Essays in Macroeconomics

by

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Abstract

This thesis explores macroeconomic issues broadly relating to monetary policy. The first chapter studies how the monetary authority should respond to shocks when labour productivity depends on past levels of employment (learning by doing). In this context, the appropriate inflation-output trade-off is between inflation today and the present value of deviations in the output gap. I find that learning induces an increase in the importance of the output gap under a cost-push shock for the (more realistic case) of a distorted steady state. The welfare costs of business cycles are shown to be significantly larger even under the optimal policy.

The second chapter introduces noisy news shocks into a model of exchange rate determination to study the importance of these shocks in explaining deviations from uncovered interest parity (UIP). Agents in the foreign exchange market make decisions with imperfect information about economic fundamentals driving interest rate differences across currencies in that they must rely on a noisy signal of future interest rates. Results show that noise shocks are roughly twice as important as news shocks in explaining UIP deviations, with the impact of noise shocks being more pronounced during periods of changing monetary policy.

The third chapter develops a new index of economic uncertainty for South Africa for the period 1990-2014 and analyses the macroeconomic impact of changes in this measure. The index is constructed from three sources: (1) forecaster disagreement, (2) a count of international and local newspaper articles discussing economic uncertainty in South Africa and (3) mentions of uncertainty in the quarterly economic review of the South African Reserve Bank. The uncertainty index is a leading indicator of a recession. An unanticipated increase in the index is associated with a fall in GDP, investment, industrial production and private sector employment. Contrary to evidence for the U.S.A and U.K., uncertainty shocks are inflationary. These results are robust to controlling for consumer confidence, a corporate credit spread proxy as well as global risk shocks (VIX index).
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Statement of Originality

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This book is dedicated to Guy Standley, my greatest friend.

Everything I’ve done and everything I will do has a piece of you in it.
Chapter 1

Monetary policy and endogenous productivity

Abstract

I study the implications of learning by doing in production for optimal monetary policy using a basic New Keynesian model. Learning-by-doing is modeled as a stock of skills that accumulates based on past employment. The presence of this learning-by-doing externality breaks the ‘divine coincidence’ result that by stabilising inflation the output gap will automatically be closed, for a variety of shocks that are important in explaining the business cycle. In this context, the appropriate inflation-output trade-off is between inflation today and the present value of deviations in the output gap. Optimal policy is approached as a Ramsey problem permitting a study of variations in key parameters and steady-states which is uncommon in a literature that relies on a quadratic approximation to the utility function. Exploiting this variation I find that learning induces an increase in the importance of the output gap under a cost-push shock for the (more realistic case) of a distorted steady-state. The welfare costs of business cycles are shown to be significantly larger even under the optimal policy.


Keywords: Monetary Policy, Labor Productivity, Inflation.

1.1 Introduction

What is the role of output in monetary policy? The Basic New Keynesian model (BNKM), that is a real business cycle model augmented only with nominal frictions, does not imply any role for managing output with monetary policy. Rather, optimal monetary policy is strict inflation targeting. This is because eliminating inflation entails that output behaves as it does in a flexible price world and absent any real rigidities or market imperfections the flexible price level of output is potential output, meaning the output gap is zero. It is as if by “divine coincidence” (Blanchard and Gali (2005)) that by only caring about inflation the policy maker can avoid welfare losses emanating from changes in the real economy.

To induce a meaningful trade-off between output and inflation some non-trivial real imperfection is needed. New Keynesian models used in policy analysis, e.g. (Smets and Wouters (2003)), include real imperfections such as nominal wage rigidities (Erceg et al. (2000)) and/or consumption habits (Leith et al. (2012)) to augment the BNKM. In this paper, I study the implications for optimal monetary policy of an alternative imperfection, a learning-by-doing (LBD) externality, in an otherwise standard Basic New Keynesian model.
1.1. Introduction

Learning-by-doing externalities have been widely studied in the literature on economic growth. However, there is a growing literature documenting their relevance to the business cycle, indicating that they induce more realistic dynamics in both a closed and open economy setting. Learning-by-doing creates a link between the levels of production or employment in the economy today and its ability to produce tomorrow. The productivity of the economy tomorrow affects marginal costs and thus inflation tomorrow. Thus it is plausible that such a mechanism may have policy implications for the trade-off between output and inflation.

In this paper I model LBD following Chang et al. (2002), whereby the productivity of workers depends on both an exogenous technology as well as an endogenous economy-wide stock of skills of the workforce. These skills depend on the past levels of employment capturing the notion that workers skills may depreciate during periods of low employment.

The LBD externality acts through two channels. The first is a marginal cost channel, whereby lower output and employment today entail lower skill levels in the future. With lower skill levels, worker productivity falls raising marginal costs and inflation. If the policy maker wants to neutralise the effect of a shock on marginal costs today by engineering a drop in output that fully absorbs the impact then she must accept that future marginal costs will rise. Since inflation is nothing but expected discounted marginal costs this induces higher inflation today. In this way LBD induces a trade-off between inflation and output today and so breaks the divine coincidence. In the BNKM, marginal costs depend only on current output implying that letting output fully absorb the effects of a shock in order to neutralise deviations in marginal costs does not have additional inflationary effects through higher future marginal costs.

The second channel is the direct impact of skills on the utility of the household. Higher levels of skills mean that households need to work fewer hours to produce a unit of output. Lower output today means households have to work more in the future due to lower productivity to produce a given unit of output. The policy maker that reduces output to stabilise marginal costs today must consider that the household will have to work harder in the future creating an additional

---

1 In a closed economy setting, Chang et al. (2002) show improved persistence in an RBC model following exogenous technology shocks. Tsuruga (2007a) documents hump-shaped responses in output to a monetary policy shock while d’Alessandro (2015) has shown that consumption rises following a positive shock to government spending when LBD is present but this is counterfactually negative without it in a BNKM. In an open economy setting, Johri and Lahiri (2008) show that LBD at the firm level implies greater persistence in real exchange rates and Benigno and Formaro (2012) have shown that LBD in the import sector can rationalise large build-ups of foreign exchange reserves.

2 For tractability, these skills are a property of a representative household that makes a labour supply decision and thus skills do not vary across workers or firms.

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3 An exception is a technology shock when the intertemporal elasticity of substitution is 1 \((\sigma = 1)\) as discussed in section 4.
cost to reducing output in order to stabilise inflation. The optimal policy considers the impact of both these channels when deciding the appropriate inflation-output trade-off today.

I approach the monetary policy problem as a Ramsey problem following Schmitt-Grohe and Uribe (2005) and Woodford (2010). This approach is consistent with the more popular method of deriving a purely quadratic approximation to the households utility function and then combining this with the first order approximation to the model’s equilibrium equations to solve a linear-quadratic control problem (see for example, Gali (2008)). The linear-quadratic approach is insightful as it allows a simple expression of the dependence of welfare on parameters of the model. However, it is not always simple or feasible to derive a second order approximation to utility\(^4\) and it may restrict the feasibility of studying parameter variation, for example (Gali and Monacelli (2005); Wren-Lewis and Leith (2007)) restrict the elasticity of intertemporal substitution to be unity to be able to study the linear-quadratic solution in an open economy environment. Moreover, it is common to make those approximations to the model around a steady state where the effects of all distortions have been removed (the efficient steady state). This efficient steady state is achieved via a subsidy that, for example induces firms to produce the pareto optimal level of output, thus neutralising the distortionary effects of monopolistic competition on steady state output. In its favour, this convention avoids conflating issues relating to long run economic growth with policies concerned with stabilising the business cycle\(^6\), however, approximating the model around the steady state where frictions are present (the distorted steady state) can yield policy relevant insights\(^7\). The quadratic approximation approach is ill suited to this purpose as it requires tedious derivations of the model equations to second order to study the behaviour of policy away from the efficient steady state (Benigno and Woodford (2005)). This is relevant, as introducing LBD leads to large deviations of output from its efficient steady state value in the absence of the appropriate subsidy.

Using this framework I study 4 shocks in both the efficient and distorted steady

---

\(^4\) There can be no first order terms in the approximation to have a solution to the linear-quadratic problem that is accurate to second order - see Woodford (2003), Chapter 6.

\(^5\) I found this approach infeasible in the context of LBD. In particular it was not possible to replace all linear terms from the objective function via second order approximations to the model equations due to the presence of the law of motion for the stock of endogenous skills.

\(^6\) The classic inflationary-bias result of Barro and Gordon (1983) is a study where inflation results from the policy maker trying to raise output above its steady state level, as if to raise long run growth.

\(^7\) DePaoli (2009), exploring parameter variation in an open economy New Keynesian model with a distorted steady state, has found conditions under which exchange rate stabilisation should be part of the goals of monetary policy.
1.1. Introduction

state: three shocks where the divine coincidence holds in the Basic New Keynesian Model (BNKM), labour supply, technology and demand or preference shock; and a cost-push shock which creates an inflation-output trade-off even in the BNKM. I find that the LBD externality implies that the flexible price allocation is no longer optimal, breaking the divine coincidence result. The ability to costlessly achieve zero inflation depends on how marginal costs move with output and the shocks hitting the economy. In the BNKM, firms’ marginal costs are determined within-period by the output level relative to technology, the preferences of households to consume and supply labour. Having output adjust to offset these changes entails that marginal costs do not deviate from steady state under these shocks. Since inflation is nothing but the present value of expected deviations in marginal costs in these models inflation is zero in every period. In a flexible price world output responds in exactly this way. I show that the LBD externality implies an additional forward looking component to policy decisions, considering the impact tomorrow of todays choices on output (beyond inflation expectations). This is summarised in an inflation-output trade-off where the costs of inflation today are weighted against the present value of future output gaps. This entails a smoothing motive for the policy maker which depends on how households value current versus future consumption (the elasticity of intertemporal substitution).

I show that LBD introduces a greater role for stabilising the output gap. Under contractionary demand and labour supply shocks optimal policy calls for a decline in the interest rate whereas a rise would be optimal without LBD. This is to reduce both the inflationary consequences of a decline in skills as well as the rise in the disutility of labour such a skills drop entails. A similar result obtains for cost-push shocks but only in the distorted steady state. Diminishing returns to labour induce lower levels of steady state output. Learning amplifies this steady state effect by further reducing the productivity of workers when output is low. As shown in section 3.2.1, these effects are large relative to the case without learning. The larger is the gap between efficient and steady state output the greater the weight the policy maker must give to the impact of changes in output on marginal costs of firms operating in the inefficient steady state. This is due to an interaction between time-consistent policy choices and the size of distortions in the steady state. A time-consistent policy maker must place a positive weight on the marginal cost and revenue conditions that firms face in the distorted steady state but the size of these distortions are significantly larger in the presence of LBD than without. These effects are not present in the efficient steady state since by assumption the appropriate production subsidy ensures firms produce the maximal level of steady state output. The welfare costs of shocks increase by up to twice the levels seen in the BNKM depending on the shock. This is due to additional costs from a non-trivial policy trade-off and, mechanically, from the endogenous propagation of shocks through LBD.
1.1. Introduction

This paper is related to the literature on optimal monetary policy addressing the divine coincidence, studies of monetary policy decisions in the distorted steady state and attempts to merge endogenous growth theories with business cycles. Blanchard and Gali (2005) originally noted the problem of the “divine coincidence” and introduced rigid real wages as a means of creating a wedge between the flexible price level of output and the efficient level of output. A similar approach that applies the Calvo (1983) mechanism, developed by Ermeg et al. (2000), to induce wage rigidity has become widespread in medium scale macro models such as Smets and Wouters (2003). Consumption habits are another addition to the core New Keynesian model popular in medium scale models that introduces a real imperfection capable of breaking the divine coincidence (Leith et al. (2012)). The present study is similar in spirit to Leith et al. (2012), however, I study the implications of alternative dynamic externality on optimal policy.

The analysis of the optimal policy in New Keynesian models rarely includes a study of the case of the distorted steady state. Either, these distortions are assumed to be small enough so that they would not materially alter policy makers response to shocks (Gali (2008); Woodford (2010)) or a production subsidy capable of supporting the efficient level of steady state output is assumed. Studies that have attempted to analyse monetary policy with steady state distortions have yielded important insights: The classic inflationary-bias result of Barro and Gordon (1983) is such a study and DePaoli (2009), exploring parameter variation in an open economy New Keynesian model with a distorted steady state, has found the conditions under which exchange rate stability should be part of the goals of monetary policy. Similarly, Benigno and Benigno (2008) have shown under which shocks and steady states there are gains from cooperation in monetary policy between countries. Production subsidies are relatively rare in practice and a real imperfection can lead to large steady state distortions requiring implausibly high subsidies.

Learning-by-doing externalities have been studied extensively in the growth literature (e.g. Grossman and Helpman (1993)) and there is evidence that skills depreciation is an important cost of low levels of employment (Altug and Miller (1998); Sparber (2011); Hansen and Imrohoroglu (2009)). However, the cyclical implications are less well understood. Studies of the cyclical implications of LBD are largely positive exercises in matching empirical regularities in the business cycle data. Chang et al. (2002) developed the reduced form LBD mechanism employed here where they showed that this mechanism delivers improved persistence

\footnote{For context these are 10% in the BNKM when the elasticity of substitution between varieties of goods is 11 rising to 29% with moderate learning and 46% when learning is strong (these qualitative descriptions of the strength of learning are made concrete in Section 4 below). Clearly the imperfection of LBD introduces makes the supposition of production subsidies of this scale more implausible and helps to motivate a check of the results in the case where the subsidy is zero.}
in a real business cycle model based on Bayesian estimation on U.S. data. Tsuruga (2007b) employed a similar mechanism to capture the hump-shaped response of output to a monetary policy shock. Recently, d’Alessandro (2015) has shown that this mechanism can generate a positive comovement between consumption and government spending in a New Keynesian Model. LBD has been fruitfully employed in an open economy setting. Johri and Lahiri (2008) have shown that learning at the firm level can help produce the persistence of real exchange rates found in the data. Benigno and Fornaro (2012) have shown that learning by doing generated by importing intermediate goods can help to explain the significant reserve accumulation seen in developing Asian countries in the last decade. The latter authors also analyse the normative implications of their learning mechanism showing that such reserve accumulation is optimal from the perspective of the accumulating country. The current work is more closely related to Benigno and Fornaro (2015) who introduce nominal frictions into an endogenous growth model where investment levels influence productivity. They find that this mechanism creates a very strong feedback between output and expectations of firm profits leading to multiple equilibria. Thus they focus on a steady state analysis showing that the level of the subsidies provided to entrepreneurs is key in avoiding a liquidity trap. The current paper also highlights the role of efficient subsidies and illustrates how they are much more potent in an environment with endogenous productivity than without. Both Benigno and Fornaro (2015) and the current paper capture, in an optimising framework, Summers (2015) “inverse Say’s law”, a channel through which a lack of demand can influence potential output. As shown in section 3.3, potential output and the output gap is dependent on the stock of skills in the economy and this then implies that the natural rate of interest is a function of the stock of skills. Shocks leading to lower output can then imply a decline in the natural rate of interest, when no such decline would occur without LBD. Indeed, with negative demand and labour supply shocks, calibrated as in Smets and Wouters (2003), I find declines in the natural rate of interest rather than the increase seen without LBD. Thus this mechanism is relevant to the secular stagnation debate explaining the protracted decline in potential output after the financial crisis of 2008.

The remainder of the paper is organised as follows. Section 2 presents a New Keynesian model with learning-by-doing and highlights the channels through which learning affects firms costs and household’s utility. Section 3 discussed the Ramsey optimal policy problem and presents the implications of learning for the steady state output and the welfare-relevant output gap. Section 4 discusses the quantitative results of the optimal monetary policy. Section 5 presents the size welfare losses induced by the externality. Section 6 presents results on the optimal inflation-output trade-off when the policy maker follows a Taylor rule. Section 7 concludes.
1.2 Model

The model presented here follows the canonical New Keynesian model of Woodford (2010) where learning-by-doing in production in the spirit of Chang et al. (2002) is introduced.

1.2.1 Households

The economy is cashless (Woodford (2003)) and populated by identical infinitely-lived households who choose their consumption, labour supply and holdings of nominal bonds to solve:

$$\max_{\{C_t;H_t;B_t\}} U_0 = E_0 \sum_{t=t_0}^{\infty} \beta^{t-t_0} \{ u(C_t;\xi^C_t) - v(H_t;\xi^H_t) \}$$

$$= E_0 \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ \frac{C_t^{1-\frac{1}{\sigma}} (\xi^C_t)^{\frac{1}{\sigma}}}{1 - \frac{1}{\sigma}} - \frac{H_t^{1+\frac{1}{\psi}} (\xi^H_t)^{\frac{1}{\psi}}}{1 + \frac{1}{\psi}} \right\}$$

s.t. $P_tC_t + Q_{t,t+1}B_t \leq B_{t-1} + W_tH_t + \Upsilon_t + T_t$

$C_t$ is an index of aggregate consumption, $H_t$ is hours of labour supplied, $W_t$ is the nominal wage, $T_t$ are net government transfers, $\Upsilon_t$ are profits from firms; and $\xi^C_t$ and $\xi^H_t$ are shocks to preferences for consumption and labour supply respectively. Households have access to complete asset markets where they can trade one-period bonds, $B_t$, at a price $Q_{t,t+1}$. Aggregate consumption, $C_t$, and the price level, $P_t$, are defined with the Dixit-Stiglitz aggregators over individual consumption good varieties, $C_t(i)$, and their prices, $P_t(i)$:

$$C_t = \left[ \int_0^1 C_t(i)^{1-\frac{1}{\tau}} di \right]^{\frac{1}{1-\tau}}$$

$$P_t = \left[ \int_0^1 P_t(i)^{1-t} di \right]^{\frac{1}{1-t}}$$

(1.2)

Where $\epsilon$ is the elasticity of substitution between varieties of goods. The solution to the households problem, (1.1), entails the following intra-temporal labour supply condition and bond price:

$$\frac{W_t}{P_t} = \frac{v_h}{u_c} = \left( \frac{C_t}{\xi^C_t} \right)^{\frac{1}{\sigma}} \left( \frac{H_t}{\xi^H_t} \right)^{\frac{1}{\psi}}$$

(1.3)

$$Q_{t,t+1} = \beta \frac{u_C(C_{t+1},\xi^C_{t+1}) P_t}{u_C(C_t,\xi^C_t) P_{t+1}} = \beta \left( \frac{Y_t}{Y_{t+1}} \right)^{\frac{1}{\sigma}} \frac{P_t}{P_{t+1}}$$

(1.4)
1.2. Model

Where the last equality follows from imposing the market clearing which requires that $Y_t = C_t$.

1.2.2 Firms

1.2.2.1 Production

There is a continuum of monopolistically competitive firms where each variety of good, indexed by $i \in [0, 1]$, is supplied by a single producer. The $i^{th}$ firms buys labour hours, $N(i)$, from households on a competitive labour market. The productivity of workers depends on the aggregate level of exogenous technology, $A_t$, as well as the level of endogenous aggregate worker skills, $X_t$, that alters the effective unit of labour supplied by households. Firms face diminishing returns to production, governed by $\alpha$, in employing this effective unit of labour:

$$Y_t(i) = A_t(X_tN_t(i))^{1-\alpha}$$

(1.5)

The aggregate stock of workers' skills $X_t$ evolves depending on past levels of employment, a form of learning-by-doing, as in Chang et al. (2002):\footnote{This nests the BNKM analysed in Woodford (2010) when $\mu \to 0$}

$$X_t = X_{t-1}^{\phi_x}N_{t-1}^{\mu}$$

(1.6)

I will follow Chang et al. (2002) in that this learning is external to firms, that is, they do not internalise the effects of employment on productivity of their workers\footnote{If a single producer or a few large producers supplied output and internalised the impact of employment levels today on their future marginal costs then the labour demand would no longer be static as here, but would instead include a dynamic term similar to the second term on the RHS of the planners first order condition (1.31) given below in section 3.1. This is not considered since it would mean the externality poses no policy problem for the monetary policy maker to counteract.}. In the current context this can be motivated by the fact that worker productivity depends on the economy-wide level of skills and each producer’s employment decision, $N_t(i)$, contributes only infinitesimally to the aggregate stock of skills, $X_t$.

Each producer faces a downward sloping demand curve for their variety of goods based on the Dixit-Stiglitz preferences described above:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} Y_t$$

(1.7)

$Y_t$ is the aggregate demand for the consumption basket $C_t$ defined in (1.2). Producers are subject to Calvo (1983) price rigidities which implies that $P_t(i)$ need not equal the aggregate price level $P_t$ as only a subset of firms are able to
reset prices each period giving rise to a measure of cross-sectional price dispersion:

\[
\Delta_t \equiv \int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{-\epsilon(1+\eta)} di \geq 1
\]  

(1.8)

Where \( \eta = \frac{\sigma}{1-\sigma} \). Using (1.5) and (1.7), this now allows us to relate the level of skills to the past levels of output and the dispersion of prices:

\[
N_t(i) = \left( \frac{Y_t(i)}{A_t} \right)^{1+\eta} \frac{1}{X_t}
\]  

(1.9)

\[
N_t \equiv \int_0^1 N_t(i) di = \left( \frac{1}{A_t} \right)^{1+\eta} \frac{1}{X_t} \int_0^1 Y_t(i)^{(1+\eta)} di = \left( \frac{Y_t}{A_t} \right)^{1+\eta} \frac{\Delta_t}{X_t}
\]  

(1.10)

(1.10) indicates that for a given level of output, \( Y_t \), improved technology or skills requires fewer workers whereas greater price dispersion acts to reduce labour productivity. Why? Consumers want to consume an equal amount of each variety of goods produced by the different firms (since Dixit-Stiglitz preferences entail that each these differentiated goods have an equal weight in the consumption basket). Thus the demand for aggregate output falls when this output exhibits a greater price dispersion across varieties of goods. Even if technology and skills are unchanged, dispersed output levels will aggregate up to a lower \( Y_t \)\(^{11}\).

Thus (lagged) price dispersion in affecting the level of aggregate output, affects the demand for labour and thus employment altering the evolution of skills:

\[
X_t = X_t^{\phi_{t-1}-\mu} \left( \frac{Y_{t-1}}{A_{t-1}} \right)^{\mu(1+\eta)} \Delta_{t-1}^\mu
\]  

(1.11)

Here we can see that higher output raises skill levels but so does price dispersion. This is due to the requirement for more labour to produce a given level of output when prices are dispersed.

1.2.2.2 Price setting

Producers are subject to Calvo (1983) price rigidities whereby they face a fixed probability, \( \omega \), of being able to reset their price each period. Thus each firm takes into account that the price chosen today, \( t \), has a probability of survival of \( \omega^{T-t} \), after \( T \) periods have passed. Thus a firm able to reset their price at time \( t \) will solve the following problem:

\(^{11}\)This is not simply a feature of Calvo pricing. Rotemberg pricing implies the same so long as the price level today is not identical to the price level last period, that is, it will matter in the presence of shocks. There is no effect of price dispersion or price adjustment costs on labour productivity in the steady state.
\[
\max_{\{P_t(i)\}} \sum_{T=t}^{\infty} \omega^{T-t} Q_t T \Pi(P_t(i), P_T, Y_T, X_T; \xi_T) 
\]  

(1.12)

\(\xi_T\) refers to the entire collection of shocks that affect firms' pricing decision, \(\xi_T = \left[ A_T \ \xi_T^C \ \xi_T^H \ \mu_T^P \right]\). \(\mu_T^P\) refers to a shock to firms desired steady state mark-up, \(\frac{\epsilon}{\epsilon - 1}\) and \(A_T\) refers to the level of exogenous technology. \(Q_{t, T}\) is the value placed on nominal profits returned to the household \(T\) periods hence (see equation (1.4)). For the \(i^{th}\) firm nominal profits in period \(T\) are simply nominal revenues less costs:

\[(1 - \tau) P_t(i) \left( \frac{P_t(i)}{P_T} \right)^{-\epsilon} Y_T - W_T N_T(i) \]  

(1.13)

Nominal revenue is \((1 - \tau) P_t(i) Y_t(i)\) where I have used (1.7) and \(\tau\) is a production tax or subsidy levied by the government. In order to see how (1.13) can be written as a function of \(P_t(i), P_T, Y_T, X_T, \xi_T\) only, as in (1.12), I make use of (1.3) and (1.10) to decompose the nominal cost term \(W_T N_T(i)\). Applying the intratemporal optimality condition of households, (1.3), nominal wages must equal the price level times the marginal rate of substitution:

\[W_T = P_T \left( \frac{C_T}{\xi_T^C} \right)^{\frac{1}{\sigma}} \left( \frac{N_T}{\xi_T^H} \right)^{\frac{1}{\psi}} \]  

(1.14)

Market clearing in this closed economy requires that \(Y_t = C_t\) and \(\int_0^1 N_t(i) di = N_t = H_t\). Using the latter and (1.3) and (1.10), the nominal wage becomes:

\[W_T = P_T \left( \frac{Y_T}{\xi_T^C} \right)^{\frac{1}{\sigma}} \left( \frac{1}{\xi_T^H} \right)^{\frac{1}{\psi}} \left( \frac{Y_T}{A_T} \right)^{\frac{1+\eta}{\psi}} \left( \frac{\Delta_T}{X_T} \right)^{\frac{1}{\psi}} \]  

(1.15)

The required number of employees, \(N_T(i)\):

\[N_T(i) = \left( \frac{Y_T(i)}{A_T} \right)^{1+\eta} \frac{1}{X_T} = \left( \frac{Y_T}{A_T} \right)^{1+\eta} \frac{1}{X_T} \left( \frac{P_t(i)}{P_T} \right)^{-\epsilon(1+\eta)} \]  

(1.16)

Thus \(Q_{t, T} \Pi(P_t(i), P_T, Y_T, X_T; \xi_T)\) in (1.12) can be written as a function of \(P_t(i), P_T, Y_T, X_T, \xi_T\) only. The first order condition for profit maximisation is:

\[E_t \sum_{T=t}^{\infty} \omega^{T-t} Q_t T \Pi_1(P_t(i), P_T; Y_T, \xi_T) = 0 \]  

(1.17)

All firms able to reset their price will make the same choice (as they are iden-
1.2. Model

(1.2) thus \( P_t(i) = P_t^* \). Given the assumptions made above a convenient closed form relationship characterising aggregate supply in the economy can be derived (see appendix):

\[
\left( \frac{P_t^*}{P_t} \right) = \left( \frac{F_t}{K_t} \right)^{\frac{1}{1+\gamma}}
\]

(1.18)

\( F_t \) captures the expected future nominal (marginal) revenue and \( K_t \) captures expected future nominal (marginal) costs. These functions are the key forward looking variables in the model that lead to the New Keynesian Phillips curve. They are defined by:

\[
F_t = E_t \sum_{T=t}^{\infty} (\omega \beta)^{T-t} f(Y_T; \xi_T^C) \left( \frac{P_T}{P_t} \right)^{1-\epsilon}
\]

(1.19)

\[
K_t = E_t \sum_{T=t}^{\infty} (\omega \beta)^{T-t} k(Y_T, X_T, \Delta_T; \xi_T) \left( \frac{P_T}{P_t} \right)^{\epsilon(1+\eta)}
\]

(1.20)

\[
f(Y_T; \xi_T) = (1 - \tau) Y_T^{1-\frac{1}{\sigma}} \left( \xi_T^C \right)^{\frac{1}{\sigma}}
\]

(1.21)

\[
k(Y_T, X_T, \Delta_T; \xi_T) = \mu_P^P (1 + \eta) \left( \frac{Y_T}{A_T} \right)^{1+\chi} \left( \frac{1}{X_T} \right)^{1+\frac{1}{\psi}} \left( \frac{\Delta_T}{\xi_T^H} \right)^{\frac{1}{\psi}}
\]

(1.22)

Exogenous variations in \( \mu_P^P \) will be studied as cost-push shocks below and \( \chi \equiv (1 + \frac{1}{\psi})(1 + \eta) - 1 \). From (1.22) we can see that a higher skills will induce lower marginal costs for firms, that is higher levels of worker skills \( (X_T) \) raises their marginal product which in turn requires firms to hire fewer workers at the given wage, reducing marginal costs. This formulation is very convenient in that (1.19) and (1.20) can be written recursively, where \( \Pi_t = P_t/P_{t-1} \):

\[
F_t = f(Y_t; \xi_t) + \omega \beta E_t \Pi_t^{1-\frac{1}{\epsilon}} F_{t+1}
\]

(1.23)

\[
K_t = k(Y_t, X_{t}, \Delta_t; \xi_t) + \omega \beta E_t \Pi_t^{(1+\eta)} K_{t+1}
\]

(1.24)

The Calvo scheme entails that the price index evolves according to:

\[
P_t^{1-\epsilon} = (1 - \omega) \left( P_t^* \right)^{1-\epsilon} + \omega P_t^{1-\epsilon}
\]

(1.25)

Which can be used in conjunction with (1.18) to yield an equation governing the behaviour of inflation each period, analogous to an aggregate supply relation:

\[
\frac{1 - \omega \Pi_t^{1-\frac{1}{\epsilon}}}{1 - \omega} = \left( \frac{F_t}{K_t} \right)^{\frac{1}{1+\eta}}
\]

(1.26)
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As noted by Woodford (2010) this description is equivalent to the New Keynesian Phillips Curve (NKPC) if one log-linearises (1.19), (1.20) and (1.26):

\[
\pi_t = \kappa \left[ \hat{Y}_t - \left( \frac{1 + \psi^{-1}}{\chi + \sigma^{-1}} \right) \hat{X}_t + \left( \frac{\psi^{-1}}{\chi + \sigma^{-1}} \right) \hat{\Delta}_t + \beta E_t \pi_{t+1} + \mathcal{O}(||\xi||) \right] \\
\kappa = \frac{(1 - \omega)(1 - \omega \beta)(\chi + \sigma^{-1})}{\omega(1 + \epsilon \eta)}; \quad u_\xi = (\chi + \frac{1}{\sigma})^{-1} \left[ -(1 + \chi) \quad -\sigma^{-1} \quad -\psi^{-1} \quad 1 \right] \\
\xi_t = \left[ A_t \quad \xi_t^C \quad \xi_t^H \quad \mu_t^P \right]
\]

Where for each variable \( \hat{Z}_t \equiv \ln Z_t - \ln \bar{Z} \) and \( \hat{Z}_t = Z_t - \bar{Z} \) and \( \pi_t = \bar{\Pi}_t \). Since the log-linearised version of the law of motion for price dispersion, see equation (1.28) in section 2.3, is the deterministic equation \( \hat{\Delta}_t = \omega \hat{\Delta}_{t-1} \) if \( \hat{\Delta}_{t_0-1} = \mathcal{O}(||\xi||) \) then \( \hat{\Delta}_t = \mathcal{O}(||\xi||) \forall t \). Thus terms in \( \hat{\Delta}_t \) can be ignored to first order. If additionally \( \hat{X}_t = 0 \) then this is simply the standard NKPC. The channel from higher skills to lower marginal costs and thus inflation is evident from (1.27). A higher stock of skills reduces firms marginal costs and thus reduces inflation.

