

# Broadband THz transmission within the symmetrical plastic film coated parallel-plate waveguide

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**Abstract:** We report the broadband THz transmission within the symmetrical plastic film coated parallel-plate waveguide. We theoretically study the antiresonant reflecting mechanism of the waveguide and we find that the broadband THz wave can transmit in this waveguide with ultra-low loss. The loss of TM mode in this waveguide can be 4 orders of magnitude lower than the uncoated parallel-plate waveguide. The transmission bandwidth of this waveguide is up to 5.12 THz. We further show the mode field distributions which explain the loss mechanism.

**OCIS codes:** 230.7370, 230.7390, 260.3090.

## 1. Introduction

The development of low-loss waveguides is essential for facilitating terahertz (THz) science and applications [1-4] but it is limited by the high ohmic loss of metals and the high absorption of dielectric materials. As we know that the dielectric-coated metallic hollow fibers [5-8] and parallel-plate waveguides [9-11] had proposed long before, but few articles theoretically describe the dielectric-coated parallel-plate waveguides for efficient transmission of THz wave. The parallel-plate waveguide [9-11] can guide THz with low losses for TE mode and the loss coefficients of TE mode are as low as 2.6 dB/km. However, the loss of TM mode in parallel-plate waveguide is always 4 – 5 orders of magnitude larger than TE mode. On the other side, the bandwidth of transmission windows for antiresonant reflecting mechanism of the dielectric pipe waveguide [12-15] is always narrow (the biggest bandwidth  $\Delta f$  is 1.1THz in [12]) which could limit the interests for these waveguides.

In this paper, we present a theoretical study of the waveguide which is coated with symmetrical plastic film inside the parallel-plate waveguide. We numerically study the relationships between the propagating loss coefficient and the waveguide structure as well as the THz wave frequency. Furthermore, the mode field distribution characteristics are studied. The obtained results show that this waveguide can guide broadband THz wave and it has the loss of TM mode 4 orders of magnitude lower than that of uncoated parallel-plate waveguide. We anticipate that the present waveguide plays an important role in many THz applications

such as spectroscopy, sensing, detection, communication and imaging.

## 2. Mode dispersion equation of the waveguide

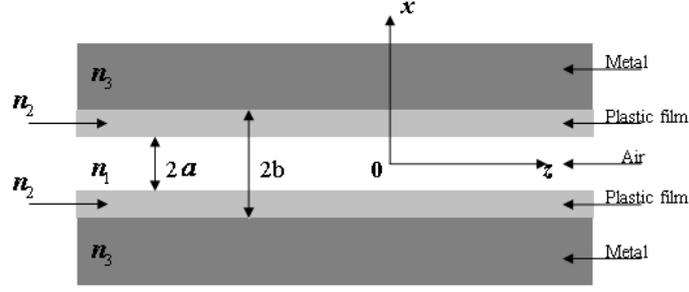


Fig. 1. The symmetrical plastic film coated parallel-plate waveguide structure.

The waveguide structure and coordinate system are shown in Fig. 1, and the waveguide width in  $y$  direction is infinity.  $d = 2a$  is the air interval, and  $t = b - a$  is thickness of the plastic film. When transverse magnetic (TM) modes propagate in the positive  $z$ -direction, field components of the even mode can be written as [16]:

$$H_y(x) = \begin{cases} A \cos(h_1 x) & |x| \leq a \\ A \left[ \cos(h_1 a) \cosh_2(|x| - a) - \frac{h_1 \varepsilon_2}{h_2 \varepsilon_1} \sin(h_1 a) \sinh_2(|x| - a) \right] & a \leq |x| \leq b \\ A \left[ \cos(h_1 a) \cosh_2(b - a) - \frac{h_1 \varepsilon_2}{h_2 \varepsilon_1} \sin(h_1 a) \sinh_2(b - a) \right] e^{-h_3(|x| - b)} & |x| \geq b \end{cases} \quad (1)$$

where  $A$  is a coefficient related with the mode power,  $h_1 = (\varepsilon_1 k_0^2 - \beta^2)^{1/2}$ ,  $h_2 = (\varepsilon_2 k_0^2 - \beta^2)^{1/2}$ ,  $h_3 = (\beta^2 - \varepsilon_3 k_0^2)^{1/2}$ ;  $\varepsilon_1 = n_1^2$ ,  $\varepsilon_2 = n_2^2$ ,  $\varepsilon_3 = n_3^2$  are relative permittivities,  $k_0$  is the wave vector in vacuum.  $\beta = \beta_1 - i^* \alpha$  is the complex propagation constant, where real part  $\beta_1$  is the phase propagation constant and it is related to effective refractive index  $n_{eff} = \beta_1 / k_0$  and the imaginary part  $\alpha$  represents the loss coefficient of the mode.

