ESSAYS ON BEHAVIOURAL ECONOMIC THEORY

by

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Declaration

I declare that the work presented in this thesis is my own.

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ABSTRACT

The chapters of this work lie at the intersection between classical choice theory and experimental data on decision making.

In chapter 2 I study necessary and sufficient conditions for a choice function to be rationalized in the following sense: there exists a complete asymmetric relation T (a tournament) such that, for each feasible (finite) set, the choice set coincides with the uncovered set of T restricted to that feasible set. This notion of 'maximization' may offer testable restrictions on observable choice behavior. In chapter 3 Mariotti and I give a group revealed preference interpretation to the concept of uncovered set, and we provide a characterization of uncovered bargaining solutions of a Pareto-consistent tournament.

In chapter 4 I study the rationalizability of reason-based choice correspondences axiomatically. A reason-based choice correspondence rationalizes choice behaviour in terms of a two stage choice procedure. Given a feasible set S, the individual eliminates in the first step all of the dominated alternatives according to her fixed (not necessarily complete) strict preference relation. In the second step, she first constructs for each maximal alternative identified in the first step its *lower contour set*, and then she eliminates from the maximal set all of those alternatives so that the following justification holds: there exists another maximal alternative whose lower contour set strictly contains that of another maximal alternative. This procedural model captures the basic idea behind the experimental finding known as "attraction effect".

Finally, in chapter 5 I build a connection between the behavioral property expressed by the weak axiom of revealed non-inferiority and a new weak notion of rationality.

This notion is weaker than that characterized by the weak axiom of revealed preference (WARP).

CONTENTS

LIST OF FIGURES

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CHAPTER 1

INTRODUCTION

1.1 The General Problem

In economics a decision maker is rational if her choices are made in accordance to the maximization of some binary preference relation. To guarantee that not only a decision maker chooses, but also that her choices are rational, the goal of rational choice theory has been to identify which rationality postulates formulated in terms of the properties of choices (and hence directly testable on observable market or non-market behaviours) are characteristic of rational decisions. The standard approach is to posit these rationality postulates of choices (henceforth, choice consistency conditions) on choice behaviours, which, in turn, allow scholars to reconstruct the underlying preferences of decision makers according to the *revelation principle* pioneered by Samuelson (1938).

A very basic choice consistency condition is the so-called **Property α** (Sen, 1970) (also known as Chernoff's axiom or basic contraction consistency). **Property α** is necessarily satisfied by any rational choice as it rules out cyclical and context dependent patterns of choices. A cyclical pattern of choices is observed when only x is chosen from the two-element set $\{x, y\}$, only y from $\{y, z\}$, and only z from $\{x, z\}$; whereas a context dependent pattern of choices is observed when an alternative is chosen while a distinct one is discarded from a feasible set and a reverse choice is made from a different feasible set to which they both belong.

Yet, in the last three decades, a large body of experimental findings on individual choice behaviour has been obtained, and a number of persistent violations of standard choice consistency conditions have been observed (see, e.g., Camerer (1994)). Contrary to what is prescribed by **Property** α, choices may be cyclical and context dependent. These startling and regular observed violations of **Property** α reveal that the standard economic interpretation of rational choice does not have a very satisfactory descriptive power, and this, in turn, motivates a substantial analytical rethink of the meaning of individual rational choice.

The main objective of this work is to develop some weakened notions of rationality of individual choice in a way that they are consonant with some robust experimental data on decision making. First, I formulate choice consistency conditions, some of which are weakened versions of standard rationality postulates, whilst others are motivated by the empirical research which has established the importance of the violation that I will be interested in. Second, I posit these choice consistency conditions on the choice behaviour of a decision maker in accordance with the standard choice theory, and then by analyzing what kind of pattern of choices she is allowed under the conjunct operation of standard and non-standard choice consistency conditions, I presume the basic procedural choice model which guides her choices. This allows me to suggest some *boundedly rational choice procedures* which offer directly testable restrictions on observable choices.

1.2 An Overview of the Results

The purpose of Chapter 2 (forthcoming in *Social Choice and Welfare*) is to study necessary and sufficient conditions for a choice correspondence to be rationalized in the following sense: there exists a complete asymmetric relation T (*a tournament*) such that, for each feasible (finite) set, the choice set coincides with the uncovered set of T restricted to that feasible set.[1] A choice correspondence behaving according to the uncovered set is named uncovered set choice rule.

The uncovered set has been extensively studied in social choice theory. It represents a weak form of maximality which may be able to explain cyclic choices and context dependent choices. This is due to the fact that the uncovered set corresponds to the idea of the existence of a dominance hierarchy among alternatives of a given set, which depends on a number of attributes and, above all, on what are the alternatives of the set under consideration.

My characterization result is provided by means of simple choice consistency conditions: two of them appear to be new to the best of my knowledge. I label them **Weakened Chernoff (WC)** and **Non-Discrimination (ND)**.

In Sen's words the Chernoff condition states "If the world champion of some game is a Pakistani, then he must also be the champion in Pakistan" (Sen, 1970, p.17) (equivalently, if in some game a Pakistani does not win in a competition, then he cannot be a candidate for world champion). In sport terminology, my weakening of the Chernoff condition is as follows: if in some game an Italian athlete never wins if a Pakistani athlete participates to competitions, then the Italian athlete cannot be a candidate for becoming the world champion.

ND states that if only x is chosen from $\{x, y\}$, only y from $\{y, z\}$, and only z from $\{x, z\}$, then all alternatives are choosable from $\{x, y, z\}$. It corresponds to the idea that if a decision maker is trapped in a three cycle, he must deem all alternatives equally adequate when called to choose from $\{x, y, z\}$. This property is implied by the Chernoff condition.

[1] This chapter is extract from my paper entitled "Uncovered set choice rules", which is forthcoming in *Social Choice and Welfare*.

As a tournament is a non-standard type of individual preference (lacking acyclicity), but it seems more appropriate to consider such non-standard preference for a group rather than for an individual. In Chapter 3, my advisor Marco Mariotti and I ask the following question: given a bargaining solution, does there exist a complete and strict relation T (a tournament) such that, for each feasible set, the bargaining solution set coincides with the uncovered set of T restricted to that set? We find a positive answer under the restrictive assumption of a Pareto-consistent tournament, and we name the bargaining solution as uncovered bargaining solution.

Our characterization result holds for the class of resolute bargaining solutions: loosely speaking, the bargaining solution chooses a unique alternative from (the comprehensive hull of) any pair of alternatives.

We offer two (related) motivations. First, a bargaining solution can be interpreted as a fair arbitration scheme (as argued for instance in Mariotti (1999)). In this sense, we may think of a bargaining solution as being ratified (or ratifiable) by a committee. In this interpretation, the tournament expresses the majority preferences of the committee, and the uncovered set is the solution to the majority aggregation problem.

A second interpretation follows the 'group revealed preference' interpretation pioneered by Peters and Wakker (1991). As they argue, 'the agreements reached in bargaining games may be thought to reveal the preferences of the bargainers as a group' (p. 1787).

The characterization uses four axioms: Strong Pareto Optimality; a standard Expansion property (if an alternative is in the solution set of a collection of problems, it is in the solution set of their union); a generalization of the 'Condorcet' property (if an alternative is chosen in 'binary' comparisons over each alternative in a collection, then it is the solution of the problem including all the alternatives in the collection); and a weak contraction consistency property (implied by Arrow's choice independence axiom).

My interest between decision theory and experimental data also leads me to the result of Chapter 4. A sizeable amount of experimental findings show that when added to a set a new relatively inferior alternative this alternative can increase the attractiveness of one of the alternatives obtainable from the original set (see, e.g., Rieskamp, Busemeyer, and Mellers, (2006)).

This systematic observed choice behaviour is known as "asymmetric dominance effect" or "attraction effect" and is explained in terms of bounded rationality: in a difficult and conflict-filled decision, where there is no escape from choosing, individuals choose by tallying defensible reasons for one alternative versus the other, rather than by trading off costs and benefits.

In this Chapter I propose a procedural choice model which is able to capture the basic idea behind the experimental finding of "attraction effect". A choice correspondence able to explain this context dependent anomaly is said to be a reason-based choice correspondence.

A reason-based choice correspondence rationalizes choice behaviour in terms of a two stage choice procedure. Given a feasible set S, the individual eliminates from it all of the dominated alternatives according to her fixed (not necessarily complete) strict preference relation, in the first step. In the second step, first she constructs for each maximal alternative identified in the first step its *lower contour set* (i.e., the set of alternatives which are dominated by it in S), and then she eliminates from the maximal set all of those alternatives, so that the following justification holds: there exists another maximal alternative whose lower contour set strictly contains that of another maximal alternative.

Most of the choice consistency conditions which are characteristic of reason-based choice correspondences appear to be new to the best of my knowledge and they relates to standard choice consistency proprieties. A key role in the development of this chapter is played by a choice consistency condition that I label **Reason-Based Bias** (**RBB**). It posits that for three distinct alternatives obtainable from a universal set, say x, y, and z, if x is strictly better than y and not worse than z, and y is not worse than z, then x must be the only choice from $\{x, y, z\}$. This property is motivated by the empirical research which established the importance of the attraction effect in decision making. This property captures this phenomenon requiring a bias toward the most defensible alternative in term of reasons.

The purpose of Chapter 5 is to study what kind of preference maximization characterizes the weak axiom of revealed non-inferiority (**WARNI**) introduced by Eliaz and Ok (2006). These authors accommodate preference incompleteness in revealed preference theory by studying the implications of weakening Arrow (1959)'s weak axiom of revealed preference (**WARP**) in **WARNI**.

This behavioural postulate entirely corresponds to maximizing behaviour on suitable domains. However, a choice correspondence rationalized by the maximization of a preference relation (not necessarily complete) may fail to satisfy **WARNI** on an arbitrary choice domain. This is due to the fact that **WARNI** characterizes a particular type of rationality. My concern is to spell out the form of maximality of choice characterized by this behavioural postulate on an arbitrary choice domain, and then I contrast this form of maximality with that characterized by **WARP**.

A choice correspondence is weak justified if there exists a binary relation J (dubbed weak justification) such that, for all feasible sets, no available alternative is J-related to any chosen alternative, for each rejected alternative there is *some* chosen alternative which is J-related to it. Therefore, the binary relation J is a strict (not necessarily complete) preference relation.

A decision maker makes weakly justified choices if she can assert that no chosen alternative is dominated by any other obtainable one, and for each discarded alternative there is *some* chosen alternative which dominates it.

My notion of rationality differs from that provided by Mariotti (2007), according to which choices are justified if there exists a binary relation J such that, for all feasible sets, no two chosen alternatives are J-related to each other, and each chosen alternative is J-related to *all* of the rejected alternatives. Mariotti (2007) shows that choices satisfy **WARP** if and only if they are justified by an asymmetric relation.

The result of this chapter is that choices satisfy **WARNI** if and only if they are weakly justified by an asymmetric relation.

CHAPTER 2

Uncovered Set Choice Rules

2.1 Introduction

There is evidence from psychological and marketing literature that in a choice task involving goods that vary along several attributes (e.g. TVs, digital cameras, job offers) a decision maker may prefer to use the majority rule (see May (1954) and Russo and Dosher (1983) for an early contribution, and Zhang, Hsee, and Xiao (2006) for a recent study).[1] The majority rule requires that in pairwise choice problems the decision maker has to deem choosable that option which is majority preferred in terms of her preferences over the relevant attributes.

The majority rule has been extensively studied in voting and social choice theory. It is well-known that this rule may produce cyclic majority relations (phenomenon known as *Condorcet paradox*). Of the existing literature on the *Condorcet paradox* two works are particularly relevant here. Miller (1977) shows that the majority relation may be represented as a tournament T. Moreover, to accommodate the general absence of undominated proposals as a solution set for majority voting (i.e. a tournament T), Miller (1980) suggested the solution concept of the *Uncovered Set*: a social state is in the uncovered set of T if it dominates every other state in at most two steps.[2]

The purpose of this chapter is to address the following question: given an individual

[1] The majority rule is applied in choice problems involving risky alternatives as well (see Fishburn and La Valle (1988)). For a discussion on the attraction of the majority rule in different choice contexts see Bar-Hillel and Margalit (1988). Fishburn (1991) also offers an example in which a decision maker may reasonably hold cyclical preferences using the absolute majority rule (pp. 120-121).

[2] For a clear and in-depth study of tournament solutions and majority voting see Laslier (1997).

choice function, does there exist a tournament T such that, for each feasible finite set, the choice set coincides with the uncovered set of T restricted to that feasible set? I find a positive answer. I label the choice function as uncovered set choice rule.

In individual choice theory, the choice behavior of a decision maker is rational if it is the outcome of the maximization of some preference relation over every feasible set (equivalently, there exists a preference relation R such that a good is R-maximal if there does not exist another available good which strictly dominates it in terms of R).[3] This implies that a decision maker should at the very least rank goods acyclically. Since the main cost that a decision maker has to bear if she uses the majority rule in a multi-attribute setting is to be trapped in intransitive cycles, there are sound reasons for developing a descriptive model of choice around a weak notion of maximality. I show that the uncovered set can be used for this purpose.

I provide a complete characterization of the class of uncovered set choice rules by means of simple consistency conditions: these provide testable restrictions on observable choice behavior. Two of them are new to the best of my knowledge, and I label them *Weakened Chernoff* (**WC**) and *Non-Discrimination* (**ND**).

The *Chernoff* condition (sometime known as *property* α or *contraction consistency*) demands that a good that is chosen from a set A and belongs to a subset B of A must be chosen from B. In Sen's words: "If the world champion of some game is a Pakistani, then he must also be the champion in Pakistan" (Sen, 1970, p.17) (equivalently, if in some game a Pakistani does not win in a competition, then he cannot be a candidate for world champion). In sport terminology, my weakening of the Chernoff condition is as follows: if in some game an Italian athlete never wins if a Pakistani athlete participates

[3] See Suzumura (1983).

to competitions, then the Italian athlete cannot be a candidate for world champion.

Since the majority rule can yield intransitive choices, the Non-Discrimination property states that if only x is chosen from $\{x, y\}$, only y from $\{y, z\}$, and only z from $\{x, z\}$, then all states are choosable from $\{x, y, z\}$. It corresponds to the idea that if a decision maker is trapped in a three cycle, he must deem all states equally adequate when called to choose from $\{x, y, z\}$. Non-Discrimination is implied by the Chernoff condition.[4]

In the next two sections I formalize and characterize the class of uncovered set choice rules, in section 1.3 I compare the suggested class with another one recently proposed in the literature (i.e. the class of top-cycle rules), while section 1.4 concludes.

2.2 Preliminaries

Let X be a universal finite set of cardinality $|X| \geq 2$. Let $\mathcal{X}$ be the collection of all subsets of X containing at least two distinct states. A *choice rule* f is a correspondence defined on $\mathcal{X}$ that assigns a nonempty subset $f(A)$ of A to every $A \in \mathcal{X}$. Therefore, as standard in axiomatic choice theory, I postulate that the choice rule f is decisive: $X \neq \emptyset \Rightarrow |f(X)| \geq 1$. The following abuse of notation will be repeated throughout the chapter: $f(xy) = x$ instead of $f(\{x, y\}) = \{x\}$, $f(xyz) = xyz$ instead of $f(\{x, y, z\}) = \{x, y, z\}$, $A \backslash x$ instead of $A \backslash \{x\}$, and $A \cup x$ instead of $A \cup \{x\}$. I say that f is resolute if $x, y \in X$, $x \neq y \Rightarrow |f(xy)| = 1$.[5]

A binary relation T on X is a *tournament* if it is asymmetric (for all distinct $x, y \in X$, $(x, y) \in T \Rightarrow (y, x) \notin T$) and complete (for all distinct $x, y \in X$, $(x, y) \in T$ or $(y, x) \in T$).

[4] To see this, let Chernoff condition hold. Assume that x, y, z are three distinct goods, and let $\{x, y, z\} = X$. Suppose that $f(\{x, y\}) = \{x\}$, $f(\{y, z\}) = \{y\}$, and $f(\{x, z\}) = \{z\}$, where f is a standard choice function. I show that $f(X) = X$. Assume, to the contrary, that $f(X) \neq X$. Then $f(X) \subsetneq X$. Let $x \notin f(X)$. Since $f(\{x, y\}) = \{x\}$ and Chernoff condition holds, I have that $y \notin f(X)$. By a similar argument, I have that $z \notin f(X)$. Therefore, $f(X) = \emptyset$, a contradiction.

[5] This condition appears in Ehlers and Sprumont (2007).

As usual, I write xTy for $(x,y) \in T$. For every $A \in \mathcal{X}$, define $T \cap (A \times A) = T|A$. This restriction of T is a tournament on A. For every $x \in X$, let

$$\begin{aligned} T^{-1}(x) &= \{y \in X | xTy\} \\ T(x) &= \{y \in X | yTx\} \end{aligned}$$

denote the lower and upper sections of T at x, respectively.

$B \subseteq A$ means that every element of B is in A, whilst $B \subsetneq A$ means that $B \subseteq A$ and $B \neq A$.

Following Miller (1980) I say x *covers* y in $A \in \mathcal{X}$, denoted $xC_{T|A}y$, if $T^{-1}(y) \cap A \subsetneq T^{-1}(x) \cap A$.[6] Observe that $C_{T|A}$ is a strict partial order (i.e. irreflexive and transitive), but it is not complete unless $T|A$ itself is transitive.

Given a tournament T on X and $A \in \mathcal{X}$, the *uncovered set of T in A*, denoted by $UC(T|A)$, is the set of maximal alternatives of $C_{T|A}$ in A:

$$UC(T|A) = \left\{x \in A | \text{for every } y \in A, x \neq y, \text{ not } yC_{T|A}x\right\}.$$

Equivalently, $x \in UC(T|A)$ if and only if the following *two-step principle* holds:

$$\text{for every } y \in A,\ x \neq y:\ xTy \text{ or for some } z \in A,\ xTzTy.$$

Note that $UC(T|A)$ is nonempty as $C_{T|A}$ is transitive and A is finite.

