $H_{zy}$ so the same holds true for $\sigma_x$ and $\sigma_x^*$. To distinguish different PML mediums the following nomenclature is defined for those two cases. In the first case the material is called a PML($\sigma_x, \sigma_x^*, 0, 0$). In the second case PML(0, 0, $\sigma_y$, $\sigma_y^*$).

The absorption of the waves travelling through PML material is now assessed. A plane wave with an electric field of magnitude $E_0$ forms an angle $\varphi$ with the y-axis as shown in fig. 3.4. The magnitudes of the magnetic components $H_{zx}$ and $H_{zy}$ are $H_{zx0}$ and $H_{zy0}$. The four field components can be expressed as:

\[
E_x = -E_0 \sin(\varphi)e^{i \alpha t - \beta y} \tag{3.27}
\]

\[
E_y = E_0 \cos(\varphi)e^{i \alpha t - \beta y} \tag{3.28}
\]

\[
H_{zx} = H_{zx0}e^{i \alpha t - \beta y} \tag{3.29}
\]

\[
H_{zy} = H_{zy0}e^{i \alpha t - \beta y} \tag{3.30}
\]

where:

$\omega$ : Radian frequency
$t$ : Time
$\alpha, \beta$ : Complex attenuation and phase constants

The angle $\varphi$ is defined as shown in figure 3.4.

Since the magnitude $E_0$ is known the equations (3.27) to (3.30) involve four unknown quantities to be determined, $\alpha$, $\beta$, $H_{zx0}$ and $H_{zy0}$. Enforcing $E_x, E_y, H_{zx}, H_{zy}$ from (3.27) to (3.30) in the PML equations (3.23) to (3.26) yields:
\[ \alpha = \sqrt{\frac{\epsilon_0 \mu_0}{G}} \left( 1 - j \frac{\sigma_x}{\epsilon_0 \omega} \right) \cos(\varphi) \]  
(3.31)

\[ \beta = \sqrt{\frac{\epsilon_0 \mu_0}{G}} \left( 1 - j \frac{\sigma_y}{\epsilon_0 \omega} \right) \sin(\varphi) \]  
(3.32)

where:

\[ G = \sqrt{w_x \cos^2(\varphi) + w_y \sin^2(\varphi)} \]  
(3.33)

with

\[ w_x = \frac{1 - j \left( \frac{\sigma_x}{\epsilon_0 \omega} \right)}{1 - j \left( \frac{\sigma_x^*}{\mu_0 \omega} \right)} \]

\[ w_y = \frac{1 - j \left( \frac{\sigma_y}{\epsilon_0 \omega} \right)}{1 - j \left( \frac{\sigma_y^*}{\mu_0 \omega} \right)} \]

As all four field components have a similar expression for their propagation through the material \( \Omega \) is denoted as any component which yields:

\[ \Omega = \Omega_0 \exp \left( j \omega \left( t - \frac{x \cos(\varphi) + y \sin(\varphi)}{G_c} \right) \right) \exp \left( - \frac{\sigma_x \cos(\varphi)}{G \epsilon_0 \mu_0} x \right) \]

\[ \times \exp \left( - \frac{\sigma_y \sin(\varphi)}{G \epsilon_0 \mu_0} y \right) \]  
(3.34)

Expressions for the complex constants are derived already so the remaining two unknowns are \( H_{zxo} \) and \( H_{zyo} \). Taking into account (3.31) and (3.32):

\[ H_{zxo} = E_0 \sqrt{\frac{\epsilon_0}{\mu_0}} \frac{1}{G} w_x \cos^2(\varphi) \]  
(3.35)

\[ H_{zyo} = E_0 \sqrt{\frac{\epsilon_0}{\mu_0}} \frac{1}{G} w_y \sin^2(\varphi) \]  
(3.36)
Considering (3.33) the summation of the two H-field components (3.35) and (3.36) are equal to:

\[ H_0 = E_0 \sqrt{\frac{\varepsilon_0}{\mu_0}} G \]  

(3.37)

and the wave impedance is therefore:

\[ Z = \sqrt{\frac{\mu_0}{\varepsilon_0} G} \]  

(3.38)

When (3.22) is satisfied for both \((\sigma_x, \sigma_x^*)\) and \((\sigma_y, \sigma_y^*)\) the quantities \(w_x, w_y, \) and \(G, \) equal unity at any frequency and are independent of the incidence angle. It is, therefore, possible to rewrite (3.34) and (3.38):

\[ \Omega = \Omega_0 \exp \left( j \omega \left( t - \frac{x \cos(\varphi) + y \sin(\varphi)}{c} \right) \right) \exp \left( -\frac{\sigma_x \cos(\varphi)}{\varepsilon_0 c} x \right) \times \exp \left( -\frac{\sigma_y \sin(\varphi)}{\varepsilon_0 c} y \right) \]  

(3.39)

\[ Z = \sqrt{\frac{\mu_0}{\varepsilon_0}} \]  

(3.40)

Analysing the above two expressions reveals information about propagation of plane waves through the PML material. The first exponential of (3.39) shows that the phase of the wave propagates normal to the electric fields. Therefore \(\varphi = \omega t\) in fig. 3.4. The propagation speed is that of light. The last two exponentials show that the magnitude of the wave decreases exponentially along \(x\) and \(y\).

Furthermore, equation (3.34) shows that the wave impedance of the PML is identical to that of a wave in a vacuum. Thus the matching condition for the TE case (3.22) is also a matching condition for the PML medium. In the case of PML two sets of conductivities must satisfy (3.22) (i.e. both \((\sigma_x, \sigma_x^*)\) and \((\sigma_y, \sigma_y^*)\)).
This PML medium seems, therefore, ideally suited to terminate the FDTD spatial domain. Berenger proposed a two dimensional TE case FDTD grid surrounded by PML which is backed by perfectly conducting walls, fig. 3.5.

The termination of the grid on the left and right hand side is built of PML where \((\sigma_x, \sigma_x^*)\) are matched according to (3.22) along with \((\sigma_y = \sigma_y^* = 0)\) to permit reflectionless transmission across the vacuum-PML interface. On top and bottom of the spatial domain the PML material is defined such that \((\sigma_y, \sigma_y^*)\) is matched according to (3.22) and \((\sigma_x = \sigma_x^* = 0)\).

At the four corners two PML mediums overlap. In this case all four losses are present \((\sigma_x, \sigma_x^*, \sigma_y, \sigma_y^*)\) and set equal to those of the adjacent PMLs.

Based on (3.39) a reflection factor can be defined as:

\[
R(\theta) = \exp \left( -2 \frac{\sigma \cos(\theta)}{\varepsilon_0 c} \delta \right) \tag{3.41}
\]
where:

\[\theta\]: Incidence angle with respect to the normal vector of the interface
\[\delta\]: Thickness of PML layer

For waves of grazing incidence the reflection factor is equal to one. This does not influence the practical value of the method as absorbing boundary condition for FDTD since a wave which is reflected on the right hand boundary will be absorbed either at top or bottom of the spatial domain. Similar considerations apply for the other boundaries.

The absorption depends on the electric loss and the thickness of the layer. In theory the layer could be as thin as needed in order to reduce the computational effort and memory requirement. In practise, there are some limitations to the thickness and the choice for the conductivity of the PML which Berenger investigated [72, 73].

In order to implement the PML ABC into the FDTD method the set of formulas (3.23)-(3.27)) has to be discretised in the same fashion as Maxwell's equations. The equations for \(E_y\) and \(H_{2x}\) are given as an example:

\[
E_{y}^{i+1/2}(i, j+1/2) = e^{-\sigma_x(i)\Delta t/\varepsilon_0} E_y^i(i, j+1/2) - \frac{\sigma_x(i)\Delta t}{\varepsilon_0} \frac{\sigma_x(i)\Delta t}{\varepsilon_0} \\
\times [H_{2x}^{i+1/2}(i+1/2, j+1/2) + H_{2x}^{i+1/2}(i+1/2, j+1/2) \\
-H_{2x}^{i+1/2}(i-1/2, j+1/2) - H_{2x}^{i+1/2}(i-1/2, j+1/2)]
\]

\[
H_{2x}^{i+1/2}(i+1/2, j+1/2) = e^{-\sigma_x(i+1/2)\mu_0\Delta t} H_{2x}^{i+1/2}(i+1/2, j+1/2) - \frac{\sigma_x(i+1/2)\mu_0\Delta t}{\varepsilon_0} \frac{\sigma_x(i+1/2)\mu_0\Delta t}{\varepsilon_0} \\
\times [E_y^i(i+1, j+1/2) - E_y^i(i, j+1/2)]
\]
where \( \sigma_x \) and \( \sigma_y \) are functions of \( x(i) \), in the left, right, and corner layers, and are equal to zero, in the upper and lower layers.

The \( E_y \) component on the interface has one magnetic component \( H_Z \) on one side and two subcomponents \( H_{ZX}, H_{ZY} \) on the other. The field components at the upper right corner of the two dimensional FDTD grid for a TE case is shown in fig. 3.6.

![Fig. 3.6: Upper right part of the FDTD grid](image-url)
The finite difference equations have to be modified. At the right side interface normal to \( x \) (3.42) becomes:

\[
E_{\gamma}^{n+1}(i\ell, j+1/2) = e^{-\frac{\sigma_s(i\ell)\Delta t}{\varepsilon_0}} E_{\gamma}^{n}(i\ell, j+1/2) - \frac{1-e^{-\frac{\sigma_s(i\ell)\Delta t}{\varepsilon_0}}}{\sigma_s(i\ell)\Delta x} \nabla[H_{\gamma}^{n+1/2}(i\ell + 1/2, j+1/2) + H_{\gamma}^{n+1/2}(i\ell + 1/2, j+1/2) - H_{\gamma}^{n+1/2}(i\ell - 1/2, j+1/2)]
\]  

(3.44)

The same approach can be applied to the TM case. The fundamental equations for this case are:

\[
\varepsilon_0 \frac{\partial E_{\alpha}}{\partial t} + \sigma_s E_{\alpha} = \frac{\partial H_y}{\partial x}
\]  

(3.45)

\[
\varepsilon_0 \frac{\partial E_{\gamma}}{\partial t} + \sigma_s E_{\gamma} = -\frac{\partial H_x}{\partial y}
\]  

(3.46)

\[
\mu_0 \frac{\partial H_x}{\partial t} + \sigma_x H_x = -\frac{\partial(E_{\alpha} + E_{\gamma})}{\partial y}
\]  

(3.47)

\[
\mu_0 \frac{\partial H_y}{\partial t} + \sigma_y H_y = \frac{\partial(E_{\alpha} + E_{\gamma})}{\partial x}
\]  

(3.48)

Taflove [71] presented the extension of the method to three dimensions. For completeness the equations for this case are given below:

\[
\mu_0 \frac{\partial H_{xy}}{\partial t} + \sigma_x H_{xy} = -\frac{\partial(E_{\alpha} + E_{\gamma})}{\partial y}
\]  

(3.49)

\[
\mu_0 \frac{\partial H_{xz}}{\partial t} + \sigma_x H_{xz} = \frac{\partial(E_{\alpha} + E_{\gamma})}{\partial z}
\]  

(3.50)

\[
\mu_0 \frac{\partial H_{yz}}{\partial t} + \sigma_x H_{yz} = -\frac{\partial(E_{\alpha} + E_{\gamma})}{\partial z}
\]  

(3.51)

\[
\mu_0 \frac{\partial H_{yx}}{\partial t} + \sigma_x H_{yx} = \frac{\partial(E_{\alpha} + E_{\gamma})}{\partial x}
\]  

(3.52)
\begin{align*}
\mu_0 \frac{\partial H_{zx}}{\partial t} + \sigma_x H_{zx} &= -\frac{\partial (E_{yx} + E_{yz})}{\partial x} \quad (3.53) \\
\mu_0 \frac{\partial H_{zy}}{\partial t} + \sigma_y H_{zy} &= \frac{\partial (E_{xy} + E_{xz})}{\partial y} \quad (3.54) \\
\varepsilon_0 \frac{\partial E_{xz}}{\partial t} + \sigma_x E_{xz} &= \frac{\partial (H_{yx} + H_{yz})}{\partial y} \quad (3.55) \\
\varepsilon_0 \frac{\partial E_{yz}}{\partial t} + \sigma_y E_{yz} &= -\frac{\partial (H_{xy} + H_{yz})}{\partial z} \quad (3.56) \\
\varepsilon_0 \frac{\partial E_{yx}}{\partial t} + \sigma_y E_{yx} &= \frac{\partial (H_{xy} + H_{xz})}{\partial z} \quad (3.57) \\
\varepsilon_0 \frac{\partial E_{zx}}{\partial t} + \sigma_x E_{zx} &= -\frac{\partial (H_{zx} + H_{yz})}{\partial x} \quad (3.58) \\
\varepsilon_0 \frac{\partial E_{xy}}{\partial t} + \sigma_x E_{xy} &= \frac{\partial (H_{yx} + H_{yz})}{\partial x} \quad (3.59) \\
\varepsilon_0 \frac{\partial E_{yz}}{\partial t} + \sigma_y E_{yz} &= -\frac{\partial (H_{xy} + H_{xz})}{\partial y} \quad (3.60)
\end{align*}

The advantages of very low reflection plane waves transmission regardless of incidence angle and frequency are still valid in the three dimensional case. The mechanism of attenuating the waves before reflection by the perfect conductor surrounding the PML is the same as in two dimensions.

3.3.3. Comparison Mur and PML ABCs

A comparison between the two methods is the subject of several publications, 70 to 73. The reduction of reflections allow the PML ABCs to be much closer to the structure than is possible for the Mur ABCs. The computational effort can be reduced in some cases and at the same time the accuracy of the simulations is increased. The PML ABCs are able to produce a predictive dynamic range of >70dB which is an
improvement on the Mur ABCs of about 40dB. The dynamic range of FDTD is therefore comparable with the quite zones of anechoic chambers.

A comprehensive study by Rayner et al [75] concludes that despite obvious advantages of the more accurate Berenger ABC the Mur ABC approximation is adequate for most antenna problems. A comparison which proves this point for corner reflector antennas is given in chapter 4.

However, for some applications the improved accuracy is necessary. It is likely that the PML ABCs will take over from the Mur ABCs as the most used ABC for FDTD simulations.

3.4. Far-Field Radiation Pattern Calculation

The radiation from an antenna is a result of the electric currents that flow in the metallic conductors. If the current distribution on an antenna and structure is known, the radiated electromagnetic fields can be calculated at any point in space using the radiation integral. This is the principle employed in the Method of Moments. In the case of FDTD the fields over a surface enclosing the sources are known and it is therefore possible to derive an equivalent set of currents. Based on the currents it is then possible to employ the ‘Field Equivalence Principle’ to compute the radiated fields.

3.4.1. Time Domain to Frequency Domain Conversion

The FDTD method determines the time domain near-field electromagnetic fields in the finite volume of space modelled by the spatial domain. The calculation of the far-field radiation patterns is accomplished using a set of equivalent currents calculated over an imaginary cuboid surface placed around the antenna geometry. These currents are then used to perform a near-field to far-field (NF-FF) transformation. The currents can be computed in either the time domain or the frequency domain. If results over a wide frequency band are required, it is necessary to use a pulsed excitation, usually
Gaussian, and to compute the time domain electric and magnetic vector potentials, from which the time domain far-fields at selected angles can be computed. The frequency domain far-fields can then be computed from the time domain far-fields via the Fourier Transform. If results are only required at a single frequency, a monochromatic sinusoidal varying electric field source is used. The computation of the far-fields could still be performed from the time domain, but since there is only one frequency component in the near-field data, it is a relatively simple task to transform the time domain near-fields to the frequency domain. This permits the frequency domain currents to be computed from which the far-fields are calculated via the frequency domain electric and magnetic vector potentials.

Since the work in this thesis required results to be obtained at only a few discrete frequencies, the sinusoidal source option was used. An idealised time domain response at a single point in the spatial domain is shown in fig. 3.7.

\[ E_1 = E_0 \sin(\omega t_1 + \phi) \]
\[ E_2 = E_0 \sin(\omega t_2 + \phi) \]

Based on two samples:

the time domain fields are converted to frequency domain with magnitude (3.61) and phase (3.62):

\[ E = \frac{E_1}{\sin(\omega t_1 + \phi)} \]

(3.61)
\[ \tan \phi = \frac{E_1 \sin(\omega t_2) - E_2 \sin(\omega t_1)}{E_2 \cos(\omega t_1) - E_1 \cos(\omega t_2)} \] (3.62)

where:

- \( E_1 \) : E field at time \( t_1 \)
- \( E_2 \) : E field at time \( t_2 \)
- \( \phi \) : Phase in respect to the source point
- \( E \) : Magnitude of the field point

The equivalent is valid for the H-components. The frequency domain fields are then used in the Field Equivalence principle described in the next section.

### 3.4.2. Field Equivalence principle

The Field Equivalence principle is illustrated in figure 3.8.

![Fig. 3.8: Field Equivalence principle](image)

The principle states that if a source radiates a set of electric and magnetic fields, then the same fields will be radiated if the original source is replaced by an imaginary conducting surface on which electric and magnetic currents, \( J \) and \( K \) respectively, exist. The electric and magnetic currents are given by:

\[ J = \hat{n} \times H \] (3.63)

\[ K = -\hat{n} \times E \] (3.64)
where $E$ and $H$ are the tangential components of the electric and magnetic fields in the frequency domain and $\mathbf{n}$ is the unit vector normal to the surface in the outwards direction. These surface current densities are used in conjunction with the vector potential radiation integrals

$$A = \frac{\mu}{4\pi} \int_{S'} J e^{-jkr} \frac{e^{-jkr}}{4\pi r} \, ds'$$  \hspace{1cm} (3.65)$$

$$F = \frac{\varepsilon}{4\pi} \int_{S'} K e^{-jkr} \frac{e^{-jkr}}{4\pi r} \, ds'$$  \hspace{1cm} (3.66)$$

where they represent the magnetic and electric vector potentials respectively. From these the radiated fields can be found. Equations (3.65) and (3.66) are only valid if the currents radiate into an unbounded space, implying that the medium inside and outside the surface $S$ must be the same. Only the tangential components of the fields are required on the surface, since taking the cross product of any normal component of the vector field on the surface with the vector normal to the surface will be zero.

The electric and magnetic vector potentials are computed using equations (3.65) and (3.66) where $k$ is the free space wave number, $J$ is the vector electric current distribution, $K$ is the vector magnetic current distribution and $r$ is the distance from the source point to the observation point.

From these vector potentials, the radiated fields can be calculated by substituting them into Maxwell’s Equations. Since the fields required are in the far-field, the spherical radial component of both the fields are considered to be negligible, when compared to the components in the $\theta$ and $\phi$ directions.

For the magnetic vector potential, the fields are given by the equation

$$E_A = -j\omega A$$  \hspace{1cm} (3.67)$$

If equation (3.65) is rewritten in terms of spherical components, the equations for the radiated electric field components are
\[ E_r = 0 \]  
\[ E_\theta = -j\omega A_\theta \]  
\[ E_\phi = -j\omega A_\phi \]  

The magnetic field components can be calculated directly from the electric field components above, and yield the following expressions:

\[ H_r = 0 \]  
\[ H_\theta = j\frac{\omega}{\eta} A_\phi = -\frac{E_\phi}{\eta} \]  
\[ H_\phi = -j\frac{\omega}{\eta} A_\theta = \frac{E_\theta}{\eta} \]  

where \( \eta \) is the intrinsic impedance of free space.

In a similar manner, the fields can be calculated using the electric vector potential. For this case, the dual of equation (3.39) is used, and the equations are as follows:

\[ H_r = 0 \]  
\[ H_\theta = -j\omega F_\theta \]  
\[ H_\phi = -j\omega F_\phi \]  
\[ E_r = 0 \]  
\[ E_\theta = -j\omega F_\theta = \eta H_\phi \]  
\[ E_\phi = j\omega F_\phi = -\eta H_\theta \]  

The total radiated vector electromagnetic fields are obtained by summing the contributions from both the electric and magnetic vector potentials.
3.5. Specification of 3-D FDTD Modelling Code

The 3-D FDTD code used for the simulations was developed at Queen Mary and Westfield College, London. The code was written in FORTRAN77. A detailed description of the code's development is given by Rayner [76].

A second order FDTD algorithm in a Cartesian co-ordinate system is implemented. The Courant stability criteria is employed to define the time stepping. The sources are excited by either a sinusoidal in the case of simulation at one frequency or a Gaussian pulse for wide band computations.

The code uses the first and second order Mur ABCs. Those ABCs were state of the art when the project started in 1995 and proofed to be sufficient for the problems solved within this study. A discussion of reducing reflections from the ABCs by implementing the Berenger PML [70] is given in chapter 4.

The near-field surface on which the electric and magnetic currents are computed is a cuboid. The time to frequency domain transformation is conducted as described in the previous section. The equivalence principle is employed to compute the far-fields at any given \( \theta, \phi \) position.

3.6. FDTD Code Verification

There are two different ways to verify the code. The easier one is to compare simulation results with analytical solutions. Since the half-wave dipole is used as the source for the corner reflector antenna and the analytical solution for this antenna is known the first verification is conducted using a half-wave dipole.

The second, and more complicated, way is to build and test the antenna under investigation. This is obviously more time consuming but the only way to verify the code for antennas, such as the corner reflector antenna, which have no analytical
solution. For this purpose two corner reflector antennas were built and measured in the Compact Antenna Test Range (CATR) of Queen Mary and Westfield College.

### 3.6.1. Analytical verification

The size of the Yee cells was set to \(\lambda/20\) and the domain consisted of 40x40x40 cells in the \(x, y, z\) directions respectively. The dipole was symmetrically positioned in the centre of the spatial domain. The antenna source point was modelled using two field points. The time step was taken as the maximum permitted by the Courant stability condition which for the chosen cell size equates to \(0.09622 \times 10^{-9}\)s. The excitation was a 1GHz sinusoid. This corresponds to 34.9 time steps per cycle for the exciting wave. The simulation was executed for 150 time steps allowing approximately 5 cycles of the input sinusoid for the near-field to reach steady state. The far-field radiation patterns in the E- and H-planes were calculated in \(1^\circ\) steps. The radiation patterns are shown in fig. 3.9, and 3.10.

![Fig. 3.9: E-plane radiation pattern for a half wave dipole](image)
The agreement is excellent over the full range of angles. The H-plane should be omnidirectional but the derivations are of the order of only 0.06dB (1.4%).

3.6.2. Measurement validation

As the subject of the study is the corner reflector antenna the measurement validation was done with two corner reflector antennas designed, manufactured and measured at Queen Mary and Westfield College.

3.6.2.1. Design of the corner reflector antennas

Two antennas were constructed and tested at 5GHz to verify the code by measurements. The dipole separation was varied from 30mm (0.5λ) to 75mm (1.25λ). This design resulted in two different scenarios for the diffraction from the edges which should have emphasised any possible problems. The apex angles were chosen so that the main lobe was in direction of boresight. A sketch of the two antennas is shown in fig. 3.11.
Fig. 3.11: Construction of the investigated antennas

The plates were made of brass and are 1mm ($\lambda/60$) thick. They are soldered at the apex. A hole was drilled to insert the dipole which was made using solid coaxial cable.

3.6.2.2. Measurement set up

The measurements were performed at 5GHz in the Compact Antenna Test Range (CATR) at Queen Mary and Westfield College, London. A plan of the CATR is shown in Fig. 3.12.

The walls and the ceiling of the chamber are lined with microwave absorbing material. The offset paraboloidal reflector consists of 18 panels. The reflector is illuminated by a feed horn positioned in the feed tower. For this measurements a C-Band horn of aperture dimensions 235mm x 175mm was used as a feed. This horn results in -8dB and -12dB edge illumination for the near and far edges of the reflector respectively. The phase centre of the feed horn was placed at the focus of the reflector.
in the feed tower to produce pseudo plane waves in the quiet zone, which is about 1m in size. The test antenna was mounted on a precision azimuth-over-elevation turntable, Orbit AL4373.1, positioned in the centre of the quiet zone. The precise movement of the turntable and all test and measurement equipment were controlled remotely by an HP workstation computer via an IEEE-488 bus (HPIB).