As the tangential components of the electromagnetic fields on interfaces are continuous, we can obtain the dispersion equation of the even mode as follows [16]:

$$h_1 a = \tan^{-1} \left[ \frac{1 - \frac{h_2 \varepsilon_3}{h_3 \varepsilon_2} \tanh_2(b - a)}{\frac{h_1 \varepsilon_3}{h_3 \varepsilon_1} + \frac{h_1 \varepsilon_2}{h_2 \varepsilon_1} \tanh_2(b - a)} \right] \quad (2)$$

By using Eq. (2), we can calculate the propagation constant  $\beta$ . Then the loss coefficient

$\alpha$  and effective refractive index  $n_{eff}$  can be derived accordingly. Based on the propagation constant, we can also obtain the field distribution of the mode by using Eq. (1).

### 3. The transmission characteristics of THz wave in the waveguide

In order to avoid the influence of the relative permeability, copper is adopted here for the parallel plates. We choose plastic material for the film with a refractive index of  $n_2 = \sqrt{\varepsilon_2} = 1.5 - 0.001 * i$  [12], and the space between films is air with a relative permittivity of  $\varepsilon_1 = 1$ . Relative permittivity of copper  $\varepsilon_3$  can be calculated by the Drude model:

$$\varepsilon_3 = \varepsilon_\infty - \frac{\omega_p^2}{\omega^2 - i\omega\omega_\tau} \quad (3)$$

where  $\varepsilon_\infty$  is the high frequency permittivity of copper which is always negligible in THz region,  $\omega_p = 1.1234 * 10^{16}$  Hz [17] is the plasma oscillation frequency of copper,  $\omega_\tau = 1.3798 * 10^{13}$  Hz [17] is the damping frequency of copper, and  $\omega$  is the angular frequency of the THz wave.

#### 3.1 The loss characteristics of THz wave in the waveguide

When air interval is  $d = 2a = 2$  mm and THz frequency is 5 THz, the dependence of  $\alpha$  on the film thickness  $t$  is derived by numerical calculation of Eq. (2) as shown in Fig. 2:

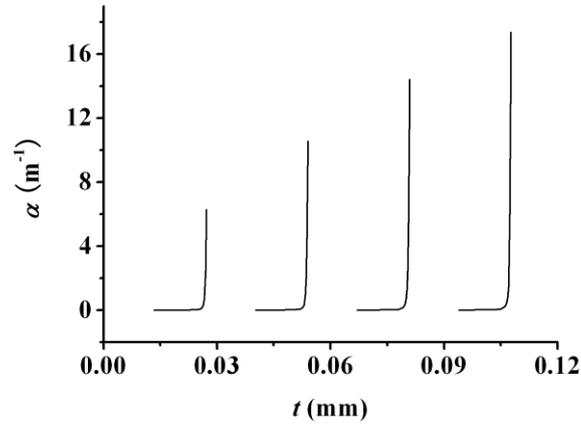


Fig. 2. The dependence of amplitude loss coefficient on the film thickness when THz frequency is 5 THz and air interval is 2 mm. From left to right in turn, the curve is corresponding to the resonance order of  $m = 1, 2, 3, 4$ , respectively.

By Fig. 2, we find that when the film thicknesses are close to 0.027228 mm, 0.054063 mm, 0.080894mm and 0.10771mm, the loss coefficient increases sharply. This is caused by the antiresonant reflection mechanism. When 5 THz becomes the resonance frequency for the film thickness, the energy of THz wave will transmit through the plastic film and much more energy will distribute in the copper plate. As we know that the ohmic loss of metals is very high, the loss increases sharply. Resonance frequency can be accurately predicted with the following relation [18]:

$$f_m = \frac{mc}{2t\sqrt{n^2 - 1}} \quad (4)$$

where  $m$  is the resonance order,  $t$  and  $n$  are the thickness and refractive index of the plastic film respectively and  $c$  is the speed of light in vacuum. Here we substitute  $f = 5 \text{ THz}$ ,

