PHYSICAL REVIEW D **88,** 026007 (2013)

# Deconstructing (2,0) proposals

N. Lambert,[1,*,†] C. Papageorgakis,[2,‡] and M. Schmidt-Sommerfeld[3,4,§]

[1]*Theory Division, CERN, 1211 Geneva 23, Switzerland*
[2]*NHETC and Department of Physics and Astronomy, Rutgers University, Piscataway, New Jersey 08854-8019, USA*
[3]*Arnold-Sommerfeld-Center für Theoretische Physik, LMU München, Theresienstraße 37, 80333 München, Germany*
[4]*Excellence Cluster Universe, Technische Universität München, Boltzmannstraße 2, 85748 Garching, Germany*

(Received 8 April 2013; published 12 July 2013)

We examine the relationships between three proposals for the six-dimensional (2,0) theory: the discrete light-cone quantization (DLCQ) of Aharony *et al.* [Adv. Theor. Math. Phys. **1**, 148 (1998); Adv. Theor. Math. Phys. **2**, 119 (1998)], the deconstruction prescription of Arkani-Hamed *et al.* [J. High Energy Phys. 01 (**2003**) 083], and the five-dimensional maximally supersymmetric Yang-Mills proposal of Douglas and Lambert *et al.* [J. High Energy Phys. 02 (**2011**) 011; J. High Energy Phys. 01 (**2011**) 083]. We show that Arkani-Hamed *et al.* gives a deconstruction of five-dimensional maximally supersymmetric Yang-Mills. The proposal of Aharony *et al.* uses a subset of the degrees of freedom of five-dimensional Yang-Mills, and we show that compactification of it on a circle of finite radius agrees with the DLCQ arising from the proposal of Douglas and Lambert *et al.* or from the deconstruction proposal of Arkani-Hamed *et al*.

DOI: 10.1103/PhysRevD.88.026007 PACS numbers: 11.25.Yb

## I. INTRODUCTION

In this paper we wish to examine the relationships between various proposals for the six-dimensional (2,0) theory. In particular, these include the discrete light-cone quantization (DLCQ) definition based on the instanton quantum mechanics [1,2], a definition via deconstruction from a family of four-dimensional $\mathcal{N} = 2$ superconformal field theories based on circular quivers [3] and the more recent conjecture that the (2,0) theory on an $S^1$ of radius $R_5$ is equivalent to five-dimensional maximally supersymmetric Yang-Mills (5D MSYM) with coupling $g^2_{\rm YM} = 4\pi^2 R_5$ [4,5].

The 5D MSYM conjecture relies on 5D MSYM being a consistent quantum theory at the nonperturbative level and not just an effective theory valid below some cutoff. Without this the conjecture is devoid of meaning since either 5D MSYM simply does not exist as a complete quantum theory, so that the conjecture is manifestly false, or it can only be defined as the (2,0) theory on $S^1$, so that the conjecture is tautological. On the other hand this is perhaps one of the more interesting aspects of this proposal: namely, that a perturbatively nonrenormalizable and divergent [6] field theory is in fact nonperturbatively well defined without additional UV degrees of freedom. Recently, several highly nontrivial tests of this proposal have been performed [7–15].

From the traditional, Wilsonian, viewpoint 5D MSYM is a nonrenormalizable effective theory obtained by integrating out degrees of freedom above some scale (proportional to $1/g^2_{\rm YM}$). The UV is then described by a conformal field theory, which in this case is the six-dimensional (2,0) theory. The viewpoint that we explore here is different. In particular, the proposal of [4,5] is that all the states of the UV theory are already present in 5D MSYM. This may seem paradoxical, however the issue is that there is no physically well-defined separation of the theory into perturbative, i.e. power series in $g^2_{\rm YM}$, and nonperturbative sectors. Perturbative calculations should only be viewed as low energy approximations where the effective coupling $g_{\rm eff} = g^2_{\rm YM} E$ is small and hence do not probe the UV behaviour.

Thus we seek other ways to define 5D MSYM. A method that comes to mind is that of deconstruction [16]. We will show how 5D MSYM on a (discretized) circle of radius $R_4$ can be deconstructed from an $\mathcal{N} = 2$ superconformal circular quiver gauge theory with $N$ nodes. In particular, for any process involving KK modes up to some finite level $L$ the correlation functions of 5D MSYM can be reproduced to arbitrary accuracy by taking $N$ suitably large compared to $L$. One could then think of the deconstruction as providing a quantum definition of 5D MSYM in terms of a well-defined theory. Indeed we will see that this relates directly the proposal of [3] to that of [4,5]. In other words, an alternative interpretation of the proposal of [3] is that one cannot deconstruct 5D MSYM on an $S^1$ of radius $R_4$ and coupling $g^2_{\rm YM}$ without also deconstructing the (2,0) theory on a torus with radii $R_4$, $R_5$ where $R_5 = g^2_{\rm YM}/4\pi^2$, keeping all KK modes.

Another method to define a theory is to consider DLCQ and we revisit the proposal of [1,2]. This proposal has the miraculous feature that it only requires knowing the dynamics of the (2,0) theory on $S^1$ in the limit that $R_5 \to 0$. Thus it does not require knowledge of the theory at finite $R_5 = g^2_{\rm YM}/4\pi^2$. We will show that this DLCQ of the (2,0) theory on a circle of finite size agrees with a DLCQ obtained from 5D MSYM defined using deconstruction or assuming it is the (2,0) theory on $S^1$.

*On leave from King's College London.
†neil.lambert@cern.ch
‡papageorgakis@physics.rutgers.edu
§m.schmidtSommerfeld@physik.uni-muenchen.de

 

The rest of this paper is organized as follows. In Sec. II we perform a discretization of one dimension in 5D MSYM and show explicitly that the resulting action is in the same universality class as the four-dimensional deconstructed quiver theory of [3], both leading to the action of 5D MSYM on $S^1$ in the limit where the spacing goes to zero. Furthermore, in the spirit of deconstruction, we argue that the quantum theory of the quiver conformal field theory can be made to be arbitrarily close to that of the discretized 5D MSYM theory on a circle of radius $R_4$. In Sec. III we review the infinite momentum frame (IMF) and DLCQ descriptions of the (2,0) theory and argue that, unlike for the DLCQ, there is no obvious simplification of the theory in the IMF. On the other hand we show that a reduction of the DLCQ prescription [1,2] of the (2,0) theory on a circle of finite radius agrees with the DLCQ description obtained from either 5D MSYM (assuming the conjecture of [4,5]) or the deconstruction proposal [3]. Finally, Sec. IV contains our conclusions and further comments.

## II. DECONSTRUCTING 5D MSYM

Our aim in this section is to deconstruct 5D MSYM starting from a well-defined four-dimensional quiver gauge theory. The idea of deconstruction is that the quiver or theory space can, in the Higgs phase of the 4D theory, be interpreted as a discretized physical direction with spacing $a = 1/vG$. Here $v$ is the Higgs vev and $G$ the 4D coupling. *A priori*, the 5D theory emerges only at energies below $1/a$ and is UV-completed by the well-defined 4D quiver theory [16]. However, for a superconformal theory one can attempt to take the spacing to zero, or in other words the UV cutoff to infinity. For this one needs to start with a 4D theory which does not experience a phase transition at strong coupling [3]. We will show that the superconformal quiver gauge theory introduced in Sec. II D exactly reproduces 5D MSYM on a discretized circle by matching the two actions. Note that our discretization process is not quite the same as replacing the circle by a lattice; for a discussion on how to latticize a theory while preserving some degree of supersymmetry, see [17]. Rather, we will replace functions of the circle by piecewise constant functions. We will then proceed to discuss the relation of [3] to the proposal of [4,5].

### A. Discretized 5D MSYM: gauge fields

Let us begin with the bosonic part of the action of 5D MSYM with gauge group SU($K$)

$$S^B_{5D} = \frac{1}{g^2_{YM}} \int d^5x \,\mathrm{Tr}\Big[ -\frac{1}{4} F_{\mu\nu}F^{\mu\nu} - \frac{1}{2} D_\mu X^I D^\mu X^I + \frac{1}{4}[X^I, X^J][X^I, X^J]\Big], \tag{2.1}$$

where $\mu, \nu = 0,\ldots,4$ and $I, J = 1,\ldots,5$. In view of discretizing and compactifying the four-direction, we will write

$$\begin{aligned} F_{mn} &= \partial_m A_n - \partial_n A_m - i[A_m, A_n] \\ F_{4m} &= \partial_4 A_m - D_m X^6 \\ D_4 X^I &= \partial_4 X^I - i[X^6, X^I], \end{aligned} \tag{2.2}$$

where we have renamed $A_4 = X^6$, and $m, n = 0,\ldots,3$.

In order to proceed we first discretize the line whose coordinate is $x^4$ by splitting it into an infinite number of equal segments of length $a$ and take the fields to be constant along each segment. This has the effect of reducing the gauge symmetries to those of four-dimensions. In the limit that $a \to 0$ we expect that the full five-dimensional gauge symmetry is restored. The integral over $x^4$ becomes a Riemann sum, which approximates the integral as $a \to 0$. Keeping only terms which will be relevant for the gauge field $A_m$, this gives

$$S^{\text{B-Gauge}}_{\text{5D-Discr}} = \frac{a}{g^2_{YM}} \int d^4x \sum_{k=-\infty}^{\infty} \mathrm{Tr}\Big[ -\frac{1}{4} F^{(k)}_{mn} F^{(k)mn} - \frac{1}{2} \tilde{\partial}_4 A^{(k)}_m \tilde{\partial}^4 A^{(k)m} \Big], \tag{2.3}$$

where $\tilde{\partial}_4$ is a discretized version of the derivative involving the forward difference operator

$$\tilde{\partial}_4 f^{(k)} = \frac{f^{(k+1)} - f^{(k)}}{a}. \tag{2.4}$$

We then compactify the discretized direction by identifying $f^{(N+k)} \equiv f^{(k)}$ and truncating the sum such that $Na = 2\pi R_4$

$$S^{\text{B-Gauge}}_{\text{5D-Disc}} = \frac{a}{g^2_{YM}} \int d^4x \sum_{k=[\frac{N}{2}]-N+1}^{[\frac{N}{2}]} \mathrm{Tr}\Big[ -\frac{1}{4} F^{(k)}_{mn} F^{(k)mn} - \frac{1}{2} \tilde{\partial}_4 A^{(k)}_m \tilde{\partial}^4 A^{(k)m} \Big]. \tag{2.5}$$

As the last step we perform a discrete Fourier transform

$$A^{(k)}_m = \frac{1}{\sqrt{N}} \sum_{p=[\frac{N}{2}]-N+1}^{[\frac{N}{2}]} B^{(p)}_m q^{kp}, \tag{2.6}$$

with $q = e^{2\pi i/N}$; Note that the reality condition on $A_m$ imposes $B^{(-k)}_m = B^\dagger_{m(k)}$. From now on we will omit the sum ranges over Fourier mode indices, to be understood as above.

