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Perturbative 3-charge microstate geometries in six dimensions

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Abstract: We construct a set of supersymmetric geometries that represent regular microstates of the D1-D5-P 3-charge system, using the solution generating technique of [1]. These solutions are constructed as perturbations around the maximally rotating D1-D5 solution at the linear order, and depend on the coordinate of S^1 on which the D1- and D5-branes are wrapped. In the framework of six-dimensional supergravity developed by Gutowski, Martelli and Reall [2], these solutions have a 4-dimensional base that depend on the S^1 coordinate v. The v-dependent base is expected of the superstratum solutions which are parametrized by arbitrary surfaces, and these solutions give a modest step toward their explicit construction.

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Contents

1 Introduction

The microphysics of black holes has been one of the most important subjects in string theory which purports to be a consistent theory of quantum gravity. Since the pioneering work of Strominger and Vafa [3] on the supersymmetric D1-D5-P black hole, much has been learned about the structures of black hole microstates.

The fuzzball conjecture [4–9] is about the gravitational description of the black hole microstates. The conjecture claims that black hole microstates are made of stringy/quantum gravity fuzz that extends over the horizon scale. The example for which this conjecture is actually true is the supersymmetric D1-D5 system (2-charge system). For this system, fuzzball microstates were explicitly constructed as smooth solutions in classical supergravity, known as microstate geometries [10, 11], which were shown [12] to correctly reproduce the asymptotic scaling of the entropy expected from microscopic computation. However, the 2-charge system is not really a black hole, the horizon area vanishing classically.

The supersymmetric D1-D5-P system (3-charge black hole) has a finite horizon and provides an ideal system in which to examine the fuzzball conjecture. The D1-D5-P system is obtained by compactifying type IIB string theory on $S^1 \times M_4$ with $M_4 = T^4$ or K3, wrapping N_1 D1-branes on S^1 and N_5 D5-branes on $S^1 \times M_4$, and putting N_p units of momentum along S^1. Even if the fuzzball conjecture is true, there is no a priori reason to expect that the black hole microstates are describable in classical supergravity as smooth solutions; they can be intrinsically stringy and have no supergravity description at all. Nonetheless, much effort has been made for constructing microstate geometries for this system within supergravity and, quite remarkably, many smooth solutions have been discovered.

In particular, a large family of smooth microstate geometries has been explicitly constructed within supergravity in [13, 14] (see also [15–17] for earlier work). This family can be characterized by the fact that they are independent of the compact S^1 coordinate which we call v. Actually, however, there is growing evidence that this family is far from the most generic microstates, even within supergravity. As we mentioned above, the D1-D5-P system has momentum charge along v, which can be naturally carried by traveling waves of the D1-D5 worldvolume depending on v and, therefore, the corresponding solution must be v-*dependent*. So, the family of v-independent solutions in [13, 14] must not be the most generic solutions. Also, in [18, 19], it was argued that placing supertubes in the throat region of v-independent solutions can enhance entropy. This also suggests that v-dependence is important for getting more generic solutions, because entropy of supertubes comes from v-dependent fluctuations of the worldvolume which, upon backreaction, turn v-independent background geometry into v-independent ones. Furthermore, it has been shown that the v-independent solutions are insufficient to account for the entropy of the D1-D5-P black hole [20].[1] For these reasons, it is worthwhile to look for v-dependent microstate solutions in supergravity in order to figure out whether the fuzzball conjecture applies to the D1-D5-P system or not. Considering non-trivial dependence on the S^1 coordinate v means that we must consider six-dimensional solutions.

Some v-dependent solutions of supergravity have already been constructed previously in the literature [1, 21–27] and were shown to represent smooth microstates of the D1-D5-P system. However, a systematic way to solve the relevant field equations in general has not been found yet. In this paper, we try to make a modest progress in this direction, by studying v-dependent solutions in the context of six-dimensional supergravity. The supersymmetric solutions of this theory have been classified in [2, 28] and, more recently, in ref. [29], the field equations that solutions should satisfy have been recast into a form in which a linear structure is manifest.[2,3] The solutions are constructed based on a four-dimensional almost hyperkähler base $\mathcal{B}$ which can generally depend on v. *If the base* $\mathcal{B}$ *is given* and the source distribution of branes is given, all one has to do in principle to obtain the backreacted solution is to solve the linear system of differential equations. However, the problem is that the base $\mathcal{B}$ must satisfy certain non-linear differential equations and we do not know how to solve them in general. Namely, we lack a systematic method to construct the base $\mathcal{B}$.

[1] Of course, it is fair to say that this might instead be evidence that generic microstates are not describable in supergravity.

[2] For recent applications of the linear structure for constructing supergravity solutions, see [30–33].

[3] Ref. [27] discusses embedding of supersymmetric solutions in a general class of 6D theory into 10D supergravity.

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Most of the v-dependent solutions constructed thus far [1, 21, 23–25, 27] have v-independent base $\mathcal{B}$ (note however the exceptions [22, 26] which we comment on below). So, it is important to work out more explicit examples of v-dependent base $\mathcal{B}$ in detail, and that is what we will do in this paper.

One may think that v-dependence of $\mathcal{B}$ may not be crucial for reproducing the correct scaling of the black hole entropy, just as for the D1-D5 system where fluctuations in the $\mathbb{R}^4$ directions were sufficient for the purpose of reproducing the entropy scaling and fluctuations in the T^4 directions [34] were not needed. However, for the D1-D5-P system, there is an argument based on the possibility of "double bubbling" that the v-dependence is essential for getting the right entropy scaling. This is a possibility that the D1-D5-P system undergoes supertube transition multiple times [35, 36] and its generic microstates are represented by a brane configuration with v-dependent worldvolume, dubbed the superstratum. This double bubbling picture is supported by a supersymmetry analysis [37]. If this is true, we will generically have a fluctuating distribution of KK monopoles [29, 37] which is described by a v-dependent base $\mathcal{B}$, and we need to take them into account to reproduce the entropy scaling.

In more detail, what we do in the current paper is to use the solution generating technique [1] to construct a solution with v-dependent base. In [1], they took the pure $\mathrm{AdS}_3\times S^3$ geometry which corresponds in boundary CFT to the NSNS ground state. Around that background, they considered small fluctuation of fields that corresponds to a chiral primary in CFT. On the fluctuation fields, they acted by a transformation which corresponds in the bulk to a rotation in S^3 and which corresponds on the boundary to an R-symmetry rotation. This transformation changes the linear and angular momenta carried by the fields. Being just a rotation, this transformation leaves smooth geometries smooth. After spectral flow to the RR sector, this procedure gives a solution that carries non-vanishing momentum charge. Although they obtained a v-dependent solution by this technique, their base was not v-dependent. In this paper, we consider more general fluctuations around $\mathrm{AdS}_3 \times S^3$ and apply their solution generating technique to obtain a v-dependent base.[4]

It is appropriate here to mention the difference between our solution and the solutions constructed in [22, 26] which also have v-dependent base. Ref. [22] discussed geometries obtained by the spectral flow of the Lunin-Mathur geometries [10, 11] and correspond to CFT states on the unitarity bound. On the other hand, our solution is above the unitarity bound and represent a different class of v-dependent solutions. Ref. [26] constructed supergravity solutions by computing perturbative open string amplitudes for certain brane bound states of the D1-D5 system as the boundary states. This worldsheet-based method has the advantage of being applicable to general boundary states but the regularity of resulting solutions is difficult to study. On the other hand, in our approach, the regularity of the solution is easier to analyze, although it is special to fluctuations around $\mathrm{AdS}_3 \times S^3$.

[4] Note that all we do is an S^3 rotation which is merely a coordinate transformation. So, in this sense, whether the base is v-dependent or not is just a matter of the coordinate system one uses. However, what is important is that this coordinate transformation does not vanish at the boundary of AdS_3. This means that this coordinate transformation generates genuinely new states in the CFT, and that is what is important for microstate counting.

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Some comments on the relevance of smooth geometries for black hole microstates are in order. First, it is possible that a solution which looks supersymmetric at the supergravity level may not be supersymmetric in full string theory [38, 39] (see also [40]). So, a given supergravity solution might not actually represent a microstate of the black hole in question. Second, the analysis of quiver quantum mechanics [41] representing multi-center black holes in 4D suggests that the black hole microstates may correspond to "pure Higgs" states with vanishing angular momentum, which is rather unnatural from the viewpoint of microstate geometries. Note that these two facts are *not* necessarily pointing toward the irrelevance of microstate geometries for the fuzzball conjecture; it may instead be completely opposite. Namely, it seems natural to interpret them as saying that microstate geometries are generally lifted by an amount invisible in supergravity except for ones with vanishing angular momentum. This would nicely explain the fact that the angular momentum of supergravity microstates is not restricted to zero whereas quiver quantum mechanics suggests that the truly supersymmetric states have vanishing angular momentum. Further investigations are needed to clarify the relevance of microstate geometries, including the ones constructed in the current paper, as the true microstates of the supersymmetric D1-D5-P black hole. In particular, v-dependent solutions are expected to play an important role.