$n = 1.5$ ,  $c = 3.0 \times 10^8 \text{ m/s}$  into Eq. (4) and we find that when  $m = 1, 2, 3$  and  $4$ , thicknesses of the plastic film are  $0.026833 \text{ mm}$ ,  $0.053665 \text{ mm}$ ,  $0.080498 \text{ mm}$  and  $0.10733 \text{ mm}$ , respectively. The simulation results are in good agreement with the theoretical results. Fig. 2 shows when the thickness of the coating film is far away from the resonance thickness, the waveguide loss is extremely low. When  $d = 2a = 2 \text{ mm}$  and  $t = 0.01337 \text{ mm}$ , the minimum loss of TM mode is as low as  $0.1398 \times 10^{-2} \text{ m}^{-1}$ . However, at the same conditions, the loss

of TM mode in the uncoated parallel-plate waveguide is  $0.3678 \text{ m}^{-1}$ , which is 2 orders of magnitude larger than the coated waveguide. The loss at the resonance thickness increases as the thickness increases. It is worth to point out that when the  $t$  is below  $0.01337 \text{ mm}$ ,  $0.04020 \text{ mm}$ ,  $0.06703 \text{ mm}$ ,  $0.09386 \text{ mm}$ , for each  $m$  respectively, the mode is cut off. It is found that the model effective refractive index is always very close to 1, which shows that the phase velocity of the guiding mode is always close to the speed of light in vacuum, hence the phase velocity dispersion of this waveguide is very low.

Taking  $t = 0.01337 \text{ mm}$  and  $f = 5 \text{ THz}$ , the dependence of  $\alpha$  on the air interval  $d = 2a$  is derived by numerical calculation of Eq. (2) as shown in Fig. 3:

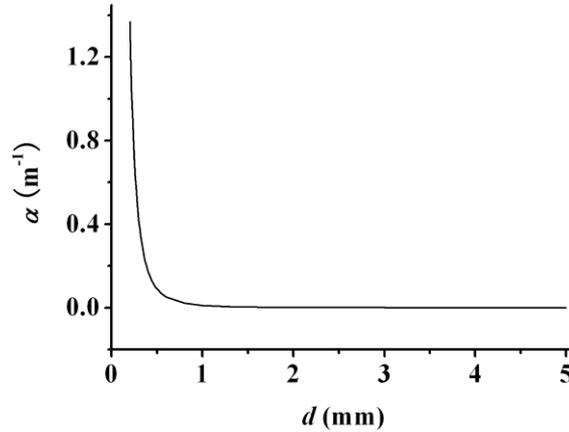


Fig. 3. The dependence of amplitude loss coefficient on the air interval  $d$  when THz frequency is 5 THz and the film thickness is  $0.01337 \text{ mm}$

From the figure, we can see that the loss decreases monotonously as  $d$  increases. The decrease in loss is much sharper when the  $d$  is very small. When  $d = 5 \text{ mm}$ , the loss is as low as  $0.78 \text{ dB/km}$ . However, at the same conditions, the loss of TM mode in the uncoated parallel-plate waveguide is  $0.2010 \text{ m}^{-1}$ , which is 4 orders of magnitude larger than the coated waveguide.

When  $t = 0.075$  mm and  $d = 2$  mm, we get the law of  $\alpha$  changing with the THz frequency  $f$  as shown in Fig. 4:

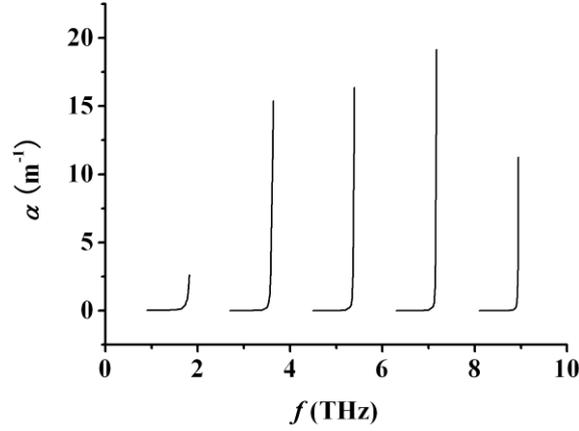


Fig. 4. The dependence of amplitude loss coefficient on the THz frequency when air interval is  $d = 2$  mm and film thickness is  $t = 0.075$  mm. From left to right in turn, the curve is corresponding to the resonance order of  $m = 1, 2, 3, 4, 5$ , respectively.

