Super-resolution orbital angular momentum based radar targets detection
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Introduction: Orbital Angular Momentum (OAM) has been widely studied in optics regime regarding imaging, microscopic particle and communication [1,2]. While radio OAM had not been explored until Bo. Thide et al. first proved the effectiveness of OAM generating in a communication [1,2]. Meanwhile, OAM multiplexing from communication and enriching radar targets more information. Recent low-frequency band in [3]. The numerous orthogonal helical beams were in demand. This paper built the model and derived spatial smoothing technique was subsequently utilized in OAM regime to tackle it. Simulation results showed the super-resolution capacity of MUSIC to detect objectives compared to the traditional Fast Fourier Transform (FFT) method.

Radiating twisted beams, Orbital Angular Momentum (OAM) based radar provides a new perspective for present radar techniques. However, estimation methods now used has a demerit of resolution. Thus, we raised Multiple Signal Classification (MUSIC) algorithm to improve resolution ability based on this innovative concept. The echo model based on uniform circular array (UCA) for MUSIC was first built. In contrast to uncorrelated signals in classical MUSIC algorithm, echo signals from targets are fully coherent with each other. Spatial smoothing technique was subsequently utilized in OAM regime to tackle it. Simulation results showed the super-resolution capacity of MUSIC to detect objectives compared to the traditional Fast Fourier Transform (FFT) method.

Detection model:

For MISO mode, the normalized real-time echo signals from M objectives in the received terminal can be extended from [14] and written as:

\[ E(\alpha,t) = \sum_{\alpha} e^{i2\pi \frac{k}{r_c} (\cos k a \sin \theta - \frac{\alpha \pi}{2})} n(\alpha,t) \]  

where \( n(\alpha,t) \) is the noise power, \( \sigma_\alpha, r_c, \theta \) and \( \varphi_\alpha \) link to radar cross section (RCS), distance and direction information of \( m \)th target, \( J_\ell \) refers to \( \ell \)th first kind Bessel function. Similar FFT transform relation between \( \alpha \) and \( \varphi \) domain can be observed from (1). Based on this, existing means to estimate azimuthal information just make a FFT transform or back projection [13,14]. According to [13], such OAM based radar technique has no capacity to identify elevation angles of targets.

For \( ka \sin \theta \gg 1 \), approximation can be made as follows:

\[ e^{i2\pi \frac{k}{r_c} (\cos k a \sin \theta - \frac{\alpha \pi}{2})} \approx 1 \]

Therefore, \( N \) discrete samples vector of \( E(\alpha,t) \) in \( \alpha \) regime can be depicted as:

\[ \mathbf{E} = \begin{bmatrix} E(\alpha_1,t) \\ E(\alpha_2,t) \\ \vdots \\ E(\alpha_M,t) \end{bmatrix} = \mathbf{A} \mathbf{S} + \mathbf{n} \]

\[ \mathbf{S}(t) = \begin{bmatrix} S_{1}(t) \\ S_{2}(t) \\ \vdots \\ S_{M}(t) \end{bmatrix} = \begin{bmatrix} n(\alpha_1,t) \\ n(\alpha_2,t) \\ \vdots \\ n(\alpha_M,t) \end{bmatrix} \]

where \( \mathbf{A} = \begin{bmatrix} a(\alpha_1) \\ a(\alpha_2) \\ \vdots \\ a(\alpha_M) \end{bmatrix} \) and \( \mathbf{S}(t) \) is the sample length of \( \alpha \)th target, \( L \) is the sample length of the \( \alpha \)th target. The amplitude of the \( \alpha \)th target, \( A \in \mathbb{C}^{M \times 2M} \) is the steering matrix, \( \mathbf{S} \in \mathbb{C}^{2M \times 1} \) indicates reformed echo signals vector, \( \mathbf{a}(\alpha) \) and \( \mathbf{S}(t) \) are modified steering vector and echo signal of \( \alpha \)th target. The amened model in (3) indicates 2M targets, implying appearance of ambiguities. For presenting Gaussian noise, covariance matrix of received signals under different orders can be acquired by:

\[ \mathbf{R}_{\mathbf{EE}} = \mathbf{A} \mathbf{R}_{\mathbf{AA}} \mathbf{A}^\mathsf{T} + \rho_n \mathbf{I} \]

where \( \rho_n \) is the noise power, \( \mathbf{R} \) involves covariance matrix of reformed echo signals, \( \mathbf{I} \) is the unitary matrix. Similar to DOA estimation, columns of \( \mathbf{A} \) are linear independently, however echo signals of multiple targets are fully coherent, in contrast to uncorrelated incident signals in classical MUSIC algorithm. For uniform samples in \( \alpha \) field with sample rate \( f_s \), front spatial smoothing technique [16] are capable of tackling this problem. According to smoothing theory, divide \( N \) samples in \( \alpha \) field to \( p \) mixed sub-sample blocks as shown in Fig.2, each block has \( h \) samples, then \( N=p+h-1 \). To achieve full rank, the number of blocks should meet \( p \geq 2M \).

