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SIMULATIONS OF SPACE PLASMA INSTABILITIES

Jonathan Peter Woodcock

Thesis submitted for the degree of
Doctor of Philosophy (PhD)
of the University of London

Queen Mary and Westfield College
1997
ABSTRACT

This work describes computer simulations of the behaviour of plasmas similar to those observed in the near Earth environment. The work can be split into three main threads.

Firstly, we have developed a set of algorithms to allow the implementation of particle-type simulation models on parallel computer architectures ranging from small workstation clusters to massively parallel supercomputers. These algorithms allow large simulations with many particles to be performed. We address the problems of efficient use of available computational resources and the scaling of algorithms as computers get larger.

Secondly, we use a parallel implementation of a two-dimensional hybrid simulation code with periodic boundaries to explore the evolution of ion beam distributions similar to those observed upstream of the Earth’s bow shock. We follow the evolution of the resonant instabilities of these cool tenuous proton beams, both isotropic and anisotropic in temperature, into the non-linear regime. We examine the waves generated, their effects on the ion distribution function, the phenomenon of gyrophase bunching and describe the life cycles of two-dimensional magnetic features including oblique propagating shocklets. We suggest that such two-dimensional structures may play a role in the saturation of beam instabilities. Coherence lengths of the waves are calculated. We see some evidence of anisotropy driven mirror waves late on in these simulations.

Thirdly, we explore the nature of parametric instabilities in two-dimensions. We examine the role of parametric, or wave-wave, instabilities in the late evolution of beam instability generated waves. We find little evidence of any parametric instability in this case. The two-dimensional evolution of a wave known to be unstable to one-dimensional parametric instability is described. We find that in this case the instability develops in a manner similar to that found in one-dimensional simulations, although with some angular broadening in wavevector space. There is some evidence of anisotropy driven instabilities later in the simulation.
Acknowledgements

There are many people who deserve thanks for their assistance in the completion of this work. Without David Burgess I would never have entered the fascinating world of the plasma, this thesis would never have come into being and the work described never begun. I thank him for his ideas, help, careful proof reading and comments.

I must thank my parents for their obvious but vital role. They have always encouraged and supported me in all my projects, however ill conceived. The same goes for my two big sisters, who have also unwittingly shaped many aspects of my life by having such a good time doing things which I wasn’t old enough to do. Many thanks also to my Grandmother, who has always expected the best.

My career in physics has been shaped by a string of remarkable people, to all of whom I owe an enormous debt; Mr. Henderson, who believed it should be fun; Mr. Firth, who told it how it was; Dr. Edmonds, for just being Dr. Edmonds; Dr. Brooker, for making me want to do a PhD; David Ko, for giving tutorials in the pub, and Prof. Ross, for being one of the good guys. I also raise a beer to those who shared that road at many points; Dillon, Turkey and Spinner in the very early days, Barry, Rupert and Gramps later on, the Wadham physmes — Jeff, Leo, Ben, Paula, Paul, Mike, Jai, Simon and Mark, and, of course, Nicola — we staggered through the Clarendon together and the rest is history.

At QMW I have to thank all those I’ve been taught by, worked with and drunk coffee with. Most notably Ann Cook, without whom I would never have made it; Claire and Elaine, who have always been nice to me; the numerous members of Plasma Group, who made it fun and helped me to understand it; the SSD Group, for letting me play, and Andy Lawrence, for his wise words. Thanks to Don Ellison for his excellent hospitality at NCSU. Cheers to PPARC for paying up.

Then we come to those without whom it would have been sad, bad, probably without fruit and altogether more coherent. Going back a bit we have strange times with Steveeee, Alan, Bob, Daniel and co. Eamonn, Richard and Fluffy showed how the game is supposed to be played. All those who’ve come and gone — Mitch (who’s hard), the strange world of Tolis, Simon and Terry (with special thanks to Terry for learning Postscript), Vaggelllis, Jon G., Matt B., Barry, Matty P., John, Vasso, Gary, Andy W., Henk, Jim, Amanda, Sarah, Tom B., Rory, Ricardo, Gerardo, Helena, Othon, Sylvia, etc. etc. — you’ve all added your own special madness to the process. Thank you to my
comrades in arms: Xochi, who proved it was possible, Ros, who read the whole thing(!), and Garry, a casualty but looking well on it. Ian M., for driving me up the wall. To Clare H./L. ever supportive, Sean ever inspiring and Tom ever ready — what can I say? You put up with me moaning about the food for longer than you should. I thank you all.

To Jim F. (mover and shaker), Dougie D. (beacon of hope) and Jon G. (the other one), who dragged me away from it all when I needed it, I say mine’s a whisky.

Words fail me when I try to thank you Nicola for your endless patience, support and love. Without you I could never have got this far. You even typed some of it.

Once again, I thank you all, even the ones I’ve forgotten.
To Mum and Dad, for everything, and Nicola, for making sense of it all.
‘The first ten million years were the worst,’ said Marvin, ‘and the second ten million years, they were the worst too. The third ten million I didn’t enjoy at all. After that I went into a bit of a decline.’

The Restaurant at the End of the Universe by Douglas Adams.
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Chapter 1

Introduction

Almost all of the observable universe is in the plasma state. Therefore, the study of the, often complex, behaviours exhibited by plasma systems has applications in many fields.

In this chapter we describe the near Earth plasma environment and the phenomena observed within it which provide the motivation for the work presented in this thesis. We introduce the subjects we explore in subsequent chapters and the methods used for this exploration.

1.1 Space Plasmas

Space plasma physics is the study of the near-Earth plasma environment with its wide variety of plasma and magnetic field conditions.

The supersonic plasma outflow from the solar corona, the solar wind, is observed throughout the Solar System. Looking at the properties of the solar wind, a typical space plasma, we find that it has a mean free path which is of the order of the distance from the Earth to the Sun. Such a plasma is thus collisionless when considered on the scales applicable to the study of the Earth’s plasma environment. A collisionless plasma’s behaviour is determined by long range interactions with binary collisions being largely irrelevant. The solar wind at 1AU is composed of a mixture of electrons and protons each at a number density of \( \lesssim 10 \text{cm}^{-3} \) at a temperature \( \sim 10^{5} \text{K} \), with an admixture of ionised helium and heavier elements (totaling less than 5% by number). The bulk flow of the solar wind at this radius is near radial at \( \sim 450 \text{km} \text{s}^{-1} \) with an embedded magnetic field of \( \sim 10^{-8} \text{T} \).
As they are electrically charged, the particles of a plasma interact with electromagnetic fields. Thus, the Earth and its intrinsic magnetic field constitute an obstacle to the solar wind, due to which, a bow shock forms as the solar wind makes the transition from super-sonic to sub-sonic speed, upstream of the Earth. The pile up of the magnetic field in this region leads to large field amplitudes and gradients which can cause effects such as particle acceleration and heating of the plasma. These phenomena represent free energy which can drive the unstable growth of waves. In turn, these waves can alter the characteristics of the plasma which generated them, through processes such as scattering. This thesis studies these instabilities, the waves they generate and the subsequent evolution of the wave-plasma system.

1.2 Plasma Instabilities

A magnetized thermal plasma with a Maxwellian distribution function consisting of protons and electrons is stable in the sense that apart from thermal noise no oscillations will spontaneously grow within the plasma. Most modes of plasma oscillation within such a system are damped, and as the population is thermally relaxed it is in a high entropy state with no free energy with which to generate waves.

We may now consider what happens if we introduce a source of free energy into such a system. Sources of free energy may be temperature anisotropies, density inhomogeneities, or the bulk motion of a particle beam. Here we will consider this last case. A particle beam’s ordered motion represents free energy which can, through the process of resonant interaction, provide modes of the system with energy for growth. Waves grow in amplitude using the beam kinetic energy lost as the beam slows down.

Various type of beam instability exist, the dominant type in a given plasma is dictated by the properties of both the beam and the background plasma. Beams can give rise to many different types of waves, many of which are observed in the near-Earth environment. Linear theory can predict the properties of low amplitude waves initially generated by such instabilities, e.g. a slow moving beam with large thermal spread will excite LH circularly polarized waves travelling in the same direction as the beam, whereas a fast moving ion beam with little thermal spread will tend to excite RH circularly polarized waves travelling in the same direction as the beam or waves with a poorly defined sense
CHAPTER 1. INTRODUCTION

of circular polarization travelling in the opposite direction depending on its exact speed and density.

1.3 Parametric Instabilities

In both spacecraft observations and simulations of ion beam instabilities waves are seen to appear which are not thought to be unstable to the types of free energy driven instabilities discussed in the previous section. One possibility is that these waves are the products of parametric instabilities (or wave—wave instabilities), where several waves couple and energy is passed between them. Such processes are not found in linear theory. Their role in the post-linear evolution of resonant instabilities is one that we explore in this thesis. We elaborate on the properties of these instabilities in Chapter 5.

A parametric instability is one in which some mode of a system is driven unstable by an externally applied oscillation of one of the parameters of the system (e.g. the magnetic field) at a different frequency. For example, in a plasma a large amplitude Alfvén wave can couple nonlinearly to excite waves of other types. Parametric instabilities have been explored in detail using fluid models (e.g. Viñas & Goldstein, 1991a,b), however, these models do not include kinetic effects and though kinetic models with various approximations have been explored (e.g. Inhester, 1990), fully kinetic treatments are very difficult, so a simulational approach is often taken, e.g. Terasawa et al. (1986) and Vasquez (1995). Treatments in two dimensions (e.g. Viñas & Goldstein, 1992) or three dimensions are useful as they open many more possible decay channels than a one-dimensional treatment, channels which are, of course, available in a real system.

1.4 Methods of Enquiry into Plasma Behaviour

In this thesis we use two main methods to predict the evolution of plasma instabilities: linear theory (using the WHAMP program) and computer simulation.

1.4.1 Linear Theory

The properties of waves supported by a particular plasma are described by its dispersion relation $\omega = \omega(k)$. Taking plane harmonic waves described in the form $e^{i(kx-\omega t)}$ the real and imaginary components of the frequency $\omega = \omega_r + i\gamma$ specify the real frequency
and the wave growth or damping at a particular wavenumber vector, $k$, a vector $|k| = 2\pi/\lambda$ lying perpendicular to the phase fronts of the wave.

The linear dispersion relation, describing the behaviour of waves of small amplitude ($\delta B/B_0 \ll 1$), can be calculated using the linearized Vlasov-Maxwell equations and the initial distribution function, see e.g. Gary (1991). This dispersion relation then predicts the unstable waves in such a plasma. Once the dispersion relation for a wave is known it is a simple matter to calculate other quantities which describe various properties of the wave—so called transport ratios. The transport ratios used in this thesis are described in Appendix A.

Various plasma models can be used to calculate the dispersion relation, however the work of Krauss-Varban et al. (1994) indicates that large discrepancies are found when the results obtained from a two-fluid model and the kinetic model are compared. This result implies that a kinetic treatment must be used to obtain reliable results over the full range of plasma parameters.

In a fully kinetic treatment, such as Gary (1991), the linear dispersion equation is obtained by combining the linear Vlasov equation with a zeroth order distribution function to obtain an expression for the conductivity of the plasma ($\sigma$). This can then be used in the wave equation, derived from Maxwell’s equations, to obtain a dispersion relation which gives the properties of the wave modes supported by the plasma specified by the zeroth order distribution function.

1.4.2 WHAMP

The program WHAMP (Waves in Homogeneous, Anisotropic, Multicomponent Plasmas) developed by Rönnmark (1982) is a program which numerically solves the linear dispersion relation of waves in a homogeneous magnetized collisionless plasma. It allows a zeroth order distribution function to be constructed from linear combinations of Maxwellian velocity distributions, each of which may have its own density, particle mass, temperature, anisotropy and drift velocity parallel to the background magnetic field. The particles must be non-relativistic.

The program works by introducing a Padé approximant for the plasma dispersion function which allows the infinite sum of modified Bessel functions in the dielectric tensor ($\epsilon$) to be reduced to summable form, which is then solved numerically by the
application of one of a variety of methods, depending on the particular parameters. The approximation will not solve the dispersion relation for heavily damped modes $(\text{Im}(\omega) > -k||, -\text{Re}(\omega - n\Omega))$. A full description of the algorithm can be found in Rönnmark (1982).

The beam instability dispersion relations we present in this thesis are all calculated using WHAMP.

1.4.3 Computer Simulations

In order to examine the behaviour of plasmas beyond the limits of analytical theories, computer simulations of plasma behaviour are often used. Many of the subject areas examined in this thesis have been simulated in the past using two broad approaches; fluid and particle. In the fluid approach a plasma is simulated by self-consistently solving the time evolution of a subset of Maxwell's equations and the fluid equations. However such an approach misses crucial aspects of the plasma's behaviour and it is often advantageous to perform a particle simulation by replacing the fluid equations with a set of equations of particle motion. This particle approach is physically accurate but computationally intensive as the ion and electron dynamics have vastly different timescales, by virtue of their large mass ratio, the shortest of which must be resolved either by using a very short timestep or a more computationally complex algorithm such as implicit methods. So when only ion dynamics are important, as is often the case in space plasmas, a hybrid simulation technique is used, where the ions are treated as particles and the electrons as a massless charge neutralising fluid. In such a scheme ion dynamics are well modelled at low computational cost as only the longer ion timescales must be resolved.

A real plasma will consist of far more particles than it is possible to model on any current computer, so any particle or hybrid simulation applied to a plasma will have to group these particles into macro-particles representative of the motions of many real particles with similar phase space trajectories. However, if many interesting and physically important regions of parameter space are to be examined then simulations must still be run with many macro-particles, and this requires the use of powerful computer resources. It is now the norm that such resources are in the form of parallel computers and existing serial algorithms will either not run or are inefficient on such machines. This has driven the need for suitable parallel algorithms. The development of a parallel algorithms for
plasma simulations is one of the topics of this thesis.

1.5 The Earth’s Plasma Environment

In this section we will describe the interaction of the Earth and the Sun and how this leads to the observed morphology of the near Earth plasma environment. We shall pay particular attention to the region just upstream of the terrestrial bow shock, the foreshock, and the behaviour of ions and waves therein, as these form much of the motivation for the work presented in this thesis.

The Sun possesses an intrinsic magnetic field, however the solar atmosphere extends and expands into interplanetary space in the form of the solar wind, a collisionless plasma formed from protons and electrons with a small proportion of heavier ions (\(\lesssim 20\%\) by mass). As the solar wind is a good conductor the theory of ideal MHD is applicable and
the magnetic field is ‘frozen in’ to the solar wind outflow. As a result of the combination
of this near radial motion with the sun’s rotation (~25 days) the solar field forms a spiral
pattern through interplanetary space. The ‘Parker Spiral’ leads to an angle between the
Sun-Earth line and the interplanetary magnetic field (IMF) of typically 45° at 1AU.

The Earth also possesses an intrinsic magnetic field, which is ablated by the effect
of the solar wind ram pressure at its sub-solar point and the drawing out of the dipole
into a tail (the ‘geotail’) through magnetic connection with the IMF. As the solar wind is
supersonic at 1AU a collisionless bow shock is formed. As the upstream and downstream
sides of this bow shock are magnetically connected energetic particles can travel back
upstream into the unshocked solar wind, allowing leakage and multi-crossing acceleration
mechanisms to generate particle beams travelling back upstream, away from the shock,
into the region known as the foreshock. The nature of the shock varies dramatically with
the changing shock geometry associated with a curved bow shock. The most important
parameter is the angle between the upstream magnetic field and the normal to the shock
\( \theta_{Bn} \). From \( \theta_{Bn} \) the shock can be described as parallel \( \theta_{Bn} = 0° \), quasi-parallel \( 0° < \theta_{Bn} < 45° \), quasi-perpendicular \( 45° < \theta_{Bn} < 90° \) and perpendicular \( \theta_{Bn} = 90° \). All these
features can be seen in Figure 1.1. A thorough review of the properties of the solar wind,
IMF and the acceleration mechanisms at work at the bow shock can be found in, e.g.,

Over the last few decades in situ spacecraft have provided observations which have
allowed the identification of this morphology. Recently, multi-spacecraft missions such
as ISEE and AMPTE have enabled us to separate the temporal and spatial variations
in this complex dynamic system, crucial to the correct identification of the properties of
observed waves, a program that will continue with the rebirth of CLUSTER.

Many sources of free energy, capable of generating waves, exist in the solar-terrestrial
environment, one of the most ubiquitous being particle beams. Such beams have been
observed in the solar wind, at and around the terrestrial bow shock and within the
geotail’s plasma sheet boundary layer. Elsewhere in the solar system particle beams are
found near comets, interplanetary shocks and at other bodies’ bow shocks. A thorough
review of these observations can be found in Gary (1991) and references therein.
1.5.1 The Earth's Foreshock Region

Upstream of the Earth's bow shock is the foreshock region; the portion of the upstream region which is accessible to energetic ions from the bow shock. The foreshock is partially organised by the IMF orientation, the bow shock's shape and the velocity filter due to the ratio of the crossfield drift to the speed of particles upstream along the magnetic field lines. The upstream boundaries of these regions are delineated by how far upstream backstreaming particles of a particular species are observed, this being determined by the ratio of the speed of the most energetic particles upstream along a field line to the \( \mathbf{E} \times \mathbf{B} \) drift velocity due to the solar wind's motional electric field and the interplanetary magnetic field (IMF). The large ion/electron mass ratio means that, at the same energies, electrons have a much higher velocity than ions along a field line and thus remain close to the tangent line of the IMF to the bow shock surface, whilst the slower ions convect further downstream. Thus the foreshock can be divided into the electron foreshock and the ion foreshock. With the IMF cone angle \( \sim 45^\circ \) as in a typical Parker spiral configuration the foreshock volume will be located upstream of the dawnside bow shock as illustrated in Figure 1.1. The location of the ion foreshock is complicated by the different ion reflection and acceleration mechanisms which depend on \( \theta_{Bn} \). The ion foreshock is observed to be anchored to the bow shock behind the electron foreshock boundary at \( \theta_{Bn} \approx 50^\circ \) (Le & Russell, 1992b).

Below we examine the different regions of the terrestrial foreshock in turn. In addition to the work cited in those sections a recent review providing an overview can be found in Onsager & Thomsen (1991).

1.5.2 Observed Waves and Particles in the Terrestrial Foreshock

Wave and particle observations are closely related as particles can generate waves through instabilities and waves can disrupt the particle distribution function.

Most of the observations described here are from ISEE 1 and 2 (Tsurutani & Rodriguez, 1986). The close proximity of the two spacecraft due to their similar orbits has allowed the separation of the spatial and temporal variations in the data allowing e.g. the rest frame polarizations and frequencies of waves to be calculated (Hoppe & Russell, 1983).

The spatial ordering of the foreshock leads to observations of strongly differing particle
distribution functions and waves at different points. Waves are both generated locally and propagate in from other regions of the foreshock, assuming that they remain undamped under changing foreshock characteristics. This makes the determination of foreshock wave origins a complex exercise.

A recent review of the waves observed in the foreshock can be found in Burgess (in press).

**Electron Foreshock**

The electron distribution function can vary from an energetic beam at the foreshock boundary to a backstreaming heat flux deep within the foreshock. Here we describe the waves commonly observed in the electron foreshock.

Electron plasma oscillations, or Langmuir waves, are seen throughout the electron foreshock, with frequencies near $\omega_e$, (e.g. Etcheto & Faucheaux, 1984; Lacombe et al., 1985; Onsager et al., 1989). At the upstream edge of the electron foreshock they are intense and narrowband, but with distance from the boundary they become more broadband and downshifted in frequency. Electron beam instabilities are accepted as the source of the waves, a view supported by theoretical work (e.g. Dum, 1990a,b).

Harmonics of the electron plasma frequency, most prominently at $2\omega_e$, are observed both upstream and within the electron foreshock as freely propagating electromagnetic radiation (Lacombe et al., 1988). They are commonly believed to be the product of the non-linear interaction of oppositely directed Langmuir waves (Cairns, 1988). Higher harmonics are more rarely observed (see Klimas, 1990, and references therein).

Whistler bursts with a frequency of 40-100Hz are observed for intervals of 2-10s (Anderson, 1981; Hayashi et al., 1994). The source of these bursts requires further investigation as heat flux, electron beams or ion beams could all be responsible (Tokar & Gurnett, 1985).

Reviews of the properties of the electron foreshock can be found in, e.g., Klimas (1985) and Fitzenreiter (1995).

**Ion Foreshock**

In this section we will describe the variety of suprathermal ion distributions and associated waves which have been observed in the ion foreshock, as depicted in Figure 1.1.
Reviews of this material can be found in, e.g., Thomsen (1985), Fuselier (1995) and Greenstadt et al. (1995). Note that all symbols used are defined in Appendix C.

Field aligned beams can be found at the upstream edge of the ion foreshock (Asbridge et al., 1968). These beams are typically tenuous with a density $\sim 1\%$ of that of the solar wind, energetic with speeds $\sim 2 \sim 3v_{SW}$, cool with temperatures of $\sim 7 \times 10^6$K such that $v_e < v_b < v_{th}$ (Bonifazi & Moreno, 1981a,b) and beam temperature anisotropies of $T_{\perp b}/T_{\parallel b} \sim 2 - 3$ (Paschmann et al., 1981, note that in this paper the thermal speed anisotropy is erroneously given as the temperature value). The origin of this population is believed to be a combination of the multiple-reflection and shock drift acceleration of a small fraction of the incident solar wind ions (these include the most energetic beam ions) and leakage upstream of magnetosheath ions (Schwartz & Burgess, 1984; Thomsen, 1985).

Gyrating ions (Thomsen et al., 1985) have peaks in the velocity distribution function at nonzero pitch angles. They can be split into the gyrotrropic ions (ring beams), which are discussed below with reference to specularly reflected ions, and the non-gyrotrropic ions (gyrophase bunched) which are found further into the foreshock and are a characteristic signature of resonant wave growth with the associated ULF waves and will be discussed at length later in this work. Gyrating ions have been misclassified on occasion as intermediate ions (Thomsen et al., 1985; Fuselier et al., 1986a,b).

Intermediate ions (Paschmann et al., 1979) resemble field aligned beams but slowed to $\sim 1.75v_{SW}$ and with a larger thermal spread ($T \sim 2 \times 10^7$K), especially in pitch angle, to form a 'kidney beam' type distribution in velocity space. They are generally believed to be the product of the evolution of the field aligned beam through scattering by the associated ULF waves, as will be discussed later in this thesis. Leakage upstream from the magnetosheath may produce intermediate distributions on occasion (Edmiston et al., 1982).

Diffuse ions (Gosling et al., 1978) are observed upstream of the quasi-parallel bow shock, deep in the foreshock. They are tenuous with a density $\sim 1\%$ of that of the solar wind, slow with speeds $\sim 1.2v_{SW}$, hot with temperatures of $\sim 4 \times 10^7$K such that $v_e < v_{th} \ll v_b$, and near isotropic with $T_{\perp b} \sim T_{\parallel b}$ (Paschmann et al., 1981; Bonifazi & Moreno, 1981a,b). It has been suggested that such diffuse distributions were the end point of field aligned beam evolution through their interaction with associated ULF waves.
(e.g. Gosling et al., 1984), however, recent observations of the foreshock when the IMF cone angle is close to 0°, when no field aligned beams enter the quasi-parallel foreshock, still show evidence of diffuse ions (Ellison & Mobius, 1987). Additionally, measurements of composition show the diffuse population to have a concentration of He$^{2+}$ near to that found in the solar wind, whereas field aligned beams contain little He$^{2+}$ (Ipavich et al., 1984). These observations are used to support the suggestion that field aligned beams are not the most important source of diffuse ions in the foreshock (Fuselier, 1995, and references therein).

Specular reflection of solar wind ions (Gosling et al., 1982; Paschmann et al., 1982) can produce gyrophase bunched distributions of up to 20% of the incident ion density. Such distributions are trapped within a gyroradius of the quasi-perpendicular shock but at the quasi-parallel shock they can escape upstream and evolve into a gyrotropic distribution through gyrophase mixing in the turbulent foreshock to give a ring beam (Gurgiolo et al., 1983). Such specularly reflected ions are suggested as a source of the diffuse population as it could provide explanations for the observations described in the previous section (Fuselier, 1995, and references therein).

