On the Mechanics of New-Keynesian Models*

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Abstract

The monetary transmission mechanism in New-Keynesian models is put to scrutiny. We show that, contrary to the conventional view, the transmission mechanism does not operate through the real interest rate channel. Instead, equilibrium inflation is approximately determined as in a flexible-price model; output is then pinned down by the New-Keynesian Phillips curve. The real rate only reflects the feasibility to keep consumption smooth when income changes. Contractionary monetary policy shocks reducing output and inflation are consistent with an increase, decline, or no change in the real rate. Consistency with the real rate channel is observational, not structural.

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1 Introduction

The New-Keynesian model—a dynamic stochastic general equilibrium (DSGE) model with sticky prices—has become a workhorse in the analysis of monetary policy. It has grown in popularity at tremendous speed both in academia and at central banks around the world. From a basic framework, consisting of an Euler equation, a New-Keynesian Phillips curve, and a Taylor rule, it has quickly grown into a model with endogenous capital a lá Real Business Cycle (RBC) theory, a number of different frictions, adjustment costs, and other features required to fit the data. The basic three-equation model is typically used to illustrate optimal monetary policy (e.g., Clarida, Galí, and Gertler, 1999), whereas the extended framework—often referred to as a medium-scale DSGE model—is used for practical monetary policy and forecasting (e.g., Christiano, Eichenbaum, and Evans, 2005; Smets and Wouters, 2007).

Unfortunately, widespread understanding of the monetary transmission mechanism in New-Keynesian models—i.e., how unexpected changes in monetary policy transmit into the real economy—appears to have been lost along the fast track to popularity.[1] In contrast, in the RBC literature, King, Plosser, and Rebelo (1988a) and King, Plosser, and Rebelo (1988b), among others, have carefully laid out the inner workings of the RBC model and showed, in a step-by-step fashion, how each additional feature affects the model's properties. A description given by Ireland (2015) in the New Pelgrave Dictionary of Economics is representative of the typical exposition of the New-Keynesian transmission mechanism:

> A monetary tightening, in the form of a shock to the Taylor rule, that increases the short-term nominal interest rate translates into an increase in the real interest rate as well when nominal prices move sluggishly due to costly or staggered price setting. This rise in the real interest rate then causes households to cut back on their spending as summarized by the IS curve. Finally, through the Phillips

[1] The responses of the real economy to unexpected changes in nominal variables controlled by the central bank are a subject of investigation by a large literature. Such interest stems from the desire to discriminate across potential channels of transmission when guiding monetary policy (Christiano, Eichenbaum, and Evans, 1999).

curve, the decline in output puts downward pressure on inflation, which adjusts only gradually after the shock.

The standard description is thus based on the traditional *real interest rate channel* of monetary policy transmission, described for instance by Bernanke and Gertler (1995), Taylor (1995), and Mishkin (1996). According to the real interest rate channel, the central bank—controlling the short-term nominal interest rate—has leverage over the ex-ante real interest rate because nominal prices are sticky. As a result, an increase in the nominal rate leads to an increase in the ex-ante real rate—an intertemporal price—which induces households and firms to cut consumption and investment, thus reducing aggregate demand and output. This puts pressure on firms to gradually adjust prices to a lower level.[2] This channel is at the core of the textbook IS-LM and AS-AD models and served as a motivation for the inception of New-Keynesian models in the wake of the rational expectations revolution (Ireland, 2015); see also Goodfriend and King (1997) for the connections between IS-LM and New-Keynesian models and between RBC and New-Keynesian models. Perhaps for these reasons, the real effects of monetary policy in New-Keynesian models are typically described in the context of the real interest rate channel (e.g., Woodford, 2003; Galí, 2015, among many others).

The main message of this paper is that the transmission mechanism of monetary policy in New-Keynesian models *does not* operate through the real interest rate channel. Any consistency with the real interest rate channel is purely observational, not structural, due to a specific parameterization. Our goal, however, is to carry out constructive scrutiny of this important class of models, rather than their critique. To this end, we transparently lay out their mechanics in relation to the monetary transmission mechanism, focusing, for reasons explained below, on the implications of endogenous capital.

Naturally, unlike in the above narrative of the real interest rate channel, in New-Keynesian

[2] The theoretical literature on the real interest rate channel has been reinforced by a large empirical literature documenting that, broadly speaking, in response to a positive innovation in a short-term nominal interest rate in a VAR model: (i) the nominal interest rate increases, (ii) output declines, and (iii) inflation (persistently) declines, but less than output; e.g., Christiano et al. (1999). The ex-ante real interest rate increases as a result of (i) and (iii).

models all variables are simultaneously determined in a dynamic general equilibrium. Therefore any description of the model in terms of the traditional real rate channel can only be a cursory way to provide intuition, rather than an accurate characterization of the model's properties. Nevertheless, we should observe declines in output and inflation, in response to a contractionary monetary policy shock, to always coincide in equilibrium with an increase in the ex-ante real interest rate, if real activity and prices indeed decline in these models as a consequence of intertemporal substitution by households and firms in the face of higher real interest rates.

We demonstrate that while this outcome always holds in the basic three-equation model, it is not generally the case once endogenous capital is introduced. In the extended model, the declines in output and inflation, in response to a contractionary monetary policy shock, are consistent with an increase, decline, or no change in the ex-ante real interest rate, depending on the parameterization of the shock persistence (this applies to both short and long rates). In a vast majority of cases, in fact, the ex-ante real interest rate *declines*.[3]

It is important to demonstrate the monetary transmission mechanism in the presence of endogenous capital for at least three reasons. First, it is investment, rather than consumption, that plays a key role in the traditional real interest rate channel, which the New-Keynesian models are meant to capture.[4] Second, endogenous capital is a key ingredient in the transition from the basic three-equation framework to the medium-scale DSGE models, containing both New-Keynesian and RBC features. And third, the basic three-equation model is a limiting case of the more general endogenous capital setup, when capital adjustment costs are infinite.

If not through the real interest rate channel, how does monetary policy transmit into output and inflation in New-Keynesian models? We demonstrate that equilibrium inflation

[3]It is well known that even in the basic model without capital the *nominal* interest rate can decline in response to a contractionary monetary policy shock, if the shock is sufficiently persistent (Woodford, 2003; Galí, 2015). This is due to a persistent decline in inflation and thus inflation expectations. The ex-ante real rate in the basic model, however, always increases, irrespective of the shock persistence.

[4]Woodford (2003), for instance, regards the basic model as a shortcut for capturing the effects of monetary policy on aggregate expenditures working primarily through investment.

is approximately determined as in a flexible-price model; output is then pinned down by the New-Keynesian Phillips curve (that is, each individual firm that cannot optimally adjust prices to keep up with the equilibrium path of the aggregate inflation rate adjusts output). In the model with endogenous capital, when output temporarily drops, as a result of temporarily low inflation, households can keep consumption relatively smooth by reducing investment. The ex-ante real rate only reflects the feasibility to do so by adjusting the capital stock. *As a first pass*, households can adjust capital with almost no effect on the real rate, given the large size of the capital stock, relative to investment. In contrast, in the model without capital, consumption smoothing in the aggregate is not possible (effectively, the costs of adjusting capital are infinite). As the representative household tries to keep consumption smooth by borrowing, the real interest rate has to increase to restore equilibrium. The equilibrium outcome of the basic model thus makes it appear *as if* monetary policy affected output and inflation through the real interest rate channel. Indeed, moving from zero to infinite capital adjustment costs makes the model gradually behave in the "standard" way.

There are a number of reasons to be aware of the points in this paper. The first reason is purely academic. It is critical to understand how the model works when attempting to extend it in various directions. We hope that our exposition will be helpful in this respect. Second, there are policy implications. Misunderstanding the transmission mechanism can lead to policy mistakes. Essentially, while the model can be parameterized to be observationally equivalent to the real interest rate channel, this channel is not a structural relationship in the model. As such, it is subject to Lucas critique when policy parameters change. We provide an example to illustrate this point. And finally, the findings contribute to the on-going debate on the relevance of the real interest rate channel for the conduct of monetary policy (e.g., Kaplan, Moll, and Violante, 2018). We demonstrate that the real rate channel is generally not the channel of monetary policy transmission in New-Keynesian models.

Our position that the basic model without capital can be misleading in understanding the monetary transmission mechanism in the presence of sticky prices is shared by

Barsky, House, and Kimball (2007). Their point, however, is different. Barsky et al. (2007) show that if long-lived and non-durable goods are produced by separate sectors, prices must be sticky in the long-lived goods sector, in order for monetary policy to have aggregate real effects. We carry out the analysis in a more common setup where all goods are produced by a single sector. Furthermore, monetary policy shocks in the model studied here always have real effects, as long as prices are sticky. The point is that such real effects are consistent with the behavior of the ex-ante real interest rate that is at odds with the real interest rate channel of monetary policy transmission.

The paper proceeds as follows. Section 2 deals with the basic model without capital. Section 3 covers the model with endogenous capital, either without or with capital adjustment costs. Section 4 briefly relates our findings to recent explorations and critiques of New-Keynesian models. Section 5 concludes. Secondary material is contained in an Appendix.

2 The basic model without capital

In the interest of a clear, self-contained exposition, the paper proceeds in a step-by-step fashion, starting with the basic three-equation model. In order not to duplicate well-known derivations from first principles (e.g., Woodford, 2003; Walsh, 2010; Galí, 2015), the starting point of our analysis is a set of equilibrium conditions; we follow closely Galí (2015).[5] The basic model serves the purpose of establishing standard results to be contrasted with the results derived for the model with endogenous capital. Given our interest in the monetary transmission mechanism, we focus only on the responses of the model to monetary policy

[5]The New-Keynesian model is usually studied in its log-linear form in the neighborhood of a deterministic steady state, under the assumption that the nominal interest rate can increase or decrease without any constraint. A number of recent studies started to explore the model's behavior at the zero lower bound (e.g., Cochrane, 2016; Kocharlakota, 2016). Among policymakers, Bullard (2015) builds on Cochrane's analysis in his description of the recent nominal environment in G7 countries. We follow the traditional analysis and abstract from the issues arising due to the zero lower bound.

shocks. The exposition is based on a standard per-period utility function

$$u = \log c - \frac{l^{1+\eta}}{1+\eta}, \quad \eta \geq 0,$$

and an intermediate goods aggregator

$$y = \left[\int y(j)^{\varepsilon} dj \right]^{\frac{1}{\varepsilon}}, \quad \varepsilon \in (0,1),$$

of the typical intermediate-final good setup of the model. In the above, c is consumption, l is labor, η is a parameter governing labor supply elasticity, y is aggregate output of the final good, $y(j)$ is output of the intermediate good j, and $\varepsilon \in (0,1)$ is a parameter governing the elasticity of substitution between intermediate goods. The only input in the production of intermediate goods is labor and the production function is linear. The economy is cashless and monetary policy is formulated as a Taylor-type rule.