\[ \mathbf{D} = \begin{bmatrix} a_1 & a_2 \cdots & a_{p-1} & a_p \\ \vdots & \vdots & \ddots & \vdots \\ a_{N-h} & a_{N-h+1} \cdots & a_{N-1} & a_N \end{bmatrix} \]

where \( \mathbf{D} \) is the deformed steering matrix, \( \mathbf{D} \) indicates modified steering matrix.

Subsequently, calculate the modified covariance matrix as follows:

\[ \mathbf{R}_{\mathbf{EE}} = \mathbf{D} \mathbf{R}_{\mathbf{AA}} \mathbf{D}^\mathsf{T} + \rho_n \mathbf{I} \]
where $\mathbf{R}_i$ is covariance matrix of $i$th sub-block $\mathbf{E}(\alpha, \tau)$. 

To make use of MUSIC, $\mathbf{M}$ is assumed known or already accurately estimated. In accordance with MUSIC algorithm [15], the whole procedure to estimate azimuth angles can be listed as follows:

1. Obtain $p$ original covariance matrix $\mathbf{R}_i$ ($i=1, 2, \ldots, p$);
2. Gain $\mathbf{R}'$ based on (5);
3. Make engine value decomposition of $\mathbf{R}'$;
4. Search azimuthal spectrum

$$P(\phi) = \frac{1}{\mathbf{a}'(\phi) \mathbf{V}_h \mathbf{V}_h \mathbf{a}(\phi)}$$

where $\mathbf{V}_h = [\mathbf{q}_3, \mathbf{q}_4, \ldots, \mathbf{q}_h] \ (h \leq 2M)$ is the noise subspace vectors, $\mathbf{a}(\phi) = [\mathbf{e}^{j \pi \phi}, \mathbf{e}^{j 2 \pi \phi}, \ldots, \mathbf{e}^{j h \pi \phi}]'$ refers to searching steering. It can be predicted from (3) that symmetric peaks at $\phi - 90^\circ$ and $\phi + 90^\circ$ directions will be obtained in the spectrum. To acquire a more visible result, replace $\phi$ with $\phi + 90^\circ$ in (6). Similar conclusions for MIMO mode can be similarly derived, here we would not state again.

**Simulation and results:** Use UCA with radius $a = 50 \lambda$ to illuminate two targets at distances $r_1 = 782.4 \lambda$ and $r_2 = 781.5 \lambda$ with directions $(\theta_1, \phi_1) = (60^\circ, 60^\circ)$ and $(\theta_2, \phi_2) = (70^\circ, 70^\circ)$ specifically. Received signals under $N=20$ discrete orders with $f_d = 1$ are captured in a SNR=20dB environment. Each order, $L=500$ snapshots of echo signals in time domain are recorded. For MUSIC algorithm, $p=2$ sub-blocks are divided to solve coherent problem. Estimation based on FFT are implemented on an average of $L=500$ FFT results, each time 1024-point FFT is conducted. Spectrums of both methods are described in Fig.3, from which we can observe that MUSIC can easily distinguish these two close targets well with two sharp peaks while only one real blunt peak can be viewed by FFT method. For both methods, symmetric ambiguities appear, consistent with previous analysis. Fig.4 demonstrates resolution angle of two methods against SNR and sample size $N$. As exhibited in Fig4a, the larger SNR, the smaller resolution ability at SNR=40 dB almost 7 times that of FFT. The asterisk in Fig4a denotes disability of FFT method to discriminate any two targets when SNR=0 dB. Similar results can be viewed in Fig4b that the larger size of sample in $a$ regime, the higher resolution ability for both methods, with MUSIC always outperforming FFT specifically.

**Fig.3 Azimuthal estimations of two targets using (a) FFT (b) MUSIC**

**Fig.4 Resolution angle for different (a) SNR and (b) sample size N.**

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