Households optimal holding of one period bonds, as described by (1.4), is linked to the choice of the short term nominal interest rate set by the policy maker via the no arbitrage relation on bonds, \( 1 + i_t = [E_t Q_t]^{-1} \). The combination of these conditions yields the New Keynesian IS curve (when log-linearised). The final component required to close the model is a statement of how the interest rate will be chosen, this is implied by the Ramsey planners choice of allocations described below in section 3.2.

1.2.3 Price dispersion, skills and welfare

A law of motion linking cross-sectional price dispersion, \( \Delta_t \), to aggregate inflation, \( \Pi_t \), can be derived from (1.8) and (1.25):

\[
\Delta_t = \omega \Delta_{t-1} \Pi_t^{(1+\eta)} + (1 - \omega) \left( \frac{1 - \omega \Pi_t^{(-1)}}{1 - \omega} \right)^{(1+\eta)} (1.28)
\]

This link is key to explaining the welfare implications of inflation in New Keynesian models. The impact of skills and price dispersion on welfare can now be seen by imposing market clearing and substitution of (1.10) into the utility function of the household:

\[
U_{t_0} = E_0 \sum_{t=t_0}^{\infty} \beta^{t-t_0} \{ U(Y_t, X_t, \Delta_t; \xi_t) \}
\]
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\begin{equation}
U_{t_0} = E_0 \sum_{t=t_0}^{\infty} \beta^{t-t_0} \{ u(Y_t; \xi^C_t) - v(Y_t, X_t, \Delta_t; \xi_t) \}
\end{equation}

From (1.29) we can see that the impact of both skills and price dispersion on period utility is equal but opposite in the sense that \( U_{X_t}X_t = -U_{\Delta_t}\Delta_t \). The intuition behind this result is that greater price dispersion implies greater output dispersion which leads to a composition of output that is less valued by households (see discussion in section 2.2.1). To produce a given unit output, more labour hours are required with higher is price dispersion. This effect reduces utility through the disutility of work. Higher levels of productivity, either exogenous \((A_t)\) or endogenous \((X_t)\), directly increase the output of workers requiring fewer hours of work to produce a given unit of output. Thus the higher is endogenous productivity (say from high level of activity in the past), the lower is the disutility created from producing a unit of output. The value of this channel in reducing the negative impact of inflation on household welfare is evaluated by solving the Ramsey problem for this economy.

A competitive equilibrium in this economy is a sequence of allocations and prices such that markets clear and household’s utility (1.1) and firms profits (1.12) are maximised. This is summarised by \( \{F_t, K_t, \Delta_t, \Pi_t, Y_t, X_t\}_{t=0}^{\infty} \) satisfying the households optimality conditions (1.3) and (1.4); the law of motion for skills (1.11); the definition of the forward looking measures of marginal costs and revenue for firms (1.23) and (1.24); the firms first order condition summarised in aggregate supply relation (1.26); the law of motion for price dispersion (1.28) together with a description for the exogenous stochastic processes \( \xi_t \).

1.3 Optimal monetary policies with learning by doing

1.3.1 Social planners problem

The social planner maximises the households utility subject to the resource constraints captured in equation (1.10) and the law of motion for skills (1.11). However the social planner will never choose to have any price dispersion in this economy since this requires the household to work harder\(^{12} \). Thus the social

\(^{12}\)In the BNKM the only source of welfare loss are due to higher price dispersion. However, in the presence of LBD there may be offsetting gains due to the positive effect of higher skills and thus a lower disutility of labour. In fact, however, this effect is not large enough to compensate
1.3. Optimal monetary policies with learning by doing

 planner solves:

$$\max_{\{Y_t\}_{t=0}^{\infty}} \ U_0 = E_0 \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ u(Y_t; \xi_t^C) - v(Y_t, X_t; \xi_t) \right\}$$  \hspace{1cm} (1.30)

s.t. \( X_t = X_{t-1}^\phi - \mu_{t-1} (Y_{t-1} - A_{t-1}) \mu (1+\eta) \)

In the standard New Keynesian model the optimal rule requires that the marginal benefit of an additional unit of output is just compensated by the additional disutility of producing that output: \( u_{Y,t} = v_{Y,t} \). In the presence of learning this static rule becomes dynamic with the additional benefit today of working being a lower disutility of work tomorrow:

$$u_Y(Y_t; \xi_t^C) + \beta E_t \left\{ -v_X(Y_{t+1}, X_{t+1}; \xi_{t+1}^C) \frac{\partial X_{t+1}(Y_t, X_t, \Delta_t; \xi_t)}{\partial Y_t} \right\} = v_Y(Y_t, X_t; \xi_t)$$  \hspace{1cm} (1.31)

Where \( v_X < 0 \). This optimal rule is used to characterise the efficient level of output, \( Y_t^e \). This level of output will not, in general, be feasible when the policy maker must make her choices subject to competitive equilibrium\(^\text{13}\). Optimal choices subject to competitive equilibrium are Ramsey Policies.

1.3.2 Ramsey policies

The Ramsey policy is a choice of \( \{F_t, K_t, \Delta_t, \Pi_t, Y_t\} \) for all \( t \geq t_0 \) to maximise (1.29) while satisfying (1.23),(1.24), the forward looking relations capturing marginal costs and revenues for firms; (1.26), the aggregate supply relation relating these forward looking variables to inflation; the law of motion for skills (1.11); and (1.28), which links inflation to the welfare relevant measure of cross-sectional price dispersion; given \( \Delta_{t_0-1} \) and \( X_{t_0-1} \). \( X_t \) is not an explicit choice variable for the policy maker since the choice of \( Y_{t-1} \) and \( \Delta_{t-1} \) take into account their influence on \( X_t \). This facilitates comparison with the literature where the for the costs, in terms of disutility of labour, that it attracts. Higher price dispersion can induce higher skills tomorrow (see equation (1.11)). However, to a first order approximation, any increase in \( \Delta_t \) will induce an increase of \( \mu \) on \( X_{t+1} \). To a first order, the law of motion for \( \Delta_t \), around the zero inflation steady state, is \( \Delta_t = \omega \Delta_{t-1} \). Thus this will induce an increase in \( \Delta_{t+1} \) of \( \omega \). Since \( \omega > \mu \) for all reasonable parametrisations, the social planner will never find it optimal to create price dispersion to increase productivity as this is always overwhelmed by the decrease in productivity induced by more price dispersion.

\(^\text{13}\)Contrasting (1.31) with (1.39) illustrates why this level of output may not be feasible: the planner would need to take account for the forward looking behaviour of firms captured by the Lagrange multipliers associated with firms’ forward looking constraints to the Ramsey problem.

\(^\text{14}\)The New Keynesian IS curve will not enter the Ramsey problem as the nominal interest rate does not appear in the utility function of the household nor in any of the constraints to the problem. Thus output can be thought of as chosen directly by the policy maker and the nominal interest rate (the instrument of monetary policy) that this requires backed out of this IS relation.
policy maker is thought of as choosing \( \{ \Pi_t, Y_t \} \) only. As currently described this problem is time inconsistent. Due to the forward looking conditions (1.23)-(1.24) policy makers know that the choices made today will have an effect on expectations formed in the previous period, since firms will form expectations based on these policies, and this constrains the choices they can make. However this constraint is not binding when \( t = t_0 \) leading to time inconsistent policy choices between the initial point \( t_0 \) and periods thereafter \( t > t_0 \). I adopt the solution proposed by Woodford (2010) which he calls optimal policy from a “timeless perspective”. The policy maker undertakes precommitments, at \( t_0 - 1 \), to certain values of the forward looking variables in the next period, \( K_{t_0} \) and \( F_{t_0} \), that are consistent with the optimal choices in future periods. These precommitments are captured in the values of the Lagrange multipliers (see problem statement below) \( \phi_{1,t_0-1} \) and \( \phi_{2,t_0-1} \) which govern whether the forward looking constraints bind; or in other words, they control the temptation to raise \( \Pi_{t_0} \) above a level consistent with the forward looking constraints. The precommitment values for \( K_{t_0} \) and \( F_{t_0} \) are the steady state values of \( F_t \) and \( K_t \), denoted by \( \bar{K} \) and \( \bar{F} \), for the Ramsey problem below. The choices of \( \bar{K} \) and \( \bar{F} \) are a function of steady state output and have been constrained to be consistent with future choices, that is choices made under the same constraints the Ramsey planner faces in future periods (see steady state solution in Section 3.2.1 and Appendix II for details). Thus, the Ramsey plan from a “timeless perspective” requires that the Ramsey planner treat forward looking behaviour in a way that is consistent with the initial conditions (or steady state) of the Ramsey problem.

The Ramsey problem outlined above can be described by the Lagrangian:

\[
\max_{\{F_t, K_t, \Delta_t, \Pi_t, Y_t\}_{t=t_0}} \mathcal{L}_{t_0} = E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} L(Y_t, X_t, F_t, K_t, \Pi_t, \Delta_t; \theta_t, \phi_t, \xi_t) \quad (1.32)
\]

Where

\[
L(Y_t, X_t, F_t, K_t, \Pi_t, \Delta_t; \theta_t, \phi_t, \xi_t) = u(Y_t; \xi_t^C) - v(Y_t, X_t, \Delta_t; \xi_t) + \theta_t \left[ \Delta_t - \omega \Delta_{t-1} \Pi_t^{(1+\eta)} - (1 - \omega) \left( \frac{1-\omega\Pi_t^{-1}}{1-\omega} \right)^{\frac{1+\eta}{\eta+1}} \right]
\]

As shown in section 2.2.2 on price setting, combining the two forward looking equations for \( F_t \) and \( K_t \) along with the aggregate supply curve (1.26) and log-linearising produces the standard New Keynesian Phillips Curve (1.27). In a standard linear quadratic approach to optimal monetary policy, e.g. Woodford (2003); Gali (2008), this would allow the problem to be stated in terms of just 2 variables aggregate inflation, \( \Pi_t \), and aggregate output, \( Y_t \). Since here I do not pursue the linear quadratic approach I use a full set of equilibrium conditions which includes variables \( \{ F_t, K_t, \Delta_t, \Pi_t, Y_t \} \). Where LBD absent this would be identical to the linear quadratic approach typically used, as shown in Woodford (2010).
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\[ \phi_{1,t} [F_t - f(Y_t; \xi_t)] - \omega \phi_{1,t-1} [\Pi_t^{e-1} F_t] + \phi_{2,t} [K_t - k(Y_t, X_t, \Delta_t; \xi_t)] + \omega \phi_{2,t-1} [\Pi_t^{(1+\eta)} K_t] + \phi_{3,t} \left[ \frac{1 - \omega \Pi_t^{e-1}}{1 - \omega} - \left( \frac{P_t}{K_t} \right)^{\frac{1}{1+\eta}} \right] \]

Where \( \phi^t = [\phi_{1,t} \ \phi_{2,t} \ \phi_{3,t}] \). The multipliers \( \phi_{1,t-1}, \phi_{2,t-1} \) will capture the precommitments i.e. they will be the values consistent with the steady state solution of the model under the same constraints the Ramsey planner faces for \( t > t_0 \). I consider the local dynamics near the zero inflation steady state of the model. The complete first-order conditions (FOCs) are described in the appendix. Here I will focus on the optimal rule for output and price dispersion since these rules involve additional terms due to LBD. The optimal rule for the choice of output is:

\[ u_Y(Y_t; \xi_t^C) (1 - \phi_{1,t}(1 - \tau)(1 - \sigma^{-1})) + \beta E_t \{-v_X(Y_{t+1}, X_{t+1}, \Delta_{t+1}; \xi_{t+1}) - \phi_{2,t+1} kX(Y_{t+1}, X_{t+1}, \Delta_{t+1}; \xi_{t+1})\} \frac{\partial X_{t+1}(Y_t, X_t, \Delta_t; \xi_t)}{\partial Y_t} = (1 + \phi_{2,t} \mu_t^P (1 - \chi)(1 + \psi^{-1}) \Delta_t^{-1}) v_Y(Y_t, X_t, \Delta_t; \xi_t) \]

(1.33)

This optimal rule is analogous to that of the social planner, (1.31), however when choosing \( Y_t \) in addition to considering the reduction in labour disutility tomorrow, \(-v_{X,t+1}\), the Ramsey planner must also consider the reduction in the marginal costs of firms tomorrow, \(-k_{X,t+1}\) as well as additional terms linked to the marginal revenue, \((1 - \tau)(1 - \sigma^{-1})\), and marginal cost, \(\mu_t^P (1 - \chi)(1 + \psi^{-1}) \Delta_t^{-1}\), of producing output in a decentralised economy which are weighted by the degree to which firms pricing decisions, captured in \( F_t \) and \( K_t \), are a binding constraint on the planner’s choice of \( Y_t \). These weights are \( \phi_{2,t} \) and \( \phi_{1,t} \).

The optimal choice for \( \Delta_t \) is:

\[ \theta + \beta \{-v_X(Y_{t+1}, X_{t+1}, \Delta_{t+1}; \xi_{t+1}) - \phi_{2,t+1} kX(Y_{t+1}, X_{t+1}, \Delta_{t+1}; \xi_{t+1})\} \frac{\partial X_{t+1}(Y_t, X_t, \Delta_t; \xi_t)}{\partial \Delta_t} = + v_{\Delta}(Y_t, X_t, \Delta_t; \xi_t) + \phi_{2,t} k_\Delta(Y_t, X_t, \Delta_t; \xi_t) + \theta_{t+1} \omega \Pi_{t+1}^{(1+\eta)} \]

(1.34)

A similar pattern is seen as with output; when deciding how much price dispersion to tolerate the Ramsey planner considers the benefit of increased price dispersion on future skills resulting in a benefit to having greater price dispersion today. However the increase in skills due to greater price dispersion (from a higher labour input requirement for firms) is in general smaller than the increase in skills from greater output since:
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\[ D_t \equiv \frac{\partial X_{t+1}}{\partial X_t} \frac{\partial Y_t}{\partial Y_t} = (1 + \eta) \frac{\Delta_t}{Y_t} \geq 1 \]

This is true since \( D_t \to 1 \) when \( Y_t \to 1 + \eta \) and \( \Delta_t \to 1 \). However it can be seen from (1.27) that for output to rise to such a high level above it’s steady state value\(^{16}\), inflation would have to rise which would push \( \Delta_t \) above 1 (1.28). Thus the key channel through which skills operates is via it’s role on increasing the productivity of households through higher levels of past output.

1.3.2.1 Steady state

In order to study the optimal response to shocks I linearise these conditions around the zero inflation steady state consistent with the optimality conditions of the Ramsey problem\(^ {17}\). This is the steady state associated with the above 5 FOCs and the 4 constraints. The steady state is characterised by \( \{ \bar{F}, \bar{K}, \bar{\Delta}, \bar{\Pi}, \bar{Y}, \bar{X}, \bar{\theta}, \bar{\phi} \} \) that solves these 9 equations when \( \xi_t = \bar{\xi} \). The zero inflation steady state has \( \bar{\Delta} = \bar{\Pi} = 1 \). This immediately implies that \( \bar{K} = \bar{F} \) from the aggregate supply relation (1.26). From this result we can find the relationship between steady state output and the production subsidy \( \tau \) as well as the steadystate stock of skills, using (1.11):

\[
f(\bar{Y}) = k(\bar{Y}, \bar{X}, \bar{\Delta}) \iff \frac{v_h(\bar{Y}, \bar{X}, \bar{\Delta})}{u_c(\bar{Y})} = \frac{1 - \bar{\tau}}{\epsilon} \iff \bar{Y} = \left( \frac{1 - \bar{\tau}}{\epsilon \gamma(1 - \phi_x + \mu) - \mu(1 + \chi)} \right) \left[ \frac{1 - \phi_x + \mu}{\epsilon(1 - \phi_x + \mu) - \mu(1 + \chi)} \right]^{\frac{1}{\epsilon(1 + \eta)}}
\]

\[ (1.35) \]

\[ \bar{X} = \bar{Y} \left( \frac{1 + \eta}{\phi_x + \mu} \right) \]  

\[ (1.36) \]

This result illustrates the role of the production subsidy in determining whether this steady state is the (constrained) efficient or inefficient one. From this we can see that the production subsidy will offset any distortion due to monopolistic competition if \( \tau = 1 - \left( \frac{\epsilon}{\epsilon - 1} \right) \). If this is the case and we further assume that \( \alpha = 0 \), that is, constant returns to scale then \( \bar{Y} = 1 \) regardless of the learning parameters \( \mu \) and \( \phi_x \). However when diminishing returns to scale are present (\( \alpha > 0 \)) or if the production subsidy is too small to neutralise the steadystate effects of monopolistic competition then these learning parameters can affect \( \bar{Y} \). To illustrate, suppose the production subsidy neutralises monopolistic competition and \( \alpha = \frac{1}{3} \). The impact of stronger learning (higher \( \mu \)) in this environment is shown in Figure 1 by

\(^{16}\)For context, steady state output, \( \bar{Y} = 1 + \eta \) would require a production subsidy larger than the output of the entire economy (a subsidy of approximately 158% given the baseline parametrisation).

\(^{17}\)Thus this is not the optimal steady state of the social planner, but only the optimal steady state from the perspective of a planner that must take actions subject to competitive equilibrium.
1.3. Optimal monetary policies with learning by doing

Figure 1.1: Steady-state output and learning

![Figure 1.1: Steady-state output and learning](image)

Figure plots the value of $\bar{Y}$ for 2 different values of $\tau$. The efficient case of $\tau = \tau^e$ which rises with $\mu$ and the usual subsidy that offsets only monopolistic competition $\tau = 1 - \left(\frac{\epsilon}{\epsilon - 1}\right)$. The values for other parameters are as assumed in Table 1.

Thus the effect of stronger learning is to reduce $\bar{Y}^{ce}$ even when $\tau = 1 - \frac{\epsilon}{\epsilon - 1}$. In competitive equilibrium firms don’t internalise the impact on future productivity of lower output today this has the effect when combined with diminishing returns to labour of amplifying the decline in steady state output that these diminishing returns induce\(^\text{18}\). This can be contrasted with the level of efficient output consistent with the social planners solution: equation (1.31) in steady state requires the following production subsidy for $\bar{Y}^e = \bar{Y}^{ce}$:

$$\tau^e = 1 - \left(\frac{\epsilon}{\epsilon - 1}\right) \left(\frac{1}{1 - \beta \mu}\right) \quad (1.37)$$

When this subsidy is in place and the determination of output thus includes a forwarding looking element we see that the presence of learning ($\mu$) acts to undo the contractionary impact of diminishing returns if learning is strong enough. The trade-off between the strength of diminishing returns and learning leading to higher output is described in Figure 2. The graph shows the level of $\mu$ whereby an increase in $\mu$ induces higher output. For low levels of $\alpha$ output rises for all increases in $\mu$, however as diminishing returns becomes stronger a higher level of $\mu$ is needed before learning induces an increase in output.

\(^{18}\)Without diminishing returns an increase in $\mu$ induces higher steadystate output
1.3. Optimal monetary policies with learning by doing

Figure 1.2: Parameter combinations where LBD dominates diminishing returns to labour

This combination of \((\mu, \alpha)\) are those where increases in the strength of learning from past employment will raise the efficient level steady state output (shaded region). Said differently, any further increases in \(\mu\) will increase \(\bar{Y}^e\). For the unshaded region increases in \(\mu\) lead to a decline in \(\bar{Y}^e\).

Note that if an efficient subsidy is not in place then the size of the distortions in output in competitive equilibrium relative to the efficient equilibrium are very large with LBD and relatively small for the standard model. To illustrate suppose \(\tau = 0\). In the standard model this will induce a distortion of output of approximately 4.5\% - which can be seen from (1.35) when the learning parameters are set to 0. However with \(\mu = 0.15\) and \(\phi_x = 0.8\) this gap becomes 13\% and 20\% if \(\mu = 0.25\).

Using the FOCs it is also possible to derive the values of the Lagrange multipliers in (1.32). The following results will be useful in the section to follow:

\[
\bar{\phi}_2 = -\bar{\phi}_1 = \frac{U_Y + \beta U_X \frac{\partial X}{\partial Y}}{k_Y - f_Y + \beta k_X \frac{\partial X}{\partial Y}} \tag{1.38}
\]

These are the steadystate levels that the Ramsey planner commits to at \(t = t_0 - 1\) which bind her for future periods. The precommitments of the planner, which constrain her to behave in a consistent way at \(t_0\) and all other periods, entail that the shadow value of the forward looking constraints (\(\phi_1\) and \(\phi_2\)) are relevant to the steady state level of output. Intuitively, both steady state output and the path of output when it evolves according to the choice of a planner acting from a “timeless perspective” reduce the influence of time in a particular way. The connection between the level of the production subsidy, \(\tau\), and the level of these Lagrange multipliers on the forward looking constraints (\(F_t\) and \(K_t\)) can be seen by recognising that the numerator in (1.38) is in fact the steadystate version of equation (1.31) which states \(U_Y + \beta U_X \frac{\partial X}{\partial Y} = 0\). Thus when the optimal subsidy is in place the value of these multipliers is zero. When \(\tau \leq \tau^e\) then \(\bar{\phi}_2 \geq 0\) and the precommitment made by the policy maker bind for \(t \geq t_0\). How the presence of a steady state distortion influences the policy makers target level of output and
thus response to shocks is discussed next.

1.3.3 The target level of output and the output gap

1.3.3.1 Basic New Keynesian Model

What is the appropriate level of output for the monetary authority to target? Consider the case without learning. The target level of output ought to be the level of output that is consistent with price stability (the steady state assumption) as well as the constraints on the monetary authority in achieving this. Following Benigno and Woodford (2005) this can be seen to be the first order condition for the Ramsey problem, (1.33) with the omission of the terms depending on $X_t$, when prices are flexible:

$$U_Y(Y_t^*, 1; \xi_t) = \bar{\phi}_1 f_y(Y_t^*, \xi_t) + \bar{\phi}_2 k_Y(Y_t^*, 1; \xi_t) \quad (1.39)$$

The Lagrange multipliers governing the behaviour of (1.23) and (1.24) take on their steady state values as under flexible prices since firms pricing decisions no longer have any dynamic considerations implying $f(Y_t; \xi_t) = k(Y_t, 1; \xi_t) \forall t$. This equation states the target output is a function of exogenous shocks only.\(^{19}\)

Intuitively, since the only friction present in the model (nominal rigidities) is removed when prices are flexible, the only driver of the target level of output are shocks hitting the economy. In the case with an additional frictions (learning) discussed below this will not be the case as the target level of output will depend on the optimal evolution of workers’ skills. This allows us to easily relate the efficient and natural (flexible price) levels of output to this target level based on the steady-state production subsidy.