Definition 1 *A choice rule f is said to be an uncovered set choice rule if there exists a tournament T on X such that $f(A) = UC(T|A)$ for every $A \in \mathcal{X}$.*

2.3 Main Theorem

I characterize the class of uncovered set choice rules by means of the following properties:

[6] Observe that C_T is a subrelation of T. Different definitions of C_T exist in the literature, but they are equivalent when T is a tournament. My main result holds for all of them.

Weak Expansion (WE). If $A_k \in \mathcal{X}$, with $k = 1, ..., K$, then $\cap_{k=1}^{K} f(A_k) \subseteq f(\cup_{k=1}^{K} A_k)$.

Binary Dominance Consistency (BDC). If $A \in \mathcal{X}$, $x \in A$, & $f(xy) = x$ for every $y \in A \backslash x$, then $f(A) = x$.

Weakened Chernoff (WC). For all $A \in \mathcal{X}$ such that $|A| > 3$, if $x \in f(A)$ & $y \in A \backslash x$, then $x \in \cup_{B \subsetneq A: x, y \in B} f(B)$.

Non-Discrimination (ND). For all distinct $x, y, z \in X$, if $f(xy) = x$, $f(yz) = y$, & $f(xz) = z$, then $f(xyz) = xyz$.

WE and **BDC** have a long history in choice theory. **WE** (also known as *property* γ or *expansion consistency*) asserts that if a state is chosen from every element of a given nonempty collection of choice problems then it must still be chosen from their union, whilst **BDC** is the choice formulation of the so called "Condorcet winner principle" and asserts that if a state is chosen over every other single state then it must be uniquely chosen from the choice problem containing all states.

WC asserts that in a set A containing more than three distinct states if x is chosen from A and y is a distinct available state in A, then the decision maker has to deem x choosable from some strict subset B of A containing y. Equivalently, it assures that if x is not chosen from any $B \subsetneq A$, with $x, y \in B$, then x cannot be chosen from A. **WC** is much weaker than the Chernoff condition (if $B \subseteq A$, then $f(A) \cap B \subseteq f(B)$). To see it observe that the Chernoff condition implies:

$$\text{for all } A \in \mathcal{X} \text{ such that } |A| > 3, \text{ if } x \in f(A) \ \& \ y \in A \backslash x, \text{ then } x \in \cap_{B \subsetneq A: x, y \in B} f(B).$$

ND states if an agent has a clear mind on three distinct pairwise choices, but his choices cycle, then when called to make a choice from their union he must deem all alternatives equally adequate. I have already observed that **ND** is implied by Chernoff condition.

Theorem 1. *A choice rule f is an uncovered set choice rule if, and only if, it is resolute and satisfies Weak Expansion (**WE**), Binary Dominance Consistency (**BDC**), Weakened Chernoff (**WC**), and Non-Discrimination (**ND**).*

Proof. (Only if) Let f be an uncovered set choice rule. Obviously, f is resolute, and satisfies **ND**. Next, I check **WE** , **BDC**, **WC**.

To see that f satisfies **WE** , let $A_k \in \mathcal{X}$, with $k = 1, ..., K$, and $x \in \cap f(A_k)$. Let $S = \cup A_k$. Then $x \in UC(T|A_k)$ for all A_k. As x reaches every other state in $S \backslash x$ in at most two steps, it follows that not $yC_{T|S}x$ for all $y \in S \backslash x$. Hence, $x \in UC(T|S)$.

To verify that **BDC** is satisfied, assume that $A \in \mathcal{X}$, $x \in A$, and $f(xy) = x$ for every $y \in A \backslash x$. I show that $x = f(A)$. Since $f(xy) = x$ for every $y \in A \backslash x$, I have that xTy for all $y \in A \backslash x$. Thus $xC_{T|A}y$ for all $y \in A \backslash x$. Since $T(x) \cap A = \emptyset$, not $yC_{T|A}x$ for all $y \in A \backslash x$. It follows that $x = UC(T|A)$.

To see that f satisfies **WC**, let $A \in \mathcal{X}$, with $|A| > 3$. Suppose that $y \in A \backslash x$ and $x \notin f(B)$ for all $B \subsetneq A$, with $x, y \in B$; I prove that $x \notin f(A)$. Assume, to the contrary, that $x \in f(A) = UC(T|A)$. By the two-step principle, I have that for all $y \in A \backslash x$: xTy or for some $z \in A$, $xTzTy$. Suppose xTy, and let $B = \{x, y\} \subsetneq A$. Since f is an uncovered set choice rule it follows that $f(B) = x$, a contradiction. Thus, let yTx. Because $x \in UC(T|A)$ it follows that there exists $z \in A$ such that $xTzTy$. Let $B = \{x, y, z\} \subsetneq A$. Because $x \in UC(T|B)$, and f is an uncovered set choice rule it follows that $x \in f(B)$ yielding a contradiction.

(If). Let f be resolute and satisfy **WE** , **BDC**, **WC**, and **ND**. Given X, define the relation T on X as follows:

$$\text{for } x, y \in X, \text{ with } x \neq y\text{: } xTy \;\Leftrightarrow f(xy) = x.$$

By resoluteness of f, it follows that T is asymmetric. Given that f is defined on a universal domain, I have that T is complete. Therefore, T is a tournament on X.

I claim that

$$f(A) = UC(T|A) \text{ for all } A \in \mathcal{X}. \tag{2.1}$$

A proof by induction based on the cardinality of A is provided.

Clearly, resoluteness, **BDC**, **ND**, and the construction of T, imply that (2.1) is true for every $A \in \mathcal{X}$, with $|A| \leq 3$. Assume that (2.1) holds for each $A \in \mathcal{X}$, with $|A| = k \geq 3$. I prove that (2.1) is true for $A \in \mathcal{X}$, with $|A| = k+1$.

Let $x \in f(A)$, and assume, to the contrary, that $x \notin UC(T|A)$. Therefore, there exists $y \in A \backslash x$ such that $yC_{T|A}x$. It follows that for $B \subsetneq A$, with $x, y \in B$, $x \notin UC(T|B)$. By the inductive hypothesis, $x \notin f(B)$. Because it is true for all $B \subsetneq A$, with $x, y \in B$, **WC** implies that $x \notin f(A)$, a contradiction.

Conversely, let $x \in UC(T|A)$, and partition A in $T^{-1}(x) \cap A$, $T(x) \cap A$, and $\{x\}$.

Since $x \in UC(T|A)$, it follows that for every $y \in A \backslash x$: xTy or for some $z \in A$, $xTzTy$. This implies that $T^{-1}(x) \cap A \neq \emptyset$. If $T(x) \cap A = \emptyset$, it follows from the construction of T and **BDC** that $x = f(A)$. Otherwise, consider $T(x) \cap A \neq \emptyset$. Take any $y \in T(x) \cap A$. Because $x \in UC(T|A)$ and yTx, there exists $z \in T^{-1}(x) \cap A$ such that $xTzTy$, by the two-step principle. Therefore, $x \in UC(T|\{x, y, z\})$. The inductive hypothesis implies $x \in f(xyz)$. Because it is true for any $y \in T(x) \cap A$, it follows that $x \in \cap_{y \in T(x) \cap A} f(xyz)$ for some $z \in T^{-1}(x) \cap A$. Moreover, as xTz for all $z \in T^{-1}(x) \cap A$, the construction of T and **BDC** imply $x = f(A \backslash T(x))$. As

$$x \in \left[\cap_{y \in T(x) \cap A} f(xyz)\right] \cap \left[f(A \backslash T(x))\right]$$

WE implies $x \in f(A)$.

Hence, (2.1) is true for every $A \in \mathcal{X}$, by the principle of mathematical induction. ■

The properties in theorem 1 are tight, as argued next.

In figure 1.1 $a \rightarrow b$ stands for $f(ab) = a$.

For an example violating only **WE** , fix $X = \{x, y, z, w\}$, and let choice in pairs be those displayed in figure 1.1. Let $f(A) = UC(T|A)$ for every $A \in \mathcal{X} \backslash X$, and $f(X) = xy$. Because w is an uncovered state it follows that f is not an uncovered set choice rule. **WE** is violated because $w \in f(xyw)$ and $w \in f(xzw)$, but $w \notin f(X)$. Observe that $f(X) \subsetneq UC(T|X)$ and **WC** is satisfied as $z \notin f(X)$.

For an example violating only **BDC**, fix $X = \{x, y, z\}$, assume that choice in pairs are those displayed in figure 1.1, and let $f(X) = xy$. Clearly $f(X) \neq UC(T|X)$, and **BDC** is violated because $y \in f(X)$.

For an example violating only **WC**, fix $X = \{x, y, z, w\}$, and let choice in pairs be those displayed in figure 1.1. Moreover, let $f(A) = UC(T|A)$ for every $A \in \mathcal{X} \backslash X$, and $f(X) = X$. Observe that $UC(T|X) \neq f(X)$, and **WC** is violated because z is not chosen in any proper subset of X containing y, but $z \in f(X)$.

For an example violating only **ND**, fix $X = \{x, y, w\}$, and assume that choice in pairs be those displayed in figure 1.1, and $f(X) = xy$. Obviously, $f(X) \neq UC(T|X)$ as $w \notin f(X)$ in violation of **ND**.

Finally, for an example violating only resoluteness, fix $X = \{x, y\}$, and let $f(X) = X$. As T is not defined, $UC(T|X) = \emptyset$. It follows that f is not uncovered set choice

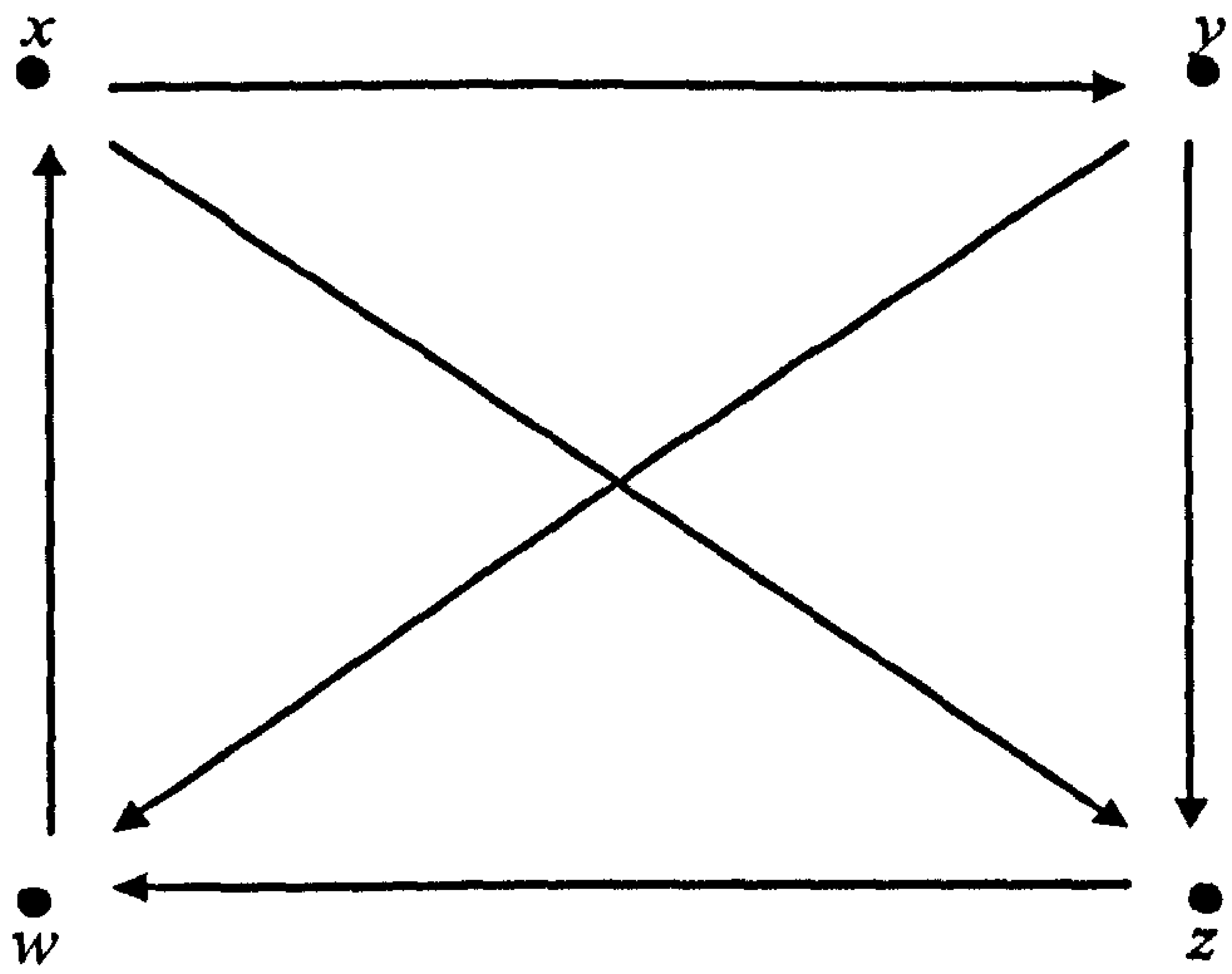

Figure 2.1: Revealed preferences in pairwise choice problems for independence of axioms used in Theorem 1

rule. Resoluteness is violated because $|f(X)| \neq 1$.

2.4 Discussion

Since the uncovered set of a tournament is a subset of its top-cycle, it seems interesting to investigate the relationship between the axiomatization provided in Theorem 1 and that of top-cycle choice rules which appears in Ehlers and Sprumont (2006). They axiomatize the class of top-cycle choice rules using resoluteness, **BDC**, and two more behavioral regularities: *Weakened Weak Axiom of Revealed Preference* (**WWARP**), and *Weak Contraction Consistency* (**WCC**).

Weakened Weak Axiom of Revealed Preference (WWARP). Let $x, y \in X$ & $A \in \mathcal{X}$. If $x \in f(A)$ & $y \in A \backslash f(A)$, then for no $B \in \mathcal{X} : y \in f(B)$ & $x \in B \backslash f(B)$.

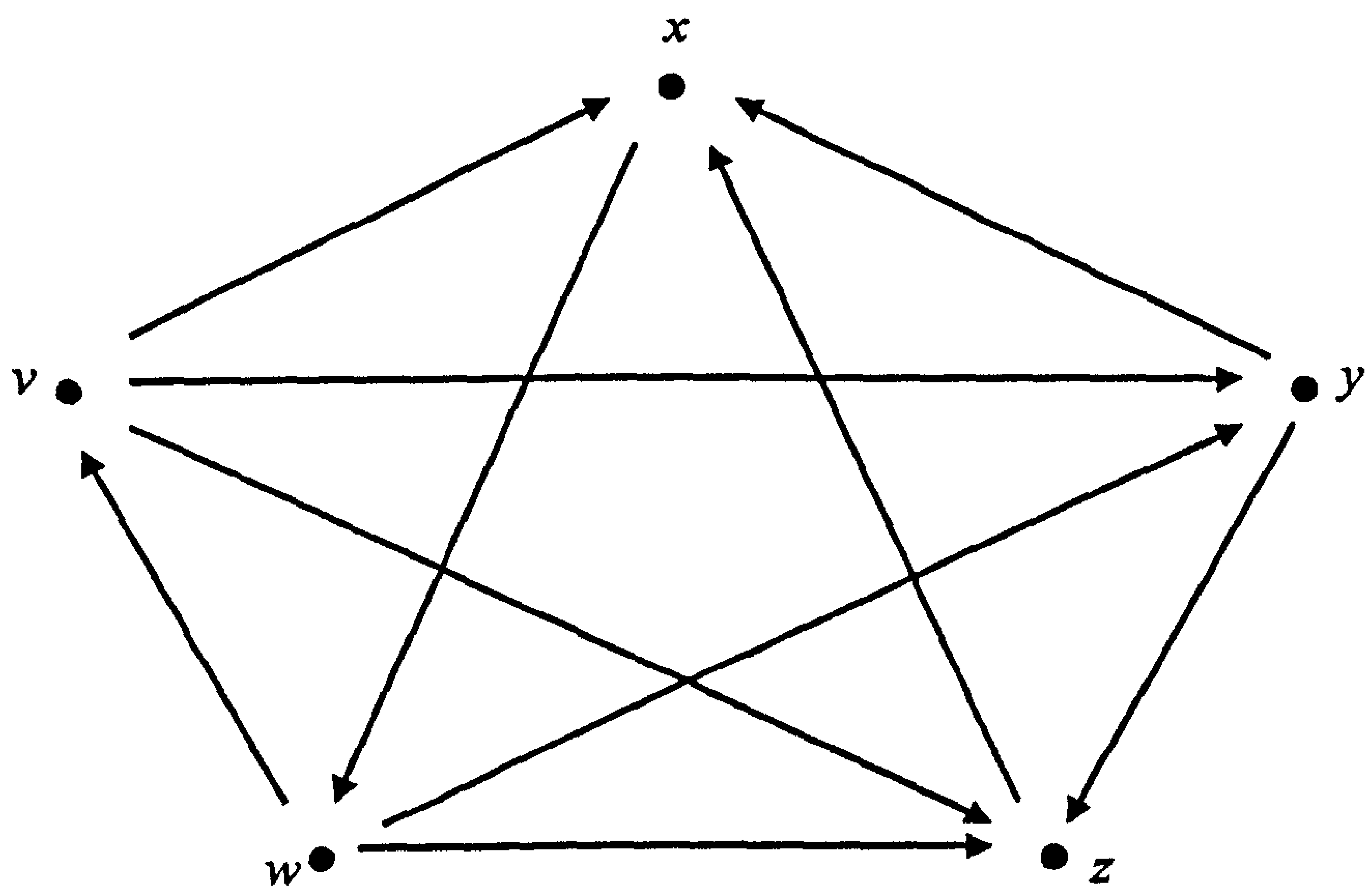

Figure 2.2: Revealed preferences in pairwise choice problems for Remark 1.

Weak Contraction Consistency (WCC). If $A \in \mathcal{X}$ and $|A| \geqslant 2$, then $f(A) \subseteq \cup_{x \in A} f(A \backslash x)$.