In terms of the Fourier modes the derivatives on the gauge fields become

$$\tilde{\partial}_4 A^{(k)}_m = \frac{1}{\sqrt{N}a} \sum_s B^{(s)}_m q^{ks}(q^s - 1), \tag{2.7}$$

whereas the gauge field strengths can be organized as

$$F_{mn}F^{mn} = (\partial^m A^n - \partial^n A^m)^2 - 2i[A_m, A_n](\partial^m A^n - \partial^n A^m) - [A_m, A_n][A^m, A^n]$$
$$= \frac{1}{N}\sum_{k,s,s'} q^{k(s-s')}(\partial_m B_n^{(-s)} - \partial_n B_m^{(-s)})(\partial^m B^{(s')n} - \partial^n B^{(s')m}) - \frac{2i}{N^{3/2}}\sum_{k,s,s',s''} q^{k(s-s'-s'')}[B_m^{(-s)}, B_n^{(-s')}](\partial^m B^{(s'')n} - \partial^n B^{(s'')m})$$
$$-\frac{1}{N^2}\sum_{k,s,s',s'',s'''} q^{k(s+s'-s''-s''')}[B_m^{(-s)}, B_n^{(-s')}][B^{(s'')m}, B^{(s''')n}]. \tag{2.8}$$

Plugging these expressions into (2.3) and performing the sums over $k$ and some of the $s$-indices, we obtain

$$S^{\text{B-Gauge}}_{\text{5D-Disc}} = \frac{a}{g^2_{\text{YM}}}\int d^4x \text{Tr}\Big[-\frac{1}{4}\sum_s(\partial_m B_n^{(-s)} - \partial_n B_m^{(-s)})(\partial^m B^{(s)n} - \partial^n B^{(s)m}) + \frac{i}{2N^{1/2}}\sum_{s,s'}[B_m^{(-s)}, B_n^{(-s')}](\partial^m B^{(s+s')n} - \partial^n B^{(s+s')m})$$
$$+\frac{1}{4N}\sum_{s,s',s''}[B_m^{(-s)}, B_n^{(-s')}][B^{(s'')m}, B^{(s+s'-s'')n}] - \frac{1}{2}\frac{1}{a^2}\sum_s |q^s - 1|^2 B_m^{(-s)} B^{(s)m}\Big]. \tag{2.9}$$

In the above we used $\sum_{k=[\frac{N}{2}]-N+1}^{[\frac{N}{2}]} q^{k(p-s)} = N\delta_{p,s}$.

### B. Discretized 5D MSYM: scalars

We proceed to consider the scalar part of 5D MSYM compactified on a discretized circle. Following the same steps as for the gauge fields and defining the Fourier transforms in terms of

$$X_A^{(i)} = \frac{1}{\sqrt{N}}\sum_s q^{is} Y_A^{(s)}, \tag{2.10}$$

we arrive at

$$S^{\text{B-Scalar}}_{\text{5D-Disc}} = -\frac{a}{2g^2_{\text{YM}}}\int d^4x\Big[\sum_s \partial_m Y_A^{(s)}\partial^m Y_A^{(-s)} - \frac{2i}{\sqrt{N}}\sum_{s,s'}[B_m^{(s)}, Y_A^{(s')}]\partial^m Y_A^{(-s-s')} - \frac{1}{N}\sum_{s,s',s''}[B_m^{(s)}, Y_A^{(s')}][B^{(-s'')m}, Y_A^{(s''-s-s')}]\Big]$$
$$-\frac{a}{4Ng^2_{\text{YM}}}\int d^4x\sum_{s,s',s''}[Y_A^{(s)}, Y_B^{(s')}][Y_A^{(-s'')}, Y_B^{(s''-s-s')}] + \frac{1}{g^2_{\text{YM}}}\int d^4x\sum_s \partial_m Y_6^{(s)} B^{(-s)m}(q^{-s} - 1)$$
$$-\frac{i}{\sqrt{N}g^2_{\text{YM}}}\int d^4x\sum_{s,s'}[B_m^{(s)}, Y_6^{(s')}]B^{(-s-s')m}(q^{-s-s'} - 1) + \frac{i}{\sqrt{N}g^2_{\text{YM}}}\sum_{s,s'}[Y_6^{(s)}, Y_I^{(s')}]Y_I^{(-s-s')}(q^{-s-s'} - 1)$$
$$-\frac{2}{ag^2_{\text{YM}}}\int d^4x\sum_s \sin^2\Big(\frac{\pi s}{N}\Big)Y_I^{(s)}Y_I^{(-s)}, \tag{2.11}$$

where $A \in \{I, 6\}$. Note that there is an asymmetry between the $A = I$ and $A = 6$ terms. In particular, there is no KK mass for $Y_6$.

### C. Discretized 5D MSYM: fermions

The fermionic part of the 5D MSYM action is naturally given by

$$S^F_{\text{5D}} = \frac{1}{g^2_{\text{YM}}}\int d^5x \text{Tr}\Big(-\frac{i}{2}\bar\psi_i\gamma^\mu D_\mu\psi_i + \frac{i}{2}\bar\psi_i\lambda^I_{ij}[X_I, \psi_j]\Big), \tag{2.12}$$

where $\mu = 0, \ldots, 4$, $I = 1, \ldots 5$, $i, j = 1, \ldots, 4$. The $\psi$'s are complex four-component spinors of Spin(1, 4) satisfying a symplectic Majorana condition and transforming in the **4** of Spin(5).[1]

However, it will be convenient to rewrite this in terms of complex two-component 4D Weyl spinors, such that we are able to compare with the action obtained from the 4D quiver theory via the deconstruction description. We decompose

$$\psi_1 = \begin{pmatrix}\zeta_1\\ -\sigma^2\zeta_3^*\end{pmatrix}, \qquad \psi_2 = \begin{pmatrix}\zeta_2\\ -\sigma^2\zeta_4^*\end{pmatrix},$$
$$\psi_3 = \begin{pmatrix}-i\zeta_4\\ -i\sigma^2\zeta_2^*\end{pmatrix}, \qquad \psi_4 = \begin{pmatrix}i\zeta_3\\ i\sigma^2\zeta_1^*\end{pmatrix}, \tag{2.13}$$

such that the symplectic Majorana condition is satisfied. Note that the action written in terms of the $\zeta$'s will not have manifest 5D Lorentz invariance.

The kinetic terms will give

$$S^{\text{F-Kin}}_{\text{5D}} = \frac{1}{g^2_{\text{YM}}}\int d^5x \text{Tr}(i\bar\zeta_1\bar\sigma^m D_m\zeta_1 + i\bar\zeta_2\bar\sigma^m D_m\zeta_2$$
$$+ i\bar\zeta_3\bar\sigma^m D_m\zeta_3 + i\bar\zeta_4\bar\sigma^m D_m\zeta_4 + \zeta_3 D_4\zeta_1$$
$$- i\bar\zeta_3 D_4\bar\zeta_1 + i\zeta_4 D_4\zeta_2 - i\bar\zeta_4 D_4\bar\zeta_2), \tag{2.14}$$

where $\bar\zeta = \zeta^\dagger$. Use has been made of the identities $(i\sigma^2)\zeta^* = \bar\zeta$ and $\zeta^T(i\sigma^2) = -\zeta$, as well as integration by parts.

[1]For our spinor conventions, we defer the reader to the Appendix.

We can also work out the Yukawa interactions. We will only explicitly write down the terms involving $X^5$. They are

$$S_{\text{5D}}^{\text{F-Int}} = \frac{1}{g_{\text{YM}}^2}\int d^5x\,\text{Tr}\Big(-\frac{i}{2}\bar{\psi}_1[X_5,\psi_1]+\frac{i}{2}\bar{\psi}_2[X_5,\psi_2]+\frac{i}{2}\bar{\psi}_3[X_5,\psi_3]-\frac{i}{2}\bar{\psi}_4[X_5,\psi_4]\Big)$$
$$= \frac{1}{g_{\text{YM}}^2}\int d^5x\,\text{Tr}(-i[\zeta_1,\zeta_3]X_5 - i[\bar{\zeta}_1,\bar{\zeta}_3]X_5 + i[\zeta_2,\zeta_4]X_5 + i[\bar{\zeta}_2,\bar{\zeta}_4]X_5). \tag{2.15}$$

Similarly to the bosonic case, we can turn the four-direction into a discretized one with spacing $a$, so that the integral becomes a sum, the derivative becomes a forward difference operator, etc. The discretization procedure produces the following action for the quadratic terms that we found above

$$S_{\text{5D-Disc}}^{\text{F-Kin}} = \frac{a}{g_{\text{YM}}^2}\int d^4x\sum_{k=-\infty}^{\infty}\text{Tr}(\zeta_3^{(k)}\tilde{\partial}_4\zeta_1^{(k)} - i\bar{\zeta}_3^{(k)}\tilde{\partial}_4\bar{\zeta}_1^{(k)} + i\zeta_4^{(k)}\tilde{\partial}_4\zeta_2^{(k)} - i\bar{\zeta}_4^{(k)}\tilde{\partial}_4\bar{\zeta}_2^{(k)} + i\bar{\zeta}_1^{(k)}\bar{\sigma}^m\partial_m\zeta_1^{(k)} + i\bar{\zeta}_2^{(k)}\bar{\sigma}^m\partial_m\zeta_2^{(k)}$$
$$+ i\bar{\zeta}_3^{(k)}\bar{\sigma}^m\partial_m\zeta_3^{(k)} + i\bar{\zeta}_4^{(k)}\bar{\sigma}^m\partial_m\zeta_4^{(k)} + \zeta_3^{(k)}[X_6^{(k)},\zeta_1^{(k)}] - \bar{\zeta}_3^{(k)}[X_6^{(k)},\bar{\zeta}_1^{(k)}] + \zeta_4^{(k)}[X_6^{(k)},\zeta_2^{(k)}] - \bar{\zeta}_4^{(k)}[X_6^{(k)},\bar{\zeta}_2^{(k)}]$$
$$+ \bar{\zeta}_1^{(k)}\bar{\sigma}^m[A_m^{(k)},\zeta_1^{(k)}] + \bar{\zeta}_2^{(k)}\bar{\sigma}^m[A_m^{(k)},\zeta_2^{(k)}] + \bar{\zeta}_3^{(k)}\bar{\sigma}^m[A_m^{(k)},\zeta_3^{(k)}] + \bar{\zeta}_4^{(k)}\bar{\sigma}^m[A_m^{(k)},\zeta_4^{(k)}]). \tag{2.16}$$