ULF electromagnetic waves (‘30s waves’) are observed in the ion foreshock with large amplitudes ($\delta B/B_0 \sim 1$), frequencies between 5mHz-0.1Hz, peaking around 0.02Hz ($\sim 0.1\Omega_i$) and wavelengths of order $1R_E$. These waves are almost monochromatic near the ion foreshock boundary, where they are seen in association with field aligned beams, gyrophase bunched and intermediate ion distribution. They becoming more broadband and compressional, deeper into the foreshock, as the quasi-parallel is approached, with steepened shocklets with whistler wave trains and diffuse ion distributions are observed (Le & Russell, 1992a; Hoppe et al., 1981). The polarization of the more monochromatic modes are almost always RH in the plasma frame, where as the compressive modes exhibit a mixture of LH and RH polarizations (Blanco-Cano & Schwartz, in press; Hoppe & Russell, 1983), results consistent with the observed association of waves and particles (Sentman et al., 1981). ULF waves are often seen to propagate at quasi-parallel oblique angles (typically $20^\circ - 40^\circ$, Hoppe et al., 1981) to the background magnetic field, apparently at odds with the findings of linear theory for ion beams where maximum growth rates are found to be at parallel propagation (e.g. Gary, 1991). Mechanisms such as refraction have been suggested to explain this (Hada et al., 1987). The problem of oblique
waves will be examined later in this thesis.

Large amplitude pulsations appear as monolithic coherent structures propagating upstream but being swept back by the solar wind, with durations 10–20 s, large magnetic fields ($\delta B/B_0 \sim 5$) and mixed polarizations (e.g. Schwartz et al., 1992; Schwartz, 1991). Amongst these phenomena are those that have been termed ‘short large amplitude magnetic structures’ or SLAMS by Schwartz & Burgess (1991). Large amplitude pulsations appear embedded in the type of compressive ULF waves associated with diffuse ion populations out of which they appear to have grown, possibly through some effect associated with proximity to the quasi-parallel shock of which they are believed to form an integral part (Burgess, 1995, and references therein).

The $\delta$s waves are a recent addition to the bestiary of waves (Le et al., 1992) appearing only in certain solar wind configurations (high $\beta$). They consist of packets of circularly polarized near planar waves, like shocklet associated whistlers but LH polarized. They propagate within $\sim 20^\circ$ of the magnetic field direction but are convected back towards the shock by the solar wind. Their source is unknown.

1Hz whistlers are small amplitude ($\delta B/B_0 \sim 0.2$) whistlers in the 0.5–4 Hz frequency range with wavelengths $\sim 100$ km propagating within $20^\circ - 40^\circ$ of the background field in either direction (e.g. Orłowski & Russell, 1995). They are associated with the quasi-perpendicular shock and it has been suggested that they are emitted by the bow shock (Krauss-Varban et al., 1995) either through a shock ramp instability or some instability driven by reflected gyrating ions.

Ion acoustic waves are observed throughout the diffuse ion foreshock with frequencies in the 1–10 kHz range (Gurnett & Frank, 1978; Rodríguez, 1981). There is no clear mechanism for their generation.

1.6 Overview of Thesis

In this thesis we present a record of our development of novel simulation tools and the results of new simulations that these tools have allowed us to perform in the area of ion beam and parametric instabilities.

In Chapter 2 we give a summary of the simulation of plasmas with computers and the factors which determine the choice of model for such simulations. We present the
algorithms which we have developed to allow simulations to be performed on computers with parallel architectures such as massively parallel processors (MPPs) like the CRAY T3D or clusters of networked workstations, work which has enabled us to perform two-dimensional simulations of ion beam instabilities at sufficiently fine spatial scales and low numerical noise levels to allow us to observe phenomena previously hidden at unresolved scales or lurking below the noise level. The PVM system used to implement our parallel algorithms is described in Appendix B.

In Chapter 3 we describe the physics of ion beam instabilities and some approaches available to the theoretician for their exploration, including linear theory and previous simulation work in the field. Appendix A defines the transport ratios we use to describe plasma and wave properties.

In Chapter 4 we give a detailed account of the results of our simulations of the instabilities of cool tenuous proton beams in magnetized plasmas, in both the isotropic and anisotropic beam regimes. We examine the evolution of waves and particles, correlations between the two and the wave coherence properties. We go on to examine in detail the properties of two phenomena, a type of short scale magnetic field feature and shocklets, previously unresolved in two-dimensional beam simulations, and examine their role in instability saturation and their consequences for the interpretation of spacecraft data.

Chapter 5 explores the importance of parametric, or wave–wave, instabilities in the evolution of beam instability generated waves and extends the work of Terasawa et al. (1986) into two dimensions.

Finally in Chapter 6 we summarise our work and its conclusions.

Definitions of transport ratios we use, a description of the PVM message passing interface and tables of symbols and acronyms used in this work are included as appendices.
Chapter 2

Computer Simulation of Plasmas

In this chapter we present the background to the simulation methods used to produce the results presented elsewhere in the thesis. After a brief justification for the use of simulation we outline some of the methods used and present in detail the CAM-CL algorithm used in our work. We give a description of the techniques we have developed to facilitate the move to running simulations on computers with parallel architectures.

2.1 Why is Simulation Necessary or Useful?

The computer simulation of plasmas is useful as it allows the exploration of plasma behaviour beyond analytically tractable models and into regions of parameter space that are inaccessible in the laboratory. It serves useful roles in the analysis of complex spacecraft data, the evaluation of models of observed phenomena and in driving theoretical work.

Plasma simulations have proved their worth in many areas: observations of cometary environments have been explained in a self consistent way, instabilities have been followed into the non-linear regime where theory is invalid, inaccessible astrophysical plasma behaviour has been modelled and the problems of controlling fusion plasmas in Tokomaks have been addressed.

A simulation’s closeness to the real world is limited by temporal and spatial resolutions dictated by available computational resources, the limitations of computer arithmetic and the fact that only a limited subset of known physics is included.
2.2 Types of Simulations

Many algorithms exist for the simulation of plasma behaviour. The available computational resources and the physics considered important in the situation to be modelled dictate the type of simulation chosen. Reviews of the many available schemes can be found in Tajima (1989), Birdsall & Langdon (1985) and Lembege & Eastwood (1989). Below we follow the classification used in Eastwood (1993).

The design of an plasma simulation algorithm must proceed through several steps. The physical phenomena to be simulated must be mathematically modelled in such a way that the important physical processes are included—simplifying assumptions are made. This model will usually be expressed as a set of coupled differential equations describing the temporal and spatial evolution of the plasma from initial and/or boundary conditions. These differential equations must then be discretized to give algebraic approximations suitable for solution by numerical methods. This design process is always a compromise between the quality of the representation of the physical processes and the available computational resources.

Available mathematical models include: Vlasov — the Vlasov equation and various approximations to Maxwell's equations, suitable for collisionless plasmas. Kinetic — collisional terms are added replacing the Vlasov equation with the Fokker-Planck equation. Two-fluid — The plasma components are described using their first two moments: mean velocity and thermal velocity. Magnetohydrodynamic (MHD) — the electron mass is set to zero so the electron equation of motion becomes an Ohm's Law, the moment equations become those of MHD.

In choosing a model we must consider three factors: What processes must be included? What are their dimensionality? What are their length and time scales? Scales of interest must be modelled, processes on longer timescales can be ignored but those on shorter timescales cannot, they must be suppressed by the model, integrated using an implicit scheme or timescales must be compressed (e.g. by artificially lowering the ion/electron mass ratio).

Once a suitable model has been chosen the equations in continuous variables must be approximated to a set of discrete algebraic equations for sets of values. These equations form the basis for the implementation of the simulation algorithm.

Various approaches to this discretization process exist, each suited to certain types
of problem. The methods span a wide range of complexity and accuracy, so the issue of available computational resources will often dictate the method chosen and thus the accuracy.

Finite Difference Approximation (FDA) — the continuum is replaced by values on a lattice of points, the derivatives by value differences and the equations by difference equations on the lattice. FDA has the advantage of simplicity in derivation and computation, but introduces undesirable numerical dispersion and diffusion and is susceptible to nonlinear instabilities. Almost all time dependent calculations use FDA for temporal calculations.

Finite Element Methods (FEM) — more sophisticated than FDA. The continuum is replaced by local piecewise polynomials. Leads to optimal schemes, easily generalized to high accuracies, but computationally expensive.

Spectral Methods — the continuum is replaced by expansions using global orthogonal polynomial basis functions, the derivatives by the derivatives of the approximating polynomials and the equations by projections onto the basis functions (spectral) or onto meshes (pseudo-spectral). If simple boundary conditions allow fast transforms to be used spectral methods can be very fast and accurate.

Particle (Particle In Cell (PIC)) — the continuum is replaced by a set of sample points (particles), the derivatives by one of the above methods, the hyperbolic equations by ODEs for particle trajectories. PIC is a combination Eulerian and Lagrangian methods.

The hyperbolic terms in the differential equations are advanced in a Lagrangian way with a set of random sample points (particles) carrying attributes such as mass, position and charge. Parabolic terms and elliptic terms are dealt with using Eulerian meshes by way of one of the methods outlined above. The two parts of the calculation are linked using interpolation and distribution: mesh point values are interpolated to particle positions and vice versa.

2.2.1 Choice of Model

Ion beam instabilities in collisionless plasmas depend upon effects due to wave-particle resonance, so models where only moments of the particle distributions are considered are incapable of modelling them correctly. A Vlasov model is thus required. The Vlasov model inherently lends itself to a PIC treatment as advection is correctly modelled and
it avoids the need for a computationally expensive velocity grid. The Vlasov equation becomes simply the equations of motion for charged massive clouds of particles (macroparticles) and Maxwell's equations can be solved on a mesh.

This simple application of the PIC method to a Vlasov model of an ion/electron plasma has certain difficulties associated with it. The ion/electron mass ratio is at least three orders of magnitude, leading to a similar ratio in the characteristic frequencies. However, as noted above, we must follow the evolution on the shortest timescale of the system, the electron motion, even if we are only actually interested in the vastly longer timescale associated with the ions, this is clearly an inefficient method.

We can alleviate this problem by use of a “hybrid” method; one that models different species in the problem using different methods. If we can show the electron dynamics are unimportant to the evolution of an instability we can model the electrons as a massless charge neutralizing fluid, retaining only the ion macroparticles. Such a system can be advanced on a timestep related to ion timescales. Additionally, free space electromagnetic waves (light) are of little interest as they typically pass through a simulation box in a time far shorter than any timescale of interest with little interaction, so they may be treated as waves with infinite velocity. This is achieved by neglecting the displacement current term in Ampere’s Law (the Darwin Approximation).

Computational considerations have forced this last approach upon many previous kinetic simulations and to some degree on the work presented in this thesis. However, this is not particularly problematic as the beam generated phenomena under examination are dominated by ion dynamics and ULF waves ($\omega \ll \omega_e, \Omega_e$).

2.3 CAM-CL: a Hybrid Code

Our chosen algorithm is CAM-CL (Matthews, 1994) the main features of which are outlined below. CAM-CL uses a standard hybrid model, as described in the next section, advanced using a moment method (CAM - Current Advance Method) similar to that described by Winske & Quest (1988) and substepping as described by Terasawa et al. (1986) combined with a modified midpoint method (CL - Cyclic Leapfrog).
2.3.1 The Hybrid Model

A normal hybrid code has two time dependent components: a number \(N\) of ion macroparticles, each with position \((x)\) and velocity \((v)\), and the magnetic field specified at the nodes of a grid. The source terms (charge density and current density) are calculated from the particle moments. To advance these components the following set of equations is used:

\[
\begin{align*}
\dot{x}_i &= v_i \\
\dot{v}_i &= \frac{q}{m}(E + v_i \times B) \\
\frac{\partial B}{\partial t} &= -\nabla \times E \\
\nabla \times B &= \mu_0 J
\end{align*}
\]

with \(i = 1..N\), giving \(N\) sets of equations (2.1) and (2.2).

Note that the displacement current term \((\partial E/\partial t)\) has been neglected in 2.4 (the Darwin Approximation), so there is no equation for the time evolution of the electric field. Instead, \(E\) is calculated from the balance of forces on the massless electron fluid, as outlined below.

For the electron fluid, with density \(n_e\) and velocity \(u_e\), we have the equation of motion:

\[
n_e m_e \dot{u}_e = -n_e eE + J_e \times B - \nabla p_e
\]

where \(m_e = 0\), \(en_e = \rho_e\), \(J_e = -n_e e u_e\) and \(J = J_i + J_e\). Various electron equations of state can be used to obtain the electron pressure \((p_e)\), e.g.:

\[
p_e = n_e kT_e \quad \text{(Isothermal)} \\
p_e = n_e^\gamma kT_e \quad \text{(Adiabatic)}
\]

Using (2.5) and (2.4) we obtain an expression for the electric field

\[
E = -\frac{J_i \times B}{\rho_e} + \frac{(\nabla \times B) \times B}{\mu_0 \rho_e} - \frac{\nabla p_e}{\rho_e}
\]

which is used to calculate \(E\) in the hybrid model, as \(E = E(\rho_e, J_i, B, T_e)\) is clearly a state function.
Combining (2.8) with (2.3) we obtain the equation for the time evolution of the magnetic field:

\[
\frac{\partial B}{\partial t} = \nabla \times \frac{J_i \times B}{\rho_e} - \nabla \times \frac{(\nabla \times B) \times B}{\mu_0 \rho_e}
\]  

(2.9)

the first term describes induction, the second dispersion. Note that the electron pressure does not directly influence magnetic field evolution.

### 2.3.2 Current Advance Method

A problem is encountered when advancing the particles: The set of equations used to advance the macroparticles, 2.10 and 2.11, is clearly implicit in \( v \) (\( E = f(v) \) via \( J \)).

\[
x^{1/2} = x^{-1/2} + \Delta t v^0
\]  

(2.10)

\[
v^1 = v^0 + \Delta t (E^{1/2}(x^{1/2}) + v^{1/2} \times B^{1/2}(x^{1/2}))
\]  

(2.11)

In order to get around this the midpoint method makes a first-order half-step:

\[
v^{1/2} = v^0 + \frac{\Delta t}{2} (E^{1/2}(x^{1/2}) + v^0 \times B^{1/2}(x^{1/2})).
\]  

(2.12)

A suitable estimate of \( E^{1/2} \) is required. One solution is to perform a pre-push where a mixed time-level electric field (\( E^* = E(\rho^{1/2}, J^0, B^{1/2}, T_c) \)) is used to obtain an estimate of \( v^{1/2} \) accurate to first order in \( \Delta t \), but this requires two passes through the particle tables and is thus computationally inefficient. An alternative is a moment method (Winske & Quest, 1988) where the pre-push is eliminated by advancing an appropriate moment of the system using an equation of motion. The Current Advance Method is just such a method where the advanced moment is current density. This is more straightforward than previously proposed moment methods. In this case the ion current is advanced using a free-streaming current (\( J^i \)):

\[
J_i^{1/2} = J_i^0 (x^{1/2}, v^0) + \frac{\Delta t}{2} \frac{q_i^2}{m} (E^* + v^0 \times B^{1/2}).
\]  

(2.13)

From this estimate of \( J_i^{1/2} \) we can calculate the required estimate of \( v^{1/2} \).

### 2.3.3 Cyclic Leapfrog

The fields are advanced using the modified midpoint method (Press et al., 1986) where two copies of the fields are leapfrogged over each other \( B^{i+1} = f(B^{i-1}, E^i) \) on a substep
timestep. Matthews (1994) refers to such a scheme as Cyclic Leapfrog. Substepping allows fields and particles to be advanced on different timesteps in order to reflect the differing timescales in each, a technique introduced to hybrid codes by Terasawa et al. (1986).

2.3.4 Implementation

For the simulations presented elsewhere in this thesis we use a two-dimensional version of the CAM-CL algorithm implemented in parallel form, using the PVM system, on a cluster of Sun Workstations. This version has doubly-periodic boundary conditions, the adiabatic electron equation of state (2.7) and zero phenomenological resistivity.

2.4 Parallel Algorithms

2.4.1 The Need for Parallel Codes

Interesting kinetic plasma simulations increasingly require the use of large configuration spaces, three dimensional grids, and consequently, a large number of simulation particles. It is recognised that in future the only way to carry out extremely large simulations will be to use a parallel computer.

PVM or Parallel Virtual Machine provides a software system to allow a group of heterogeneous UNIX machines to work as a single distributed memory message passing parallel computer. The PVM system has been developed by a team at Oak Ridge National Laboratory since 1989 and is described in detail in Geist et al. (1994), a more detailed description of PVM can be found in Appendix B. PVM allows a network of varied workstations to operate in parallel, co-operating via the passing of messages between them. It can also be used on massively parallel processors (MPPs), where many identical processing nodes are linked by fast communication channels (e.g. the CRAY T3D). This offers a route to the solution of larger problems through parallel approaches, beginning on workstations which are often already in place for other purposes with easy conversion to specialized parallel machines. Here, we shall outline the application of parallel techniques to plasma particle codes through the use of PVM, though the approach described is valid for any message passing system.
2.4.2 Message Passing : a Parallel Paradigm

The history of parallel processing has been one of unfulfilled promise, in part because computational problems must often be reformulated in a manner totally unlike those suited to the serial approach required for traditional scalar processes. Recently, message passing between a number of computation nodes has become an an increasingly attractive paradigm for parallel processing. Each processor (node) of a parallel machine runs a process. These processes must cooperate to achieve some goal (such as running a large plasma simulation). In a message passing system this cooperation is arranged through the exchange of messages between processes. These messages typically contain data or requests for particular actions. Such an approach is attractive for problems where their exist operations with outcomes that are independent of one another and can thus be performed concurrently. Subroutines from existing serial code can be reused but running as separate processes concurrently on different nodes. The message passing approach has been encouraged by the appearance of computers with architectures tailored to its efficient implementation, such as the CRAY T3D and T3E.

In one possible structure for a message passing algorithm a master process controls a team of slave processes. Each slave has a portion of the total data involved in a problem, the master sends messages requesting operations to be carried out upon this data by each slave which then returns a message containing the results of that processing. This can be likened to a ‘subroutine call in parallel’. As a trivial example imagine that the mean of a vast list of numbers needs to be calculated. Each slave could be given an equal subset of the data which it would sum and return this subtotal to the master which would sum these subtotals and divide by the total number of data items to give the mean. In comparison to a single processor calculating the mean the whole process would occur faster by a factor of the number of slaves minus the communications overhead.

2.4.3 Application to Plasma Simulation

A typical particle simulation algorithm advances time by stepping through a sequence of operations: Field solving → Particle moving → Moment collection. Typically, particle movement and moment collection take much longer than field solving since there are many more particles than grid points. Thus, in a master-slave algorithm the most straightforward approach is to allocate a fraction of the total number of particles to each
CHAPTER 2. COMPUTER SIMULATION OF PLASMAS

SLAVE

\[ \begin{align*}
SOLVE & \rightarrow MOVE \\
E, B & \rightarrow \{x, v\}_\text{particles} \\
\end{align*} \]

\[ \begin{align*}
\{x, v\}_\text{particles} & \rightarrow COLLECT \\
\rho, J & \rightarrow SOLVE FIELDS \\
\end{align*} \]

MASTER

\[ \begin{align*}
\text{\textcircled{}} & = \text{Data passed as message} \\
\end{align*} \]

Figure 2.1: The flow of data (arrows) between the operations (boxes) in a hybrid simulation loop. The dotted line shows how the operations are split in a particle parallel scheme. Note that particle data remains within the slave’s domain — only field and moment data needs to be passed.

slave so that operations on the particles occur in parallel, the number of concurrent particle operations being equal to the number of slave processes. The master performs the field solving and provides each slave with a complete set of new field data at each time step. Particles are not grouped spatially - every slave has to have a complete set of field grids. This scheme is illustrated in Figure 2.1.

This is a particle parallel/field serial approach. We have used PVM to implement such a scheme for the CAM-CL two-dimensional hybrid code.

2.4.4 Parallelising a Serial Code Using PVM

We converted the two-dimensional code CAM-CL to a particle parallel program, code-name Barbie and Ken, running on a small group of networked (Ethernet) Sun workstations. Each slave (the Kens) moves a subset of the particles (a particle’s data remains on one machine — it is not passed around) and has a complete copy of the field data.

All field solving is carried out by the master (Barbie). The master and slave are set such that the master requests services which the slave provides — no requests are made by the slave. All master to slave messages prompt an immediate acknowledgement from the slave. This is not altogether necessary but eases debugging. The processing performed by the master and the slaves and the messages passed between them are shown in Table 2.1.
## Table 2.1

<table>
<thead>
<tr>
<th>MASTER ACTIONS</th>
<th>MESSAGES</th>
<th>SLAVE ACTIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>MAIN LOOP BEGIN</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Distribute field data</td>
<td>(E,B) ⇒</td>
<td>Unpack &amp; store fields</td>
</tr>
<tr>
<td>(Broadcast)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Await reply from all slaves</td>
<td>⇐ Received</td>
<td>Acknowledge fields</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Request particle push</td>
<td>Do particles! ⇒</td>
<td></td>
</tr>
<tr>
<td>&amp; moment collect</td>
<td>(Broadcast)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Await reply from all slaves</td>
<td>⇐ Doing</td>
<td>Acknowledge push &amp; collect start</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Particle push</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Moment collect</td>
</tr>
<tr>
<td></td>
<td></td>
<td>⇐ Done</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>REPEAT</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Request moment data</td>
<td>Send moments! ⇒</td>
<td></td>
</tr>
<tr>
<td>from a ‘DONE’ slave</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>⇐ (ρ,J)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Unpack and sum moments</td>
</tr>
<tr>
<td></td>
<td></td>
<td>UNTIL all slaves done</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>If it’s output time</td>
<td>Write output! ⇒</td>
<td>Output to file</td>
</tr>
<tr>
<td>(Broadcast)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Solve fields</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>MAIN LOOP END</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
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2.4.5 Efficiency

In testing, the field solver to particle pusher execution time ratio was around 1 : 10 — so around 90\% parallel efficiency was obtained. Note that efficiency is highest with the field solver on the fastest machine. A static load balancing scheme was used, with each slave allocated a proportion of the particles calculated from the speed of its host. This scheme is inefficient if one host becomes heavily loaded by other users since the other hosts have to wait for it. This is not a problem when hosts are supercomputer nodes.

2.4.6 The Problems of Scaling to Big Machines

A large massively parallel processor (MPP) such as the CRAY T3D typically has 128–2048 nodes each comparable in power to a workstation. Simply scaling the particle parallel approach to such a machine generates various problems and inefficiencies, arising from two main sources: firstly, having only a single master forms a communication bottleneck and means that field solving will be slow for large problems, with slave nodes waiting idle; secondly, large grids impose a high network load and grid storage on each machine will occupy a large amount of each slave’s available memory.

The solution to these problems lies with some kind of field parallel algorithm. Such an algorithm would split particles and grid spatially and then solve the fields on different sections of the grid in parallel. A particle moving out of one slave’s spatial domain would be passed to the slave dealing with the section of the grid containing its new position. Problems arise when density inhomogeneities develop, as equal grid spaces will not contain equal numbers of particles, leading to a load imbalance. In practice more complex (hierarchical and/or dynamic) decompositions are required. Such problems are complex and appear in many different situations, so the finding of efficient solutions to them has attracted a large amount of effort in recent years, see e.g. Fox et al. (1988). Liewer & Decyk (1989) and Ferraro et al. (1993) present work on non-uniform spatial domain decompositions to achieve load balance. However, their approach leads to large amounts of particle data being passed between processors as the particles move in space, accompanied by frequent repartitioning of the simulation space between processors requiring yet more movement of particle data. If a load imbalance at the field solving stage is too be avoided the spatial domain must again be repartitioned with further exchanges of field and moment data.
In an attempt to produce a load balanced algorithm which avoids these problems, we have devised a new semi-spatial decomposition in which the fields are spatially split equally between the processors, the particles are arranged to keep the number per processor the same and a message passing structure is implemented such that the movement of particles between processors is unnecessary. A processor can hold particles which are not located in the section of space for which the processor holds the field and moment data, this data must therefore be passed from processor to processor when it is required for particle pushes and field solving respectively.

If a simulation contains particles ($i = 1 \ldots N$) and is split across a number of processors ($j = 1 \ldots M$) we would proceed as follows. The spatial domain is split regularly across the processors such that each processor manages an equal portion of space. A processor knows which other processors hold adjacent portions of space so that boundary information can be exchanged in a pre-determined way without the need for extra messages. The spatial decomposition never changes.

Particles are split equally by number between the processors and a particle’s data remains in the memory of that processor even if it is not within the portion of space managed by that processor. In this way both the particle and field operations are load balanced. It can be imagined that a particle is labelled $P_{ijk}$, where $i$ is its identity (constant), $j$ is the processor holding its data (constant) and $k$ is the processor within whose portion of simulation space the particle is located (changes as the particle moves in simulation space). Thus when a particle is pushed or its moment summed messages need to be passed between processors $j$ and $k$, the holder of the particle’s data and the holder of the data on the portion of space where the particle is currently located in the simulation. In effect, load balance problems are replaced by a communications overhead.

