

A time varying DSGE model with financial frictions*

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Abstract

We build a time varying DSGE model with financial frictions in order to evaluate changes in the responses of the macroeconomy to financial friction shocks. Using US data, we find that the transmission of the financial friction shock to economic variables, such as output growth, has not changed in the last 30 years. The volatility of the financial friction shock, however, has changed, so that output responses to a one-standard deviation of the shock increase twofold in the 2007-2011 period in comparison with the 1985-2006 period. The time varying DSGE model with financial frictions improves the accuracy of forecasts of output growth and inflation during the tranquil period of 2000-2006, while delivering similar performance to the fixed coefficient DSGE model for the 2007-2012 period.

JEL codes: C11, C53, E27, E52

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1 Introduction

Dynamic stochastic general equilibrium (DSGE) models are popular tools used for policy analysis and macroeconomic forecasting. Their success is a result of their capacity to combine economic microfoundations derived from the optimisation decisions of rational agents with business cycle fluctuations. Traditionally, the consensus in the macroeconomic literature has been an apparent trade-off between theoretical coherence, relating to model outcomes being explained by reference to some well-established theory, and empirical coherence, relating to the ability of a model to fit and explain macro data well. Models that exhibit both theoretical and empirical coherence were deemed infeasible. DSGE models were traditionally at the theoretical end of this trade-off curve. At the empirical end, we find non-structural reduced-form models, such as VARs, which exploit correlations in time series with little reliance on macroeconomic theory. It was the work of Smets and Wouters (2003, 2005, 2007), based on earlier work of Rotemberg and Woodford (1997) and Christiano, Eichenbaum and Evans (2005), that changed such perception and demonstrated that medium-sized DSGE models are not too abstract to be taken to the data and successfully employed in forecasting. Following Smets and Wouters (2007), many authors evaluated the DSGE models's forecasting performance, providing evidence that they can produce accurate forecasts of output growth and inflation in real time (Edge and Guerkaaynak (2010), Wouters (2012) and Del Negro and Schorfheide (2013)).

However, the recent financial crisis has posed a serious challenge to macroeconomic modelling. Perhaps the most important aspect of this challenge is the inability of standard DSGE models to accommodate the impact of developments in the financial sector on the rest of the economy. Based on the seminal work of Bernanke, Gertler and Gilchrist (1999), various authors have exploited financial channels in a DSGE structure as a way of improving the fit of the DSGE model to the 2008-2009 global financial crisis, including Christiano, Motto and Rostagno (2014), Del Negro and Schorfheide (2013) and Del Negro, Hasegawa and Schorfheide (2014). Interestingly, Del Negro et al. (2014) find that the Smets and Wouters (2007) model with financial frictions, while delivering relatively better forecasts during the crisis, performs worse in tranquil periods than the model without financial frictions. This is consistent with evidence of the changing predictive power of various economic and financial indicators on U.S. output and inflation (Stock and Watson (2003)). Even if asset prices are, on average, poor indicators of economic activity, their predictive power should have increased during the recent financial crisis. For example, Gilchrist and Zakrajsek

(2011) and Philippon (2009) argue that the predictive power of corporate bond credit spread for the business cycle and economic activity reveal the potential of bond markets to signal (even more accurately than stock markets) the decline in fundamentals prior to the 2007-2008 business cycle downturn.

Incorporating a financial channel in a DSGE model may not be enough to address the effect that structural changes in the underlying economy might have on preference parameters and on exogenous shock processes. A standard assumption in the literature is that the DSGE parameters are structural in the Lucas sense, that is, they are invariant to both policy and structural shocks. However, this does not imply that they are constant at all time scales. Long term cultural or technological shifts might result in slow parameter variation. While DSGE analysis focuses primarily on business cycle frequency, parameter drift is potentially of great importance when considering sample periods of over 40 years, which are routinely used for estimation and calibration of DSGE models. A related issue is the extent to which all parameters of medium-sized DSGE models are equally immune to the Lucas critique. While parameters such as households' discount factor with distinct microfoundations may be unaffected to long run change, other parameters associated with rigidity dynamics have a reduced-form flavour and may be more vulnerable to technological or social change, or other factors. Even if one believes in the structural nature of DSGE parameters, it is important that one recognises at least the possibility of time variation in the parameters when estimated over long time periods.

Time variation in the preference parameters or in the volatility of structural shocks of a DSGE model have been modelled by specifying a stochastic process for a small subset of the parameters (Justiniano and Primiceri (2008), Fernandez-Villaverde and Rubio-Ramirez (2008)). For instance, Fernandez-Villaverde and Rubio-Ramirez (2008) assume that agents in the model take into account future parameter variation when forming their expectations. Similar assumptions are made by Schorfheide (2005), Bianchi (2013), Foerster, Rubio-Ramirez, Waggoner and Zha (2014), but the parameters are modelled as Markov-switching processes.

In contrast, Canova (2006), Canova and Sala (2009) and Castelnuovo (2012) allow for parameter variation by estimating DSGE models over rolling samples. In recent work, Galvão, Giraitis, Kapetanios and Petrova (2015a) have provided a new approach that allows time varying estimation of Bayesian models, used for the time varying estimation of the Smets and Wouters (2007) DSGE model in Galvão, Giraitis, Kapetanios and Petrova (2015b). Their approach is an extension and formalisation of rolling window estimation, generalised by combining kernel-generated local

likelihoods with appropriately chosen priors to generate a sequence of posterior distributions for the objects of interest over time, following the methodology developed in Giraitis, Kapetanios and Yates (2014) and Giraitis, Kapetanios, Wetherilt and Zikes (2016). Both the kernel and the rolling window approaches, when applied to structural models, assume that, instead of being endowed with perfect knowledge about the economy’s data generating process, agents take parameter variation as exogenous when forming their expectations about the future. This assumption facilitates estimation and can be rationalised from the perspective of models featuring learning problems, where agents form beliefs about the parameters based on observing past data. For example, Cogley and Sargent (2009) utilise Kreps (1998)’s anticipated utility approach, where in each period agents employ their current beliefs as the true (time invariant) parameters. They show that in the presence of parameter uncertainty, the anticipated utility approach outperforms the rational expectation approximation. A recent application of the anticipated utility approach is Johannes, Lochstoer and Mou (2015), where assets are priced at each point in time, using current posterior means for the parameters and assuming that current values will last indefinitely in the future. At each period, agents learn the new parameter values and adjust their expectations¹.

In this paper, we employ the approach of Galvão et al. (2015b) to investigate the changing nature of the effect of financial frictions to the rest of the economy. The model we investigate is a Smets and Wouters (2007) model with an added financial sector as in Bernanke et al. (1999) and Del Negro and Schorfheide (2013). The advantage of the specification discussed in this paper is that the importance of the financial frictions for macroeconomic variables depends on a preference parameter and on the stochastic properties of the new financial friction shock. By looking at the possibility of time variation in these parameters, while also allowing all other DSGE parameters to change over time, we can measure whether the significance of financial frictions change over time. We find that the parameter that triggers the transmission of financial frictions to the economy remains relatively constant during the entire sample period we analyse. However, the volatility of the financial friction shock rises dramatically during the 2007-2011 period. This new finding contributes to the debate between ‘Good Luck’ versus ‘Good Policy’ when explaining the Great Moderation (Gali and Gambetti (2009), Benati and Surico (2009), Sims and Zha (2006)). We provide evidence that the financial frictions shock was muted during the 1985-2007 period. Note

¹A similar outcome can be achieved in a linearised DSGE model with random walk processes for the drifting parameters (a frequently made assumption in the time varying parameter VAR literature), where rational expectations on the side of agents would imply that the future values of the parameters are equal to the current posterior means.

that our model presents arguments in favour of changes in the volatility of the shocks while also allowing for changes in the parameters of the policy rule. As a consequence, this paper produces a new source of evidence of ‘Good Luck’ during the Great Moderation period while also allowing for a ‘Good Policy’ channel. Related investigation of changes in the volatility of financial shocks over time is presented in Fuentes-Albero (2014), where a Smets and Wouters (2007) DSGE model with financial frictions is estimated with constant parameters over different subsamples and the breaks in the volatility of the residuals of the model are dated. The author finds that the size of the financial shocks has increased over time, while their importance in explaining non-financial variables has remained relatively unchanged. Cardani, Paccagnini and Villa (2015) estimate a Smets and Wouters (2007) model with banking intermediation and report the posterior means of the parameters over time implied by their recursive forecasting scheme, providing further evidence of the changing importance of the financial shock in their model.

This paper also exploits the forecasting performance of the time-varying DSGE model with financial frictions, extending the results of Del Negro and Schorfheide (2013) and Kolasa and Rubaszek (2015), who use only fixed parameters specifications.

The paper is organised as follows. Section 2 contains a brief account of the Bayesian Local Likelihood (BLL) approach for DSGE models proposed by Galvão et al. (2015b). Section 3 describes the DSGE model used in the empirical applications in Section 4. Finally, Section 5 concludes.

2 Modelling time variation in DSGE parameters

This section outlines the estimation strategy used in our local Bayesian Local Likelihood (BLL) method. The linearized rational expectation model can be written in the form

$$A(\theta_t)x_{t+1} = B(\theta_t)x_t + C(\theta_t)v_t + D(\theta_t)\eta_{t+1}, \quad v_t \sim N(0, Q(\theta_t))$$

where x_t is a $n \times 1$ the vector containing the model’s endogenous and exogenous variables, v_t is a $k \times 1$ vector of structural shocks, η_{t+1} is an $l \times 1$ vector of expectation errors, θ_t is a vector of parameters, including parameters governing preferences and the shocks’ stochastic processes, A, B, C and D are matrix functions of θ_t , and $Q(\theta_t)$ is a diagonal covariance matrix. Observe that we have one such equation *for each point in time* t .

A numerical solution of the rational expectations model can be obtained by one of the available methods (for instance, Blanchard and Kahn (1980) or Sims (2002)). The resulting state equation

is given by

$$x_t = F(\theta_t)x_{t-1} + G(\theta_t)v_t \quad (1)$$

where the $n \times n$ matrix F and the $n \times k$ matrix G can be computed numerically for a given parameter vector θ_t . The system is augmented with a measurement equation:

$$Y_t = K(\theta_t) + Z(\theta_t)x_t \quad (2)$$

where Y_t is a $m \times 1$ vector of observables, normally of a smaller dimension than x_t (i.e. $m < n$) and Z is a $m \times n$ matrix that links those observables to the latent variables in the model x_t .

Equations (1) and (2) define the state space representation of the model, which is linear and Gaussian. Therefore, the Kalman filter can be employed to recursively build the likelihood of the sample of observables $\{Y_j\}_{j=1}^T$. The local likelihood of the sample - the weighted product of the likelihood functions of each observation - is given by

$$L_t(Y|\theta_t) = \prod_{j=1}^T L(Y_j|Y^{j-1}, \theta_t)^{w_{tj}} \text{ for } t = 1, \dots, T$$

where w_{tj} is an element of the $T \times T$ weighting matrix $W = [w_{tj}]$, computed using a kernel function

$$\tilde{w}_{tj} = K((t-j)/H) \text{ for } j, t = 1, \dots, T \quad (3)$$

with a bandwidth parameter H . The weights are then normalised to sum to $2H + 1$ for each t , i.e.,

$$w_{tj} = (2H + 1) \left(\tilde{w}_{tj} / \sum_{j=1}^T \tilde{w}_{tj} \right) \text{ for } j, t = 1, \dots, T.$$

In the fixed parameter case, the weights on each likelihood sum up to T . The difference comes from the rolling window methodology and reflects its use of downweighting/subsampling. The normalisation is employed to maintain the relative balance between the likelihood and the prior. In this paper, the normal kernel function

$$K(x) = \frac{1}{\sqrt{2\pi}} \exp \left\{ -\frac{1}{2}x^2 \right\}$$

is used to generate the weights w_{tj} . We set $H = \sqrt{T}$, in line with the optimal bandwidth parameter choice used for inference of time varying random coefficient models in Giraitis et al. (2014).

The local likelihood of the DSGE model at time t , denoted by $L_t(Y|\theta_t)$, is augmented with the prior distribution of the structural parameters, $p(\theta_t)$, to get the posterior at time t , $p(\theta_t|Y)$:

$$p(\theta_t|Y) = \frac{L_t(Y|\theta_t)p(\theta_t)}{p(Y)} \propto \prod_{j=1}^T L(Y_j|Y^{j-1}, \theta_t)^{w_{tj}} p(\theta_t).$$

It should be noted that for our DSGE investigation, we assume the prior $p(\theta_t)$ to be fixed over time, i.e., $p(\theta_t) = p(\theta)$ for all t .

One could potentially allow the prior to be time varying, exploring further the idea that the posterior yesterday can be used for a prior today. However, since we focus only on the possibility of parameter change driven by the data, we assume that the prior is constant over time.

To obtain the joint posterior distribution of the parameters, we use numerical methods since the matrices F and G are non-linear functions of θ and hence the posterior does not fall into families of known distributions and we cannot derive moments of that posterior analytically. The most commonly used procedure to generate draws from the posterior distribution of θ is the Metropolis, Rosenbluth, Rosenbluth, Teller and Teller (1953) algorithm, and its generalisation by Hastings (1970). The algorithm described here is the Schorfheide (2000)'s Random Walk Metropolis (RWM) modified to include the kernel weights. For each point in time $t = 1, \dots, T$, the algorithm implements the following steps.

Step 1 The posterior is log-linearised and passed to a numerical optimisation routine. Optimisation with respect to θ_t is performed to obtain the posterior mode,

$$\hat{\theta}_t = \arg \min_{\theta} \left(- \sum_{j=1}^T w_{tj} \log L(Y_j | Y^{j-1}, \theta_t) - \log p(\theta_t) \right).$$

Step 2 Numerically compute $\hat{\Sigma}_t$, the inverse of the (negative) Hessian, evaluated at the posterior mode, $\hat{\theta}_t$.

Step 3 Draw an initial value θ_t^0 from $N(\hat{\theta}_t, c_0^2 \hat{\Sigma}_t)$.

Step 4 For $k = 1, \dots, n_{sim}$, draw ζ_t from the proposal distribution $N(\theta_t^{(k-1)}, c^2 \hat{\Sigma}_t)$.

Compute

$$r(\theta_t^{k-1}, \zeta_t | Y_{1:T}) = p(\zeta_t | Y) / p(\theta_t^{k-1} | Y) = \prod_{j=1}^T L(Y_j | Y^{j-1}, \zeta_t)^{w_{tj}} p(\zeta_t) / \prod_{j=1}^T L(Y_j | Y^{j-1}, \theta_t^{k-1})^{w_{tj}} p(\theta_t^{k-1}),$$

which is the ratio between the *weighted* posterior at the proposal draw ζ_t and the previous draw θ_t^{k-1} .

The draw ζ_t is accepted (setting $\theta_t^k = \zeta_t$) with probability $\tau = \min\{1, r(\theta_t^{k-1}, \zeta_t | Y_{1:T})\}$ and rejected ($\theta_t^k = \theta_t^{k-1}$) with probability $1 - \tau$.