2.1 Equilibrium conditions

The starting point of our analysis is the system of equations describing the general equilibrium, with the New-Keynesian Phillips curve (NKPC) already in its linearized form, around the zero steady-state inflation rate, the usual approximation point in the literature.[6] The equilibrium conditions are as follows

$$\frac{w_t}{c_t} = l_t^{\eta}, \tag{1}$$

$$\frac{1}{c_t} = \beta E_t \left(\frac{1}{c_{t+1}} \frac{1+i_t}{1+\pi_{t+1}} \right), \tag{2}$$

$$y_t = l_t, \tag{3}$$

$$\chi_t = w_t, \tag{4}$$

[6] Most of the literature works with approximation around zero inflation steady state as this yields a simple-looking NKPC allowing a straightforward interpretation. Throughout the paper, we therefore proceed in this tradition. Nevertheless, all the results presented in this paper were numerically cross-checked against results obtained under a nonzero inflation steady-state, without detecting any significant differences.

$$\pi_t = -\frac{1}{\phi(\varepsilon - 1)}(\chi_t - \chi) + \beta E_t \pi_{t+1}, \tag{5}$$

$$i_t = i + \nu \pi_t + \xi_t, \tag{6}$$

$$y_t = c_t. \tag{7}$$

Here, in addition to the notation already introduced, w_t is a real wage rate, i_t is a one-period nominal interest rate, π_t is the inflation rate between periods $t-1$ and t, χ_t is the real marginal cost, and ξ_t is a standard mean-zero monetary policy shock. Equation (1) is the household's first-order condition for labor, equation (2) is the Euler equation for a one-period nominal bond, which is in zero net supply, equation (3) is a production function, equation (4) gives the marginal cost, equation (5) is the NKPC (for the Rotemberg, 1982, quadratic price adjustment cost specification), equation (6) is the Taylor rule, and equation (7) is the goods market clearing condition. In the NKPC, $\phi \geq 0$ is the Rotemberg cost parameter. Further, $\beta \in (0, 1)$ is a discount factor and $\nu > 1$ is a parameter describing the response of monetary policy to inflation. Variables without a time subscript denote steady-state values (the steady-state value of the inflation rate is equal to zero). As in Cochrane (2011) or Galí (2015) the exposition is cleaner when the weight on output in the Taylor rule is set equal to zero, as implicitly assumed in equation (6).

The linearized NKPC is derived for the Rotemberg specification. It is, however, well-known that the same form is obtained also for the Calvo (1983) specification.[7] Namely, under Calvo specification,

$$\pi_t = \frac{(1-\theta)(1-\theta\beta)}{\theta}(\chi_t - \chi) + \beta E_t \pi_{t+1}, \tag{8}$$

where $\theta \in [0, 1]$ is the fraction of producers not adjusting prices in a given period. The

[7] The Calvo specification leads to an aggregation bias that shows up as total factor productivity in the production function (3). This bias, however, disappears once the model is linearized around the zero inflation steady state. The Rotemberg specification, on the other hand, leads to a resource loss that shows up in the goods market clearing condition (7). Again, it disappears in a linearized version of the model. For these reasons, the above general equilibrium system abstracts from these two details.

mapping between Rotemberg and Calvo NKPC is thus

$$\frac{(1-\theta)(1-\theta\beta)}{\theta} = -\frac{1}{\phi(\varepsilon-1)} > 0.$$

The endogenous variables in the system (1)-(7) are c_t, w_t, l_t, i_t, π_t, y_t, and χ_t. The exogenous variable is the monetary policy shock ξ_t. In the model without capital, the shock is the only state variable. In a linear solution, the dynamics of the endogenous variables are thus fully governed by the exogenous process for the shock; the model parameters affect only the sign and size of the responses of the endogenous variables to the shock.

Eliminating equations (1), (3), (4), and (7) by substitution for c_t, l_t, w_t, and χ_t, the system can be reduced to a three-equation system, which when log-linearized around a steady state (with $y = 1$) becomes

$$-\widehat{y}_t = -E_t\widehat{y}_{t+1} + \widehat{i}_t - E_t\pi_{t+1}, \tag{9}$$

$$\pi_t = \Omega\widehat{y}_t + \beta E_t\pi_{t+1}, \tag{10}$$

$$\widehat{i}_t = \nu\pi_t + \xi_t. \tag{11}$$

Here, $\widehat{i}_t \equiv i_t - i$ and $\widehat{y}_t \equiv (y_t - y)/y$. Further, $\Omega \equiv -(1+\eta)/[\phi(\varepsilon-1)] = (1+\eta)(1-\theta)(1-\theta\beta)/\theta > 0$, depending on whether Rotemberg or Calvo NKPC is used. The system (9)-(11) is the common three-equation representation of the basic New-Keynesian model.[8] The ex-ante real interest rate is defined as

$$\widehat{R}_t \equiv \widehat{i}_t - E_t\pi_{t+1},$$

which from equation (9) implies $\widehat{R}_t = E_t\widehat{y}_{t+1} - \widehat{y}_t$.

[8] As we are concerned only with monetary policy shocks, it is not necessary to further normalize the variables as deviations from flexible-price levels, as the nominal shock does not affect the flexible-price equilibrium. Deviations from steady state and output/inflation gaps are thus in our analysis the same thing.

2.2 Equilibrium output, inflation, and the real interest rate

It is convenient to reduce the above system further by substituting out i_t from equation (11) to get two first-order difference equations in two endogenous variables, π_t and y_t,

$$-\widehat{y}_t = -E_t\widehat{y}_{t+1} + \nu\pi_t + \xi_t - E_t\pi_{t+1} \tag{12}$$

$$\pi_t = \Omega\widehat{y}_t + \beta E_t\pi_{t+1}. \tag{13}$$

The system (12)-(13) can be solved by the method of undetermined coefficients. Assume that the equilibrium decision rule for $\widehat{y}_t$ and the pricing function for π_t are linear functions of the state variable

$$\widehat{y}_t = a\xi_t \quad \text{and} \quad \pi_t = b\xi_t,$$

where a and b are unknown. The guesses are linear, rather than affine, functions of the state as all variables are expressed as deviations from steady state, and thus are equal to zero when $\xi_t = 0$. Suppose that the monetary policy shock follows a stationary AR(1) process

$$\xi_{t+1} = \rho\xi_t + \epsilon_{t+1}, \quad \rho \in [0, 1),$$

where ϵ_{t+1} is an innovation. Substituting the guesses into the system (12)-(13), evaluating the expectations using the AR(1) process, and aligning terms gives unique equilibrium coefficients[9]

$$a = -\frac{1-\beta\rho}{(1-\rho)(1-\beta\rho)+\Omega(\nu-\rho)} < 0, \tag{14}$$

$$b = -\frac{1}{(1-\rho)\frac{1-\beta\rho}{\Omega}+(\nu-\rho)} < 0. \tag{15}$$

[9]This is a particular solution that implicitly excludes explosive paths of inflation and output, a common assumption in the literature under $\nu > 1$. See, e.g., Cochrane (2011)

2.2.1 Flexible prices

It is illustrative to consider two extreme cases of price stickiness. First, suppose that prices are fully flexible ($\theta \to 0$ or $\phi \to 0 \Rightarrow \Omega \to \infty$). The reason for considering this case is that, as shown below, the solution for inflation under this assumption is approximately the same as in the case of *sticky* prices and endogenous capital. Under flexible prices, the solutions (14) and (15) become

$$a \to 0 \quad \text{and} \quad b \to -\frac{1}{\nu - \rho} < 0.$$

Output is thus unaffected by the monetary policy shock and the response of inflation is maximised, in absolute value.

The equilibrium coefficient b can alternatively be obtained by solving forward equation (12), with $\widehat{y}_t = 0$, and excluding explosive paths for inflation (the 'bubble term')

$$\pi_t = -\frac{1}{\nu} \sum_{j=0}^{\infty} \left(\frac{1}{\nu}\right)^j E_t \xi_{t+j} = -\frac{1}{\nu - \rho} \xi_t. \tag{16}$$

This alternative way makes it clear that under flexible prices inflation is determined only by the expected path of the monetary policy shock, with the real rate playing no role in its determination.

Why is the response of inflation to a positive monetary policy shock negative under flexible prices? To understand this, it is helpful to rewrite the monetary policy rule (6) as

$$i_t = (i + \zeta_t) + \nu(\pi_t - \zeta_t), \tag{17}$$

where the new shock ζ_t is related to the original shock as $\zeta_t \equiv -(\nu - 1)^{-1} \xi_t$. The shock ζ_t thus inherits the persistence of the original shock but the two shocks are negatively related. When the policy rule is rewritten as equation (17), the shock ζ_t has typically an interpretation as an inflation target shock (e.g., Smets and Wouters, 2003; Ireland, 2007). This reformulation provides an intuitive explanation of the result that the equilibrium inflation rate declines, when ξ_t increases (the inflation target declines).

2.2.2 Fixed prices

Next, suppose that prices are completely fixed ($\theta \to 1$ or $\phi \to \infty \Rightarrow \Omega \to 0$). This case is useful as it shows why the New-Keynesian model can be perceived as working through the real interest rate channel (Kaplan et al., 2018, , for instance, addopt this assumption to illustrate the decomposition of monetary policy effects in representative agent New-Keynesian models into direct and indirect effects of interest rates). When prices are completely fixed

$$a \to -\frac{1}{1-\rho} < 0 \quad \text{and} \quad b \to 0.$$

Now, inflation is unaffected by monetary policy and the response of output is maximized, in absolute value. Observe that output is fully determined by the Euler equation (9) and the monetary policy rule (11), both of which have $\pi_t = 0 \; \forall \; t$ (on the production side, as $\Omega \to 0$, producers become increasingly sensitive to any given change in inflation and, in the limit, find any output level optimal; see the NKPC). Equation (9) can be written as

$$E_t \widehat{y}_{t+1} - \widehat{y}_t = \widehat{R}_t, \tag{18}$$

where the monetary policy shock translates one-for-one to the ex-ante real interest rate, $\xi_t = \widehat{R}_t$. Monetary policy thus affects output through the real interest rate channel.

Why is the response of output to a positive ξ_t shock negative? According to equation (18), output is expected to grow as long as ξ_t is positive (the ex-ante real interest rate is above steady state). Because the solution to the model is stationary, and ξ_t is governed by a stationary AR(1) process, the only way output can grow is if it falls, on the impact of the shock, below its steady state level.

2.2.3 Sticky prices

In general, the combination of the Euler equation (9) and the Taylor rule (11) yields

$$\pi_t = -\frac{1}{\nu}\sum_{j=0}^{\infty}\left(\frac{1}{\nu}\right)^j E_t\xi_{t+j} + \frac{1}{\nu}\sum_{j=0}^{\infty}\left(\frac{1}{\nu}\right)^j E_t\widehat{R}_{t+j}. \tag{19}$$

Under flexible prices, only the first infinite sum determines inflation, as in equation (16). Under fixed prices, the two infinite sums exactly offset each other. In the intermediate case, the first infinite sum dominates and the response of inflation to a positive monetary policy shock is negative, given by equation (15). Observe that the parameter Ω in equations (14) and (15), reflecting price stickiness, works like a weight shifting the equilibrium coefficients a and b between the fully flexible and completely fixed price solutions.