Such an algorithm takes advantage of the real strength of MPP architectures such as that of the CRAY T3D: very high bandwidth bidirectional communication across the machine, allowing a single node to hold particles located in the spaces whose fields are managed by many other nodes, and thus requiring many node to node messages to be passed at very little communications overhead in computation time. However, such an algorithm must possess a degree of sophistication to ensure that all necessary messages are passed whilst maintaining synchronisation across the whole machine, avoiding problems such as deadlocking, where two nodes both await messages from each other before
continuing their processing.

With this algorithm, from the point of view of each individual node, a moment method simulation proceeds as outlined below:

- **Push particles:**
  - Make list of all nodes you need field data from in order to push your particles
  - Circulate this list to all other nodes
  - From lists received from other nodes compile list of which nodes require field data from you
  - Push particles and make field data requests to other nodes as required
  - Simultaneously answer requests for your field data from other nodes
  - After your particle push is finished answer all remaining field data requests until all the requests on your list are answered

- **Collect moments:**
  - Make list of all nodes managing the spatial domains for which you now hold particles
  - Circulate this list to all other nodes
  - From lists received from other nodes compile list of which nodes you require moment data from
  - Collect moments for own spatial domain
  - Collect moments for other nodes spatial domains in turn and send to that node
  - Simultaneously receive incoming moment data for your own spatial domain and sum it
  - Wait until all moment data you are expecting has arrived

- **Field solve:**
  - Exchange boundary cell information with spatial neighbours
  - Solve field for own spatial domain from accumulated moments
CHAPTER 2. COMPUTER SIMULATION OF PLASMAS

If the communications overhead becomes too great then periodically some sort of attempt to re-sort the particles to correspond more closely to their spatial arrangement, whilst maintaining load balance, may be required. Such a process could be triggered by some threshold in the number of communications taking place, which might be calculated at the “list-passing” stage. The mechanics this represent a tricky optimisation problem, possibly requiring one node to make global allocations or more than one attempt at the reorganisation. Our work suggests that on a 256 node T3D with communication between every possible pair of nodes at every step the communications overhead is insignificant—the T3D has very rapid communications.

2.4.7 Implementation on the EPCC CRAY T3D

The algorithm described in the previous section is based on an algorithm developed for the CRAY T3D at the Edinburgh Parallel Computing Centre (EPCC). The “particle push” section of the algorithm was implemented in full and was shown to work with an efficiency very close to that expected from a simple division of the total task by the number of participating nodes, i.e. communications overheads where negligible. This section of the algorithm tests all the mechanisms required by the moment collection section, so we believe that there would have been no fundamental difficulties with the implementation of this second stage or that of the relatively straightforward process of solving the fields.

On the basis of this work we believed that this algorithm could have been used to perform simulations of upwards of $10^8$ particles on timescales which, for a typical time award on the machine, would allow several runs a year. However, after this promising start, we reluctantly abandoned the project. The development work required was judged to be too great in comparison to the results we felt achievable in the very much reduced time awarded to us on the machine during the second round of allocations.

2.5 Summary

In this chapter we have presented simulation as a useful method in the exploration of plasma behaviour. We have described a broad selection of the simulation methods available to the prospective simulator and present in detail CAM-CL (Matthews, 1994),
a hybrid algorithm, with some appealing properties, which we have chosen to use in our simulation work.

Future big simulations will need to use parallel computers. Message passing libraries, like PVM, provide both emulation of a message passing distributed memory parallel computer on a collection of separate machines and supercomputer support, thus allowing straightforward conversion of serial codes to parallel operation, and providing a route for the development and testing of codes for supercomputers.

Parallel algorithms create various problems to be solved, e.g., load balancing and scaling to big machines, which demand the development of efficient field parallel algorithms.
Chapter 3

Electromagnetic Ion Beam Instabilities

3.1 Low Frequency Linear Instabilities of Ion Beams

In a homogeneous, magnetized, collisionless plasma with an ion distribution function consisting of two drifting Maxwellian ion distributions (a beam and a core, the core being denser than the beam) and a Maxwellian electron distribution, the free energy possessed by the beam can drive wave growth. Instability can occur when there is sufficient free energy in the system, e.g. the beam speed or temperature exceeds some threshold value. The nature of the waves so generated depends upon the properties of the plasma: beam density, beam speed, component betas, temperatures and anisotropies.

A given beam is described as hot or cold depending on the ordering of its various velocities. A cold beam has a definite separation between the beam and core ions \( v_c < v_b < v_{0b} \). For a hot beam these will overlap \( v_c < v_{0b} \ll v_b \). All symbols are defined in Appendix C.

Gary (1991) has reviewed the properties of the instabilities identified by linear theory for a large range of types of beams. Here we will follow the nomenclature outlined in that paper.

For tenuous isotropic beams \( n_b/n_0 \ll 1 \) three linear instabilities have been identified. Sentman et al. (1981) and Gary (1985) outline the linear theory for hot beams, whilst Gary et al. (1984) presents the linear theory for cooler beams. First we briefly describe instability properties at parallel propagation. The situation at oblique propagation is
more complex.

Resonance

To obtain some insight into the physical nature of an instability it is common to introduce a resonance parameter. For the particles of species $j$, with bulk velocity $v_{0j}$, thermal velocity $v_j$, interacting with a wave with frequency and wavevector $(\omega, k)$, the resonance parameter is defined as:

$$\zeta_j^m = \frac{(\omega - k \cdot v_{0j} + m \Omega_j)}{|k||v_j|}$$

(3.1)

with $m = (0, \pm 1, \pm 2, \ldots)$ corresponding to the order of the resonance (e.g. Gary, 1991). When $|\zeta_j^m| \lesssim 1$ a significant number of particles of species $j$ feel a strong resonance with the wave $(\omega, k)$ during its linear growth. The resonant velocity lies within the thermal peak of the distribution function. An instability for which this is the case is termed resonant. When $|\zeta_j^m| \gg 1$ all particles feel only a weak wave-particle interaction during the linear growth of the wave. An instability for which this is the case is termed non-resonant.

A wave-particle resonance occurs when particles are travelling with a velocity such that they have a constant relationship with the electric field of a wave—energy can then be transferred between the wave and particle. Consider the simplest case, that of Landau damping. In the presence of a longitudinal electrostatic wave a particle which travels near the phase velocity of the wave will experience a constant parallel electric field and will thus be either accelerated or decelerated according to its phase relationship with the wave. However, a particle travelling at a velocity very different to the phase velocity of the wave will feel a rapidly varying field with no net effect. The form of the distribution function of the interacting particles will determine whether the particles, on average, gain energy from the wave (damping) or lose energy to the wave (instability).

In the more complex case of circularly polarized waves in a magnetic medium the field vectors of the wave rotate around the background field vector so a resonantly interacting particle must have a combination of cyclotron gyromotion and parallel velocity, a helical motion, such that it sees a near constant field. This is cyclotron resonance and its more complex geometry, as compared with Landau resonance, makes it possible for high order resonances ($|m| > 1$) to occur at oblique angles and harmonics of the wave frequency.
Properties of Instabilities

We now describe the properties of various ion beam instabilities.

The ion/ion right-hand resonant instability excites waves on a modified magnetosonic/whistler branch; the presence of the beam modifies the stable plasma dispersion relation to be highly dispersive at small $k$. The ions in the beam are resonant with the generated waves, the core ions and electrons are not, so the relation $\omega_r = k \|v_\Omega - \Omega_i$ is approximately satisfied. The waves are right-handed and travel in a pro-beam direction and thus have a positive helicity. As described in Appendix A, the helicity represents the sense of field rotation in space at a fixed time. This instability is dominant in the range $n_i/n_0 \approx 0.01 \rightarrow 0.1$, with $\omega_r \approx \gamma$ at maximum growth and a threshold beam velocity of around the Alfvén speed.

The ion/ion non-resonant instability excites waves with negative helicity propagating in the anti-beam direction, with a small phase velocity, so, although nominally right-hand polarized, the polarization is not a good observational discriminant. No wave—particle resonance take place with any components so the instability is fluid/reactive in nature. It has a higher beam velocity threshold than the resonant instabilities but can dominate if beam velocity and density are sufficiently large (Gary et al., 1984). As the density of the beam rises the non-resonant instability is found to become more beam-resonant in character (Akimoto et al., 1993).

In Figure 3.1 we show the dispersion relation for parallel propagating waves in the presence of a cool ($\beta = 1$) proton beam travelling at $10v_A$ containing 2% of the total plasma mass (see Table 4.1, simulation [B1]). The complex frequency has been split into real frequency and growth rate ($\omega = \omega_r + i\gamma$). Note that we are using the convention whereby $k \geq 0$ and the direction of wave propagation is determined by the sign of $\omega_r$, with $\omega_r > 0$ representing propagation in the forward, $+x$-direction. The ion/ion right-hand resonant instability is the forward propagating dispersive mode with the largest growth rate. The ion/ion non-resonant instability is the backward propagating mode with low real frequency and growth rate. The straight lines seen in real frequency represent ion acoustic (sound) waves. This is the most heavily damped mode.

The ion/ion left-hand resonant instability excites waves on a modified Alfvén/Ion cyclotron branch. The waves are left-hand polarized and pro-beam propagating and thus have negative helicity. The resonant velocity for this instability typically lies in a region
Figure 3.1: Dispersion relation for parallel propagating waves for a tenuous cool beam (Case B1, Table 4.1).
with few particles and the growth rate of the instability remains small compared with those of the previous two instabilities unless the beam is very hot. It has a beam velocity threshold around the Alfvén speed. This mode is not seen in the dispersion relation in Figure 3.1 as the beam from which it is calculated is not sufficiently hot to excite the LH resonant instability.

All these instabilities have maximum growth rates at parallel propagation. The resonant instabilities possess higher order harmonic branches at oblique propagation. The oblique properties of the resonant instabilities were examined by Gary et al. (1981), Gary et al. (1984) and Hada et al. (1987). Analytical approaches are limited in what they can reveal about oblique propagation of these instabilities and useful results are usually obtained from numerical solution of the dispersion relation. Such results will be presented later in this thesis and a more detailed analysis of the oblique modes excited by beams will be postponed until then.

Also of particular interest are cool tenuous anisotropic beams as they closely resemble the field aligned beams observed in the quasi-perpendicular region of the Earth’s foreshock. For an anisotropic beam with sufficiently large intrinsic $T_{\perp b}/T_{\parallel b}$ the left-hand cyclotron anisotropy instability can be excited at relatively small beam velocity. As the beam velocity rises the instability is Doppler shifted to a RH resonant instability, which exhibits enhanced growth rates (Gary & Schriver, 1987). This growth rate enhancement is found to extend to oblique modes (Smith & Gary, 1987), however, though local maxima are found at oblique angle of propagation these never exceed the maximum growth rate at parallel propagation (Brinca & Tsurutani, 1989). As an instability proceeds the beam may evolve into an anisotropic state. In this way, the properties of anisotropic beams may become relevant even if the initial beam is isotropic.

### 3.2 Previous Simulation Work

Winske & Leroy (1984) attempted to explain the origin of the diffuse ion population observed in the Earth’s foreshock by the evolution of an unstable field aligned beam. In so doing they established the now familiar evolutionary sequence from field aligned beam, through intermediate, to diffuse ion distribution. They performed a set of one-dimensional periodic hybrid simulations of isotropic ($\beta_b = \beta_c = 1$) beam/core distri-
butions with a total drift velocity of $10v_A$, for beam densities of $1.5\%$ and $10\%$. The lower density beam is in the region of parameter space where the resonant instability is dominant, the latter is in the non-resonant instability regime. These one-dimensional simulations dealt only with parallel propagating waves.

From these studies they concluded that linear theory gives a good prediction of the initial growth rates observed in simulations. Thus, the resonant instability is expected to dominate in typical foreshock solar wind conditions, unless the beam density was unusually high. The instability was observed to evolve through linear wave growth, scattering of the beam in the perpendicular direction along constant energy surfaces (in good agreement with the predictions of quasi-linear theory) followed by strong non-linear scattering to a diffuse type distribution and relaxation to near isotropy with only a low level of waves remaining. It was noted that the appearance of shorter wavelength modes occurs as non-linear scattering takes hold, first observed in the density and then in transverse waves, which leads to the development of highly non-linear waveforms including large compressions in density and $B$ and chaotic particle motion in phase space. Winske & Leroy (1984) suggested these short wavelength features could be the products of parametric decay of the large amplitude waves into electromagnetic and electrostatic modes which facilitate the generation of parallel propagating compressive waves. However, no features bearing strong resemblance to shocklets as seen in the foreshock were observed, possibly due to the resolution and statistical limitations of their simulations. Parametric instabilities are discussed in Chapter 5.

Winske & Quest (1986) performed hybrid simulations of the same two instability regimes, resonant and non-resonant, but compared results in one and two dimensions. Again, poor spatial resolution and poor particle statistics (8 particles per cell per species) make the results quantitatively unreliable. In the resonant regime they observed lower levels of wave activity at saturation in the two-dimensional case, as compared with the one-dimensional case. They explained this by the fact that in the two-dimensional case far more modes are near-resonant and thus the free energy is more thinly spread. Also, the two-dimensional case allows the growth of compressive modes at oblique angles, so that unlike in one dimension, parametric instabilities are not required to explain compressive features. They found the magnetic field and density to be, in general, uncorrelated. They concluded that, except for scaling purposes, one-dimensional simulations are quite
adequate to model electromagnetic instabilities of this type, the exceptions being when parametric instabilities are believed to be important or more exotic instabilities with off axis growth maxima. These conclusions and the large computer resources required for higher dimensional studies have discouraged work on beam instabilities in two dimensions.

Akimoto et al. (1993) examined the non-linear stages of the evolution of the parallel propagating electromagnetic ion beam instabilities for a range of ion beam densities using a one-dimensional periodic hybrid simulation code. Keeping the beam speed at $10v_A$ and the component betas equal to one, the beam density was set at 2%, 10% and 25% of the background density. The resulting waves were followed past saturation into the non-linear regime. A simulation domain of length $256c/\omega_i$ ($\Delta x = 0.5$ for the 2% case and 0.25 for the others) was used, with beam and core represented by 50 macroparticles per cell each. The parameters chosen span the region of parameter space where the dominant instability moves from the ion/ion RH resonant instability to the ion/ion non-resonant instability with increasing density. From linear theory Akimoto et al. (1993) note that as the density increases the value of the resonance parameter indicates that the non-resonant instability becomes more resonant in character.

Most relevant to the study of the foreshock and solar wind region is the 2% beam case. Resolution of the waves generated in the simulation into positive and negative helicity components revealed the expected dominance of the resonant instability which generates pro-beam propagating, RH polarized waves with, consequently, positive helicity. Initially circularly polarized waves grow to large amplitude at long wavelength and later break up into a broad spectrum of shorter wavelength waves all travelling at approximately the same super-Alfvénic phase velocity. Steepened features with associated whistler wave trains are generated by a clumping process as outlined by Terasawa (1988). During the linear phase, growing modes beat together to form a linear wave packet, the large amplitude portions of which trap particles which are decelerated, gyrophase bunched and scattered in perpendicular velocity. These regions locally alter the distribution function in such a way as to destabilize certain wave modes, leading to rapid local wave growth. The observed steepened features and their associated wave trains are parallel propagating.

Most wave energy in the system (> 95%) remains in positive helicity waves throughout the simulation, the only significant negative helicity being generated when steepened wavefronts split to give anti-beam propagating features. Modes initially grow at
rates close to those predicted by linear theory, but in the late pre-saturation era explosive growth occurs of shorter wavelength modes which, according to linear theory, are damped or stable. This is due to either modification of the initial distribution function destabilizing these modes or some parametric instability mechanism.

Initially there exists little or no correlation between magnetic fluctuations and density fluctuations \( C_{nB} \) as expected when circularly polarized waves dominate. However, as the instability approaches saturation \( C_{nB} \) rises to around 0.5 indicating a move to more compressional waves as particles become trapped. Post-saturation, a turbulent cascade to higher wavenumbers occurs. Spiky pulsations with a width of \( \sim 3 - 6 \omega_i / c \) and an amplitude \( | \delta B / B_0 | \sim 1 \) appear, with a characteristic half-wave signature as described in Akimoto et al. (1991) and reminiscent of a large amplitude pulsation (Schwartz & Burgess, 1991).

At higher beam density (10%) the non-resonant instability becomes competitive and the nature of the waves generated is initially dictated by the non-resonant instability, with the resonant instability appearing only later (but with enhanced growth rates). \( C_{nB} \) remains close to zero. In the highest density simulations (25%) the non-resonant instability dominates with a majority of the energy in negative helicity modes. \( C_{nB} < 0 \) indicates an anticorrelation between field and density. These results are of less importance to the foreshock than the lower density beam simulations which are closer to observations. Indeed, it is difficult to imagine a situation which could produce such a dense beam in any steady fashion.

More recently, a set of simulations has been performed by Killen et al. (1995) to examine shocklet formation, in two-dimensional hybrid simulations, by ring beams with characteristics similar to those observed in the foreshock (gyrating ion populations seen upstream of the quasi-parallel shock) and upstream from comets (oxygen pickup ions). The results of these simulations are compared with those of field aligned beams. The proton field aligned beam simulations were performed as initial value problems in a doubly-periodic spatial domain \( 700 \times 700 \Delta x = \Delta y = c / \omega_i \), with a beam density of 1% of the background density and a beam speed of \( 12 v_A \)—a region of parameter space where the resonant beam instability dominates. The background plasma was represented by 25 particles per cell.

They demonstrated wave growth through a wide range of angles \( (0^\circ \leq \theta_{kB0} \lesssim 60^\circ) \)
with maximum power at parallel propagation. They found sinusoidal ULF waves which
lacked the characteristic leading edge steepening of shocklets. They argue that as parallel
propagating waves have no compressional component they cannot form shocklets and
that the dominance of parallel propagating modes rules out the generation of shocklets
at the observed $\sim 30^\circ$. This appears to ignore possible non-linear evolution such as that
demonstrated by Akimoto et al. (1993), where during the non-linear phase particles are
trapped and thus steepen large amplitude field features by a snowplough type process.
This may be due to the low grid resolution of Killen et al. (1995) suppressing the formation
of short scale features. It is unclear how far beyond the linear regime these simulations
were taken as few quantitative results are given for field aligned beams.

For ring beams they find that off-axis wave growth is boosted by an anisotropy driven
instability with maximum growth rates at oblique angles. This means that most energy
goes into oblique modes and oblique propagating compressional waves dominate. Their
simulations of oxygen ring beams imply that this effect is enhanced in driven simulations
(where a beam is injected along a non-periodic boundary). They conclude that gyrating
populations in the quasi-parallel shock region are likely to be the source of shocklets
observed in the diffuse region.

Other simulations have examined different aspects of beam instabilities. Zachary
et al. (1989) used a novel hybrid code, with the core population treated as a fluid but
the beam population treated as particles, to investigate the evolution of very low density
beams ($n_b/n_0 = 10^{-5} - 3 \times 10^{-3}$) over long periods ($t \approx 2000\Omega_i^{-1}$). This work clarified
the process of saturation through trapping and scattering. Thomas & Brecht (1988) and
Akimoto et al. (1991) explore the evolution of finite ion beams, finding local instability
generated waves with a strong tendency to steepen through a snowplough effect. Various
simulations to examine the formation of non-gyrotropic distributions have been performed
(e.g. Hoshino & Terasawa, 1985; Gary et al., 1986b,a). These will be discussed in detail
in Section 4.2.4.

It has been argued (e.g. Jokipii et al., 1993) that a one-dimensional or two-dimensional
simulation prevents any cross-field diffusion from occurring. In other words, a particle is
tied to a particular field line which could have consequences for the physical validity of
simulations with less than three spatial dimensions. However, the recent work on three-
dimensional hybrid simulations of beam instabilities by Kucharek & Scholer (1995) finds
little evidence for cross-field diffusion, from which we can conclude, tentatively, that its lack in our two-dimensional simulations has little impact on our results. We await larger three-dimensional simulations with higher resolution to confirm this to be the case.
Chapter 4

Simulation of Ion Beam Instabilities in Two Dimensions

Here we present an analysis of our two-dimensional simulations of the evolution of cool tenuous beams within a magnetized background plasma. We have applied the parallel hybrid code Barbie and Ken, described in Chapter 2, to both isotropic and anisotropic proton beams with properties similar to those observed in the ion foreshock.

4.1 Parameters for Beam Simulations

The selection of the parameters describing the initial beam distributions in these simulations has been driven by three main factors. Firstly, the types of proton beams observed in the Earth’s ion foreshock region, especially in the anisotropic beam cases. Secondly, the need to compare our work with previous one-dimensional and lower resolution two-dimensional work by other authors has necessitated a certain amount of conformity with their initial conditions, however, they too are often motivated by foreshock observations. Thirdly, we are constrained by what it is possible for us to simulate due to computational restrictions, both numerical and in terms of available processing power. Very hot or high speed beams are difficult to correctly model numerically, but fortunately observed beams are usually in the subset of parameter space where this is not the case.

The simulation parameters used in this work are given in Tables 4.1 and 4.3.

In an ideal world the variation of non-physical simulation parameters (cell size, timestep, etc.) would have no effect on the results of a simulation. Obviously, in the real
### Chapter 4. Simulation of Ion Beam Instabilities in 2D

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<td>0.75</td>
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<td>Cell size ( (y) ), ( \Delta y (c/\omega_i) )</td>
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<td>0.65</td>
<td>1.30</td>
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<td>100</td>
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<td>80</td>
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<tr>
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<td>25</td>
<td>80</td>
</tr>
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<td>~0.2</td>
</tr>
<tr>
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</tr>
<tr>
<td>Ion timestep ( \Delta t (\Omega_i^{-1}) )</td>
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<td>0.05</td>
<td>0.03</td>
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</tr>
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<tr>
<td>Field divergence test (frequency in iterations)</td>
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<tr>
<td>Total time ( (\Omega_i^{-1}) )</td>
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<td>200</td>
<td>250</td>
<td>150</td>
<td>100</td>
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Table 4.1: Isotropic beam simulation parameters.
world, this is not the case. Degrading the spatial resolution of a simulation to beyond
the scale length of some phenomenon will remove that phenomenon from the simulation
with unpredictable side effects. Similarly the timestep must be chosen with regard to the
shortest important timescale of the system and numerical stability criteria.

In the case of the isotropic beam simulations described in Section 4.2 we have explored
the non-physical parameter space for one-dimensional simulations and the spatial
and temporal resolutions of our simulations have been chosen to be as coarse as possible
whilst still accurately modelling the processes of interest. In this way the available com-
puting resources are used efficiently. Other factors such as field substepping have been
examined to ensure physically meaningful, robust results and it is found that in hybrid
codes the particular electron model chosen has little effect on the results (e.g. Winske,
1985).

The statistical noise associated with the small number of macroparticles in computer
simulations, which is much greater than the thermal noise in the real plasma, can mask
important effects. For example, whistler wave trains are not seen in isotropic beam
simulations with low numbers of particles per cell. Problems such as this drive the
desire for large numbers of macroparticles, and therefore the work on parallel algorithms
discussed in Chapter 2.

A more complex issue is that of competition between different modes for the available
free energy in the system. A doubly periodic code, such as ours, can resolve only a finite
number of modes of the system, those which fit complete wavelengths into the simulation
box, such that $\lambda_{xi} = L_x/i$, where $i$ is an integer. In a real system a continuum of modes
compete for the free energy, whereas in our system only a discrete set compete. This
is obviously most important where the mode spacing is greatest, namely at the largest
wavelengths. Effects such as beating between different waves will also differ in the two
cases. Computational resources limit the size of our simulation box so that for the results
presented here, typically only about five parallel propagating modes are resolved in the
range predicted to be growing by linear theory. However, we tailor our box to exactly
resolve the fastest linear growing mode, which, in a real system, will often dominate the
evolution of the system.

We discuss in detail the effects of the variation of non-physical simulation parameters
on our simulation results in Section 4.4.
4.2 Instability of a Cool Tenuous Beam (Case [B1])

In this section we describe the linear dispersion relation and the evolution of a hybrid simulation of a cool tenuous beam with the parameters labelled [B1] in Table 4.1. Our simulation box, of dimensions $L_x$ by $L_y$, lies in the $x$-$y$ plane, has doubly-periodic boundary conditions and an initial background field $B_0 = B_0 \hat{x}$. The initial bulk velocity of the beam ($v_0$) lies parallel to the $x$-axis.

