Geometric Optimisation of Piezoelectric Fan Arrays for Low Energy Cooling

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Abstract—A numerical model of a typical piezoelectric fan blade is derived and validated against experimental data. Numerical error is found to be 5.4% and 9.8% using two data comparison methods. The model is used to explore the variation of pitch as a function of amplitude, A , for a confined two-blade piezoelectric fan array in face-to-face orientation, with the blades oscillating both in-phase and counter-phase. It has been reported that in-phase oscillation is optimal for generating maximum downstream velocity and flow rate in unconfined conditions, due at least in part to the beneficial coupling between the adjacent blades that leads to an increased oscillation amplitude. The present model demonstrates that confinement has a significant detrimental effect on in-phase oscillation. Even at low pitch, counter-phase oscillation produces enhanced downstream air velocities and flow rates. Downstream air velocity from counter-phase oscillation can be maximally enhanced, relative to that generated from a single blade, by 17.7% at $P = 8A$. Flow rate enhancement at the same pitch is found to be 18.6%. By comparison, in-phase oscillation at the same pitch outputs 23.9% and 24.8% reductions in peak downstream air velocity and flow rate, relative to that generated from a single blade. This optimal pitch, equivalent to those reported in the literature, suggests that counter-phase oscillation is less affected by confinement. The optimal pitch for generating bulk airflow from counter-phase oscillation is large, $P > 16A$, due to the small but significant downstream velocity across the span between adjacent blades. However, by considering design in a confined space, counter-phase pitch should be minimised to maximise the bulk airflow generated from a certain cross-sectional area within a channel flow application. Quantitative values are found to deviate to a small degree as other geometric and operational parameters are varied, but the established relationships are maintained.

Keywords—Piezoelectric fans, low energy cooling, power electronics, computational fluid dynamics.

I. INTRODUCTION

PIEZOELECTRIC (PE) fans were first considered at the end of the 1970s [1], and a PE fan device was patented in 1985 [2]. PE fans are ultra-low power air movers made up of a fan blade, a lead zirconate titanate (PZT) actuator and a clamp [3]. In operation, very small levels of expansion and contraction are generated in the actuator by an alternating electrical current [4]. The power is supplied to the actuator at the blade's first natural frequency to induce resonance, and the resulting blade tip oscillation amplitude is several orders of magnitude greater than the initial motion imparted [5]. The high oscillation amplitude and frequency disturbs the

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Fig. 1. Geometric parameters of piezoelectric fan and a fan blade array with face-to-face orientation

surrounding air sufficiently for vortex formation and creates significant downstream wind velocities [6] [7] [8].

Research and innovation in the field is driven by the power consumption of electrical components which increased three-fold between 2000 and 2010 [9], and is still rising year upon year [10]. Innovation in thermal management has fallen behind this trend. Business areas with high computational demand have been shown to put 33% of all energy towards cooling [11]. PE fans are excellent air-movers for moderate heat flux situations (1000 to 10,000 W.m⁻²) [9], and have been shown to provide sufficient cooling capabilities using just 50% of the power demand required by conventional thermal management systems [12] [13] [14].

A. Piezoelectric Fan Arrays

PE fan arrays can be categorised by two primary characteristics, orientation and phase. Face-to-face (FTF) orientation is preferable for implementation in confined spaces due to the geometry of the assembled blade array, and shall be solely considered in the present study. The geometric parameters defining a FTF array are shown in Figure 1, where length, L , width, W , and thickness, t_{BL} , define an individual blade and pitch, P , defines the distance between adjacent blade faces. Oscillation amplitude, A , and frequency, f , defined as operational characteristics, are also affected by material characteristics and input power. Therefore different materials or driving voltages can be used to create the desired motion for any sized blade [15] [16] [17].

Oscillation phase dramatically affects the generated airflow from a FTF blade array. Interaction between the blades is at a minimum when in-phase (IP), progressively increasing to a maximum level when in counter-phase (CP) [18].

IP, a single vortex, rotating as the vortex behind the trailing blade, is formed between the two blades through each oscillation half cycle [19]. In unconfined conditions, the generated

Fig. 2. Velocity profile of the downstream airflow at three distances downstream of oscillating fan blades [18]

airflow when the pitch is small is enhanced compared to the oscillation of a single PE fan blade: greater than double the downstream airflow is generated from a two-blade array [7]. This enhancement must be primarily driven by the blades' increased oscillation amplitude, resulting from the momentum gained by the air moving between adjacent blade faces, a phenomenon known as beneficial coupling [20]. A blade's oscillation amplitude may increase by 16% [21]. At larger pitch, above the region of beneficial coupling, IP oscillation is not optimal. Figures 2a and 2b show inferior downstream air velocity is created from the IP array at $P = 8A$, compared to a single fan blade.

