AN OSCILLATORY TURBULENT BOUNDARY LAYER
IN AN ADVERSE PRESSURE GRADIENT

BY

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ABSTRACT

A turbulent boundary layer experiencing a time mean adverse pressure gradient and a controllable travelling wave periodic oscillation, was examined experimentally. An open return low speed wind-tunnel with a semi-open working section was used for this purpose, with oscillating flaps at its exit inducing the oscillations. The boundary layer on a specially designed "S" shaped model of chord 2m and thickness/chord ratio of 3.6% was investigated, for a range of frequencies from 1 to 6Hz, and amplitudes of the order of 10% of the time mean freestream velocity. The turbulent boundary layer evolved naturally around x/c= .23, and measurements were taken for a Reynolds number Re_c=3x10^6. The effect of flap amplitude was examined for a range of amplitudes, from 2 to 4 inches. Unsteady velocity and pressure quantities were measured using Hot-wire techniques and pressure transducers, with the aid of a digital sampling system. Boundary layer mean values, were found to be invariant with both frequency and amplitude of oscillation, while unsteady components were predominantly affected by frequency and downstream position but not amplitude. Unsteady velocities in the boundary layer lagged the freestream oscillations by as much as 150° in some cases, while amplitudes exceeded freestream values by as much as 70%.
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## CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABSTRACT</td>
<td>2</td>
</tr>
<tr>
<td>ACKNOWLEDGMENTS</td>
<td>3</td>
</tr>
<tr>
<td>1. INTRODUCTION</td>
<td>7</td>
</tr>
<tr>
<td>2. REVIEW ON UNSTEADY BOUNDARY LAYERS</td>
<td></td>
</tr>
<tr>
<td>2.1 Introduction</td>
<td>10</td>
</tr>
<tr>
<td>2.2 Steady Streaming in Oscillatory Motion</td>
<td>12</td>
</tr>
<tr>
<td>2.3 Unsteady Boundary Layers</td>
<td>14</td>
</tr>
<tr>
<td>2.3a The oscillatory laminar boundary layer</td>
<td>14</td>
</tr>
<tr>
<td>2.3b The oscillatory turbulent boundary layer</td>
<td>16</td>
</tr>
<tr>
<td>3. THE OSCILLATORY FLOW FACILITY</td>
<td></td>
</tr>
<tr>
<td>3.1 The Tunnel</td>
<td>25</td>
</tr>
<tr>
<td>3.2 The Oscillatory Flow Exciter</td>
<td>26</td>
</tr>
<tr>
<td>3.3 Frequency Measurement</td>
<td>27</td>
</tr>
<tr>
<td>3.4 The Oscillatory Flow Mechanism</td>
<td>27</td>
</tr>
<tr>
<td>4. THE MODEL</td>
<td></td>
</tr>
<tr>
<td>4.1 Preliminary Considerations</td>
<td>32</td>
</tr>
<tr>
<td>4.2 Full Scale Model Construction</td>
<td>35</td>
</tr>
<tr>
<td>4.3 Steady Flow Pressure Distribution</td>
<td>37</td>
</tr>
<tr>
<td>5. UNSTEADY FLOW MEASURING SYSTEM</td>
<td></td>
</tr>
<tr>
<td>5.1 Traversing Gears</td>
<td>40</td>
</tr>
<tr>
<td>5.2 Pressure Transducers</td>
<td>42</td>
</tr>
<tr>
<td>5.3 Hot-wire Calibration</td>
<td>42</td>
</tr>
<tr>
<td>5.4 The digital Measuring System</td>
<td>44</td>
</tr>
<tr>
<td>6. INVISCID FLOW MEASUREMENTS-RESULTS</td>
<td></td>
</tr>
<tr>
<td>6.1 Introduction</td>
<td>51</td>
</tr>
<tr>
<td>6.2 The Pressure and Velocity Amplitudes</td>
<td>51</td>
</tr>
<tr>
<td>6.3 Pressure and Velocity Phase Angles</td>
<td>58</td>
</tr>
<tr>
<td>7. BOUNDARY LAYER MEASUREMENTS-RESULTS</td>
<td></td>
</tr>
<tr>
<td>7.1 Introduction</td>
<td>61</td>
</tr>
<tr>
<td>7.2 The Steady Boundary Layer</td>
<td>62</td>
</tr>
<tr>
<td>7.3 Unsteady Boundary Layer Results</td>
<td>64</td>
</tr>
<tr>
<td>7.3.1 Velocity amplitudes and wall phase angles</td>
<td>64</td>
</tr>
</tbody>
</table>
7.3.2 Boundary layer mean values
7.3.3 Boundary layer turbulence
7.3.4 Freestream amplitude effect

8. DISCUSSION
8.1 The Freestream
8.2 The Boundary Layer

9. CONCLUSIONS

10. REFERENCES

APPENDIX I:
Frequency Response of a Pressure Transducer and associated Tubing.

APPENDIX II:
The sampling System Frequency Response.

FIGURES

PLATES
## LIST OF PLATES

<table>
<thead>
<tr>
<th>Plate Number</th>
<th>Title</th>
<th>Facing Page No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Top Flap and Drive Mechanism</td>
<td>226</td>
</tr>
<tr>
<td>II</td>
<td>The Model</td>
<td>229</td>
</tr>
<tr>
<td>III</td>
<td>Hot-wire Arrangement</td>
<td>232</td>
</tr>
<tr>
<td>IV</td>
<td>Working Section-Model in Situ</td>
<td>235</td>
</tr>
<tr>
<td>V</td>
<td>Signal Processing System</td>
<td>238</td>
</tr>
</tbody>
</table>
INTRODUCTION

Unsteady flow fields occur in a great variety of situations, both in nature and numerous engineering applications. Thus a continuously increasing importance is attached to the study of such flows, as indicated by the recent increase of research effort in the field.

In many engineering problems, the understanding of unsteady flow effects could lead to improved design of components for better performance, reliability and cost effectiveness. Furthermore, the fundamental knowledge gained as a consequence could be even more beneficial in the long run, leading to the solution of many problems encountered in nature not only in the engineering domain but also such diverse subjects as biology and zoology. We will now highlight some of the problems encountered in the engineering field.

The flow through an isolated turbine disc is nominally steady, in a coordinate system moving with the rotating blades; but in a real situation with many parallel stages of rotating and stationary components, this is no longer true. Some periodic unsteadiness is introduced which has to be taken into account, as it leads to flutter and premature fatigue of the blades, stall, surge and even noise. A similar problem is vividly presented when one studies the aerodynamics of helicopter rotors. In this case, as the rotor translates nearly parallel to its plane of rotation than axially, the oncoming flow varies periodically in magnitude, yaw and incidence with reference to any single blade. This leads to such undesirable effects, as transonic flow on the advancing and dynamic stall on the retreating blade. Flutter prediction and prevention on flexible wings, remains an important study topic in the aircraft industry, and such modern applications as the space shuttle and control configured vehicles, require a knowledge of unsteady aerodynamics. In
other industrial situations pulsating jets find applications in breaking rocks, combustion chambers, car silencers and in oscillating jet flaps. For the civil engineer, bluff body separation and vortex shedding from tall buildings and flexible bridges, lead to considerable worries and sometimes spectacular catastrophies that need to be resolved. All these examples highlight the importance and diversity of the problem.

From the above examples it is obvious that unsteady flow situations can arise, either due to unsteadiness or periodicity in the motion of a component necessary to fulfil its function, e.g. a helicopter blade, or due to an unsteady freestream passing a stationary body. Also unsteady flow situations can arise in a nominally steady freestream passing over an initially steady obstacle, but inducing oscillatory forces and motion of the body.

Freestream unsteadiness can be classified for analytical purposes into two main categories, impulsive and periodic. Impulsive flow implies a transition from one state of affairs to another, such as a sudden change in velocity in a finite interval of time. It usually leads to transient effects which die away with time.

The second category involves a periodic or cyclic relative motion between freestream and surface. The simplest example of a periodic flow, is that of a sinusoidal freestream passing a body with or without a mean flow superimposed to it, given by the equation,

\[ U(x,t) = U_1(x) \cdot (1 + N \sin \omega t) \]  

where \( U_1(x) \) is the mean flow velocity, \( N \) is the ratio of amplitude of oscillation to the mean flow and \( \omega \) is the radial frequency.

If we are looking for the effects on a body induced by such a flow, we can always measure them experimentally, by measuring the forces on the body. But if we are concerned with the cause for these effects, then we have to study the resulting flowfield around the body.
Unsteady potential flow theory is already well understood and readily adaptable to many unsteady flow problems. However the inviscid approach proves inadequate where large departures from the ideal flow occur, as in the case of separation or where the boundary layer thickness cannot be neglected. Then we have to turn our attention to the boundary layer. The problem of the laminar boundary layer has been studied exhaustively both theoretically and experimentally as shown in the following chapter and essentially the problem is now well understood. Unfortunately the turbulent boundary layer presents still many difficulties due to its non-linearity, and although it has received in recent years a renewed attention, little has been so far achieved.

Although with the advent of modern computers, numerous numerical methods for solving the problem have evolved, the lack of experimental data is acute. A comparison of some of these methods, shown in figure (1), shows large quantitative differences and even trends, and the present state of the art still leaves a lot to be desired, even for the simple case of a flat plate.

In cases where a severe adverse pressure gradient is present, the problem is mainly untackled and no reliable experimental evidence exists. In this area we hope to provide some data, by studying experimentally the case of a turbulent boundary layer in a mean adverse pressure gradient, subjected to a travelling wave type of oscillation of the form,

\[ U(x,t) = U_t(x) \times (1 + \sqrt{2} \sin \omega(t/a)) \]

(2)

where \( Q \) is the wave convection velocity. This type of oscillation introduces the added complexity of an oscillating streamwise pressure gradient.
2. REVIEW ON UNSTEADY BOUNDARY LAYERS.

2.1 Introduction.

The inclusion of a brief review of previous work in this volume was considered necessary, both for laying the foundations and drawing the objectives of this work.

Unsteadiness in boundary layer flow, can produce many unusual phenomena, sometimes completely different from those arising in the steady case. This has stipulated a lot of research on the subject, both theoretical and experimental. A recent review on the more general aspects of unsteady flow and comprehensive bibliography, is given by W.J. McCroskey (1977), while Riley (1975) provides a review of the state of the art for the case of the unsteady laminar boundary layer, both in oscillatory and impulsively started flows. In this work, specific emphasis will be given to the case of the oscillatory turbulent boundary layer.

The concept of a boundary layer as a rational mathematical and physical theory, stems from Prandtl (1904); however the existence of unsteady boundary layers was reported even further back, in the mid 19th century work of Stokes (1851) and Rayleigh (1884). If we consider the flow relative to a flat plate in the plane \( y=0 \), and independent of the \( x \) and \( z \) directions in an orthogonal system of coordinates, then the Navier-Stokes and continuity equations are reduced to:

\[
\begin{align*}
\frac{\partial u}{\partial t} & = - \frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2} \\
0 & = - \frac{1}{\rho} \frac{\partial p}{\partial y} - \frac{1}{\rho} \frac{\partial p}{\partial z}
\end{align*}
\]

(3)

with \( u=u(y,t) \), the velocity component in the \( x \) direction only. If the pressure gradient is zero, we get the simpler diffusion equation,

\[
\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2}
\]

(4)
The Stokes layer is periodic in $t$, with boundary conditions:

$$
\begin{align*}
    u &= u_w \cos \omega t , \quad y = 0 \\
    u &= 0 \quad , \quad y = \infty
\end{align*}
$$

(5)

and the solution is,

$$
    u = u_w e^{-\eta} \cos(\omega t - \eta)
$$

(6)

where $\eta = y(\omega/2\nu)^\frac{1}{2}$

This is the solution arising from the oscillation of the plate in its own plane, in a stationary flow field.

The Rayleigh layer only differs in the boundary conditions, which are:

$$
\begin{align*}
    u(y,0) &= 0 , \quad y > 0 \\
    u(0,t) &= U_0 \\
    u(\infty, t) &= 0 \\
    u (-, t) &= 0 \quad t > 0
\end{align*}
$$

(7)

and the solution is, $u = U_0 \text{erfc}(y/2(\nu t)^\frac{1}{2})$

(8)

It represents the flow generated by a sudden movement of the plane.

By introducing a virtual pressure field to account for inertial effects, we can transform, with a suitable $x$-coordinate transformation both (6) and (8) to situations where the wall is kept at rest, but the fluid is allowed to have motion at infinity. Thus the periodic solution of (3) with

$$
-\omega U_\infty \sin \omega t = -\frac{1}{\rho} \frac{\partial p}{\partial x}
$$

is,

$$
    u = U_\infty \cos \omega t , \quad y = \infty
$$

(9)

and from (4), with

$$
\begin{align*}
    u(y,0) &= 0 , \quad y > 0 \\
    u(0,t) &= 0 \quad t > 0 \\
    u(\infty, t) &= U_0
\end{align*}
$$

(11)

we get,

$$
    u = U_0 \text{erf}(y/2(\nu t)^\frac{1}{2})
$$

(12)

Both these layers indicate, that although shear waves are propagated from the surface into the fluid, they decay exponentially, and there is no mean outflow from them. However it is important to note
that if we let \( y \rightarrow 0 \) in equation (10), then the limiting velocity at the wall (or the skin friction) leads the velocity at the outer edge of the layer by \( \pi/4 \).

The pioneering work summarised above, helps to classify the unsteady boundary layer problem into two main sections, impulsive boundary layers, and oscillating boundary layers, corresponding to the Rayleigh and Stokes layers respectively.

On the problem of impulsive boundary layers, although important in itself, we shall comment no further. We shall rather restrict ourselves to oscillatory boundary layers for the rest of this chapter.

### 2.2 Steady streaming in oscillatory motion

As indicated in the previous section, there is no outflow from the Stokes layer in the form so far presented. However this is only true if the fluid velocity does not depend on the \( x \)-coordinate. In many cases where such a dependence exists, the Stokes layer of thickness \( (V/\omega)\frac{1}{2} \) is still relevant and the appropriate phase differences between the velocity components will be present. The non-linearity of the system though leads to the development of harmonics of the given oscillation and of a steady outflow from the layer. This steady outflow does not die away with \( y \) as in the previous case, but rather has a non-zero limit.

An example of such a flow, over an infinite circular cylinder oscillating in its own axis, was investigated theoretically by Stokes himself (1886) and Coster (1919), and experimentally by Winny (1932). A notable contribution to the subject was made by Schlichting (1932), who analysed the streaming around a cylinder and verified the results experimentally with flow visualisation in water.

Assuming as a first approximation equation (3), with \( U(x,t) = U_0(x) \cos t \) and taking the \( x \) and \( y \) coordinates parallel and perpendicular to the surface, with \( (u_a, v_a) \) the first approximation to the velocities
Schlichting derived an equation containing a second approximation to the velocities \((u_b, v_b)\), in the form

\[
\frac{a}{\partial t} + v \frac{\partial^2 u}{\partial y^2} = \frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} - v \frac{\partial u}{\partial x} - a x - a y - v_a - v_b - v_c - v_d - v_e - v_f - v_g - v_h - v_i - v_j - v_k - v_l - v_m - v_n - v_o - v_p - v_q - v_r - v_s - v_t - v_u - v_v - v_w - v_x - v_y - v_z
\]

and assuming \(\frac{\partial u}{\partial x}\) is of the second order compared to \(\frac{\partial u}{\partial t}\), which is true if the amplitude of oscillation is much smaller than the cylinder diameter he obtained a solution of (13), in the form:

\[
u = \frac{1}{n} U_o \frac{dU}{dt} \left\{ C_1(\eta) \cos 2\omega t + C_2(\eta) \sin 2\omega t + C_3(\eta) \right\}
\]

where \(C_2(\eta), C_1(\eta)\) and \(C_3(\eta)\) are known functions. This last equation satisfies the conditions at the wall only \((y = 0)\), and not an appropriate condition at infinity, since \(u(\infty) = -\frac{3}{4\omega} \frac{dU}{dx}\), a finite value. The reason for this is that whereas the unsteady components of the flow, the acceleration and viscous terms, develop a balance in which \(u \to 0\) at infinity, this balance is not possible for the steady streaming within the Stokes layer, because the acceleration is zero. The existence of the steady streaming outside the Stokes layer is a natural consequence of the presence of the Reynolds stress within the Stokes layer.

If we assume that \(u\) should be bounded at infinity, then

\[C_3(\eta) = 0\] in (14).

Andres & Ingard (1953) extended Schlichting's work to third and fourth order effects. Stuart (1963), suggested an appropriate Reynolds number for steady streaming, as \(R_S = \frac{U_\infty}{\omega^2}\), where \(U_\infty\) is a characteristic velocity of the flow. For large values of \(R_S\), he stipulated on the existence of an outer non linear boundary layer, within which the steady streaming goes to zero at infinity. Calculations of the structure of this layer have been made by Stuart (1966) and Riley (1965).
2.3 Unsteady boundary layers.

2.3a The oscillatory laminar boundary layer.

The subject of oscillatory laminar boundary layers, being susceptible to analytical treatment, has received considerable attention in recent years. Thus an essentially complete understanding of the problem exists, at least in the absence of strong adverse pressure gradients.

One of the most powerful contributions to the subject was made by Lighthill (1954), who studied an oscillatory laminar boundary layer on a cylindrical body, when the velocity of the oncoming stream relative to the body, oscillates in magnitude but not in direction, with a free-stream velocity,

\[ U = U_0(x) \cdot (1 + N \exp(i\omega t)) \]  \hspace{1cm} (15)

where \( N = \frac{U}{U_0} \), the amplitude parameter, which is assumed to be small. This restriction allows the spatial and temporal dependence of the boundary layer velocities to be separated, giving:

\[
\begin{align*}
  u(x,y,t) &= u_0(x,y) + u_1(x,y) \cdot \exp(i\omega t) \\
  v(x,y,t) &= v_0(x,y) + v_1(x,y) \cdot \exp(i\omega t)
\end{align*}
\]  \hspace{1cm} (16)

By substituting these values in the laminar boundary layer and continuity equations and by separating zeroth and first order terms, Lighthill obtained a set of simultaneous equations which he solved for low and high frequencies. Thus for each point on the body surface there is a critical frequency \( \omega_0 \), such that for \( \omega_0 < \omega \) the oscillations are to a good approximation "shear waves" unaffected by the mean flow; the phase advance in the skin friction is then 45°. For \( \omega > \omega_0 \) the oscillations are closely approximated as the sum of parts proportional to the instantaneous velocity and acceleration of the oncoming stream; the phase advance of the skin friction is then \( \tan^{-1}(\omega/\omega_0) \). This part of the solution is called quasi-steady, and it is important as an anchoring point for higher frequency investigations. The critical frequency \( \omega_0 \) was defined by Lighthill as \( \omega_0 = 0.6 \frac{U_0}{x} \). Here again the Stokes layer
is of importance. If the steady layer was not present, then the thickness of the Stokes layer would be $(v/\omega)^{1/2}$. If the boundary layer thickness $\delta$, is comparable in magnitude to the Stokes layer, i.e. $\omega \delta^2/v \approx 0(1)$ and the Stokes layer is spatially mixed with the steady layer, then the boundary layer would respond as a whole to the imposed oscillation. (The quasi-steady case, with the relevant phase shifts.) If however $\omega \delta^2/v \gg 1$, then the Stokes layer is very thin on the wall, and essentially exists independently of the steady flow. Lighthill presented his results as plots of $u/U$ and versus $y/\sqrt{\omega x/2v}$. (see figure (2))

Similar calculations were also performed by Lin (1956) for the high frequency case, quite independently, producing the same results. 

For the boundary layer near a stagnation point (Hiemenz layer), results have been given by S&A Ghoshal (1970) and H. Ishigaki (1970), although there is some discrepancy between them. Hill & Stenning (1960) confirmed Lighthill's theory for both the Blasius and Howarth layers ($U_0=1-Cx$). The comparison is shown in figure (2). 

Among numerous other workers in the field of oscillatory laminar boundary layers it is worth mentioning the work done by Hall (1969) and Phillips &Ackerberg (1973), who solved the problem numerically producing very good results.

The travelling wave type of oscillation which is more relevant to this work, has been recently studied experimentally and theoretically by M. H. Patel (1974). Patel made comprehensive measurements on a flat plate for both laminar and turbulent boundary layers, for a range of frequencies from 4 to 12 Hz, and freestream amplitudes from 2% to 10% of the freestream velocity. He analysed his results using Lighthill's method with a freestream velocity of the form, 

$$U(x,t) = U_0(x)\{1+N\sin(\omega(t-x))\}$$

where $Q$ is the wave convection velocity. His analysis was for a gene-
ral Q, although his experimental results were for \( Q = 0.77U_0 \). His results in the laminar case present broad similarities with his turbulent boundary layer results, and in that context we will discuss them in the next section.