We then compactify the discretized direction, which truncates the sum, and also perform a discrete Fourier transform such that

$$\zeta^{(k)} = \frac{1}{\sqrt{N}}\sum_{p=[\frac{N}{2}]-N+1}^{[\frac{N}{2}]}\eta^{(p)}q^{kp} \quad\text{and}\quad \bar{\zeta}^{(k)} = \frac{1}{\sqrt{N}}\sum_{p=[\frac{N}{2}]-N+1}^{[\frac{N}{2}]}\bar{\eta}^{(p)}q^{-kp}, \tag{2.17}$$

which lets us write the $\tilde{\partial}_4\zeta^{(k)}$ derivatives as

$$\tilde{\partial}_4\zeta^{(k)} = \frac{1}{\sqrt{N}a}\sum_s\eta^{(s)}q^{ks}(q^s-1). \tag{2.18}$$

The final answer for the kinetic and mass terms is

$$S_{\text{5D-Disc}}^{\text{F-Kin}} = \frac{a}{g_{\text{YM}}^2}\int d^4x\sum_s\text{Tr}\Big[-\frac{i}{a}(1-q^{-s})(\eta_3^{(s)}\eta_1^{(-s)} - \bar{\eta}_3^{(-s)}\bar{\eta}_1^{(s)} + \eta_4^{(s)}\eta_2^{(-s)} - \bar{\eta}_4^{(-s)}\bar{\eta}_2^{(s)} + i\bar{\eta}_1^{(s)}\bar{\sigma}^m\partial_m\eta_1^{(s)}$$
$$+ i\bar{\eta}_2^{(s)}\bar{\sigma}^m\partial_m\eta_2^{(s)} + i\bar{\eta}_3^{(s)}\bar{\sigma}^m\partial_m\eta_3^{(s)} + i\bar{\eta}_4^{(s)}\bar{\sigma}^m\partial_m\eta_4^{(s)})\Big] + \frac{a}{g_{\text{YM}}^2\sqrt{N}}\int d^4x\sum_{s,s'}\text{Tr}(\bar{\eta}_1^{(s)}\bar{\sigma}^m[B_m^{(s-s')},\eta_1^{(s')}]$$
$$+ \bar{\eta}_2^{(s)}\bar{\sigma}^m[B_m^{(s-s')},\eta_2^{(s')}] + \bar{\eta}_3^{(s)}\bar{\sigma}^m[B_m^{(s-s')},\eta_3^{(s')}] + \bar{\eta}_4^{(s)}\bar{\sigma}^m[B_m^{(s-s')},\eta_4^{(s')}] + [\eta_1^{(s)},\eta_3^{(s')}]Y_6^{(-s-s')}$$
$$- [\bar{\eta}_1^{(s)},\bar{\eta}_3^{(s')}]Y_6^{(s+s')} + [\eta_2^{(s)},\eta_4^{(s')}]Y_6^{(-s-s')} - [\bar{\eta}_2^{(s)},\bar{\eta}_4^{(s')}]Y_6^{(s+s')}). \tag{2.19}$$

The Yukawa interactions are dealt with in a similar way. We will once again discuss the sample term (2.15). Upon discretizing we get

$$S_{\text{5D-Disc}}^{\text{F-Int}} = -\frac{ia}{g_{\text{YM}^2}}\int d^4x\sum_{k=-\infty}^{\infty}\text{Tr}([\zeta_1^{(k)},\zeta_3^{(k)}]X^{5(k)} + [\bar{\zeta}_1^{(k)},\bar{\zeta}_3^{(k)}]X_5^{(k)} - [\zeta_2^{(k)},\zeta_4^{(k)}]X_5^{(k)} - [\bar{\zeta}_2^{(k)},\bar{\zeta}_4^{(k)}]X_5^{(k)}). \tag{2.20}$$

After compactifying and Fourier transforming we end up with

$$S_{\text{5D-Disc}}^{\text{F-Int}} = -\frac{ia}{g_{\text{YM}}^2\sqrt{N}}\sum_{s,s'}\text{Tr}\int d^4x[([\bar{\eta}_1^{(s)},\bar{\eta}_3^{(s')}] - [\bar{\eta}_2^{(s)},\bar{\eta}_4^{(s')}])Y_5^{(s+s')} + ([\eta_1^{(s)},\eta_3^{(s')}] - [\eta_2^{(s)},\eta_4^{(s')}])Y_5^{(-s-s')}] \tag{2.21}$$

and similar expressions for other interaction terms involving different scalar components. This concludes our discussion of 5D MSYM on a discretized circle.

### D. Deconstruction: setup

We now turn to the deconstruction picture. As the four-dimensional starting point we will use the $\mathcal{N}=2$ superconformal $A_N$ (circular) quiver theory, in the large $N$ limit. The expectation from [3] is to obtain, upon Higgsing, a theory with enhanced supersymmetry in 5D.

The full action of the $\mathcal{N}=2$ $A_N$ quiver theory, written in terms of $\mathcal{N}=1$ superfields, is given by

$$S_{4D} = \sum_{i=[\frac{N}{2}]-N+1}^{[\frac{N}{2}]} \mathrm{tr} \int d^4x \Big[ \frac{1}{8\pi} \mathrm{Im}\Big( \tau \int d^2\theta W^{\alpha(i)} W^{(i)}_{\alpha} \Big) - \int d^2\theta d^2\bar{\theta} e^{2V^{(i)}} \Phi^{\dagger(i)} e^{-2V^{(i)}} \Phi^{(i)} - \int d^2\theta d^2\bar{\theta} e^{2V^{(i+1)}} Q^{\dagger(i)} e^{-2V^{(i)}} Q^{(i)} - \int d^2\theta d^2\bar{\theta} e^{-2V^{(i+1)}} \tilde{Q}^{(i)} e^{2V^{(i)}} \tilde{Q}^{\dagger(i)} + \int d^2\theta \mathcal{W}^{(i)} + \int d^2\bar{\theta} \bar{\mathcal{W}}^{(i)} \Big], \tag{2.22}$$

with $\tau = \theta/2\pi + 4\pi i/G^2$, where $G$ is the four-dimensional gauge coupling. Note that the range of the sum (i.e. the labeling of the nodes of the quiver) has been conveniently chosen so as to match the discrete mode expansion of the previous sections. With that in mind, we will again suppress the sum ranges from now on, for brevity.

Each node has an SU($K$) gauge field and is connected to its neighbours by bifundamental and anti-bifundamental matter fields. The trace should accordingly be thought of as being over each term in the respective representation of SU$(K)^{(i)}$. The superpotential encodes the matter structure and is given by

$$\mathcal{W}^{(i)} = -i\sqrt{2} G \,\mathrm{tr}[\tilde{Q}^{(i)} \Phi^{(i)} Q^{(i)} - Q^{(i)} \Phi^{(i+1)} \tilde{Q}^{(i)}]. \tag{2.23}$$

In terms of components,[2] the bosonic part of the action is then

$$S^B_{4D} = \sum_i \int d^4x \,\mathrm{tr}\Big[ -\frac{1}{4G^2} F^{(i)}_{mn} F^{(i)mn} - D_m \Phi^{(i)} D^m \Phi^{(i)\dagger} - D_m Q^{(i)} D^m Q^{(i)\dagger} - D_m \tilde{Q}^{(i)} D^m \tilde{Q}^{(i)\dagger} - V_S \Big], \tag{2.24}$$

where $m, n = 0, \ldots, 3$ and the covariant derivatives are defined as

$$\begin{aligned} D_m \Phi^{(i)} &= \partial_m \Phi^{(i)} - i[A^{(i)}_m, \Phi^{(i)}] \\ D_m Q^{(i)} &= \partial_m Q^{(i)} - i A^{(i)}_m Q^{(i)} + i Q^{(i)} A^{(i+1)}_m \\ D_m \tilde{Q}^{(i)} &= \partial_m \tilde{Q}^{(i)} - i A^{(i+1)}_m \tilde{Q}^{(i)} + i \tilde{Q}^{(i)} A^{(i)}_m. \end{aligned} \tag{2.25}$$

The scalar potential $V_S$ is given by

$$V_S = V_F + V_D, \tag{2.26}$$

where

$$\begin{aligned} V_F &= \sum_i \mathrm{tr}(F^{\dagger}_{Q^{(i)}} F_{Q^{(i)}} + F^{\dagger}_{\tilde{Q}^{(i)}} F_{\tilde{Q}^{(i)}} + F^{\dagger}_{\Phi^{(i)}} F_{\Phi^{(i)}}), \\ V_D &= \frac{G^2}{2} \sum_i D^{(i)A} D^{(i)}_A, \end{aligned} \tag{2.27}$$

with $A$ an adjoint gauge symmetry index. In turn, one has that

$$\begin{aligned} F_{Q^{(i)}} &= -i\sqrt{2} G (\tilde{Q}^{(i)} \Phi^{(i)} - \Phi^{(i+1)} \tilde{Q}^{(i)}) \\ F_{\tilde{Q}^{(i)}} &= -i\sqrt{2} G (\Phi^{(i)} Q^{(i)} - Q^{(i)} \Phi^{(i+1)}) \\ F_{\Phi^{(i)}} &= -i\sqrt{2} G (Q^{(i)} \tilde{Q}^{(i)} - \tilde{Q}^{(i-1)} Q^{(i-1)}), \end{aligned} \tag{2.28}$$

for the $F$-terms and

$$\begin{aligned} D^{(i)A} = \mathrm{tr}[T^A ([\Phi^{(i)}, \Phi^{(i)\dagger}] + Q^{(i)} Q^{(i)\dagger} - \tilde{Q}^{(i)\dagger} \tilde{Q}^{(i)} \\ - Q^{(i-1)\dagger} Q^{(i-1)} + \tilde{Q}^{(i-1)} \tilde{Q}^{(i-1)\dagger})], \end{aligned} \tag{2.29}$$

for the $D$-terms. Note that since we are working with SU($K$) gauge groups, the $D$-term potential involves both single and double-trace terms coming from

$$(T^A)^i{}_j (T^A)^k{}_l = \delta^i_l \delta^k_j - \frac{1}{K} \delta^i_j \delta^k_l, \tag{2.30}$$

where our normalization for the generators is $\mathrm{tr}(T^A T^B) = \delta^{AB}$.