In CP, the vortices induced on the outside faces of the two blades are similar to those observed with a single blade, but the flow field between the blades is complex [19]. Two vortices are formed during the opening period of the oscillation cycle behind the trailing face of each blade, counter-rotating to one another. Fully grown, these vortices are $3A$ to $4A$ in diameter [22], and their interaction creates an adverse flow during the cycle's closing period [19]. Figure 2c shows a negative axial velocity between the two blades [18]. This adverse flow is not observed when pitch is minimised ($P = 2A$), but otherwise is [7]. At a low pitch, detrimental coupling, in the form of reduced oscillation amplitude, occurs due to the entrapped air's resistance to expansion (in the opening period) and compression (in the closing period) [21].

B. Fan Blade Confinement

The vast majority of research in the field has been undertaken assuming unconfined positions. Confinement of PE fan arrays is less well documented. Of particular interest is confinement close to the blade edges, as it severely inhibits the airflow around the edges from the leading to trailing blade face [23] [24]. The setup presented, effectively one of channel flow, has been investigated experimentally by Kim *et al.* [25], and shall be introduced in Section II-C. Considering a single blade, when the side walls are less than 2 mm from the blade edge, substantial downstream airflow enhancement has been observed [24], but a larger gap has a negligible effect on the generated thrust [13] [24].

In a tightly pack FTF array, the blades are effectively confined on their faces by the adjacent blades' faces. This confinement is reported to be highly detrimental to the generated airflow when coupled with blade edge confinement, with an 80% reduction reported [16]. Air entrapment is fundamental to this reduced performance. The close confinement above and below the blade edges in a FTF orientated array is therefore likely to present an additional hindrance to its operation of PE fan arrays, as the air propelled from the volume between the blade arrays is less easily replaced. Air must therefore be drawn from the upstream direction and into the volume between the blades, before being driven downstream by the oscillating blade tips. It can be hypothesised, therefore, that low pitched arrays will become less effective as the level of confinement is increased. This will be considered throughout the discussion and analysis.

C. Numerical Modelling of Piezoelectric Fans

Numerical modelling has become an important tool across the thermal management topic [26], and PE fan research has followed this trend. 2D modelling, considering the plane through the mid-section of the blade, neglects the movement of air in the third dimension around the blade edge [27], and this is shown to contribute to the error during validation [28]. However, these errors can be reduced firstly by considering a wide blade, where edge effects are minimal [29], and secondly assessing the specific case of channel flow, where the clearance between the blade edges and domain boundaries is small, reducing the flow of air through the gap [23] [24]. The given arguments, coupled with examples of strong experimental validation for a 2D numerical model [6] [30], provides good evidence that a channel flow problem including PE fan blades can be effectively modelled in a 2D flow domain.

Methods of transiently resolving the Navier-Stokes equations in the PE fan research field are established. An incompressibility assumption is used across the wider literature since the pressure gradients in the domain are insubstantial [30], and this is validated in studies specifically considering blade confinement [31]. The energy equation is often neglected when thermal and buoyancy effects are considered to be negligible, for example when no heat source is associated with the study [12], or when gravity acts normal to the 2D plane upon which the flow domain is modelled [30] [32]. Discretisation is often first order accurate, but noted in recent studies to be improved

by the inclusion of a second order upwind term [27] [33]. The presence of turbulence in the flow domain is widely accepted, and the turbulent viscosity has been approximated using a number of models including the SST $k-\omega$ model [34] [35] [36], RNG $k-\epsilon$ model [27] and LES model [29]. The standard $k-\epsilon$ model is well suited for highly turbulent models [37], and chosen when the investigation is focussed on the near vicinity of the blade tip and oscillation frequency is high [6] [18].

The size of the domain mesh is largely dependent on the desired resolution and the computational capacity of the hardware resolving the numerical model. As few as 14,500 [12] and as many as 1,500,000 [29] elements have been used for 2D flow domains. Triangular elements are most often implemented as they are better suited for skewing and re-meshing when a dynamic mesh is used to accommodate the moving blade boundary [6] [27]. At least 100 time steps per oscillation are reported across the reviewed literature [33] [38], and more have improved the quality of convergence in certain instances [29] [35]. The number of completed blade oscillations to reach a repeatable solution has varied substantially in the reviewed work, with as few as 20 [27] and as many as 100 [36] [38] reported. The required number of oscillations is most commonly determined in preliminary work, by studying the changes to the flow field from one oscillation to the next.