The efficient level of output maximises utility subject only to technology and exogenous shocks hitting the economy, restating (1.31):

$$U_Y(Y^*_e, 1; \xi_t) = 0 \quad (1.40)$$

The natural level of output must be consistent with firms (static) price setting decision when prices are flexible. The aggregate supply relation, (1.26), entails that $F_t = K_t \forall t$ which in turn requires:

$$f(Y^*_n, \xi_t) = k(Y^*_n, 1; \xi_t) \forall t. \quad (1.41)$$

\(^{19}\)A log-linearised version of the equation around the zero inflation steady state reveals that:

$$Y_t^* = \frac{1}{(U_{YY} - \bar{\phi}_1 f_{YY} + \bar{\phi}_2 k_{YY})} (\bar{\phi}_1 f_{Y \xi} + \bar{\phi}_2 k_{Y \xi} - U_{Y \xi})' \xi_t$$
1.3. Optimal monetary policies with learning by doing

\begin{equation}
(1 - \tau)u_Y(Y^n_t; \xi_t) = \left( \frac{\epsilon}{\epsilon - 1} \right) v_Y(Y^n_t, 1; \xi_t) \tag{1.42}
\end{equation}

Now the links between \( Y^e_t, Y^n_t \) and \( Y^*_t \) can be clarified. If \( \tau = 1 - \frac{\epsilon}{\epsilon - 1} \) then (1.42) is just the statement \( U_Y(Y^n_t, 1, \xi_t) = 0 \) which means \( Y^e_t = Y^n_t \), that is, the efficient and natural (or flexible price) level of output are the same since there is no additional friction to drive a wedge between them e.g. a learning externality (see the next section). This then also implies \( \phi_1 = -\phi_2 = \frac{U_Y}{k_Y - k_Y} = 0 \) and thus \( Y^e_t = Y^n_t = Y^*_t \) from (1.39).

1.3.3.2 Learning-by-doing

The target level of output is given by (1.33) when prices are fully flexible:

\begin{equation}
U_Y(Y^*_t, X^*_t, 1; \xi_t) + \beta E_t \left\{ U_X(Y^*_t, X^*_t, 1; \xi_{t+1}) - \bar{\phi}_2 k_X(Y^*_t + 1, X^*_t, 1; \xi_{t+1}) \right\} \frac{\partial X^*_t}{\partial Y^*_t} = \phi_1 f_Y(Y^*_t; \xi_t) + \bar{\phi}_2 k_Y(Y^*_t, X^*_t, 1; \xi_t) \tag{1.43}
\end{equation}

Comparison with (1.39) shows that targeting output is no longer a function of \( \xi_t \) only but now includes dynamic consideration of \((X^*_t, Y^*_t, \xi_t)\). Moreover the monetary authority cannot know the target level of output unless they know the target level of skills, \( X^*_t \). Of course, \( X^*_t \) is nothing but a summary of \( \{Y^*_t\}^T_{T=0} \). As such the target level of skills is simply given by \( X^*_t = (X^*_t)^{\phi_2 - \mu} \left( \frac{Y^*_t}{Y^*_{t-1}} \right)^{\mu(1+\eta)} \). The efficient level of output would see a social planner choosing output according to the rule:

\begin{equation}
U_Y(Y^e_t, X^e_t, 1; \xi_t) + \beta E_t \left\{ U_X(Y^e_t, X^e_t, 1; \xi_{t+1}) \right\} \frac{\partial X^e_t}{\partial Y^e_t} = 0
\end{equation}

where skills are \( X^e_t = (X^e_t)^{\phi_2 - \mu} \left( \frac{Y^e_t}{Y^*_{t-1}} \right)^{\mu(1+\eta)} \). We can see that the result of the previous section continues to hold: if \( \phi_1 = \phi_2 = 0 \), due to the appropriate subsidy, then \( Y^e_t = Y^*_t \). The natural level of output must be consistent with the analogous versions of (1.42) and (1.41):

\begin{equation}
f(Y^n_t; \xi_t) = k(Y^n_t, X^n_t, 1; \xi_t) \forall t. \tag{1.44}
\end{equation}

\begin{equation}
(1 - \tau)u_Y(Y^n_t; \xi_t) = \left( \frac{\epsilon}{\epsilon - 1} \right) v_Y(Y^n_t, X^n_t; \xi_t) \tag{1.45}
\end{equation}

Similarly, \( X^n_t = (X^n_t)^{\phi_2 - \mu} \left( \frac{Y^n_t}{Y^n_{t-1}} \right)^{\mu(1+\eta)} \). Comparison between \( Y^*_t \) and \( Y^n_t \) shows that even if \( \tau = \tau^e = 1 - \left( \frac{\epsilon}{\epsilon - 1} \right) \left( \frac{1}{1-\beta \mu} \right) \Rightarrow \phi_1 = \phi_2 = 0 \), (1.45) is not
1.3. Optimal monetary policies with learning by doing

a restatement of (1.43) as in the BNKM. This is because firms do not take into account the dynamic effects of their hiring decision today on costs tomorrow. To define an output gap that is zero in the zero inflation steady state regardless of the production subsidy, i.e. even in the case of the distorted steady state that is studied below, I define the output gap as \( y_t^g = \hat{Y}_t - \hat{Y}_t^* \) and the skills gap as \( x_t^g = \hat{X}_t - \hat{X}_t^* \).

To more clearly see the implications of the forward looking nature of optimal policy I log-linearise (1.33) and (1.43) to find the following inflation-output gap trade-off\(^{20}\) (derivation in the appendix):

\[
\zeta_\pi \pi_t = \lambda E_t \sum_{j=0}^{\infty} \beta^j \left( \frac{k_X}{f_Y - k_Y} \right)^j \left\{ \bar{Y} (\Omega_Y \Delta y_{t+j}^g + \beta \gamma_Y \Delta y_{t+j+1}^g) + \bar{X} (\Omega_X \Delta x_{t+j}^g + \beta \gamma_X \Delta x_{t+j+1}^g) \right\}
\]

\[
\lambda = \frac{1}{(f_Y - k_Y)} < 0
\]

Where \( \zeta_\pi < 0, \Omega_Y < 0, \Omega_X > 0, \gamma_Y < 0 & \gamma_X > 0 \) (under the baseline calibration in Table 3) are functions of the model parameters and the steady state level of output (details are in the appendix). Note that \( \left( \frac{k_X}{f_Y - k_Y} \right) > 0 \). Without LBD this rule would be that found by Woodford (2010) p.60:

\[
\zeta_\pi \pi_t = \Upsilon_Y \Delta y_t^g
\]

\[
\Upsilon_Y = \lambda \bar{Y} \left( U_{YY} - \phi_1 (f_Y - k_Y) \right) > 0
\]

In the BNKM the optimal inflation-output gap trade-off\(^{21}\) is between inflation and the growth in the contemporaneous output gap. We can now compare these to rules to highlight how LBD changes the policy makers behaviour. The presence of skills creates a link between today’s decisions and tomorrows state of the world making the inflation-output trade-off one where the entire present value of output gap changes is considered when thinking about what is the appropriate level of inflation. The forward looking nature of this rule introduces an output gap smoothing motive not present in the BNKM. The effective discount rate \( \beta \left( \frac{k_X}{f_Y - k_Y} \right) \) depends on how strong is the influence of skills on marginal costs, \( k_X \), relative to the gap between the marginal revenue, \( f_Y \), and marginal cost, \( k_Y \),

\(^{20}\)I have here assumed for simplicity that \( \Delta y_{t+1} = O(||\xi||) \Rightarrow \hat{\Delta}_t = O(||\xi||) \forall t \) (as in the discussion of the NKPC deviation, equation (1.27)).

\(^{21}\) Note that both (1.46) and (1.47) capture a trade-off between \( \pi_t \) and \( \Delta y_t^g \) since \( \frac{\Upsilon_Y \Omega_Y}{\zeta_\pi} < 0 \) for the former and \( \frac{\Upsilon_Y}{\zeta_\pi} < 0 \). Thus the coefficients have opposite signs.
of producing output. How much does policy change with this forward looking rule? This is discussed in the following 2 sections.

1.3.4 The case for price stability

When target output moves in proportion with the flexible-price level of output then the goal of monetary policy is maximal price stability (i.e. to attempt to replicate the flexible price equilibrium response to shocks). Woodford (2010); Benigno and Woodford (2005) have shown that this is true even for the case of the distorted steady state, \( \tau < \tau^e = 1 - \frac{\epsilon}{\epsilon - 1} \) in the BNKM. Here I show that this result does not hold under LBD.

The response of \( Y^n_t \) to shocks is governed by (1.44) which can be rewritten as:

\[
(1 - \tau)u_Y(Y^n_t; \xi_t) = \left( \frac{\epsilon}{\epsilon - 1} \right) v_Y(Y^n_t, X^n_t, 1; \xi_t)
\]

and in log-linear form is:

\[
\dot{Y}^n_t - \frac{1 + \psi^{-1}}{\sigma^{-1} + \chi} \dot{X}^n_t = -u'_\xi \xi_t \quad (1.48)
\]

Where \( u_\xi \) is as defined in (1.27). The response of \( Y^*_t \) is governed by (1.43) which can be rewritten:

\[
\frac{1 + \bar{\phi}_2(1 - \tau)(1 - \sigma^{-1})}{1 + \bar{\phi}_2(1 + \chi)} u_Y(Y^*_t; \xi_t) - \beta E_t v_X(Y^*_t, X^*_t, 1; \xi_{t+1}) \frac{\partial X^*_{t+1}}{\partial Y^*_{t+1}} = v_Y(Y^*_t, X^*_t, 1; \xi_t)
\]

(1.49)

Using the steadystate value of \( \bar{\phi}_2 \) and under the assumption that LBD is zero (i.e. the BNKM), these equations show that \( Y^*_t = Y^n_t \) and the result of price stability holds as in Woodford (2010). However learning entails an additional term focusing on the implications for the disutility of labour tomorrow. The presence of \( \xi_{t+1}, Y^*_{t+1}, X^*_{t+1} \) will create a smoothing motive for target output that is absent in the behaviour of \( Y^n_t \). The role of LBD depends importantly on the elasticity of intertemporal substitution, \( \sigma \), as expected when a smoothing motive is present. If \( \sigma > 1 \) then \( \dot{Y}^*_t < \dot{Y}^n_t \) whereas for \( \sigma < 1 \) then \( \dot{Y}^*_t > \dot{Y}^n_t \). However the quantitative impact is relatively small (Figure 3). The reasons for this behaviour are discussed in the following section.
1.4. The optimal response to shocks

The optimal response of the Ramsey planner is studied by log-linearising the 5 FOCs and 4 constraints around the steady state described in section 3.2.1. A perturbation approach is pursued as described in Schmitt-Grohe and Uribe (2004). The Ramsey solution is studied under shocks to $\xi_t' = \begin{bmatrix} A_t & \xi_t^C & \xi_t^H & \mu_t^P \end{bmatrix}$. The model is calibrated to a quarterly frequency where shocks are temporary but persistent AR(1) processes (see calibration in Table 3). The simulations compare three models, the Basic New Keynesian model (i.e. the model with $\mu = 0$), a model with moderate LBD ($\mu = 0.15$) and a model with strong LBD ($\mu = 0.25$); in two steadystates, the distorted steady state without any production subsidy and the Pareto efficient steady state supported by an efficient production subsidy.

The findings are as follows. The LBD mechanism undoes the 'divine coincidence' (Blanchard and Gali (2005)) that closing the output gap is entailed by complete price stability. This is because the path of output has stronger implications for marginal costs and thus inflation. This implies that labour supply, preference/demand and technology shocks are non-trivial matters for monetary policy. However the strength of this effect is determined by the divergence between the target level of output, $Y^*_t$, and the flexible price level of output, $Y^n_t$. Since this divergence is relatively small (as seen in the preceding section) the resultant inflation and output gap deviations are small also. In addition there is a motive to reduce the fall in the output gap in the face of cost-push shocks however this effect is only present in the distorted steady state. Near the efficient steady state output falls but the target level of output remains unchanged. Near the distorted steady state, both output and the target level of output fall, leading to a smaller output gap. The reasons for this drop in the target level of output are discussed in detail below.
1.4. The optimal response to shocks

The results for a technology shock, $A_t$, indicate a departure from optimal policy in the standard model: so long as $\sigma \neq 1$ there is an inflation-output trade-off (see figure 4). The 'divine coincidence' that there is no such trade-off in the standard model is broken by a real imperfection. Without such a real imperfection monetary policy is trivial in response to a variety of shocks (Woodford (2010)): labour supply, demand/prefence shocks and technology shocks. The policy maker simply ensures that the nominal interest rate matches the path of the real interest rate consistent with the flexible price equilibrium (the target real interest rate) and the economy will replicate that flexible price equilibrium with an output gap and inflation rate of zero. How the policy maker responds to this dynamic externality depends on how future consumption is valued relative to consumption today, parametrised by $\sigma$. When $\sigma > 1$ households place more value on output growth. The planner achieves this by returning output to steady state more quickly which entails returning skills to steady state more quickly. This has the cost of inducing deflation via the marginal cost channel as skills recover more quickly; but this cost is more than compensated for by the faster output growth. The reverse holds when $\sigma < 1$. When $\sigma = 1$ the planner chooses to have output match the path of technology. This entails that the current level of employment is sufficient to produce this level of output as exogenous productivity changes exactly proportionally with the required amount these workers must produce. This requires no change in hours and so no change in the level of skills.

A cost-push shocks creates inflationary pressure as firms raise their desired mark-ups (see Figure 5). This forces the planner to face a trade-off between stabilising the output gap and inflation even in the Basic New Keynesian model. The optimal response is price level targeting with initial inflation and a small subsequent deflation. The presence of positive inflation permits a smaller negative output gap than if the policy maker cared only about inflation. As is well known, to make this gap smaller the policy maker must tolerate higher inflation (Gali
1.4. The optimal response to shocks

Figure 1.5: Optimal policy under a cost-push shock changes with steady state

The top set of graphs relates to the solution around the efficient steady state, the bottom to the distorted steady state.

(2008)). How does LBD affect these results? Near the efficient steady state the optimal response is broadly similar with LBD: price-level targeting is achieved by engineering a hump-shaped drop in output (see figure 5). There are additional costs to this choice of output in the case of LBD whereby output falls further as skills depreciate. However these costs are not large enough to make accepting larger swings in inflation worthwhile. An interesting pattern emerges when we consider the response of the policy maker who operates near the distorted steady state. When \( \tau < \tau^e \) the behaviour of the target level of output changes to take account of the precommitments made in the steady state by the policy maker operating from a 'timeless perspective' (see equation 1.39). These precommitments are captured in \( \bar{\phi}_1 \) and \( \bar{\phi}_2 \). The larger the steady state gap between output and its efficient level the larger these weights become. These weights bind the policy maker to make decisions taking into account the impact of changes in output on the marginal costs and revenues of firms that operate near \( \bar{Y} \). Marginal revenue is proportional to \( f_Y(\bar{Y}) \) and marginal costs are described by \( k_Y(\bar{Y}, \bar{X}, \bar{I}) \) and \( k_X(\bar{Y}, \bar{X}, \bar{I}) \). \( f_Y(\bar{Y}) \) is the same in both the BNKM and LBD models. The differences in results are driven by the behaviour of marginal costs. When \( \bar{Y} = 1 \) marginal costs are the same in the standard and LBD model since \( \bar{X} = 1 \), as can be seen from the steady state result (1.36). When \( \bar{Y} > 1 \) marginal costs are lower in the LBD model due to the beneficial effects of higher skills on productivity however, \( \bar{Y} < 1 \) induces low levels of skills pushing marginal costs above those in the standard model (see Figure 6). Due to this endogenous productivity channel steady state output is much lower when \( \tau < \tau^e \) with LBD than without (see figure
1.4. The optimal response to shocks

Figure 1.6: Marginal cost behaviour and steady state output

1. For these monopolistically competitive firms a drop in output lowers marginal costs and raises marginal revenue. Thus the planner aims to accommodate this drop in output more in the distorted steady state. For this reason $Y_t^*$ falls with the cost-push shock near a distorted steady state but is unchanged near the efficient steady state. The drop in $Y_t^*$ combined with a similar path for $Y_t$ as in the efficient steady state case implies a smaller negative output gap. This mechanism is large with LBD and inconsequential without it.

Shocks to preferences over consumption and hours worked do not induce any inflation-output trade-off in the BNKM. As with a technology shock this ‘divine coincidence’ result does not hold with LBD. Moreover the nature of the shocks makes offsetting the influence of skills more challenging than the case of technology shocks (where output matching movements in exogenous technology result in no change in employment). Here the planner wishes to make output fall in response to a negative shock to consumption or labour supply (see Figures 1.10 & 1.11) since output is less valued by households. In the case of the preference shock to consumption the results are similar to those of a technology shock (with $\sigma < 1$) with positive inflation and a larger output decline. The policy maker now faces a negative output gap to achieve price level targeting.

The labour supply shock is slightly different in that deflation is experienced on impact with subsequent inflation (the reverse of the consumption shock experience). Why? The Ramsey plan involves engineering a small positive skills gap as skills value in reducing the disutility of labour is now higher. The same effect applies to the value of skills in controlling firms marginal costs as hiring labour

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From equation (1.29) we can see that $\frac{\partial^2 U(Y_t, X_t, \Delta_t, x_t)}{\partial X_t \partial c_t} > 0$
1.5 Welfare

Welfare comparisons are based on steady state output changes required to make the household indifferent between experiencing the shock and enjoying that level of output: a value for $\zeta$ satisfying:

$$\sum_{t=t_0}^{\infty} \beta^{t-t_0} U(\bar{Y}(1-\zeta), \bar{X}, 1; \bar{\xi}) = E_0 \sum_{t=t_0}^{\infty} \beta^{t-t_0} U(Y_t, X_t, \Delta_t; \xi_t)$$

Table 1.1: Welfare with $\tau = \tau^e$

<table>
<thead>
<tr>
<th>Model / Shocks</th>
<th>Cost-push</th>
<th>Technology</th>
<th>Preference</th>
<th>Labour Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Standard NK model</td>
<td>0.0078</td>
<td>0.2687</td>
<td>0.3541</td>
<td>0.2531</td>
</tr>
<tr>
<td>(2) LBD $\mu = 0.15$</td>
<td>0.0138</td>
<td>0.2827</td>
<td>0.3726</td>
<td>0.4465</td>
</tr>
<tr>
<td>(3) LBD $\mu = 0.25$</td>
<td>0.0182</td>
<td>0.2926</td>
<td>0.3858</td>
<td>0.5864</td>
</tr>
</tbody>
</table>

(b) Relative to Standard NK model

<table>
<thead>
<tr>
<th>Model / Shocks</th>
<th>Cost-push</th>
<th>Technology</th>
<th>Preference</th>
<th>Labour Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>(2) LBD $\mu = 0.15$</td>
<td>1.7740</td>
<td>1.0518</td>
<td>1.0524</td>
<td>1.7644</td>
</tr>
<tr>
<td>(3) LBD $\mu = 0.25$</td>
<td>2.3447</td>
<td>1.0889</td>
<td>1.0898</td>
<td>2.3172</td>
</tr>
</tbody>
</table>

The results are presented in Table 1 & Table 2, with absolute losses in percent of steady state output. The baseline calibration in Table 3 is used. As may be expected the welfare costs of introducing an additional imperfection to the NK model raises the costs of business cycles. This can be ascribed to the fact that the divine coincidence no longer holds meaning that additional costs need to be incurred either in terms of higher inflation or output volatility. The increase in the welfare cost is most notable for cost push and labour supply shocks.
1.6 Optimal Simple Rules

Describing monetary policy in terms of Taylor rules is useful for (at least) two reasons: first, it directly and simply explains the trade-off between output and inflation in terms of interest rate policy and, second, it provides a comparison to a large body of empirical work estimating these Taylor Rules for central banks (Taylor (1993) and Clarida et al. (1997)).

To find the optimal weights for inflation and output in the interest rule for the policy maker I solve the competitive equilibrium described in the last paragraph of section 2 where monetary policy is described by the Taylor Rule $i_t = \gamma_y \left( y_t / \bar{y} \right) + \gamma_\pi \pi_t$ (following Leith et al. (2012)). This means adding the Taylor rule to the system comprised of the households optimality conditions (1.3) and (1.4), the law of motion for skills (1.11), the definition of the forward looking measures of marginal costs and revenue for firms (1.23) and (1.24), the firms first order condition summarised in aggregate supply relation (1.26), the law of motion for price dispersion (1.28) and the description for the exogenous stochastic processes $\xi_t$. I assume that an optimal production subsidy is in place, thus the model is solved around the efficient steady state. The optimal weights $\gamma_y$ and $\gamma_\pi$ are found from a minimising the welfare loss described in (1.50) by solving the model at each point on a grid of 300 values for $\gamma_y \in [0, 10]$ and $\gamma_\pi \in (1, 2]$\textsuperscript{23}. This is done for each value of the parameter $\mu$ governing the strength of the feedback from employment to skills ranging from a low of 0.1 to a high of 0.4. Welfare losses are measured as an average all shocks, i.e. technology, preference, labour supply and cost-push. Thus these weights are ones that are optimal from the perspective of a policy maker concerned equally with each of these shocks.

For low levels of the learning parameter, $\mu \leq 0.11$, $\gamma_y / \gamma_\pi \to 0$ as in the BNKM. For increases in the learning parameter in the range $\mu \in [0.11, 0.27]$, it is optimal for the policy maker to put a higher (but small) weight on output variations ($\gamma_y / \gamma_\pi = 2\%$). High dependence of skills on past hours worked, $\mu \in [0.28, 0.33]$ substantially raise the value of output fluctuations to the policy maker where the weight on output is half that of inflation. For very high levels of learning, $\mu > 0.34$, output variations matter almost as much as movements in inflation with $\gamma_y / \gamma_\pi = 87\%$.

This exercise suggests that for learning by doing to have a quantitatively significant impact on the operating policy of a central bank that uses a Taylor Rule, learning needs to be much stronger than that measured by Chang et al. (2002) for the US economy. However, it may well be that this learning effect is stronger

\textsuperscript{23}Repeated simulations with $\gamma_\pi \in (1, 10]$ showed that the optimal weight on inflation is always near 1 when $\mu > 0.11$ and for $\mu < 0.11$, $\gamma_y / \gamma_\pi \to 0$. Since the latter result is preserved with a smaller grid I use it to gain better accuracy with fewer simulations over the range of interest for $\mu$. 

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in other economies and is plausibly increasing in relevance as exogenous technical progress leads to higher levels of depreciation in worker skills.

![Figure 1.7: The optimal weight on output relative to inflation based on a Taylor Rule](image)

Optimal weights $\gamma_y$ and $\gamma_\pi$ are found from a minimising the welfare loss described in (1.50) by searching across a grid of 300 values for $\gamma_y \in [0, 10]$ and $\gamma_\pi \in (1, 2]$. For each value these weights the competitive equilibrium described in the last paragraph of section 2 is solved where monetary policy is described by the Taylor Rule $i_t = \gamma_y \left( \frac{y_t}{\bar{y}} \right) + \gamma_\pi \pi_t$. This is done for values of the learning feedback parameter from employment to skills, $\mu \in [0.1, 0.4]$. This requires 9300 simulations.

1.7 Conclusion

This paper studies the implications for monetary policy of introducing learning-by-doing in production into an otherwise standard Basic New Keynesian model. The time-consistent Ramsey policies are studied in the neighbourhood of the distorted and efficient steady state. The presence of learning introduces two new channels through which output matters for the policy maker. Firstly a marginal cost channel whereby changes in output today lead to proportionate changes in worker productivity tomorrow. Secondly, the presence of learning directly affects the disutility of labour creating an incentive for the policy maker to avoid raising this disutility by letting skills depreciate. The presence of this LBD externality breaks the 'divine coincidence' result, that by stabilising inflation the output gap will automatically be closed, for a variety of shocks that are considered impor-
1.7. Conclusion

tant in explaining business cycles. I find that skills induce a small increase in the importance of the output gap under a cost-push shock but only for the (more realistic case) of a distorted steady state. The reason for this is due to an interaction between time-consistent policy choices and significant steady state distortions to output due to the presence of LBD on productivity. The welfare costs of business cycles are shown to be significantly larger when learning effects are strong even under the optimal policy.

The approach to optimal monetary policy pursued here, following Woodford (2010), allows for convenient study of different steady states by avoiding the need to derive a purely quadratic approximation to the representative household’s utility. This approach may be fruitful in studying time-consistent policy choices where steady state distortions can be large, due to significant real imperfections. Korinek (2010) and Benigno and Fornaro (2012) have shown that the results of the growth literature where imported technology drives learning externalities can have large welfare effects in an open economy setting. Neither of these studies have nominal rigidities and thus do not study the implications for monetary policy in the context of an open economy. Building on these results drawing on the framework applied here to study monetary policy for the small open economy may be a fruitful avenue for further research.
1.8 Appendix I: Additional Figures & Tables

1.8.1 Figures

All simulations presented here have used the baseline calibration presented in Table 3.

1.8.1.1 Optimal Response to shocks with efficient subsidy

The efficient subsidy for the the standard model is $\tau^e = 1 - \frac{\epsilon}{\epsilon + 1}$ and $\tau^e = 1 - \left(\frac{\epsilon}{\epsilon + 1}\right) \left(\frac{1}{1 - \beta \mu}\right)$ for the model with learning by doing. These values are assumed in simulations below.

Figure 1.8: Cost-push shock with $\tau = \tau^e$
Figure 1.9: Technology shock with $\tau = \tau^c$
Figure 1.10: Labour Supply shock with $\tau = \tau^e$
Figure 1.11: Preference shock with $\tau = \tau^e$
1.8.1.2 Optimal Response to shocks with $\tau = 0$

Only the response to a cost-push shock is significantly altered thus the IRFs for the remaining shocks are not reported here.