Once the posterior distribution of the parameters is obtained, out-of-sample forecasts can be generated. For each forecast, we only need the posterior distribution at the end of the corresponding in-sample period. Therefore, for generating DSGE-based forecasts, our method is no more

computationally intensive than a standard fixed parameter DSGE forecasting: it requires the computation of the posterior only once. The predictive distribution of the sample $p(Y_{T+1:T+h}|Y_{1:T})$, 1 to h horizons ahead, is given by the conditional probability of the forecasts, averaged over all possible values of the parameters, the variables in the state vector at the end of the sample x_T , and all possible future paths of the variables in the state vector $x_{T+1:T+h}$: $p(Y_{T+1:T+h}|Y_{1:T}) =$

$$\int_{(x_T, \theta_T)} \left[\int_{s=x_{T+1:T+h}} p(Y_{T+1:T+h}|s)p(s|x_T, \theta_T, Y_{1:T})ds \right] p(x_T|\theta_T, Y_{1:T})p(\theta_T|Y_{1:T})d(x_T, \theta_T)$$

where $p(\theta_T|Y_{1:T})$ is the posterior of the parameters at the end point T of the sample used. We employ a slightly modified version of the algorithm for generating draws from the predictive distribution outlined in Del Negro and Schorfheide (2013). The algorithm is as follows.

Step 1 Using the saved draws from the posterior at the end of the sample $p(\theta_T|Y_{1:T})$, for every draw $k = 1, \dots, n_{sim}$ (or for every i^{th} draw), use the Kalman filter to compute the moments of the unobserved variables at T : $p(x_T|\theta_T^k, Y_{1:T})$.

Step 2 Draw a sequence of shocks $v_{T+1:T+h}^k$ from a $N(0, Q(\theta_T^k))$, where $Q(\theta_T^k)$ is a draw from the estimated posterior distribution of the diagonal variance-covariance matrix of the shocks at T . For each draw k from $p(\theta_T|Y_{1:T})$ and from $p(x_T|\theta_T^k, Y_{1:T})$, use the state equation to obtain forecasts for the state variables

$$\hat{x}_{T+1:T+h}^k = F(\theta_T^k)x_{T:T+h-1}^k + G(\theta_T^k)v_{T+1:T+h}^k.$$

Step 3 Use the forecast simulations for $\hat{x}_{T+1:T+h}^k$ in the measurement equation

$$\hat{Y}_{T+1:T+h}^k = K(\theta_T^k) + Z(\theta_T^k)\hat{x}_{T+1:T+h}^k.$$

Using the above algorithm, we obtain a predictive density of n_{sim} draws of $\hat{Y}_{T+1:T+h}^k$, which can be used to obtain numerical approximations of moments, quantiles and densities of the out-of-sample forecasts. Finally, point forecasts can be computed as the mean of the predictive density for each forecasting horizon.

3 The DSGE model with financial frictions

The DSGE model with financial frictions combines the Smets and Wouters (2007) model (SW), which extends a small-scale monetary RBC model with sticky prices (such as Goodfriend and King (1997), Rotemberg and Woodford (1997), Woodford (2003), Ireland (2004) and Christiano et al.

(2005)), with financial frictions as in Bernanke et al. (1999). In addition to the sticky prices, the SW model also includes additional shocks and frictions, featuring sticky nominal price and wage settings with backward inflation indexation, investment adjustment costs, fixed costs in production, habit formation in consumption and capital utilization. Our complete log-linearised specification of the model is described in Appendix 6.1. It differs from the financial friction specification in Kolasa and Rubaszek (2015) and Del Negro and Schorfheide (2013) in that we are using a deterministic rather than stochastic trend in productivity.

In comparison with the SW model, the main difference of the model discussed in this paper is the inclusion of a financial sector from where entrepreneurs borrow funds to finance their projects. To prevent entrepreneurs to accumulate enough for self-financing, the model assumes that a constant proportion of them dies each period. The success of the entrepreneurs' projects depend on both aggregate and idiosyncratic shocks. While entrepreneurs observe the impact of both types of shocks, the banks do not observe idiosyncratic shocks. The financial intermediary faces a standard agency problem in writing the optimal contract to lend to the entrepreneurs. The bank charges a finance premium in order to cover its monitoring costs. The first order condition from the expected return maximisation of the entrepreneurs, subject to the bank contract, gives rise to one of the three key equations in the financial frictions block together with the evolution of the net worth of entrepreneurs and the arbitrage equation for capital. The most important impact of the financial friction is that it 'accelerates' the impact of negative shocks, since the default risk increases during recessions, which has a negative impact on net worth and investment, that further rises the default risk as a consequence of the corporate bond spread.

The log-linearised equation, assuming a deterministic trend in productivity, that links the financial friction shock ε_t^ω and the expected spread is written as

$$\mathbb{E}_t \left[\tilde{R}_{t+1}^k - r_t \right] = \frac{(1 - \lambda/\gamma)}{(1 + \lambda/\gamma)\sigma_c} \varepsilon_t^b + \varsigma_{sp,b}(q_t + \bar{k}_t - n_t) + \varepsilon_t^\omega,$$

where ε_t^b is the risk premium shock, λ describes the habit formation on consumption, γ is the long-run growth rate, σ_c is the elasticity of intertemporal substitution. The transmission of the financial shock to aggregate investment via Tobin's q_t depends crucially of the parameter $\varsigma_{sp,b}$. If this parameter collapses to zero (in the absence of the financial friction shock ε_t^ω), the model is equivalent to one with no financial frictions. The financial friction shock follows an AR(1) process

$$\varepsilon_t^\omega = \rho_\omega \varepsilon_{t-1}^\omega + \sigma_\omega \eta_t^\omega,$$

with variance σ_ω^2 . This implies that the DSGE model with financial frictions has eight stochastic

shocks. We are particularly interested in how the parameters $\varsigma_{sp,b}$, ρ_ω and σ_ω evolve over time since they have an impact on how the ‘accelerator’ mechanism, created by allowing for financial frictions, changes over time.

Our full set of measurement equations is described in Appendix 6.2. In addition to the seven observables employed by Smets and Wouters (2007), we add a time series of the corporate bond spread, Spread_t , measured as the difference between the BAA Corporate Bond Yield over the 10 Year Treasury Note Yield. This time series is linked to the financial friction block above by equation

$$\text{Spread}_t = SP^* + 100 \times \mathbb{E}_t[\tilde{R}_{t+1}^k - r_t],$$

where r_t is the policy rate.

4 Empirical results

In this section, we apply the Bayesian local likelihood (BLL) method outlined in Section 2 to the DSGE model with financial frictions described in Section 3. We compare our results with the model estimated assuming fixed parameters. The BLL method is applied with the weights w_{tj} generated by the normal kernel function and a bandwidth \sqrt{T} . The parameter prior distributions can be found in Appendix 6.2. These priors are the same as in Smets and Wouters (2007); for the financial friction block parameters we tried different prior specifications (see Appendix 6.2). The number of draws of the MH algorithm is 150,000, from which we drop the first 15,000. The scaling parameter for the MH has been adjusted in order to obtain rejection rates of 20%-30%². We use U.S. data on eight observables described in Appendix 6.2 from 1970Q1 up to 2014Q2.

Figures 1 and 2 present the estimates of selected parameters. The remaining parameters can be found in Appendix 6.3, and they are qualitatively similar to Galvão et al. (2015b). In Figures 1 and 2, the blue solid line is the posterior mean obtained by BLL, with the black dotted lines displaying 5% and 95% posterior confidence intervals, and the pink dash-dotted line is the posterior mode obtained by BLL. Finally, the dashed blue line is the posterior mean obtained by standard Bayesian methods with fixed coefficients, and the green dashed lines are the 5% and 95% posterior quantiles. We would judge informally whether a parameter’s variation is substantial by checking whether our estimates are outside the 5% and 95% quantiles of the posterior distribution of the

²Roberts, Gelman and Gilks (1997) show that the optimal acceptance rate is 0.234 and their result serves as a rough benchmark in the literature; however, it is asymptotic and rests on the assumption that the elements of each chain are independent.

fixed parameter model. We expect that the time-varying parameters, estimated by BLL, will move slowly over time, in agreement with their variation representing stable and gradually changing relationships between the variables of the model, caused by smooth structural change. Parameters that vary in a erratic way would suggest that there exists no stable relationship between variables over time which might indicate model misspecification of a different nature to that arising out of smooth structural change.

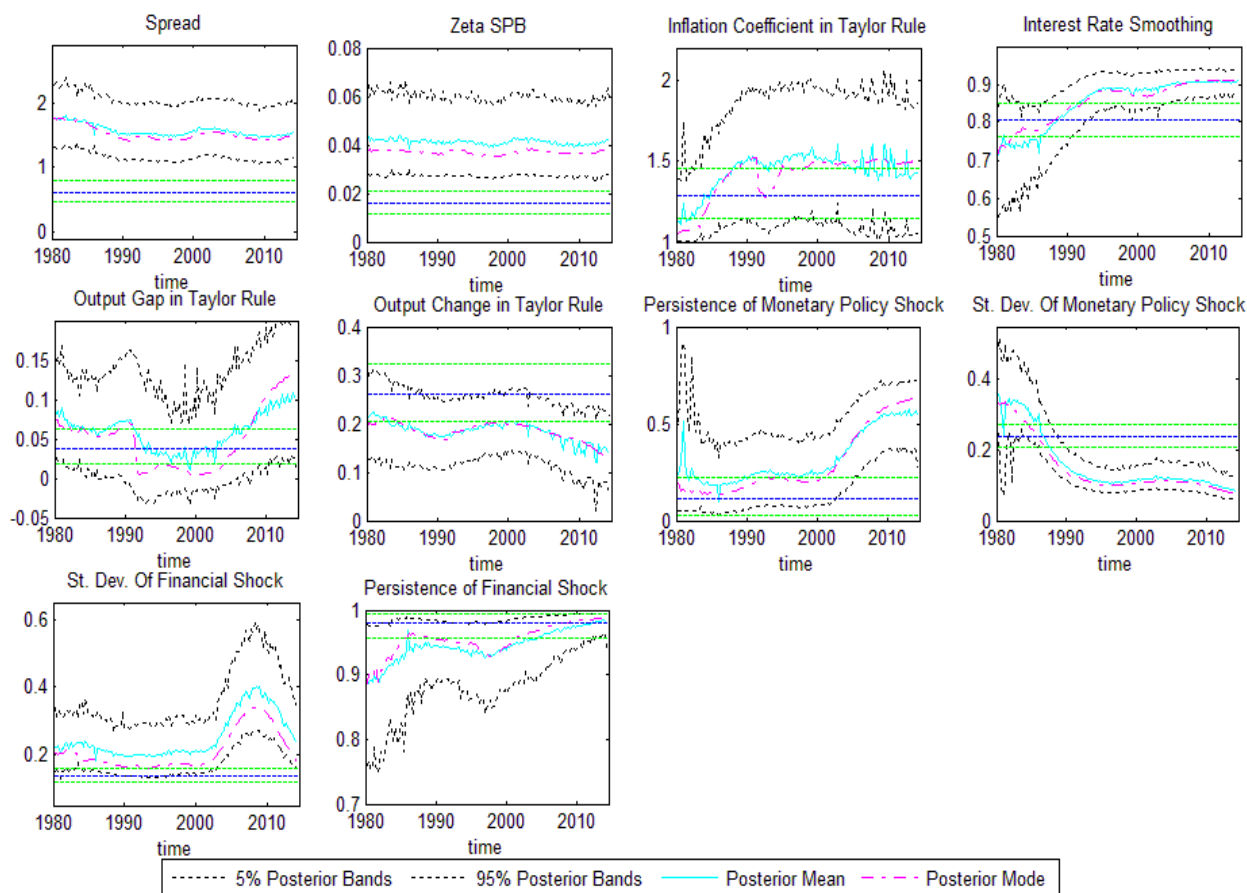


Figure 1: Estimates of DSGE model with FF parameters. The posterior mean obtained by BLL (blue solid line), the 5% and 95% posterior quantile values (black dotted lines), the posterior mode obtained by BLL (pink dash-dotted line), the posterior mean obtained by fixed parameter Bayesian estimation (dashed blue line) and the 5% and 95% posterior quantiles (green dashed lines).

Figure 1 displays the parameters of the Taylor rule, that is, interest rate smoothing and the relative impacts of inflation, output gap, and output growth on the policy rate. Estimated values are broadly in agreement with previous studies (Clarida, Gali and Gertler (2000), Cogley and Sargent (2002), Fernandez-Villaverde and Rubio-Ramirez (2008)) and suggest that the Federal Reserve has

shifted the priority of its policy from output towards inflation in the mid-1980s. In particular, the Taylor rule inflation parameter starts increasing considerably especially after 1983, Paul Volcker’s second term as a Chairman of the Federal Reserve, while the output gap coefficient falls during that period. Interest rate smoothing seems to have been low in the 1980s with tackling inflation being a priority and becomes higher through the second half of the sample. More interesting, we observe an increase in the output gap coefficient during the recent crisis, providing evidence that U.S. monetary authorities shifted attention to the sharply declining output. Monetary policy shock becomes more persistent during the crisis with interest rates near the Zero Lower Bound.

Figure 1 also includes the parameters of the financial friction block. Both the measurement equation parameter, SP^* , and the coefficient that measures the impact of financial frictions on Tobin’s q , $\zeta_{sp,b}$, have posterior means obtained with BLL that are larger than assuming fixed parameters, but their values are in general stable over time³. The parameter measuring the persistence of the financial friction shock obtained with BLL is in line with the one obtained with the fixed coefficient model since the BLL estimate has large posterior confidence bands. In contrast, the volatility of financial frictions shocks increases twofold in period between 2007 and 2011 in comparison with the previous period and also with the fixed parameter posterior estimates. One might worry that this large increase may be caused by the inadequacy of the DSGE model to fit the data during the financial crisis period.

Figure 2 compares the time variation of the financial friction volatility with the volatilities of the other seven shocks. The results are broadly consistent with findings of low volatility during the Great Moderation (e.g. Primiceri (2005) and Sims and Zha (2006)). In particular, the volatilities of all shocks (except the wage mark up shock) fall in the late 1980s and remain low throughout the 1990s. The standard deviation of the monetary policy shock, for instance, is twice as large in the 1980s than in the 1990s. The volatilities of productivity and spending shocks are also small during the 1985-2005 period. These results suggest that the BLL approach applied to the DSGE with financial frictions is able to reproduce previously documented changes in the variance of the

³In the presence of time variation, fixed parameter estimation is inconsistent and hence results can be very different under both schemes especially in a non-linear setup. To address the question of why the confidence intervals of SP^* and $\zeta_{sp,b}$ obtained by BLL do not overlap with the fixed parameter ones, we offer a different explanation. Since the BLL approach uses a smaller sample (taking into account the down-weighting), it gives larger relative weight to the prior. For most parameters, this makes little difference, as the priors are not very tight. However, for SP^* and $\zeta_{sp,b}$, the BLL approach delivers estimates much more narrowly concentrated around the prior means than the fixed parameter model which uses a larger sample.

shocks. Our new finding, however, is the clear increase in the financial friction shock variance from 0.2 to 0.4 during the 2007-2011 period. Similar relative size increases are not found for the other shocks since the total factor productivity shock only slightly exceeds the fixed parameter estimate during the most recent period.

