Supplementary materials for
"Real option valuation of reserve capacity"
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Appendix A Plots of UK system prices

Figure A.1 provides a histogram of the UK main system price, which is used in the imbalance mechanism, between 2nd June 2013 and 12th January 2016. The boxplot is displayed in Figure A.2. The data was obtained from the *ELEXON Portal*.

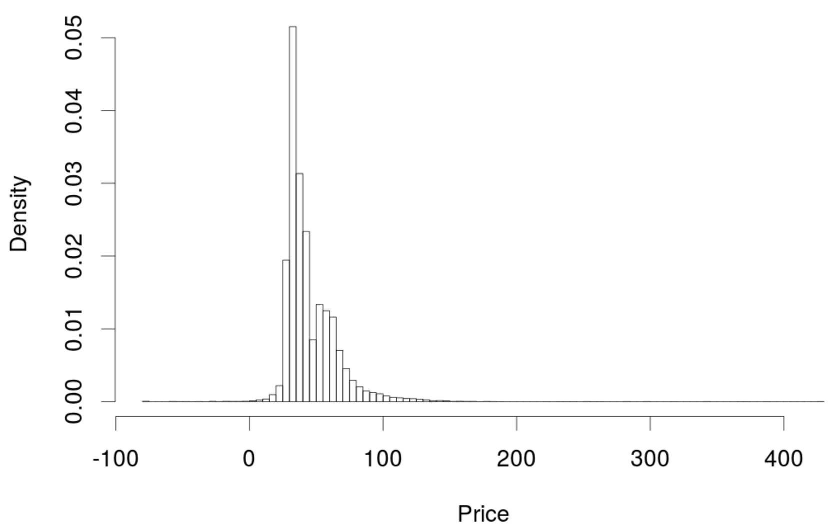


Figure A.1: Histogram of main UK system price, 2nd June 2013 to 12th January 2016.

Appendix B Auxiliary results for smooth fit

This appendix provides several results concerning the existence of points of smooth fit.

Lemma B.1. *Let* $h : [x, z] \to \mathbb{R}$ *for some* $x > 0$ *and* $z \in (x, \infty]$ *satisfy* $h(x) = 0$ *and* $h'(y) \geq h(y)/y$ *for* $y \in (x, z)$. *Then* $h \geq 0$.

Figure A.2: Boxplot of main UK system price, 2nd June 2013 to 12th January 2016.

Proof. Let $g = -h$. Then $g'(y) \le g(y)/y$ and Gronwall's lemma yields $g(y) \le g(x)e^{\int_x^y u^{-1}du} = 0$.

□

Lemma B.2. *Let f be a continuously differentiable function on $[0, A)$ for $A \in (0, \infty]$. Assume that $f(y) = 0$ for some $0 < y < A$, $\lim_{y\to A} f(y) > 0$ and $\lim_{y\to A} f'(y) \le 0$. Then there is a point $z \in [y, A)$ such that $f(z)/z = f'(z) \ge 0$. Moreover, there is at most one such point on each interval of strict concavity of f (concavity is sufficient at the ends of the interval).*

Proof. Let $y_1 < A$ be the largest root of f (its existence is guaranteed by $\lim_{y\to A} f(y) > 0$). Then $y_1 > 0$ and $f(y) > 0$ on (y_1, A). Define $\xi(y) = f(y) - f'(y)y$. Clearly, $\xi(y_1) \le 0$. If there is $y_2 > y_1$ such that $\xi(y_2) > 0$ then by continuity ξ must have a root z between y_1 and y_2. Since $f(z) \ge 0$ then $f'(z) \ge 0$.

Assume, for a contradiction, that $\xi(y) \le 0$ on $[y_1, A)$ and take any $x \in (y_1, A)$. Let g be the solution to the ODE: $g(y) - g'(y)y = 0$ for $y \in [x, A)$, $g(x) = f(x)$, i.e., $g(y) = yf(x)/x \ge f(x)$. Let $h = f - g$. By Lemma B.1, $h \ge 0$, i.e., $f \ge g$ on $[x, A)$. When $A < \infty$ then since $\lim_{y\to A} f'(y) \le 0$ we have $\lim_{y\to A} \xi(y) \ge \lim_{y\to A} f(y) \ge \lim_{y\to A} g(y) \ge f(x) > 0$, a contradiction. Otherwise $A = \infty$ and $\lim_{y\to\infty} h'(y) \le -f(x)/x < 0$, which contradicts the positivity of h.

