Stringy Spacetime Uncertainty as an Alternative to Inflation

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Abstract

In this paper we point out that the spacetime uncertainty relation proposed for string theory has strong cosmological implications that can solve the flatness problem and the horizon problem without the need of inflation. We make minimal assumptions about the very early universe.

Inflation is becoming one of the most important ingredients in modern cosmology. It provides solutions to the flatness problem, the horizon problem and the monopole problem. It also provides a framework to construct models that can explain existing experimental observations, in particular the spectrum of cosmic microwave background radiation (CMB) anisotropies. However, inflationary models are frequently involved with physics at very high energy scales, such as the GUT scale or even the Planck scale, far above the energy scale which has been experimentally confirmed (see for example [1]). In the light of the brane world scenario [2], it is possible that the fundamental scale of quantum gravity, or the scale of string theory, is much smaller than the 3+1 dimensional Planck scale. This means that certain new physics very different from conventional field theory may play a crucial role in the early universe, and thus our current understanding of cosmology may have to be greatly modified accordingly. The fact that the physics at very short distances can be amplified to observable cosmological effects is in fact one of the reasons why cosmology can be a crucial arena for testing string theory.

In this paper we point out such a possibility. We find that the spacetime uncertainty relation appearing in string theory have cosmological implications so significant that it may even resolve two of the above-mentioned puzzles without inflation. Remarkably, as we will see below, even if the energy scale associated with this spacetime uncertainty relation is as high as the Planck scale, it may still have equally strong cosmological implications.

In string theory it is proposed [3, 4] that there is a spactime uncertainty relation

$$\Delta x \Delta t \geq l_s^2, \tag{1}$$

where Δx and Δt are the uncertainties in the measurement of length in any spatial direction and that of time, and l_s is the string length scale. This relation is of a stringy origin. The simplest argument for it is the following. In quantum mechanics we have the uncertainty relation

$$\Delta E \Delta t > 1. \tag{2}$$

For a string the uncertainty ΔE in energy is related to an uncertainty Δx in its spatial extension

$$\Delta E \sim T_s \Delta x, \tag{3}$$

where $T_s = l_s^{-2}$ is the tension of the string. Combining the two relations above we obtain (1). It follows that when we use strings to probe the spacetime geometry, the precision in our measurement is limited by this uncertainty relation.

Note that in string theory, the property of spacetime is defined by the dynamics of the strings. The meaning of Δx in (1) is different from that of Δx in Heisenberg's uncertainty relation. In quantum mechanics, each measurement of x gives a definite value of x. Heisenberg's relation means that the distribution of possible values of x is of scale Δx. On the other hand, for the spacetime uncertainty relation, what Δx means is that at the string scale, one can no longer think of the spacetime as a classical manifold. Rather, the spacetime has to be described by certain noncommutative geometry or quantum geometry, for which it doesn't make sense to talk about two points separated by a distance smaller than Δx. For a field theory living on such a quantum space, it is therefore impossible to associate independent physical degrees of freedom within a range smaller than Δx. The meaning of Δx in (1) is thus more fundamental than that in Heisenberg's relation.

If the string length scale l_s is smaller than the scale currently accessible in experiments, say, smaller than $(TeV)^{-1}$, we might expect that we need to build more powerful colliders in order to see the effects of the uncertainty relation. However, as we will see below, it can leave clear cosmological imprint to be readily observed in the sky.

To fix our notation, recall that in standard cosmology, the Friedmann-Robertson-Walker (FRW) metric of the universe is

$$ds^2 = dt^2 - R^2(t)\left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega\right), \tag{4}$$

where $k = 0, 1$, or -1, corresponding to a flat, closed or open universe. The metric takes this form whenever the universe is isotropic and homogeneous. For $k = \pm 1$, $R(t)$ is also the radius of curvature (up to a constant numerical factor). For $0 < t < t_{EQ}$, the universe is radiation dominated, and $R(t) \propto t^{1/2}$. For $t_{EQ} < t$, the universe is matter dominated, and $R(t) \propto t^{2/3}$. At $t = t_{EQ} \sim 10^{10} sec$, the radiation density equals the matter density. The Hubble radius l_H and the particle horizon d_H are roughly the same $l_H(t) \sim d_H(t) \sim t$, up to an overall constant factor of order one. As a classical solution to the Einstein equation for four dimensional (4D) spacetime, we expect that the FRW metric be modified when the temperature exceeds the energy scale of new physics, say the string scale. However, the qualitative discussion below are robust to changes in the exact solution, as long as the universe starts with a big bang at a certain instant of time.