Figure 1.12: Cost-push shock with $\tau = 0$
1.8.2 Tables

Table 1.2: Baseline Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.99</td>
<td>Consistent with 4% annual interest rate</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.33</td>
<td>Consistent with a labour share of $\frac{2}{3}$</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.8</td>
<td>Attanasio (1999). Unless stated otherwise.</td>
</tr>
<tr>
<td>$\psi$</td>
<td>1</td>
<td>Dyrdal et al. (2012)</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>11</td>
<td>Leith et al. (2012)</td>
</tr>
<tr>
<td>$\omega$</td>
<td>0.75</td>
<td>Average price duration of 4 quarters, Klenow and Malin (2010)</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.15</td>
<td>Consistent with range in Chang et al. (2002). Unless stated otherwise.</td>
</tr>
<tr>
<td>$\phi_x$</td>
<td>0.79</td>
<td>Chang et al. (2002)</td>
</tr>
<tr>
<td>$\sigma_x^\mu$</td>
<td>0.0016</td>
<td>Smets and Wouters (2003)</td>
</tr>
<tr>
<td>$\sigma_A^x$</td>
<td>0.0071</td>
<td>Gali and Monacelli (2005)</td>
</tr>
<tr>
<td>$\sigma_H^x$</td>
<td>0.0166</td>
<td>Smets and Wouters (2003)</td>
</tr>
<tr>
<td>$\sigma_C^x$</td>
<td>0.0028</td>
<td>Smets and Wouters (2003)</td>
</tr>
<tr>
<td>$\rho_\mu$</td>
<td>0.8</td>
<td>Leith et al. (2012)</td>
</tr>
<tr>
<td>$\rho_A$</td>
<td>0.91</td>
<td>Adolfson et al. (2007a)</td>
</tr>
<tr>
<td>$\rho_H$</td>
<td>0.93</td>
<td>Smets and Wouters (2003)</td>
</tr>
<tr>
<td>$\rho_C$</td>
<td>0.88</td>
<td>Smets and Wouters (2003)</td>
</tr>
</tbody>
</table>

Shocks are assumed to be uncorrelated and follow an AR(1) process with persistence $\rho$ and standard deviation $\sigma^x$. The model is calibrated to quarterly frequency.
1.9 Appendix II: Derivations

1.9.1 Derivation of Aggregate Supply relation (1.26)

The firm able to reset their price at time $t$ will solve the following problem:

$$\max_{\{P_t(i)\}, t=0} \sum_{T=t}^{\infty} \omega^{T-t} Q_{t,T} \Pi(P_t(i), P_T, Y_T, X_T; \xi_T)$$  \hspace{1cm} (1.51)

$\xi_T$ refers to the entire collection of shocks that affect firms pricing decision, 
$\xi_T' = \begin{bmatrix} A_T & \xi_T^C & \xi_T^H & \mu_T \end{bmatrix}.$  $\mu_T$ refers to a shock to firms desired steady state mark-up. $Q_{t,T}$ is the value placed on nominal profits returned to the household $T$ periods hence:

$$Q_{t,T} = \beta^{T-t} \frac{u_C(C_T, \xi_T^C)}{u_C(C_t, \xi_t^C)} P_t = \beta^{T-t} \left( \frac{Y_t \xi_T^C}{Y_T \xi_t^C} \right)^{\frac{1}{\sigma}} \frac{P_t}{P_T}$$  \hspace{1cm} (1.52)

For the $i^{th}$ firm nominal profits in period $T$ are simply nominal revenues less costs:

$$(1 - \tau)P_t(i) \left( \frac{P_t(i)}{P_T} \right)^{-\epsilon} Y_T - W_T N_T(i)$$  \hspace{1cm} (1.53)

Substituting $W_T N_T(i)$ using (1.15), (1.16) and (1.4), the firms problem is:

$$\max_{\{P_t(i)\}, t=0} \sum_{T=t}^{\infty} (\omega \beta)^{T-t} \left( \frac{Y_t}{\xi_T^C} \right)^{\frac{1}{\sigma}} \left\{ (1 - \tau)P_t(i)^{1-\epsilon} \left( \frac{P_t}{P_T} \right)^{\epsilon} \left( \frac{Y_T}{A_T} \right)^{(1+\chi)} \frac{\Delta_T^{\frac{1}{\psi}}}{X_T^{(1+\frac{1}{\psi})}} \frac{1}{(\xi_T^H)^{\frac{1}{\psi}}} \right\}$$

The first order conditions for profit maximisation is:

$$\sum_{T=t}^{\infty} (\omega \beta)^{T-t} \left\{ (1 - \tau) \left( \frac{P_t(i)}{P_T} \right)^{-\epsilon(1+\eta)} \left( \frac{Y_T}{A_T} \right)^{(1+\chi)} \frac{\Delta_T^{\frac{1}{\psi}}}{X_T^{(1+\frac{1}{\psi})}} \frac{1}{(\xi_T^H)^{\frac{1}{\psi}}} \right\}$$

$$\left( \frac{\epsilon}{\epsilon - 1} \right) (1 + \eta) \left( \frac{P_t(i)}{P_T} \right)^{-\epsilon(1+\eta)-1} \left( \frac{Y_T}{A_T} \right)^{(1+\chi)} \frac{\Delta_T^{\frac{1}{\psi}}}{X_T^{(1+\frac{1}{\psi})}} \frac{1}{(\xi_T^H)^{\frac{1}{\psi}}} \right\}$$

Which can be rearranged as, after imposing $P_t(i) = P_t^*$:
1.9. Appendix II: Derivations

\[ E_t \sum_{T=t}^{\infty} (\omega \beta)^{T-t} \left\{ \left( 1 - \tau \right) \left( \frac{P_t^*}{P_t} \right)^{\epsilon - \eta} \left( \frac{P_t}{P_t} \right)^{1 - \frac{1}{\sigma}} \left( \frac{\xi T}{\xi t} \right)^{\frac{1}{\sigma}} \right\} = 1 \]

Multiplying by \( \left( \frac{P_t^*}{P_t} \right)^{-(1+\epsilon)} \) and rearranging we have:

\[ \left( \frac{P_t^*}{P_t} \right)^{-(1+\epsilon)} = \frac{E_t \sum_{T=t}^{\infty} (\omega \beta)^{T-t} \left\{ \left( 1 - \tau \right) Y_t^{(1 - \frac{1}{\sigma})} \left( \xi T \right)^{\frac{1}{\sigma}} \right\} \left( \frac{P_t}{P_t} \right)^{(\epsilon-1)}}{E_t \sum_{T=t}^{\infty} (\omega \beta)^{T-t} \left\{ \left( \frac{\varepsilon}{\varepsilon - 1} \right) (1 + \eta) \left( \frac{Y_t}{X_t} \right)^{(1+\chi)} \Delta T \left( \xi T \right)^{\frac{1}{\sigma}} \frac{1}{X_t} \left( \xi T \right)^{\frac{1}{\sigma}} \right\} \left( \frac{P_t}{P_t} \right)^{(1+\eta)}} \]

Which rearranged gives equation (1.26):

\[ \left( \frac{P_t^*}{P_t} \right) = \left( \frac{F_t}{K_t} \right)^{-\frac{1}{1+\epsilon}} \]  

1.9.2 Linearised New Keynesian Phillips Curve (1.27)

The linearisation is around the zero inflation steady state described by the Ramsey solution (fully described in 8.3 of this appendix). The law of motion for the price dispersion (equation 1.28), the forward looking relations \( F_t, K_t \) (equations 1.23, 1.24) and the aggregate supply relation (equation 1.26); can be linearised as:

\[ \hat{\Delta}_t = \omega \hat{\Delta}_{t-1} \]  

\[ F_t = (1 - \omega \beta) \left[ f_0 \hat{Y}_t + f_1 \hat{\xi}_t \right] + \omega \beta E_t \left[ (\epsilon - 1) \hat{\Pi}_{t+1} + \hat{F}_{t+1} \right] \]  

\[ K_t = (1 - \omega \beta) \left[ k_0 \hat{Y}_t + k_1 \hat{X}_t + k_2 \hat{\Delta}_t + k_3 \hat{\xi}_t \right] + \omega \beta E_t \left[ (\epsilon + \eta) \hat{\Pi}_{t+1} + \hat{K}_{t+1} \right] \]

\[ \hat{\Pi}_t = \frac{1 - \omega}{\omega} \frac{1}{1 - \epsilon(1-\epsilon)} (\hat{K}_t - \hat{F}_t) \]  

From (1.57) it can be seen that if \( \Delta_{t_{0-1}} = O(||\xi T||) \) then \( \hat{\Delta}_t = O(||\xi T||) \forall t \). 

1.9.3 Solution to Ramsey problem in section 3.2

The Ramsey problem outlined above can be described by the Lagrangian:

\[ \max_{\{F_t, K_t, \Delta_t, \Pi_t, Y_t\}} L_{t_0} = E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} L(Y_t, X_t, F_t, K_t, \Pi_t, \Delta_t; \theta_t, \phi_t, \xi_t) \]  

48
Where

\[ L(Y_t, X_t, F_t, K_t, \Pi_t, \Delta_t; \theta_t, \phi_t, \xi_t) = U(Y_t, X_t, \Delta_t; \xi_t) + \phi_{1,t} [F_t - f(Y_t; \xi_t) - \omega \beta E_t [\Pi_{t+1}^{(1+\eta)} F_{t+1}] + \phi_{2,t} \left[ K_t - k(Y_t, X_t, \Delta_t; \xi_t) - \omega \beta E_t [\Pi_{t+1}^{(1+\eta)} K_{t+1}] \right] + \theta_t \left[ \Delta_t - \omega \Delta_{t-1} \Pi_t^{(1+\eta)} - (1 - \omega) \left( \frac{1 - \omega \Pi_t^{(1+\eta)} - 1}{e-1} \right) \right] + \phi_{3,t} \left[ \frac{1 - \omega \Pi_t^{(1+\eta)} - 1}{e-1} - (F_t - K_t) \right] \]

Or equivalently

\[ L(Y_t, X_t, F_t, K_t, \Pi_t, \Delta_t; \theta_t, \phi_t, \xi_t) = U(Y_t, X_t, \Delta_t; \xi_t) + \theta_t \left[ \Delta_t - \omega \Delta_{t-1} \Pi_t^{(1+\eta)} - (1 - \omega) \left( \frac{1 - \omega \Pi_t^{(1+\eta)} - 1}{e-1} \right) \right] \]

\[ \phi_{1,t} [F_t - f(Y_t; \xi_t)] - \omega \phi_{1,t-1} \left[ \Pi_t^{(1+\eta)} F_t \right] + \phi_{2,t} \left[ K_t - k(Y_t, X_t, \Delta_t; \xi_t) \right] - \omega \phi_{2,t-1} \left[ \Pi_t^{(1+\eta)} K_t \right] + \phi_{3,t} \left[ K_t \left( \frac{1 - \omega \Pi_t^{(1+\eta)} - 1}{e-1} \right) - F_t \right] \]

Where the multipliers \( \phi_{1,t_0-1}, \phi_{2,t_0-1} \) will capture the precommitments made at \( t_0 \) i.e. they will be the values consistent with the steady state solution of the model (which holds for \( t_0 - 1 \)) under the same constraints the Ramsey planner faces for \( t > t_0 \).

The first-order conditions (FOCs) of the above problem when the Ramsey planner chooses \( \{F_t, K_t, \Delta_t, \Pi_t, Y_t\} \) are:

\[ \frac{\partial L_{t_0}}{\partial Y_t} = 0: \ U_Y(Y_t, X_t, \Delta_t; \xi_t) + \beta \{U_X(Y_{t+1}, X_{t+1}, \Delta_{t+1}; \xi_{t+1}) - \phi_{2,t+1} k_X(Y_t, X_t, \Delta_t; \xi_t+1)\} \frac{\partial X_{t+1}}{\partial Y_t} = \]

\[ + \phi_{1,t} f_y(Y_t; \xi_t) + \phi_{2,t} k_y(Y_t, X_t, \Delta_t; \xi_t) \quad (1.62) \]

\[ \frac{\partial L_{t_0}}{\partial \Delta_t} = 0: \ U_{\Delta}(Y_t, X_t, \Delta_t; \xi_t) + \theta_t + \beta \{U_X(Y_{t+1}, X_{t+1}, \Delta_{t+1}; \xi_{t+1}) - \phi_{2,t+1} k_X(Y_t, X_t, \Delta_t; \xi_t+1)\} \frac{\partial X_{t+1}}{\partial \Delta_t} = \]

\[ + \phi_{2,t} k_{\Delta}(Y_t, X_t, \Delta_t; \xi_t) + \theta_{t+1} \omega \Pi_{t+1}^{(1+\eta)} \]

\[ (1.63) \]

\[ \frac{\partial L_{t_0}}{\partial K_t} = 0: \ \phi_{3,t} \frac{\omega (1+\eta)}{1-\omega} P(\Pi_t) \left( \frac{(1+\eta)(1)}{e-1} \right) \Pi_t^{(1+\eta)-2} K_t + \phi_{1,t-1} \omega (1-1) \Pi_t^{(1+\eta)-2} F_t + \phi_{2,t-1} \omega (1+\eta) \Pi_t^{(1+\eta)-1} K_t \]

\[ + \omega (1+\eta) \Delta_t \Pi_t^{(1+\eta)-1} - \omega \epsilon (1+\eta) P(\Pi_t) \left( \frac{(1+\eta)(1)}{e-1} \right) \Pi_t^{(1+\eta)-2} = 0 \]

\[ (1.64) \]

\[ \frac{\partial L_{t_0}}{\partial F_t} = 0: \ \phi_{3,t} P(\Pi_t) \left( \frac{(1+\eta)(1)}{e-1} \right) + \phi_{2,t} - \phi_{2,t-1} \omega \beta \Pi_t^{(1+\eta)} = 0 \]

\[ (1.65) \]
\[
\frac{\partial L_{t_0}}{\partial F_t} = 0 : -\phi_{3,t} + \phi_{1,t} - \phi_{1,t-1}\omega\Pi_t^{e-1} = 0 \quad (1.66)
\]

Where \( P(\Pi_t) = \left( \frac{1-\omega\Pi_t^{e-1}}{1-\omega} \right) \). In order to study the optimal response to shocks I linearise these conditions around the optimal steady state. This is the steady state associated with the above 5 FOCs and the 4 constraints. The optimal steady state is characterised by \( \{ \bar{F}, \bar{K}, \bar{\Delta}, \bar{\Pi}, \bar{Y}, \bar{X}, \bar{\theta}, \bar{\phi} \} \) that solves these 9 equations when \( \xi_t = \bar{\xi} \). This steady state includes the following conditions:

\[
f(\bar{Y}) = k(\bar{Y}, \bar{X}, \bar{\Delta}) \iff \frac{v_h(\bar{Y}, \bar{X}, \bar{\Delta})}{u_c(\bar{Y})} = \frac{1 - \bar{\tau}}{\bar{\epsilon}^{-1}} \iff \bar{Y} = \left( \frac{1 - \bar{\tau}}{\bar{\epsilon}^{-1}(1+\eta)} \right)^{\frac{1-\phi x + \mu}{\gamma(1+\phi x + \mu)(1+\eta)}} \quad (1.67)
\]

\[
\bar{X} = \bar{Y}^{\frac{\mu(1+\gamma)}{1-\phi x + \mu}} \quad (1.68)
\]

\[
\bar{\Delta} = \bar{\Pi} = 1 \quad (1.69)
\]

\[
\bar{K} = \bar{F} = (1 - \omega\beta)^{-1}f(\bar{Y}) \quad (1.70)
\]

\[
\bar{\phi}_2 = -\bar{\phi}_1 = \frac{U_Y + \beta U_X \frac{\partial X}{\partial Y}}{k_Y - f_Y + \beta \frac{\partial X}{\partial Y} k_X} \quad (1.71)
\]

\[
\bar{\theta} = \frac{U_\Delta + \beta U_X \frac{\partial X}{\partial \Delta}}{1 - \omega} + \frac{(k_\Delta + \beta k_X \frac{\partial X}{\partial \Delta}) (U_Y + \beta U_X \frac{\partial X}{\partial Y})}{1 - \omega (k_Y - f_Y + \beta k_X \frac{\partial X}{\partial Y})} \quad (1.73)
\]

These are the steady state levels that the Ramsey planner commits to at \( t = t_0 \) which bind her for future periods.

### 1.9.4 Derivation of \( \pi - y_t^\sigma \) trade-off of Section 3.3.2

The FOC for output in the Ramsey problem (1.32) is:

\[
\frac{\partial \xi_{t_0}}{\partial Y_t} = 0 : U_Y(Y_t, X_t, \Delta_t; \xi_t) + \beta E_t \{ U_X(Y_{t+1}, X_{t+1}, \Delta_{t+1}; \xi_{t+1}) - \phi_{2,t+1}k_X(Y_t, X_t, \Delta_t; \xi_t) \} \frac{\partial X_{t+1}}{\partial Y_t} = \\
+ \phi_{1,t} f_y(Y_t; \xi_t) + \phi_{2,t} k_y(Y_t, X_t, \Delta_t; \xi_t) \quad (1.74)
\]

\^24Thus this is not the optimal steady state of the social planner, but only the optimal steady state from the perspective of a planner that must take actions subject to competitive equilibrium.
1.9. Appendix II: Derivations

Linearised around the steady state described in Section 8.3 of this appendix this becomes:

\[
\bar{Y} \Omega_{y} \dot{Y} + \bar{X} \Omega_{x} \dot{X} + \Omega_{\Delta} \dot{\Delta} + \Omega_{\xi} \dot{\xi}
\]

\[
\beta E_t \left\{ \bar{Y} \gamma_{y} Y_{t+1} + \bar{X} \gamma_{x} X_{t+1} + \gamma_{\Delta} \Delta_{t+1} + \gamma_{\xi} \xi_{t+1} \right\}
\]

\[
(k_{Y} - f_{Y}) \tilde{\phi}_{1,t} - \beta E_t k_x \tilde{\phi}_{2,t+1} = 0 \quad (1.75)
\]

Where for each variable \( \bar{Z}_t \equiv \ln Z_t - \ln \bar{Z} \) and \( \bar{Z}_t = Z_t - \bar{Z} \). The equation defining the target level of output is:

\[
U_Y (Y_{t}^*, X_{t}^*, 1; \xi_t) + \beta E_t \left\{ U_X (Y_{t+1}^*, X_{t+1}^*, 1; \xi_{t+1}) - \tilde{\phi}_2 k_x (Y_{t+1}^*, X_{t+1}^*, 1; \xi_{t+1}) \right\} \frac{\partial X_{t+1}^*}{\partial Y_{t}^*} =
\]

\[
\tilde{\phi}_1 f_y (Y_t^*; \xi_t) + \tilde{\phi}_2 k_y (Y_t^*; X_t^*, 1; \xi_t) \quad (1.76)
\]

Linearising this gives:

\[
U_{t_0} = E_0 \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ U(Y_t, X_t, \Delta_t; \xi_t) \right\}
\]

\[
\bar{Y} \Omega_{y} \dot{Y}_t + \bar{X} \Omega_{x} \dot{X}_t + \Omega_{\xi} \dot{\xi} + \beta E_t \left\{ \bar{Y} \gamma_{y} Y_{t+1} + \bar{X} \gamma_{x} X_{t+1} + \gamma_{\xi} \xi_{t+1} \right\} = 0 \quad (1.77)
\]

The coefficients in both (1.75) and (1.77) are defined by:

\[
\Omega_{y} = U_{yy} + \beta \frac{\partial^2 X}{\partial Y^2} (U_x - \tilde{\phi}_2 k_x) - \tilde{\phi}_1 (f_{yy} - k_{yY}) < 0; \quad \Omega_{x} = U_{yx} + \beta \frac{\partial^2 X}{\partial Y \partial X} (U_x - \tilde{\phi}_2 k_x) + \tilde{\phi}_1 k_{yY} > 0
\]

\[
\Omega_{\xi} = U_{\xi \xi} + \beta \frac{\partial^2 X}{\partial Y \partial \xi} (U_x - \tilde{\phi}_2 k_x) - \tilde{\phi}_1 (f_{y\xi} - k_{\xi})
\]

\[
\gamma_{y} = \frac{\partial X}{\partial Y} (U_{yy} - \tilde{\phi}_2 k_{yY}) > 0; \quad \gamma_{x} = \frac{\partial X}{\partial Y} (U_{yx} - \tilde{\phi}_2 k_{yX}) < 0; \quad \gamma_{\xi} = \frac{\partial X}{\partial Y} (U_{\xi \xi} - \tilde{\phi}_2 k_{\xi X})
\]

The signs for these coefficients assume the baseline calibration of the model given in Table 3. (1.75) and (1.77) can be combined into a statement in terms of the output gap \( y_t^* = Y_t - \bar{Y}_t \) and the skills gap as \( x_t^* = X_t - X_t^* \):

\[
\bar{Y} \Omega_{y} y_t^* + \bar{X} \Omega_{x} x_t^* + \beta E_t \left\{ \bar{Y} \gamma_{y} y_{t+1}^* + \bar{X} \gamma_{x} x_{t+1}^* - k_x \frac{\partial X}{\partial Y} \phi_{2,t+1} \right\}
\]

\[
= (f_{Y} - k_{Y}) \tilde{\phi}_{1,t} \quad (1.78)
\]

Where for simplicity I have again used the assumption that \( \Delta_{t_0-1} = O(||\xi^2||) \Rightarrow \Delta_t = O(||\xi^2||) \forall t \) and ignored price dispersion. For ease of notation call \( A_t = \bar{Y} \Omega_{y} y_t^* + \bar{X} \Omega_{x} x_t^* \) and \( B_{t+1} = \bar{Y} \gamma_{y} y_{t+1}^* + \bar{X} \gamma_{x} x_{t+1}^* \). Thus (1.78) can be written as:

\[
(f_{Y} - k_{Y}) \tilde{\phi}_{1,t} = \beta \left( k_x \frac{\partial X}{\partial Y} \right) E_t \tilde{\phi}_{1,t+1} + A_t + \beta E_t B_{t+1} \quad (1.79)
\]
Where I have used the fact that $\tilde{\phi}_{1,t} = -\tilde{\phi}_{2,t}$. Iterating on (1.79) and assuming no bubble solutions we have:

$$\tilde{\phi}_{1,t} = \frac{1}{(f_Y - k_Y)} E \sum_{j=0}^{\infty} \beta^j \left( \frac{k_X \partial X}{f_Y - k_Y} \right)^j \left\{ A_{t+j} + \beta E_t B_{t+j+1} \right\}$$ \hspace{1cm} (1.80)

Note that $\left( \frac{k_X \partial X}{f_Y - k_Y} \right) > 0$. Linearising the conditions (1.64), (1.65) and (1.66) yields the following relationship between inflation and the multiplier $\tilde{\phi}_{1,t}$:

$$\zeta_{\pi} \pi_t = \Delta \tilde{\phi}_{1,t}$$ \hspace{1cm} (1.81)

$$\zeta_{\pi} = -\frac{\bar{\theta}}{K} \epsilon(1 + \chi) - \frac{\omega}{1 - \omega}(1 + \epsilon \eta)$$

Taking first differences of (1.80) to replace the term $\Delta \tilde{\phi}_{1,t}$ in (1.81) we have:

$$\zeta_{\pi} \pi_t = \frac{1}{(f_Y - k_Y)} E_t \sum_{j=0}^{\infty} \beta^j \left( \frac{k_X \partial X}{f_Y - k_Y} \right)^j \left\{ \Delta A_{t+j} + \beta E_t \Delta B_{t+j+1} \right\}$$ \hspace{1cm} (1.82)

Which can be rearranged using the definitions of $A_t$ and $B_{t+1}$ as equation (1.46) in the main text:

$$\zeta_{\pi} \pi_t = \lambda E_t \sum_{j=0}^{\infty} \beta^j \left( \frac{k_X \partial X}{f_Y - k_Y} \right)^j \left\{ \bar{Y}(\Omega_Y \Delta y^Y_{t+j} + \beta \gamma_Y \Delta y^Y_{t+j+1}) + \bar{X}(\Omega_X \Delta x^X_{t+j} + \beta \gamma_X \Delta x^X_{t+j+1}) \right\}$$ \hspace{1cm} (1.83)

$$\lambda = \frac{1}{(f_Y - k_Y)} < 0$$

---

25. This follows from a linearisation of (1.64), (1.65) and (1.66) as proved by Woodford (2010), page 58.
Chapter 2

Noisy news and exchange rates: a SVAR approach

Abstract

This paper introduces noisy news shocks into a model of exchange rate determination to measure the impact of these shocks using a SVAR. Agents in the foreign exchange market make decisions with imperfect information about economic fundamentals driving interest rate differentials between countries in that they must rely on a noisy signal of future interest rates. I apply the framework to the USD/GBP nominal exchange rate for the period 1986-2013. Results show that noisy-news explains approximately one fifth of the forecast error variance in the nominal exchange rate, with noise accounting for double (12%) that of news (6%). A historical decomposition of the exchange rate indicates that noise shocks are especially important during periods of changing monetary policy e.g. the 1990 easing and 2001 tightening of U.S. monetary policy and the unconventional monetary policies surrounding the financial crisis of 2008.

JEL classification: C32, F31, F41, G15, D84.

Keywords: Exchange rates, SVAR, News, Noise, nonfundamentalness, invertibility.

2.1 Introduction

A large empirical literature exists on explaining the movements in exchange rates based on shocks to macroeconomic fundamentals (see, for example, Eichenbaum and Evans (1993), Chari et al. (2002a) and Scholl and Uhlig (2008)). There is, however, strong evidence that exchange rates are not driven by the same shocks that drive other macroeconomic variables: exchange rates lack the cyclical pattern of macro variables (Baxter and Stockman (1989)), have a surprisingly weak relationship with those variables past and present values (Flood and Rose (1995)) and, famously, are forecast more reliably by a random walk than a model based on economic fundamentals (Meese and Rogoff (1983) and Rossi (2013)). Recent theoretical work has addressed this exchange rate disconnect puzzle by focusing on the kind information that agents use to make decisions in asset markets and in particular on news about macroeconomic conditions.

Duarte and Stockman (2005) deliver a model where news shocks lead agents to rationally update their beliefs about risk premia leading to exchange rate beh-
behavior that is independent of changes in macro variables. Ilut (2012), building on the models of Gourinchas and Tornell (2004) and Baccetta and Van Wincoop (2006), models agents as ambiguity averse investors who receive noisy news about productivity. His model is consistent with the empirical regularities of delayed appreciation following an interest rate shock, a higher likelihood of large rapid depreciation or “crash risk”, and momentum trading profits. While news based models of exchange rates are theoretically appealing they entail two difficulties in identifying news shocks in the data.

News entails that agents decisions depend on an unobservable state variable - the time lapse from when news arrived to when the shock is realised. This anticipation will be reflected in the data that agents generate. For example, pound sterling may appreciate prior, and respond less on impact, to an increase in U.K. interest rates if agents receive news. An econometrician using the set of observable macroeconomic variables, such as interest rates, exchange rates, GDP, prices and so on, will not be able to distinguish the anticipatory effects from the direct effects of shocks. This informational gap between the agents and the econometrician can be closed by increasing the information set of the econometrician using, for example, a Factor Augmented VAR (Bernanke et al. (2005)).

Noise in the signal of future shocks deepens the problem: agents make decisions without knowing whether innovations will be realized (news) or not (noise). Thus even the information set of the agents is not sufficient to identify news and noise shocks. The solution pursued in this paper uses the fact that agents learn in subsequent periods whether a signal received in the past is borne out (news) or not (noise) and correct their behaviour using the signal. This correction, reflected in observables, then distinguishes past news from noise. In this way news and noise shocks are identified using past, present and future values of observables.

In the structural VAR (SVAR) literature, identification problems due to anticipation are a subset of well known problems due to insufficient information leading to non-invertible moving average representations (Sargent and Hansen (1990), Lippi and Reichlin (1994) and Fernandez-Villaverde et al. (2007)). Identification problems due to noisy news have been addressed by resorting to estimation of a fully specified DSGE model as pursued by Blanchard et al. (2009) and Barsky and Sims (2012) who, respectively, find evidence of noise and news in driving the business cycle. However, a significant cost to this estimation strategy, especially given the short comings of DSGE models of exchange rate determination, is the sensitivity to modeling assumptions.

The aim of this paper is to study an alternative, less restrictive, scheme to identify noisy-news shocks affecting exchange rates. I pursue a SVAR using the a scheme recently proposed by Forni et al. (2013b) to identify the impact of noisy news about interest rates on the USD/GBP exchange rate. This identification procedure involves two steps. First, SVAR identification based on standard meth-
2.1. Introduction

methods to identify shocks using the agents information set\textsuperscript{27}. Second, restrictions are imposed on the relationship between the shocks identified in the first step and noise and news shocks. These restrictions are derived from a simple model of exchange rate determination under noisy news. The restrictions are expressed as dynamic rotations of the shocks identified in the first step. This dynamic rotation makes use of Blaschke factors (Lippi and Reichlin (1994)) which result in news and noise shocks that are linear combinations of the past, present and future values of shocks from the first step. The restrictions imposed by the theory are sufficient to identify the autoregressive relationship between news and interest rates at the first step allowing identification of the appropriate Blaschke factor to use in the second step\textsuperscript{28}. Forni et al. (2013b) developed this methodology to identify noisy news about total factor productivity and applied it to measure the role of noise shocks on stock prices in the U.S (Forni et al. (2013a)). Mertens and Ravn (2010) have used a related identification strategy to identify the effects of anticipated government spending shocks.