2.3b The oscillatory turbulent boundary layer.

As mentioned above, the subject of oscillatory laminar boundary layers is at present well understood mathematically. Unfortunately the same cannot be said for the turbulent case, which presents inherent mathematical difficulties, especially in the modelling of the unsteady Reynolds stresses, which is essential for closure of the problem. The fact that the problem facing us is three-dimensional, in \( x, y, t \), does not help the situation, since only recently some progress has been made in the calculation of steady three-dimensional turbulent boundary layers. Recent attempts to analyse the flat plate problem are in qualitative agreement especially for the quasi-steady case, but when it comes to higher frequencies large quantitative differences and even trends appear, which need to be resolved (see figure (1)).

Due to the difficulties mentioned, the first ever attempts to calculate the unsteady turbulent boundary layer, were simple extensions of the methods used in calculating the three-dimensional steady layer with \( t \) instead of \( z \) in the three-dimensional equations, and steady flow models for the Reynolds stresses.

Hence Cebeci & Keller (1971), introduced an eddy viscosity concept to account for the Reynolds stresses in the boundary layer. To do this, they split the boundary layer into an inner and outer region. In the inner region they specified an eddy viscosity \( \varepsilon_i \), based on Prandtl's mixing length, with modifications by Van Driest and Cebeci. In the outer region an eddy viscosity \( \varepsilon_0 \) modified by Klebanoff's intermittency factor was used. Solution was confined to \( y, t \) dimensions (infinite plate) for convenience, and a two point finite difference
method was used for calculations. A freestream velocity oscillating about a mean value was used, and the instantaneous values of the skin friction $C_f$ were obtained for frequencies of 10 and 100 Hz, and various freestream velocity amplitudes. Results showed $C_f$ increasing both with frequency and amplitude. Mean velocity profiles were found to vary little with frequency.

Bradshaw a few years earlier (1969) extended his method for calculating three-dimensional turbulent boundary layers to include the unsteady case, using the turbulent energy equation together with the Momentum and Continuity equations to obtain closure of the problem. Again he used a finite difference scheme for solution with the computer and long computer run times restricted the solution to the infinite flat plate. His results for skin friction in a decelerating unsteady freestream agree closely with Cebesiu's results in the prediction of the separation position, but elsewhere the agreement is not very good. A shortcoming of both methods is that the shear stress at the wall should remain positive.

Patel & Nash (1971), calculated the problem of a two-dimensional unsteady turbulent boundary layer, by using a rate equation of $u'v'$ derived from the kinetic energy equation, in the manner described by Bradshaw, Ferris and Atwell (1967). The resulting system of equations was solved numerically in an explicit finite difference scheme, also used by Nash (1969) for the three-dimensional steady turbulent boundary layer. The solution of the entire flow field of interest, evolved at each instant of time and the calculation proceeded until after a large number of steps the different solutions converged to the true value. The problem of a purely sinusoidal oscillation on a flat plate with both zero and adverse pressure gradient was solved. Results were presented as instantaneous velocity and shear stress profiles and instantaneous boundary layer integral values. A study of the results
for the velocity and shear profiles indicates waves being generated continuously at the wall and propagated outwards with a distinct velocity. Also a phase difference between freestream and boundary layer values is apparent. Comparison of different profiles for the same freestream velocity \( U \) but \( \frac{du}{dt} \) of opposite sign shows significant differences, which invalidate any quasi steady assumptions. In general the mean values of the integral parameters \( \Theta \delta \) and \( \tau_w/\rho U_0^2 \) appear to be larger than the steady values by as much as 10%.

Although the above effort appears to be a more realistic approach to the solution of the problem, it still suffers from the same basic drawbacks as the previous methods, in as far as it uses empirical functions which seek to model convection, dissipation and diffusion of turbulent kinetic energy, applicable in the steady case; while there use in the unsteady case is still uncertain. Figure (1) compares the method with Karlsson's results for the phase lead in the boundary layer.

McDonald & Shamroth (1971) used an integral method to solve the problem for small time dependent disturbances, by assuming a Cole's velocity profile matched to Karlsson's (1958) experimental mean profile as representative for an unsteady turbulent boundary layer. The same profile was used for the in-phase velocity components, while the out of phase component was assumed identically zero. A mixing length concept similar to Cebeci & Keller's was used to evaluate the turbulent shear stresses. Results were computed for a travelling wave type of oscillation,

\[
U = U_1(1 + 0.1\sin(2\pi x/\lambda + \omega t))
\]

(17)

A plot of wavelength versus \( C_f \), shows that a decrease in wavelength \( \lambda \), results in an increase in skin friction, until it levels off at a wave length \( \lambda/\delta < 10 \). Now since \( \lambda = Q/\xi \), if the convection velocity \( Q \) is kept constant, decreasing \( \lambda \) means increasing frequency, or alternatively if \( f \) is kept constant, decreasing \( \lambda \) has the same effect as reducing the travelling wave velocity \( Q \). The phase relation between \( C_f \) and \( U \),
shows a zero lead for very low frequencies, and at high frequencies $C_f$ leads $U$ by about $45^\circ$.

Nash & Singleton (1974) used the method of Patel & Nash, refined by the inclusion of a more accurate numerical scheme to calculate two-dimensional oscillatory turbulent boundary layers, with specific emphasis on the effects on the onset of flow reversal at the wall. Flows studied included a flat plate, and a mean retarded boundary layer. Results were compared with quasi-steady solutions. Agreement for $\tau_w$ was surprisingly good, although the quasi-steady calculation overestimated the minimum value of $\tau_w$. The agreement for the instantaneous values of $\delta^*$ was not so good, even for the low frequency test case, ($\omega = 1.57$).

Kuhn & Nielson (1975), integrated the unsteady boundary layer equations using a small perturbation technique, for a restricted range of frequencies. As highest frequency limit, $\omega = U/\delta$ was taken, following McDonald & Shamroth, so that the shear stress integral could be neglected. Results were compared with Nash & Singleton for frequencies of 1.57 and 15.7 Hz, showing a good agreement for $\tau_w$ but large differences in $\delta^*$. Agreement was generally better for the highest frequency.

Telionis & Tsahalis (1976) were the first people to attempt extensive comparison with experimental data (Karlsson's flat plate boundary layer). They integrated the time dependent turbulent boundary layer equations numerically using a two layer eddy viscosity concept (Cebesi), for transient and oscillatory freestream. Calculations were extended beyond the zero skin friction point and into regions of partially reversed flow. Comparison with experimental results proved satisfactory for the higher frequency parameters for both mean velocity profiles and wall phase angles. Agreement with averaged fluctuating components was not though very good. Calculations in the laminar sublayer, indicated similar trends to Lighthill's laminar boundary layer
analytic solution.

All the above methods with the possible exception of the last one, can only be considered as interesting numerical exercises if no comparison with experimental data is available to verify them.

Until recently the only experimental data available were those obtained by Karlsson, in 1958. He used a low speed (7.65 ft/sec) blow-down boundary layer tunnel with a 20 ft long working section, to investigate the response of a tripped turbulent boundary layer, to a sinusoidally varying freestream.

The oscillation was produced by means of shutters varying periodically the working section exit area. Assuming the laminar and turbulent boundary layers were sufficiently related, so that a laminar boundary layer analysis could give us at least the qualitative behaviour in the turbulent case, Karlsson extended Lighthill's quasi-steady analysis for large oscillation amplitudes (up to 50% of $U_\infty$). Thus he indicated that the non-linear interaction between freestream and boundary layer, could give rise to harmonics of the driving frequency in the boundary layer. Also the skin friction was found to increase with amplitude, although mean and steady velocity profiles deviated only slightly from each other, even at the highest amplitude.

Assuming a boundary layer response of the form,

$$u(x, y, z, t) = u^1(x, y)\cos(\omega t + \phi(x, y)) + r(x, y, z, t)$$

$$= u^1\cos\phi\cos\omega t - u^1\sin\phi\sin\omega t + r$$

and a freestream fluctuation,

$$u_\infty = u^1\cos\omega t$$

Karlsson measured electronically $u^1\cos\phi$, $u^1\sin\phi$, $\overline{u^2}$ and $\overline{r^2}$, by using two hot-wire probes, one placed in the freestream and the other traversing the boundary layer. Here $^1$ denotes only the driving frequency component, and $r$ the total turbulence in the boundary layer, including higher harmonics. The in and out of phase components were
obtained by suitably shifting the freestream signal through 90° multiplying and averaging. Thus,

\[ \bar{u}_\infty u = u_\infty u^1 \cos \phi/2 \]

and

\[ \bar{u}_\infty (\omega t + \pi/2) u = u_\infty u^1 \sin \phi/2 \]  \hspace{1cm} (20)

Measurements were taken for a range of frequencies from 0.33Hz to 48Hz, and for amplitudes varying from 8% to 34% of the freestream mean velocity. The results were presented as plots of in-phase, out of phase and mean velocity profiles. Also the R.M.S. value of the turbulence was plotted against \(y\), the distance from the wall. A typical experimental result is shown in figure (3). As predicted, no systematic variation in the mean profiles with fluctuation amplitude or frequency could be detected, not surprising since the effect on the quasi steady profile was found to be very small. The in-phase velocity component was found to increase rapidly near the wall and more gradually afterwards, reaching a maximum above the freestream value in the boundary layer. The effect of increasing frequency, was to move the maximum nearer to the wall. The out of phase component was always positive near the wall, indicating a maximum phase lead of about 35° at 7.65Hz; although in this region the results exhibited considerable scatter, presumably due to hot-wire cooling and flow reversal during part of the cycle. The results for the turbulence level, showed even higher scatter and seemed to be amplitude dependent probably due to the presence of harmonics.

Telionis e.a. used Karlsson’s results to calculate the phase angle \(\phi(x, y)\) through the boundary layer. It was found to vary little through the boundary layer, except at the region near the wall where phase leads increased rapidly, showing a tendency to approach \(\pi/4\), the Lighthill value. His findings are shown in figure (1) for the wall phase lead at different frequencies.

The problem of a turbulent boundary layer experiencing a time
mean adverse pressure gradient was examined recently (1976) by A.A. Schachenmann and D.O. Rockwell, in a conical diffuser. For this purpose a blowdown tunnel was used with a fibre-glass conical diffuser fairied to the exit. An upstream sliding plate valve produced a sinusoidal core oscillation, with amplitudes less than 10%. Velocities were measured using a hot-wire probe and pressure fluctuations using a DISA microphone transducer, fixed on the diffuser wall. To eliminate the random from the deterministic signal, a phase averaging process was used, triggered by a photocell reacting to the motion of the plate valve, and also doubling as a frequency counter. Mean and fluctuating pressures and velocities were measured along the core of the diffuser and boundary layer traverses were made at different x stations. Results again showed no measurable effect of frequency on mean values. Results at the inlet were given for the extremes of the frequency range, i.e. St_L=1.0 and St_L=7.33, where St_L = \frac{fL}{U_0}, the Strouhal number based on the diffuser length, and U_0 is the mean velocity at the inlet core. For the lower frequency the amplitude of oscillation showed a decrease in the boundary layer with a corresponding phase lead of about 30°. The higher frequency instead, showed a peak of about 20% in the boundary layer and a corresponding small phase lag.

Measurements in the core of the diffuser, showed a decay of the amplitude of oscillation for both the velocity and pressure with x. For St_L = 1.0, the phase angle between the velocity at the inlet and subsequent downstream positions showed an almost linear variation, and the velocity at the exit lagged the velocity at the inlet by about 20°. Corresponding phase lag for the pressure was very small. A similar response was obtained for the highest frequency used, i.e. St_L = 7.33, although in this case the velocity phase lag increased sharply near the exit to about 30°. Boundary layer traverses at different downstream positions indicated that the boundary layer fluctuations lead
the core fluctuations for the lowest frequencies, reaching a phase lead of 80° at the exit. The highest frequencies exhibited a phase lag, eventually spreading through the whole of the boundary layer. The fluctuation amplitude was found to be very much a frequency dependent quantity, showing a "fluctuation defect" for the upstream half of the diffuser, turning into a "fluctuation amplification" for the downstream half. In some cases the fluctuation was amplified by as much as 100% in the boundary layer, while for the highest frequencies double peak responses were obtained. Boundary layer integral values were found to vary little with frequency and amplitude of freestream oscillation.

As mentioned in the previous section (2.3a), Patel studied the problem of a flat plate boundary layer subjected to a travelling wave oscillation, characterised by equation (2). The experimental equipment and travelling wave facility used in his case are very similar to the ones used in the present investigation and as such they will be described in detail in subsequent chapters. He presented his results as plots of mean and boundary layer to freestream amplitude ratio velocity profiles, and velocity phase angles. From Patel's results, several general characteristics can be deduced, which apply to both laminar and turbulent boundary layers. The mean velocity profiles were found to be identical to the steady ones for all frequencies and amplitudes used, an observation supported by Karlsson. Furthermore, amplitude ratio and phase angle profiles were unaffected by freestream oscillation amplitudes, although in the case of the turbulent boundary layer for amplitudes of oscillation greater than 5% of \( U_0 \), the phase lag across the layer was found to increase with freestream amplitude.

Amplitude ratio and phase angle profiles were affected both by frequency and downstream position. The different curves were similar in shape and results could be collapsed together by plotting maximum
amplitude ratio and wall phase angles against a frequency parameter \( \frac{\omega x}{U_0} \) as shown in figure (4).

The general response to \( \frac{\omega x}{U_0} \) of both turbulent and laminar boundary layers was very similar, with the laminar boundary layer producing higher amplitude ratios and wall phase lags. The turbulent boundary layer produced fuller mean and amplitude ratio profiles.

Phase angles were found to change more rapidly in the outer part of the layer, while amplitude ratios increased more rapidly near the wall.

The effect of the travelling wave velocity \( Q \) was studied analytically and showed good agreement with experimental results at \( Q = 0.77U_0 \) especially for the laminar case. It was found that \( Q \) affects drastically both phase angles and velocity amplitude ratios, as shown in figures (5) & (6).

It can be seen that considerable work is still required before the problem of the unsteady boundary layer can be solved. Numerous analytical or numerical solutions have been devised in recent years, but experimental evidence is scarce. This and the fact that theoretical problems cannot often be simulated adequately in the laboratory in cases as complex as this, prevent the necessary comparison between theory and experiment. The need therefore for more experimental data is obvious, as is the need for closer cooperation between experimentalists and theoreticians. The present work hopes to provide some of the experimental results, so urgently needed.
3. THE OSCILLATORY FLOW FACILITY

3.1 The tunnel.

The windtunnel used in this investigation is shown schematically in figure (7). It is located at the "Marshgate lane" annexe of the Queen Mary College aeronautical laboratories and it was designed by Dr. L.G. Whitehead. It is a straight-through, open return blow-down tunnel, of contraction ratio 5.6/1, with a semi-open working section.

Two 75Kw electric motors drive two contrarotating fans, rated at a maximum of 800 r.p.m. The contra-rotating configuration eliminates large scale swirl in the flow. A coarse wire grid at the inlet protects the wooden blades from large particle injection. Irregularities and non-uniformities in the flow are smoothed out by means of wire gauzes and a honeycomb screen. Downstream of the screens, the settling chamber has a 2.86m side square cross-section, which tapers to a nozzle exit 1.4m high and 1.0m across. The last .72m of the upper and lower nozzle walls are flexible. The working section downstream of the nozzle is "semi-open" as it has only side walls. The walls are held into place by means of five parallel steel frames, placed at 48 inch intervals and bolted onto the laboratory floor. Although special attention was given to rigidity when the working section was constructed, and substantial steel box-beams were used throughout, the structure as a whole was not vibration free. The flow oscillation excited transverse vibrations of the side walls, with amplitudes being frequency dependent. With the model in situ bridging the two walls, rigidity was reinforced and the effect was less pronounced. In future projects some form of lateral support for the whole structure, might be advisable. Three pairs of ports on the side walls facilitated the
mounting of the models.

The flow velocity \( U_m \) could be infinitely adjusted to a maximum of about 26m/s at the limiting fan speed, although this value was somewhat reduced with the model in place. A Betz water manometer was used for measuring flow velocities at the tunnel exit.

3.2 The oscillatory flow exciter.

The flexible nozzle ends, or flaps, provided the mechanism for introducing unsteadiness in the mean flow. They were machined aluminium sections of graded stiffness, so that with the upstream edge rigidly connected to the tunnel and the free edge deflected, no slope discontinuities were introduced in the nozzle. The flaps could be deflected periodically about a mean zero position by means of a suitable crank arrangement, as shown in figure (8). Two such parallel arrangements were connected on each flap, in order to provide a two-dimensional displacement, and they were coupled on a common shaft. Power was provided by a .75HP electric motor and transmitted via a system of toothed belts and pulleys, on both flaps. The speed of rotation \( \Omega \), could be varied continuously providing a usable range of frequencies, from 0 to 6Hz. Balancing weights had to be added for smoother operation and true sinusoidal deflection, especially for the lower frequency range. With the slider C at the centre of the slotted disc (figure 8) the link dimension L was adjusted for zero flap deflection. The required amplitude could be chosen by adjusting the eccentric distance R with the slot vertical, against a graduated scale on the tunnel wall at the edge of each flap.

The phase angle between the flaps, was determined by the relative position of the sliders. Their operation was tested both in the in-phase and 180\(^\circ\) out of phase positions, to determine the most suitable mode for measurements. The latter mode was later rejected, although
it induced larger flow amplitudes, due to an undesirable feedback effect in the settling chamber.

3.3 Frequency measurement.

A Mullard vane switched detector (VSD) was used for measuring the frequency of oscillation. The VSD comprises a contactless switch capable of detecting a metallic vane, which switches an oscillator circuit. The layout of the oscillator is such that when a suitable piece of metal (the vane) is inserted in a gap between the oscillator coil windings, the oscillation stops and the D.C. output of the VSD falls to zero. For this reason, a semicircular aluminium vane coupled onto the rotating shaft was used. This caused the oscillator to switch on and off once in a cycle, providing a square wave output, of amplitude equal to 6V and frequency identical to the frequency of the flaps. An "Advance Instruments" timer counter (model TC12) was used to measure this output, to an accuracy of .05%. Frequency measurement accuracy was found to be critical, as small deviations in frequency, caused large changes in boundary layer phase angles.

3.4 The oscillatory flow mechanism.

With the flaps undeflected, the flow in the working section away from the walls, is that of a two-dimensional jet, with a potential core, and upper and lower mixing regions at the constant pressure boundaries. These regions were studied experimentally by Parker (1970) and they were found to converge into the core at 5°, and diverge at 10° to the horizontal. Their behaviour is very important in connection to the mechanism producing the unsteadiness in the potential core. Thus, when a wave-like disturbance is introduced into the shear layers defining the mixing regions, the pressure differences set up, tend to increase its amplitude until the wave contour becomes more and more distorted and is soon unsymmetrical (figure (9b&c)). The waves finally
overly each other, and roll-up like vortices (figure 9d,e). The mechanism is reminiscent of the creation of water waves and it was first investigated by Lord Rayleigh. The presence of these vortices changes drastically the potential flow in the core. Thus a sinusoidal disturbance introduced by the oscillating flaps, induces a harmonic oscillation in the potential core. The eventual distribution and strength of these vortices, determines the frequency and amplitude of oscillation. The rolling up process is also of importance, as amplitude increases in the shear layer are accompanied by corresponding increases in the core. It is expected that an increase in the flap amplitude, will speed up the rolling up process.

Two parallel streets of vortices thus develop, one for each shear layer. Stability considerations dictate that each vortex in one street is opposite the centre of the interval between two consecutive vortices in the other row, as shown in figure (10a). The complex potential due to an infinite row of equidistant vortices, each of strength $\gamma$ will be given by,

$$w = \frac{i\gamma}{2\pi} \ln(\sin \frac{\pi z}{a})$$

(21)

where $a$ is the spacing between them, and $z = x + iy$.