Eq.(4) describes the evolution of the universe for sufficiently late times. Near $t = 0$, the universe may have a completely different description, involving stringy effects that patch to an earlier contracting phase [5], or a gauge theoretical dual, or a phase

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dominated by D-branes [6], or a spacetime "foam" of virtual black holes [7]. We assume it makes sense to define the extent in time, from $t = 0$ to $t = l_*$, of such an early phase up to an uncertainty Δt. Clearly $\Delta t < l_*$. This relation together with (1) implies that

$$\Delta x > \frac{l_s^2}{l_*}. \tag{5}$$

As a fundamental uncertainty in the measurement of length, Δx is the lower bound on essentially any physical length variable.

In addition to the spacetime uncertainty relation, it was often thought that another uncertainty is true which claims that Δx and Δt should by themselves always be greater than l_s. This claim is now believed to be incorrect, after developments showing that $D0$-branes can probe very short distances [8]. We are not assuming such fundamental uncertainties separately on time and space, in agreement with [4]. In our application, l_* will depend on the nature of the evolution of the early universe. Possible candidates include l_p, the 4D Planck scale, or l_{11}, the 11D Planck scale. It is important that it is different from l_s, and further, for our application to cosmology, it is taken to be less than l_s. For simplicity we first focus on the case $l_* = 0$.

The flatness problem is to explain why the universe is observed to be so close to flat, or equivalently, why the energy density is so close to the critical density. Another way to describe the problem is to note that the radius of curvature $R(t)$ is about 10^{30} times larger than the characteristic length scale l_H or d_H at Planck time $t_P \sim 10^{-44} sec$. The uncertainty relation solves the flatness problem in a very simple way. At any time t, the radius of curvature $R(t)$ has to be larger than Δx. Since $t > \Delta t$, it follows from the uncertainty relation that $R(t) > l_s^2/t$ at any time. As we take t all the way down to $t = 0$, we see that the radius of curvature has to be infinite, and so the universe is completely flat at $t = 0$. The metric (4) then implies that $k = 0$, and the universe is flat at all times. Note that $R(t)$ is now only a scaling factor and is not a physical length scale for the case $k = 0$. It does not have to satisfy $R(t) > \Delta x$. This argument works equally well no matter how small l_s is, as long as it is not zero. In addition, it works for any dependence of $R(t)$ on t.

The spacetime uncertainty relation also solves the horizon problem, which is essentially asking why the CMB radiation is isotropic on the last scattering surface while the particle horizon at the decoupling time ($t_d \sim 10^{12} sec$) can only explain a very small fraction (10^{-5}) of the observed region. According to (5), contrary to the ordinary expectation that at earlier times the fluctuations are more violent, the universe is actually much smoother. Although the physics in different horizons are not causally related at t_d, they still have strong correlation because fluctuations of short wavelength can not

exist when time t is small. In fact, at $t = 0$, the size of particle horizon is zero, but the whole universe is uniform, with total correlation between any two point in space no matter how far they are separated. In some sense, the uncertainty relation with $l_* = 0$ dictates the universe to start with a very peculiar initial state. From the viewpoint of ordinary field theories, such states would be considered almost impossible to occur because they are so rare that they constitute a set of measure zero [9].

As an example of nonzero l_*, let

$$l_s \sim (TeV)^{-1}, \quad l_* \sim l_p \sim (10^{19} GeV)^{-1} \sim 10^{-44} sec, \tag{6}$$

where l_p denotes the 4D Planck length. We choose l_s to be of this value because it is roughly the largest possible value without contradiction to particle experiments. Although l_s enters the spacetime uncertainty principle since we expect stringy degrees of freedom to become relevant at that scale, l_* has been identified with l_p since this is where one might expect a four dimensional description which ignores the dynamics of the extra dimensions to break down completely. For this case (6), one has $\Delta x \simeq 10^{-2} cm$ at $t = l_*$.

For the flatness problem, using $R(l_*) > l_s^2/l_*$ we find

$$\frac{R(l_p)}{d_H(l_p)} > \frac{l_s^2}{l_p^{1/2} l_*^{3/2}}. \tag{7}$$

It follows that this ratio is about 10^{32} for the example (6), in agreement with the experimental bound (10^{30}). For the horizon problem, fluctuations can exist only at a length scale larger than l_s^2/l_* when $t = l_*$. At decoupling time t_d, this smoothness scale is amplified into the size of

$$L \simeq \frac{l_s^2}{l_*} \frac{R(t_d)}{R(l_*)}. \tag{8}$$

Causal interactions will result in a correlation length larger than L by the size of the particle horizon at t_d, but it is negligible compared to the smoothness we need to account for. If l_* is sufficiently smaller than l_s, it is possible to have L large enough to agree with CMB observations. For the example (6), L is roughly the same as the size of today's horizon. and suffices to account for the homogeneity we observe in CMB today. For fixed l_s, the smaller l_* is, the longer we will have to wait until we see comparable anisotropies in CMB. One can say that we are replacing the two problems by the new hierarchy problem why l_s/l_* is so large. This may turn out to be a dynamical result of the evolution of the early universe, or an initial condition problem.

