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Dual-Loop Adaptive Iterative Learning Control for a Timoshenko Beam With Output Constraint and Input Backlash

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***Abstract*—In this paper, vibration control and output constraint are considered for a Timoshenko beam system with input backlash and external disturbances. By integrating iterative learning control (ILC) into adaptive control, two dual-loop adaptive ILC schemes are proposed in the presence of the input backlash. Two observers are designed to estimate two bounded terms, which are divided from the backlash inputs. Based on the defined barrier composite energy function, all the signals are proved to be bounded in each iteration. Along the iteration axis: 1) the endpoint transverse displacements and the endpoint angle displacements are restrained; 2) the transverse vibrations and the rotation vibrations are suppressed to zero; and 3) the spatiotemporally varying disturbance and the time-varying disturbances are rejected. Simulations are provided to manifest the effectiveness of the proposed control laws.**

***Index Terms*—Adaptive control, distributed parameter system, disturbance rejection, flexible structure, input backlash, iterative learning control (ILC), output constraint, vibration control.**

I. Introduction

LEARNING plays an essential role in autonomous control systems, including neural learning control [1]–[7], [46], [80], learning impendence control [8]–[10], and iterative learning control (ILC) [11], [12]. For the sake of simple structure and model-free nature, *P*-type ILC, PD-type ILC, PID-type ILC, etc., are widely used to track prescribed trajectories, suppress undesired vibrations, reject time-varying external disturbances, and tackle nonlinear inputs and outputs [13], [14]. In [11], an adaptive robust ILC law is proposed in the presence of input dead-zone for an ordinary differential equation (ODE) system. Based on the defined composite energy function, a *P*-type ILC scheme is proposed subject to the input saturation in [12], where the system state is regulated to track a certain time-varying trajectory from iteration to iteration. In [13], a PID-type adaptive ILC (AILC) law is proposed in the presence of input saturation for finite-dimensional systems. In [15], an AILC scheme with input saturation is designed to guarantee the convergence of the tracking error.

In engineering, backlash is frequently encountered in sensors and actuators, such as gearboxes, mechanical connections, and so on [16]. Different from input saturation and input dead-zone [17]–[20], the input backlash is nondifferentiable and dynamic nonlinear [21]. The input backlash may generate delays, vibrations, and even system paralysis. Therefore, it is meaningful to tackle the nonlinearities of the input backlash. Until now, there have been many papers addressing the input backlash through adaptive control. In [22], by estimating the bounded "disturbance-like" term of input backlash, vibrations are suppressed by employing adaptive control. In [23], by constructing an input backlash inverse, an adaptive control scheme is designed to asymptotically stabilize the target system. However, to the best of our knowledge, no paper proposes AILC to tackle the input backlash for a distributed parameter system.

In order to guarantee personal security and system performance, system states have to be bounded [24]–[27]. Otherwise, it is of possibility to give rise to undesired vibrations and even result in the system paralysis [28], [29]. Some control methodologies, including ILC, boundary control [30]–[35], adaptive control [32], [36]–[40], [81], [82], neural control [41]–[46], sliding mode control [47], [48], fuzzy control [49]–[51], switched control [52], [53], fault diagnosis method [54]–[57], etc., have been proposed for various systems [58], [59]. In [60] and [61], logarithmic functions are adopted in the defined Lyapunov function to asymptotically guarantee the output constraint.

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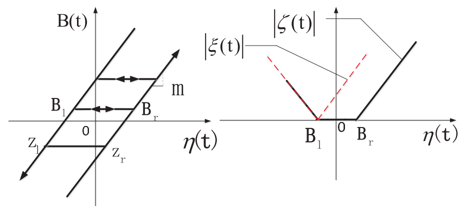

Fig. 1. Input backlash.

In this paper, the input backlash, the output constraint, and the external disturbances are considered in a Timoshenko beam system, which is presented by two second-order governing equations and four boundary equations. Similar to Euler–Bernoulli beam [62]–[69] and flexible string [70]–[73], the considered Timoshenko beam is sensitive to the external disturbances, making the vibration control indispensable. Observers are often constructed to tackle the uncertainties of the target system and the external disturbances [74]–[76]. In the presence of spatiotemporally varying disturbances, the target system is merely stabilized to be uniformly bounded along the time axis. In order to address such challenge, a dual-loop ILC method is utilized to form two controllers, which, respectively, contain a pure ILC loop and a pure adaptive boundary control loop. The remainder of this paper is organized as follows. Section II proposes the Timoshenko beam system model and perliminaries. In Section III, two dual-loop AILC laws are proposed based on the defined barrier composite energy function (BCEF). Section IV proves Theorems 1 and 2. Section V presents some simulation results. The main results are followed in Section VI.

II. Problem Formulation

In this section, the input backlash is divided into a linear input and an unknown bounded term, which is estimated by an observer. The Timoshenko beam system is described by a second-order distributed parameter system.

A. Input Backlash

As shown in Fig. 1, $B(t)$ is an input backlash [16], which is defined as

$$B(t) = B(\eta(t)) = \begin{cases} m(\eta(t) - B_l), & \text{if} \quad \eta(t) \le z_l \\ m(\eta(t) - B_r), & \text{if} \quad \eta(t) \ge z_r \\ B\big(t_{\text{pre}}\big), & \text{if} \quad z_l < \eta(t) < z_r \end{cases} \tag{1}$$

where m denotes the slope. $(B_l, 0)$ and $(B_r, 0)$ are two intersections on the horizontal axis. $B(t_{\text{pre}})$ represents the $B(t)$-axis value in the previous time.

z_l is the abscissas of the intersections of two lines $B(t) = m(\eta(t) - B_l)$ and $B(t) = B(t_{\text{pre}})$, which is expressed

$$z_l = \frac{B\big(t_{\text{pre}}\big)}{m} + B_l, \quad z_r = \frac{B\big(t_{\text{pre}}\big)}{m} + B_r. \tag{2}$$

Define $\xi(t) = \xi(\eta(t)) = m(\eta(t) - \min\{B_r, B_l\}) = m(\eta(t) - B_l)$ and then $B(t)$ is reconstructed as

$$B(t) = \xi(t) + d_b(t). \tag{3}$$

$d_b(t)$ is obtained as follows:

$$d_b(t) = d_b(\eta(t)) = \begin{cases} 0, & \text{if} \quad \eta(t) \le z_l \\ m(B_l - B_r), & \text{if} \quad \eta(t) \ge z_r \\ B\big(t_{\text{pre}}\big) - \xi(t), & \text{if} \quad z_l < \eta(t) < z_r \end{cases} \tag{4}$$

which implies $d_b(t)$ is bounded and unknown. A positive constant exists with $|d_b(t)| \le \bar{d}_b$.

We define $\zeta(t)$ as follows:

$$\zeta(t) = \begin{cases} m(\eta(t) - B_l), & \text{if} \quad \eta(t) \le B_l \\ m(\eta(t) - B_r), & \text{if} \quad \eta(t) \ge B_r \\ 0, & \text{if} \quad B_l < \eta(t) < B_r. \end{cases} \tag{5}$$

Then, we have $|\xi(t)| \ge |\zeta(t)|$.

Assumption 1: For the input backlash $B(t)$, $m > 0$, $B_r > 0$ and $B_l < 0$ are unknown and further $m_{\min} \le m \le m_{\max}$, $B_{r\min} \le B_r \le B_{r\max}$, and $B_{l\min} \le B_l \le B_{l\max}$, where $m_{\min}$, $m_{\max}$, $B_{r\min}$, $B_{r\max}$, $B_{l\min}$, and $B_{l\max}$ are unknown constants.

B. System Model

Let L and ρ represent the length and the unit mass per unit length of the Timoshenko beam. I_ρ denotes the uniform mass moment of inertia of the cross section of the Timoshenko beam. M represents the mass of the tip payload and J is the inertia of the tip payload. EI expresses the bending stiffness. $K = kAG$, where $k > 0$, A is the cross sectional area of the Timoshenko beam and G denotes the modulus of elasticity in shear.