The model assumes agents have imperfect information on economic fundamentals that will determine the path of interest rates and rely on a noisy signal of their value. This implies the exchange rate can diverge from the level determined by the present value of economic fundamentals during periods where noise shocks predominate. However, as the interest rate differential is a function of lagged economic fundamentals, agents learn which signals represented news vs. noise once future interest rates are realized - ensuring the exchange rate returns to a level consistent with economic fundamentals.

The main contribution of this paper is to measure the contribution of shocks to interest rate expectations to movements in the USD/GBP over the October 1986 to February 2013 period. I find that a positive shock to expectations of the U.S. - U.K. interest rate differential results in delayed dollar appreciation, where agents purchase dollars but not in sufficiently large quantities that the entire appreciation takes place on impact. This is consistent with the Bayesian updating behaviour assumed in the simple model put forward in section 2. Using yields on 10 year bonds and contrary to previous SVAR studies using short term yields that do not include expectations of interest rates (for example, Eichenbaum and Evans (1993), Chari et al. (2002a); Scholl and Uhlig (2008)), I do not find evidence of delayed dollar appreciation in response to a shock to the interest rate differential but rather a depreciation on impact followed by a mild appreciation. Decomposing shocks to

\textsuperscript{27}This first step will suffer from the non-invertibility problems mentioned above given anticipation by agents. A factor augmented VAR is pursued in the empirical section to ensure sufficient information to identify shocks to the agents information set. This is tested using the procedure of Forni and Gambetti (2014) see section 5.2.

\textsuperscript{28}Blaschke factors are not unique. They can be used to illustrate the multiplicity of MA representations when, for example, anticipation is important see e.g. Canova (2007). However, the relationship between news and interest rates in the model above pins down a unique Blaschke factor after estimation of the first step.
interest rate expectations into news and noise, I find that that between 3-6% of the variation in the exchange rate is accounted for by news shocks, whereas 12-14% is due to noise present in that news (depending on identification scheme employed). News shocks induce a period of delayed dollar depreciation in the first 3 months after the shock followed by a protracted period of mild appreciation. The effect of noise shocks is the opposite, a period of dollar appreciation lasting approximately 3 months followed by mild depreciation. Noise shocks are largest during periods of changing monetary policy, when agents find it harder to gauge future interest rates and expectational errors are larger, e.g. the 1990 easing and 2001 tightening of U.S. monetary policy; and the unconventional monetary policies surrounding the financial crisis of 2008.

These findings relate to the theoretical exchange rate literature in a number of ways. Firstly, Reproducing the empirical regularity of a negative correlation between exchange rate depreciation and interest rate differentials, found in both single equation UIP tests and the SVAR literature documenting delayed dollar appreciation, is a key goal of theoretical models of exchange rate determination (Engel (2014)). Using yields on 10 year bonds, I find that delayed dollar appreciation is not robust to controlling for interest rate expectations suggesting that this goal may not be appropriate for models of exchange rates based on dispersed information or models using longer term interest rates. Secondly, New Keynesian models of exchange rate determination have struggled to replicate exchange rate dynamics and volatility without resorting to exogenous risk premium shocks to the UIP equation (see Chari et al. (2002b); Jung (2007) and Adolfson et al. (2007b)). Benigno et al. (2012) provide a recent attempt to endogenise these UIP risk shocks by incorporating time-varying uncertainty in monetary policy in a New Keynesian model. Through this mechanism they successfully replicate the negative correlation of exchange rate depreciation and short term interest rate differentials. The finding in the current paper, that the role of noise shocks is greatest during periods of changing monetary policy complements those findings and suggests an alternative mechanism (expectational errors) through which monetary policy uncertainty may influence exchange rates. Thirdly, this paper complements the findings of high frequency news studies, such as Clarida and Waldman (2008), showing that expectational errors, that are independent of changes in macroeconomic fundamentals, are important drivers of exchange rates. However, I document these effects at the frequency targeted by theoretical models of exchange rate determination (monthly or quarterly). Finally, the theoretical (and empirical) literature on exchange rates has focused almost exclusively on short term yields (3 months or less) however I find that modelling the role of longer term yields may be important for understanding exchange rate behaviour.

An earlier version of this paper employed short term interest rates finding that the impact of noisy news explained only 9.5% of the forecast error variance of changes in the USD/GBP where as using long rates doubles this figure. I am am grateful to an anonymous referee for suggesting...
2.2. Model

The remainder of the paper is organized as follows. Section 2 presents a simple model of exchange rate determination under imperfect information and relates it to the identification scheme employed in estimation. Section 3 describes the estimation of the SVAR. Section 4 describes the data, section 5 presents the empirical results and section 6 concludes.

2.2 Model

This section describes a simple, partial equilibrium model of exchange rate determination based on uncovered interest rate parity (UIP) and noisy exogenous interest rates. UIP requires that the expected return on assets abroad \( i^f_t + E_t \Delta s_{t+1} \) be the same as the return on assets of similar risk at home \( i^h_t \): \( i^h_t = i^f_t + E_t \Delta s_{t+1} \), where \( s_t \equiv USD/GBP \). This can be solved forward to link the value of the exchange rate to expected future interest rates:

\[
s_t = k - E_t \sum_{j=0}^{\infty} i_{t+j}
\]  

(2.1)

Where \( i_t \equiv i^h_t - i^f_t \). I take the long run level of the nominal exchange rate, as given, \( k = \lim_{T \to \infty} E_t s_{t+T+1} \). This formulation of UIP emphasises the role of the term structure of interest rates in determining exchange rates as recently emphasised by Anderson et al. (2010) and Sarno et al. (2012). Interest rates are a function of monetary policy which in turn is a function of economic fundamentals such as inflation, the output gap, past interest rates, technology, monetary policy shocks and so on. News, \( a_t \), about these economic fundamentals drive future interest rates: \( i_t = c(L)a_t \). Where \( c(L) \) is a lag polynomial and \( c(0) = 0 \) since \( a_t \) is a news shock. This restriction also ensures that agents don’t know \( a_t \) at time \( t \) via the observability of \( i_t \). Agents know the coefficients in \( c(L) \), however, they have imperfect information on the value of \( a_t \) and must rely on a noisy signal, \( z_t \) of this news: \( z_t = a_t + e_t \). Where \( a_t \sim iid N(0, \sigma_a^2) \) and \( e_t \sim iid N(0, \sigma_e^2) \) are news and noise shocks respectively. Agents make the best use of their limited information set, \( \Omega_t \), to predict the behavior of \( a_t \) by using the signal \( z_t \):

---

30 Purchasing power parity motivates the claim that \( \lim_{T \to \infty} E_t \left( s_{t+T+1} + p^f_{t+T+1} - p^h_{t+T+1} \right) = \eta \), a constant. Bimannual IMF 5 year ahead forecast data for 2008-2014 supports the claim that long run inflation forecast differences for the U.S-U.K. are approximately constant: the differences average 10 basis points with a standard deviation of 20 basis points. Supporting the claim that \( \lim_{T \to \infty} E_t \left( p^f_{t+T+1} - p^h_{t+T+1} \right) \) is constant and so \( \lim_{T \to \infty} E_t \left( s_{t+T+1} \right) \) must be for PPP to hold. Additionally, I assume that the long run interest rate gap converges to zero in expectation, i.e. \( \lim_{T \to \infty} E_t \left[ i_t - i^h_t \right] = 0 \).

31 As opposed to developing a New Keynesian model where the exchange rate is determined as the present value of variables found in the Taylor rules of two countries; as is common in the forecasting literature (see Rossi (2013) for a review). The benefit being that the model used above is consistent with a larger set of more fully specified models.
Where \( \gamma \equiv \frac{\sigma_a^2}{\sigma_z^2} \), \( 0 < \gamma < 1 \), reflecting the usefulness of the signal in predicting \( a_t \). Thus the full information case is where \( \gamma = 1 \) and \( z_t = a_z \). In this environment, agents have two sources of information which they use to determine the value of the exchange rate. Firstly, they form expectations about the level of future interest rates using information from the signal, \( z_t \). Secondly, the realized value of interest rates today, \( i_t \), reveals information about the level of economic fundamentals which agents predicted in the past. Agents use this information to correct their previous predictions about the exchange rate.

### 2.2.1 Identification

The relationship between the interest rate differential (\( i_t \)), the signal (\( z_t \)) and changes in the exchange rate (\( \Delta s_t \)) can be summarised as the the MA representation:

\[
x_t = C(L)w_t
\]

\[
\begin{pmatrix}
i_t \\
z_t \\
\Delta s_t
\end{pmatrix} = \begin{pmatrix}
c(L) & 0 & 0 \\
1 & 1 & 0 \\
s_a(L) & s_e(L) & 1
\end{pmatrix} \begin{pmatrix}
a_t \\
e_t \\
\eta_t
\end{pmatrix}
\]

Where \( c(L), s_a(L) \) and \( s_e(L) \) are lag polynomials, whose coefficients are known to agents; \( a_t \) and \( e_t \) are news and noise shocks respectively; and a shock to the exchange rate has been added, \( \eta_t \). Using the UIP equation (2.1) and the Bayesian updating behaviour captured in (2.2), the solution for the exchange rate under an arbitrary \( c(L) \), such that \( c(0) = 0 \), is given by \( s_a(L) \) and \( s_e(L) \) (see appendix). In the simple case where \( c(L) = L \) so that \( i_{t+1} = a_t \) we have that \( s_a(L) = -\gamma - (1 - \gamma)L + L^2 \) and \( s_e(L) = -\gamma(1 - L) \). This case be used to illustrate how the model captures two relevant empirical regularities of exchange rates: slow incorporation of news and exchange rate disconnect. The first can be seen form the impact response of the exchange rate to news under full information (\(-1\)) compared to imperfect information (\(-\gamma\)): Agents respond only in proportion to the reliability of the signal, \( \gamma \). However, news is fully incorporated after one period (\(-\gamma - (1 - \gamma) = -1\)) when \( i_{t+1} \) reveals \( a_t \). Exchange rate disconnect results in the short run from a noise shock (\(-\gamma\)) but does not have long run effects (\(-\gamma + \gamma = 0\)).

Unfortunately, this representation cannot be recovered by inverting a VAR on data for the observables \((i_t, z_t, \Delta s_t)\) since the determinant of this matrix vanishes inside the unit circle entailing a non-invertible MA representation (Lippi and Reichlin (1994), Blanchard et al. (2009)). The shocks above are not innovations to agents information set at time \( t \) since agents cannot distinguish between news and
2.2. Model

noise. Thus the data that they generate from decisions at time $t$ cannot reveal these values. Instead, agents make decisions based on shocks to the signal, $z_t$, and new information revealed by the realised interest rate about past interest rate predictions which I call, following Forni et al. (2013a), a learning shock, $u_t$. These shocks can be recovered from data generated by agents at time $t$. The model above imposes a sufficiently close relationship between the learning and signal shocks and the noise and news shocks to identify the latter. That relationship is illustrated by a Wold decomposition of the observables in terms of learning and signal shocks.

The relationship between the interest rate and the signal is given by the assumption that at time $t - 1$, agents use the signal to predict $i_t$ as $E[i_t|\Omega_{t-1}] = \gamma c(L)z_t$. There are two restrictions that identify the relationship between learning shocks and the interest rate. Firstly, by the definition of $u_t$ as an innovation to the agents information set, it must be known by agents at time $t$. Secondly, the way agents use the signal to predict $i_t$ combined with the observability of $i_t$ at time $t$, imposes the restriction that $u_t$ must ensure that agents update their predictions to match the observed $i_t$ at $t$. The decomposition is achieved by separating $c(L)$ into two parts using a Blaschke factor (Lippi and Reichlin (1994); Forni et al. (2013b); Leeper et al. (2013)). The Blaschke factors are given by:

$$b(L) = \prod_{j=1}^{J} \frac{L - r_j}{1 - \bar{r}_j L}$$

Where $r_j = 1, \ldots, J$ are the roots of $c(L)$ that are smaller than one in modulus and $\bar{r}_j$ is the complex conjugate. Using Blaschke factors it is possible to factorise $c(L)$ into a lag polynomial with roots inside the unit circle, $b(L)$, and one that has all its root outside the unit circle, $g(L)$: $c(L) = b(L)g(L)$. Thus the lag polynomial $\frac{c(L)}{b(L)} = g(L)$ does not vanish inside the unit circle. This implies that $g(L)$ satisfies the first identifying restriction since it captures the relationship between the interest rate and news via information that agents have received at time $t$, that is $u_t$. The second restriction imposes that when the information about past values of $a_t$ and $e_t$ come to light via $u_t$, that the interest rate differential fully reflects these values, that is $i_t = c(L)a_t$ in each period. For this to occur learning shocks must provide the the right re-weighting to noise and news that accounts for the downward bias implied by Bayesian updating (i.e. $\gamma$) and the information in $c(L)$ that is only available to agents from future realisations of $i_t$ i.e. the information not included in $g(L)$, that is, $b(L)^{32}$. This information captured in $b(L)$ is not known at time $t$ (leading to non-invertibility via roots inside the unit circle).

However, the structure of the model entails that agents learn this information from future realisations of $i_t$. This depends on the dynamic relationship between news and the interest rate and may take many periods for all information to become

\[32\text{In the 1 period case above the factorisation of } c(L) \text{ is given by applying (2.5), } b(L) = L \text{ and } g(L) = \frac{L}{L} = 1\]
available. This learning time is captured by the order of \( b(L) \). This implies that \( u_t = (1 - \gamma)b(L)u_t - \gamma b(L)e_t \) and the decomposition of the interest rate is:

\[
i_t = g(L)u_t + \gamma c(L)z_t
\]  

(2.6)

\[
i_t = \underbrace{\gamma c(L)u_t + c(L)}_{b(L)} (1 - \gamma)b(L)u_t + \gamma c(L)e_t - \underbrace{\frac{c(L)}{b(L)} \gamma b(L)e_t}_e = c(L)a_t
\]  

(2.7)

\[\text{prediction correction prediction correction}\]

The full Wold decomposition is then:

\[
x_t = D(L)v_t
\]  

(2.8)

\[
\begin{pmatrix}
i_t \\
z_t \\
\Delta s_t
\end{pmatrix} = \begin{pmatrix}
g(L) & \gamma c(L) & 0 \\
0 & 1 & 0 \\
s_u(L) & s_z(L) & 1
\end{pmatrix}
\begin{pmatrix}
u_t \\
z_t \\
\eta_t
\end{pmatrix}
\]  

(2.9)

Where

\[
v_t = B(L)w_t
\]  

(2.10)

\[
\begin{pmatrix}
u_t \\
z_t \\
\eta_t
\end{pmatrix} = \begin{pmatrix}
(1 - \gamma)b(L) & -\gamma b(L) & 0 \\
1 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
a_t \\
e_t \\
\eta_t
\end{pmatrix}
\]  

(2.11)

Where \( s_z(L) \) and \( s_u(L) \) are derived in the appendix. Unlike the MA representation in news and noise (2.4), the MA representation in learning and signal shocks (2.9) is recovered by inverting a VAR on data for the observables \((i_t, z_t, \Delta s_t)\). By construction, \( g(L) \) has no roots inside the unit circle leading to an invertible MA representation. The Bayesian updating behaviour of agents assumed in the model means that the relationship between the interest rate and the signal, \( \gamma c(L) \), can also be recovered. This allows \( c(L) \) to be found and thus \( b(L) \) constructed as per (2.5). \( b(L) \) and an estimate of \( \gamma \) (see section 3 below) allows the the construction of the dynamic rotation matrix (2.11). Equation (2.11) illustrates the relationship between this identification scheme and the standard SVAR identification. The structural shocks, \( w_t \), are recovered from the shocks, \( v_t \), using the restriction matrix \( B(L)^{-1} \). In standard SVAR identification this matrix is constant\(^{33} \) whereas here it is a dynamic relationship:

\(^{33}\)That is \( w_t = Rv_t \) where choosing \( R = B^{-1} \) identifies the model. A variety of restrictions have been employed in the literature, e.g. recursive ordering between shocks, short-run restrictions, long-run restrictions or sign restrictions on the impact of structural shocks (see for example Lutkepohl (2011)).
\[ v_t = B(L)w_t \Rightarrow w_t = B(L)^{-1}v_t \] (2.12)

\[
\begin{pmatrix}
    a_t \\
    c_t \\
    \eta_t
\end{pmatrix} =
\begin{pmatrix}
    b(F) & \gamma & 0 \\
    -b(F) & (1-\gamma) & 0 \\
    0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
    u_t \\
    z_t \\
    \eta_t
\end{pmatrix} \tag{2.13}
\]

Where \( b(L)^{-1} = b(L^{-1}) = b(F) \) and \( F \) is the forward operator. Thus to recover the news and noise shocks future values of the learning and signal shocks are required. The intuition is that agents don’t observe these shocks but learn their value in the future, thus once agents have sufficient information available to identify what shocks took place in the past, the econometrician can recover them. Impulse responses from the innovations to news and noise can be recovered from \( D(L)B(L) = [C(L)B(L)^{-1}]B(L) = C(L) \).

Figure 2.1: Level of exchange rate under AR(1) process for interest rate differential

\[ \Delta s_t = s_u(L)u_t + s_z(L)z_t \] and \( \gamma = 0.5 \). The interest rate \( i_t \) follows AR(1) process in news:

\[ i = \sum_{k=1}^{\infty} c_k a_{t-k} \], where \( c = 0.66 \).

To illustrate these mechanisms, the case of \( i_t \) following an AR(1) process, subject to the requirement that \( c(0) = 0 \), is given in figure 2.1 - showing the deviation of the level of the exchange rate from steady state. The full information solution shows that a news shock requires the USD/GBP exchange rate to appreciate fully on impact, followed by exponential depreciation following the AR(1) process for \( i_t \). On impact, under imperfect information, agents underestimate the news shock in proportion to \( \gamma = 0.5 \). However, new information arrives in period 1 and 2 from \( u_t \) indicating further appreciation. After 2 periods all information about the shock has been revealed and the imperfect information path for the exchange rate is identical to the full information case. For a noise shock, agents with full information ignore noise, however agents making predictions using the signal pre-
dict the same outcome for the exchange rate as under the news shock. Updates received in period 1 and 2 indicate that this was in fact a noise shock and thus a required depreciation relative to the prediction so that after period 2 the noise shock has no impact.

2.3 Estimation of News and Noise Shocks

Estimation of news and noise shocks follows the two step identification scheme of Forni et al. (2013a). In the first step, I identify the learning and signal shocks corresponding to equation (2.9) using standard SVAR methods. In the second stage, I impose restrictions on the relationship between those shocks and the news and noise shocks using the framework outlined in section 2.2.

In order to capture the contemporaneous influence of other macroeconomic variables on the exchange rate (as opposed to news about them) and to ensure that the VAR contains sufficient information to recover the learning and signal shocks, I add a vector of variables $\mathbf{w}_t$ with additional shocks in the system given by $\epsilon_t$. Agents observe the shocks $\epsilon_t$. For ease of presentation $\Delta s_t$ is included in the set $\mathbf{w}_t$. This vector includes additional variables thought to be important to exchange rate behaviour: cyclical output differences and inflation rates (Scholl and Uhlig (2008)) as well as a set of factors capturing the principal components of a large set of US and UK macroeconomic variables (see Table 2.1, Table 2.2 and the additional appendix for the data set used to construct the factors). The signal shocks, $z_t$, are not directly observable. I assume that market expectations of interest rates, denoted by $x_t$, reveal these shocks in the form of innovations to $x_t$. This augments the representation (2.9) as a 10 variable FAVAR (Bernanke et al. (2005)):

$$\mathbf{x}_t = \mathbf{D}(L)\mathbf{v}_t$$

$$\begin{pmatrix} i_t \\ x_t \\ \mathbf{w}_t \end{pmatrix} = \begin{pmatrix} g(L)\sigma_u & c(L)\gamma & k(L) \\ d(L)\sigma_u & f(L)\sigma_z & p(L) \\ q(L) & h(L) & M(L) \end{pmatrix} \begin{pmatrix} u_t/\sigma_u \\ z_t/\sigma_z \\ \epsilon_t \end{pmatrix} \quad (2.15)$$

Where $\mathbf{q}(L),\mathbf{h}(L),\mathbf{k}(L),\mathbf{p}(L)$ and $\mathbf{M}(L)$ are matrix polynomials in the lag operator. $\mathbf{M}(L)$ relates the vector $\mathbf{w}_t$ to its own shocks, $\epsilon_t$, $\mathbf{k}(L)$ and $\mathbf{p}(L)$ indicate the response of $i_t$ and $x_t$, respectively, to the shocks $\epsilon_t$. $\mathbf{q}(L)$ and $\mathbf{h}(L)$ give the response of the vector $\mathbf{w}_t$ to the learning, $u_t$, and signal shocks, $z_t$. This is the system I will take to the data for the first step which reveals the response of the system to learning and signal shocks. With an estimate of $\mathbf{D}(L)$ and $\mathbf{v}_t$, I can use the appropriate $\mathbf{B}(L)$ matrix\footnote{62} to recover $\mathbf{C}(L)$ as the second step to reveal the
2.3. Estimation of News and Noise Shocks

structural shocks $w_t$ as described in section 2.1. The structural representation is then:

$$x_t = C(L)w_t$$

(2.16)

$$
\begin{pmatrix}
i_t \\
x_t \\
w_t
\end{pmatrix} =
\begin{pmatrix}
c(L)\sigma_a & -\gamma g(L)b(L)\sigma_e + \gamma c(L)\sigma_e & k(L) \\
(1-\gamma)d(L)b(L)\sigma_a + f(L)\sigma_a & -\gamma d(L)b(L)\sigma_e + f(L)\sigma_e & p(L) \\
q(L)(1-\gamma)b(L)\frac{\sigma_a}{\sigma_z} + h(L)\frac{\sigma_a}{\sigma_z} & -\gamma q(L)b(L)\frac{\sigma_e}{\sigma_z} + h(L)\frac{\sigma_e}{\sigma_z} & M(L)
\end{pmatrix}
\begin{pmatrix}
a_t/\sigma_a \\
e_t/\sigma_e \\
\epsilon_t
\end{pmatrix}
$$

(2.17)

The model outlined in section 2 interprets the response of the interest rate to interest rate shocks as learning shocks. To the degree that this interpretation holds in the data, the estimate for $g(L)$ will be related to the response of the interest rate to signal shocks such that $g(L) = \frac{c(L)}{b(L)}$. In this case the interest rate will not respond to noise shocks at any lag: $-\gamma g(L)b(L)\sigma_e + \gamma c(L)\sigma_e = -\gamma \frac{c(L)}{b(L)}b(L)\sigma_e + \gamma c(L)\sigma_e = 0$. However, estimation only imposes restrictions via $b(L)$ which in turn depends solely on the estimated $c(L)$, the relationship between interest rate expectations and the actual interest rate. For example, $b(0) = c(0) = 0$ means an impact effect of zero. The estimated impulse response function (IRF) of the interest rate to noise provides a test of how well this model holds in the data. The response of $\Delta s_t$ to learning shocks is unrestricted but I impose the long run restriction that signal shocks have no long run effect on the level of the exchange rate $s_t$ - i.e. I impose that the sum of the IRF of $\Delta s_t$ to $x_t$ is zero. The degree to which the behaviour posited by the model of section 2 is borne out depends only on $b(L)$ and this long run restriction. To estimate the IRFs I undertake the following steps in the Forni et al. (2013a) identification scheme.

2.3.1 Step 1: Identify learning and signal shocks

Estimate the unrestricted VAR in (2.15) to recover the MA representation. The learning and signal shocks, $u_t$ and $z_t$ are recovered using standard SVAR techniques. I use a combination of short and long run restrictions following Lutkepohl (2005). Consider the model as three blocks: the slow moving variables, the block comprising the noisy-news model , and a block of fast moving variables, see Table 2.2. I impose a Cholesky recursive short run restrictions on the variables in the first block i.e. the output gap can respond only with a lag to all other shocks

Which is:

$$
\begin{pmatrix}
(1-\gamma)b(L)\sigma_a/\sigma_u & -\gamma b(L)\sigma_e/\sigma_u & 0' \\
\sigma_a/\sigma_z & \sigma_e/\sigma_z & 0' \\
0 & 0 & I_{n-2}
\end{pmatrix}
$$

, where $n$ is the number of variables in the VAR.
2.4. Data

I also impose short run restrictions that no shocks to variables outside the slow moving block can have a contemporaneous effect on the variables in the slow moving block. This is motivated by the convention in the SVAR literature that slower moving macroeconomic variables such as output and prices are ordered before fast moving expectational and financial variables (see for example Popescu and Smets (2010)). For the noisy-news model block, following the model outlined above I impose no contemporaneous effect of \( x_t \) shocks on \( i_t \), i.e. \( c(0) = 0 \). However, I allow shocks to \( \Delta s_t \) to influence both \( i_t \) and \( x_t \) and the fast moving variables. As noted above, I impose the long run restriction that signal shocks have no long run effect on \( s_t \). This restriction is sufficient to ensure that noise shocks have no long run effect on \( s_t \). The final block is composed of 4 leading principal components of a large dataset of fast moving U.K. and U.S. macro variables, primarily financial. I allow shocks to these fast moving variables to influence each other, interest rate expectations and the exchange rate contemporaneously.

2.3.2 Step 2: Identify news and noise shocks

The structural representation (2.17) requires an estimate of \( c(L) \) from which \( b(L) \) can be constructed. This can be recovered from the first step estimate of the response of \( i_t \) to \( z_t \), see (2.15). An estimate of the news-to-noise variance ratio \( \hat{\sigma}_a/\hat{\sigma}_e \) can be recovered from the ratio of the sum of the coefficients for the IRF of the interest rate response to learning and to signal shocks:

\[
\frac{g(1)\sigma_u}{c(1)\gamma\sigma_z} = \frac{c(1)\gamma\sigma_z}{c(1)\gamma\sigma_z} = \frac{1}{b(1)\sigma_e}.
\]

Using that \( \frac{\sigma_a}{\sigma_z} = \frac{\sigma_u}{\sigma_z} = \frac{\sigma_a}{\sigma_e} \) and that from (2.5), \( b(1) = \frac{1}{35} \). From this estimate of \( \frac{\sigma_a}{\sigma_e} \), estimates of the news-to-signal and noise-to-signal variance ratios can be recovered using the structure of news, noise and signal assumed in (2.4):

\[
\left( \frac{\sigma_a}{\sigma_z} \right)^2 + \left( \frac{\sigma_a}{\sigma_z} \right)^2 = 1.
\]

This then implies that \( \frac{\sigma_a}{\sigma_z} = \sin(\arctan(\frac{\sigma_a}{\sigma_e})) \) and \( \frac{\sigma_a}{\sigma_z} = \cos(\arctan(\frac{\sigma_a}{\sigma_e})) \). This, along with the IRFs from step 1, gives all the elements required to describe the IRFs for news and noise (2.17). The news and noise shocks can then be recovered in using the inversion procedure in (2.12) as a function of the future values of the learning and signal shocks.

