Super-resolution orbital angular momentum based radar targets detection
Lin, M; Gao, Y; Liu, P; Liu, J

“The final publication is available at http://digital-library.theiet.org/content/journals/10.1049/el.2016.0237”

For additional information about this publication click this link.
http://qmro.qmul.ac.uk/xmlui/handle/123456789/13041

Information about this research object was correct at the time of download; we occasionally make corrections to records, please therefore check the published record when citing. For more information contact scholarlycommunications@qmul.ac.uk
Radiating twisted beams, Orbital Angular Momentum (OAM) based radar provides a new perspective for present radar techniques. However, estimation methods now used has a demerit of resolution. Thus, we raised Multiple Signal Classification (MUSIC) algorithm to improve resolution ability based on this innovative concept. The echo model based on uniform circular array (UCA) for MUSIC was first built. In contrast to uncorrelated signals in classical MUSIC algorithm, echo signals from targets are fully coherent with each other. Spatial smoothing technique was subsequently utilized in OAM regime to tackle it. Simulation results showed the super-resolution capacity of MUSIC to detect objectives compared to the traditional Fast Fourier Transform (FFT) method.

Introduction: Orbital Angular Momentum (OAM) has been widely studied in optic regime regarding imaging, microscopic particle and communication [1,2]. While radio OAM had not been explored until Guo et al. [13] first proposed OAM based target detection model, opening a new perspective for existing radar techniques. Guo et al. [13] reformed echo signals, implying appearance of ambiguities. Later, [14] further extended the detection model to Multiple-in-multiple-out (MIMO) and Multiple-in-single-out (MISO) modes. Besides, two-dimension imaging method by exploiting OAM and frequency diversity was studied as well. Both papers utilized traditional methods like FFT and black projection to make estimations of azimuth angle. To gain a high resolution, numerous samples in OAM domain are required, thus increasing complexity of the system. Therefore, super-resolution techniques based on OAM radar detection are in demand. This paper built the model and derived spatial smoothing multiple signal classification (MUSIC) [15] algorithm to gain super-resolution estimations of targets.

\[ E(\alpha,t) = \sum_{\nu=1}^{N} e^{i \nu\phi} + \sum_{l=1}^{L} e^{i \nu\phi} \frac{1}{2\pi} J_\nu(k \sigma \sin \theta) s(t) + n(\alpha,t) \]  

For MISO mode, the normalized real-time echo signals from \( M \) objectives in the received terminal can be extended from [14] and written as:

\[ E(\alpha,t) = \sum_{\nu=1}^{N} e^{i \nu\phi} + \sum_{l=1}^{L} e^{i \nu\phi} \frac{1}{2\pi} J_\nu(k \sigma \sin \theta) s(t) + n(\alpha,t) \]  

where \( E(\alpha,t) \) denotes the OAM topological order, \( k \) is wave number, \( \sigma, \rho, \theta \) and \( \varphi \) link to radar cross section (RCS), distance and direction information of \( m \)th target, \( J_\nu \) refers to \( \nu \)th first kind Bessel function. Similar FFT transform relation between \( \alpha \) and \( \varphi \) domain can be observed from (1). Based on this, existing means to estimate azimuthal information just make a FFT transform or back projection [13,14]. According to [13], such OAM based radar technique has no capacity to identify elevation angles of targets.

For \( k \sigma \sin \theta > 1 \), approximation can be made as follows:

\[ e^{i \nu \theta} \approx \frac{1}{2\pi} \cos(k \sigma \sin \theta - \alpha) \approx \frac{\pi}{4} \]

Therefore, \( N \) discrete samples vector of \( E(\alpha,t) \) in \( \alpha \) regime can be depicted as:

\[ E = \begin{bmatrix} E(\alpha,1) \\ E(\alpha,2) \\ \vdots \end{bmatrix} = AS + n \]

where \( A \) denotes reformed echo signals vector, \( n \) and \( S(t) \) are modified steering vector and echo signal of \( l \)th target, \( L \) is the sample length of \( s(t) \).