Remark 1: Throughout this paper, we give the definitions such that $(*)' = (\partial(*)/\partial x)$, $(*)'' = (\partial^2(*)/\partial x^2)$, $(\dot{*}) = (\partial(*)/\partial t)$, and $(\ddot{*}) = (\partial^2(*)/\partial t^2)$.

As shown in [77], the Timoshenko beam system in jth iteration is described by the governing equations

$$I_\rho \ddot{\phi}_j(x,t) - EI\phi_j''(x,t) + K\Big[\phi_j(x,t) - w_j'(x,t)\Big] = 0 \tag{6}$$
$$\rho\ddot{w}_j(x,t) + K\Big[\phi_j'(x,t) - w_j''(x,t)\Big] = f_{jw}(x,t) \tag{7}$$
$$w_j(0,t) = 0 \tag{8}$$
$$\phi_j(0,t) = 0 \tag{9}$$
$$J\ddot{\phi}_j(L,t) + EI\phi_j'(L,t) = d_{j\phi}(t) + B\big(\tau_{j0}(t)\big) \tag{10}$$
$$M\ddot{w}_j(L,t) - K\Big[\phi_j(L,t) - w_j'(L,t)\Big] = d_{jw}(t) + B\big(u_{j0}(t)\big) \tag{11}$$

for $\forall t \in [0, T_b]$ and $j \in \mathbb{N}$. $B(\tau_{j0}(t))$ and $B(u_{j0}(t))$ are the backlash inputs and defined in (1). $w_j(x,t)$ and $\phi_j(x,t)$ describe the transverse displacement and the angle displacement for the position x, the time t and the iteration j. $f_{jw}(x,t)$, $d_{jw}(t)$, and $d_{j\phi}(t)$ express the external disturbances.

Considering (3), we can obtain $B(u_{j0}(t)) = \xi(u_{j0}(t)) + d(u_{j0}(t))$ and $B(\tau_{j0}(t)) = \xi(\tau_{j0}(t)) + d(\tau_{j0}(t))$. In order to make it easy to understand, define $\xi_{ju_0}(t) = \xi(u_{j0}(t))$, $\xi_{j\tau_0}(t) = \xi(\tau_{j0}(t))$, $d_{1j}(t) = d(u_{j0}(t))$, and $d_{2j}(t) = d(\tau_{j0}(t))$.

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Therefore, (10) and (11) can be rewritten as

$$J\ddot{\phi}_j(L,t) + EI\phi_j'(L,t) = d_{j\phi}(t) + \xi_{j\tau_0}(t) + d_{2j}(t) \tag{12}$$

$$M\ddot{w}_j(L,t) - K\Big[\phi_j(L,t) - w_j'(L,t)\Big] = d_{jw}(t) + \xi_{ju_0}(t) + d_{1j}(t) \tag{13}$$

$|d_{1j}(t)| \leq \bar{d}_1$ and $|d_{2j}(t)| \leq \bar{d}_2$, where $\bar{d}_1$ and $\bar{d}_2$ are two positive constants.

Remark 2: In this paper, $d_{1j}(t)$ and $d_{2j}(t)$ are separated from $B(u_{j0}(t))$ and $B(\tau_{j0}(t))$, respectively. $d_{jw}(t)$ and $d_{j\phi}(t)$ are external boundary disturbances. According to Assumption 3, the boundary disturbances are bounded with two known positive constants $\bar{d}_w$ and $\bar{d}_\phi$. However, $d_{1j}(t)$ and $d_{2j}(t)$ are bounded but unknown, as shown in Assumption 1. Therefore, two different ways are used to reject the external disturbances and to tackle the uncertainties of $d_{1j}(t)$ and $d_{2j}(t)$. For the unknown $d_{1j}(t)$ and $d_{2j}(t)$, two adaptive laws are designed in (16) and (18). For the boundary disturbances, $\alpha_2\bar{d}_w\text{sgn}(\dot{w}_j(L,t))$ and $\alpha_5\bar{d}_\phi\text{sgn}(\dot{\phi}_j(L,t))$ are adopted in the AILC laws (15) and (17), respectively.

For the Timoshenko beam system, some preliminaries are given to facilitate the subsequent context.

Property 1 [78]: If the kinetic energy of the Timoshenko beam system $E_{kj}(t) = (J/2)[\dot{\phi}_j(L,t)]^2 + (M/2)[\dot{w}_j(L,t)]^2 + (1/2)\int_0^L \rho[\dot{w}_j(x,t)]^2 + I_\rho[\dot{\phi}_j(x,t)]^2 dx$ is bounded for $\forall t \in [0, T_b]$ and $j \in \mathbb{N}$, we can then obtain $\dot{w}_j(x,t)$, $\dot{w}_j'(x,t)$, $\dot{\phi}_j(x,t)$, and $\dot{\phi}_j'(x,t)$ are bounded for $j \in \mathbb{N}$.

Property 2 [78]: If the potential energy of the Timoshenko beam system $E_{pj}(t) = (EI/2)\int_0^L [\phi_j'(x,t)]^2 dx + (K/2)\int_0^L [\phi_j(x,t) - w_j'(x,t)]^2 dx$ is bounded for $\forall t \in [0, T_b]$ and $j \in \mathbb{N}$, we can obtain $w_j(x,t)$, $w_j'(x,t)$, $w_j''(x,t)$, $\phi_j(x,t)$, $\phi_j'(x,t)$, and $\phi_j''(x,t)$ are bounded for $j \in \mathbb{N}$.

Assumption 2: For the Timoshenko beam system, the alignment condition is assumed, $w_j(x,0) = w_{j-1}(x,T_b)$, $\dot{w}_j(x,0) = \dot{w}_{j-1}(x,T_b)$, $\phi_j(x,0) = \phi_{j-1}(x,T_b)$, and $\dot{\phi}_j(x,0) = \dot{\phi}_{j-1}(x,T_b)$ for $\forall j \in \mathbb{N}$.

Assumption 3: Considering the finite energies of the boundary disturbances, there exist two known positive constants $\bar{d}_w$ and $\bar{d}_\phi$, satisfying $|d_{jw}(t)| \leq \bar{d}_w$ and $|d_{j\phi}(t)| \leq \bar{d}_\phi$ for $\forall t \in [0, T_b]$ and $j \in \mathcal{N}$.

Assumption 4: The distributed disturbance has the finite energy, and then a positive constant exists with $|f_{jw}(x,t)| \leq \bar{f}_w$ for $\forall (x,t) \in [0,L] \times [0,T_b]$ and $j \in \mathcal{N}$.

Assumption 5: For the Timoshenko beam system, we assume $\dot{w}_j(L,t) \not\equiv 0$ and $\dot{\phi}_j(L,t) \not\equiv 0$, including the special case $\dot{w}_0(L,0) \neq 0$ and $\dot{\phi}_0(L,0) \neq 0$.

Lemma 1 [79]: Let $\phi(x,t)$ be a function on $(x,t) \in [0,L] \times [0,+\infty)$ with $\phi(0,t) = 0$ for $t \in [0,+\infty)$. For $\forall x \in [0,L]$, we can obtain

$$[\phi(x,t)]^2 \leq L\int_0^L [\phi'(x,t)]^2 dx. \tag{14}$$

III. Control Design

In this section, two dual-loop AILC laws are designed in the presence of input backlash, aiming to suppress the undesired vibrations, reject the external disturbances and restrain the endpoint transverse displacement and the endpoint angle displacement.