2.2.4 Equilibrium real interest rate

The equilibrium function for the ex-ante real interest rates is $\widehat{R}_t \equiv \widehat{i}_t - E_t\pi_{t+1} = (1 + \nu b - \rho b)\,\xi_t$, where $b < 0$ is given by (15). The three terms loading onto ξ_t reflect, respectively: a direct effect of the shock in the Taylor rule, an indirect effect due to the response of monetary policy to the equilibrium inflation rate, and expected inflation. Substituting in for b and rearranging terms gives

$$\widehat{R}_t = \left(1 - \frac{1}{1 + \frac{1-\rho}{\nu-\rho}\frac{1-\beta\rho}{\Omega}}\right)\xi_t, \tag{20}$$

where the expression in the brackets is positive, as the term in the denominator is greater than one.

Alternatively, one can see that the ex-ante real interest rate always responds positively to the ξ_t shock by recalling that output always declines on impact of the shock (a is always negative) and converges back to its original steady state from that point on (i.e., it is growing). This can only happen, according to the Euler equation, if the deviation from steady state of the ex-ante real interest rate is positive.

The nominal interest rate depends on the above direct effect of the shock in the Taylor rule and the response of monetary policy to the equilibrium inflation rate, $\widehat{i}_t = (1 + \nu b)\xi_t$. It is well-known (e.g., Woodford, 2003) that the second effect dominates for sufficiently persistent monetary policy shocks (observe that b increases in absolute value as a function of ρ), making the response of the nominal interest rate in that case negative. The ex-ante real rate, however, always increases, irrespective of the persistence of the shock.

3 The model with capital

We now introduce endogenous capital into the above framework. Existing literature offers limited help in isolating the effects of endogenous capital on the properties of New-Keynesian models. Textbooks (e.g., Walsh, 2010; Galí, 2015) stop at the basic model, while research based on the medium-scale DSGE models (e.g., Christiano et al., 2005; Smets and Wouters, 2007) starts straight away with the full-blown version containing many additional features.[10]

When endogenous capital is introduced into the model, it allows households to adjust their savings in the aggregate. Further, intermediate goods are produced with both labor and capital, with implications for the marginal cost function. The general equilibrium becomes characterized by the following system

$$\frac{w_t}{c_t} = l_t^{\eta}, \tag{21}$$

$$\frac{1}{c_t} = \beta E_t \left[\frac{1}{c_{t+1}} \left(\frac{1 + i_t}{1 + \pi_{t+1}} \right) \right], \tag{22}$$

$$\frac{1}{c_t} = \beta E_t \left[\frac{1}{c_{t+1}} (1 + r_{t+1} - \delta) \right], \tag{23}$$

[10] McCandless (2008) comes closest to bridging the gap between the basic model and the medium-scale DSGE models, but his treatment is carried out in the context of a model with money and a monetary policy rule formulated as a money growth rule (as in, e.g., Hairault and Portier, 1993; Kimball, 1995; Yun, 1996; Ellison and Scott, 2000). The New-Keynesian literature, instead, follows Woodford (2003) by abstracting from money and formulates monetary policy as a Taylor rule. Woodford (2003), Chapter 5, extends the basic model to include endogenous capital, but presents results for parameterizations leading to only a subset of the possible outcomes documented here, thus obscuring the mechanism at work (Woodford, 2005, corrects some mistakes contained in that chapter). Galí and Gertler (2007) also extend the basic model to include capital, but do not study the transmission mechanism.

$$y_t = k_t^\alpha l_t^{1-\alpha}, \tag{24}$$

$$\frac{w_t}{r_t} = \frac{1-\alpha}{\alpha}\left(\frac{k_t}{l_t}\right), \tag{25}$$

$$\chi_t = \left(\frac{r_t}{\alpha}\right)^\alpha \left(\frac{w_t}{1-\alpha}\right)^{1-\alpha}, \tag{26}$$

$$\pi_t = -\frac{1}{\phi(\varepsilon-1)}(\chi_t - \chi) + \beta E_t \pi_{t+1}, \tag{27}$$

$$i_t = i + \nu\pi_t + \xi_t, \tag{28}$$

$$y_t = c_t + k_{t+1} - (1-\delta)k_t. \tag{29}$$

Here, in addition to the notation introduced so far, k_t is capital, r_t is the capital rental rate, and $\delta \in (0,1)$ is a depreciation rate; investment can be defined residually as $x_t \equiv k_{t+1} - (1-\delta)k_t$. In the NKPC, the mapping between Rotemberg and Calvo specifications still holds: $-1/\phi(\varepsilon-1) = (1-\theta)(1-\theta\beta)/\theta > 0$. The endogenous variables are c_t, w_t, l_t, i_t, π_t, y_t, χ_t, r_t, k_{t+1}; the exogenous variables are ξ_t and k_0.[11]

Notice that (21), (22), (27), and (28) are the same as before. Further, (24) and (26) are the same as before for $\alpha = 0$ and (29) is the same for $k_t = 0\ \forall t$. The truly new equations are equations (23) and (25), which add the two new endogenous variables, k_{t+1} and r_t. Equation (23) is the Euler equation for capital and equation (25) is a condition for the optimal mix of capital and labor in production; it equates the marginal rate of technological substitution with the relative factor prices (a first-order condition of a cost minimization problem of each firm j).[12]

The model contains the key element of a prototypical RBC model: capital accumulation

[11] As in the baseline model, for the reasons discussed earlier, the exposition abstracts from the aggregation bias in the case of Calvo pricing and the resource loss in the case of Rotemberg pricing.

[12] We follow the simpler setup in which capital can be rented each period on an economy-wide rental market. Woodford (2005) considers the opposite case in which capital is firm specific. In equilibrium, the two environments differ only in the form of the elasticity of marginal costs to inflation in the NKPC. The Woodford (2005) setup implies that a given value of the elasticity is consistent with prices being sticky for a shorter period of time than in the model with a common rental market. While this implication has a clear empirical appeal, the distinction between the two environments is unimportant for the points made in this paper.

as a means for the economy as a whole to smooth out consumption in the presence of fluctuations in income (output). In fact, under flexible prices, the model collapses into a RBC model with two additions, the Euler equation for bonds and the Taylor rule. To see this, note that under flexible prices (either $\phi = 0$ or $\theta = 0$) the NKPC (27) implies $\chi_t = \chi$. If, in addition, $\varepsilon = 1$ (perfect competition), $\chi = 1$; see Galí (2015), Chapter 3. This is a standard profit maximization condition under perfect competition, stating that the marginal cost is equal to the good's relative price, which is equal to one, as all goods are perfect substitutes. When this condition is used in equation (26), and the resulting equation is combined with the cost minimization condition (25), we get the standard RBC conditions equalizing marginal products to factor prices: $r_t = \alpha k_t^{\alpha-1} l_t^{1-\alpha}$ and $w_t = (1-\alpha) k_t^{\alpha} l_t^{-\alpha}$. Under flexible prices, these two conditions replace equations (25) and (26) in the above system. Notice that the system becomes recursive (a classical dichotomy): equations (21), (23), (24), (29), and the above two marginal product conditions—the standard RBC system—determine c_t, w_t, l_t, y_t, r_t, and k_{t+1}, given k_0, independently of ξ_t (in addition, $\chi_t = 1$ from the NKPC). Equations (22) and (28) then pin down i_t and π_t. The NKPC (27), with either $\phi > 0$ or $\theta \in (0, 1]$, is what breaks the classical dichotomy under sticky prices.

3.1 The log-linear system

In what follows it is convenient to substitute in for r_t in the expression for the marginal cost (26) from the cost minimization condition (25). The marginal cost then becomes

$$\chi_t = \frac{w_t}{1-\alpha} \left(\frac{y_t}{k_t} \right)^{\frac{\alpha}{1-\alpha}} .$$

Observe that for $\alpha = 0$ this expression becomes the same as in the model without capital. Further, substitute in for l_t in the first-order condition for labor (21) from the production function (24). This gives the first-order condition for labor as

$$\frac{w_t}{c_t} = \left(\frac{y_t}{k_t^{\alpha}} \right)^{\frac{\eta}{1-\alpha}} .$$

Again, for $\alpha = 0$, this condition is the same as in the model without capital.

With the above two substitutions, the we can log-linearize the general equilibrium system to get

$$-\widehat{c}_t + \widehat{w}_t = \frac{\eta}{1-\alpha}\widehat{y}_t - \frac{\alpha\eta}{1-\alpha}\widehat{k}_t$$

$$-\widehat{c}_t = -E_t\widehat{c}_{t+1} + \widehat{i}_t - E_t\pi_{t+1},$$

$$-\widehat{c}_t = -E_t\widehat{c}_{t+1} + E_t\widehat{r}_{t+1},$$

$$\widehat{l}_t = \frac{1}{1-\alpha}\widehat{y}_t - \frac{\alpha}{1-\alpha}\widehat{k}_t,$$

$$\widehat{r}_t = r(\widehat{l}_t - \widehat{k}_t + \widehat{w}_t),$$

$$\widehat{\chi}_t = \widehat{w}_t + \frac{\alpha}{1-\alpha}\widehat{y}_t - \frac{\alpha}{1-\alpha}\widehat{k}_t,$$

$$\pi_t = \Psi\widehat{\chi}_t + \beta E_t\pi_{t+1},$$

$$\widehat{i}_t = \nu\pi_t + \xi_t,$$

$$\widehat{y}_t = \frac{c}{y}\widehat{c}_t + \frac{k}{y}\widehat{k}_{t+1} - (1-\delta)\frac{k}{y}\widehat{k}_t.$$

This system may seem complex, but as shown below, it has fairly intuitive implications for the model's dynamics. As before, variables without a time subscript are steady-state values. Interest rates are expressed as percentage point deviations from steady state, $\widehat{r}_t \equiv r_t - r$, $\widehat{i}_t \equiv i_t - i$, and all other variables as percentage deviations from steady state, e.g., $\widehat{c}_t \equiv (c_t - c)/c$. Eliminating $\widehat{r}_t$, $\widehat{\chi}_t$, $\widehat{w}_t$, $\widehat{i}_t$, and $\widehat{l}_t$ we get a final system of four equilibrium first-order difference equations in four endogenous variables $\widehat{c}_t$, $\widehat{y}_t$, $\widehat{k}_{t+1}$, and $\widehat{\pi}_t$

$$-\widehat{c}_t = -E_t\widehat{c}_{t+1} + \nu\pi_t - E_t\pi_{t+1} + \xi_t, \tag{30}$$

$$-\widehat{c}_t = -E_t\widehat{c}_{t+1} + rE_t\left(\widehat{c}_{t+1} + \frac{1+\eta}{1-\alpha}\widehat{y}_{t+1} - \frac{1+\alpha\eta}{1-\alpha}\widehat{k}_{t+1}\right), \tag{31}$$

$$\pi_t = \Psi\left[\frac{\eta+\alpha}{1-\alpha}\widehat{y}_t - \frac{\alpha(1+\eta)}{1-\alpha}\widehat{k}_t + \widehat{c}_t\right] + \beta E_t\pi_{t+1}, \tag{32}$$

$$\widehat{y}_t = \frac{c}{y}\widehat{c}_t + \frac{k}{y}\widehat{k}_{t+1} - (1-\delta)\frac{k}{y}\widehat{k}_t, \tag{33}$$

where, $\Psi \equiv -\chi/\phi(\varepsilon - 1) = \chi(1-\theta)(1-\theta\beta)/\theta$.[13] Relative to the basic model (12)-(13), the number of equations has increased to four. This is because of the presence of the extra variable $\widehat{k}_{t+1}$ and because $\widehat{c}_t \neq \widehat{y}_t$. Here, (30) is the same as in the model without capital, (32) is the same as in the model without capital for $\widehat{k}_t = 0$ and $\alpha = 0$, and (33) is the same as in the model without capital for $k = 0$. Only equation (31), the Euler equation for capital, is genuinely new.