The fermionic part of the four-dimensional theory is given in component form by the expression

$$\begin{aligned} S^F_{4D} = \sum_i \mathrm{tr} \int d^4x \Big[ & \frac{i}{G^2} \bar{\lambda}^{(i)} \bar{\sigma}^m D_m \lambda^{(i)} + i \bar{\chi}^{(i)} \bar{\sigma}^m D_m \chi^{(i)} + i \bar{\psi}^{(i)} \bar{\sigma}^m D_m \psi^{(i)} + i \bar{\tilde{\psi}}^{(i)} \bar{\sigma}^m D_m \tilde{\psi}^{(i)} \\ & - i\sqrt{2} (\bar{\lambda}^{(i+1)} \bar{\psi}^{(i)} - \bar{\psi}^{(i)} \bar{\lambda}^{(i)}) Q^{(i)} - i\sqrt{2} (\lambda^{(i)} \psi^{(i)} - \psi^{(i)} \lambda^{(i+1)}) Q^{(i)\dagger} \\ & - i\sqrt{2} (\lambda^{(i+1)} \tilde{\psi}^{(i)} - \tilde{\psi}^{(i)} \lambda^{(i)}) \tilde{Q}^{(i)\dagger} - i\sqrt{2} (\bar{\lambda}^{(i)} \bar{\tilde{\psi}}^{(i)} - \bar{\tilde{\psi}}^{(i)} \bar{\lambda}^{(i+1)}) \tilde{Q}^{(i)} \\ & - i\sqrt{2} G (\tilde{\psi}^{(i)} \chi^{(i)} - \chi^{(i+1)} \tilde{\psi}^{(i)}) Q^{(i)} + i\sqrt{2} G (\bar{\chi}^{(i)} \bar{\tilde{\psi}}^{(i)} - \bar{\tilde{\psi}}^{(i)} \bar{\chi}^{(i+1)}) Q^{(i)\dagger} - i\sqrt{2} G (\chi^{(i)} \psi^{(i)} - \psi^{(i)} \chi^{(i+1)}) \tilde{Q}^{(i)} \\ & + i\sqrt{2} G (\bar{\psi}^{(i)} \bar{\chi}^{(i)} - \bar{\chi}^{(i+1)} \bar{\psi}^{(i)}) \tilde{Q}^{(i)\dagger} - i\sqrt{2} G (\psi^{(i)} \tilde{\psi}^{(i)} - \tilde{\psi}^{(i-1)} \psi^{(i-1)}) \Phi^{(i)} \\ & + i\sqrt{2} G (\bar{\tilde{\psi}}^{(i)} \bar{\psi}^{(i)} - \bar{\psi}^{(i-1)} \bar{\tilde{\psi}}^{(i-1)}) \Phi^{(i)\dagger} - i\sqrt{2} [\bar{\lambda}^{(i)}, \bar{\chi}^{(i)}] \Phi^{(i)} - i\sqrt{2} [\lambda^{(i)}, \chi^{(i)}] \Phi^{(i)\dagger} \Big]. \end{aligned} \tag{2.31}$$

[2]We follow the conventions of [18]. The superfield expansions can be found in the Appendix.

### E. Deconstruction: gauge fields

Deconstruction instructs us to expand the above theory around a real hypermultiplet vev, $\langle Q^{(i)}\rangle = \frac{v}{\sqrt{2}}\mathbb{1}_{K\times K}$. This leads to a Higgsing of the gauge group down to the diagonal subgroup $\mathrm{SU}(K)^N \to \mathrm{SU}(K)$; hence the trace (now denoted by Tr) will be over the latter gauge group.

Let us explicitly describe the setup of the calculation for the gauge fields. As a result of Higgsing (2.24), we get

$$S^{\text{B-Gauge}}_{\text{4D-Higgs}} = \frac{1}{G^2}\sum_i \int d^4x \,\mathrm{Tr}\Big[-\frac{1}{4}F^{(i)}_{mn}F^{(i)mn} - \frac{1}{2}v^2G^2(2A^{(i)}_m A^{(i)m} - A^{(i)}_m A^{(i+1)m} - A^{(i+1)}_m A^{(i)m})\Big]. \tag{2.32}$$

Note that the gauge fields have acquired a mass, but that the mass matrix is off-diagonal,

$$A^{(i)}M_{ij}A^{(j)}, \tag{2.33}$$

where

$$M = v^2G^2\begin{pmatrix} 2 & -1 & 0 & 0 & 0 & \dots & -1 \\ -1 & 2 & -1 & 0 & 0 & \dots & 0 \\ 0 & -1 & 2 & -1 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots & \vdots \\ 0 & \dots & 0 & 0 & -1 & 2 & -1 \\ -1 & \dots & 0 & 0 & 0 & -1 & 2 \end{pmatrix}. \tag{2.34}$$

In more compact notation the above can be expressed as

$$M = v^2G^2[2\mathbb{1}_{N\times N} - (\Omega + \Omega^{-1})], \tag{2.35}$$

with $\Omega_{ij} = \delta_{i+1,j}$ the so-called $N\times N$ "shift" matrix. The latter can be straightforwardly diagonalized into a "clock" matrix (see e.g. [19–21])

$$Q = \mathrm{diag}(q^{[\frac{N}{2}]-N+1}, \dots, q^{-1}, q^0, q, \dots, q^{[\frac{N}{2}]}), \tag{2.36}$$

with $q = e^{2\pi i/N}$. To be specific,

$$O^{-1}\Omega O = Q \quad \text{and} \quad O^{-1}\Omega^{-1}O = Q^{-1}. \tag{2.37}$$

The precise form of $O$ is given by

$$O_{ss'} = \frac{1}{\sqrt{N}}q^{ss'} \quad \text{and} \quad (O^{-1})_{ss'} = \frac{1}{\sqrt{N}}q^{-ss'}, \tag{2.38}$$

where the exponent on the RHS is a product of the two labels. Note that $O^\dagger = O^{-1}$.

We can use this to diagonalize the mass matrices

$$\tilde{M} = O^{-1}MO = v^2G^2[2\mathbb{1}_{N\times N} - (Q + Q^{-1})], \tag{2.39}$$

and the mass-matrix eigenvalues can be read off easily

$$\tilde{M}_{kk} = v^2G^2[2 - (e^{2\pi ik/N} + e^{-2\pi ik/N})] = v^2G^2\Big[2 - 2\cos\Big(\frac{2\pi k}{N}\Big)\Big] = 4v^2G^2\sin^2\Big(\frac{\pi k}{N}\Big). \tag{2.40}$$

In order to implement the above at the level of the action, one needs to redefine the gauge fields by the same matrix $O$, such that

$$A^{(i)} = \frac{1}{\sqrt{N}}q^{ij}B^{(j)} \quad \text{and} \quad A_{(i)} = \frac{1}{\sqrt{N}}q^{-ij}B^\dagger_{(j)}. \tag{2.41}$$

Note that the unitarity of $O$ and the reality of $A$ imply a reality condition for the $B$'s

$$\sum_j q^{-ij}B^\dagger_{(j)} = \sum_j q^{ij}B^{(j)} = \sum_j q^{-ij}B^{(-j)}, \tag{2.42}$$

where in the last step we have taken $j \to -j$ which does not affect the sum, and hence

$$B^\dagger_{(j)} = B^{(-j)}. \tag{2.43}$$

Then for the mass term appearing in (2.32) we have

$$A_{(i)}M^i_jA^{(j)} = B^\dagger_{(k)}O^{\dagger k}_i M^i_j O^j_l B^{(l)} = B^{(-k)}\tilde{M}_{kl}B^{(l)}, \tag{2.44}$$

while for the field strength

$$\begin{aligned}\sum_i F^{(i)}_{mn}F^{(i)mn} &= \sum_i[(\partial_mA^{(i)}_n - \partial_nA^{(i)}_m)^2 - 2i[A^{(i)}_m, A^{(i)}_n](\partial^mA^{(i)n} - \partial^nA^{(i)m}) - [A^{(i)}_m, A^{(i)}_n][A^{(i)m}, A^{(i)n}]] \\ &= \sum_s(\partial_mB^{(-s)}_n - \partial_nB^{(-s)}_m)(\partial^mB^{(s)n} - \partial^nB^{(s)m}) - \frac{2i}{N^{1/2}}\sum_{s,s'}[B^{(-s)}_m, B^{(-s')}_n](\partial^mB^{(s+s')n} - \partial^nB^{(s+s')m}) \\ &\quad - \frac{1}{N}\sum_{s,s',s''}[B^{(-s)}_m, B^{(-s')}_n][B^{(s'')m}, B^{(s+s'-s'')n}].\end{aligned} \tag{2.45}$$

In the intermediate steps of the above, one obtains sums similar to (2.8), some of which can be explicitly performed.