Assume further that f is concave on $[a, b]$ and strictly concave inside of this interval. Roots of ξ define tangents to f of the form $x \mapsto x\, f(y)/y$. Due to concavity the function f is majorised by its tangents. Hence, if there are two roots y_1, y_2 of ξ then these tangents have to coincide. This is impossible due to strict concavity. □

Corollary B.3. *Point z in the above lemma can be chosen such that f is not strictly convex in its neighbourhood.*

Proof. Assume that f is strictly convex on (l, r). This implies $f(y_1) > f(y_2) + f'(y_2)(y_1 - y_2)$ for any $y_1, y_2 \in (l, r)$ and $y_1 < y_2$. Rearranging the terms yields

$$\xi(y_2) = f(y_2) - f'(y_2)y_2 < f(y_1) - f'(y_2)y_1 < f(y_1) - f'(y_1)y_1 = \xi(y_1),$$

where we used the fact that $f'(y_1) < f'(y_2)$ following from strict convexity. Hence, ξ is strictly decreasing on intervals of strict convexity of f. Similarly,

ξ is non-increasing on intervals of convexity of f and non-decreasing on the intervals of concavity of f.

Let z be the point constructed in the proof of Lemma B.2. Assume that f is strictly convex in the neighbourhood (l, r) of z. Then $\xi(y) < 0$ on $(z, r]$. This implies that there is a root of ξ on (r, A). Let $\hat{z}$ be the root on (r, A) closest to r. Then $\xi < 0$ on $(z, \hat{z})$ and if f were strictly convex around $\hat{z}$ then f would decrease to 0 at $\hat{z}$, a contradiction. This implies that f is not stricty convex around $\hat{z}$. □

Corollary B.4. *Assume that f is continuously differentiable on $[0, \infty)$ and strictly convex on $(0, r)$. If $f(0) = 0$, $\lim_{y\to\infty} f(y) > 0$ and $\lim_{y\to\infty} f'(y) \le 0$ then there exists $z > 0$ such that $f(z)/z = f'(z) \ge 0$.*

Proof. If there is $y > 0$ such that $f(y) = 0$ then the result follows from Lemma B.2. Otherwise, assume that $f > 0$ on $(0, \infty)$. Define $\xi(y) = f(y) - f'(y)y$. Then $\xi(0) = 0$ and ξ is decreasing on the interval of strict convexity $(0, r)$. Hence $\xi(r) < 0$. Arguments from the proof of Lemma B.2 imply that there is $y_2 > r$ such that $\xi(y_2) > 0$. This combined with the continuity of ξ yields that there is a root z of ξ on (r, y_2). Recalling that $f(z) > 0$ we obtain $f'(z) > 0$. □

Lemma B.5. *Assume that function f is continuously differentiable on $[0, \infty)$, convex on $[0, \bar{y}]$ and increasing on $[\bar{y}, \infty)$ for some $\bar{y} > 0$ and that the following hold:*

$$
\begin{aligned}
f(0) &= 0, \\
\lim_{y\to\infty} f(y) &> 0, \\
\lim_{y\to\infty} f'(y) &= 0.
\end{aligned}
$$

Then there is a point $y \in [\bar{y}, \infty)$ such that $f(y)/y = f'(y)$. Moreover, if f is strictly concave on $(\bar{y}, \infty)$ then this point is unique.