The organization of the rest of the paper is as follows. In section 2, we review the supersymmetric solutions in the six-dimensional supergravity theory of our interest, and how they can be embedded in 10D supergravity. After reviewing the solution generating technique of [1] in section 3, we present the construction of the solution in section 4. We will only describe the outline and the result, referring to the appendix for details. In section 5, we discuss possible future directions.

2 Review of supersymmetric solutions in 6D

Here we review the supersymmetric solutions in 6D supergravity as presented in [29]. We will be brief here; for more details the reader is referred to [2, 28, 29].

The classification of supersymmetric solutions in 6D $\mathcal{N}=1$ supergravity was first done by Gutowski, Martelli, and Reall (GMR) [2] for minimal supergravity and later generalized in [28] to include vector multiplets. The supergravity theory we consider here is $\mathcal{N}=1$ theory with an anti-self-dual tensor multiplet [29], and its bosonic field content consists of the metric $g_{\mu\nu}$, an unconstrained 2-form B_2 with field strength $G=dB_2$, and a dilaton ϕ. The most general supersymmetric solutions for this theory have a null Killing direction u, of which all fields are independent. However, the fields can in general depend on the remaining five coordinates. Because null Killing vector introduces a $2+4$ split in the geometry, it is natural to introduce a second retarded time coordinate v and a four-dimensional, and generically v-dependent, spatial base $\mathcal{B}$ with coordinates x^m, $m=1,\dots,4$.

The six-dimensional metric is given by

$$ds_6^2 = 2H^{-1}(dv+\beta)\left[du+\omega+\frac{1}{2}\mathcal{F}(dv+\beta)\right] - ds_4^2, \tag{2.1}$$

where $H, \mathcal{F}$ are functions and β, ω are 1-forms in $\mathcal{B}$. $H, \mathcal{F}, \beta, \omega$ in general depend on v, x^m. The base $\mathcal{B}$ has the metric

$$ds_4^2 = h_{mn} dx^m dx^n \tag{2.2}$$

and equipped with almost hyperkähler structure 2-forms $J^{(A)}$, $A = 1, 2, 3$, which are anti-self-dual,

$$*_4 J^{(A)} = -J^{(A)}, \tag{2.3}$$

and satisfy the quaternionic relation

$$J^{(A)m}{}_n J^{(B)n}{}_l = -\delta^{AB} \delta_l^m + \epsilon^{ABC} J^{(C)m}{}_l, \qquad J^{(A)m}{}_n \equiv g^{ml} J_{ln}^{(A)}. \tag{2.4}$$

Here, $*_4$ is the Hodge star with respect to the four-dimensional metric (2.2). For our convention of differential forms and Hodge star, see appendix A. The 2-forms $J^{(A)}$ are not closed but its non-closure is related to β as

$$\tilde{d} J^{(A)} = \partial_v (\beta \wedge J^{(A)}), \tag{2.5}$$

with $\tilde{d}$ being the exterior derivative restricted to the base, $\tilde{d} = dx^m \partial_m$. The 1-form β must satisfy the condition

$$D\beta = *_4 D\beta \tag{2.6}$$

where

$$D \equiv \tilde{d} - \beta \wedge \partial_v \ . \tag{2.7}$$

We also introduce the 2-form

$$\widehat{\psi} \equiv \frac{1}{16} \epsilon^{ABC} J^{(A)ij} \dot{J}_{ij}^{(B)} J^{(C)}, \tag{2.8}$$

which measures the rotation of $J^{(A)}$ as v varies. Here, we defined $\dot{} \equiv \partial_v$.

Given the base $\mathcal{B}$ and the 1-form β satisfying the above equations, we can determine H, ω in the metric, the dilaton ϕ, and the flux $G = dB_2$ by solving a linear system as follows. We introduce functions Z_1, Z_2 by

$$Z_1 = He^{\sqrt{2}\phi}, \quad Z_2 = He^{-\sqrt{2}\phi}, \tag{2.9}$$

and 2-forms Θ_1, Θ_2. Then they satisfy the following linear equations:

$$\begin{aligned} D *_4 (DZ_1 + \dot{\beta} Z_1) &= -2\Theta_2 \wedge D\beta, \qquad \tilde{d}\Theta_2 = \partial_v \left[\frac{1}{2} *_4 (DZ_1 + \dot{\beta} Z_1) + \beta \wedge \Theta_2 \right], \\ D *_4 (DZ_2 + \dot{\beta} Z_2) &= -2\Theta_1 \wedge D\beta, \qquad \tilde{d}\Theta_1 = \partial_v \left[\frac{1}{2} *_4 (DZ_2 + \dot{\beta} Z_2) + \beta \wedge \Theta_1 \right] \end{aligned} \tag{2.10}$$

The $\Theta_{1,2}$ are not quite self-dual but the failure is related to $\widehat{\psi}$ as

$$*_4 \Theta_1 = \Theta_1 - 2Z_2 \widehat{\psi}, \qquad *_4 \Theta_2 = \Theta_2 - 2Z_1 \widehat{\psi}. \tag{2.11}$$

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Once Z, Θ are known, the field strength $G = dB_2$ is given by

$$G = d\left[-\frac{1}{2}Z_1^{-1}(du+\omega)\wedge(dv+\beta)\right] + \widehat{G}_1, \tag{2.12}$$

$$e^{2\sqrt{2}\phi} *_6 G = d\left[-\frac{1}{2}Z_2^{-1}(du+\omega)\wedge(dv+\beta)\right] + \widehat{G}_2, \tag{2.13}$$

where

$$\widehat{G}_1 \equiv \frac{1}{2} *_4 (D+\dot{\beta})Z_2 + (dv+\beta)\wedge\Theta_1, \tag{2.14}$$

$$\widehat{G}_2 \equiv \frac{1}{2} *_4 (D+\dot{\beta})Z_1 + (dv+\beta)\wedge\Theta_2. \tag{2.15}$$

The 1-form ω is found by solving the equation

$$(1+*_4)D\omega = 2(Z_1\Theta_1 + Z_2\Theta_2) - \mathcal{F}D\beta - 4Z_1Z_2\widehat{\psi}. \tag{2.16}$$

Finally, $\mathcal{F}$ is determined by

$$\begin{aligned} *_4 D *_4 L = \frac{1}{2}Hh^{ij}\partial_v^2(Hh_{ij}) + \frac{1}{4}\partial_v(Hh^{ij})\partial_v(Hh_{ij}) \\ - 2\dot{\beta}_i L^i + 2H^2\dot{\phi}^2 - 2 *_4 [\Theta_1\wedge\Theta_2 - \widehat{\psi}\wedge D\omega], \end{aligned} \tag{2.17}$$

where

$$L \equiv \dot{\omega} + \frac{1}{2}\mathcal{F}\dot{\beta} - \frac{1}{2}D\mathcal{F}. \tag{2.18}$$

We embed the above 6D theory into 10D type IIB supergravity as follows [34, 42] (note that embedding is not unique). We identify the 2-form B_2 with the RR 2-form potential C_2 and the 6D dilaton ϕ with the 10D dilaton Φ as

$$B_2 = \frac{1}{2}C_2, \qquad \phi = \frac{1}{\sqrt{2}}\Phi. \tag{2.19}$$

Then the relation of $G = dB_2$ to the RR 3-form flux $F_3 = dC_2$ and the dual $F_7 = *_{10}F_3$ is

$$G = \frac{1}{2}F_3 = \frac{1}{2}dC_2, \qquad e^{2\sqrt{2}\phi} *_6 G = \frac{1}{2}F_7|_6. \tag{2.20}$$

Here, $[\dots]|_6$ means to strip off the M_4 part of the differential form. Because $F_3 \propto G$ couples electrically to D1 and magnetically to D5, the first term $d[\dots]$ of G in (2.12) corresponds to D1(u,v) and the function Z_1 is the potential for it. The first term of $\widehat{G}_1$ in (2.14) corresponds to D5(u,v,M_4) and the second term in $\widehat{G}_1$ to D5(u,ψ,M_4) where ψ is some curve in $\mathcal{B}$. Inside $\mathcal{B}$, D5(u,ψ,M_4) is a 1-brane along ψ and we can measure its charge by integrating Θ_1 over a 2-surface going around it. From $e^{2\sqrt{2}\phi}*_6 G \propto F_7$, we can similarly read off charges, setting D1$\leftrightarrow$D5. Also, β_m, ω_m correspond to linear combinations of momentum charge along x^m and KK monopole charge along $x^m \times M_4$ with special circle v.