Putting everything together, we arrive at the final answer for the gauge fields,

$$S^{\text{B-Gauge}}_{\text{4D-Higgs}} = \frac{1}{G^2}\int d^4x\,\mathrm{Tr}\Big[-\frac{1}{4}\sum_s(\partial_mB^{(-s)}_n - \partial_nB^{(-s)}_m)(\partial^mB^{(s)n} - \partial^nB^{(s)m}) + \frac{i}{2N^{1/2}}\sum_{s,s'}[B^{(-s)}_m, B^{-(s')}_n](\partial^mB^{(s+s')n} - \partial^nB^{(s+s')m}) + \frac{1}{4N}\sum_{s,s',s''}[B^{(-s)}_m, B^{(-s')}_n][B^{(s'')m}, B^{(s+s'-s'')n}] - \frac{1}{2}(4v^2G^2)\sum_s\sin^2\Big(\frac{\pi s}{N}\Big)B^{(-s)}_mB^{(s)m}\Big]. \tag{2.46}$$

### F. Deconstruction: scalar fields

We continue by considering the scalar field terms in the action. In particular we have upon Higgsing (2.28)

$$\begin{aligned} F_{Q^{(i)}} &= -i\sqrt{2}G(\tilde{Q}^{(i)}\Phi^{(i)} - \Phi^{(i+1)}\tilde{Q}^{(i)}) \\ F_{\tilde{Q}^{(i)}} &= -iv G(\Phi^{(i)} - \Phi^{(i+1)}) - i\sqrt{2}G(\Phi^{(i)}Q^{(i)} - Q^{(i)}\Phi^{(i+1)}) \\ F_{\Phi^{(i)}} &= -iv G(\tilde{Q}^{(i)} - \tilde{Q}^{(i-1)}) - i\sqrt{2}G(Q^{(i)}\tilde{Q}^{(i)} - \tilde{Q}^{(i-1)}Q^{(i-1)}) \end{aligned} \tag{2.47}$$

and from (2.29)

$$\begin{aligned} D^{(i)A} = \mathrm{Tr}\Big[ T^A\Big([\Phi^{(i)}, \Phi^{(i)\dagger}] - \tilde{Q}^{(i)\dagger}\tilde{Q}^{(i)} + \tilde{Q}^{(i-1)}\tilde{Q}^{(i-1)\dagger} \\ + Q^{(i)}Q^{(i)\dagger} - Q^{(i-1)\dagger}Q^{(i-1)} + \frac{v}{\sqrt{2}}(Q^{(i)} + Q^{(i)\dagger}) \\ - \frac{v}{\sqrt{2}}(Q^{(i-1)} + Q^{(i-1)\dagger})\Big)\Big]. \end{aligned} \tag{2.48}$$

The covariant derivatives will give

$$\begin{aligned} D_m\Phi^{(i)} &= \partial_m\Phi^{(i)} - i[A_m^{(i)}, \Phi^{(i)}] \\ D_m Q^{(i)} &= \partial_m Q^{(i)} - \frac{i}{\sqrt{2}}v(A_m^{(i)} - A_m^{(i+1)}) - iA_m^{(i)}Q^{(i)} + iQ^{(i)}A_m^{(i+1)} \\ D_m\tilde{Q}^{(i)} &= \partial_m\tilde{Q}^{(i)} - iA_m^{(i+1)}\tilde{Q}^{(i)} + i\tilde{Q}^{(i)}A_m^{(i)}. \end{aligned} \tag{2.49}$$

Combining the above will lead to a variety of mass and interaction terms in addition to contributions coming from the kinetic terms. Similar to the gauge field example, the mass matrices can be diagonalized by working with redefined fields

$$\Phi^{(i)} = \frac{1}{\sqrt{N}}q^{ij}\hat{\Phi}^{(j)} \quad \tilde{Q}^{(i)} = \frac{1}{\sqrt{N}}q^{ij}\hat{\tilde{Q}}^{(j)} \quad Q^{(i)} = \frac{1}{\sqrt{N}}q^{ij}\hat{Q}^{(j)}. \tag{2.50}$$

At this stage we would like to bring the reader's attention to the following fact: in the subsequent calculation one finds that for cubic and quartic interactions involving matter fields with different node indices there is disagreement with the discretized 5D description for generic values of $N$. This is no cause for concern since we have already mentioned that the prescription of [3,16] requires large $N$. In fact, in the large-$N$ limit there is a simplification arising from the redefinitions (2.50). Note that in terms of the hatted fields, one has e.g.

$$\tilde{Q}^{(i)}\tilde{Q}^{(i-1)} = q^{ij}q^{(i-1)k}\hat{\tilde{Q}}^{(j)}\hat{\tilde{Q}}^{(k)} \simeq q^{ij}q^{ik}\hat{\tilde{Q}}^{(j)}\hat{\tilde{Q}}^{(k)}, \tag{2.51}$$

for each fixed $k \ll N$. Thus, provided that we restrict attention to processes involving KK modes up to some finite level $L$, there is no difference between $\tilde{Q}^{(i)}\tilde{Q}^{(i-1)}$ and $\tilde{Q}^{(i)}\tilde{Q}^{(i)}$ to leading order in $N \gg L$. Hence, ignoring all $1/N$ corrections, one can write

$$\begin{aligned} F_{Q^{(i)}} &\simeq -i\sqrt{2}G[\tilde{Q}^{(i)}, \Phi^{(i)}] \\ F_{\tilde{Q}^{(i)}} &\simeq -iv G(\Phi^{(i)} - \Phi^{(i+1)}) - i\sqrt{2}G[\Phi^{(i)}, Q^{(i)}] \\ F_{\Phi^{(i)}} &\simeq -iv G(\tilde{Q}^{(i)} - \tilde{Q}^{(i-1)}) - i\sqrt{2}G[Q^{(i)}, \tilde{Q}^{(i)}] \end{aligned} \tag{2.52}$$

and

$$\begin{aligned} D^{(i)A} \simeq \mathrm{Tr}\Big[ T^A\Big([\Phi^{(i)}, \Phi^{(i)\dagger}] + [\tilde{Q}^{(i)}, \tilde{Q}^{(i)\dagger}] + [Q^{(i)}, Q^{(i)\dagger}] \\ + \frac{v}{\sqrt{2}}(Q^{(i)} + Q^{(i)\dagger}) - \frac{v}{\sqrt{2}}(Q^{(i)} + Q^{(i)\dagger})\Big)\Big]. \end{aligned} \tag{2.53}$$

Moreover, the covariant derivatives will now be

$$\begin{aligned} D_m\Phi^{(i)} &= \partial_m\Phi^{(i)} - i[A_m^{(i)}, \Phi^{(i)}] \\ D_m Q^{(i)} &\simeq \partial_m Q^{(i)} - i[A_m^{(i)}, Q^{(i)}] - \frac{i}{\sqrt{2}}v(A_m^{(i)} - A_m^{(i+1)}) \\ D_m\tilde{Q}^{(i)} &\simeq \partial_m\tilde{Q}^{(i)} - i[A_m^{(i)}, \tilde{Q}^{(i)}], \end{aligned} \tag{2.54}$$

and to leading order in $1/N$, the bifundamental scalars behave as adjoints of the diagonal SU($K$).

The above simplification also dictates that to leading order we can ignore both the trace parts of $Q$ and $\tilde{Q}$ as well as the double-trace terms coming from the $D$-terms: First note that the commutator structure of the $F$- and $D$-terms above is going to eliminate the trace part of the $Q$s and $\tilde{Q}$s. Furthermore, any double-trace expressions coming from the second term of $(T^A)^i{}_j(T^A)^k{}_l = \delta^i_l\delta^k_j - \frac{1}{K}\delta^i_j\delta^k_l$ in the $D$-term potential are also going to vanish.

With this in mind, we can treat the $\Phi$, $Q$ and $\tilde{Q}$ on equal footing. It will be useful to express the complex scalars in terms of their real and imaginary parts. So we write

$$\begin{aligned} \hat{\Phi}^{(i)} &= \frac{1}{G\sqrt{2}}(Y_1^{(i)} - iY_2^{(i)}) \\ \hat{\tilde{Q}}^{(i)} &= \frac{1}{G\sqrt{2}}(Y_3^{(i)} - iY_4^{(i)}) \\ \hat{Q}^{(i)} &= \frac{1}{G\sqrt{2}}(Y_5^{(i)} - iY_6^{(i)}). \end{aligned} \tag{2.55}$$

Now, consider terms involving only the adjoint scalars $\Phi$. The only contribution to their mass is going to come from the $F$-term potential, while the quartic interaction will come from the $D$-term,

$$\begin{aligned} S^{\mathrm{B}\text{-}\Phi}_{\text{4D-Higgs}} = \sum_i \int d^4x\, \mathrm{Tr}\Big[ -D_m\Phi^{(i)}D^m\Phi^{(i)\dagger} \\ - \frac{G^2}{2}[\Phi^{(i)}, \Phi^{(i)\dagger}]^2 - v^2G^2(2\Phi^{(i)}\Phi^{(i)\dagger} \\ - \Phi^{(i)}\Phi^{(i+1)\dagger} - \Phi^{(i+1)}\Phi^{(i)\dagger})\Big]. \end{aligned} \tag{2.56}$$

This expression is identical in structure to the one for the gauge fields (2.32) and we can proceed analogously. Our redefinition in terms of hatted fields diagonalizes the mass matrix, and in terms of real components one obtains, e.g. for the real part of $\Phi$,

$$S^{\text{B-}Y_1}_{\text{4D-Higgs}} = \frac{1}{G^2}\int d^4x\,\mathrm{Tr}\Big[-\frac{1}{2}\sum_s \partial_m Y_1^{(-s)}\partial^m Y_1^{(s)} + \frac{i}{N^{1/2}}\sum_{s,s'}[B_m^{(-s)}, Y_1^{(-s')}]\partial^m Y_1^{(s+s')}$$
$$+\frac{1}{2N}\sum_{s,s',s''}[B_m^{(-s)}, Y_1^{(-s')}][B^{(s'')m}, Y_1^{(s+s'-s'')}] + \frac{1}{4N}\sum_{s,s',s''}[Y_1^{(-s)}, Y_1^{(-s')}][Y_1^{(s'')}, Y_1^{(s+s'-s'')}]$$
$$-\frac{1}{2}(4v^2G^2)\sum_s \sin^2\Big(\frac{\pi s}{N}\Big)Y_1^{(-s)}Y_1^{(s)}\Big]. \tag{2.57}$$