Proof. If there is $y \ge \bar{y}$ such that $f(y) \le 0$, then the result follows from Lemma B.2. Otherwise, $f > 0$ on $[\bar{y}, \infty)$. Define $\xi(y) = f(y) - f'(y)y$. By convexity of f, $f(0) \ge f(\bar{y}) + f'(\bar{y})(0 - \bar{y})$. Hence, $\xi(\bar{y}) \le 0$. Existence of y such that $\xi(y) > 0$ completes the proof due to continuity of ξ. Assume, by contradiction, that $\xi \le 0$ on $[\bar{y}, \infty)$. Let g be the solution to the ODE: $g(y) - g'(y)y = 0, g(\bar{y}) = f(\bar{y})$, i.e., $g(y) = yf(\bar{y})/\bar{y}$. Let $h = f - g$. By Lemma B.1, $h \ge 0$, i.e., $f \ge g$ on $[\bar{y}, \infty)$. But then $\lim_{y\to\infty} f'(y) \ge f(\bar{y})/\bar{y} > 0$, a contradiction. Uniqueness is proved identically as in Lemma B.2. □

Appendix C Single option: case-by-case analysis

Notice that $g(y^*) = \sqrt{y^*}\big(p_c + K_c - f(x^*)\big)$, hence its sign is determined by the relation between $p_c + K_c$ and $f(x^*)$. This will be useful in interpreting the conditions arising in the analysis below.

Table C.1: Stopping regions for the single option when $b < a$. Whenever the stopping region is trivial we write $V_c = 0$.

Stopping regions in the case $b < a$.					
Parameter range				Stopping region	Figure C.2
$p_c \le D$	$K_c + p_c \le D$			$V_c = 0$	
	$K_c + p_c > D$		$K_c + p_c \le f(x^*)$	$V_c = 0$	
	$K_c + p_c > D$		$K_c + p_c > f(x^*)$	$\hat{\Gamma} = [\min\{\hat{y}_b, y^*\}, y^*]$	a
$p_c > D$	$K_c + p_c \le f(x^*)$			$\hat{\Gamma} = [y_b, \infty)$	d
	$K_c + p_c > f(x^*)$	Case A	$y^* \ge Y_c$	$\hat{\Gamma} = [\min(\hat{y}_b, y^*), \infty)$	e
	$K_c + p_c > f(x^*)$	Case A	$y^* < Y_c$	$\hat{\Gamma} = [\min(\hat{y}_b, y^*), y^*] \cup [y_b^{(1)}, \infty)$	f & Fig. 2
	$K_c + p_c > f(x^*)$	Case A^c		$\hat{\Gamma} = [y_b, \infty)$	d

For the convenience of the reader we state the derivatives of $g, \hat{g}$ and η:

$$
\begin{aligned}
\eta'(y) &= \frac{1}{2} y^{-\frac{1}{2}} \left[p_c - D - d\left(1 - \frac{b^2}{a^2}\right) y^{-\frac{b}{2a}} \right], \\
g'(y) &= \frac{1}{2} y^{-\frac{1}{2}} \left[p_c - D - d\left(1 - \frac{b}{a}\right) y^{-\frac{b}{2a}} \right], \\
\hat{g}'(y) &= \frac{1}{2} y^{-\frac{1}{2}} \left[K_c + p_c - D - d\left(1 - \frac{b}{a}\right) y^{-\frac{b}{2a}} \right], \\
g''(y) &= \frac{1}{4} y^{-\frac{3}{2}} \left[D - p_c + d\left(1 - \frac{b^2}{a^2}\right) y^{-\frac{b}{2a}} \right], \\
\hat{g}''(y) &= \frac{1}{4} y^{-\frac{3}{2}} \left[D - K_c - p_c + d\left(1 - \frac{b^2}{a^2}\right) y^{-\frac{b}{2a}} \right].
\end{aligned}
\tag{C.1}
$$

A summary of the results for each case is collected in Tables C.1-C.3. Graphs showing the shape of the obstacle and related stopping regions are located in Figures C.1 and C.2 with links in the last column of the aforementioned tables for guidance. For futher clarity, the graphs in Figure C.1 display the smallest concave majorant of the obstacle in red and blue. The blue region, which is where the majorant coincides with the obstacle, defines the stopping region.