This makes

$$u - iv = -\frac{dw}{dz} = -\frac{i\gamma}{2a} \cot \frac{\pi z}{a}$$

(22)

whence

$$u = -\frac{\gamma}{2a} \frac{\sinh(2\pi y/a)}{\cosh(2\pi y/a) - \cos(2\pi x/a)}$$

$$v = \frac{\gamma}{2a} \frac{\sin(2\pi x/a)}{\cosh(2\pi y/a) - \cos(2\pi x/a)}$$

(23)

It follows that for the parallel rows considered, symmetrical with respect to the plane $y = 0$, the strengths being $\gamma$ for the upper and $-\gamma$ for the lower row, the whole system will advance with a uniform velocity,

$$Q = \frac{\gamma}{2a} \tanh \left( \frac{mh}{a} \right)$$

(24)

$h$ being the distance between the two rows, inducing a velocity at the plane of symmetry, $U_0 = \frac{\gamma}{a}$; assuming vorticity is derived from the
shear layer only. By considering figure (9), we can also see that $a$ is the flap oscillation period; i.e. $a = Q/f$.

Thus (24) gives,

$$\frac{Q}{U_0} = \frac{C}{2} \cdot \tanh \left( \frac{\pi f h}{Q} \right)$$

(25)

the constant $C$, being introduced to account for the additional vorticity introduced by the flaps.

The effect of the alternating vortices is to produce an oscillation in the $y$ direction, near $y = 0$, resulting in oscillatory changes in incidence described by:

$$\varepsilon(t) = \varepsilon_{\text{max}} \sin \omega (t - \frac{x}{Q})$$

(26)

This oscillatory "gust flow" is characterised by a phase shift (lag) in the $x$ direction. This is due to the vortices convecting downstream with a velocity $Q$, thus creating a travelling wave oscillation.

Therefore, the phase lag $\phi = \frac{\omega x}{Q} = \frac{\omega x}{U_0} \frac{U_0}{Q}$. The non-dimensional frequency $\frac{\omega x}{U_0}$ is a convenient frequency parameter, which we will meet again in subsequent sections of this work, together with the Strouhal number $St_h = \frac{f h}{U_0}$ appearing in (25).

With $Q = \infty$ the oscillation, (26), reduces to a purely time-dependent motion. Parker (1970), studying delta wings in unsteady flow and Wasserson (1971) studying a circular cylinder in oscillatory flow, produced streamwise phase lag results for a range of non-dimensional frequencies from 0.2 to 2.2, indicating a travelling wave velocity $Q = 0.6U_0$ invariant with frequency and flap amplitude. Patel (1974), used this value to calculate $C$ in equation (25). For $f = 8Hz$, $St_h = 0.308$ he obtained $C = 1.3$.

The presence of a rigid plane boundary at $y = 0$ (a splitter plate of chord greater than $h$), changes drastically the streamline pattern near the boundary, since the flow now is constrained to oscillate in the $x$-direction only in accordance with equation (2). This type of flow can be induced by the presence of a virtual image row of vortices, symmetrical to the one present in the half-plane under consideration. This leads to an equation for the travelling wave velocity, of the form:
\[ \frac{Q}{U_0} = \frac{C}{2} \coth \left( \frac{\pi fh}{Q} \right) \]  

(27)

Patel calculated the modified value of \( Q \) for the experimental arrangement shown schematically in figure (10b) and for a non-dimensional frequency range from 0.15 to 2.5, finding \( Q = 0.77U_0 \) again invariant with frequency and flap amplitude. He confirmed the experimental result using equation (27), with \( C = 1.3 \) and \( f = 8 \)Hz.

Patel's result together with Parker and Wasserson's, are shown in figure (11) with experimental results obtained in this investigation. These results were taken for a frequency parameter range from 0.06 to 2.7. Figure (11) shows phase angles measured just outside the boundary layer, \( y = 5 \text{cm} \) in a system of coordinates \((x, y)\) parallel and perpendicular to the model surface. Figure (12) shows results from measurements taken traversing the working section in a direction parallel to the model chord-line, at a height \( y = \frac{h}{4} \) above it.

Both sets of results portray a dependence of \( Q \) on frequency, not observed by previous workers. A "least square" fit on phase lags for different frequencies, reveals \( Q \) increasing with frequency rapidly at first, and more gradually for frequencies greater than 3Hz. It finally reaches a value very close to the one obtained by Patel, as shown in figure (11). It should be noted though, especially for the first set of results, that scatter increases with decreasing frequency and values obtained for \( Q \) at 1 and 2Hz, should be treated with caution. Away from the model surface \((y = 35 \text{cm})\), there is much less scatter in the results and frequency dependence of \( Q \) is more pronounced.

Calculations are shown in tables (1a) and (1b). From these it can be seen that for frequencies greater than 4Hz \( (St_h = 0.255) \) the assumption for a constant \( Q \) holds well.

The frequency dependence of \( Q \) is inherent in equations (25) and (27) and the experimental trend is portrayed well by (25), although one should realise the limitations in both equations. They both assu-
me an infinite row of vortices of equal strength, thus neglecting the rolling up process which is bound to have an increasingly important effect at the lowest frequencies. Also any variation of vortex strength with frequency and vorticity diffusion in the shear layers was not taken into account. Both equations have a limiting value, \( Q = 0.65 \), for \( f > 6 \text{Hz} \), being reached from below by (25) and from above by (27) the constant \( C \) taken as 1.3.

A frequency spectrum analysis revealed that the flow not only responded at the flap forcing frequency, but also at half and double its value. Previous workers observed the flow increasingly responding to the double frequency harmonic with increasing frequency, due to laminar separation from the flaps. This was corrected by inserting a row of vortex generators in the flow, at each flap edge. As a consequence, half and double harmonics were an order of magnitude lower than the forcing values.

Errors due to this phenomenon will only affect total boundary layer turbulence measurements, since the digital sampling system used acted as a narrow band filter around the forcing frequency.

<table>
<thead>
<tr>
<th>Table 1a. ((y = 5\text{cm}))</th>
<th>Table 1b. ((y = 35\text{cm}))</th>
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<td>( f/\text{Hz} )</td>
<td>( 0/U_o )</td>
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<td>6</td>
<td>0.750</td>
</tr>
</tbody>
</table>
4. **THE MODEL**

4.1 Preliminary considerations.

A model had to be designed, fulfilling the necessary requirements imposed by the problem; i.e. a large adverse pressure gradient and a well developed turbulent boundary layer thick enough for hot-wire traverses. The presence of the model in the tunnel working section should not alter too much the development of the shear layers.

The large adverse pressure gradient requirement, necessitated the use of an aerofoil model decelerating the flow up to the point of separation and then relaxing again. An ordinary highly cambered aerofoil or an aerofoil with flaps was rejected, as it would induce undesirable flow deflections which might change drastically the upper and lower shear layer distributions by leading the flow into the region outside the tunnel walls and through the cross member supports. Instead, an "S" shaped aerofoil was envisaged, ideally imposing a favourable, adverse and then relaxing pressure gradient on the measuring surface, and a symmetrically opposite pressure distribution on the other surface. This configuration produced a net zero lift and hence zero flow deflection. To test its feasibility and determine the possible range of velocity reduction attainable before separation, a small pilot model was constructed having a flat plate profile with its leading and trailing edges deflected so that they lied parallel to the freestream direction. A small blow-down tunnel of jet exit area 12x12cm$^2$, contraction ratio 14/1 & a maximum speed close to 50m/s was used, with side-walls added to produce a scale version of the full scale working section. A 17cm chord was chosen, to give the same h/c ratio as the full scale facility, for a 2m chord model. Thickness to chord ratio was 3.6% and 20 pressure tappings (dia. .020inch) on the upper surfa-
ce centreline, recorded the pressure distribution on a multitube alcohol manometer. The tunnel was run to its maximum speed, achieving a Reynolds number $Re_c = 6 \times 10^5$. Results indicated that a continuous pressure increase could be achieved on the flat plate portion, ranging from a minimum $Cp = -0.75$, to a maximum $Cp = 0.35$. By increasing the model incidence, the maximum suction point changed little, while the maximum pressure was reduced due to a thickening boundary layer, until any further increase in incidence caused separation.

With this pressure range in mind, an "S" shaped aerofoil was designed, using thin aerofoil theory. (ref 21) To account for the constant pressure boundaries, an image system of cascade aerofoils spaced at a distance $h$ above and below the x-axis was used as the simplest approximation.

Consistent with thin aerofoil theory, each aerofoil was replaced by a vorticity distribution $\gamma(x)$, on the chordline. This is a reasonable approximation for a camber to chord ratio less than 10%.
aerofoil AB due to an elemental vortex $\gamma \delta x$ located on the $n^{th}$ aerofoil at $x$, is given by:

$$\delta w = \frac{\gamma \delta x}{2\pi r} \sin e$$

with $\sin e = (x_0 - x)/r$ and $r^2 = (x_0 - x)^2 + (nh)^2$.

Thus,

$$w(x_0) = \frac{1}{2\pi} \int_0^\infty \left[ \sum_{n=-\infty}^{\infty} (x_0 - x) \gamma(x) \right] dx$$

$$= \int_0^\infty \left[ \sum_{n=-\infty}^{\infty} (x_0 - x) \right] dx$$

$$= \int_0^\infty \left[ \sum_{n=-\infty}^{\infty} (x_0 - x) \right] dx$$

and in order that the mean camberline may be a boundary,

$$w(x_0) = U_\infty \left[ \alpha - \left( \frac{dy}{dx} \right)_x \right]$$

where $\alpha$ is the angle of attack and $y_C$ is the camberline displacement from the chordline. To evaluate (28), we change the variables so that $x = \frac{c}{2}(1-\cos \theta)$. Then we get:

$$w(\phi) = \frac{1}{2\pi} \int_0^\infty \sin \theta (\cos \theta - \cos \phi) \times \left( \sum_{n=-\infty}^{\infty} \left( \frac{c}{2h} \right)^2 \left( \frac{\sin \theta}{\cos \theta - \cos \phi} \right) \right) d\theta$$

The doubly infinite sum can be evaluated using contour integral methods. If we let $B = \frac{c}{2h}(\cos \phi - \cos \phi)$ and noting $\frac{1}{\pi^2 + B^2} = \frac{1}{B} \cot (\pi B)$

$$= \frac{\pi}{B} \coth (\pi B),$$

then

$$w(\phi) = \frac{c}{4h} \int_0^\infty \sin \theta \coth \left( \frac{\pi c}{2h} \right) (\cos \theta - \cos \phi) d\theta$$

The problem can now be solved for $y_C$, if a suitable $y(x)$ distribution is chosen. If $y = \sum_{n=1}^{\infty} A_n \sin (n\theta)$, then we have stagnation points at the leading and trailing edges. For an antisymmetrical loading on each surface, even Fourier coefficients have to be used, and in this case $y$ was taken as,

$$y = A_2 \sin 2\theta + A_4 \sin 4\theta$$

with $A_2 = 0.7121$ and $A_4 = -1.161$, calculated from the relation

$$\Delta C_p = C_{p_{upper}} - C_{p_{lower}} = 2C_{p_{max}} = \frac{2\gamma}{U},$$

$C_{p_{max}} = \frac{\pi}{2} - 0.75$ at $x/c = 0.21$ and $x/c = 0.81$ respectively. The position of maximum suction was chosen as close to the leading edge as possible for early transition to turbulence, without causing laminar separation.
Then \( y_c(\phi) = \frac{c}{2u} \int \phi w(\phi) \sin \phi d\phi + \frac{U_\infty \alpha \cdot c}{2}(1 - \cos \phi) \) (31)

Thus integrating (30) numerically and substituting in (31), the required camberline was obtained as shown in figure (13), together with the assumed load distribution.

Using this camberline a second pilot model was constructed with the same dimensions. The number of pressure tappings was now increased to 26. Results for the pressure and velocity distributions are shown in figure (14). Small changes in incidence affected the pressure distribution only near the leading edge of the aerofoil, consistent with the cascade assumption. Results agreed well with the assumed load distribution between \( x/c = 0.15 \) and \( x/c = 0.6 \). Although the maximum loading positions were predicted accurately, the measured \( C_p \) at \( x/c = 0.81 \) was lower than expected at \( C_p = 0.4 \). To eliminate the possibility of local separation at this position a flow visualisation experiment was performed. For this purpose green fluorescent paint was used mixed with methylated spirit, to reveal the surface streamline pattern. No separation was detected and the discrepancy was attributed to the presence of a relatively thick boundary layer in this position, changing the aerofoil profile. Halving the tunnel speed did not affect the pressure distribution significantly, and so the results were expected to hold well, apart from boundary layer effects, for the full-scale model at a Reynolds number \( Re_c = 3 \cdot 10^6 \).

4.2 Full scale model construction.

A full scale model was constructed using the developed camberline, having a chord of 2.04m and a thickness to chord ratio of 3.6% (see figure 15). A strong wooden frame provided the structural skeleton on which the wooden nose and tail sections were bolted. The main frame was covered on both sides with a 1/16 inch thick, steel sheet.
The wooden frame consisted of six 4.7cm-thick longitudinal sections and five transverse sections, of which the end sections were 4.7cm thick and the three intermediate ones 2.4cm. Wooden blocks at the joints enhanced rigidity. The nose had an elliptic profile and the tail was contoured (see figure (20)) so as to delay separation at the lower surface trailing edge. Both nose and tail sections were made of hard plywood planks, sandwiched together and contoured to shape. The sharp trailing edge was made of "Tufnol" to prevent chipping and to facilitate the mounting of pressure tappings very close to the trailing edge. A total of 59 pressure tappings were used on the model centre-line; 39 on the upper surface and 20 on the lower, including the leading edge. At intervals of about 25cm, a row of inserts was placed on the first off-centre longitudinal member, to provide the facility for a surface mounted boundary layer traversing gear. A scaled view of the model is shown in figure (16), indicating the general arrangement of pressure tappings and tubing, the inserts and the internal structure. (Also see plate II).

For unsteady pressure measurements, "Statham" pressure transducers were used. As it was desirable to eliminate any phase lags and attenuations between the measuring point and the transducer diaphragm, the transducer had to be mounted as close as possible to the pressure tapping. Due to the small thickness of the model, mounting the transducers directly underneath the measuring position was impracticable and therefore the pressure transducers had to be mounted outside the working section. Pressure tubing had to be employed to transmit the information. The presence of the pressure tubing introduces undesirable time lags in the measuring system, and this called for a careful design of all the components involved, for minimum signal distortion. For this reason a simple experiment was designed, treating the pressure tapping, tubing and transducer series combination, as a single
degree of freedom mass-spring system responding to a known pressure step input. The experimental setup and some results are given in Appendix I. As the best compromise, a rather large orifice diameter was chosen (.04 inch), with an orifice length equal to 3/32 inch. All pressure tubes from the measuring surface were kept to a minimum practical length of 50 inches.

While the measuring surface was glued onto the wooden frame, the opposite surface could be removed, both for mounting the model in the wind-tunnel and for access to the internal components.

4.3 Steady flow pressure distribution.

The model was mounted in the working section with the leading edge (L.E.) well into the nozzle, to prevent laminar separation on either side of the leading edge due to large oscillating pressure gradients introduced by the flaps. The measuring surface was placed facing downwards for easier boundary layer traverses.

Initially both surfaces and the tunnel walls near the model were extensively tufted, to give the surface streamline pattern and reveal possible separation regions. Boundary layer separation from the tunnel walls was also possible, due to the imposed severe adverse pressure gradients. With the tunnel run at a variety of speeds and different flap settings, no separation was observed on the measuring surface, but the flow separated on the other surface near the trailing edge (T.E.) at \( x/c = .945 \). The surface flow pattern indicated a two-dimensional flow for a considerable span on either side of the tappings and also a separation position invariant with span.

With the flaps deflected upwards in-phase, a small separation bubble was detected between \( x/c = .05 \) and \( .1 \), for a flap deflection equal to 2 inches. The bubble disappeared for smaller deflection
amplitudes.

Separation of the flow near the T.E. was not expected to influence measurements upstream since it only produced a very thin wake. Nevertheless it impeded intended measurements around the T.E. to establish the oscillatory Kutta condition. The boundary layer transition position was found to be near x/c = 0.23, using a stethoscope.

A series of steady pressure measurements were performed, using a bank of 25-tube alcohol manometers, for different flap deflections with the flaps both in and out of phase. Results are shown in figures (18) & (19) with the flaps at their maximum deflection of 2 inches. Figure (17) shows the pressure distribution on the model, with the flaps undeflected. The result is compared with that obtained using a camberline singularity method developed by B.C Basu (1975), for calculating the potential flow about an arbitrary aerofoil both isolated or in a cascade. It is essentially a derivative of the A.M.O. Smith surface singularity method, with the singularities (sources, sinks and vortices) placed on the camberline. The boundary condition of tangency of the flow is of course still satisfied on the surface of the aerofoil. The method offers advantages over the A.M.O. Smith method, as it requires only about half the number of unknown singularities required by the latter, thus reducing computing time considerably. This method was used for the cascade problem, and the solution shown was derived for an unstaggered cascade of ten aerofoils on either side of the model, and 30 camberline singularities. Physical agreement with experiment is excellent, although the method underestimates somewhat the Cp values at the maximum suction positions. The disagreement near the leading edge on the upper surface is probably due to the presence of the flaps, up to about x/c = 0.2, which were not accounted for in the theory. The pressure distribution shows a well behaved flow with $C_{p_{\text{min}}} = -0.88$ at $x/c = 0.19$ and $C_{p_{\text{max}}} = 0.4$ at $x/c = 0.76$ on the
measuring surface. Separation is confirmed at $x/c = .945$, on the opposite surface. Figure (18) shows the pressure distribution varying with the flaps deflected up or down, in-phase. The displacement of the flaps essentially changes the incidence of the oncoming stream and the effects are again prominent at the front part of the aerofoil, up to $x/c = .4$, with the rest of the pressure distribution not duly affected. Hence with the flaps down, $C_{p_{\text{min}}} = -1.08$ at $x/c = .175$, while with the flaps up, $C_{p_{\text{min}}} = -.68$ at $x/c = .21$. The change in the maximum suction position might be quite important as it affects the transition position, which with the flaps oscillating will move in a cyclic fashion accordingly. This could not be verified experimentally, as measurements in the transition region were not taken.

From the change in pressure coefficient at the leading edge, it is obvious that the stagnation point moves considerably with flap position.

Figure (19) shows the effect of deflecting the flaps in the out of phase mode, forming either a convergent or divergent nozzle at the tunnel exit. This time, the whole pressure distribution is displaced either upwards or downwards, although the pressure gradient is affected again only near the front end of the aerofoil. In this region, each surface is affected mainly by the position of the adjacent flap, in a manner similar to figure (18). The displacement of the pressure distribution can be attributed to the change in $h$, the separation between the shear layers, the effect being more pronounced for smaller values of $h/c$ than larger ones.
5. UNSTEADY FLOW MEASURING SYSTEM

5.1 Traversing gears.

In order to measure velocity phase angle changes both in the freestream along x and in the boundary layer, two hot-wires and therefore two traversing gears had to be used; one suitable for B.L. traverses and the other for measuring freestream velocities. The boundary layer traversing gear, shown in detail in figure (21) (and plate III) had to be fixed on the model surface, so as to prevent any relative displacement between model and measuring probe. For this purpose inserts were provided on the model surface as described in the previous chapter, so that the traversing gear could be rigidly bolted onto the model.

The basic slide and slider assembly was bought commercially, and a "Unislide" unit was chosen (model A1500P40), having an overall length of 6 inches and an accuracy of .001 per foot. This came complete with a micrometer drum dial reading to .001" and progressing .025" with each revolution. A linear index scale could also be used, for .025" gross location. The base of the unit was modified to accommodate the model fastening bolt C, and wooden nose and tail sections were added to provide a streamlined body. The hot-wire mounting probe was fastened on an aerofoil shaped extension (section AA'), so that measurements could be taken on the model centreline. The whole assembly was then pivoted on the slider at P, so that not only the height of the probe above the surface could be chosen, but also its incidence against a scale graduated in degrees, with a range of ±20°. This was done in an effort to keep the boundary layer traverses as nearly perpendicular to the surface as possible. The hot-wire mounting tube had
a total length of 45 cm with a constant diameter for the first 20 cm and then tapering linearly to a diameter of 4 mm at the probe end. The constant diameter section could be slid along its support, giving a streamwise measuring span, from a minimum of 25 cm to a maximum of 45 cm ahead of the traversing gear body. This ensured that upstream interference was kept to a minimum. The minimum value of 25 cm was chosen from a previous investigation by M. H. Patel. The requirement for a non-vibrating hot-wire mount, even at the maximum extended position, directed the use of an initially thick tube (8 mm outside diameter) finally tapering to the required thickness.