The range is thoroughly validated by many measurements at 10GHz. However, to make the dimensions of the test antennas large enough to handle, a frequency of 5GHz ($\lambda=6\text{cm}$) was chosen. The reflectivity level at this frequency is about -40dB.

The antenna itself was supported by the set up as shown in fig. 3.13. The material and design of the supports were optimised to avoid distraction of the fields as much as possible. Measurements show that the support structure changed the fields in the back region which accounts partly for discrepancy between prediction and measurement results.

It was possible to rotate the reflector so that the E- and H-planes were both measured using azimuth rotation. A measurement for almost 360° was possible using the turntable. The whole structure was wrapped in absorber material to minimise reflections.

As the Orbit turn-table is a very large piece of metal it is necessary to put as much space as possible between the turntable and the antenna. Therefore the antenna was mounted on a 2 foot dielectric pole. If the antenna was placed too close to the turntable, reflections from the turntable and the supporting structure would interfere with the radiation pattern. The turntable was covered with absorber material to further reduce the possible reflections that could occur. The dielectric support was chosen to limit the likelihood of interference. The dielectric support provided a shielding of the feed cable by fixing it to the rear end of the tube and then wrapping the tube and cable in aluminium foil and a thin layer of absorber material. The perspex tube was bored in two positions to allow for measuring the E- and H-plane patterns in azimuth.
Fig. 3.12: Compact antenna range
The theoretical design did not allow for the influence of the balun. The task of the balun is to make sure that the currents in both arms of the dipole are equal and that there is no residual current flowing back down the feed cable.

To build a balun so that it is operating correctly at the desired frequency is not easy. During the measurements it became obvious that a slightly unbalanced dipole influences the E-plane pattern of the antenna such that it is not symmetric. The coaxial balun is asymmetric as shown in Fig. 3.14.
Due to the connection of the inner and outer shield of the coaxial line the currents on both arms are equal. If the auxiliary line is $\lambda/4$ then currents cancel each other and there is no net-current flow on the outer shield.

Switching to the Bazooka balun described by Balanis [52], fig. 3.15, leads to a more symmetric pattern.

The principal of the bazooka balun is to produce a very high (ideally infinity) input impedance at the open end of the $\lambda/4$ shorted transmission line. Therefore there is no current on the outer shield. The symmetric structure of the bazooka balun is more likely to lead to a symmetric pattern.

Measurements performed during the PhD project highlighted the difficulties of manufacturing a balanced dipole at 5GHz. It is advisable to measure a dipole first on its own to make sure that the radiation in the E-plane is symmetric which should be the case for a balanced dipole. If the dipole is already mounted into a structure and is suspected of causing problems a change of frequency in the order of $\pm 5\%$ of the
operating frequency should give some indication on whether or not the problem is caused by the dipole.

3.6.2.3. Results for antenna 1

Antenna 1 has an apex angle of 60°, a dipole separation of 0.5\( \lambda \), the length of plates was 2\( \lambda \), and the width was 2\( \lambda \).

The details of the FDTD simulation are:

- **Grid size (x, y, z):** 60 60 56
- **Cell size (cubic):** \( \lambda/20 \)
- **Sources:** Two x-directed sinusoidal gap sources
- **Frequency:** 5GHz
- **Position of source (y, z):** (30, 20)
- **Position of apex (y, z):** (30, 10)
- **Time step:** 5.2ps
- **Number of time steps:** 450
- **Time steps per cycle:** 38.5
- **Number of cycles:** 11.7
- **NF-FF surface (x, y, z):** (8-52, 8-52, 8-48)

The measured H-plane pattern, fig. 3.16, is almost symmetric about zero degrees which indicates that there are no problems with reflections from the range. The shoulders at around 75° are at a level of -20dB. The -3dB beam-width is 33.4°. The comparison with the predicted result in the same figure shows an excellent agreement up to an angle of 120°.
The slight difference for greater angles are at such low level that they do not pose any doubt on the usefulness of the FDTD method. At these angles they effect of the supporting structure, re-radiation of the cable and the noise can distort the measured signal.

The measured and predicted E-plane radiation pattern can be seen in fig. 3.17

The measured radiation pattern is asymmetric about 0°. The reason is believed to be the non-perfect balanced dipole (see section 3.6.2.2 for details). The -3dB beam-width is 35.4°. The comparison with the predictions of the 3-D FDTD code obviously suffers from the effects of the unbalanced dipole.

The agreement is good for positive angles. The appearance of disagreement at low levels and wide angles are due to the reasons mentioned above.
3.6.2.4. Results for antenna 2

Antenna 1 has an apex angle of 65°, a dipole separation of 1.25λ, the length of plates was 2λ, and the width was 2λ.

The details of the FDTD simulation are:

Grid size (x, y, z): 60 64 54
Cell size (cubic): λ/20
Sources: Two x-directed sinusoidal gap sources
Frequency: 5GHz
Position of source (y, z): (32, 35)
Position of apex (y, z): (32, 10)
Time step: 5.2ps
Number of time steps: 450

Time steps per cycle: 38.5

Number of cycles: 11.7

NF-FF surface (x, y, z): (8-52, 8-56, 8-46)

The measured H-plane radiation pattern is shown in fig. 3.18. The result is again symmetric around zero degrees. The -3dB beam-width is 25.3°. The first side lobe is at 45° with a level of -8dB. This is much higher than for the closer dipole separation in antenna 1.

![Fig. 3.18: Comparison of measured and predicted pattern of H-plane for φ=65° and ρ₀=1.25λ](image)

The comparison with the results of the 3-D FDTD code shows an excellent agreement over the whole range of angles.

The measured E-plane radiation pattern is shown in fig. 3.19. The asymmetry is smaller since this time the Bazooka balun was used. The -3dB beam-width is 31.9°.
The comparison shows that the agreement is better than for the E-plane pattern of antenna 1. This backs up the argument that the unbalanced dipole caused the problems.

3.6.3. Summary

The predicted pattern are in excellent agreement with the measured and analytically computed results. The differences in the case of the measured results are believed to be caused by the unbalanced dipoles. Discrepancies at wide angles are due to the influence of the supporting structure, re-radiation of the cable and noise which falsify the measured signal.

The conclusion can be drawn that the 3-D FDTD method is well suited to predict radiation patterns of corner reflector antennas in E- and H-planes. Furthermore, a separation between the structure and the ABCs of half a wavelength (10 cells) is found to be sufficient to achieve a good agreement.
3.7. Development of a 2-D FDTD code

There are several reasons why a 2-D FDTD simulation of a corner reflector antenna is advantageous over a 3-D FDTD simulation. As pointed out in chapter 1 one of the main application for the corner reflector antenna is in mobile communication systems as a base station antenna. In this scenario the shaping of the beam in the H-plane is achieved by variation of the apex angle, plate length and dipole position whereas the E-plane is shaped by the array factor of several corner reflector antennas on top of each other. Therefore, it would be beneficial if the influence of the finite width of the corner reflector antenna plates did not show up in the simulation results.

This is achieved by employing a two dimensional model of the reflector plates, assuming that they are infinite in the direction parallel to the dipole. Neglecting the third dimension also reduces both the memory requirement and the computational effort. Consequently, the use of a two dimensional version of the FDTD method allows parametric studies which show the shaping of the H-plane pattern by changing the length of the plates, the dipole position, and the apex angle. The possibilities of near-field cuts and inclusion of dielectric material into the model can still be done without the third dimension.

A 2-D FDTD code was developed and implemented by the author. The programming language chosen is FORTRAN90 which offers very sophisticated array functions and has the advantage of dynamic memory allocation over FORTRAN77. This is important for FDTD implementations since the change of the spatial domain results in a change of array sizes. With FORTRAN77 the size of the arrays has to be specified at compilation time. Dynamic memory allocation enables the size of arrays to be changed by changing the input data. Re-compiling the code when the spatial domain is changed can consequently be avoided. It is also possible to allocate and de-allocate memory for arrays at any time during the runtime which offers an additional opportunity to reduce the memory requirement. The availability of FORTRAN90 compilers on mainframes is still restricted but more computer centres are deciding to
move towards FORTRAN90. The compiler used by the author to develop the code is the Salford FTN90 for Windows. The code was developed on a 133MHz Pentium PC. It is possible to use virtual memory to increase the capability of the PC to handle bigger problems but this increases the runtime very substantially. The available RAM was 32MByte which is sufficient to solve problems with a grid size of up to 700*700 cells without using virtual memory.

3.7.1. The FDTD spatial domain

If the corner reflector antenna is arranged such that the dipole is along the z-axis it means that there is no change in the z-direction and therefore:

\[ \frac{\partial E_z}{\partial z} = 0 \] \hspace{1cm} (3.80)

\[ \frac{\partial E_x}{\partial z} = 0 \] \hspace{1cm} (3.81)

If (3.80) and (3.81) are substituted into equations (3.3)-(3.8) they reduce to:

\[ \frac{\partial B_z}{\partial t} = \frac{\partial E_z}{\partial y} \] \hspace{1cm} (3.82)

\[ \frac{\partial B_y}{\partial t} = \frac{\partial E_z}{\partial x} \] \hspace{1cm} (3.83)

\[ \frac{\partial B_x}{\partial t} = 0 \] \hspace{1cm} (3.84)

\[ \frac{\partial D_x}{\partial t} = 0 \] \hspace{1cm} (3.85)

\[ \frac{\partial D_y}{\partial t} = 0 \] \hspace{1cm} (3.86)

\[ \frac{\partial D_z}{\partial t} = \frac{\partial H_x}{\partial x} - \frac{\partial H_y}{\partial y} - J_z \] \hspace{1cm} (3.87)

The equations (3.82)-(3.87) describe the propagation of waves in free space where only one electric field component \( E_z \) is present. This is called the TM case. For definition of the field components see fig. 3.3. With the existence of \( E_z \) it is possible
to model an electric source. Due to the definition the size of structures in the z-direction is infinite and therefore the source is an infinitely long wire. The perfectly conducting walls of the reflector are represented by forcing the corresponding $E_z$ field points to zero which results in infinite reflector plates as intended. Now it is only necessary to set the tangential electric fields to zero since the normal components of the magnetic fields will be forced to zero automatically by the finite difference equations.

There are only three field components which have to be computed: $E_z$, $H_x$, $H_y$. Thus it is obvious that the computational effort is reduced. At the same time the formulations for each of the field components is simpler than in the 3-D case.

Equation (3.10) reduces to:

$$
\frac{B_x^{n+\frac{1}{2}}(i, j + \frac{1}{2}, k + \frac{1}{2}) - B_x^{n-\frac{1}{2}}(i, j + \frac{1}{2}, k + \frac{1}{2})}{\Delta t} = \frac{E_z^n(i, j + 1, k + \frac{1}{2}) - E_z^n(i, j, k + \frac{1}{2})}{\Delta y}
$$

(3.88)

The spatial domain is no longer a 3-D cuboid but a cut in the x-y plane. The representation of the fields can be seen in fig. 3.20.

Fig. 3.20: 2-D Yee cell grid for the TM case
The spatial domain reduces from a cuboid to a plane. The computational effort is therefore reduced for three reasons:

- Only three field components have to be computed.
- The equation for each component is simpler than in the 3-D formulation.
- The spatial domain is two dimensional and therefore there are fewer field points to compute.

The numerical stability and dispersion are similar to the three dimensional case and therefore not discussed again.

The Courant stability criteria for two dimensions reduces to:

\[
\Delta t \leq \frac{v^{-1}}{\sqrt{\left(\frac{1}{(\Delta x)^2} + \frac{1}{(\Delta y)^2}\right)}}
\]  
(3.89)

For the case of square cells (i.e. \(\Delta x = \Delta y = \Delta\) (3.89) can be written as:

\[
\Delta t \leq \frac{\Delta}{v\sqrt{2}}
\]  
(3.90)

### 3.7.2. The Absorbing Boundary Conditions

The Mur ABCs which are described earlier in this chapter are employed since at the time the PhD project started the PML by Berenger was not established. The first order Mur boundary condition for the 2-D case is applied at the four corners of the grid and is given by the first three terms of (3.18). The second order Mur ABC is applied elsewhere at the domain boundaries using (3.18).

### 3.7.3. Far-field radiation pattern calculation

The far-field radiation pattern based on the fields gained by the 2-D FDTD is restricted to the H-plane pattern (\(\phi\)-cut). Since the structure is assumed to be infinite
in the z-direction there is no change of the fields in the E-plane (θ-cut). The equivalence principle for the 3-D case, described earlier in this chapter, can still be employed. The chosen arbitrary surface is a rectangle as shown in fig. 3.21. The origin of the co-ordinate system for the far-field is not at the same position as the origin for co-ordinate system of the field components. All far-field positions are in terms of the co-ordinate system with the origin in the middle of the FDTD grid. Since the fields are all in the x-y plane the far-field is computed in the so called H-plane.

The time- to frequency domain conversion of the currents is performed in exactly the same way as for the 3-D FDTD, see equations (3.61)-(3.62) and fig. 3.7.

Fig. 3.21: Near-field surface for the 2-D FDTD
The starting point for the termination of the radiated E-field are the magnetic and electric vector potentials given by Balanis [52]:

\[
A = \frac{\mu}{4\pi} \int_{S'} J \frac{e^{-jkr}}{r} \, ds' \equiv \frac{\mu e^{-jkr}}{4\pi r} N
\]  \hspace{1cm} (3.91)

\[
N = \int_{S'} J e^{ikr \cos \psi} \, ds'
\]  \hspace{1cm} (3.92)

\[
F = \frac{E}{4\pi} \int_{S'} K \frac{e^{-jkr}}{r} \, ds' \equiv \frac{E e^{-jkr}}{4\pi r} L
\]  \hspace{1cm} (3.93)

\[
L = \int_{S'} K e^{ikr \cos \psi} \, ds'
\]  \hspace{1cm} (3.94)

where:

\( N \): Fourier transform of the electric currents  
\( L \): Fourier transform of the magnetic currents  
\( r' \cos \psi \): Differential path

In the case of two dimensions the surface integral reduces to a ring integral. The Fourier transform of the electric and magnetic currents for the 2-D case are:

\[
L = \oint [-K_x \sin \phi + K_y \cos \phi] * e^{jk'(r \cos \psi)} \, ds'
\]  \hspace{1cm} (3.95)

\[
N = \oint [-J_z] * e^{jk'(r \cos \psi)} \, ds'
\]  \hspace{1cm} (3.96)

\[
r' \cos \psi = x' \cos \phi + y' \sin \phi
\]  \hspace{1cm} (3.97)

The theta component of the electric far-field is:

\[
E_\theta = L_\theta + \eta N_\theta
\]  \hspace{1cm} (3.98)

Note, there is no \( E\phi \) component in the TM case. Combining (3.95) and (3.96) with (3.98) leads to:

\[
E_\theta = \oint \left[-K_x \sin \phi + K_y \cos \phi - \eta J_z \right] * e^{jk'(r' \cos \psi + y' \sin \phi)} \, ds'
\]  \hspace{1cm} (3.99)

The far-field of the antenna can be computed at any given \( \phi \) angle.
3.7.4. Considerations for parametric studies.

In order to conduct a parametric study it is necessary to vary one parameter of the investigated antenna and keep all other parameters fixed. If more than one parameter is varied at the same time it is not possible to trace back which parameter caused the change in the investigated quantity. There are several parameters for the corner reflector antenna which can be varied.

In the two dimensional case those parameters are:

- Plate length \( l \).
- Apex angle \( \psi \).
- Source separation \( \rho_0 \).

In order to conduct a comprehensive study a large number of simulations are necessary. The number of simulations can be limited by classifying which possible combinations of parameters are sensible. The classification is based on the results for the infinite corner reflector antenna using Wait's method and experience. To avoid confusion caused by the amount of data and to reduce the possibility of human error the procedure of conducting the parametric study has to be carefully considered.

For the parametric studies which were conducted during the PhD project a two staged process was employed.

As a first step, a file was created which consists of the information needed to run a FDTD simulation for the particular structure. These are:

- Grid dimension
- Cell size
- Frequency of simulation
- Number of time steps
- Position of near-field to far-field surface
• Apex angle
• Length of plates
• Position of source
• Position of the apex

Each of those files has an individual file name. The name was composed of the apex angle, source distance, and plate length. To mark the files as input file for the FDTD simulation the suffix '_in' was added to the name. A file for a corner reflector antenna with an apex angle of 60°, source distance to apex of 0.5λ, and plate length of 2λ was named '60_0.5_2.0_in'.

Those input files were generated automatically by software which needed the following input parameters:
• Cell size
• Start, stop and step size of apex angles
• Start, stop and step size of plate lengths
• Start, stop and step size of dipole positions
• Number of time steps

For each combination the program produced an input file as described.

The grid size and near-field to far-field surface were defined such that the reflector is 10 cells from the ABCs and the near-field to far-field surface was 2 cells away from the structure. The position of the ABC and the near-field to far-field surface was chosen based on experience with the code.

Rather than storing the actual field points where the boundary condition for the reflector plates has to be enforced in the FDTD grid they were computed at the beginning of the FDTD simulation. This procedure kept the size of the input files small and therefore saved disk space. The field points where the \( E_z \) values had been
forced to zero were written into a file to enable checking of the automatic generation of the reflector.

The frequency and cell size were always kept at 5GHz and \( \lambda/20 \) respectively if not stated otherwise. The number of time steps is a crucial figure in terms of run time. Obviously, it is not feasible to check for every given simulation if the fields have stabilised. The number of time steps was set to values which were based on experience with the code. To ensure that the fields had stabilised and therefore the results correct, samples were checked to make sure that these numbers were sufficiently high to allow the fields to converge.

The results were stored in files with similar names to the input file, only the suffix was changed from '_in' to '_out'. Therefore, for the previous example the filename was '60_0.5_2.0_out'.

Each output file stores all relevant data of each FDTD simulation:

- Definition of the corner reflector antenna (i.e. plate length \( l \), apex angle \( \psi \), source separation \( \rho_0 \))
- Position of the source
- Grid size
- Frequency
- Cell size
- Number of time steps
- Runtime
- H-plane radiation pattern
- Angle of maximum radiation
- -3-dB beam-width
- Level of the sidelobes,
• Position of the sidelobes,
• Number of sidelobes
• Level of nulls
• Positions of nulls
• Front-to-back ratio
• Aperture efficiency
• H-plane directivity (see chapter 4).

This scheme was very useful in handling the large quantity of data. The unique filename made it very easy to find results. The sorting algorithms of UNIX editors, such as 'awk', were used to filter certain information.

The automatic generation of the input files reduced the chance of human error and consequently less checks were necessary. Even though it took quite a long time to develop such a scheme the benefit of a clear order in the computed data and the guarantee of a consistent input file generation made the effort worthwhile.

3.7.5. Validation of the Code

No validation of the code can be obtained with measurement results since no structures exist which are infinite in one dimension. In this instance the validation is based on analytical data. Chapter 5 shows that for structures where the influence of one dimension can be almost neglected the results of 2-D FDTD matches well with measurement results. However, these results are not considered here to validate the code since it is always questionable whether the influence of the third dimension can be completely neglected. As a test structure an infinite line source with an omnidirectional H-plane pattern is chosen. Any distortion from the ideal pattern is due either to reflections of the boundaries or numerical dispersion.
The line source is placed at the centre of a 20*20 cells FDTD grid. The cell size is \( \lambda/20 \). Therefore the distance between the excited \( E_z \) field point and the ABCs is half a wavelength. The near-field to far-field surface is two cells from the antenna and the number of time steps is 400. This setup is identical with the distances for the simulation of the corner reflector antenna. Since the line source radiates omnidirectional outgoing waves hit the ABCs at all possible incidence angles.

The use of the first order Mur ABC in the corners of the FDTD grid causes some reflection as expected and, therefore, the radiation pattern differs at 45°, 135°, 225°, and 315° from the ideal pattern. The deviation is only 0.001dB. Two conclusions are drawn from this result. The developed code is working and a distance of half a wavelength between the radiator and the ABCs is sufficient to reduce the reflections to an acceptable level.

### 3.7.6. Summary

The two dimensional FDTD method offers the opportunity to investigate the shaping of the H-plane far-field pattern without the influence of finite width of the reflector plates. At the same time the computational effort is reduced very significantly since the number of field components is halved and the equation for each component is simpler compared to the three dimensional case. Thus, the 2-D FDTD code was used to conduct the parametric studies which are described in chapter 4-6.

The chosen programming language FORTRAN90 is ideally suited for implementations of the FDTD algorithm. Memory requirements can be optimised by using dynamic memory allocation. The biggest advantage over FORTRAN77, however, is that the code does not have to be re-compiled if the size of the FDTD grid is changed.
3.8. Other techniques

Several other techniques exist as table 3.1 already indicates which can be employed to predict the radiation from corner reflector antennas. The Method of Moments (MM) and GTD are described briefly in the following section since they are the two methods that can easily be employed to solve the problem. References giving detailed descriptions are given below.

Both methods have been successfully used for modelling corner reflector antennas. Elkamchouchi et al [15] demonstrated the use of MM for a three dimensional corner reflector antenna. Radiation patterns for corner reflector antennas are computed with the help of GTD in the paper by Zhang et al [36]. Despite the fact that these methods produce good results the possibility of near-field cuts, the observation of transient wave propagation, and the ease with which dielectric material can be implemented makes FDTD the preferred method.

3.8.1. Method of Moments (MM)

The Method of Moments was developed in the 1960's by Kantorovich and Akilov, [77]. It was expanded to deal with electromagnetic problems in 1968 by Harrington [78] and is widely used all over the world. The availability of software for both desktop and mainframe computers supports the implementation of the method.

The basics of the Method of Moments is an integral equation approach. This can be applied to determine the electromagnetic response of antennas and scatterers. The complexity of the equation increases if the problem also involves dielectric. The general proceedings are the same with or without dielectric material in the model. The structure under investigation is split up into small parts called segments. To determine the induced current distribution flowing in each of the segments the matrix equation:

\[ I_{[i]} = [Z_{[i,j]}]^{-1}[V_{[i]}] \]  

(3.100)
where

\[
\begin{align*}
[I_{[i]}) & : \text{column matrix of resultant induced surface current on each segment} \\
[Z_{[i,j]}) & : \text{square matrix of self or mutual impedance between each segment} \\
[V_{[i]}) & : \text{column matrix of the voltage excitation at each segment}
\end{align*}
\]

has to be solved. This process requires the solution of one of two integral equations. One is known as the Electric Field Integral Equation (EFIE):

\[
E(r) = \frac{-j\eta}{4\pi k} \int \int J(r') \cdot G(r, r') dV'
\]  \hspace{1cm} (3.101)

where:

\[
\begin{align*}
G(r, r') & : \text{Greens function operator} \\
J(r') & : \text{electric current density} \\
k & : \text{free space wave number} \\
\eta & : \text{intrinsic impedance of free space}
\end{align*}
\]

The other equation is known as the Magnetic Field Integral Equation (MFIE):

\[
H_s(r) = \frac{1}{4\pi} \int \int J_s(r') \times \nabla g(r, r') dA'
\]  \hspace{1cm} (3.102)

where:

\[
\begin{align*}
g(r, r') & : \text{Greens function operator} \\
J_s(r') & : \text{electric surface current density}
\end{align*}
\]

The segmentation can be achieved in two ways. The first approach is the 'wire grid' modelling. This technique uses short wire sections to model the structure. The other technique is the 'surface patch' modelling. This method sub-divides the structure into small patches. The solution of the Green's function is different for both cases and therefore the EFIE (3.101) is better for the 'wire grid' model and MFIE (3.102) fails for thin wires but is suited for problems with smooth surfaces which can be segmented using the 'surface patch' model. For many structures is may be necessary to
use a combination of the two modelling techniques. For such problems equation (3.101) and (3.102) may be coupled as proposed by Albertsen et al [79].

The method can be applied to scattering and radiation problems. The choice of excitation function is dependent upon the type of problem being considered.

The induced currents flowing on the structure are found by solving the matrix equations (3.99). From the resulting surface current distribution all of the required antenna characteristics including input impedance and radiated fields can be calculated using suitable formulations.

The drawback of the Moment Method is that it requires high computation power when the structure is even moderately large electrically.