The functions that define the oscillating PE fan blade in the numerical model are of note. In cases, the blade is assumed to be a clamped uniform cantilever beam oscillating in its first resonance mode. The mode shape defined as in Equation 1. Y is the deflection at maximum amplitude of any point at distance x along a beam of length L , and A_{XC} is the cross-sectional area ($W \cdot t_{BL}$). The coefficient β is defined in Equation 2 [39], where m , I and f_r are the mass, second moment of area and first resonant frequency of the blade, and E is the blade material's Young's modulus. In reality, the PZT actuator affects the blade shape, limiting deflection in the region [35]. Equation 3 is an empirical function derived from experimental work to better define the mode shape [12] for a blade. Y_{L65} with x_{L65} are used because the function in this state is limited in use, only applicable to a blade with a length of 65 mm. For globalisation, a coefficient must be applied to the x_{L65} terms, as described in Section II-B.

$$Y(x) = A_{XC} \cdot \left[(\sin(\beta \cdot L) - \sinh(\beta \cdot L)) \cdot ((\sin(\beta \cdot x) - \sinh(\beta \cdot x)) + (\cos(\beta \cdot L) - \cosh(\beta \cdot L)) \cdot ((\cos(\beta \cdot x) - \cosh(\beta \cdot x))) \right] \quad (1)$$

$$\beta = \sqrt[4]{\frac{2\pi \cdot f_r \cdot m}{L \cdot I \cdot E}} \quad (2)$$

$$Y_{L65}(x_{L65}) = -42.3402 \cdot x_{L65}^2 + 33587.5 \cdot x_{L65}^3 - 2.7317 \cdot 10^6 \cdot x_{L65}^4 + 9.05342 \cdot 10^7 \cdot x_{L65}^5 - 1.2653 \cdot 10^9 \cdot x_{L65}^6 + 6.34496 \cdot 10^9 \cdot x_{L65}^7 \quad (3)$$

Transiently, the displacement from the centre of oscillation, y , at any given time, t , along the PE fan blade is defined by Equation 4 [35]. A drive coefficient, D_c , is required to set

the amplitude to the desired magnitude. The drive coefficient is therefore equivalent to a magnitude of power input. The derivation of the drive constant is described in Section II-B.

$$y(x, t) = D_c \cdot Y(x) \cdot \sin(2\pi \cdot f_r \cdot t) \quad (4)$$

D. Experimental Validation of Numerical Results

Two primary methods of experimental validation have been established in previous research. In both cases a numerical model is set up to accurately replicate the geometric and operational conditions used in the experimental study [6] [27]. Simple quantitative validation can be achieved through comparison of numerically derived and experimentally determined heat transfer coefficients from boundaries and surfaces. Such methods have been widely used in previous research for both 2D [30] and 3D domains, with error values ranging from 8% [8] to 17% [4] [33]. Choi *et al.* considered a different approach by analysing the generation of airflow from the blade tips in experimental and computation setups [6] [18] [25]. Vortex characteristics, specifically size variation and downstream trajectory, and downstream velocity profiles were compared and considered to be well-matched. Unlike the former process introduced, which compares the effects induced by airflow generation, this method analyses the fundamental airflow. Beneficially, the model can be used to analyse thermally and geometrically dissimilar systems, without the need for further validation.

E. Aims and Objectives

Pitch, as a function of amplitude, is highlighted as the key parameter defining the optimal geometry for downstream airflow generation from a FTF PE fan blade array in a state of negligible confinement. At a small pitch IP oscillation has been shown to be preferable, due at least in part to the increased amplitude achieved through beneficial coupling [21]. It is clear however that CP oscillation is beneficial at larger pitch [18]. The effect of confinement on these relationships has not previously been documented, and this will be analysed in detail in the present investigation. Research will aim to evaluate these pitches as a function of amplitude as well as considering other relevant array characteristics.

II. NUMERICAL METHODS

The investigation was undertaken using various 2D numerical models, validated against published experimental results. Model verification, described in Section II-C, is essential for high quality numerical analysis and therefore the model is designed to replicate the experimental conditions.

A. Numerical Setup and Boundary Conditions

The 2D flow domain was modelled on a plane through the oscillating PE fan blade's mid-span. The model was setup with reference to methods established in the reviewed literature. The continuity and momentum equations were maintained, but the resolution of the Navier-Stokes equations was simplified by eliminating the energy equation, justifiable given the lack of

Fig. 3. Geometry and boundaries of the present 2D model's flow domain

a heat source in the flow domain. The simplified 2D, incompressible equations were solved using the SIMPLE scheme defining the pressure-velocity coupling. The equations were discretised using second order upwind methods, and a first order implicit method was used for formulation of the transient solution. Turbulence was assumed in the near vicinity of the blades, and turbulent viscosity was approximated through the standard $k-\varepsilon$ model. Convergence for each time step of the transient solution was assured by iterating the simplified governing equations until the monitored values' (continuity, velocity and k and ε coefficients) residuals were each $\leq 10^{-4}$.