For remote operation, the micrometer drum was removed, and placed on a detachable 1.5 m long extension, which could be removed every time a new "y" position was chosen during a traverse. Thus again any unnecessary interference with the flow was avoided.

Finally the probe support, carried a single cranked B.L. hot-wire probe (DISA 55P05).

The freestream traversing gear was already available for the tunnel in use, and had a three degree of freedom movement, although it was mainly used only for rough positioning in the freestream. It carried a single straight hot-wire probe (DISA 55P01). A bracket was added to support an N.P.L. pitot-static tube, used for calibrating the hot-wires.

Both hot-wires were carried on standard 4 mm insulated supports, with standard 5 m coaxial cables transmitting the signals. Probe and cable had a mean cold resistance of about 3.7 Ω. The sensor element was made of platinum-plated tungsten wire of 5 μm diameter and overall length of 3 mm, although the temperature sensitive portion was only 1.25 mm confined at the centre. The end portions were gold and copper plated to a diameter of approximately 30 μm.

Plate IV, gives an overall view of the wind-tunnel working section
with the model and both traversing gears in situ.

5.2 Pressure Transducers.

Pressure transducers were used both for measuring unsteady pressures, and for hot-wire calibration, in conjunction with the pitot-static tube. A pair of "Statham" differential transducers were used (model PM283TC), incorporating a strain-gaged diaphragm as a part of a bridge circuit, from which the out of balance voltage provided an output proportional to the applied pressure difference. They operated on a 5V d.c. voltage, supplied by a suitable stabilised source. Their calibration characteristic was linear and repeated recalibration showed no changes with time. However they were found to be attitude sensitive; thus changing the transducer's attitude, shifted the calibration curve. For this reason they had to be permanently fixed in the vertical position throughout the experiments.

The problem of transducer-pressure tubing time response, a vital parameter in unsteady measurements, was treated in detail (see Appendix 1) and suitable corrections were applied to the experimental results.

5.3 Hot-Wire Calibration.

The two hot-wires provided the dual signal path necessary for the evaluation of flow velocity phase angles. Calibration of the reference signal path was not necessary, as it was only used to provide the Trigger signal for periodic signal sampling. To calibrate the boundary layer probe, the traversing gear was adjusted so that the hot-wire was in line with the N.P.L. pitot-static tube, without being too close to it, in a region of low turbulence outside the boundary layer. Running the tunnel throughout its speed range, the pressure transducer connected to the pitot-static tube gave a series of voltages $E_p$, directly proportional to $\frac{1}{2} \rho U^2$, while the hot-wire anemometer output voltage $E$, 

was read on a Solatron digital voltmeter (DVM) after being processed through the system of filters shown in figure (22).

A calibration law was used of the form,

\[ E^2 = A + BU + CU \]  

(32)
suggested by Siddal & Davies (1972). The values of the constants A, B and C were calculated on a Hewlett-Packard mini-computer (model 9820A) using a least squares curve-fit program. The calibration process took about 15 minutes, a short enough time interval to allow frequent recalibration, so that dirt accumulation and temperature variations encountered in this type of tunnel facility, could be properly taken into account.

Once the probe angle was set before calibration, it was kept constant during a boundary layer traverse so as to prevent changes in A, B and C due to prong interference.

Using equation (32), a formula can be derived for estimating velocity turbulence values. Thus if \( E(t) = \bar{E} + e(t) \) and \( U(t) = \bar{U} + u(t) \), where the bar denotes mean values and the lower case symbols the instantaneous fluctuation values, then substituting in (32) we get

\[(\bar{E} + e)^2 = A + B(\bar{U} + u) + C(U + u)\]

Neglecting small order terms \( (e^2 \text{ and } u^2) \) and rearranging, we get

\[2\bar{E}e = u(C + \frac{1}{2}BU^{-1/2})\]

Taking the root mean square on both sides we get

\[u_{rms} = \sqrt{\frac{2\bar{E}}{(C + \frac{1}{2}BU^{-1/2})}}\]  

(33)

As a precaution, the hot-wire was also calibrated against a voltage drawn directly from the anemometer output, the normal channel for measuring the total flow turbulence, and the new values of \( E \) and \( e \) were used for calculating \( u_{rms} \). Comparison with values obtained using the previous calibration curve, showed differences of up to 3%.
5.4 The Digital Measuring System.

The velocity fluctuations in the turbulent boundary layer subjected to an oscillatory freestream, contain both periodic and randomly fluctuating terms. To obtain therefore any intelligible results the velocities (or pressures) at the forcing frequency had first to be extracted from the overall turbulence spectrum. The presence of half and second harmonics of the driving frequency in the freestream and consequent harmonics in the boundary layer due to its non-linearity, complicate matters even further. The use of a conventional narrow band filter was also not possible, as it would introduce unacceptable phase shifts in the readings. A digital periodic sampling system was used instead, designed by Dr. L.G. Whitehead for extracting the required information at the driving frequency only.

The unit (termed a phase shifter) is similar to the one described by M.H. Patel. Sample voltages are taken from a signal wave-form at selected phase intervals, over a number of oscillation cycles. An oscillatory signal of the same amplitude and frequency is needed to provide the time reference. This signal fed to the input terminal, passes through a high gain amplifier which converts it into a reference square wave. This triggers a reading at the beginning of the first cycle, and at intervals of \((1 + \frac{1}{n})\) of each subsequent cycle, up to \(n\) cycles; thus composing a single cycle of the measured signal for every \(n\) cycles, giving enough time to the associated equipment to read and record the samples.

The number of cycles \(n\), can be chosen from two fixed values, 8 and 16 and the process can be repeated automatically in steps of 10, up to 100 times.

Figure (22), shows in block diagram form the parallel paths for both reference and measured signals. Using this diagram as reference,
the function of the various units used, will now be explained. The
suffix 1 on the diagram refers to the reference values, accented quan-
tities denote randomly fluctuating terms, and the wavy overbar the
organised frequency components. (Any harmonics are included in the tur-
bulent terms) First the signal from the two hot-wires is fed to ane-
mometers 1 & 2, DISA 55D01 and 55M01 respectively. On both units the
resistance compensation dials were set at 1.8 times the probe-lead
resistance value, to achieve a high signal to noise ratio and optimum
sensitivity. Voltages from the two anemometers are made up of mean
and fluctuating components, i.e. $E(t) = \bar{E} + \varepsilon + e^t$ and $E_1(t) = \bar{E}_1 + \varepsilon_1$, neglecting harmonics in the reference signal. The mean value $\bar{E}$, is
read directly on a DISA 55D30 digital voltmeter (DVM), averaged over
8 seconds and the total turbulence R.M.S. value, on a DISA 55D35 R.M.S.
meter, operative from 1Hz to 400kHz, but having an accuracy of ±7% (fsd)
for the lower frequency range, from 1 to 10Hz. This unit was later
substituted by a "Solatron" JM1860 time domain analyser (TDA) which
had an accuracy of ±1% from 3Hz upwards, again deteriorating to ±3% of
the displayed value at 1Hz. Both channels were then fed through a
"Barr & Stroud" variable filter (unit EF2), which was used in the low
pass mode with a frequency cut-off at 25Hz. Signal attenuation in this
mode was within 0±.5dB, increasing to 3±.5dB at the cut-off frequency.
For higher frequencies attenuation increases at approximately 36dB/octave
down to noise level. Although the working frequency maximum was only
6Hz, the signal was not filtered below 25Hz, to ensure no attenuation,
while the identical cut-off frequency for both channels ensured no re-
relative phase errors.

From the filter, the measurement signal passes through a DISA
auxiliary unit (AU) (55D25) where it is negatively biased and fed to
a "Solatron" DVM for measuring the instantaneous voltage values.

The digital voltmeter (LM1420.2), had its polarity relay by-
passed, to increase digitising speed from 8, to 22 conversions per sec.

A negative programmed polarity was chosen for greatest common mode rejection, and so that the internal calibration cell (connected as a negative voltage of \(-1.019\text{V}\)) could be operated normally. For this reason the voltage input to the DVM had to be negative . After a digitise command is received, digitisation is complete in 20ms. This introduces a small error, due to averaging the signal during this period.

Thus if 
\[ f(t) = f_s + f_o e^{i\omega t}, \]
then averaging over \( 2t_1 (=20\text{ms}) \)
we get:
\[
\bar{f}(t) = \frac{1}{2t_1} \int_{-t_1}^{t_1} (f_s + f_o e^{i\omega t}) dt \\
= f_s + f_o \frac{\sin \omega t_1}{\sin \omega t_1} e^{i\omega t}
\]

Thus although the mean component of the signal \((f_s)\) remains unchanged, the amplitude of the oscillatory component \(f_o\), has to be increased by a factor \(\frac{\omega t_1}{\sin \omega t_1}\), to give the correct value.

Table 2. shows the correction factors for the working frequency range. A beneficial outcome of this, is the total exclusion from the signal of any 50c/s mains hum and its harmonics, and also a further suppression of turbulence frequencies higher than the working range.

Returning now to the reference signal path, from the filter the signal is fed through a DISA 55D26 signal conditioner, where the d.c. component is removed by shifting the zero volts position and the "cleaned" reference signal \(\bar{\varepsilon}_1\) is now ready to be fed to the phase shifter input. To ensure that the d.c. component always remains zero, the signal is monitored on a cathode ray oscilloscope (CRO), so that small adjustments can be made during a boundary layer traverse.

In the phase shifter, as explained previously it is converted into a square wave. The ensuing trigger signal is then fed to a "Schlumberger" data transfer unit (DTU), consisting of an interface
module and an output driver, thus making a single channel data logger. The trigger signal causes the interface module to issue a "digitise" command to the DVM via a digital data link, record the information and translate it into a form acceptable to the DTU output driver. From the output driver the information is stored in the Hewlett-Packard mini-computer, which issues back a "ready to record" or "recording complete" signal, initialising or terminating the whole process.

A simple computer program analyses the stored information using a Fourier summation technique. Thus if \( f(t) \) is the signal to be measured, for each oscillation \( n \) samples comprising a full cycle were taken. If \( T_0 \) is the period of oscillation of \( f(t) \), then \( T = \frac{n+1}{n} T_0 \) is the sampling period. The \( n \) samples are: \( f(0), f(T), \ldots, f(sT), \ldots, f(n-1 \cdot T) \).

If this process is repeated over \( m \) cycles, then for the \( m \)th repetition we have: \( f((m-1 \cdot nT), f((m-1 \cdot n+1)T), \ldots, f((m-1 \cdot n+s)T), \ldots, f(mn-1 \cdot T) \).

Summing now in columns and taking the mean values, we get a mean estimate of a full cycle of \( f(t) \).

\[
\text{i.e. } f_s = \frac{1}{m} \sum_{r=0}^{m-1} f((rn+s)T) & s=1,2,\ldots,n
\]

These \( n \) values (\( f_1 \) to \( f_n \)) can be analysed to give the mean value, the in-phase and the quadrature components of \( f(t) \).

Thus,

- the mean value \( F_0 = \frac{1}{n} \sum_{s=1}^{n} f_s \)
- the in-phase component \( F_{\text{in}} = \frac{2}{n} \sum_{s=1}^{n} f_s \sin \alpha_s \) \hspace{1cm} (34)
- the out of phase component \( F_{\text{out}} = \frac{2}{n} \sum_{s=1}^{n} f_s \cos \alpha_s \)

Then amplitude = \( \sqrt{(F_{\text{in}}^2 + F_{\text{out}}^2)} \) and phase angle = \( \tan^{-1}(F_{\text{out}}/F_{\text{in}}) \), with \( \alpha_s = \frac{2\pi}{n}(s-1) \), giving \( \alpha_1 = 0 \) at the beginning of a cycle, and subsequent intervals of \( \frac{2\pi}{n} \). The number \( n \) was chosen as 8 for the present experiments.

The effect of \( m \), the number of repetitions of \( n \) readings requi-
ires particular attention, as it does not only affect the variance of the results (especially the phase angles), but also the system bandwidth, which has to be kept as narrow as possible, to eliminate any half or double frequency components inherent in the flow and also the remaining turbulence which was unfiltered below 25Hz. The latter could be especially significant for the thicker regions of the boundary layer where the increase in scale of turbulence was expected to shift the turbulence spectrum towards the lower frequency end.

To determine the actual frequency response of the system, the problem was treated analytically in Appendix II, by assuming a general wave-form \( f(t) = a \sin \omega t + b \cos \omega t \), in the preceding Fourier analysis. The results revealed a striking pattern of resonance peaks, not only at the driving frequency \( \omega_0 \), but also at \( \frac{\omega}{\omega_0} = \frac{1}{n+1}, \frac{n-1}{n+1}, \frac{2n-1}{n+1}, \frac{2n+1}{n+1} \) and at intervals of \( \frac{n}{n+1} \) thereafter. The \( \frac{n-1}{n+1} \) and \( \frac{2n-1}{n+1} \) values, exhibited a negative resonance peak.

For the rest of the frequency spectrum the response was suppressed to near zero, this being especially true for the half and double frequency cases. Striking as it was, this response did not affect the measurements significantly, as the system bandwidth at the resonant points was very narrow, varying from about 1% of the driving frequency at \( m=15 \), down to .4% for \( m=40 \). The in and out of phase response to the assumed signal are shown in figure (23), near \( \frac{\omega}{\omega_0} = 1 \) and \( m=40 \). The behaviour is similar at the other resonance peaks. The values of the constants \( a \) and \( b \), were chosen for normalised amplitude at resonance, for the limiting case \( \frac{\omega}{\omega_0} = 1 + \epsilon \) with \( \epsilon \to 0 \).

The phase shifter was initially tuned for zero phase shift at the driving frequency, with the help of a "Muirhead" decade oscillator (D-880-A), feeding an identical sinusoidal signal to both reference and measuring channels. The tuner dial settings are shown in table 2, together with the DVM correction factors. After tuning, the value of
b should be zero for \(\omega = \omega_0\), but a small value was introduced, equivalent to a phase error of 10°, to examine the effect of signal distortion on the frequency response. No apparent change was observed, and even for values of \(b = a\), the pattern was not significantly altered. Of course, phase shifts are introduced where \(\omega \neq \omega_0\), but this is academic since the turbulence contribution there is minimal.

The maximum number of repetitions \(m\), was restricted from a practical point of view, by the time required for a complete boundary layer traverse, before hot-wire recalibration became necessary; especially for the lowest experimental frequency of 1Hz. Thus \(m = 40\) was chosen for the majority of measurements, with \(m = 30\) used at the lower turbulence regions, near the edge of the boundary layer. This achieved an accuracy better than ±3°, with the exception of the viscous sub-layer, where other factors such as wall proximity and possible flow reversal during part of the cycle creep in. The largest deviations occur elsewhere, at the lowest freestream amplitudes (1Hz) where turbulence and organised oscillation become comparable in magnitude.

The block diagram in figure (22) (and plate V) give an overall picture of the processing equipment. Figure (22) also gives the alternative path used for pressure velocity phase angles.
Table 2.

<table>
<thead>
<tr>
<th>Frequency/Hz</th>
<th>Tuner dial position</th>
<th>Amplitude correction factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>67.25</td>
<td>1.0002</td>
</tr>
<tr>
<td>1</td>
<td>141.35</td>
<td>1.0007</td>
</tr>
<tr>
<td>2</td>
<td>300.50</td>
<td>1.0026</td>
</tr>
<tr>
<td>3</td>
<td>463.28</td>
<td>1.0059</td>
</tr>
<tr>
<td>4</td>
<td>620.18</td>
<td>1.0106</td>
</tr>
<tr>
<td>5</td>
<td>764.30</td>
<td>1.0166</td>
</tr>
<tr>
<td>6</td>
<td>892.94</td>
<td>1.0241</td>
</tr>
</tbody>
</table>
6. INVISCID FLOW MEASUREMENTS-RESULTS

6.1 Introduction.

In this section, a detailed account of experiments and results is given in the inviscid region of the flow, outside the boundary layer.

With the flaps oscillating in-phase, mean and R.M.S. values of pressure and velocity were recorded, together with the corresponding phase changes in the streamwise direction. The flow was investigated for a range of input velocity, amplitude and frequency parameters, at different streamwise positions. Initial measurements with the flaps oscillating in the out of phase mode were finally rejected, due to a feedback effect in the settling chamber which made the interpretation of the results difficult.

The following sections constitute an effort to determine the overall calibration characteristics of the oscillatory flow facility, in the neighbourhood of the model.

6.2 The Pressure and Velocity Amplitudes.

Measurements were taken at eight chordwise positions, as shown in table 3, so that the full range of pressure gradients available could be exploited. Although the subsequent boundary layer measurements were performed in regions of adverse pressure gradient only, downstream of the flaps, for the freestream case measurements were also taken between the flaps (x/c = .1323) and at the flap trailing edge position (x/c = .2328).

At each measuring station, flow velocities were calculated using the model-fixed traversing gear with the probe set just outside the boundary layer. Pressures were recorded by connecting the pressure
### Table 3.

<table>
<thead>
<tr>
<th>Measuring Station</th>
<th>Parameter Range</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Pressure Tapping Number</strong></td>
<td><strong>x = x/c</strong></td>
</tr>
<tr>
<td>30</td>
<td>0.1323</td>
</tr>
<tr>
<td>26</td>
<td>0.2328</td>
</tr>
<tr>
<td>22</td>
<td>0.3779</td>
</tr>
<tr>
<td>19</td>
<td>0.5069</td>
</tr>
<tr>
<td>18</td>
<td>0.5657</td>
</tr>
<tr>
<td>16</td>
<td>0.6137</td>
</tr>
<tr>
<td>13</td>
<td>0.6838</td>
</tr>
<tr>
<td>9</td>
<td>0.7819</td>
</tr>
</tbody>
</table>
transducer to the corresponding pressure tube outlet. After calibration, measurements were taken for the full range of frequencies available, \( f = 1 \) to 6Hz. The effect of flap amplitude was checked at two positions (tappings 16 & 19) together with the effect of varying the freestream velocity. Three flap amplitudes were used, \( A = 2, 3 \) and 4 in. and three velocity settings, \( U_\infty = 17.9, 21.9 \) and 25.3m/s. For the rest of the measurements, flap amplitude was kept at 4" and freestream velocity at 21.9m/s.

Velocity mean and R.M.S. values were measured, using the digital system described in the previous chapter. Both filtered and unfiltered velocities were recorded, to show the effects of freestream turbulence, and the contribution due to the harmonics present.

Consulting figure (22), the oscillating pressure provided the reference signal during these measurements, also giving a measure of the pressure velocity phase angle at the measuring point. For this reason, one side of the transducer was connected to the pressure-tube outlet, while the other was open to the atmosphere. The resulting pressure difference \( (P - P_{at}) \), mean and R.M.S. values, were read on a "Solatron" time domain analyser (TDA) operated in the "true time" mode for averaging and a.c. coupled for measuring R.M.S. values.

Thus, assuming \( P(t) = F + P_{at} \), then

\[
P - P_{at} = (P - P_{at}) + P_{at} + \sin(2\pi ft)
\]

and with the instrument a.c. coupled, the first term on the R.H.S of equation (35) disappears giving:

\[
(P - P_{at})_{rms} = \sqrt{\frac{1}{f} \int_0^f (\sin(2\pi ft))^2 dt} = \frac{P}{\sqrt{2}} = P_{rms}
\]

since \( P_{at} \) is assumed constant.

To calculate the potential head \( \frac{1}{2} \rho U^2 \) at the measuring station, the difference \( P - P_{total} \) was required, with \( P_{total} \) measured at the tunnel settling chamber. This could be achieved by connecting the open
end of the pressure transducer to the settling chamber pressure outlet, but it was avoided since the resulting long tubing could amplify small fluctuations in the settling chamber, leading to erroneous results. Assuming instead that the mean and steady values of $P_{at}$ and $P_{total}$ remain unchanged with the flow oscillating, $P_{total}-P_{at}$ was recorded in the steady case. Then, $\overline{P_{total}-P_{at}} = \overline{(P_{total}-P_{at})_{steady}} - \overline{(P-P_{at})}$.

Results presented in graphical form (figures 24 to 40) for each measuring station, show the pressure and velocity amplitudes as a function of frequency. Pressures are non-dimensionalised with respect to $\frac{1}{2} \rho U^2$, while velocity amplitudes are presented as percentages of the local mean velocity.