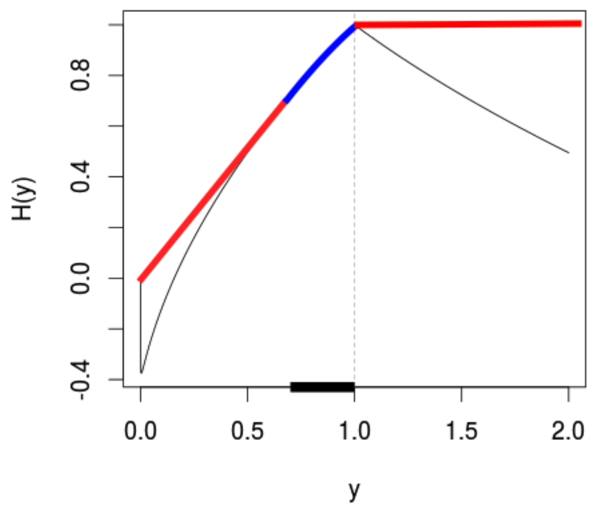
C.1 Solutions in the case $b < a$

A summary of the results of this subsection is presented in Table C.1.

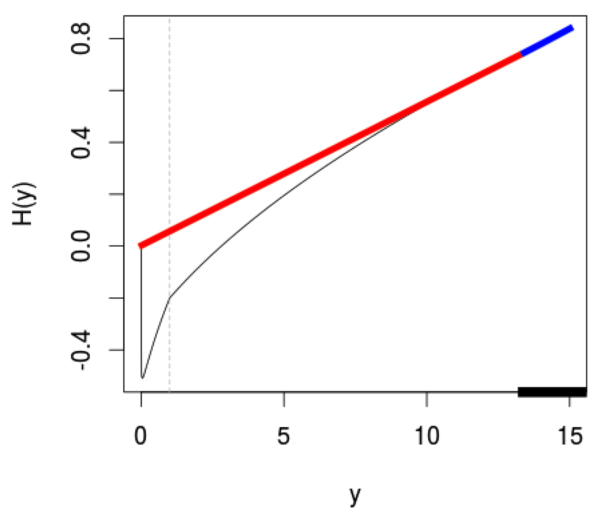
C.1.1 Case $p_c \le D$

When $K_c + p_c \le D$: Each of $Y_m, Y_c, \hat{Y}_m, \hat{Y}_c$ are equal to positive infinity, hence g and $\hat{g}$ are decreasing on $(0, \infty)$. Combining this with $\hat{g}(0) = 0$ makes H non-positive and W zero everywhere, giving $V_c = 0$.
When $K_c + p_c > D$: $\hat{g}$ is 0 at 0, then decreases and, if $\hat{Y}_m < y^*$, later increases, to meet g at y^*. Also g is decreasing everywhere as $Y_m = \infty$. Hence, $g(y^*) \le 0$

Figure C.1: Illustrative plots for the single option obstacle and stopping region (thick horizontal line). The dashed vertical lines mark y^*. The least nonnegative concave majorant W is shown in blue (where W coincides with H) and red (otherwise).