3.8.2. Geometrical Theory of Diffraction (GTD)

The radiated field of an antenna consists of electromagnetic waves which are similar to light waves but at a different wavelength. Visual light has wavelengths in the range of 380nm - 780nm whereas the wavelength for a 5GHz wave is 6cm. Despite this difference, Geometric Optics (GO) provides solutions to scattering problems and is especially useful for electrically large structures. A detailed description of the method is given by Rudge et al [80] and James [81].

Fig. 3.22: Shadow boundaries
As the energy is constrained, in homogenous medium, to travel in straight lines it is not possible to calculate the fields around the back of the scatterer, see fig. 3.22.

Therefore, it is necessary to expand GO by the 'Geometrical Theory of Diffraction' (GTD) which takes into account the diffraction on the edges. The expansion was first introduced by Keller [82].

Since GTD assumes localised diffraction it is not suited for small reflectors.

3.9. Summary

A detailed description of the FDTD method has been given. Analytical and measurement data are compared with predictions of the three dimensional FDTD code which was developed at Queen Mary and Westfield College. The good agreement of the results validated the code and at the same time proved that the FDTD method is suitable to model corner reflector antennas. The specifications of the code are given so that it is possible to reproduce the results. The 2-D FDTD method can be employed to avoid the influence of finite size of the reflector plates. A code implementing the 2-D FDTD algorithm was developed by the author. The computational effort for the 2-D FDTD is such that parametric studies can be handled with a 133MHz Pentium PC. The chosen programming language, FORTRAN90, offers the possibility of dynamic memory allocation which reduces the memory requirements of the method. It is also possible to change the grid size of the simulation without re-compiling the code as it would be necessary if FORTRAN77 is used.

An outline of the method used for the parametric studies has been presented. It is shown that the simulations can be automated so the user only has to specify the parameters of the corner reflector antenna. This concept proved to be very helpful in order to analyse the data.

To contrast the different approach of the FDTD method two other methods are briefly outlined. Even though both methods are capable of simulating the corner reflector antenna the possibility of visualising the near-fields and the ease with which dielectric
materials can be included in the model using the FDTD method caused the author to employ it as the modelling tool in the reminder of the study.
4. SOLID CORNER REFLECTOR ANTENNA

The parametric studies conducted to date on the corner reflector antenna were mainly those of Harris [6] and Cottony and Wilson [8], [9]. They were based on measurements and hence were very expensive and time consuming to do. Each antenna had to be built and measured. It was therefore not possible to cover the same range of corner reflector antenna designs as in this study. This chapter represents to the best knowledge of the author the first study on infinite and finite corner reflector antennas which covers all apex angles from 30° to 180°. Additionally it is also the first time that a comparison between the data for the infinite sized corner reflector antenna and finite corner reflector antenna is used to establish which length of plates is necessary so that the H-plane radiation pattern for the two cases converge.

The objective of this chapter is to investigate the influence of the geometric parameters of a corner reflector antenna on its radiation characteristic. The parameters which were varied are:

- Dipole distance from apex
- Apex angle
- Length and width of the reflector plates

In order to assess the influence of the parameters the 2-D and 3-D FDTD method as described in chapter 2 were employed. Results computed using these methods were compared with those computed by using Wait's method for the infinite sized reflector plates. To establish the influence of finite long reflector plates by keeping the width of the plates infinite the 2-D FDTD method was used which assumes infinite wide reflector plates.

The 2-D FDTD is restricted to the H-plane so it was necessary to adapt the quantities 'directivity' and 'aperture efficiency' so that they could be applied to two dimensional
problems. For the directivity a new quantity was defined. The so called 'H-plane directivity' corresponds to the directivity in three dimensions but is purely based on
the radiation pattern in the H-plane. By defining this quantity it is possible to compare
the ability of different corner reflector antenna designs to direct energy in the desired
direction. The two dimensional aperture efficiency is a measure of how efficient the
corner reflector antenna illuminates a line aperture.

The range of corner reflector antenna designs covered in this study is:

- **Apex angle:** 30° - 180°
- **Plate length:** 2λ - 15λ and infinite
- **Plate width:** 2λ - 4λ and infinite
- **Dipole position:** 0.5λ - 1.5λ

Those parameters were chosen to cover all designs which gave a compact corner reflector antenna with a H-plane radiation pattern suitable for base station antennas of mobile communication systems.

Within this chapter a detailed discussion on how to model the corner reflector antenna using the FDTD method is given. Special attention is paid to the apex region of the corner reflector, the determination of the dipole position, and the influence of the stair-case approximation of the reflector plates.

The influence of the Mur ABCs is assessed by comparing the H-plane radiation pattern with a reference gained by increasing the grid such that there are no reflections from the ABCs which reach the near-field to far-field surface during run-time. The superior performance of Berenger ABC is shown by comparing results using this ABC with the previous two results.

Based on the parametric study the novel idea of a corner reflector antenna with variable apex angle was developed. Such an antenna can be used to adapt base station
antennas of mobile communication networks to varying traffic in cells. A practical design together with simulation- and measurement results for this antenna are given.

At the end of the chapter the possibility is discussed of replacing the dipole as a feed for the corner reflector antenna by directive feeds or to introduce additional parasitic elements in order to increase directivity and bandwidth.

4.1. Finite difference time domain method applied for corner reflector antenna

The FDTD method has been used for a variety of antennas. Thiele and Taflove [83] modelled Vivaldi slot antennas and arrays. Penny and Luebbers [84] computed the input impedance, radiation pattern, and radar cross section of spiral antennas using FDTD. Reinex and Jecko [85] used FDTD for analysing microstrip patch antennas, Rayner et al [86] investigated the short back fire antenna. Even though the method is now widely used for several years there are still a number of antennas which have not been modelled using FDTD. To the knowledge of the author one of those structures is the corner reflector antenna. Whenever a new structure is investigated it is necessary to establish in which form the method has to be applied. A suitable co-ordinate system and how to map the corner reflector onto the grid has to be established. For the corner reflector antenna the structure is built completely of perfect conductors (PEC) and therefore no dielectric material has to be included into the model and that is why the implementation of material other than PEC is not discussed in this section. The H-plane radiation patterns of two corner reflector antennas employing Mur ABCs or Berenger ABCs in two dimensions are presented to assess the influence of the reflections from the ABCs on the pattern. The results are compared with a reference gained by increasing the size of the FDTD grid such that no reflections from the ABCs reach the near-field to far-field surface during the runtime.
4.1.1. Modelling the reflector

The FDTD algorithm implemented in the code is second-order accurate as shown in chapter 3. This error is only associated with the differential operators. Additional errors are encountered when the boundary conditions cannot be enforced directly on the boundary, but rather on an auxiliary boundary, which is a staircase approximation. For all apex angles other than $90^\circ$ and $180^\circ$ the corner reflector has to be mapped onto the Cartesian co-ordinate system. This yields a serrated surface for the reflector plates. The serration of the plates could be avoided if a conformal grid was used as described by Taflove [87].

A relatively simple method to assess the influence of the serration is to change the way the plates are modelled. In fig. 4.1 the auxiliary boundary of a corner reflector is shown with an apex angle of $60^\circ$ and a plate length of $2\lambda$ where the cell size is $\lambda/20$. The corner reflector antenna is aligned that way so that it radiates in the z-direction which is important for the definition of co- and cross-polar. The model also applies for 2-D and 3-D simulation, the only difference is that in the 3-D case the plates are finite in the x-direction. Each plate subtends a $30^\circ$ angle with the z-axis. In fig. 4.2 the plates of the same corner reflector were arranged differently. One plate is now flat along the z-axis. The other plate forms a $60^\circ$ angle with the z-axis. Therefore, the plates of the corner reflector can be modelled in three different ways with varying degree of serration.

The waves are reflected differently in the two cases. If the serration of the plates influences the computed radiation pattern, it will show in a direct comparison between patterns computed using the model where both plates are serrated and the model in which only one plate is serrated.
Both scenarios were modelled using 2-D FDTD. The H-plane radiation patterns are shown in fig. 4.3. The figure has three curves. The first curve is called 'Both plates serrated' and represents the H-plane from $0^\circ$ to $180^\circ$ for the case where the corner
reflector was modelled with both plates serrated as shown in fig. 4.1. In this case the H-plane between 0° and 180° is the same as from 0° to -180°. The second curve is entitled 'Flat plate half pattern'. It represents the H-plane pattern from 0° to -180° when the corner reflector is modelled such that one plate is aligned along the z-axis as shown in fig. 4.2. The last curve represents the H-plane pattern of this case for angles from 0° to 180° and is called 'Serrated plate half pattern'. In order to compare all three curves the H-plane for negative angles is shifted to positive values. The zero degree direction in both cases is from the apex towards the dipole which is placed at the bisector of the apex angle.

![Graph showing H-plane pattern comparison](image)

**Fig. 4.3: Comparison of 2-D FDTD H-plane pattern for θ=60° l=2λ, ρ₀=0.5λ and different reflector models**

The results for the H-plane radiation patterns are shown in fig 4.3. The comparison between the three curves shows that the difference is small. All three curves are on top of each other up to an angle of 100°. The discrepancy for greater angles occurs below -35dB. One reason for the disagreement is that both the near-field to far-field surfaces are not in the same position relative to the reflector plates of the two different
models. The surface is of rectangular shape in the y-z plane and is kept 2 cells from the extreme points of the corner reflector in each direction. The extreme points are the apex and the ends of the two plates. Additionally the distance between the ABCs and the structure is different in both cases which also accounts for some of the difference in the results.

The serration can be avoided for the special cases of corner reflector with 90° and 180°. A 90° corner reflector antenna was investigated to further assess the influence of the serration. Similar to the previous case two different ways of modelling the plates were simulated. The length of the plates was again 2λ.

The first model was such that both plates were serrated with a 45° angle to the z-axis (similar to what is shown in fig. 4.1 for an apex angle of 60°). In the second model both plates were aligned along the axis. One along the y-axis and the other along the z-axis. Both models of the corner reflector were simulated using 2-D FDTD. The H-plane radiation patterns are shown in fig. 4.4.

![Graph showing comparison of 2-D FDTD H-plane pattern for different reflector models.](image)

**Fig. 4.4:** Comparison of 2-D FDTD H-plane pattern for $\psi=90^\circ$, $l=2\lambda$, $p_0=0.5\lambda$ and different reflector models.
In the figure are two curves. The first curve is called 'Both plates serrated' and represents the H-plane radiation pattern from 0° to 180° for the case where the corner reflector was modelled with both plates serrated. In this case the H-plane between 0° and 180° is the same as from 0° to -180°. The second curve is entitled 'Flat plates'. Since both plates are now along the axis the H-plane radiation pattern is the same for positive and negative angles.

For the 2-D FDTD simulation shown in fig. 4.4 the two curves are on top of each other up to an angle of 120°. There is only a slight discrepancy further out at the levels well below -40dB.

It can be concluded that the serration of the plates has no significant influence on the H-plane radiation pattern of the corner reflector antenna.

Another important question is how to model the apex correctly. There are two possibilities how to model the apex which are indicated in fig. 4.5. Firstly, the apex can be modelled pointed. The dipole to apex distance $\rho_o$ is measured from cell number 10 in z-direction. Therefore, a source half a wavelength away from the apex had to be placed at cell position 20 in z-direction assuming a cell size of $\lambda/20$.

The second possibility is to model the apex flat. According to fig. 4.5 the reflector would start at the z-position 11 and the pointed extension towards cell position 10 would be omitted. A source half a wavelength away from the apex in this instance had to be placed at cell position 21 in z-direction.

To establish which is the correct option it is necessary to compare the two for a corner reflector antenna where the pattern is very sensitive to the position of the source. This is not so much the case when peak directivity is at boresight as it is when destructive interference produces a null in this direction.
The H-plane directivity at boresight for a $\psi=60^\circ$ corner reflector antenna where the apex was modelled pointed is shown in fig. 4.6 with plate lengths of $10\lambda$ and dipole positions from $0.6\lambda$ to $1.05\lambda$ in $0.05\lambda$ steps. The length of the plates were chosen such that the influence of the finite size of the reflector does not influence the position of the minimum directivity.

The H-plane directivity has a sharp drop if the dipole position is changed from $0.95\lambda$ to $1.00\lambda$ and rises sharply for a dipole position of $1.05\lambda$. This offers the possibility to establish which of the two apex model is correct. The flat shape of the curve seen at shorter dipole distances continues to $\rho_0=0.1\lambda$. This underlines the difficulty in establishing the correct model of the reflector for dipole positions at which maximum directivity is at boresight. On the other hand it shows that the position is not crucial if set to a value where the curve is flat. Later in the chapter this will be of interest when the bandwidth of the antenna is discussed.
If the apex was modelled flat the dip in the H-plane directivity curve was maintained but the dip occurred at $\rho_0=1.05\lambda$. According to Klopfenstein's expression for the directivity of an infinite size corner reflector antenna the dip in H-plane directivity has to be at $\rho_0=1.00\lambda$.

It is interesting to investigate how a drop of 9.5dB for the H-plane directivity at boresight can be caused by moving the dipole by 0.05$\lambda$. Firstly, the H-plane radiation pattern for the three dipole positions are given in fig. 4.7.

One of the strengths of FDTD is that near-field cuts can be computed to visualise the fields within the spatial domain. The near-field cuts in figures 4.8 - 4.13 show the magnitude of $E_x$ within the spatial domain of the 2-D FDTD simulation. The grid size was 194 cells in $z$-direction and 222 cells in $y$-direction. The source was at $y$-position 111 and $z$-position 29, 30, 31 for $\rho_0=0.95\lambda$, $\rho_0=1.00\lambda$, and $\rho_0=1.05\lambda$, respectively. The brighter colours indicate higher intensity. The position of the apex is $z=10$, 

**Fig. 4.6: H-plane directivity at boresight for pointed apex model with different dipole positions and $\psi=60^\circ$, $l=10\lambda$.**
y=111. One plate reaches from the apex to $z=184$, $y=10$ and the other plate to the same $z$-position and $y=212$. They can be located in the near-field cuts at the border between the bright and purple coloured cuts. The scale for the intensity was chosen such that it is the same for all three cases to enable comparison between the near-field. For each corner reflector antenna the near-field cut at the aperture, $z=184$, is given in a graph in order to provide a detailed presentation of the fields at the position where they leave the reflector.

![Graph](image)

**Fig. 4.7: H-plane pattern for different dipole positions $\psi=60^\circ$, $l=10\lambda$.**

The radiation pattern for the closest dipole position, $\rho_0=0.95\lambda$, exhibits a broad main beam with a -3dB beam-width of $37^\circ$. There is no side lobe or shoulder. Analysing the near-field cut for this dipole position in fig. 4.8 reveals that the omnidirectional radiation from the line source is directed by the reflector towards the aperture without any sign of destructive interference. The main power is directed along boresight and the intensity decreases towards the plates. The aperture illumination is given in fig. 4.9. The field distribution consists of one lobe with the centre at the middle of the aperture.
Fig. 4.8: Near-field cut for $\psi=60^\circ$ $l=w=2\lambda$ $\rho_0=0.95\lambda$.

Fig. 4.9: Near-field cut at aperture for $\psi=60^\circ$ $l=w=2\lambda$ $\rho_0=0.95\lambda$. 
Fig. 4.10: Near-field cut for \( \psi=60^\circ \ l=2\lambda \ \rho_0=1.0\lambda \).

Fig. 4.11: Near-field cut at aperture for \( \psi=60^\circ \ l=2\lambda \ \rho_0=1.0\lambda \).
Fig. 4.12: Near-field cut for $\psi=60^\circ$, $l=w=2\lambda$, $\rho_0=1.05\lambda$.

Fig. 4.13: Near-field cut at aperture for $\psi=60^\circ$, $l=w=2\lambda$, $\rho_0=1.05\lambda$. 
Moving the dipole by 0.05\(\lambda\) away from the dipole so that \(r_0=1.00\lambda\) yields to a H-plane radiation pattern shown in fig. 4.7. The dip at boresight is 5.8dB and the main beam is at 14\(^\circ\). Between 14\(^\circ\) and 120\(^\circ\) the curve is similar to the one for the smaller dipole separation but at an about 4dB higher level. In the rear region the two curves converge even further. The near-field cut in fig. 4.10 shows a significant change to the previous case. Destructive interference tails the fields at \(z=45\). The fields along the bisection die out due to destructive interference and two lobes along the plates built up in the near-field. The illumination of the aperture shown in fig. 4.11 exhibits a lower level for the magnitude of \(E_x\) as in the previous example. The dip in the magnitude is in the middle of the aperture.

The H-plane radiation pattern for the dipole position furthest away from the apex with \(r_0=1.05\lambda\) is shown in fig. 4.7. The main beam is narrow with a -3dB beam-width of 14\(^\circ\). The side lobe is at 23\(^\circ\) at a level of -14.2dB. From 15\(^\circ\) till 120\(^\circ\) the shape of the curve is similar to the previous two but 1dB below the first curve. Analysing the near-field cut in fig. 4.12 shows that a destructive interference cancel out the fields at the bisection at \(x\)-position 50. Three lobes built up and cease out towards the aperture. Additionally to the two lobes close to the plates there is another lobe at the bisection line. Fig. 4.13 shows the aperture illumination for this corner reflector antenna. The magnitude of \(E_x\) is in the same order as for a dipole position of \(r_0=1.00\lambda\) but this time there is a main lobe in the near-field in the middle of the aperture with a side lobe either side of it.

The influence of how to model the corner reflector and the distance from the apex to the source in a FDTD simulation was assessed. It was shown that the pointed shape of the reflector produces a minima in H-plane directivity at the dipole position at which it is predicted by Klopfenstein's method for the infinite size corner reflector plates.

The previous discussion demonstrated how valuable the near-field cuts are for analysing a radiation mechanism of antennas. Another helpful option of the FDTD
method is to combine near-field cuts of the time domain fields from a sequence of time steps to an animation. In that way the building up of the fields, diffraction, resonance, and trapped power can be identified.

The next point is to show how the H-plane patterns differ if the length of the plate is finite as in the 2-D FDTD or infinite case as assumed in Wait's approach.

4.1.2. Comparison between 2-D FDTD and Wait's method

Wait's method assumes infinite size of the reflector plates as shown in chapter 2. The study of the corner reflector antenna with finite size presented in this thesis is based on the results gained by employing Wait's method to establish combinations of apex angles and dipole positions which produce high H-plane directivity. To prove that these combinations also produce high H-plane directivity in the 2-D case H-plane radiation pattern predicted using Wait's method and 2-D FDTD are compared. If the patterns are similar the search for high H-plane directivity can be reduced to the combination which was assessed using Wait's method. This offers the advantage to narrow down the huge amount of possible combinations using the computational less intensive method by Wait.

In order to reach a solidly founded conclusion the comparison was done for a number of corner reflector antennas. The apex angle was varied from 30° to 90° in 10° steps. The dipole position was varied from 0.5λ to 1.2λ. The plate length for the 2-D FDTD model was 2λ, 4λ, and 15λ in order to show the influence of increasing the plates length. Since Wait's method assumes infinite long plates the longer the plates are the more similar the H-plane radiation pattern for the 2-D FDTD simulations should be to the ones of Wait's method. In the following figures the H-plane radiation pattern for the various combinations are given. In order to keep the number of figures to reasonable number only the results for apex angles of 30°, and 90° are presented. This does not influence the conclusion since the comparison of the H-plane radiation patterns for the other apex angles are similar in agreement as those presented. Note,
Wait's method predicts the H-plane radiation pattern only within the plates, hence only for angles from $0^\circ$ to half the apex angle.

Fig. 4.14: H-plane patterns for $\psi=30^\circ$, $\rho_o=0.5\lambda$ and different plate lengths

Fig. 4.15: H-plane patterns for $\psi=30^\circ$, $\rho_o=1.2\lambda$ and different plate lengths
Fig 4.14 and fig. 4.15 show the various H-plane radiation pattern for an apex angle of 30° and a dipole position of $\rho_0=0.5\lambda$ and $\rho_0=1.2\lambda$ respectively. There is no change in the pattern when the dipole is moved. Based on Wait's expression this can be explained since the relevant Bessel function term $J_6$ is almost zero and does not vary much for these dipole positions and, therefore, the H-plane radiation pattern stays the same. The longer the plates are the better becomes the match between the 2-D FDTD model and Wait's method. The main beam becomes narrower and the radiation level for higher angles reduces when the length of the plates is increased.

In fig. 4.16 and fig. 4.17 the various H-plane radiation patterns are presented for an apex angle of 90° and a dipole position of $\rho_0=0.5\lambda$ and $\rho_0=1.2\lambda$ respectively. Compared with the previous cases it is obvious that an even shorter length of the plates is sufficient to reach agreement between the two methods. The dipole position of $\rho_0=1.2\lambda$ proves that the agreement is not only given for the case of maximum radiation at boresight but also for corner reflector antennas with the main beam off boresight.
From the results discussed above the following conclusions can be drawn:

- The H-plane radiation pattern of a corner reflector antenna with finite length plates converges to the predictions of Wait's method for the infinite long plates. The agreement depends on the apex angle and the position of the dipole. The smaller the apex angle and the further away the dipole the longer the plates have to be in order to produce a similar radiation as an infinite sized corner reflector antenna. Table 4.1 shows the -3dB beam-width of the investigated corner reflector antennas. There is no -3dB beam-width for $\psi=90^\circ$ and $\rho_0=1.2\lambda$ since the main beam is off boresight.

- The results gained using Wait's method can be utilised to distinguish between corner reflector antenna designs which produce high H-plane directivity and those producing low H-plane directivity. Therefore, the parametric study using the 2-D and 3-D FDTD simulation technique can be reduced significantly.
Table 4.1: -3dB Beam-width for various long plates and apex angles of 30°, 60° and 90°

Even though this study focuses on the H-plane shaping without considering the influence of the finite width of the plates it is of interest to compare H-plane radiation patterns of finite and infinite width plates.

4.1.3. Comparison between 2-D and 3-D FDTD corner reflector antenna simulations

Several H-plane radiation patterns of corner reflector antennas predicted by 3-D and 2-D FDTD have already been presented in this study. In this section the cause is explained of the apparent difference in the H-plane radiation pattern shape between the prediction of the 2-D and 3-D FDTD. As shown in chapter 3.7 the plates of the corner reflector antenna are assumed to be infinite in width when modelled with 2-D FDTD as supposed to be of finite width in the 3-D FDTD.

One corner reflector antenna with an apex angle of 65° and a dipole to apex distance of 1.25λ (i.e. it is the same antenna which was used for validation of the code in section 3.6.2.4) was modelled using 2-D and 3-D FDTD. The width of the plates was changed from 2λ to 4λ and eventually, using 2-D FDTD, to infinity. The resulting three H-plane radiation patterns are shown in fig. 4.18.
Fig. 4.18: Comparison of H-plane pattern for $\psi=65^\circ, \rho_0=1.25\lambda$ and different wide reflector plates

Focusing first on the difference between finite and infinite width of the plates in general it can be observed that the main beam does not change. The first side lobe at 40° rises from -8.6dB for a width of $2\lambda$ to -5.9dB for $w=4\lambda$. For a infinite width the side lobe level is -7dB. Where the two finite width H-plane radiation patterns exhibit more side lobes the radiation pattern for the infinite wide plates declines steadily 165° before rising again towards 180°.

The waves contributing to the main beam are not influenced by the finite width of the reflector plates. The three curves begin to split at an angle of 25°. However, the main differences occur at angles where there is no direct radiation from the source. In the case of 2-D FDTD the only way the waves can reach this region is by diffraction on the front edges of the plates. This is not the case for the 3-D FDTD. Additionally to the diffraction at the front of the plates there is also diffraction from top and bottom of the plates which contributes to the radiation at those angles. The two contributions can add together or cancel out. Therefore the H-plane radiation pattern is different using...
either the 2-D or 3-D FDTD method. The shape of the H-plane pattern curve for a
width of $4\lambda$ is closer to the H-plane pattern for the infinite width case as the curve for
$w=2\lambda$. The difference is not a shortcoming of the 2-D FDTD, it is due to the fact that
one models a finite- and the other an infinite wide corner reflector antenna.

4.1.4. Establishing the influence of the ABCs

An inherit limitation to the FDTD method is the reflections caused by the ABCs
which terminate the spatial domain of the simulation model. Currently two different
methods are in use, Mur and Berenger ABCs. The later allows the amount of free
space required around the antenna to be reduced compared to that required for the
Mur ABCs. However, one must remember that the spatial domain size includes the
PML layer itself. The two methods are described in chapter 3. A discussion of the
memory saving aspect can be found in [75]. For the present study it is of interest to
show that the Mur ABCs placed half a wavelength away from the structure does
reduce reflections sufficiently.