It should be noted that the particle statistics in our simulations are far superior to that found in previous works, so our results are more robust and features previously lost in numerical noise become visible. The simulation [B1], discussed here has 80 particles per cell per species (ppcps) with $\Delta x = \Delta y = 0.65c/\omega_i$ in comparison with the one-dimensional simulations of Winske & Leroy (1984) 40ppcps, assuming $\Delta x = 1c/\omega_i$ and Akimoto et al. (1993) 50ppcps, $\Delta x = 0.5c/\omega_i$ and the two-dimensional results of Winske & Quest (1986) 8ppcps, $\Delta x = \Delta y = 2c/\omega_i$ and Killen et al. (1995) 25ppcps, $\Delta x = \Delta y = 1c/\omega_i$.

4.2.1 Linear Dispersion Relation

The two-dimensional linear dispersion relation for an ion beam distribution as described by the parameter set [B1] is shown in Figure 4.1. This figure shows only the wavenumbers with growing ($\gamma > 0$) modes. If more than one mode grows at a given wavenumber then only the fastest growing is shown. This dispersion relation was calculated numerically using WHAMP as described in Section 1.4.2. Note that this figure is not square in $k$-space, i.e. angles are distorted, with modes far more oblique than they appear in this figure. This is done to illustrate the harmonic structure of the growing modes. Figure 4.2 shows a smaller range in $k_y$ with a 1:1 aspect ratio.

The dispersion relation shows two instabilities, the ion/ion RH resonant instability in the forward propagating ($k > 0$) direction and the ion/ion non-resonant instability in the backwards propagating ($k < 0$) direction. The growth rate of the former can be seen to be far greater that of the latter, so our discussion will concentrate on the resonant instability.

From the dispersion relation in Figure 4.1 it is clear that the nature of the dominant modes excited by the resonant instability can be split into two groups. Those which
emerge under modification of the distribution function by the beam from the parallel propagating modes of the RH whistler/magnetosonic mode of the unmodified Maxwellian plasma and the highly oblique modes emerging from the LH Alfvénic mode. Gary et al. (1984) named these two groups the right oblique branch and left oblique branch respectively, from the polarizations of their parent modes. The differences are clearly seen in Figure 4.3 where the magnetic compression ratio, $C_{BB}$, is plotted (see definition in Appendix A). The right oblique branch has $C_{BB} \geq 0.5$, increasing with $\theta_{kB}$ (the angle between $B_0$ a wave’s $k$). This represents a move from a near circular polarized wave at parallel propagation to a highly compressive one at oblique propagation. Conversely, the left oblique branch has $C_{BB} \ll 0.5$ representing a wave where $\delta B$ lies near perpendicular to the $B_0-k$ plane, i.e., the wave is Alfvénic. This is reinforced by the phase velocities of the two branches which differ dramatically, as is seen in the lower half of Figure 4.3. The right oblique branch is restricted to $\theta_{kB} \lesssim 70^\circ$, with the fastest growing group of modes within $\theta_{kB} \lesssim 60^\circ$, however the left oblique branch has growth rates of $\sim 75\%$ of the maximum (parallel) growth rate up to obliquities $\theta_{kB} > 85^\circ$. The helicity of both modes (not shown) is positive, a consequence of the transition of the polarization of the Alfvén mode from LH to RH at high obliquities, as noted by Gary (1986) and explored by Krauss-Varban et al. (1994).

The maximum growth rate ($\gamma_m = 0.20, \omega_r = 0.21$) is associated with the first harmonic of the right oblique branch, $k = (0.113, 0)$. The much weaker second harmonic is seen as a small group of wavenumbers showing wave growth centred on $k \simeq (0.25, 0.25)$. By the third harmonic no growth is observed. The first three growing harmonics of the left oblique branch can be seen at $k_0 \simeq 0.11, 0.22, 0.34$ respectively. The first harmonic of this branch has a growth rate $\gamma \simeq \omega_r \simeq 0.15$, remaining almost constant with $k_\perp$.

A periodic simulation only supports waves at wavenumbers where an integer number of wavelengths fit into the simulation box. Thus in the following discussion we will often refer to waves from the basis set of two-dimensional discrete Fourier transforms using the mode numbers $(m_x, m_y)$, where the first number is the number of wavelengths in the $x$-direction (parallel to $B_0$) and the second is the number of wavelengths in the $y$-direction (perpendicular).
Figure 4.1: Linear dispersion relation for the case of a tenuous cool beam [B1]. The wavevector ($k$) is in units of $\omega_i/c$, frequencies and growth rates are in units of $\Omega_i$. Note that the aspect ratio is not 1:1 so modes are much more oblique than they appear.
Figure 4.2: Linear dispersion relation for the case of a tenuous cool beam [B1]. The wavevector \((k)\) is in units of \(\omega_i/c_i\), frequencies and growth rates are in units of \(\Omega_i\). Note that this figure is identical to Figure 4.1 except it has an aspect ratio of 1:1.
Figure 4.3: Linear magnetic compression ratio, $C_{BB}$, and phase velocity (expressed in Alfvén speeds) relation for the case [B1]. The wavevector ($k$) is in units of $\omega_l/c$, with axes as in Figure 4.1.
Figure 4.4: Time evolution of magnetic fluctuation energy, component drift velocities, component perpendicular and parallel temperatures (beam solid line, core dashed line) for a tenuous cool beam instability [B1].
4.2.2 Overview of the Simulation

In this section we outline the important stages in the overall evolution of the system. In broad terms, we find a sequence similar to that found in the work of Winske & Quest (1986).

In Figure 4.4 we show the evolution with time of various global properties of the simulation plasma: the energy in magnetic fluctuations, the drift velocities of the core and beam components and the parallel and perpendicular temperatures of the components. Various features are immediately obvious. The energy in magnetic fluctuations, i.e. the energy in waves, rises nearly exponentially until $t = 50\Omega_i^{-1}$ when it peaks. This is the saturation of the instability; the free energy contained in the ordered drift of the beam is converted into wave energy up to this point in time, corresponding to a fall in the drift energy of the beam. After saturation energy continues to be passed between the the waves and particles, but the process is more complex, with energy being passed between the waves, beam ions and the core ions.

The parallel and perpendicular temperatures indicate how much the initial ion distribution function has altered. As the wave amplitude grows particles begin to be pitch angle scattered and their perpendicular temperature rises. Initially this scattering will take place along constant energy surfaces in the propagating wave frame (quasi-linear scattering). As this process continues particles spread more in parallel velocity and the parallel temperature rises. The parallel temperature rises as the particle scattering leaves the quasi-linear phase as the wave amplitude becomes larger and the scattering becomes more non-linear and particles diffuse in energy. These changes are clearly seen in the temperature histories as the perpendicular temperature rises at the same time as the wave energy, shortly followed by a more gradual rise in parallel temperature.

The energy in the growing wave modes behaves as predicted by linear theory from $t = 0$ to $t \simeq 25\Omega_i^{-1}$. The observed growth rates are consistent with those predicted by linear theory as little evolution of the distribution function has occurred and the initial dispersion relation is not significantly perturbed. Growth rates, even during the linear phase, can be difficult to measure from simulation data as the initial noise spectrum takes some time to redistribute amongst the available modes and modes exchange energy through weak couplings in an oscillatory manner. Over this period the fastest growing modes $(m_x, m_y) = (3, 0)$ and $(3,1)$ have growth rates matching those predicted to within
a few percent, continuing almost to saturation. The energy in the other \( m_x = 3 \) modes tends to be more oscillatory, and thus less well defined, but still shows agreement with theory to within \( \lesssim 20\% \). There is a tendency for the growth rates of modes with higher \( m_y \), corresponding to the left oblique branch, to show the greatest suppression. This suppression is probably due to competition for free energy with the dominant modes. The \( m_x = 4 \) modes show a similar effect. The \( m_x = 2 \) modes show good agreement with linear theory.

Around \( t \approx 30 \Omega_i^{-1} \) various changes occur, most notably the destabilization of previously near stable modes and step-changes in the observed growth rates of some modes, which can be taken to indicate significant changes in the distribution function have begun to take place. This is discussed in more detail in Section 4.2.3.

Post-saturation trapping oscillations are observed in the magnetic fluctuation energy with a period of approximately \( 28 \Omega_i^{-1} \) as energy is exchanged between trapped particles and waves. The frequency of trapping oscillations due to a single wave is given by

\[
\frac{\omega_B}{\Omega_i} = \left( \frac{\delta B}{B_0} \right)^{1/2} \left( \frac{k v_\perp}{\Omega_i} \right)^{1/2}
\]  

(e.g. Gary et al., 1986a). With values typical of post-saturation \( (k = 0.113 \omega_i/c, v_\perp = 10v_A, \delta B = 0.1B_0) \) this gives a trapping period \( T_B \approx 18.7 \Omega_i^{-1} \). This discrepancy might be attributable to the effects of particles interacting with multiple waves. The value observed here is consistent with the results of Winske & Leroy (1984). It should be noted that Winske & Quest (1986) observed no trapping oscillations in their two-dimensional simulations, a fact probably explained by their poor particle statistics. Note that the trapping oscillations have a signature in the drift velocity and temperatures: a fall in wave energy corresponds to a small rise in drift velocity and perpendicular temperature and a small fall in parallel temperature. This can be explained by phase coherence between spatial modulation in drift velocity and the waves.

The magnitude of the wave energy and the trapping oscillations fall with time as the more ordered energy is converted into thermal energy through particle scattering. By the end of the simulation the beam is hot and isotropic, its temperature having risen to approximately 40 times its initial value in both the perpendicular and parallel directions and the drift velocity is close to the instability threshold. By this time, little free energy remains, no more waves are being generated through resonant instability and
those remaining are weakly damped. The possibility of parametric instabilities at this stage is discussed in Chapter 5.

Examination of the total kinetic energies of the two components (not shown) shows not only that the core ions are most active in the observed trapping oscillations, but, as pointed out by Gary (1978), the generation of waves is merely an intermediate stage in the transfer of the, initially large, beam momentum and energy to the core population through scattering. Though the core temperature rise is small compared with that of beam, its greater density leads it to dominate in terms of energy.

4.2.3 Evolution of Waves

The types of waves and structures that are observed as the beam instability evolves are best illustrated by looking at density and magnetic field data from the simulation at particular points in time. These can be seen in Figures 4.5–4.10 along with the spatial discrete Fourier transform (FT) for each field component which allow easy identification of the wavenumbers of the dominant waves in the system. Fourier modes will be identified using the notation \((m_x, m_y)\). The mode numbers are equivalent to the wavevector \(k = (m_x2\pi/L_x, m_y2\pi/L_y)\). In the simulation [B1] this gives \(k = (0.0378m_x, 0.0378m_y)(\omega_i/c)\). Note that the \((0,0)\) modes are suppressed in all FTs in this work as they are of little interest and distort the greyscale.

The spatial domain of the simulation was chosen such that it would accommodate exactly three complete wavelengths of the fastest growing mode for this instability in the parallel direction.

At early times the growth is exponential in nature with the growth rate of each mode of the system agreeing closely with those predicted by the linear dispersion relation. Figure 4.5 shows the mass density, the magnitude of the magnetic field fluctuations and the field components at \(t = 25\Omega_i^{-1}\). The distribution function is close to its initial form (though as will be seen in Section 4.2.4 the beam has become gyrophase bunched), the field fluctuations are still small \((\delta B/B_0 \sim 0.2)\) and the wave modes which dominate are those which linear theory predicts, with \(m_z = 2, 3, 4\). The Alfvénic component, \(B_z\), shows growth at highly oblique \(k\) as expected from the Alfvénic nature of the more oblique modes predicted from linear theory. These left oblique branch modes show near uniform growth rates with increasing \(k_\perp\), as in Figure 4.1. The magnetosonic component,
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Figure 4.5: Summary plots of density and magnetic field components at $t = 25\Omega_i^{-1}$ for a tenuous cool beam instability [B1].
By, shows growth within the cone predicted for the right oblique branch by linear theory, \( \theta_{kB} \lesssim 60^\circ \). The approximately uniform value of \( \delta B \) indicates that the dominant waves are circularly polarized, a view supported by the amplitudes of the dominant near-parallel modes in the \( B_y \) and \( B_z \) components of the magnetic field being approximately equal. \( B_x \) confirms this view with little activity in the near-parallel direction \( \theta_{kB} \lesssim 30^\circ \), but growing modes are seen, corresponding to the compressive magnetosonic modes. Oblique modes corresponding to those with an Alfvénic nature are, somewhat surprisingly, seen in the density at this point. However, Krauss-Varban et al. (1994) found that, when oblique, the Alfvénic mode in a Maxwellian plasma has an appreciable compressibility \((C_nB)\), which increases with wavenumber. The near-parallel propagating modes have little associated density disturbance.

At around \( t = 300t^{-1} \) some wave modes show a modification in their behaviour. Modes, some originally damped, show a step-change in growth rate and in some cases show very rapid growth, in excess of that of the initially fastest growing modes. For example, modes (4,0) and (4,1) change from \( \gamma \simeq 0.1 \) to \( \gamma \simeq 0.2 \); modes (5,0), (6,0) and (7,0) change from close to stable to \( \gamma \simeq 0.25, 0.4, 0.45 \) respectively. Similar behaviour was observed in the one-dimensional work of Akimoto et al. (1993). The most obvious explanation is that a modification to the distribution function has driven these waves unstable. Examination of the beam temperatures at this time shows that \( T_{\perp} \) has risen without any significant rise in \( T_{\parallel} \) or fall in \( v_\parallel \). It is therefore possible that this anisotropy of the beam is responsible for the sudden change in growth rate for these modes. However, an examination of Figure 4.24 showing the dispersion relation for a beam with an anisotropy of 10 — close to the parameters of the modified beam at this stage in this simulation, shows growth rates nowhere near as large as those observed. Another explanation might be that the beam is no longer even bi-Maxwellian and that some more complex instability is at work, for example, the beam being driven non-gyrotropic through the action of the dominant waves in the system. Brinca et al. (1993) found that for some instabilities non-gyrotropy leads to an enhanced instability in terms of both increased growth rate and enlarged range of unstable wavenumbers, exactly as is seen here. The gyrophase bunching observed in this simulation (see Section 4.2.4) indicates that this is almost certainly the case. Another possibility that can probably be discounted is that parametric instabilities are at work, since the observed growth rates are far in excess of those expected for these
types of instabilities. This will be examined in detail in Chapter 5.

By \( t = 40\Omega_i^{-1} \) rapid wave growth is underway, as can be seen in Figure 4.6, with a peak field amplitude of \( \delta B / B_0 \approx 1.2 \). The dominant waves are, as expected, those with the highest growth rates (3,0) and (3,1). Since these are near parallel propagating they are near circularly polarized. The \( B_x \) component shows strong short-wavelength activity at oblique \( k \), the sum of many resonantly excited highly oblique modes. The spectra show a strong resemblance to the linear dispersion relation: most power clustered around the the fastest growing \( m_x = 3 \) modes and the Alfvénic and magnetosonic field components reflecting the different characteristics of the different branches. Also visible is a deficit of activity at shorter wavelengths (modes in the triangular regions of Fourier space extending beyond \( (\pm 6, 0) \)) with angles below about \( 45^\circ \), giving the spectra a ‘V’ shape, in both density and magnetic field components (except \( B_y \) which remains confined to low \( k_y \)). As the modes within this region of wavenumber space have become more active, their evolution is difficult to interpret as there is evidence of growth both with and without an exponential (instability) nature. Most likely, this is a combination of newly unstable modes from distribution function evolution, together with artifacts of steepening which appears in the higher harmonics of the steepening mode. In particular, evidence of activity at \( m_x = 6 \) and 9 is seen, some of which is due to steepening of the dominant \( m_x = 3 \) modes. The magnetic fluctuation \( | \delta B | \) shows that various waves are beating together to produce local maxima, a noted departure from the uniformity seen earlier and a potential source of ion scattering.

Figure 4.7 shows the simulation at saturation when \( t = 50\Omega_i^{-1} \). \( B_z \) and \( B_y \) are still visibly dominated by the near parallel \( m_x = 3 \) waves, with \( B_z \) still showing remnants of highly oblique short wavelength activity. Short scale structures associated with longer wavelength waves are sometimes seen in the field, e.g. the oblique features centered around \( x = 70, y = 120 \). Visible in \( B_y \) as highly positive and negative regions on the leading and trailing edges of a larger feature, these features also appear in \( B_z \) as transitions in polarity and as near nulls in \( B_x \). They are responsible for a large magnetic fluctuation \( \delta B / B_0 \gtrsim 1 \) and are correlated with a slight density enhancement suggesting a pile up of particles trapped by the field feature. Not every long wavelength wave has associated short scale features of this type. Note that the minimum value of \( B_x \) has dropped by over 0.5\( B_0 \) since \( t = 40\Omega_i^{-1} \) and falls rapidly to a minimum of \( B_x \simeq -0.2B_0 \) at \( t \simeq 51.5\Omega_i^{-1} \).
Figure 4.6: Summary plots of density and magnetic field components at $t = 40\Omega_{\text{ci}}^{-1}$ for a tenuous cool beam instability [B1]
before it begins to rise again. Curved wavefronts are also observed, formed from many superposed modes. A set of thin compressional features can be seen in $B_x$ and density with a width of approximately $10c/\omega_i$ at $45^\circ$ associated with the right oblique branch mode (3,3). These features are coincident with clumping in the ion beam density seen in Figure 4.8. The magnetic fluctuation ($\delta B$) shows the simulation box to be threaded with non-linear perturbations $\delta B/B_0 \gg 1$ appearing almost turbulent with little regular structure, providing a good source of ion scattering. The possible role of these features in the saturation of the instability is discussed in Section 4.2.7.

By $t = 700\Omega_i^{-1}$ the short scale field features have become more pronounced, narrowing and becoming more structured despite a decrease in amplitude. This can be seen in Figure 4.9. They move in a pro-beam direction and are lead in a majority of cases by a whistler wave train of decreasing amplitude in the forward direction. These features bear a close resemblance to those observed in the one-dimensional simulations of Omidi & Winske (1990). The whistler wave trains dissipate energy from the high field features and are probably responsible for their observed fall in amplitude. The nature and evolution of the whistler wave packets is explored in detail in Section 4.2.8. Perturbations in $\delta B$ are now more localized and of decreasing magnitude, though there still exist widespread peaks with $\delta B/B_0 > 1$. The density shows little structure, except the decaying remnants of the previously seen lattice structure, which is also still faintly seen in the field. The amplitudes of the previously dominant modes has fallen to a fraction of their saturation value and the Fourier space representations have now filled out to shorter wavelengths, reflecting the relative increase in short scale activity and providing evidence that some linearly stable modes have been driven unstable by the evolution of the distribution function.

Over the next $500\Omega_i^{-1}$ the whistlers spread and dissipate along with the features which generated them, with the occasional weak new feature appearing and quickly dissipating. The plasma relaxes further, wave amplitudes fall, as does $\delta B/B_0$ as wave energy goes into further scattering of the core ions and thermalization of the beam. The dominant remaining modes are those near parallel propagation. The end point of the simulation at $t = 2000\Omega_i^{-1}$ is shown in Figure 4.10. Short wavelength ($\lesssim 10c/\omega_i$), highly oblique wavevector waves are seen at this time in the density and field components (except $B_y$). Though these waves resemble those growing due to linear instability in the early part of
Figure 4.7: Summary plots of density and magnetic field components at $t = 50\Omega_i^{-1}$ for a tenuous cool beam instability [B1].
the simulation, they appear to be stationary ($\omega_r = 0$). Chapter 5 explores the possibility that these features are due to an anisotropy driven mirror instability.

### 4.2.4 Gyrophase Bunching

The phenomenon of non-gyrotropic particle distributions, or gyrophase bunching, has been noted in observations of ions upstream of the terrestrial bow shock (Gurgiolo et al., 1981; Eastman et al., 1981; Fuselier et al., 1986b; Fazakerley et al., 1995). Gurgiolo et al. (1983) proposed that such observations could be explained by the creation of gyrophase bunched ions through reflection at the bow shock. However, rapid gyrophase mixing was also predicted which would lead to such distributions being short lived and therefore only expected to be observed within a few gyroradii of the shock.

Simultaneous observations of large amplitude magnetic waves and gyrophase bunched ions in the foreshock were presented by Thomsen et al. (1985). This led Hoshino & Terasawa (1985) to propose a mechanism which linked the two, by suggesting that gyrophase bunching of ions is a natural consequence of the generation of large amplitude magnetic waves through ion beam instabilities. They explored this mechanism through one-dimensional hybrid simulations.
Figure 4.9: Summary plots of density and magnetic field components at $t = 70\Omega_i^{-1}$ for a tenuous cool beam instability [B1].
Figure 4.10: Summary plots of density and magnetic field components at $t = 200\Omega_i^{-1}$ for a tenuous cool beam instability [B1].
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The phase relationship between particles and wave, in particular the wave electric field, determines the rate and direction of energy transfer between the two. For a parallel propagating wave linear theory predicts the following: if a particular population of particles are cyclotron resonant with a wave then, in the frame of reference where a particle has zero velocity parallel to the background magnetic field \((B_0)\), the vectors \(\delta B\), \(\delta E\) and \(v_{\perp}\) all lie in a plane perpendicular to \(B_0\), and rotate about \(B_0\) at an angular velocity \(\Omega_0\) with a constant phase relationship. Figure 4.11 illustrates this relationship and defines the angle \(\psi_j\) between \(\delta B\) and \(v_{\perp}\).

The work done on a particle by the electric field is given by \(q_jv_{\perp}\delta E = q_jv_{\perp}\delta E \sin \psi_j\). For a group of particles this averages to \(q_j\langle v_{\perp} \rangle \cdot \delta E = j \cdot \delta E\), where \(\langle \rangle\) represents an average over these particles. So, for an ion population \((q > 0)\), if \(\langle v_{\perp} \rangle \cdot \delta E > 0\) then the particles will gain energy at the expense of the wave, which will consequently be damped. Conversely, if \(\langle v_{\perp} \rangle \cdot \delta E < 0\) then the particles will lose energy to the wave, which will be unstable and grow. Clearly in steady state, i.e., with a wave neither growing nor damped, \(\langle v_{\perp} \rangle \cdot \delta E = 0\), so that \(\psi_s = 0^\circ, 180^\circ\). At exact ion resonance with an unstable wave \(\psi_s = 90^\circ\). This discussion excludes unusual cases where \(\langle v_{\perp} \rangle\) is unrepresentative of the actual distribution, (e.g. if many separate particle clumps exist in velocity space).

Gary et al. (1986b) used linear theory to predict the angle between the core and beam ion populations. They found that for a beam—core distribution with properties as in simulation [B1] that the wave with maximum growth rate should cause bunching centred on \(\psi_b \approx 90^\circ\) and \(\psi_c \approx 150^\circ\). For this bunching to be visible the wave amplitude must be sufficiently large that the thermal speed of the bunched component is smaller than the velocity perturbation generated by the wave. In the case [B1] we find this to be true and bunching is thus visible above the thermal spread.

In Figure 4.12 we plot for each spatial cell, at various times, the phase angles calculated relative to the initial background magnetic field, of the magnetic field perturbations \(\phi_B = \tan^{-1}(B_z/B_y)\), and, for the mean velocities of particles when spatially binned, \(\phi_s = \tan^{-1}(\langle v_{\perp} \rangle_s/\langle v_y \rangle_s)\). From this plot we can see both the degree of spatial coherence of the waves, which we shall discuss in detail in Section 4.2.5, and the correlation of the phase angles of the beam and core ions with the phase angle of the magnetic field.

Over the period shown \((100\Omega_1^{-1})\) several trends can be identified. The phase coherence of the waves moves from a low level, where the plot is seen to be noisy with a low degree
Figure 4.11: The relationships between $B_0$ and the $E$ and $B$ perturbations due to the wave and the perpendicular ion velocity for a RH circularly polarized wave travelling in the $+B_0$ direction.
Figure 4.12: Phase angles (as defined in Section 4.2.4) are shown for the field and particle velocities at various times in the cool tenuous beam simulation [B1]. The phase angle is calculated across the $x$-$y$ simulation box (units $c/\omega_i$) and shown as a grey scale from white $= +\pi$ to black $= -\pi$. 
of coherence perpendicular to the background field, through to a more ordered state which then goes on to lose coherence. The particle velocity phase angles show that the beam ions are quickly ordered in phase but this rapidly disappears post-saturation, as the waves and core ions approach equilibrium and as the beam temperature increases. The core ions begin with little organisation in phase but over time become more ordered, this trend continuing into the post-saturation regime. Comparing the phase of the magnetic field with that of each particle species suggests a constant phase relationship between the two, however the actual values of the phase differences are unclear from this figure.