The following AILC force is designed:

$$\begin{cases} \xi_{ju_0}(t) = \xi_{ju}(t) - \text{sgn}\big(\dot{w}_j(L,t)\big)\hat{d}_{1j}(t) - 2\alpha_1\bar{f}_w L \\ \qquad \times\ \text{sgn}\big(\dot{w}_j(L,t)\big) - \alpha_2\bar{d}_w\text{sgn}\big(\dot{w}_j(L,t)\big) \\ \qquad -\ \alpha_3\frac{w_j(L,t)}{C_1^2 - [w_j(L,t)]^2}\ln\left(\frac{C_1^2}{C_1^2 - [w_j(L,t)]^2}\right) \\ \xi_{ju}(t) = \zeta_{(j-1)u}(t) - \alpha_4\dot{w}_j(L,t) \end{cases} \tag{15}$$

where $\zeta_{(-1)u}(t) = 0$, $\alpha_1 > 0$, $\alpha_2 \geq 1$, $\alpha_3 > 0$, $\alpha_4 > 0$, and $C_1 > 0$. $\hat{d}_{1j}(t)$ is an observer to estimate the upper bound of $d_{1j}(t)$ and the estimation error is $\tilde{d}_{1j}(t) = \bar{d}_1 - \hat{d}_{1j}(t)$. The observer is designed as

$$\hat{d}_{1j}(t) = \hat{d}_{1(j-1)}(t) + \alpha_8\big|\dot{w}_j(L,t)\big| \tag{16}$$

where $\hat{d}_{1(-1)}(t) = 0$ and α_8 is a positive constant.

An AILC law is proposed

$$\begin{cases} \xi_{j\tau_0}(t) = \xi_{j\tau}(t) - \text{sgn}\big(\dot{\phi}_j(L,t)\big)\hat{d}_{2j}(t) - \alpha_5\bar{d}_\phi \\ \qquad \times\ \text{sgn}\big(\dot{\phi}_j(L,t)\big) - \alpha_6\frac{\phi_j(L,t)}{C_2^2 - [\phi_j(L,t)]^2} \\ \qquad \times\ \ln\left(\frac{C_2^2}{C_2^2 - [\phi_j(L,t)]^2}\right) \\ \xi_{j\tau}(t) = \zeta_{(j-1)\tau}(t) - \alpha_7\dot{\phi}_j(L,t) \end{cases} \tag{17}$$

where $\zeta_{(-1)\tau}(t) = 0$, $\alpha_5 \geq 1$, and α_6, α_7, and C_2 are positive constants.

In order to estimate the unknown term $\bar{d}_2$, an observer is designed as

$$\hat{d}_{2j}(t) = \hat{d}_{2(j-1)}(t) + \alpha_9\big|\dot{\phi}_j(L,t)\big| \tag{18}$$

where $\hat{d}_{2(-1)}(t) = 0$ and α_9 is a positive constant. Let $\tilde{d}_{2j}(t) = \bar{d}_2 - \hat{d}_{2j}(t)$ denote the estimation error.

Remark 3: The difficulties confronted in this paper are summarized as follows.

1) *How to Tackle the Input Backlash:* Input backlash has been addressed frequently by adaptive control for distributed parameter systems. The common way used to tackle the input backlash $u_0(t)$ is dividing into the linear input $u(t) = m\eta(t)$ and the bounded term $d(t)$, namely

$$d(t) = \begin{cases} -mB_l, & \text{if}\ \ \eta(t) \leq z_l \\ -mB_r, & \text{if}\ \ \eta(t) \geq z_r \\ B\big(t_{\text{pre}}\big) - m\eta(t), & \text{if}\ \ z_l < \eta(t) < z_r. \end{cases} \tag{19}$$

However, such common means is not directly applicable for the ILC methodology. Moreover, no works address the input backlash for distributed parameter systems.

2) *How to Reject the External Disturbances:* In practice, the Timoshenko beam system with the distributed disturbance and the boundary disturbance has been considered in many papers. Confronted with the vibration suppressing and the trajectory tracking, the closed-loop system is frequently stabilized not toward zero, but within a small interval of zero, as the time goes to infinity. In other words, it is difficult to obtain the exponential stability or asymptotic stability for the system under the distributed disturbance and the boundary disturbance. In the literature of ILC, time-varying disturbances have been rejected for ODE system, but a few works achieve the learning convergence under the spatiotemporally varying disturbances.

3) *How to Tackle the Output Constraints:* ILC is mostly designed to suppress the vibrations, tackling input saturation, and input dead-zone, rejecting periodic time-varying disturbances or constraining time-varying outputs. Confronted with complex objectives, including restraining output constraint, tackling nondifferentiable input, rejecting aperiodic distributed disturbances, and stabilizing the infinite-dimensional system, it is of large difficulty to propose an ILC law to effectively guarantee such requirements.
4) *How to Propose the AILC Laws:* Subject to a 3-D coordinate system of the space, the time and the iteration, a positive definite BCEF is defined with respect to time and iteration. AILC schemes are proposed to ensure that its derivative with respect to time is bounded in each iteration and its difference with respect to iteration is negative along the iteration axis. The closed-loop system with the designed AILC laws in each iteration is then proved to be bounded in the time domain and meanwhile converges to zero in the domain. It is cumbersome but important to find such proper BCEF and AILC schemes.

Remark 4: This paper mainly considers a second-order PDE system with input backlash, external boundary disturbances, external distributed disturbance, and output constraint. The contributions mainly include the following.

1) Comparing with common objectives in the ILC literature, such as convergence of ODE systems, rejection of time-varying disturbances, tackling the nonlinearities of input saturation and input dead-zone, etc., it is a novel challenge to extend nonlinear inputs to nondifferentiable input backlash and to extend time-varying disturbances to spatiotemporally varying disturbance.
2) Different from the common ILC scheme in the forms of *P*-type, *D*-type, PD-type, PID-type, etc., a dual-loop ILC law in this paper is utilized by integrating an ILC loop into an adaptive control loop.
3) For the Timoshenko beam system under the distributed disturbance and the boundary disturbance, rather than suppressing the vibrations into a neighborhood of zero, the designed AILC laws regulate the transverse displacements and rotate displacements to zero along the iteration axis.

Remark 5: In (15), the ILC loop is constructed with $\zeta_{(j-1)u}(t)$ and $\dot{w}_j(L,t)$. In (17), $\zeta_{(j-1)\tau}(t)$ and $\dot{\phi}_j(L,t)$ are used to form $\xi_{j\tau}(t)$. Such loops are the pure ILC laws, aiming to suppress the transverse vibrations and the rotation vibrations. As shown in Fig. 2, the pure ILC loop is represented by the red lines. The main loops are the pure adaptive boundary control laws, aiming to reject the disturbances, tackle the input backlash, and prevent the violation of the constraint. By adopting $\xi_{ju}(t)$ and $\xi_{j\tau}(t)$, the ILC loop is then embedded into the adaptive control loop.

IV. Convergence Analysis

In this section, the convergence is proved for the closed-loop system with the proposed AILC laws (15) and (17).

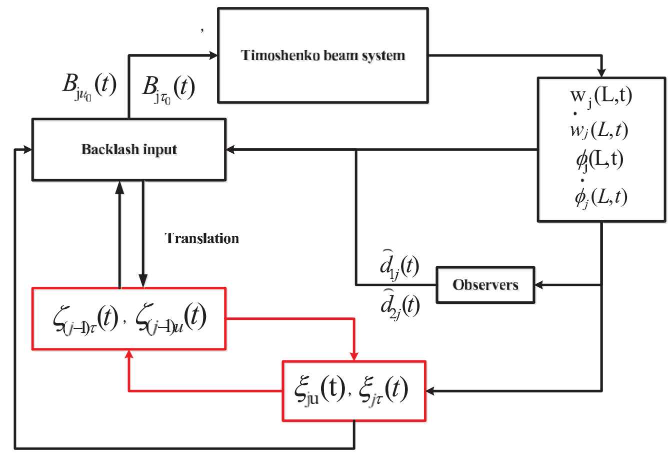

Fig. 2. Block diagram of the control design.