3 Solution generating technique

In this section, we review the solution generating technique by Mathur, Saxena, and Srivastava (MSS) [1], which allows one to construct a solution carrying momentum charge starting with a seed solution carrying no momentum charge.

The Lunin-Mathur (LM) geometry [10, 11] is a family of smooth geometries in 6D describing microstates of the D1-D5 system. They are parametrized by continuous functions $F_m(w)$ called the profile function which parametrizes the closed curve in $\mathbb{R}^4$ along which the D1-D5 worldvolume is extending.[5] They represent the ground states in the RR sector of the D1-D5 CFT and the dictionary between the geometries and CFT states is well established [42]. Expressed in the GMR form of section 2, the LM geometry is given by the following v-independent functions and forms [22]:

$$\begin{aligned}
&Z_1 = \frac{Q_5}{L}\int_0^L \frac{|\dot{\vec{F}}(w)|^2 dw}{|\vec{x}-\vec{F}(w)|^2}, \qquad Z_2 = \frac{Q_5}{L}\int_0^L \frac{dw}{|\vec{x}-\vec{F}(w)|^2},\\
&ds_4^2 = \delta_{mn}dx^m dx^n, \qquad \mathcal{F}=0, \qquad \beta = -\frac{A+B}{\sqrt{2}}, \qquad \omega = -\frac{A-B}{\sqrt{2}},\\
&A_m = -\frac{Q_5}{L}\int_0^L \frac{\dot{F}_m(w)dw}{|\vec{x}-\vec{F}(w)|^2}, \qquad dB = *_4 dA, \qquad \Theta_1=\Theta_2=\widehat{\psi}=0. \qquad (3.1)
\end{aligned}$$

where L is a constant defined in (B.1), Q_5 is the D5 charge proportional to N_5 (see (B.3)) and the D1 charge Q_1 is given in (B.2). The profile function satisfies the periodicity condition $F_m(w+L) = F_m(w)$. The RR 2-form C_2, which is related to the 2-form B_2 by (2.19), is given by

$$C_2 = -Z_1^{-1}(du+\omega)\wedge(dv+\beta)+\mathcal{C}_2, \qquad d\mathcal{C}_2 = *_4 dZ_2, \qquad (3.2)$$

which is nothing but (2.12), (2.14). Note that we dropped "1" in the harmonic functions $Z_{1,2}$ so that the above solution describes asymptotically AdS space. Extending our computation to asymptotically flat space would be interesting but we will not attempt to do it in this paper. See appendix B for more about the LM geometry.

In [1], MSS constructed a v-dependent 3-charge configurations by considering small fluctuations around maximally rotating LM geometry [43, 44]. This geometry is given by a circular profile function,

$$F_1 + iF_2 = ae^{i\omega w}, \qquad F_3=F_4=0, \qquad \omega = \frac{2\pi}{L}, \qquad a = \frac{\sqrt{Q_1 Q_5}}{R}, \qquad (3.3)$$

with R being the radius of S^1, and represents a particular RR ground state of the D1-D5 CFT with maximal possible R-charge. In this case, the GMR data (3.1) and (3.2) are

[5] We do not discuss the generalization for the profile function to describe fluctuations in the T^4 directions [34].

computed to be

$$\begin{aligned} Z_1 &= \frac{Q_1}{h}, \qquad Z_2 = \frac{Q_5}{h}, \qquad h \equiv r^2 + a^2\cos^2\theta, \\ A &= -\frac{\sqrt{Q_1Q_5}\, a\, \sin^2\theta}{h} d\phi, \qquad B = \frac{\sqrt{Q_1Q_5}\, a\, \cos^2\theta}{h} d\psi, \\ \mathcal{C}_2 &= -\frac{Q_5(r^2+a^2)\cos^2\theta}{h} d\phi\wedge d\psi. \end{aligned} \tag{3.4}$$

Here, we introduced the coordinates r, θ, ϕ, ψ by [11]

$$\begin{aligned} x^1 + ix^2 &= s e^{i\phi}, \qquad x^3 + ix^4 = w e^{i\psi}, \\ s &= \sqrt{r^2+a^2}\ \sin\theta, \qquad w = r\cos\theta, \\ s, w, r &\in [0,\infty), \qquad \phi,\psi \in [0, 2\pi), \qquad \theta \in \left[0, \frac{\pi}{2}\right], \end{aligned} \tag{3.5}$$

in terms of which the metric for the flat 4D base becomes

$$ds_4^2 = h\left(\frac{dr^2}{r^2+a^2} + d\theta^2\right) + (r^2+a^2)\sin^2\theta\, d\phi^2 + r^2\cos^2\theta\, d\psi^2. \tag{3.6}$$

By the spectral flow transformation of the CFT, this state can be mapped into the ground state in the NS-NS sector. In the bulk, the spectral flow corresponds to a simple coordinate transformation

$$\widetilde{\phi} = \phi - \frac{t}{R}, \qquad \widetilde{\psi} = \psi + \frac{y}{R}, \tag{3.7}$$

where

$$t = \frac{u+v}{\sqrt{2}}, \qquad y = \frac{u-v}{\sqrt{2}}. \tag{3.8}$$

One can show that this brings the 6D metric (2.1) into $\mathrm{AdS}_3 \times S^3$:

$$ds_6^2 = -ds^2_{\mathrm{AdS}_3} - \sqrt{Q_1Q_5}\, ds^2_{S^3}, \tag{3.9a}$$

$$ds^2_{\mathrm{AdS}_3} = \frac{1}{\sqrt{Q_1Q_5}}\left[-(r^2+a^2)dt^2 + r^2dy^2 + \frac{Q_1Q_5}{r^2+a^2}dr^2\right], \tag{3.9b}$$

$$ds^2_{S^3} = d\theta^2 + \sin^2\theta\, d\widetilde{\phi}^2 + \cos^2\theta\, d\widetilde{\psi}^2, \tag{3.9c}$$

$$C_2 = \frac{r^2+a^2}{Q_1}dt\wedge dy + \sqrt{\frac{Q_5}{Q_1}}\, a\, d\widetilde{\phi}\wedge dy - Q_5\cos^2\theta\, d\widetilde{\phi}\wedge d\widetilde{\psi}. \tag{3.9d}$$

Around this $\mathrm{AdS}_3 \times S^3$ background, MSS considered a fluctuation of the fields that corresponds to a chiral primary with

$$(h^{\mathrm{NS}}, j^{\mathrm{NS}}) = (k,k), \qquad (\bar{h}^{\mathrm{NS}}, \bar{\jmath}^{\mathrm{NS}}) = (k,k), \tag{3.10}$$

where $h, \bar{h}$ are the eigenvalues of the Virasoro generators $L_0, \bar{L}_0$ while $j, \bar{\jmath}$ are the eigenvalues of the $\mathrm{SU}(2) \times \widetilde{\mathrm{SU}(2)}$ R-symmetry generators $J_0^3, \bar{J}_0^3$. The subscript NS denotes the NS sector. The corresponding bulk fields can be worked out using the field equations of 6D supergravity. If one did the inverse spectral flow transformation to this state, the one would obtain an RR ground state which has less than maximal R-charge and no momentum charge. In order to generate a new solution, they instead acted by $(J_0^-)^{\rm NS}$ on the state (3.10) to get an NSNS state with

$$(h^{\rm NS}, j^{\rm NS}) = (k, k-1), \qquad (\bar{h}^{\rm NS}, \bar{j}^{\rm NS}) = (k, k), \tag{3.11}$$

and then did the inverse spectral flow. In the bulk, $(J_0^-)^{\rm NS}$ corresponds to one of the generators of the $\mathrm{SO}(4) = \mathrm{SU}(2) \times \widetilde{\mathrm{SU}(2)}$ rotation group of S^3 and is represented by a simple differential operator. So, it is easy to work out the fields corresponding to (3.11). After inverse spectral flow transformation[6]

$$h^{\rm R} = h^{\rm NS} - j^{\rm NS}, \qquad j^{\rm R} = j^{\rm NS}, \tag{3.12}$$

we end up with an RR state with

$$(h^{\rm R}, j^{\rm R}) = (1, k-1), \qquad (\bar{h}^{\rm R}, \bar{j}^{\rm R}) = (0, k), \tag{3.13}$$

which has non-vanishing momentum charge

$$N_p = h^{\rm R} - \bar{h}^{\rm R} = 1. \tag{3.14}$$

Being a simple SU(2) rotation of the original solution, this solution is guaranteed to be smooth and represents a microstate of the D1-D5-P system.