A 2D model was preferred over a 3D model because of the specific situation under consideration. Previous work, discussed in Section I-C, has shown 2D modelling to be valid for channel flow applications. The simplification allowed for greater detail in other aspects of the model and overall investigation, such as mesh resolution, time-step size and variables adjusted.

Figure 3 summarises the flow domain geometry. The PE fan blades were geometrically similar to that present in the experiments used to validate the model: 31 mm in length and 0.13 mm thick. The validation process showed strong correlation with experimental results using a relatively small domain, which substantially reduced the computation demand of the model. The domain side walls were $1L$, 31 mm, from the outer blade face, the downstream boundary, a pressure-outlet, was $2L$ downstream from the blade tip and the upstream, pressure-inlet, boundary was $0.5L$ from the clamped end of the blade. In control cases where amplitude was doubled, the outer dimensions of the domain were also doubled.

A focus of the investigation was to achieve strong levels of validation by analysing the flow field created by the oscillating fan blade. Therefore, a very fine mesh was used, with initial node spacing of $1t_{BL}$. The domain used to validate the model, the smallest of all used, included

95,413 elements. The deforming blade boundaries presented an additional complication. Other published investigations in the field have summarised a division of the mesh into a near-field domain (close to the blade boundary), which dynamically deforms with the blade motion, and far-field (far from the blade boundary), which is static [6]. The aim is to reduce computational complexity by reducing the number of elements that much be re-meshed for each transient time step.

In the present study, a dynamic mesh was implemented across the entire domain. In order to reduce the computational demand, a relatively low spring constant was set for the individual elements, thus confining deformation and re-meshing to the near vicinity of the deforming boundaries. This was found to have the additional benefit of improving continuity of modelled vortices as they moved from the near-field to far-field. Solely triangular elements were used in the mesh for better compatibility with the chosen dynamic meshing technique. Quadrilateral elements were found to deform and re-mesh poorly. This is in-line with discussion in previous literature [6] [18].

B. Piezoelectric Fan Blade Motion

Functions were used to define the blade's deformed shape and motion. Equation 3 describes the deformed blade shape specifically of a 65 mm long blade. The x-coordinate of each node on the derived blade is therefore defined as x_{L65} , for clarity. In order to globalise Equation 3 a conversion factor, C_{XM} , was required to make the function relevant for a blade of any length, L . The conversion factor, C_{XM} , is detailed in Equation 5, and therefore the global x-coordinate, x , of any node on a blade of length L maybe be defined by Equation 6. The derived mode shape at various points in an oscillation cycle, for the case of a 31 mm long blade oscillating to an amplitude of 1.3716 mm, is plotted in Figure 4. As can be observed, the x-coordinates of each node along the blade boundaries is also varied, to best replicate a real oscillating fan blade. The significance of this additional calculation increases as amplitude is increased, but it is included for all present models for continuity. The updated x-coordinates are calculated assuming their unique distances from the point $x = 0.32L$ is constant, regardless of progress through the oscillation cycle. This 'datum' point was determined iteratively to achieve the most realistic blade shape.

$$C_{XM} = \frac{0.065}{L} \quad (5)$$

$$x_{L65} = x \cdot C_{XM} \quad (6)$$

Equation 4 was used to describe the transient blade boundary deformation. The magnitude of the drive coefficient, D_C , had to be predetermined in order to achieve the desired tip oscillation amplitude of 1.3716 mm. This was achieved rearranging Equation 4 to make D_C the function of the equation, and setting $\sin(2\pi \cdot f_r \cdot t) = 1$, and blade tip displacement, y , to 1.3716 mm. x_{L65} , from Equation 3, was determined through Equation 6, where $x = 0.032$, i.e. at the tip of the 32 mm blade.

Fig. 4. Modelled fan blade mode shape at instances in an oscillation cycle

For the instance where an oscillation amplitude of 1.3716 mm is desired, $D_C = 1.364$. In order to globalise the drive coefficient, Equation 7 defines D_C as a function of desired amplitude. This is achieved in the same manner as described above, instead setting $y=A$.

$$D_c = \frac{A}{1.0054} \quad (7)$$

A total of 8000 time steps were resolved with a time step size of $3.472 \cdot 10^{-5}$ s. This allowed for 50 complete oscillations of the blade with 160 time steps per oscillation. 50 oscillations were considered necessary and sufficient following analysis from preliminary validation models. After 50 oscillations, the time-averaged downstream air velocity was increasing at a low rate of 0.02% per oscillation, which can be compared to the rate of increase after 25 oscillations, 0.45% per oscillation.