In order to accurately determine the angles between the phases of the magnetic field and the perturbation to the particles due to the wave we have taken the cross-correlation between the phase angles calculated for these two vectors. As the wave is approximately circularly polarized the ratio of the lag to the peak of the correlation coefficient to the dominant mode's wavelength is equivalent to the angular difference in the phases of the magnetic perturbation vector and the mean gyration velocity vector of the particles, i.e., when the lag is such that the two appear in phase, the cross-correlation is large. In Figure 4.13 we plot, at various different times, both the autocorrelation in the parallel direction of the magnetic field and the cross-correlation between magnetic field and particle species' phases, averaged over the y-direction. The ranges from which the cross-correlation is calculated are selected to have the same length whatever the lag—this is possible as the simulation box is periodic. However, this leads to the situation where a lag of $L_x/2$ is equivalent to a lag of $-L_x/2$. From these calculations plot we obtain the values for $\psi_{\text{core}}$ and $\psi_{\text{beam}}$ shown in Table 4.2. Note that the estimates of error for these angles are 1σ.

<table>
<thead>
<tr>
<th>Time ($\Omega^{-1}_i$)</th>
<th>$\psi_{\text{core}} \pm \sigma$</th>
<th>$\psi_{\text{beam}} \pm \sigma$</th>
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<tr>
<td>25.0</td>
<td>133° ± 8°</td>
<td>74° ± 1°</td>
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<tr>
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<td>7° ± 7°</td>
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<tr>
<td>100.0</td>
<td>115° ± 6°</td>
<td>17° ± 10°</td>
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Table 4.2: Phase angles for gyrophase-bunching centres with respect to the magnetic field fluctuation for simulation [B1].
Figure 4.13: Parallel autocorrelation of the magnetic field phase angle (solid line) and cross-correlation of magnetic field phase angle with core population phase angle (dotted) and beam population phase angle (dashed) for [B1]. The box is treated as periodic and is averaged in the perpendicular (\(y\)) direction.
In the linear growth rate regime our results are close to those found by Gary et al. (1986b), representing a resonance driven cyclotron instability supplying energy to the wave, with energy supplied by both beam and core. At saturation \((t = 50\Omega_i^{-1})\) the phase relationship is as predicted by our simple physical argument, the beam ions lying perpendicular to the electric field perturbation and core ions approaching the same state \(180^\circ\) out of phase. Thereafter the beam ions remain in this steady-state configuration, whilst the core ions appear to move towards driving energy into the wave, though this could simply be the effect of trapping oscillations about the steady-state (Hoshino & Terasawa, 1985) being viewed at a similar point in the trapping oscillation cycle at both these later times. If this were the case then between these two points in time the core ions would appear to shift from driving to damping and back again.

4.2.5 Coherence Lengths

Simple linear theory assumes waves which are coherent in the transverse direction. Thus it is interesting to study how valid this assumption is both using spacecraft data and simulation results. Wave coherence lengths are also important in the analysis of data from spacecraft. In order to use multiple spacecraft data to calculate wave properties such as polarization and frequency in the wave frame all the spacecraft must be observing the same wave and thus be separated by less than the coherence length of that wave.

Le & Russell (1990) and Le et al. (1993) used ISEE 1 and 2 observations to calculate the coherence lengths of ULF waves in the foreshock. For the ‘30-s waves,’ which correspond to the waves generated here by the RH resonant instability, the dominant factor determining the coherence length was found to be the separation of the spacecraft perpendicular to the solar wind. This is mainly due to a geometric effect connected with solar wind convection. The coherence length in the direction perpendicular to the solar wind flow was found to be of the order of \(1R_E\), close to one wavelength. They found coherence lengths parallel to the solar wind of several wavelengths. Similar coherence lengths were found for directions parallel and perpendicular to the wave vector, when determined from the subset of their observations for which principal axis analysis indicated that such a vector was well defined.

In order to calculate the correlation lengths of the dominant parallel propagating waves in our simulation we took the autocorrelation function of the components of mag-
netic field within the simulation box at a particular point in time, at various parallel and perpendicular lags (allowing the box to wrap around periodically). We obtained a set of autocorrelation coefficients which allowed us to make an estimate of the coherence length in the two directions parallel and perpendicular to $B_0$.

In Figure 4.14 we show the autocorrelation of the field component data against lag in the parallel direction at four times through the simulation. This autocorrelation has three peaks. The peak value of 1.0 at zero lag must occur by definition. This is equivalent to a lag of one box length ($166.4c/\omega_i$ in this case) for this periodically calculated autocorrelation. The other two peaks of approximately 0.2 occur at lags very close to 1 and 2 (or -1) wavelengths at the early time of $t = 25\Omega_i^{-1}$ when the waves are still growing and of low amplitude. This indicates a weak correlation between adjacent growing waveforms. Later ($t \geq 50\Omega_i^{-1}$) this correlation becomes stronger with values near to 0.6 for the transverse components, $B_y$ and $B_z$, whilst the weaker $B_x$ component is also correlated, but at a lower value due to its higher noise level.

In Figure 4.15 we show the autocorrelation of the field component data against lag in the perpendicular direction again at the same four times through the simulation. At the earliest time there is almost no correlation. By $t = 50\Omega_i^{-1}$ when the waves have significant amplitude the correlation can be seen to fall away in an exponential manner on scales of the order of $1/4$ of a wavelength, with this coherence length dropping slightly as the simulation proceeds. The components are ordered in their correlation lengths with $B_z$ the longest approaching $1/2$ wavelength, $B_y$ around $1/4$ wavelength and $B_x$ lower than this for the reasons outlined in the parallel case. The slight peaks superimposed on the general fall of the autocorrelation probably relate to long wavelength modes just off the field axis, also connected with the strong anticorrelation of the $B_z$ component around saturation.

Comparing with the related plot of magnetic field phase angles in Figure 4.12 one sees visual agreement with these results. In the parallel direction there is a strong correlation seen in the parallel direction, but this is to be expected of a simulation with a dominant parallel propagating mode from the nature of its growth in a periodic box. In the perpendicular direction the coherence length appears to the eye to be longer, up to the order of a wavelength, but in our results this value is depressed by the averaging over the entire box length with its pollution by oblique modes. These effects help to explain
CHAPTER 4. SIMULATION OF ION BEAM INSTABILITIES IN 2D

Figure 4.14: Parallel autocorrelation of the magnetic field components: $B_x$ (solid) and $B_y$ (dotted) and $B_z$ (dashed) for simulation [B1]. The box is treated as periodic and is averaged in the perpendicular ($y$) direction.
Figure 4.15: Perpendicular autocorrelation of the magnetic field components: $B_x$ (solid) and $B_y$ (dotted) and $B_z$ (dashed) for simulation [B1]. The box is treated as periodic and is averaged in the parallel ($x$) direction.
why we obtain shorter coherence lengths than those calculated from observational results by Le & Russell (1990) and Le et al. (1993).

### 4.2.6 Compressive Waves: Correlation Between $B$ and $n$

In Figure 4.16 we plot the time evolution of the correlation between magnetic field and density ($C_{nB}$ as defined in Appendix A) for our two-dimensional simulation [B1]. Initially, as expected for predominantly circularly polarized waves, there is little correlation ($C_{nB} \approx 0$), but as we approach saturation $C_{nB}$ rises to approximately 0.1 indicating that the waves are slightly compressive. This was noted earlier from Figure 4.6 at $t = 40\Omega_i^{-1}$, where the density was seen to be correlated with features in the field. We note that $C_{nB}$ begins to increase above zero at $t \approx 25\Omega_i^{-1}$, which, from Figure 4.4, is when the perpendicular beam temperature begins to increase rapidly. When saturation is reached the correlation coefficient actually becomes weakly negative, suggesting that field features have arisen which are partially excluding ions, possibly by a weak snowplow effect. After saturation the correlation fluctuates around $C_{nB} \approx 0 \pm 0.05$.

In one dimensional simulations, Akimoto et al. (1993) observed a correlation of upto $C_{nB} \approx 0.6$, growing from near zero to this value with increasing wave energy and remaining high in the post-saturation phase. Simulations using our code with a long narrow box to give a pseudo-one-dimensional simulation show a similar behaviour but at a lower level of correlation, probably a consequence of our better modelling of density due to better particle statistics. The lack of significant field and density correlations is in agreement with the qualitative findings in two dimensions of Winske & Quest (1986), and suggests a qualitative difference in the evolution of the instability in one and two dimensions, possibly linked to the two-dimensional magnetic structures examined in Section 4.2.7.

There is evidence of some beam clumping, as can be seen in the plots of core and beam ion densities (Figure 4.8) showing the production of spatially bunched ions. Terasawa (1988) outlines a mechanism for the generation of such clumps by ion beam instabilities. The initial linear instability causes waves to grow at various wavenumbers, forming waves which beat together to generate large spatial fluctuations in the magnetic field amplitude which are not present for a single wave, which, being circularly polarized, has uniform amplitude. Beam ions are then decelerated by the mirror force at these field enhancements, giving a local increase in beam density. This leads to an increased growth rate for
waves generated by this portion of slowed beam. There thus exists a positive feedback loop where the increasing field becomes more efficient at decelerating beam particles. Such clumps will be gyrophase bunched and so could destabilize new wave modes.

4.2.7 Short Scale Magnetic Field Features

It was noted in Section 4.2.3 that around saturation short scale magnetic features become visible scattered throughout the plasma. Figure 4.17 illustrates one such feature centred around \( x = 63c/\omega_i, \ y = 12c/\omega_i, \) at \( t = 50\Omega_i^{-1} \) (see Figure 4.7). We also show the beam velocity distribution function in the neighbourhood of the structure.

A diagonal cross section across the box from \((50,0)\) to \((80,30)\) \(c/\omega_i\) is shown in Figure 4.18. The angle of this cut to the initial background field is 45°, very close to the minimum variance direction. Minimum variance analysis is described by, e.g., Sonnerup & Scheible (In press). The field components transformed into the minimum variance frame are shown in the lower three panels of this figure, and a corresponding hodogram of \( B_{\text{max}} \) versus \( B_{\text{int}} \) is shown in Figure 4.19.

The feature is a magnetic perturbation with dimensions of approximately \( 5 \times 25c/\omega_i \) oriented at about 45° to \( B_0 \). The feature is marked by a fall in \( B_x \), a rise in \( B_y \) and a reversal of \( B_z \). A slight enhancement in the total field is observed at the feature and the field magnitude rises from around 1.0\( B_0 \) approaching from the left to 1.5\( B_0 \) to the right.

Transforming to the minimum variance frame we find that the field components represent a field reversal in the maximum variance component with a corresponding increase.
Figure 4.17: Enlargement of short scale magnetic field feature at $t = 50\Omega_{ci}^{-1}$ in [B1]. Magnetic field components are shown (top) with 3 boxed regions, and (below) the projected beam particle velocity distributions within each of these boxes.
Figure 4.18: Cross-section of short scale magnetic field feature in Figure 4.17 from (50,0) to (80,30) \( c/\omega_i \) at \( t = 50\Omega_i^{-1} \). Shown are the magnetic field components, total field, total density and the magnetic field components in the minimum variance frame.
Figure 4.19: Hodogram in the plane of maximum and intermediate variance along the cross section of Figure 4.18. The hodogram runs clockwise from (50,0) to (80,30) $c/\omega_i$. 
in the intermediate variance component, as dictated by $\nabla \cdot \mathbf{B} = 0$. This can be clearly seen in Figure 4.19 where the hodogram in the minimum variance frame shows a LH rotation moving along the line from $(50,0)$ to $(80,30) c/\omega_i$ of $180^\circ$ centred on $(-0.5,-0.5)B_0$ with a radius of approximately $1.0B_0$.

Little overall density change above the fluctuation level is observed to be associated with the feature, however a significant beam density enhancement (coincident with a slight fall in the core density) is observed at the field feature which we discuss later. Both beam and core ions appear to be reflected inside the feature, with their local drift velocities negative.

Figure 4.17 shows the local beam velocity distribution in the vicinity of the feature. In Box 1, the top row of velocity space scatter diagrams show the beam to the left of the feature where the beam is seen to have slowed and be weakly pitch angle scattered and gyrophase bunched with the magnetic field. Some $-x$-direction travelling beam ions are present, having been backscattered by the feature, these are visible as the tenuous arc across the top of the $v_y - v_z$ plot and they are not seen in Box 3.

Within the feature (Box 2) we see a very different picture with a majority of the beam ions travelling in an $-x$-direction, having been reflected within the feature. The signature of the unscattered beam remains, leading into the scattered arc visible in the $v_x - v_z$ plot, the beam particles following the rapid change in the field phase angle (due to the change in polarity of $B_z$). Within the feature the field direction no longer points predominantly along the initial direction so the ions’ gyration takes the form of ‘pitch angle scattering’ about the mean field direction.

To the right of the feature (Box 3) we again observe the signature of the beam, $+x$-direction moving bunched ions, which must have travelled around the feature on field lines unconnected to the feature. The more diffuse $-x$-direction ions must have been reflected, probably by a similar feature to the right of Box 3.

The feature examined here is generated during the saturation phase of the instability and lasts for approximately a gyroperiod $(2\pi\Omega_i^{-1})$ before fading into the wave background. During this period it changes in appearance, spreading and losing amplitude whilst being refracted to higher $\theta_{k_B}$. This particular field feature has dissipated before the phase in which whistlers are generated by high field features. It is not a steepened wave in the sense that Omidi & Winske (1990) discuss shocklets where they believe the beam plays
little or no role in the evolution.

Other similar features can be seen at saturation throughout the simulation box. We can estimate the proportion of beam particles interacting with such features over their lifetime. Taking the beam speed to be $5v_A$ and estimating the number and size of similar features from area of the simulation box showing suppression of the $B_x$ component below some threshold (say 0.5), we obtain an estimate that around 10-20% of the beam particles interact with these features. This appears to be only a small proportion, however comparing Figure 4.8 and the $B_x$ component at the same time as shown in Figure 4.7, the beam ion density is seen to be enhanced by a large factor ($\lesssim 6$) in regions of low $B_x$ (and high $\delta B$), suggesting that a similar effect is taking place even at much weaker perturbations to the field, which have a similar nature to the strongly enhanced feature explored in detail here. In this case a far larger proportion of the beam ions than we estimate undergo this type of scattering and it becomes important to the evolution of the instability, most notably its saturation mechanism.

The role of these features in the saturation process may shed light on the result of Winske & Quest (1986) that beam simulations in two dimensions saturate at a approximately half the level of magnetic fluctuation energy than equivalent one-dimensional simulations. Could this be due to a more rapid denaturing of the beam via a more efficient pitch angle scattering process being at work in two dimensions, a process not present in one dimension? If this is the case then could the process we describe here with short scale field features efficiently reflecting beam particles play this role?

These features have no immediate analogue in one dimension. Around saturation in a comparable pseudo-one-dimensional (parallel) simulation $B_x$ has a maximum variation from the initial value of $\sim 10\%$, far lower than that observed in any of the two-dimensional runs. In a true one-dimensional simulation $\delta B_x \equiv 0$ (from $\nabla \cdot B = 0$), however, our pseudo-one-dimensional simulation was performed in two dimensions with a very narrow box so the observed variations in $B_x$ must be due to modes with a slight $\theta_{k_B}$. Though the feature has an apparently one-dimensional structure orientated at near $45^\circ$, a one-dimensional simulation at this angle would, we believe, not replicate these features as they originate in the beating of waves propagating at different angles, which are obviously not resolved in a single one-dimensional simulation. It therefore appears likely that these features and their more ubiquitous weaker versions play a role in the
saturation of the instability in two dimensions. An inspection of lower spatial resolution two-dimensional results [B3] and poorer particle statistics two-dimensional results [B2] show similar features, but weaker and less well defined.

In three-dimensional simulations a similar effect would occur as the beam particles are rapidly reversed and thus travel only a short way in the $z$-direction (not spatially resolved in our two-dimensional simulations) and would thus require only a short spatial coherence of the feature in the third direction.

4.2.8 Steepened Waves and Whistler Wave Trains

As shown in Figure 4.9, whistlers appear at around $t = 60\Omega_i^{-1}$ associated with localized high field features. These bear a strong resemblance to those described by Omidi & Winske (1990). Here we describe the characteristics of such features as previously identified by other authors and examine how the features observed in our simulations compare.

Steepened magnetosonic waves, or shocklets, have been observed both at planetary foreshocks (Hoppe et al., 1981) and upstream of comets (Tsurutani & Smith, 1986). A shocklet forms when the leading edge of an initially sinusoidal wave becomes steepened. Associated with the steepened edge of the shocklet is a high frequency RH circularly polarized whistler wave packet.

A shocklet has several characteristic features, as described by Omidi & Winske (1990). The steepened wave typically moves from a circular to more linear polarization as it steepens. This shift shows up as a characteristic signature in the field phase angle $\phi = \tan^{-1}(B_z/B_y)$, which remains constant through a region of linear polarization as the total field strength increases. The steepened edge itself is RH polarized as it forms an integral part of the whistler wave packet. The shift in polarization can be explained by the spreading of a large amplitude local field feature in a dispersive medium, such that the field remains approximately constant in transverse components in a region of space, giving linear polarization. The wave packet of short wavelength whistlers is seen to originate at the steepened edge of the wave. The whistler wave train moves ahead of the field feature by virtue of the dispersion of the whistler branch and has almost linear decay in space ahead of the steepened front. More pulses appear with time and the original feature loses amplitude, an indication that the whistler is carrying energy away
from the field feature. The transient nature of the whistler is further demonstrated by
the linear fall off of the wave train. If it were simply being emitted into a damping region
then an exponential fall off would be observed, however, the linear fall off indicates a
constant leakage of energy away from the original peak.

In our simulations we observe shocklet like features with all these characteristics.
This is the first time that such features have been observed in simulations with isotropic
proton beams. These features are visible only when the numerical noise level in the
simulation is sufficiently low, i.e. a large number of simulation particles is needed and the
spatial resolution must be sufficient to resolve the waves’ short wavelengths. The example
described below is not a “clean” as would be found in a single wave simulation because
the feature is found near to saturation and consequently both off-axis and backwards
travelling modes contribute as “noise”.

In Figure 4.20 we show the evolution in time of the transverse components of the
magnetic field along a slice of the simulation spatial domain from the isotropic beam
simulation [B1]. Figure 4.21 shows the magnetic field amplitude and total density at the
same times. The magnetic phase angle, \( \phi_B = \tan^{-1}(B_z/B_y) \), is shown in Figure 4.22.
The slice is taken at \((80c/\omega_i \leq x \leq 176c/\omega_i, y = 35c/\omega_i)\), note that the plots wrap around
the RH boundary. The long wavelength, near monochromatic, wave seen at \( t = 50\Omega_i^{-1} \)
begin to steepen at \( t \approx 55\Omega_i^{-1} \) near \( x \approx 30c/\omega_i \) (the \( x \) co-ordinate of the plot has its
origin at \( x = 80c/\omega_i \) in the simulation box). A steepened wavefront rapidly develops in
front of a region where the magnetic polarization has shifted from circular to linear, as
can be seen in the magnetic phase angle (Figure 4.22). The phase angle is seen to shift
from a falling sawtooth characteristic of circular polarization to a more constant value
indicating linear polarization. A slight density enhancement trails the steepened front
(Figure 4.21), which is itself associated with a magnetic field enhancement, similar to
the observations of Elaoufir et al. (1990). The steepened wavefront moves with a phase
velocity \( \approx 1.3v_A \), though accurate measurement is made difficult by the spreading of the
original field feature.

Shortly after the wavefront has steepened a whistler wavetrain is observed leading
the steepened wavefront, e.g. at \( t \approx 55\Omega_i^{-1} \). Adjacent whistler peaks have a separation,
in this \( x \)-direction cut, of \( 4 - 5c/\omega_i \). This value is close to the true wavelength which is
dependent on the angle of propagation as will be discussed shortly. These shocklet like
features lie on the whistler branch as they are RH polarized and have a group velocity greater than the other waves in the system. The whistler peaks travel away from the steepened wavefront as they have a phase velocity $\simeq 1.8v_A$, and this also leads to a spreading of the extent of the shocklet. The whistler wave train is thus carrying energy away from the steepened wavefront which can be seen to die steadily away over the next $200\Omega_i^{-1}$. The wavetrain is linearly attenuated with $x$ as expected, with the additional implication that the wavetrain is not as monochromatic as it first appears. The magnetic phase angle plots in Figure 4.22 confirm that the whistler is RH polarized. Figure 4.21 shows that the whistler has associated modulations of the density and total field.

The shocklet shown in Figures 4.20-4.22 slows with time, possibly due to its propagation into a region with different local characteristics. Other whistlers scattered around the box do not show this particular behaviour, having more constant velocities close to those initially seen in this feature. Any variations in the properties of these shocklets may be explained by different local plasma conditions. All shocklets eventually decay, and by $t \simeq 100\Omega_i^{-1}$ no obvious sign of these features remains.

This analysis tends to ignore the inherent two-dimensional nature observed of such structures which can be seen in Figure 4.23, where the finite extent across the structure and the non-perpendicularity of the phase fronts can be clearly seen. To examine this aspect we used minimum variance analysis of strips taken across the shocklets in the $x$-direction, to determine the minimum variance direction and thus the likely principal axis of the shocklet. This process was made difficult by the small spatial extent of the features leading to noisy results. At $t = 67.5\Omega_i^{-1}$ the shocklets showed angles between the background field and the minimum variance direction of between approximately $0^\circ$ and $30^\circ$, with the more reliable values lying at about $15^\circ \pm 3^\circ$. Clearly, the shocklets have an oblique nature, as do those observed in the foreshock (Hoppe et al., 1981). This angle is close to that of the $(3,1)$ mode of the simulation, the origin of many of the steepened features, however it is unclear whether the observed oblique nature is inherent to the shocklets or a product of a refraction type mechanism in a medium with spatial variations in its characteristics. A closer look at Figure 4.23 suggests that the latter is more likely the case. In time the structure moves in the parallel direction despite the apparent orientation of the phase fronts, suggesting that the group velocity lies in the parallel direction. In addition, at later times the phase fronts become more perpendicular
to the direction of motion, a fact confirmed by minimum variance analysis at $t = 70.0\Omega_i^{-1}$.

4.3 Instability of a Cool Tenuous Anisotropic Beam (Case [AB1])

In this section we present the results of a simulation of a cool tenuous beam with a temperature anisotropy $T_{\perp}/T_{\parallel} = 10$. A density of 2% of the total ion density and a velocity $10v_A$ make this beam typical of those observed in the terrestrial foreshock. The parameters used are given in Table 4.3.

The beam parameters used here resemble those of the ring beams of Killen et al. (1995), however, here we present results for a non-ring anisotropic beam, including the two-dimensional linear dispersion relation and details revealed by our better resolution and statistics.

4.3.1 Linear Dispersion Relation

The two-dimensional linear dispersion relation for an anisotropic ion beam distribution as described by the parameter set [AB1] is shown in Figure 4.24. This figure shows only the wavenumbers with growing ($\gamma > 0$) modes. If more than one mode grows at a given wavenumber then the fastest growing is shown. This dispersion relation was calculated numerically using WHAMP as described in Section 1.4.2.

Comparing the dispersion relation in this case with that of the isotropic beam in Figure 4.2 we find that the peak in the growth rate has shifted to slightly shorter wavelength with $k \simeq 0.125\omega_i/c$. The local maxima of the other growing harmonics shift by a similar amount. The region of wavevector space over which these harmonics grow spreads and they have larger growth rates due to the greater free energy associated with an anisotropy in addition to a bulk velocity.

The mode structure remains much as in the isotropic case, with clearly defined left and right oblique branches, as can be seen in Figure 4.25.

4.3.2 Overview of the Simulation

The time histories of various quantities from simulation [AB1], are shown in Figure 4.26. Comparing this figure to that for an isotropic beam (Figure 4.4) we observe the follow-
Figure 4.20: Perpendicular field components $B_y$ (solid) and $B_z$ (dotted) from simulation [B1], showing the time evolution of a shocklet. Slices along the line $80c/\omega_i \leq x \leq 176c/\omega_i$, $y = 35c/\omega_i$ are shown at the times marked.
Figure 4.21: Total field $|B|$ (solid) and total density (dotted) from simulation [B1], showing the time evolution of a shocklet. Slices along the line $80c/\omega_i \leq x \leq 176c/\omega_i$, $y = 35c/\omega_i$ are shown at various times.
Figure 4.22: Magnetic field phase angle from simulation [B1], showing the time evolution of a shocklet. Slices along the line $80c/\omega_i \leq x \leq 176c/\omega_i$, $y = 35c/\omega_i$ are shown at various times.
Figure 4.23: The time evolution of the transverse magnetic field components \((B_y, B_z)\) of a shocklet in two dimensions shown in an enlarged cutout from the \(x-y\) simulation box of \([B1]\). The spatial coordinates are given in \(c/\omega_i\).
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Table 4.3: Anisotropic beam simulation parameters.
Figure 4.24: Linear dispersion relation for the case of a tenuous cool beam with a temperature anisotropy $T_\perp/T_\parallel = 10$ [AB1]. The wavevector $(k)$ is in units of $\omega_i/c$, frequencies and growth rates are in units of $\Omega_i$. 
Figure 4.25: Linear magnetic compression ratio and phase velocity (expressed in Alfvén speeds) relation for the case of a tenuous cool beam with a temperature anisotropy $T_\perp/T_\parallel = 10$ [AB1]. The wavevector $(k)$ is in units of $\omega_i/c$. 
ing: The $\delta B$ fluctuation energy rises to saturation at the same time in both cases, with the anisotropic beam case saturating at some 15% lower wave energy. The subsequent trapping oscillations have a period of $\approx 25\Omega_i^{-1}$. Comparison with the value observed for the isotropic beam show this period to be slightly shorter, a result expected from equation (4.1), given the anisotropic case’s higher wavenumber and perpendicular velocity, but very similar post-saturation field level. The fluctuation energy in both cases falls to $\lesssim 30\%$ saturation value by the end of the simulation at $t = 200\Omega_i^{-1}$.