Observe that when prices are fully flexible ($\Psi \to \infty$), the NKPC (32) implies that, given an initial steady-state condition (i.e., $\widehat{k}_t = 0$), $\widehat{c}_t = \widehat{y}_t = 0$. Equation (33) then implies $\widehat{k}_{t+1} = 0$. Monetary policy is neutral and inflation is determined from equation (30) in exactly the same way as under flexible prices in the basic model. The addition of capital thus does not change the dynamics of inflation in response to the monetary policy shock when prices are fully flexible.

Consider now the other extreme, when prices are completely fixed ($\Psi \to 0$). Now, like in the basic model, the NKPC (32) implies that inflation is equal to zero. Further, equation (30) determines consumption as a function of the monetary policy shock in exactly the same way as in the basic model and a positive monetary policy shock reduces consumption. The presence of capital thus has no effect on equilibrium consumption or inflation when prices are completely fixed either (it, however, affects output differently than in the basic model because $c_t \neq y_t$). The interesting case is the case of sticky prices, to which we turn next.

3.2 The monetary transmission mechanism—a first look

Here we take a first look at the transmission mechanism in the presence of capital and sticky prices and revisit it in more detail in the sections that follow. The main insight, however, can be obtained from this section.

As in equilibrium all variables are determined simultaneously by the system of the dif-

[13] In the basic model, $\chi = w = 1$.

ference equations (30)-(33), it is difficult to gain insight into the exact inner workings of the model. However, an approximate description can be provided once we realize that for any plausible calibration, the steady-state capital rental rate r is close to zero (under standard calibrations it is between 0.01 and 0.03, given by $1/\beta - 1 + \delta$). Let us therefore proceed under the assumption that the steady-state capital rental rate is in fact equal to zero. This greatly simplifies the analysis and provides a useful insight, as the system (30)-(33) becomes recursive.[14] Under the above assumption, equation (31) implies $\widehat{c}_t = E_t\widehat{c}_{t+1}$; that is, the presence of capital allows perfect consumption smoothing across time. Further, as the model is stationary (see below), it has to be the case that $\widehat{c}_t = E_t\widehat{c}_{t+1} = 0$. Otherwise, under $\widehat{c}_t = E_t\widehat{c}_{t+1}$, a given shock would lead to a permanent shift of consumption (in expectations) away from the steady state, violating stationarity. With consumption thus determined, equation (30) then determines the equilibrium inflation rate, which depends only on the monetary policy shock. The solution for the inflation rate is therefore the same as in the basic model under flexible prices, $\pi_t = -[1/(\nu - \rho)]\xi_t$, even though prices here are sticky, though not completely fixed. The inflation rate falls on the impact of the shock and converges back to zero from below. Along this path, $\pi_t - E_t\pi_{t+1}$ is negative. Thus, for β close to one, equation (32) implies that on the impact of the shock output has to decline. This is because $\widehat{c}_t = 0$ for the above reasons and $\widehat{k}_t = 0$, as in the impact period the existing capital stock is assumed to be in steady state. From equation (33) then, $\widehat{k}_{t+1}$ has to decline; i.e., the decline in output is fully absorbed by a decline in investment.

What is going on? Essentially, the presence of endogenous capital, as summarized by equation (31), allows the economy as a whole to smooth out fluctuations in output (income) brought about by the monetary policy shock in the presence of sticky prices. Because investment makes up only a small fraction of the capital stock, consumption can be kept smooth by adjusting investment with only a small effect on the expected rate of return on capital, and thus—through arbitrage—on the ex-ante real interest rate (the exact responses

[14] We check that the description of the mechanism that follows under this assumption is consistent with the actual workings of the model by computing, in the next section, impulse responses for the exact calibrated value of r.

of the ex-ante real rate are demonstrated below). Effectively, the presence of endogenous capital makes consumption 'sticky' and *equilibrium* prices behave as in the basic model with flexible prices, even though at the level of individual producers prices are sticky. Given the equilibrium inflation, each individual firm that cannot optimally adjust prices (in a Calvo sense) to keep up with the equilibrium path of the aggregate inflation rate adjusts output. When equilibrium inflation temporarily drops, those firms that cannot adjust prices reduce output. A temporary drop in households' income brought about by the lower output is then smoothed out by a drop in investment.

While this description of the transmission mechanism holds only approximately (we have assumed $r = 0$), the numerical examples below show that it provides a good way of describing how the model works. This description shows that monetary policy affects output in the model with capital even if the ex-ante real interest rate does not change. And even though the model with capital may produce responses of consumption that are from empirical perspective too smooth, it helps uncovers the underlying monetary transmission mechanism in New-Keynesian models. After confirming this description with numerical examples, we show how increasingly higher capital adjustment costs, which make consumption smoothing in the aggregate harder, bring the model's behavior gradually back in line with that of the basic model.

3.3 Numerical examples

The model can again be solved by the method of undetermined coefficients (this is done for the general non-zero value of r). There are two state variables, k_t and ξ_t. The solution thus has the form: $\widehat{c}_t = a_0\widehat{k}_t + a_1\xi_t$, $\pi_t = b_0\widehat{k}_t + b_1\xi_t$, $\widehat{y}_t = d_0\widehat{k}_t + d_1\xi_t$, and $\widehat{k}_{t+1} = f_0\widehat{k}_t + f_1\xi_t$. Substituting these functions into the system (30)-(33), evaluating expectations, and collecting terms yields a system of eight equations in eight unknowns, the coefficients of the above four linear functions. The resulting system is provided in the Appendix. It is block recursive, whereby a_0, b_0, d_0, and f_0 can be solved for independently of a_1, b_1, d_1, and f_1 and the persistence of the shock, ρ, shows up only in the equations for the latter four coefficients. The shock persistence thus has no effect on the equilibrium coefficients loading onto $\widehat{k}_t$. In other words, the internal dynamics of the model are unaffected by the dynamics of the shock. The equilibrium coefficients loading onto $\widehat{k}_t$, however, affect a_1, b_1, d_1, and f_1 and thus the responses of the endogenous variables to the monetary policy shock. In other words, the presence of capital affects the responses of the endogenous variables to the shock.

From here we proceed numerically, using standard parameter values: $\beta = 0.99$, $\eta = 1$, $\theta = 0.7$, $\nu = 1.5$, $\delta = 0.025$, $\alpha = 0.3$, and $\varepsilon = 0.83$. The persistence of the monetary policy shock is treated as a free parameter and we consider four values, $\rho \in \{0, 0.1, 0.5, 0.95\}$.[15] Figures 1-4 display responses to a 1 percentage point increase in ξ_t in period $t = 1$ for the above four values of ρ. Interest and inflation rates are reported as percentage point deviations from steady state; all other variables as percentage deviations from steady state. We are not concerned with the exact values of the responses, rather with their qualitative properties.

The impulse-response functions confirm our conjecture from the previous subsection that the real effects of monetary policy shocks in the model with capital do not transmit through the real interest rate channel. In all cases but $\rho = 0$, output and inflation fall, in response to the positive monetary policy shock, while the ex-ante real interest rate declines. Experimentation reveals that the real interest rate increases only for $\rho \in [0, 0.04]$; output and inflation, however, decline for *all* values of $\rho \in [0, 1)$.[16] Regarding consumption and investment, in all four cases both variables fall, although consumption falls only a little, in line with our discussion in the previous subsection (in the case of $\rho = 0.95$, consumption on impact somewhat increases, before falling persistently). Most of the decline in output is thus absorbed

[15]Previous studies have investigated how endogenous capital affects the determinacy of equilibria (e.g., Dupor, 2001; Carlstrom and Fuerst, 2005; Kurozumi and Van Zandweghe, 2008). In our setup the findings of Carlstrom and Fuerst (2005) and Kurozumi and Van Zandweghe (2008) hold and the Taylor principle with respect to current inflation ensures a unique nonexplosive equilibrium.

[16]In all cases, the real interest rate increases above its steady-state level several periods after the impact of the shock due to the decline in capital—once the effect of sticky prices (time-varying markup) dies off, the dynamics of the real rate become governed by the marginal product of capital, as in a real business cycle model. The decline in capital increases its marginal product and thus the real rate.

by investment.

It is sometimes argued that it is the long-term real interest rate, rather than the one-period real interest rate, that is crucial for the workings of the real interest rate channel. The case of $\rho = 0.95$, however, shows that in the model with endogenous capital, output and inflation can decline even when both short- and long-term ex-ante real interest rates on impact decline. The decline of the short rate can be observed directly in the figure. To see that also the long rate declines, notice that by forward substitution of the log-linear Euler equation for bonds, and imposing stationarity,

$$\widehat{c}_t = -\sum_{j=0}^{\infty} \widehat{R}_{t+j}.$$

Here, the right-hand side is usually interpreted as the ex-ante long-term real interest rate (as in the expectations hypothesis). Because in the case of $\rho = 0.95$ consumption increases on the impact of the shock, this equation implies that the long-term real interest rate declines. Thus, in the model with endogenous capital, inflation and output decline, in response to the monetary policy shock, even though the ex-ante short- and long-term real interest rates decline.

3.4 Explaining the responses of the ex-ante real interest rate

How is the ex-ante real interest rate determined and why does the model have such a hard time generating its increase in response to the monetary policy shock? We offer explanations from two perspectives.

First, use the log-linear Euler equation for bonds and the equilibrium decision rule for consumption to write

$$\begin{aligned} \widehat{R}_t &= E_t\widehat{c}_{t+1} - \widehat{c}_t \\ &= a_0(f_0\widehat{k}_t + f_1\xi_t) + a_1\rho\xi_t - (a_0\widehat{k}_t + a_1\xi_t) \\ &= a_0(f_0 - 1)\widehat{k}_t + (\rho a_1 - a_1 + a_0 f_1)\xi_t. \end{aligned} \tag{34}$$

Focus on the immediate response from steady state, thus setting $\widehat{k}_t = 0$, and observe from the figures that in most cases a_1 is negative—i.e., consumption declines on the impact of the shock. The coefficient loading onto ξ_t in equation (34) consists of three terms. The first two terms are the same as in the basic model, although the value of a_1 may now be different—this is the 'indirect' effect of capital, working through the effect of a_0 on a_1, as explained in the preceding subsection. We have shown in Section 2 that $\rho a_1 - a_1$ in the basic model is always positive, generating a positive response of $\widehat{R}_t$ to ξ_t. The third term, $a_0 f_1$, is related to the 'direct' effect of endogenous capital. Here, f_1 gives the equilibrium response of $\widehat{k}_{t+1}$ to ξ_t. It is negative, reflecting a decline in investment—due to consumption smoothing—in response to a drop in output brought about by a positive ξ_t shock. The coefficient a_0 gives the equilibrium response of $\widehat{c}_{t+1}$ to $\widehat{k}_{t+1}$. This coefficient is positive, prescribing a lower consumption when capital is lower.[17] The product of the two coefficients $(a_0 f_1)$ thus prescribes a drop in tomorrow's consumption, in response to a positive ξ_t shock today. The consumer is effectively trading off not having to adjust consumption in line with the fall in income on the impact of the shock, for a slightly lower consumption in the immediate future due to a lower capital stock. If the direct effect of endogenous capital is strong enough, the real rate declines even if $\rho a_1 - a_1$ is positive as in the basic model.