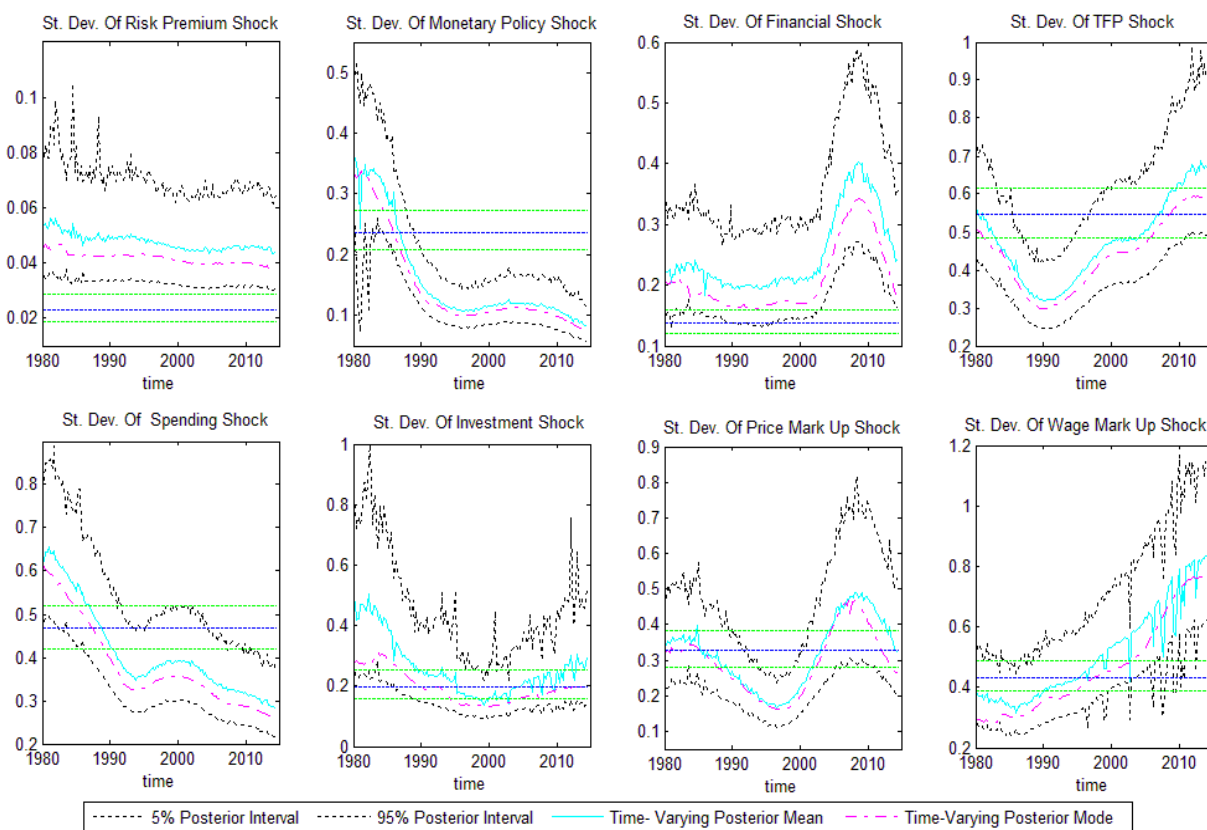


Figure 2: Posterior Estimates of the Volatility of DSGE shocks. The posterior mean obtained by BLL (blue solid line), 5% and 95% posterior quantile values (black dotted lines), the posterior mode obtained by BLL (pink dash-dotted line), the posterior mean obtained by fixed Bayesian estimation (dashed blue line), and the 5% and 95% posterior quantiles (green dashed lines).

In summary, the application of the BLL approach to the DSGE model with financial frictions suggests that the volatility of the financial shock increased in the 2007-2011 but was small in the previous period. This adds a ‘Good Luck’ component to the interpretation of the Great Moderation period (1985-2006) since previous papers (Gali and Gambetti (2009), Benati and Surico (2009), Sims and Zha (2006)) looked at DSGE models that did not include a financial sector, and as consequence had no financial shocks. Moreover, we find that the volatility of the financial friction shock starts

falling in 2012 and returns to pre-crisis levels in the end of 2014, suggesting a recovery of the economy from the financial crisis. An alternative explanation for the uncovered variation in the financial volatility is that the DSGE model we consider is too stylised and cannot capture fully the linkages between the financial sector and the rest of the economy and consequently, the impact of the events of the financial crisis appear in the variance of the financial friction shock in our empirical investigation.

4.1 Robustness Checks

In this section, we report a number of robustness checks we performed in order to test the validity of the results presented in the previous section. First, we checked the robustness of our findings by trying different prior specifications (see Appendix 6.2 for details) and by changing the trend assumption on productivity from deterministic to stochastic⁴. In all these specifications, we confirmed the results presented in Figure 1 and 2.

In addition, in Figure 3, we provide a comparison of the BLL estimates of selected parameters⁵ with ones generated with a simple rolling window scheme⁶. It is clear from Figure 3 that while the general pattern of the parameters does not change, the estimates obtained using the rolling window are considerably noisier. This is the case as at each point, a new observation is added and another one is thrown away. On the other hand, the BLL, due to its capacity to reweight past observations without completely discarding any information, delivers smoother time variation. Noisy time variation in the DSGE parameters is not desirable for at least two reasons. First, if moving one observation forward causes large shifts in the values of some parameters, this might distort forecasting performance. Second, as argued in the previous section, we believe that the variation in the DSGE parameters should be gradual, because it implies stable and gradually changing relationships between the variables of the model. The normal kernel has been found in the Monte Carlo study of Giraitis et al. (2014) to provide estimators with lower MSE compared to the flat kernel.

Furthermore, in order to assess the robustness of our results with respect to different spread variables, we estimated the model using the difference between the BAA corporate bond yield and

⁴These additional results are available upon request.

⁵The remaining parameters can be found in Appendix 6.5.

⁶For computational considerations, we only present the posterior mode estimates. To make the results comparable with the BLL results from the previous section, we use window size of $[2H + 1]$ observations, where H is the bandwidth used for the normal kernel.

the Fed Funds rate. Figure 4 displays the posterior modes of selected parameter estimates⁷. We discover that the results with this alternative spread specification do not alter our main conclusions.

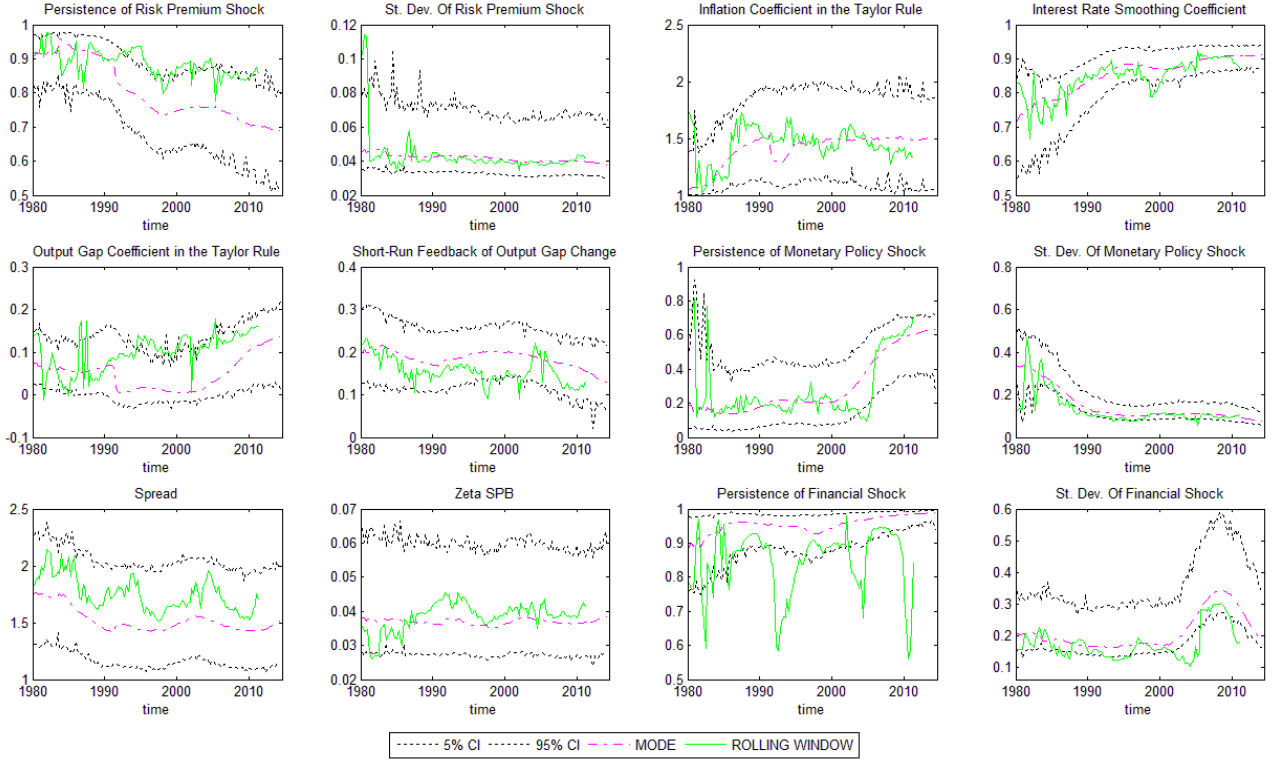


Figure 3: Robustness Check. The posterior mode obtained by BLL (pink dash-dotted line), the 5% and 95% posterior quantile values (black dotted lines), the posterior mode obtained by rolling window (solid green line).

In addition, we also ran a small simulation exercise⁸ in order to check if the BLL approach works even in the absence of parameter time variation. We generated data from a fixed parameter DSGE model with financial frictions, using as a parameter vector the prior means⁹. Then, we applied our approach to these artificial data. Figure 5 displays the resulting estimates from a representative replication for selected parameters¹⁰ and demonstrates how the BLL approach recovers the true parameters with virtually no time variation. This suggests that the method is valid even in the absence of time variation and therefore the uncovered variation in the model's parameters in our empirical application is not spurious but is instead a feature of the US data used for estimation.

⁷The remaining parameters can be found in Appendix 6.6.

⁸Due to computational time considerations, we only ran 10 replications, each with a sample size of 1000.

⁹We set the standard deviations to 0.1, as the prior mean for these is infinite.

¹⁰The remaining parameters can be found in Appendix 6.7.

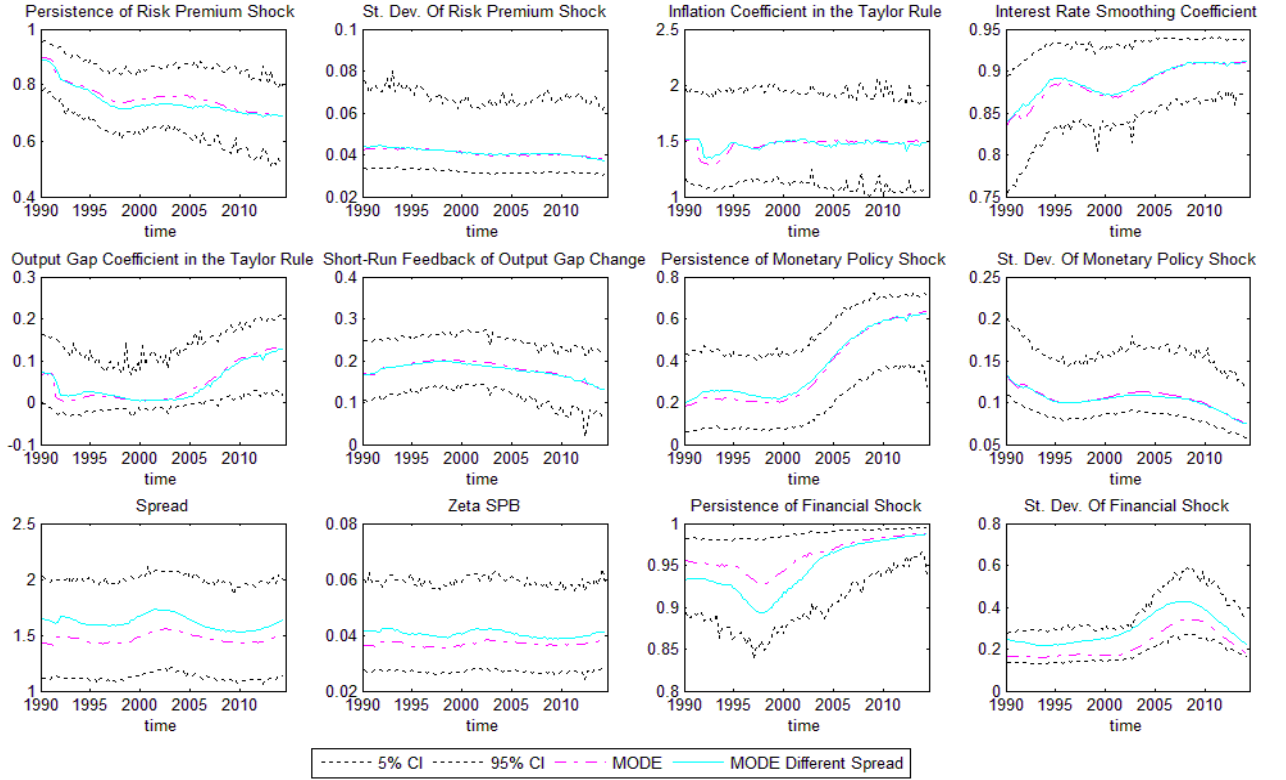


Figure 4: Robustness Check. The posterior mode obtained by BLL with spread BAA corporate bond yield over 10 year Treasury note (pink dash-dotted line), the 5% and 95% posterior quantile values (black dotted lines), the posterior mode obtained by BLL with spread BAA corporate bond yield minus Fed Funds Rate (solid blue line).

Finally, our choice of bandwidth in the empirical application in the previous section is motivated by the optimal bandwidth choice used for inference in time varying random coefficient models in Giraitis et al. (2014). In addition, Galvão et al. (2015a) and (2015b) perform a number of robustness checks with respect to different values of H . Here, we report some of their main findings. Galvão et al. (2015a) show that $H = \sqrt{T}$ in fact delivers the best mean squared errors in Monte Carlo evidence. Moreover, in an application to the Smets and Wouters (2007) model without financial frictions, Galvão et al. (2015b) show that $H = \sqrt{T}$ delivers the best forecast performance for most variables.

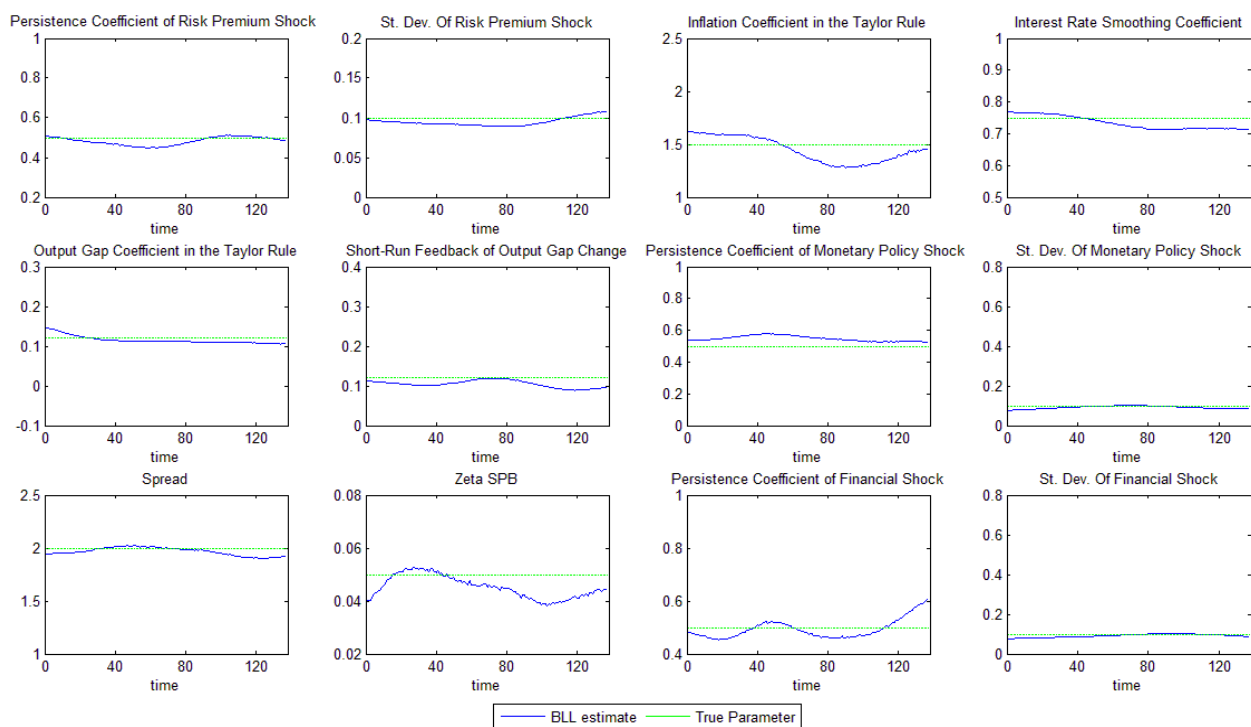


Figure 5: Simulation Exercise. The posterior mode obtained by BLL when the DGP is a model with fixed parameters (solid blue line) and the true parameter values at which the data are generated (dotted green line).