Starting with figure (24), we see the results obtained at $x/c = 0.1328$ well within the nozzle region, in a favourable pressure gradient. The pressure amplitude is seen to increase rapidly, from a steady value of 1.2% to almost 11% at 1Hz; varies little with frequency up to 3Hz and then decreases rapidly to 3.4% at 6Hz. This behaviour can be explained with reference to figure (18), which represents the quasi-steady behaviour of the pressure distribution around the model, to a slow change in flap position. Thus any movement of the flaps causes large changes in pressure distribution, giving a $C_P$ amplitude of about 12.5% of the steady value. These changes are reflected in figure (24), with the almost instantaneous increase in pressure amplitude to a large value, until inertia effects take over for frequencies higher than 3Hz. From there on, pressure variations do not have time to adjust to the rapid change in flow geometry.

Velocity fluctuations follow a similar behaviour, with amplitudes up to 5% of the freestream velocity reached, diminishing to about 2.5% at 6Hz. Digital, and analogue unfiltered signals indicate a similar behaviour, with the two curves running parallel to each other. This suggests that turbulence is almost independent of frequency, well in
accordance with the assumption that any harmonics present have negligible effects. The maximum deviation due to turbulence between the two curves is about 10%.

Assuming no correlation between turbulence and organised oscillation, we can calculate the freestream turbulence using

\[ u_{\text{turb}}^2 = u_{\text{total}}^2 - \bar{U}^2 \]  (36)

with \( u_{\text{total}} \) obtained from the unfiltered signal and \( \bar{U} \) from the digital signal. A turbulence value of 2% of the freestream velocity was obtained, varying little with frequency but considerably higher than the steady value of .2%. It can be inferred that the additional turbulence is a measure of the error in using equation (36) as we are subtracting two quantities which are considerably larger than their difference, since no additional source of turbulence is obvious.

The same procedure (equation 36) could be followed for the pressure signal. But as this was filtered down to 12Hz to preclude any resonances due to the pressure tubing, the remaining pressure fluctuations could only have a small effect on the amplitude characteristic. Thus only the analogue signal was measured.

Results for \( x/c = .2328 \) corresponding to the flap trailing edge are shown in figure (25). Again pressure and velocity curves show a similar behaviour, although now changes with frequency are more gradual. The response is parabolic, with a clear maximum at 3.5Hz for the pressure and 3.75Hz for the velocity amplitudes, with values of 8% and 5.5% respectively. The main difference between this and the previous station, is that turbulence now becomes frequency dependent, changing from 1.6% at 1Hz, to 2.5% at 6Hz. This could be due to the presence of the vortex generators at the flap trailing edge.

Proceeding to the next station (\( x/c = .378 \)), pressure and velocity amplitudes now reach 8.5% and 7% respectively. (see figure 26) The oscillations are now less dependent on nozzle geometry, and results
can be explained with reference to the shear layer rolling up process. This is enhanced by the fact that turbulence has now again decreased to a mean of 1.7%, quite independent of frequency. This is in contrast with an increase in steady flow turbulence to .5%. Since the difference between the two stations is only 30cm, the rapid disappearance of the frequency dependent component suggests that no significant large scale unsteadiness is introduced by the presence of the vortex generators. Maximum response for both pressure and velocity oscillations is now near 4Hz, corresponding to a Struhal number $St_h = .28$.

Figures (27)-(30), show the effect of varying the freestream velocity and flap amplitude on the pressure and velocity curves, at $x/c = .5069$. Figures (27) & (29) show the effect of a velocity change, which only alters the curves in the $f$-direction, by a factor inversely proportional to the freestream velocity. Thus non-dimensionalising the frequency by a frequency parameter $\nu = fx/U$, results can be collapsed onto one curve, shown in figure (31) for the pressure case. A maximum response for both pressure and velocity was again achieved at $St_h = .28$.

Figures (28) & (30) show the effect of changing the flap amplitude, for a mean tunnel speed of 21.9m/s. By lowering the flap amplitude from 4 to 3 inches, the response maximum is delayed to 4.8Hz and at 2 inches is only reached at 5.5Hz. Velocity and pressure amplitudes vary linearly with amplitude until a maximum is reached, from there on deviating further and further from the linear condition as frequency increases. This behaviour can be explained if we refer again to the rolling up process (figure 9). The rolling up of the shear layers is speeded up as the input amplitude increases. Thus the small amplitudes at the lowest flap deflection are due to the process still being incomplete, while a maximum is reached as the process is completed. Any further decrease in amplitude with frequency after the maximum is reached is due to a combination of many factors depending on frequency,
such as the vortex spacing, vortex strength and rate of vorticity diffusion.

The same comments applied to the previous two stations also apply at x/c = 0.5657 (figure 32). Turbulence here increases with (U/U) from 0.3% at zero frequency, to a maximum of 3.4% at 4Hz and drops again to 3% at 6Hz. As detailed spectral analysis of the signal was not performed, it is difficult to say whether this frequency dependence is due to the harmonics present, or an actual effect on the random turbulence spectrum.

Figures (33)-(36) again show the effects of changing the flap amplitude and freestream velocity at x/c = 0.614. The same comments apply as for x/c = 0.5069, although amplitudes are now larger, reaching 10% for u/U and 11.5% for the pressure fluctuations. Figures (37) & (38) show the velocity and pressure amplitudes respectively, plotted against a frequency parameter f/U. Commenting on figure (37) for the velocity amplitudes, we observe the total turbulence u_{total} and the organised fluctuations U deviating from each other at the highest frequency parameters, a fact which can only be attributed to the presence of harmonics, which in this position can be as large as 15% of the forced oscillation values. Thus while the narrowly filtered digital signal decreases progressively with frequency after a maximum is reached, the total turbulence tends to level off, showing that the total energy of turbulence is conserved by the creation of harmonics. Use of equation (36) in this region will produce erroneous results, since now there is a definite correlation between random and forcing signals.

Figures (39) & (40) show this behaviour even more pronounced even though frequency parameters recorded are now lower (only for U_{in} = 21.9m/s). Both pressure and velocity amplitudes still increase to 15.4% and 9.7% respectively (figure 39), and 15.3 and 11% (figure 40). After the maximum is reached, considerable scatter is observed for
frequencies higher than 4Hz in both the pressure and velocity analogue results, although digital results are not affected. This instability is related to the apparent increase in total turbulence. Oscilloscope traces of the signal in this region, failed to show any characteristic distortion of the signal at higher frequencies, to explain this phenomenon.

Figures (41) and (42) give an overall picture of the flow behaviour with frequency, at different downstream positions. Figure (41) for the pressure fluctuations (results affected directly by the flaps shown dotted) displays a continuous increase in amplitude downstream. Maximum response occurs roughly at 4Hz downstream of the flaps. All curves for the velocity amplitudes are similar outside the flap region with a maximum response obtained between 4 and 5Hz, and seemingly unaffected by x.

Figures (43) & (44) give the chordwise amplitude distribution for pressure and velocity respectively, for the range of frequencies available. Pressure amplitudes as high as 16% can be obtained at the position of maximum Cp. It is notable that this is also the position of maximum turbulence in the steady case. A curious peak also occurs between 30 and 40% chord, for 5 and 6Hz, but no explanation can be offered for this. No corresponding peaks are observed for the velocity amplitude case, but after the direct influence of the flaps wears out, a continuous increase in amplitude in the downstream direction is observed. Graphs fold over at 4Hz, with the distributions for 4 and 5Hz being almost identical. Although in figure (44) only the total fluctuation signal is displayed, the picture is very similar for the digital signal, representing the forcing oscillation only.

6.3 Pressure and Velocity Phase Angles.

To calculate the phase velocity angles in the downstream direction,
one traversing gear was positioned at a station downstream of the flaps, to provide the reference signal, while the other was positioned at various x-stations.

The calculated phase angles were plotted against a frequency parameter $f(x-x_{\text{ref}})/U$, as shown in figures (10) & (11). From these graphs the travelling wave velocity was obtained as explained in chapter 3, and results are shown in tables (1a) & (1b).

The relative phase angle between the pressure, at a certain pressure tapping and the velocity just outside the boundary layer at the same chordwise position, was also measured. To eliminate any phase errors due to the pressure and velocity signals following different paths, the digital system was first calibrated. This was done by introducing an identical sinusoid through both signal paths and measuring the resulting phase shifts, from $f=1$ to $6\text{Hz}$. Using these, the results could be corrected by a simple addition or subtraction. A further correction was also necessary, to allow for the phase lags introduced by the pressure tubing (see Appendix I). Since pressure was measured at the wall, the phase angle between velocity and pressure at the wall was required. Hence the phase angles were finally corrected for the velocity phase lag through the boundary layer. This last factor introduced some uncertainties, because as it will be explained in chapter 7, the velocity phase angle at the wall could not be measured accurately. Errors as high as $10^{0}$ are therefore possible.

The resulting phase angles are plotted in figure 44a, for a range of chordwise positions and for frequencies from 1 to $6\text{Hz}$. At each x-station, the pressure leads the velocity by an angle decreasing with frequency. Extrapolating the results to 0Hz, we see that the curves pass through, or near, $180^0$, a result which was expected since pressure and velocity have opposite signs in the steady Bernoulli's equation.
Hence intersects with the \( \phi \)-axis, give also a measure of the possible error due to wall velocity phase angle uncertainties.

The slope of the curves increases from \( x/c = 0.2328 \), to \( x/c = 0.5657 \) and decreases again as the pressure gradient changes from zero to adverse, and then declines gradually to favourable at the last station, \( x/c = 0.7819 \).
7. BOUNDARY LAYER MEASUREMENTS-RESULTS

7.1 Introduction.

In the previous chapter we dealt with measurements taken in the inviscid region of the flow; here we concentrate on the boundary layer region, at the model top surface. All results are presented in graphical form, in figures (45)-(118). As pointed out before, in these experiments we are presented with a large number of controllable parameters, such as frequency, flap amplitude, tunnel velocity and of course streamwise position. Since for each boundary layer traverse at least twenty experimental points were required for adequate profile definition and each point was produced from a sample of 30 or more cycles, (up to 40 in high turbulence regions) for an adequately narrow system bandwidth, measurements had to be critically selective rather than comprehensive.

Thus wind-tunnel velocity was kept constant at \( U_{\infty} = 21.9 \text{m/s} \) throughout the measurements as the first constraint. This was decided, since on the strength of measurements in the freestream region, any change in \( U_{\infty} \), only affected the effective frequency parameter. Flap amplitude was kept largely at its maximum value of 4 inches peak to peak, in order to get the largest possible flow oscillation amplitudes, although amplitude effects on the boundary layer were also investigated at a chosen number of cases.

Seven chordwise positions were selected for boundary layer investigation, where a well developed turbulent boundary layer was present, of adequate thickness for the traversing apparatus used.

The pressure gradient in the chosen region, varied from severe adverse at the foremost upstream position to a small favourable at the last position \( (x/c = 0.7819) \), changing gradually downstream.
The whole frequency range from 1 to 6Hz, in steps of 1Hz, was used, although results at 1Hz were largely rejected due to poor accuracy.

The measurement range was thus restricted, but it is hoped that the following results are truly representative of the more general spectrum and reasonably complete.

7.2 The Steady Boundary Layer.

Steady boundary layer turbulence and velocity profiles are shown in figures (45) to (51) for the seven x-stations, namely \( x = .3779, .5069, .5657, .6137, .6838, .7328 \) and \( .7819 \). A study of these graphs shows the velocity increasing rapidly near the wall, up to about \( \eta = .1 \), then following a nearly linear increase up to \( \eta = .7 \), and then slowly reaching the freestream value at \( \eta = 1 \), with \( \delta \) defined at \( U(y) = .99U_1 \). The form of the velocity profiles and also the accompanying turbulence profiles confirm a well developed turbulent boundary layer, with maximum turbulence values ranging from 12 to 17% of the freestream velocity, increasing in the streamwise direction. (Flat plate values obtained from M.H. Patel, show a maximum turbulence equal to 10% of the freestream velocity.) Excluding the first position at \( x = .3779 \) which presents some unique characteristics, for the rest of the profiles maximum turbulence lies between .3 and .46.

Figure (52) shows how the boundary layer develops in the streamwise direction. The velocity defect due to the boundary layer, increases from its value at \( x = .5069 \) up to \( x = .6838 \) and then decreases again to a minimum at \( x = .7819 \), showing that the small favourable pressure gradient there is already causing the boundary layer to accelerate. The dotted line on the same graph indicates the velocity profile at the first station, \( x = .3779 \). At this position, as pointed above, the boundary layer appears quite different from the rest. Although quite clear
of the transition region, the velocity profile is less full and the turbulence profile has a different shape. It is worth noting that at this station the pressure gradient is quite large, while the thickness of the boundary layer is small at \( \delta^+ \). Thus probe positioning errors can be relatively large, and measurements near the wall must be treated with caution due to the wall cooling effect on the hot-wire probe.

Figures (53a, b & c) show the boundary layer integral values, and the variation in skin friction \( C_f \) along the model. Values obtained for \( \delta \) and \( \theta \) (53b & c) confirm the existence of a turbulent boundary layer. The form factor \( H \), reaches a maximum of 2 near \( \bar{x} = .7 \), indicating that the boundary layer approaches separation, and then decreases again as the pressure gradient becomes favourable.

The boundary layer integral values were also calculated theoretically, and results are provided for comparison on the same graphs. Three distinctly different methods were used for the calculation. First an equilibrium method was tried, by J. F. Nash and A. G. J. McDonald (1966). This was found to predict the momentum thickness value reasonably well, up to the point where the pressure gradient relaxes. The form factor \( H \) though, was found to be very high, reaching a peak of 2.8 at \( \bar{x} = .77 \). As a consequence of this, the skin friction (53a) became zero, predicting separation. As this was not true, two alternative methods were tried, one by E. Truckenbrodt (1952) and the other by H. P. Horton (1969).

Truckenbrodt's method calculates both the laminar and turbulent problem. The momentum thickness and the shape parameter of the velocity profile, are both evaluated by simple quadrature. This method was found very convenient to use, with computer run times well below 100 seconds, and it worked quite well for both \( C_f \) and \( \theta \). Values of \( H \) were again rather high, reaching a maximum of 2.2, although the correct trends were established, and the boundary layer was clearly shown to
accelerate again after $x = .8$.

The final method by H.P. Horton, takes into account the entrainment history of the boundary layer, and is therefore suitable for both equilibrium and non-equilibrium boundary layers. It produced the best fit to the experimental results, although exact agreement with experiment was not achieved. The reason for this, is that the theoretical rather than experimental pressure distribution was used for calculations and the transition position was fixed rather arbitrarily at $x = .23$, the position just after maximum suction. Due to this, values of $\theta$ are rather overestimated and the position of maximum $H$, is predicted further downstream than where it should be. Also a rather more drastic reduction in $H$ was achieved at $x = .7819$, than expected.

It must be pointed out that all these methods are two-dimensional which might not be strictly true for the flow in question. Nevertheless, both theory and experiment prove that the design conditions of a nearly separating and then relaxing turbulent boundary layer, have been met.

7.3 Unsteady-Boundary Layer Results.

Measurements described in this section, constituted the main effort of this research, and therefore results are given comprehensively, in figures (54)-(118). Graph presentation, follows for the most part the work of previous investigators. (i.e. S.K.F. Karlsson and M.H. Patel)

7.3.1 Velocity amplitudes & Wall phase angles.

These quantities are plotted in figures (54) to (101). For each frequency, velocity phase angles and velocity amplitude ratios are presented on the same graph. The overall frequency effect for each station is given in two graphs at the end of each set, e.g. figures (59) and (60); for the velocity amplitude ratio and the phase angle respecti-
vely. The procedure is repeated for each station, up to $\bar{x} = .7819$.

Results for $f = 1$Hz were rejected, because at this frequency freestream amplitudes were very small, sometimes of the same order as the local random turbulence. This resulted in a poor forcing to random signal ratio. Also long wind-tunnel run times were necessary for each boundary layer traverse (up to 30 minutes), introducing additional errors due to hot-wire temperature drift.

(a) $\bar{x} = .5069$

Starting with figure (54), for $\bar{x} = .5069$ and $f = 2$Hz, the velocity amplitude ratio shows an initial dip below the $u/u_1$ line in the outer layer region, reaching a minimum value of 0.8. From then on increases to a maximum of 1.2 at $\eta = .4$, decreasing again gradually near the wall. The curve should pass through the origin at the wall, but wall proximity effects in this region prevented reliable measurements.

Velocity phase lag through the boundary layer increases rapidly in the outer layer up to $\eta = .4$, reaching a maximum of $120^\circ$. From there on decreases again slightly, to increase finally at the wall back to $120^\circ$.

Figure (55) for $f = 3$Hz, portrays the same general behaviour for $u/u_1$, with the dip now only marginally below $u=u_1$, and an overshoot of 1.2. The phase angle again increases rapidly in the outer layer region but this time it levels off in the inner region, to a mean value of only $68^\circ$. Behaviour very near the wall is rather uncertain, due to the viscous nature of the laminar sublayer, although the few experimental points available indicate a large reduction in phase lag, by almost $15^\circ$.

Figures (56) to (58) for the rest of the frequencies up to 6Hz, show a similar behaviour for both phase angle and amplitude ratio curves as figure (55). The dip initially observed in the outer layer region, now disappears with the increase in frequency, and phase angles at the wall tend to decrease after levelling off in the inner region.

The relative frequency effects are more easily studied in figures
The amplitude ratio overshoot at $n = 0.4$, increases with frequency to a maximum of 1.5, while the dip slowly disappears. Velocity phase angles, shown for all frequencies higher than 2Hz, have similar profiles, suggesting that a suitable parameter could be used to collapse them onto one graph. For example, phase angles could be normalised with respect to the value of $\Phi$ at the flat portion of the curves. Phase lags decrease in the sense 2, 3, 4, 5Hz, and then increase again at 6Hz.

(b) $x = 0.5657$

The next set of figures (61)-(67), show the results for $x = 0.5657$. Amplitude ratio and phase lag curves remain similar in shape with changing frequency, except for f=1Hz. The dip in the outer region is again present, although it now disappears at 5Hz. The result for 1Hz, shows the reason why it was considered unacceptable; the significant proportion of turbulence present masks the forcing signal, giving unacceptably high amplitude ratios throughout the boundary layer. It was not though possible to measure the magnitude of this random component present in the signal, at the forcing frequency. Amplitude ratios at 2Hz, are still quite high, with a maximum of 1.6. This drops to 1.15 at 3Hz and rises thereafter to 1.35 at 6Hz.

Figure (67) shows how the velocity phase angles through the boundary layer, vary from 2 to 6Hz. Again at 2Hz a large phase lag is observed reaching 85°, and dropping again slightly in the inner region. The slope of all curves in the outer layer decreases with frequency, and maximum phase lag decreases in the sense 2, 3 to 4Hz, increasing thereafter at 6Hz.

(c) $x = 0.6137$

The same remarks apply for the next set of graphs (68)-(73), with a more pronounced dip in the amplitude ratio curves. Maximum amplitude decreases to a minimum at 4Hz and increases again for 5 and
6Hz, while phase lags decrease in the sense 2, 3, 4, 5Hz, increasing again at 6Hz.

(d) \( x = 0.6838 \)

Figures (74) to (80), display the results at \( x = 0.6838 \). The most significant difference between this and the previous set, is that although the large amplitude ratios at 2Hz are conserved, (up to \( u/u_l = 1.7 \)) for frequencies above 4Hz no overshoot is observed in the graphs. This indicates a damping of the oscillation in the boundary layer, above a certain frequency.

Graphs for \( \phi \) again fold over at 4Hz, while a tendency for the phase lag to decrease near the wall is obvious for the lower frequencies 1, 2, and 3Hz. Wall phase lag increases again for 5 and 6Hz. This apparent dependence of the behaviour near the wall on frequency cannot be easily explained, but it is worth noting that wall phase lag tendencies reverse at the particular frequency (4Hz), where the curves fold-over. Phase lags as high as 130° are possible, as demonstrated by the 1Hz result.

(e) \( x = 0.7323 \)

Proceeding to \( x = 0.7323 \), figures (81)-(87)) where the adverse pressure gradient is now easing off, all amplitude ratio curves except the one at 2Hz, lie below the \( u = u_l \) line and furthermore, at 4 and 5Hz where the maximum freestream amplitude occurs, \( u/u_l \) never rises above its value at the initial dip in the outer layer region. The overshoot at 2Hz is now less pronounced, reaching a maximum of 1.35. The phase lag curves exhibit the same characteristics as in the previous station, although now the curves fold over twice, at 3 and 4Hz. A significant decrease in phase lag persists for the lower frequency values. Maximum phase lag is achieved at 1Hz, and it reaches 150°!