MSS studied particular chiral primaries which are represented in 6D supergravity [45][7] by fluctuations only of 6D dilaton and gauge fields but does not change the background metric from $\mathrm{AdS}_3 \times S^3$.[8] The latter fact greatly simplified their analysis but at the same time implies that, when recast in the GMR form, the solution has a v-independent base.

4 Construction of the v-dependent solution

In this section, we use the solution generating technique reviewed above to construct a 3-charge solution with v-dependent base $\mathcal{B}$. Here we will outline the main computations, followed by a summary of the results, relegating some details to appendix C.

4.1 The seed solution and spectral flow

We would like to use the solution generating technique of MSS reviewed above in order to obtain a solution with a v-dependent base. For that we need fluctuations more general

[6]Note that this is for the weight and R-charge of the perturbation, not including that of the background.

[7]There are different ways to embed the 6D fields into 10D fields, and they correspond to different chiral primaries. For particular ways to embed solutions in 6D supergravity into 10D supergravity, see e.g. [27, 34, 42].

[8]This is true only at the first order in the fluctuation. At higher order, the fields backreact on the metric and the background will change.

than was considered by MSS. Specifically, as the "seed", we take the following fluctuation of the LM profile,

$$\delta F_1 + i\,\delta F_2 = b e^{i(k+1)\omega w + i\alpha}, \qquad \delta F_3 = \delta F_4 = 0, \tag{4.1}$$

around the circular profile (3.3). Here, b is a small number and we will work only at the linear order in expansions in b. α is an arbitrary constant phase while

$$k \in \mathbb{Z}, \qquad k \leq -2 \quad \text{or} \quad 1 \leq k. \tag{4.2}$$

$k = -1$ is excluded because it would correspond to translating the entire profile, while $k = 0$ is excluded because it would correspond to changing the background radius a and change the D1 charge Q_1. The change in the GMR data, such as δZ_1, can be computed readily by plugging $F + \delta F$ into (3.1) and expanding it in the small parameter b (see (C.1)). Actually, it is more convenient to take a suitable linear combination of fluctuations with different phase α, which we are permitted to do in the linear approximation. Specifically, taking the linear combination $(\alpha = 0) + i(\alpha = -\pi/2)$, we find that the change in the GMR data is

$$\begin{aligned} \delta Z_1 &= 2Q_5 ab\,\omega^2 \left[a(sI_2(k+1) - aI_2(k)) + (k+1)I_1(k)\right] e^{ik\phi}, \\ \delta Z_2 &= 2Q_5 b\,(sI_2(k+1) - aI_2(k)) e^{ik\phi}, \\ \delta A &= Q_5 b\omega\,(-iX_-\,ds - sX_+\,d\phi) e^{ik\phi}, \qquad \delta B = 2Q_5 ab\omega w^2 e^{ik\phi} I_2(k) d\psi, \end{aligned} \tag{4.3}$$

where $I_n(k), X_\pm$ are defined in (C.3), (C.6). $\Theta_{1,2}, \mathcal{F}, \widehat{\psi}$ still vanish, because we are dealing with the LM geometry anyway.[9]

The GMR data (4.3) represent a small fluctuation around the maximally rotating LM geometry. This solution still belongs to the LM geometries (3.1) and therefore corresponds to a certain RR ground state of the D1-D5 CFT. To use the solution generating technique of MSS, let us do a spectral flow transformation to the NS sector, so that we have fluctuating fields around $\mathrm{AdS}_3 \times S^3$. To the zeroth order, the spectral flow transformation is implemented by the coordinate transformation (3.7) but, in the presence of the fluctuation on top, we have the freedom to do a further coordinate transformation at the same order in b. Let us use this freedom to bring the fluctuation of the metric into the canonical form of Deger et al. [45]. Concretely, we apply the following coordinate transformation[10]

$$\begin{aligned} \xi^\mu &= (\xi^t, \xi^y, \xi^r, \xi^\theta, \xi^{\widetilde{\phi}}, \xi^{\widetilde{\psi}}) \\ &= \frac{ba^{|k|} e^{ik(t/R + \widetilde{\phi})} \sin^{|k|}\theta}{(r^2 + a^2)^{|k|/2}} \left(\mp i \frac{\sqrt{Q_1 Q_5}}{r^2 + a^2}, 0, \frac{ar\sin^2\theta}{h}, \frac{a\sin\theta\cos\theta}{h}, 0, 0 \right), \\ g_{\mu\nu} &\to g_{\mu\nu} + \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu, \end{aligned} \tag{4.4}$$

[9]We can identify the fluctuation studied in MSS [1], which does not change the 6D metric, with a linear combination of the fluctuation (4.3). Specifically, if we denote the fields in (4.3) depending on k collectively by $F(k)$, then the fluctuation in [1] corresponds to $\frac{1}{2}(F(k) - F(-k)^*)$. In terms of the profile function $F_m(w)$, this is a "longitudinal" fluctuation that does not change the shape but only the parametrization. More precisely, one can show that it corresponds to $(F_1 + iF_2) + (\delta F_1 + i\delta F_2) = a\exp[i\omega(w + (b/a\omega)\sin(k\omega w))]$.

[10]Part of this coordinate transformation has been written down in [46]. This is a generalization so that the full 6D metric is in the form given in [45], not just the S^3 part.

where the $\mp$ signs correspond to $k \gtrless 0$, respectively. Then the change in the 6D metric, relative to the $\mathrm{AdS}_3 \times S^3$ metric (3.9), takes a rather simple form as follows:

$$\begin{aligned}\delta(ds_6^2) = (|k|+1)\, a^{|k|-1} b\, \widehat{B}\, \widehat{Y} \Bigg[&\frac{r^2 - a^2}{\sqrt{Q_1 Q_5}} dt^2 - \frac{r^2 dy^2}{\sqrt{Q_1 Q_5}} + \frac{\sqrt{Q_1 Q_5}(r^2 - a^2) dr^2}{(r^2 + a^2)^2} \\ &\mp \frac{4iar}{r^2 + a^2} dt dr + \sqrt{Q_1 Q_5}(d\theta^2 + \sin^2\theta d\widetilde{\phi}^2 + \cos^2\theta d\widetilde{\psi}^2) \Bigg], \end{aligned} \tag{4.5}$$

where

$$\widehat{B} \equiv \frac{e^{ikt/R}}{(r^2 + a^2)^{|k|/2}}, \qquad \widehat{Y} \equiv e^{ik\widetilde{\phi}} \sin^{|k|}\theta. \tag{4.6}$$

We can also find the change in dilaton to be

$$\delta\Phi = \sqrt{2}\,\delta\phi = (k+1) a^{|k|-1} b \widehat{B}\widehat{Y}. \tag{4.7}$$

Also, the change in the RR 2-form relative to (3.9) can be written in the canonical form of [45] as

$$\delta C_2 = \begin{cases} -\dfrac{2(k+1)a^{k-1}b}{Q_1 \omega} \widehat{B}\widehat{Y} \left[r^2 \omega\, dt \wedge dy + i Q_1 \dfrac{r\, dy \wedge dr}{r^2 + a^2} \right] & (k > 0), \\ -\dfrac{2a^{l-1}b}{Q_1\omega} \widehat{B}\widehat{Y} \Bigg[r^2 \omega\, dt \wedge dy - i Q_1 \dfrac{r\, dy \wedge dr}{r^2 + a^2} & \\ \qquad -ilQ_1Q_5\omega \cot\theta (d\theta - i \sin\theta\cos\theta d\widetilde{\phi}) \wedge d\widetilde{\psi} \Bigg] & (k = -l < 0). \end{cases} \tag{4.8}$$

See appendix C.2 for details.