4.2 Time-varying impulse response functions

The main objective of this subsection is to evaluate how the financial frictions shock propagates to the rest of the economy over time. Our previous results suggest that the size of the financial shock is larger in the 2007-2011 period. However, the parameter that governs the transmission of the shock to macro variables, $\zeta_{sp,b}$, does not change very much. Figure 6 displays the impulse response functions of output and investment. The top panel of Figure 6 describes the responses to one-standard deviation of the shock, so it captures the effect of the shock on the desired variables over time while also taking into account its changing size. The bottom panel of Figure 6 describes the responses to 25-basis-points shock, which are useful for investigating the changes in the transmission while keeping the size of the shock constant over time.

We can see that the negative responses of output 10 quarters after the shock are 0.1% during 2008-2011 period instead of 0.05% prior to 2008. Similarly, investment, which is the main channel through which the financial shock affects output, responds much more sharply during the 2007-2011 period, with an accumulated response of minus 6.5% in the five years after the shock hits,

as supposed to minus 1.5% in the pre-crisis period. If we consider the impact of a fixed-sized shock instead, the resulting responses of both output and investment are virtually the same across periods. This confirms our conjecture that what has changed over time is the size of the financial shocks rather than the way in which financial markets operate in the model.

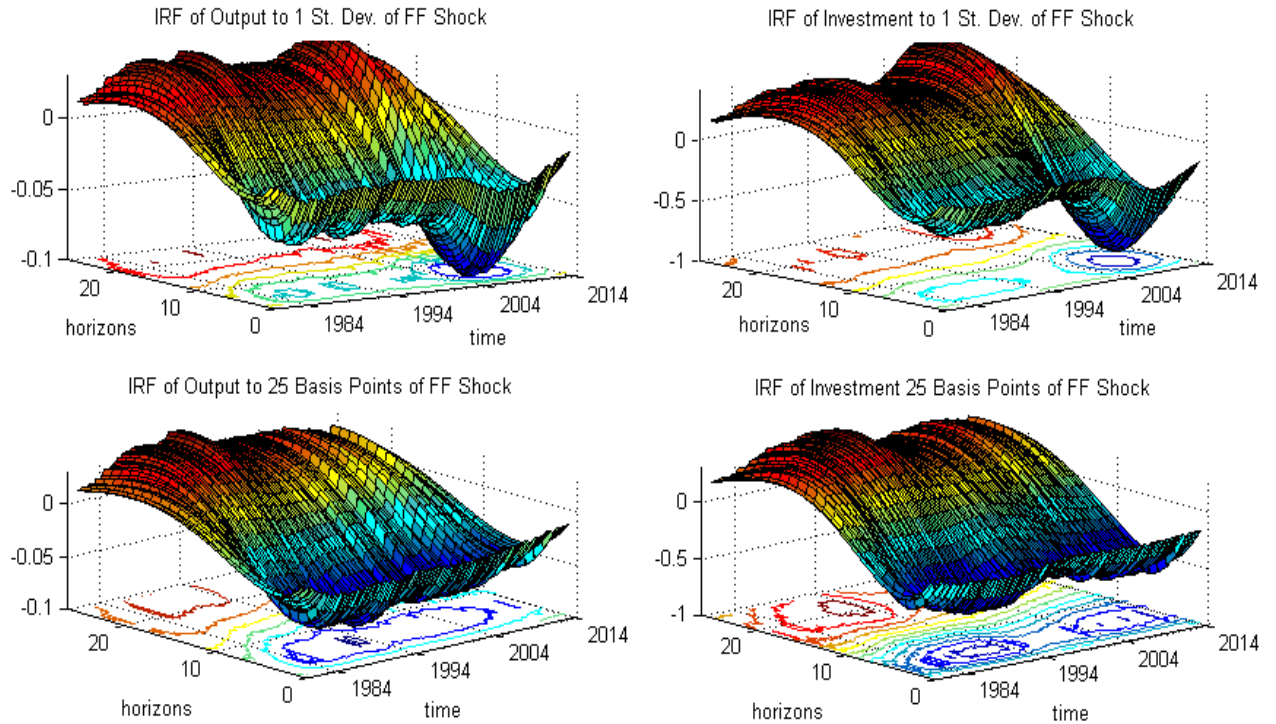


Figure 6: Responses to 1 st. dev. and 25 basis points of financial friction shock

4.3 Forecasting

Our previous results indicate that the BLL approach applied to the DSGE model with financial frictions is able to capture important variation of the parameters over time. In particular, Figure 2 provides exhaustive evidence of changes in the volatility of the shocks. The literature on forecasting with time-varying volatilities (e.g., Carriero, Clark and Marcellino (2015)) suggests that we should expect improvements in forecasting accuracy in particularly when evaluating the predictive densities. In this subsection, we use the algorithm outlined in Section 2 to generate density forecasts for the observables, using the posterior distribution of the parameters at the last period T of the in-sample to generate the out-of-sample predictions. Our forecast origins are 2000Q1-2012Q2 and we generate projections 1 to 8 quarters ahead. In addition to our time-varying DSGE model with financial frictions (TV FF), we compute forecasts for the DSGE model with (fixed FF) and

without financial frictions assuming fixed parameters. The standard Smets and Wouters (2007) (SW) model has been evaluated by Edge and Guerkaaynak (2010), Wouters (2012) and Del Negro and Schorfheide (2013), and it is able to perform well at long forecast horizons for output growth and inflation, so we use it as a benchmark in Table 1.

	2000Q1-2006Q4				2007Q1-2012Q2			
	TV FF relative to SW				TV FF relative to SW			
	h=1	h=2	h=4	h=8	h=1	h=2	h=4	h=8
Output Growth	1.00	1.20	1.26	1.27	0.90**	0.90*	0.95	0.97
Investment Growth	0.87	0.74*	0.68*	0.88	0.95	0.99	1.12	1.17
Inflation	0.99	0.98	0.97	0.90	1.03	1.05	1.05	0.93
Fed Funds Rate	0.89**	0.89**	0.86**	0.79	0.64**	0.66*	0.86	1.05
	Fixed FF relative to SW				Fixed FF relative to SW			
Output Growth	1.21	1.53	1.77	1.65	0.92	0.86*	0.86**	0.84*
Investment Growth	0.93	0.84	0.80	0.95	0.94	0.93	1.06	1.19
Inflation	1.02	1.05	1.09	1.01	0.98*	0.98*	0.99	1.10
Fed Funds Rate	0.82**	0.79**	0.84*	0.89	0.97	0.95	1.00	0.98

Table 1: RMSFEs. The table reports ratios of RMSFEs relative to the SW model RMSFEs. ‘*’, ‘**’ and ‘***’ indicate rejection of the null of equal performance against the one-sided alternative at 10%, 5% and 1% respectively, using a Diebold and Mariano test.

Table 1 evaluates the performance of point forecasts using root mean squared forecast errors (RMSFEs) for output growth, inflation and the Fed Funds rate, since these variables are of prime interest. In addition, we also report the forecasts for investment growth¹¹ as it is the channel through which the financial markets enter the model¹². Entries are ratios with respect to the SW model benchmark. Values smaller than one imply that the model (either the TV FF or fixed FF) is more accurate than the benchmark. Table 2 presents the relative forecasting performance in terms of log predictive scores. The log score is computed as the value of the predictive density evaluated at the realised target variable and is therefore a measure of the precision of the density forecasts. We test whether a model is statistically more accurate than the SW benchmark with the Diebold and Mariano (1995) statistic computed with Newey West estimator for the standard errors. One, two and three stars indicate rejection of the null of equal performance against the one-sided alternative of better performance over the SW benchmark at 10%, 5% and 1% respectively. Both

¹¹The forecasts for the remaining variables can be found in Appendix 6.4. They lead to qualitatively similar conclusions.

¹²A forecasting comparison with an AR(1) and a TVP AR(1) models can be found in Appendix 6.4. The autoregressive models are included because it is important to verify that the DSGE model is at least as accurate as univariate statistical models.

tables present results for two sub-periods: 2000Q1-2006Q4 and 2007Q1-2012Q2. The first period is relatively tranquil in comparison with the second one.

	2000Q1-2006Q4				2007Q1-2012Q2			
	Density TV FF relative to SW				Density TV FF relative to SW			
	h=1	h=2	h=4	h=8	h=1	h=2	h=4	h=8
Output Growth	0.13***	0.10*	0.10*	0.05	0.05	0.05	-0.09	-0.09
Investment Growth	0.19*	0.29*	0.35*	0.11	-0.04	0.31	-0.27	-0.22
Inflation	-0.08	-0.07	0.03	0.11	5.56	3.60	2.52	-0.36
Fed Funds Rate	0.49***	0.18	-0.16	-0.12	0.47***	0.31	-0.08	-0.48
	Density Fixed FF relative to SW				Density Fixed FF relative to SW			
Output Growth	-0.10	-0.15	-0.19	-0.19	0.01	0.08	0.12*	0.05
Investment Growth	0.01	0.01	0.01	-0.06	0.16	0.52	0.14	-0.11
Inflation	0.00	-0.04	-0.11	-0.09	-2.92	0.59	1.58	-0.09
Fed Funds Rate	0.03**	-0.02	-0.08	-0.05	0.02	0.01	-0.03	0.11

Table 2: Log Scores. The table reports differences of log predictive scores from to the SW model log scores. ‘*’, ‘**’ and ‘***’ indicate rejection of the null of equal performance against the one-sided alternative at 10%, 5% and 1% respectively, using a Diebold and Mariano test.

Del Negro et al. (2014) and Kolasa and Rubaszek (2015) documented that the DSGE model with financial frictions does not improve forecasts in comparison with the Smets and Wouters (2007) model in the period before 2007. The results in Table 1 confirm this since the inclusion of financial frictions worsens the forecasts of inflation, output and investment growth in the period 2000-2006, while improving forecasts of the interest rate. The TV FF model brings the forecasting performance at similar levels to the SW model during this tranquil period. In addition, during the volatile period of 2007-2012, the TV FF model confirms previous findings of relatively good performance of financial friction models in comparison to the standard SW model. For the Fed Funds Rate, the TV FF model delivers statistically significant improvements over the SW model for both point and density forecasts. One explanation is that by allowing for time variation in the coefficients of the Taylor rule, we obtain a value for the smoothing parameter that is close to one; that is, the forecasts from the Taylor rule resemble random walk forecasts, which is an adequate model for forecasting the Fed Funds Rate in the vicinity of the Zero Lower Bound. Table 2 presents similar results for density forecast performance for selected variables¹³ using log scores. Qualitatively, the results are similar to the ones using RMSFEs.

¹³The density forecast for the remaining variables, as well as density forecast comparison with an AR(1) and a TVP AR(1) models can be found in Appendix 6.4.

5 Conclusion

This paper employs the Bayesian Local Likelihood approach developed previously by Galvão et al. (2015a) and Galvão et al. (2015b) to a DSGE model that combines the Smets and Wouters (2007) model with financial frictions as in Bernanke et al. (1999). As a consequence, this paper proposes a time varying DSGE model with financial frictions. Our results suggest that the parameter governing how financial friction shocks affect investment decisions is stable over time, but the volatility of the financial shock jumps in the period 2007-2012 and returns to the pre-crisis values after 2012. Moreover, when looking at the impulse response functions, we find that the responses of output and investment to 25 basis points of the financial shock do not change over time. In contrast, when we consider the responses to a standard deviation of the shock, taking into account the changing volatility over time, we observe a substantial change in the way these variables respond to financial shocks during the period of the recent crisis. This evidence leads us to provide an interpretation of the recent financial crisis as a ‘Bad Luck’ event, that is, it is caused by changes in the volatility of financial shocks while taking into account policy changes. An alternative explanation of the results presented in this paper is that the DSGE model we consider is perhaps too stylised to fully account for the connection between the financial sector and the macroeconomy and, as a consequence, the events of the 2008 crisis appear in the variance of the financial friction shock instead. Finally, our forecasting exercise demonstrates that the time varying model with financial frictions improves the forecasting performance of the financial friction model especially in the tranquil 2000-2006 period.

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6 Appendix

6.1 The Smets and Wouters (2007) model with financial frictions

The model we use is a Smets and Wouters (2007) model with a deterministic trend, modified to include a financial friction block, as in Bernanke et al. (1999). We refer the reader to the original paper, Smets and Wouters (2007), for discussion and derivation of the model's equation and for completeness, we list here the linearised equations. See also the Technical Appendix in Smets and Wouters (2007) available at: http://www.aeaweb.org/aer/data/june07/20041254_app.pdf. For expressions of the FF block parameters and steady states, see:

http://sites.sas.upenn.edu/schorf/files/hb_forecasting_appendix.pdf, pp 37-39.