WWARP is a weak version of Arrow's **WARP**, and it asserts that if x and y are two available states in A, and x is chosen while y is actually rejected, then there does not exist any distinct conceivable set B, with $x, y \in B$, such that y is chosen and x is rejected. Conversely, **WCC** states that given a feasible set A, with $|A| = k$, a decision maker can deem x choosable from A only if he has deemed x choosable from some sets $|A| = k - 1$. **WCC** implies a well known consistency property, namely, Never Chosen (**NC**): if $A \in \mathcal{X}$, $x \neq f(xy)$ for all $y \in A \backslash x$, then $f(A) \subseteq A \backslash x$.

As resoluteness of f and **BDC** are necessary and sufficient for the uncovered set choice rules as well, I will focus on **WWARP** and **WCC**. Let me begin with **WWARP**.

Remark 1. *WWARP is not necessary for the uncovered set choice rule.*

To see this, fix $X = \{v, w, x, y, z\}$. Binary choices are visualized in figure 1.2, where $a \to b$ stands for $a = f(ab)$. Let $f(A) = UC(T|A)$ for every $A \in \mathcal{X}$. It is easy to see that $x \in UC(T|X)$ and $y \in X \backslash UC(T|X)$, while $y \in UC(T|\{x, y, z\})$ and $x \notin UC(T|\{x, y, z\})$.

The example above also shows that $x \in f(A)$ as long as $w \in A$. Thus the desirability of x is conditional to the availability of w. From this perspective, Remark 1 suggests that the class of uncovered set choice rules captures some notion of context dependency of choices. The example above also shows that the uncovered set choice rule violates the so called dual Chernoff axiom: If $A, B \in \mathcal{X}$, $B \subseteq A$, then $f(A) \cap B = \emptyset$ or $f(B) \subseteq f(A) \cap B$ (Suzumura, 1983). It is easy to see that **WWARP** implies **ND.**

Next, I show that resoluteness, **BDC, WE** , and **WC** are sufficient for f to satisfy **WCC.**

Proposition 1. *Let f be resolute and satisfy Weak Expansion (WE), Binary Dominance Consistency (BDC), and Weakened Chernoff (WC). Then f satisfies Weak Contraction Consistency (WCC).*

Proof. Let f be resolute and satisfy **WE** , **BDC,** and **WC.** Let $A \in \mathcal{X}$, with $|A| \geq 2$, and $x \in f(A)$. I show that $x \in f(A \backslash y)$ for some $y \in A \backslash x$.

Define $D_x = \{y \in A | x \neq y, f(xy) = y\}$ and $S_x = \{y \in A | x \neq y, f(xy) = x\}$. Since f is resolute, A is partitioned in $\{x\}$, D_x, and S_x.

Suppose $D_x = \emptyset$. It follows from **BDC** that $x = f(A \backslash y)$ for all $y \in A \backslash x$. Otherwise, consider $D_x \neq \emptyset$. I proceed according to whether $|A| \leq 3$ or $|A| > 3$.

Case 1. $|A| \leq 3$

The case $|A| = 2$ is not possible as f is resolute and $x \in f(A)$. Thus, let $|A| = 3$. Because $x \in f(A)$, f is resolute and satisfies **BDC**, it follows that $|D_x| = 1$. Then $S_x \neq \emptyset$, as desired.

Case 2. $|A| > 3$

Suppose that $|D_x| = 1$, and so let $d_1 \in D_x$. **BDC** implies that $x = f(A \backslash d_1)$, as desired. Otherwise, let $|D_x| = n$, with $n > 1$. Take any $d_i \in D_x$, for $i \in \{1, ..., n\}$. **WC** implies that there exists $B \subsetneq A$, with $x, d_i \in B$, such that $x \in f(B)$. Whenever $|B| > 3$, I can iterate the application of **WC** until getting $x \in f(B_i')$, with $d_i \in B_i' \subsetneq B$, and $|B_i'| = 3$. Because $|D_x| = n$, there are n sets $B_i' \subsetneq A$ such that $d_i \in B_i'$, $|B_i'| = 3$, and $x \in f(B_i')$. Let $\mathcal{B}$ be a class made of sets B_i', for $i \in \{1, ..., n\}$.

As $|B_i'| = 3$, let $s = B_i' \backslash x d_i$. Since f is resolute either $f(sx) = s$ or $f(sx) = x$. If $f(sx) = s$, resoluteness of f, combined with **BDC**, implies that either $s = f(B_i')$ or $d_i = f(B_i')$ yielding a contradiction. Then, it must be the case that $x = f(xs)$, and so $s \in S_x \neq \emptyset$. Therefore, for every $B_i' \in \mathcal{B}$, we established that: $B_i' = \{d_i, s, x\}$, with $d_i \in D_x$ and $s \in S_x$; and $d_i \notin B_j'$ for all $B_j' \in \mathcal{B} \backslash B_i'$. Now, fix any $B_i' \in \mathcal{B}$. As $x \in f\left(B_j'\right)$ for every $B_j' \in \mathcal{B} \backslash B_i'$, it follows that $x \in \cap_{B_j' \in \mathcal{B} \backslash B_i'} f\left(B_j'\right)$. Moreover, as $S_x \neq \emptyset$, **BDC** implies that $x = f(x \cup S_x)$. Because $D_x \backslash d_i \subsetneq \cup_{B_j' \in \mathcal{B} \backslash B_i'} B_j'$, and

$$x \in \left[\cap_{B_j' \in \mathcal{B} \backslash B_i'} f\left(B_j'\right)\right] \cap \left[f\left(x \cup S_x\right)\right]$$

WE implies $x \in f(A \backslash d_i)$. ∎

Corollary 1. *Let $A \in X$, and let f satisfy resoluteness, Binary Dominance Consistency (**BDC**), Weak Expansion (**WE**), and Weakened Chernoff (**WC**). Then f satisfies Never Chosen (**NC**).*

Proof. It directly follows from Proposition 1 combined with the fact that **WCC**

implies **NC**. ∎

Under resoluteness, **WE** and **BDC**, **WC** and **WCC** are not equivalent. To see it, fix $X = \{w, x, y, z\}$. Assume that binary choices are those displayed in figure 1.1, where $a \to b$ stands for $a = f(ab)$. Let $f(xyz) = x$, $f(xyw) = xyw$, $f(xzw) = xzw$, $f(yzw) = y$, and $f(X) = X$. As it is easy to check, f is resolute and satisfies **WE** , **BDC**, and **WCC**. However, f does not satisfy **WC** as $z \in f(X)$ and there does not exist any $A \subsetneq X$, with $z, y \in A$, such that $z \in f(A)$. It also follows that **WCC** cannot replace **WC** in Theorem 1.

2.5 Concluding Remarks

There is evidence that choice cycles and menu dependence of choices can be displayed by decision makers that use a consistent and deliberate rule of choice. One of this choice rule is the majority rule. Borrowing from Miller's solution concept, I characterize the class of uncovered set choice rules. It is different from that offered by Moulin (1986) because I use only properties relating choices across feasible sets. This has the advantage of offering behaviorists testable restrictions on observable choice behavior. Moreover, the class of rules characterized here may explain choice cycles.

Ehlers and Sprumont (2007) characterize the class of top-cycle choice rules. They introduce the property of Weakened Weak Axiom of Revealed Preference which assures context independency of choices. **WWARP** is violated by my class of rules: this suggests that the uncovered set choice rule captures some notion of context dependency of choices. Manzini and Mariotti (2007) characterize the choice as the outcome of the implementation of a fixed ordered pair of asymmetric (and possibly incomplete) preference relations. This choice procedure is able to explain both choice cycles and menu dependency of choices, but it does not necessarily preserve the notion of maximality suggested here.

I conclude by observing that in a companion paper of Moulin's work, Dutta (1988) suggests the minimal covering set as solution concept (which is finer than the ultimated uncovered set) and offers a characterization by means of consistency requirements of different nature. A natural step forward would be a characterization of the minimal covering set by means of consistency properties relating only choices across sets. This open question is left for future research.

CHAPTER 3

Uncovered Bargaining Solutions

3.1 Introduction

A bargaining solution expresses 'reasonable' compromises on the division of a surplus within a group. In this chapter my advisor prof. Marco Mariotti and I ask the following question: given a bargaining solution, does there exist a complete and strict relation T (a tournament) such that, for each feasible set A, the bargaining solution set coincides with the uncovered set of T restricted to A? If the answer is positive, we call the bargaining solution an *uncovered bargaining solution.*

We offer two (related) motivations. First, a bargaining solution can be interpreted as a fair arbitration scheme (as argued for instance in Mariotti (1999)). In this sense, we may think of a bargaining solution as being ratified (or ratifiable) by a *committee.* In this interpretation, the tournament expresses the majority preferences of the committee, and the uncovered set is the solution to the majority aggregation problem. A bargaining solution that does not coincide with the solution of any tournament is certainly not fair in the described sense: it could not be ratified by any committee.

A second interpretation follows the 'group revealed preference' interpretation pioneered by Peters and Wakker (1991). As they argue, 'the agreements reached in bargaining games may be thought to *reveal* the preferences of the bargainers as a group' (p. 1787). A tournament is a non-standard type of preference (lacking transitivity), which has recently been considered in individual choice theory (Ehlers and Sprumont (2007), Lombardi (2007)). It seems even more appropriate to consider such non-standard preference for a group than for an individual.

For *single valued* solutions the issue under study has essentially been solved, since a single valued uncovered bargaining solution *maximizes* (if certain regularity conditions are met)[1] a binary relation (in other words, the solution point is a Condorcet winner of the underlying tournament). For the domain of convex problems, Peters and Wakker (1991) have shown that this is the case if and only if the solution satisfies Nash's Independence of Irrelevant Alternatives[2]. Denicolò and Mariotti (2000) show that the same holds for certain domains of non-convex problems, provided that Strong Pareto Optimality is assumed. In this latter case the binary relation is transitive. Therefore, the problem under study is new and interesting only for multi-valued solutions. It is thus natural to look at a domain of nonconvex problems, as many notable solutions (such as the Nash Bargaining Solution) are single-valued on a domain of convex problems.

We focus on solutions which satisfy a 'resoluteness' condition: loosely speaking, when only two feasible alternatives x and y are Pareto optimal (so the bargaining problem is essentially binary), the solution picks either x or y. For this class of solutions, we provide a complete characterization of uncovered bargaining solutions for which the underlying tournament satisfies certain Paretian properties. The characterization uses four axioms: Strong Pareto Optimality; a standard Expansion property (if an alternative is in the solution set of a collection of problems, it is in the solution set of their union); a generalization of the 'Condorcet' property (if an alternative is chosen in 'binary' comparisons over each alternative in a collection, then it is the solution of the problem including all

[1] See the end of the next section for a discussion of this point.

[2] Peters and Wakker work with a weak relation. However it is easy to show - by using elementary duality properties in the maximization of binary relations - that a strict relation could be used instead. See e.g. Kim and Richter (1986) or Aleskerov and Monjardet (2002) for discussions of this issue in abstract choice theory.

the alternatives in the collection); and a weak contraction consistency property (implied by Arrow's choice independence axiom).

3.2 Preliminaries

An n-person bargaining problem is a pair (A, d), with $d \in A$ and $A \subseteq \Re^n$, where A represents the set of feasible *alternatives* and d is the *disagreement point.*

The null-vector is denoted $\mathbf{0} \in \Re^n$. The vector inequalities in $\Re^n$ are: $x > y$ (resp.: $x \geqslant y$) if and only if $x_i > y_i$ (resp.: $x_i \geqslant y_i$) for every i. We view, as usual, $x \in \Re^n$ as a utility or welfare vector for n agents.

A domain of bargaining problem $\mathcal{B}$ is said to be *admissible* if:

D1 For every pair $(A, d) \in \mathcal{B}$: A is compact, and there exists $x \in A$ such that $x > d$.

D2 For all $x, y \in \Re^n_d$, where $x \neq y$ and $\Re^n_d = \{x \in \Re^n | x > d\}$, there exists a unique $(M(x, y), d) \in \mathcal{B}$ such that:

1) $x, y \in M(x, y)$ and for every $z \in M(x, y)$ such that $z \notin \{x, y\}$, $x \geqslant z$ or $y \geqslant z$;

2) for every $(A, d) \in \mathcal{B}$ such that $x, y \in A$: $M(x, y) \subseteq A$.

D3 For all $(A, d), (B, d) \in \mathcal{B}$: $(A \cup B, d) \in \mathcal{B}$.

Many bargaining domains considered in the literature are particular cases of admissible domains[3]. For example the set of comprehensive problems (Zhou (1997), Peters and Vermeulen (2006)), the set of finite problems (Mariotti (1998), Peters and Vermeulen (2006)), the set of all problems satisfying D1 (Kaneko (1980)), the set of d-star shaped problems[4]. D2 guarantees the existence of a 'minimal' problem containing any two given

[3] This class was essentially introduced in Denicolò and Mariotti (2000).

[4] That is, those problems (A, d) for which the convex hull of $\{d, x\}$ is in A for all $x \in A$.

alternatives x and y, and such that x and y are the only strongly Pareto optimal feasible alternatives.

Unless specified otherwise, $\mathcal{B}$ is from now on a class of n-person admissible bargaining problems. A bargaining *solution* on $\mathcal{B}$ is a nonempty correspondence $f : \mathcal{B} \rightrightarrows \Re^n$ such that $f(A, d) \subseteq A$ for all $(A, d) \in \mathcal{B}$.

Given a bargaining solution f, we say that an alternative $x \in A$ is the f-*Condorcet winner* in $(A, d) \in \mathcal{B}$, denoted by $x = CW(A, d)$, if $x = f(M(x, y), d)$ for all $y \in A$, with $y \neq x$. Moreover, $x \in A$ is said to be an f-*Condorcet loser* in (A, d), denoted by $x \in CL(A, d)$, if $y = f(M(x, y), d)$ for all $y \in A$, with $y \neq x$.

Finally, the following abuses of notation will be repeated throughout this note: $f(A, d) = x$ instead of $f(A, d) = \{x\}$, $A \cup x$ instead of $A \cup \{x\}$, $A \backslash x$ instead of $A \backslash \{x\}$.

We consider only *resolute* solutions, that is those which satisfy the following property. For all $x, y \in \Re^n$, with $x \neq y$, for all $A \subseteq \Re^n$:

Resoluteness: $|f(M(x, y))| = 1$.

Resoluteness is analogous to a property with the same name imposed by Ehlers and Sprumont (2007) and Lombardi (2007) for individual choice functions over finite choice sets, given that (in the presence of Strong Pareto Optimality, defined below) the minimal problem $M(x, y)$ involves essentially a choice between only two alternatives. For standard solutions that are obtained by maximizing a quasiconcave 'social welfare function' (e.g. the Nash Bargaining Solution or the Utilitarian solution) this involves adding a tie-breaking criterion on minimal problems.

In addition the following properties will be used in the characterization result.

Axiom 2 *(Strong Pareto Optimality)* $x \geqslant y$ *and* $x \neq y \in f(A, d) \Rightarrow x \notin A$.

Axiom 3 *$A, B \in \mathcal{B}$, $x = CW(A,d)$ and $y \in CL(B \cup x, d) \Rightarrow y \notin f(A \cup B, d)$*

Axiom 4 *$x, y, z \in \Re^n$, with $x \neq y \neq z$, $x = f(M(x,y), d)$ and $y = f(M(y,z), d)$ $\Rightarrow x \in f(M(x,y) \cup M(y,z), d)$*

Axiom 5 *Given a class of problems $\{A_k, d\}$, then $\cap_k f(A_k, d) \subseteq f(\cup_k A_k, d)$*

Strong Pareto Optimality is standard. Axioms 3 is a generalization of the natural 'Condorcet Winner Principle'

$$x = CW(A,d) \Rightarrow x = f(A,d)$$

which is implied by setting $B = \emptyset$ in axiom 3.

Axiom 4 is a weak independence property. It says that if an alternative x is the unique solution point in a minimal problem where the only other Pareto optimal feasible alternative is y, and if y is the unique solution point in a minimal problem where the only other Pareto optimal feasible alternative is z, then x is a solution point of a minimal problem where the only other Pareto optimal feasible alternatives are y and z. Consider the following standard contraction consistency axiom[5]: $R \subset S$ & $f(S,d) \cap R \neq \varnothing \Rightarrow f(R,d) = f(S,d) \cap R$. Suppose $x \notin f(M(x,y) \cup M(y,z), d)$. If f is Pareto optimal then $f(M(x,y) \cup M(y,z), d) \subseteq \{y, z\}$. Suppose $y \in f(M(x,y) \cup M(y,z), d)$. If contraction consistency holds, then $f(M(x,y), d) = y$. If on the other hand $y \notin f(M(x,y) \cup M(y,z), d)$, that is $z = f(M(x,y) \cup M(y,z), d)$, and if f satisfies contraction consistency, then $z = f(M(y,z), d)$. In either case the premise of axiom 4 is violated. This shows that, in the presence of Pareto optimality, axiom 4 is a very special implication of contraction consistency.

[5] This is also called Arrow's choice independence axiom.

Finally Axiom 5 is standard in choice theory: if an alternative is a solution point for every element of a given collection of bargaining problems, then it is still a solution point of their union.

We are, as usual, only interested in solutions that satisfy translation invariance. Then, we can set $d \equiv 0$. A bargaining problem simply becomes a subset of $\Re^n$ containing the null-vector and the notation is simplified accordingly.

A binary relation $T \subseteq \Re^n \times \Re^n$ is a *tournament* if it is asymmetric (i.e., for every $x, y \in \Re^n$, $x \neq y$, $(x, y) \in T \Rightarrow (y, x) \notin T$) and weakly connected (i.e., for every $x, y \in \Re^n$ with $x \neq y$, $\{(x, y), (y, x)\} \cap T \neq \emptyset$). We denote by $\mathcal{T}$ the set of all tournaments on $\Re^n$. A restriction of T to $A \subseteq \Re^n$, denoted by $T|A$, is a tournament.