A BCEF is given as

$$E_j(t) = E_{1j}(t) + E_{2j}(t) + E_{3j}(t) + E_{4j}(t) + E_{5j}(t) \quad (20)$$

where $E_{1j}(t)$ and $E_{2j}(t)$ are relative to the system energy and are defined as follows:

$$E_{1j}(t) = \frac{\mu\rho}{2}\int_0^L e^{-\lambda t}\big[\dot{w}_j(x,t)\big]^2 dx + \frac{\mu I_p}{2}\int_0^L e^{-\lambda t} \times \big[\dot{\phi}_j(x,t)\big]^2 dx + \frac{\mu EI}{2}\int_0^L e^{-\lambda t}\big[\phi_j'(x,t)\big]^2 dx + \frac{\mu K}{2}\int_0^L e^{-\lambda t}\big[\phi_j(x,t) - w_j'(x,t)\big]^2 dx \quad (21)$$

$$E_{2j}(t) = \frac{\mu M}{2} e^{-\lambda t}\big[\dot{w}_j(L,t)\big]^2 + \frac{\mu J}{2} e^{-\lambda t}\big[\dot{\phi}_j(L,t)\big]^2 \quad (22)$$

where $\mu > 0$ and $\lambda > 0$.

To restrain the system outputs, including $w_j(L,t)$ and $\phi_j(L,t)$, $E_{3j}(t)$ is expressed by

$$E_{3j}(t) = \frac{\alpha_3\mu}{4} e^{-\lambda t}\left[\ln\frac{C_1^2}{C_1^2 - \big[w_j(L,t)\big]^2}\right]^2 + \frac{\alpha_6\mu}{4} e^{-\lambda t}\left[\ln\frac{C_2^2}{C_2^2 - \big[\phi_j(L,t)\big]^2}\right]^2. \quad (23)$$

In order to tackle the input backlash, $E_{4j}(t)$ and $E_{5j}(t)$ are defined

$$E_{4j}(t) = \frac{\mu}{2\alpha_4}\int_0^t e^{-\lambda r}\big[\xi_{ju}(r)\big]^2 dr + \frac{\mu}{2\alpha_7}\int_0^t e^{-\lambda r}\big[\xi_{j\tau}(r)\big]^2 dr \quad (24)$$

$$E_{5j}(t) = \frac{\mu}{2\alpha_8}\int_0^t e^{-\lambda r}\big[\tilde{d}_{1j}(r)\big]^2 dr + \frac{\mu}{2\alpha_9}\int_0^t e^{-\lambda r}\big[\tilde{d}_{2j}(r)\big]^2 dr. \quad (25)$$

Based on the defined BCEF, Theorems 1 and 2 are proved through the designed AILC schemes.

Theorem 1: For the Timoshenko beam system with the input backlash, at the initial time assuming all the signals are bounded, $|w_0(L,0)| \le C_1$ and $|\phi_0(L,0)| \le C_2$, by using Properties 1 and 2, Assumptions 1–5 and the proposed AILC laws (15) and (17), all the signals are proved to be bounded for $\forall t \in [0, T_b]$ in each iteration.

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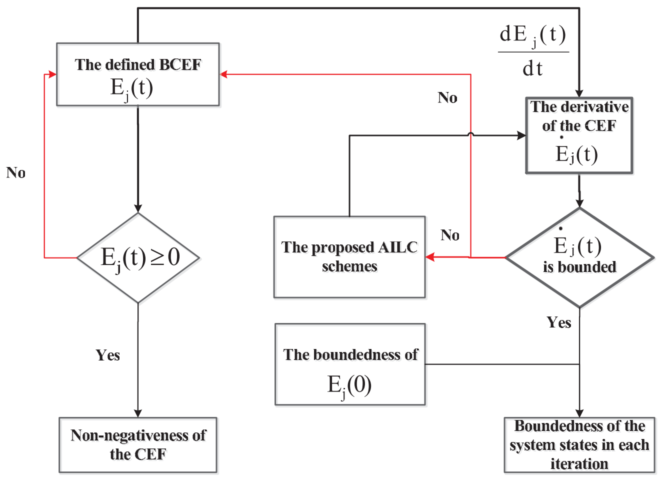

Fig. 3. Flow chart of how to prove Theorem 1.

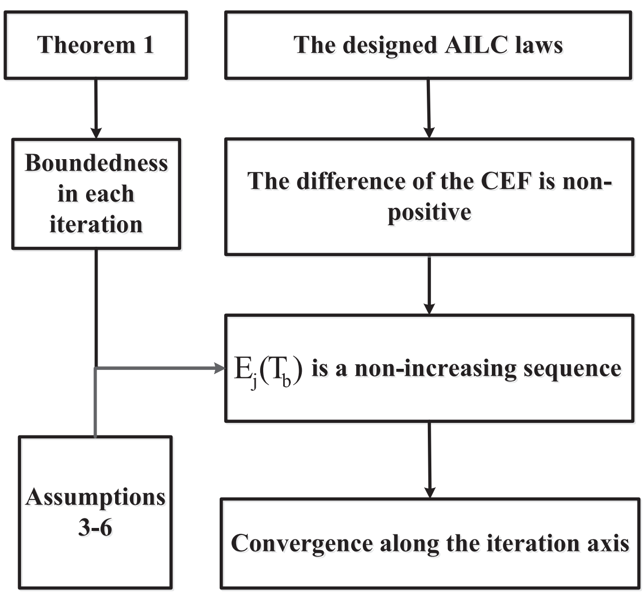

Fig. 4. Flow chart of how to prove Theorem 2.

Proof: Please see Appendix A. ■

Theorem 2: For the Timoshenko beam system with the input backlash, assuming $|w_0(L,0)| \leq C_1$ and $|\phi_0(L,0)| \leq C_2$ at the initial time, by using Properties 1 and 2, Assumptions 1–5, Theorem 1 and the proposed AILC laws (15) and (17), the following properties are proved.

1) The convergence of $w_j(x,t)$ and $\phi_j(x,t)$ are proved along the iteration axis.
2) $w_j(L,t)$ and $\phi_j(L,t)$ are restrained, namely, $|w_j(L,t)| < C_1$ and $|\phi_j(L,t)| < C_2$ for $\forall j \in \mathbb{N}$ and $t \in [0, T_b]$.
3) Through the designed AILC laws subject to the input backlash, the spatiotemporally varying disturbance is rejected from trail to trail, together with the boundary disturbances.

Proof: Please see Appendix B. ■

V. SIMULATION

Through the comparison of the performance without control and that with the AILC laws (15) and (17), the above theoretical conclusion is manifested and revealed. The system parameters of the Timoshenko beam system (see Table I) are chosen as follows:

1) Boundary output-feedback stabilization of a Timoshenko beam using disturbance observer.
2) Free vibrations of a stepped, spinning Timoshenko beam.

The external disturbances are chosen as

$$d_{jw}(t) = \frac{1}{10}[\cos((j+1)\pi t) + \cos(2(j+1)\pi t) + \cos(3(j+1)\pi t)] \tag{26}$$

$$d_{j\phi}(t) = \frac{1}{10}[\cos(2(j+1)\pi t) + \cos(4(j+1)\pi t) + \cos(6(j+1)\pi t)] \tag{27}$$

$$f_{jw}(x,t) = \frac{x}{20}[\sin((j+1)\pi xt) + \sin(2(j+1)\pi xt) + \sin(3(j+1)\pi xt)] \tag{28}$$

where $T_b = 2$ s, $j = \{0, 1, 2, \ldots, 23, 24\}$. The output constraints are chosen as $C_1 = 0.2$ m and $C_2 = 0.2$ rad. Let $\bar{d}_w = \bar{d}_\phi = 0.3$ N and $\bar{f}_w = 0.15$ N. The initial states are given as $w_0(x,0) = 0.16x$, $\phi_0(x,0) = 0.18x$, $\dot{w}_0(x,0) = 0.1$ and $\dot{\phi}_0(x,0) = 0.1$. The parameters of the input backlash are set as $m = 0.3$, $B_r = 0.1$, and $B_l = -0.2$. By employing the finite difference method, the continuous target systems (6)–(13) in each iteration is then discretized into a series of rectangular grids with the length $\Delta t = [T_b/(nt-1)]$ and the width $\Delta x = [L/(nx-1)]$, where $(x,t) \in [0,L]\times[0,T_b]$, $nt > 1$ and $nx > 1$. By changing the iteration number from $j = 0$ to $j = 24$, the discrete iteration is then intertwined with the 2-D system of space and time, which matches to the target system model (6)–(13).