Second, combining the Euler equation for bonds with that for capital, (30) and (31), the ex-ante real interest rate can be written as

$$\widehat{R}_t = rE_t \left(\widehat{c}_{t+1} + \frac{1+\eta}{1-\alpha}\widehat{y}_{t+1} - \frac{1+\alpha\eta}{1-\alpha}\widehat{k}_{t+1} \right),$$

where the expression in the bracket is a combination of markups and the marginal product of capital. To provide a useful description of the mechanics of the model, we have set the steady-state value of the capital rental rate r equal to zero, on the basis that in plausible calibrations it is close to zero (our calibration implies $r = 0.035$). To explain the actual behavior of $\widehat{R}_t$ in the numerical examples, we naturally cannot adopt the same simplification. Thus, when

[17] The signs of the coefficients are established numerically.

r is small but not equal to zero, the responses of the ex-ante real rate are determined by the responses of $\widehat{c}_{t+1}$, $\widehat{y}_{t+1}$, and $\widehat{k}_{t+1}$. Given that consumption moves only a little, the main determinants are future output and capital stock. For temporary monetary policy shocks (such as $\rho = 0$), only future capital is affected. Its decline thus pushes the the ex-ante real rate up. For persistent monetary policy shocks, future output declines as well. If the decline is sufficiently large, to couterweight the decline in the capital stock, the ex-ante real rate declines. This is the case for most of the values of ρ considered, at least in the immediate future after the impact of the shock.

3.5 Adjustment costs

One way to reduce consumption smoothing is to introduce into the model capital adjustment costs. Suppose that whenever the household changes the capital stock, it has to incur a cost in terms of foregone real income. The simplest form of capital adjustment costs is a quadratic cost function

$$-\frac{\kappa}{2}(k_{t+1} - k_t)^2, \quad \kappa \geq 0.$$

In steady state, the adjustment cost is equal to zero. Further, as the adjustment cost is quadratic, it does not affect the resource constraint of the economy in a log-linear approximation of the model around a steady state. The Euler equation for capital now becomes

$$1 = \beta E_t \left[\frac{c_t}{c_{t+1}} \left(\frac{r_{t+1} - \delta}{q_t} + \frac{q_{t+1}}{q_t} \right) \right],$$

where $q_t \equiv 1 + \kappa(k_{t+1} - k_t)$ is Tobin's q, the price of capital in terms of current consumption. Notice that for $\kappa = 0$, the Euler equation collapses into the Euler equation in the version without adjustment costs. The expression in the round brackets in the Euler equation has an interpretation as a sum of a dividend yield and a capital gain. Denote the capital gain by $G_{t+1} \equiv q_{t+1}/q_t$. In a log-linearized form the Euler equation can be written as

$$-\widehat{c}_t = -E_t\widehat{c}_{t+1} + E_t\widehat{G}_{t+1} + rE_t\left(\widehat{c}_{t+1} + \frac{1+\eta}{1-\alpha}\widehat{y}_{t+1} - \frac{1+\alpha\eta}{1-\alpha}\widehat{k}_{t+1}\right), \tag{35}$$

where $\widehat{G}_{t+1} = \widehat{q}_{t+1} - \widehat{q}_t = \overline{\kappa}(\widehat{k}_{t+2} - \widehat{k}_{t+1}) - \overline{\kappa}(\widehat{k}_{t+1} - \widehat{k}_t)$ is a percentage deviation of capital gains from steady state and $\overline{\kappa} \equiv \kappa k$. Combining equations (35) and (30), the ex-ante real interest rate can be written as a sum of expected capital gain and dividend yield

$$\begin{aligned}\widehat{R}_t &= \nu\pi_t + \xi_t - E_t\pi_{t+1} \\ &= E_t\widehat{G}_{t+1} + rE_t\left(\widehat{c}_{t+1} + \frac{1+\eta}{1-\alpha}\widehat{y}_{t+1} - \frac{1+\alpha\eta}{1-\alpha}\widehat{k}_{t+1}\right).\end{aligned} \tag{36}$$

The exposition proceeds again under the simplifying assumption that $r \approx 0$. Under this assumption, the dividend term in equations (35) and (36) drops out. Now, however, the presence of the capital gains does not allow us to conclude that $\widehat{c}_t = E_t\widehat{c}_{t+1} = 0$. Capital adjustment costs prevent perfect consumption smoothing, resulting in $\widehat{c}_t \neq E_t\widehat{c}_{t+1} \neq 0$. Any drop in output dictated by inflation dynamics and the NKPC has to be, at least partially, accommodated by a drop in consumption. The higher is κ, the more any given change in output is accounted for by a change in consumption, rather than investment. Increasing κ thus brings the response of consumption closer to the response of output and thus closer to the response of consumption in the basic model; i.e., consumption falls on impact and converges back to steady state from below. As a result, $\widehat{R}_t = E_t\widehat{c}_{t+1} - \widehat{c}_t > 0$. By bringing the consumption response closer to the basic model, capital adjustment costs also bring the response of inflation closer to the basic model: from equation (30), $\pi_t = \nu^{-1}\left(-\xi_t + E_t\pi_{t+1} + E_t\widehat{c}_{t+1} - \widehat{c}_t\right)$, which is the same as equation (12) and the response of consumption is now similar to that in the basic model. When $\kappa = \infty$, the responses in the model with capital coincide with those in the basic model.

The model can again be solved by the method of undetermined coefficients, guessing $\widehat{c}_t$, π_t, $\widehat{y}_t$, and $\widehat{k}_{t+1}$ as linear functions of $\widehat{k}_t$ and ξ_t. Relative to the system of restrictions in the model without adjustment costs, only the restrictions resulting from equation (35) are different. These are contained in the Appendix.

Figures 5-7 show the responses of the model under $\rho = 0.5$ and $\kappa \in \{0.1, 0.2, 0.5\}$. The rest of the parameterization is as before. The figures show that as κ increases, the model starts to produce responses consistent with the real interest rate channel. Specifically, at $\kappa = 0.1$ the model still suffers from producing a decline in the nominal interest rate and only a gradual increase in the ex-ante real interest rate. At $\kappa = 0.2$, the ex-ante real interest rate increases on impact, but the nominal interest rate still falls. At $\kappa = 0.5$, finally, both the ex-ante real interest rate and the nominal interest rate increase on impact. Throughout these experiments, the increase in the ex-ante real interest rate occurs due to expected capital gains. Sufficiently high capital adjustment costs, as in Figure 7, thus make the model consistent with the real interest rate channel. As before, consumers want to smooth consumption when income declines. To prevent consumption smoothing in equilibrium, expected capital gains—and thus the ex-ante real interest rate—have to sufficiently increase.

3.6 Observational equivalence

The consistency with the real interest rate channel, however, is only observational and thus subject to change when policy parameters change. To illustrate this point, we contrast Figure 7, which is observationally equivalent to the real interest rate channel, with Figure 8. Figure 8 plots again the responses for $\kappa = 0.5$, but under a shock persistence $\rho = 0.85$, instead of $\rho = 0.5$. Under $\rho = 0.85$, both inflation and output again decline, yet the ex-ante real interest rate declines as well (the decline in output is in fact as large as in Figure 7, but more persistent). An econometrician estimating a VAR on data generated by the model with $\rho = 0.5$ would conclude a presence of the real interest rate channel. A policy advice based on such evidence would then be that the central bank needs to increase the ex-ante real interest rate in order to reduce inflation and output. Such advice, however, would be misguided—the same model, but with a higher persistence parameter for the policy shock, predicts that about the same effect on inflation and output can be accompanied with a decline in the real rate. The reason for such a contrasting policy implications is that the whole focus on the ability to affect the ex-ante real interest rate in the transmission of monetary policy is misguided in the context of the New-Keynesian model.

4 Relating the findings to recent critiques

Although this paper is meant to be a constructive exploration of the monetary transmission mechanism in New-Keynesian models, it is worth relating our findings to some recent critiques. Some earlier studies have been skeptical about the plausibility of the real interest rate channel itself, as it relies on a sensitivity of expenditures, especially consumption, to real interest rates perceived to be unrealistic (Bernanke and Gertler, 1995; Taylor, 1995, contain references). By invoking this channel, the New-Keynesian literature exposes itself to the same criticism. A recent critique along these lines has been developed by Kaplan et al. (2018). Our analysis, however, shows that the monetary transmission mechanism in New-Keynesian models does not operate through the real interest rate channel. The Kaplan et al. (2018) critique, however, still applies in the sense that the features they emphasize—household heterogeneity, illiquid assets, and borrowing constraints—are important for generating empirically plausible responses of consumption, especially in the cross-section of households.

New-Keynesian models have also been critiqued on a number of other grounds. Cochrane (2011) attacks New-Keynesian models on the basis that the way inflation is determined under a Taylor rule is ad hoc. Nekarda and Ramey (2013) point out that New-Keynesian models exhibit counterfactual behavior of markups. Broer, Hansen, Krusell, and Oberg (2015) highlight the models' difficulties in a worker-capitalist setup. Finally, Chari, Kehoe, and McGrattan (2009) question a number of the additional features of the medium-scale models that we have abstracted from.

5 Conclusion

How does monetary policy affect inflation and output in the economy? A widely accepted view is that it is through its effect on the ex-ante real interest rate. In this paradigm, a common justification for the transmission from the nominal interest rate, the policy instrument, to the real interest rate, a price that ultimately affects decisions of the private sector, rests on nominal price rigidities. Introducing this channel into a modern dynamic stochastic general equilibrium environment was one of the motivations for the development of New-Keynesian models. These models, both in their basic and medium-scale DSGE forms are routinely used at central banks around the world to guide monetary policy. This paper scrutinizes the inner workings of the monetary transmission mechanism in this important class of models.

We demonstrate that the monetary transmission mechanism does not operate through the real interest rate channel. Instead, as a first pass, inflation is determined as in a flexible price model, through current and expected future monetary policy shocks, while output is then pinned down by the New-Keynesian Phillips curve. According to the New-Keynesian Phillips curve, output temporarily drops, when inflation temporarily declines, as firms that cannot adjust prices to keep pace with the decline in the aggregate price level reduce output. The ex-ante real rate only reflects the desire and ability of households to keep consumption smooth in face of such temporary changes in output (income). An increase, decline, or no change in the ex-ante real interest rate are all consistent with declines in output and inflation in response to a standard contractionary monetary policy shock. High enough capital adjustment costs make the model appear as if it operated through the real interest rate channel. This relationship, however, is not structural and is subject to a change when policy parameters change. Understanding the inner workings of this important class of models is key for their future development in most fruitful directions.