4.2 SU(2) rotation

Now we would like to do a transformation to the fluctuation (4.5), (4.7), (4.8) to generate a new solution. The S^3 is parametrized by $\theta, \widetilde{\psi}, \widetilde{\phi}$, and its isometry group $\mathrm{SO}(4) = \mathrm{SU}(2) \times \widetilde{\mathrm{SU}(2)}$ is generated by[11]

$$\begin{aligned} J^{\pm} &= \frac{i}{2} e^{\pm i(\widetilde{\phi}+\widetilde{\psi})} (\mp i\partial_\theta + \cot\theta\, \partial_{\widetilde{\phi}} - \tan\theta\, \partial_{\widetilde{\psi}}), & J^3 &= -\frac{i}{2}(\partial_{\widetilde{\phi}} + \partial_{\widetilde{\psi}}), \\ \bar{J}^{\pm} &= \frac{i}{2} e^{\pm i(\widetilde{\phi}-\widetilde{\psi})} (\mp i\partial_\theta + \cot\theta\, \partial_{\widetilde{\phi}} + \tan\theta\, \partial_{\widetilde{\psi}}), & \bar{J}^3 &= -\frac{i}{2}(\partial_{\widetilde{\phi}} - \partial_{\widetilde{\psi}}). \end{aligned} \tag{4.9}$$

For $k > 0$, all the fluctuation fields (4.5), (4.7), (4.8) are proportional to the scalar spherical harmonic with the highest weight $(k, k; k, k)$ of $\mathrm{SU}(2) \times \widetilde{\mathrm{SU}(2)}$,

$$\widehat{Y} = e^{ik\widetilde{\phi}} \sin^k\theta, \qquad k > 0, \tag{4.10}$$

which is killed by $J^+, \bar{J}^+$. This means that the fluctuation fields have

$$(h^{\mathrm{NS}}, j^{\mathrm{NS}}) = (k, k), \qquad (\bar{h}^{\mathrm{NS}}, \bar{j}^{\mathrm{NS}}) = (k, k). \tag{4.11}$$

[11] The SO(4) generators $J^{mn} = -i(x_m\partial_n - x_n\partial_m)$, $m, n = 1, 2, 3, 4$ can be split into $\mathrm{SU}(2) \times \widetilde{\mathrm{SU}(2)}$ generators as $J^a = J_+^{a4}$, $\bar{J}^a = J_-^{a4}$, $a = 1, 2, 3$, where $J_\pm^{mn} = \frac{1}{2}(\tilde{J}^{mn} \pm J^{mn})$, $\tilde{J}^{mn} = \frac{1}{2}\epsilon_{mnpq} J^{pq}$.

Since the background preserves the $SU(2) \times \widetilde{SU(2)}$ symmetry, the above solution remains a solution even if we replace $\widehat{Y}$ with the $(k, k-m; k, k)$ state,

$$(J^-)^m \widehat{Y} \propto e^{i(k-m)\widetilde{\phi} - im\widetilde{\psi}} \sin^{k-m}\theta \cos^m \theta \equiv \widetilde{Y}, \tag{4.12}$$

which has

$$(h^{\mathrm{NS}}, j^{\mathrm{NS}}) = (k, k-m), \qquad (\bar{h}^{\mathrm{NS}}, \bar{\jmath}^{\mathrm{NS}}) = (k, k). \tag{4.13}$$

After this replacement $\widehat{Y} \to \widetilde{Y}$, we go back to the RR sector by the spectral flow transformation (3.7). (Note that we do not do a coordinate transformation similar to (4.4) before spectral flowing back.) The resulting configuration has

$$(h^{\mathrm{R}}, j^{\mathrm{R}}) = (m, k-m), \qquad (\bar{h}^{\mathrm{R}}, \bar{\jmath}^{\mathrm{R}}) = (0, k) \tag{4.14}$$

and therefore the momentum charge

$$N_p = h^{\mathrm{R}} - \bar{h}^{\mathrm{R}} = m. \tag{4.15}$$

The resulting fields can be rewritten in the GMR form, as summarized in the next subsection.

For $k = -l < 0$, on the other hand, the fields are proportional to

$$\widehat{Y} = e^{-il\widetilde{\phi}} \sin^l \theta, \qquad l > 0, \tag{4.16}$$

which is the lowest state $(l, -l; l, -l)$. The corresponding CFT charges are

$$(h^{\mathrm{NS}}, j^{\mathrm{NS}}) = (l, -l), \qquad (\bar{h}^{\mathrm{NS}}, \bar{\jmath}^{\mathrm{NS}}) = (l, -l). \tag{4.17}$$

Acting on the state by $(J^+)^n$, $n > 0$, we obtain the $(l, -(l-n); l, -l)$ state

$$\widetilde{Y} \propto (J^+)^n \widehat{Y} \propto e^{i(-l+n)\widetilde{\phi} + in\widetilde{\psi}} \sin^{l-n}\theta \cos^n \theta. \tag{4.18}$$

After inverse spectral flow, we end up with an RR state with

$$(h^{\mathrm{R}}, j^{\mathrm{R}}) = (2l - n, -l + n), \qquad (\bar{h}^{\mathrm{R}}, \bar{\jmath}^{\mathrm{R}}) = (2l, -l), \tag{4.19}$$

$$N_p = h^{\mathrm{R}} - \bar{h}^{\mathrm{R}} = -n. \tag{4.20}$$

The expression for $\widetilde{Y}$ that works for both $k > 0, k < 0$ is

$$\widetilde{Y} = e^{i(k-m)\widetilde{\phi} - im\widetilde{\psi}} \sin^{|k|-|m|}\theta \cos^{|m|}\theta, \tag{4.21}$$

where for $k < 0$ we take $m = -n < 0$. The value of m is restricted to $0 \leq |m| \leq |k|$.

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4.3 The v-dependent solution

As the result of the procedure outlined above, we obtain the following GMR fields representing a microstate of the D1-D5-P system:

$$\delta H = \frac{c\sqrt{Q_1 Q_5}(r^2 - a^2\cos^2\theta)}{h^2} F, \qquad \sqrt{2}\,\delta\phi = \delta\Phi = (k+1)a^{|k|-1} b F \tag{4.22a}$$

$$\delta Z_1 = \frac{a^{|k|-1} b Q_1}{h^2}\left[r^2(k+|k|+2) + a^2(k-|k|)\cos^2\theta\right] F, \tag{4.22b}$$

$$\delta Z_2 = -\frac{a^{|k|-1} b Q_5}{h^2}\left[r^2(k-|k|) + a^2(k+|k|+2)\cos^2\theta\right] F, \tag{4.22c}$$

$$\delta\beta = \frac{ac\sqrt{2Q_1Q_5}}{h} F\left[\pm\frac{ir\,dr}{r^2+a^2} + \frac{r^2}{h}(\sin^2\theta d\phi - \cos^2\theta d\psi)\right], \tag{4.22d}$$

$$\delta\omega = \frac{ac\sqrt{2Q_1Q_5}}{h} F\left[\pm\frac{ir\,dr}{r^2+a^2} + \frac{r^2}{h}(\sin^2\theta d\phi + \cos^2\theta d\psi)\right], \tag{4.22e}$$

$$\mathcal{F} = 0, \tag{4.22f}$$

$$\delta(ds_4^2) = 2a^2 cF\left[\sin^2\theta\left(d\phi \pm \frac{ir\,dr}{r^2+a^2}\right)^2 + \cos^2\theta\, d\theta^2\right], \tag{4.22g}$$

where

$$c \equiv (|k|+1)a^{|k|-1}b, \qquad F \equiv \frac{e^{i\sqrt{2}mv/R + i(k-m)\phi - im\psi}\sin^{|k|-|m|}\theta\cos^{|m|}\theta}{(r^2+a^2)^{|k|/2}}. \tag{4.23}$$

Here $k \in \mathbb{Z}$ ($k \neq -1, 0$) and $|m| \leq |k|$. The sign of m is also correlated to that of k, namely, $\mathrm{sign}(m) = \mathrm{sign}(k)$. The $\pm$ signs above correspond to $k \gtrless 0$. We can see that the base metric is v-dependent as we wanted. This solution carries non-vanishing momentum

$$N_p = m. \tag{4.24}$$

Note that, in our approximation at first order in perturbation, we have $\mathcal{F} = 0$ and we cannot read off the momentum charge from the asymptotic behavior of g_{uv}. This is because the metric starts to feel momentum only at the quadratic order, because the energy-momentum tensor $T_{\mu\nu}$ is quadratic in fields.

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The Θ fields can be read off from (2.12) and (2.13) as

$$\delta\Theta_1 = \begin{cases} (k+1)ma^{k+2}b\sqrt{\frac{2Q_5}{Q_1}}\frac{F}{h^2}\cos^2\theta\left(\frac{rh}{r^2+a^2}dr - ir^2\sin^2\theta\, d\phi\right)\wedge d\psi & (k>0) \\ |m|a^{|k|}b\sqrt{\frac{2Q_5}{Q_1}}F\Big[|k|\tan\theta\left(-\frac{ir\,dr}{r^2+a^2}+d\phi\right)\wedge d\theta & \\ \qquad +\frac{(-|k|r^2+a^2\cos^2\theta)r}{h}\left(\frac{dr}{r^2+a^2}+\frac{ir}{h}\sin^2\theta d\phi\right)\wedge d\psi\Big], & (k<0). \end{cases} \tag{4.25}$$

$$\delta\Theta_2 = \begin{cases} (k+1)ma^{k}b\sqrt{\frac{2Q_5}{Q_1}}F\Big[\tan\theta\left(\frac{ir\,dr}{r^2+a^2}+d\phi\right)\wedge d\theta & \\ \qquad +\frac{r^3}{h}\left(-\frac{dr}{r^2+a^2}+\frac{ir}{h}\sin^2\theta d\phi\right)\wedge d\psi\Big] & (k>0). \\ |m|a^{|k|}b\sqrt{\frac{2Q_5}{Q_1}}F\Big[\tan\theta\left(-\frac{ir\,dr}{r^2+a^2}+d\phi\right)\wedge d\theta & \\ \qquad +\frac{(a^2|k|\cos^2\theta-r^2)r}{h}\left(\frac{dr}{r^2+a^2}+\frac{ir}{h}\sin^2\theta d\phi\right)\wedge d\psi\Big] & (k<0). \end{cases} \tag{4.26}$$

It is a good consistency check that these vanish for $m=0$, because Θ_I vanishes for the original LM geometries. Using (2.11), we can compute $\widehat{\psi}$:

$$\delta\widehat{\psi} = -\frac{(|k|+1)\,|m|\,a^{|k|+2}\,b}{\sqrt{2Q_1Q_5}}F\Big[\sin\theta\cos\theta\left(\mp\frac{irdr}{r^2+a^2}-d\phi\right)\wedge d\theta + \cos^2\theta\left(-\frac{rdr}{r^2+a^2}\pm\frac{ir^2\sin^2\theta}{h}d\phi\right)\wedge d\psi\Big]. \tag{4.27}$$

Both Θ_1 and Θ_2 give the same $\widehat{\psi}$, as they should.