2.4 Data

I estimate (2.15) on U.S.-U.K. monthly data for October 1986 to February 2013. The data sources are described in table 2.1. I use shocks to a proxy for market interest rate expectations, \( x_t \), to correspond to the signal shocks, \( z_t \), described in the model of section 2. I measure U.S. interest rate expectations as the 1 year ahead median federal funds rate forecast from the Philadelphia Federal Reserve’s Survey of Professional Forecasters (SPF). For the U.K., I use 1 year ahead median

\[\text{In these calculations I truncate the IRFs at a lag length of 60 periods.}\]
forecasts for the Bank of England bank rate drawn from Her Majesty’s Treasury survey of independent forecasters. This choice of proxy is based on comparability of each dataset (since they both are survey measures of similar rates with the same horizon) and this measure leads to signal shocks with a sizeable news component. Alternative measures are considered in section 5.6 on robustness.

The baseline FAVAR includes 10 variables: five macro variables and five factors derived from a large data set of monthly macroeconomic data from the U.S. and U.K (see table 2.2). The factor variables are extracted as leading principal components. These factors are divided into slow moving (responding to interest rates with a lag, e.g. output and prices) and fast moving (contemporaneous response to interest rates, e.g. financial variables). The inclusion of these factors is based on passing a test of sufficient information after being included in the VAR in order to capture the structural shocks as outlined in Forni and Gambetti (2014) - described fully in section 5.2. This test was passed once 1 slow moving and 4 fast moving factors were included.

Table 2.1: Data Description

<table>
<thead>
<tr>
<th>Variable</th>
<th>$\Delta \pi_t$</th>
<th>$\pi_t$</th>
<th>$y_t$</th>
<th>$x_t$</th>
<th>$i_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>US</td>
<td></td>
<td>US GDP</td>
<td>Phil. Fed SPF 1 year</td>
<td>10 year</td>
<td></td>
</tr>
<tr>
<td></td>
<td>% change</td>
<td>(cyclical)</td>
<td>abandonment forecast for the Federal funds rate</td>
<td>bond yield</td>
<td></td>
</tr>
<tr>
<td>UK Nominal USD/GBP</td>
<td>% change</td>
<td>UK GDP</td>
<td>HM Treasury survey 1 year</td>
<td>10 year</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(cyclical)</td>
<td></td>
<td>abandonment forecast for the Bank of England bank rate</td>
<td>bond yield</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Source</th>
<th>FRED</th>
<th>FRED / NISER</th>
<th>Phil. Fed / HM Treasury</th>
<th>FRED</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transformation</td>
<td>Difference</td>
<td>US - UK in levels</td>
<td>U.S. - UK in levels</td>
<td>U.S. - UK in levels</td>
</tr>
</tbody>
</table>

Data period is October 1986 to February 2013. *Quarterly data temp orally disaggregated to monthly using the method of Santos Silva and Cardoso (2001). †Cyclical component recovered from HP filter with smoothing parameter $\lambda = 129600$ (Ravn and Uhlig (2002)). *Kindly provided by Haroon Mumtaz, individual surveys available at https://www.gov.uk/government/collections/data-forecasts

In order to run a monthly VAR some quarterly data must be temporally disaggregated. This data is: market expectations of T-Bill rates found in the Philadelphia Fed Survey of Professional forecasters, the U.K. GDP deflator and U.S. GDP. Temporal disaggregation was achieved via the method of Santos Silva and Cardoso (2001) which updates the Chow-Lin best linear interpolation procedure (Chow and Lin (1971)) for dynamic models. This method temporarily disaggregates the quarterly data using a linear regression model based on monthly indicators. I used industrial production and private consumption expenditure for U.S. GDP, industrial production and retail sales for the U.K. GDP deflator and implied interest rates from the Eurodollar Futures contract prices on the Chicago Mercantile Ex-

---

36 See the additional appendix for a full description of this data set - which is an updated version of the Stock and Watson (2001) data for the U.S. and similar monthly series for the U.K.
change (CME) for the SPF data. The impact of temporal disaggregation is checked by estimation using quarterly data, see robustness section below.

<table>
<thead>
<tr>
<th>Slow moving</th>
<th>$y_t$</th>
<th>U.S. less U.K. cyclical output gap</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\pi_t$</td>
<td>U.S. less U.K. inflation rate</td>
</tr>
<tr>
<td></td>
<td>$F_{1,t}$</td>
<td>Slow moving factor (first principal component)</td>
</tr>
<tr>
<td>Noisy-news model</td>
<td>$i_t$</td>
<td>U.S. less U.K 10 year government bond yield</td>
</tr>
<tr>
<td></td>
<td>$x_t$</td>
<td>U.S. less U.K survey expectations of 1 year ahead central bank rate</td>
</tr>
<tr>
<td></td>
<td>$\Delta s_t$</td>
<td>Change in nominal exchange rate (USD/GBP)</td>
</tr>
<tr>
<td>Fast moving</td>
<td>$F_{2,t}$</td>
<td>Fast moving factors (first 4 principal components)</td>
</tr>
</tbody>
</table>

### 2.5 Results

#### 2.5.1 FAVAR estimation

The FAVAR is estimated in step 1 of section 3.1 using both short and long run restrictions. I estimate the model in levels with 3 lags (1 quarter) based on the Akaike information criteria.

#### 2.5.2 Test of sufficient information to identify structural shocks

The model of section 2 showed that if UIP holds and agents form expectations about future interest rates using a noisy signal then the news and noise shocks moving agents expectations will lead to a MA representation of the model that is non-invertible in past observables, see equation (2.4). It was shown that it is possible to recover these shocks from future values of observables, see equations (2.9) and (2.11). However, it is still possible that the shocks recovered in step 1 (learning and signal) and step 2 (news and noise) of the identification scheme are correlated with observables that agents can use to make decisions. This would be due to the VAR estimated in step 1 containing insufficient information to capture the information set used by agents to make decisions that in turn result in the observed data. This insufficient information problem at step 1 would then affect the shocks recovered in step 2. To include as much information as possible a FAVAR is used in step 1. I perform the Forni and Gambetti (2014) test of sufficient information on the shocks recovered as a check that these problems don’t affect the recovery of the news and noise shocks. This procedure tests if the shocks identified with the FAVAR are orthogonal to a large set of macroeconomic variables:

$$v_{i,t} = \alpha(L) F_t + \epsilon_{i,t}$$  \hspace{1cm} (2.18)

Where $v_{i,t}$ are the shocks identified from the VAR and $i$ denotes learning.

---

37 All variables in Table 2.2 are stationary. The 3 lag FAVAR passed standard diagnostic tests. The results of these are provided in the additional appendix.
2.5. Results

signal, news and noise. $F_t$ is a vector of principal components. $\alpha$ are the OLS estimates. The orthogonality test is an F-test that all $\alpha$’s are 0. If the shocks identified in (2.15) and (2.17) are uncorrelated with these data then all relevant information is contained within the VAR model and we can proceed assuming there is enough information to be able estimate the true structural shocks. This regression is implemented for 10 principal components with 2 lags. The F-test results reveal that there is sufficient information in the 10 variable FAVAR to identify the structural shocks (see table 2.3).

Table 2.3: F-test for fundamentalness of shocks (p-values)

<table>
<thead>
<tr>
<th>Principal components included in regression</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Learning shock</td>
<td>1.00</td>
<td>0.90</td>
<td>0.94</td>
<td>0.97</td>
<td>0.98</td>
<td>1.00</td>
</tr>
<tr>
<td>Signal shock</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>0.97</td>
<td>0.48</td>
<td>0.68</td>
</tr>
<tr>
<td>News shock</td>
<td>0.97</td>
<td>0.97</td>
<td>0.72</td>
<td>0.51</td>
<td>0.60</td>
<td>0.66</td>
</tr>
<tr>
<td>Noise shock</td>
<td>0.84</td>
<td>0.79</td>
<td>0.95</td>
<td>0.90</td>
<td>0.39</td>
<td>0.58</td>
</tr>
</tbody>
</table>

2.5.3 Impulse response functions

Figure 2.2: Impulse response functions to learning & signal shocks

The IRFs corresponding to equation (2.15) reveal the impact of learning and signal shocks on the system (see figure 2.2 and 2.4 and note that a shock to $i_t$ is a learning shock and a shock to $x_t$, a signal shock). All confidence intervals are 68% and 95% from nonparametric bootstrap procedure with 10 000 replications. Market expectations of interest rates are a useful predictor of $i_t$: a signal shock is associated with dollar returns exceeding that of the pound of approximately 7 basis points, after 1.5 years. Moreover, Market expectations of interest rates ($x_t$)
2.5. Results

respond positively to learning shocks, with a peak impact around 1.5 years. The latter can be interpreted, in light of the model in section 2, as an update agents make to their previous forecasts of interest rates.

A signal shock, indicating higher dollar returns, results in delayed dollar appreciation, as found in empirical studies of exchange rate dynamics (Chari et al. (2002a); Scholl and Uhlig (2008)) - see figure 2.4. Agents purchase dollars but not in sufficiently large quantities that the entire appreciation takes place on impact - consistent with underestimating future returns due to uncertainty about information conveyed by the signal. In periods following the signal shock realized values of the interest rate serve to confirm the signal of future rates and continued dollar purchases induce further appreciation. The appreciation is relatively short lived lasting 5 months followed by an extended period of appreciation. This relatively fast appreciation supports the evidence in Kim and Roubini (2000) and Bjornland (2009) who find that the delayed appreciation is much more mild (2-3 months) than the original Eichenbaum and Evans (1993) finding of 2-3 years before peak appreciation occurs.

Figure 2.3: Impulse response functions to news and noise shocks

Median impulse response functions to orthogonalized one standard deviation innovations. Confidence intervals are 68% and 95% from nonparametric bootstrap with 10,000 replications.

In contrast to previous VAR studies of exchange rate dynamics (Chari et al. (2002a); Scholl and Uhlig (2008)), this study distinguishes between a shock to today’s interest rates and how that shock changes market expectations of future interest rates. The former is the more dominant driver of exchange rate dynamics on impact. In response to a learning shock the dollar depreciates for 3 months followed by a protracted period of mild appreciation. Depreciation on impact contradicts the UIP equation when agents have full information. However, under limited information, shocks to current interest rates matter primarily as updates
2.5. Results

to previous forecasts of the exchange rate made using the signal. Since agents respond to signal shocks with delayed dollar appreciation, if the signal shocks are predominately driven by noise shocks, the appropriate update would be a correction in the form of a depreciation of the exchange rate.

Figure 2.4: Impulse response functions of $\Delta s_t$ to learning & signal shocks

![Impulse response functions](image)

Median impulse response functions to orthogonalised one standard deviation innovations. Confidence intervals are 68% and 95% from nonparametric bootstrap with 10,000 replications.

Indeed, I find that while news shocks are important for explaining the interest rate differential (see figure 2.3) as evinced by an estimate of $\left(\frac{\sigma_a}{\sigma_z}\right)^2 = 0.42$, noise shocks drive more of the changes in signal shocks with $\left(\frac{\sigma_e}{\sigma_z}\right)^2 = 0.58$. Market interest rate expectations, $x_t$, respond similarly to both news and noise shocks, however news has a more persistent impact. This is consistent with agents learning later whether the jump in the signal was driven by news or noise resulting in expectations adjusting more quickly to their original position after a noise shock.

The model motivating the identification scheme requires that noise shocks don’t influence the interest rate differential, $i_t$. The response of interest rates to noise shocks provides some support for this claim in that this claim cannot be rejected at a 95% level of confidence. However, there is some evidence that noise may influence rates at short horizons.

Noise shocks result in delayed dollar appreciation in the first 3 months after the shock followed by a protracted period of mild depreciation (figure 2.5). The limited information model provides qualitatively similar results to a noise shock: appreciation followed by a period of depreciation such that the noise shock has no long run effect. The restriction that signal shocks don’t have long run effects on $s_t$, in combination with the restrictions imposed by the dynamic relationship between learning shocks and noise shocks is sufficient to result in a period of depreciation.
that approximately offsets the appreciation on impact. The interpretation being that agents are correcting past forecast errors until the impact of those errors is removed.

These movements in the USD/GBP correspond to periods when current and future realised interest rates do not motivate any revaluation in the currency and is evidence of expectational errors driving the exchange rate disconnect puzzle. A related empirical finding, documented in Brunnermeier et al. (2009) and Burnside et al. (2011), is “crash risk” where carry trade pay-offs have a negatively skewed distribution. In the months following a noise shock, traders start to realise that the value of the dollar vis-a-vis the pound is based on false expectations of a higher U.S. relative to U.K. interest rates. The corrections that ensue have the potential to generate crash-like dynamics if they are large, however I find that the depreciation is relatively mild.

The impact of news shocks is roughly the opposite of noise shocks: a period of dollar depreciation lasting approximately 3 months followed by mild appreciation. This depreciation follows from the way agents respond to learning shocks. The theory predicts, following UIP, an appreciation since in the model agents respond to learning shocks with an appreciation of the exchange rate in the one period and AR(1) cases examined. The identification imposes that the response to a news shock is proportional to the news-signal variance ratio \( \left( \frac{\sigma_a}{\sigma_z} \right) \), resulting in a small appreciation on impact. There is, however, some evidence of appreciation in response to a news shock in the case where the proxy for interest rate expectations is taken from futures contracts (see section 5.6 on robustness).

Figure 2.5: Impulse response functions of \( \Delta s_t \) to news & noise shocks

\[\text{Median impulse response functions to orthogonalised one standard deviation innovations. Confidence intervals are 68\% and 95\% from nonparametric bootstrap with 10,000 replications.}\]
2.5.4 Variance decomposition

In combination noise and news shocks explain 18% of the forecast error variance of $\Delta s_t$, see figure (2.6). Noise shocks account for around double the variation in $\Delta s_t$ compared to news shocks (12% vs. 6%). This result is robust to identification scheme for noise but not for news - discussed in section 5.6. For this reason I focus in the role of noise. As agents learn to distinguish reliable from unreliable signals regarding the future path interest rates, finding that they responded to false information leads to a larger response than finding that this information was correct. Part of this response is due to the predominance of noise in the signal (58%) meaning that more corrections take place due to noise relative to news. Additionally, it is plausible that trading losses are larger in the case of a noise shock, where the interest rate doesn’t respond in the future, than in the case of a news shock where the interest rate follows a more similar path to that originally predicted by the signal. With the exception of the shock to the exchange rate itself, noisy news explains more of the variation in $\Delta s_t$ than other variables in the model. Individually only fast factor 3, the third leading principal component of fast moving macro variables, explains more of that variation than noise (which has its highest factor loadings on Moody’s Aaa and Baa corporate bond yields). The importance of noise relative to news shocks is suggestive as to why the UIP puzzle is so robust: realised shocks to the interest rate differential are substantially less important than expectational errors about those interest rates in driving exchange rates. This result complements high-frequency studies, such as Clarida and Waldman (2008), finding expectational errors are important in driving exchange rates. However, it documents such effects at the same frequency at which macroeconomic data is available and thus shows that expectational errors are important even when those macro fundamentals have the potential to drive exchange rates.
2.5. Results

Figure 2.6: Variance decomposition of $\Delta s_t$

The contribution of each shock, excluding the $\Delta s_t$ shock, is measured on the left axis. The contribution of the $\Delta s_t$ shock to the variable $\Delta s_t$ is measured on the right axis.

2.5.5 Historical decomposition

The historical decomposition follows Lutkepohl (2011). The contribution of each structural shock to each observed series is described by:

$$x_t = \sum_{i=0}^{t-1} \Psi_i v_{t-i} + A_1^t x_0 + A_2^t x_{-1} + A_3^t x_{-2}$$  (2.19)

Where $x_t$ is the vector of observables in (2.17) and $A_j$ are the coefficients from the estimated VAR for lag $j$, $v_t$ is the vector of structural shocks. $\Psi_i$ is the MA representation for the model at horizon $i$, in months. The contribution of the news and noise shocks to the nominal exchange rate are displayed in figure 2.7. As equation (2.13) illustrates, to recover noise and news shocks requires future values of learning and signals shocks. In order to estimate $\Psi_{24}$, that is a horizon of 2 years, the last date for which we can recover $v_t$ is October 2010.
During periods of increased uncertainty about monetary policy, expectational errors regarding interest rates have been large, resulting in noise shocks having had a more pronounced impact (see figure 2.7). The period of 1988-1993 saw the U.K. central bank rate rise above the U.S. federal funds rate with a depreciation of the dollar. The recession of 1990 lead to easing of central bank rates in both countries but with a much faster decline in U.K. rates. Market expectations misread the speed of the decline in this gap, initially underestimating and then overestimating the size of the interest rate differential. This period was also characterised by large movements in the USD/GBP. Similarly, in 2000, market expectations of rates did not correctly forecast the easing of U.S. monetary policy relative to that of the U.K. during the 2001 recession, with expectations that U.S. rates would remain above U.K. rates (this gap was negative until 2005). The uncertainty associated with the financial crisis of 2008 and the subsequent unconventional monetary policies on both sides of the Atlantic saw volatile interest rate expectations. Early 2008 saw a positive gap open up between U.K.-U.S. rates that closed by the end of the year, however market expectations forecast the opposite then quickly corrected. The post-crisis uncertainty in monetary policy actions saw expectations that U.K. rates would rise above U.S. rates around 2009 and 2010, which were not borne out. The largest news shock on the USD/GBP took place in the last quarter of 2001 with market expectations reliably forecasting the U.K.-U.S. rates gap that opened up between 2001-2004. A delayed but protracted rally in the U.K. pound followed.

2.5.6 Robustness

The choice of proxy for market interest rate expectations to measure signal
shocks plays a central role in the results above. Forward looking variables that are useful leading indicators of interest rate changes are reasonable candidates. Robustness of the above results is tested by considering alternative signals of future interest rates in the form of interest rate futures and uncertainty indices for the U.S. and U.K (see additional appendix for data description and additional figures). In addition I compare the results with the baseline proxy for interest rate expectations but using a Cholesky recursive identification scheme at step 1 of the estimation. The variables are ordered as in table (2.2). This recursive ordering follows the convention in the SVAR literature that slower moving macroeconomic variables such as output and prices are ordered before fast moving expectational and financial variables (see for example Popescu and Smets (2010)). The results are presented in figure 2.8.

The results for noise shocks are robust. There is broad agreement across alternative proxies and identification schemes as to the timing and direction of the effects, however they disagree on the size of the impact of noise shocks. The two alternative proxies indicate more mild appreciation than the baseline case. The more muted effect is due in part to a lower degree of news content in these proxies, 29% for future rates, and only 5% for the policy uncertainty indices compared to 42% for the baseline proxy of survey expectations.

There is more uncertainty surrounding the response to a news shock. The alternative proxies and identification scheme agree on the direction of the response with depreciation followed by mild appreciation, however the timing of these effects and their size is quite variable. The largest deviation is for the case of a Choleski decomposition at step 1 with only a small appreciation in the first and third month after impact. Unsurprisingly, this translates into a much smaller proportion of the forecast errors variance of $\Delta s_t$ explained by news: 3% instead of 8%. However, the contribution of noise shocks remains high, and slightly above the baseline figure of 12%, at 14% (details of variance decomposition results are given in the additional appendix).
2.5. Results

Figure 2.8: Impulse response functions for $\Delta s_t$ under alternative proxies for interest rate expectations

The baseline results, with survey expectations as a signal, were also tested with quarterly data as a check of the influence of the interpolation procedures used on the data that is only available at a quarterly frequency. Given the coarseness of quarterly data only the broad movements in the monthly IRFs is captured (see figure 2.9). News shocks exhibit delayed depreciation in the quarters following impact, followed by a mild appreciation. The size of the depreciation as well as the presence of a delay are in contrast to the monthly results. The identification scheme imposes that agents respond to news in proportion to the news content of the signal. This restriction entails appreciation on impact since agents respond to signal shocks with an appreciation of the USD/GBP. This restriction then inhibits the 3 month period of depreciation seen in the monthly data, since it is part of the impact period in quarterly data, leading to a much more mild depreciation overall. Noise shocks lead to appreciation on impact, followed by a period of depreciation in line with the monthly results.
2.6 Conclusion

This paper employs the SVAR identification scheme of Forni et al. (2013b) to incorporate noisy news into an imperfect information model of exchange rates. That scheme uses dynamic rotations to recover structural shocks despite the presence of noisy news. I employ a factor augmented VAR using survey expectations of professional forecasters as a signal of future interest rates in the U.S. and the U.K. The results indicate that news and noise shocks are important drivers of the USD/GBP exchange rate over the sample period. Noise shocks induce a period of delayed dollar appreciation in the first 3 months after the shock followed by a protracted period of mild depreciation. The effect of news shocks is the opposite, a period of dollar depreciation lasting approximately 3 months followed by mild appreciation. These patterns partly accord with the model of exchange rate determination under imperfect information: both news and noise shocks matter when a useful signal is available, delayed depreciation followed by an equal and opposite period of appreciation under noise shocks are found. The model predicts delayed appreciation following a news shock whereas I find delayed appreciation. However, there is evidence that this appreciation effect is milder when alternative proxies for interest rate expectations and quarterly data are used.

Variance decompositions show that noisy news is an important driver of the exchange rate explaining approximately a fifth of its variation. Noise shocks have a larger effect (12-14%) than news shocks (3-6%). This is partly due to the larger noise component measured in the proxy for interest rate expectations. These re-
2.6. Conclusion

Results provide further evidence that expectational errors are an important source of deviations in UIP and relevant to any resolution of the exchange rate disconnect puzzle. A historical decomposition of the exchange rate indicates that noise shocks are more important during periods of changing monetary policy e.g. the 1990 easing and 2001 tightening of U.S. monetary policy and the unconventional monetary policies surrounding the financial crisis of 2008.

Future research could extend this study to developing countries where the carry trade is known to be significant, policy making less transparent and thus where expectations about future interest rate differentials are likely to be important drivers of exchange rate dynamics.
2.7 Appendix

2.7.1 Limited information model of exchange rate determination

The values of the lag polynomials $s_u(L)$ and $s_z(L)$ relating the value of the $s_t$ to learning, $u_t$, and signal, $x_t$, shocks in (2.9) can be found by using the relationship between $i_t$ and these shocks:

$$E[i_{t+j}|\Omega_t] = E_t i_{t+j} = \left\{ \begin{array}{l} \gamma c(L) z_t + g(L) u_t ; j = 0 \\ \gamma \sum_{k=j}^{\infty} c_k L^k z_{t+j} + \sum_{k=j}^{m} g_k L^k u_{t+j} ; j > 0 \end{array} \right.$$

(2.20)

Where $n$ is the order of the $c(z)$ and $m$ is the order of $g(z)$, $m \leq n$, and the fact that $E_t z_{t+j} = E_t u_{t+j} = 0 \forall j \geq 1$. The value of $s_t$ is then:

$$s_t = -E_t \sum_{j=0}^{\infty} i_{t+j} = -s_z(L) \gamma z_t - s_u(L) u_t$$

(2.21)

Where,

$$s_z(L) = \sum_{k=1}^{n} c_k + \sum_{k=1}^{n} c_k L + \sum_{k=2}^{n} c_k L^2 + \ldots + \sum_{k=n-1}^{n} c_k L^{n-1} + c_n L^n$$

$$s_u(L) = \sum_{k=1}^{m} g_k + \sum_{k=1}^{m} g_k L + \sum_{k=2}^{m} g_k L^2 + \ldots + \sum_{k=m-1}^{m} g_m L^{m-1} + g_m L^m$$

The value for the change in the nominal exchange rate is simpler:

$$\Delta s_t = -s_z(L) (1-L) \gamma z_t - s_u(L) (1-L) u_t = s_z(L) \gamma z_t + s_u(L) u_t$$

(2.22)

Where,

$$s_z(L) = \begin{cases} \{ -c(1) - \sum_{k=1}^{n} c_k L^{k+1} \} & \text{if } n \geq 1 \\ \{ -(1-L)c(1) \} & \text{if } n = 0 \end{cases}$$

$$s_u(L) = \begin{cases} \{ -g(1) - \sum_{k=1}^{m} g_k L^{k+1} \} & \text{if } m \geq 1 \\ \{ -(1-L)g(1) \} & \text{if } m = 0 \end{cases}$$

This the solution for $\Delta s_t$ in terms of the learning and signal shocks. To get the solution in terms of the news and noise shocks, the relationship in (2.11) is used:

$$\Delta s_t = s_z(L) \gamma z_t + s_u(L) u_t = s_z(L) \gamma (a_t + e_t) + s_u(L) ((1-\gamma)b(L) a_t - \gamma b(L) e_t)$$

(2.23)

$$\Delta s_t = s_a(L) a_t + s_e(L) e_t$$

(2.24)

Where,

$$s_a(L) = s_z(L) \gamma + (1-\gamma) s_u(L) b(L)$$

$$s_e(L) = \gamma (s_z(L) - s_u(L) b(L))$$
2.8 Additional Appendix

Figure 2.10: Alternative Signals: $x_t = \text{CME 3-month Eurodollar futures rate - BoE option implies rates}$

(a) Median impulse response functions to orthogonalised one standard deviation innovations. Confidence intervals are 68% and 95% from nonparametric bootstrap with 10,000 replications.

(b) Median impulse response functions to orthogonalised one standard deviation innovations. Confidence intervals are 68% and 95% from nonparametric bootstrap with 10,000 replications.
Figure 2.11: Alternative Signals: $x_t = \text{Dendy et al. (2013a)}$ index of uncertainty for U.K. - Baker et al. (2012) index of policy uncertainty for U.S.*

(a) Median impulse response functions to orthogonalised one standard deviation innovations. Confidence intervals are 68% and 95% from nonparametric bootstrap with 10,000 replications.