C. Numerical Model Validation

The present model was validated against experimental data published by Kim *et al.* [25]. The experimental apparatus was set up as shown in Figure 5. The PE fan blade had dimensions of 31 mm $\times$ 38.1 mm $\times$ 0.13 mm ($L \times W \times t_{BL}$), and oscillated at a frequency of 180 Hz, to an amplitude of 1.3716 mm. The far side walls were 140.9 mm from either face of the undeflected blade, making the channel width, W_{CH} , 282 mm. The blade was closely confined at its edges by the near side walls which, as discussed, will have reduced the blade edge effects that are neglected in the present 2D model. The flow field was analysed using laser Doppler velocimetry, which allowed for time-averaged airflow measurements as well as vortex formation and propagation analysis.

Validation, compared the flow fields generated by a single fan blade. Figure 6 shows the resolved velocity vector fields from the present model at $\frac{\pi}{4}$ intervals for one complete oscillation. Graphically, they are very well matched to those recorded by Kim *et al.* [25] in experimental conditions.

Vortex size and trajectory were compared, and the results were well matched. At $\frac{\pi}{4}$ and $\frac{\pi}{2}$ after separation, the error in vortex diameter was 7.2% and 6.8% respectively, and up to $\frac{3\pi}{2}$ after separation, the average error in vortex diameter was 11.5%. It was found that vortex dispersion after $\frac{3\pi}{2}$ from

Fig. 5. Geometry and layout of the experimental channel flow equipment used by Kim *et al.* [25]

separation occurred more rapidly in the model's data, a limitation that has previously been observed in similar validation processes [6]. The downstream trajectories of the vortices can be observed in Figure 7. Here, error in downstream displacement was 10.6% and 5.3% for clockwise and counter-clockwise vortices, respectively. A similar validation process was used by Choi *et al.* [6] to analyse the accuracy of their numerical model. The vortex trajectories computed by their computational model is also included for visual comparison. Additionally, volumetric flow rate of air over a complete cycle was compared, in a region of width $6A$, centred on the oscillation midpoint. Modelled error was 3.0%.

The blade mode shape is not determined from the original experimental work, and therefore discrepancy between that mode shape and the one used in the present model is a likely contributor to the determined error. This is not considered to be an issue, as the aim of this investigation is to understand the importance of parameter variance on the induced airflow, not quantify the airflow for a specific PE fan blade.

III. METHODOLOGY AND RESULTS

Models were set up maintaining the same blade geometric and operational parameters as that considered for experimental validation. Pitch and phase of oscillation were the independent variables. Pitch was varied $1A$ – $14A$, although the minimum achievable pitch for CP oscillation was $2.1A$ to avoid contact between the two blade tips. CP and IP oscillation phases were trialled. The velocity vector field generated from two blades oscillating in CP and IP, for $P = 8A$, are shown in Figures 8 and 9 respectively.

Figure 10 shows the time-averaged downstream velocity, v_x , profiles, recorded along the line $x = L + A$, generated by two blades with variable pitch oscillating in CP and IP.

Fig. 6. Velocity vector fields resolved from the single blade model at $\frac{\pi}{4}$ intervals in the near vicinity of the blade tip

Fig. 7. The present model's vortex trajectories, compared with previous experimental data [25] and previous numerically modelled results [6]

Fig. 8. Velocity vector fields resolved from the $P = 8.4$, counter-phase, two-blade array model at $\frac{\pi}{4}$ intervals in the near vicinity of the blade tips

v_x is calculated as the mean of the instantaneous downstream velocities through the period of one complete blade oscillation. The velocity profile generated from a single blade is included for reference. Figure 11 displays the maximum time-averaged downstream velocity, $v_{x,max}$, generated from each blade pitch,

Fig. 9. Velocity vector fields resolved from the $P = 8A$, in-phase, two-blade array model at $\frac{\pi}{4}$ intervals in the near vicinity of the blade tips

for CP and IP oscillation. $v_{x,max}$ is taken as the mean of the peak v_x values from Figure 10. This averaging process reduces any asymmetrical inaccuracies in the flow domain.

Figure 12 shows the downstream volume* flow rate, $\dot{V}_x$,

generated by the different blade pitches for CP and IP oscillation. $\dot{V}_x$ is determined by taking the area beneath the time-averaged velocity profiles plotted in Figure 10, and multiplying the value by the blade width. Given the 2D domain, blade width is not defined at the outset and therefore unity, 1 m, is assumed for simplicity. The inclusion of a volume* flow rate is essential for analysis and discussion, as the entire velocity profile is summarised to a single value which may be plotted against a geometric parameter.