Finally, let us turn to the almost hyperkähler structure 2-forms, $J^{(A)}$. To consider their fluctuation, we must first fix the zeroth order expression. The flat metric (3.6) can be rewritten in the Gibbons-Hawking form as follows:

$$ds_4^2 = V^{-1}(d\chi+\xi)^2 + V ds_3^2, \tag{4.28}$$

where

$$\begin{gathered} V=\frac{1}{\rho}, \qquad ds_3^2 = d\rho^2+\rho^2(d\vartheta^2+\sin^2\vartheta\, d\varphi^2), \\ \sqrt{r^2+a^2} = 2\sqrt{\rho}\cos\frac{\vartheta}{2}, \qquad r = 2\sqrt{\rho}\sin\frac{\vartheta}{2}, \\ \phi = \frac{\chi}{2}-\varphi, \qquad \psi=\frac{\chi}{2}, \qquad \xi = (1+\cos\vartheta)d\varphi. \end{gathered} \tag{4.29}$$

As the zeroth order basis, let us take

$$J^{(A)} = e^1\wedge e^{A+1} - \frac{1}{2}\epsilon^{ABC}e^{B+1}\wedge e^{C+1}, \tag{4.30}$$

where $A, B, C = 1, 2, 3$ and

$$\begin{aligned} e^1 &= V^{-\frac{1}{2}}(d\chi + \xi), \qquad e^2 = V^{\frac{1}{2}} d(\rho \sin\vartheta \cos\varphi), \\ e^3 &= V^{\frac{1}{2}} d(\rho \sin\vartheta \sin\varphi), \qquad e^4 = V^{\frac{1}{2}} d(\rho \cos\vartheta). \end{aligned} \tag{4.31}$$

e^2, e^3, e^4 give the Cartesian coordinate basis of the base $\mathbb{R}^3$. We could have instead taken the four Cartesian coordinate basis forms of $\mathcal{B}_4 = \mathbb{R}^4$ as the zeroth order, but the above choice is more in line with the circular profile function of the background LM geometry.

With the above choice of $J^{(A)}$, the fluctuation $\delta J^{(A)}$ are found to be

$$\begin{aligned} \delta J^{(1)} =& \frac{(1+|k|)\, a^{1+|k|}\, b\, F e^{\pm i(\phi-\psi)}}{(r^2+a^2)^{3/2}} \\ &\times \Bigg[\pm i \left(\frac{1}{2}\left[a^2 + (a^2+2r^2)\cos(2\theta)\right] dr\wedge d\theta - r(r^2+a^2)\sin\theta\cos\theta\, d\phi\wedge d\psi \right) \\ &\quad - \cos\theta\sin\theta\, dr \wedge \left[(r^2+a^2)d\phi - r^2 d\psi\right] + r(r^2+a^2) d\theta \wedge (\sin^2\theta\, d\phi - \cos^2\theta\, d\psi) \Bigg], \\ \delta J^{(2)} =& \frac{(1+|k|)\, a^{1+|k|}\, b\, F e^{\pm i(\phi-\psi)}}{(r^2+a^2)^{3/2}} \\ &\times \Bigg[\left(\frac{1}{2}\left[a^2 + (a^2+2r^2)\cos(2\theta)\right] dr\wedge d\theta - r(r^2+a^2)\sin\theta\cos\theta\, d\phi\wedge d\psi \right) \\ &\quad \pm i \cos\theta\sin\theta\, dr \wedge \left[(r^2+a^2)d\phi - r^2 d\psi\right] \mp i r(r^2+a^2) d\theta \wedge (\sin^2\theta\, d\phi - \cos^2\theta\, d\psi) \Bigg], \\ \delta J^{(3)} =& (1+|k|)\, a^{1+|k|}\, b\, F \sin(2\theta) \left(\frac{\pm i r}{r^2+a^2} dr\wedge d\theta - d\theta \wedge d\phi \right). \end{aligned} \tag{4.32}$$

For details of the computation, see appendix C.3. One can check that the above $\delta J^{(A)}$ correctly give $\delta\widehat{\psi}$ given in (4.27) using the definition (2.8).

5 Future directions

In this paper, we perturbatively constructed supersymmetric configurations of the D1-D5-P system as solutions of 6D supergravity at the linear order. An important characteristic of our solutions is that they has v-dependent base space $\mathcal{B}_4$. This is a feature expected of superstratum solutions [29] and we hope that our solutions are useful for constructing general superstrata.

Our solutions have AdS asymptotics, because we used the solution generating technique of [1]. It would be interesting if our solutions can be generalized to flat asymptotics. This is a non-trivial problem, because adding "1" to the harmonic functions $Z_{1,2}$ affect other equations in section 2 and finding $\Theta_{1,2}, \omega, \mathcal{F}$ that satisfy them is not an obvious task. Also, it is interesting to see how our solutions fit in the framework of [33], which discusses v- and χ-dependent fluctuations on top of v- and χ-independent Gibbons-Hawking base. Finally, our solutions are constructed as linear perturbations around the maximally rotating Lunin-Mathur geometry. It would be interesting to see if this perturbative solution can be non-linearly completed to finite deformations of the LM geometry [27]. This will make it easier to see the location of the brane sources in our solutions, which should be useful for finding general smooth solutions of the 6D system.

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A Convention

We define the following operators

$$D \equiv \tilde{d} - \beta \wedge \partial_v, \tag{A.1}$$

$$\dot{} \equiv \partial_v \equiv \mathcal{L}_{\frac{\partial}{\partial v}} = \iota_{\frac{\partial}{\partial v}} d + d\iota_{\frac{\partial}{\partial v}} . \tag{A.2}$$

The Hodge star is defined by

$$*_d \left(dx^{m_1} \wedge \cdots \wedge dx^{m_p}\right) = \frac{1}{(d-p)!} dx^{n_1} \wedge \cdots \wedge dx^{n_{d-p}} \, \epsilon_{n_1 \ldots n_{d-p}}{}^{m_1 \ldots m_p}. \tag{A.3}$$

Our choice for the 6D ϵ tensor is [29]

$$\epsilon^{vu1234} = \epsilon^{ty1234} = +\frac{1}{\sqrt{|g|}}, \qquad \epsilon_{ty1234} = -\sqrt{|g|}. \tag{A.4}$$

B Lunin-Mathur geometry

Here we summarize relations relevant for the Lunin-Mathur solutions presented in (3.1).

The periodicity of the profile functions, L, is related to the radius R of the S^1 and the quantized D5 charge N_5 as

$$L = \frac{2\pi g_s \alpha' N_5}{R}. \tag{B.1}$$

Given the profile function $F_m(w)$, D1 charge is given by

$$Q_1 = \frac{Q_5}{L} \int_0^L |\dot{F}|^2 dw. \tag{B.2}$$

D1 charge Q_1 and D5 charge Q_5 are related to quantized charges N_1, N_5 by

$$Q_1 = g_s \alpha' N_1, \qquad Q_5 = \frac{g_s \alpha'^3}{v_4} N_5, \tag{B.3}$$

where the coordinate volume of T^4 is $(2\pi)^4 v_4$.