(f) \( x = 0.7819 \)

The final station at \( x = 0.7819 \) (figures (88)-(92)), is now sub-
jected to a small favourable pressure gradient. For all frequencies above 2Hz, all amplitude ratio curves now decrease monotonically below their freestream value, indicating that for the given conditions, the boundary layer exercises a strong damping effect on the oscillations.

The overshoot at 2Hz now barely exceeds the \( u=U_1 \) line at \( u/\lambda_1 = 1.07 \). Phase angles again tend to decrease near the wall for all frequencies, although in some cases measurements too close to the wall were avoided as they proved destructive on hot-wire probes.

To separate frequency from chordwise position effects; keeping the frequency constant, results at different \( x \)-stations were plotted on the same graph. Thus figures (93) & (96) give the effect of \( x \), on the velocity phase angle for frequencies from 2 to 5Hz, while figures (97)-(101) give amplitude ratios from 2 to 6Hz.

Commenting first on the phase angles \( \phi \), phase lag tends to increase in an orderly fashion in the downstream direction. The behaviour near the wall although consistent with frequency, seems to change in an unpredictable manner with \( x \).

Starting from figure (97) (at 2Hz) for the amplitude ratios, maximum values increase considerably from \( x = 0.7819 \) through to \( x = 0.6837 \) in an increasingly adverse pressure gradient, and decrease again for \( x = 0.5657 \) and 0.3779. Changing the frequency to 3Hz, pronounces this effect even more, with very small amplitude ratios at \( x = 0.7819 \), increasing again through to \( x = 0.6137 \) and dropping slightly for upstream positions. The same behaviour persists for the rest of the frequencies up to 6Hz, indicating a strong pressure gradient effect on the damping or amplification of the imposed oscillations.

Oscillation amplitudes have also been plotted in an alternative form as a percentage of the local boundary layer mean velocity, \( U(y) \). Figure (102a), shows results at \( x = 0.5657 \), as frequency varies from 1 to 6Hz, while figure (102b) shows results at five chordwise stations for
a frequency of 6Hz. A comparison of the effects in the two graphs shows that for all stations with a mean adverse pressure gradient, frequency rather than downstream position seems to be the most influential parameter.

Thus for all stations except $\bar{x} = .7819$, all graphs lie close together, while frequency seems to shift the graphs relative to one another, changing the percentage of oscillation near the wall, from 10% at 1 and 2Hz, to as much as 23% at 5Hz. The result for 1Hz, demonstrates that although the freestream amplitude is only 1.69% of the freestream velocity, it can reach apparent values of 11% of the local velocity in the boundary layer, thus explaining the very high amplitude ratios previously observed.

The initial dip in the outer layer, although not obvious in figure (102a), is still there when other stations are investigated. (see (102b). The boundary layer damping effect in a favourable pressure gradient region, is demonstrated convincingly at $\bar{x} = .7819$ and $f = 6$Hz, with percentage values as low as 7% being recorded near the wall.

Some investigators have presented their results, as in and out of phase velocity components, rather than amplitudes and phase angles. (e.g. Karlsson, figure (3)) A few representative results have also been plotted here in the same fashion, and they help to clarify some of the rather unique results observed, such as the high amplitude ratio overshoots and the outer layer dip.

First we consult figure (103), showing variations in the freestream direction for a frequency of 5Hz. There the in-phase component decreases continuously through the boundary layer, except for a small overshoot at $\bar{x} = .3779$. This overshoot was also observed by Karlsson (fig. 3) and is present in the laminar case (Lighthill). The rate of decrease through the boundary layer, seems to be governed by downstream position. Thus while at $\bar{x} = .3779$ the result is very similar to the
flat plate case, it decreases much more rapidly as we move downstream, increasing again slightly at the last station where the pressure gradient is favourable. It is interesting to note that an initially positive in-phase component can reverse and become negative, as is the case at the last two stations, for $\eta < 0.3$.

The initially small negative out of phase component in the outer layer, is rapidly increased through the boundary layer, and it can reach values greater than $u_1$, as demonstrated by the results at $\bar{x} = 0.5657$ and $0.6137$. This helps to explain the very large overshoots observed in the amplitude ratio curves. The reason for the negative out of phase component, is the presence of a travelling wave type of oscillation in the freestream leading to phase lags in the boundary layer. The fact that the freestream is also decelerating, explains why the positive in-phase component decays rapidly in an adverse pressure gradient environment, while the initially small negative out of phase component will tend to increase, since its direction is reversed. Of course both the local pressure gradient and the boundary layer upstream history will have a strong effect on the boundary layer development, and the above explanation is rather oversimplified. Thus the maximum value of the out of phase component increases from $-0.3u_1$ at $\bar{x} = 0.3779$ to almost $-1.2$ at $\bar{x} = 0.6137$, decreasing again through to $-0.6u_1$ at $\bar{x} = 0.684$. Although it was expected to decrease even further at $\bar{x} = 0.7819$, on the above assumptions, it increases again to a maximum of $-0.8u_1$ the pressure gradient being now favourable.

Figures (104) & (105), show the effect of frequency at two stations, $\bar{x} = 0.5657$ and $\bar{x} = 0.6828$. For both cases, in-phase components increase with frequency, while out of phase components tend to decrease (in an absolute sense). Figure (105) demonstrates that quite high negative in-phase values are possible at the lowest frequencies, as a consequence of the large phase lags observed. This again explains the
higher overshoots at low frequencies.

The relative rate of change in magnitude between the in and out of phase components, determines whether there is a dip or not, in the amplitude ratio profiles. Thus if the in-phase component decays more rapidly than the out of phase component increases, a dip will develop in the outer layer region.

7.3.2 Boundary Layer Mean Values.

Figures (106) to (109), show the boundary layer mean velocity profiles at four measuring stations. A comparison with the steady profiles previously obtained, shows that frequency effects are negligible on mean boundary layer values. This is not surprising since the mean pressure gradient also remains unchanged.

Small discrepancies between curves can be attributed to traversing errors rather than actual deviations. The same conclusions were drawn by S.K. Karlsson who used oscillation amplitudes as high as 37% for a purely sinusoidal freestream, and also by M.H. Patel for the travelling wave case.

7.3.3 Boundary Layer Turbulence.

Figures (110)-(112), show the effect of frequency on the boundary layer random turbulence values. Results are given for three positions, \( \bar{x} = 0.5657, 0.6137 \) and \( 0.6838 \).

A general increase is observed at the forcing frequency, above the steady values. Figure (110) shows the turbulence increasing with frequency up to 6Hz. Most of the change seems to occur in the outer layer region, but it is not obvious whether this large change in freestream turbulence is an actual phenomenon due to the unsteadiness in the freestream, or a measure of the error in using equation (36) in chapter 6. It is though certain that some of the change is due to
harmonics and subharmonics of the forcing frequency, either in the free-stream, or developing in the boundary layer itself, due to its non-linearity. Since the R.M.S. meter used (Solatron JM1860) was only linear above 3Hz, some of the subharmonics were attenuated especially at the lowest frequencies, which partly explains the higher turbulence at 5 and 6Hz.

The same remarks apply for the other two stations, although in some cases (figure 111) a considerable scatter is observed in the results.

7.3.4 Freestream Amplitude Effect.

So far we investigated the effects of frequency on the boundary layer; but since a change in frequency is followed by a corresponding change in freestream amplitude, we must also examine possible amplitude effects, which might mask effects attributed to a pure change in frequency of the forcing oscillation. Thus amplitude ratios, phase angles, mean velocity profiles and boundary layer turbulence were plotted for a range of flap amplitudes, from 2 to 4 inches, for a restricted number of x-stations and frequencies.

Figures (113) & (116) show the effects on amplitude ratio, for \( x = 0.6137 \) and \( x = 0.6838 \), and for a frequency of 5Hz. The total turbulence values (\( u_{\text{total}} \)) were also normalised with respect to \( u_1 \), and plotted on the same graphs. This gives a visual indication of the effectiveness of the sampling system, as a narrow band filter. In general total turbulence amplitude ratios for the lowest flap amplitude (2") were found to be much larger than corresponding values for 3 and 4 inches. This again indicates that although freestream amplitude is quite small, the random turbulence component in the boundary layer is still appreciable, leading to large amplitude ratios. Results for 3 and 4 inches coincide, which suggests that the forcing oscillation amplitude is now...
an order of magnitude higher than the local turbulence, which has a small influence on the results.

When the signal is processed through the digital sampling system, again both higher amplitude results coincide, indicating that after a certain magnitude, amplitude ratios in the boundary layer are unaffected by amplitude changes in the freestream. Results for Δ = 2" deviate considerably from the rest, especially in the outer layer region. For this reason lower amplitude cases, (i.e. 1Hz and possibly some 2Hz results) should be viewed with caution.

Figure (118), shows the effect of amplitude on boundary layer phase angles. Phase lags appear to be slightly higher at the largest amplitude of 4 inches, but the deviation is too small to cause concern.

Mean velocity profiles remain unchanged with freestream amplitude, a result which was expected, since no change was observed with frequency either. An example is given in figure (114).

Boundary layer turbulence values are again slightly higher than steady values, but the role of freestream amplitude is not quite clear. Again the lowest amplitude leads to higher turbulence, an observation which could be attributed to a poor signal to noise ratio. A considerable scatter is observed in the results, which could be due to the poor accuracy resulting by the use of equation (36).

It is now certain that both frequency and amplitude affect the boundary layer turbulence, and for more conclusive results the two effects should be separated. It should though be noted, that at certain amplitudes subharmonics and harmonics of the forcing frequency are created, which could explain the increase observed.
8. DISCUSSION

Here we recollect the main findings of previous chapters and with the benefit of the overall view gained, draw some general conclusions on the behaviour of the oscillatory turbulent boundary layer in a mean adverse pressure gradient.

It is useful if the findings of chapter 6, dealing with the freestream are discussed first.

8.1 The Freestream.

Chapter 6, deals with measurements and results obtained in the inviscid region of the flow, outside the boundary layer.

Both pressure and velocity amplitudes were measured for a variety of chordwise positions, and the full range of frequencies available was exploited. (see figures (24)-(44)) The effects of tunnel speed and flap amplitude were also investigated in a chosen number of cases. Thus a thorough calibration of the oscillatory flow facility was performed in the neighbourhood of the model, both for greater understanding of the vortex induced type of oscillation, and for determining suitable positions for boundary layer investigation.

The response of both pressure and velocity fluctuations to frequency appear to be very similar and consequently in the following discussion, were not stated, comments applying to pressure fluctuations apply equally well to velocity and vice versa.

The form of the results suggests that a distinct difference in behaviour of the flow upstream and downstream of the flap trailing edge exists. Hence in the first case (figures (24) & (25)), the oscillations are created mainly due to large changes in the pressure distribution around the model during a cycle, induced by the motion of the flaps.
These changes are shown in a quasi steady form in figure (18), and they mainly affect the region of the flow upstream of $x/c = .3$. This type of excitation leads to relatively high pressure amplitudes, while corresponding velocity amplitudes remain quite small. Even at the lowest frequency used (1Hz), quite large pressure amplitudes are achieved, as high as 11% at $x = .1323$, dropping rapidly to 5% at the flap trailing edge. As we move further downstream past the flaps ($x = .3779$), this value is reduced even further to just over 2%. As frequency increases amplitudes also increase, up to 3Hz for $x = .1323$ and 4Hz for $x = .2327$.

If we increase the frequency further than these values, amplitudes decline rapidly as inertia effects set in, and the flow has no time to follow the rapid changes in nozzle geometry.

The above behaviour can also be explained mathematically, if we consult the unsteady Bernoulli's equation, relating the pressure and velocity fluctuations on a streamline. This equation is set out in section 8.2 (equation 37), together with an approximation for small disturbances (equation 38). Thus while downstream of the flaps the travelling wave velocity of the disturbance ($Q$) has a finite value slightly lower than the freestream velocity, between the flaps the disturbance is purely time dependent, i.e. $Q = \infty$. If we substitute this value of $Q$ in equation (38), the travelling wave contribution to the pressure fluctuations, $\frac{\omega U_0}{Q}$ disappears. Since this term is comparable in magnitude with the unsteady inertia term $\omega U_0$ but of opposite sign, the relative magnitude between the pressure and velocity fluctuations upstream of the flaps will change, leading to the high pressure values observed.

For very low frequencies, the quasi-steady term ($U_1 \frac{\partial U_0}{\partial x} + U_0 \frac{\partial U_1}{\partial x}$) becomes dominant, and since $U_1$ increases rapidly in the favourable pressure gradient region between the flaps, this explains the large amplitudes obtained at 1Hz.
Downstream of the flaps, the oscillations are created by the rolling up of the shear layers on either side of the semi-open jet section, perturbed by the action of the flaps. (see also chapter 3) This changes the frequency response of the flow considerably, since now Q is finite. Characteristically, velocity amplitudes can now reach quite high values (up to 12% of the freestream recorded) and they are comparable in magnitude to pressure amplitudes. Also the change with frequency is now more gradual, to a well defined maximum around 4Hz ($U_\infty = 21.9\text{m/s}$, $\Delta = 4^\circ$). The rolling up process in the shear layers can be employed to explain this behaviour.

If we consult figure (9), showing schematically the rolling up process, we see that a certain time is required for the rolling up of the shear layers into discrete vortices. This time is governed both by frequency and flap amplitude. Also the distance between adjacent vortices is governed by frequency. Thus for low frequencies, the separation between adjacent vortices is quite large, and the rolling up process is far from complete in the small chordwise distances used. The resulting weak unsteady velocity field near the model, leads to small oscillation amplitudes. As the frequency increases vortex separation becomes progressively smaller and their strength increases as the rolling up process is completed. A maximum amplitude is observed when an optimum condition is reached. Any further increase in frequency will cause the vortices to move closer together, but their strength diminishes as diffusion takes place, leading to ever decreasing amplitudes.

The vorticity shed by the flaps is also expected to have an effect on the above development, as it accounts for as much as 30% of the total vorticity in the shear layers (Patel).

The effects of the tunnel velocity and flap amplitude, can be proved to support the modelling of figure (9). They can be described uniquely by observing the changes at the position of maximum amplitude.
The various independent variables, i.e. $U_\infty$, $h$ the separation between the shear layers and $f$ the frequency, can be grouped together in a non-dimensional parameter, the Strouhal number ($St_h = \frac{fh}{U_\infty}$). The separation between the shear layers $h$, depends mainly on the tunnel used, and in our case is the distance between the two flaps. It is assumed that $h$ remains constant in the streamwise direction.

Thus for a tunnel speed $U_\infty = 21.9$ m/s and $\Delta = 4''$, the maximum response occurs at a relatively constant $St_h = 0.28$ for all $x$, as demonstrated in figures (43) & (44) ($f=4$ Hz). If the tunnel speed is changed, the maximum response moves to either a higher or a lower frequency, so that the Strouhal number always remains constant. (figures 27, 29, 34 & 36). The optimum condition will not be reached, if the reduction in $U_\infty$ impedes the completion of the rolling up process, for the range of frequencies considered.

Reducing the flap amplitude (figures 28, 30, 33 and 35) delays in effect the rolling up process, leading to poor oscillation amplitudes. The maximum response is also delayed appreciably, while maximum amplitudes are reduced. This reduction can be partly attributed to a reduction in that part of the vorticity contributed by the flaps. For the amplitudes tested, most of the change occurs between 2 and 3 inches rather than 3 and 4 inches, indicating that at 2'' the amplitude of the initial disturbance is too low to cause a complete rolling up of the shear layers, even at the highest frequency of 6 Hz.

Amplitudes can also be plotted against a non-dimensional frequency parameter dependent on $x$, $\nu = \frac{fx}{U}$. Figures (31), (37) and (38) show both pressure and velocity amplitudes plotted in this form. The results for different velocities thus merge into one, demonstrating the argument that changes in freestream velocity only change the wavelength of the disturbance and therefore the frequency. A similar correlation for different $x$ could not be obtained, since the local amplitude also
depends on the local pressure gradient, in the neighbourhood of the model.

Figures (41) & (42) show this variation with frequency for different \( x \), while (43) & (44) show the amplitude change along the model for different frequencies.

Together with the forcing oscillation amplitudes, the total velocity amplitudes were also plotted on the same graphs. Then the freestream turbulence at each station was determined from,

\[ u_{\text{total}}^2 = \bar{u}^2 + u_{\text{turb}}^2 \]  

assuming no correlation exists between the forcing component \( \bar{u} \) and the random component \( u_{\text{turb}} \) of velocity. Unfortunately this method of calculation has certain drawbacks, which we can list, not necessarily in order of importance.

(i) The use of the Solatron time domain analyser in the a.c. mode (see chapter 5) for calculating the total turbulence intensity, \( (u_{\text{total}}) \) filters out some of the turbulence below 3Hz. The error is of course largest at the lowest frequency of 1Hz, and the indicated value must be increased by 3%.

(ii) The forcing oscillation component is slightly overestimated as some of the random turbulence passes through the digital sampling system at the resonant peaks. Errors of this type are expected to be very small in the freestream, where \( u' \) is small. They can be a source of concern in the boundary layer, especially where high turbulence is accompanied by low oscillation amplitudes (e.g. at 1Hz), and even more so where the boundary layer is thickest and a turbulence shift is expected towards the lower frequency spectrum.

(iii) The nature of the oscillatory flow facility creates oscillations not only at the driving frequency, but also at half and double its value, in the form of harmonics. The magnitude of these harmonics
increases both with flap amplitude and frequency, and they can reach 10% of the driving frequency component. The subharmonics will be reduced by the low frequency cut-off characteristic of the TDA in (i), but the nett effect will be a serious overestimation of $u_{total}$, by as much as 10%.

(iv) The accuracy of $u'$ (and harmonics) derived from equation (36) is rather poor, as we are subtracting two quantities which are considerably larger than their difference. This is especially true in the freestream and the outer portion of the boundary layer.

Without detailed spectral analysis it is difficult to estimate the cumulative effect of all these errors on the calculated value of $u'$. However even if quantitative estimates are suspect, the correct trends are given, and they help to provide an insight into the mechanism of the flow. Starting from well inside the flaps at $x = 0.1323$, the steady turbulence is very low, at 0.2% of the freestream velocity. With the flaps in motion, this value increases to 2%, and is quite independent of frequency. Since there is no apparent reason for this increase it can be attributed to errors under (iv). Any harmonics must have a negligible effect, since they are frequency dependent. At the flap trailing edge ($x = 0.2328$), the freestream turbulence changes from 1.6% at 1Hz to 2.5Hz at 6Hz. Again it is quite small and the frequency dependence must be due to the presence of the vortex generators in the flow. This change with frequency disappears not far downstream at $x = 0.3779$, suggesting that the presence of the vortex generators does not introduce any large scale turbulence in the flow.

Further downstream (figure 37) we see both $u_{total}$ and $U$, plotted against $fx/U$. Again turbulence is invariant with frequency, up to $fx/U = 18$. From then on there is an apparent increase in turbulence, suggesting that the effect of subharmonics and double harmonics of the driving frequency is now important. The same applies for positions
further downstream (figures (39) & (40)), where a comparison between the narrowly filtered signal $\bar{u}$, and $u_{\text{total}}$ for $f>4\text{Hz}$, reveals that a significant proportion of the applied oscillation is conserved in the form of harmonics. Any estimates of turbulence in this region are obviously in error.

Another quantity measured in the freestream, was the streamwise variation of the velocity phase shift, produced by the travelling wave velocity $Q$. To determine $Q$ for different frequencies the results were plotted against a non-dimensional frequency parameter $\frac{f(x-x_{\text{ref}})}{U}$. The results are given in figures (11) & (12), and the values of $Q$ in tables Ia & Ib. In contrast with Patel's and Parker's results also shown in figure (11), a distinct variation of $Q$ with frequency was detected, especially for the flow away from the model, at $y=h/4$. Frequency dependence is more pronounced for the lower frequencies of 1 and 2Hz, showing a significant reduction in the travelling wave velocity. In accordance with Patel's theoretical results, (figure 5) this change in $Q$ will have a marked effect on the boundary layer response to the oscillation. The decrease in $Q$ in a direction perpendicular to the model also suggests that for $f=1$ and 2Hz, there is a velocity phase shift across the working section.

The variation in $Q$ was partly explained in chapter 3, by comparing the experimental results with the solutions of equations (25) & (27), again given in tables Ia and Ib respectively. The comparison is somewhat limited, owing to uncertainties as to the extend of any additional vorticity introduced by the flaps, and the change in strength of the vortices in the shear layers, either with frequency or downstream position. Also the model bound vorticity is expected to have an effect especially near the model.