Since at leading order in $1/N$ we can treat $\Phi$, $Q$ and $\tilde{Q}$ similarly, it is straightforward to evaluate the rest of the scalar terms. The only point of special interest is that the field $Y_6$ does not pick up a mass during the Higgsing process and there is an asymmetry between the $A = I$ and $A = 6$ terms. The final result is

$$S^{\text{B-Scalars}}_{\text{4D-Higgs}} = \frac{1}{G^2}\int d^4x\,\mathrm{Tr}\Big[-\frac{1}{2}\sum_s \partial_m Y_A^{(-s)}\partial^m Y_A^{(s)} + \frac{i}{N^{1/2}}\sum_{s,s'}[B_m^{(-s)}, Y_A^{(-s')}]\partial^m Y_A^{(s+s')}$$
$$+\frac{1}{2N}\sum_{s,s',s''}[B_m^{(-s)}, Y_A^{(-s')}][B^{(s'')m}, Y_A^{(s+s'-s'')}] + \frac{1}{4N}\sum_{s,s',s''}[Y_A^{(-s)}, Y_B^{(-s')}][Y_A^{(s'')}, Y_B^{(s+s'-s'')}]$$
$$-\frac{1}{4NG^2}\int d^4x\sum_{s,s',s''}[Y_A^{(s)}, Y_B^{(s')}][Y_A^{(-s'')}, Y_B^{(s''-s-s')}]$$
$$+\frac{v}{G}\int d^4x\sum_s \partial_m Y_6^{(s)}B^{(-s)m}(q^{-s}-1) - \frac{iv}{\sqrt{N}G}\int d^4x\sum_{s,s'}[B_m^{(s)}, Y_6^{(s')}]B^{(-s-s')m}(q^{-s-s'}-1)$$
$$+\frac{iv}{\sqrt{N}G}\sum_{s,s'}[Y_6^{(s)}, Y_I^{(s')}]Y_I^{(-s-s')}(q^{-s-s'}-1) - \frac{1}{2}(4v^2G^2)\sum_s \sin^2\Big(\frac{\pi s}{N}\Big)Y_I^{(-s)}Y_I^{(s)}\Big]. \tag{2.58}$$

### G. Deconstruction: fermions

We now proceed to study the effect of expanding the fermionic part of the action (2.31) around $\langle Q^{(i)}\rangle = \frac{v}{\sqrt{2}}\mathbb{1}_{K\times K}$. This gives rise to the following mass terms:

$$S^{\text{F-mass}}_{\text{4D-Higgs}} = \sum_i \mathrm{Tr}\int d^4x[-iv(\bar{\lambda}^{(i+1)}\bar{\psi}^{(i)} - \bar{\psi}^{(i)}\bar{\lambda}^{(i)}) - iv(\lambda^{(i)}\psi^{(i)} - \psi^{(i)}\lambda^{(i+1)}) - ivG(\tilde{\psi}^{(i)}\chi^{(i)} - \chi^{(i+1)}\tilde{\psi}^{(i)})$$
$$- ivG(\bar{\tilde{\psi}}^{(i)}\bar{\chi}^{(i+1)} - \bar{\chi}^{(i)}\bar{\tilde{\psi}}^{(i)})] \tag{2.59}$$

In order to diagonalize the fermion mass matrices, define

$$(\lambda^{(i)}, \chi^{(i)}, \psi^{(i)}, \tilde{\psi}^{(i)}) = \frac{1}{G\sqrt{N}}\sum_s q^{is}(G\eta_1^{(s)}, \eta_2^{(s)}, \eta_3^{(s)}, \eta_4^{(s)}), \tag{2.60}$$

and note that the large-$N$ simplifications which we used in (2.51) will also apply for products of bifundamental fermions.

The fermion mass terms then become

$$S^{\text{F-Mass}}_{\text{4D-Higgs}} = -\frac{iv}{G}\,\mathrm{Tr}\int d^4x\sum_s(1-q^{-s})[\eta_1^{(-s)}\eta_3^{(s)} - \bar{\eta}_1^{(s)}\bar{\eta}_3^{(-s)} + \eta_2^{(-s)}\eta_4^{(s)} - \bar{\eta}_2^{(s)}\bar{\eta}_4^{(-s)}]. \tag{2.61}$$

For the fermion kinetic terms we have

$$S^{\text{F-Kin}}_{\text{4D-Higgs}} = \frac{i}{G^2}\sum_s \mathrm{Tr}\int d^4x[\bar{\eta}_1^{(s)}\bar{\sigma}^m\partial_m\eta_1^{(s)} + \bar{\eta}_2^{(s)}\bar{\sigma}^m\partial_m\eta_2^{(s)} + \bar{\eta}_3^{(s)}\bar{\sigma}^m\partial_m\eta_3^{(s)} + \bar{\eta}_4^{(s)}\bar{\sigma}^m\partial_m\eta_4^{(s)}]$$
$$+\frac{1}{G^2\sqrt{N}}\sum_{s,s'}\mathrm{Tr}\int d^4x[\bar{\eta}_1^{(s)}\bar{\sigma}^m[B_m^{(s-s')}, \eta_1^{(s')}] + \bar{\eta}_2^{(s)}\bar{\sigma}^m[B_m^{(s-s')}, \eta_2^{(s')}] + \bar{\eta}_3^{(s)}\bar{\sigma}^m(B_m^{(s-s')}\eta_3^{(s')} - q^{s-s'}\eta_3^{(s')}B_m^{(s-s')})$$
$$+\bar{\eta}_4^{(s)}\bar{\sigma}^m(q^{s-s'}B_m^{(s-s')}\eta_4^{(s')} - \eta_4^{(s')}B_m^{(s-s')})], \tag{2.62}$$

while the terms involving $\Phi$ become in the large-$N$ limit

$$S^{\text{F-Int-}\Phi}_{\text{4D-Higgs}} = \frac{1}{G^2\sqrt{N}}\sum_{s,s'}\text{Tr}\int d^4x[-i[\eta^{3(s)},\eta^{4(s')}](Y_1^{(-s-s')}+iY_2^{(-s-s')})+i[\bar\eta_4^{(s')},\bar\eta_3^{(s)}](Y_1^{(s+s')}-iY_2^{(s+s')}) - i[\bar\eta_1^{(s)},\bar\eta_2^{(s')}](Y_1^{(s+s')}+iY_2^{(s+s')})-i[\eta_1^{(s)},\eta_2^{(s')}](Y_1^{(-s-s')}-iY_2^{(-s-s')})]. \tag{2.63}$$

Finally, for the terms involving $Q$ we will have, again in the large-$N$ limit

$$S^{\text{F-Int-Q}}_{\text{4D-Higgs}} = -\frac{i}{G^2\sqrt{N}}\sum_{s,s'}\text{Tr}\int d^4x[([\bar\eta_1^{(s)},\bar\eta_3^{(s')}]-[\bar\eta_2^{(s)},\bar\eta_4^{(s')}])Y_5^{(s+s')}+([\eta_1^{(s)},\eta_3^{(s')}]-[\eta_2^{(s)},\eta_4^{(s')}])Y_5^{(-s-s')}] + \frac{1}{G^2\sqrt{N}}\sum_{s,s'}\text{Tr}\int d^4x[([\bar\eta_1^{(s)},\bar\eta_3^{(s')}]+[\bar\eta_2^{(s)},\bar\eta_4^{(s')}])Y_6^{(s+s')}-([\eta_1^{(s)},\eta_3^{(s')}]+[\eta_2^{(s)},\eta_4^{(s')}])Y_6^{(-s-s')}]. \tag{2.64}$$

and similar expressions for terms involving $\tilde{Q}$.

### H. Comparing discretized 5D MSYM to deconstruction and the (2,0) theory

Having obtained explicit expressions for all terms in both discretized and compactified 5D MSYM, as well as the Higgsed $\mathcal{N}=2$ $A_N$ quiver theory, we are now in a position to compare the two. We see that the kinetic and mass terms for the gauge field in the expressions (2.9) and (2.46) fix the relations

$$\frac{1}{G^2}=\frac{a}{g_{YM}^2} \quad \text{and} \quad G^2v^2=\frac{1}{a^2}, \tag{2.65}$$

since $|e^{2\pi is/N}-1|^2=4\sin^2(\frac{\pi s}{N})$. These further yield

$$a=\frac{1}{Gv} \quad \text{and} \quad g_{\text{YM}}^2=\frac{G}{v}. \tag{2.66}$$

It is straightforward to check that, with these identifications, all terms between the 5D and 4D calculations match exactly, that is (2.11) with (2.58) and (2.19) with (2.61), (2.62), and (2.21) with (2.64). Thus, we arrive at the conclusion that the $A_N$ quiver theory at large $N$ deconstructs 5D MSYM on a discretized circle with spacing $a$.[3]

During the course of the 4D calculation, we noted (and just confirmed) that in order to get agreement between the two descriptions we needed to ignore corrections of $\mathcal{O}(1/N)$. This is not surprising. The claim of deconstruction is not that one finds 5D MSYM exactly, at all scales. Rather, if we restrict to observables that only involve KK modes up to some level $L$ then correlation functions of these can be computed to arbitrary accuracy in the deconstructed theory by choosing $N \gg L$ [16].

[3]Alternatively, one can compare the deconstructed theory to 5D MSYM compactified on a circle without prescribing any kind of discretization but truncating the KK tower at level $N$. This leads to a four-dimensional action which is similar to the one we have obtained by discretization but with two differences. First, the mass spectrum of the fundamental fields is the familiar KK pattern of $M^2=L^2/R^2$. Second, one finds no powers of $q$ in the interaction terms. Nevertheless, the discussion that follows is similar.

Let us expand upon this point. For fixed $N$ we have two theories at hand. One is the superconformal quiver gauge theory with $N$ nodes and coupling $G^2=1/v^2a^2$. The other is the discretized 5D MSYM on a circle of radius $R_4=Na/2\pi$. This latter theory is a truncated version of the full 5D MSYM on a circle, analogous to a KK reduction keeping the first $N$ levels. It is a deformation of 4D MSYM obtained by adding a finite number of massive fields which form complete short $\mathcal{N}=4$ multiplets at each level. It is renormalizable and we view it as an effective field theory below the scale $N/R_4$. If we examine physical processes up to some scale $L/R_4$, we then expect the effect of any modes of 5D MSYM with energy above $N/R_4$ that we neglected in the discretization to be suppressed by powers of $L/N$.