For $x \in \Re^n$, let $T^{-1}(x)$ and $T(x)$ denote the *lower* and *upper sections* of T at x, respectively, that is:

$$\begin{aligned} T^{-1}(x) &= \{y \in \Re^n | \, (x, y) \in T\}, \text{ and} \\ T(x) &= \{y \in \Re^n | \, (y, x) \in T\}. \end{aligned}$$

For any tournament $T \in \mathcal{T}$ and $A \subseteq \Re^n$, define its covering relation $C|A$ on A by:

$$(x, y) \in C|A \text{ iff } (x, y) \in T|A \text{ and } T^{-1}(y) \cap A \subset T^{-1}(x) \cap A$$

The uncovered set of $T|A$, denoted $UC(T|A)$, consists of the $C|A-$maximal elements of A, that is:

$$UC(T|A) = \{x \in A | \, (y, x) \notin C|A \text{ for all } y \in A\}.$$

The *Strong Pareto* relation P on $\Re^n$ is defined by

$$\text{for } x, y \in \Re^n,\ x \neq y\text{: } (x, y) \in P \Leftrightarrow x_i \geq y_i \text{ for all } i, \text{ and } x_j > y_j \text{ for some } j.$$

We say that a tournament $T \in \mathcal{T}$ is *Pareto consistent* if for $x, y, z \in \Re^n$, with $x \neq y \neq z$:

$$(x,y) \in P \Rightarrow (x,y) \in T,$$

$$(x,y) \in P \;\&\; (y,z) \in T \Rightarrow (x,z) \in T.$$

So, a Pareto consistent tournament includes the Strong Pareto relation and satisfies a form of 'Pareto transitivity': any x which Pareto dominates y will beat any alternative z which is beaten by y.

Definition. *A bargaining solution f is an uncovered set bargaining solution (UCBS) if there exists $T \in T$ such that, for every $A \in B$, $f(A) = UC(T|A)$. In this case we say that T rationalizes f.*

As an example of an UCBS which does not coincide with a standard solution, consider the following class. Let F be a asymmetric transitive and weakly connected relation, which here we interpret as 'fairness'[6]. Recall that P is the Strong Pareto relation. Then define the solution f by: $x \in f(A)$ iff for all $y \in A\backslash x$: either $(x,y) \in P$; or $[(y,x) \notin P \;\&\; (x,y) \in F]$; or $[(x,z) \in P \;\&\; (z,y) \in F \;\&\; (y,z) \notin P$ for some $z \in A]$; or $[(x,z) \in F \;\&\; (z,x) \notin P \;\&\; (z,y) \in P$ for some $z \in A]$. In words, fairness is ignored if and only if a Pareto ranking is possible, and given this constraint, for any other alternative y, the chosen alternative x must either dominate y directly in terms of Pareto or fairness, or indirectly via an intermediate alternative z, applying the Pareto and fairness (or vice versa) criteria in succession. The solution f is, in each problem, the uncovered set of the tournament T defined by: $(x,y) \in T$ iff either $(x,y) \in P$; or $[(y,x) \notin P$ and $(x,y) \in F]$ (note that T is weakly connected and asymmetric); or both.

[6] F could be constructed for example on the basis of the Euclidean distance to the 45^0 line, with the addition of a tie-breaking criterion.

Finally, we come back briefly to the issue of single-valued solutions alluded to in the introduction. Let T be a tournament on A, and suppose $UC(T|A)) = \{x\}$ for some $x \in A$. If x is not a Condorcet winner, $T(x)$ is nonempty. Let $y \in UC(T|T(x) \cup x)$. Then $y \in UC(T|A)$, since for any $z \in T^{-1}(x)$ we have $(y,x), (x,z) \in T$. But this contradicts the assumption that $UC(T|A) = \{x\}$. So x must be a Condorcet winner of A if it is the unique uncovered element of A. In this reasoning, however, it assumed that the uncovered set of $T(x) \cup x$ is nonempty, which is not necessarily true if $T(x)$ is not finite. For conditions guaranteeing the nonemptiness of the uncovered set on general topological spaces see Banks, Duggan and Le Breton (2006).

3.3 Characterization

We show below that in the presence of Resoluteness, axioms 2-5 characterize uncovered bargaining solutions for which the rationalizing tournament is Pareto consistent.

Theorem 2. *Let f be a resolute bargaining solution. Then f is an UCBS, rationalized by a Pareto consistent tournament, if, and only if, it satisfies axioms 2-5.*

Proof. (*Only if*). Let f be a resolute UCBS. Obviously f satisfies Strong Pareto Optimality and Weak Expansion. Next, we check axioms 3-4.

To verify axiom 3, let $x = CW(A)$, and $y \in CL(B \cup x)$, with $x \neq y$. The existence of a Pareto consistent T implies that $(x,z) \in T$ for all $z \in A \backslash x \cup y$. Moreover, as $y \notin f(M(y,w))$ for all $w \in B \backslash y$, there exists $w' \in M(w,y) \backslash y$ which covers y. If $w' = w$, then $(w,y) \in T$. Otherwise, consider $w' \neq w$. Since w' is not strongly Pareto dominated by y, it must be the case that $(w,w') \in P$, by D2. It follows from Pareto consistency of T that $(w,y) \in T$. Therefore, whether or not $w = w'$ we have that $(w,y) \in T$. Since

$(x,z) \in T$ for all $z \in A\backslash x \cup y$ and $(w,y) \in T$ for all $w \in B\backslash y$, it follows that x covers y, and so $y \notin UC(T|A \cup B)$ as desired.

For axiom 4, let $x,y,z \in \Re^n$, with $x \neq y \neq z$, and let $x = f(M(x,y))$ and $y = f(M(y,z))$. We show that $x \in f(M(x,y) \cup M(y,z))$. Since $x = f(M(x,y))$ and $y = f(M(y,z))$, there exists a Pareto consistent T such that $(x,x') \in T$ for all $x' \in M(x,y)\backslash x$ and $(y,y') \in T$ for all $y' \in M(y,z)\backslash y$. Observe $M(x,y) \cup M(y,z) \in \mathcal{B}$, by D3. Since no point in $M(x,y) \cup M(y,z)\backslash x$ covers x, it follows that $x \in f(M(x,y) \cup M(y,z))$.

(If). Let f be a resolute bargaining solution satisfying the axioms. Define the relation T on $\Re^n$ as follows:

$$\text{for all } x,y \in \Re^n, \text{ with } x \neq y,\ (x,z) \in T \text{ iff } x = f(M(x,y)).$$

For all $x,y \in \Re^n$, with $x \neq y$, there exists a minimal problem $M(x,y)$, by D2. It follows from Strong Pareto Optimality and Resoluteness that either $x = f(M(x,y))$ or $y = f(M(x,y))$. Then, T is weakly connected and asymmetric, and so $T \in \mathcal{T}$. To see that T is Pareto consistent as well, let $x,y,z \in \Re^n$, with $x \neq y \neq z$. We show that *i)* $xPy \Rightarrow xTy$, and *ii)* $(x,y) \in P$ & $(y,z) \in T \Rightarrow (x,z) \in T$. Case *i)* directly follows from Strong Pareto Optimality. Next, we show case *ii)*. Since $x = f(M(x,y))$ and $y = f(M(y,z))$, it follows from axiom 4 combined with D3 that $x \in f(M(x,y) \cup M(y,z))$. Since $M(x,y) \cup M(y,z) = M(x,z)$, Resoluteness implies that $x = f(M(x,z))$, and we are done.

We claim that

$$f(A) = UC(T|A) \text{ for all } A \in \mathcal{B}.$$

Fix $A \in \mathcal{B}$. For any $x \in A$ partition A in $T(x)$, $T^{-1}(x)$ and $\{x\}$.

Let $x \in f(A)$ and assume, to the contrary, that x is a covered point. Then for some $y \in A \backslash x$ it must be the case that $(y, x) \in T$ and $T^{-1}(x) \subset T^{-1}(y)$. Therefore $y = CW\left(T^{-1}(x) \cup \{x, y\}\right)$. Let $z \in T(x)$, and consider the minimal bargaining problem $M(x, z)$. By definition of T, we have that $z = f(M(x, z))$ for all $z \in T(x)$, and so $x \in CL(T(x) \cup x)$. It follows from axiom 3 that $x \notin f(A)$, a contradiction.

Conversely, let $x \in UC(T|A)$. Take any $y \in T^{-1}(x)$, and consider the minimal bargaining problem $M(x, y)$. By definition of T it follows that $x = f(M(x, y))$. Because it is true for any $y \in T^{-1}(x)$, we have that $x = CW\left(T^{-1}(x) \cup x\right)$. If $T(x) = \varnothing$, it follows from the Condorcet Winner Principle implied by axiom 3 that $x \in f(A)$. Otherwise, take any $z \in T(x)$. Since T is Pareto consistent and $z \in T(x)$, there exists $y \in T^{-1}(x)$ which is not strongly Pareto dominated either by x nor by z such that $(y, z) \in T$. Axiom 4, combined with D3, implies that $x \in f(M(x, y) \cup M(y, z))$. Because this holds for any $z \in T(x)$, axiom 5 implies that $x \in f(A)$.

3.4 Independence of the Axioms

The axioms used in Theorem 2 are tight, as argued next.

For an example violating only Strong Pareto Optimality, consider the disagreement point d as the solution of any admissible bargaining problem, that is, $f(A, d) = d$ for every $(A, d) \in \mathcal{B}$. Clearly, f is resolute and satisfies axioms 3-5, but not Strong Pareto Optimality.

Next, let us consider for simplicity only 2-person bargaining problems.

For an example violating only axiom 3, define, for every $x, y \in \Re^2_+$, with $x \neq y$:

$$f(M(x, y)) = x \text{ if } x_1 + x_2 > y_1 + y_2 \text{ or } x_1 + x_2 = y_1 + y_2 \;\&\; x_1 > y_1,$$

whilst, for any non-minimal problem $A \in \mathcal{B}$, define the bargaining solution f as:

$$f(A) = \arg\max_{s \in A}(s_1 + s_2).$$

To see that axiom 3 is contradicted, consider the domain of finite problems, and let $x, y, z \in A$, where $x = (2,1)$, $y = (1,2)$, and $z = (1,0)$. By definition, $f(xy) = f(xz) = x$, and $f(yz) = y$, but $f(xyz) = xy$, which violates axiom 3. Obviously, the bargaining solution is resolute, and it satisfies axioms 2 and 4-5.

For an example violating only axiom 4, fix $y, z \in \Re^2_{++}$, with $y \neq z$, such that $y_1 + y_2 = z_1 + z_2$. Fix $f(M(z,y)) = z$.

Given any other bargaining problem $A \in \mathcal{B}$, define the bargaining solution f as the following:

$$f(A) = \begin{cases} \arg\max_{s_1}\{\arg\max_{s \in A}(s_1 + s_2)\} & \text{if} \quad y \notin A \text{ or } z \notin A \\ \arg\max_{s_1}\{\arg\max_{s \in A}(s_1 + s_2) - \{y\}\} & \text{otherwise} \end{cases}.$$

To see that axiom 4 is contradicted, consider the domain of finite problems, and let $x, y, z \in A$, where $x = (2,2)$, $y = (3,1)$, and $z = (1,3)$. We have that $f(xy) = y$, $f(xz) = x$, and $f(yz) = z$. Consider the bargaining problem $A' = \{x, y, z\}$. Given that $y, z \in A'$, it follows from definition of f that $x = f(A')$, which violates axiom 4. Clearly, the bargaining solution is resolute and satisfies axioms 2 and 5. It is easy but tedious to check that it satisfies axioms 3 as well (details has been relegated to the Appendix A).

Finally, for an example violating only axiom 5, fix $x, y, z \in \Re^2_{++}$, with $x \neq y \neq z$ and $x_1 + x_2 = y_1 + y_2 = z_1 + z_2$, and let $M(x,y) \cup M(y,z) = C \in \mathcal{B}$ with $f(M(x,y)) = x$, $f(M(y,z)) = y$, and $f(M(x,z)) = z$. Define for any $a, b \in \Re^2_{+}\backslash\{x, y, z\}$, with $a \neq b$: $f(M(a,b)) = a$ if $a_1 + a_2 > b_1 + b_2$ or $a_1 + a_2 = b_1 + b_2$ & $a_1 > b_1$, whilst let for any

$a \in \Re^2_+ \backslash \{x, y, z\}$ and $b \in \{x, y, z\}$:

$$f(M(a,b)) = a \quad \text{if} \quad a_1 + a_2 > b_1 + b_2$$
$$f(M(a,b)) = b \quad \text{if} \quad a_1 + a_2 \leq b_1 + b_2$$

Define the following set of alternatives S_a:

$$S_a = \{b \in \Re^2_+ \backslash a | f(M(a,b)) = a\}$$

and for any bargaining problem $A \in \mathcal{B}$ not yet considered define the bargaining solution f as:

$$f(A) = \begin{cases} \arg\max_{s_1}(\arg\max_{s \in A}(s_1 + s_2)) & \text{if} \quad A \cap \{x, y, z\} = \emptyset \\ \arg\max_{s_1}(\arg\max_{s \in A}(s_1 + s_2) - S_a) & \text{if} \quad A \cap \{x, y, z\} = \{a\} \\ \arg\max_{s_1}(\arg\max_{s \in A}(s_1 + s_2) - S_a) & \text{if} \quad A \cap \{x, y, z\} = \{a, b\} \ \& \ f(M(a,b)) = a \\ x, y, z & \text{if} \quad A = C \\ \arg\max_{s_1}(\arg\max_{s \in A}(s_1 + s_2) - S_y) & \text{otherwise} \end{cases}$$

To see that axiom 5 is contradicted, consider the domain of finite problems, and let $A = \{x, y, z, w\}$, where $x = (2,2)$, $y = (3,1)$, $z = (1,3)$, and $w = (1,1)$. By construction $f(xy) = x$, $f(yz) = y$, $f(xz) = z$, and $f(xyz) = xyz$; furthermore, we have that $f(xw) = x$, $f(yw) = y$, and $f(zw) = z$. Let us consider the bargaining problem $\{x, z, w\} = B$. Since $x, z \in B$ and $f(xz) = z$, it follows from the definition of f that $z = f(B)$. However, we have that $z \notin f(A)$, by definition of f, which violates axiom 5. The bargaining solution as defined above is obviously resolute and it satisfies 2. Moroeover, it can be checked that it satisfies axioms 3-4 (the tedious analysis has been relegated to the Appendix A).

3.5 Concluding Remarks

Lombardi (2007) studies choice correspondences on the domain of all subsets of an abstract finite set, and poses the same question as this chapter. At the technical level, the main difficulty here is that bargaining sets are not always finite. This necessitates the different axioms and argument of proof presented in this chapter, as well as the restriction to Pareto consistent tournaments. These arguments exploit heavily the ordering structure of $\Re^n$ and the natural Strong Pareto Optimality assumption, which is instead meaningless on the domain considered by Lombardi.

Ehlers and Sprumont (2007), on the same domain as Lombardi, characterize choice correspondences for which there exists a tournament such that, for each choice set, the choice is the *top cycle* of the tournament. It is natural to seek a similar characterization in the context of bargaining solutions, as we have done for the uncovered set. This remains an open question for future research.

CHAPTER 4

REASON-BASED CHOICE CORRESPONDENCES

4.1 Introduction

Rationality of choice behaviour cannot be assessed without seeing it in the context in which a choice is made (Sen, (1993) and (1997)). This view is confirmed by a sizeable amount of experimental findings which show that when added to a choice set a new relatively inferior alternative can increase the attractiveness of one of the alternatives obtainable from the original set (see, Rieskamp, Busemeyer, and Mellers, (2006)).

This systematic observed choice behaviour, known as "asymmetric dominance effect" or "attraction effect",[1] is explained in terms of bounded rationality. In a difficult and conflict-filled decision, where there is no escape from choosing, individuals choose by tallying defensible reasons for one alternative versus the other, rather than by trading off costs and benefits. Furthermore, in this respect, the dominant structure of alternatives in the choice set provides the decision maker with good reasons for her choice (see, Simonson (1989), Tversky and Simonson (1993), and Shafir, Simonson, and Tversky (1993), and the references cited therein).[2]

Let me give an example. Suppose that an individual wishes to buy herself a digital camera for next holiday in Rome, and she has a choice among three competing models, say, x, y, and x', where each model is characterized by exactly two equally important

[1] Strictly speaking these two effects are slightly different, and the difference refers to the attributed levels of the new alternative that is added to the choice set. In this chapter I will refer only to the attraction effect since the asymmetric dominance effect is a special case.

[2] A first formalization of how reasons affect the individual's decisions in a game theoretical framework appears in Spiegler (2002).

dimensions, say, price and quality. She may find the choice between x (resp., x') and y hard because x (resp., x') is better than y on one dimension (say, price) while y is better than x (resp., x') on the other dimension (say, quality). She would find the choice between x and x' an easy one because the former dominates the latter with respect to both dimensions. Thus, while she has a clear and indisputable reason for choosing x over x', she cannot hold any compelling reason for choosing only x (resp., x') from $\{x, y\}$ (resp., $\{x', y\}$) or only y from $\{x, y\}$ and $\{x', y\}$. However, the fact that x' is obtainable from $\{x, y, x'\}$ and x is better priced and of higher quality than x', whilst y is only of higher quality, may provide her with a reason for choosing only x from $\{x, y, x'\}$.

This pattern of observed choices - which is not confined to consumer products, but also extends to choices among gambles, job applicants, political candidates (Rieskamp, Busemeyer, and Mellers, (2006)) - is partially consistent with the standard economic interpretation of rationality which is preference maximization.

In our example, the individual has an incomplete preference relation on $\{x, y, x'\}$ because she deems x and y choosable from $\{x, y\}$, x' and y from $\{x', y\}$, and only x from $\{x, x'\}$ and $\{x, y, x'\}$. For any feasible set she faces, she chooses undominated alternatives relative to her preferences in that set. However, contrary to what is envisaged from the standard preference maximization hypothesis, she discards y from her choice. This suggests that our individual may have refined her choice by using the information available from the entire choice set (given her preferences) as a tie-breaking rule: As x dominates x', but y does not, the set of alternatives dominated by x strictly contains that dominated by y, providing the individual with a convincing reason for choosing only x from $\{x, y, x'\}$ (see, e.g., Tversky and Simonson, (1993), p. 1185).

The idea of rationalizing choice correspondences in terms of a two-stage choice procedure whereby the individual arrives at a choice by using the information obtainable from the entire set in the second round of elimination appears in Ok (2004), who identifies in these terms all the choice correspondences satisfying the canonical Property α (also known as Chernoff choice-consistency condition or basic contraction consistency). Property α requires that an alternative that is deemed choosable from a feasible set T and belongs to a subset S of T must be deemed choosable from S (Sen, (1971)).