(b) Median impulse response functions to orthogonalised one standard deviation innovations. Confidence intervals are 68% and 95% from nonparametric bootstrap with 10,000 replications.
2.8. Additional Appendix

Figure 2.12: Cumulative IRFs to a noise shock

Red dotted lines represent 68% CIs

Table 2.4: Data for alternative signals

<table>
<thead>
<tr>
<th>Variable</th>
<th>Futures</th>
<th>Uncertainty index</th>
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<tr>
<td>US</td>
<td>Chicago Mercantile Exchange 3-month Eurodollar Futures</td>
<td>Policy uncertainty index of Baker et al. (2012)</td>
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<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>UK</td>
<td>Option implies interest rates</td>
<td>Uncertainty index developed by Dendy et al. (2013a)</td>
</tr>
<tr>
<td>Source</td>
<td>Chicago Mercantile Exchange / Bank of England</td>
<td>policyuncertainty.com / Kindly provided by Haroon Mumtaz</td>
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<td>Data</td>
<td>January 1988 to February 2013</td>
<td>October 1986 to June 2012</td>
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<tr>
<td>Period</td>
<td></td>
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</table>
2.8. Additional Appendix

Table 2.5: Variance decomposition: baseline Identification at step 1

(a) Learning and signal shocks

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<tr>
<th>Variable</th>
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<tr>
<td>Δs_t</td>
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<tr>
<td>Impact</td>
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</tr>
<tr>
<td>3 month</td>
<td>2.114995</td>
</tr>
<tr>
<td>6 months</td>
<td>2.137271</td>
</tr>
<tr>
<td>1 year</td>
<td>2.198675</td>
</tr>
<tr>
<td>5 years</td>
<td>2.238433</td>
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</table>

(b) News and noise shocks

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<th>Variable</th>
<th>Shock</th>
</tr>
</thead>
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Table 2.6: Variance decomposition: recursive Cholesky identification at step 1

(a) Learning and signal shocks

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<td>Δs_t</td>
<td>$\epsilon_t^u$</td>
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<tr>
<td>Impact</td>
<td>0.145995</td>
</tr>
<tr>
<td>3 month</td>
<td>2.114995</td>
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<td>2.198675</td>
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(b) News and noise shocks

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<td>5 years</td>
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### Table 2.7: U.K. Macroeconomic data set used to extract principal components

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<td>Business Tendency Surveys for Construction: Confidence Indicators:</td>
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<td>2</td>
<td>BSCICP03GBM665S</td>
<td>Business Tendency Surveys for Manufacturing: Confidence Indicators:</td>
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<td>4</td>
<td>CCUSMA02GBM618N</td>
<td>National Currency to US Dollar Exchange Rate: Average of Daily Rates</td>
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<tr>
<td>5</td>
<td>CSCICP02GBM660S</td>
<td>Consumer Opinion Surveys: Confidence Indicators: Composite Indicators:</td>
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<td>6</td>
<td>CSCICP03GBM665S</td>
<td>Consumer Opinion Surveys: Confidence Indicators: Composite Indicators:</td>
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<td>Consumer Opinion Surveys: Economic Situation: Future Tendency:</td>
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<td>Consumer Price Index: Energy for United Kingdom</td>
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<td>GBPRPOINDMISMEI</td>
<td>Production of Total Industry in the United Kingdom</td>
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<td>12</td>
<td>GBPRPOMANMISMEI</td>
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<td>13</td>
<td>GBRRECM</td>
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# 2.8. Additional Appendix

## Table 2.8: U.S. Macroeconomic Data set used to extract principal components

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<td>AWHI</td>
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<td>PERMIT</td>
<td>New Private Housing Units Authorized by Building Permits</td>
<td>0</td>
</tr>
<tr>
<td>35</td>
<td>PPIFGS</td>
<td>Producer Price Index: Finished Goods</td>
<td>1</td>
</tr>
<tr>
<td>36</td>
<td>PPIITM</td>
<td>Producer Price Index: Intermediate Materials: Supplies &amp; Components</td>
<td>1</td>
</tr>
<tr>
<td>37</td>
<td>RPI</td>
<td>Real Personal Income</td>
<td>1</td>
</tr>
<tr>
<td>38</td>
<td>TB6MS</td>
<td>6-Month Treasury Bill: Secondary Market Rate</td>
<td>0</td>
</tr>
<tr>
<td>39</td>
<td>TCU</td>
<td>Capacity Utilization: Total Industry</td>
<td>0</td>
</tr>
<tr>
<td>40</td>
<td>TOTRESNS</td>
<td>Total Reserves of Depository Institutions</td>
<td>0</td>
</tr>
<tr>
<td>41</td>
<td>UEMP27OV</td>
<td>Civilians Unemployed for 27 Weeks and Over</td>
<td>1</td>
</tr>
</tbody>
</table>
2.8. Additional Appendix

Table 2.9: U.S. Macroeconomic Data set used to extract principal components

<table>
<thead>
<tr>
<th>No</th>
<th>Mnemonic</th>
<th>Name</th>
<th>Slow</th>
</tr>
</thead>
<tbody>
<tr>
<td>42</td>
<td>UEMPSTO14</td>
<td>Civilian Unemployed for 5-14 Weeks</td>
<td>1</td>
</tr>
<tr>
<td>43</td>
<td>UEMPST5</td>
<td>Civilian Unemployed - Less Than 5 Weeks</td>
<td>1</td>
</tr>
<tr>
<td>44</td>
<td>UEMPMEAN</td>
<td>Average (Mean) Duration of Unemployment</td>
<td>1</td>
</tr>
<tr>
<td>45</td>
<td>UNRATE</td>
<td>Civilian Unemployment Rate</td>
<td>1</td>
</tr>
<tr>
<td>46</td>
<td>USCONS</td>
<td>All Employees: Construction</td>
<td>1</td>
</tr>
<tr>
<td>47</td>
<td>USFIRE</td>
<td>All Employees: Financial Activities</td>
<td>1</td>
</tr>
<tr>
<td>48</td>
<td>USGOOD</td>
<td>All Employees: Goods-Producing Industries</td>
<td>1</td>
</tr>
<tr>
<td>49</td>
<td>USCIV</td>
<td>All Employees: Government</td>
<td>1</td>
</tr>
<tr>
<td>50</td>
<td>USFIRE</td>
<td>All Employees: Trade, Transportation &amp; Utilities</td>
<td>1</td>
</tr>
<tr>
<td>51</td>
<td>USFIRE</td>
<td>All Employees: Retail Trade</td>
<td>1</td>
</tr>
<tr>
<td>52</td>
<td>USFIRE</td>
<td>All Employees: Wholesale Trade</td>
<td>1</td>
</tr>
<tr>
<td>53</td>
<td>SP500</td>
<td>S&amp;P 500 Stock Price Index</td>
<td>0</td>
</tr>
<tr>
<td>54</td>
<td>DJIA</td>
<td>Dow Jones Industrial Average</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 2.10: Autocorrelation of residuals

<table>
<thead>
<tr>
<th>Equation residual</th>
<th>$y_t$</th>
<th>$z_1$</th>
<th>$s_f$</th>
<th>$t_1$</th>
<th>$s_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Durbin-Watson statistic</td>
<td>2.012259</td>
<td>2.110185</td>
<td>2.030214</td>
<td>1.905895</td>
<td>1.967177</td>
</tr>
<tr>
<td>p-value</td>
<td>0.611867</td>
<td>0.685519</td>
<td>0.738761</td>
<td>0.507733</td>
<td>0.353702</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Equation residual</th>
<th>$f_1$</th>
<th>$f_2$</th>
<th>$f_3$</th>
<th>$f_4$</th>
<th>$\Delta s_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Durbin-Watson statistic</td>
<td>1.967102</td>
<td>2.026991</td>
<td>2.004667</td>
<td>2.076374</td>
<td>1.994003</td>
</tr>
<tr>
<td>p-value</td>
<td>0.353338</td>
<td>0.711270</td>
<td>0.563164</td>
<td>0.263396</td>
<td>0.498091</td>
</tr>
</tbody>
</table>

The null of no autocorrelation cannot be rejected for all equations in the VAR

Table 2.11: Unit root test of VAR residuals

<table>
<thead>
<tr>
<th>Equation residual</th>
<th>$y_t$</th>
<th>$z_1$</th>
<th>$s_f$</th>
<th>$t_1$</th>
<th>$s_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Augmented Dickey-Fuller statistic</td>
<td>-17.831912</td>
<td>-18.089707</td>
<td>-17.006910</td>
<td>-17.038773</td>
<td>-17.377136</td>
</tr>
<tr>
<td>p-value</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Equation residual</th>
<th>$f_1$</th>
<th>$f_2$</th>
<th>$f_3$</th>
<th>$f_4$</th>
<th>$\Delta s_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Augmented Dickey-Fuller statistic</td>
<td>-17.377212</td>
<td>-17.042401</td>
<td>-17.760491</td>
<td>-18.354603</td>
<td>-17.610157</td>
</tr>
<tr>
<td>p-value</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
</tr>
</tbody>
</table>

The null hypothesis of a unit root is rejected for all equations in the VAR
Chapter 3

Macroeconomic Uncertainty in South Africa

Abstract

This paper develops a new index of economic uncertainty for South Africa for the period 1990-2014 and analyses the macroeconomic impact of changes in this measure. The index is constructed from three sources: (1) Disagreement among professional forecasters about macroeconomic conditions using novel data from a forecasting competition run by a national newspaper, (2) a count of international and local newspaper articles discussing economic uncertainty in South Africa and (3) mentions of uncertainty in the quarterly economic review of the South African Reserve Bank. The index shows high levels of uncertainty around the period of democratic transition in 1992-4, the large depreciation of the currency in 2001 and the financial crisis of 2008. The uncertainty index is a leading indicator of a recession. An unanticipated increase in the index is associated with a fall in GDP, investment, industrial production and private sector employment. Contrary to evidence for the U.S.A and U.K., uncertainty shocks are inflationary. These results are robust to controlling for global shocks (VIX index), consumer confidence and a measure of financial stress.

Keywords: economic uncertainty, business cycles, inflation, South Africa.

JEL classification numbers: D80, E32, E31, E66, N17.

3.1 Introduction

The Great Recession has renewed interest in the question of the sources of business cycle fluctuations. Traditional sources of fluctuations, such as technology and labour supply shocks, are less plausible explanations of this episode than of previous recessions. I study a new driver proposed by Baker and Bloom (2013): fluctuations in uncertainty. These authors develop a proxy for economic policy uncertainty based on news articles discussing policy uncertainty, the number of federal taxes set to expire and disagreement among professional forecasters over the future values of government purchases and inflation. They show, using a vector autoregression, that an increase in their proxy equivalent to the rise seen during the financial crisis is associated with a loss of around 2 million jobs and a decline in industrial production of 2.5% for the U.S.A. Moreover, Bloom (2009) show that uncertainty rises by around 50% during a typical U.S. recession. Studies following
Baker and Bloom (2013) have provided similar evidence that uncertainty shocks are important drivers of the business cycle, e.g. Dendy et al. (2013b) for the U.K.

Despite some cross-country work relating uncertainty to growth by Baker and Bloom (2013), there is little evidence of the effects of such proxies for economic uncertainty for developing countries. Given that developing countries experience much higher levels of realised volatility than developed nations (Fernandez-Villaverde et al. (2011b) and Bloom (2014)) it is plausible that fluctuations in uncertainty are important drivers of business cycles in these regions. It has been argued by Leduc and Liu (2012) that shocks to uncertainty have a central role to play in understanding business cycles as they are prototypical aggregate demand shocks, with lower output and inflation. However, recent papers by Popescu and Smets (2010) and Gilchrist et al. (2013a) have challenged the relevance of uncertainty shocks once their correlation with credit spreads is accounted for, suggesting that uncertainty shocks only matter when acting through a financial channel. Extending studies of uncertainty beyond developed nations can help disentangle the effects of financial shocks from uncertainty shocks. During the Great Recession many developing countries experienced increases in uncertainty, as the impact of a large recession in trading partner countries took hold, yet they did not experience the same levels of financial stress and instability as in the developed world.

This paper makes two contributes to this literature. Firstly, it extends the evidence that uncertainty shocks generate drops in real activity to a developing country. Secondly, it provides evidence that uncertainty shocks have real effects even when controlling for financial stress (credit spreads).

I construct an index of economic uncertainty following Dendy et al. (2013b) for the period 1990-2014. The index is constructed from three sources: (1) Disagreement among professional forecasters about macroeconomic conditions using novel data from a forecasting competition run by a national newspaper, (2) a count of international and local newspaper articles discussing economic uncertainty in South Africa and (3) mentions of uncertainty in the quarterly economic review of the South African Reserve Bank (SARB). The index is positively correlated with other proxies for uncertainty, i.e. realised and option implied volatility of the stock market. The index shows high levels of uncertainty around the period of democratic transition in 1992-4, the large depreciation of the currency in 2001 as well as the financial crisis of 2008.

To measure the impact of uncertainty shocks I use a structural VAR. The results show that economic uncertainty is a leading indicator of a recession in South Africa. An unanticipated increase in the index is associated with a fall in GDP, investment, industrial production, capital inflows and private sector employment. Contrary to evidence for the U.S.A. and U.K., uncertainty shocks are inflationary.
I show that this result is robust to the inclusion of a proxy for credit spreads as well as alternative methods of construction for the index.

The remainder of the paper is organised as follows. Section 1.1 reviews the literature on uncertainty shocks. Section 2 describes the construction of the index and compares it to alternative proxies in South Africa. Section 3 presents the VAR results and robustness checks and section 5 concludes.

3.1.1 Literature

Why should uncertainty matter? There are (at least) three broad reasons identified in the literature: real options, risk aversion and growth options effects (Bloom (2014)).

The real options approach to uncertainty (Bernanke (1983)) envisages that firms face a number of projects which they may pause if prospects diminish. However, for this to have macroeconomic effects a number of preconditions are needed: firms must be subject to fixed costs or partially irreversibilities in investment, be able to wait to bring its products to market (e.g. not in a patent race with other firms) and operate in an environment where today’s investment decision affects tomorrow’s actions e.g. through increasing-returns-to-scale technology. These effects have the potential to weaken productivity-enhancing reallocation of resources as productive firms expand less and unproductive firms contract less as both wait for uncertainty to clear. This can generate pro-cyclical productivity as in Bloom et al. (2012) and link these shocks to the business cycle.

Greater uncertainty directly increases risk premia if investors are risk averse and will increase the probability of default among lenders, leading to higher default premia. An important channel through which uncertainty operates is its ability to generate and amplify financial stress (Arellano et al. (2012); Christiano et al. (2014); Gilchrist et al. (2013a)). Risk averse households respond with precautionary savings which is contractionary in the short run but may stimulate long run growth through. For small open economies much of this savings flows abroad leading to large reductions in domestic demand (Fernandez-Villaverde et al. (2011b)). If nominal rigidities are strong, the drop in demand will not be met with sufficiently reduced prices leading to a recession even in large economies (Leduc and Liu (2012); Fernandez-Villaverde et al. (2011a)).

It is not clear why an increase in uncertainty should be interpreted always as equivalent to bad news. Growth options refer to the idea that entrepreneurs can only lose their investment but the upside of an increase in potential outcomes is unbounded. Thus uncertainty creates call option effects. However, the empirical literature has consistently found non-positive responses to increases in uncertainty on a macroeconomic level. A potential reason for the bad news interpretation of increased uncertainty is that agents are “ambiguity averse”. Such agents cannot assign a probability distribution over the future and respond by assuming the
3.1. Introduction

worst-case-scenario of the possible distributions they consider (Ilut and Schneider (2014)). Thus any increase in the possible range of outcomes acts only to worsen expectations of the future.

The large increases in uncertainty during the 2008 recession has stimulated research into better proxies to measure uncertainty. These focus on macroeconomic estimates of time varying volatility, cross-sectional dispersion of firms earnings or productivity, and direct measures of perceived uncertainty in the form of forecast distributions from surveys of professional forecasters.

The literature developing proxies for uncertainty was initiated by Bloom (2009) who uses large shifts in U.S. stock market volatility as a proxy for exogenous changes in uncertainty. He finds this measure is a leading indicator for declines in industrial output and employment with a short recessionary effect and a subsequent period of recovery and positive catch-up growth. This pattern is explained as due to drops in real activity as investment and hiring plans are paused in response to higher uncertainty but can be quickly rekindled as this uncertainty dissipates. Baker et al. (2012) develop an economic policy uncertainty index for the U.S. comprised of a frequency count of news stories on uncertainty about the economy or fiscal and monetary policy, the number and revenue impact of scheduled federal taxes set to expire, and the extent of disagreement among economic forecasters over future government purchases and future inflation. Dendy et al. (2013b) pursue a similar methodology for the U.K. focusing on economic rather than policy uncertainty with an index composed of a newspaper search, variation in forecasts of economic variables and mentions of uncertainty in the Bank of England Monetary Policy Committee (MPC) minutes and Financial Stability Reports (FSRs). Both studies find similar results to Bloom’s original study, although without the positive growth catch-up phase, with large negative real effects on employment and industrial production which peak after 1 year to 18 months, respectively, after the shock.

Studies that make exclusive use of forecaster disagreement from surveys of professional forecasters include Dovern et al. (2012), who find that these measures matter more for nominal than real variables in the G7, Bachmann et al. (2013), who use German business climate surveys and find significant (but short lived) decline in production, and Leduc and Liu (2012) who measure perceived uncertainty directly as the fraction of respondents in surveys of businesses and consumers indicating uncertainty about the future as a factor limiting their spending plans (cars for consumers or capital expenditure for firms). The latter find evidence that uncertainty shocks are prototypical aggregate demand shocks with delayed declines in inflation, employment and short term interest rates.

Other studies aim to measure the role of uncertainty through econometric techniques to estimate the time varying volatility of macroeconomic time series.
Mumtaz and Zanetti (2013), studying U.S. data, augment a standard SVAR model to allow for time variation in the volatility of identified monetary policy shocks where the level of endogenous variable included in the VAR and this time varying volatility dynamically interact. They find similar results to Leduc and Liu (2012) with a demand shock like response of falling output, interest rates and inflation. Mumtaz and Surico (2013) extended this to measures of fiscal policy uncertainty using the same methodology. They find that uncertainty about public debt sustainability, government spending and tax changes all have significant contractionary effects on GDP. Using a more structural econometric approach Fernandez-Villaverde et al. (2011a) estimate volatility of government spending and taxes and feed this series of volatility estimates into a general equilibrium model finding similar contractionary patterns for real variables, however, their model indicates that fiscal uncertainty shocks have potentially inflationary effects. Using a similar methodology, Fernandez-Villaverde et al. (2011b), study time-varying volatility in the real interest rates of four emerging small open economies: Argentina, Ecuador, Venezuela, and Brazil. They find that real interest rate volatility leads to a fall in output, consumption, investment, and hours worked. A recent alternative econometric approach pursued by Jurado et al. (2015) measures macroeconomic and financial uncertainty as the conditional variance of the unforecastable component common to a large number of firm-level, macroeconomic and financial variables. This approach indicates uncertainty episodes are less common than the above proxies tend to indicate but that when spikes in uncertainty do occur they are larger and more persistent. These authors find the real macroeconomic effects of their measure of uncertainty lead to a large and protracted drop in real activity (production, hours, employment) without the growth catch-up period found in Bloom (2009). Their results do agree with the results of Bloom (2009); Bloom et al. (2012) in finding a countercyclical pattern to cross-sectional dispersion in firm earnings and productivity however they only find a recessionary effect for an increase in productivity dispersion.

The above results have been challenged by Popescu and Smets (2010) and Gilchrist et al. (2013a), who argue that once a measure of financial stress, as proxied by credit spreads, is included in these regressions the independent role of uncertainty shocks becomes minimal. Popescu and Smets (2010), studying German data, use a VAR with forecaster dispersion as a proxy for uncertainty and credit spreads (corporate and mortgage bond rates to government bonds rates) as a measure of financial stress. They show that the real effects of financial stress are much larger and persistent than those of uncertainty with lower inflation and GDP, and higher unemployment. In contrast to the findings above, they find uncertainty shocks are inflationary once financial stress is controlled for. Similarly, Gilchrist et al. (2013a) seek to discriminate between financial and uncertainty
3.2. Measuring macroeconomic uncertainty

I construct a index of economic uncertainty following Dendy et al. (2013b) for the period 1990-2014. The index is constructed from three sources: (1) Disagreement among professional forecasters about macroeconomic conditions, (2) a count of international and local newspaper articles discussing economic uncertainty in South Africa and (3) mentions of uncertainty in the quarterly economic review of the SARB.

Figure 3.1: Forecaster Disagreement

Source: Die Beeld Newspaper Economist of the Year Competitions. Normalised standard deviation of forecasts across forecasters. Normalised to have a mean and standard of 100 for each variable for the sample period of 1990-2014.

3.2.1 Forecaster disagreement

I use a novel data source to capture forecaster disagreement. Since 1988 the South African national daily newspaper Die Beeld has run a forecasting competition for professional forecasters from the private and public sector. Contestants

shocks role in the business cycle. Their identification procedure uses a penalty function method of Uhlig (2005) to (1) extract the shock explaining the largest forecast error variance of corporate credit spreads (adjusted for predictable default) then (2) do the same for an uncertainty (realised volatility of cross-sectional stock market returns) conditional on the financial shock identified in the first step. They repeat this but reversing the order of shocks. The first identification strategy makes it hard for uncertainty shocks to matter, but it extracts the most powerful financial shock in the system and the second strategy delivers the most powerful uncertainty shock by minimizing the role played by financial shocks. They find that financial shocks are important drivers of the business cycle but that uncertainty shocks are not unless they have their effect through a financial channel i.e. by tightening credit conditions.
are asked to make nowcasts (estimates of current year) and forecasts (estimates for next year) for real GDP growth, CPI inflation, the short term interest rate, gold price, Rand/Dollar exchange rate and the level of current account. The newspaper reports both the mean and standard deviation of across forecasters. I use the reported standard deviation for nowcasts across forecasters as my measure of forecaster disagreement. I use nowcasts since one year ahead forecasts are only available for GDP and CPI from 1996. Gaps in availability of the monthly publication of this data is overcome by aggregating to quarterly through averaging. Unfortunately there remain gaps in this data for 1990 and 1993Q4-1995Q4. To make these comparable I standardise each series to have a mean and standard deviation of 100: $y_q = 100 + 100 \left( \frac{x_q - \bar{x}}{\sigma_x} \right)$. Where $x_q$ is the quarterly data, $y_q$ is the standardised value; $\bar{x}$ is the mean and $\sigma_x$ the standard deviation calculated from the entire sample.

Forecaster disagreement is higher across all variables during the 2008 recession with the most pronounced response for GDP, CPI inflation and the Gold Price (see figure 3.1). The Asian crisis of 1999 and subsequent financial distress associated with Russia’s default on its sovereign debt along with the collapse of Long Term Capital Management had a contagion effect on the Rand with a substantial depreciation in 2001. This appears to introduce greater exchange rate and inflation uncertainty in the next 5 years following this episode. While domestic uncertainty (that over GDP, Interest rates and CPI) has decreased after the great recession, external uncertainty (Gold Price, Exchange Rate, Current Account) remains elevated. The pattern in uncertainty in the gold price and current account mimics the levels of these variables.

Two alternative methods of construction were explored. The first considered use of forecasts instead of nowcast estimates which resulted in a highly similar series (see appendix) and almost no change in the final index to measure uncertainty. The second was using the adjustment of Dovern et al. (2012) to convert the fixed event forecasts in the data to approximate fixed horizon forecasts which are better suited to the notion of uncertainty. Fixed event forecasts are forecast made regarding a fixed event, such as GDP growth in 1992 and forecasters are surveyed as they approach this date. Fixed horizon forecasts are when forecasters give an estimate a fixed time horizon away e.g. forecasters give their estimate for GDP growth 1 year from the time they are surveyed regardless of when they are surveyed. I describe the approximation in the appendix and show that the results are very similar.

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38 The gold price rose from lows of around $300 in the early 2000s to over $1750 in 2011, falling thereafter back down to $1250 by the end of 2014. Similarly the current account to GDP ratio has deteriorated from a surplus of in the early 1990s to a consistent deficit since with a declining trend (around -5% in 2014). Similar depreciation trend is relevant for the Rand with a spike in 2001 and 2009.
3.2.2 News and policy uncertainty

To measure economic uncertainty by news stories I use the Nexis U.K. database of national and international newspapers. I searched for stories based on inclusion of the word stem “econ*” within 10 words of the stem “uncert*” within 10 words of “South Africa”. An informal audit of these results showed that the stories were, in general, about economic uncertainty in South Africa rather than unrelated stories that happen to contain these words. Since the number of articles produced and archived varies over time I normalise the number of articles found in the previous step by the number of articles found that include the term “today” within 10 words of “South Africa”. This is similar to the normalisation used in Baker et al. (2012) where they normalise by the number of articles in the database each month and Dendy et al. (2013b) who normalise by the use of the key word “the” for the U.K. newspapers in their archive. This series is normalised to have a mean and standard deviation of 100 as before.

The results show peaks in uncertainty around 1992Q2, 1996Q1, 1999Q1, 2003, 2008Q3 and 2010Q1 (see figure 3.2). The spike in 1992Q2 relates to news about political and economic change surrounding the end of Apartheid and its potential to extend the protracted recession that began 1989Q4. The rise in 1996Q1 relates to EU/South African free trade area talks, 1999Q1 relates to sharp movement in the Rand, 2003 relates to stagflation induced by the large and persistent exchange rate depreciation in 2001. The spike in 2008Q3 relates to political uncertainty surrounding the resignation of President Thabo Mbeki and potential corruption charges for the leading candidate to succeed him, Jacob Zuma; as well as concerns about global financial developments affecting the domestic economy. News in 2010Q1 was dominated by discussion relating to economic recovery after the 2008 global financial crisis, further deterioration of neighbouring Zimbabwe and concerns over political stability under President Jacob Zuma.

Uncertainty from the perspective of policy makers is measured by searching for mentions of the word stem “uncert*” in the Quarterly Economic Review found in the Quarterly Bulletin of the SARB for 1990-2014. Although published by the monetary authority, this review is broad and covers a range of developments including domestic production, labour markets, housing markets, foreign trade and payments, financial markets and public finance. This is done using the free text analysis software AntConc (Anthony (2014)). This series is normalised to have a mean and standard deviation of 100 as before.

Periods of outstanding uncertainty are 1994Q2, 1996, 2002Q1, 2008Q2 and the period from 2011Q2 onward (see figure 3.1). April 1994 saw the first democratic

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39 The use of the stem econ* means that terms like uncertain, uncertainty, uncertainties, etc. will all be included in the search.
40 Fiscal policy documents, such as the annual budget, and analysis from international organisations, such as the IMF Article IV country reports, are not available at the required frequency (quarterly) and for the sample period.
3.2. Measuring macroeconomic uncertainty

elections in South Africa. Unsurprisingly, policy makers policy were unsure of the political and regulatory environment to follow. 1996 saw elevated levels of turbulence in the demand for South African sovereign bonds, leading to a SARB injection of liquidity by taking 2/3 of a Treasury Bill tender in May. Political uncertainty and labour market unrest helped amplify these concerns leading to bond yield and exchange rate volatility. Uncertainties surrounding the U.S recession, domestic equity market volatility and the large depreciation of the currency are responsible for the peak in 2002Q1. 2008Q2 relates to concerns due to the global financial crisis. The period after 2011 is driven by the chicanery around raising the U.S. federal debt ceiling, the earthquake in Japan, continued uncertainty regarding the stability of the Euro and concerns over the impact of rising interest rates in the U.S..