Let us now compare the quantum theories arising from these two actions. We restrict attention to correlation functions of local operators composed from the fields that appear in the Lagrangians but only up to some scale $L/R_4$. The correlation functions obtained in the two theories will agree up to powers of $L/N$, arising from the differing interaction terms in the action. By taking $N \gg L$ we can match the computations of the discretized 5D MSYM arbitrarily well by using the quiver theory and in particular we could take the limit $N\to\infty$ with $R_4$ fixed. This allows us to compute a large class of local correlators of 5D MSYM on a circle of radius $R_4$ as a limit of the superconformal quiver gauge theories.

The deconstruction that we have just obtained is precisely the deconstruction of the (2,0) theory that was proposed in [3]. These authors noted that the four-dimensional SCFT has an SL$(2,\mathbb{Z})$ $S$-duality which maps $G\to 2\pi N/G$. Hence, in addition to the perturbative spectrum of states, arising from the deconstruction of the four-direction, with masses

$$M_{KK1}^2=4G^2v^2\sin^2\left(\frac{\pi L}{N}\right)=\frac{4}{a^2}\sin^2\left(\frac{\pi L}{N}\right) \quad L\in\mathbb{Z}, \tag{2.67}$$

there is also a dual tower of magnetically charged soliton states with

$$M_{KK2}^2 = 4\frac{(2\pi)^2 v^2 N^2}{G^2}\sin^2\left(\frac{\pi L}{N}\right) = \frac{4(2\pi)^2 N^2}{g_{YM}^4}\sin^2\left(\frac{\pi L}{N}\right)$$
$$L \in \mathbb{Z}. \tag{2.68}$$

In the limit $N \gg L$ these can be identified as two KK towers corresponding to compactification of a six-dimensional theory with radii $R_4 = Na/2\pi$ and $R_5 = g_{\rm YM}^2/4\pi^2$, both of which are freely adjustable parameters. In addition there will be a complete $SL(2,\mathbb{Z})$ invariant spectrum of states carrying both types of KK momenta. Thus the deconstruction argument of [3] shows that one cannot deconstruct 5D MSYM without also simultaneously deconstructing a six-dimensional theory with 16 supersymmetries and an SO(5) $R$-symmetry—presumably the (2,0) theory.

In the limit $a \to 0$, $N \to \infty$, with $R_4 \to \infty$ the 4D theory results are matched to uncompactified 5D MSYM with arbitrary value for the coupling $g_{\rm YM}^2$, which suggests that deconstruction in principle provides a quantum definition of 5D MSYM at all scales. Finally, the remaining tower of KK modes of masses $M_{KK2}$ is nothing but the instanton-soliton tower of [22,23]. This matches the content of the conjecture of [4,5].

We note that since 5D MSYM is not well defined (at least naively), we cannot claim that the deconstructed theory is 5D MSYM. Rather, our discussion shows that deconstruction provides a controlled definition of 5D MSYM. Our purpose here was to find a way of identifying parameters between the 4D and 5D descriptions and this approach has enabled us to do so in a natural way.

## III. THE DLCQ OF THE (2,0) THEORY AND 5D MSYM

We now shift gears and turn our attention to the (2,0) proposals of [1,2]. In order to compare the latter to the proposal that the (2,0) theory on $S^1$ is equivalent to 5D MSYM, we will need to quickly review the philosophy behind the infinite momentum frame (IMF) and the related discrete light-cone quantization (DLCQ). There are various outstanding conceptual and technical issues with the IMF, and especially DLCQ, which need to be addressed before one can claim to have a complete understanding of a theory that is defined using these methods. However, it is not our intention to resolve or discuss these issues here. Rather, we will accept the IMF and DLCQ prescriptions at face value and focus on the arguments leading to the proposal of [1,2].

### A. IMF and DLCQ

The basis for the IMF is that, since we are considering a Lorentz-invariant field theory, we can examine it in any frame we like. By a judicious choice of frame the physics might be simpler to analyze. To this end let us consider an M5-brane wrapped on an $S^1$ of radius $R_5$ and boost it along the compact $x^5$ direction. The energies and momenta transform as ($E = P_0$)

$$E' = \frac{1}{\sqrt{1-u^2}}(E - uP_5) \quad P_5' = \frac{1}{\sqrt{1-u^2}}(P_5 - uE)$$
$$P_i' = P_i, \tag{3.1}$$

where $i = 1, \ldots, 4$. If we introduce light-cone coordinates

$$x^{\pm} = \frac{1}{\sqrt{2}}(t \pm x^5), \qquad P_{\pm} = \frac{1}{\sqrt{2}}(E \pm P_5),$$

then (3.1) can be written as

$$P_+' = \sqrt{\frac{1-u}{1+u}}P_+ \quad P_-' = \sqrt{\frac{1+u}{1-u}}P_- \quad P_i' = P_i. \tag{3.2}$$

Let us write $u = (1-\epsilon^2)/(1+\epsilon^2)$ so that an infinite boost corresponds to $\epsilon \to 0$. This limit defines the IMF. In what follows we always only consider the term of leading order in $\epsilon$. We find that to leading order (3.1) becomes

$$P_+' = \epsilon P_+ \qquad P_-' = \frac{1}{\epsilon}P_- \qquad P_i' = P_i. \tag{3.3}$$

Thus if we view the original $S^1$ as an orbifold,

$$(t, x^i, x^5) \cong (t, x^i, x^5 + 2\pi R_5), \tag{3.4}$$

then in the IMF we have

$$(x^{+\prime}, x^{-\prime}, x^{i\prime}) \cong (x^{+\prime} + 2\pi R_+, x^{-\prime}, x^{i\prime}), \tag{3.5}$$

where

$$R_+ = R_5/\sqrt{2}\epsilon. \tag{3.6}$$

Next let us consider on-shell modes in the unboosted frame with momentum $P_5 = n/R_5$ for some integer $n$. These have energy

$$E = \sqrt{P_5^2 + P_i^2 + m^2} = \frac{|n|}{R_5}\left(1 + F\left(\frac{R_5^2 P_\perp^2}{n^2}\right)\right), \tag{3.7}$$

where we have denoted $P_\perp^2 = P_i^2 + m^2$ and $F(x) = \sqrt{1+x} - 1 = \frac{1}{2}x + \cdots$. Here $m$ allows for the possibility of massive states that can arise on the Coulomb branch. We see that

$$P_+' = \frac{|n| + n}{\sqrt{2}R_5}\epsilon + \frac{|n|\epsilon}{\sqrt{2}R_5}F\left(\frac{R_5^2 P_\perp^2}{n^2}\right)$$
$$P_-' = \frac{|n| - n}{\sqrt{2}\epsilon R_5} + \frac{|n|}{\sqrt{2}R_5\epsilon}F\left(\frac{R_5^2 P_\perp^2}{n^2}\right) \tag{3.8}$$
$$P_i' = P_i.$$

We find that modes with $n < 0$ have diverging $P_-'$ in the IMF and thus decouple. Therefore, we can simply look at the effective theory with these modes integrated out. This can be made arbitrarily precise by taking $\epsilon$ suitably small. Therefore, we restrict to $n > 0$ for which

$$P'_+ = \frac{\sqrt{2}n\epsilon}{R_5} + \frac{n\epsilon}{\sqrt{2}R_5} F\left(\frac{R_5^2 P_\perp^2}{n^2}\right)$$
$$P'_- = \frac{n}{\sqrt{2}R_5\epsilon} F\left(\frac{R_5^2 P_\perp^2}{n^2}\right) \tag{3.9}$$
$$P'_i = P_i.$$

Now in the original theory we have states with all values of $n$. However for fixed $R_5$, in the $\epsilon \to 0$ limit the finite momentum states are those that have large $n$, with $n\epsilon$ finite. This is the traditional IMF picture (as used e.g. in [24]) and is valid for any finite $R_5$ but takes $n \to \infty$. Physically this corresponds to the fact that the only modes left in the infinite momentum frame are those that were moving sufficiently fast against the boost so that they have finite velocity after the boost. Note also that for any given $\epsilon$ there are still infinitely many $n$ that must be included if one wishes to describe the full theory.

There is another possibility which is to take $R_5$ small with $R_+ = R_5/\sqrt{2}\epsilon$ fixed. This is the DLCQ construction and does not require large $n$.[4] Fixing $n$ here simply means truncating to a fixed momentum sector of the theory. One must then still allow $n$ to be arbitrary in order to describe the full theory.

In either case we find

$$P'_+ = \frac{\sqrt{2}n\epsilon}{R_5} \quad P'_- = -\frac{R_5}{2\sqrt{2}n\epsilon}(P_i^2 + m^2) \quad P'_i = P_i. \tag{3.10}$$

Note that we have the three parameters $R_5$, $n$ and $\epsilon$ and in the limit $\epsilon \to 0$ we have just one constraint, namely that $n\epsilon/R_5$ is fixed. To arrive at (3.10) from (3.9) we simply require that $P_\perp R_5/n \ll 1$ in the limit $\epsilon \to 0$.

### B. Application to the (2,0) theory

The DLCQ construction of [1,2] works as follows. In the limit that $R_5$ is small the (2,0) theory on $S^1$ is well described by weakly coupled 5D MSYM with coupling $g_{\rm YM}^2 = 4\pi^2 R_5$. As observed in [1,2], this is something of a miracle since Lagrangian field theories usually become strongly coupled when compactified on a small circle. In this limit states with $n$ units of momentum along the compact direction correspond to solitons that carry instanton number $n$. These states are heavy when $R_5$ is small so that keeping $P_\perp$ fixed means slow motion in the transverse directions. This is the Manton approximation [27] for solitons whereby the relevant degrees of freedom correspond to motion on the soliton moduli space. In the limit that $g_{\rm YM}^2 \sim R_5 \to 0$ all other interactions can be neglected. Thus we find that the theory reduces to motion on the moduli space of $n$ instantons.[5] In the IMF, this corresponds to the second possibility discussed at the end of the previous section: for fixed $R_+ = R_5/\sqrt{2}\epsilon$, sending $R_5 \to 0$ also requires $\epsilon \to 0$ and therefore the DLCQ description of the (2,0) theory is given by quantum mechanics on the $n$-instanton moduli space.