The drift velocity shows a near identical fall off in the two cases, with the anisotropic case differing only in its initially larger amplitude fluctuations. We observe the same association with trapping oscillations, as found in the isotropic beam case between fluctuations in the drift speed, temperature and magnetic fluctuation energy. The parallel and perpendicular temperatures rise to final values $\gtrsim 50$ in the anisotropic case as compared with approximately 40 in the isotropic case. The anisotropic case displays larger amplitude fluctuations in all these quantities during the presaturation phase.

Figure 4.27 shows the evolution of magnetic compressibility, $\mathcal{C}_{nB}$, as defined in equation (A.4). It behaves in a similar manner to that found in the case of an isotropic beam, rising to a maximum of around 0.1 in the pre-saturation period ($t \approx 40\Omega_i^{-1}$) before dipping sharply to below zero at saturation and remaining around this value thereafter. Thus there is little compressibility in the system after the trapping has played its role in the saturation mechanism.

4.3.3 Evolution of Waves

In this section we shall outline the evolution of the waves and structures in the simulation [AB1]. We will particularly highlight the similarities and differences with the isotropic case described in Section 4.2.

During the linear stage of the instability the evolution follows the predictions of the linear dispersion relation Figure 4.24, as can be seen in Figure 4.28 corresponding to $t = 25\Omega_i^{-1}$. A strong resonant structure can be observed especially in the Alfvénic component, $B_z$. The fastest growing modes are those at near parallel propagation with a parallel mode number of $m_x = 2, 3$. Again the more magneto-sonic parallel propagating modes dominate the magneto-sonic field component, $B_y$, with little evidence of the more oblique modes. The $B_z$ component is dominated by the resonant modes at around 45° propagation,
Figure 4.26: Time evolution of magnetic fluctuation energy, component drift velocities, component perpendicular and parallel temperatures (beam solid line, core dashed line) for the case of a tenuous cool beam with a temperature anisotropy $T_\perp/T_\parallel = 10$ [AB1]
Figure 4.27: Plot of time evolution of the correlation coefficient $C_{nB}$ for an anisotropic tenuous cool beam instability [AB1].

Although at a lower amplitude of variation than the other components; these variations are also visible in the density. The magnetic fluctuations $\delta B$ are uniform ($\lesssim 0.2$) reflecting the dominance of approximately circularly polarized, near-parallel propagating fluctuations. In comparison to the isotropic beam case [B1] we observe weaker growth confined to a narrower range of wavenumbers, as expected from the dispersion relation. In particular the anisotropic case has fewer short wavelength oblique features than seen for the isotropic beam (Figure 4.5), where their effects can be seen in the more structured and noisy magnetic fluctuations $\delta B$.

From $t \simeq 35 - 50 \Omega_i^{-1}$ (not shown) harmonics of the dominant modes become visible in the $k$-space plots, initially in the more oblique modes of $B_z$ around $m_x = 6, 7$, and $m_y \gtrsim 8$, the second harmonic, spreading down to $m_y \gtrsim 2$ and appearing in all components as saturation is approached. Traces of what could be third harmonic can also be seen. Post-saturation, $t \simeq 60 \Omega_i^{-1}$, many higher order harmonics are seen but these are more likely to be due to wave steepening. In comparison to the isotropic case, waves remain unsteepened for a longer period mainly due to the lack of short wavelength oblique modulation and show some 20–30% lower peak fluctuations in the field components at saturation. The isotropic case shows a far more turbulent looking field with more scope for scattering. However, the pattern of evolution observed in the isotropic and anisotropic beam cases is roughly the same.

Steepened structures and associated whistlers appear earlier in the anisotropic beam case than the isotropic case, with clear signs of propagating whistlers by $t \simeq 55 \Omega_i^{-1}$. 
Figure 4.28: Summary plots of density and magnetic field components at $t = 25\Omega_i^{-1}$ for an anisotropic tenuous cool beam instability [AB1].
Figure 4.29: Summary plots of density and magnetic field components at $t = 70\Omega_i^{-1}$ for an anisotropic tenuous cool beam instability [AB1].
These whistlers are predominantly associated with the $(\pm 3, 2)$ modes. Well developed steepened waves with their whistler wave trains can be seen in Figure 4.29 at $t = 70\Omega_i^{-1}$. In contrast to the shocklet like features observed in the isotropic case those observed here have both progenitor steepened waves and associated wave trains that are definitely obliquely propagating. In addition they are wider in the direction perpendicular to their motion, with the exception of a number of small, more circular, features, as can be seen at, e.g., $(90, 25)c/\omega_i$ in Figure 4.29. These latter features resemble those observed in the isotropic beam case which led to near parallel propagating whistlers. The anisotropic case has longer lived shocklets with stronger correlated disturbances in $B_x$ suggesting a more two-dimensional nature. As the wave fronts of the whistler spread their propagation angle can be seen to become more oblique. This is probably due to their propagation into a field geometry strongly perturbed by the dominant long wavelength oblique waves.

We can see the evolution of a whistler in simulation [AB1] in Figure 4.30. Both the progenitor feature and the whistler wavetrain can be seen to propagate obliquely with respect to the initial axis of the beam. Minimum variance analysis shows that the feature has an oblique minimum variance direction. Slices through the feature at $t = 65\Omega_i^{-1}$ along the lines $10 < x < 60$ and $y = 80, 90, 100$ result in minimum variance directions of $22.1^\circ, 25.5^\circ$ and $4.7^\circ$. The first two cut through the centre of the feature including the whistler, the last is on the edge of the feature. Clearly these features are oblique in nature and propagate at steeper angles than those generated by the isotropic beam.

The whistlers are no longer visible by $t \simeq 130\Omega_i^{-1}$, beyond which the waves remaining in the simulation become weaker and dominated by the largest modes remaining from linear growth, i.e. $(\pm 3, 0)$ and $(\pm 3, 1)$, which can now be seen in $B_x$ and the density. Over the period $t \simeq 100 - 200\Omega_i^{-1}$ peak field values fall by around 60%, and energy appears in the density in modes $(0, 3)$ and $(0, 5)$ which migrates to $(0, 1)$ and $(0, 2)$.

4.4 The Effects on Simulation Results of Variations in Non-physical Parameters

In order to ascertain whether or not the results of our simulations are accurate models of the real physical behaviour of ion beams it is necessary to observe how the results change
Figure 4.30: The time evolution of the transverse magnetic field components of a shocklet in two dimensions shown in an enlarged cutout from the x-y simulation box of [AB1]. The spatial coordinates are given in \( c/\omega_k \).
as we change various non-physical aspects of the simulation.

In this section we will discuss the results of the various simulations described in Table 4.1. These simulations maintain the same beam parameters but vary the following five aspects of the simulation model: one or two-dimensional, fastest growing linear mode resolved or unresolved, the total number of wavelengths of the dominant modes in the simulation box, particle statistics and field grid cell size. Factors on which these simulations' results are compared include: peak field and density values, partitioning of energy between particles and waves, the global properties of the simulations evolution, e.g. saturation level and time or $C_{nB}$, and the appearance and properties of other features such as shocklets and short scale magnetic field features.

It will become clear that the results we have discussed above are robust with respect to the changes in the simulation; discrepancies are generally improvements related to improved particle statistics and spatial resolution.

Firstly we examine the effects of moving from one to two dimensions by comparing the results of the simulations [B1] and [ONED] with the parameters as shown in Table 4.1. Both simulations fit three wavelengths of the fastest growing linear mode into the simulation box and have 256 grid cells in the $x$-direction and identical particle statistics. However the (one-dimensional) simulation has only 4 cells in the $y$-direction compared with the 256 of the two-dimensional [B1]. An important difference is the higher level of magnetic fluctuation density at saturation in the one-dimensional case, some 50% higher than in the two-dimensional case, a feature also seen in the maximum amplitudes achieved in individual modes' growths. The trapping oscillation period is correspondingly lower at around $200\tau_i^{-1}$. Also, the energy in any single mode in two dimensions is much lower than that found in the one-dimensional simulation, a consequence of the same amount of free energy being shared between many more competing modes in two dimensions. Due to the one-dimensional nature of [ONED] fluctuations in $B_x$ are low ($\lesssim 10\%$) and density fluctuations remain below 50% of those seen in [B1].

To examine the effect of choosing the simulation box such that the fastest growing linear mode is either resolved or unresolved we compare the results of [B2] and [B5] two simulations with identical numbers of grid cells and particles but with different grid cell sizes, in [B2] $\delta x = \delta y = 0.65c/\omega_i$ giving exactly three wavelengths of the dominant linear mode in the box, whereas [B5] has $\delta x = \delta y = 0.75c/\omega_i$ which gives $m_x = 3$ and $m_x = 4$
equal growth rates at about $\pm 0.02\omega_i/c$ from the wavenumber of the maximum growing mode $k_m = 0.113\omega_i/c$. The saturation of both [B2] and [B5] occur at almost identical times and levels ($t \approx 46\Omega_i^{-1}$ with the magnetic fluctuation energy density $\sim 0.5$). Peak densities and magnetic field components are very similar with one significant exception, $B_x$ has a lower minimum (-0.23) in the competing case [B5] as compared with (0.12) in the single dominant mode case [B2]. This is a possible result of the beating of the two rapidly growing modes, with possible implications for formation of short scale magnetic features. Evidence of steepening is more easily seen in the single mode case as a result of that mode reaching higher amplitude. The overall pattern of the evolution of these two runs is nearly indistinguishable. $C_{nB}$ follows the established two-dimensional pattern of a small peak ($\sim 0.1$) pre-saturation followed by a nearly equal magnitude negative peak at saturation and a post-saturation value remaining within $\pm 0.05$, its oscillation connected to the trapping oscillations.

Considering next the effect of the total number of wavelengths of the dominant modes in the simulation box, we compare [B2] and [B4] the former as described in the previous section with a simulation box containing exactly three wavelengths of $k_m$, whereas [B4] has only two. The number of grid cells remains the same in both case but the grid cell size has been adjusted to fit the appropriate number of wavelengths. The two wave simulation [B4] saturates slightly later ($t \approx 51\Omega_i^{-1}$) but at the same level as the three wave simulation [B2]. Density and field component peaks have similar amplitudes and qualitatively the evolution of the two simulations appear very similar. However, both these cases are far from a true multi-mode simulation as the modes at long wavelength are widely spaced. Such simulations are difficult for a long wavelength instability such as the RH resonant ion beam instability as computational resources normally rule out boxes large enough to support many wavelengths of the dominant mode.

The effects of particle statistics are seen in the comparison of [B1] with 80 particles per species per cell and [B2] identical except it has 25 particles per species per cell. The number of particles in a simulation determines the level of statistical noise present. This is reflected in the qualitative examination of the results of these two simulations with [B1] far cleaner and resolving features hidden in the noise in [B2]. The simulation with better statistics [B1] saturates at $t \approx 50\Omega_i^{-1}$, a little later than [B2] owing to the lower initial noise level, from which linear growth is seeded. [B1] has slightly higher peak values of
field components, but lower densities, again explicable in terms of lower noise. Features due to field feature—particle interactions such as shocklets are better resolved in [B1] and the short scale magnetic features are observed to have much lower minima in $B_x$ than observed in [B2] (-0.3 compared with 0.1), where such features are largely poorly resolved or absent. Examining $C_{nB}$ we again see the same pattern as described above but with [B1] showing a slightly larger compressional peak in the pre-saturation phase, as might be expected when density features are better resolved.

Finally we examine the effects of cell size. Comparing [B1] on a $256 \times 256$ grid with $\Delta x = \Delta y = 0.65c/\omega_i$ with [B3] on a $128 \times 128$ grid with $\Delta x = \Delta y = 1.3c/\omega_i$, we have an identical sized box but with grid spacings differing by a factor of 2. Saturation occurs at a slightly lower level and slightly earlier $t \approx 47\Omega_i^{-1}$ for the coarser grid [B3], again due to differences in the initial noise levels. Field levels remain lower in [B3] due to the smoothing over the larger grid cells and fine details are missed, as they remain simply unresolved. $C_{nB}$ exhibits very similar behaviour in the two cases.

### 4.5 Summary

In this chapter we have presented the results of two-dimensional simulations of cool tenuous proton beams, for beams isotropic and anisotropic in temperature. These simulations were conducted with a combination of spatial resolution and particle statistics superior to those previously used by other authors. Comparison of results from simulations with different non-physical simulation parameters indicates that our results are physically meaningful. Here we summarise the important results and conclusions from these simulations.

In both isotropic and anisotropic beam cases the pattern of evolution observed in previous simulations (e.g. Winske & Leroy, 1984) was followed: initial wave growth through linear instability, scattering of the ions by these waves altering the distribution function leading to saturation and the eventual relaxation of the system. In both cases the early (pre-saturation) behaviour follows that expected from the linear theory. This includes many oblique waves which can beat and play an important role in the formation of non-linear structures in post-saturation development of the system.

We observe gyrophase bunching of both the core and beam populations, in agreement
with the one-dimensional linear theory and simulation work of Gary et al. (1986a). We find coherence lengths for the waves parallel to the background magnetic field of more than a wavelength, though this is connected with the periodic nature and restricted size of our simulation box, and of between 1/2–1/4 of a wavelength perpendicular to the background magnetic field.

We observe beam-driven short scale length features not previously resolved in two-dimensional simulations. We observe magnetic features showing a field rotation and consequent depression of the parallel field component, $B_x$. These features appear to have their origin in the interaction of waves travelling at different angles beating to generate high field features which become strongly coupled with the beam ions. These features scatter ions and play an important role in the rapid saturation observed in two-dimensional simulations.

We also observe shocklets, again not previously observed in simulations of isotropic beams due to poor resolution. These shocklets bear all the characteristics typically expected of such structures. There is some evidence of these shocklets having oblique orientation and propagation, as seen in observations of foreshock waves.

In our anisotropic beam results [AB1] we see evidence of broader wavenumber growth at harmonics, and a resulting lower level of fluctuations at saturation as the free energy is more widely spread amongst the modes than in the isotropic case. The anisotropic beam also excites less short wavelength, highly oblique activity, probably due to differences in the evolution of the distribution function. The anisotropic beam instability generates many shocklets which are seen to propagate at oblique angles ($\sim 25^\circ$). This lends weight to theories that such structures in the foreshock are due to anisotropic beams.

The full two-dimensional evolution of ion beam instabilities is an incredibly complex area, and, of necessity, we have only examined a small section of the possible parameter space. The extension of this work into, for example, hot beam instabilities would allow the exploration of many interesting instabilities and phenomena, however, such distributions are broad in velocity space and their accurate representation requires numbers of particles far greater than currently available to us.
Chapter 5

Parametric Instabilities

In this chapter we investigate the susceptibility to parametric instabilities of the waves generated by ion beam instabilities and examine the two-dimensional evolution of a wave known to be unstable to parametric decay in one dimension. First we outline the nature of parametric instabilities and their classification for a fluid model. We then discuss how this scheme is modified by kinetic effects. We then go on to compare two-dimensional simulations of the evolution of an ion beam instability with that of a single, monochromatic, circularly polarized Alfvén wave. Finally, we present the results of our two-dimensional simulations of a parametrically unstable wave.

5.1 Parametric Instabilities

A parametric instability is a non-linear phase coherent coupling of waves. Such instabilities have been extensively investigated using fluid plasma models. The classification here follows that emerging from the two-dimensional two-fluid model studies of Viñas & Goldstein (1991a,b).

In such a two fluid model, a parallel propagating monochromatic circularly polarized Alfvén wave is susceptible to 4 main types of parametric instability: decay, modulational, filamentation and magneto-acoustic. In the decay instability an initial Alfvén pump wave \((\omega_0, k_0)\) decays into an acoustic daughter wave \((\omega, k)\) initially excited from the noise and a pair of oppositely propagating Alfvén sideband waves \((\omega_0 \pm \omega, k_0 \pm k)\); this occurs when \(k > k_0\). In the modulational instability a similar process occurs, but for \(k < k_0\). However, the sideband waves are very close to the pump wave in wavelength and frequency and the
acoustic daughter wave has very low frequency and long wavelength, and hence modulates the amplitudes of the other three waves. In the filamentation instability (Kuo et al., 1988), which excites near perpendicular propagating waves (i.e. for the daughter $k \perp B_0$), density perturbations of the acoustic wave propagate perpendicular to the pump wave and excite Alfvén side bands that beat with the pump wave, giving rise to a ponderomotive force, which in turn drives the growth of density fluctuations. The filamentation instability is nearly non-propagating at an angle to the background field of $\theta_{kB} = 90^\circ$ ($Re(\omega) \approx 0$) with $Re(\omega) \ll \gamma$, but by $\theta_{kB} = 80^\circ$, $Re(\omega) \approx \gamma$ giving a narrow band of propagating wavevectors (Viñas & Goldstein, 1992). The magnetoacoustic instability produces waves with both oblique and perpendicular propagation. It is characterised by propagating density fluctuations with large real frequencies ($Re(\omega) \gg \gamma$). In contrast to the filamentation instability the magneto-acoustic instability excites a broad angular range of wavevectors around perpendicular, but only in a limited range of wavenumbers.

By definition, fluid treatments neglect kinetic effects. Inhester (1990) used a drift kinetic treatment to show that kinetic effects both enlarge the unstable wavenumber range and reduce the growth rates of parametric instabilities. Such a treatment takes the ion mass, charge and magnetic moment ($\mu_B$) as constants, and uses the coordinates $(r, v_{\parallel})$ to describe the behaviour of the ion in one dimension. It is valid for any wave amplitude, however ion cyclotron effects are not included. In contrast, the Vlasov treatment of Lee & Kaw (1972) is only valid for moderate wave amplitudes, but can be generalized to include these effects. The one-dimensional simulations of Terasawa et al. (1986) and Vasquez (1995), described below, illustrate the effects of kinetic departures from fluid theory.

5.2 Previous Simulations

Previous kinetic simulation work on parametric instabilities can be divided into one and two-dimensional. We shall describe each in turn.

5.2.1 One-Dimensional Simulations

Terasawa et al. (1986) investigated the decay instability of a single monochromatic circularly polarized Alfvén wave of variable initial wavelength and amplitude using a one-
dimensional hybrid simulation. Their results indicated that the kinetic instabilities are qualitatively the same as the fluid instabilities, a view later disputed by Vasquez (1995). Terasawa et al. (1986) observed an inverse-cascading process where the Alfvén sideband waves are susceptible to further decay such that energy is continuously transferred to longer wavelengths.

Vasquez (1995) performed a similar study in which a pseudo-spectral one-dimensional hybrid code was used to examine the robustness of the claims of Terasawa et al. (1986) that the kinetic parametric instabilities differ only quantitatively to those of the corresponding fluid instabilities. To allow comparison with Terasawa et al. (1986) the same wavenumber $k_0 = 0.408\omega_i/c$ was used, but with a much larger range of wave amplitudes and plasma betas. They compared the predictions of fluid theory with their results and found large discrepancies, with the ion dynamics appearing to both stabilize, by reducing the growth rates, and, on the other, hand destabilize in new ranges of wavenumber. In some simulations they even observed the destabilization of a different instability to that predicted by fluid theory, e.g. a decay instability when a modulational instability was expected. They concluded that the physical nature of the kinetic instability must differ fundamentally from that of the fluid instability in, at least, some parameter ranges. To recognise these differences, in the kinetic case, they refer to instabilities resembling the fluid decay and modulational instabilities as the “D” and “M” instabilities respectively.

Vasquez (1995) also notes that simulations are useful in determining the evolution parametric instabilities as they include the weak non-linear coupling between modes absent in a straightforward linear parametric theory, but which will tend to modify the observed growth rates as compared with even a fully kinetic linear theory. Additionally, investigation of the mechanisms by which saturation occurs requires complex non-linear theory, and simulation can provide important insights into these processes. Large differences are predicted in the post-linear evolution of parametric instabilities (i.e., when the pump wave has diminished in amplitude by more than a few percent), when wave-particle interactions become important. This latter phenomenon is absent in the pure fluid model where only wave-wave interactions can occur. However, the growth rates associated with parametric instabilities are slow enough that, on the timescales over which we have examined beam evolution, we are unlikely to progress beyond the linear parametric regime, so will not consider these differences in any detail.
5.2.2 Two-Dimensional Simulations

Viñas & Goldstein (1992) compared the results of analytical and numerical investigations of parametric instabilities with those of hybrid simulations which were allowed to proceed into the non-linear stages of evolution. Their analytical and numerical work admitted participating waves to propagate at arbitrary angles to the background magnetic field. The dependence of these instabilities on the properties of the plasma and pump wave was then examined. The simulations with which these results were compared were performed in a doubly periodic two-dimensional box with a grid $128 \times 64$ cells $\Delta x = \Delta y = 1.96c/\omega_i, \beta = 0.2$. The pump wave used was a cold (no ion thermal velocity) large amplitude ($0.5B_0$) RH circularly polarised wave propagating along the background magnetic field ($x$-direction) with $k = 0.1\omega_i/c$, corresponding to $m_x = 4$ in their system.

For the decay instability they found evidence of daughter ion-acoustic waves in the density, with $\omega_i$ close to that predicted by their theory but with growth rates lower than predicted. They attributed this to damping through dissipation which was absent from their theory. The low beta used in their simulations leads to relatively high growth rates (0.01–0.02), especially at the higher harmonics ($m_x = 13, 15$). The origin of this latter growth is unclear. At perpendicular propagation they see excitation of the magnetoacoustic instability ($m_y = 1$) and filamentation instability ($m_y = 2, 4, 5$) in the magnetic field. The low number of modes are due to their coarse spatial grid. They found the magnetoacoustic instability to have depressed growth rate when compared with their theory, whereas the filamentation instability’s growth was accurately predicted, as were the real frequencies in both cases. However, in both of these oblique instabilities the growth rates were low ($\gamma \ll 0.01$).

In contrast to this approach Liewer et al. (1992) used a full particle code, with both electrons and ions represented as particles (taking $m_i/m_e = 100$), with a simulation grid $256 \times 256$, $k = 1.1\omega_i/c$, corresponding to $m_x = 3$, $\delta B/B_0 = 0.6$, $T_i = T_e$ with $\beta_e = 0.22$. They observed their pump wave damping into particle thermal motion, accompanied by three parallel (one-dimensional) decay instabilities, two slightly oblique decay instabilities, and a filamentation instability. They concluded that for $\delta B/B_0 > 0.1$ decay dominates, otherwise parametric decay was very slow. The simulation was hampered by poor statistics (4 particles per cell per species), and the slow operation of the full particle code even on the supercomputer at their disposal. The large wavenumber
chosen makes it of little relevance to our work.

5.3 Role of Parametric Instability in Evolution of Ion Beam Instability

As no complete kinetic treatments of parametric instabilities exist and simulations have shown that kinetic effects can dramatically alter the range and growth rate of instability, the only route available to investigate kinetic parametric instabilities for a particular wave in a given plasma is simulation.

Previous authors (e.g. Terasawa et al., 1986; Vasquez, 1995) have only examined restricted regions of wavenumber space. In one of the first simulations of electromagnetic ion beam instabilities Winske & Leroy (1984) suggested that parametric instabilities could play a role in the post-saturation behaviour. In order to test the hypothesis that waves generated by ion beam instabilities are susceptible to parametric instabilities we perform a comparison of the results of hybrid simulations of a single Alfvén wave with those of an ion beam instability. The Alfvén wave is chosen to have the same wavenumber and polarization and comparable amplitude to the fastest growing waves in the ion beam simulation [B1].

In simulating the evolution of a single large amplitude Alfvén wave we closely follow the approach of Terasawa et al. (1986), but using our two-dimensional hybrid code.