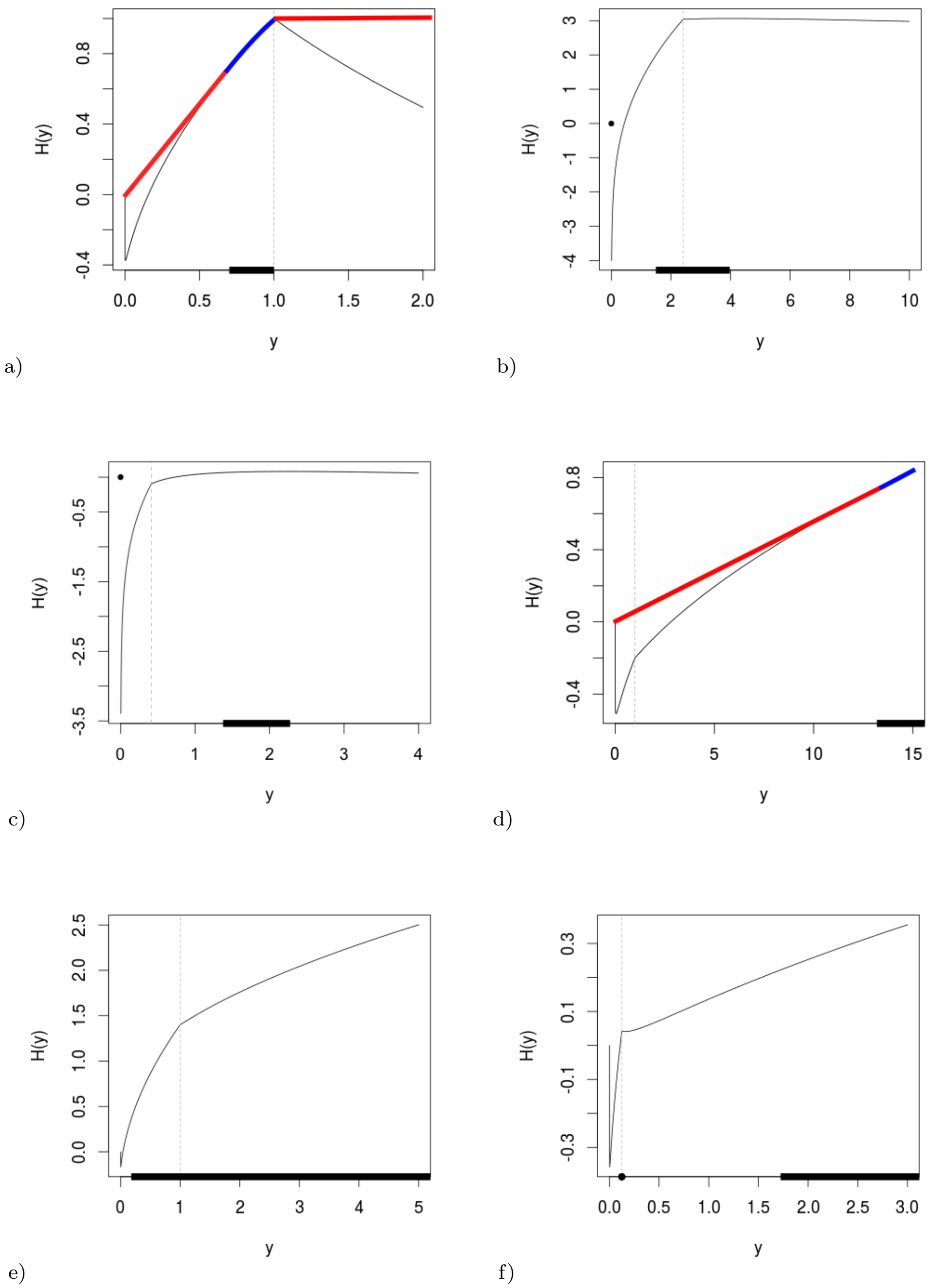


a)



b)

Figure C.2: Illustrative plots for the single option obstacle and stopping region (thick horizontal line). The dashed vertical lines mark y^*.



makes H non-positive and W zero everywhere, giving $V_c = 0$. When $g(y^*) > 0$ we have $V_c \neq 0$, the stopping region $\hat{\Gamma}$ has right endpoint y^* and exercise is profitable at y^*. We have $\hat{\Gamma} = [\min\{y^*, \hat{y}_b\}, y^*]$ (see panel (a) in Figure C.1). Note that the smooth fit condition never holds at the right end of the stopping region, and holds at the left end only if $\hat{y}_b < y^*$.

C.1.2 Case $p_c > D$

Both functions g and $\hat{g}$ are convex close to 0 (decreasing then increasing) and then concave, increasing without bound, and so the majorant W is nonzero and $V_c \neq 0$.

$g(y^*) \leq 0$: There exists $y_0 \geq y^*$ such that g is nonpositive on $(0, y_0]$ and positive on (y_0, ∞). Since $g(y)$ grows to infinity as $y \to \infty$ and $\lim_{y\to\infty} g'(y) = 0$, by Lemma B.2 in the Appendix there exists a point $y_b \geq y_0$ such that the tangent to g at y_b crosses the origin, i.e., $g'(y_b) = g(y_b)/y_b$ and $g'(y_b) > 0$ since $g(y_b) > 0$. This point can be taken on the concave part of g (so that $y_b \geq Y_c$) by Corollary B.3. Then it is unique by Lemma 2.2 and we conclude that $\hat{\Gamma} = [y_b, \infty)$.