In the 2-D FDTD the influence of the ABCs can be determined accurately. In order to
establish how much the reflections of the ABCs change the fields in the domain it is
possible to move the boundary a long distance from the antenna so that the reflections
cannot reach the near-field to far-field surface during the number of time steps which
are necessary to settle the fields. The additional grid size does increase both memory
requirement and run time but makes sure that the fields are not disturbed by any
reflections. The increased grid is called 'infinite grid' in the following discussion.

Two cases were investigated. Two corner reflector antennas were chosen since they
had significantly different H-plane radiation pattern. The first corner reflector antenna
had an apex angle of $60^\circ$, the plates were $2\lambda$ long and the dipole was $0.5\lambda$ from the
apex. The second corner reflector antenna had an apex angle of $65^\circ$ and the plates
were $2\lambda$. The dipole was $1.25\lambda$ from the apex. For each case the H-plane radiation
pattern was computed using three different simulation set-ups. Firstly, an 'infinite grid'
was used in order to get a reference. Secondly, the same problem was simulated with an eight layer Berenger ABCs with parabolic conductivity profile as ABCs. The Berenger ABCs was placed two cells from the structure. The last set-up was the Mur ABCs placed ten cells from the structure which was the set-up used for all presented data in this chapter. The details of the 'infinite grid' and the Mur ABCs is given in tables 4.2. The set-up for the Berenger ABCs is the same as for the Mur ABCs with ten cells distance from the structure.

<table>
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<th>Mur ABCs 10 cells from structure</th>
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<td></td>
<td>(\lambda/20)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sources</td>
<td>Infinite long dipole x-directed</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Frequency</td>
<td>5GHz</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Position of source (y, z)</td>
<td>(30, 20)</td>
<td>(108, 98)</td>
<td>(32, 35)</td>
<td>(119, 122)</td>
</tr>
<tr>
<td>Position of apex (y, z)</td>
<td>(30, 10)</td>
<td>(108, 88)</td>
<td>(32, 10)</td>
<td>(119, 97)</td>
</tr>
<tr>
<td>Time step</td>
<td>6.4ps</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of time steps</td>
<td>200</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Time steps per cycle</td>
<td>31.4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of cycles</td>
<td>6.4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NF-FF surface (y, z)</td>
<td>(8-52, 8-48)</td>
<td>(86-130, 86-126)</td>
<td>(8-56, 8-46)</td>
<td>(95-143, 95-133)</td>
</tr>
</tbody>
</table>

Table 4.2: Parameters of FDTD simulation of a corner reflector antenna with \(\psi=60^\circ\) and \(\psi=90^\circ\) \(\rho_0=0.5\lambda\), and \(l=2\lambda\) moving Mur ABCs from structure
The reference H-plane radiation patterns of the 60° corner reflector antenna and the error for the two other cases with ABCs are shown in the double y-scaled fig. 4.19.

![Graph showing H-plane directivity](image)

**Fig. 4.19: Comparison between Berenger-Mur ABCs and 'infinite grid' on H-plane pattern for $\psi=60^\circ$, $\rho_0=0.5\lambda$, and $l=2\lambda$.**

The comparison shows that the Mur ABCs 10 cells from the structure is sufficient to compute the H-plane radiation pattern of the corner reflector. However, it shows that the Berenger ABCs produce less reflections. The next antenna was the 65° corner reflector antenna.

The H-plane radiation patterns of the 65° corner reflector antenna and the error for the two other cases with ABCs are shown in the double y-scaled fig. 4.20.

The comparison of the H-plane radiation pattern shows good agreement between the three simulations. The agreement for the Mur ABCs is not as good as in the previous case which is due to the fact that rays hit the ABCs at smaller angles which can be deducted from the high side-lobe in the H-plane radiation pattern. However, waves hitting the ABCs at smaller angels result in higher reflections from the Mur ABCs as
shown in chapter 3. Additionally, the corners of the domain are more illuminated in this case. At the corners the first order ABCs are implemented which have a higher reflection coefficient. The Berenger ABCs produce an equally good result as in the previous case.

![Graph showing comparison between Berenger-Mur ABCs and infinite grid on H-plane pattern for psi=65°, rho=1.25\lambda, and l=2\lambda.]

In the region of most interest from 0° to 90° the PML ABC is significantly better than the Mur ABC but the error introduced by the Mur ABC is acceptable for antenna computations.

4.2. The H-plane Directivity

4.2.1. Definition of H-plane directivity

The H-plane directivity is a measure for the ability of the antenna to direct radiating power into a given direction. Therefore it is similar to the directivity discussed in section 2.4.3. Considering equation (2.11) it is clear that this directivity cannot be computed for a two dimensional model.
For the purpose of this study it was decided to define a quantity which carries the
same information for a cylindrical model as the directivity does for a spherical model.
The computation of the radiated power is restricted to the power radiated at θ=90°, the
so called H-plane. The quantity is called 'H-plane directivity' and is defined as the
ratio of radiated power in the direction (90°, φ1) to the total radiated power in this
plane:

\[
\text{H - plane directivity}(90^\circ, \phi_1) = \frac{2\pi \text{power radiated per unit angle in direction } (90^\circ, \phi)}{\text{Total power radiated in H-plane}} = \frac{2\pi I(90^\circ, \phi_1)}{\int_0^\pi I(90^\circ, \phi) d\phi}
\]  

(4.1)

where:

- H - plane directivity \((90^\circ, \phi_1)\): H-plane directivity in direction \((90^\circ, \phi_1)\)
- I: Radiation Intensity in any direction \((90^\circ, \phi)\)

### 4.2.2. Influence of the plate length on H-plane directivity in case of infinite wide plates

The H-plane directivity can be used to compare different corner reflector antennas for
the ability to concentrate power in any direction. This ability should differ from the
case of infinite plates only because of the finite length of the plates of the corner
reflector. It is therefore interesting to compare the results of the 2-D FDTD with the
computed H-plane directivity results of infinite corner reflector antennas by using
Wait's method.

Substituting Wait's formulation (2.7) for the H-plane radiation pattern into (4.1)
yields:

\[
\text{H - plane directivity}(90^\circ, \phi_1) = \frac{2\pi |G(90^\circ, \phi_1)|^2}{2 \int_0^\pi |G(90^\circ, \phi)|^2 d\phi}
\]

(4.2)
where:

\[ G = \sum_{n=1}^{\infty} e^{-jn\pi^2/2w} \sin(n\pi\phi_0 / \psi) \sin(n\pi\phi / \psi) J_{\text{max}}(k\rho_0 \sin \theta) \]

The integral in the nominator is computed only within the plates since the fields are assumed to be zero elsewhere.

In order to compare the results of the 2-D FDTD simulation and Wait's method the length of the plates has to be sufficient in size to prevent any influence from the finite size plates. A plate length of 10\(\lambda\) was found to be sufficient for all apex angles between 30° and 180°.

The study was performed on plate lengths of 2\(\lambda\), 3\(\lambda\), 4\(\lambda\), 10\(\lambda\) and covered an apex angle range from 30° to 180° in 1° steps. The dipole distance was varied from 0.5\(\lambda\) to 1.5\(\lambda\) in steps of 0.1\(\lambda\). The maximum H-plane directivity for each angle can be seen in fig. 4.21. The curve for the infinite plates was predicted using Wait's method.

**Fig. 4.21:** Maximum H-plane directivity comparison for different plate lengths

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The two curves for finite length and $l=10\lambda$ show good agreement. This match gives ground for the conclusion that the model of the corner reflector antenna in the 2-D FDTD simulation is accurate in respect of the apex angle. The staircase approximation does represent a valid model for the flat plates. It also shows that the restriction of the power integration to between the plates, for Wait's method, does not influence the H-plane directivity. This is due to the low power levels for radiation angles outside the apex angle range.

The results for the maximum H-plane directivity are in agreement with the prediction based on Image Theory. For apex angles which are submultiple of $\pi$ it is possible to arrange the images such that the main beam becomes narrow and therefore the H-plane directivity high. It is therefore not surprising that the maximum H-plane directivity has peaks for the corner reflector antenna with those apex angles. The drastic change of the maximum H-plane directivity is unexpected. The fact that the peaks are present for both methods tends to confirm that the peaks are not artefacts of the model.

In table 4.3 the positions of the peaks for the maximum H-plane directivity together with the corresponding positions of the dipole are given for plate lengths of $10\lambda$ and infinite.

<table>
<thead>
<tr>
<th>Apex angle</th>
<th>36°</th>
<th>45°</th>
<th>60°</th>
<th>90°</th>
</tr>
</thead>
<tbody>
<tr>
<td>H-plane directivity infinite length</td>
<td>14.2dB</td>
<td>14.4dB</td>
<td>13.8dB</td>
<td>13.7dB</td>
</tr>
<tr>
<td>Position of the dipole for infinite length</td>
<td>$1.4\lambda$</td>
<td>$1.2\lambda$</td>
<td>$1.4\lambda$</td>
<td>$1.5\lambda$</td>
</tr>
<tr>
<td>H-plane directivity $l=10\lambda$</td>
<td>13.8dB</td>
<td>14.0dB</td>
<td>13.7dB</td>
<td>13.6dB</td>
</tr>
<tr>
<td>Position of the dipole $l=10\lambda$</td>
<td>$1.4\lambda$</td>
<td>$1.2\lambda$</td>
<td>$1.4\lambda$</td>
<td>$1.5\lambda$</td>
</tr>
</tbody>
</table>

Table 4.3: Maximum H-plane directivity for various apex angles computed by 2-FDTD and Wait's method

The values of the dipole positions are identical for the two different methods. Thus, it is believed that the peaks of the maximum H-plane directivity are due to the nature of
the corner reflector antenna. For some apex angles the images of the dipole can be arranged by moving the dipole such that the radiation of the individual images add up. In this case a sharp main beam is achieved. According to Image Theory this is only possible for apex angles which are submultiple of π.

The next step in the parametric study is to investigate the influence of finite long plates of the corner reflector. The lengths were chosen to be \( 2\lambda, 3\lambda, 4\lambda, 10\lambda \) in order to show the transition from finite to infinite length. The focus was on the smaller lengths since the objective of the study is to find compact designs of the corner reflector antenna with high H-plane directivity.

The maximum H-plane directivity for different plate lengths converges with an increasing apex angle. A comparison between the H-plane directivity of Wait's theory for infinite length of plates and the 2-D FDTD pattern shows that for apex angles smaller than 65° a plate length of 10\( \lambda \) is necessary to achieve good agreement between the two. For a plate length of 4\( \lambda \) the difference vanishes at an apex angle of 65°, for \( l=3\lambda \) this happens at \( \psi=90° \), whereas for an apex angle of 100° a plate length of 2\( \lambda \) is satisfactory. This implies that for smaller apex angles the plates have to be longer in order to guide the waves sufficiently. A small apex angle is therefore not suitable to achieve a compact antenna design. The peaks of the maximum H-plane directivity are not as pronounced for smaller lengths of the plates. Again, this varies with increasing the apex angle.

A steady decrease of H-plane directivity is shown for apex angles exceeding 90°. There are no further peaks, which is in agreement with the Image Theory. All curves converge towards 6dB for an apex angle of 180° which corresponds to the H-plane directivity of a dipole in front of a flat plate. Since the H-plane directivity exhibits no distinctive feature for apex angles beyond 90° the remainder of the study concentrates on apex angles in the range of 30° to 90°.
Another indication of the influence of the finite length of the plates on the performance of the corner reflector antenna is given by the position of the dipole for maximum H-plane directivity. A comparison of the dipole position which produced maximum H-plane directivity for different plate lengths at different apex angles is presented in fig. 4.22.

![Fig. 4.22: Dipole position for maximum H-plane directivity](image)

The discrepancies of apex angles smaller than 50° are not significant since the H-plane directivity varies for dipole positions between $\rho_0=0.5\lambda$ and $\rho_0=1.5\lambda$ less than 0.5dB for those apex angles. The differences at angles higher than 75° appear for plate lengths smaller than $10\lambda$ because the optimal position of the dipole is too far away from the apex to allow for guidance with the short plates. Therefore, the benefit of the dipole position at $1.5\lambda$ can only be exploited if either the apex angle is large enough or the plate length is sufficient.

If the dipole position is kept constant and the apex angle varied the consequence of the dipole position on the guidance of waves becomes clearer. Two different dipole positions were chosen to investigate the change of the H-plane directivity with
The dipole positions were chosen to be 0.5λ and 1.5λ in order to cover the two extremes.

If the dipole was fixed to a position 0.5λ away from the apex and the apex angle was varied from 30° to 90° in 1° steps the H-plane directivity is given by fig. 4.23.

The convergence of the curves begins at smaller apex angles as it was the case when the dipole position was chosen to achieve the maximum H-plane directivity. The exact values are given in table 4.4. The start of the curves is exactly the same as it was for the maximum H-plane directivity since at this apex angles the H-plane directivity varies very little with the dipole position. For higher apex angles the peaks of the H-plane directivity are not present since the dipole is not positioned to arrange the images such that they sharpen the main beam. It is shown in fig. 4.22 that this only happens for dipole positions at least 1.2λ from the apex. The H-plane directivity for ρ₀=1.5λ and various plate lengths is shown in fig. 4.24 to confirm this argument.
In this case the dipole is at times further away than necessary in order to achieve the maximum H-plane directivity. Therefore, the discrepancy between the curves of finite length and infinite length is for certain combinations of the apex angles and plate lengths even bigger than for maximum H-plane directivity. The peaks of the H-plane directivity at apex angles of 60° and 90° are this time clearly to observe. The apex angle at which the curves of H-plane directivity for different plate lengths converge with the curve of infinite length for a given dipole positions are given in table 4.4.

<table>
<thead>
<tr>
<th></th>
<th>l=2λ</th>
<th>l=3λ</th>
<th>l=4λ</th>
</tr>
</thead>
<tbody>
<tr>
<td>ρ₀=0.5λ</td>
<td>70°</td>
<td>60°</td>
<td>50°</td>
</tr>
<tr>
<td>ρ₀=1.5λ</td>
<td>100°</td>
<td>90°</td>
<td>65°</td>
</tr>
<tr>
<td>Dipole pos. for max. H-plane directivity</td>
<td>100°</td>
<td>90°</td>
<td>65°</td>
</tr>
</tbody>
</table>

Table 4.4: Apex angle at which curves for H-plane directivity converge

An explanation of the relation of the apex angle of convergence, plate length, and dipole position is given in section 4.4 using the ray tracing technique.
The influence of the plate length varies with the apex angle and dipole position and has been assessed in this section for apex angles from 30° to 180° and dipole positions from 0.5λ to 1.5λ. The results for the H-plane directivity are a guideline of how to design a compact corner reflector antenna in order to meet a given H-plane directivity specification.

4.3. Aperture Efficiency

4.3.1. Definition of aperture efficiency

Similar to the normal definition of directivity the aperture efficiency is normally defined in three dimensions. It relates the maximum effective area $A_{em}$ to the physical area $A_p$. The formula to calculate the efficiency is:

$$\varepsilon_{ap} = \frac{D_o \lambda^2}{4\pi A_p}$$  \hspace{1cm} (4.3)

where:

$\varepsilon_{ap}$ : Aperture efficiency

$D_o$ : Peak Directivity

$A_p$ : Physical area

For the two dimensional case there is no aperture area. This is replaced by an aperture width which is defined by the distance between the front edges of the corner reflector plates. The peak directivity is replaced by the H-plane directivity and since the radiation is not into a sphere but into one plane the $4\pi$ term is replaced by $2\pi$. A aperture efficiency for two dimensions can be defined as:

$$\varepsilon_{ap} = \frac{\text{Peak H-plane directivity} \cdot \lambda}{2\pi \cdot W_p}$$  \hspace{1cm} (4.4)

where:

$W_p$ : Physical width of aperture

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For the corner reflector antenna the aperture efficiency (4.4) becomes:

\[ \varepsilon_{ap} = \frac{\text{Peak H-plane directivity} \cdot \lambda}{2\pi \cdot 2l \sin \left( \frac{\psi}{2} \right)} \]  \hspace{1cm} (4.5)

where:

- \( l \) : length of plates
- \( \psi \) : Apex angle

The aperture efficiency for the two dimensional corner reflector has the same meaning as it has in the three dimensional case. It is a measure of how uniform the aperture is illuminated. For a perfectly uniform illuminate aperture the maximum H-plane directivity is achieved and the aperture efficiency is 1.0. Since the aim of the study is to find compact corner reflector antenna with high H-plane directivity the aperture efficiency is an important parameter.

**4.3.2. Influence of plate length on aperture efficiency in case of infinite wide plates**

In fig. 4.25 the aperture efficiency of corner reflector antenna is presented with an apex angle of 30°-90° for plates lengths of \( 2\lambda, 3\lambda, 4\lambda, \) and \( 10\lambda \). The position of the dipole was varied from \( 0.5\lambda \) to \( 1.5\lambda \) such that maximum H-plane directivity and therefore maximum aperture efficiency was gained. The aperture efficiency is based on the results of the H-plane directivity which are described in the previous section.

The aperture efficiency shows that the high H-plane directivity achieved by increasing the length of the plates does not yield to a uniform illumination of the aperture. The plate length has to be short to get the highest aperture efficiency. For short plate lengths the aperture width remains small and according to equation (4.5) a lower H-plane directivity still gives a high aperture efficiency. The difference of aperture efficiency between a plate length of \( 2\lambda \) or \( 3\lambda \) is significant, whereas the further drop
of aperture efficiency is only minor if the length is increased to $4\lambda$. The graph also shows that a plate length of $10\lambda$ is too long to achieve high aperture efficiency.

![Graph](image)

**Fig. 4.25:** Aperture efficiency for different apex angles and plate lengths

The highest aperture efficiency is 0.99 for a corner reflector antenna with an apex angle of $50^\circ$, a plate length of $2\lambda$, a dipole position of $1.5\lambda$, and a H-plane directivity of 10.2 dB. The second highest aperture efficiency is 0.95 for corner reflector antenna with an apex angle of $36^\circ$, a plate length of $2\lambda$, a dipole position of $0.5\lambda$, and a H-plane directivity of 8.7 dB.

The results in fig. 4.25 indicate that in order to achieve a high aperture efficiency the plate length has to be limited to $2\lambda$. However, if the H-plane directivity produced by this antenna is not high enough the plate length might be increased at the expense of aperture efficiency.
4.4. Ray tracing model

In the previous sections the influence of the finite length plates of the corner reflector antenna on the H-plane radiation pattern was investigated. It is of some interest to back up the theories developed with a ray tracing model. The aim of this section is to explain why reducing the length of the plates has a different influence on the pattern depending on the apex angle and dipole position.

Ray tracing includes power conservation and is based on Snell's law of reflection which states that a plane wave hitting a perfect conductor is reflected such that the angle of reflection is equal to the angle of incidence. By applying Snell's law of reflection it is possible to determine angles and positions of rays which are bouncing within the corner reflector. Angles and geometry are shown in fig. 4.26.

It is assumed that the source radiates omnidirectional which is the case for an infinite line source as was used in the two dimensional FDTD simulation. The rays leave the source at an angle $\beta$ which is called the 'initial radiation angle'.

![Fig. 4.26: Sketch of rays propagation](image)

- $\psi$: Apex angle
- $\beta$: Initial radiation angle
- $\alpha_n$: Incidence angle of n'th reflection
- $l_n$: Position of first n'th reflection
- $\gamma_n$: Reflection angle of n'th reflection

Fig. 4.26: Sketch of rays propagation
If the initial radiation angle is smaller than $180^\circ - \psi/2$ the rays hit the plate first time at the position $l_1$ at an angle $\alpha_f$ called 'incidence angle'. This angle is measured between the incident ray and the plate starting from the left hand side. According to Snell's law of reflection the 'reflection angle' $\gamma$ is equal the incidence angle.

4.4.1. Basic expressions

In this section formulas to compute the necessary quantities are developed in order to determine the position of reflections on the plates for given apex angles and dipole distances. All expressions are based on geometry. The derivation is not given but is trivial.

The general expression for $\alpha_n$ is:

$$\alpha_n = 180^\circ - \left(\beta + \frac{(2n-1)\psi}{2}\right)$$  \hspace{1cm} (4.6)

where $n$ is the number of reflections.

Two different kinds of reflections are to be distinguished. The first kind are those which take the rays closer to the apex. They are called 'backward reflections'. The second kind take the rays further away and are called 'forward reflections'. The reason why it is necessary to make this distinction is because the backward reflections are not significant for determining the influence of the finite length of the plates whereas the forward reflections are significant.

A forward reflection of rays of initial radiation angles between $0^\circ$ to $180^\circ$ occurs for a range of angles defined by:

$$90^\circ - \psi < \beta < 180^\circ - \frac{\psi}{2}$$  \hspace{1cm} (4.7)

For angles larger than those defined by (4.7) there is no reflection at all whereas for smaller angles the ray is reflected backwards. No matter whether the ray is reflected forwards or backwards the incidence angle gets smaller after each reflection until at
one point the reflection angle is less than half the apex angle in which case there is no further reflection.

The number of forward reflections depends on the apex angle and the initial radiation angle. Based on (4.6) it is obvious that for smaller apex angles there are more reflections since the angle of incidence is reduced with each reflection by the smaller amount of the apex angle. The following expression gives the borders between the regions of 0, 1, 2, 3 or 4 forwards reflections for a given $\beta$.

$$\beta > 180^\circ - \frac{(2n + 1)}{2} \psi$$

$n = 0, 1, 2, 3, 4$

\[\text{(4.8)}\]

Figure 4.27 shows the borders for a certain number of possible reflections depending on the initial radiation angle $\beta$ for a range of apex angles from $30^\circ$-$90^\circ$. For example, there are two forward reflections for an apex angle of $\psi=60^\circ$ and an initial radiation angle of $\beta=50^\circ$. The incidence angles of the plane waves are:

$n=1 \quad \alpha=100^\circ$
\[ n=2 \quad \alpha = 40^\circ \]

Since \(40^\circ < 60^\circ\) (apex angle) there will be no further reflection.

Rays which leave the source at an angle smaller than the border of backward radiation are reflected once or twice backwards before being reflected forwards. This is described in detail with the help of the following examples:

A wave is reflected backwards if the incidence angle is below a certain value which is defined by:

\[ \alpha < 90 - \frac{\psi}{2} \quad (4.9) \]

With each reflection the reflection angle gets steeper until at one point the angle will be such that the ray is reflected in the forward direction.

The number of backward reflections depends on the incidence angle as well as on the apex angle. As for the forward reflected rays the number of reflections are higher if the apex angle is small. The reason is the same as for the forward reflection. The following equation gives the borders between the regions of 1 or 2 backward reflections for a given \( \beta \).

\[ \beta > 90^\circ - n\psi \]
\[ n = 1, 2 \quad (4.10) \]

In fig. 4.28 the boundaries are given for different numbers of backward reflections depending on the apex angle and initial angle of radiation.

A comparison with fig. 4.27 highlights that the number of forward reflections is higher than the number of backward reflections. There is no backward reflected ray for an apex angle of \(90^\circ\) and only for apex angles smaller than \(45^\circ\) there is a second reflection backwards. The possibility of modifying the apex and therefore the backward reflection is addressed in chapter 6 within the section on a modified corner reflector.
For a discussion on how the finite length of the plates influences the H-plane radiation pattern of a corner reflector antenna it is now important to determine the position of the reflection on the plates. Based on the angles defined in this section the positions are computed as:

\[
l_i = \rho_0 \frac{\sin(\beta)}{\sin\left(180^\circ - \left(\frac{\psi + \beta}{2}\right)\right)}
\]

(4.11)

\[
l_n = l_{n-1} \frac{\sin\left(\frac{2n-3}{2} - \psi + \beta\right)}{\sin\left(180^\circ - \left(\frac{2n-1}{2}\right)\psi + \beta\right)}
\]

(4.12)

where:

\[n = 2, 3, \ldots\]
With the expressions given in this section it is now possible to argue the case of finite length plates by taking into account the proportion of rays which are affected when the plates are reduced in length.