When the control inputs are zero, Figs. 5–8 are used to describe the performance of the Timoshenko beam system with the external disturbances. In Figs. 5 and 6, $w(x,t)$ and $\phi(x,t)$ vibrate largely, in spite of the small initial states. Moreover, $w(L,t)$ exceeds the prescribed constraints $C_1 = 0.2$ m as shown in Fig. 7 and in Fig. 8 there is no convergence for $\phi(L,t)$.

TABLE I
PARAMETERS OF THE TIMOSHENKO BEAM SYSTEM

Parameter	Description	Value
ρ	Unit mass per unit length	1.00 kg/m
L	Length	1.00 m
M	Mass	0.1 kg
EI	Bending stiffness	35 N m^2
I_ρ	Mass moment of inertia	2 kg/m
J	Inertia	0.1 kg m^2
K	kAG	15 N

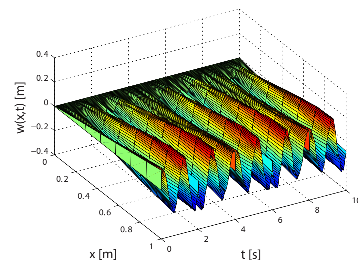

Fig. 5. $w(x,t)$ without control.

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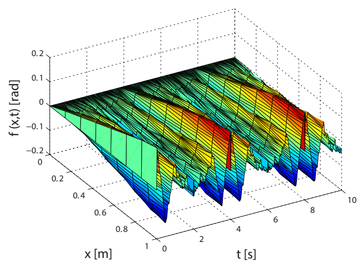

Fig. 6. $\phi(x, t)$ without control.

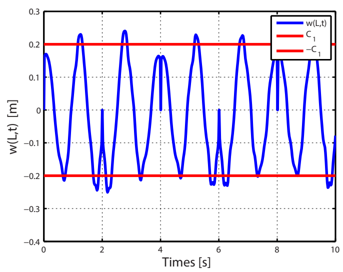

Fig. 7. $w(L, t)$ without control.

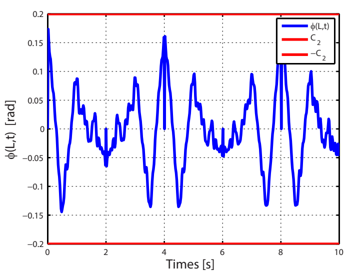

Fig. 8. $\phi(L, t)$ without control.

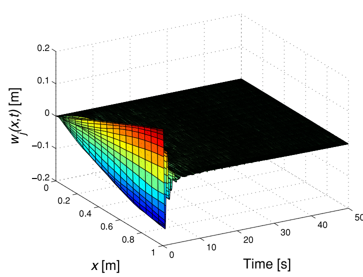

Fig. 9. $w_j(x, t)$ with the AILC laws.

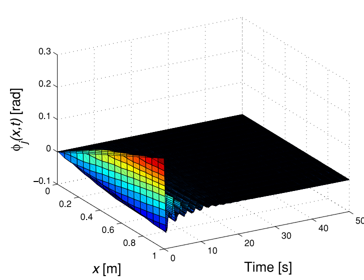

Fig. 10. $\phi_j(x, t)$ with the AILC laws.

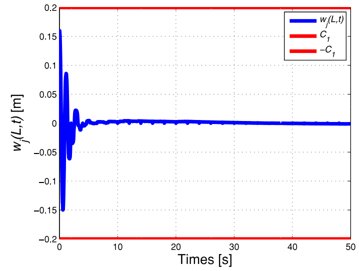

Fig. 11. $w_j(L, t)$ with the AILC laws.

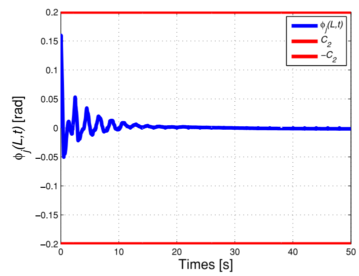

Fig. 12. $\phi_j(L, t)$ with the AILC laws.

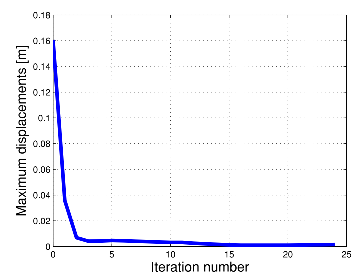

Fig. 13. $\max\{|w_j(x, t)|\}$ along the iteration axis.

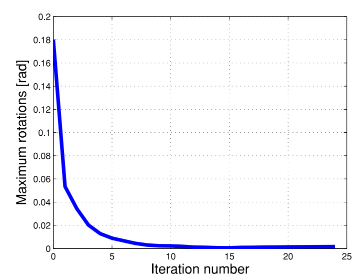

Fig. 14. $\max\{|\phi_j(x, t)|\}$ along the iteration axis.

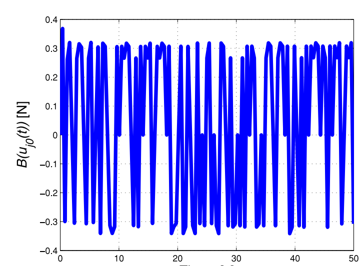

Fig. 15. AILC law $B(u_{j0}(t))$ in (15).

The proposed AILC laws (15) and (17) in the MATLAB code are conducted by choosing $\alpha_1 = 0.01$, $\alpha_2 = 1$, $\alpha_3 = 0.1$, $\alpha_4 = 0.05$, $\alpha_5 = 1$, $\alpha_6 = 0.1$ and $\alpha_7 = 5$, $\alpha_8 = 0.1$, and $\alpha_9 = 0.1$. Figs. 9–16 are used to present the effectiveness of (15) and (17) in suppressing the vibrations, restraining the endpoint displacements and rejecting the disturbances. In

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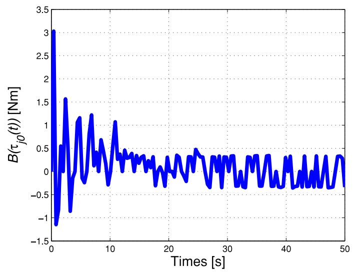

Fig. 16. AILC law $B(\tau_{j0}(t))$ in (17).

Fig. 9, through the dual-loop AILC laws, the transverse displacements are reduced and regulated to zero as time increases. From Fig. 10, the angle displacement $\phi_j(x,t)$ reduces and converges to zero within 50 s. As shown in Fig. 11, by designing the AILC laws (15) and (17), $w_j(L,t)$ is restrained during the whole simulation process, $|w_j(L,t)| < 0.2$ m. The endpoint angle displacement $\phi_j(L,t)$ is also constrained, namely, $|\phi_j(L,t)| < 0.2$ rad, as shown in Fig. 12. From Fig. 13, the maximal vibration of the transverse movement in each iteration is regulated to zero as the iteration increases. From Fig. 14, the maximum of $|\phi_j(x,t)|$ in each iteration is reduced to zero within 15 iterations. The backlash inputs $B(u_{j0}(t))$ and $B(\tau_{j0}(t))$ are bounded, as shown in Figs. 15 and 16.

VI. Conclusion

A Timoshenko beam system has been considered with external disturbances, the input backlash, and the output constraints. Two dual-loop AILC laws have been proposed based on the defined BCEF. By using the designed AILC schemes, the boundedness of all the signals has been proved in each iteration. Furthermore, the convergence of the transverse displacements and the angle displacements has been guaranteed along the iteration axis. In addition, the external disturbances have been rejected and the endpoint transverse displacements and the endpoint angle displacements have been restrained. The proved theoretical results have matched with the simulation results, which are manifested through a comparison of the target system with no control and with the designed AILC schemes.