A uniform velocity profile across the width of the blade is therefore assumed. In a real-world application, this may not be justifiable, since the air velocity would reduce close to the confined top and bottom walls of the channel. The present investigation is a comparative study as geometric properties of the PE fan array are varied. Therefore, comparative volumetric flow rates are desired, as opposed to absolute values for real world application. The simplification to a 1 m blade width (unrealistic for real world application) highlights this. The uniform velocity profile across the blade width is justified for this case because the near-wall effects on the velocity profile will be the constant for each array under test.

Oscillation amplitude was doubled for three pitches independently, and the time-averaged velocity profiles outputted by these models are presented in Figure 13. The $v_{x,max}$ and $\dot{V}_x$ data from each of these models is shown in Figure 14. The equivalent data at the original pitch is included in each for reference.

IV. DISCUSSION

CP oscillation is shown to be superior for generating downstream velocity and flow rate in Figures 11 and 12 respectively, for all trialled pitches. Compared the single blade data, the two-blade array oscillating in CP generates superior $v_{x,max}$ values when $P \geq 3A$, and greater than double $\dot{V}_x$ values when $P \geq 6A$ (i.e. greater $\dot{V}_x$ per blade). For IP oscillation, performance in terms of $v_{x,max}$ is always inferior to a single blade, and double a single blade's $\dot{V}_x$ is only recorded when $P \geq 12A$.

The results for CP oscillation largely fit with conclusions drawn from the literature [7] [18]. Performance is poorer at low pitch, and the optimal pitch, certainly for generating high $v_{x,max}$ in the flow field, is found at $P = 8A$. This has been attributed to the size of the vortices formed between the blades during the opening period, and can be demonstrated through comparison of various pitches at $\frac{\pi}{2}$ through the oscillation cycle, when the distance between the blade tips are at a maximum, as shown in Figure 15. This represents a vortices' points of separation. In the case of $P = 4A$ and, to a lesser extent, $P = 6A$, the vortices are compressed by the constriction imparted by the blades, and are oval in shape. This inhibition to reach full size has an adverse effect on their ability to drive airflow downstream. At $P = 8A$, the vortices are circular in shape, suggesting they have fully formed, and this is optimal for generating downstream airflow. Circular vortices are also present when $P = 10A$, but the interaction between the vortices is weaker and therefore the benefits of CP oscillation are felt to a lesser extent. For the prescribed

(a) Counter-phase oscillation

(b) In-phase oscillation

Fig. 10. Time-averaged downstream velocity profiles at a distance of $1A$ downstream of oscillating fan blades across a range of array pitches, when $A = 1.3716$ mm

parameters, the $P = 8A$ array uniquely allows full vortex formation and strong vortex interaction.

Negative v_x values are observed between the blades oscillating in CP when $4A \leq P \leq 8A$, but not outside of this range. This is in-line with established relationships in the literature, and is due to the vortex interaction observable in Figure 15. The counter-rotating nature of the adjacent vortices, even though they are moving as a whole downstream, creates an upstream flow between them. The varying flow direction, close to the $y = 0$ centreline, is shown in Figure 8. During the open period of the blades, when the vortices between the blades are growing, an upstream velocity is observed. This is strongest just before separation. During the closing period of the blades, the vortices forming on the outer faces of blades are able to induce a positive flow direction between the blades.

This phenomenon summarises the most critical difference between CP and IP oscillation, and observation of the flow field between the blades in Figure 9 reveals a downstream

Fig. 11. Peak time-averaged downstream velocities recorded at a distance of $1A$ downstream of oscillating fan blades across a range of array pitches, when $A = 1.3716$ mm

Fig. 12. Volume of downstream airflow recorded at a distance of $1A$ downstream of oscillating fan blades across a range of array pitches, when $A = 1.3716$ mm

velocity throughout an IP oscillation cycle. Negative v_x values are only observed when $P \leq 3A$, for IP oscillation. The stark contrast between the two flow fields is summarised in Figure 16, which plots the average of the instantaneous downstream velocity, $v_{ix,av}$, values recorded across the pitch between the blades at the clamped end, through a complete oscillation cycle for CP and IP oscillation. The volume* flow rate through the region is 5.5% greater for CP oscillation.