The policy implications of our analysis are that (i) either monetary policy in actual economies does transmit through the real interest rate channel, but then the New-Keynesian model—in the form currently used in the literature—may be a misleading tool for its analysis or (ii) the New-Keynesian model—for its elegant micro-foundations of the price-setting behavior and internal consistency—is a useful tool in its own right, but then policy makers need to rethink the channel through which monetary policy affects inflation and real activity. This paper provides a way how to think more accurately about the transmission mechanism in New Keynesian models.

Appendix

This Appendix contains the systems that determine the equilibrium coefficients in the version with capital, both without and with capital adjustment costs.

The equilibrium functions in the model with capital take the form: $\widehat{c}_t = a_0\widehat{k}_t + a_1\xi_t$, $\pi_t = b_0\widehat{k}_t + b_1\xi_t$, $\widehat{y}_t = d_0\widehat{k}_t + d_1\xi_t$, and $\widehat{k}_{t+1} = f_0\widehat{k}_t + f_1\xi_t$. In the version without adjustment costs, using these functions in the system (30)-(33) and aligning terms yields a system of eight equations in eight unknowns, a_0,a_1, b_0, b_1, d_0, d_1, f_0, f_1. From equation (30) we get:

$$-a_0 = -a_0 f_0 + \nu b_0 - b_0 f_0,$$

$$-a_1 = -a_0 f_1 - a_1\rho + \nu b_1 - b_0 f_1 - b_1\rho + 1.$$

From equation (31):

$$-a_0 = -(1-r)a_0 f_0 + \frac{r(1+\eta)}{1-\alpha} d_0 f_0 - \frac{r(1+\alpha\eta)}{1-\alpha} f_0,$$

$$-a_1 = -(1-r)a_0 f_1 - (1-r)a_1\rho + \frac{r(1+\eta)}{1-\alpha} d_0 f_1 + \frac{r(1+\eta)}{1-\alpha} d_1\rho - \frac{r(1+\alpha\eta)}{1-\alpha} f_1.$$

From equation (32):

$$b_0 = -\psi\frac{\eta+\alpha}{1-\alpha} d_0 + \psi\frac{\alpha\eta+\alpha}{1-\alpha} - \psi a_0 + \beta b_0 f_0,$$

$$b_1 = -\psi\frac{\eta+\alpha}{1-\alpha} d_1 - \psi a_1 + \beta b_0 f_1 + \beta b_1\rho.$$

And from equation (33):

$$f_0 = \frac{y}{k}d_0 - \frac{c}{y}a_0 + (1-\delta),$$

$$f_1 = \frac{y}{k}d_1 - \frac{c}{y}a_1.$$

With capital adjustment costs, the second pair of equations becomes

$$-\overline{\kappa} - a_0 + (1-r)a_0 f_0 - \frac{r(1+\eta)}{1-\alpha}d_0 f_0 + \frac{r(1+\alpha\eta)}{1-\alpha}f_0 + 2\overline{\kappa}f_0 - \overline{\kappa}f_0 = 0,$$

$$-a_1 + (1-r)a_0 f_1 + (1-r)a_1\rho - \frac{r(1+\eta)}{1-\alpha}d_0 f_1 - \frac{r(1+\eta)}{1-\alpha}d_1\rho + \frac{r(1+\alpha\eta)}{1-\alpha}f_1 + 2\overline{\kappa}f_1 - \overline{\kappa}f_0 f_1 - \overline{\kappa}f_1\rho = 0.$$

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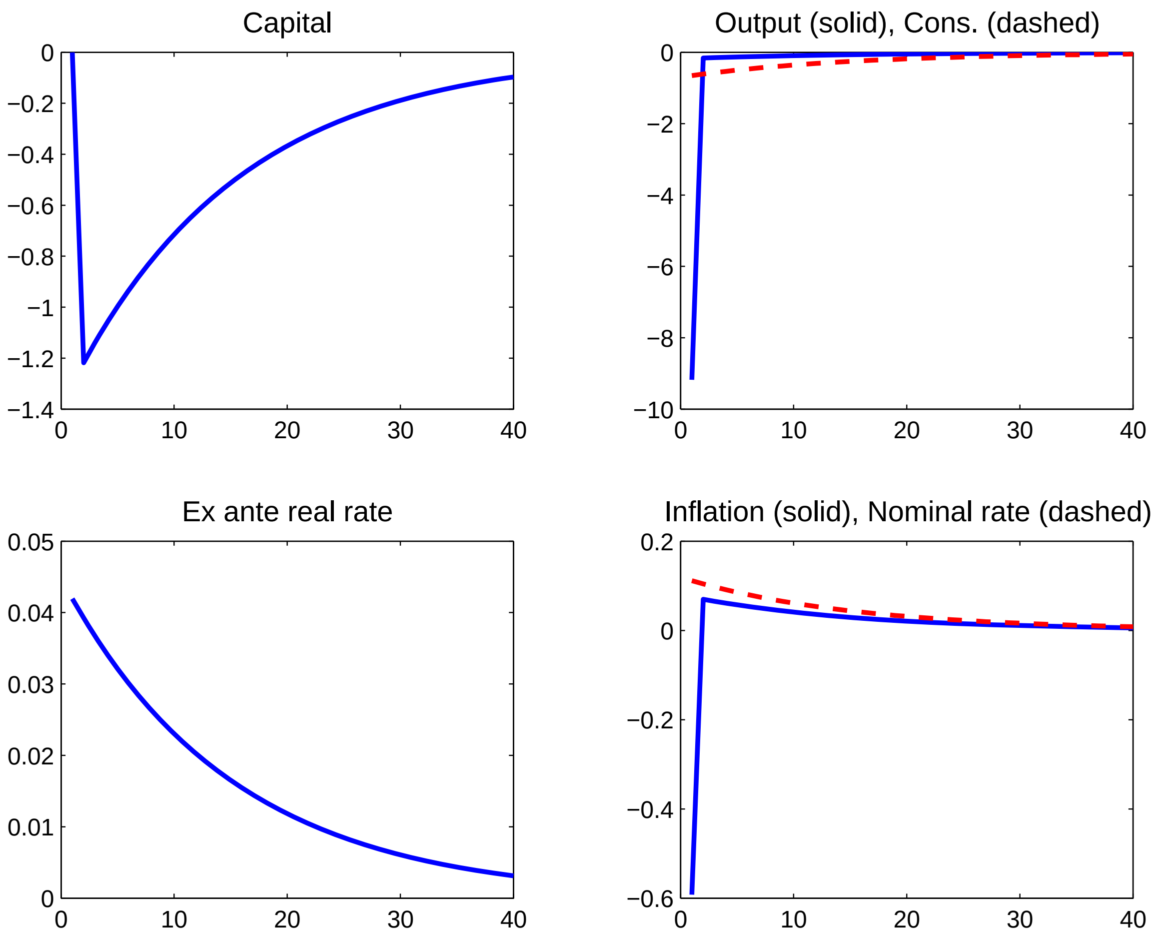

Figure 1: The model with capital, $\rho = 0$. Responses to 1 percentage point increase in ξ_t. The remaining parameterization is: $\beta = 0.99$, $\eta = 1$, $\theta = 0.7$, $\varepsilon = 0.83$, $\nu = 1.5$, $\alpha = 0.3$, and $\delta = 0.025$.

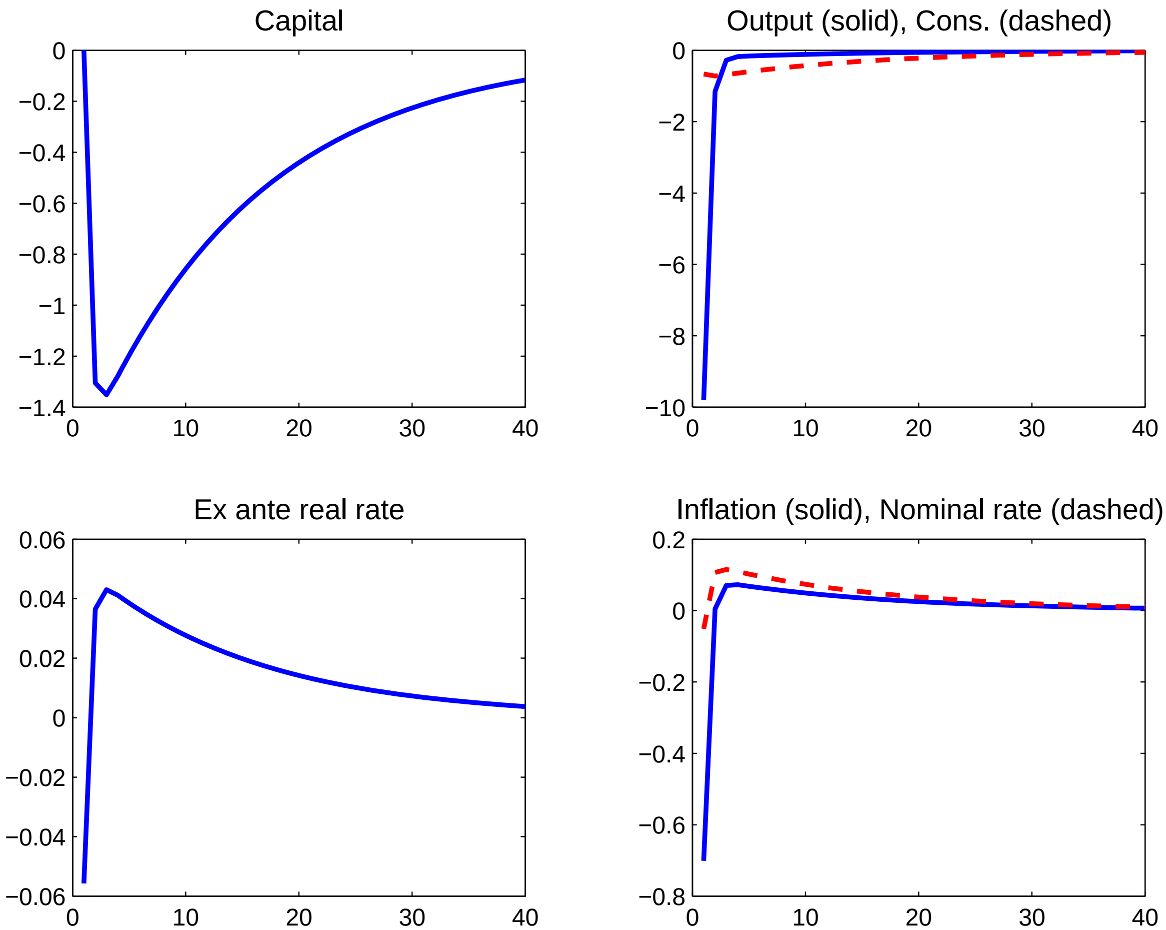

Figure 2: The model with capital, $\rho = 0.1$. Responses to 1 percentage point increase in ξ_t. The remaining parameterization is: $\beta = 0.99$, $\eta = 1$, $\theta = 0.7$, $\varepsilon = 0.83$, $\nu = 1.5$, $\alpha = 0.3$, and $\delta = 0.025$.