- The resource constraint in the model is given by equation,

$$y_t = (1 - g_y - i_y)c_t + ((\gamma - 1 - \delta)k_y)i_t + (R_*^k k_y)z_t + \varepsilon_t^g,$$

- the consumption Euler equation,

$$c_t = \frac{(\lambda/\gamma)}{(1 + \lambda/\gamma)}c_{t-1} + \frac{1}{(1 + \lambda/\gamma)}\mathbb{E}_t c_{t+1} + \frac{(\sigma_c - 1)W_*^h L_*/C_*}{\sigma_c(1 + \lambda/\gamma)}\mathbb{E}_t(l_t - l_{t+1}) - \frac{(1 - \lambda/\gamma)}{(1 + \lambda/\gamma)\sigma_c}(r_t - \mathbb{E}_t \pi_{t+1} + \varepsilon_t^b),$$

- the investment Euler equation,

$$i_t = \frac{1}{1 + \beta\gamma^{1-\sigma_c}}i_{t-1} + \left(1 - \frac{1}{1 + \beta\gamma^{1-\sigma_c}}\right)\mathbb{E}_t i_{t+1} + \frac{1}{(1 + \beta\gamma^{1-\sigma_c})\gamma^2\varphi}q_t + \varepsilon_t^i,$$

- the aggregate production function, $y_t = \phi(\alpha k_t^s + (1 - \alpha)l_t + \varepsilon_t^a)$,

- the relation between effectively rented capital and capital, $k_t^s = k_{t-1} + z_t$,

- the degree of capital utilization, $z_t = \frac{1-\psi}{\psi}r_t^k$,

- the capital accumulation equation, $k_t = \frac{1-\delta}{\gamma}k_{t-1} + (1 - \frac{1-\delta}{\gamma})i_t + (1 - \frac{1-\delta}{\gamma})((1 + \beta\gamma^{1-\sigma_c})\gamma^2\varphi)\varepsilon_t^i$,

- the price mark-up, $\mu_t^p = \alpha(k_t^s - l_t) + \varepsilon_t^a - w_t$,

- the new Keynesian Phillips curve,

$$\pi_t = \frac{\iota_p}{1 + \beta\gamma^{(1-\sigma_c)\iota_p}}\pi_{t-1} + \frac{\beta\gamma^{(1-\sigma_c)}}{1 + \beta\gamma^{(1-\sigma_c)\iota_p}}\mathbb{E}_t \pi_{t+1} - \frac{1}{1 + \beta\gamma^{(1-\sigma_c)\iota_p}} \left\{ \frac{(1 - \beta\gamma^{(1-\sigma_c)}\xi_p)(1 - \xi_p)}{\xi_p((\phi - 1)\varepsilon_p + 1)} \right\} \mu_t^p + \varepsilon_t^p,$$

- the rental rate of capital, $r_t^k = -(k_t - l_t) + w_t$,

- the wage mark-up, $\mu_t^w = w_t - (\sigma_l l_t + \frac{1}{1-\lambda/\gamma}(c_t - \lambda/\gamma c_{t-1}))$,

- the wage equation,

$$w_t = \frac{1}{1 + \beta\gamma^{(1-\sigma_c)}} w_{t-1} + \left(1 - \frac{1}{1 + \beta\gamma^{(1-\sigma_c)}}\right) (\mathbb{E}_t w_{t+1} + \mathbb{E}_t \pi_{t+1}) - \frac{1 + \beta\gamma^{(1-\sigma_c)} \iota_w}{1 + \beta\gamma^{(1-\sigma_c)}} \pi_t \\ + \frac{\iota_w}{1 + \beta\gamma^{(1-\sigma_c)}} \pi_{t-1} - \frac{1}{1 + \beta\gamma^{(1-\sigma_c)}} \left\{ \frac{(1 - \beta\gamma^{(1-\sigma_c)} \xi_w)(1 - \xi_w)}{\xi_w((\phi_w - 1)\varepsilon_w + 1)} \right\} \mu_t^w + \varepsilon_t^w,$$

- the Taylor Rule,

$$r_t = \rho r_{t-1} + (1 - \rho) \{r_\pi \pi_t + r_y (y_t - y_t^p)\} + r_{\Delta y} ((y_t - y_t^p) - (y_{t-1} - y_{t-1}^p)) + \varepsilon_t^r.$$

6.1.1 The financial friction block

- The corporate spread is defined as

$$\mathbb{E}_t [\tilde{R}_{t+1}^k - r_t] = \frac{(1 - \lambda/\gamma)}{(1 + \lambda/\gamma)\sigma_c} \varepsilon_t^b + \varsigma_{sp,b}(q_t + \bar{k}_t - n_t) + \varepsilon_t^\omega.$$

- The arbitrage condition between the return to capital and the riskless rate in Smets and Wouters (2007) is now replaced by

$$\tilde{R}_t^k - \pi_t = \frac{r_*^k}{r_*^k + (1 - \delta)} r_t^k + \frac{(1 - \delta)}{r_*^k + (1 - \delta)} q_t - q_{t-1}.$$

- Finally, the entrepreneurs' net worth evolution is defined as

$$n_t = \varsigma_{n,RK}(\tilde{R}_t^k - \pi_t) - \varsigma_{n,R}(r_{t-1} - \pi_t) + \varsigma_{n,q}(q_{t-1} + \bar{k}_{t-1}) + \varsigma_{n,n} n_{t-1} - \frac{\varsigma_{n,\omega}}{\varsigma_{sp,\omega}} \varepsilon_{t-1}^\omega.$$

6.1.2 Stochastic processes of exogenous shocks

- Exogenous government spending is defined as $\varepsilon_t^g = \rho_g \varepsilon_{t-1}^g + \sigma_g \eta_t^g + \rho_{ga} \sigma_z \eta_t^z$,
- TFP shock, $\varepsilon_t^a = \rho_a \varepsilon_{t-1}^a + \sigma_a \eta_t^a$,
- risk premium shock, $\varepsilon_t^b = \rho_b \varepsilon_{t-1}^b + \sigma_b \eta_t^b$,
- investment-specific technology shock, $\varepsilon_t^i = \rho_i \varepsilon_{t-1}^i + \sigma_i \eta_t^i$,
- monetary policy shock, $\varepsilon_t^r = \rho_r \varepsilon_{t-1}^r + \sigma_r \eta_t^r$,
- price mark-up shock, $\varepsilon_t^p = \rho_p \varepsilon_{t-1}^p + \sigma_p \eta_t^p + \mu_p \sigma_p \eta_{t-1}^p$,
- wage mark-up shock, $\varepsilon_t^w = \rho_w \varepsilon_{t-1}^w + \sigma_w \eta_t^w + \mu_w \sigma_w \eta_{t-1}^w$,
- financial friction shock, $\varepsilon_t^\omega = \rho_\omega \varepsilon_{t-1}^\omega + \sigma_\omega \eta_t^\omega$.

6.2 Measurement equation, data description and transformations

6.2.1 Measurement equation

$$Y_t = \begin{bmatrix} \text{Output Growth}_t \\ \text{Consumption Growth}_t \\ \text{Investment Growth}_t \\ \text{Wage Growth}_t \\ \text{Hours Worked}_t \\ \text{Inflation}_t \\ \text{Policy Rate}_t \\ \text{Spread}_t \end{bmatrix} = \begin{bmatrix} \bar{\gamma} \\ \bar{\gamma} \\ \bar{\gamma} \\ \bar{\gamma} \\ \bar{l} \\ \bar{\pi} \\ \bar{r} \\ SP^* \end{bmatrix} + \begin{bmatrix} y_t - y_{t-1} \\ c_t - c_{t-1} \\ i_t - i_{t-1} \\ w_t - w_{t-1} \\ l_t \\ \pi_t \\ r_t \\ 100 * \mathbb{E}_t(\tilde{R}_{t+1}^k - r_t) \end{bmatrix}.$$

6.2.2 Data description

Data Description	
Variable	Source
GDP, Total, Constant Prices, AR, SA, USD, 2009 chnd prices	U.S. Bureau of Economic Analysis
PCE, Total, Constant Prices, AR, SA, USD, 2009 chnd prices	U.S. Bureau of Economic Analysis
Private Fixed Investment, Total, Current Prices, AR, SA, USD	U.S. Bureau of Economic Analysis
Consumer price index, AR, SA, Index, 2005=100	U.S. Bureau of Economic Analysis
Real hourly compensation, nonfarm business, index, SA, Index, 2009=100	U.S. Bureau of Labor Statistics
Hours worked per employee, AR	U.S. Bureau of Labor Statistics
Employment, all persons (ages 15 and over), SA	U.S. Bureau of Labor Statistics
Popultaion Total (Estimates Used in National Accounts)	U.S. Bureau of Economic Analysis
Federal Funds Rate (Monthly Average)	Federal Reserve, U.S.
Moody's Baa-Rated Long-Term, Yield, Average, USD	Reuters
Constant Maturity Yields, 10 Year, USD	Federal Reserve, U.S.

6.2.3 Data transformations:

$$\text{Output Growth}_t = 100 * \Delta \ln(GDP_t / POP_t)$$

$$\text{Consumption Growth}_t = 100 * \Delta \ln(CON_t / POP_t)$$

$$\text{Investment Growth}_t = 100 * \Delta \ln((INV_t / CPI_t) / POP_t)$$

$$\text{Wage Growth}_t = 100 * \Delta \ln(WAGE_t)$$

$$\text{Hours Worked}_t = 100 * \ln \left[\left((EMPL_t * HOURS_t) / POP_t \right) - \overline{(EMPL_t * HOURS_t) / POP_t} \right]$$

$$\text{Inflation}_t = 100 * \Delta \ln(CPI_t)$$

$$\text{Policy Rate}_t = 1/4 * FFR_t$$

$$\text{Spread}_t = 1/4 * (BAA_Yield_t - 10YTreasury_Yield_t)$$

6.2.4 Priors¹⁴

Table 3: Prior distributions for the structural parameters.

	Parameter Name	Prior Distribution		
		Distribution	Mean	St. Dev.
φ	Elasticity of Capital Adjustment Cost Function	Normal	4	1.5
σ_c	Elasticity of Intertemporal Substitution	Normal	1.5	0.37
λ	External Habit Formation	Beta	0.7	0.1
ξ_w	Calvo Probability in Labour Markets	Beta	0.5	0.1
σ_l	Elasticity of Labour Supply to Real Wage	Normal	2	0.75
ξ_p	Calvo Probability in Goods Markets	Beta	0.5	0.1
ι_w	Degree of Wage Indexation	Beta	0.5	0.15
ι_w	Degree of Price Indexation	Beta	0.5	0.15
ψ	Normalized Elasticity of Capital	Beta	0.5	0.2
Φ	Fixed Costs of Intermediate Goods Producers	Normal	1.25	0.12
r_π	Inflation Coefficient in the Taylor Rule	Normal	1.5	0.25
ρ	Interest Rate Smoothing Coefficient	Beta	0.75	0.1
r_y	Output Gap Coefficient in the Taylor Rule	Normal	0.12	0.05
$r_{\Delta y}$	Short-Run Feedback of Output Gap Change	Normal	0.12	0.05
$100(\beta^{-1}-1)$	Normalized Households' Discount Factor	Gamma	0.25	0.1
π^*	Steady State Inflation Rate	Gamma	0.62	0.1
l^*	Steady State Hours Worked	Normal	0	2
γ^*	Steady State Quarterly Growth Rate	Normal	0.4	0.1
α	Capital Share	Normal	0.3	0.05
SP^*	Steady State Spread	Gamma	2	0.3
$\zeta_{sp,b}$	Effect of spread on Tobin's Q, capital and network	Beta	0.05	0.015

Table 4: Prior distributions for the parameters of the exogenous processes.

	Parameter Name	Prior Distribution		
		Distribution	Mean	St. Dev.
σ_a	St. Dev. Of TFP Shock	Inverse Gamma	0.1	2
σ_b	St. Dev. of Risk Premium Shock	Inverse Gamma	0.1	2
σ_g	St. Dev. of Exogenous Spending Shock	Inverse Gamma	0.1	2
σ_l	St. Dev. of Investment-Specific Technology Shock	Inverse Gamma	0.1	2
σ_r	St. Dev. of Monetary Policy Shock	Inverse Gamma	0.1	2
σ_p	St. Dev. of Price Mark-Up Shock	Inverse Gamma	0.1	2
σ_w	St. Dev. of Wage Mark-Up Shock	Inverse Gamma	0.1	2
σ_ω	St. Dev. of Financial Friction Shock	Inverse Gamma	0.1	2
ρ_a	Persistence Coefficient of TFP Shock	Beta	0.5	0.2
ρ_b	Persistence Coefficient of Risk Premium Shock	Beta	0.5	0.2
ρ_g	Persistence Coefficient of Spending Shock	Beta	0.5	0.2
ρ_l	Persistence Coefficient of Investment Shock	Beta	0.3	0.2
ρ_r	Persistence Coefficient of Monetary Policy Shock	Beta	0.3	0.2
ρ_p	Persistence Coefficient of Price Mark Up Shock	Beta	0.3	0.2
ρ_w	Persistence Coefficient of Wage Mark Up Shock	Beta	0.5	0.2
ρ_ω	Persistence Coefficient of Financial Friction Shock	Beta	0.5	0.2
μ_p	MA Coefficient of Price Mark Up Shock	Beta	0.5	0.2
μ_w	MA Coefficient of Wage Mark Up Shock	Beta	0.5	0.2
ρ_{ga}	Coefficient for TFP Shock in the Spending Equation	Normal	0.5	0.2

¹⁴We also have upon request time varying estimation results with prior standard deviations 0.1 and 0.005 for SP^* and $\zeta_{sp,b}$ respectively and with prior standard deviations 1 and 0.1 for SP^* and $\zeta_{sp,b}$ respectively. Results on the FF block remain robust to these two specifications.

6.3 Additional results

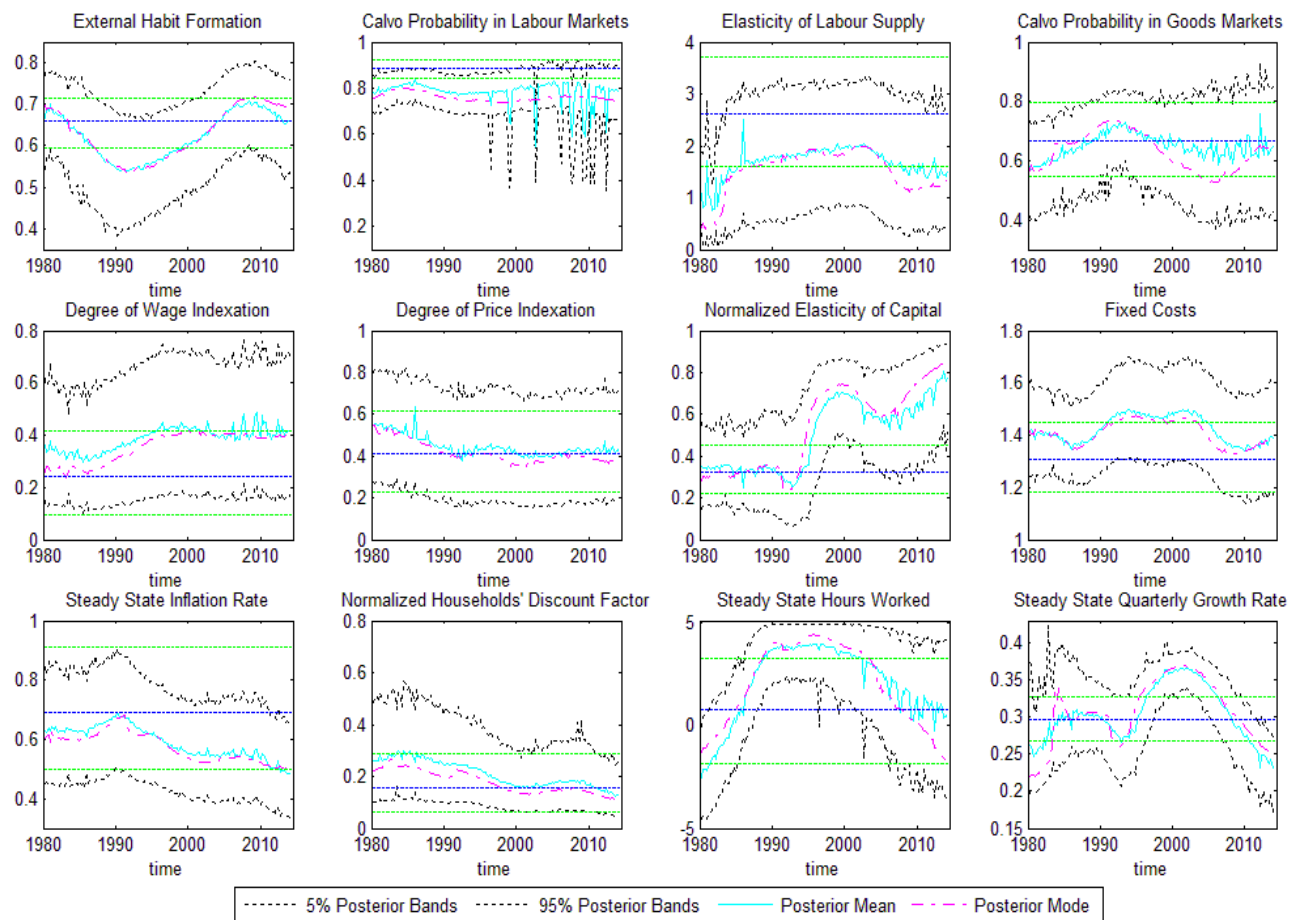


Figure 7: Posterior Estimates of Additional Parameters. The posterior mean obtained by BLL (blue solid line), 5% and 95% posterior quantile values (black dotted lines), the posterior mode obtained by BLL (pink dash-dotted line), the posterior mean obtained by fixed Bayesian estimation (dashed blue line), and the 5% and 95% posterior quantiles (green dashed lines).