4.4.2. Results of ray tracing on corner reflector antenna

Two representative apex angles were chosen to investigate the problem. In order to cover the two extremes the apex angles are $30^\circ$ and $90^\circ$. The dipole was moved by $1\lambda$ from $0.5\lambda$ to $1.5\lambda$ to further highlight the difference between the two extremes. The length of the plates was $2\lambda$, $3\lambda$, $4\lambda$, $10\lambda$. The upper limit of the length of the plates was set to $10\lambda$ since for this value the H-plane directivity was similar to the results when using Wait's method for the infinite sized plates case. Therefore, it is assumed that reflections further away from the apex are not significant in terms of the radiation pattern.

Figures 4.29, 4.31, 4.32, 4.34 compare the location of reflections for $\psi=30^\circ$ and $90^\circ$ and $\rho_0=0.5\lambda$, $1.5\lambda$ based on (4.11) and (4.12).
Fig. 4.29 shows that the position of the reflections are mainly up to $4\lambda$ away from the apex of the plates. A closer look reveals that the first reflection up to an initial radiation angle of $60^\circ$ is backwards. The sharp rise of all curves indicates that the location of the reflection moves quickly away from the apex within a very small range of $\beta$. Furthermore, it becomes clear that reflections further away than $1\lambda$ from the apex are not reflected again, i.e. their incidence angle is equal or below the apex angle. Reducing the plate length to less than $4\lambda$ influences a high proportion of rays which contribute to the main beam in the case of infinite long reflector plates.

Fig. 4.30 reveals that the assumption for the H-plane pattern based on the ray tracing model holds true.

![Fig. 4.30: H-plane pattern for $\psi=30^\circ \rho_{0}=0.5\lambda$.](image)

The H-plane pattern shows a significant change when the length of the plate is increased: The main beam becomes narrower when the length is increased. The front-to-back ratio remains very much the same. It is interesting to note that the change from $1=2\lambda$ to $1=3\lambda$ is bigger than for the next step to $4\lambda$. An explanation for this behaviour is difficult to find due to the complexity of the reflections in the case of
small apex angles. A clearer connection between reflections and H-plane pattern is possible if the number of reflections is reduced as is the case for bigger apex angles.

In fig. 4.31 the reflections of a 30° apex angle and a dipole separation of 1.5λ are given. The positions of the reflections are further away and the curves do not rise as sharply as in the previous case. Considering the positions of the reflections it is expected that longer plates are needed to achieve the same degree of the waves' guidance of the waves as for a dipole separation closer to the apex. Therefore, it is surprising to find that the H-plane patterns are exactly the same as for the previous case.

An explanation is within the nature of the ray tracing model. The distance of the dipole to the apex does not play a part in determining the angles for the reflections. It is only when the location of the reflection arises that the distance of the dipole has to be taken into account. The dipole position is merely a scaling factor for the problem. The changes of the radiation pattern for different dipole positions are due to the path difference which is imposed on the rays for different dipole separations.
In the case of small apex angles the change of the path for different dipole positions is not as significant for bigger apex angles since the distance between the plates and the dipole does not change as quickly as for the small apex angles.

It is now interesting to compare this results with the scenario of a 90° apex angle. Remember that for this case a weaker dependence of the radiation pattern on the plate length was noticed. From fig 4.28 it becomes clear that there are no backward reflected rays in that case. This fact reduces the complexity of the ray tracing model and, therefore, conclusions can be drawn more easily.

Fig. 4.32 shows the positions of the reflections of a corner reflector antenna with an apex angle of 90° and a dipole separation of 0.5λ. The number of reflections reduces to two which is four down on the previous case. The shape of the curve is similar to the case of ψ=30° with a very sharp rise of the curves once the position of the reflection exceeds 4λ.

![Diagram showing the positions of the reflections of a corner reflector antenna with an apex angle of 90° and a dipole separation of 0.5λ.](image)

Fig. 4.32: Position of forward reflection for ψ=90°, ρ₀=0.5λ.
Based on the curves of fig. 4.32 the effect of increasing the plates from $2\lambda$ to $3\lambda$ should be more significant than from $3\lambda$ to $4\lambda$. In fig 4.33 the corresponding H-plane radiation pattern are presented.

![Graph showing H-plane pattern for $\psi=90^\circ$, $\rho_0=0.5\lambda$.](image)

This time the expectations are met. The change of the pattern is not very significant. The main beam barely changes. The change from a plate length of $3\lambda$ to $4\lambda$ is also minor as is concluded from the results of fig. 4.32.

The last case investigated was a corner reflector antenna with an apex angle of $90^\circ$ and $\rho_0=1.5\lambda$. The number of reflections is obviously the same as in the previous case but the shape of the curves and the positions change, see fig. 4.34. The reflections occur further away from the apex and the curves are not as steep as they have been for the closer separated dipole. Thus, it is expected that the length variation of the plates does result in a significant change in the radiation pattern.
The radiation pattern for $\psi=90^\circ \rho_0=1.5\lambda$ for different length of plates is given in fig. 4.35.
The change of the H-plane pattern is substantial. The pattern is sensitive for an increase of the plates. This was expected based on the positions of the reflected rays as shown in fig. 4.34.

The problems encountered using the ray tracing model to explain the radiation from the corner reflector antenna proved that even though the corner reflector antenna is a simple structure to build the radiation mechanism is far from being simple.

4.5. Influence of finite sized reflector plates in three dimensions

The corner reflector antenna is arranged in arrays as mentioned already in one of its most common applications as base station antenna for mobile communication systems. Nevertheless, for other applications such as point-to-point communications single corner reflector antennas are employed. For these applications and for the top and bottom corner reflector antenna of an array the influence of finite wide reflector plates on the radiation pattern in H- and E-plane is also of importance. The 3-D FDTD code described in chapter 3 was used to investigate the influence of the finite width on the radiation pattern and gain.

The investigation focused on two designs which were a 60° corner reflector antenna and the half wave dipole 0.5λ from the apex. The other design was a 65° corner reflector antenna and the half wave dipole 1.25λ from the apex. The length and the width of the plates was varied from 1λ, 2λ, to 4λ. Whenever one dimension was varied the other was kept to 2λ. The designs were chosen in order to provide two different scenarios for the diffraction on the plates since the dipole is further away in the second design and therefore the waves hit the edges differently. Diffraction on top and bottom of the plates should be dominant over the diffraction on the front edge since the E-vector is perpendicular at these locations which results in higher diffraction.
As examples the E-plane radiation pattern for the corner reflector antenna with $\psi=60^\circ$, $w=2\lambda$, $\rho_0=0.5\lambda$ a various length and widths of the plates is given in fig. 4.36 and 4.37 respectively.

Fig. 4.36: E-plane pattern for $\psi=60^\circ$, $w=2\lambda$, $\rho_0=0.5\lambda$ and different lengths of plates

Fig. 4.37: E-plane pattern for $\psi=60^\circ$, $l=2\lambda$, $\rho_0=0.5\lambda$ and different widths of plates
The E-plane radiation pattern shown in fig 4.36 and 4.37 indicates that the change in length of the plates did not influence the radiation in this plane. Tables 4.5 and 4.6 give the -3dB beam-width of the E- and H-plane and the gain for the two corner reflector antennas investigated.

<table>
<thead>
<tr>
<th>width*length</th>
<th>$\psi=60^\circ \rho_0=0.5\lambda$</th>
<th>$\psi=65^\circ \rho_0=1.25\lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>gain</td>
<td>efficiency</td>
</tr>
<tr>
<td>$1\lambda+2\lambda$</td>
<td>12.0dBi</td>
<td>0.63</td>
</tr>
<tr>
<td>$2\lambda+2\lambda$</td>
<td>14.2dBi</td>
<td>0.53</td>
</tr>
<tr>
<td>$4\lambda+2\lambda$</td>
<td>13.4dBi</td>
<td>0.22</td>
</tr>
<tr>
<td>$2\lambda+1\lambda$</td>
<td>11.7dBi</td>
<td>0.58</td>
</tr>
<tr>
<td>$2\lambda+4\lambda$</td>
<td>15.8dBi</td>
<td>0.38</td>
</tr>
</tbody>
</table>

Table 4.5: Gain and efficiency for variation of length and width of the plates, $\psi=60^\circ$ and $65^\circ$

<table>
<thead>
<tr>
<th>width*length</th>
<th>$\psi=60^\circ \rho_0=0.5\lambda$</th>
<th>$\psi=65^\circ \rho_0=1.25\lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>BW E-plane</td>
<td>BW H-plane</td>
</tr>
<tr>
<td>$1\lambda+2\lambda$</td>
<td>50°</td>
<td>32°</td>
</tr>
<tr>
<td>$2\lambda+2\lambda$</td>
<td>34°</td>
<td>32°</td>
</tr>
<tr>
<td>$4\lambda+2\lambda$</td>
<td>48°</td>
<td>34°</td>
</tr>
<tr>
<td>$2\lambda+1\lambda$</td>
<td>40°</td>
<td>58°</td>
</tr>
<tr>
<td>$2\lambda+4\lambda$</td>
<td>30°</td>
<td>22°</td>
</tr>
</tbody>
</table>

Table 4.6: Beam-width for E- and H-plane for variation of length and width of the plates, $\psi=60^\circ$ and $65^\circ$

From the data presented it becomes obvious that the E-plane radiation pattern is influenced more by the width of the plates than by the length. It is also shown that the aperture efficiency can fall drastically by increasing the width or the length of the plates. There are optimum lengths and widths for the plates which yield higher H-plane directivity than larger plates.
4.6. Variable beam-width corner reflector antenna

As mentioned several times already the corner reflector antenna has been used at base stations for mobile communication systems. The amount of traffic which occurs within a cell of such communication systems varies with time. On one hand, a cell that includes Wembley stadium has, for instance, a low traffic rate when nobody is in the stadium. On the other hand, the traffic rate is very high during events with more than 70000 people around. Changing demands on the capacity of the cells can best be handled if the cells can be adapted to the actual situation.

One possibility to adapt the cells is to adapt the radiation pattern of the base station antennas. In this section the variation of the H-plane pattern through a change of the apex angle is investigated and a design which is easy to build is presented.

Fig. 4.38 and 4.39 shows a variable beam-width corner reflector antenna which was constructed and measured at Queen Mary Westfield College. The main difficulty for the design was to ensure that the two plates move synchronously such that the dipole was always at the bisector. The additional structure to enable the movement of the plates should not influence the radiation pattern but should be easy to build, and enable the apex angle to be changed precisely.

Fig. 4.38: Picture of variable apex angle corner reflector antenna
The synchronous movement of the plates was realised through two struts from a sliding ring at the rear of the reflector to the front end of the plates. The location of the fixing point of the struts was chosen for two reasons. Firstly, if the struts were fixed closer to the hinge the force which is transmitted onto the plates would have acted mainly along the plates rather than normal to them which is necessary to change the angle. This would yield extra strain on the hinge and additional power would be needed to change the angle.

Secondly, a change of the apex angle translated to less movement at the apex than at the front end of the plates. Therefore, a more precise control over the apex angle was achieved when the struts were fixed at the front end of the plates.

Special care was taken at the hinge. A normal hinge would produce a slot between the plates when the apex angle is increased. This is avoided by using the kind of hinge which is shown in fig 4.39. All metal parts are made of brass.

The parameters for the antenna, which was designed for 5GHz, were a half wave dipole as source separated by 30mm (0.5λ) from the apex, and the plates were 120mm (2λ) long and 120mm (2λ) wide.
The FDTD simulations were conducted in 3-D. The cell size was $\lambda/20$, and the size of the grid was chosen such that the structure was half a wave length from the ABCs. The near-field to far-field transformation surface was 2 cells from the apex and the edges of the structure. The simulation frequency was 5GHz and the number of time steps was set such that the field settled down.

In fig. 4.40 the change of the gain and the -3dB beam-width for apex angles in the range from $30^\circ$ to $120^\circ$ are given.

![Graph](image-url)

**Fig. 4.40:** -3dB beam-width and gain over apex angle, $\rho_0=0.5\lambda$, $l=w=2\lambda$. Gain does not include mismatch loss.

The beam-width changed from $54^\circ$ for an apex angle of $30^\circ$ to a low of $30^\circ$ between $\psi=70^\circ$ and $90^\circ$. When the plates were opened up even more the beam-width increases to $60^\circ$ once the apex angle reached $120^\circ$. The beam-width changed $30^\circ$ by changing the apex angle. For a larger apex angle the beam started splitting into two main beams. This ties in with the Image Theory which predicts a null at boresight for a dipole position of $\rho_0=0.5\lambda$ in front of a flat plate (i.e. apex angle $180^\circ$).
The gain exhibited the opposite trend. It climbed from 12.6dBi at $\psi=30^\circ$ to 14.2dBi at $\psi=60^\circ$ and fell again at $\psi=120^\circ$ to 10dBi. The gain variation was 4.2dB for a change of 90° of the apex angle.

The same investigation conducted with the 2-D FDTD proved that the prediction of the main beam matched between 2-D and 3-D FDTD. The difference in beam-width between 2-D and 3-D varied by no more than 4° over the whole range and showed overall the same shape as the curve in the three dimensional case. Again, this was expected since the main beam is not much influenced by the diffraction which occurs on top and bottom of the plates.

The influence of the width of the plates is not important and therefore a parametric study was carried out in two dimensions. To avoid the beam splitting for apex angles bigger than 120° the dipole position was reduced to 0.25$\lambda$ from the apex. Increasing the length of the plates from 2$\lambda$ over 4$\lambda$ to 6$\lambda$ showed that the H-plane directivity increased for apex angles below 70°, fig. 4.41. In the same way the beam-width did decrease for the same apex angles. This fully ties in with what was expected.

![Fig. 4.41: Beam-width and H-plane directivity over apex angle for various plate lengths](image)
Thus, it was not only possible to adapt the beam-width of a corner reflector antenna by changing the apex angle but also to determine the amount of variation by choosing the plate length appropriately.

4.6.1. Measurement results

An antenna was built and measured at 5GHz in the Compact Antenna Test Range (CATR) at Queen Mary and Westfield College to further validate the code and the results presented.

The two examples for the measured and computed H-plane radiation patterns for apex angles of 70° and 170° are shown in fig 4.42 and 4.43 respectively.

![Graph showing comparison between measured and predicted H-plane for ψ=70°](image)

**Fig. 4.42:** Comparison between measured and predicted H-plane for ψ=70°
The predicted results are in very good agreement with the measurements.
A series of computed H-plane patterns for apex angles of 30°, 70°, 120° and 180° is given in fig. 4.44. For an apex angle of 30° the pattern consists of a broad main beam due to the lack of guidance from the short plates. Increasing the apex angle to 70° sharpens the main beam which is why the gain for this apex angle is highest.

Further increasing of the apex angle weakened the guidance of the waves again which resulted in broadening the main beam and eventually it splits up. This was due to destructive interference on boresight for a dipole which is half a wavelength in front of a flat plate.

4.6.2. Gain measurements

Gain measurements can be conducted in several ways. The most suitable one for a given problem depends on the expected gain, the required precision, and the available equipment. In the present case the gain-transfer (gain-comparison) method was chosen.

4.6.3. Gain transfer measurement method

The method most commonly used to measure the gain of an antenna is the gain-transfer method. In order to utilise this technique a standard antenna (with a known gain) is necessary. Relative gain measurements are performed, which when compared with the known gain of the standard antenna, yield gain transfer values.

Two sets of measurements are required. The first set uses the test antenna as the receiving antenna, the received power (PT) into a matched load is recorded. This power is compared with the received power of the standard antenna (PS) into a matched load which is measured in the second set.

The absolute gain of the test antenna is then computed with the help of:

\[ G_T = G_s + 10 \times \log \left( \frac{P_T}{P_s} \right) \]  

(4.13)
where:

\[ G_T : \text{Gain of test antenna} \]
\[ G_s : \text{Gain of standard antenna} \]
\[ P_T : \text{Power measured for test antenna} \]
\[ P_s : \text{Power measured for standard antenna} \]

If the measured power is not into a matched load there are reflections which reduce the measured power. Those reflections give a reduction in the measured gain and must be taken into account. Therefore it is necessary to determine the reflection loss (\( S_{11} \)) of both the standard and the test antenna. The reflection loss was measured using a \( S_{11} \) test set. The power which would be received into a matched load can be obtained from the measured power if the reflected power is added to the measured power. The reflected power is:

\[
\text{Reflected power} = 10 \times \log \left( \frac{1}{S_{11}} \right) \quad (4.14)
\]

After adding the reflected to the measured power the power levels are inserted into equation (4.13).

### 4.6.4. Gain measurement set-up

A practical measurement set-up is shown in fig. 4.45. The standard antenna was a C-Band horn with an aperture of 231*170mm and a gain of 19.7dBi. The \( S_{11} \) was -8.7dB which results in a reflected power of 0.6dB which has to be added to the measured standard power. As feed a Yagi antenna was used.

A HP8360 synthesiser produced a 5GHz sinusoid which was very stable in both frequency and amplitude. A splitter was used to supply the feed antenna and a mixer. The mixer has the task of translating the radio frequency (rf) signal to a lower frequency of 45MHz (intermediate frequency IF) at which the attenuation of the cable is smaller. The IF signal was the reference of the HP8510 network analyser. The signal of the receiving antenna was mixed down to the IF at the second mixer. This
should be done as close as possible to the antenna and possibly without disturbing the received signal. The reason for doing that was the high attenuation of the cable at the frequency of 5GHz. The test IF was fed to the network analyser as a test signal.

The results were known to be in error through the non-linearity of the network analyser. The correct procedure would be to introduce a precision variable attenuation into the test channel which is adjusted such that the received power is the same for all measurements. Such a device was not available for this frequency. However, the network analyser was specified to have an uncertainty of less than 0.05dB at -30dB where the measurements took place.

4.6.5. Gain measurement results

A comparison between the measured and the 3-D FDTD predicted gain over a range of apex angles from 30° to 120° is shown in fig. 4.46. The error bars of ±1dB represent the accuracy of gain measurements for low gain antennas in the CATR of Queen Mary and Westfield College at 5GHz when a set-up as described is used. Again, measurement and prediction match well as long as the mismatch of the test
antenna is not too high. For values of $S_{11}$ bigger than -2dB, more than 63% of the input power is reflected towards the source, hence measurements of the gain are not very precise.

![Graph showing predicted and measured gain and $S_{11}$]

**Fig. 4.46: Gain and $S_{11}$ for different apex angles. Gain does not include mismatch loss**

### 4.6.6. Discussion of the results

This section has shown that the beam-width of the corner reflector antenna can be adapted to the needs of a communication network by altering the apex angle. The amount of change depends on the length of the plates. For a length of $2\lambda$, a variation of the beam-width of 30° was shown while the gain varied by 4.2dB. A limitation is the matching problem which arises when the apex angle is decreased below 50°. Even though it is possible to compensate mismatching with a matching network it becomes more complicated for very high $S_{11}$, especially when the matching has to be provided over a wide range of frequencies.

The good agreement between measurements and 3-D FDTD prediction validate the code again and shows that the gain prediction is valid.
4.7. Feed study

The feed used in this study has been a half wave dipole in the 3-D case and an infinite long wire for the 2-D case. An omnidirectional feed has the advantage that it supports the radiation mechanism of the corner reflector antenna. The rays reflected by the reflector plates interfere with the rays which are radiated directly from the source. Whether the interference is of constructive or destructive nature depends on the path difference which in turn depends on the dipole position relative to the apex and the apex angle. Given the complexity of the radiation mechanism of the corner reflector antenna (see section on ray tracing in this chapter) it is far from simple to determine in which way the omnidirectional pattern of a dipole has to be changed in order to achieve higher directivity. On the other hand it is well-known that a dipole has a small bandwidth. It is therefore necessary to investigate if this restricts the bandwidth of the corner reflector antenna and if so what can be done to overcome this restriction.

4.7.1. Feeds and directivity

In general there are two ways of increasing the directivity of the feed. Additional parasitic elements can be placed such that they support the guidance of the waves in the desired direction, or the dipole can be replaced by a directive source.

The problem with introducing a more directive source (i.e. a horn or a patch antenna) is blockage. The aim of the design is to get a compact corner reflector antenna and therefore the source has to be placed either inside the aperture or very close to the aperture. This places the feed in the main beam of radiation which leads to blockage as an undesired effect. For a small corner reflector antenna of the order of a few wavelengths in size the blockage caused by a directive antenna cannot be tolerated. Unlike the parabolic antenna there is no possibility of an offset mode to prevent the blockage. The idea of a directive source was not further pursued due to this reason.

Placing additional parasitic elements in the proximity of the dipole did not show the desired effect. There is ground to believe that there are combinations of parasitic
elements which could enhance the performance of the corner reflector antenna in respect of directivity since clearly not all reflection contributes towards the shaping of the main beam. The complexity of the reflections prevents an assessment on whether a systematic strategy for the placement of the parasitic elements could be narrowed down to a level where it is feasible to conduct a parametric study. This might be overcome by employing genetic algorithms as was demonstrated in the case of the Yagi antenna by Jones and Joines [88].

4.7.2. Feeds and bandwidth

It is a well known fact that the dipole has a small bandwidth since the input reactance varies with frequency. However, the rate of change of the input resistance is dependent on the diameter of the dipole. If the ratio of length to diameter decreases the bandwidth of the dipole increases up to several percent. The H-plane radiation pattern stays omnidirectional. The E-plane radiation pattern is not much influenced by increasing the diameter. The areas of intense radiation do not change, but side lobes are reduced and nulls are filled up. It is interesting to assess whether this causes a restriction on the bandwidth of the corner reflector antenna.

The apex angle stays constant if the frequency is changed. However, it is obvious that the electrical size of the plates varies proportionally with frequency. Thus if the frequency is doubled the electrical size of the plates is doubled as well. The dipole position in terms of wavelength is influenced in the same way. As was shown in fig. 2.14 a change in dipole position could lead to a split of the main beam. Fig. 2.14 is therefore an important guideline for the question where to place the dipole such that the corner reflector antenna offers a high bandwidth.

The dipole should be placed closely to the apex if high bandwidth is needed because the change in directivity shown in fig. 2.14 is much less for dipole variations in the region of $\rho_0=0.5\lambda$ than it is for dipole separation in the order of $1\lambda$. If the dipole is placed closely to the apex care has to be taken that for lower frequencies the dipole is
electrically close to the reflector plates which results in matching problems. This is especially the case for small apex angle corner reflector antennas.

In order to assess the bandwidth which is offered by the corner reflector antenna the 2-D FDTD method is best suited since it does not take into account the change of the radiation pattern in the dipole’s E-plane but merely gives a picture on how the H-plane radiation pattern is influenced.

Two corner reflector antennas were investigated. The apex angles were 60° and 90°, the dipole was 30mm from the apex and the plates were 120mm long. This corresponds to $\rho_0=0.5\lambda$ and $l=2\lambda$ at 5GHz. Analysing the results is not straightforward as two mechanisms are acting at the same time. Increasing the electrical length of the plates does also increase the H-plane directivity, as shown in section 4.2, but at the same time the change of the electrical position of the dipole influences the H-plane directivity. It was shown that increasing the length of the plates influences the corner reflector antenna with small apex angles more than those with bigger ones but still the overlaid influence of the modified dipole position has to be taken into account.

In order to be able to consider the influence of the electrical dipole position, the H-plane directivity (HPD) for corner reflector antennas for those two apex angles with plate length $2\lambda$ is given in table 4.7.

<table>
<thead>
<tr>
<th>Frequency</th>
<th>3GHz</th>
<th>4GHz</th>
<th>4.5GHz</th>
<th>5GHz</th>
<th>5.5GHz</th>
<th>6GHz</th>
<th>7GHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dipole pos.</td>
<td>0.30\lambda</td>
<td>0.40\lambda</td>
<td>0.45\lambda</td>
<td>0.50\lambda</td>
<td>0.55\lambda</td>
<td>0.60\lambda</td>
<td>0.70\lambda</td>
</tr>
<tr>
<td>HPD for $\psi=60^\circ$</td>
<td>9.7dB</td>
<td>9.7dB</td>
<td>9.7dB</td>
<td>9.7dB</td>
<td>9.7dB</td>
<td>9.7dB</td>
<td>9.7dB</td>
</tr>
<tr>
<td>HPD for $\psi=90^\circ$</td>
<td>9.4dB</td>
<td>9.5dB</td>
<td>9.5dB</td>
<td>9.6dB</td>
<td>9.8dB</td>
<td>10.0dB</td>
<td>11.0dB</td>
</tr>
</tbody>
</table>

Table 4.7: H-plane directivity for $\psi=60^\circ$ and $90^\circ$ plate length $2\lambda$ at 3GHz - 7GHz

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Table 4.7 shows that it is important to consider the fact that the H-plane directivity for a corner reflector antenna with an apex angle of 90° increases when the dipole is moved from $\rho_0=0.6\lambda$ to $\rho_0=0.7\lambda$ even if the electrical plate length is kept constant.