Fig. 4 illustrates that the loss of plastic film coated parallel-plate waveguide is very low when THz frequency is far away from the resonance frequency and the loss at the resonance frequency has increased dramatically. The resonance frequencies of the simulation are: 1.82 THz, 3.64 THz, 5.39 THz, 7.17 THz, 8.95 THz, respectively. Here we substitute  $t = 0.075$  mm,  $n = 1.5$ ,  $c = 3.0 \times 10^8$  m/s into Eq. (4), and we get that the resonance frequencies as: 1.79 THz, 3.58 THz, 5.37 THz, 7.15 THz, 8.94 THz, corresponding to  $m = 1, 2, 3, 4, 5$ , respectively. The simulation results are in good agreement with the theoretical results. However, this waveguide also has a low frequency cutoff and this is because of the parallel-plate waveguide structure. From Fig. 4 we can see the cutoff frequencies are 0.89 THz, 2.68 THz, 4.48 THz, 6.26 THz, 8.05 THz, respectively. The cutoff frequency of the parallel-plate waveguide can be accurately predicted as [9]  $f_m = Nc / 2d$ . We substitute  $d = 2$  mm,  $c = 3.0 \times 10^8$  m/s into it and we get that the low cutoff frequencies are: 0.90 THz, 2.70 THz, 4.50 THz, 6.30 THz, 8.10 THz, corresponding to the  $N = 12, 36, 60, 84, 108$ , respectively. We can see that at these conditions, the bandwidth of the transmission window of the waveguide is 0.9 THz. Actually when  $d = 2$  mm and  $t = 0.01337$  mm, the bandwidth of the transmission window is up to 5.12 THz as shown in Fig.5:

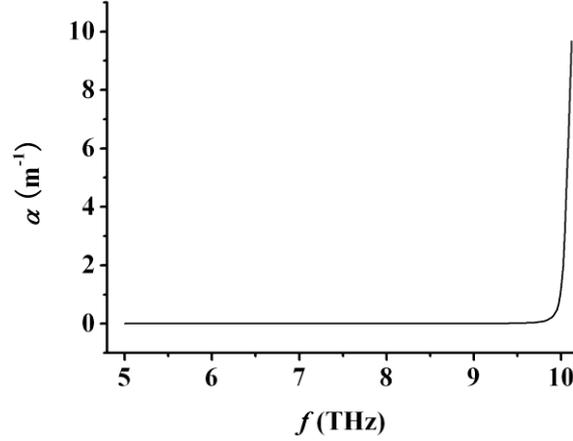


Fig. 5. The dependence of amplitude loss coefficient on the THz frequency when air interval is  $d = 2$  mm and film thickness is  $t = 0.01337$  mm and the curve is corresponding to the resonance order of  $m = 1$ .

From Fig. 5, we can see that the low cutoff frequency is 5.00 THz and the resonance frequency is 10.12 THz. We can see the bandwidth is up to 5.12 THz. It is worth to point out that in the range of 5.00 THz – 9.70 THz, the loss of the TM mode is lower than  $0.05 m^{-1}$ .

We also calculate the group velocity of the guiding mode according to the formula

$$v_g = \frac{c}{n_{eff} \left(1 + \frac{k_0}{n_{eff}} \frac{dn_{eff}}{dk_0}\right)},$$

and then we obtain the dependence of the group velocity on the

frequency  $f$  at  $d = 2$  mm and  $t = 0.01337$  mm, as shown in Fig. 6. From Fig. 6, we can observe that the group velocity is very near the speed of light in vacuum in the frequency range of 5.00 THz – 9.70 THz, which indicates that the group velocity dispersion of the mode is very low when the THz wave is far from the resonance frequency. However, the group velocity decreases sharply near the resonance frequency, and this is because the mode loss is very high and much THz wave energy is in the plastic film.

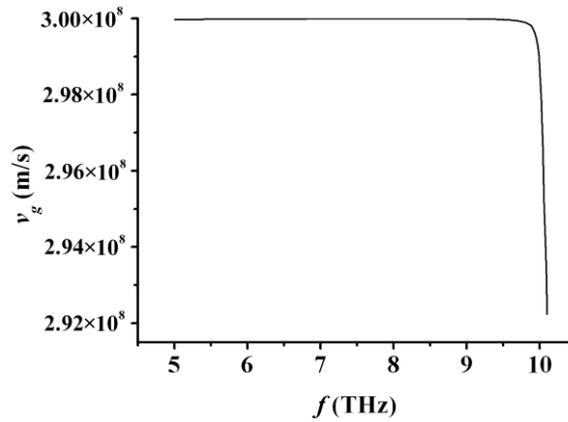


Fig. 6. The dependence of the group velocity of the guiding mode on the frequency when the air interval is  $d = 2$  mm and the film thickness is  $t = 0.01337$  mm.