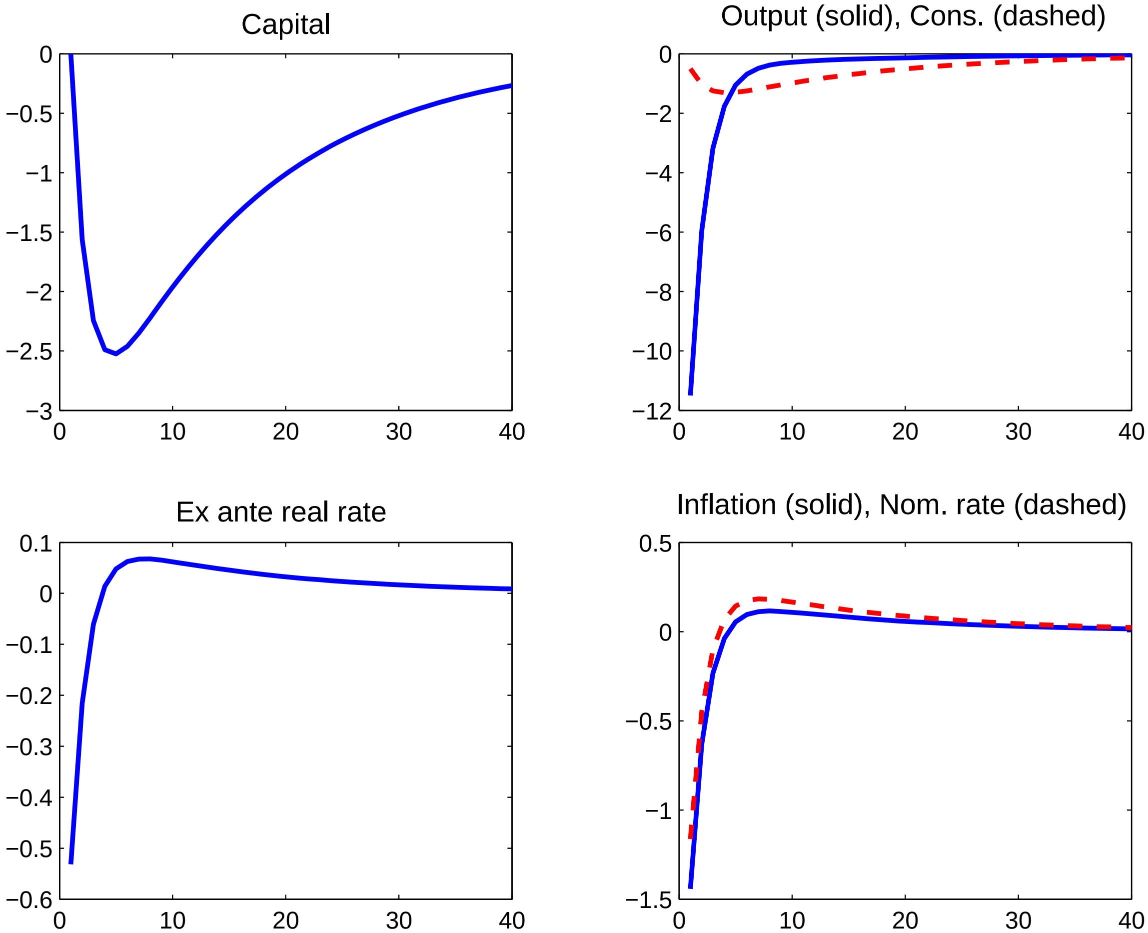

Figure 3: The model with capital, $\rho = 0.5$. Responses to 1 percentage point increase in ξ_t. The remaining parameterization is: $\beta = 0.99$, $\eta = 1$, $\theta = 0.7$, $\varepsilon = 0.83$, $\nu = 1.5$, $\alpha = 0.3$, and $\delta = 0.025$.

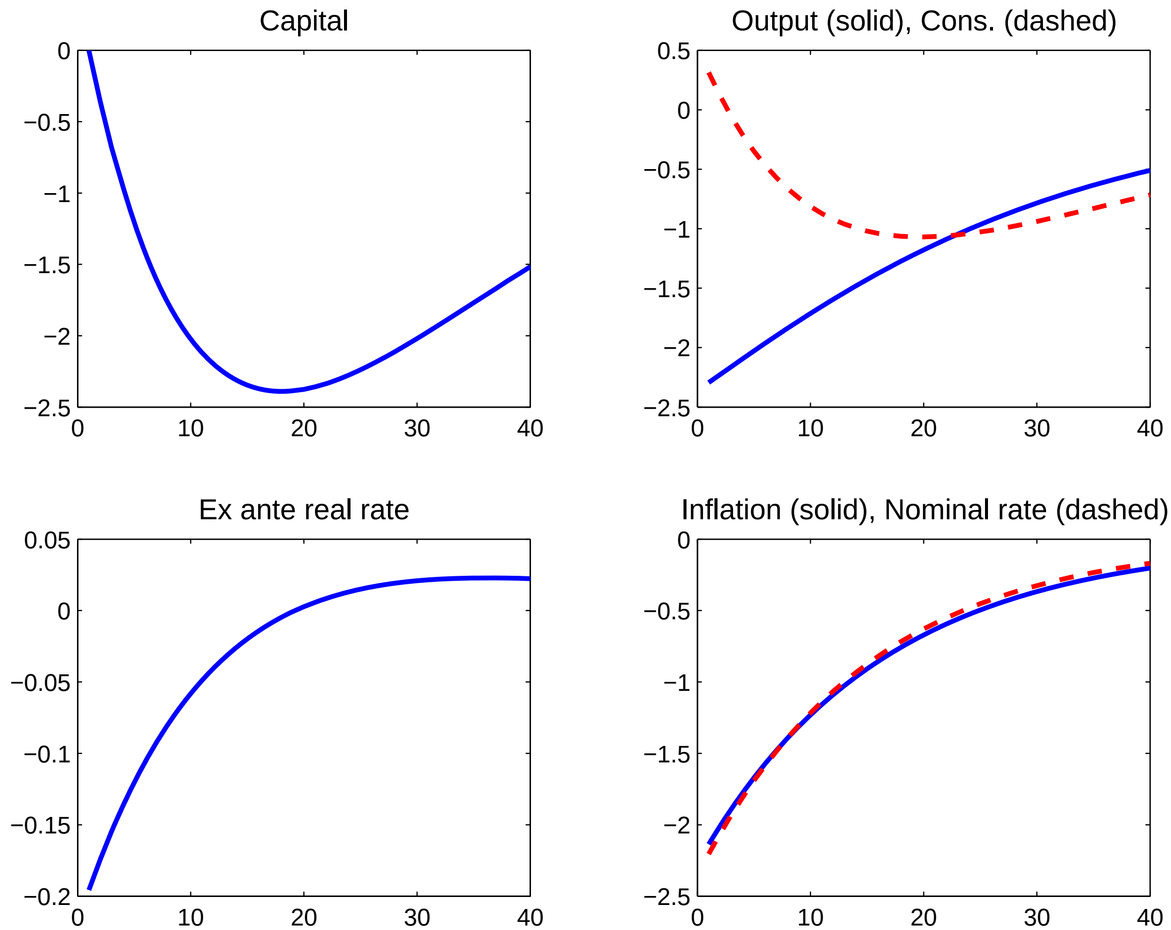

Figure 4: The model with capital, $\rho = 0.95$. Responses to 1 percentage point increase in ξ_t. The remaining parameterization is: $\beta = 0.99$, $\eta = 1$, $\theta = 0.7$, $\varepsilon = 0.83$, $\nu = 1.5$, $\alpha = 0.3$, and $\delta = 0.025$.

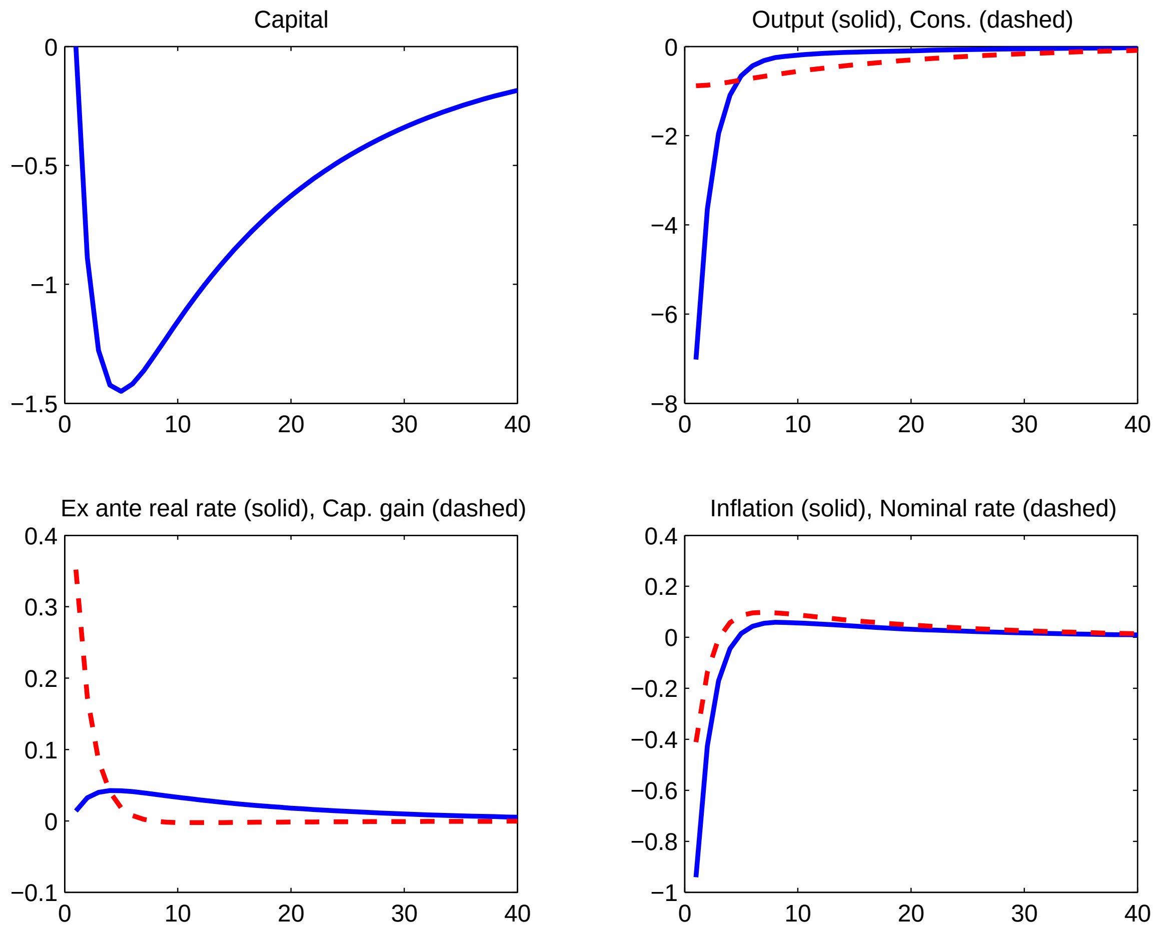

Figure 5: The model with capital adjustment costs, $\kappa = 0.1$. Responses to 1 percentage point increase in ξ_t. The remaining parameterization is: $\beta = 0.99$, $\eta = 1$, $\theta = 0.7$, $\varepsilon = 0.83$, $\nu = 1.5$, $\alpha = 0.3$, $\delta = 0.025$, and $\rho = 0.5$.