Indeed many contexts of choice which lead individuals to violate the normatively appealing Property α, and so the weak axiom of revealed preference (**WARP**) proposed by Samuelson (1938),[3] and the ways in which they interact, await further investigation.

Returning to our consumer, suppose that another camera model y' - which is dominated by y with respect to both of the dimensions, whilst it is of higher quality than x and x' and worse priced than them – is added to the set $\{x, y, x'\}$. In this new choice-context, the individual loses the compelling reason which led her to choose x from $\{x, y, x'\}$ because the set of alternatives dominated by x does not contain that dominated by y, and vice versa. The presence of y' (which indeed should be irrelevant for her choice) makes x and y reasonably choosable from the grand set, whereas its absence makes only x choosable from $\{x, y, x'\}$. The combination of these choices violates Property α even though there is nothing particularly "unreasonable" in this pair of choices.

What is more, the described tie-breaking rule may lead an individual to suffer from certain framing manipulations. Let me give another example. Suppose an employee spends her lunch vouchers in one of her local restaurants. Assuming that her preferences may be incomplete and that her vouchers are enough to get any kind of luncheon served

[3] Analyses of these and related choice-consistency conditions can be found, among others, in Moulin (1985), Sen (1971), Suzumura (1983). For a recent study of what kind of preference maximization WARP characterizes, see Mariotti (2007).

Day	Menu	Choice
1	{luncheon 1, luncheon 1', luncheon 2}	{luncheon 1}
2	{luncheon 2, luncheon 2', luncheon 3}	{luncheon 2}
3	{luncheon 3, luncheon 3', luncheon 1}	{luncheon 3}

Figure 4.1: Framing Effect

at any chosen local restaurant, on day 1 she steps into one of the local restaurants finding three kinds of luncheon on the menu (say, *luncheon 1*, *luncheon 1'*, and *luncheon 2*). Our employee strictly prefers *luncheon 1* to *luncheon 1'*, whereas she cannot make up her mind between *luncheon 1* (resp., *luncheon 1'*) and *luncheon 2*. To satisfy one's hunger she goes for *luncheon 1* as it dominates *luncheon 1'*, but *luncheon 2* does not. The day after (day 2) she steps into another available local restaurant to explore her range of choices, and it is serving three luncheons (say, *luncheon 2*, *luncheon 2'* and *luncheon 3*, where *luncheon 2* is the same luncheon served from the restaurant of day 1). Because she dithers between *luncheon 2* (resp., *luncheon 2'*) and *luncheon 3*, whereas she strictly prefers *luncheon 2* to *luncheon 2'*, she goes for *luncheon 2* as it seems the most "attractive" according to the menu of the day (i.e., *luncheon 2* dominates *luncheon 2'*, but *luncheon 3* does not). On day 3, she decides to return to the restaurant of day 1 which is serving only *luncheon 3*, *luncheon 3'* and *luncheon 1* (*luncheon 1* is the same luncheon served on day 1, and *luncheon 3* is the same luncheon served from the restaurant of day 2). Since she strictly prefers *luncheon 3* to *luncheon 3'*, while she cannot make up her mind between *luncheon 3* (resp., *luncheon 3'*) and *luncheon 1*, she goes for *luncheon 3* because it dominates *luncheon 3'*, but *luncheon 1* does not. Her choices are displayed in figure 3.1.

The choices made over the three days may appear weird from an economic perspective, but they are not as irrational in any minimal significant sense. The reason for this is that the employee's preferences are insufficient to solve the decision problem that she faces, and so she constructs a reason on the basis of the problem that she faces by using her known preferences. Since each day there is a maximal luncheon (i.e., *luncheon 1* on day 1, *luncheon 2* on day 2, and *luncheon 3* on day 3) which outperforms the other maximal one (i.e., *luncheon 2*, *luncheon 3*, and *luncheon 1*, respectively), this allows her to complete her preferences by knocking the latter off.

Motivated by these observations, I believe that there is a need to shed more light on the phenomenon of how individuals use the set under consideration to identify the most "reasonable" alternatives following the revealed preference approach introduced by Samuelson (1938), the importance of which has been recently emphasized by Rubinstein and Salant (2006, 2007).[4]

With this aim I provide a full characterization of a choice correspondence as exemplified above in terms of a two-stage choice procedure. Given a feasible finite set, the individual eliminates from the decision all of the dominated alternatives according to her fixed (not necessarily complete) strict preference relation, in the first step. In the second step, she eliminates from the maximal set, identified in the first step, those alternatives which have the set of dominated alternatives strictly contained in that of another undominated alternative. Whenever a choice correspondence can be rationalized with the described two-stage rationalization, I say that the choice correspondence is a reason-based choice correspondence.

The rest of the chapter is organized as follows. I begin by outlining our axiomatic

[4] For a set theoretical foundation of the revealed preference theory see Richter (1966). For a recent survey on revealed preference see Varian (2005).

framework, delineating the behavioural consistency properties used in our characterization result. Next, I provide our characterization of reason-based choice correspondences. I conclude with a brief discussion of our result in relation to the literature.

4.2 Preliminaries

Let X be a universal finite set of conceivable alternatives that is fixed from now on. Let $\mathcal{S}$ be a collection of all nonempty subsets of X. By a *choice correspondence* C on $\mathcal{S}$ I mean a map C which assigns a nonempty subset $C(S)$ of S to every $S \in \mathcal{S}$. Following Sen (1993), I read $x \in C(S)$ as x is choosable from S. Moreover, given $x, y \in X$, with $x \neq y$, $x, y \in C(S)$ for some $S \in \mathcal{S}$ does not necessarily mean that x is indifferent to y, but I interpret it as both of them are choosable from S.

Two distinct alternatives x and y in X are said to be *indistinguishable* on a set $S \in \mathcal{S}$, $x, y \notin S$, if, for all $z \in S$, one of the following holds:

1. $\{x\} = C(\{x, z\}) \Rightarrow \{y\} = C(\{y, z\})$;
2. $\{z\} = C(\{x, z\}) \Rightarrow \{z\} = C(\{y, z\})$;
3. $\{x, z\} = C(\{x, z\}) \Rightarrow \{y, z\} = C(\{y, z\})$.

Then x and y are indistinguishable one another if they behave in the same way with respect to direct choice comparisons with other alternatives. Observe that if x and y are not indistinguishable it does not necessarily mean that they are $C-$incomparable as I am silent on $C(\{x, y\})$.[5]

The set of positive integers is denoted by $\mathbb{N} = \{1, 2, ...\}$. Let $\succ \subseteq X \times X$ be a binary relation on X which represents the individual preference relation. As usual I

[5] For a choice theoretical study of incomplete preferences, see Eliaz and Ok (2006).

write $x \succ y$ for $(x,y) \in \succ$, and $x \nsucc y$ for $(x,y) \notin \succ$. A relation $\succ \subseteq X \times X$ is *acyclical* if, for all $t \in \mathbb{N}$ and for all $x^1, ..., x^t \in X$, $x^\tau \succ x^{\tau+1}$ for all $\tau \in \{1, 2, ..., t-1\}$ implies $x^t \nsucc x^1$. For any $S \in \mathcal{S}$, $\prec(x, S)$ denotes the lower section of $\succ$ restricted to S at x, i.e., $\prec(x, S) = \{y \in S | x \in S \backslash \{y\}, y \prec x\}$. $S \subseteq T$ means that every alternative in S is in T, whilst $S \subset T$ means that $S \subseteq T$ and $S \neq T$.

For $S \in \mathcal{S}$ and a binary relation $\succ$ on X, the set of $\succ$-maximal alternatives in S is $M(S, \succ) = \{x \in S | y \nsucc x \text{ for all } y \in S \backslash \{x\}\}$. Whenever a choice correspondence C on $\mathcal{S}$ has an acyclical relation $\succ$ on X such that, for all $S \in \mathcal{S}$,

$$C(S) = \{x \in M(S, \succ) \mid \prec(x, S) \subset \prec(y, S) \text{ for no } y \in M(S, \succ) \backslash \{x\}\},$$

I say that C is a *reason-based choice correspondence.*

Now I define some choice-consistency conditions of interest. The first is borrowed by Sen (1977) which is much weaker than Property α.

Property $\alpha 2$ ($\alpha 2$). For all $S \in \mathcal{S}$: $x \in C(S) \Rightarrow x \in C(\{x, y\})$ for all $y \in S$.

The second property is a weakening of Sen's (1971) Property β. Property β demands that for all pair of feasible sets, say S and T, and for all pair of alternatives, say x and y, if x and y are choosable from S, a subset of T, then y is choosable from T if and only if x is choosable from T. Our Weak Property β on the other hand requires Sen's Property β to hold if x and y are indistinguishable one another on $T \backslash \{x, y\}$.

Weak Property β (Wβ). For all $S, T \in \mathcal{S} : S \subseteq T$, $x, y \in C(S)$, and x, y indistinguishable on $T \backslash \{x, y\} \Rightarrow [y \in C(T) \Leftrightarrow x \in C(T)]$.

The third property is a weakening of Samuelson's (1938) Weak Axiom of Revealed Preference (**WARP**), according to which if $x \in C(S)$ and $y \in S \backslash C(S)$, then there is no feasible set T, with $x \in T$, such that $y \in C(T)$. Our Weak **WARP** on the other hand demands Samuelson's **WARP** to hold if x is uniquely chosen from S, and I add to the set S a feasible set T such that x and y are indistinguishable one another on T, with $x, y \notin T$.

Weak WARP (WWARP). For all $S, T \in \mathcal{S} : \{x\} = C(S)$ and $y \in S \backslash C(S) \Rightarrow$ $[x, y$ indistinguishable on T, $x, y \notin T \Rightarrow y \notin C(S \cup T)]$.

The following property is a straightforward strengthening of the choice formulation of the so called "*Condorcet Winner Principle*" - labeled Binary Dominance Consistency (**BDC**) by Ehlers and Sprumont (2007) -, according to which for a feasible set, say T, with $x \in T$, if x is uniquely chosen over every other alternative obtainable from T, then x must be the only choice from T. Our Strong **BDC** demands that for all pairs of feasible sets, say S and T, if x is the only choice from S and it is uniquely chosen over every other alternative obtainable from T, then x must be the only choice from $S \cup T$. Obviously, if our property holds, **BDC** follows.

Strong BDC (SBDC). For all $S, T \in \mathcal{S} : \{x\} = C(S)$ and $\{x\} = C(\{x, y\})$ for all $y \in T \Rightarrow \{x\} = C(S \cup T)$.

Our next property is a particular weakening of Weak Axiom of Revealed Non-Inferiority (**WARNI**) of Eliaz and Ok (2006), according to which for any feasible set, say S, if for every $y \in C(S)$ there exists a feasible set, say T, such that $x \in C(T)$ and $y \in T$, then $x \in C(S)$. Our Weak **WARNI** on the other hand demands **WARNI** to hold if there exists a T, with $\{x, y\} \subset T \subset S$, such that x is the only choice from T, and x is choosable over every other $y \in S$.

Weak WARNI (WWARNI). For all $S \in \mathcal{S}$, $x \in S$: for all $y \in C(S)$ there exists

$T \subset S : \{x\} = C(T)$ and $\{x,y\} \subset T$, and $x \in C(\{x,y\})$ for all $y \in S \Rightarrow x \in C(S)$.

The final property that I will consider here for reason-based choice correspondences plays a key role in the development of this chapter. It posits that for three distinct alternatives obtainable from a universal set, say x, y, and z, if x is strictly better than y and not worse than z, and y is not worse than z, then x must be the only choice from $\{x, y, z\}$. This property is motivated by the empirical research which established the importance of the attraction effect in decision making. Our property captures this phenomenon requiring a bias toward the most defensible alternative in term of reasons.

Reason-Based Bias (RBB). For all distinct $x, y, z \in X : \{x\} = C(\{x,y\})$, $x \in C(\{x,z\})$, and $y \in C(\{y,z\}) \Rightarrow \{x\} = C(\{x,y,z\})$.

4.3 Reason-Based Choice Correspondences

The following theorem shows that whenever X is a universal finite set of alternatives, the axioms above characterize completely a reason-based choice correspondence.

Theorem 3. *A choice correspondence C on $\mathcal{S}$ is a reason-based choice correspondence if and only if it satisfies Property $\alpha 2$ ($\boldsymbol{\alpha 2}$), Weak Property β (Wβ), Weak WARP (WWARP), Strong BDC (SBDC), Weak WARNI (WWARNI), and Reason-Based Bias (RBB).*

Proof. Suppose that C is a reason-based choice correspondence on $\mathcal{S}$. That C satisfies $\boldsymbol{\alpha 2}$ is straightforward, thus omitted. I show that C satisfies the remaining choice-consistency conditions listed above.

To prove that C satisfies Wβ, take any $S, T \in \mathcal{S}$, such that $S \subseteq T$, and assume that $x, y \in C(S)$ and x and y are indistinguishable on $T \backslash \{x,y\}$. Let $y \in C(T)$. I show that $x \in C(T)$. Because $x, y \in M(S, \succ)$, neither $x \succ y$ nor $y \succ x$. Since $y \in M(T, \succ)$, then $z \succ y$ for no $z \in T \backslash \{x, y\}$. As x and y are indistinguishable on $T \backslash \{x, y\}$, $z \not\succ x$ for all $z \in T \backslash \{x, y\}$. It follows that $x \in M(T, \succ)$. Moreover, there does not exist $z \in M(T, \succ) \backslash \{y\}$ such that $\prec (y, T) \subset \prec (z, T)$, by our supposition. Because x and y are indistinguishable on $T \backslash \{x, y\} \supseteq S \backslash \{x, y\}$, and neither $x \succ y$ nor $y \succ x$, I have that $\prec (y, T) = \prec (x, T)$. It follows from our supposition that $x \in C(T)$. Suppose that $y \notin C(T)$. I show that $x \notin C(T)$. Assume, to the contrary, that $x \in C(T)$. By an argument similar to the case above, I have that $y \in C(T)$, a contradiction.

To show that C satisfies **WWARP**, let $x, y \in X$ be two distinct alternatives, and take any $S, T \in \mathcal{S}$ such that $x, y \in S$ and $x, y \notin T$. Suppose that $\{x\} = C(S)$, and x and y are indistinguishable on T. Then $y \notin M(S, \succ)$ or $\prec (y, S) \subset \prec (x, S)$. If $y \notin M(S, \succ)$, then $y \notin M(S \cup T, \succ)$. As C is a reason-based choice correspondence it follows that $y \notin C(S \cup T)$. Otherwise, let consider $\prec (y, S) \subset \prec (x, S)$. If $\{z\} = C(\{y, z\})$ for some $z \in T \cup S$, then $y \notin M(S \cup T, \succ)$, and so $y \notin C(S \cup T)$, by our supposition. Otherwise, suppose $\{z\} \neq C(\{y, z\})$ for all $z \in T \cup S$. It follows that $y \in M(S \cup T, \succ)$. As x and y are indistinguishable on T, and $\prec (y, S) \subset \prec (x, S)$, it follows that $\prec (y, S \cup T) \subset \prec (x, S \cup T)$. Therefore, I have that $y \notin C(S \cup T)$, as desired.

To show that C satisfies **SBDC**, take any $S, T \in \mathcal{S}$, and suppose that $\{x\} = C(S)$ and $\{x\} = C(\{x, y\})$ for all $y \in T$. Because $\{x\} = C(S)$, it follows that either $\{x\} = M(S, \succ)$ or $\prec (z, S) \subset \prec (x, S)$ for all $z \in M(S, \succ) \backslash \{x\}$. As $x \succ y$ for all $y \in T \backslash \{x\}$ it follows that $x \in M(S \cup T, \succ)$. Suppose that $M(S \cup T, \succ) \neq \{x\}$. Then, the only possible case is that $z \in M(S \cup T, \succ) \backslash \{x\}$ for some $z \in M(S, \succ) \backslash \{x\}$. Because $\prec (z, S) \subset \prec (x, S)$ and $x \succ y$ for all $y \in T \backslash \{x\}$, it follows that $\prec (z, S \cup T) \subset \prec (x, S \cup T)$. Because this holds for any $z \in M(S \cup T, \succ) \backslash \{x\}$, with $z \in M(S, \succ) \backslash \{x\}$, our supposition implies that $\{x\} = C(S \cup T)$. Otherwise, let $M(S \cup T, \succ) = \{x\}$. It follows from our supposition that $\{x\} = C(S \cup T)$.

To prove that C satisfies **WWARNI**, take any $S \in \mathcal{S}$, with $x \in S$, and suppose that for every $y \in C(S)$ there exists $T \subset S$ such that $\{x\} = C(T)$ and $\{x, y\} \subset T$, and $x \in C(\{x, y\})$ for all $y \in S$. I show that $x \in C(S)$. By the way of contradiction, let $x \notin C(S)$. Thus $x \notin M(S, \succ)$ or $\prec(x, S) \subset \prec(z, S)$ for some $z \in M(S, \succ) \setminus \{x\}$. As $x \in C(\{x, y\})$ for all $y \in S$, it follows that $x \in M(S, \succ)$. Thus, it must be the case that $\prec(x, S) \subset \prec(z, S)$ for some $z \in M(S, \succ) \setminus \{x\}$. If $z \in C(S)$, it follows from our supposition that for no $T \subset S$ it can be that $\{x\} = C(T)$ and $\{x, z\} \subset T$, a contradiction. Otherwise, let $z \notin C(S)$. As S is finite and C is a reason-based choice correspondence, there exists $y \in C(S)$ such that $y \in M(S, \succ)$ and $\prec(z, S) \subset \prec(y, S)$. By the transitivity of set inclusion, $\prec(x, S) \subset \prec(y, S)$. Therefore, by our supposition, I have that for no $T \subset S$ it can be that $\{x\} = C(T)$ and $\{x, y\} \subset T$, a contradiction.