Fig. 4.47: H-plane pattern for $\psi=60^\circ$, $\rho_0=30\text{mm}$, $l=120\text{mm}$ at 3GHz, 5GHz and 7GHz

Fig. 4.48: H-plane pattern for $\psi=90^\circ$, $\rho_0=30\text{mm}$, $l=120\text{mm}$ at 3GHz, 5GHz and 7GHz
In fig. 4.47 and 4.48 the H-plane radiation pattern for the corner reflector antennas with the two apex angles are shown for frequencies of 3GHz, 5GHz, and 7GHz. For both corner reflector antennas it becomes obvious that the main beam becomes narrower if the frequency is increased and the radiation at higher angles is suppressed. This is to be expected since the electrical length of the plates is increased. The values for the H-plane directivity for both antennas for 3GHz, 4GHz, 4.5GHz, 5GHz, 5.5GHz, 6GHz, and 7GHz are shown in table 4.8. The H-plane directivity increases by 2.7dB for the 60° corner reflector antenna when the frequency is changed from 3GHz to 7GHz. In the case of a 90° corner reflector antenna the H-plane directivity increases for the same frequencies by 2.5dB.

![Graph](image)

**Fig. 4.49: E-plane pattern of 30mm long dipole for 3GHz, 5GHz and 3GHz**

The same antennas were modelled with a finite width of 120mm and a 30mm dipole using the 3-D FDTD method. For this model the change of the source's radiation has to be acknowledged. Varying the frequency does not influence the E-plane radiation pattern of the dipole much. The E-plane radiation characteristic for 3GHz, 5GHz, and
7GHz is shown in fig. 4.49. This should not significantly influence the E-plane radiation pattern of the corner reflector antenna.

The H-plane radiation pattern of the dipole remains omnidirectional for all frequencies so that all changes in this plane are caused by the change of the electrical size of the plates and the electrical position of the dipole relative to the apex plus the contribution of diffraction. The E-and H-plane radiation pattern for 3GHz, 5GHz and 7GHz for the two corner reflector antennas can be seen in fig. 4.50 to 4.53.

![Graph showing H-plane pattern for various frequencies](image)

*Fig. 4.50: H-plane pattern for $\psi=60^\circ$, $\rho_o=30\text{mm}$, $l=120\text{mm}$, $w=120\text{mm}$ at 3GHz, 5GHz and 7GHz*
Fig. 4.51: E-plane pattern for $\psi=60^\circ$, $\rho_0=30\text{ mm}$, $l=120\text{ mm}$, $w=120\text{ mm}$ at 3GHz, 5GHz and 7GHz

Fig. 4.52: H-plane pattern for $\psi=90^\circ$, $\rho_0=30\text{ mm}$, $l=120\text{ mm}$, $w=120\text{ mm}$ at 3GHz, 5GHz and 7GHz
It is interesting to note that the E-plane radiation pattern is more influenced by the frequency change for the corner reflector antenna with the bigger apex angle. However, the main change in the radiation of the corner reflector antennas is in the H-plane. The directivity for the finite width corner reflector antennas can be seen in table 4.8. For an apex angle of 60° the directivity increased by 3.7dB from 3GHz to 7GHz whereas for the corner reflector antenna with an apex angle of 90° the directivity decreased by 0.6dB when the frequency was increased. This is caused by a change in the E-plane radiation pattern which becomes broader with frequency and splits eventually so that the main radiation is no longer at θ=90°

The results show that the corner reflector antenna is capable of wide band if carefully designed.
Table 4.8: Directivity and H-plane directivity for corner reflector antennas with apex angle of 60° and 90° and various frequencies

Wong et al [20] - [22] investigated the possibility of optimising the impedance of the corner reflector antenna by varying the parameters of an open sleeve dipole feed. A broad band source such as the log-periodic dipole array (LPDA) was investigated by Hasan [24], Stephenson and Finley [25], and Kosat et al [26]. For both feeds the input impedance was less frequency dependent when they were in free space. However, the proximity of the plates of the corner reflector antenna reduced the bandwidth of those sources.

The design of a wide band corner reflector antenna is a compromise between the need to place the dipole close to the apex so that the directivity does not change very much and the desire to place the dipole sufficiently far away from the plates so as not to reduce the bandwidth of the source.
4.8. Summary

This chapter studied the design of the corner reflector antenna with the help of the FDTD method. The correct model of the apex region and therefore the position of the dipole within the FDTD grid was found by comparing FDTD results with results gained by Klopfenstein's method. The influence of the plates' serration due to the staircase approximation was established and proved to be negligible.

The influence of the ABCs in the two dimensional case was investigated. The results of different ABCs, Mur and Berenger, were compared with a reference gained by increasing the spatial domain so that no reflection reach the near-field to far-field surface during run-time. It was shown that the lower reflectivity of the Berenger does still yield changes in the far-field pattern but the difference is very little. The comparison showed that the Mur ABCs placed 10 cells from the structure does reduce the reflections sufficiently.

In order to establish the influence of finite wide reflector plates on the H-plane pattern of corner reflector antennas 2-D and 3-D FDTD simulation results were compared. The difference in the H-plane radiation pattern was caused by diffraction on top and bottom of the plates.

In the parametric study the maximum H-plane directivity and aperture efficiency for apex angles from 30° to 180°, plate lengths of $2\lambda$, $3\lambda$, $4\lambda$ and $10\lambda$ and dipole positions from $0.5\lambda$ to $1.5\lambda$ were found. It was established that the influence of the plate length on the H-plane radiation pattern is dependent on the apex angle as well as on the dipole position. The further the dipole is away from the apex and the smaller the apex angle the more sensitive is the H-plane radiation pattern to the plate length. The ray tracing technique was used to explain why this is the case.

An investigation of the corner reflector antenna in three dimensions included the radiation pattern in the E-and H-plane, gain and aperture efficiency. It was found that
the width of the plates did not much influence the H-plane radiation pattern. However, increasing the width of the plates can yield a reduction of gain. This is due to the fact that the diffraction on top and bottom of the plates is higher than on the front edges of the plates since the electric field vector is perpendicular to the edge. This strong diffraction changes when the width of the plates is changed and therefore influences the radiation of the corner reflector antenna. It was found that a plate width of $2\lambda$ is a local optimum.

The influence of the plate length on the H-plane radiation pattern was similar to what was established in the two dimensional case. As in the two dimensional case it was found that the smaller the apex angle and the further away the dipole is from the apex the bigger the influence on the H-plane radiation pattern. The E-plane radiation pattern is not much influenced by the plate length once the plates are longer than $2\lambda$.

The novel idea of a variable apex angle corner reflector antenna was presented together with a practical design. It was shown that the beam-width and gain of the corner reflector antenna can be varied by varying the apex angle. Measurement results were presented in order to validate the predictions.

The final point of the chapter was the question of the feed. It was explained why directive sources are not suited to enhance the performance of the corner reflector antenna. However, introducing parasitic elements could increase the gain and front-to-back ratio.

The bandwidth offered by the corner reflector antenna was presented. It was shown that for a high bandwidth of the corner reflector antenna the dipole should be placed close to the apex since then the H-plane radiation pattern is not changed much when the electrical dipole position is changed as it is the case when the frequency is varied. On the other hand it was indicated that the proximity of the plates does influence the input resistance of the feed which limits the bandwidth. This can be partly countered
by employing sleeve dipoles or log-periodic antennas which exhibit less frequency sensitive input resistance.
5. **GRIDDED CORNER REFLECTOR ANTENNA**

5.1. **Introduction**

The gridded corner reflector antenna is a corner reflector antenna where the solid plates are replaced by circular rods as can be seen in fig. 5.1. The replacement of the plates reduces the weight and the wind resistance of the antenna as well as reducing the costs of manufacturing. A drawback is that the performance of the antenna will have deteriorated. In this chapter the influence of the rod spacing on the H-plane directivity, the front-to-back ratio and the -3dB beam-width is investigated using a 2-D FDTD code written by the author and 3-D FDTD code developed at QMW. A detailed investigation on how to model round rods using the FDTD method implemented in the Cartesian co-ordinate system is undertaken. Results of measurements conducted in the CATR of QMW will be presented to verify the methods and assumptions.

![Graph of antenna](image-url)

**Fig. 5.1: Graph of antenna**
5.2. FDTD modelling of rods

This section investigates the limitations of the FDTD method in its ability to model circular structures and the minimum realisable spacing between two rods. Since the Cartesian co-ordinate system was chosen for the implementation of the FDTD algorithm the modelled structure is composed of rectangular cells in the two dimensional case and cuboids in the three dimensional. It was shown in the previous chapter that this approximation was valid for the solid corner reflector antenna. However, the question of how accurate a round rod can be modelled using a Cartesian co-ordinate system has to be investigated.

Consider the 2-D case. If the diameter of a circle is bigger than 4 cells an approximation in form of a cross can be used to model the structure. The cell size for radiation problems in FDTD is usually chosen to be \( \lambda/20 \). The diameter of the rods used to replace the solid plates does not in practice exceed \( \lambda/20 \). Therefore, throughout this study a single square cell was modelled as perfect conductor (PEC) to represent a round rod, see fig 5.2.

A way to judge how accurate the model works is to compare the radar cross section computed by FDTD with analytical results. The RCS is a measure of the power that is
returned or scattered in a given direction, normalised with respect to the power
density of the incident field, Knott et al [89]. This can be expressed as:

$$\sigma = 10 \log \left( \lim_{r \to \infty} \left( 4\pi r^2 \frac{|E'_S|^2}{|E'|^2} \right) \right)$$ (5.1)

where

- $\sigma$: RCS
- $E'_S, E'$: scattered, incident electric fields
- $r$: distance from scatterer to observation point

The unit of the RCS is one meter squared (dBsm). When the transmitter and the
receiver are at the same location the RCS is usually referred to as monostatic (or
backscattered) and it is referred to as bistatic when the transmitter and receiver are at
different locations.

5.2.1. Monostatic RCS of small sphere

In order to show that the approximation of a round rod by a square model is valid the
monostatic RCS of two small spheres was computed using the three dimensional
FDTD method. The sphere was chosen since the analytical solution of the monostatic
RCS for small spheres is (Balanis [54]):

$$RCS(\text{monostatic}) \equiv 10 \ast \log \left( \frac{9\lambda^2}{4\pi} \left( \frac{2\pi}{\lambda} a \right)^6 \right)$$ (5.2)

where:

- $a$: radius of sphere

A sphere is considered to be electrically very small if the phase change of the
incidence wave over the length of the scatterer is less than $\pi/8$ radians (22.5°) which
corresponds to a radius of the sphere of $a=\lambda/8$. 

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The two cases investigated were a $\lambda/40$ and a $\lambda/80$ cuboid. Both structures satisfy the assumption that they are electrically very small and the monostatic RCS can therefore be computed using equation (5.2).

To make use of this equation the actual size of a cell filled with a PEC has to be considered. To fill a cell with PEC the four $E_x$-field points at the corners of the cell are set to zero. However, the $E$-and $H$-field points in the FDTD grid are half a cell size apart. Since the $H$-field points next to the PEC filled cell are computed as in free space the area between those two points could be assumed to be either PEC or free space. Taflove [87] showed that the actual cell size for a PEC filled cell appears a quarter of a cell bigger at each boundary. This can be neglected when the structure under investigation is several cells thick but has to be taken into consideration when the structure consists of one cell only. In this case the cell appears to be one and a half cells big since the quarter of cell has to be added at each boundaries as shown in fig. 5.3.

![Fig. 5.3: Actual size of a PEC filled cell](image)

When considering the cuboid structure it is necessary to compute the radius of a sphere with equivalent volume. It was shown by Knott et al [89] that: "For low-
frequency scattering, the entire body participates in the scattering process. Details of the shape are not important and, therefore, only a basic or crude geometric description is required because the volume of the scatterer is important.

In table 5.1 the predicted 3-D FDTD monostatic RCS is compared with the analytical values. In order to show the significance of the actual cell size the analytical value is given for two radii. The first value is for a sphere with the equivalent volume of a cuboid with each side one cell long (case 1). The second value is for a sphere were the volume is such that it corresponds to a cuboid of one and a half cell size side length (case 2).

<table>
<thead>
<tr>
<th>Cell size</th>
<th>FDTD predicted</th>
<th>analytical (case 1)</th>
<th>analytical (case 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda/40$</td>
<td>-61.1dBsm</td>
<td>-71.0dBsm</td>
<td>-60.4dBsm</td>
</tr>
<tr>
<td>$\lambda/80$</td>
<td>-78.6dBsm</td>
<td>-89.0dBsm</td>
<td>-78.5dBsm</td>
</tr>
</tbody>
</table>

Table 5.1: Monostatic RCS for small spheres, comparison analytical and FDTD predicted

The results make two facts obvious. For the problem considered it is crucial to take the actual size of the filled cell into account. The monostatic RCS of the analytical solution increased by almost 10dBsm when the bigger radius was considered. The monostatic RCS values for case 2 are in very good agreement to the FDTD predicted values. This shows that the statement of Knott et al about the significance of the volume of the scatterer is valid for the investigated case. Since the RCS of a cuboid model is in good agreement to the RCS of a small sphere it can be assumed that it reflects incident waves in the same way as a sphere.

5.2.2. Bistatic RCS of a rod

The bistatic RCS of a round rod is constant for different angles of incidence. If the round rod is approximated by a cuboid rod the bistatic RCS is bound to vary if the
angle of incidence is changed. The amount of variation is another indication of the error which is introduced by the approximation.

Fig. 5.4: Comparison of RCS for a cuboid at different angles of incidence

Fig. 5.4 shows the comparison of the RCS for a TM polarised wave hitting a rod of $l=2\lambda$ and a diameter of $0.085\lambda$ (cell size $\lambda/20$). The angle of incidence was $0^{\circ}$ (solid line) and $45^{\circ}$ (dashed line). The rod was modelled using a cuboid as an approximation of the cylinder. The wave hits the rod parallel to one plane face if the angle of incidence is $0^{\circ}$ and at a corner of the cuboid if the angle of incidence is $45^{\circ}$. These two cases represent the two extremes and it can be assumed that the maximum difference in bistatic RCS for any combination of incident angles occurs between these two cases.

The scattering in forward direction differs for the two cases by $0.08\text{dB}$. The forward scattering for a incidence angle of $0^{\circ}$ is at $180^{\circ}$ and for $45^{\circ}$ incidence it is at $225^{\circ}$. Since the bistatic RCS patterns are very similar for the two cases it is valid to conclude that the error introduced by the approximation of the rods in the model of the gridded corner reflector antenna can be neglected.
An inspection of fig 5.2 gives insight into the phase distortion caused by different path lengths. A square of 0.075\(\lambda\) side length (i.e. 0.05\(\lambda\) plus a quarter of cell size added either side) has the same area as a circle of \(a=0.0423\lambda\). The maximum difference from the centre to the surface occurs in direction of the corner and is 0.0107\(\lambda\) which corresponds to a phase error of 3.8°.

For a reflected wave this would add up to an maximum error of 7.7° which is not expected to influence the interference pattern significantly.

The results for the monostatic RCS for the sphere and the bistatic RCS for the rod both indicate that the error introduced by modelling a round rod using a cuboid approximation is small. If the reflected waves from a round rod and a cuboid rod are almost the same it is valid to assume that a gridded corner reflector antenna with round rods can be modelled using a cuboid approximation on the condition that the cell size is chosen according to the previous investigations.

In conformation of this result, Furse et al [90] reported that the RCS for a perfect conducting infinite cylinder can be computed within an error of 10% using a cell size between \(\lambda/20\) and \(\lambda/30\) for a TM polarised wave. To achieve the same precision in the TE case a minimum cell size of \(\lambda/30\) must be employed.

5.2.3. Minimum distance between rods

Fig. 5.5 shows a section of a 2-D FDTD grid where on each intersection of lines the \(E_x\)-field points are placed. The dots indicate those field points which are set to zero. According to electromagnetic theory the tangential \(E\)-fields at a perfect conductor are zero. Therefore setting those field points to zero makes them appear as field points on the surface of a perfect conductor. Having set the four corners of a square to zero the area enclosed is assumed to be filled with a perfect conductor. As previous described the actual size of the perfect conductor appears to be a quarter of a cell size bigger on each boundary.
To introduce a gap between a row of filled squares there has to be at least one 2-D column of $E_x$-field points which are non zero and computed using the FDTD algorithm. In doing so there will be a gap of two cells which can be seen in fig. 5.5. Since the spacing between the rods is measured from centre to centre the minimum spacing which can be modelled with FDTD is three cells.

This is not important since a rod separation of only two cells is not a practical solution to the problem. In the parametric study later in this chapter it is shown that the interesting region is a separation of several cells between the rods.

### 5.3. Experimental verification of the methods

The FDTD simulations were verified by constructing an experimental model and measuring the radiation patterns at 5GHz ($\lambda=60$mm) in the CATR of QMW. The grid spacing was larger than might be used in practice, but served to emphasise the characteristics of a gridded corner reflector antenna.

#### 5.3.1. Design of the gridded corner reflector antenna

The gridded corner reflector antenna with an apex angle of 60° and a dipole spacing of 30mm (0.5$\lambda$) was constructed in such a way that the rods could be moved. The spacing of the rods could be either 15 mm (0.25$\lambda$), 30mm (0.5$\lambda$) or irregular. Each
rod was 3mm (0.05\(\lambda\)) in diameter. The height and the length of the 'plates' were 120mm (2\(\lambda\)). A frame to support the rods was constructed using perspex with an \(\varepsilon_r=2.5\). The dielectric frame was not symmetric in respect to the dipole and caused a slight asymmetry in the E-plane radiation pattern. A picture of the antenna is shown in fig. 5.6.

\begin{figure}
\centering
\includegraphics[width=0.5\textwidth]{image}
\caption{Picture of gridded corner reflector antenna}
\end{figure}

The FDTD simulations were conducted using the code described in chapter 3. In order to model the size of the rods according to the considerations in section 5.2 the cells size was chosen to be 2mm (\(\lambda/30\)). Since the cell size determines the time stepping, the number of time steps had to be increased compared to previous simulations where the cell size was \(\lambda/20\). This ensures that the simulation time stays constant and the fields can settle down. The rods were modelled by enforcing four \(E_x\)-field points to zero as discussed in section 5.2. The parameters of the FDTD simulation are shown in table 5.2.
### Table 5.2: Parameters of FDTD simulation of gridded corner reflector

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Apex angle</td>
<td>$60^\circ$</td>
</tr>
<tr>
<td>Dipole distance</td>
<td>$0.5\lambda$</td>
</tr>
<tr>
<td>Plate lengths</td>
<td>$2\lambda$</td>
</tr>
<tr>
<td>Plate widths</td>
<td>$2\lambda$</td>
</tr>
<tr>
<td>Grid size (x, y, z)</td>
<td>90 90 84</td>
</tr>
<tr>
<td>Cell size (cubic)</td>
<td>$\lambda/30$</td>
</tr>
<tr>
<td>Sources</td>
<td>Two x-directed sinusoidal gap sources</td>
</tr>
<tr>
<td>Frequency</td>
<td>5GHz</td>
</tr>
<tr>
<td>Position of source (y, z)</td>
<td>(45, 30)</td>
</tr>
<tr>
<td>Position of apex (y, z)</td>
<td>(45, 15)</td>
</tr>
<tr>
<td>Time step</td>
<td>3.5ps</td>
</tr>
<tr>
<td>Number of time steps</td>
<td>675</td>
</tr>
<tr>
<td>Time steps per cycle</td>
<td>57.8</td>
</tr>
<tr>
<td>Number of cycles</td>
<td>11.7</td>
</tr>
<tr>
<td>NF-FF surface (x, y, z)</td>
<td>(13-77, 13-77, 13-71)</td>
</tr>
</tbody>
</table>

The comparison between the predicted and the measured radiation pattern for H- and E-plane for $0.25\lambda$ spacing of the rods is shown in fig. 5.7 and 5.8 respectively.

The agreement is good. The gain is 13.8dBi which is 0.4dB lower than in the case of equivalent size solid plates.

Based on the results shown it can be concluded that the gridded corner reflector antenna can be modelled using an FDTD algorithm implemented in the Cartesian coordinate system.
Fig. 5.7: H-plane radiation pattern for $\Psi=60^\circ$, $\rho_0=0.5\lambda$, $l=2\lambda$, $w=2\lambda$, rod diameter $0.05\lambda$ and rod separation of $0.25\lambda$.

Fig. 5.8: E-plane radiation pattern for $\Psi=60^\circ$, $\rho_0=0.5\lambda$, $l=2\lambda$, $w=2\lambda$, rod diameter $0.05\lambda$ and rod separation of $0.25\lambda$. 
5.4. Comparison of gridded- solid- and two dimensional solid corner reflector antenna

A comparison between the radiation pattern of a solid and a gridded corner reflector antenna shows in which way the radiation patterns are changed when the solid plates are replaced by rods. This is interesting in order to see if the gridded corner reflector antenna is a practical alternative to its solid counterpart.

The comparison of the H- and E-plane radiation pattern between the solid and the gridded reflector for the case of a separation of 0.25\(\lambda\) and a rod size of 0.05\(\lambda\) can be seen in fig. 5.9 and 5.10 respectively. Note, the data for the gridded corner reflector antenna are from a model without the perspex frame.

![Graph of H-plane radiation pattern comparison](image)

**Fig. 5.9:** H-plane radiation pattern comparison for \(\psi=60^\circ\), \(\rho_0=0.5\lambda\), \(l=2\lambda\), \(w=2\lambda\) between solid case with rods 0.05\(\lambda\) in diameter and rod separation of 0.25\(\lambda\).

The main beam of the H-plane radiation pattern is almost not influenced. The slight broadening indicates that the guidance of the waves is weaker as power penetrates
through the grid. The increase of the side-lobes and especially the level in the back region is also due to the penetration of power through the grid.

Fig. 5.10: E-plane radiation pattern comparison for $\psi=60^\circ$, $p_0=0.5\lambda$, $l=2\lambda$, $w=2\lambda$ between solid case with rods $0.05\lambda$ in diameter and rod separation of $0.25\lambda$.

Fig. 5.10 shows that the influence of the grid on the E-plane radiation pattern is negligible out to about $140^\circ$.

In chapter 4 the difference in the H-plane radiation pattern between a two- and three dimensional model was investigated. It was found that the difference between the two radiation patterns are caused by diffraction on top and bottom of the reflector plates. If this is the case this difference should be smaller when the solid plates are replaced by rods.

A comparison between measured-, 3-D FDTD-, and 2-D FDTD data for a corner reflector antenna with $\psi=60^\circ$, $p_0=0.5\lambda$, $l=2\lambda$, $w=2\lambda$ (infinite in case of 2-D FDTD). is shown in fig. 5.11. The rods are $0.05\lambda$ in diameter and the rod separation is $0.25\lambda$. 

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The match between the 3-D FDTD- and 2-D FDTD result is better than it was in the case of solid plates. This confirms the assumption that the difference between 2-D and 3-D FDTD is mainly due to the diffraction of the top and bottom of the plates. Once the penetration of power through the grid governs the diffraction on the top and bottom of the rods the difference is smaller and therefore the results are becoming more similar.

5.5. Band-width considerations

In chapter 4 a thorough investigation on the band-width for the solid corner reflector antenna is presented. However, the gridded corner reflector antenna has an additional source of influence on the band-width which has to be addressed. The rod separation and diameter varies with frequency.
Despite the fact that the physical size of the antenna is constant the electrical size of the antenna varies if the frequency is changed. Decreasing the frequency by 5% causes the electrical size to be 5% smaller.