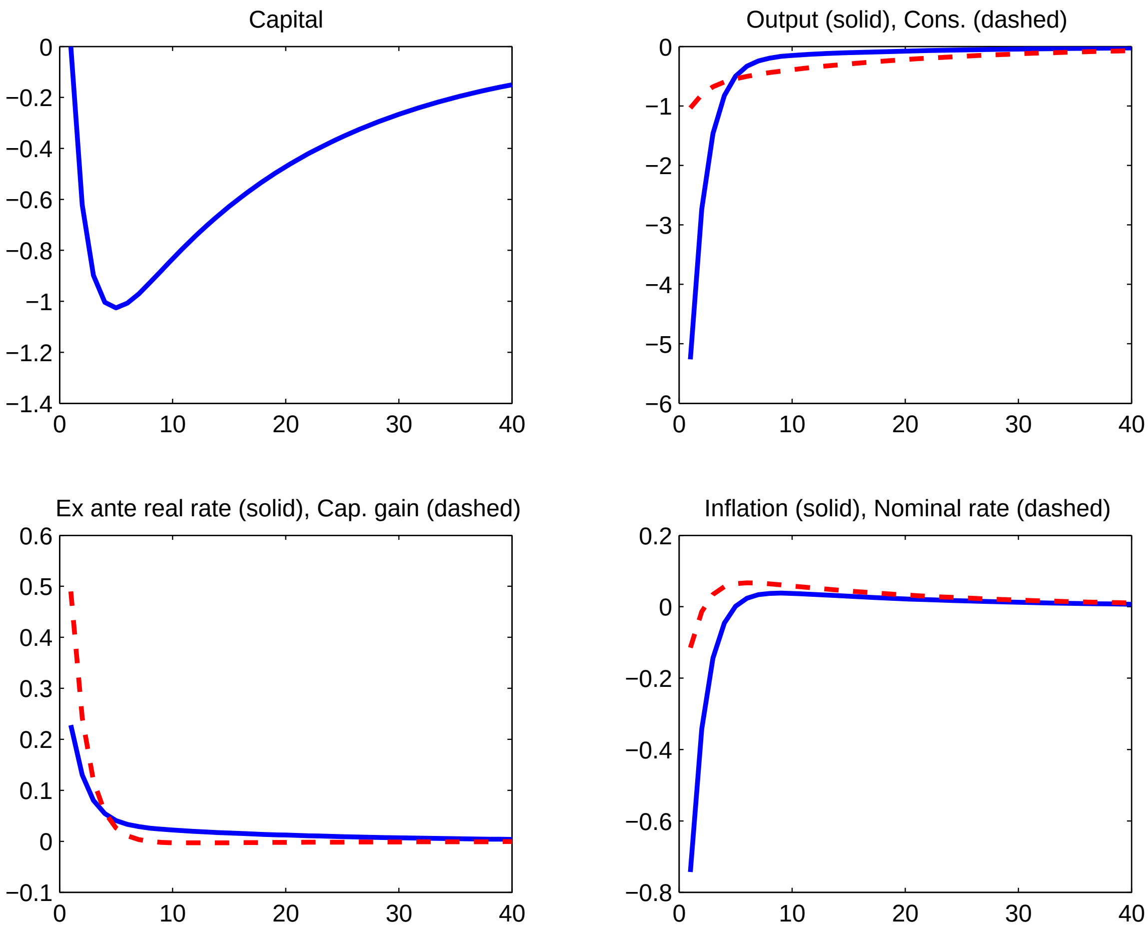

Figure 6: The model with capital adjustment costs, $\kappa = 0.2$. Responses to 1 percentage point increase in ξ_t. The remaining parameterization is: $\beta = 0.99$, $\eta = 1$, $\theta = 0.7$, $\varepsilon = 0.83$, $\nu = 1.5$, $\alpha = 0.3$, $\delta = 0.025$, and $\rho = 0.5$.

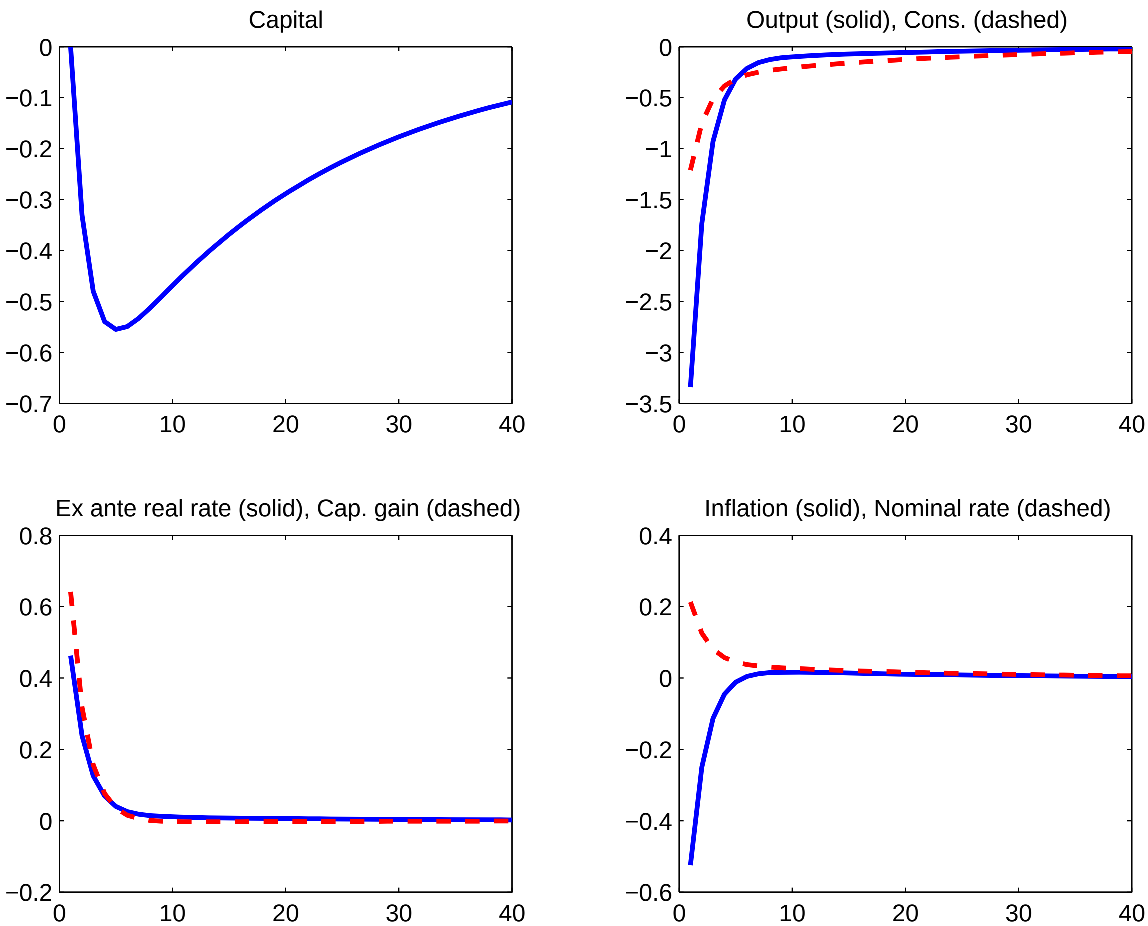

Figure 7: The model with capital adjustment costs, $\kappa = 0.5$. Responses to 1 percentage point increase in ξ_t. The remaining parameterization is: $\beta = 0.99$, $\eta = 1$, $\theta = 0.7$, $\varepsilon = 0.83$, $\nu = 1.5$, $\alpha = 0.3$, $\delta = 0.025$, and $\rho = 0.5$.

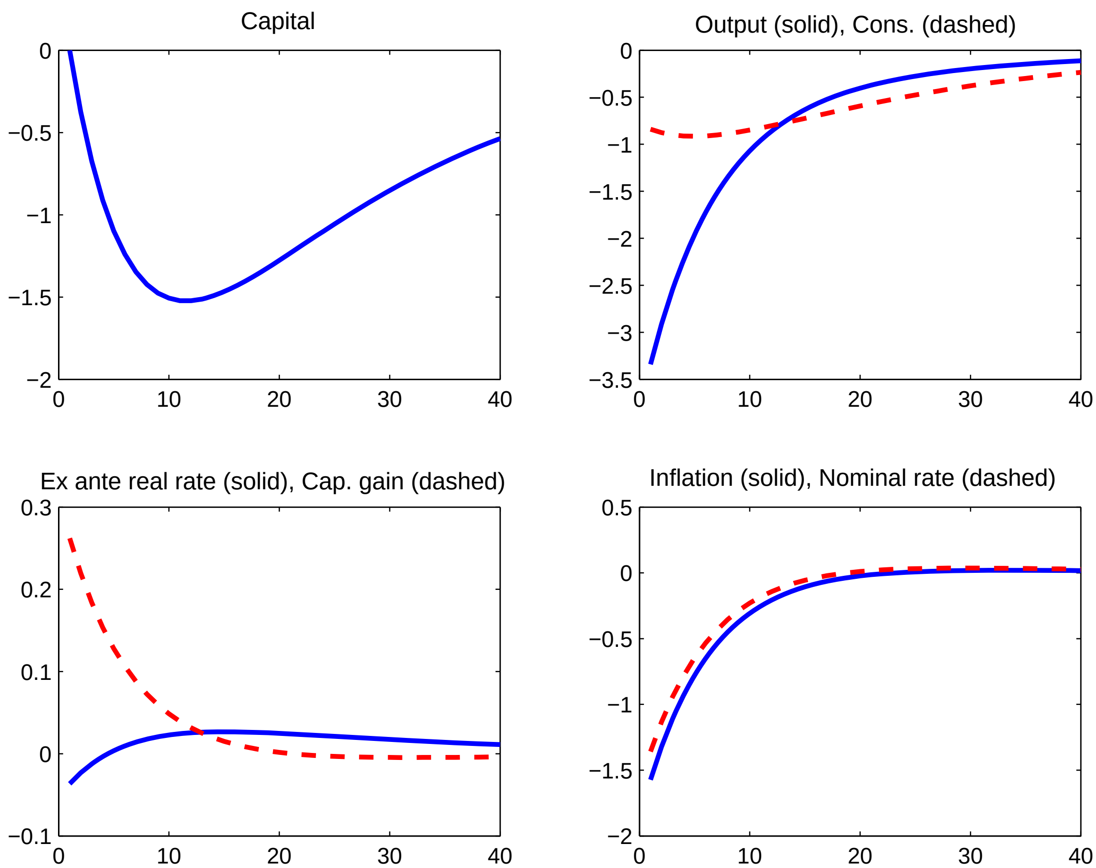

Figure 8: The model with capital adjustment costs, $\kappa = 0.5$, but higher shock persistence, $\rho = 0.85$. Responses to 1 percentage point increase in ξ_t. The remaining parameterization is: $\beta = 0.99$, $\eta = 1$, $\theta = 0.7$, $\varepsilon = 0.83$, $\nu = 1.5$, $\alpha = 0.3$, and $\delta = 0.025$.