When $g(y^*) > 0$ let $y' := \min(\hat{y}_b, y^*)$. We distinguish between the following two cases:

- Case A: $g(y)/y \leq \hat{g}(y')/y'$ for all $y > y^*$ and hence the majorant coincides with H at y' (note that $\hat{g}$ is concave at $\hat{y}_b$)
- Case A^C: there exists $\tilde{y} > y^*$ with $g(\tilde{y})/\tilde{y} > \hat{g}(y')/y'$ and so the majorant does not coincide with H anywhere on $[0, y^*]$. If the majorant touches H then it must do so to the right of y^* and then smooth fit holds.

Lemma C.1. *Case A^C holds if and only if both of the following conditions hold:*

1. *η has a root $y_b > \max(y^*, Y_c)$,*
2. *$g'(y_b) > \hat{g}(y')/y'$.*

Proof. Suppose first that case A^C holds. For condition 1, apply Lemma B.5 to the function $g^{(1)}(y) = g(y^*+y) - g(y^*)$ on $[0, \infty)$ (taking $\bar{y} = \max(0, Y_c - y^*)$) to establish the existence of a smooth fit point $\tilde{y}_b^{(1)}$. Let $y_b^{(1)} = \tilde{y}_b^{(1)} + y^*$. Since we are in case A^C, it is easy to see that the tangent to g at $y_b^{(1)}$ has a negative intercept at the vertical axis, i.e., $\eta(y_b^{(1)}) < 0$ and since $\eta \to \infty$ as $y \to \infty$ it follows that η has a root y_b in $[y_b^{(1)}, \infty)$. Since the tangent at y_b must strictly dominate H on $[0, y^*]$, condition 2 follows.

Conversely, if conditions 1 and 2 hold then the conclusion follows by the definition of η. □

Case A and $y^* \geq Y_c$: H is concave at every point in $[y', \infty)$ (because $\hat{g}$ is steeper at y^* than g and both are concave there) so $\hat{\Gamma} = [y', \infty)$.

Case A and $y^* < Y_c$: Then H is convex on (y^*, Y_c). The problem decomposes into (i) finding the smallest non-negative concave majorant of $\hat{g}$ on $[0, y^*]$ and (ii) finding the smallest non-negative concave majorant of the function $g^{(1)}(y) = g(y^* + y) - g(y^*)$ on $[0, \infty)$. The majorant in (i) coincides with $\hat{g}$ on $[y', y^*]$ and

Table C.2: Stopping regions for the single option when $b > a$. When the stopping region is trivial we simply write $V_c = 0$.

Stopping regions in the case $b > a$.					
Parameter range				Stopping region	Figure C.2
$p_c \geq D$	$\hat{y}_b \leq y^*$			$\hat{\Gamma} = [\hat{y}_b, \infty)$	e
	$\hat{y}_b > y^*$			$\hat{\Gamma} = [\max(y_b, y^*), \infty)$	d
$p_c < D$	$K_c + p_c < D$			$V_c = 0$	
	$K_c + p_c \geq D$	$K_c + p_c \leq f(x^*)$	$Y_m \leq y^*$ or $g(Y_m) \leq 0$	$V_c = 0$	
			$Y_m > y^*$ and $g(Y_m) > 0$	$\hat{\Gamma} = [y_b, Y_m]$	c
		$K_c + p_c > f(x^*)$	$g'(y^*) > g(y^*)/y^*$	$\hat{\Gamma} = [y_b, Y_m]$	c
			$g'(y^*) \leq g(y^*)/y^*$	$\hat{\Gamma} = [\min(\hat{y}_b, y^*), \max(Y_m, y^*)]$	b

is linear on $(0, y')$. Since $\lim_{y\to\infty} g^{(1)}(y) = \infty$ and the derivative converges to 0 as $y \to \infty$, there exists a unique point z_b such that $g^{(1)}$ and its smallest nonnegative concave majorant coincide exactly on $[z_b, \infty)$ (apply Corollaries B.3 and B.4 in the Appendix and recall that $Y_c > y^*$). Note that $z_b > 0$ since $g^{(1)}$ is strictly convex on $(0, Y_c - y^*)$. Clearly, $y_b^{(1)} := y^* + z_b$ is a unique solution of $\frac{g(y)-g(y^*)}{y-y^*} = g'(y) > 0$. We will show that the smallest nonnegative concave majorant of H is given by

$$W(y) = \begin{cases} \frac{\hat{g}(y')}{y'} y, & y < y', \\ \hat{g}(y), & y' \leq y \leq y^*, \\ g(y^*) + g'(y_b^{(1)})(y - y^*), & y^* < y < y_b^{(1)}, \\ g(y), & y_b^{(1)} < y. \end{cases}$$