Appendix A

Differentiating (21) and substituting (6) and (7), we have

$$\begin{aligned}\dot{E}_{1j}(t) =& -\frac{\mu\rho\lambda}{2}\int_0^L e^{-\lambda t}\big[\dot{w}_j(x,t)\big]^2dx - \frac{\mu\lambda I_p}{2}\\ &\times\int_0^L e^{-\lambda t}\big[\dot{\phi}_j(x,t)\big]^2dx - \frac{\mu\lambda EI}{2}\int_0^L e^{-\lambda t}\\ &\times\Big[\phi_j'(x,t)\Big]^2dx - \frac{\mu\lambda K}{2}\int_0^L e^{-\lambda t}\Big[\phi_j(x,t) - w_j'(x,t)\Big]^2dx\\ &+\mu\int_0^L e^{-\lambda t}\dot{w}_j(x,t)\times f_{jw}(x,t)dx\\ &+\mu EIe^{-\lambda t}\phi_j'(L,t)\dot{\phi}_j(L,t)\\ &-\mu Ke^{-\lambda t}\Big[\phi_j(L,t) - w_j'(L,t)\Big]\dot{w}_j(L,t). \end{aligned}\tag{29}$$

By considering (12), (13), (15), and (17), $\dot{E}_{2j}(t)$ is expressed by

$$\begin{aligned}\dot{E}_{2j}(t) \le& -\frac{\mu\lambda M}{2}e^{-\lambda t}\big[\dot{w}_j(L,t)\big]^2 - \frac{\mu\lambda J}{2}e^{-\lambda t}\big[\dot{\phi}_j(L,t)\big]^2\\ &+\mu e^{-\lambda t}\dot{w}_j(L,t)\Bigg[K[\phi_j(L,t) - w_j'(L,t)] + \xi_{ju}(t)\\ &+\operatorname{sgn}\big(\dot{w}_j(L,t)\big)\tilde{d}_{1j}(t)\\ &-2\alpha_1\bar{f}_wL\operatorname{sgn}\big(\dot{w}_j(L,t)\big)\\ &-\alpha_3\frac{w_j(L,t)}{C_1^2-\big[w_j(L,t)\big]^2}\\ &\times\ln\Bigg(\frac{C_1^2}{C_1^2-\big[w_j(L,t)\big]^2}\Bigg)\Bigg]\\ &+\mu e^{-\lambda t}\dot{\phi}_j(L,t)\Bigg[\xi_{j\tau}(t) + \operatorname{sgn}\big(\dot{\phi}_j(L,t)\big)\tilde{d}_{2j}(t)\\ &-EI\phi_j'(L,t) - \alpha_6\frac{\phi_j(L,t)}{C_2^2-\big[\phi_j(L,t)\big]^2}\\ &\times\ln\Bigg(\frac{C_2^2}{C_2^2-\big[\phi_j(L,t)\big]^2}\Bigg)\Bigg]. \end{aligned}\tag{30}$$

Taking the time derivative of $E_{3j}(t)$, we have

$$\begin{aligned}\dot{E}_{3j}(t) =& -\frac{\alpha_3\mu\lambda}{4}e^{-\lambda t}\Bigg[\ln\frac{C_1^2}{C_1^2-\big[w_j(L,t)\big]^2}\Bigg]^2 - \frac{\alpha_6\mu\lambda}{4}\\ &\times e^{-\lambda t}\Bigg[\ln\frac{C_2^2}{C_2^2-\big[\phi_j(L,t)\big]^2}\Bigg]^2 + \mu\alpha_3e^{-\lambda t}\\ &\times\frac{w_j(L,t)\dot{w}_j(L,t)}{C_1^2-\big[w_j(L,t)\big]^2}\ln\Bigg(\frac{C_1^2}{C_1^2-\big[w_j(L,t)\big]^2}\Bigg) + \mu\alpha_6\\ &\times e^{-\lambda t}\frac{\phi_j(L,t)\dot{\phi}_j(L,t)}{C_2^2-\big[\phi_j(L,t)\big]^2}\ln\Bigg(\frac{C_2^2}{C_2^2-\big[\phi_j(L,t)\big]^2}\Bigg). \end{aligned}\tag{31}$$

Substituting the designed AILC laws (15) and (17), $\dot{E}_{4j}(t)$ is obtained

$$\begin{aligned}\dot{E}_{4j}(t) =& \epsilon_{uj}(t) - \mu e^{-\lambda t}\xi_{ju}(t)\dot{w}_j(L,t) - \frac{\mu\alpha_4}{2}\\ &\times e^{-\lambda t}\big[\dot{w}_j(L,t)\big]^2 - \mu e^{-\lambda t}\xi_{j\tau}(t)\dot{\phi}_j(L,t)\\ &-\frac{\mu\alpha_7}{2}e^{-\lambda t}\big[\dot{\phi}_j(L,t)\big]^2 \end{aligned}\tag{32}$$

where $\epsilon_{uj} = (\mu/2\alpha_4)e^{-\lambda t}[\zeta_{(j-1)u}(t)]^2 + (\mu/2\alpha_7)e^{-\lambda t}[\zeta_{(j-1)\tau}(t)]^2$.

Substituting the proposed observers (16) and (18), we can obtain

$$\begin{aligned}\dot{E}_{5j}(t) =& \frac{\mu}{2\alpha_8}e^{-\lambda t}\Big[\tilde{d}_{1j}(t)\Big]^2 + \frac{\mu}{2\alpha_9}e^{-\lambda t}\Big[\tilde{d}_{2j}(t)\Big]^2\\ =& \epsilon_{dj}(t) - \mu e^{-\lambda t}\tilde{d}_{1j}(t)\big|\dot{w}_j(L,t)\big| - \frac{\mu\alpha_8}{2}e^{-\lambda t}\\ &\times\big[\dot{w}_j(L,t)\big]^2 - \mu e^{-\lambda t}\tilde{d}_{2j}(t)\big|\dot{\phi}_j(L,t)\big|\\ &-\frac{\mu\alpha_9}{2}e^{-\lambda t}\big[\dot{\phi}_j(L,t)\big]^2 \end{aligned}\tag{33}$$

where

$$\epsilon_{dj} = (\mu/2\alpha_8)e^{-\lambda t}[\tilde{d}_{1(j-1)}(t)]^2 + (\mu/2\alpha_9)e^{-\lambda t}[\tilde{d}_{2(j-1)}(t)]^2.$$

By substituting (29)–(33), the time derivative of $E_j(t)$ is obtained

$$\begin{aligned}\dot{E}_j(t) \leq\ & \epsilon_{uj}(t) + \epsilon_{dj}(t) + \frac{\mu \bar{f}_w^2 L}{\delta_1} - \left(\frac{\mu\rho\lambda}{2} - \mu\delta_1\right) \\ & \times \int_0^L e^{-\lambda t}\big[\dot{w}_j(x,t)\big]^2 dx - \frac{\mu\lambda I_p}{2}\int_0^L e^{-\lambda t} \\ & \times \big[\dot{\phi}_j(x,t)\big]^2 dx - \frac{\mu\lambda EI}{2}\int_0^L e^{-\lambda t}\Big[\phi_j'(x,t)\Big]^2 dx \\ & - \frac{\mu\lambda K}{2}\int_0^L e^{-\lambda t}\Big[\phi_j(x,t) - w_j'(x,t)\Big]^2 dx \\ & - \left(\frac{\mu\alpha_8}{2} + \frac{\mu\alpha_4}{2} + \frac{\mu\lambda M}{2}\right)e^{-\lambda t}\big[\dot{w}_j(L,t)\big]^2 \\ & - \left(\frac{\mu\alpha_9}{2} + \frac{\mu\alpha_7}{2} + \frac{\mu\lambda J}{2}\right)e^{-\lambda t}\big[\dot{\phi}_j(L,t)\big]^2 \\ & - \frac{\alpha_3\mu\lambda}{2}e^{-\lambda t}\left[\ln\frac{C_1^2}{C_1^2 - \big[w_j(L,t)\big]^2}\right]^2 \\ & - \frac{\alpha_6\mu\lambda}{2}e^{-\lambda t}\left[\ln\frac{C_2^2}{C_2^2 - \big[\phi_j(L,t)\big]^2}\right]^2 \end{aligned} \tag{34}$$