5.3.1 Initial Conditions

An initial parallel propagating monochromatic Alfvén wave of a given amplitude, wavelength and polarization is generated using appropriately phase shifted sinusoidal $B_y$ and $B_z$ components superimposed on the initial background field. $B_0$ lies in the $x$-direction. We follow Terasawa et al. (1986) by calculating the frequency of the wave from the cold plasma dispersion relation ($k_0^2 = \omega_0^2/(1 \pm \omega_0)$, where $+/− = \text{RH/LH}$). Particles are given a bulk perpendicular velocity using the Alfvén condition

$$v_\perp = -\frac{k_0}{\omega_0} B_\perp,$$

upon which is superimposed a thermal velocity. For a wave travelling in the $+x$ direction the particle velocity is thus opposite to the wave field and corresponds to the phase relationship between the dominant waves and the supporting core population in an isotropic
### PHYSICAL PARAMETERS

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</tr>
<tr>
<td>Frequency ($\Omega_i$)</td>
<td>0.120</td>
<td>0.120</td>
<td>0.120</td>
<td>0.5</td>
</tr>
<tr>
<td>Helicity</td>
<td>+1</td>
<td>+1</td>
<td>+1</td>
<td>+1</td>
</tr>
<tr>
<td>Simulation box:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$L_x$ ($c/\omega_i$)</td>
<td>166.40</td>
<td>166.40</td>
<td>166.40</td>
<td>123.14</td>
</tr>
<tr>
<td>$L_y$ ($c/\omega_i$)</td>
<td>166.40</td>
<td>166.40</td>
<td>166.40</td>
<td>123.14</td>
</tr>
</tbody>
</table>

### SIMULATION PARAMETERS

| Grid properties: |      |      |      |       |
| No. cells ($x$), $N_x$ (cells) | 128 | 128 | 128 | 128 |
| No. cells ($y$), $N_y$ (cells) | 128 | 128 | 128 | 128 |
| Cell size ($x$), $\Delta x$ ($c/\omega_i$) | 1.3 | 1.3 | 1.3 | 0.962 |
| Cell size ($y$), $\Delta y$ ($c/\omega_i$) | 1.3 | 1.3 | 1.3 | 0.962 |
| Particles per cell: | 100 | 100 | 100 | 100 |
| Total particles ($\times 10^6$): | $\sim 1.7$ | $\sim 1.7$ | $\sim 1.7$ | $\sim 1.7$ |
| Timesteps: |      |      |      |       |
| Ion timesteps $\Delta t$ ($\Omega_i^{-1}$) | 0.05 | 0.05 | 0.05 | 0.05 |
| Field Substeps per $\Delta t$ | 20 | 20 | 20 | 20 |
| Field divergence test (frequency in iterations) | 10 | 10 | 10 | 10 |
| Total time ($\Omega_i^{-1}$) | 200 | 200 | 500 | 500 |

Table 5.1: Parametric instability simulation parameters.
beam instability as seen in Figure 4.12. This initial state is identical to that used by Terasawa et al. (1986) and Vasquez (1995). Vasquez noted that the initial particle distribution is not an exact solution of the Vlasov equations, so that within a wave period the ions relax to a slightly anisotropic state with $T_{i\perp}/T_{i\parallel} > 1(< 1)$ for L(R)H modes, this being a true equilibrium state. This agrees with Abraham-Shrauner & Feldman (1977) who find steady, non-dispersive, non-linear wave train solutions for the Vlasov-Maxwell equations with anisotropies. This effect is seen after $t \approx 60\Omega_{i}^{-1}$ (i.e. within one wave period) with a temperature anisotropy of about 10%.

Vasquez (1995) also notes an oscillation of the transverse magnetic field and ion energy densities, with their total remaining constant, an effect not due to ion kinetics as it is seen at $T_{i}/T_{e} = 0$, which is interpreted as a purely numerical effect due to finite size particles. The transverse magnetic field fluctuation is not exactly parallel to the transverse velocity moment calculated from finite size particles, as it should be for a wave train solution. This error causes oscillation of the ions around an equilibrium position (parallel to the field) with the observed oscillatory exchange of energy between the field and particles. These oscillations appear not to affect the physics of the system because they are small (a few percent of total energy) and occur as an amplitude variation of the whole wave train and are thus in mode 0 which cannot drive an instability.

The parameters describing our initial waves and the initial plasma conditions are given in Table 5.1. The waves resemble those generated in simulation [B1], the cool, tenuous, isotropic beam case. Three wavelengths are included in the box, as in the case of the beam instability, where three wavelengths of the mode with maximum growth rate fitted exactly into the simulation box. The hypothesis to test is that this initial wave will couple to waves in the numerical noise of the system to excite parametric instabilities.

5.3.2 Parallel Propagating Waves in Two Dimensions: Simulation Results

Cool Tenuous Ion Beam (Case [B1])

First we examine our two-dimensional beam instability simulations for evidence of the previously well explored parallel propagating parametric instabilities. In order to detect the presence and the types of such instabilities we filter our simulation results in $k$-space so that only those wavevectors lying in the direction parallel to the initial magnetic
Figure 5.1: Time evolution of the tenuous cool beam simulation [B1] filtered to leave only the parallel propagating modes, showing $B_y$, the magnetic field decomposed by helicity and $E_x$, together with their spatial Fourier Transform.
CHAPTER 5. PARAMETRIC INSTABILITIES

field \( \mathbf{B}_0 \) remain. This allows us to determine whether the well understood parametric instabilities of parallel propagating waves play any role. The application of this filtering process in the case of the cool, tenuous proton beam simulation \([B1]\) can be seen in Figure 5.1, where the one-dimensional field data and its spatial FT are shown stacked in time. The four plots shown are for a transverse field component, the transverse field decomposed into its two helicity components (as described in appendix B of Terasawa et al. (1986)) and the longitudinal component of the electric field \( E_x \) (which represents the acoustic wave). This decomposition should allow the waves involved in a parametric instability to be identified.

It can be seen that a large amplitude positive helicity wave grows through linear instability of the mode \( m_x = 3 \), the instability saturating at \( t \approx 50 \Omega_i^{-1} \). This wave travels in the \(+x\)-direction and splits into shorter wavelength faster moving waves, the dominant wave remains strong and can be seen to fluctuate in amplitude due to the exchange of energy with particles during trapping oscillations. In the positive helicity plot we see the dominant mode and the two adjacent modes unstable according to linear theory and some low level activity appearing shortly afterwards in higher mode numbers, comparison with the \( B_y \) plot shows the dominance of the positive helicity waves. Also clearly visible are waves at \( m_x = 3 \) but travelling in the \(-x\)-direction, these appear to originate with initially growing large amplitude wave, but by virtue of their appearance in the positive helicity component must be LH polarized. Their origin could be in the destabilization of LH modes through beam clumping or non-gyrotropy in the distribution function (Terasawa, 1988).

The negative helicity components exist at a much lower amplitude and are dominated again by the \( m_x = 3 \) component as the steepening process shifts its polarization from circular to more linear. A purely linear polarized wave would appear equally in the two helicities. Looking in the real space plots of the negative helicity data, waves can be seen to propagate in both directions, notably post-saturation growth of LH polarized \( m_x = 4 \) waves travelling in the \(+x\)-direction and weaker RH polarized \( m_x = 7 \) waves travelling in the \(-x\)-direction. The electric field \( m_x = 1 \) mode can be seen to grow oscillating over the trapping period, before dying away.

The waves observed to appear after saturation do not appear to fit into a pattern associated with a decay type instability, appearing more modulational instability-like
with $m_x = 1$ acoustic waves and $m_x = 4$ negative helicity waves apparently associated. However, there is no clear transfer of energy between these waves as would be expected if a parametric instability were operating.

**Parametric Instability of Single Waves [P1-3]**

To examine in isolation the effects of parametric instabilities on the dominant wave in these beam simulations a single large amplitude circularly polarized wave, with properties similar to those of that dominant wave, was allowed to evolve in isolation. Three amplitudes of wave were used as described in Table 5.1. Each wave had a Maxwellian ion distribution superimposed on the ion velocity due to the wave, representing the near isotropic core population in a beam simulation. These simulations were performed in two dimensions but first we will describe the results after filtration in the same way as described above.

The first simulation [P1] (Figure 5.2) uses a wave with an initial amplitude of $0.2B_0$ corresponding to that of the later post-saturation dominant wave amplitude in the beam case. Little evolution of the initial wave occurs and the qualities of the negative helicity and $E_x$ do not change from those of the initial noise and merely show modulation by trapping effects. Thus, in one dimension no evidence of any parametric instability is seen, suggesting that if such effects are present it is at a very low level.

The second simulation [P2] (Figure 5.3) uses a wave with an initial amplitude of $0.4B_0$, corresponding to that of the dominant wave amplitude at saturation in the beam case. Again, little visible evolution of the initial wave occurs, but in the negative helicity and $E_x$ components, the signature of a decay-type instability is present at low level. Growth of the $E_x$ modes $m_x = 5 - 8$ can be seen to correspond with the growth of the negative helicity modes $m_x = 2 - 5$. However, the weakness of these waves and their low growth rates, combined with the rapid post-saturation drop in amplitude of the dominant beam-generated wave, make it unlikely that they would be either visible or present in the beam instability case.

Though the third simulation [P3] (Figure 5.4) uses a wave with an initial amplitude equal to that of $B_0$, a wave of far higher amplitude than occurs in our beam simulations, it is useful to examine its results to obtain a clear idea of the properties of a decay-type instability, and how high growth rates might be. Also, it has some interesting features
Figure 5.2: Results of the single wave simulation [P1] filtered to leave only the parallel propagating modes, showing the time evolution of $B_y$, the magnetic field decomposed by helicity and $E_x$, plus their spatial FTs. Note: $m_x = 3$ has been removed from the positive helicity plots to make small amplitude activity visible.
Figure 5.3: Results of the single wave simulation [P2] filtered to leave only the parallel propagating modes, showing the time evolution of $B_y$, the magnetic field decomposed by helicity and $E_x$, plus their spatial FTs. Note: $m_x = 3$ has been removed from the positive helicity plots to make small amplitude activity visible.
Figure 5.4: Results of the single wave simulation [P3] filtered to leave only the parallel propagating modes, showing the time evolution of $B_y$, the magnetic field decomposed by helicity and $E_x$, plus their spatial FTs. Note: $m_x = 3$ has been removed from the positive helicity plots to make small amplitude activity visible.
which might help to explain aspects of the later evolution of the beam simulation in two dimensions (Section 5.3.3).

In the negative helicity and $E_x$ components, the signature of a decay-type instability appears to be present at some low level. Growth of the $E_x$ mode $m_x = 5$ can be seen to correspond with the growth of the negative helicity mode $m_x = 2$. Both these modes are visible in the real space plots. Even this large amplitude wave produces growth of only a few percent of its amplitude over the time period studied, suggesting that unless in a given situation there is some enhancement of the effect through, say, some unusual feature of the particle distribution function, this decay-type instability is unlikely to play a large role in the real post-saturation wave field of the ion beam instability.

In summary, we find that in our beam simulation we see no parametric effects of the decay type predicted from our single wave simulations. We see some traces of what might be a modulational type instability but this could equally be due to the effects of destabilization of other instabilities by the evolution of the beam distribution function.

### 5.3.3 Evolution of Single Waves in Two Dimensions

We now present the two-dimensional results of the single initial wave simulations. For each of the three simulations we give a description of the two-dimensional evolution of the wave and show a summary plot of the density, magnetic field components and total magnetic fluctuation for the end state of the simulation at $t = 200\Omega_i^{-1}$. For clarity, the large components associated with the initial wave seen in $B_y$ and $B_z$, are suppressed.

From these results we estimate to what extent kinetic effects influence the evolution of parametric instabilities in two dimensions. The existence of any evidence for these instabilities in the cool tenuous beam simulation [B1] will be discussed.

For the case [P1] (Figure 5.5) no mode in any component has a level exceeding 0.003$B_0$, two orders of magnitude below that of the initial wave. Notable however is the activity at $m_x = \pm 3$ over the range of $m_y = 2 - 12$ seen in $B_z$. Activity is also seen in $|\delta B|$ centred at (0,4), also associated with that seen in density.

For the case [P2] (Figure 5.6) again we find very weak activity. No mode in any component has a level exceeding 0.005$B_0$. The most notable features are those observed in $B_z$ and $|\delta B|$ at ($\pm 3, 3$) and ($\pm 6, 3$). The ($\pm 3, 3$) appear as early as $t \approx 50\Omega_i^{-1}$, ($\pm 6, 3$) appearing later, suggesting that the first may excite the second. The ($\pm 6, 0$) component
Figure 5.5: Summary plots of density and magnetic field components at $t = 200\Omega_i^{-1}$ for the case of an initial single wave of amplitude $0.2B_0$ [P1]. Note: The pump wave $(\pm 3, 0)$ has been removed from the perpendicular field components $(B_y, B_z)$ spatial FTs for clarity.
visible in \(|\delta B|\) is simply a product of slight irregularities produced in the initial wave due to trapping oscillations. Some perpendicular activity in \(|\delta B|\) is also visible.

For the case [P3] (Figures 5.7 and 5.8) no mode in any component has a level exceeding \(0.009B_0\). The earliest growth takes place in modes \((\pm 3, 1)\) in all components but not density and \((0, 1)\) in \(B_x\), \(|\delta B|\) and density, all these modes showing oscillations superimposed on a growing trend from the beginning of the simulation. At the end of the simulation there are notable peaks in \((\pm 3, 1 – 4)\), \((\pm 6, 1 – 4)\), \((\pm 9, 0)\) and \((0, 1 – 3)\), various combinations of components representing each mode as can be seen in Figure 5.8. Most interestingly in this figure we also see strong short wavelength activity, predominantly in the \(B_x\) component. In Figure 5.9 we show the total magnetic field and its spatial FT up to modes with a wavelength of two cells \((m_x = m_y = 64)\). Here activity can be seen to centre around the four cell level in two strips orientated at around \(60^\circ\). They can be seen to have structure related to gaps in \(m_x\) of 3, which must in some way be related to the initial wave. From the configuration space data it is clear that we are seeing a short wavelength wave with a wave vector locally modulated by the underlying \(m_x = 3\) wave. We believe that these waves are mirror waves probably generated by the proton anisotropy driven by the large amplitude pump wave.

We see similar structure at longer wavelengths at the end of the beam simulation [B1] however the FT shows no such structures, so it is more likely that these are waves generated early on by linear instabilities that are only weakly damped.

Proper comparison with the long term evolution of the beam simulation [B1] limits us to looking at [P1] and [P2], since [P3] has wave amplitudes not found for waves in [B1]. Though there are possible similarities in [P2] and [B1], e.g. in the \((\pm 3, 3)\) and \((\pm 6, 3)\) modes, the parametric simulations have such low growth rates that they appear too late and at too low a level to impact upon the beam generated waves’ evolution. This conclusion is reinforced when we note that the \(m_x = 3\) modes seen in [P1] are at levels of less than 10% those seen in [B1], so that they are unlikely to be either important or noticed.

5.3.4 Difficulties in Comparisons of Parametric/Beam Simulations

Various problems exist when comparing simulations of beams where one suspects that parametric instabilities play a role and single waves allowed to evolve. Firstly there is
Figure 5.6: Summary plots of density and magnetic field components at $t = 200\Omega_e^{-1}$ for the case of an initial single wave of amplitude $0.4B_0$ [P2]. Note: The pump wave $(\pm 3, 0)$ has been removed from the perpendicular field components ($B_y, B_z$) spatial FTs for clarity.
Figure 5.7: Summary plots of density and magnetic field components at $t = 200\Omega^{-1}$ for the case of an initial single wave of amplitude $1.0B_0$ [P3]. Note: The pump wave $(\pm 3, 0)$ has been removed from the perpendicular field components ($B_y, B_z$) spatial FTs for clarity.
Figure 5.8: Summary plots of density and magnetic field components at $t = 200\Omega_i^{-1}$ for the case of an initial single wave of amplitude $1.0B_0$ [P3]. Note: The pump wave $(\pm 3, 0)$ has been removed from the perpendicular field components ($B_y, B_z$) spatial FTs for clarity.
the problem that the beam alters the plasma dispersion relation and so the waves being compared will have different properties as with those parametrically excited. In addition we have only examined the case of a single wave embedded in a noisy background whereas the beam case has many waves providing a developed spectrum of turbulence for waves to interact with. We have not explored the case of an obliquely propagating initial wave.

Plasma conditions continue to evolve in the beam case rather than the near equilibrium situation in the parametric case. Also there are problems associated with the simulation system. A finite length two-dimensional box provides only a discrete set of modes that can couple, in reality there is a three-dimensional continuum. Some of these deficiencies are inherent to numerical simulation, some can be addressed in future simulation work.

5.4 Simulation of a Wave Unstable to Parametric Instability (Case [T2D])

So far our investigations have not revealed any important parametric instability. In order to investigate the modification of a parametric instability when it is allowed to occur in two dimensions rather than one we have used our two-dimensional hybrid code to follow the evolution of a wave which is known to be unstable to a decay instability. The wave
Figure 5.10: Results of the single wave simulation [T2D] filtered to leave only the parallel propagating modes, showing the time evolution of $B_y$, the magnetic field decomposed by helicity and $E_x$, together with their spatial FTs.
parameters were taken from the set examined by Terasawa et al. (1986), who used a one-dimensional hybrid code. The parameters used for this simulation [T2D] are given in Table 5.1.

We will first confirm that we recover the results of Terasawa et al. (1986) when our two-dimensional results are filtered such that only the parallel propagating modes remain. We shall then explore the full two-dimensional results.

### 5.4.1 Behaviour at Parallel Propagation

As before, we first filter in \( k \)-space to leave only parallel propagating waves and then decompose the transverse magnetic components by helicity. Our results (Figure 5.10) are very similar to those of Terasawa et al. (1986).

The initial pump wave \((m_x = 8)\), a RH circularly polarized wave travelling in the \(+x\)-direction, appears in the positive helicity plot \((R^+)\) and is seen to fall off in amplitude as sideband waves grow up from \(t \gtrsim 50\Omega_i^{-1}\) at \(m_x = 3, 4\). These waves are also RH circularly polarized, travelling in the \(-x\)-direction at around \(1.3v_A\) and therefore have negative helicity \((R^-)\). Corresponding daughter sound waves can be seen in \(E_x\) at \(m_x = 11, 12\), which damp rapidly through ion heating. A brief burst of weak sideband activity occurs between \(t \simeq 150 - 250\Omega_i^{-1}\) in \(m_x = 6\) \((R^-)\), with \(E_x\) activity seen in \(m_x = 14\). This later activity could be the product of parametric instability of the initial decay products, as could the beginnings of growth visible in the sideband at \(m_x = 1, 2\) \((R^-)\).

Terasawa et al. (1986) found that the initially dominant sideband mode (seen in \(R^-\)) was susceptible to an inverse cascade process with energy moving, through successive decay instabilities, to longer wavelengths (lower mode numbers). They also observed a more rapid damping of the daughter sound waves, though we have confirmed this to be due to poor particle statistics in separate one-dimensional experiments.

### 5.4.2 \( \omega - k \) Power Spectrum

In order to further compare our parallel results with the one-dimensional simulation of Terasawa et al. (1986), we now construct the \((\omega-k)\) power spectrum for the magnetic field parallel propagating modes. This is done by collecting the full (two dimensional) data for a number of different time steps. For the results we present here, we take 256 different time steps, equally spaced every 10 time steps (i.e., every \(0.5\Omega_i^{-1}\)). In order to reduce the
Figure 5.11: Wave power in parallel propagating magnetic field modes analyzed by angular frequency and wave mode number (equivalent to wavevector). The field components have been decomposed into RH and LH helicities, and then arranged according to the sign of $\omega$ so that the plot resembles Fig. 8 of Terasawa et al. (1986). The top right hand quadrant represents $R^+$ waves, the top left hand quadrant $R^-$ waves, the bottom left hand quadrant $L^+$ waves, and the bottom right hand quadrant $L^-$ waves. The time period used for this data corresponds to when the parametric decay instability is active.
computational load in the Fourier analysis, we lower the spatial resolution by averaging over four cell squares. Then a full three dimensional (two space, and one time) FFT is performed, and hence the frequency-wavenumber power spectrum can be assembled for any set of mode numbers, e.g., for the parallel modes. Figure 5.11 shows the result for the positive and negative helicity magnetic field components. This figure has been arranged so that it is directly comparable to Figure 8 of Terasawa et al. (1986). The top right hand quadrant represents $R^+$ waves, the top left hand quadrant $R^-$ waves, the bottom left hand quadrant $L^+$ waves, and the bottom right hand quadrant $L^-$ waves. The period over which the power spectrum is computed is $t = 100.50_i^{-1} - 228.50_i^{-1}$, which corresponds to when the parametric decay instability is active (see Figure 5.10). Note that in Figure 5.11, and subsequent similar figures, the data is shown twice, once (top) with the grey scale covering the full range of power, and second (bottom) with a restricted range of grey scale to illustrate the most prominent features. In all cases the logarithm of the power (in arbitrary units) is calculated before mapping to a grey scale.

Figure 5.11 is remarkably similar to the result of Terasawa et al. (1986). However there are a number of discernible differences. Because the time series used for the analysis are far from time-stationary (e.g., the amplitude of the parent driver wave is decreasing dramatically), we have used a cosine data window (e.g. Press et al., 1986). This will emphasize the data in the middle of the sample period, but results in much better (narrower) spectral features. For our purposes, it means that the $m_x = 8$ and $m_x = -3, -4$ modes appear at clearly defined frequencies, and are not broadband features as found by Terasawa et al. (1986). Next, the power in the natural $R^+$ and $R^-$ modes, with $|\omega|$ increasing with $|k|$, can be seen, with maxima at the parent mode ($R^+, m_x = 8$) and the daughter modes ($R^-, m_x = -3, -4$).

One of the major differences between our result and that of Terasawa et al. (1986), is the very clear set of slanted features ("lines"). These are present in Figure 8 of Terasawa et al. (1986), but not so clearly. Also, our Figure 5.11, shows that these features extend to higher mode numbers and frequencies than shown in Terasawa et al. (1986). These features only appear when the parametric instability is operating, up to about $t = 2000_i^{-1}$. Considering the top right $R^+$ quadrant for which $\omega$ is positive, one sees that the features have increasing frequency for decreasing wavenumber, i.e., negative group velocity. This means that it is completely improbable, given the low frequencies
Figure 5.12: Similar to Figure 5.11, but for the $E_x$ electric field component. The sign of the frequency corresponds to the direction of propagation of the waves.

involved, that they correspond to a natural mode of the plasma. Terasawa et al. (1986) only mention these features briefly: “Waves around the shaded area labeled P’ [i.e., the features we are discussing] are interpreted to be the waves resulting from the interaction between the thermal R waves and daughter sound waves.”

Before further discussion, we show the $(\omega-k)$ power spectra for the $E_x$ electric field component (Figure 5.12) and the total magnetic field amplitude (Figure 5.13). In Figure 5.13 one can see, at a low level, the natural forward propagating mode(s) (in the positive $\omega$ region, with frequency increasing with wave mode number). This indicates that there is a compressive component, so that the modes are not exactly circularly po-
Figure 5.13: Similar to Figure 5.12, but for the total magnetic field amplitude.
larized. But, in both these figures, the dominant feature is the appearance of curious slanted line features as in Figure 5.11. In the $E_x$ electric field component there is a clear maximum around $m_x = 12$ corresponding to the daughter sound wave in the parametric decay instability theory. It is interesting to note from Figure 5.12 that there is no sign of the acoustic $\omega - k$ mode relation as used by the parametric theory, $\omega_s = kc_s$. A natural explanation is that the mode is heavily damped, as would be predicted by linear kinetic theory. Even so, one might expect to see the mode “lit up” as the energy damped into it, since the idea is that the $m_x = 12$ mode is a sound wave, and slight variations or steepening (etc.) would shift that energy to different $(\omega, k)$, but still on the same mode. It seems that this is not happening, even at very low power levels. Also, linear kinetic theory would predict that, at parallel propagation, the acoustic wave should have no magnetic component. But it is clear from Figure 5.13 that there is an associated magnetic component. This indicates that what we are seeing is a modification of the mode in the presence of the large amplitude parent wave (i.e., a quasi-mode). This would invalidate the assumption used in linear parametric theory. This begins to throw some doubt on the full details of the linear parametric theory used to explain the simulation results. On the other hand, there remains substantial agreement between the simulation results and the predictions of linear parametric theory.