To prove that C meets **RBB**, let $x, y, z \in X$ be three distinct alternatives such that $\{x\} = C(\{x, y\})$, $x \in C(\{x, z\})$, and $y \in C(\{y, z\})$. I show that $\{x\} = C(\{x, y, z\})$. Assume, to the contrary, that $\{x\} \neq C(\{x, y, z\})$. As C is a reason-based choice rule and $y \not\succ x$ and $z \not\succ x$, I have that $x \in M(\{x, y, z\}, \succ)$. If $x \succ z$, then $z \notin M(\{x, y, z\}, \succ)$, and so that $\{x\} = M(\{x, y, z\}, \succ)$. It follows from our supposition that $\{x\} = C(\{x, y, z\})$, a contradiction. Otherwise, consider $x, z \in C(\{x, z\})$. If $\{y\} = C(\{y, z\})$, then $z \notin M(\{x, y, z\}, \succ)$. Because $\{x\} = M(\{x, y, z\}, \succ)$, it follows that $\{x\} = C(\{x, y, z\})$, a contradiction. Therefore, let $y, z \in C(\{y, z\})$. So, I have that $x, z \in M(\{x, y, z\}, \succ)$. Because $\prec(z, \{x, y, z\}) \subset \prec(x, \{x, y, z\})$, it follows from our supposition that $\{x\} = C(\{x, y, z\})$, a contradiction.

For the converse, assume that C satisfies **α2**, **Wβ**, **WWARNI**, **SBDC**, **WWARP**, and **RBB**. Given X, define the relation $\succ$ on X as follows:

$$\text{for } x, y \in X, \text{ with } x \neq y\text{: } x \succ y \Leftrightarrow C(\{x, y\}) = \{x\}.$$

I have to prove that, for all $S \in \mathcal{S}$,

$$C(S) = \{x \in M(S, \succ) \mid \prec (x, S) \subset \prec (y, S) \text{ for no } y \in M(S, \succ) \setminus \{x\}\}$$

holds true and that $\succ$ is acyclic.

To show acyclicity of $\succ$, suppose $x^1, x^2, ..., x^t \in X$ are such that $x^{\tau-1} \succ x^\tau$ for $\tau \in \{2, ..., t\}$, that is, $C(\{x^{\tau-1}, x^\tau\}) = \{x^{\tau-1}\}$ for $\tau \in \{2, ..., t\}$. Let $S = \{x^1, x^2, ..., x^t\} \subseteq \mathcal{S}$. Suppose that $x^\tau \in C(S)$ for $\tau \in \{2, ..., t\}$. As $\boldsymbol{\alpha 2}$ holds, I have $x^\tau \in C(\{x^{\tau-1}, x^\tau\})$, and so $x^{\tau-1} \not\succ x^\tau$, a contradiction. Then $x^\tau \notin C(S)$ for $\tau \in \{2, ..., t\}$. It follows from the nonemptiness of C that $\{x^1\} = C(S)$. Because $x^t \in S \setminus \{x^1\}$ and $\boldsymbol{\alpha 2}$ holds, I have $x^1 \in C(\{x^1, x^t\})$. This implies $x^t \not\succ x^1$, as desired.

Take any $S \in \mathcal{S}$, and let $x \in C(S)$. I show that $x \in M(S, \succ)$ and $\prec (x, S) \subset \prec (y, S)$ for no $y \in M(S, \succ) \setminus \{x\}$. Assume, to the contrary, that $x \notin M(S, \succ)$ or there exists $y \in M(S, \succ) \setminus \{x\}$ such that $\prec (x, S) \subset \prec (y, S)$. As $x \in C(\{x, y\})$ for all $y \in S$, by $\boldsymbol{\alpha 2}$, the case $x \notin M(S, \succ)$ is not possible. Thus, let $x \in M(S, \succ)$ and $\prec (x, S) \subset \prec (y, S)$ for some $y \in M(S, \succ) \setminus \{x\}$. Take any $z \in \prec (y, S) \setminus \prec (x, S)$. Because $\{y\} = C(\{y, z\})$, $y \in C(\{x, y\})$, and $z \in C(\{x, z\})$, **RBB** implies $\{y\} = C(\{x, y, z\})$. It follows from **SBDC** that $\{y\} = C(\prec (y, S) \cup \{x, y\})$. If $S \setminus (\prec (y, S) \cup \{x, y\})$ is empty, then $x \notin C(S)$, a contradiction. Otherwise, let $S \setminus \{\prec (y, S) \cup \{x, y\}\}$ be a nonempty set. Because x and y are indistinguishable on $S \setminus (\prec (y, S) \cup \{x, y\})$ and $\{y\} = C(\prec (y, S) \cup \{x, y\})$, **WWARP** implies $x \notin C(S)$, a contradiction.

Assume that $x \in M(S, \succ)$ and $\prec (x, S) \subset \prec (y, S)$ for no $y \in M(S, \succ) \setminus \{x\}$. I show that $x \in C(S)$. Because $x \in M(S, \succ)$, it follows that $x \in C(\{x, y\})$ for all $y \in S$.

If $\{x\} = M(S,\succ)$, it is clear, by **α2** and the nonemptiness of C, that $\{x\} = C(S)$. Otherwise, consider $\{x\} \neq M(S,\succ)$. By the nonemptiness of C, $\{x\} = C(S)$ whenever $y \notin C(S)$ for all $y \in S \backslash \{x\}$. Thus, let $y \in C(S)$ for some $y \in S \backslash \{x\}$. It follows from the paragraph above that $y \in M(S,\succ)$ and $\prec(y,S) \subset \prec(z,S)$ for no $z \in M(S,\succ) \backslash \{y\}$. Therefore, $\{x,y\} = C(\{x,y\})$. If $\prec(y,S)$ is empty, then $\prec(x,S)$ must be empty, and so **Wβ** implies $x \in C(S)$. Thus, let $\prec(y,S)$ be a nonempty set. It follows that $\prec(x,S)$ is nonempty as well. If $\prec(x,S) = \prec(y,S)$ for some $y \in C(S)$, **Wβ** implies $x \in C(S)$. So, let $\prec(x,S) \neq \prec(y,S)$ for all $y \in C(S)$. Thus, for any $y \in C(S)$, there exists $z \in \prec(x,S) \setminus \prec(y,S)$ and $w \in \prec(y,S) \setminus \prec(x,S)$. Therefore, for all $y \in C(S)$, $\{x,y,z\} \subset S$ for some $z \in \prec(x,S) \setminus \prec(y,S)$. Since $\{x\} = C(\{x,z\})$, $z \in C(\{y,z\})$, and $x \in C(\{x,y\})$ it follows from **RBB** that $\{x\} = C(\{x,y,z\})$. Because this holds for any $y \in C(S)$, **WWARNI** implies $x \in C(S)$. ■

The mutual independence of choice-consistency conditions used in Theorem 3 has been relegated to the Appendix B.

Observe that a reason-based choice correspondence does not meet Sen's (1977) Property $\gamma 2$, according to which for any feasible set, say S, if $x \in C(\{x,y\})$ for all $y \in S$, then $x \in C(S)$. To see it, suppose that an individual has the following preferences among three distinct alternatives: $x \succ x'$, $x \not\succ y$ and $y \not\succ x$, and $x' \not\succ y$ and $y \not\succ x'$. If our individual follows the reason-based procedural choice, then her choices are $y \in C(\{x,y\})$, $y \in C(\{x',y\})$, and $\{x\} = C(\{x,x',y\})$ which contradicts Property $\gamma 2$. It follows that Property $\alpha 2$ is not equivalent to Property α in our framework (that is, Sen' s (1977, p.65) Propositon 10 is not fulfilled). Needless to say, Sen' s (1971) Property γ is not necessary for a reason-based choice correspondence. It also follows that our reason-based choice correspondences do not necessarily meet the standard 'binariness' property.

4.4 Concluding Remarks

Motivated by the vast literature on the attraction effect, I provide a characterization of reason-based choice correspondences which captures the basic idea behind this choice 'anomaly'.

Our characterization result is obtained by using the standard revealed preference methodology. Thus I suppose that our individual possesses a (not necessarily complete) preference relation which is revealed by her choices. From the normative point of view our choice-consistency conditions are appealing because they never lead an individual to make 'bad' choices (i.e., dominated alternatives) even though most of them are weaker than the conventional choice-consistency conditions (i.e., Property α, Property β, Samuelson's **WARP**). Nonetheless, a reason-based choice correspondence lends itself to certain framing manipulations that are hard to explain only from the point of view of preference maximization. However, it differs from other rules which allow an individual to reveal a fixed cyclic preference relation (see, Ehlers and Sprumont (2007), Lombardi (2007)).

In this work I have attempted to analyze how offered sets may induce an individual to follow a particular guidance in her decisions, and how this affects her choices across sets. Needless to say, reasons that guide decisions are likely to be diverse. In this respect, for example, Baigent and Gaertner (1996) characterize a choice procedure in which the individual's choices are guided by a self-imposed constraint of "choosing a non unique largest or otherwise a second largest alternative" from each offered set.

Our two-stage choice model can be contrasted with other decision-making procedures recently suggested in the literature. Closer to our reason-based choice rules are the two-stage procedural choice models suggested by Houy (2006), Manzini and Mariotti (2006, 2007), Rubinstein and Salant (2006). The first author proposes a choice model in which an individual eliminates maximal alternatives identified in the first-stage according to her conservative mood. In contrast to the history dependent choice model of Houy, in our model the choices of an individual are driven by the set under consideration whenever her preferences are insufficient to solve the decision problem that she deals with. Manzini and Mariotti (2006, 2007) and Rubinstein and Salant characterize slightly different two-stage choice models which have the following property in common: the individual arrives at a choice by eliminating some of the shortlisted alternatives identified in the first stage according to a fixed asymmetric (and not transitive) preference relation.[6] In contrast to their two-stage choice procedures, in our model an individual constructs in the second-stage a (not necessarily complete) binary relation (i.e., the strict set inclusion) according to her known preferences on the set of alternatives under consideration to knock off some of the alternatives that survive the first round of elimination.

To conclude I observe that Masatlioglu and Ok (2006) axiomatize a reference-dependent procedural choice model in which an individual, endowed with an objective utility function, solves sequentially a two-stage constrained utility maximization problem where the constraints depend on her status quo alternative - if she cannot find an alternative yielding her a higher utility level than that brought to her by the status quo alternative, she keeps the latter. Our model differs from the choice model of Masatlioglu and Ok in many respects. Mainly I have investigated how in the second-stage a convincing reason (i.e., the information inferable from the entire offered set) plays a role in decision making rather than investigating how a reference alternative affects the

[6] Indeed, Manzini and Mariotti (2007) suggest a choice model in which an individual may use several asymmetric preference relations to arrive at a choice.

decisions that an individual makes.

CHAPTER 5

WHAT KIND OF PREFERENCE MAXIMIZATION DOES THE WEAK AXIOM OF REVEALED NON-INFERIORITY CHARACTERIZE?

5.1 Introduction

Eliaz and Ok (2006) accommodate preference incompleteness in revealed preference theory by studying the implications of weakening the fundamental choice-consistency condition of the weak axiom of revealed preference (**WARP**) in the weak axiom of revealed non-inferiority (**WARNI**).[1] This behavioural postulate entirely corresponds to maximizing behaviour on suitable domains. However, a choice function rationalized by the maximization of a preference relation (not necessarily complete) may fail to satisfy **WARNI** on an arbitrary choice domain. This is due to the fact that **WARNI** characterizes a particular type of rationality. Our concern is to spell out the form of maximality of choice characterized by this behavioural postulate on an arbitrary choice domain, and then I contrast this form of maximality with that characterized by **WARP**.

A choice function is weak justified if there exists a binary relation J (dubbed weak justification) such that, for all feasible sets, no available alternative is J-related to any chosen alternative, for each rejected alternative there is *some* chosen alternative which is J-related to it. Therefore, the binary relation J is a strict (not necessarily complete) preference relation.

A decision maker makes weakly justified choices if she can assert that no chosen alternative is dominated by any other obtainable one, and for each discarded alternative

[1] On a finite universal set WARNI is identical to one of the behavioral properties suggested by Bandyopadhay and Sengupta (1993).

there is some chosen alternative which dominates it.

My rationality hypothesis differs from that provided by Mariotti (2007), according to which choices are justified if there exists a binary relation J such that, for all feasible sets, no two chosen alternatives are J-related to each other, and each chosen alternative is J-related to *all* of the rejected alternatives. Mariotti (2007) shows that choices satisfy **WARP** if and only if they are justified by an asymmetric relation.

The result of this chapter is that choices satisfy **WARNI** if and only if they are weakly justified by an asymmetric relation.

5.2 Analysis

Let X be a nonempty set of alternatives. Let $\mathcal{S}$ be a collection of nonempty subsets of X. By a *choice function* C on $\mathcal{S}$ I mean a map C which assigns a nonempty subset $C(S)$ of S to every $S \in \mathcal{S}$.

A binary relation $J \subseteq X \times X$ is said to be asymmetric if, for all $x, y \in X$, $(x, y) \in J$ implies $(y, x) \notin J$.

If there exists a binary relation J on X such that, for all $S \in \mathcal{S}$:

$$1) \qquad \forall x \in C(S), \forall y \in S : (y, x) \notin J$$

$$2) \quad \forall y \in S \backslash C(S) : (x, y) \in J \text{ for some } x \in C(S)$$

then I say that J is a weak justification for C. If C has a weak justification, I say that C is weak justified. I will call J an asymmetric weak justification if J is asymmetric. Our notion is weaker than that provided by Mariotti (2007), according to which choices are justified if, for all $S \in \mathcal{S}$, for all $x, y \in C(S)$ it holds that $(x, y) \notin J$, and for all $x \in C(S)$ and for all $y \in S \backslash C(S)$ it holds that $(x, y) \in J$.

Eliaz and Ok (2006) suggested to read the statement "$x \in C(S)$" as "x is revealed not to be inferior to any other obtainable alternative in S" rather than to follow the classic interpretation of "x is revealed to be at least as good as all other available alternatives in S". Under this interpretation of revealed preferences, they propose the weak axiom of revealed non-inferiority (**WARNI**). The idea behind this behavioural regularity is quite mild. It asserts that if an obtainable alternative from a set S is revealed not to be inferior to all of other chosen alternatives from S, then it must be chosen from S as well.

WARNI: $\forall S \in \mathcal{S}, y \in S : [\forall x \in C(S) \exists T \in \mathcal{S} : y \in C(T) \text{ and } x \in T] \Rightarrow y \in C(S)$.

This behavioral postulate is weaker than **WARP** which asserts that if $x \in C(S)$ and there exists a feasible set T such that $y \in C(T)$ and $x \in T$, then $y \in C(S)$. Furthermore, **WARNI** implies the canonical Property α (also known as Chernoff choice-consistency condition or basic contraction consistency), according to which an alternative that is deemed choosable from a feasible set T and belongs to a subset S of T must be deemed choosable from S (i.e., $x \in T \subseteq S$ and $x \in C(S) \Rightarrow x \in C(T)$).[2]

Theorem 4. *There exists a choice that is not weak justified.*

Proof. Let X be the set consisting of three distinct alternatives: x, y, and z. Let $\mathcal{S} = \{\{x,y\}, X\}$, and suppose that $C(\{x,y\}) = \{x\}$ and $C(X) = \{x,y\}$. It is easy to see that C cannot be weak justified. For suppose that C is weak justified. Then, since $C(X) = \{x,y\}$, I must have $(x,y) \notin J$. Since $C(\{x,y\}) = \{x\}$, I must have $(x,y) \in J$, contradicting the definition of weak justification of C. ■

Theorem 5. *A choice function C on $\mathcal{S}$ is asymmetric weak justified if and only if it satisfies WARNI.*

[2] See Eliaz and Ok (2006, lemma 1, p. 81).

Proof. Assume that C is asymmetric weak justified. I show that C satisfies **WARNI**. Suppose that for all $S \in \mathcal{S}$, with $y \in S$, it holds that for every $x \in C(S)$ there exists $T \in \mathcal{S}$ such that $y \in C(T)$ and $x \in T$. As C is asymmetric weak justified it follows that for all $x \in C(S)$ it holds that $(x,y) \notin J$. By way of contradiction, let $y \notin C(S)$. Because C is asymmetric weak justified it follows that there exists $x \in C(S)$ such that $(x,y) \in J$ yielding a contradiction.

For the converse, let C satisfy **WARNI**. I show that C is asymmetric weak justified. Define for all distinct $x, y \in X$:

$$(x,y) \in J \Leftrightarrow \exists S \in \mathcal{S} : x \in C(S),\, y \in S \backslash C(S),\text{ and } \nexists T \in \mathcal{S},\ x \in T : y \in C(T).$$

Then J is asymmetric. To show that C satisfies property 1), let $x \in C(S)$ and $y \in S$ for some $S \in \mathcal{S}$. By way of contradiction, let $(y,x) \in J$. Then there exists $S' \in \mathcal{S}$ such that $y \in C(S')$, $x \in S' \backslash C(S')$, and for no $T \in \mathcal{S}$, with $y \in T$, it holds that $x \in C(T)$, which contradicts that $x \in C(S)$ and $y \in S$. Finally, I show that C meets property 2). Suppose that $y \in S \backslash C(S)$ for some $S \in \mathcal{S}$. **WARNI** implies that there exists $x \in C(S)$ such that for all $T \in \mathcal{S}$ it holds true $y \notin C(T)$ if $x \in T$. It follows that $(x,y) \in J$. ■

Theorem 2 clarifies how much rationality in terms of preference maximization I give up in passing from **WARP** to **WARNI**. Both properties require no chosen alternative is dominated by any other available alternative. However, while **WARP** demands that each chosen alternative has to dominate *all* of the discarded alternatives, **WARNI** requires that for each rejected alternative there exists *some* chosen alternative which dominates it. Obviously there are choices which are weak justified but not justified, as argued next.

Theorem 6. *There exists a choice that is weak justified but not justified.*

Proof. Let X be the set consisting of three distinct alternatives: x, y, and z. Suppose that $\mathcal{S} = \{\{x,y\}, \{z,y\}, X\}$. Define the choice C on $\mathcal{S}$ by $C(X) = \{x,z\}$, $C(\{x,y\}) = \{x\}$, and $C(\{z,y\}) = \{z,y\}$. It is easy to see that C is weak justified, but not justified. For suppose that C is justified. Then, since $C(X) = \{x,z\}$, I must have $(x,y), (z,y) \in J$. But $C(\{z,y\}) = \{z,y\}$ implies that $(z,y) \notin J$ yielding a contradiction.