So far we have not addressed the problem of monopole abundance. A treatment of this problem requires an understanding of the thermal history of the universe, and more

details of the new physics at the string scale. It cannot be solved by the uncertainty relation alone. Nevertheless, roughly speaking, the uncertainty relation forbids high momentum modes from appearing at early times. It makes it harder to generate a lot of heavy particles. On the other hand, there are many moduli fields in string theory which may show up as relics for us to worry about, just like monopoles.

Another important issue is the energy density perturbation. The primordial energy density perturbation is found to have a scale invariant power spectrum, and to satisfy the Gaussian distribution. Just like for inflationary scenarios one needs to construct specific models to account for these experimental data, here we need to first construct a model, which obeys the uncertainty relation (1).

There are two obvious choices to realize (1) in a field theory. One way is to consider a noncommutative spacetime which satisfies some nontrivial commutative relations from which (1) can be derived. This may require some care since noncommutativity between time and space variables is expected to cause many difficulties in field theories, such as the violation of causality and unitarity [10], although it may be allowed in string theory [11]. However non-commutativity in both space-space and space-time directions appears to be possible for decoupled theories [12]. Non-commutativity in spatial as well as space-time directions has also been related to dual CFTs in the context of the ADS/CFT correspondence [13, 14, 15]. Generalizations of Riemannian geometry and general relativity to noncommutative spaces have been proposed [16, 17], but a better understanding is still needed.

Another possibility is to modify the canonical commutation relation between x and p, such that from this relation one can derive

$$\Delta x > \frac{l_s^2}{t}. \tag{9}$$

The case of a time-invariant uncertainty for Δx is considered in [18]. It is straightforward to modify it to have the uncertainty (9). The drawback of this approach is that we are not realizing the fundamental uncertainty relation (1) but only a consequence of that. Its merit is that it is very close to a traditional formulation. In this approach the Hilbert space consists of fluctuations of all frequencies all the time, but the Hamiltonian operator vanishes on higher Fourier modes until the time when Δx is smaller than its wavelength. In other words, those fluctuations violating the uncertainty relation are spectators which are decoupled from everything else.

An exploration of these possibilities is left for future study [19]. Here we shall only discuss what are the implications of the observed properties of the energy density perturbations. The fluctuation $\delta \rho$ in energy density with comoving momentum k has

physical momentum $k/R(t)$ at time t, and thus is forbidden for $t < (kl_s^2/a)^{2/3}$ for $R(t) = at^{1/2}$. It follows that fluctuations of different length scale we see today emerge at different t. The fluctuations of interest to us are those of sizes $1Mpc$ to $1000Mpc$. They emerge out of the uncertainty constraint between $t \sim 10^{-42}sec$ and $10^{-40}sec$ for the example (6). As in inflation, longer wavelengths appear earlier in time.

At first glance, the fact that fluctuations of length scale smaller than $\Delta x(t) \sim l_s^2/t$ are forbidden may seem to violate causality. One may argue that if someone interferes with a field at a certain place, the field will have to react simultaneously at another place Δx away from it. This is however not a valid argument. As we have tried to emphasize ealier, the quantum geometry of spacetime is such that it does not make sense to talk about independent physical degrees of freedom at two points within a range of Δx. Any interference one can perform to a physical system is done over a region of size Δx simultaneously.

Our discussion of the horizon and flatness problems has made minimal assumptions about the very early universe, except for associating with it a small uncertainty in time, in addition to the stringy uncertainty principle. We presented scenarios where these problems are solved without any need for inflation, but it is also possible to use this mechanism in conjunction with inflation.

In the brane world scenario, it is also of interest to consider cosmological effects of noncommutative geometry associated to the worldvolume of the brane, as in ref.[20]. This possibility is also motivated from string theory, where it was found that a D-brane worldvolume becomes noncommutative when the NS-NS B field background is turned on [21]. It was also pointed out that even if the vacuum expectation value of the B field vanishes, its quantum fluctuations will result in an effective uncertainty relation for the measurement of spacetime coordinates on the D-brane [22]. It would also be interesting to consider the cosmological effects of these uncertainty relations.

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