The 1-form B can be found by solving the differential equation $dB = *_4 dA$ in (3.1). The explicit solution is

$$B = -\frac{Q\epsilon_{ijkl}}{L}\int_0^L dw \int_0^1 dt\, \frac{t\dot{F}_k F_l (y_i dx_j - y_j dx_i)}{|\vec{y}|^4}, \qquad y_i \equiv x_i - tF_i(w). \tag{B.4}$$

This can be derived as follows. Let us rewrite the expression for A in (3.1) by decomposing the closed curve $\vec{x} = \vec{F}(w)$ into sum of many closed curves, just like one does in Stokes' theorem.

$$\begin{aligned} A &= -\frac{Q_5}{L}\int_0^L dw \int_0^1 dt \frac{\partial}{\partial t}\left[\frac{t\dot{F}_i(w)dx_i}{|\vec{x} - t\vec{F}(w)|^2}\right] \\ &= -\frac{Q_5}{L}\int_0^L dw \int_0^1 dt \left[\frac{\dot{F}_i(w)dx_i}{|\vec{x} - t\vec{F}(w)|^2} + \frac{2((\vec{x} - t\vec{F})\cdot\vec{F})\, t\dot{F}_i(w)dx_i}{|\vec{x} - t\vec{F}(w)|^2}\right] \end{aligned} \tag{B.5}$$

This corresponds to decomposing the closed curve $\vec{x} = \vec{F}(w)$ as a sum of many curves $\vec{x} = (t + dt)\vec{F}(w)$ and $\vec{x} = -t\vec{F}(w)$. The curves are along w, but we further want to divide them by adding segments along t, so that now we have infinitesimal curves along both t,w directions. This can be done by adding a total derivative in w (which integrates to zero upon $\int dw$) as follows:

$$A = -\frac{Q_5}{L}\int_0^L dw \int_0^1 dt \left[\frac{\dot{F}_i(w)dx_i}{|\vec{x} - t\vec{F}(w)|^2} + \frac{2((\vec{x} - t\vec{F})\cdot\vec{F})\, t\dot{F}_i(w)dx_i}{|\vec{x} - t\vec{F}(w)|^2} - \frac{\partial}{\partial w}\left(\frac{F_i(w)dx_i}{|\vec{x} - t\vec{F}(w)|^2}\right)\right]. \tag{B.6}$$

After some manipulation, this can be written as

$$A = \frac{2Q}{L}\int_0^L dw \int_0^1 dt\, \frac{t\dot{F}_i F_j (y_i dx_j - y_j dx_i)}{|\vec{y}|^4}, \qquad y_i \equiv x_i - tF_i(w). \tag{B.7}$$

Now, if we have a 1-form

$$a = a_{ij}\frac{x_i dx_j - x_j dx_i}{|\vec{x}|^4}, \tag{B.8}$$

where a_{ij} is constant and antisymmetric, then the 1-form b that satisfies

$$da = *_4 db \tag{B.9}$$

is given by

$$b = b_{ij}\frac{x_i dx_j - x_j dx_i}{|\vec{x}|^4}, \qquad b_{ij} = -\frac{1}{2}\epsilon_{ijkl}a_{kl} = -\tilde{a}_{ij}. \tag{B.10}$$

Therefore, (B.4) is the solution to $dB = *_4 dA$.

C Details of calculations

Here we describe some details of the computation in section 4.

C.1 Fluctuation of Lunin-Mathur geometry

We study fluctuations of the LM geometry corresponding to the fluctuation $\delta F_m(w)$ of the profile function around the background profile $F_m(w)$. The change in the harmonic functions in (3.1) is given by

$$\begin{aligned}
\delta Z_1 &= \frac{2Q_5}{L}\int_0^L dw \left[\frac{((\vec{x}-\vec{F})\cdot\delta\vec{F})\dot{F}^2}{|\vec{x}-\vec{F}|^4}+\frac{\dot{F}\cdot\delta\dot{\vec{F}}}{|\vec{x}-\vec{F}|^2}\right],\\
\delta Z_2 &= \frac{2Q_5}{L}\int_0^L dw \frac{(\vec{x}-\vec{F})\cdot\delta\vec{F}}{|\vec{x}-\vec{F}|^4},\\
\delta A_i &= -\frac{Q_5}{L}\int_0^L dw \left[\frac{2((\vec{x}-\vec{F})\cdot\delta\vec{F})\dot{F}_i}{|\vec{x}-\vec{F}|^4}+\frac{\delta\dot{F}_i}{|\vec{x}-\vec{F}|^2}\right].
\end{aligned} \tag{C.1}$$

Also, from (B.4), The change in D1 charge Q_1 defined in (B.2) is

$$\delta Q_1 = \frac{2Q_5}{L}\int_0^L dw\, \vec{F}\cdot\delta\vec{F}. \tag{C.2}$$

For studying fluctuations around the maximally rotating LM solution (3.3), it is useful to define

$$I_n(k) \equiv \frac{1}{2\pi}\int_0^{2\pi}\frac{\cos(k\gamma)\,d\gamma}{(s^2+a^2+w^2-2as\cos\gamma)^n} = I_n(-k), \tag{C.3}$$

for $k\in\mathbb{Z}$ and $n=1,2,\ldots$. Explicitly,

$$I_1(k) = \frac{a^{|k|}\sin^{|k|}\theta}{h\,(r^2+a^2)^{|k|/2}}, \tag{C.4}$$

$$I_2(k) = \frac{\left[(|k|+1)r^2+((|k|-1)\cos^2\theta+2)a^2\right]a^{|k|}\sin^{|k|}\theta}{h^3\,(r^2+a^2)^{|k|/2}}. \tag{C.5}$$

We also define

$$X_\pm \equiv as[I_2(k+2)\pm I_2(k)]+a^2[\mp I_2(k-1)-I_2(k+1)]+(k+1)I_1(k+1). \tag{C.6}$$

More explicitly,

$$X_+ = \begin{cases} \dfrac{\left[|k|\left((1-2\cos^2\theta)r^2-a^2\cos^2\theta\right)h+2(r^2+a^2)(r^2-a^2\cos^2\theta)\sin^2\theta\right]a^{|k|+1}\sin^{|k|-1}\theta}{(r^2+a^2)^{\frac{|k|+1}{2}}h^3} & (k\neq 0),\\[2ex] \dfrac{2\sqrt{r^2+a^2}(r^2-a^2\cos^2\theta)a\sin\theta}{h^3} & (k=0). \end{cases} \tag{C.7}$$

$$X_- = \frac{ka^{|k|+1}\sin^{|k|-1}\theta}{(r^2+a^2)^{\frac{|k|+1}{2}}h}. \tag{C.8}$$

C.2 The seed solution and spectral flow

In section 4.1, we considered the fluctuation (4.1) around the maximally rotating LM geometry and computed the change in the GMR data. The change in Z_1, Z_2, A, B is

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straightforward to compute using the formulas (C.1) and (B.4). The change in the RR 2-form (3.2), δC_2, has contributions $\delta C_{2,\mathrm{elec}}$ and $\delta C_{2,\mathrm{mag}}$:

$$\delta C_{2,\mathrm{elec}} = -\frac{\delta Z_1}{Z_1^2}(dt - A) \wedge (dy + B) + Z_1^{-1}[-\delta A \wedge (dy + B) + (dt - A) \wedge \delta B], \tag{C.9}$$

$$\delta C_{2,\mathrm{mag}} = \delta \mathcal{C}_2, \qquad d\delta\mathcal{C}_2 = *_4 dZ_1. \tag{C.10}$$

If we carry out the spectral flow (3.7) followed by the coordinate transformation (4.4), we have an additional contribution:

$$(\delta C_{2,\mathrm{diff}})_{\mu\nu} = (\mathcal{L}_\xi C_2)_{\mu\nu} = \xi^\rho \partial_\rho C_{\mu\nu} + \partial_\mu \xi^\rho C_{\rho\nu} + \partial_\nu \xi^\rho C_{\mu\rho}, \tag{C.11}$$

where $\mathcal{L}_\xi$ is the Lie derivative. The total change in C_2 is given by

$$\delta C_2 = \delta C_{2,\mathrm{elec}} + \delta C_{2,\mathrm{mag}} + \delta C_{2,\mathrm{diff}}. \tag{C.12}$$

In order to find δC_2 in the canonical form of [45], it is easier to first compute $\delta F_3 = d\delta C_2$, because then we do not have to know $\delta C_{2,\mathrm{mag}}$ but only its exterior derivative $d\delta C_{2,\mathrm{mag}} = *_4 dZ_1$. After some tedious computation, we find, for $k > 0$,

$$\begin{aligned}\delta F_3 = \frac{2(k+1)a^{k-1} b e^{ik(t/R+\widetilde{\phi})} \sin^k\theta}{Q_1 (r^2+a^2)^{k/2}} \Bigg[&-(k-2) r\, dt \wedge dr \\ &+ kr^2 \left(dt - \frac{ia\sqrt{Q_1 Q_5}}{r(r^2+a^2)} dr \right) \wedge (\cot\theta d\theta + i d\widetilde{\phi}) \Bigg] \wedge dy \end{aligned} \tag{C.13}$$

while, for $k = -l < 0$,

$$\begin{aligned}\delta F_3 = \frac{2a^{l-1} b e^{-il(t/R+\widetilde{\phi})} \sin^l\theta}{Q_1 (r^2+a^2)^{l/2}} \Bigg[&(l-2) r\, dt \wedge dy \wedge dr + l(l+2) Q_1 Q_5 \sin\theta \cos\theta\, d\theta \wedge d\widetilde{\phi} \wedge d\widetilde{\psi} \\ &- lr^2 \left(dt + \frac{i\sqrt{Q_1 Q_5}\, a}{r(r^2+a^2)} dr \right) \wedge dy \wedge (\cot\theta\, d\theta - i d\widetilde{\phi}) \\ &- l^2 a \sqrt{Q_1 Q_5} \left(dt - \frac{i\sqrt{Q_1 Q_5}\, r}{a\,(r^2+a^2)} dr \right) \wedge \left(\cot\theta\, d\theta - i \cos^2\theta\, d\widetilde{\phi} \right) \wedge d\widetilde{\psi} \Bigg]. \end{aligned} \tag{C.14}$$

Note that the expression for $k = -l < 0$ is not simply obtained from the one for $k > 0$ by replacing $k \to l$.