In order to investigate the bandwidth performance, measurements were conducted changing the frequency by ±5%. Table 5.3 gives the values of the front-to-back (F/B), side lobe (SL) level, and the -3dB beam-width (BW) for the two different cases of rod separation at the three frequencies.

<table>
<thead>
<tr>
<th>Separ.</th>
<th>4.75GHz</th>
<th>5GHz</th>
<th>5.25GHz</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>BW</td>
<td>F/B</td>
<td>SL</td>
</tr>
<tr>
<td>15mm</td>
<td>35°</td>
<td>-30dB</td>
<td>-16dB</td>
</tr>
<tr>
<td>30mm</td>
<td>37°</td>
<td>-11dB</td>
<td>-5dB</td>
</tr>
</tbody>
</table>

Table 5.3: -3dB beam-width, front-to-back ratio and side lobe level for $\psi=60^\circ$, $\rho_0=0.5\lambda$, $l=2\lambda$, $w=2\lambda$ with rods $0.05\lambda$ in diameter and a rod separation of $0.25\lambda$ and $0.5\lambda$ (at 5GHz)

The front-to-back ratio decreased when the frequency was increased due to the electrically larger separation of the rods. At the same time the electrical length of the 'plates' increased with frequency. As shown in chapter 4 longer plates results in a narrower beam which ties in with what can be observed here for increasing frequency. There is no apparent impact on the side-lobes.

5.6. Influence of rod spacing

It is conceivable that a non-uniform distribution of rods could be beneficial. For example a higher front-to-back ratio could be achieved by a small rod separation close to the apex and larger separation further away from the apex. This could yield a reduction of the number of rods for a required specification.
In this section several rod arrangements which were measured are discussed. The arrangements are defined in fig. 5.12. The gridded corner reflector antenna under consideration is the same as in the previous sections.

![Graph showing rod arrangement](image)

**Fig. 5.12 : Arrangement of the rods for a 60° corner reflector antenna**

Six different arrangements were measured. Rods 1 and 9 were permanently placed. The arrangements were:

- **A** is without rod 6, 8
- **B** is without rod 4, 6, 8
- **C** is without rod 3, 6, 8
- **D** is without rod 4, 5, 7, 8
- **E** with all rods
- **F** without 2, 4, 6, 8
For orientation it should be noted that arrangement E corresponds with uniform spacing of the rods by 0.25\(\lambda\) and arrangement F corresponded to a uniform spacing of 0.5\(\lambda\). The results for the -3dB beam-width, front-to-back ratio and the side lobe level for the different arrangements are given in table 5.4. To give a complete picture the data for a solid corner reflector antenna is also given.

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>Solid</th>
</tr>
</thead>
<tbody>
<tr>
<td>BW</td>
<td>37°</td>
<td>36°</td>
<td>33°</td>
<td>43°</td>
<td>34°</td>
<td>33°</td>
<td>33°</td>
</tr>
<tr>
<td>F/B</td>
<td>-17dB</td>
<td>-17dB</td>
<td>-11dB</td>
<td>-16dB</td>
<td>-22dB</td>
<td>-9dB</td>
<td>-48dB</td>
</tr>
<tr>
<td>SL</td>
<td>-16dB</td>
<td>-10dB</td>
<td>-9dB</td>
<td>-7dB</td>
<td>-15dB</td>
<td>-5dB</td>
<td>-20dB</td>
</tr>
</tbody>
</table>

Table 5.4: -3dB beam-width, front-to-back ratio and side lobe level for \(\psi=60^\circ\), \(\rho_0=0.5\lambda\), \(l=2\lambda\), \(w=2\lambda\) with rods \(0.05\lambda\) in diameter and different arrangements of rods

Arrangement A shows that the rods at the end opposite the apex were also important for the front-to-back ratio. The front-to-back ratio dropped by 5dB compared to the scenario when all rods were placed. Most surprisingly is the effect of arrangement C. The main beam was sharper if a rod was taken away compared to arrangement A, however, the side lobe level was increasing and the front-to-back level dropped. Arrangement C had, together with arrangement F, the lowest front-to-back ratio which shows that rod 2 and 3 were crucial for the radiation into the back region.

Arrangement B was the uniform spacing of 0.5\(\lambda\) plus rod number two. This left the main beam almost the same (beam-width increases by 3") but decreased the level of side lobes by 5dB and increased the front-to-back ratio by 8dB.

Arrangement D has a higher front-to-back ratio as arrangement F. This proved again what was found for arrangement C, rods 2 and 3 were the important rods for the front-to-back ratio.

It is clear from those results as well as from the ray tracing in chapter 4 that the rods near to the apex have a higher influence on the patterns. Rod number 4 is important
for the beam shaping and rods number 2 and 3 are crucial for the front-to-back ratio. The significance of the individual rods varies if the dipole is moved or the apex angle is changed.

Comparing the results of the different arrangements with the solid corner reflector antenna shows that the main beam can be maintained with several combinations (C, E, F) but the front-to-back ratio and the side lobe level varies. It depends on the required specification which arrangement suits the purpose best.

5.7. Parametric study of H-plane directivity and front-to-back ratio

A parametric study was conducted to establish the influence of the spacing of rods on the H-plane radiation pattern. Four antennas were investigated. The characteristics of the corner reflector antennas were an apex angle of 60° with \( \rho_o = 0.5\lambda \), 1.2\( \lambda \) and an apex angle of 90° with \( \rho_o = 0.5\lambda \), 1.4\( \lambda \). The length of the 'plates' was as close to 2\( \lambda \) as possible for given rod separation. In the 2-D FDTD model the width of the rods is assumed to be infinite. The cell size was chosen to be \( \lambda/40 \) (\( \lambda/60 \)) and the rods were modelled by filling one cell with PEC which corresponded to a radius of 0.021\( \lambda \) (0.014\( \lambda \)) respectively. The separation was varied in discrete steps of 0.025\( \lambda \) (0.017\( \lambda \)) starting from 0.075\( \lambda \) (0.05\( \lambda \)), which was the smallest possible separation (see fig. 5.5) to 0.7\( \lambda \). The first point for zero separation was the case of solid plates.

To minimise the reflections from the Mur ABCs the distance between the corner reflector antenna and the ABC was 60 cells for a cell size of \( \lambda/40 \) and 90 cells for the smaller cell size of \( \lambda/60 \). This leaves the distance between the corner reflector antenna and the ABCs at 1.5\( \lambda \) for both cases. In order to compensate for the shorter time steps due to the smaller cell size the number of time steps had to be increased from 600 to 900 to simulate the same period of time.
Fig. 5.13: H-plane directivity and front-to-back ratio for $\psi=60^\circ$, $\rho_0=0.5\lambda$, $l=2\lambda$ with rods of radius 1.25mm, 0.85mm and various separation

Fig. 5.14: H-plane directivity and front-to-back ratio for $\psi=60^\circ$, $\rho_0=1.2\lambda$, $l=2\lambda$ with rods of radius 1.25mm, 0.85mm and various separation
Fig. 5.15: H-plane directivity and front-to-back ratio for $\psi=90^\circ$, $\rho_o=0.5\lambda$, $l=2\lambda$ with rods of radius 1.25mm, 0.85mm and various separation

Fig. 5.16: H-plane directivity and front-to-back ratio for $\psi=90^\circ$, $\rho_o=1.4\lambda$, $l=2\lambda$ with rods of radius 1.25mm, 0.85mm and various separation
The angles and dipole positions were chosen to give a comparison between different scenarios where the distance between the 'plates' and the dipole is different. For an apex angle of 90° the dipole is further from the 'plates' than for the 60° corner reflector antenna with the same dipole position. The dipole for the 90° corner reflector antenna was slightly further from the apex at 1.4λ to avoid the splitting of the main beam which occurs at 1.2λ. The results of the H-plane directivity and the front-to-back ratio can be seen in fig. 5.13 -5.16.

From the data presented it is clear that the front-to-back ratio is much more sensitive to the rod separation than the H-plane directivity. It is therefore conceivable that this parameter is the restricting factor for most applications.

The size of the rods is also important for the performance of the antenna. Increasing the rod size allows a bigger separation to achieve the same performance as with a smaller separation in case of thinner rods.

It seems to be surprising that the H-plane directivity of the 90° corner reflector antenna with the dipole 1.4λ from the apex increases when the rods separation is increased. To investigate the reason behind the phenomena the near-field cuts from the FDTD method were plotted. They show how the field propagated through the space between the rods.

In figures 5.17 a-c the near-field cut of the 90° corner reflector antenna with ρ₀=1.4λ is shown. Fig. 5.17a shows the near-field for a solid corner reflector antenna. The radius of the rods is 0.02λ and the separation was increased to 0.25λ (fig. 5.17b) and to 0.4λ (fig. 5.17c). The cell size was λ/40. To be able to observe details of the fields in the shadow region of the 'plates' the scale was chosen so that all Ex values >30% of the maximum are represented with the colour red. The scale for all three figures is shown on the left hand side of figure 5.17a.
Fig. 5.17: Near-field cut for $\psi=90^\circ$, $\rho_0=1.4\lambda$, $l=2\lambda$ with rods of radius 1.25mm separated by a) $0\lambda$, b) $0.25\lambda$ and c) $0.4\lambda$. 
The fig. 5.17a shows that in the case of solid plates there is virtually no $E_X$-fields in the shadow region of the plates. The fields which do propagate into the shadow region are due to diffraction at the ending of the plates. The two lobes just inside the plates result in side lobes in the far-field pattern which is shown in fig. 5.18. If the solid plates were replaced by rods which were $0.25\lambda$ apart the fields could propagate through the space in-between the rods which reduced the energy in the two lobes just inside the 'plates' as shown in fig. 5.17b. This results in a reduction of the side lobe level as shown in fig. 5.18 which in return increased the H-plane directivity. The rods can be identified as the purple dots at the positions where in the previous figure the plates have been.

![Graph showing comparison of radiation patterns](image)

**Fig. 5.18: Comparison of H-plane radiation pattern for $\psi=90^\circ$, $\rho_o=1.2\lambda$, $l=2\lambda$ with rods of radius 0.02$\lambda$ and 0.25$\lambda$ separated and solid plates**

If the separation between the rods is increased to $0.4\lambda$ the propagation of the $E_X$-fields into the shadow region of the 'plates' became more obvious, fig. 5.17c. It is clear from this picture that this separation of the rods was too far apart and there is a leakage of power between the rod at the apex and its neighbours. Between those and the next...
rods there seems to be no propagation. This indicates that a reasonable increase of the front-to-back ratio could be achieved by placing an additional rod between the first rod at the apex and its neighbours. The near-field pictures can be utilised in order to find the location where additional rods would be most useful.

From the previous H-plane radiation pattern it becomes obvious that the main lobe is not influenced very much by the replacement of the solid plates. The main change of the pattern is at those angles which are in the shadow region of the plates. The influence is summarised for four corner reflectors in tables 5.5 and 5.6. The H-plane directivity (HPD), the front-to-back ratio (F/B) and the -3dB beam-width (BW) for the corner reflector antennas with solid plates and rod replacement are listed.

<table>
<thead>
<tr>
<th>( \psi=60^\circ, \rho_0=0.5\lambda, l=2\lambda )</th>
<th>( \psi=60^\circ, \rho_0=1.2\lambda, l=2\lambda )</th>
</tr>
</thead>
<tbody>
<tr>
<td>solid</td>
<td>(0.25\lambda) sep.</td>
</tr>
<tr>
<td>HPD</td>
<td>9.7dB</td>
</tr>
<tr>
<td>F/B</td>
<td>-48.3dB</td>
</tr>
<tr>
<td>BW</td>
<td>36°</td>
</tr>
</tbody>
</table>

Table 5.5: Comparison of H-plane directivity, front-to-back ratio and -3dB beam-width for solid and gridded corner reflector antenna with \( \psi=60^\circ \)

<table>
<thead>
<tr>
<th>( \psi=90^\circ, \rho_0=0.5\lambda, l=2\lambda )</th>
<th>( \psi=90^\circ, \rho_0=1.4\lambda, l=2\lambda )</th>
</tr>
</thead>
<tbody>
<tr>
<td>solid</td>
<td>(0.25\lambda) sep.</td>
</tr>
<tr>
<td>HPD</td>
<td>9.6dB</td>
</tr>
<tr>
<td>F/B</td>
<td>-55.8dB</td>
</tr>
<tr>
<td>BW</td>
<td>36°</td>
</tr>
</tbody>
</table>

Table 5.6: Comparison of H-plane directivity, front-to-back ratio and -3dB beam-width for solid and gridded corner reflector antenna with \( \psi=90^\circ \)

The tables 5.5 and 5.6 show that the H-plane directivity of the corner reflector antennas does not decrease by more than 1dB when the rods are used but the front-to-back ratio decreases by up to 35.6dB. The -3dB beam-width does not vary by more
than 4° which underlines that the main beam is not influenced to the same degree as the front-to-back ratio.

5.8. Summary

The influence of the replacement of the solid plates by rods have been investigated using 2-D and 3-D FDTD. In a first step the minimum spacing which can be realised using FDTD was determined and the aspect of modelling small circular structures using a rectangular FDTD grid was investigated.

The 2-D and 3-D FDTD results have been verified by measurements conducted in the CATR at QMW. The agreement between the simulation and the measurements were very good.

The influence of frequency change was investigated with the view of band-width considerations. It was shown that the front-to-back ratio is very sensitive to a variation of the frequency. This is caused by the change of electrical separation between the rods. The higher the frequency the more energy can propagate through the spacing between the rods.

Several rod arrangements were measured to emphasise the contribution of single elements to the pattern shaping. The possibility of near-field cuts proved to be a very good tool to localise where additional rods have to be placed to achieve a higher front-to-back ratio.

The predictions of the H-plane directivity, front-to-back ratio and the -3dB beam-width of four different corner reflector antennas for two different rod sizes and various spacing were presented. The data indicate that the H-plane directivity of the corner reflector antennas is less influenced by the replacement of the solid plates than the front-to-back ratio.
6. SHAPING OF THE REFLECTOR PLATES

In this chapter the performance of the corner reflector antenna is compared with two other shapes for the reflector plates. The objective of the investigation is to establish if either an improvement in the performance of the corner reflector antenna by shaping the reflector plates can be achieved or the overall depth of the corner reflector antenna can be reduced.

There are two main alternatives to the flat reflector shape of the corner reflector. The first alternative is to replace the apex by a flat plate at right angle to the axis. This reduces the overall depth of the reflector. However the performance of the antenna deteriorates when the apex is replaced. The other alternative is the parabolic reflector antenna. The parabolic reflector is the most common reflector antenna and is widely used in satellite communication and directional radio systems. In those applications the aperture of the reflector is usually electrically large. In this chapter the similarities between the corner reflector and the parabolic reflector in both, geometry and performance, for small aperture sizes are investigated.

For applications where the H-plane directivity does not have to be higher than 8dB but a high front-to-back ratio is needed a box shaped reflector is a possible option.

6.1. Modified corner reflector antenna

In order to reduce the depth of the corner reflector antenna the apex can be replaced by a flat plate. This arrangement is called here the *modified corner reflector antenna*.

6.1.1. Geometry of modified corner reflector antenna

A comparison between the geometry of a modified and a standard corner reflector is shown in fig. 6.1. The additional plate creates a virtual apex which is indicated by the dotted line. The apex angle in both cases is 90° and the plate length is $3\lambda$ for the standard corner reflector. The flat plate is mounted $0.75\lambda$ from the virtual apex and
thereby the overall depth of the reflector is reduced by $0.75\lambda$. The distance is measured from the virtual apex in the direction of the dipole along the $z$-axis and is called $t$. It follows that for different apex angles the width of the plate will be different for the same $t$ (i.e. for $\psi=60^\circ$ the width will be smaller than for $\psi=90^\circ$). The length of the reflector plates is measured from the virtual apex of the corner reflector. Therefore the actual length of the modified corner reflector plates reduce by:

$$\text{reduction} = \frac{t}{\cos\left(\frac{\psi}{2}\right)}$$

(6.1)

The reduction in actual length is bigger if $\psi$ is bigger. The length is defined this way so as to keep the aperture width constant as $t$ is varied. The benefit from modifying the reflector is a reduction in depth and a possible easier option to mount the antenna onto a mast.

Fig. 6.1: Sketch of a modified corner reflector

6.1.2. H-plane directivity and front-to-back ratio for modified corner reflector antenna

The two figures of interest in this investigation are the front-to-back ratio and the H-plane directivity. The aim of the investigation was to find designs of modified corner
reflectors which do not deteriorate the H-plane directivity and should not decrease the front-to-back ratio.

In order to be able to introduce an additional plate the dipole has to be some distance away from the apex. In chapter 4 it is shown that the further the dipole is away from the apex the longer the plates to achieve guidance of the waves. Therefore, the investigation was conducted for corner reflector antennas with apex angles of 60° and 90° and dipole positions of 1.0λ and 1.5λ. The length of the plates was chosen to be 3λ and 5λ. The aperture widths for the investigated corner reflector antennas are shown in table 6.1.

<table>
<thead>
<tr>
<th>Apex angle 60°</th>
<th>Apex angle 90°</th>
</tr>
</thead>
<tbody>
<tr>
<td>l=3λ, 1=5λ</td>
<td>l=3λ, 1=5λ</td>
</tr>
<tr>
<td>Aperture width</td>
<td>3λ, 5λ</td>
</tr>
</tbody>
</table>

Table 6.1: Aperture width for corner reflector of different apex angle

The distance of the additional plate l is increased in steps of 0.1λ starting from 0.0λ (standard corner reflector antenna) to within 0.1λ of the dipole. The impedance of the antenna falls drastically when the dipole is closer than 0.2λ from an infinite plane electric conductor, Balanis [52]. This gives a matching problem since the smaller the impedance the more difficult it becomes to match the antenna over a wide frequency range.

In fig. 6.2 and 6.3 the H-plane directivity for pO=1λ, and 1.5λ respectively for apex angles of 60°, and 90° and different l are shown. The plate length is varied from 3λ to 5λ.
The results show that it is possible to achieve a high H-plane directivity with a modified reflector if $t$ is chosen carefully. For the dipole position further away from
the apex the H-plane directivity exhibits a periodic behaviour with a period of roughly 0.5λ. For instance in the case of ρ₀=1.5λ and ψ=90° there is a drop in the H-plane directivity at around t=0.5λ, it then rises towards t=0.8λ, and drops at t=1.0λ. At these positions the additional plate is 1.0λ, 0.7λ and 0.5λ away from the dipole. To establish why there are positions of the additional plate which do not change, the H-plane radiation pattern the rays within the reflector are investigated, fig. 6.4.

The angle under which the ray starts off from the source is called β (initial radiation angle). The ray bounces off the plate and hits the other plate. This is repeated until the leaving angle α is smaller than the apex angle. The leaving angle relates to the initial radiation angle as follows:

\[ α_n = \frac{\psi}{2} - \left(180° - \frac{2n - 1}{2} \psi - β\right) \]  

(6.2)

where:

n: number of reflections
If
\[ \frac{2n-1}{2} \psi = 180^\circ - \frac{\psi}{2} \]
\[ n = \frac{180^\circ}{\psi} \]  \hspace{1cm} (6.3)

then the leaving angle of the last reflection is equal to the initial radiation angle
\[ \alpha = \beta \]  \hspace{1cm} (6.4)

Therefore, if an additional plate is placed halfway between the dipole and the apex, the leaving angle \( \alpha \) is maintained for all rays but the path length changes for those rays which are reflected by the additional plates. This can be interpreted as a rearrangement of the images. The reason why the optimum position of the additional plate for a corner reflector antenna with an apex angle of \( \psi = 60^\circ \) is not halfway between the apex and the dipole is due to the fact that the resulting image behind the reflector is out of phase. However, for the case of an 90\(^\circ\) apex angle the image is in phase and the optimal position is found to be halfway between the dipole and the apex. The different phase of the image behind the apex results in a shift of the optimum position of 0.25\(\lambda\) as can be seen in fig. 6.3.

<table>
<thead>
<tr>
<th>( \psi = 60^\circ )</th>
<th>( \rho_0 = 1.0\lambda )</th>
<th>( \rho_0 = 1.5\lambda )</th>
</tr>
</thead>
<tbody>
<tr>
<td>HPD. st.</td>
<td>6.9dB</td>
<td>10.3dB</td>
</tr>
<tr>
<td>f/b st.</td>
<td>-47.4dB</td>
<td>-38.4dB</td>
</tr>
<tr>
<td>t</td>
<td>0.9(\lambda)</td>
<td>1.0(\lambda)</td>
</tr>
<tr>
<td>HPD mod.</td>
<td>11.4dB</td>
<td>11.6dB</td>
</tr>
<tr>
<td>f/b mod.</td>
<td>-42.9dB</td>
<td>-41.8dB</td>
</tr>
<tr>
<td>Reduction of plate length</td>
<td>1.1(\lambda)</td>
<td>1.2(\lambda)</td>
</tr>
<tr>
<td>Width of add. plate</td>
<td>1.0(\lambda)</td>
<td>1.2(\lambda)</td>
</tr>
</tbody>
</table>

Table 6.2: Summary of results for standard and modified corner reflector antennas with \( \psi = 60^\circ \)
Table 6.3: Summary of results for standard and modified corner reflector antennas with $\psi=90^\circ$

<table>
<thead>
<tr>
<th>$\psi=90^\circ$</th>
<th>$\rho_O=1.0\lambda$</th>
<th>$\rho_O=1.0\lambda$</th>
<th>$\rho_O=1.5\lambda$</th>
<th>$\rho_O=1.5\lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$l=3\lambda$</td>
<td>$l=5\lambda$</td>
<td>$l=3\lambda$</td>
<td>$l=5\lambda$</td>
</tr>
<tr>
<td>HPD st.</td>
<td>-13dB</td>
<td>-20.6dB</td>
<td>13.3dB</td>
<td>13.8dB</td>
</tr>
<tr>
<td>f/b st.</td>
<td>-43.1dB</td>
<td>-49.7dB</td>
<td>-41.5dB</td>
<td>-49.6dB</td>
</tr>
<tr>
<td>$t$ ($\lambda$)</td>
<td>0.4$\lambda$</td>
<td>0.4$\lambda$</td>
<td>0.8$\lambda$</td>
<td>0.8$\lambda$</td>
</tr>
<tr>
<td>HPD mod.</td>
<td>10.6dB</td>
<td>9.8dB</td>
<td>13.3dB</td>
<td>13.3dB</td>
</tr>
<tr>
<td>f/b mod.</td>
<td>-47.6dB</td>
<td>-41.8dB</td>
<td>-42.3dB</td>
<td>-46.7dB</td>
</tr>
<tr>
<td>Reduction of plate length</td>
<td>0.6$\lambda$</td>
<td>0.6$\lambda$</td>
<td>1.1$\lambda$</td>
<td>1.1$\lambda$</td>
</tr>
<tr>
<td>Width of add. plate</td>
<td>0.8$\lambda$</td>
<td>0.8$\lambda$</td>
<td>1.6$\lambda$</td>
<td>1.6$\lambda$</td>
</tr>
</tbody>
</table>

A closer look on the results for the H-plane directivity (HPD) and the front-to-back ratio (f/b) for the standard (st.) and the modified (mod.) corner reflector antennas is given in table 6.2 and 6.3 for $\psi=60^\circ$ and $\psi=90^\circ$, respectively. The data for the modified corner reflector is given for the optimum position of the additional plate to achieve a high H-plane directivity. The reduction of the actual plate length and the width of the additional plate is also given.

The data shows that a dipole position of $\rho_O=1.5\lambda$ gives a higher H-plane directivity compared to a dipole position of $\rho_O=1.0\lambda$. Therefore, the following discussion of the data is reduced to designs with $\rho_O=1.5\lambda$.

For the corner reflector with an apex angle of $60^\circ$ the introduction of the additional plate increases the H-plane directivity and the front-to-back ratio. Increasing the plate length from $3\lambda$ to $5\lambda$ does improve the performance of the standard and modified corner reflector antenna. In order to achieve a high H-plane directivity the additional plate has be introduced $1.0\lambda$ from the virtual apex. For this position of the additional plate the reduction in the actual plate length is $1.2\lambda$ and the width of the added plate is $1.2\lambda$.  