Pressure-velocity phase angles were also measured at different chordwise positions and the results are shown in figure (44a). Pressure
leads the velocity by $180^0$ at $0Hz$ as expected, but this lead is linearly reduced as frequency increases, so that in some cases it is as low as $50^0$ at $6Hz$.

The slope of the deviation seems to be governed by the local pressure gradient, in as favourable or zero pressure gradients cause a smaller change in phase lead, while adverse pressure gradients pronounce the change. Velocity phase angles near the wall introduce some uncertainties, the effect of which appears as a deviation from $180^0$ in the steady case, when the results are extrapolated to $0Hz$.

Both pressure-velocity phase angles and oscillation amplitudes outside the boundary layer are interrelated, and can be described by the unsteady Bernoulli's equation. An attempt to calculate these values using a constant $Q$, failed to produce reasonable results.

8.2 The Boundary Layer.

Measurements and results in the unsteady boundary layer, are given in chapter 7 and figures (54)-(118). Measurements were taken at 7 chordwise stations, chosen for a well developed turbulent boundary layer of adequate thickness. The pressure gradient was large adverse at first, ($X = .3779$) decreasing continuously downstream until it became favourable at the last downstream station, $X = .7819$.

Unsteady velocity components in the boundary layer were presented comprehensively as velocity phase angles and amplitude ratios (figures (54)-(101)) for the complete frequency range above $2Hz$ inclusive. Results at $1Hz$ were in their majority rejected, due to small amplitudes and correspondingly poor signal to noise ratio.

Graphs were discussed individually in chapter 7 and here we will try to point out some general characteristics, without going into too much individual detail.

Examining first the amplitude ratios, $u=u_1$ by definition at the
boundary layer edge. As we move inwards, a dip is observed in the outer layer, which in general increases as we move downstream, sometimes reducing the freestream amplitude by 60%. As \( y \) decreases even further the oscillations are amplified again, reaching a maximum around \( \eta = 0.4 \). It is noteworthy that at this position the random turbulence in the steady boundary layer is also a maximum, in most cases.

Overshoots at this position can reach \( 1.7u_1 \) for the lowest frequency of 2Hz. Overshoots at 1Hz are even higher, but as these contain a significant proportion of random turbulence they were rejected. As we move downstream, the overshoot decreases progressively for \( f > 3\text{Hz} \), until finally at \( x = 0.7819 \) they disappear completely and the boundary layer dampens out most of the imposed oscillation. This damping is accompanied by a decrease in the mean adverse pressure gradient, which is finally lifted at \( x = 0.7819 \).

As we move nearer to the wall, amplitude ratios continue to fall gradually at first, and more rapidly in the sublayer region.

The form of the graphs is similar to the ones obtained by Patel for the flat plate case, although overshoots are higher, and the dip in the outer layer was not observed. This suggests that it is a pressure gradient effect. The large overshoots at 1 and 2Hz, can also be explained by looking at Patel's theoretical study of the effects of \( Q \) on the results (figure 5). Thus since at 1 and 2Hz the travelling wave velocity is appreciably lower than in higher frequencies, the auxiliary pressure gradient due to the travelling wave disturbance becomes dominant leading to high amplitude ratios.

Velocity phase lags shown on the same figures are all similar in shape. Starting from zero at the boundary layer edge, they increase rapidly in the outer layer region, levelling off as we move further towards the wall. Therefore for most of its thickness the boundary layer oscillates in phase.
The behaviour near the wall is not at all certain, mostly for the lag of adequate experimental data in this region. The signal also here is highly distorted, making phase angle results rather unreliable. Hot-wire cooling in this region is not expected to affect the velocity phase angles, which do not depend on the temperature sensitive calibration constants. It will though affect the velocity amplitudes very near the wall.

No general trends were observed applicable in all cases in the sublayer, but changes in phase lag were sometimes large, either increasing or decreasing the velocity phase angle by as much as 15°. Since the phase angle is given by the ratio of the in and out of phase velocity components, the rapid change near the wall shows that either the in-phase or the out of phase component decays more rapidly in the laminar sublayer, depending on whether the phase lag increases or decreases.

The fact that the velocity fluctuations in the boundary layer lag the freestream values, is the single most important parameter affecting the boundary layer response. This lag is initially introduced partly by the adverse pressure gradient, and partly by the travelling wave type of oscillation.

Thus the unsteady Bernoulli's equation is,

\[
\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x} = \frac{1}{\rho} \frac{\partial P}{\partial x} \tag{37}
\]

where \( U = U(x,t) \) and \( P = P(x,t) \).

Assuming we can separate spatial and temporal dependence, we get

\[
U(x,t) = U_1(x) + U_0(x)e^{i\omega(t-x/\lambda)} , \text{ with } U_1 \gg U_0
\]

also let \( \phi = \omega(t-x/\lambda) \)

Substituting in (37), after a little reduction and by neglecting terms of order \( U_0^2 \), the L.H.S. of the equation becomes:
Thus while the unsteady inertia term leads the oscillations by 90°, the travelling wave contribution will introduce a lag of 90°. The relative magnitude of these two terms will decide whether there is a net lead or lag in the oscillations. Now since Q is always smaller than U, for the range of x considered, the effect is a net lag in the boundary layer. The quasi-steady pressure gradient term will be either in phase or 180° out of phase to the oscillations, depending on its sign. In a mean adverse pressure gradient, the term $\frac{\partial U}{\partial x}$ will be negative, while $U_0 \frac{\partial U}{\partial x}$ depends on the rate of growth of $U_0$ in the x direction.

The unsteady inertia term $\frac{\partial u}{\partial t}$ in the boundary layer will introduce further phase lags, together with the viscous contribution.

The above argument explains why previous investigators observed a phase lead in the boundary layer, with a purely sinusoidal ($Q=\infty$) freestream. (see Karlsson, Lighthill, Rakowsky). Phase lags are of course possible in the purely sinusoidal case, if the pressure gradient is adverse, as demonstrated by Schachenmann, studying the oscillatory flow in a conical diffuser. Thus phase lag is the cause for the very large overshoots observed in this investigation, and also in Schachenmann's work.

We can study the cause and effect more clearly, if we consider the oscillations as in-phase and quadrature components, rather than total amplitudes and velocity phase angles. Figures (103)-(106) give a set of results presented in this way.

In figure (103), the in and out of phase components at $x=3.779$ are seen to behave as in Karlsson's work, with the in-phase component presenting an initial small overshoot, and then decreasing rapidly towards the wall. The out of phase component also increases initially
to a maximum at $\eta = .4$ before it drops again towards the wall. This behaviour was also observed for the laminar case in Lighthill's theoretical work and Hill & Stenning's experiments. The main difference between the laminar and turbulent case being that the turbulent results lead to fuller profiles.

The one significant difference between the present results and the ones mentioned above, lies in the fact that the out of phase component is negative throughout the boundary layer.

In the case of the flat plate travelling wave problem studied by Patel, the results are again similar, and they retain their similarity in the downstream direction due to the absence of the pressure gradient term. However here as we move downstream, the boundary layer decelerates. This damps down the positive in-phase component very quickly, while the reversed out of phase component gains prominence, as the adverse pressure gradient in the positive $x$-direction becomes favourable in the upstream sense. As the pressure gradient is lifted ($X = .7819$) the in-phase component is already beginning to recover, as shown in figure (103).

The rapid decay of the in-phase component accompanied by a rapid increase in the quadrature component, will sometimes cause the dips observed in the amplitude curves near the edge of the boundary layer. Also the presence of out of phase components as large as $-1.6u_1$ in some cases, will lead to the large overshoots observed.

The reason for the large amplitudes at $f = 2$Hz, is shown in figure (105). Although the in-phase component is zero before it reaches $\eta = .6$, the out of phase component is always quite large; as $y$ decreases even further, the in-phase component also becomes negative, and as such is also amplified. So by $\eta = .4$, both components are large, giving rise to very large overshoots. This ties in well with Patel's predictions in cases of low travelling wave velocity, as mentioned before.
Apart from the unsteady velocity components, the mean velocity profiles were also measured in the boundary layer for comparison with the steady ones. A series of graphs (figures (106)-(109)) demonstrate that mean and steady values are identical within experimental error, even with oscillation amplitudes as high as 12% of the freestream velocity. Any small deviations can be attributed to small traversing errors and difficulties as to the actual edge of the boundary layer.

This invariance of mean from steady values with frequency was expected from the results of previous investigators. Thus Karlsson first, for the flat plate case, proved both theoretically and experimentally that even for amplitudes as high as 35%, the mean values deviate from the steady ones only slightly, due to the non-linearity of the turbulent boundary layer. The same result was confirmed by Rakowsky, for frequencies of the order of 200Hz, but of small amplitude. Patel also deduced the same result for both the turbulent and laminar case, for the travelling wave type of oscillation on a flat plate. This work therefore, extends the result to cases with an adverse freestream.

The effect of the imposed oscillation on the boundary layer turbulence spectrum was also investigated, following the procedure used for freestream turbulence (equation 36). The drawbacks are still the same (see section 8.1) and the results should be treated with caution.

Figures (110)-(112) give the results at three streamwise positions, for a range of frequencies including the steady case. In all graphs, unsteady turbulence exceeds the steady one, the deviation increasing with frequency. As mentioned in chapter 7, this increase could well be due to the effects of harmonics of the driving frequency. For a better estimate of \( u' \), these harmonics should be filtered out of the total turbulence values, by using suitable narrow band filters. The increase of turbulence with frequency was expected, both because
the amplitude of the harmonics increases with frequency and because of some filtering of half harmonics below 3Hz (see chapter 5). Karlsson's results on turbulence also show a considerable amount of scatter, which makes any deductions inconclusive. A considerable amount of scatter is also obvious in Patel's results, who proceeds to conclude rather intuitively that turbulence in the boundary layer is only affected at the forcing frequency.

To determine whether the apparent increase in turbulence is an amplitude rather than frequency effect, the results of figures (115) & (117) should be consulted. From these it is possible to say that there is an amplitude effect, although small, and turbulence increases with amplitude. The effect is more pronounced in the outer layer, but there also the error is largest since $u'$ is very small compared with $u_{total}$ and $\bar{u}$. The effect of amplitude on the mean velocity profiles was negligible as expected (see figure 114).

Freestream amplitude effects on the unsteady velocity components i.e. amplitude ratio and phase angles were investigated for a few chosen cases, and the results are given in figures (113) & (116) for the amplitude ratio, and (118) for the phase angles.

In figures (113) & (116), total amplitude ratios are also shown with $\bar{U}/u_1$. For the smallest flap amplitude (2"), $u_{total}/u_1$ is appreciably higher than the 3 and 4" cases, which agree fairly well within experimental error. This is due to the poor signal to noise ratio at the smallest oscillation amplitudes. The sampled signal also reveals that while both the 3 and 4" results remain the same, the 2" result shows higher amplitude ratios especially in the outer layer, for the same reason. Due to this, small freestream amplitude results should be viewed with caution, as in the case of 1Hz and sometimes even 2Hz.

For higher frequencies we can deduce that amplitude ratios are unaffected by freestream amplitude. The phase lag through the boundary
layer is affected only slightly by freestream amplitude, although a small increase in phase lag is observed for $\Delta=4^\circ$. This can be explained if we consider the quasi steady component of equation (38). Thus for the adverse pressure gradient case the term $U_0 \frac{\partial U_1}{\partial x}$ is negative, thus lagging the oscillations by $180^\circ$. The contribution due to this term will increase or decrease, according to the value of $U_0$. Thus for higher freestream amplitudes this term becomes more significant, increasing slightly the phase lag through the boundary layer.

We have seen so far that the effect of the travelling wave velocity $Q$ on the boundary layer behaviour is quite significant, affecting the flow much more than it was expected from the simple approach of equation (38). Concluding therefore, it is useful to suggest as a continuation to this project, the study of the effects of a controllable $Q$ on the boundary layer behaviour. This can be done by varying the velocity in the core of the jet, without duly affecting the shear layer distribution. Thus suitably designed grids can be inserted in the wind-tunnel downstream of the settling chamber, reducing the velocity in the core of the tunnel, but not the velocity near the upper and lower walls which determines the shear layer distribution.
9. CONCLUSIONS

The main conclusions arising from this investigation are outlined in broad terms here.

The wind-tunnel facility used, proved eminently suitable for unsteady flow work, producing velocity amplitudes in the freestream up to 12% of the freestream velocity, and having a working range from 1 to 6 Hz. Amplitudes at 1 Hz were found to be too low for turbulent boundary layer work.

The travelling wave velocity of the oscillations varied with frequency, for frequencies below 3 Hz. The rolling up mechanism of the shear layers was used to explain this variation, and also the frequency response of the flow in the freestream. The response maximum was found to be at $St_n = 0.28$, irrespective of $x$.

The digital sampling system designed for this work, proved accurate in measuring mean and unsteady velocity components in the boundary layer, and also velocity phase angles. Some improvement is necessary for the measurement of both boundary layer and freestream unsteady turbulence.

Mean and steady turbulent boundary layer values were found to be identical, irrespective of frequency or amplitude of oscillation.

Velocity amplitude ratios depended on both frequency and downstream position. The adverse pressure gradient produced a dip in the outer layer response, due to the differing rates of change of the in and out of phase velocity components. Amplitude ratios up to $1.7u_1$ were recorded at $\eta = 0.4$, due to the large negative out of phase components. Proceeding downstream in a direction of everdiminishing adverse pressure gradient, had a damping effect on the oscillations, especially for frequencies higher than 2 Hz. High overshoots at 2 Hz can be explained
by the lower travelling wave velocity, and also physically by the reversal and subsequent increase of the in-phase component.

The effect of the travelling wave velocity, together with the adverse pressure gradient induced large phase lags throughout the boundary layer. These phase lags increased rapidly in the outer layer region, and more slowly further towards the wall. In the laminar sublayer, large changes in phase lag were observed, and the phenomenon warrants further investigation.

The lowest frequencies induced higher phase lags, again an effect of Q, although in some cases an increase in phase lag was observed at the highest frequency of 6Hz. Phase lag also increased downstream with the thickness of the boundary layer, reaching values as high as 150°.

Variations in freestream amplitude did not affect the velocity amplitude ratio considerably, although phase lags were found to increase slightly with amplitude.

Boundary layer turbulence was found to increase in general with both frequency and amplitude, although the results here suffer from poor accuracy. Driving frequency harmonics affect the results significantly, especially in the outer layer region and the freestream. The error increases with frequency, amplitude, and x.
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APPENDIX I

Frequency Response of a Pressure Transducer and Associated Tubing

The frequency response of a transducer, pressure tubing and orifice series combination, was determined using a semidynamic calibration technique.

The system shown schematically in figure Ia, is representative of the one used in the model, for deducing R.M.S. values of pressure fluctuations and relative pressure-velocity phase shifts. Referring to Ia, dimensions \( l_t, d_t, l_o \) and \( d_o \) had to be chosen, for an optimum signal reproduction, keeping amplitude and phase distortions to a minimum.

![Figure Ia](image-url)

To determine experimentally the optimum values, a semi-dynamic transducer calibration technique was employed; a technique normally used in supersonic work. Figure Ib, shows the semidynamic calibrator, which employs the "shared volume" principle of operation. The pressure tapping under investigation was bored in a suitable adaptor (A), and connected to the pressure transducer through a plastic tube. The tapping orifice was exposed to a small cavity, this latter being separated from a much larger volume, or reservoir, by a quick acting solenoid valve. The reservoir could be raised to a pressure \( P \), measured
by a Betz water manometer, using a hand pump. The pressure in the two volumes was then equilised by opening the valve, using an overrun solenoid rated at 12V. A suitable condenser bank of capacity 10000\mu F, storing energy at 45V was discharged through the solenoid by a thyristor switching circuit.

The resulting pressure step at the orifice, had rise times of the order of 1ms, depending on the size of the step.

Before each measurement, the small cavity was brought to atmospheric pressure, through a small orifice at the base of the valve. The free end of the transducer was also at atmospheric pressure.

The solenoid firing unit was also used to trigger a "Datalab" transient recorder (DL905), which digitised and stored the pressure transducer output, which could then be processed at leisure. A typical output is shown in figure Ic.

An obvious peculiarity of this response, is the dip at A, which is due to the operation of the valve lowering momentarily the small cavity pressure below the atmospheric. The dotted line shows the ideal system response. The actual output overshoots the step line at B, after an initial time lag, and settles back to it, after a few decaying oscillations.

Apart from the fault at A, (the small cavity is normally evacuated for supersonic work) the rest of the response resembles that of a simple, single degree of freedom mass spring and damper system, subjected
to a step input. Thus with the usual notation we have,

\[ m\ddot{p} + \beta \dot{p} + kp = P \quad (i) \]

where \( m \) is the effective mass,
\( \beta \) the viscous resistance
and \( k \) the stiffness of the system.

Using a Laplace transform approach, and normalising with respect to \( k \), we get:

\[ L(p) = \frac{P\omega_0^2}{s(s^2 + 2\zeta \omega_0 + \omega_0^2)} \]

or \[ p(t) = P 1\left(1 - e^{-\omega_0 \zeta t}(\cos\omega t + \frac{\omega_0}{\omega} \sin\omega t)\right) \quad (ii) \]

where \( \zeta = \beta/\beta_0 \) the damping ratio, and \( \omega = \omega_0 \sqrt{1 - \zeta^2} \); \( \omega_0 \) and \( \beta_0 \) are the resonant values. Then if we let \( T = 2\pi/\omega \), be the time between successive peaks and \( \delta = \omega_0 \zeta T \) the logarithmic decrement, (ii) becomes:

\[ p(t) = P 1\left(1 - e^{-\delta t/T}\left(\cos\left(\frac{2\pi t}{T}\right) + \frac{\delta}{2\pi} \sin\left(\frac{2\pi t}{T}\right)\right)\right) \quad (iii) \]

Now \( \delta \) and \( T \) can be determined directly from \( I_c \), and using \( \omega_0 = \sqrt{k/m} \) and \( \beta_0 = 2\sqrt{km} \) we get,

\[ m = \frac{T^2}{\delta^2(1 + 4\pi^2/\delta^2)} \]
\[ \text{and} \quad \beta = 2\frac{T}{\delta(1 + 4\pi^2/\delta^2)} \quad (iv) \]

substituting (iv) into (i), we get

\[ \frac{T^2}{\delta^2(1 + 4\pi^2/\delta^2)} \dddot{p} + \frac{2T}{\delta(1 + 4\pi^2/\delta^2)} \ddot{p} + \dot{p} + p = \text{Input} \quad (v) \]

For a sinusoidal input, \( P(t) = P_0 e^{i\omega t} \), and \( G = P/P_0 \) the magnification factor, then
\[ G = \left[(1 - m\beta^2)^2 + \omega^2 \beta^2\right]^{-\frac{1}{2}} \quad (vi) \]

phase lag \( \phi = \tan^{-1}(\frac{\omega \beta}{1 - m\beta^2}) \]
whence for any given frequency \( \omega \), the magnification \( G \), and phase lag \( \phi \) due to the system can be calculated, assuming \( m \) and \( \beta \) remain as in (iv) for a step input.

The diameter of the plastic tubing used was dictated by availabili-
lity at \( d_t = 0.080 \) inch. The effect of changing tube length was investigated, from \( l_t = 10 \) to 60 inches in steps of 10". A range of tapping holes were also used from \( 0.020" \) to \( 0.080" \) diameter, together with a range of orifice lengths, from \( l_o = 3/64 \) to \( 3/16 \) inches.

In the space of this appendix, only a brief summary of the main results will be given, supported by figures Ia to If.

(a) The length of the tube \( (l_t) \): This is by far the most influential parameter in the system investigated. Increasing the length of the plastic tubing, lowers the resonant frequency \( f_o \) significantly, as shown in figure Ie. Thus from 82Hz at 10", \( f_o \) drops rapidly to 27Hz at 60", with corresponding changes in \( \phi \) and \( G \). Figure Id, shows the linear increase of \( T \), with \( l_t \).

(b) The step magnitude, \( P-P_{at} \): This does not affect the resonant frequency significantly, as demonstrated in figure Ig, although amplification near resonance increases considerably with step magnitude. Corresponding changes in \( \phi \), were found to be very small. Figure Id, shows that \( T \) is also not affected by the change in \( P \).

By lowering the reservoir pressure below the atmospheric, a negative pressure step could be applied. Both negative and positive pressure steps of equal magnitude, produced identical results.