Variation in $v_{ix,av}$ is driven by a pressure distributions in the channel, and a qualitative comparison of the pressure gradients between the blades at the start of an oscillation cycle is presented in Figure 17, for CP and IP oscillation. Unlike previous work, which has not considered the specific case of channel flow: close confinement above and below the blades' edges, these contour plots offer a complete picture of the how CP oscillation is able to generate a pressure gradient between the blades that is directionally downstream, whilst IP oscillation generates minimal gradient in the same region. The superior gradient forms the basis of the enhanced performance

(a) Counter-phase oscillation

(b) In-phase oscillation

Fig. 13. Time-averaged downstream velocity profiles at a distance of $1A$ downstream of oscillating fan blades across a range of array pitches and two oscillation amplitudes

in CP.

The optimal pitch for peak $\dot{V}_x$ is not found in the range under test for either CP or IP oscillation. An explanation for this can be found in Figure 10. For $P \geq 10A$ (in CP), and for $P \geq 8A$ (in IP), a positive v_x is present between the blades, and this contributes to the gradually increasing $\dot{V}_x$ values as pitch is increased. In CP, this effect proves to be more significant than the diminishing $v_{x,max}$ values. In IP, the reducing dominance of detrimental coupling means the $v_{x,max}$ values increase with pitch for the entire range of pitches trialed and a greater rate of increase in $\dot{V}_x$ at $P = 14A$ is observed.

A. Array Size Considerations

The pitch for optimal $\dot{V}_x$, however, does not represent the optimal pitch for performance in the large majority of potential applications for a PE fan array. In reality, the close confinement demands airflow generation from a small unit.

(a) Peak time-averaged downstream velocities

(b) Volume of downstream airflow

Fig. 14. Data recorded at a distance of $1A$ downstream of oscillating fan blades across a range of array pitches and two oscillation amplitudes

For example, an array set at $P = 14A$ is not practical, when $\dot{V}_x$ generated from it is just 12.4% greater than that generated from an array just over half its size, at $P = 8A$.

Figure 18 relates $\dot{V}_x$ to the physical dimensions of an array by displaying pitch plotted against $\dot{V}_{xd}$, calculated from Equation 8. As a result, the flow rate generated as a function of the size of the arrays' oscillation envelopes may be assessed. It is apparent that CP outperforms IP oscillation at all pitches, and that if sufficient airflow can be generated from a single fan blade in a given circumstance, this represents the optimal design. These results further highlight the poor performance of IP oscillation, particularly at low pitch. IP oscillation has a key advantage over CP regarding the design of unconfined blade arrays, as pitch can be reduced below $P = 2A$, and more blades can be fitted into an air-moving component of set dimensions. It is clear however that this is of no benefit for design in the presented confinement, and in fact IP oscillation performs optimally, with regard to $\dot{V}_{xd}$, when $P = 4A$.

$$\dot{V}_{xd} = \frac{\dot{V}_x}{\frac{P}{A} + 2} \quad (8)$$

Fig. 15. Velocity vector fields resolved from counter-phase, two-blade array models across a range of pitches, at the $\frac{\pi}{2}$ instance in the oscillation cycle

Fig. 16. Instantaneous downstream velocity averaged across the pitch between the blades for the $P = 8A$, two-blade model oscillating counter-phase and in-phase. Recorded at the clamped end of the blades over one complete oscillation cycle

Considering the CP oscillation case, these results demonstrate that in spite of inferior performance at low pitch, the detrimental effects caused by close confinement never outweigh the benefits of reduced array dimensions, in terms of producing the greatest downstream airflow per oscillation envelope size. Comparing the case of $P = 8A$ to $P = 2.1A$, performance, in terms of $\dot{V}_{xd}$, is enhanced by 78.1%. Assuming, as was set out in Section I-A, that multiple blades are necessary for sufficient airflow generation, pitch should be minimised to as close to $2A$ as tolerance limits allow.

B. Oscillation Amplitude Variation

The effect of doubling oscillation amplitude can be seen in Figures 13 and 14, where data is displayed for $A = 2.7432$ mm as well as $A = 1.3716$ mm. The time-averaged velocity

Fig. 17. Qualitative pressure contour plot for the $P = 8A$, two-blade model oscillating counter-phase and in-phase, at the start of the oscillation cycle

Fig. 18. Volume of downstream airflow as a function of oscillation envelope size recorded at a distance of $1A$ downstream of oscillating fan blades across a range of array pitches, when $A = 1.3716$ mm

profile shapes remain largely consistent as amplitude is altered. Considering CP oscillation, negative v_x are replicated for the $P = 4A$ and $P = 6A$ cases, but are not at $P = 8A$. This discrepancy can be related back to vortex formation and growth. The counter-rotating vortices growing between the blades in the opening period of oscillation are unable to reach $4A$ in diameter and therefore, when $P = 8A$, do not interact with sufficient strength to generate a negative v_x values along the $y = 0$ centreline. The effect of the reduced vortex growth, in terms of A , is evident in Figure 14a: the case $P = 6A$ outperformed the case $P = 8A$ in terms of generating peak $v_{x,max}$. For IP oscillation, relationships regarding $v_{x,max}$ remain consistent as amplitude is varied: the effect of detrimental coupling reduces as pitch is increased. Evidence suggests, therefore, that oscillation amplitude is also a variable defining the size of a fully formed vortex, and therefore a characteristic to be considered for PE fan array optimisation.