■

REFERENCES

Aleskerov, F., and B. Monjardet (2002): "Utility, Maximization, Choice and Preference", Springer, Berlin.

Arrow, K. (1959): "Rational Choice Functions and Orderings", *Economica*, 26(102): 121-127.

Baigent, N., and W. Gaertner (1996): "Never Choose the Uniquely Largest: A characterization", *Economic Theory*, 8(2): 239-249.

Banks, J.S., J. Duggan, and M. Le Breton (2006): "Social Choice and Electoral Competition in the General Spatial Model", *Journal of Economic Theory*, 126(1): 194-234.

Bar-Hillel, M., and A. Margalit (1988): "How vicious are cycles of intransitive choice?", *Theory and Decision*, 24(2): 119-145.

Bandyopadhyay, T., and K. Sengupta (1993): "Characterization of Generalized Weak Orders and Revealed Preference", *Economic Theory*, 3(3): 571-576.

Camerer, C. (1994): "Individual decision making" in Kagel, J., and A. Roth (Eds), *Handbook of Experimental Economics*, Princeton University Press, Princeton, 587-704.

Denicolò, V., and M. Mariotti (2000): "Nash Bargaining Theory, Nonconvex Problems and Social Welfare Ordering", Theory and Decision, 48(4): 351-358.

Dutta, B. (1988): "Covering Sets and a New Condorcet Choice Correspondence", *Journal of Economic Theory*, 44(1): 63-80.

Ehlers, L., and Y. Sprumont (2007): "Weakened WARP and Top-Cycle Choice Rules", forthcoming in *Journal of Mathematical Economics.*

Eliaz, K., and E.A. Ok (2006): "Indifference or indecisiveness? Choice-theoretic foundations of incomplete preferences", *Games and Economic Behavior*, 56(1): 61-86.

Fishburn, P.C. (1991): "Nontransitive Preferences in Decision Theory", *Journal of Risk and Uncertainty*, 4(2): 113-134.

Fishburn, P.C., and I.H. LaValle (1988): "Context-Dependent Choice with Nonlinear and Nontransitive Preferences", *Econometrica*, 56(5): 1221-1239.

Houy, N. (2006): "Choice Correspondence with States of Mind", mimeo.

Kaneko, M. (1980): "An extension of the Nash bargaining problem and the Nash social welfare function", *Theory and Decision*, 12: 135-148.

Kim, T., and M.K. Richter (1986): "Nontransitive-nontotal consumer theory", *Journal of Economic Theory*, vol. 38(2): 324-363.

Laslier, J.-F. (1997). *Tournament Solutions and Majority Voting*, Volume 7 in the series "Studies in Economic Theory", Springer Verlag, Heidelberg, New York.

Lombardi, M. (2007): "Uncovered Set Choice Rules", forthcoming in *Social Choice and Welfare.*

Manzini, P., and M. Mariotti (2006): "Two-Stage Boundedly Rational Choice Procedures: Theory and Experimental Evidence", Queen Mary Working Paper n. 561.

Manzini, P., and M. Mariotti (2007): "Sequentially Rationalizable Choice", forthcoming in *American Economic Review.*

Mariotti, M. (1998): "Nash bargaining theory when the number of alternatives can be finite", *Social Choice and Welfare,* 15(1): 413-421.

Mariotti, M.(1999): "Fair Bargains: Distributive Justice and Nash Bargaining Theory", *Review of Economic Studies,* 66(3): 733-41.

Mariotti, M. (2007): "What kind of preference maximization does the weak axiom of revealed preference characterize?", forthcoming in *Economic Theory.*

Masatlioglu, Y., and E.A. Ok (2006): "Reference-Dependent Procedural Decision Making", Department of Economics, New York University.

May, O.K. (1954): "Intransitivity, Utility, and the Aggregation of Preference Patterns", *Econometrica,* 22(1): 1-13.

Miller, N.R. (1977): "Graph-Theoretical Approaches to the Theory of Voting", *American Journal of Political Science,* 21(4): 769-803.

Miller, N.R. (1980): "A New Solution Set for Tournaments and Majority: Further Graph-Theoretical Approaches to the Theory of Voting", *American Journal of Political Science,* 24(1): 68-96.

Moulin, H. (1985): "Choice Functions over a Finite Set: A Summary," *Social Choice and Welfare,* 2(2): 147-160.

Moulin, H. (1986): "Choosing from a Tournament", *Social Choice and Welfare,* 3(4): 271-291.

Ok, E.A. (2004): "Independence of Irrelevant Alternatives and Individual Choice", mimeo.

Peters, H., and D. Vermeulen (2006): "WPO, COV and IIA bargaining solutions", mimeo.

Peters, H., and P. Wakker (1991): "Independence of Irrelevant Alternatives and revealed group preferences", *Econometrica,* 59(6): 1787-1801.

Richter, M.K. (1966): "Revealed Preference Theory", *Econometrica,* 34(3): 635-645.

Rieskamp, J., J.R. Busemeyer, and B.A. Mellers (2006): "Extending the Bounds of Rationality: Evidence and Theories of Preferential Choice", *Journal of Economic Literature*, 44(3): 631-661.

Rubinstein, A., and Y. Salant (2006): "Two Comments on the Principle of Revealed Preference", mimeo, availaible online at: http://arielrubinstein.tau.ac.il/papers/frames.pdf.

Rubinstein, A., and Y. Salant (2007): "Some thoughts on the principle of revealed preferences", forthcoming in *Foundations of Positive and Normative Economics* (Andrew Caplin and Andrew Schotter, eds).

Russo, J.E., and B.A. Dosher (1983): "Strategies for Multiattribute Binary Choice", *Journal of Experimental Psychology: Learning, Memory, and Cognition*, 9(4): 676-696.

Samuelson, P. (1938): "A Note on the Pure Theory of Consumer's Behaviour", *Economica*, 5(17): 61-71.

Sen, A.K. (1970): *Collective Choice and Social Welfare*, Edinburg: Oliver and Boyd.

Sen, A.K. (1971): "Choice Functions and Revealed Preference", *Review of Economic Studies*, 38(3): 307-317.

Sen, A.K. (1977): "Social Choice Theory: A Re-Examination", *Econometrica*, 45(1): 53-88.

Sen, A.K. (1993): "Internal Consistency of Choice", *Econometrica*, 61(3): 495-521.

Sen, A.K. (1997): "Maximization and the Act of Choice", *Econometrica*, 65(4): 745-779.

Simonson, I. (1989): "Choice Based on Reasons: The Case of Attraction and Compromise Effects", *Journal of Consumer Research*, 16(2): 158-174.

Shafir, E., I. Simonson, and A. Tversky (1993): "Reason-based Choice", *Cognition*, 49(1-2): 11-36.

Spiegler, R. (2002): "Equilibrium in Justifiable Strategies: A Model of Reason-Based Choice in Extensive-Form Games", *Review of Economic Studies*, 69(3): 691-706.

Suzumura, K. (1983): *Rational Choice, Collective Decisions, and Social Welfare*, Cambridge University Press, Cambridge U.K..

Tversky, A., and I. Simonson (1993): "Context-dependent Preferences", *Management Science*, 13(10): 1179-1189.

Varian, H.R (2005): "Revealed Preference", http://www.ischool.berkeley.edu/~hal/Papers/2005/revp

Zhang, J., C.K. Hsee, and Z. Xiao (2006): "The majority rule in individual decision making", *Organizational Behavior and Human Decision Processes*, 99(1): 102-111.

Zhou, L. (1997): "The Nash bargaining theory with non-convex problems", *Econometrica*, 65(3): 681-685.

A. Independence of Axioms used in Theorem 2

Independence of axiom 4.

Since it is easy to check that the bargaining solution is resolute, and satisfies axioms 2 and 5, next we show that axiom 3 is satisfied as well.

Axiom 3. $A, B \in \mathcal{B}$, $a = CW(A)$ & $b \in CL(B \cup a) \Rightarrow b \notin f(A \cup B)$

Let $A, B \in \mathcal{B}$, and $a = CW(A)$ and $b \in CL(B \cup a)$, with $a \neq b$. Obviously axiom 2 is satisfied whenever $y \notin A \cup B$ or $z \notin A \cup B$. Let us consider the case that $y, z \in A \cup B$.

It may be useful to distinguish the following subcases: *i)* $a = z$ & $b = y$, *ii)* $a = y$ & $b = z$, *iii)* $a = x \neq z$ & $b = y$, *iv)* $a = x \neq y$ & $b = z$, *v)* $a = y$ & $b = x \neq z$, *vi)* $a = z$ & $b = x \neq y$, *vii)* $a, b \notin \{y, z\}$.

Next, we check all the subcases.

Subcase i). $a = z$ & $b = y$

By definition $z_1 + z_2 = y_1 + y_2$. If there exists $s \in A \cup B \backslash yz$ such that $s_1 + s_2 > z_1 + z_2$, we have that $y \notin \arg\max_{s \in A \cup B}(s_1 + s_2)$, and so $y \notin f(A \cup B)$. Otherwise, let $z_1 + z_2 \not< s_1 + s_2$ for any $s \in A \cup B \backslash yz$. Then $y \notin \{\arg\max_{s \in A \cup B}(s_1 + s_2) - \{y\}\}$, but $z \in \{\arg\max_{s \in A \cup B}(s_1 + s_2) - \{y\}\}$. It follows that $y \notin f(A \cup B)$.

Subcase ii). $a = y$ & $b = z$

Since this case violates the premise of axiom 2, it does not apply.

Subcase iii). $a = x \neq z$ & $b = y$

Then $x_1 + x_2 > y_1 + y_2$ or $x_1 + x_2 = y_1 + y_2$ and $x_1 > y_1$. If there exists $s \in A \cup B \backslash xy$ such that $s_1 + s_2 > x_1 + x_2$, we have that $y \notin f(A \cup B)$. If $x_1 + x_2 \not< s_1 + s_2$ for any

$s \in A \cup B \backslash xy$, then we distinguish whether $x_1 + x_2 > y_1 + y_2$ or $x_1 + x_2 = y_1 + y_2$. In either case we have that $y \notin f(A \cup B)$.

Subcase iv). $a = x \neq y$ & $b = z$

Then $x_1+x_2 > z_1+z_2$ or $x_1+x_2 = z_1+z_2$ and $x_1 > z_1$. If there exists $s \in A \cup B \backslash xz$ such that $s_1 + s_2 > x_1 + x_2$, we have that $z \notin f(A \cup B)$. If $x_1 + x_2 \not< s_1 + s_2$ for any $s \in A \cup B \backslash xz$, then we distinguish whether $x_1 + x_2 > z_1 + z_2$ or $x_1 + x_2 = z_1 + z_2$. Recall that $y_1 + y_2 = z_1 + z_2$, by definition. In either case we have that $z \notin f(A \cup B)$.

Subcase v). $a = y$ & $b = x \neq z$

Then $y_1+y_2 > x_1+x_2$ or $y_1+y_2 = x_1+x_2$ and $y_1 > x_1$. If there exists $s \in A \cup B \backslash xy$ such that $s_1 + s_2 > y_1 + y_2$, we have that $x \notin f(A \cup B)$. Otherwise, consider $y_1 + y_2 \not< s_1 + s_2$ for any $s \in A \cup B \backslash xy$. Recall that $y_1 + y_2 = z_1 + z_2$ and $f(M(y,z)) = z$, by definition. It follows that $z \notin A$, and so $f(M(x,z)) = z \in B \backslash x$. We distinguish whether $y_1 + y_2 > x_1 + x_2$ or $y_1 + y_2 = x_1 + x_2$. Consider the case $y_1 + y_2 > x_1 + x_2$. Then $x \notin f(A \cup B)$ and $f(A \cup B)$ is nonempty as $z \in \{\arg\max_{s \in A \cup B}(s_1 + s_2) - \{y\}\}$. Consider the case $y_1 + y_2 = x_1 + x_2$. Then $x, z \in \{\arg\max_{s \in A \cup B}(s_1 + s_2) - \{y\}\}$, and so $x \notin f(A \cup B)$ given that $f(M(x,z)) = z$.

Subcase vi). $a = z$ & $b = x \neq y$

Then $z_1 + z_2 > x_1 + x_2$ or $z_1 + z_2 = x_1 + x_2$ and $z_1 > x_1$. If there exists $s \in A \cup B \backslash xz$ such that $s_1 + s_2 > z_1 + z_2$, we have that $x \notin f(A \cup B)$. Otherwise, consider $z_1 + z_2 \not< s_1 + s_2$ for any $s \in A \cup B \backslash xz$. Recall that $y_1 + y_2 = z_1 + z_2$, by definition. We distinguish whether $z_1 + z_2 > x_1 + x_2$ or $z_1 + z_2 = x_1 + x_2$. Obviously $x \notin f(A \cup B)$ if $z_1 + z_2 > x_1 + x_2$. Otherwise, consider the case that $z_1 + z_2 = x_1 + x_2$. Therefore, $x, z \in \{\arg\max_{s \in A \cup B}(s_1 + s_2) - \{y\}\}$. It follows from $z_1 > x_1$ that $x \notin f(A \cup B)$.

Subcase vii). a,$b \notin \{y, z\}$

It is easy to check as $f(M(a,b)) = a$ if $a_1 + a_2 > b_1 + b_2$ or $a_1 + a_2 = b_1 + b_2$ & $a_1 > b_1$, and $f(A) = \arg\max_{s_1} \{\arg\max_{s \in A} (s_1 + s_2) - \{y\}\}$.

Independence of axiom 5.

Since it can easily be checked that the bargaining solution f is resolute and satisfies axiom 1, we check here that it satisfies axioms 3-4.

Axiom 3. $A, B \in \mathcal{B}$, $a = CW(A)$ & $b \in CL(B \cup a) \Rightarrow b \notin f(A \cup B)$

To see that f satisfies axiom 2, we distinguish the following cases:

Case 1. $|(A \cup B) \cap \{x,y,z\}| = 0$

Case 2. $|(A \cup B) \cap \{x,y,z\}| = 1$

Case 3. $|(A \cup B) \cap \{x,y,z\}| = 2$

Case 4. $(A \cup B) \cap \{x,y,z\} = \{x,y,z\}$

Case 1 is obvious and so omitted.

Case 2. $|(A \cup B) \cap \{x,y,z\}| = 1$

Wlog let $(A \cup B) \cap \{x,y,z\} = \{x\}$. We distinguish whether *i)* $a = x$, *ii)* $b = x$, or *iii)* $a, b \neq x$.

Subcase i). $a = x$

Then $x_1 + x_2 \geq b_1 + b_2$. It follows from $b \in S_x$ that $b \notin f(A \cup B)$, as desired. Observe that $f(A \cup B) \neq \emptyset$.

Subcase ii). $b = x$

Then $a_1 + a_2 > x_1 + x_2$. It follows that $x \notin f(A \cup B)$. Observe that $f(A \cup B) \neq \emptyset$.

Subcase iii). $a, b \neq x$

Then $a_1 + a_2 > b_1 + b_2$ or $a_1 + a_2 = b_1 + b_2$ & $a_1 > b_1$. Since $x \in A \cup B \backslash ab$, we proceed according to:

1. $f(M(a,x)) = a$ & $f(M(b,x)) = b$

2. $f(M(a,x)) = a$ & $f(M(b,x)) = x$

3. $f(M(a,x)) = x$ & $f(M(b,x)) = x$

4. $f(M(a,x)) = x$ & $f(M(b,x)) = b$

Subcase iii.1. $f(M(a,x)) = a$ & $f(M(b,x)) = b$

Then $a, b \notin S_x$. If $a_1 + a_2 > b_1 + b_2$, then $b \notin f(A \cup B)$, as desired. Otherwise, let $a_1 + a_2 = b_1 + b_2$ & $a_1 > b_1$. Since $a_1 > b_1$ we have that $b \notin f(A \cup B)$. Observe that $f(A \cup B) \neq \emptyset$.

Subcase iii.2. $f(M(a,x)) = a$ & $f(M(b,x)) = x$

Then $a_1 + a_2 > x_1 + x_2 \geq b_1 + b_2$. Thus $b \in S_x$, and so $b \notin f(A \cup B)$. Observe that $f(A \cup B) \neq \emptyset$.

Subcase iii.3. $f(M(a,x)) = x$ & $f(M(b,x)) = x$

Then $x_1 + x_2 \geq a_1 + a_2 \geq b_1 + b_2$, and so $a, b \in S_x$. Clearly, $b \notin f(A \cup B)$. If there exists $s \in A \cup B \backslash abx$ such that $s_1 + s_2 > x_1 + x_2$, we have that $f(A \cup B) \neq \emptyset$. Otherwise, consider $x_1 + x_2 \not< s_1 + s_2$ for any $s \in A \cup B \backslash x$. It follows that $x = f(A \cup B)$.

Subcase iii.4. $f(M(a,x)) = x$ & $f(M(b,x)) = b$

This case is not possible as it contradicts $f(M(a,b)) = a$.

Case 3. $|(A \cup B) \cap \{x, y, z\}| = 2$

Wlog let $(A \cup B) \cap \{x, y, z\} = \{x, y\}$. It may be useful to distinguish the following subcases: *i)* $a = x$ & $b = y$, *ii)* $a = y$ & $b = x$, *iii)* $a \neq x$ & $b = y$, *iv).* $a \neq y$ & $b = x$, *v)* $a = y$ & $b \neq x$, *vi)* $a = x$ & $b \neq y$, *vii)* $a, b \notin \{y, x\}$.

Observe that in all cases $y \in S_x$.

Subcase i). $a = x$ & $b = y$

By construction $x_1 + x_2 = y_1 + y_2$. It follows that $y \notin f(A \cup B)$. Observe that

$f(A \cup B) \neq \emptyset$.

Subcase ii). $a = y$ & $b = x$

Since this case violates the premise of axiom 2, it does not apply.

Subcase iii). $a \neq x$ & $b = y$

Then $a_1 + a_2 > x_1 + x_2 \geq y_1 + y_2$. We have that $y \notin f(A \cup B)$. Observe that $f(A \cup B) \neq \emptyset$.

Subcase iv). $a \neq y$ & $b = x$

Then $a_1 + a_2 > x_1 + x_2$. It follows that $x \notin f(A \cup B)$. Observe that $f(A \cup B) \neq \emptyset$.