As for the slanted line features, they could be explained as basically the forced response of the system, and a basic signature of the damping of the parent wave. They are several orders of magnitude weaker than the signal at $m_x = 12$, and so probably only make a minor contribution to the decay of the parent wave. One suggestion is that their form in the $\omega - k$ diagram is simply from the locus of points that satisfy the three wave energy and momentum conservation equations, given that one of the waves is the original daughter sound wave. This would give a relation of the form $\omega(k) = \omega_s - \omega_R(k)$, where $\omega_s$ is the frequency of the daughter sound wave, and $\omega_R(k)$ is the dispersion relation for the RH natural mode. This is equivalent to the explanation of Terasawa et al. (1986). One problem with this is that close inspection of Figure 5.11 and Figure 5.12 does not reveal any evidence that the line features have the same dispersion as the RH natural mode. This uncertainty, coupled with the apparent electromagnetic nature of these features, indicates that our understanding of the detailed process by which the parent wave decays is still incomplete.
5.4.3 Two-Dimensional Behaviour

Figures 5.14–5.16 are summary plots of the density and magnetic field components at the start \((t = 0.5\Omega_i^{-1})\), the middle \((t = 250\Omega_i^{-1})\) and the end \((t = 500\Omega_i^{-1})\) of the simulation [T2D].

The initial conditions show the pump wave, a RH circularly polarized wave at \(m_x = 8\), clearly visible in the transverse magnetic field components. By \(t = 250\Omega_i^{-1}\) the transverse field components show that the pump wave is approximately one third of its initial amplitude, with the parallel propagating decay products of nearly equal amplitude, as described in Section 5.4.1. The decay process is also active at slightly oblique angles as can be seen by the activity in the modes clustered around the dominant parallel propagating modes, with \(m_y \neq 0\). The real space plots of Figure 5.15 show a mixture of the pump and sideband waves in the transverse components, the daughter sound wave in the density and the magnetic field fluctuation due to the beating of the various circularly polarized waves. \(B_z\) shows a low amplitude mixture of near perpendicular short wavelength corrugations with \(m_y \sim 20\) and longer wavelength waves with \(|m_x| \lesssim 4\) and \(m_y = 1\). These latter waves are the compressive components of the sidebands of the oblique decay instabilities. The corrugations are likely to be the product of an anisotropy driven mirror instability, they are also faintly seen in \(B_z\) (Section 5.5).

By \(t = 500\Omega_i^{-1}\) the original pump wave has disappeared, leaving the \(m_x = 3\) sideband to dominate the transverse components, with some much weaker surrounding modes. In both density and \(B_x\) we see a set of modes with \(m_y = 0\), notably \((0,1)\) and \((0,3)\), these are probably the perpendicular modes of the filamentation instability, appearing late either due to their typically very low growth rates or their late destabilization by one of the decay product waves. \(B_z\) maintains a very similar structure to earlier, with similar corrugations again visible in \(B_z\). The feature at \(y > 100c/\omega_i\) is also related to the mirror instability. A band of compressive waves are excited around \(m_x = 3\), \(2 \lesssim m_y \lesssim 10\). Again, these could be mirror waves modulated by the background wave field.

5.5 Mirror waves

As we have seen, in the later stages of the present simulations, there develop waves with wavevectors perpendicular to the background magnetic field, with apparently fairly short
Figure 5.14: Summary plots of density and magnetic field components for single wave simulation [T2D] near start-up ($t = 0.5\Omega_i^{-1}$).
Figure 5.15: Summary plots of density and magnetic field components for single wave simulation [T2D] at $t = 250\Omega_k^{-1}$. 
Figure 5.16: Summary plots of density and magnetic field components for single wave simulation [T2D] at $t = 500\Omega_k^{-1}$.
wavelengths. We can use the $\omega - k$ power spectrum diagnostic used earlier to investigate these modes. In Figure 5.17 we show, for simulation [T2D] the $\omega - k$ power spectrum for the total magnetic field amplitude for the exactly perpendicular modes (i.e., $m_x = 0$). The time period used for this data, $t = 250\Omega_i^{-1} - 378\Omega_i^{-1}$, corresponds to after the parametric instability is active. One can see propagating modes with $\omega \neq 0$, travelling in both directions, and these correspond to the perpendicular fast/magnetosonic mode. But, it is also clear that there is a clear signal at zero frequency, corresponding to a non-propagating disturbance, over a range of wavenumbers, up to about $m_y = 25$. By eye, one estimates a wave mode number of about 20 from Figure 5.16, and this probably just corresponds to a break in the $\omega = 0$ spectrum in Figure 5.17.

Because of non-stationarity in the simulation time series, we wish to convince ourselves that there is truly a zero frequency signal, so in Figure 5.18 we plot a time sequence of slices through the simulation domain in the $y$ direction at a fixed $x$ position (cell 90). From this it is apparent that there is an apparent zero frequency component visible from about $t = 250\Omega_i^{-1}$. The signal, which is really only apparent by eye at the shorter wavelengths, appears to meander, but, as we shall see, that it is a result of the modulation (in the $y$ direction) of the wave fronts as seen in Figure 5.16.

It seems that the most natural explanation for these waves, given their orientation and compressive nature (with field and density anti-correlated), is that they are mirror waves. We also note that the wave magnetic field in components $B_x$ and $B_z$ agrees with what is expected for the mirror mode. If so, then they are probably generated by a linear instability driven by a perpendicular temperature anisotropy of the ion distribution. Given the large amplitude of the parent wave, and the Alfvénic velocity perturbation imposed on the ion distribution, it would be surprising if the end state did not have a perpendicular temperature anisotropy. The linear mirror instability has been much discussed recently in the context of magnetosheath turbulence. A review of recent theoretical, observational and simulational work can be found in Schwartz et al. (1996).

At weak perpendicular anisotropies the mirror instability has its maximum growth rate close to perpendicular to the background magnetic field. This provides an explanation for the modulation of these waves seen in Figure 5.16. In Figure 5.19 we plot the magnetic field amplitude and density at a late time in the simulation [T2D], together with the superimposed projection in the $x$-$y$ plane of a number of magnetic field lines. It
Figure 5.17: Similar to Figure 5.13, for the total magnetic field amplitude, but for the perpendicular propagating modes. The time period used for this data corresponds to after the parametric instability is active.
is clear that the wave fronts of the mirror waves are locally aligned with the magnetic field direction. Or, in other words, the wavectors are locally perpendicular to the magnetic field. This figure also illustrates the anti-correlation between the field amplitude and the density.

We can now return to some simulations discussed earlier, and ask if we see similar behaviour. Figure 5.20 is similar to Figure 5.19, but for the simulation [P3] at the same time as in Figure 5.9. It is apparent that the near sinusoidal modulation of the short wavelength perpendicular waves is because of the underlying large amplitude, propagating, parent wave. It is this modulation which causes the signature in k-space seen in Figure 5.9. We can even return to the beam instability simulation [B1] of Chapter 4, and note the short wavelength, perpendicular waves in $B_x$ and $B_z$ that are seen in the late stages of the simulation (Figure 4.10). Analysis (not shown) similar to Figure 5.19 shows that, once again, the modulation is because of the changing direction of the magnetic field lines associated with the long wavelength parallel propagating waves. Although in the case of Figure 4.10 the field modulation is not as sinusoidal as in Figure 5.20.

This is the first time that mirror waves have been discovered in either ion beam instability simulations or parametric decay instability simulations, and their appearance in two different contexts indicates that they may be present, but as yet unobserved, in a number of different situations. For example, our simulations of ion beam instabilities was partly motivated by the beams and waves observed in the Earth’s bow foreshock, but there have, as yet, been no reports of mirror waves in that region. Our results here indicate that it might be worthwhile investigating whether they can be observed in the foreshock.

5.6 Summary and Conclusions

In this chapter we have examined the two-dimensional kinetic evolution of parametric instabilities of single circularly polarized waves. There were two motivations: to see if the results of our proton beam simulations contain any evidence of the action of parametric evolution of beam instability generated waves, and to investigate how the parametric instability of a single wave proceeds in a two-dimensional kinetic simulation.

In the first case we initialised each of three simulations ([P1], [P2], [P3]) with a wave
Figure 5.18: The time evolution of the magnetic field amplitude in the simulation [T2D], shown as a time sequence of slices through the simulation domain in the $y$ direction at fixed $x$ position. The zero frequency perturbations are visible at later times.
Figure 5.19: Magnetic field amplitude and density at late time in the simulation [T2D], with the projection of the magnetic field lines superimposed.
Figure 5.20: Magnetic field amplitude and density at late time in the simulation [P3], with the projection of the magnetic field lines superimposed.
having properties similar to those of the dominant wave in the post-saturation phase of our beam instability simulations. Each simulation had a different initial wave amplitude but identical wavelength and polarization.

Over the timescales we observed there was some evidence of parametric instability in the single wave simulations, especially those with larger amplitudes. However, little resemblance was found to the evolution during the later stages of our beam instability simulations. We concluded from this that, in general, low growth rates and the further suppression of those growth rates as the plasma beta increases make it unlikely that parametric instabilities play any important role in the evolution of instabilities due to beams typically found in, for example, the terrestrial foreshock. However, the limited size of simulation box available to us does reduce the number of possible channels for parametric instability. It is also possible that waves other than the largest may be parametrically unstable, though it is expected that they should have low growth rates due to their lower amplitude.

In order to observe how the parametric instability of a single wave proceeds in a two-dimensional kinetic simulation we used an initial wave that was known to be unstable to the parametric decay instability (simulation [T2D]). We found behaviour in the parallel propagation direction very similar to that found in purely one-dimensional simulations. Some broadening of the decay instability into the perpendicular direction is observed but the nature of the instability is little changed. The filamentational instability appears late in the simulation despite its low growth rate as a low plasma beta lead to little suppression.

In all our two-dimensional single initial wave simulations we observed the generation of highly oblique, short wavelength (a few ion inertial lengths) waves with zero phase velocity. We attribute these to the mirror instability driven by the temperature anisotropy driven by the large amplitude waves present.
Chapter 6

Summary and Conclusions

In this thesis we have explored three areas: the design of parallel algorithms to allow large plasma simulations to be performed efficiently on modern computer architectures, the hybrid simulation of instabilities in cool tenuous proton beams in two dimensions and the hybrid simulation of parametric instabilities in two dimensions.

We have described a parallel implementation of the hybrid code CAM-CL using a message passing library, PVM. Our implementation, Barbie & Ken, uses a master-slave algorithm to perform particle operations in parallel. We have used this code to run simulations, with over $10^7$ simulation particles, on a small cluster of SUN workstations. This has allowed us to perform simulations with better particle statistics and spatial resolution than previously possible.

We also describe a plasma simulation algorithm which we have developed for massively parallel processors (MPPs) such as the CRAY T3D. This algorithm uses such an MPP efficiently, by keeping both field solution and particle operations load balanced, so that all processors remain fully utilized throughout the simulation. This algorithm adds a communications overhead to remove load imbalances, which turns out to be a good trade off given that MPPs usually have exceptionally fast inter-processor links. We have tested the critical aspects of this algorithm on the EPCC T3D and believe it would allow us to perform simulations of more than $10^8$ simulation particles on that machine. Unfortunately, insufficient time was available to us on the EPPC T3D to carry the work to completion. The algorithm scales in such a way that as larger machines become available this algorithm will allow simulations to grow in size accordingly.

We have used our two-dimensional parallel hybrid code, Barbie & Ken, to follow the
evolution of instabilities of tenuous cool proton beams, for beams both isotropic and anisotropic in temperature. We find that two-dimensional simulations reveal behaviour not present in one dimension.

For a tenuous isotropic cool proton beam we find that the linear dispersion relation accurately predicts the nature of waves generated during the early stages of the instability. Waves are observed to compete for energy, leading to the slight suppression of some growth rates. Waves at angles highly oblique to the direction of parallel propagation occur at this time. The beam is slowed and scattered, driving up the beam temperature and introducing a temperature anisotropy in the perpendicular direction. Evidence for gyrophase bunching of both core and beam particles is observed.

Around saturation obliquely orientated short scale magnetic features and more distributed structures within the field strongly scatter the beam and contribute to the saturation of the instability. Post-saturation the instability generated waves steepen non-linearly and shocklets are observed with whistler wavetrains. These shocklets lie at approximately $15^\circ$ to the initial background field.

Beyond saturation the simulation is increasingly dominated by the waves with the largest amplitude at saturation (and therefore linear growth rates) as the remaining waves are damped. At the latest times highly oblique, short wavelength (a few ion inertial lengths) waves with zero phase velocity are seen. We attribute these to the mirror instability driven by the temperature anisotropy driven by the large amplitude waves present.

We find that for a tenuous, temperature anisotropic, cool proton beam, growth rates of waves at oblique angles are enhanced and these waves play a greater role in the evolution of the instability. Shocklets are observed with angles of propagation of approximately $25^\circ$ to the initial background field, in agreement with spacecraft observations of the foreshock. In other respects the evolution of the anisotropic beam closely resembles that of the isotropic beam.

After experiments involving the variation of many non-physical simulation parameters we believe these simulation results to be physically robust.

We have used our two-dimensional parallel hybrid code, Barbie & Ken, to follow the kinetic evolution of parametric instabilities of single circularly polarized waves. There were two motivations: to see if the results of our proton beam simulations contain any
evidence of the action of parametric evolution of beam instability generated waves, and to investigate how the parametric instability of a single wave proceeds in a two-dimensional kinetic simulation.

In the first case we initialised each of three simulations ([P1], [P2], [P3]) with a wave having properties similar to those of the dominant wave in the post-saturation phase of our beam instability simulations. Each simulation had a different initial wave amplitude but identical wavelength and polarization.

Over the timescales we observed there was some evidence of parametric instability in the single wave simulations, especially those with larger amplitudes. However, little resemblance was found to the evolution during the later stages of our beam instability simulations. We concluded from this that, in general, low growth rates and the further suppression of those growth rates as the plasma beta increases make it unlikely that parametric instabilities play any important role in the evolution of instabilities due to beams typically found in, for example, the terrestrial foreshock. However, the limited size of simulation box available to us does reduce the number of possible channels for parametric instability. It is also possible that waves other than the largest may be parametrically unstable, though it is expected that they should have low growth rates due to their lower amplitude.

In order to observe how the parametric instability of a single wave proceeds in a two-dimensional kinetic simulation we used an initial wave that was known to be unstable to the parametric decay instability (simulation [T2D]). We found behaviour in the parallel propagation direction very similar to that found in purely one-dimensional simulations. Some broadening of the decay instability into the perpendicular direction is observed but the nature of the instability is little changed. The filamentational instability appears late despite its low growth rate as a low plasma beta lead to little suppression.

In all our two-dimensional single initial wave simulations we observed the generation of highly oblique, short wavelength (a few ion inertial lengths) waves with zero phase velocity. We attribute these to the mirror instability driven by the temperature anisotropy driven by the large amplitude waves present.
Appendix A

Transport Ratios

A.1 Polarization

Polarization is defined as

\[ P = \frac{E_y}{i E_z} \frac{\omega_r}{|\omega_r|} \]  \hspace{1cm} (A.1)

with \( B_0 = B_0 \hat{\phi} \) (after Gary, 1993). \( P = +1 \) corresponds to RH circular polarization. \( P = -1 \) corresponds to LH circular polarization.

For a circularly polarized wave this is equivalent to the sense of rotation in time of a fluctuating field vector, viewed from a fixed point in space in the direction parallel to the magnetic field, at positive real frequency.

A.2 Helicity

Matthaeus & Goldstein (1982) define the dimensionless helicity as

\[ \sigma = \frac{k < \delta A \cdot \delta B >}{< \delta B \cdot \delta B >} \]  \hspace{1cm} (A.2)

where \( A \) is the magnetic vector potential \( (B = \nabla \times A) \) and angle brackets denote a spatial average.

A simple interpretation of helicity for a parallel propagating wave is that it represents the handedness in space of the wave. So a RH polarized wave travelling in the \( +B_0 \) direction has positive helicity.
A.3 Magnetic Compression Ratio

The magnetic compression ratio $C_{BB}$ gives information about the degree to which the magnetic field perturbation lies in or out of the $B_0-k$ plane. It is defined as

$$C_{BB} = \frac{1}{\tan^2 \theta} \frac{|B_\parallel|^2}{|B_\perp|^2}. \quad (A.3)$$

The behaviour of the magnetic compression ratio for stable plasma modes is outlined in Krauss-Varban et al. (1994). If $C_{BB} = 0.5$ a wave is circularly polarised. If $C_{BB} > 0.5$ then the magnetic field perturbation lies in the $B_0-k$ plane and the wave is compressive or magnetosonic in character. If $C_{BB} < 0.5$ then the magnetic field perturbation lies out of the $B_0-k$ plane and the wave is more Alfvénic in character.

A.4 Compressibility, $C_{nB}$

Akimoto et al. (1993) calculate the correlation coefficient $C_{nB}$ between density and magnetic field for their one-dimensional beam simulation. This shows to what degree ion beam generated beams are compressive. They define this quantity as

$$C_{nB} = \frac{\langle \delta n_i \delta B \rangle}{\langle \delta n_i^2 \rangle^{1/2} \langle \delta B^2 \rangle^{1/2}}. \quad (A.4)$$

where angle brackets denote a spatial average.
Appendix B

An Outline of PVM

The PVM (Parallel Virtual Machine) system has been developed by a team at Oak Ridge National Laboratory since 1989, it is described in detail in Geist et al. (1994). It allows a heterogeneous group of hosts to be configured as a single virtual machine, an imaginary multiprocessor computer. These hosts can be anything from workstations to supercomputer nodes connected over any network ranging from Ethernet to Internet. First each host has a daemon installed upon it which coordinates the virtual machine’s internals, then an application is built from tasks—processes running on the hosts. User management of the virtual machine and message passing between tasks is implemented via a subroutine library (callable from C or FORTRAN). This library provides amongst its facilities (PVM Version 3.3):

- Machine/Process control — add/delete hosts, spawn tasks, etc.
- Information — error management, machine/task info, etc.
- Dynamic Process Groups — group control, barriers, intra-group communication, etc.
- Message passing — buffer control, packing/unpacking typed data, send, receive

After the virtual machine is set up and slave tasks spawned, message passing is the dominant activity. To send a message, a buffer is first initialised, then its contents are assembled (packed in PVM’s jargon) and can then be sent (to a single task) or broadcast (to multiple tasks). Messages may be labelled with an integer (the message tag) facilitating identification by receiving tasks. Messages are received by a task by
specifying the sender and the tag (or wildcards), then *unpacked* (in the same order as packed). Both blocking (wait until message arrives) or non-blocking (if no message yet then carry on with something else) receives are available. Translation of data formats between hosts with different architectures is provided.

### B.1 Example Programs

Below is an example of a simple master-slave code in FORTRAN to illustrate calls to PVM. One message is shown, sent from master to slave. A reply from slave to master is implied and takes the same form.

```fortran
C************************
C *** EXAMPLE MASTER PROGRAM ***
C************************

program master
    integer mytid, slavetid(10), info, n
    real a(1000)

    c Join PVM and find out own ID
    call pvmfmytid(mytid)

    c Spawn ten slave processes and get their IDs
    call pvmfspawn('slave',PvmDefault,'**',10,slavetid,info)

    .
    .    Do some processing setting contents of n and a
    .

    c Initialize send buffer
    call pvmfinitsend( PvmDefault, info )

    c Put some data into the buffer
    call pvmfpack( INTEGER4, n, 1, 1, info )
    call pvmfpack( REAL4, a, n, 1, info )

    c Send the message to slave 3
    call pvmfsend( slavetid(3), 1, info )
```

. Get slave replies/Do some more processing.

c Leave PVM
    call pvmfexit(info)
end

c*******************************
c *** EXAMPLE SLAVE PROGRAM ***
c*******************************
program slave
    integer mytid, ptid, info, n
    real a(1000)

c Find out own ID and parent process ID
    call pvmfmytid(mytid)
    call pvmfparent(ptid)

c Blocking receive from parent - ie wait for message
    call pvmfrecv(ptid, 1, info)

c Unpack message contents from receive buffer
    call pvmfunpack(integer4, n, 1, 1, info)
    call pvmfunpack(real4, a, n, 1, info)

    . Do some processing/reply to master.

c Leave PVM
    call pvmfexit(info)
end
Appendix C

Symbols

\( A \) Magnetic vector potential
\( B \) Magnetic induction
\( B \) Magnitude of magnetic induction \( B = |B| \)
\( B_0 \) Background or initial magnetic field
\( B_0 \) Magnitude of background or initial magnetic field \( B_0 = |B_0| \)
\( B_\perp \) Magnetic field perpendicular to \( B_0 \)
\( B_{x,y,z} \) Components of magnetic field
\( C_{BB} \) Magnetic compression ratio
\( C_{nB} \) Compressibility
\( c \) Speed of light
\( E \) Electric field strength
\( f_j \) Distribution function of species \( j \)
\( k \) Wavevector
\( k_0 \) Wavevector of pump wave in parametric instability
\( k \) Wavenumber \( k = |k| \)
\( k_\parallel \) Component of wavevector parallel to \( B_0 \)
\( k_B \) Boltzmann constant (often omitted)
\( J \) Current density
\( J_j \) Current density due to species \( j \)
\( L_{x,y} \) Total length of simulation box in given direction
\( m \) Order of resonance
\( m_j \) Mass of single particle of species \( j \)
APPENDIX C. SYMBOLS

$m_{x,y}$ Fourier mode number — often expressed as $(m_x, m_y)$
$N_{x,y}$ Number of simulation cells in given direction
$n_0$ Total number density
$n_j$ Number density of species $j$
$P$ Polarization
$T_0$ Initial temperature (may combine with other subscripts)
$T_j$ Temperature of species $j$
$T_\parallel$ Parallel temperature
$T_\perp$ Perpendicular temperature
$t$ Time
$v$ Particle velocity vector
$v_A$ Alfvén velocity
$v_{SW}$ Solar wind velocity
$v_0$ Total velocity of beam relative to core
$v_{0j}$ Bulk velocity of species $j$
$v_j$ Thermal speed of species $j$: $\frac{1}{2} m_j v_j^2 = k_B T_j$
$v_{x,y,z}$ Particle velocity components
$\mathbf{v}_\perp$ The component of a particle’s velocity perpendicular to $B_0$
$\hat{x}$ Unit vector in $x$-direction
$x$ Particle position vector
$x, y$ Position vector components, either of particle or within simulation box
$\beta_j$ Plasma Beta for species $j$
$\gamma$ Growth rate
$\Delta x, \Delta y$ Size of simulation cell
$\Delta t$ Size of simulation timestep
$\delta B$ Magnetic perturbation ($\delta B = B - B_0$)
$|\delta B|$ Magnitude of magnetic perturbation ($|\delta B| = |B - B_0|$)
$\zeta_{mj}^n$ Resonance parameter for resonance order $m$, species $j$
$\theta_{Bm}$ Angle between the upstream magnetic field and shock normal
$\theta_{kB}$ Angle between the background magnetic field and wavector
$\lambda$ Wavelength
$\phi_j$ Velocity phase angle around $B_0$ of particle $j$
APPENDIX C. SYMBOLS

$\sigma$  Helicity

$\phi_B$  Phase angle around $B_0$ of magnetic field

$\psi$  Angle between $B_\perp$ and $v_\perp$ for a particle interacting with a wave

$\Omega_j$  Cyclotron frequency of component $j$

$\omega$  Complex frequency ($\omega = \omega_r + i\gamma$)

$\omega_0$  Frequency of pump wave in parametric instability

$\omega_B$  Bounce frequency

$\omega_j$  Plasma frequency of component $j$

$\omega_r$  Real angular frequency
Appendix D

Acronyms

AMPTE Active Magnetospheric Particle Tracer Explorers.
AU Astronomical Unit. Mean distance from the Earth to the Sun.
CAM-CL Current Advance Method - Cyclic Leapfrog,
a hybrid simulation algorithm by Matthews (1994)
EPCC Edinburgh Parallel Computing Centre. Home of a CRAY T3D.
FDA Finite Difference Approximation.
FEM Finite Element Methods.
FT Fourier Transform
IMF Interplanetary Magnetic Field.
ISEE International Sun-Earth Explorer.
LH Left-Hand.
MHD Magnetohydrodynamics.
MPI Message Passing Interface. A package similar to PVM.
MPP Massive Parallel Processor. e.g. the CRAY T3D.
ODE Ordinary Differential Equation.
PIC Particle In Cell. A type of particle simulation algorithm.
ppcps Particles Per Cell Per Species. A measure of the number of macroparticles used in a simulation.
PVM Parallel Virtual Machine. A message passing library.
RH Right-Hand.
SLAMS Short Large Amplitude Magnetic Structure (Schwartz & Burgess, 1991).
ULF Ultra Low Frequency. In ion instabilities \( \omega_r \ll \Omega_i \).
WHAMP  Waves in Homogeneous Anisotropic Multicomponent Plasmas
Package by Rönnmark (1982) to calculate linear dispersion relations.
Bibliography