Figure 3.2: News and policy based measures of uncertainty

(a) News measure of uncertainty

(b) Policy based measure of uncertainty

Sources: Nexis U.K. newspaper archive (News), SARB Quarterly Bulletins (Policy). The News index is a count of articles with the word stem “econ*” within 10 words of the stem “uncert*” within 10 words of “South Africa” for international and South African Newspapers normalised by a count of articles with the term “today” within 10 words of “South Africa”. The policy index is a count of the word stem “uncert*” in the Quarterly Economic Review found in the Quarterly Bulletin of the SARB. Both series are normalised to have a mean and standard deviation of 100.
3.2.3 Macroeconomic uncertainty index

I construct 2 indices of macroeconomic uncertainty. The first uses an equally weighted average of the (standardised) values of forecaster disagreement over GDP, CPI and interest rates and the second, disagreement over the gold price, exchange rate and the current account balance. The first captures domestic issues, the second has more focus on open economy developments. Each index is an equally weighted average of forecaster disagreement with the (standardised) values of news uncertainty and policy uncertainty mentioned above (see figure 3.3). I label the index with domestic focus SAUI and the open economy analogue SAUIO.

The two peak periods of uncertainty effectively identify the key drivers of uncertainty in the first half and second half of the 1990-2014 period. The first peak in 1994, and the period of the 1990s, is principally driven by political uncertainty. The second peak around the 2008 global financial crisis is typical of the period after 2000 when developments in global economy have a contagion effect on South Africa. The 1990s was the most politically turbulent in modern South African history with the unbanning of political organisations, the release of political prisoners along with violent political unrest e.g. around the negotiations to end Apartheid at the Convention for a Democratic South Africa (CODESA). The period after 2000 saw a depreciation of the currency of almost 50% from 2000 to 2002 due to capital flight associated with destabilising effects of the earlier Asian crisis and collapse of Long Term Capital Management in 2000. The period of 2002-2007 saw the highest levels of post-Apartheid GDP growth, off high consumption levels and strong house price growth that ended with the contagion effects of the global financial crisis in 2008. Continued external uncertainty relating to the protracted
recovery from the episode, especially surrounding the eurozone (South Africa’s largest trading partner) and the potentially destabilising effects of the large interest rate differential with developed markets closing when central banks raise base rates above zero for the first time in half a decade. Due to little independent variation in the two indices, I use the SAUI index for the empirical section below.

This measure of uncertainty accords well with other proxies for uncertainty: realised daily volatility of the Johannesburg Stock Exchange All Share index (ALSIVOL) and a measure of option implied volatility based on the 40 largest shares by market value on the JSE index (SAVIT40) - see figure 3.4. The SAUI is correlated with US uncertainty indices but still exhibits independent variation (see figure 3.5). The SAUI has a correlation of 45% with the VIX and 65% with the measure put forward by Jurado et al. (2015). However, those indices are dominated by 2008 financial crisis whereas the SAUI index captures shocks of comparable magnitude in the 1990s period.

Figure 3.4: Comparison with realised and implied stock market volatility

Sources: Bloomberg, author’s calculations. SAUI is the index described in section 2.3, ALSIVOL is the standard deviation of the daily JSE All Share index over each quarter, SAVIT40 is a weighted average of call and put options on JSE Top 40 (i.e. the 40 largest shares on the JSE All Share index) expiring within 3 months and is thus a measure of expected equity market volatility. All series are normalised to have mean and standard deviation of 100.
3.3. Impact of uncertain shocks

3.3.1 VAR model

The benchmark model is given below:

\[ Y_t = A_0 + B(L)Y_{t-1} + e_t \]

Where \( B(L) \) is a matrix lag polynomial of coefficients estimated with Bayesian methods and \( e_t \sim N(0, \Sigma) \). The estimation implements a Normal Wishart prior using dummy observations following Banbura et al. (2010). The variables included in the matrix \( Y_t \) are private sector employment rate, log of Industrial production, log of investment, log of GDP, log of the CPI index, log of the JSE All Share Index, 10 year government bond yield, repurchase rate of the Reserve Bank and the SAUI index. The sample is quarterly and runs from 1990 to 2013Q4. The Schwarz information criteria calls for only 2 lag however I extend this to a lag lent of 3 is based on tests of no serial correlation and normality of the error term \( e_t \).

To identify the structural shocks I use a Cholesky decomposition of \( \hat{\Sigma} \) using a ordering as described above. This identification assumption follows the convention in the VAR literature of assuming with the slower moving macro variables are ordered before fast moving financial variables (for example Popescu and Smets (2010)). The macro bloc is ordered with quantities first and the price level afterwards. I order uncertainty last since it is predominately a measure of agents expectations (which can change very quickly). The results are robust to alternative orderings (see below).

An unanticipated rise in the uncertainty index is associated with a decline in
output, employment, asset prices and investment in the future (see figure 3.6). The effects are most marked for industrial production and investment with a peak fall in industrial production of almost 4%. These results are broadly in line with the findings in the literature where a strong response of industrial production to uncertainty (Dendy et al. (2013b); Leduc and Liu (2012); Baker et al. (2012)). Similarly the strong response of investment accords with the real options view of uncertainty whereby higher levels of uncertainty have a significant effect on investment decisions of firms (Bernanke (1983); Bloom (2009); Bloom et al. (2012)). The effects on GDP and the employment rate are more moderate but still significant with a peak impact of 1.2% and 1.9% after a year and a half, respectively. Asset prices respond with a peak decline of around 13% after a year. Similar negative responses to asset prices have been found for the U.K., although with much less persistence (Dendy et al. (2013b)). In contrast to the studies of (Leduc and Liu (2012)) for the U.S. who find that uncertainty shocks are deflationary, I find that they are associated with 1% increase in the price level after about 1 year. This result accords with the finding of Klein (2011 - IMF) that mark-ups are, contrary to international experience, countercyclical in South Africa. Variance decompositions show that almost half of the forecast error variance (43%) of industrial production is explained by fluctuations in uncertainty (figure 3.7). Similarly, the index is an important component of the variance of the stock market and investment as well as GDP.

Figure 3.6: Impulse Responses to SAUI

Response to Cholesky One S.D. Innovations with 68% and 90% credible intervals. IRFs are annualised and in percent. Sample is from 1990Q1-2013Q4. Cholesky ordering is the baseline: (1) Private Employment Rate (2) log(Industrial Production) (3) log(Investment) (4) log(GDP) (5) log(CPI) (6) log(All Share Index) (7) 10 Year Government Bond Yield (8) Repo Rate (9) Uncertainty Index SAUI

3.3.2 Robustness
These results extend the findings for developed nations that uncertainty shocks are an important source of business cycle fluctuations. In order to test the robustness of these results I augment the VAR above to include consumer confidence and a measure of the financial stress, in the form of a bank lending credit spread. The first robustness check follows Baker et al. (2012), who include consumer confidence in order to disentangle uncertainty (a mean-preserving increase in the variance of macro variables) from that of bad news (a change in the mean). Consumer confidence is the OECD consumer opinion survey composite indicator. The second is motivated by the recent debate in the literature that the effects of uncertainty shocks are primarily through their impact on financial conditions, i.e. higher uncertainty matters because it raises risk levels and credit spreads, but have little effect in themselves. Gilchrist et al. (2013b), studying the U.S., and Popescu and Smets (2010), looking at German data, find that once credit conditions are controlled for the impact of uncertainty shocks on the real economy is relatively modest. Those authors used the spread of corporate to government bond yields as a proxy for financial stress.

Figure 3.7: Contribution of SAUI to Forecast Error Variance

Unfortunately bond market in South Africa is dominated by government instruments, and features only a small number of (mostly state owned) firms (Hassan (2013)). Thus using a market based measure of corporate spreads would be undesirable. Instead I construct a measure of the bank lending conditions facing firms as the spread on new fixed-rate instalment sale credit over the 3 month Negotiable Certificate of Deposit rate at banks. The first measures credit conditions facing firms and households seeking credit on movable property and the second is closely tied to the South African Benchmark Overnight Rate (SABOR) used for interbank lending, however a longer series is available for the Negotiable Certificate
3.3. Impact of uncertainty shocks

of Deposit (NCD) rate. Both series are available from the SARB.

Figure 3.8: Uncertainty, Consumer Confidence and Credit Spreads

Source: Authors calculations, SARB, FRED database of St. Louis Federal Reserve. Consumer confidence is the OECD consumer opinion survey composite indicator from FRED (CSCICP02ZAQ460N). Credit Spread is the difference between the bank lending rate on new fixed-rate instalment sale credit (KBP1181) less the NCD rate (KBP1411).

Consumer confidence and uncertainty are negatively correlated (see figure 3.8). Consumer confidence improves during the boom years of the early 2000s and collapses in 2008 as the financial crisis hits, uncertainty follows the inverse pattern. Credit spreads are weakly positively correlated with uncertainty (28%) with generally lower spreads during the boom years and a spike in rates as the global financial crisis hits South Africa. Interestingly this spike only happens about a year after the spikes in uncertainty and consumer confidence. It took about a year for the contagion effects of dislocation in credit markets in the U.S. and Europe to translate into a recession in South Africa. This is reflected in lending conditions. This timing is helpful to distinguish the role of uncertainty from financial stress in that these two were not as highly correlated as in the markets were the global financial crisis originated.

The baseline results are robust to the inclusion of both consumer confidence and the credit spread measure (see figure 3.9)\(^{41}\). The size of the effects of uncertainty on industrial production, GDP and investment are moderated and there is evidence of a period of lower prices after about 2 years following the initial inflationary period. This result is noteworthy as, for example, Bachmann et al. (2013) finds that effects of uncertainty shocks are not robust to the inclusion of consumer

\(^{41}\)For each robustness exercise presented here IRFs with credible sets are available in the appendix
3.3. Impact of uncertainty shocks

confidence. Moreover, the evidence in developed markets that uncertainty only matters as a proxy for financial stress is not supported by these findings.

The series for forecaster disagreement has missing data for 1992, 1994 and 1995. Thus the uncertainty index is comprised only of news and policy uncertainty for these years. To check the robustness of the results to this I repeat the baseline regressions for 1996-2013. The results are very similar to the case with the credit spread and consumer confidence included in the VAR. The uncertainty index for South Africa tends to rise during times of global uncertainty in addition to domestic developments (see figure 3.3). Thus it may be that much of the affects found are due to correlation with global uncertainty shocks. To control for this I include the VIX index (ordered before the South Africa uncertainty index to reflect that SAUI will respond to the VIX on impact but not vice versa). The results broadly agree with the baseline, with some further, but moderate, evidence for deflation.

Figure 3.9: Robustness of Impulse Responses to SAUI

Response to one standard deviation shock to SAUI. IRFs are annualised and in percent.

The baseline results are robust to Cholesky ordering of the shocks, however results are more sensitive to identification with sign restrictions (see figure 3.10). I impose short run sign restrictions following previous findings for uncertainty shocks in literature cited above but leave the response of inflation and the repo (which will be strongly correlated with inflation) unrestricted (see table 3.1). The results for GDP, investment and share prices are broadly similar. The effects for inflation, industrial production and employment are more muted, somewhat in-line with the results from long run restrictions. Moreover, I find that the inflationary
3.4 Discussion of results

Table 3.1: Sign restrictions

<table>
<thead>
<tr>
<th>Employment</th>
<th>IPI</th>
<th>Inv</th>
<th>GDP</th>
<th>CPI</th>
<th>ALSI</th>
<th>Gov Bond</th>
<th>Repo</th>
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</tbody>
</table>

The effect is no longer significant.

Figure 3.10: Robustness of Impulse Responses to SAUI

Response to one standard deviation shock to SAUI. IRFs are annualised and in percent.

3.4 Discussion of results

The results discussed above generally indicate inflation following an uncertainty shock. Here I discuss three the potential causal channels relevant in understanding this effect: a precautionary savings channel, an exchange rate channel and a upward pricing channel.

The precautionary savings channel refers to the contractionary effects of high uncertainty leading to paused spending and investment projects along with higher savings by risk averse households. This drop in demand incentivises profit maximising firms to lower their prices and is the mechanism behind the finding of deflation in other studies (e.g. Leduc and Liu (2012)). It is possible that this mechanism is very weak following an uncertainty shock in South Africa reducing the likelihood of a deflationary period. Consumption drops following an uncertainty shock, with a peak decline of 1% after 1.5 years (See figure 3.11). The response of savings is weak, taking a full year before a significant rise is seen. This helps rationalise the timing of response of inflation: the inflationary impact occurs in the first year when the precautionary savings effects are weaker while deflation may occur at longer horizons when this effect is stronger.

There remains the question of why inflation occurs at shorter horizons. There is a growing body of evidence suggesting that capital flight out of emerging markets
3.4. Discussion of results

Figure 3.11: Impulse responses to SAUI: The precautionary savings channel

Response to Cholesky One S.D. Innovations with 68% and 90% credible intervals. IRFs are annualised and in percent. Sample is from 1990Q1-2013Q3. Cholesky ordering is: (1) Private Employment Rate (2) log(Industrial Production) (3) log(Investment) (4) log(GDP) (5) log(Consumption Index) (6) log(Savings in Rands) (7) log(CPI) (8) log(All Share Index) (9) 10 Year Government Bond Yield (10) Repo Rate (11) Uncertainty Index SAUI

is driven by global risk shocks as proxied by indices such as the VIX (Passari and Rey (2015) and Nier et al. (2015)). If this capital flight is associated with nominal exchange rate depreciation then import prices should rise in the short term (when the valuation effect on imports dominates expenditure switching effects on export demand). Including open economy variables to measure this effect, an import price index, the balance on the financial account (BoFA) and the nominal exchange rate (Rands/$) does lend support to this channel. While a small depreciation follows the uncertainty shock on impact and the inclusion of the exchange rate in the VAR does reduce the size of the inflationary effect to around 0.5% from 1% in the baseline, this depreciation doesn’t translate into higher import prices.

Finally the upward pricing bias channel may lead firms and wage setters to raise prices despite a drop in demand for their goods and labour. An uncertainty shock makes future demand more uncertainty for wage setting households and price setting firms. The presence of nominal rigidities means that price and wage setters can get stuck with the price they choose for many periods. It is also the case the the pay-off for both firms and workers in setting prices is asymmetric: losses due to a low relative price are much larger than gains from a relative price that is too high. These incentives combine to provide an insurance value to raising prices or wages when an uncertainty shock hits to reduce the chance that firms/workers are stuck with a relative price/wage that is too low (when output rebounds).

The upward pricing bias channel was first noted by Fernandez-Villaverde et al. (2011a) for the case of price setting firms subject to Calvo nominal rigidities in
3.5 Conclusion

Figure 3.12: Impulse responses to SAUI: The exchange rate channel

The context of uncertainty shocks to capital taxes in a calibrated New Keynesian Model. In the appendix I use a medium scale New Keynesian model to show that the same mechanism applies for wage setting in the context of uncertainty shocks to exogenous technology. Moreover, in this model the wage setting bias channel is much more important in generating positive inflation following the technology uncertainty shock than is the impact of price setters bias.

Despite the fact that the labour market in South Africa is very rigid with highly active unions supporting the plausibility of this channel, it is not supported by adding wages to the baseline VAR: wages fall following an uncertainty shock (see figure 3.13).

Whilst none of the channels discussed above is able to explain the inflationary response to uncertainty shocks an explanation may lie in the counter-cyclical behaviour of price mark-ups in South Africa - a fact documented by Klein (2011). The recessionary effect of the uncertainty shock in GDP/Industrial production then translate into higher mark-ups - the timing of this decline accords with the rise in inflation. Of course the real question is why does South Africa experience counter-cyclical mark-ups. The empirical evidence presented here along with the theoretical upward pricing channel suggests one explanation.

3.5 Conclusion

This paper develops a new index of macroeconomic uncertainty in South Africa using (1) forecaster disagreement among professional forecasters about macroeco-
Figure 3.13: Impulse responses to SAUI: Upward pricing bias channel

Response to Cholesky One S.D. Innovations with 68% and 90% credible intervals. IRFs are annualised and in percent. Sample is from 1990Q1-2013Q3. Cholesky ordering is: (1) Private Employment Rate (2) log(Industrial Production) (3) log(Investment) (4) log(GDP) (5) log(wages) (6) log(CPI) (7) log(All Share Index) (8) 10 Year Government Bond Yield (9) Repo Rate (10) Uncertainty Index SAUI

Economic conditions, (2) newspaper articles about uncertainty and (3) uncertainty from the perspective of policy makers. The impact of unanticipated increases in uncertainty is studied using a Structural VAR. These results provide evidence that a rise in uncertainty is important for the business cycle in South Africa, as has been found for the U.S.A. and the U.K., with a decline in GDP, investment, industrial production and private sector employment. However, in contrast to those developed market studies, I find that uncertainty shocks are inflationary. This effect is robust to controlling for financial stress in the form of a credit spreads, measured by bank lending rates relative deposit rates, and consumer confidence - a proxy to disentangle the effect of higher uncertainty from bad news.

These results suggest that both fiscal and monetary policy makers should monitor the levels of economic uncertainty as this may foreshadow a decline in economic activity. Moreover, the empirical and theoretical results show that uncertainty shocks are particularly pernicious for inflation targeting central banks in that they have stagflation effects. Thus it may be worthwhile for South African policy makers to survey professional forecasters as is done in the U.S.A. (Survey of Professional Forecasters by the Philadelphia Federal Reserve), U.K. (Forecasts for the UK economy by HM Treasury) and the E.U. (Survey of Professional Forecasters by the ECB). This would allow for a richer study of uncertainty, for example perceived subjective uncertainty in the form of forecast distributions by individual forecasters.

Future empirical research could formally explore the ability of this index to forecast economic activity, study firm level data to test the theoretical mechanism of precautionary pricing and develop measures of uncertainty more focused on policy and political uncertainty.
3.6 Appendix

3.6.1 Fixed event and fixed horizon forecasts

The data contains fixed event forecasts: each forecast is based on expectations over the current calendar year as opposed to a forecast for the value of a variable 1 year ahead (a fixed horizon forecast). For example, a forecast of GDP growth in quarter 1 of 1992 and quarter 4 of 1992 are both expectations of GDP growth for the year 1992\(^{42}\). Since forecasts made closer to the data of the forecast year have more information there is likely to be both less forecaster uncertainty and this may be manifest in greater seasonality in the series for forecaster dispersion. To address this issue I follow Dovern et al. (2012) in re-weighting the forecast data to approximate fixed horizon forecasts. Let \(S_{y0,q,y1}^{fe}(x)\) denote the fixed event forecast for the variable \(x\) in year \(y1\) which is made in the previous year \(y0\), \(y0 = y1 - 1\), and quarter \(q\). For example the forecast for 1992 made in quarter 1 of 1992 is \(S_{1992,1,1992}^{fe}(x)\) and forecast for 1993 made in quarter 1 of 1992 \(S_{1992,1,1993}^{fe}(x)\). The fixed horizon forecast is approximated as:

\[
S_{y0,q,y1}^{fh}(x) = \frac{4 - q + 1}{4}S_{y0,q,y0}^{fe}(x) + \frac{q - 1}{4}S_{y0,q,y0+1}^{fe}(x)
\]

For example, the forecast of GDP growth between Q4 1992 and Q4 1993 is approximated by the sum of \(S_{1992,4,1992}^{fe}(GDP)\) and \(S_{1992,4,1993}^{fe}(GDP)\) with weights of \(\frac{1}{4}\) and \(\frac{3}{4}\), respectively, since the first forecast has a 1 quarter horizon for the forecaster surveyed and the second has 3 quarter horizon. Ideally this could be done on the raw data for each forecasters for each quarter. Unfortunately, there are too many data gaps to pursue this approach and consequently I have to perform this adjustment with the standard deviations across forecasters which is available for every quarter\(^{43}\). See figure 3.14 for comparision of the Dovern adjusted and unadjusted forecaster dispersion.

\(^{42}\) The official data for Q4 GDP growth would only be released in Q1 1993, at the end of February

\(^{43}\) A significant portion of the forecast data was recovered from archives at the National Library of South Africa. These archives are both incomplete and require ordering a physical copy of each newspaper where the data is expected to be found. Since it is not known which day of the month the competition results for that month will be published, it is a challenge to find even one table of this data. Happily the task of recovering the standard deviations across forecasters was made feasible by the fact that the last table of the year included this standard deviation data for all previous months. However it does not include individual forecast data for each month.
3.6. Appendix

Figure 3.14: Fixed event and fixed horizon forecasts dispersion

3.6.2 Nowcasts vs. forecasts

Figure 3.15: GDP nowcasts vs. forecasts (+1)
3.6. Appendix

Figure 3.16: CPI nowcasts vs. forecasts (+1)

3.6.3 Upward pricing bias channel in the medium scale New Keynesian model of Fernandez-Villaverde (2006)

In this appendix I examine the channels driving the response of prices to an uncertainty shock using the medium scale closed economy New Keynesian model of Fernandez-Villaverde and Rubio-Ramirez (2006). For a full description of this model the reader is directed to those authors’ paper\(^{44}\), here I will outline the (completely standard) components of their model and focus on the impact of an uncertainty shock.

Households consume (with external habit formation), save and supply differentiated labour services subject to a Dixit-Stiglitz demand curve and Calvo staggered wage setting. These labour services are purchased and aggregated by a labour-packer firm who rents the aggregate labour input to intermediate producing firms. Intermediate firms rent capital (subject to variable capital utilisation and convex adjustment costs) and labour to manufacture their good and are subject to Calvo rigidities in setting the prices faced by the competitive final goods producer. Monetary policy operates by controlling the one-period nominal interest rate through open market operations with government bonds held by households. The model is calibrated at quarterly frequency following Fernandez-Villaverde (2010).

I study a shock to the volatility of exogenous technology of intermediate firms as a proxy for an uncertainty shock, a shock to \(\eta_t\):

\[
Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}
\]

\(^{44}\)Available here: http://economics.sas.upenn.edu/~jesusfv/benchmark_DSGE.pdf
\[
\log A_t = \rho_a \log A_{t-1} + \sigma_t^A \epsilon_t
\]

\[
\log \sigma_t^A = (1 - \rho_\sigma) \log \sigma^A + \rho_\sigma \log \sigma_t^{A-1} + \nu_\eta_t
\]

The solution to the model is found via a third-order perturbation method using Dynare 4.4.3. This is necessary since a first order solution results in certainty equivalence where an increase in the variance of a shock has no effect. A second-order solution would only include the shocks to the variance of technology as a cross-products with the shock to the level of technology, thus the variance of technology has no impact unless the level of technology is being changed at the same time. Only a third-order solution allows me to study the effect of a mean-preserving increase in the variance of technology, the appropriate proxy for an increase in uncertainty. However, higher order solutions can induce explosive terms when the model is simulated. In order to resolve this I use the pruning solution in Dynare which follows Andreasen et al. (2013). Pruning solves this problem leaving out terms in the solution that have higher-order effects than the approximation order. For example, this would occur when a second order solution to one variable is substituted into the policy function for another which is also approximated by a quadratic function of the state variables, resulting in terms of 3rd and 4th order. Pruning removes these terms from the solution inducing stability.

Figure 3.17: Technology uncertainty shock (baseline calibration)

IRFs are annualised and in percent change from the steady state.
To study the impact of a one standard deviation increase in the volatility of technology ($\eta_t$), I follow Fernandez-Villaverde et al. (2011b) in calculating the impulse response functions. This involves calculating the deviation of the model from the ergodic steady state after a one standard deviation shock to $\eta_t$. This is preferable to the deviation from the deterministic steady state since the unconditional moments of variables solved under higher-order approximations are, in general, not equal to their steady state values since these solutions include non-linear terms that correct for uncertainty (see Andreasen et al. (2013)). The computation proceeds as follows. I simulate the model with all shocks set to zero for 2048 periods starting at the deterministic steady state. I take the ergodic mean as the value each variable has converged to after 2000 iterations. I then use the last 48 periods to find the response with the volatility shock by setting the volatility shock ($\eta_t$) to one standard deviation and simulating the for 48 periods, starting at the ergodic means. The impulse responses are reported as the deviations from the ergodic mean of each variable.

Figure 3.18: Inflation response to technology uncertainty shock

IRFs are annualised and in percent change from the steady state. The proportion of labour suppliers who can reset their wage each period is $1 - \theta_{wages}$. I assume a level of $\theta_{prices} = 0.4$ (baseline is $\theta_{prices} = 0.8$).

The results from this baseline calibration accord well with the VAR results presented in the main section (see figure 3.17). This model then provides an environment to understand why inflation follows an uncertainty shock. In this closed economy environment the precautionary savings channel acts to reduce prices whilst the upward pricing bias channel acts to raise them. Thus it should be the case that when prices and wages are more flexible inflation should fall following an uncertainty shock. Moreover, the importance of firms’ upward pricing bias relative to that of labour suppliers can be studied. I examine these channels by increasing the proportion of firms that can reset prices from the baseline of 20% to 60% and consider the response of inflation when the proportion of labour
suppliers able to reset their wage moves from the baseline of 32% towards 100% (flexible wages) - see figure 3.18. When prices are more flexible and wages are fairly flexible (100% to 85% of labour suppliers can reset wages each period) the precautionary savings motive dominates upward pricing bias effects. For higher levels of nominal rigidities in wage setting, where 70% or less of labour suppliers can reset wages each period, uncertainty shocks are inflationary. Thus, in the standard macroeconomic model used by central banks, the key rigidity generating an inflationary response to uncertainty shocks in the strength of nominal wage rigidity.

As described in the main text, the upward pricing bias is due to two features of the model, firstly, workers are subject to nominal rigidities and thus may be unable to change their wages for a number of periods; secondly, the pay-off they face in choosing the relative wage strongly biased towards avoiding states with a low relative wage (see figure 3.19). This leads workers to raise wages as a precautionary measure when future demand for their labour becomes more uncertain.

Figure 3.19: Utility pay-off of $j^{th}$ household choosing $w_j$: $\alpha \left(\frac{w_j}{w}\right)^{1-\eta} - \beta \left(\frac{w_j}{w}\right)^{-\eta(1+\gamma)}$

$w_j/w$ is the relative price of the $j$th household. $\eta$ is the elasticity of substitution between labour types and $\gamma$ is the inverse of the Frisch elasticity of labour supply. $\eta = 10, \gamma = 1$ are the baseline values. $\alpha, \beta$ are functions of steady state parameters. $\alpha = \lambda w l^d$ where $\lambda$ is the marginal utility of wealth and $l^d$ is the aggregate demand for labour. $\beta = (l^d)^{1+\gamma}/1 + \gamma$. 

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3.6.4 Robustness checks: Impulse response functions

Figure 3.20: IRFs: Baseline + Confidence Index + Credit Spread

Response to Cholesky One S.D. Innovations with 68% and 90% credible intervals. IRFs are annualised and in percent. Sample is from 1990Q1-2013Q3. Cholesky ordering: (1) Private Employment Rate (2) log(Industrial Production) (3) log(Investment) (4) log(GDP) (5) log(CPI) (6) log(All Share Index) (7) 10 Year Government Bond Yield (8) Repo Rate (9) Credit Spread Uncertainty Index (10) SAUI (11) Consumer Confidence

Figure 3.21: IRFs: Short Sample: 1996-2013Q4

Response to Cholesky One S.D. Innovations with 68% and 90% credible intervals. IRFs are annualised and in percent. Sample is from 1996Q1-2013Q4. Cholesky ordering is the baseline: (1) Private Employment Rate (2) log(Industrial Production) (3) log(Investment) (4) log(GDP) (5) log(CPI) (6) log(All Share Index) (7) 10 Year Government Bond Yield (8) Repo Rate (9) Credit Spread Uncertainty Index (10) SAUI
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