The amended model in (3) indicates 2M targets, implying appearance of ambiguities. For presenting Gaussian noise, covariance matrix of received signals under different orders can be acquired by:

\[ R_{xx} = A R A^H + \rho_n I \]

where \( \rho_n \) is the noise power, \( R \) involves covariance matrix of reformed echo signals, \( I \) is the unitary matrix. Similar to DOA estimation, columns of \( A \) are linear independently, however echo signals of multiple targets are fully coherent, in contrast to uncorrelated incident signals in classical MUSIC algorithm. For uniform samples in \( \alpha \) field with sample rate \( f_s \), front spatial smoothing technique [16] are capable of tackling this problem. According to smoothing theory, divide \( N \) samples in \( \alpha \) field to \( p \) mixed subsample blocks as shown in Fig.2, each block has \( h \) samples, then \( N = p+1 \). To achieve full rank, the number of blocks should meet \( p \geq 2M \).

\[ a_0, a_1, a_2, \ldots, a_{N-1}, a_N \]

\[ E_s (\alpha,t) \]

\[ E (\alpha,t) \]

Subsequently, calculate the modified covariance matrix as follows:

\[ \text{Fig.1} \quad \text{Illuminate target with twisted beams.} \]

\[ \text{Fig.2} \quad \text{Front-spatial smoothing} \]
where $\mathbf{R}_i$ is covariance matrix of the $i$th sub-block $\mathbf{E}(\alpha, \phi)$.

To make use of MUSIC, $M$ is assumed known or already accurately estimated. In accordance with MUSIC algorithm \[8\], the whole procedure to estimate azimuth angles can be listed as follows:
1) Obtain $p$ original covariance matrix $\mathbf{R}_i$ (\(i = 1, 2, \cdots, p\));
2) Gain $\mathbf{R}'$ based on (8);
3) Make engine value decomposition of $\mathbf{R}'$;
4) Search azimuthal spectrum

$$P(\phi) = \frac{1}{\mathbf{V}_v^\top \mathbf{V}_v} \mathbf{a}(\phi)^\top$$

(6)

where $\mathbf{V}_v = [\mathbf{q}_{3h}, \cdots, \mathbf{q}_4]$ (\(h \geq 2M\)) is the noise subspace vectors, $\mathbf{a}(\phi) = [\mathbf{e}_{\sin\theta}, \cdots, \mathbf{e}_{\sin\phi}]$ refers to searching steering. It can be predicted from (3) that symmetric peaks at $\phi - 90$ and $\phi + 90$ directions will be obtained in the spectrum. To acquire a more visible result, replace $\phi$ with $\phi + 90$ in (6). Similar conclusions for MIMO mode can be similarly derived, here we would not state again.

**Simulation and results:** Use UCA with radius $\alpha = 50\lambda$ to illuminate two targets at distances $r_1 = 782.4\lambda$ and $r_2 = 78.15\lambda$ with directions $(\theta_1, \phi_1) = (60^\circ, 60^\circ)$ and $(\theta_2, \phi_2) = (70^\circ, 70^\circ)$ specifically. Received signals under $N = 20$ discrete orders with $f = 1$ are captured in a SNR=20dB environment. Each order, $L = 500$ snapshots of echo signals in time domain are recorded. For MUSIC algorithm, $p$ sub-blocks are divided to solve coherent problem. Estimation based on FFT are implemented on an average of $L = 500$ FFT results, each time 1024-point FFT is conducted. Spectrums of both methods are described in Fig.3, from which we can observe that MUSIC can easily distinguish these two close targets well with two sharp peaks while only one real blunt peak can be viewed by FFT method. For both methods, symmetric ambiguities appear, consistent with previous analysis. Fig.4 demonstrates resolution angle of two methods against SNR and sample size $N$. As exhibited in Fig.4a, the larger SNR, the smaller resolution angle MUSIC method gains, with only $2\theta$ resolution ability at SNR=40 dB almost 7 times that of FFT. The asterisk in Fig.4a denotes disability of FFT method to discriminate any two targets when SNR=0 dB. Similar results can be viewed in Fig.4b that the larger size of sample in $\alpha$ regime, the higher resolution ability for both methods, with MUSIC always outperforming FFT specifically.

**References**