If $y' < y^*$, then $\hat{g}$ is concave on $[y', y^*]$ and lies below the tangent at y^*. We infer the concavity of W at y^* from this and the fact that $\hat{g}$ majorises H. When $y' = y^*$ the concavity at y^* follows from the condition A; concavity at other points is trivial. Finally, we conclude that $\hat{\Gamma} = [y', y^*] \cup [y_b^{(1)}, \infty)$, see panel (b) in Figure C.1. The principle of smooth fit fails at y^*.

Case A^C: By Lemma C.1 y_b lies in $(\max(y^*, Y_c), \infty)$, a region in which H is equal to g, concave, and increasing. We conclude that $\Gamma = [y_b, \infty)$.

C.2 Solutions in the case $b > a$

A summary of the results of this subsection is presented in Table C.2.

C.2.1 Case $p_c \geq D$

In this case each of $Y_m, Y_c, \hat{Y}_m, \hat{Y}_c$ are equal to positive infinity. Noting that $\hat{g}'(y^*) > g'(y^*)$, H is concave and increasing without bound so that $V_c \neq 0$. The tangency points $\hat{y}_b$ and y_b are uniquely defined. Then $\hat{\Gamma} = [A, \infty)$ where $A = \hat{y}_b$ if $\hat{y}_b \leq y^*$ and $A = \max(y_b, y^*)$ otherwise. Note that there is no smooth fit at A when $y_b \leq y^* \leq \hat{y}_b$.

C.2.2 *Case $p_c < D$*

When $K_c + p_c \geq D$: $\hat{Y_c}, \hat{Y_m}$ are equal to $+\infty$ while Y_c, Y_m lie in $(0, \infty)$ so that $\hat{g}$ is increasing and concave, making H concave on $(0, \max(y^*, Y_c))$ and both convex and decreasing on $(\max(y^*, Y_c), \infty)$. Notice that if $Y_m \leq y^*$ then the stopping region $\hat{\Gamma}$ has an empty intersection with (y^*, ∞) and the value function W is constant on $[y^*, \infty)$.

If $\hat{g}(y^*) = g(y^*) \leq 0$ then the problem reduces to finding a non-negative concave majorant of g. In this case, if $Y_m \leq y^*$ or $g(Y_m) \leq 0$ then $V_c = 0$. Otherwise, $V_c > 0$ and there exists a unique solution y_b of $\eta(y) = 0$ on (y^*, Y_m) such that $g'(y_b) > 0$ (uniqueness follows from Lemma 2.2, existence is easy). The stopping region has the form $\hat{\Gamma} = [y_b, Y_m]$.

If $\hat{g}(y^*) = g(y^*) > 0$ and $g'(y^*) > g(y^*)/y^*$ then $Y_m > y^*$. Since $\eta(y) = 2(g(y) - g'(y)y)$ we have $\eta(y^*) < 0$ and $\eta(Y_m) > 0$, so by the continuity and monotonicity of η(recall that $Y_c > Y_m$) there exists a unique solution y_b of $\eta(y) = 0$ on (y^*, Y_m), and we have $g'(y_b) > 0$. It follows also that the tangent at y_b goes through 0 (has a null vertical intercept). By concavity of H on $(0, y_b)$ it majorises H there. Hence, the stopping region is $\hat{\Gamma} = [y_b, Y_m]$.