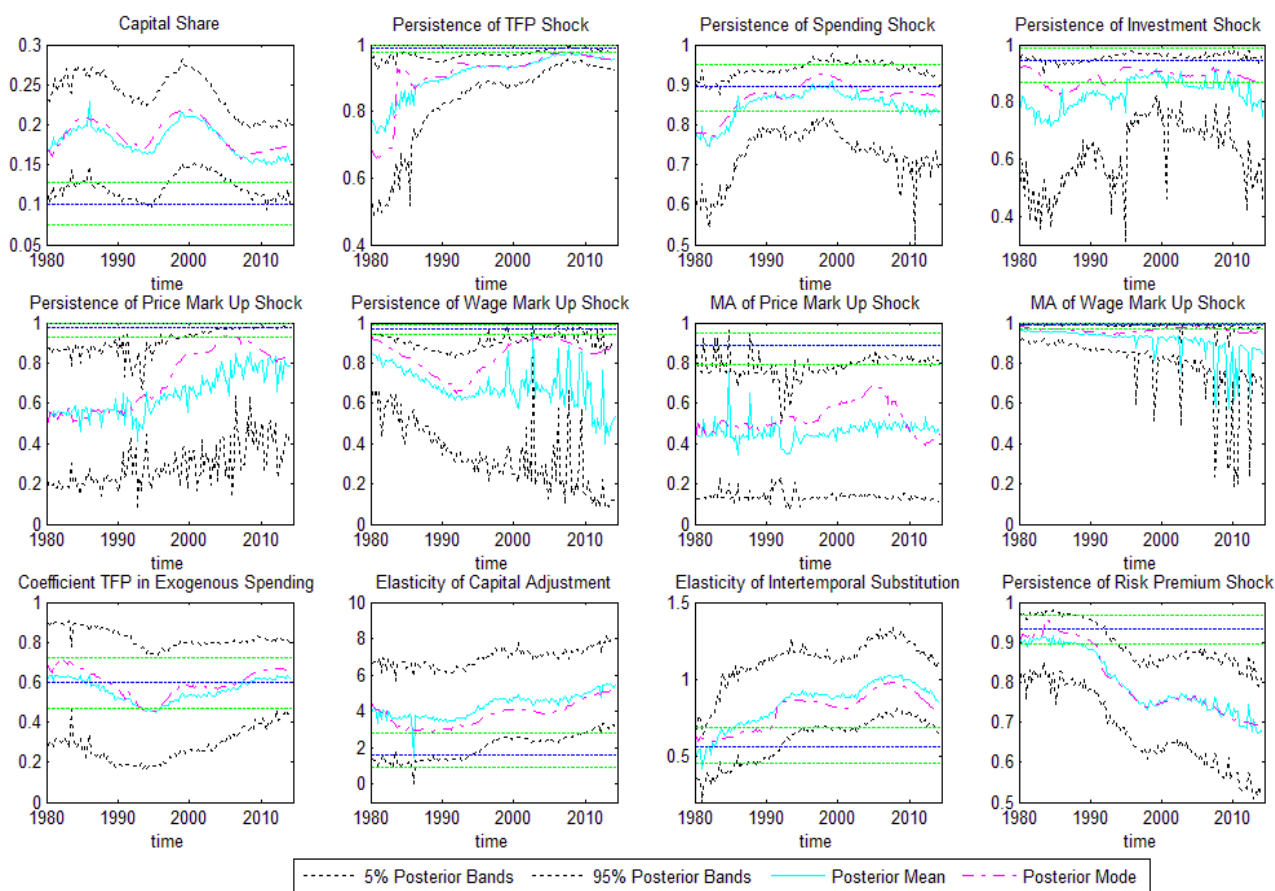


Figure 8: Posterior Estimates of Additional Parameters. The posterior mean obtained by BLL (blue solid line), the 5% and 95% posterior quantile values (black dotted lines), the posterior mode obtained by BLL (pink dash-dotted line), the posterior mean obtained by fixed Bayesian estimation (dashed blue line), and the 5% and 95% posterior quantiles (green dashed lines).

6.4 Additional forecasting results

	2000Q1-2006Q4				2007Q1-2012Q2			
	TV FF relative to SW				TV FF relative to SW			
	h=1	h=2	h=4	h=8	h=1	h=2	h=4	h=8
Consumption Growth	1.04	1.03	1.13	1.30	1.02	0.97	1.11	1.57
Wage Growth	1.10	1.03	1.03	1.02	1.14	1.13	1.04	1.04
Hours Worked	1.01	0.869*	0.668*	0.64	0.87**	0.85*	0.85	0.84
	Fixed FF relative to SW				Fixed FF relative to SW			
Consumption Growth	1.45	1.64	1.90	1.87	1.13	1.15	1.23	1.54
Wage Growth	1.06	1.04	1.00	0.99	1.02	1.02	0.99	1.01
Hours Worked	1.01	0.98	1.02	1.22	1.00	0.94	0.87*	0.81*
	2000Q1-2006Q4				2007Q1-2012Q2			
	Density TV FF relative to SW				Density TV FF relative to SW			
	h=1	h=2	h=4	h=8	h=1	h=2	h=4	h=8
Consumption Growth	-0.03	-0.04	-0.04	-0.11	-0.04	-0.06	-0.18	-0.30
Wage Growth	-0.18	0.05	0.02	0.04	0.13	0.53	0.39*	0.15
Hours Worked	0.043*	0.082*	0.210*	0.28	0.16**	0.53	2.08	2.79
	Density Fixed FF relative to SW				Density Fixed FF relative to SW			
Consumption Growth	-0.34	-0.43	-0.45	-0.43	-0.19	-0.16	-0.19	-0.26
Wage Growth	-0.15	-0.06	0.00	0.01	-0.20	0.24	-0.08	-0.02
Hours Worked	0.032*	-0.01	-0.06	-0.21	-0.12	0.11	1.76	2.88

Table 3: RMSFEs and Log Scores for additional variables. The table reports ratios of RMSFEs relative to the SW model RMSFEs and differences of log predictive scores from SW model log scores. ‘*’, ‘**’ and ‘***’ indicate rejection of the null of equal performance against the one-sided alternative at 10%, 5% and 1% respectively, using Diebold and Mariano test.

	2000Q1-2006Q4				2007Q1-2012Q2			
	TV FF relative to AR(1)				TV FF relative to AR(1)			
	h=1	h=2	h=4	h=8	h=1	h=2	h=4	h=8
Output Growth	0.85	0.98	1.04	1.16	0.82	0.84	0.91	1.07
Consumption Growth	1.32	1.49	1.34	1.13	0.777*	0.720*	0.83	1.11
Investment Growth	0.96	0.86	0.79	0.91	1.11	1.22	1.41	1.38
Wage Growth	1.04	1.06	1.05	1.07	1.20	1.18	1.12	1.08
Hours Worked	0.92	0.77	0.56	0.49	0.711**	0.754*	0.84	0.87
Inflation	0.906*	0.90	0.876*	0.94	1.02	1.01	0.99	0.730*
Fed Funds Rate	0.666***	0.704**	0.716*	0.660*	0.742*	0.713*	0.82	1.04
	Fixed FF relative to AR(1)				Fixed FF relative to AR(1)			
Output Growth	1.02	1.25	1.45	1.51	0.84	0.81	0.82	0.93
Consumption Growth	1.83	2.37	2.25	1.62	0.86	0.86	0.92	1.09
Investment Growth	1.03	0.97	0.92	0.98	1.10	1.15	1.33	1.42
Wage Growth	1.01	1.07	1.03	1.04	1.08	1.06	1.06	1.06
Hours Worked	0.93	0.87	0.86	0.94	0.817***	0.828*	0.85	0.83
Inflation	0.94	0.96	0.98	1.05	0.97	0.94	0.94	0.86
Fed Funds Rate	0.610***	0.621**	0.704*	0.74	1.13	1.03	0.95	0.98
	2000Q1-2006Q4				2007Q1-2012Q2			
	Density TV FF relative to AR(1)				Density TV FF relative to AR(1)			
	h=1	h=2	h=4	h=8	h=1	h=2	h=4	h=8
Output Growth	0.00	-0.09	-0.17	-0.24	0.06	-0.14	-0.23	-0.38
Consumption Growth	-0.18	-0.32	-0.21	-0.16	0.272*	0.19	0.03	-0.41
Investment Growth	0.11	0.248**	0.352**	0.21	-0.07	-0.32	-0.80	-0.67
Wage Growth	0.321*	0.375**	0.400**	0.380*	0.07	0.27	0.398*	0.336*
Hours Worked	0.00	0.17	0.35	0.59	0.378*	0.40	3.40	5.35
Inflation	0.251*	-0.03	0.158**	0.12	0.37	0.29	-0.19	-0.09
Fed Funds Rate	0.477***	0.250*	0.05	0.24	0.10	0.14	-0.03	-0.53
	Density Fixed FF relative to AR(1)				DensityFixed FF relative to AR(1)			
Output Growth	-0.22	-0.33	-0.45	-0.48	0.02	-0.11	-0.02	-0.24
Consumption Growth	-0.48	-0.70	-0.63	-0.48	0.12	0.09	0.02	-0.36
Investment Growth	-0.08	-0.03	0.01	0.04	0.13	-0.11	-0.39	-0.56
Wage Growth	0.351**	0.274**	0.374**	0.348*	-0.27	-0.01	-0.07	0.165*
Hours Worked	-0.01	0.08	0.09	0.10	0.10	-0.03	3.09	5.43
Inflation	0.329**	0.01	0.02	-0.08	-8.12	-2.72	-1.14	0.18
Fed Funds Rate	0.02	0.05	0.13	0.30	-0.35	-0.17	0.02	0.06

Table 4: RMSFEs and Log Scores :comparison with AR(1). The table reports ratios of RMSFEs relative to an AR(1) model RMSFEs and differences of log predictive scores from an AR(1) model log scores. ‘*’, ‘**’ and ‘***’ indicate rejection of the null of equal performance against the one-sided alternative at 10%, 5% and 1% respectively, using Diebold and Mariano test.

	2000Q1-2006Q4				2007Q1-2012Q2			
	TV FF relative to TVP AR(1)				TV FF relative to TVP AR(1)			
	h=1	h=2	h=4	h=8	h=1	h=2	h=4	h=8
Output Growth	0.759*	0.89	0.94	1.11	0.83	0.84	0.88	0.78
Investment Growth	0.811**	0.747**	0.659*	0.772*	1.09	1.14	1.09	0.87
Inflation	1.08	1.07	1.11	1.11	0.88	1.22	1.19	0.67
Fed Funds Rate	0.604***	0.602**	0.589**	0.465*	0.80	0.76	0.84	0.96
	Fixed FF relative to TVP AR(1)				Fixed FF relative to TVP AR(1)			
Output Growth	0.91	1.14	1.31	1.44	0.84	0.80	0.79	0.68
Investment Growth	0.871*	0.84	0.77	0.83	1.07	1.08	1.03	0.89
Inflation	1.12	1.15	1.24	1.24	0.84	1.14	1.13	0.79
Fed Funds Rate	0.553***	0.531**	0.579**	0.521*	1.21	1.10	0.97	0.90
	2000Q1-2006Q4				2007Q1-2012Q2			
	Density TV FF relative to TVP AR(1)				Density TV FF relative to TVP AR(1)			
	h=1	h=2	h=4	h=8	h=1	h=2	h=4	h=8
Output Growth	0.248*	0.01	-0.11	-0.20	0.12	0.10	0.02	0.00
Investment Growth	0.228*	0.390**	0.560**	0.540**	0.12	-0.18	-0.33	0.24
Inflation	-0.04	-0.14	-0.04	0.07	-2.17	-0.22	-0.45	0.07
Fed Funds Rate	0.672***	0.569**	0.44	1.010*	-0.11	-0.03	-0.14	-0.54
	Density Fixed FF relative to TVP AR(1)				DensityFixed FF relative to TVP AR(1)			
Output Growth	0.03	-0.24	-0.39	-0.44	0.07	0.13	0.23	0.14
Investment Growth	0.04	0.11	0.22	0.367*	0.31	0.03	0.07	0.36
Inflation	0.04	-0.11	-0.17	-0.13	-10.65	-3.23	-1.39	0.33
Fed Funds Rate	0.21	0.369*	0.520*	1.072**	-0.57	-0.33	-0.09	0.04

Table 5: RMSFEs and Log Scores for selected variables. The table reports ratios of RMSFEs relative to a TVP AR(1) model RMSFEs and differences of log predictive scores from a TVP AR(1) model log scores. ‘*’, ‘**’ and ‘***’ indicate rejection of the null of equal performance against the one-sided alternative at 10%, 5% and 1% respectively, using Diebold and Mariano test.

For the time varying parameter (TVP) AR(1), the model is estimated in each point in time t : $\hat{\beta}_t = (X'D_tX)^{-1}X'D_tY$ where X contains the lagged dependent variable Y and D_t is a diagonal matrix with the kernel weights of the t^{th} row of the weighting matrix in equation (3) in its main diagonal. The variance of the residuals is also time varying and computed in point t as $\hat{\sigma}_t^2 = \varepsilon'D_t\varepsilon/tr(D_t)$. Density forecasts are then generated, using wild bootstrap and the last period values $\hat{\beta}_T$ and $\hat{\sigma}_T^2$.

6.5 Robustness checks: flat kernel

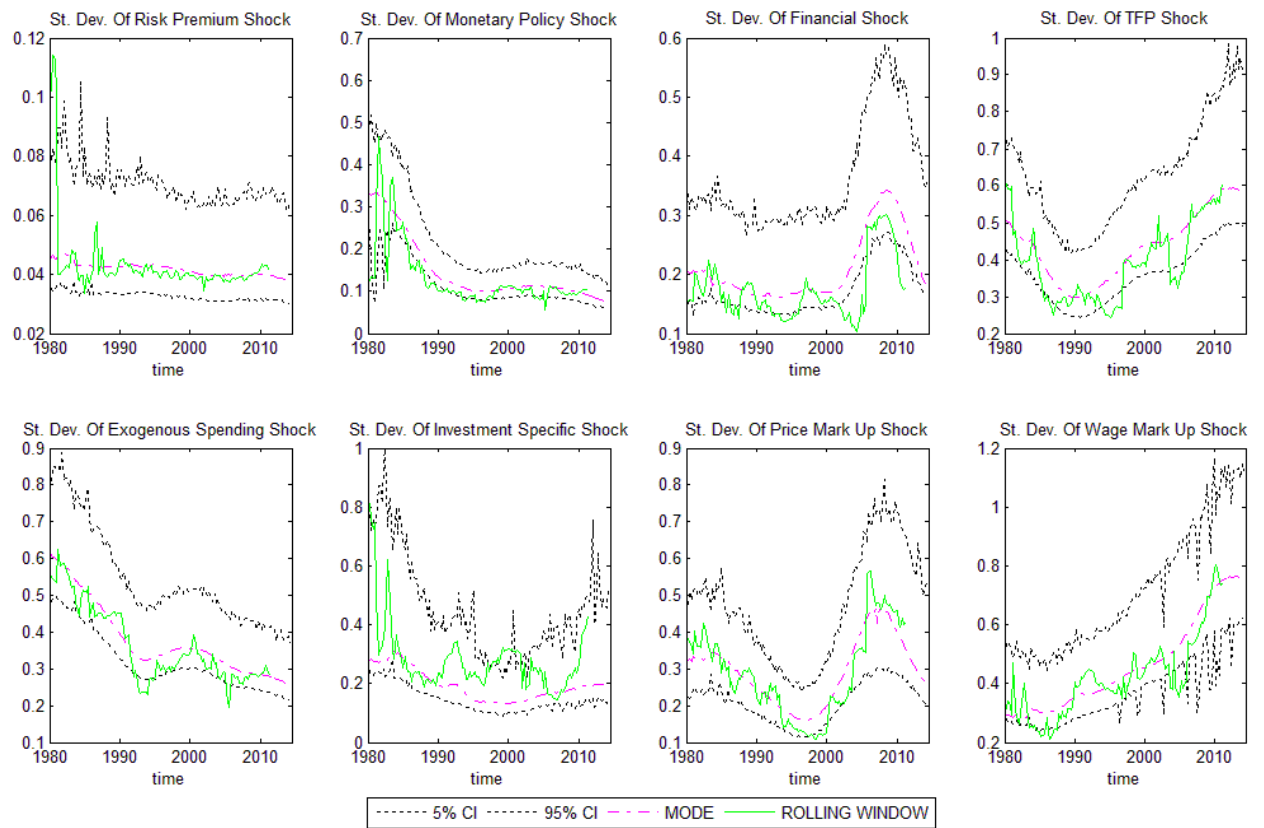


Figure 9: Robustness Check. The posterior mode obtained by BLL (pink dash-dotted line), the 5% and 95% posterior quantile values (black dotted lines), the posterior mode obtained by rolling window (solid green line).