### 3.2 Mode field distributions of the waveguide

For the air interval is 2 mm, THz frequency is 5 THz and film thicknesses are 0.09386 mm and 0.1077 mm, we obtain the transverse magnetic field amplitudes of the waveguide by

using Eq. (1). We choose these two thicknesses because when  $m = 4$ , the thickness of 0.09386 mm is the low cutoff thickness which is far away from resonance thickness of 5 THz and the other is the resonance thickness of 5 THz. The normalized mode field distributions are shown in Fig. 7 (a) and (b), respectively.

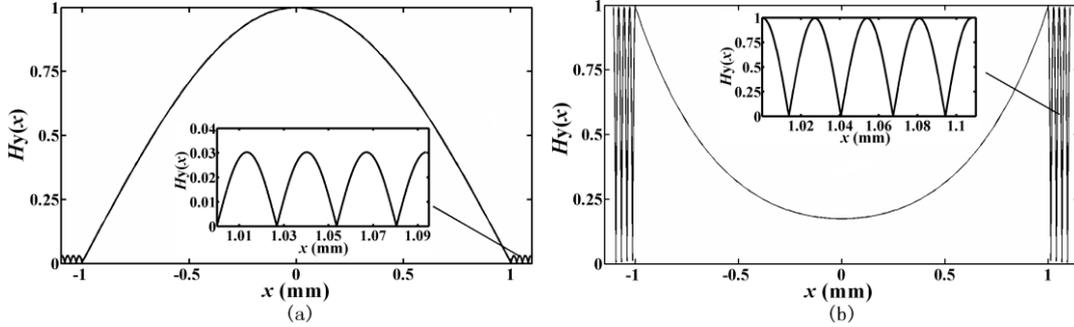


Fig. 7. The normalized mode field distributions of the waveguide when air interval is 2 mm, THz frequency is 5 THz and film thicknesses are 0.09386 mm (a) and 0.1077 mm (b), respectively. Insets present the mode field distributions in the plastic film respectively.

From Figs. 7 (a) and (b), we can see that for the film thickness of  $t = 0.09386$  mm, most of the mode field energy is distributed in the air layer. However for the film thickness of  $t = 0.1077$  mm, a much larger part of the mode field energy is distributed in the plastic film. This is because when the film thickness is far away from resonance thickness, most of the THz wave energy reflects on the interface between the air and plastic film, so that almost no energy will arrive to the plastic film and the copper plate. When the THz wave directly exposes to metals, the amplitude of the TM mode to metal is always very big and the ohmic loss of metals is very high, hence after coating a plastic film, we can lower the loss of TM mode in parallel-plate waveguide drastically. It is worth to point out that because of the amplitude of electric field to the metals is always negligible, coating cannot lower the loss of TE mode. From the insets, we can see that the mode field distributions are basically same in the film for the two film thicknesses, which both vary rapidly and present oscillation distribution. However, the relative amplitudes are different and they are 0.03 and 1, respectively. It is worth to point out that at the boundaries between the air-core and plastic film (position =  $\pm 1$  mm shown in Fig. 7), the phase of the oscillatory magnetic field is the nadir for the thickness of  $t = 0.09386$  mm, while it is peak for the thickness of  $t = 0.1077$  mm. The total phases of the oscillatory magnetic fields within the film layers associated with the low cutoff thickness and resonance thickness, respectively, equal an odd number and even number of half-oscillations, as shown in the insets of Figs. 7 (a) and (b). They are  $3.5\pi$  and  $4\pi$  in the case of  $m = 4$ . When a  $0.5\pi$ -phase-difference is introduced into the plastic film layer, it transforms the low-loss confined waveguiding mode to high-loss leaky mode. This is the reason why when the  $t$  is below 0.09386 mm, for  $m = 4$ , the mode is cut off. As the electromagnetic fields on interfaces are continuous, we can see a large part of energy come to the copper as given in Fig. 7 (b).

#### 4. Conclusion

In summary, we propose to coat symmetrical plastic film on the parallel-plate waveguide. We theoretically obtain that this waveguide can transmit broadband THz wave with ultra-low loss. In the considered scope, the minimum loss is as low as 0.78 dB/km which is 4 orders of magnitude lower than the loss of the uncoated case. The transmission mechanism of this

waveguide is antiresonant reflection. This kind of waveguide can realize broadband transmission windows and the obtained transmission bandwidth can be as high as 5.12 THz. The dispersion of this waveguide is also very low. The mode field distribution characteristics are further discussed which explain the loss mechanism. These results emphasize the high potential of this class of inexpensive waveguides for various THz applications including spectroscopy, sensing, detection, communication and imaging.

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