Alternatively, suppose that both $g(y^*) > 0$ and $g'(y^*) \leq g(y^*)/y^*$. Then since $\hat{g}'(y^*) > g'(y^*)$, the problem decomposes into (i) finding the smallest non-negative concave majorant of $\hat{g}$ on $[0, y^*]$ and (ii) finding the smallest non-negative concave majorant of the function $g^{(1)}(y) = g(y^* + y) - g(y^*)$ on $[0, \infty)$. The majorant in (i) coincides with $\hat{g}$ on $[\min(\hat{y}_b, y^*), y^*]$, whereas the majorant in (ii) coincides with $g^{(1)}$ on $[0, \max(Y_m - y^*, 0)]$. The overall stopping region and majorant are then recovered by adjoining these parts, so that $\hat{\Gamma} = [\min(\hat{y}_b, y^*), \max(Y_m, y^*)]$. Notice that when $\hat{y}_b > y^*$ there is no smooth fit at the left end of the interval $\hat{\Gamma}$.

$K_c + p_c < D$: we have $\hat{g} < 0$ on $(0, \infty)$ and $g < \hat{g}$ on (y^*, ∞), so that $H \leq 0$ and $V_c = 0$.

C.3 *Solutions in the case $b = a$*

A summary of the results of this subsection is presented in Table C.3. Although there does not seem to be any economic rationale behind this border case, the analysis simplifies:

$$
\begin{aligned}
g(y) &= (p_c - D)\sqrt{y} + K_c\sqrt{y^*} - d, \\
g'(y) &= \frac{1}{2}y^{-\frac{1}{2}}(p_c - D), \\
g''(y) &= \frac{1}{4}y^{-\frac{3}{2}}(D - p_c), \\
\hat{g}(y) &= (K_c + p_c - D)\sqrt{y} - d, \\
\hat{g}'(y) &= \frac{1}{2}y^{-\frac{1}{2}}(K_c + p_c - D), \\
\hat{g}''(y) &= \frac{1}{4}y^{-\frac{3}{2}}(D - K_c - p_c).
\end{aligned}
\tag{C.2}
$$

Table C.3: Stopping regions for the single option when $a = b$. Whenever the stopping region is trivial we write $V_c = 0$.

Stopping regions in the case $b = a$.			
Parameter range		Stopping region	Figure C.2
$p_c > D$	$y_b \geq y^*$	$\hat{\Gamma} = [y_b, \infty)$	d
	$\hat{y}_b < y^*$	$\hat{\Gamma} = [\hat{y}_b, \infty)$	e
	$y_b < y^*, \hat{y}_b \geq y^*$	$\hat{\Gamma} = [y^*, \infty)$	
$p_c = D$	$K_c\sqrt{y^*} - d > 0$	$\hat{\Gamma} = [\min(\hat{y}_b, y^*), \infty)$	e
	$K_c\sqrt{y^*} - d \leq 0$	$V_c = 0$	
$p_c < D$	$K_c + p_c \leq f(x^*)$	$V_c = 0$	
	$K_c + p_c > f(x^*)$	$\hat{\Gamma} = [\min\{\hat{y}_b, y^*\}, y^*]$	a

C.3.1 Case $p_c > D$

Here g, $\hat{g}$ and hence also H are strictly concave and increasing without bound so $V_c \neq 0$ and the stopping region $\hat{\Gamma}$ will be of the form $[A, \infty)$. We have

$$y_b = 4\left(\frac{d - K_c\sqrt{y^*}}{p_c - D}\right)^2, \qquad \hat{y}_b = 4\left(\frac{d}{K_c + p_c - D}\right)^2.$$

If $y_b \geq y^*$ then $A = y_b$ and smooth fit holds; otherwise, if $\hat{y}_b < y^*$ then $A = \hat{y}_b$ and smooth fit holds. If both $y_b < y^*$ and $\hat{y}_b \geq y^*$ then smooth fit does not hold and $A = y^*$.

C.3.2 Case $p_c = D$

The function g is constant and $\hat{g}$ is increasing and concave. Hence $V_c \neq 0$ precisely when $K_c\sqrt{y^*} - d > 0$, in which case $\hat{\Gamma} = [A, \infty)$ with $A = \min(\hat{y}_b, y^*)$.

C.3.3 Case $p_c < D$

In this case g is strictly convex and strictly decreasing and also $\hat{g}(0) < 0$, so $V_c \neq 0$ if and only if $g(y^*) > 0$. In this case the stopping region is $\hat{\Gamma} = [\min\{\hat{y}_b, y^*\}, y^*]$.