where δ_1 is a positive constant with $(\mu\rho\lambda/2) - \mu\delta_1 > 0$. Therefore, we can conclude $\dot{E}_j(t) \leq \epsilon_{dj}(t) + \epsilon_{uj}(t) + (\mu\bar{f}_w^2 L/\delta_1)$. Considering $\zeta_{(-1)u}(t) = 0$, $\zeta_{(-1)\tau}(t) = 0$, $d_{1(-1)}(t) = 0$, $d_{2(-1)}(t) = 0$ and (34), we can obtain $E_0(t) \leq (\mu\bar{f}_w^2 LT_b/\delta_1)$ for $t \in [0, T_b]$. By utilizing Properties 1 and 2, then all the signals for $j = 0$ are bounded, including $\dot{w}_0(L,t)$ and $\dot{\phi}_0(L,t)$. Considering (15) and (17), we can thus obtain the boundedness of $\xi_{0u}(t)$, $\xi_{0\tau}(t)$, $d_{10}(t)$, and $d_{20}(t)$ for $\forall t \in [0, T_b]$.

Assume $\xi_{(j-1)u}(t)$, $\xi_{(j-1)\tau}(t)$, $d_{1(j-1)}(t)$, and $d_{2(j-1)}(t)$ are bounded for $\forall j \in \mathbb{N}$ and $t \in [0, T_b]$, namely, $|\xi_{(j-1)u}(t)| \leq \bar{\xi}_u$, $|\xi_{(j-1)\tau}(t)| \leq \bar{\xi}_\tau$, $|d_{1(j-1)}(t)| \leq \bar{d}_1$ and $|d_{2(j-1)}(t)| \leq \bar{d}_2$, where $\bar{\xi}_u$, $\bar{\xi}_\tau$, $\bar{d}_1$, and $\bar{d}_2$ are positive constants. Considering (1)–(4), two positive constants exist with $|\zeta_{(j-1)u}(t)| \leq \bar{\zeta}_u$ and $|\zeta_{(j-1)\tau}(t)| \leq \bar{\zeta}_\tau$. Define

$$\bar{\epsilon}_j = \frac{\mu\bar{d}_2^2}{2\alpha_9} + \frac{\mu\bar{d}_1^2}{2\alpha_8} + \frac{\mu\bar{\zeta}_u^2}{2\alpha_4} + \frac{\mu\bar{\zeta}_\tau^2}{2\alpha_7} + \frac{\mu\bar{f}_w^2 L}{\delta_1} \tag{35}$$

$$\nu = \min\left\{\frac{\rho\lambda - 2\delta_1}{\rho}, \lambda, \frac{\lambda M + \alpha_4 + \alpha_8}{M}, \frac{\alpha_9 + \alpha_7 + \lambda J}{J}\right\} > 0. \tag{36}$$

Then, we have $\dot{A}_{Ej}(t) \leq \bar{\epsilon}_j - \nu A_{Ej}(t)$, where $A_{Ej}(t) = E_{1j}(t) + E_{2j}(t)$. Furthermore, we can obtain

$$A_{Ej}(t) \leq A_{Ej}(0)e^{-\nu t} + \frac{\bar{\epsilon}_j}{\nu}. \tag{37}$$

Considering (21), (22), and (37), we can obtain

$$\int_0^L e^{-\lambda t}\big[\dot{w}_j(x,t)\big]^2 dx \leq \frac{2}{\mu\rho}\left[A_{Ej}(0)e^{-\nu t} + \frac{\bar{\epsilon}_j}{\nu}\right] \tag{38}$$

$$\int_0^L e^{-\lambda t}\big[\dot{\phi}_j(x,t)\big]^2 dx \leq \frac{2}{\mu I_p}\left[A_{Ej}(0)e^{-\nu t} + \frac{\bar{\epsilon}_j}{\nu}\right] \tag{39}$$

$$e^{-\lambda t}\big[\dot{w}_j(L,t)\big]^2 \leq \frac{2}{\mu M}\left[A_{Ej}(0)e^{-\nu t} + \frac{\bar{\epsilon}_j}{\nu}\right] \tag{40}$$

$$e^{-\lambda t}\big[\dot{\phi}_j(L,t)\big]^2 \leq \frac{2}{\mu J}\left[A_{Ej}(0)e^{-\nu t} + \frac{\bar{\epsilon}_j}{\nu}\right] \tag{41}$$

which implies the boundedness of the kinetic energy of the closed-loop system. By using Property 1, all the signals are thus bounded in each iteration, including $\dot{w}_j(x,t)$, $\dot{w}_j'(x,t)$, $\dot{\phi}_j(x,t)$, and $\dot{\phi}_j'(x,t)$ for $\forall(x,t) \in [0,L] \times [0,T_b]$ and $\forall j \in \mathbb{N}$. Considering (1)–(4) and (15)–(18), we can further obtain $\zeta_{ju}(t)$, $\zeta_{j\tau}(t)$, $d_{1j}(t)$, and $d_{2j}(t)$ are bounded for $t \in [0, T_b]$.

Besides, we can also obtain

$$\int_0^L e^{-\lambda t}\Big[\phi_j'(x,t)\Big]^2 dx \leq A_{Ej}(0)e^{-\nu t} + \frac{\bar{\epsilon}_j}{\nu} \times \frac{2}{\mu EI} \tag{42}$$

$$\int_0^L e^{-\lambda t}\Big[\phi_j(x,t) - w_j'(x,t)\Big]^2 dx \leq A_{Ej}(0)e^{-\nu t} + \frac{\bar{\epsilon}_j}{\nu} \times \frac{2}{\mu K}. \tag{43}$$

Then, the potential energy is bounded, $\forall j \in \mathbb{N}$ and $t \in [0, T_b]$. By using Property 2, $w_j(x,t)$, $w_j'(x,t)$, $w_j''(x,t)$, $\phi_j(x,t)$, $\phi_j'(x,t)$, and $\phi_j''(x,t)$ are bounded for $\forall(x,t) \in [0,L] \times [0,T_b]$ and $j \in \mathbb{N}$.

By substituting the above steps repeatedly, in each iteration the boundedness of all the system states are proved through the designed AILC laws (15)–(17) for $\forall t \in [0, T_b]$ and $j \in \mathbb{N}$.

Appendix B

Considering $|\xi_{ju}(t)| \geq |\zeta_{ju}(t)|$, $|\xi_{j\tau}(t)| \geq |\zeta_{j\tau}(t)|$, and substituting (15) and (17), $\Delta E_{4j}(t)$ is expressed by

$$\begin{aligned}\Delta E_{4j}(T_b) \leq\ & -\mu\int_0^{T_b} e^{-\lambda r}\xi_{ju}(r)\dot{w}_j(L,r)dr \\ & - \frac{\mu\alpha_4}{2}\int_0^{T_b} e^{-\lambda r}\big[\dot{w}_j(L,r)\big]^2 dr \\ & - \mu\int_0^{T_b} e^{-\lambda r}\xi_{j\tau}(r)\dot{\phi}_j(L,r)dr \\ & - \frac{\mu\alpha_7}{2}\int_0^{T_b} e^{-\lambda r}\big[\dot{\phi}_j(L,r)\big]^2 dr. \end{aligned} \tag{44}$$

By substituting (16) and (18), we have

$$\begin{aligned}\Delta E_{5j}(T_b) =\ & -\mu\int_0^{T_b} e^{-\lambda r}\tilde{d}_{1j}(r)\big|\dot{w}_j(L,r)\big|dr \\ & - \frac{\mu\alpha_8}{2}\int_0^{T_b} e^{-\lambda r}\big[\dot{w}_j(L,r)\big]^2 dr \\ & - \mu\int_0^{T_b} e^{-\lambda r}\tilde{d}_{2j}(r)\big|\dot{\phi}_j(L,r)\big|dr \\ & - \frac{\mu\alpha_9}{2}\int_0^{T_b} e^{-\lambda r}\big[\dot{\phi}_j(L,r)\big]^2 dr. \end{aligned} \tag{45}$$