(c) Orifice diameter, \( d_o \): Again it does not affect the resonant frequency, although it greatly affects the damping of the system. Hence increasing \( d_o \), increases the peak at resonance considerably. The phase lag \( \phi \), also decreases slightly, levelling off after \( d_o/d_t = 0.5 \).

(d) Orifice length, \( l_o \): Slight changes in the resonant frequency, although magnification is not in any way affected. Phase lag variations of the order of \( 10^\circ \) at 6Hz were observed, for the range of lengths considered.

(e) Reducing the volume of air inside the transducer casing, had the same effect as lowering the length of the plastic tube, thus increasing
slightly the resonant frequency \( f_0 \) in all cases. The effect is shown in figure 1d, where the \( T_v S \& L_t \) line is translated parallel to itself towards the origin.

In the actual experimental set-up used for measuring pressures on the model surface, tube length was fixed at 50 inches, a value dictated by the dimensions of the model. Tapping diameter was chosen at \( d_0 = \frac{1}{20} \) inch, half the tube diameter, to keep phase angle errors to a minimum. Orifice length was chosen at \( L_o = \frac{3}{32} \) inch. The above set-up gave a maximum amplitude error of 3% at 6Hz, and a corresponding phase lag of 60°. Measured values, were therefore corrected accordingly.
APPENDIX II

The Sampling System Frequency Response.

Let \( f(t) = \sin \omega t + \cos \omega t \) be the signal to be sampled. Then \( n \) samples over an equal number of cycles were taken, to reconstitute a complete cycle of the processed signal. If \( T_0 \) is the period of oscillation of \( f(t) \), then \( T = \frac{n+1}{n} T_0 \) is the sampling period. Then also, \( n = \frac{T_0}{\omega_0} \).

The first \( n \) samples will be: \( f(0), f(T), f(2T), \ldots, f(nT) \). If this process is repeated over \( m \times n \) cycles, then for the \( m \)th cycle we have: \( f(mT), f((m-1 Transition repeated for all \( m \).

Summing now in columns and averaging, we get

\[
\sum_{r=0}^{m-1} f(n + rT) = \frac{1}{m} \sum_{r=0}^{m-1} f(n + rT) \]

Substituting for \( f(t) \), this becomes

\[
f_s = \frac{1}{m} \sum_{r=0}^{m-1} \left[ \sin(\omega s + r\omega T) + \cos(\omega s + r\omega T) \right]
\]

Now, \( \sum_{r=0}^{m-1} \sin(\alpha + r\beta) = \frac{\sin(\alpha + (m-1)\beta/2) \cdot \sin m\beta/2}{\sin \beta/2} \) \( \sum_{r=0}^{m-1} \cos(\alpha + r\beta) = \frac{\cos(\alpha + (m-1)\beta/2) \cdot \sin m\beta/2}{\sin \beta/2} \).

Therefore using \( \alpha = \omega sT, \beta = \omega nT \) and \( T \) from (ii), (iii) becomes:

\[
f_s = \frac{1}{m} \sin(\pi(n+1)\omega/\omega_0) \left[ \sin(2\pi s(n+1)\omega/\omega_0 + (m-1)(n+1)\pi\omega/\omega_0) + \cos(2\pi s(n+1)\omega/\omega_0 + (m-1)(n+1)\pi\omega/\omega_0) \right]
\]

Now of course we have \( n \) samples of this type, from \( f_1 \) to \( f_n \), and we can now proceed using a standard Fourier analysis technique, to evaluate the in and out of phase components of \( f(t) \), with respect to the trigger.
signal.

Thus

\[ f_{in} = \frac{1}{2n} \sum_{s=0}^{n-1} f_s \sin^2 \frac{2\pi s}{n} \]  

\[ f_{out} = \frac{1}{2n} \sum_{s=0}^{n-1} f_s \cos^2 \frac{2\pi s}{n} \]  

or,

\[ f_{in} = \frac{1}{2n} \sin \left( \frac{\pi (m+1) \omega}{\omega_0} \right) \left\{ \begin{array}{l} a \sum_{s=0}^{n-1} \sin \left( \frac{2\pi s (n+1) \omega}{n \omega_0} \right) + (m-1)(n+1)\pi \omega/\omega_0 + \\
\cos \left( \frac{2\pi s (n+1) \omega}{n \omega_0} \right) + (m-1)(n+1)\pi \omega/\omega_0 \end{array} \right\} \]  

Proceeding with the 'a' component only, we get

\[ \frac{1}{4mn} \sin \left( \frac{\pi (m+1) \omega}{\omega_0} \right) \left\{ \begin{array}{l} a \sum_{s=0}^{n-1} \cos \left( \frac{2\pi s (n+1) \omega}{n \omega_0} \right) + (m-1)(n+1)\pi \omega/\omega_0 + \\
\cos \left( \frac{2\pi s (n+1) \omega}{n \omega_0} \right) + (m-1)(n+1)\pi \omega/\omega_0 \end{array} \right\} \]  

Letting \( \alpha = (m-1)(n+1)\pi \omega/\omega_0 \),

\( \beta = \{(n+1)\omega/\omega_0-1\}2\pi/n \),

\( \beta' = \{(n+1)\omega/\omega_0+1\}2\pi/n \) and using (iv), we get:

\[ \frac{1}{4mn} \sin \left( \frac{\pi (m+1) \omega}{\omega_0} \right) \left\{ \begin{array}{l} \cos \left( \frac{\pi (m+1) \omega}{\omega_0} - \pi \right) + (m-1)(n+1)\pi \omega/\omega_0 + \\
-\cos \left( \frac{\pi (m+1) \omega}{\omega_0} + \pi \right) + (m-1)(n+1)\pi \omega/\omega_0 \end{array} \right\} \]  

This with a little reduction gives for the 'a' component,

\[ \sin(2\pi(n+1)\omega/\omega_0) \left[ \text{via} \right] \]  

similarly, the 'b' component is,

\[ \sin^2(\pi(m(n+1)\omega/\omega_0) \left[ \text{vb} \right] \]  

Combining (via) & (vb), we get:

\[ f_{in} = \frac{\sin(2\pi/n)\{a\sin(2\pi(n+1)\omega/\omega_0) - b\sin^2(\pi(m(n+1)\omega/\omega_0)\}}{2mn[\cos(2\pi/n) - \cos((2\pi(n+1)/n)\omega/\omega_0)]} \]  

The out of phase component,

\[ f_{\text{out}} = \frac{1}{4mn} \left\{ \frac{2\sin^2(\pi m(n+1)\omega/\omega_0) \cdot \sin((2\pi(n+1)/n)\omega/\omega_0)}{\cos(2\pi/n) - \cos((2\pi(n+1)/n)\omega/\omega_0)} - \sin(2\pi m(n+1)\omega/\omega_0) \right\} + b \left( \frac{\sin(2\pi m(n+1)\omega/\omega_0) \cdot \sin((2\pi(n+1)/n)\omega/\omega_0)}{\cos(2\pi/n) - \cos((2\pi(n+1)/n)\omega/\omega_0)} + 2\sin^2(\pi m(n+1)\omega/\omega_0) \right) \]  

Now we can examine (vii) and (viii) for a range of frequencies \( \omega/\omega_0 \), in order to determine the frequency response of the system. Ideally we would prefer, only one narrow positive resonance peak, at \( \omega/\omega_0 = 1 \). But the nature of the above equations, shows that it is possible to get other peaks too. Thus if we consider the denominator, 

\[ \cos(2\pi/n) - \cos((2\pi(n+1)/n)\omega/\omega_0) = 0 \]  

for resonance, we get \( \omega/\omega_0 = \frac{pn + 1}{n+1} \), where \( p = 0, 1, 2, \ldots \). For the experimental case with \( n = 8 \), we have resonance peaks at \( \omega/\omega_0 = .111, .778, 1, 1.667, 1.889 \ldots \). These frequencies alternate from \(-1\) to \(+1\), being positive at the driving frequency \( \omega/\omega_0 = 1 \).

Computed results using various values of \( a \) and \( b \), and a range of values of \( m \), show that this frequency characteristic does not change drastically. Increasing the number \( m \), narrows the width of the peaks. Figure (23), shows the response for \( m = 40 \), in the region \( \omega/\omega_0 = 1 \).
FIG. 1: Calculations of the wall shear phase lead on a flat plate with an oscillatory free stream $\frac{\omega X}{U}$.
FIG. 2: Lighthill's results, compared with Hill and Stenning (1960) (Blasius flow)

AMPLITUDE

$\omega x / U_0 = 4.99$

$N = 0.09$

PHASE LEAD

Interm. Freq. Solution

Shear Wave

Measurement
FIG. 3: Karlsson's 'in and out of phase velocity profiles' – a typical result
FIG. 4: Overall b.l. characteristics (M.H. Patel)
FIG. 6: Effect of travelling wave velocity on velocity phase angle.
FIG. 7: The Tunnel

(Dimensions in Metres)

G Grid  s Stator  f Rotor  g Gauze  h Honeycomb

Section BB

Section AA

4.85
0.72
3.80
Δ = Flap Deflection
C = Slotted Disc
Ω = Rotational Velocity
L&R: Adjustable

FIG. 8: The flap displacement (not to scale)
FIG. 9: The Rolling Up Process
FIG. 10a: 'GUST' FLOW MODE

FIG. 10b: 'U' FLOW MODE
FIG. 12: Velocity Phase Angles

\[ \phi - \phi_0 \] degrees

\[ f/Hz \] symbol

1  z
2  +
3  v
4  A
5  o
6  x

\[ y = 35cm (= h/4) \]

\[ \frac{f(x-x_0)}{U_\infty} \]
$x = 7121 \sin 2 \theta - 1160 \sin 4 \theta$

FIG. 13: Model Camberline & Load Distribution
FIG 14: Pilot Model - Pressure & Velocity distribution
FIG. 15: THE MODEL
FIG. 17: Flaps undeflected—Comparison with theory
Flap Deflection = 2 in.

- Upper Surface
- Lower Surface
- Flaps Undetected
- Flaps Up
- Flaps Down

FIG. 18: Flaps in phase (Re = 3.06 x 10^6)
FIG. 21: THE B.L. TRAVERSING GEAR
FIG. 22: The signal processing equipment.

(See also text!)
FIG. 24: Pressure & Velocity Amplitudes
\( x/c = 0.1323, U_\infty = 21.9, \Delta = 4^\circ \)
FIG. 26: Pressure & Velocity Amplitudes

\[ \frac{p_{\text{rms}}}{\frac{1}{2} \rho U^2} \quad \frac{u_{\text{rms}}}{U} \%
\]

\( \bar{x} = 0.3779, U_\infty = 21.9, \Delta = 4'' \)
FIG. 27: Velocity Amplitudes
($\sqrt{\gamma} = 0.0569, \Delta = 4^\circ$)
FIG. 28: Velocity Amplitudes
($\frac{x}{x_0}=0.5069, U_\infty=21.9 \text{m/s}$)
FIG. 29: Pressure Amplitudes
($\bar{x}=0.5069, \Delta=4^\circ$)

\[ \frac{p_{rms}}{\frac{1}{2} \rho U^2} \% \]

- $U_\infty = 17.9 \text{ m/s}$
- $U_\infty = 21.9 \text{ m/s}$
- $U_\infty = 25.3 \text{ m/s}$

\[ f/\text{Hz} \]

Range: 0 to 6 Hz.
FIG. 30: Pressure Amplitudes
($\overline{x} = 0.5069, U_g = 21.9 \text{ m/s}$)
FIG. 31: Pressure Amplitude vs. Frequency Parameter

\( x = 0.5069, \Delta = 4'' \)

- \( U_\infty (\text{m/s}) \)
  - 17.9
  - 21.9
  - 25.3

- \( \frac{p}{U} - \frac{U^2}{2} \)

- \( f \)

\( f_x \) vs. \( U \)
FIG. 32: Pressure & Velocity Amplitudes

\( \frac{P_{\text{rms}}}{\frac{1}{2} \rho U^2} \) vs. \( \frac{U_{\text{rms}}}{U} \%

\( x = 0.5657, U_\infty = 21.9 \text{ m/s}, \Delta = 4^\circ \)

- \( \circ \)- \( p(\text{total}) \)
- \( \times \)- \( u(\text{total}) \)
- \( \cdot \)- \( \bar{u} \)
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FIG. 35: Pressure Amplitudes (x = 0.6137, U_\infty = 21.9 m/s)
FIG. 37: Velocity Amplitudes $v_s$. Non-dimensional Frequency
($\bar{x} = 0.6137, \Delta = 4''$)
FIG. 38: Pressure Amplitude $V_s$, Non-dimensional Frequency

($x = 0.6137, \Delta = 4''$)
FIG. 39: Pressure & Velocity Amplitudes
($x = 0.6838, U_\infty = 21.9, \Delta = 4^\circ$)
FIG. 40: Pressure & Velocity Amplitudes
(\bar{x} = 0.7819, U_\infty = 21.9 \text{ m/s}, \Delta = 4^\circ)
FIG. 43: Chordwise Pressure Amplitude Distribution
($U_\infty = 21.9$ m/s, $\Delta = 4''$)
FIG. 44: Chordwise Velocity Amplitude Distribution

\[ \frac{u_{rms}}{U} \]

\[ f/Hz \]

\( U_\infty = 21.9 \text{ m/s}, \Delta = 4'' \)
\( x = 0.3779 \)

**FIG. 45:** Steady BL Turbulence and Velocity profiles
FIG 52: Steady BL velocity profiles - Effect of $X$
FIG. 53: B.L. Integral Values

(a) Skin Friction Coefficient

(b) Momentum Thickness

(c) Form Factor

- Truckendrot
- Nash e.a.
- Horton
- Experimental
$X = 5.069$

$f = 2H_2$

**FIG. 54.: Amplitude ratio & phase lag through the B.L.**
\( \phi \) degrees

\( u/u_0 \)

\( x = 0.569 \)
\( f = 3 \text{ Hz} \)

**FIG. 55:** Velocity Amplitude ratio and phase lag through B.L.
$X = 0.5059$

$Z = 4$ Hz

**Figure 56:** Velocity Amplitude ratio and phase lag through B.L.
FIG. 57. Velocity Amplitude ratio and phase lag through B.L.
FIG. 60: Velocity phase lag - Effect of Frequency
Fig. 62: Velocity Amplitude ratio and phase lag through the B.L.

\[ \bar{x} = 0.5657 \]
\[ f = 3\text{Hz} \]
FIG. 6.3: Velocity Amplitude ratio and phase lag through the B.L.
FIG. 65: Velocity Amplitude ratio and phase lag through the B.L.

$\bar{x} = 0.6567$

$f = 6\text{Hz}$
FIG. 66: Velocity Amplitude ratio - Effect of Frequency
FIG. 67: Velocity phase lag -
Effect of frequency

$\overline{x} = 0.5657$
Fig. 68: Velocity Amplitude ratio and phase lag through B.L.
\[ X = 0.5137 \]
\[ f = 1 \]

**Fig. 71:** Velocity amplitude ratio and phase lag through the B.L.
FIG. 72: Velocity Amplitude Ratio - Effect of Frequency
Fig. 74: Velocity Amplitude ratio and phase lag through the B.L.

\[ x = 0.6838 \]
\[ f = 2 \text{Hz} \]
\[ X = 6838 \]
\[ f = 3 \text{ Hz} \]
$\bar{X} = 58.38$

$f = 6 \text{Hz}$

**FIG. 78:** Velocity Amplitude ratio and phase lag through the B.L.
Figure 81: Velocity Amplitude ratio and phase lag through the B.L.
\( \bar{x} = 0.7323 \quad \text{f} = 3 \text{Hz} \)

**FIG. 8.2:** Velocity Amplitude ratio and phase lag through the B.L.
FIG. 83: Velocity Amplitude ratio and phase lag through the B.L.

\[ \bar{x} = 0.6838 \]
\[ f = 4 \text{ Hz} \]

\[ u = u_i \]
\[ x = 0.7323 \]
\[ f = 5 \text{ Hz} \]

**Fig. 8.4:** Velocity Amplitude ratio and phase lag through B.L.
FIG. 85: Velocity Amplitude ratio and phase lag through the B.L.

\[ \bar{X} = 7.323 \]

\[ f = 5 \text{ Hz} \]
Fig. 88: Velocity Amplitude ratio and phase lag through the B.L.

\[ \bar{X} = 0.7819 \]
\[ f = 3 \text{Hz} \]
Figure 91: Velocity Amplitude ratio and phase lag through the B.L.
FIG. 93: Phase Angles - Effect of $x$. 

$f = 2$ Hz

$X$

$0.3773$

$0.5657$

$0.6838$

$0.7323$
Fig. 94: Phase Angles—Effect of x.
FIG. 97: Amplitude ratios - Effect of $x'$
FIG. 102: Oscillatory Velocity Component, as a percentage of \( U(y) \)

(a) Effect of Frequency

\[ x = 56.57 \]

\[
\begin{array}{c|c}
\text{f/Hz} & \mu/U, \% \\
\hline
5 & 10.98 \\
4 & 10.25 \\
3 & 8.96 \\
2 & 7.86 \\
1 & 4.35 \\
\end{array}
\]

(b) Effect of \( x \)

\( f = 6 \text{Hz} \)

\[
\begin{array}{c|c}
x (\text{tapping no.}) & \mu/U, \% \\
\hline
9 & \\
11 & \\
13 & \\
18 & \\
19 & \\
\end{array}
\]
FIG. 103: In & Out of Phase Velocity Components
(effect of 'x')

\[ f = 5 \text{Hz} \]

\[ \bar{x} \]

\[ 0.3779 \]

\[ 0.5657 \]

\[ 0.6137 \]

\[ 0.6838 \]

\[ 0.7819 \]
FIG. 104: In & Out of Phase Velocity Components

$\bar{x} = 0.5657$

$u/u_i$

$f/Hz$

$u_{in}$

$u_{out}$
FIG. 106: Mean and Steady Boundary Layer velocity profiles

$\bar{x} = .5069$

- $f/\text{Hz}$
  - $0$
  - $3$
  - $6$

$U/U_1$
FIG. 108: Mean and Steady B.L. velocity profiles
FIGURE 109: Mean and Steady B.L. velocity profiles
FIG. 112: Boundary Layer Turbulence - Effect of 'f'

\( \frac{U'}{U} \% \)

\( f/Hz \)
- \( \cdot \): 3
- \( \Delta \): 4
- \( \nabla \): 5
- \( + \): 6

\( \bar{x} = 0.638 \)
Effect of Amplitude

\[ f = 5 \text{ Hz} \]
\[ \frac{\text{U}}{\text{U}'} = 0.6137 \]
FIG. 115: B.L. Turbulence - Effect of Amplitude

\( f = 5 \text{ Hz} \)
\( \bar{x} = 0.6137 \)

- \( \Delta \) 2″
- \( \times \) 3″
- \( \circ \) 4″
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FIG. Ib: The semidynamic calibrator

Volume = 0.059 m³
FIG. 1d: Effect of tube length on resonant frequency

FIG. 1e: Effect of tube length on resonant frequency

P - P_{at}/mm H_{2}O

- - - - - Modified Transducer
FIG. If: Frequency Response—Effect of P.

$P/\text{mm H}_2\text{O}$

$G$ vs. $f/\text{Hz}$

$l_t = 50''$

Plug IV ($d_o = 0.3$, $l_o = 3/32''$)

FIG. Ig: Frequency Response—Effect of $d_o$

$\phi/\text{rad}$ vs. $f/\text{Hz}$

$l_t = 30''$

$P = 100 \text{ mm H}_2\text{O}$

$l_o = 3/32''$

$d_o/\text{in.}$

$0.04$

$0.03$

$0.02$
PLATE I

TOP FLAP AND DRIVE MECHANISM

A Balancing Weights

B Drive Belt

C Crank Arrangement
PLATE II

THE MODEL

A Pressure Tubing

B Pressure Tappings
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PLATE III

HOT-WIRE ARRANGEMENT

A Model Fixed Traversing Gear
B Boundary Layer Probe
C Freestream Probe
D Pitot-Static Probe
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PLATE IV

WORKING SECTION - MODEL IN SITU

A Flaps and Vortex Generators
B Boundary Layer Traversing Gear
C Freestream Traversing Gear
PLATE V

SIGNAL PROCESSING SYSTEM

A  The "Phase Shifter"
B  Data Transfer Unit
C  "Solatron" Digital Voltmeter
D  Decade Oscillator
E  Two-Channel Anemometer
F  Two-Channel Filter
G  "Hewlett-Packard" Mini Computer
H  Transducer Amplifier and Power Supply
I  Mean and R.M.S. Meter (TDA)
J  Frequency Counter
K  Pressure Tubing