The $v_{x,max}$ values do not increase in direct proportion

Fig. 19. Volume of downstream airflow as a function of oscillation envelope size recorded at a distance of $1A$ downstream of oscillating fan blades across a range of array pitches and two oscillation amplitudes

with amplitude. At $P = 4A$, a 52.0% and 72.0% increase in $v_{x,max}$ is observed for CP and IP oscillation respectively, and these $v_{x,max}$ increase rates reduce for each phase as pitch is increased. However, assessment of Figure 14b reveals that increasing amplitude is highly effective for driving a greater magnitude of downstream airflow. For CP oscillation, $\bar{V}_x$ is increased by 469%, 462% and 444% when $P = 4A$, $P = 6A$ and $P = 8A$ respectively, whilst IP performance is enhanced by 575%, 531% and 510% for the same pitches. These values must be evaluated against the size of the oscillation envelope, which increases in direct proportion with oscillation amplitude for each pitch. Therefore, $\bar{V}_{xd}$ may once again be used to assess the increased amplitudes' performances in terms of confinement, and data for each amplitude is plotted in Figure 19. Improved performance is evident for each pitch and both oscillation phases.

For CP oscillation, the volume of downstream airflow generated from a given oscillation envelope size is increased by 42.7%, 40.1% and 35.9% for $P = 4A$, $P = 6A$ and $P = 8A$ respectively, whilst values of 61.6%, 57.9% and 52.6% are calculated for IP oscillation at the same pitches. Therefore, increasing amplitude of oscillation, even whilst maintaining pitch as a function of amplitude ($\frac{P}{A}$ ratio) is beneficial for generating larger volumes of airflow in confined environments. Once again, data suggests CP oscillation is optimal for achieving the largest magnitude of downstream airflow from a given oscillation envelope size.

V. CONCLUSION

CP oscillation is shown to outperform IP oscillation across the range of pitches tested and at two different amplitudes, for the case of channel flow when the fan blades are closely confined above and below their edges. At $P = 8A$, CP oscillation generates an additional 32.8% volume of downstream airflow. As pitch is reduced, the confinement has a more significant detrimental effect on IP oscillation, and at $P = 2.1A$, CP oscillation performs 180% better than IP oscillation considering the same measure. Compared a single blade and considering

airflow generation per blade, a two-blade array oscillating IP is only beneficial when $P \geq 12A$, whilst CP oscillation outperforms a single blade when $P \geq 6A$.

The requirement for generating the largest volume of airflow from a given size of oscillation envelope was highlighted, and results suggest CP oscillation, with a minimised pitch (close to $2A$), provides the optimal design for any array in a channel flow environment. However, if sufficient airflow can be generated from a single blade then this design setup is optimal. The benefit of no confinement on the blade's faces, provided by the adjacent blade in an array setup, means greater airflow can be driven downstream from a given size of oscillation envelope. This evaluation of design assumes no confinement of a single blade's faces by side walls, and model testing would be required should this consideration become relevant.

The amplitude was doubled for three pitches and performance (in terms of volume of airflow generated per size of oscillation envelope) was shown to improve substantially for both CP and IP oscillation. It can be assumed therefore that amplitude, as well as pitch as a function of amplitude ($\frac{P}{A}$), is an important factor in optimising the performance of a two-blade array. This must be investigated further before quantitative relationships can be fully determined.

Considering CP oscillation, evidence has highlighted the formation and growth of vortices in determining the optimal pitch for generating peak downstream air velocity. A pitch exists at which the vortices formed between two blades in their opening period are able to grow to a full size and be circular in shape, whilst still interacting with one another strongly to drive downstream air velocity. At the lower amplitude tested, 1.3716 mm, optimal v_x was found at $P = 8A$, but when amplitude was doubled to 2.7432 mm, the optimal pitch reduced to $6A$. This analysis suggests that pitch alone does not define the size of a fully formed vortex in PE fan blade theory, and that amplitude magnitude is also a defining variable. The amplitude directly affects the speed of the blade tip through the air, and this characteristic may be important for defining vortex growth. Oscillation frequency also defines blade tip speed, and is therefore another variable which must be considered.

Analysis was not carried out on arrays containing three or more blades, and further investigations in this field will help to further improve the optimal design of PE fan arrays to be used in confined environments.

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