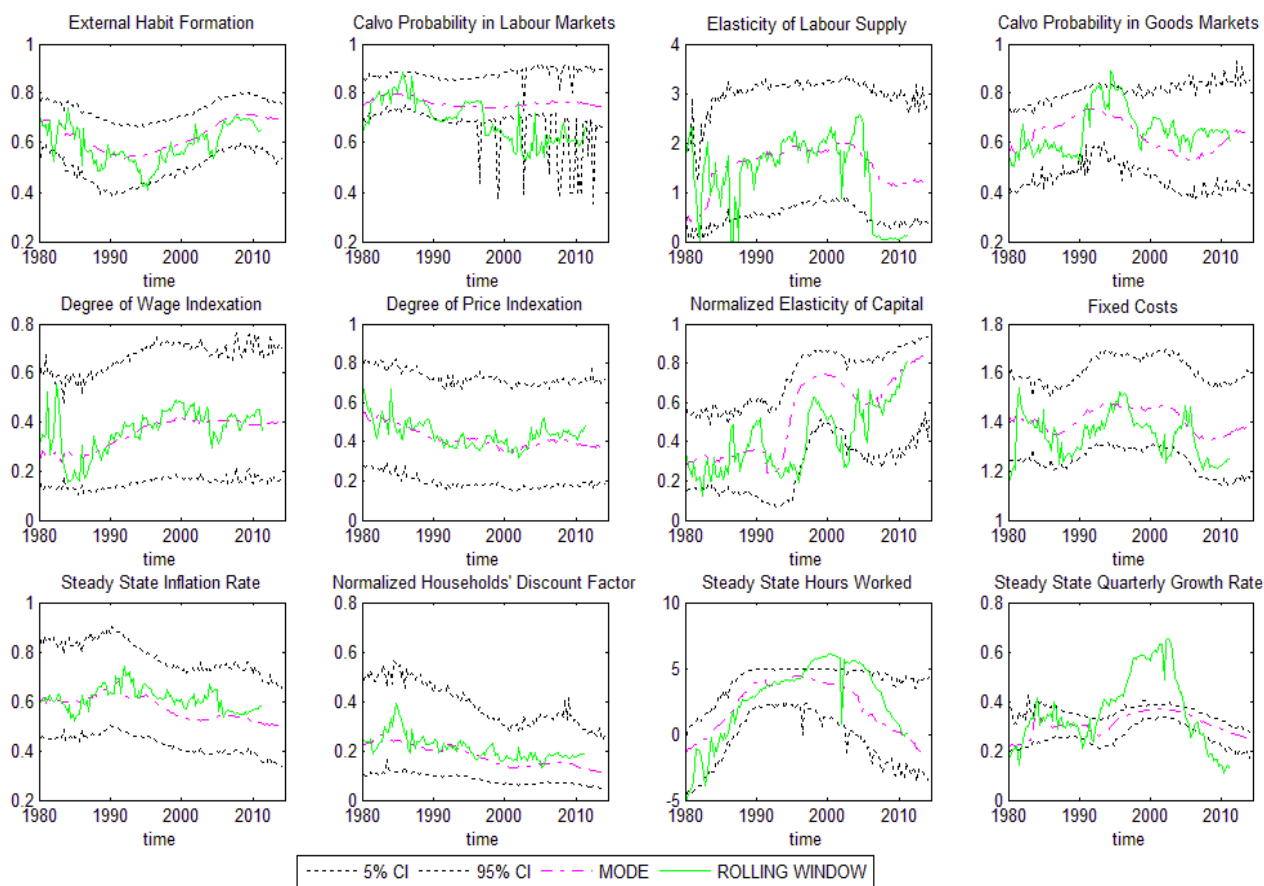


Figure 10: Robustness Check. The posterior mode obtained by BLL (pink dash-dotted line), the 5% and 95% posterior quantile values (black dotted lines), the posterior mode obtained by rolling window (solid green line).

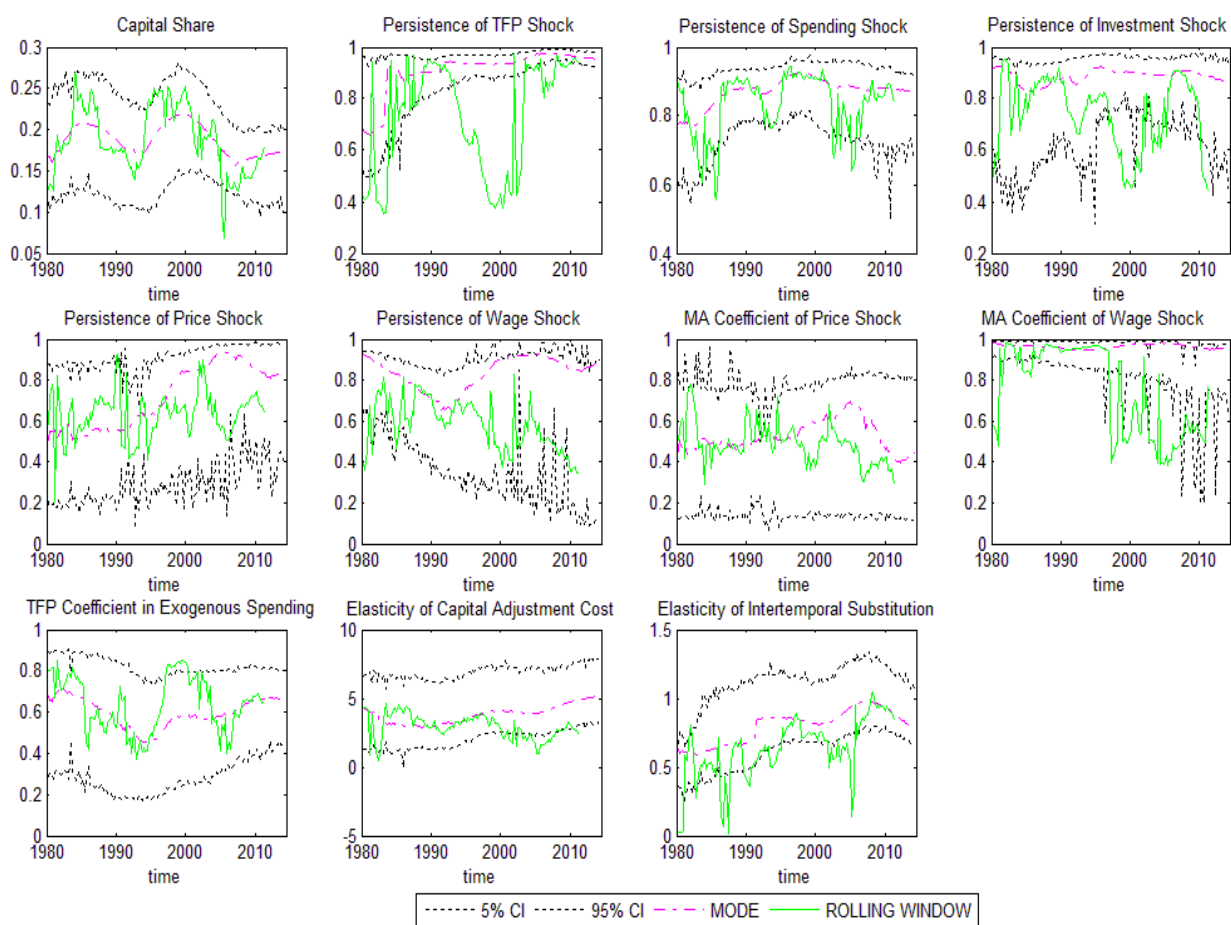


Figure 11: Robustness Check. The posterior mode obtained by BLL (pink dash-dotted line), the 5% and 95% posterior quantile values (black dotted lines), the posterior mode obtained by rolling window (solid green line).

6.6 Robustness check: different spread variable

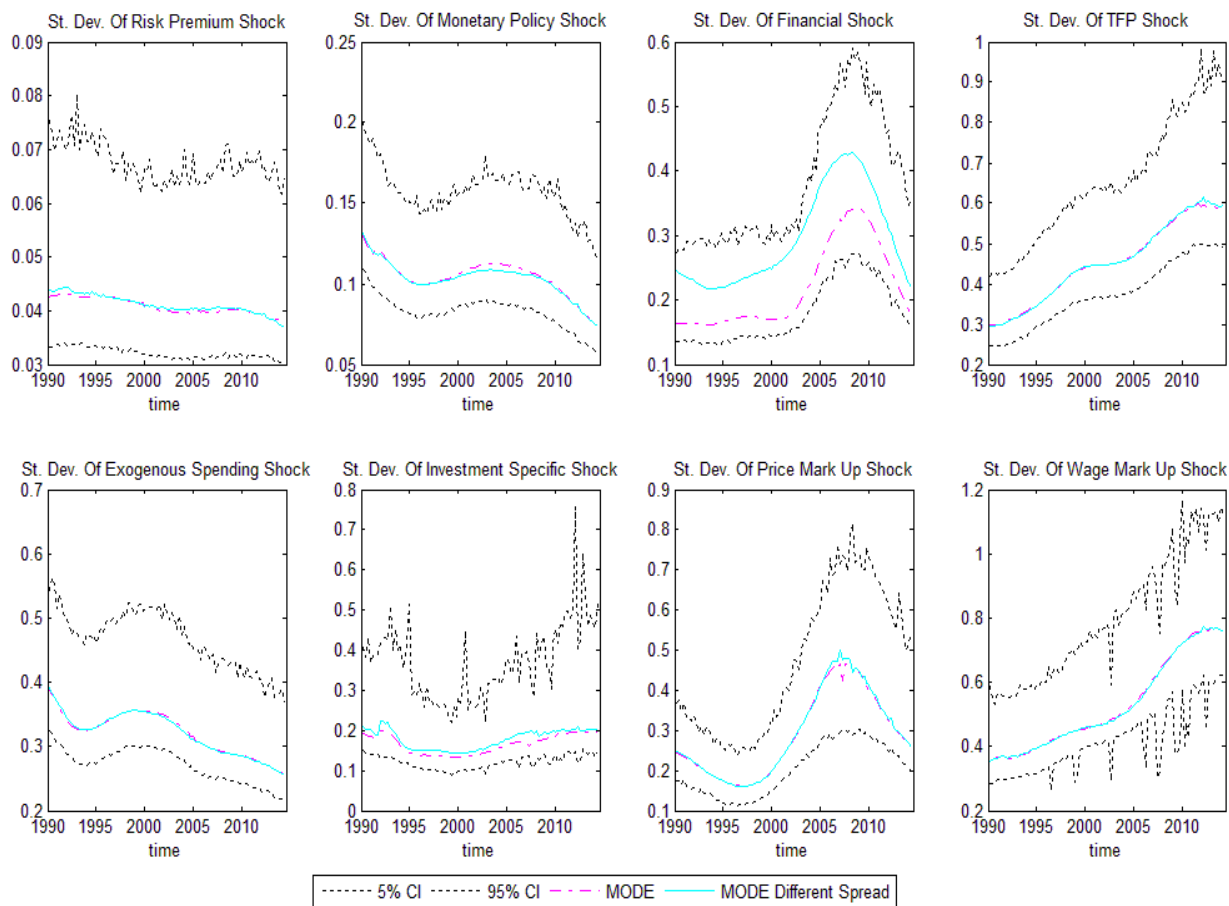


Figure 12: Robustness Check. The posterior mode obtained by BLL with spread BAA corporate bond yield over 10 year Treasury note (pink dash-dotted line), the 5% and 95% posterior quantile values (black dotted lines), the posterior mode obtained by BLL with spread BAA corporate bond yield minus Fed Funds Rate (solid blue line).

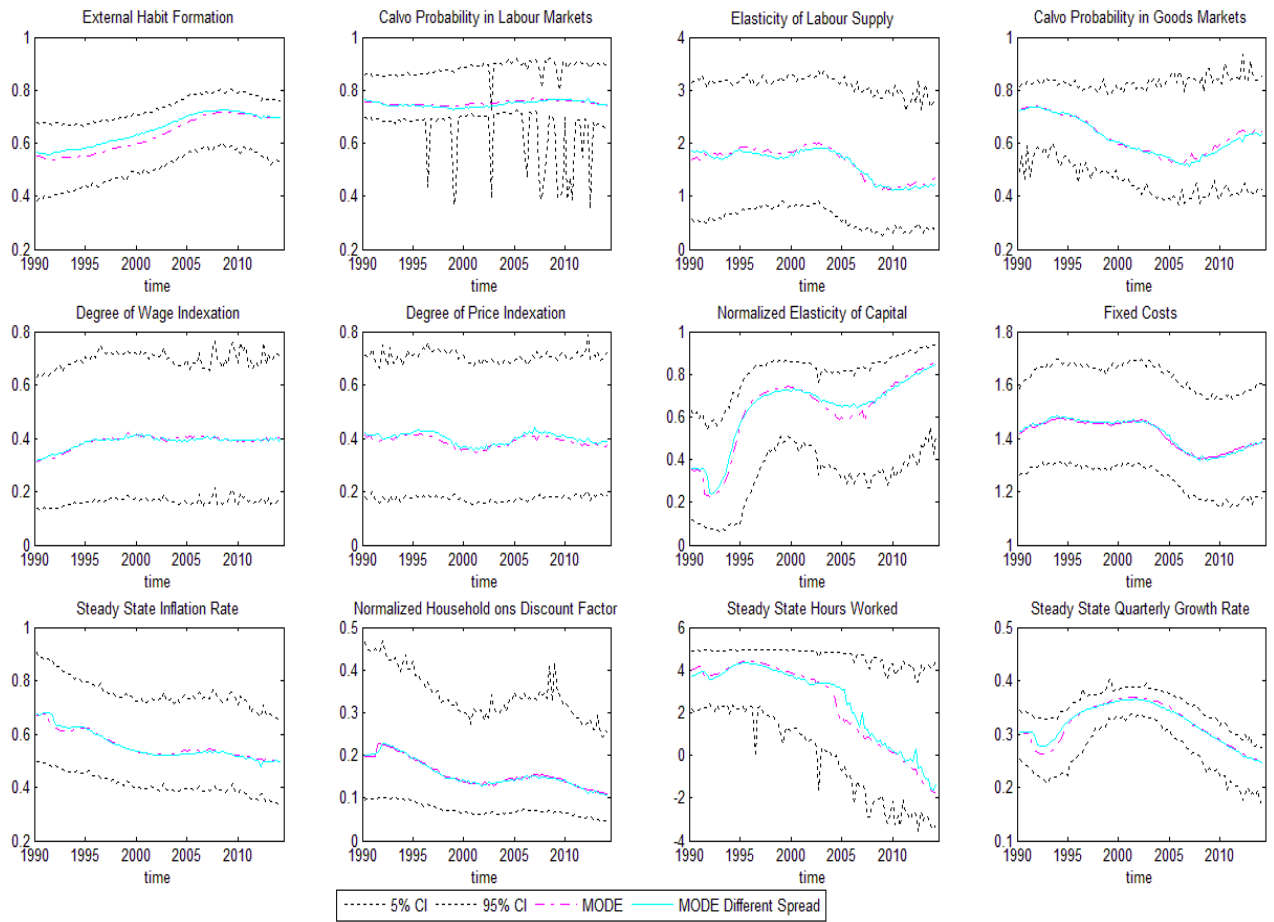


Figure 13: Robustness Check. The posterior mode obtained by BLL with spread BAA corporate bond yield over 10 year Treasury note (pink dash-dotted line), the 5% and 95% posterior quantile values (black dotted lines), the posterior mode obtained by BLL with spread BAA corporate bond yield minus Fed Funds Rate (solid blue line).