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For the corner reflector with an apex angle of 90° the introduction of the additional plate means that the H-plane directivity keeps constant for a plate length of 3λ and decreases by 0.5dB for a plate length of 5λ. The front-to-back ratio improves by 0.8dB for the shorter plate length and deteriorates by 2.9dB for the longer plate length. Increasing the length of the plate increases the H-plane directivity of the standard corner reflector antenna by 0.5dB and the front-to-back ratio by 8.1dB. For the modified corner reflector antenna the same change in plate length has no influence on the H-plane directivity but the front-to-back ratio increases by 4.4dB. In order to achieve a high H-plane directivity the additional plate has be introduced 1.1λ from the virtual apex. For this position of the additional plate the reduction of the actual plate length is 1.1λ and the width of the additional plate is 1.6λ.

6.1.3. H-plane radiation pattern of modified corner reflector antenna

In order to assess if the modified corner reflector antenna is suitable for mobile communication applications the H-plane radiation pattern is investigated. Fig. 6.5 shows the H-plane radiation pattern for ψ=60° ρ0=1.5λ with and without an additional plate at t=1.0λ. Fig. 6.6 shows the H-plane radiation pattern for ψ=90° ρ0=1.5λ with and without an additional plate at t=0.8λ. The length of the plates for these four designs is varied from 3λ to 5λ.

The H-plane radiation patterns of the ψ=60° corner reflector antenna have side lobes between -11dB and -4dB. Increasing the length of the reflector plates has a strong influence on the H-plane radiation patterns. The side lobe level and the radiation in the rear region is decreased if the length of the reflector plates is increased. The introduction of the additional plate also reduces the side lobe level and the radiation in the rear region. This is the reason for the increase of the H-plane directivity shown in table 6.2.
For the corner reflector antenna with an apex angle of $90^\circ$ the side lobe levels are between -13dB and -16dB. Increasing the length of the reflector plates influences the
H-plane radiation pattern not as much as in the previous case. The introduction of the additional plate has very little influence on the radiation pattern.

Considering the data presented, the most suited design in respect of H-plane directivity, aperture efficiency and side lobe level is the $\psi=90^\circ \rho_o=1.5\lambda l=3\lambda$ with an additional plate $0.8\lambda$ from the virtual apex.

6.1.4. Three dimensional investigation on modified corner reflector antennas

The front-to-back ratio of the modified corner reflector antenna could be influenced by the diffraction on top and bottom which occurs at the additional plate and is not accounted for in the two dimensional model. To establish the influence of the diffraction on the H-plane radiation pattern the modified corner reflector antenna was simulated with the three dimensional FDTD method.

The width of the reflector plates was set to $2\lambda$. The third plate is placed at $t=0.75\lambda$ so that it is exactly halfway between the apex and the source, in the three dimensional case a half wave dipole.

![Graph showing radiation pattern comparison between 2-D and 3-D FDTD predictions](image)

**Fig. 6.7: Pattern for $\psi=90^\circ \rho_o=1.5\lambda t=0.75\lambda l=3\lambda$, w=2\lambda, 2-D and 3-D FDTD predictions**
The comparison shows good agreement between the 2-D and 3-D FDTD predictions from 0° to 70°. The difference between the 2-D and 3-D predictions for larger angles is due to the diffraction on top and bottom of the reflector plates. The H-plane radiation patterns of the 3-D FDTD prediction for the standard- and the modified corner reflector are very similar. The front-to-back ratio is 27.4dB for the three dimensional standard corner reflector antenna and 27.8dB for the three dimensional modified corner reflector antenna. This shows that the front-to-back ratio is not influenced significantly by the introduction of the additional plate. The gain predicted by the 3-D FDTD for the modified corner reflector antenna is 17.1dBi which is 0.5dB higher than for the standard corner reflector antenna.

The data presented show that the modification of the corner reflector antenna by introducing a flat plate to replace the apex does alter the gain, the H-plane radiation pattern and the front-to-back ratio of the corner reflector antenna only insignificantly. The overall depth of the modified corner reflector antenna is 35% smaller than the overall depth of the standard corner reflector antenna.

6.2. Parabolic reflector

The most common reflector antennas is the parabolic reflector antenna. In this section a comparison of the performance of cylindrical parabolic reflector antennas and corner reflector antennas for small aperture size is presented.

6.2.1. Geometry of the parabolic antenna compared with the corner reflector antenna

The beam shaping of the corner reflector antenna is based on arranging the images so that they are in phase. A different approach to achieve high H-plane directivity is to convert the spherical wave radiated by the dipole into a plane wave. In order to achieve this conversion the reflector shape has to be, by definition, parabolic. If a line
source is used as a feed, the cylindrical parabola with a focal line parallel to the reflector can be used. The shape of the reflector is described by:

\[ y^2 = 4fz \] (6.5)

where:

\( f \): focus of the parabola

Fig. 6.8 shows two cylindrical parabolas with different focus-to-aperture-diameter ratios \((f/d)\) of 0.1 and 0.3 in comparison with a corner reflector with an apex angle of 60°. The aperture width for all three reflectors is \(2\lambda\).

As can be seen for small reflectors the shape of a parabolic reflector with \(f/d=0.1\) is not dissimilar to the shape of the corner reflector. It is therefore conceivable that the performance is similar. The position of the source in the case of the corner reflector
antenna is defined as to the position where the images add up in phase. For the parabolic reflector the feed has to be placed at the focal line of the cylindrical parabola. The cylindrical parabola converts a cylindrical wave radiated by a source at the focus into a plane wave at the aperture.

The line source which is used as a feed for the cylindrical parabolic has to be placed inside the reflector in order to allow the reflector to shape the waves radiated from the source. The f/d has therefore to be less than 0.25. The disadvantage is that with decreasing f/d the reflector becomes deeper. A f/d of 0.1 is therefore almost impractical for two reasons. If both, the aperture and the f/d is small the dipole is placed very close to the metal structure which gives a unfavourable low input impedance and the matching problems involved. The second reason is that for larger apertures the structure is very deep and is therefore contradicting with the aim to built a compact antenna.

6.2.2. H-plane directivity of parabolic reflector antenna

The comparison of the H-plane directivity for various cylindrical parabolas and corner reflector antennas is shown in fig. 6.9. Two corner reflector antennas were chosen. The 60° corner reflector antenna provides a reasonable high H-plane directivity for small aperture widths but the H-plane directivity does not increase when the aperture width is increased further than 4λ. The corner reflector antenna with a 90° apex angle does give a higher H-plane directivity if the aperture width is bigger than 2.2λ. The H-plane directivity rises until the aperture width reaches 6λ.

To exceed the H-plane directivity of a infinite flat plate, which is 6dB, the aperture width has to be at least 0.7λ. The H-plane directivity of the f/d=0.1 parabolic reflector is higher than what can be achieved for a corner reflector antenna when the aperture width is bigger than 5λ. A parabolic reflector with an aperture width of 5λ is 15.6λ deep compared with a depth of 2.5λ for the 90° corner reflector antenna.
In the case of $f/d=0.2$ the H-plane directivity is higher if the aperture width is bigger than $6\lambda$. The parabolic reflector is $11.3\lambda$ deep for an aperture width of $6\lambda$ which is almost 4 times deeper than the $90^\circ$ corner reflector antenna which is $3\lambda$ deep.

The data presented shows that with the corner reflector antenna a H-plane directivity can be achieved which is as high as what can be achieved with a parabolic reflector below an aperture width of $5\lambda$. The simple construction and the compact size makes the corner reflector antenna the more favourable antenna of the two for small apertures below $5\lambda$. The increasing H-plane directivity for the parabolic reflector is the explanation why this shape is employed for larger reflector sizes.

The oscillation of the H-plane directivity for $f/d=0.2$ and 0.3 is due to feed positions multiple of $0.25\lambda$ from the start point of the reflector and the resulting out of phase image. Due the flat shape at the start of the parabola there is an image of opposite phase on the negative z-axis.
6.2.3. **H-plane radiation pattern of parabolic reflector antenna**

The H-plane radiation pattern of a corner reflector $\psi=60^\circ$, $\rho_0=0.75\lambda$ and aperture width of $2\lambda$ is compared with a parabolic reflector with $f/d=0.1$ and the same aperture width. Fig. 6.8 shows the geometry. As can be seen in fig. 6.9 the H-plane directivity differ very little although the plates are at places $0.5\lambda$ apart and the source position differs by $0.55\lambda$ (source position for the parabolic reflector is $0.2\lambda$). If the parabolic reflector is moved along the $z$-axis by $0.55\lambda$ so that the sources of the two antennas are at the same location the reflectors are partly on top of each other, fig. 6.10.

![Diagram showing comparison of corner reflector and parabolic reflector](image)

**Fig. 6.10: Comparison of geometry of corner reflector and parabola with adjusted source position**

The geometry of the parabolic reflector is very similar to the geometry of a modified corner reflector antenna. The H-plane radiation patterns of the two antennas and that of a modified corner reflector antenna with a $60^\circ$ apex angle, $\rho_0=0.75\lambda$, $t=0.55\lambda$ and aperture width of $2\lambda$ is shown in fig. 6.11.

The H-plane radiation patterns of the three antennas are very similar.
The modified corner reflector antenna represents an approximation of the parabolic reflector consisting of three flat plates. It can be concluded that the corner reflector antenna is the more cost-effective solution over the parabola for small aperture sizes up to $6\lambda$ without any loss of H-plane directivity. Further it is possible to substitute the more difficult to manufacture parabolic reflector by three straight plate to build a modified corner reflector antenna.

![Comparison of H-plane pattern](image)

**Fig. 6.11:** Comparison of H-plane pattern between a standard corner reflector, a modified corner reflector and a parabolic reflector with an aperture of $2\lambda$.

### 6.3. Box shape reflector

The box shape reflector, fig. 6.12, will not achieve high H-plane directivity but it should increase the front-to-back ratio.

#### 6.3.1. Geometry of the box shape reflector

The geometry of a box shape reflector with a width of $1\lambda$ and a length of $0.5\lambda$ is shown in fig. 6.12. The box shape reflector can be defined as a modified corner reflector with an apex angle of $0^\circ$. The length of the plates corresponds to the depth of
the reflector. The source is placed halfway between the parallel plates and is moved along the z-axis.

![Geometry of box shaped reflector](image)

The parameters which can be varied are the length of the parallel plates, the position of the source and the width of the plate at the bottom of the reflector.

### 6.3.2. H-plane directivity of the box shape reflector

The resulting H-plane directivity for widths for the box reflector of 0.5λ, 1λ, 2λ, 3λ with the dipole 0.25λ in front of the bottom plate can be seen in fig. 6.13. The dipole position was chosen so that the image due to the bottom plate adds up in phase to the real source in the direction of boresight. Since the box structure represents a waveguide the H-plane directivity settles down as soon as the side plates exceed a length of 0.3λ for a box width of 0.5λ and 0.4λ for a width of 1λ. In the case of wider box reflectors the H-plane directivity varies if the length is increased. This means that there is a different aperture distribution of fields along the z-axis of a waveguide.

To investigate the matter further the plate length for the 2λ and 3λ wide box shape reflector is increased up to 10λ. The H-plane directivity is shown in fig. 6.14. It becomes apparent that the H-plane directivity in the case of w=2λ has a periodic behaviour and there seems to be also a periodic behaviour for the case of w=3λ but
with a longer period. To explain this phenomenon the theory of modes is employed. The wider the box shape reflector, the more modes are excited. For a width of 0.5\(\lambda\) only the TE\(_{10}\) mode is excited. Since the source is placed halfway between the plates only odd modes can be excited, this explains why there is no periodic behaviour for a box shape reflector width of 1\(\lambda\). However, if the width is further increased to 2\(\lambda\) and 3\(\lambda\) the TE\(_{30}\) and TE\(_{50}\) modes are excited, respectively.

![Fig. 6.13: H-plane directivity for different box shaped reflectors](image)

Since different modes have different velocities within the structure they add either in phase or out of phase along the z-axis which accounts for the interference patterns which repeat themselves as the length of the plates is increased. In the case of w=2\(\lambda\) there are two modes excited (TE\(_{10}\), TE\(_{30}\)) which are in phase at the maxima and out of phase at the minima of the H-plane directivity curve. In the case of w=3\(\lambda\) there are three modes excited (TE\(_{10}\), TE\(_{30}\), TE\(_{50}\)) and therefore the occurrence that all modes are in phase is rare which yields a longer period. It is therefore understandable that the H-plane directivity settles if there is only one mode present as in the case of w=0.5\(\lambda\) and 1\(\lambda\) but has a periodic behaviour for bigger widths.
The maximum H-plane directivity which can be achieved for aperture widths of $2\lambda$ is 10dB. Increasing the width of the box reflector does not always give a higher H-plane directivity. Compared with a width of $2\lambda$ the box shape reflector with a width of $1\lambda$ has a higher H-plane directivity for plate lengths between $0\lambda$ and $0.8\lambda$. For the width of $0.5\lambda$ and $1\lambda$ the H-plane directivity stays constant beyond a plate length of $0.5\lambda$.

6.3.3. Front-to-back ratio of box shaped reflector

As pointed out at the beginning the main aim of the box shape reflector is to improve the front-to-back ratio. The front-to-back ration for box shape reflectors with $w=0.5\lambda$, $1\lambda$, $2\lambda$, $3\lambda$ and lengths from $0\lambda$ to $3\lambda$ is shown in fig. 6.15. For the box shape reflector with $w=1\lambda$ of the front-to-back ratio can be improved by 14dB if $0.7\lambda$ long side plates are added. For $w=0.5\lambda$ the improvement is 10dB for the same length of plates. There is no improvement on the front-to-back ratio if the plates are increased beyond $0.7\lambda$ and the width of the box is less than $3\lambda$. 
It can be said that the front-to-back ratio can be improved considerably by adding plates onto the sides of a plate. The improvement is heavily dependent on the width of the bottom plate.

6.4. Summary

In this chapter it was shown that the modified corner reflector antenna is a suitable alternative to the conventional corner reflector antenna in order to reduce the overall depth of the reflector with the additional benefit of an easier mounting option. A comparison of H-plane radiation pattern, H-plane directivity and gain showed that the introduction of an additional plate, provided the optimum position was chosen, did hardly alter those significant parameters of the antenna. It was therefore possible to reduce the depth of the reflector by 35% without degrading the performance of the antenna. An explanation based on the ray tracing technique was given.

Changing the shape of the reflector to a parabolic shape did not yield an increase of the H-plane directivity if the aperture width was below $5\lambda$ and $f/d$ was higher than 0.1.
It is therefore to say that the easier to built corner reflector antenna is the favourable solution for a desired H-plane directivity of up to 14dB. If a higher H-plane directivity is desired it is advisable to use a parabolic reflector.

It is was also shown that the geometry of the modified corner reflector is similar to the geometry of small parabolic reflectors. The H-plane radiation pattern of a parabolic reflector of f/d=0.1 and an aperture width of 2\(\lambda\) is almost the same as the H-plane radiation pattern of a modified corner reflector with an apex angle of 60°, \(\rho_0=0.75\lambda\), \(t=0.55\lambda\) and an aperture width of 2\(\lambda\).

The last shape investigated was the box shape. It was shown that this shape is suited to enhance the front-to-back ratio. It was shown that in the case of \(w=1\lambda\) the front-to-back ratio increases by 14dB if the side plates are 0.7\(\lambda\) long. For \(w=0.5\lambda\) the improvement is 10dB for the same length of plates.
CONCLUSIONS AND FURTHER WORK

7.1. Conclusions

The thesis has identified the FDTD method as very suitable to model two and three dimensional corner reflector antennas. Comparison of measured and predicted radiation pattern of two experimental antennas showed that the method chosen and the way the antenna has been modelled were valid. The comparison showed excellent agreement.

In order to map the geometry of the reflector plates on the Cartesian co-ordinate system, which has been used for the FDTD simulation, the straight plates were approximated. It has been demonstrated that a staircase approximation of the reflector plates did not influence the radiation patterns if the cell size was chosen to be \(\lambda/20\) or smaller. Special care had to be taken to model the apex. In section 4.1.1 it has been shown that the apex had to be modelled as what was defined as 'pointed' in order to ensure the correct distance from apex to the source.

A very important aspect in the use of FDTD is the application of appropriate absorbing boundary conditions to terminate the spatial domain. A comparison of the Mur ABC and the Perfectly Matched Layer ABC has demonstrated that the latter yielded less reflections but if the separation between the antenna geometry and the absorbing boundaries was \(0.5\lambda\) the reflections produced by the Mur ABC were tolerable for radiation problems.

The 2-D FDTD has been employed to model corner reflector antennas which are infinite by wide since the influence of the finite width of the plates on the H-plane radiation pattern can often be neglected. It has been shown that the H-plane radiation pattern of the 3-D FDTD converged to the prediction of the 2-D FDTD method when the width was increased. It has been shown that a width of \(2\lambda\) was sufficient and that no significant improvement was achieved by increasing the width further. The
difference in the H-plane radiation pattern between finite wide and infinite wide corner reflector antennas was caused by diffraction of top and bottom of the plates which occurred for the finite but not for the infinite wide reflectors.

The maximum H-plane directivity for corner reflector antennas with apex angles from $30^\circ$ to $180^\circ$, plate lengths of $2\lambda$, $3\lambda$, $4\lambda$ and $10\lambda$ with dipole positions of $\rho_0=0.5\lambda$ to $1.5\lambda$ have been determined using the 2-D FDTD method. The comparison with the results for the infinite size corner reflector antennas gained by using Wait's methods has highlighted that the influence of the plate length on the H-plane radiation pattern depends on the apex angle and the dipole position. With the help of the ray tracing technique it has been shown that the smaller the apex angle the longer the plates have to be in order to guide the waves.

It has been shown that curves of maximum H-plane directivity for the corner reflector antennas with different length plates converge with the curve for infinite size corner reflector antennas. For a plate length of $10\lambda$ the curve for the maximum H-plane directivity is almost the same as for the infinite size corner reflector antenna for all apex angle from $30^\circ$ to $180^\circ$. For a plate length of $4\lambda$ the curves converge at an apex angle of $65^\circ$. If the plate length is $3\lambda$ this happens at $70^\circ$ whereas for a plate length of $2\lambda$ the curves converge at an apex angle of $100^\circ$.

The aperture efficiency for the above finite size corner reflector antennas has been computed. It has been shown that the highest efficiency of $99\%$ can be achieved by a corner reflector antenna with an apex angle of $50^\circ$, a dipole position of $\rho_0=1.5\lambda$ and a plate length of $2\lambda$. The H-plane directivity of this corner reflector antenna is $10.2\text{dB}$.

The novel idea of a variable beam-width corner reflector antenna has been developed based on the results of the parametric study. It has been demonstrated that by changing the apex angle from $70^\circ$ to $120^\circ$ the -3dB beam-width could be varied from $30^\circ$ to $60^\circ$ while the gain of the antenna changed from $14.0\text{dBi}$ to $10.1\text{dBi}$. Measurements of the H-plane radiation pattern and gain of an experimental antenna
have given very good agreement with the predicted results. The measurements revealed that the input impedance of the corner reflector antenna changes when the apex angle is changed. This causes matching problems for the corner reflector antenna with a dipole position of $\rho_0 = 0.5\lambda$ when the apex angle is smaller than 50°.

A investigation of the bandwidth of the corner reflector antenna has shown that it is limited by the input impedance of the dipole. Considering the results of the parametric study the dipole has to be placed close to the apex in order to provide little change of the H-plane directivity when the electrical separation between dipole and apex varies due to frequency change. When the dipole was close to the plates the input impedance was low and therefore the bandwidth was limited.

A parametric study on gridded corner reflector antennas has been conducted. It has been demonstrated that the round rods could be modelled by filling one Yee-cell of the spatial domain with perfect conductor. This is possible when the diameter of the rod is $\lambda/20$ or smaller. It has been shown that by choosing the size of the Yee-cell it has to be considered that the physical size of the filled Yee-cell appears to be 1.5 times the size of the Yee-cell. This actual size has to be chosen such that the volume of the square rod is equal to the volume of the round rod. The minimum distance of two rods in the FDTD model is three cells. Following this guidelines a experimental gridded corner reflector antenna has been modelled and excellent agreement between measured and predicted E- and H-plane radiation pattern has been reported.

A parametric study of two dimensional corner reflector antennas with apex angles of 60° and 90° and a dipole position of $\rho_0 = 0.5\lambda$ showed that the H-plane directivity dropped by 0.7dB and 0.1dB respectively comparing the solid corner reflector antenna with a rod separation of 0.25$\lambda$ and a rod diameter of $\lambda/24$. The front-to-back ratio decreased for the 60° corner reflector antenna from -48.3dB to -22.0dB when the solid plate was replaced by the rods. For the 90° corner reflector antenna the front-to-back ratio changed from -55.8db to -19.2dB. It can be concluded that the front-to-
back ratio is much more sensitive to the replacement of the solid plates than the H-plane directivity.

Comparing the corner reflector antenna with the cylindrical parabolic reflector fed by a line source has shown that the latter does not give a higher H-plane directivity if the f/d is greater than 0.1 and the aperture width is smaller than 5λ. Due to the more compact size the corner reflector antenna is favourable if a H-plane directivity of 14dB or less is required.

It also has been demonstrated that the depth of the corner reflector antenna can be reduced by up to 35% by replacing the apex with a flat plate. This arrangement has been called a modified corner reflector antenna. Furthermore it has been shown that a corner reflector antenna with an apex angle of 60°, ρ₀=0.75λ and an aperture width of 2λ has almost an identical H-plane radiation pattern to a cylindrical parabolic antenna with f/d=0.1 and the same aperture size.

Finally, it has been demonstrated that the front-to-back ratio of a flat plate can be significantly increased by attaching plates at the sides. For a 1λ wide plate the front-to-back ratio has been increased by 14dB by attaching 0.7λ long plates. The improvement of the front-to-back ratio for a width of 0.5λ has been 10dB when attaching the same plates.

In conclusion, very useful design guidelines for the corner reflector antenna have been developed. The influence of the finite size reflector is analysed and physical explanation for the cause of the influence was given. The agreement between predicted and measured data has been very good.

7.2. Further work

An interesting aspect for the solid corner reflector antenna is the possibility of introducing parasitic elements in order to improve the bandwidth and the gain of the corner reflector antenna. An introduction of parasitic elements for the gridded corner
reflector antenna could be combined with the rods which built the 'plates'. With increasing computer power it will soon be possible to utilise generic algorithms in connection with FDTD simulations. The most suitable number and position of parasitic elements could be established.

The increasing computer power will soon also enable arrays of corner reflector antennas to be modelled. This is necessary to determine the performance of corner reflector antennas which are arranged in arrays. It is not expected that the radiation characteristic in the H-plane radiation pattern would be much influenced compared to a single element. The E-plane radiation pattern would be shaped by the array factor. What would be expected to change is the input impedance of the single elements due to mutual coupling.

Designing an antenna for a system should ideally consider the matching with the front end of the system. At the moment the simulation of the antenna and the design of the electronic circuit of the following system is a two stepped process. With an interface between the FDTD simulation and a electric circuit simulation tool such as SPICE the design of the matching network could be integrated which would allow simulations of the front end of the system. With this approach it might be possible to improve the quality of the simulation results.

Due to the rapid growth of demand, the lack of communication channels has become a serious problem especially in metropolitan areas in the United States, Europe, and Japan. Technologies for effective frequency reuse are strongly needed. The cellular system has the advantage of reusing frequencies, but the efficiency depends significantly on the radiation pattern of the base station antenna. The novel idea of the variable beam-width corner reflector antenna could be followed up using a simulation tool for cellular communication networks in order to establish the usefulness of the idea in this area. The concept of the variable beam-width corner reflector antenna could be incorporated into research currently undertaken at QMW with the aim to use
self adjusting software in order to adapt the cells of cellular communication networks to changing traffic demands. This would help to increase the capacity of these networks.

Another aspect of the corner reflector antenna is the operation frequency. The study undertaken is restricted to 5GHz. Although all results presented are valid at other frequencies there are practical limitations. The source assumed in the thesis has been a half wave dipole. If the frequency would be in the order of several 100GHz this non printed dipoles could not be used since the physical size of the source would be too small to handle. Alternative sources could be investigated. At higher frequencies directive sources might be possible since the blockage would not be so significant as the electrical size of the reflector could be larger.
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