Subcase v). $a = y$ & $b \neq x$

Then $x_1 + x_2 = y_1 + y_2 \geq b_1 + b_2$, so $b \in S_x$. It follows that $b \notin f(A \cup B)$. Observe that $f(A \cup B) \neq \emptyset$.

Subcase vi). $a = x$ & $b \neq y$

The same reasoning of previous subcase applies.

Subcase vii). $a,b \notin \{x, y\}$

Then $a_1 + a_2 > b_1 + b_2$ or $a_1 + a_2 = b_1 + b_2$ & $a_1 > b_1$. Since $x \in A \cup B \backslash ab$, we proceed according to:

1. $f(M(a,x)) = a$ & $f(M(b,x)) = b$
2. $f(M(a,x)) = a$ & $f(M(b,x)) = x$
3. $f(M(a,x)) = x$ & $f(M(b,x)) = x$
4. $f(M(a,x)) = x$ & $f(M(b,x)) = b$

Subcase vii.1. $f(M(a,x)) = a$ & $f(M(b,x)) = b$

Then $a, b \notin S_x$. If $a_1 + a_2 > b_1 + b_2$, then $b \notin f(A \cup B)$, as desired. Otherwise, let $a_1 + a_2 = b_1 + b_2$ & $a_1 > b_1$. Since $a_1 > b_1$ we have that $b \notin f(A \cup B)$. Observe that $f(A \cup B) \neq \emptyset$.

Subcase vii.2. $f(M(a,x)) = a$ & $f(M(b,x)) = x$

Then $a_1 + a_2 > x_1 + x_2 \geq b_1 + b_2$. Thus $b \in S_x$, and so $b \notin f(A \cup B)$. Observe that $f(A \cup B) \neq \emptyset$.

Subcase vii.3. $f(M(a,x)) = x$ & $f(M(b,x)) = x$

Then $x_1 + x_2 \geq a_1 + a_2 \geq b_1 + b_2$, and so $a, b \in S_x$. Clearly, $b \notin f(A \cup B)$. If there exists $s \in A \cup B \backslash abx$ such that $s_1 + s_2 > x_1 + x_2$, we have that $f(A \cup B) \neq \emptyset$. Otherwise, consider $x_1 + x_2 \nless s_1 + s_2$ for any $s \in A \cup B \backslash x$. It follows that $x = f(A \cup B)$.

Subcase vii.4. $f(M(a,x)) = x$ & $f(M(b,x)) = b$

This case is not possible as it contradicts $f(M(a,b)) = a$

Case 4. $(A \cup B) \cap \{x,y,z\} = \{x,y,z\}$

We proceed according to: *i)* $a, b \in \{x,y,z\}$, *ii)* $a \in \{x,y,z\}$ & $b \notin \{x,y,z\}$, *iii)* $a \notin \{x,y,z\}$ & $b \in \{x,y,z\}$, *iv).* $a, b \notin \{x,y,z\}$.

Recall that $M(x,y) \cup M(y,z) = C$.

Subcase i). $a, b \in \{x,y,z\}$

By construction, $f(M(x,y)) = x$, $f(M(y,z)) = y$, and $f(M(x,z)) = z$. Let $a = x$ and $b = y$. Then $z \notin A$, and so it must be the case that $z \in B$. Thus $y \notin CL(B \cup x)$. It follows that for this case the axiom does not apply. The same reasoning applies for the remaining cases.

Subcase ii). $a \in \{x,y,z\}$ & $b \notin \{x,y,z\}$

Then $x_1 + x_2 = y_1 + y_2 = z_1 + z_2 \geq b_1 + b_2$. Hence, $b \in S_y$. If $C = A \cup B$, then $b \notin f(C)$. Otherwise, let $C \subset A \cup B$. It follows from $b \in S_y$ that $b \notin f(A \cup B)$. Observe that in either case $f(A \cup B) \neq \emptyset$.

Subcase iii). $a \notin \{x,y,z\}$ & $b \in \{x,y,z\}$

Then $a_1 + a_2 > x_1 + x_2 = y_1 + y_2 = z_1 + z_2$. Hence, it must be the case that

$C \subset A \cup B$. Let $b = x$. Since $a_1 + a_2 > y_1 + y_2$, we have that $a \notin S_y$ and $x \notin f(A \cup B)$. Observe that $f(A \cup B) \neq \emptyset$. Similar argument applies whether $b = y$ or $b = z$.

Subcase iv). $a, b \notin \{x, y, z\}$

We distinguish the following cases: 1. $f(M(a,x)) = a$ & $f(M(b,x)) = b$, 2. $f(M(a,x)) = a$ & $f(M(b,x)) = x$, 3. $f(M(a,x)) = x$ & $f(M(b,x)) = x$, 4. $f(M(a,x)) = x$ & $f(M(b,x)) = b$.

Recall that $x_1 + x_2 = y_1 + y_2 = z_1 + z_2$. First observe that *subcase iv*.4 is not possible as it contradicts $f(M(a,b)) = a$.

Subcase iv.1. $f(M(a,x)) = a$ & $f(M(b,x)) = b$

Then it must be that $C \subset A \cup B$. Since $f(M(a,b)) = a$, we have that either $a_1 + a_2 > b_1 + b_2$ or $a_1 + a_2 = b_1 + b_2$ & $a_1 > b_1$. Since $a, b \notin S_y$, we have that $b \notin f(A \cup B)$. Observe that $f(A \cup B) \neq \emptyset$.

Subcase iv.2. $f(M(a,x)) = a$ & $f(M(b,x)) = x$

Then $a_1 + a_2 > y_1 + y_2 \geq b_1 + b_2$. If $C = A \cup B$, then $b \notin f(C)$. Otherwise, let $C \subset A \cup B$. Since $b \in S_y$, it follows that $b \notin f(A \cup B)$. Observe that $f(A \cup B) \neq \emptyset$.

Subcase iv.3. $f(M(a,x)) = x$ & $f(M(b,x)) = x$

Then $x_1 + x_2 \geq a_1 + a_2 \geq b_1 + b_2$. If $A \cup B = C$, then $b \notin f(C)$, by construction. Otherwise, consider $C \subset A \cup B$. Observe that $x \notin S_y$ and $b \in S_y$. If there exists $s \in A \cup B \backslash xyz$ such that $s_1 + s_2 > x_1 + x_2$, we have that $b \notin f(A \cup B)$ and $f(A \cup B) \neq \emptyset$. Otherwise, consider $x_1 + x_2 \nless s_1 + s_2$ for any $s \in A \cup B \backslash x$. Then $b \notin f(A \cup B)$ and $f(A \cup B) = xy$.

Next we check axiom 4.

Axiom 4. $a, b, c \in \Re^n_+$, with $a \neq b \neq c$, $a = f(M(a,b))$ & $b = f(M(b,c)) \Rightarrow a \in$

$f(A)$, with $M(a,b) \cup M(b,c) = A$.

We proceed according to: 1. $a,b,c \in \{x,y,z\}$, 2. $a,b \in \{x,y,z\}$ & $c \notin \{x,y,z\}$, 3. $a,c \in \{x,y,z\}$ & $b \notin \{x,y,z\}$, 4. $a \in \{x,y,z\}$ & $b,c \notin \{x,y,z\}$, 5. $a \notin \{x,y,z\}$ & $b,c \in \{x,y,z\}$, 6. $a,b \notin \{x,y,z\}$ & $c \in \{x,y,z\}$, 7. $a,c \notin \{x,y,z\}$ & $b \in \{x,y,z\}$, 8. $a,b,c \notin \{x,y,z\}$.

Case 1 is obvious, so omitted. *Case* 3 is not admissible, by construction.

Case 2. $a,b \in \{x,y,z\}$ & $c \notin \{x,y,z\}$

Let $a = x$ and $b = y$. Observe that $A \cap \{x,y,z\} = x, y$. It follows that $y, c \in S_x$. Since for not $s \in A \backslash x$ $s_1 + s_2 > x_1 + x_2$, we have that $x \in f(A)$. Similar reasoning applies whether $a = y$ and $b = z$ or $a = z$ or $b = x$.

Case 4. $a \in \{x,y,z\}$ & $b,c \notin \{x,y,z\}$

Then $a_1 + a_2 \geq b_1 + b_2 \geq c_1 + c_2$. Let $a = x$. Observe that $A \cap \{x,y,z\} = x$. Then $b, c \in S_x$. Since for not $s \in A \backslash x$ $s_1 + s_2 > x_1 + x_2$, we have that $x \in f(A)$. The same reasoning applies whether $a = y$ or $a = z$.

Case 5. $a \notin \{x,y,z\}$ & $b,c \in \{x,y,z\}$

Then $a_1 + a_2 > b_1 + b_2$. Observe that either $|A \cap \{x,y,z\}| = 2$ or $A \cap \{x,y,z\} = \{x,y,z\}$. In either case it follows that $a = \arg\max_{s \in A}(s_1 + s_2)$. Then $a \in f(A)$. This is true in all admissible cases: $b = x$ & $c = y$; $b = y$ & $c = z$; and $b = z$ & $c = x$.

Case 6. $a,b \notin \{x,y,z\}$ & $c \in \{x,y,z\}$

Then $a_1 + a_2 \geq b_1 + b_2 > c_1 + c_2$. The following cases are possible: *i*) $A \cap \{x,y,z\} = c$, *ii*) $|A \cap \{x,y,z\}| = 2$, *iii*) $A \cap \{x,y,z\} = \{x,y,z\}$. In any case $a \in f(A)$, as desired.

Case 7. $a,c \notin \{x,y,z\}$ & $b \in \{x,y,z\}$

Then $a_1 + a_2 > b_1 + b_2 \geq c_1 + c_2$. The following cases are possible: *i*) $A \cap \{x,y,z\} = b$, *ii*) $|A \cap \{x,y,z\}| = 2$, *iii*) $A \cap \{x,y,z\} = \{x,y,z\}$. In all cases we have that $a \in f(A)$,

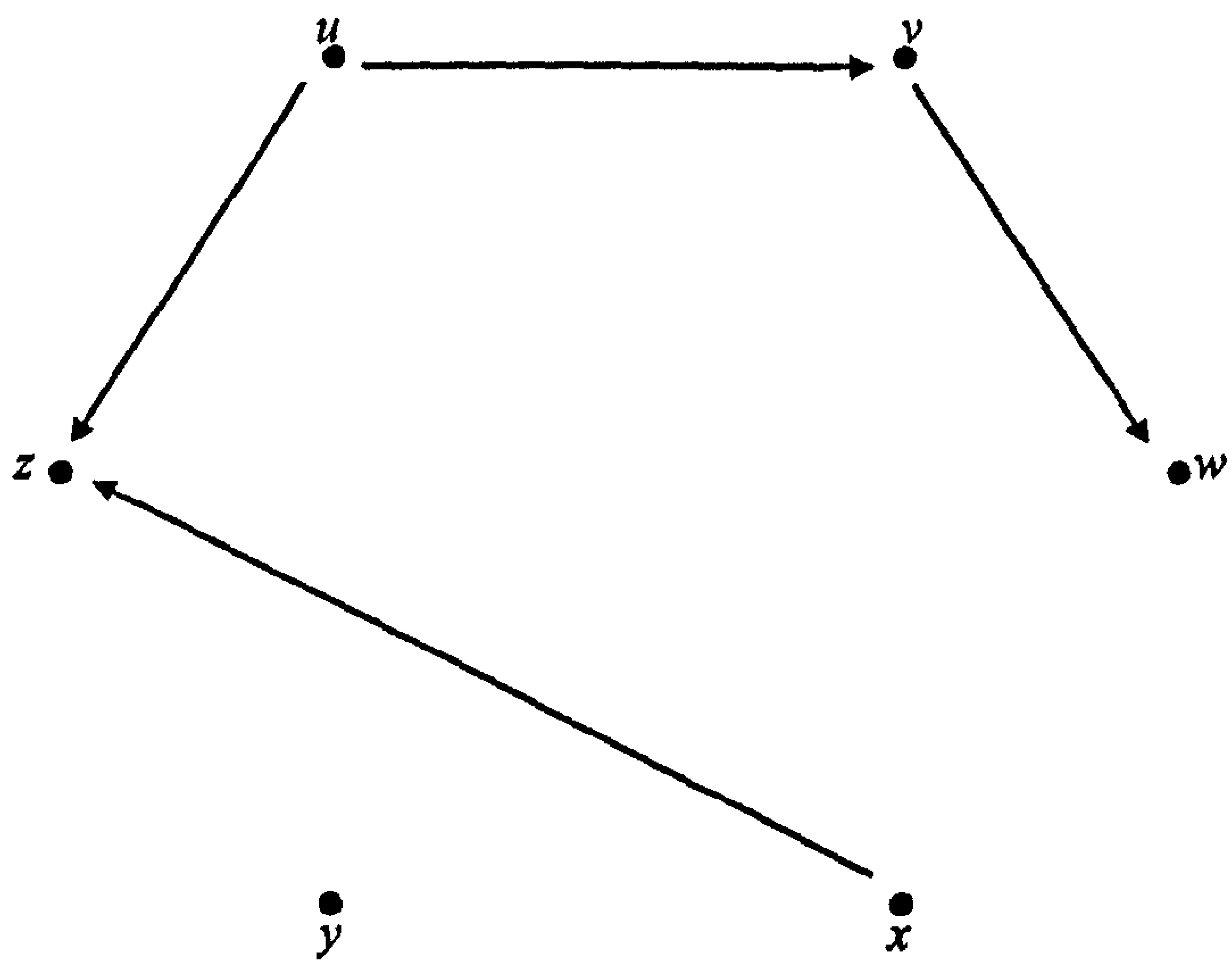

Figure 5.1: Revealed preferences in pairwise choice problems for independence of axioms used in Theorem 3

as desired.

Case 8. $a, b, c \notin \{x, y, z\}$.

The following cases are possible: *i)* $|A \cap \{x, y, z\}| = 0$, *ii)* $|A \cap \{x, y, z\}| = 1$, *iii)* $|A \cap \{x, y, z\}| = 2$, *iv)* $A \cap \{x, y, z\} = \{x, y, z\}$. It is easy to see that in all cases $a \in f(A)$, as desired.

B. Independence of Axioms used in Theorem 3

To complete the proof of Theorem 3, we show that Property $\alpha 2$ (**α2**), Weak Property β (**Wβ**), Weak **WARNI** (**WWARNI**), Strong BDC (**SBDC**), Weak WARP (**WWARP**), and Reason-Based Bias (**RBB**) are independent.

Suppose that u, v, w, x, y and z are distinct feasible alternatives, and let choice in pairs be as displayed in Figure 2, where $a \rightarrow b$ stands for $\{a\} = C(\{a, b\})$, whilst no arrow between a and b stands for $\{a, b\} = C(\{a, b\})$.

For an example violating only $\boldsymbol{\alpha}\mathbf{2}$, fix $X = \{u, x, z\}$, and suppose $C(X) = X$. C is not a reason-based choice correspondence because $z \notin M(S, \succ)$ but $z \in C(S)$. $\boldsymbol{\alpha}\mathbf{2}$ is violated as $z \notin C(\{x, z\})$. All other choice-consistency conditions are satisfied.

For an example violating only $\mathbf{W}\boldsymbol{\beta}$, fix $X = \{u, x, z\}$, and suppose $C(X) = \{u\}$. C is not a reason-based choice correspondence because $x \in M(S, \succ)$ and there does not exist $a \in M(S, \succ) \backslash \{x\}$ such that $\prec (x, S) \subset \prec (a, S)$ but $x \notin C(X)$. $\mathbf{W}\boldsymbol{\beta}$ is violated because $\{u, x\} = C(\{u, x\})$, u and x are indistinguishable on $\{z\}$, and $u \in C(X)$ but $x \notin C(X)$. All other choice-consistency conditions are satisfied.

For an example violating only **WWARNI**, fix $X = \{v, w, x, z\}$. Let $C(S)$ be a reason-based choice correspondence for all $S \in \mathcal{S} \backslash X$, and suppose $C(X) = \{v\}$. C is not a reason-based choice correspondence because $x \in M(X, \succ)$ and there does not exist $a \in M(X, \succ) \backslash \{x\}$ such that $\prec (x, X) \subset \prec (a, X)$, but $x \notin C(X)$. **WWARNI** is violated because $v \in C(X)$, and there exists $S = \{v, x, z\} \subset X$ such that $\{x\} = C(S)$ and $\{x, v\} \subset S$, $x \in C(\{a, x\})$ for all $a \in X$, but $x \notin C(X)$. All other choice-consistency conditions are satisfied.

For an example violating only **SBDC**, fix $X = \{u, v, y, z\}$. Let $C(S)$ be a reason-based choice correspondence for all $S \in \mathcal{S} \backslash X$, and suppose $C(X) = \{u, y\}$. C is not a reason-based choice correspondence because $u, y \in M(S, \succ)$ and $\prec (y, S) \subset \prec (u, S)$, but $y \in C(X)$. **SBDC** is violated because $C(\{u, v, y\}) = \{u\}$ and $\{u\} = C(\{u, z\})$ but $C(X) \neq \{u\}$. All other choice-consistency conditions are satisfied.

For an example violating only **WWARP**, fix $X = \{w, x, y, z\}$. Let $C(S)$ be a reason-based choice correspondence for all $S \in \mathcal{S} \backslash X$, and suppose $C(X) = X \backslash \{z\}$. C is not a reason-based choice correspondence because $w, x, y \in M(S, \succ)$, $\prec (w, S), \prec$

$(y, S) \subset \prec (x, S)$, but $w, y \in C(X)$. **WWARP** is violated because $C(\{x, y, z\}) = \{x\}$, $y \notin C(\{x, y, z\})$, x and y are indistinguishable on $\{w\}$, but $y \in C(X)$. All other choice-consistency conditions are satisfied.

For an example violating only **RBB**, fix $X = \{x, y, z\}$. Let $C(X) = \{x, y\}$. C is not a reason-based choice correspondence because $x, y \in M(S, \succ)$, $\prec (y, S) \subset \prec (x, S)$, but $y \in C(X)$. **RBB** is violated because $\{x\} = C(\{x, z\})$, $x \in C(\{x, y\})$, $z \in C(\{y, z\})$, but $C(\{x, y, z\}) \neq \{x\}$. All other choice-consistency conditions are satisfied.