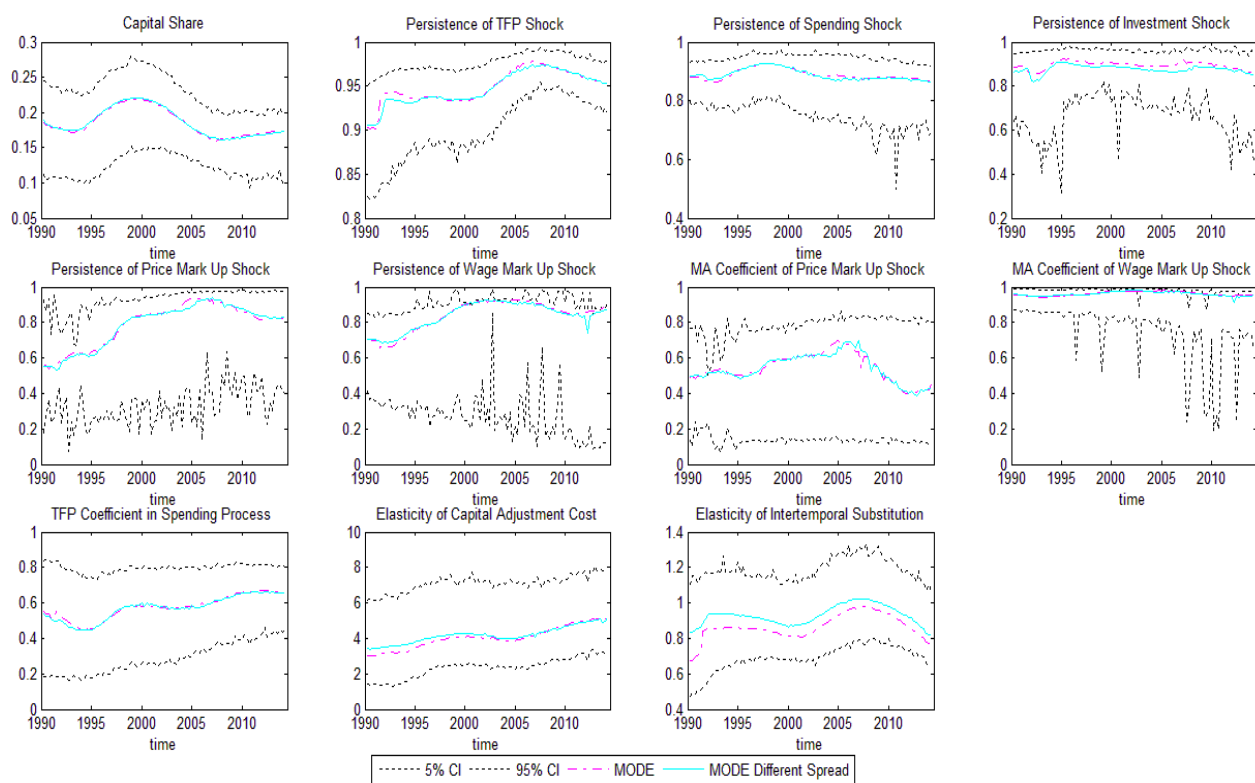


Figure 14: Robustness Check. The posterior mode obtained by BLL with spread BAA corporate bond yield over 10 year Treasury note (pink dash-dotted line), the 5% and 95% posterior quantile values (black dotted lines), the posterior mode obtained by BLL with spread BAA corporate bond yield minus Fed Funds Rate (solid blue line).

6.7 Robustness check: Simulation Exercise

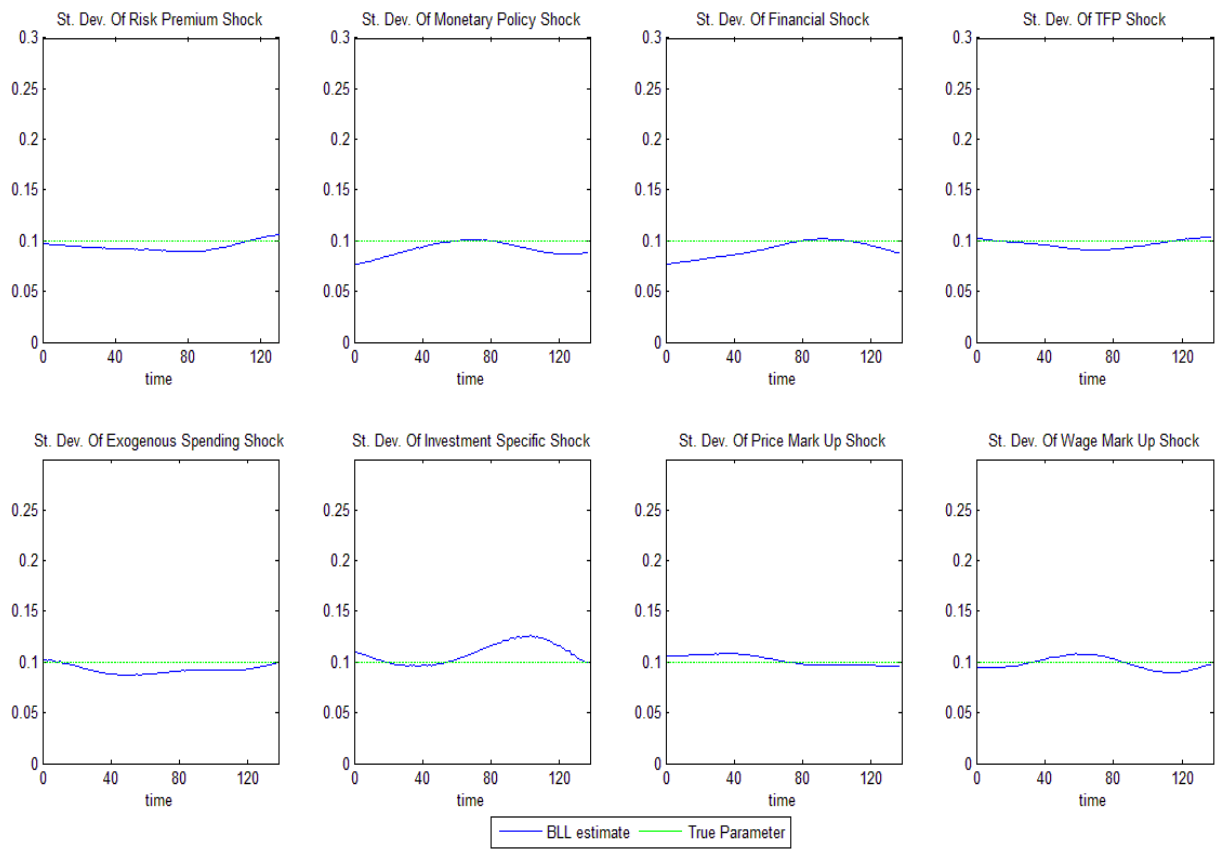


Figure 15: Simulation Exercise. The posterior mode obtained by BLL when the DGP is a model with fixed parameters (solid blue line) and the true parameter values at which the data are generated (dotted green line).

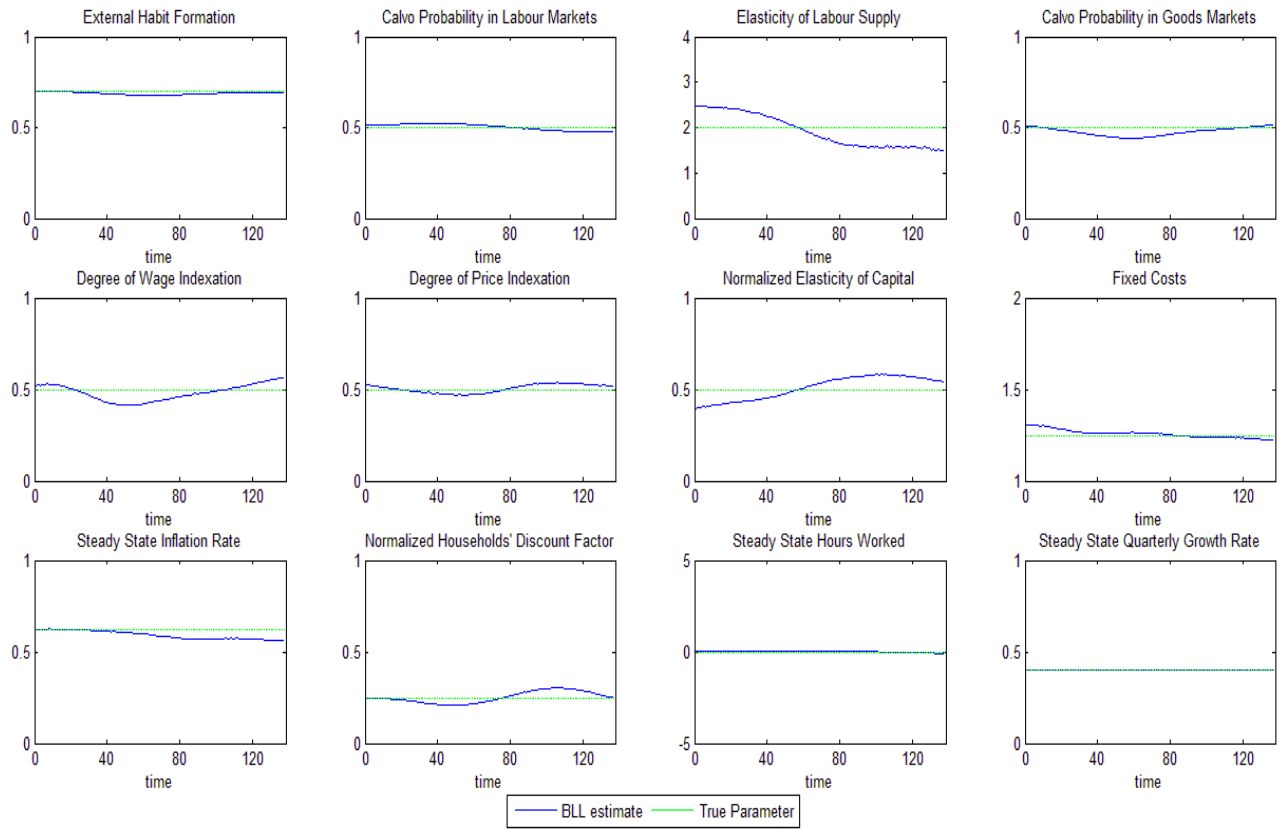


Figure 16: Simulation Exercise. The posterior mode obtained by BLL when the DGP is a model with fixed parameters (solid blue line) and the true parameter values at which the data are generated (dotted green line).

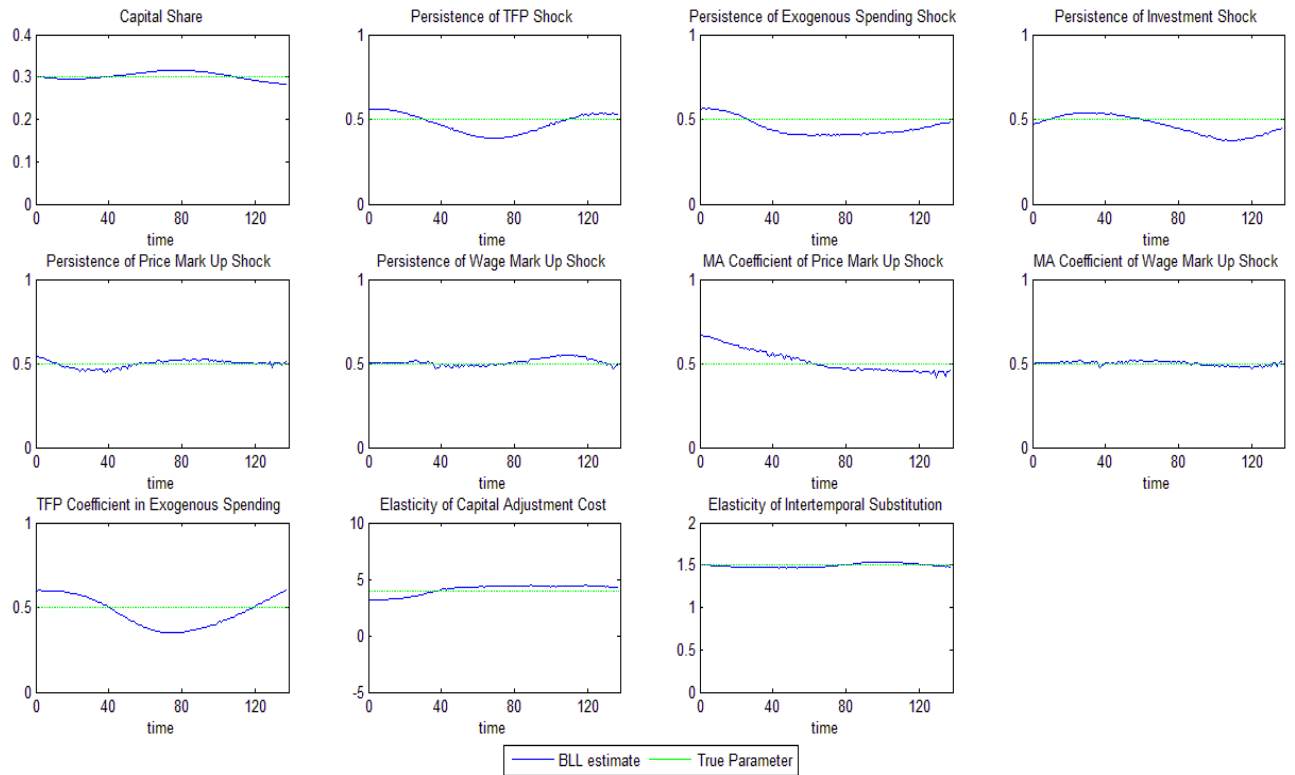


Figure 17: Simulation Exercise. The posterior mode obtained by BLL when the DGP is a model with fixed parameters (solid blue line) and the true parameter values at which the data are generated (dotted green line).