By using Assumption 2, the difference of $E_j(T_b)$ is constructed by

$$\begin{aligned}\Delta E_j(T_b) =\ & \int_0^{T_b}\big[\dot{E}_{1j}(r) + \dot{E}_{2j}(r) + \dot{E}_{3j}(r)\big]dr \\ & + \Delta E_{4j}(T_b) + \Delta E_{5j}(T_b). \end{aligned} \tag{46}$$

Substituting (29)–(31), (44), and (45), we can obtain

$$\begin{aligned}E_j(T_b) =\ & \sum_{i=1}^{j}\Delta E_j(T_b) + E_0(T_b) \\ \leq\ & E_0(T_b) - \left(\frac{\mu\alpha_8}{2} + \frac{\mu\alpha_4}{2} + \frac{\mu\lambda M}{2}\right)\end{aligned}$$

$$\times \sum_{i=1}^{j} \int_0^{T_b} e^{-\lambda r}[\dot{w}_i(L,r)]^2 dr - \frac{\mu\rho\lambda}{2} \times \sum_{i=1}^{j} \int_0^{T_b} \left[\int_0^L e^{-\lambda r}[\dot{w}_i(x,r)]^2 dx\right] dr - \left(\frac{\mu\alpha_9}{2} + \frac{\mu\alpha_7}{2} + \frac{\mu\lambda J}{2}\right) \times \sum_{i=1}^{j} \int_0^{T_b} e^{-\lambda r}\big[\dot{\phi}_i(L,r)\big]^2 dr - \frac{\mu\lambda I_p}{2} \times \sum_{i=1}^{j} \int_0^{T_b} \left[\int_0^L e^{-\lambda r}\big[\dot{\phi}_i(x,r)\big]^2 dx\right] dr - \frac{\mu\lambda EI}{2} \sum_{i=1}^{j} \int_0^{T_b} \left[\int_0^L e^{-\lambda r}\big[\phi_i'(x,r)\big]^2 dx\right] dr - \frac{\mu\lambda K}{2} \sum_{i=1}^{j} \int_0^{T_b} \left[\int_0^L e^{-\lambda r}\big[\phi_i(x,r) - w_i'(x,r)\big]^2 dx\right] dr - \frac{\alpha_3\mu\lambda}{2} \times \sum_{i=1}^{j} \int_0^{T_b} e^{-\lambda r}\left[\ln \frac{C_1^2}{C_1^2 - [w_i(L,r)]^2}\right]^2 dr - \frac{\alpha_6\mu\lambda}{2} \int_0^{T_b} e^{-\lambda r}\left[\ln \frac{C_2^2}{C_2^2 - [\phi_i(L,r)]^2}\right]^2 dr + \mu \sum_{i=1}^{j} \int_0^{T_b} \int_0^L e^{-\lambda r}\dot{w}_i(x,r) f_{iw}(x,r) dx - \mu \sum_{i=1}^{j} \int_0^{T_b} 2\alpha_1 \bar{f}_w L e^{-\lambda r}|\dot{w}_i(L,r)| dr. \tag{47}$$

By using Theorem 1 and Assumptions 3–5, we can obtain

$$\sum_{i=1}^{j} \int_0^{T_b} 2\bar{f}_w L e^{-\lambda r}|\dot{w}_i(L,r)| dr > 0.$$

There must exist a positive constant α_1 satisfying

$$\sum_{i=1}^{j} \int_0^{T_b} \int_0^L e^{-\lambda r}\dot{w}_i(x,r) f_{iw}(x,r) dx \le \sum_{i=1}^{j} \int_0^{T_b} 2\alpha_1 \bar{f}_w L e^{-\lambda r}|\dot{w}_i(L,r)| dr.$$

Therefore, (47) is simplified as

$$E_j(T_b) \le E_0(T_b) - \frac{\mu\rho\lambda}{2} \sum_{i=1}^{j} \int_0^{T_b} \left[\int_0^L e^{-\lambda r}[\dot{w}_i(x,r)]^2 dx\right] dr - \frac{\mu\lambda I_p}{2} \sum_{i=1}^{j} \int_0^{T_b} \left[\int_0^L e^{-\lambda r}\big[\dot{\phi}_i(x,r)\big]^2 dx\right] dr - \frac{\mu\lambda EI}{2} \sum_{i=1}^{j} \int_0^{T_b} \left[\int_0^L e^{-\lambda r}\big[\phi_i'(x,r)\big]^2 dx\right] dr - \frac{\mu\lambda K}{2} \sum_{i=1}^{j} \int_0^{T_b} \left[\int_0^L e^{-\lambda r}\big[\phi_i(x,r) - w_i'(x,r)\big]^2 dx\right] dr - \left(\frac{\mu\alpha_8}{2} + \frac{\mu\alpha_4}{2} + \frac{\mu\lambda M}{2}\right) \times \sum_{i=1}^{j} \int_0^{T_b} e^{-\lambda r}[\dot{w}_i(L,r)]^2 dr - \left(\frac{\mu\alpha_9}{2} + \frac{\mu\alpha_7}{2} + \frac{\mu\lambda J}{2}\right) \sum_{i=1}^{j} \int_0^{T_b} e^{-\lambda r} \times \big[\dot{\phi}_i(L,r)\big]^2 dr - \frac{\alpha_3\mu\lambda}{2} \sum_{i=1}^{j} \int_0^{T_b} e^{-\lambda r} \times \left[\ln \frac{C_1^2}{C_1^2 - [w_i(L,r)]^2}\right]^2 dr - \frac{\alpha_6\mu\lambda}{2} \times \sum_{i=1}^{j} \int_0^{T_b} e^{-\lambda r}\left[\ln \frac{C_2^2}{C_2^2 - [\phi_i(L,r)]^2}\right]^2 dr. \tag{48}$$

Therefore, $E_j(T_b)$ is a nonincreasing sequence along the iteration axis. Considering (48) and the positiveness of $E_j(T_b)$, as $j \to +\infty$, $|\dot{w}_j(x,t)|$, $|\dot{\phi}_j(x,t)|$, $|\phi_j'(x,t)|$, $|\phi_j(x,t) - w_j'(x,t)|$, $|\dot{w}_j(L,t)|$, and $|\dot{\phi}_j(L,t)|$ converge to zero along the iteration axis.

By using Lemma 1, and considering (8) and (9), we have

$$\big[\phi_j(x,t)\big]^2 \le L \int_0^L \Big[\phi_j'(x,t)\Big]^2 dx \tag{49}$$

$$\big[w_j(x,t)\big]^2 \le L \int_0^L \Big[w_j'(x,t)\Big]^2 dx \tag{50}$$

which advises $\phi_j(x,t)$ also asymptotically converges to zero. Considering $\lim_{j\to+\infty} |\phi_j(x,t) - w_j'(x,t)| = 0$ and $\lim_{j\to+\infty} |\phi_j(x,t)| = 0$, we can further prove that $|w_j(x,t)|$ is suppressed toward zero from iteration to iteration.

Therefore, by proposing the AILC laws (15) and (17), the following control objectives are achieved along the iteration axis: 1) the vibrations in the transverse movement and the rotation are suppressed; 2) the output constraints of the endpoint transverse displacements and the endpoint angle displacements are guaranteed; and 3) the external disturbances are rejected.

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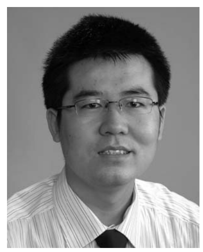

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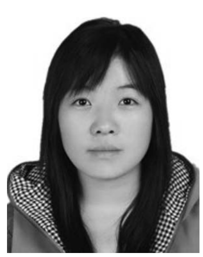

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