The 2-form potential δC_2 that gives the above δF_3 is obtained as follows. First, from [45], the AdS_3 part of the 2-form can be written as

$$C_{\mu\nu} = (\epsilon^{AdS_3})_{\mu\nu}{}^{\lambda} X_\lambda \widehat{Y}, \tag{C.15}$$

where μ, ν, λ are AdS_3 indices and ϵ^{AdS_3} is the volume form for AdS_3 with the metric (3.9b). X_λ are functions in AdS_3 while $\widehat{Y}$ is a harmonic function in S^3. On the other hand, the S^3 part can be written as

$$C_{ab} = (\epsilon^{S^3})_{ab}{}^{c} U \partial_c \widehat{Y}, \tag{C.16}$$

where a, b, c are S^3 indices, ϵ^{S^3} is the volume form for unit S^3 with the metric (3.9c), and U is a function in AdS_3. In general, there can be also mixing terms, $C_{\mu a}$, but that turns out unnecessary in the present case. So, after a bit of redefinitions, our ansatz for the 2-form is

$$\begin{aligned}\delta C_2 = \widehat{B}\left[X_t \frac{r}{r^2+a^2} dy\wedge dr + X_y \frac{dr\wedge dt}{r} + X_r \frac{r(r^2+a^2)}{Q_1Q_5} dt\wedge dy\right]\widehat{Y}\\ + \widehat{B}U\left[\sin\theta\cos\theta(\partial_\theta\widehat{Y})d\widetilde{\phi}\wedge d\widetilde{\psi} + \frac{\cos\theta}{\sin\theta}(\partial_{\widetilde{\phi}}\widehat{Y})d\widetilde{\psi}\wedge d\theta + \frac{\sin\theta}{\cos\theta}(\partial_{\psi}\widehat{Y})d\theta\wedge d\widetilde{\phi}\right],\end{aligned} \tag{C.17}$$

where $\widehat{B}, \widehat{Y}$ are defined in (4.6). By requiring that this reproduce the 3-form δF_3 in (C.13) and (C.14), we get the following simple result:

$$\begin{aligned} k>0: \quad X_t &= -\frac{2i(k+1)a^{k-1}b}{\omega}, \quad X_y = 0, \quad X_r = -\frac{2(k+1)a^{k-1}bQ_5 r}{r^2+a^2}, \quad U = 0,\\ k=-l<0: \quad X_t &= \frac{2ia^{l-1}b}{\omega}, \quad X_y = 0, \quad X_r = -\frac{2a^{l-1}bQ_5 r}{r^2+a^2}, \quad U = -2a^{l-1}bQ_5. \end{aligned} \tag{C.18}$$

Or, more explicitly,

$$\delta C_2 = \begin{cases} -\dfrac{2(k+1)a^{k-1}b}{Q_1\omega}\widehat{B}\widehat{Y}\left[r^2\omega\, dt\wedge dy + iQ_1\dfrac{r\, dy\wedge dr}{r^2+a^2}\right] & (k>0),\\ -\dfrac{2a^{l-1}b}{Q_1\omega}\widehat{B}\widehat{Y}\left[r^2\omega\, dt\wedge dy - iQ_1\dfrac{r\, dy\wedge dr}{r^2+a^2}\right.\\ \qquad\qquad \left. -ilQ_1Q_5\omega\cot\theta(d\theta - i\sin\theta\cos\theta d\widetilde{\phi})\wedge d\widetilde{\psi}\right] & (k=-l<0). \end{cases} \tag{C.19}$$

This is what we used in (4.8).

C.3 Computing $\delta J^{(A)}$

As explained in the main text, as the zeroth order solution, we used the hyperkähler structure 2-forms $J^{(A)}$ defined through the vierbein $e^I = e^I{}_i dx^i$, $I = 1,2,3,4$, as (4.30). Note that $J^{(A)}$ are genuinely hyperkähler, not almost hyperkähler, and therefore closed. Also, note that e^I are orthonormal in the sense

$$g_4^{ij} e^I{}_i e^J{}_j = \delta^{IJ}, \tag{C.20}$$

where g_4^{ij} is the inverse of the base metric $g_{4\,ij}$ defined in (4.28).

Let us assume that the corrected 2-forms $J^{(A)} + \delta J^{(A)}$ are still constructed from the corrected vierbein $e^I + \delta e^I$ by (4.30). Namely,

$$\delta J^{(A)} = \delta e^1 \wedge e^{A+1} + e^1\wedge\delta e^{A+1} - \frac{1}{2}\epsilon^{ABC}(\delta e^B\wedge e^C + e^B\wedge\delta e^C). \tag{C.21}$$

Let us expand δe^I as $\delta e^I = \delta e^I{}_i dx^i$ and raise and lower indices using the zeroth order quantities $e^I{}_j$, $g_{4\,ij}$, and g_4^{ij}. If we require that $e^I + \delta e^I$ be orthonormal with respect to the corrected metric $g_4 + \delta g_4$, then (C.20) implies that

$$\delta e_{ij} + \delta e_{ji} = \delta g_{4\,ij}. \tag{C.22}$$

Therefore, we can write δe^I in terms of the 6 independent variables $\delta e_{i<j}$ as

$$\delta e^I = \sum_{i=1}^{4} \left[\frac{1}{2} e^{Ii} \delta g_{4\,ii} + \sum_{1\le j<i} e^{Ij} \delta e_{ji} + \sum_{i<j\le 4} e^{Ij} (\delta g_{4\,ij} - \delta e_{ij}) \right] dx^i. \tag{C.23}$$

With this construction, the conditions (2.3) and (2.4) on $J^{(A)} + \delta J^{(A)}$ are automatically satisfied. However, they will not be closed any more.

In the present case, all fields (4.22), (4.25), and (4.26) depend on v through F defined in (4.23). So, let us assume that $\delta e, \delta J^{(A)}$ are also proportional to F and therefore

$$\delta \dot{J}^{(A)} = i\sqrt{\frac{2}{Q_1 Q_5}}\, am\, \delta J^{(A)}. \tag{C.24}$$

In this case, $\widehat{\psi}$ in (2.8) is given by

$$\begin{aligned} \widehat{\psi} &= i\sqrt{\frac{2}{Q_1 Q_5}}\, am \cdot \frac{1}{16} \epsilon^{ABJ} J^{(A)ij} \delta J^{(B)}_{ij} J^{(C)} \\ &= \frac{i}{\sqrt{8 Q_1 Q_5}am} (1 - *_4) M, \qquad M = e^I \wedge \delta e_I. \end{aligned} \tag{C.25}$$

If we plug the explicit expression (C.23) into (C.25) and require that it be equal to (4.27), it turns out that we can eliminate 3 out of 6 independent parameters $\delta e_{i<j}$. For example, we can take $\delta e_{12}, \delta e_{13}, \delta e_{14}$ as independent variables.

One can show that the differential condition (2.5), which reads

$$\tilde{d}\delta J = \partial_v (\beta \wedge \delta J + \delta B \wedge J) \tag{C.26}$$

$$= i\sqrt{\frac{2}{Q_1 Q_5}} ma (\beta \wedge \delta J + \delta B \wedge J), \tag{C.27}$$

is identically satisfied, whatever the values of $\delta e_{12}, \delta e_{13}, \delta e_{14}$ are. If we compute $\delta J^{(A)}$ using (C.21), we obtain (4.32), independent of $\delta e_{12}, \delta e_{13}, \delta e_{14}$.

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