There also exists an alternative derivation directly from the (2,0) system of [28]. This system is essentially 5D MSYM covariantly embedded into 6 dimensions using a nondynamical vector field $C^\mu$. Choosing $C^\mu$ spacelike, $C^\mu = g_{\rm YM}^2 \delta_5^\mu$, leads to 5D MSYM along $x^0, \ldots, x^4$ with coupling $g_{\rm YM}^2$. However one can also consider a null embedding corresponding to an infinitely boosted D4-brane. Deferring to [29] for the details, we simply wish to observe here that for the choice $C^\mu = g^2 \delta_+^\mu$, where $g^2$ is an arbitrary parameter with dimensions of length, one finds

$$\mathcal{P}_+ = -\frac{4\pi^2 n}{g^2}$$
$$\mathcal{P}_- = \frac{1}{2g^2} g_{\alpha\beta} \partial_- m^\alpha \partial_- m^\beta \tag{3.11}$$
$$\mathcal{P}_i = \frac{1}{2g^2} \mathrm{Tr} \int d^4x F_{ij} F^j_-,$$

where $n$ is the instanton number, $g_{\alpha\beta}$ is the metric on the $n$-instanton moduli space with coordinates $m^\alpha$ and $F_{ij}$, $F_{i-}$ are obtained from the field strength of the instanton and are determined by the ADHM construction.[6]

We note that the derivation of (3.11) in [29] is exact, starting from the (2,0) system of [28], and does not require taking the limit of an infinite boost. Examining the spectrum of $\mathcal{P}_+$ shows that it can be identified with that of the (2,0) theory reduced on a null circle obtained by the identification

$$x^+ \cong x^+ + \frac{g^2}{2\pi}. \tag{3.12}$$

Comparing (3.11) with (3.10) we find that $\mathcal{P}_+ = P_+$ if we identify $R_+ = R_5/\sqrt{2}\epsilon = g^2/4\pi^2$. In addition (3.12) precisely matches (3.5). Thus for any finite value of $g^2$, we obtain the DLCQ picture of the (2,0) theory. In particular, finite $g^2$ requires that $R_5 = g_{\rm YM}^2/4\pi^2 \to 0$ as $\epsilon \to 0$ and so again we only need the extreme IR of 5D MSYM.

Having obtained the DLCQ at finite $R_+$ it would appear that we can arrange for $R_+ \to \infty$ in the limit that $R_5$, $\epsilon \to 0$, leading to a description of the uncompactified (2,0) theory. At this stage one needs to be careful: In a null compactification the radius of the $x^+$ identification is not physically meaningful by itself as a Lorentz boost can rescale $R_+$. But in the IMF, when we choose a specific frame with fixed $P_+ = n/R_+$, this can be done if we also scale $n \to \infty$ [26]. This gives the DLCQ description of the

[4] See e.g. [25,26] for DLCQ in the context of matrix theory.
[5] We will come back to the details of this derivation shortly.
[6] There is also a generalization that arises on the Coulomb branch, where one finds that $\mathcal{P}_- = \frac{1}{2g^2}(g_{\alpha\beta}(\partial_- m^\alpha - L^\alpha) \times (\partial_- m^\beta - L^\beta) + V)$. Here $V \propto g_{\alpha\beta} L^\alpha L^\beta$ and $L^\alpha$ is a triholomorphic Killing vector on the instanton moduli space [29].

(2,0) theory on $\mathbb{R}^{1,5}$ as the large-$n$ limit of superquantum mechanics on the instanton moduli space [1,2].

This conclusion is a puzzling feature of the DLCQ proposal, since we have somehow managed to define the six-dimensional (2,0) theory, which is the UV of 5D MSYM, by only using information contained in the extreme IR of 5D MSYM. The origin of this is the miracle mentioned above, namely that the (2,0) theory becomes weakly coupled when compactified on a circle with a small radius. This miracle is also important for $S$-duality of four-dimensional maximally supersymmetric Yang-Mills since it ensures that the four-dimensional coupling constant is given by the ratio of the two circle radii and hence that modular transformations, such as interchanging the two circles, map strong to weak coupling [30].

In addition, one needs to keep in mind that, on top of the usual concerns about the IMF and DLCQ, the instanton moduli space has singularities which must somehow be dealt with in the quantum theory. For example the authors of [1,2] provide one resolution in terms of turning on mild noncommutativity as a regulator,[7] in view of switching it back off at the end of any explicit calculation.

### C. The (2,0) theory in the IMF

Having reviewed the DLCQ proposal, let us instead consider an IMF prescription. If we assume that the (2,0) theory on $S^1$ is equivalent to 5D MSYM, then we can also consider a traditional IMF description of the (2,0) theory with a finite value for $R_5 > 0$ but large $n$.

First, what is 5D MSYM at large $n$ and low energies[8] described by? It is given by an expansion around the ground state in the $n$th soliton sector. At low velocities this leads to the Manton approximation of slow motion on the soliton moduli space [27]. For a recent detailed discussion on instanton-solitons of 5D MSYM see [31].

However, now we would like to see what happens to this description beyond the low-energy approximation. It is well known that the Manton approximation is not exact, although in the case of monopoles in four dimensions it can be shown to be valid as long as the moduli velocities remain small [32]. Nevertheless at finite $g^2_{\mathrm{YM}}$ the Manton approximation does not capture all the dynamics of 5D MSYM. For the particular example of monopoles in four dimensions there are estimates that the radiation produced in soliton scattering scales as the third or fifth power of the velocity [33,34]. We expect that the instanton-soliton solutions relevant for 5D MSYM will behave in a similar way: at nonzero velocity these effects can only be neglected in the strict $g^2_{\mathrm{YM}} \to 0$ limit. Therefore, the IMF picture does not simply reduce to quantum mechanics on the instanton moduli space for finite $R_5 = g^2_{\mathrm{YM}}/4\pi^2$. Rather, it contains an infinite number of additional radiation modes that represent fluctuations about the soliton.

One might hope that in the large-$n$ limit there could be a further suppression of the massive modes so that the Manton approximation is again valid. For example, the solitons become heavy at large $n$ so their centre-of-mass velocity must be bounded by $1/n$ to ensure that the momentum remains small. However, this seems unlikely to extend to all moduli as the large-$n$ moduli space contains configurations where the solitons are widely separated. It would then seem that various "light" massive modes, obtained in the small-$n$ moduli space, can be lifted to the large-$n$ moduli space. For instance, one can imagine configurations of well-separated solitons where only a few are moving, in which case their velocities are not required to be small to ensure that the total excitation energy of the system is small. Therefore, the radiation and other nonzero modes seem to be insensitive to the value of $n$ and we do not expect any additional suppression at large $n$.

Hence there does not appear to be any significant simplification by considering the (2,0) in the IMF as there was with DLCQ.

### D. A DLCQ of 5D MSYM

To complete the circle of ideas we can also consider a DLCQ of 5D MSYM and compare it to a compactified version of the proposal [1,2]. To this end, let us start with 5D MSYM and compactify on $x^4$ with radius $R_4$. To construct a DLCQ it is sufficient to only consider a small $R_4$. Compactifying 5D MSYM on $S^1$ with coupling $g^2_{\mathrm{YM}}$ we find 4D MSYM with coupling $G^2 = g^2_{\mathrm{YM}}/2\pi R_4$, coupled to a tower of KK modes. For small $R_4$ this is strongly coupled but if the proposal of [4,5] is true then 5D MSYM on $S^1$ admits an $S$-duality since it is the (2,0) theory on $S^1 \times S^1$. Evidence for this can be found in [7]. Alternatively, we could use the 4D quiver deconstruction approach as the quantum definition of 5D MSYM, as argued in Sec. II H. This has a built-in $S$-duality for the theory on a (discretized) circle.

Applying $S$-duality we arrive at weakly coupled 4D Yang-Mills with gauge coupling $G^2 \propto 2\pi R_4/g^2_{\mathrm{YM}} \to 0$, but where the tower of KK modes around $x^4$ are now given by their $S$-duals, which will be some sort of monopole states. What exactly are these? From the point of view of the (2,0) theory $S$-duality corresponds to swapping $x^4$ with $x^5$. Therefore $S$-duality takes momentum modes around $x^4$ to momentum modes around $x^5$. In 5D MSYM momentum modes around $x^5$ are given by instanton-solitons along the $\mathbb{R}^4$ spanned by $x^1, \ldots, x^4$. Compactifying this $\mathbb{R}^4$ to $\mathbb{R}^3 \times S^1$ leads to so-called calorons, namely instantons that are periodic along $x^4$ (monopoles are special cases of calorons that are invariant along $S^1$). As before we decompactify the theory by taking $n$, $R_4/\epsilon \to \infty$. Thus we find that the DLCQ of 5D MSYM on $\mathbb{R}^{1,4}$ is given by quantum

[7] A noncommutative deformation corresponds to blowing up the moduli space singularities.

[8] Here, low energies means small excitation energy above the BPS bound of the $n$ instanton sector.

mechanics of the caloron moduli space (see also [35] for a related discussion). Note that the use of $S$-duality was crucial for this argument. As we mentioned this is guaranteed in deconstructed theory. If we start with 5D MSYM on its own then $S$-duality on an $S^1$ of finite size is tantamount to assuming that it is the (2,0) theory on $S^1 \times S^1$ and therefore essentially assumes the conjecture of [4,5].

Let us compare this with the DLCQ proposal constructed above for the (2,0) theory on $\mathbb{R}^{1,5}$. To obtain a DLCQ of the (2,0) theory on $\mathbb{R}^{1,4} \times S^1$ we can take an orbifold that acts as $x^5 \cong x^5 + 2\pi R_5$ and hence need to consider instantons that are periodic: once again one is lead to the moduli space of calorons. Thus we find agreement between the DLCQ of the (2,0) theory on $\mathbb{R}^{1,4} \times S^1$ at finite radius and the DLCQ of 5D MSYM on $\mathbb{R}^{1,4}$ at finite coupling.

## IV. CONCLUSIONS

In this paper we have discussed the relationships between three proposals for the (2,0) theory: deconstruction [3], DLCQ [1,2] and 5D MSYM [4,5]. In particular we explicitly showed how deconstruction leads to the action of 5D MSYM. This provides a definition of the full 5D MSYM as a limit of a family of well-defined four-dimensional superconformal field theories. Furthermore we showed how the DLCQ construction is also consistent with the view that the (2,0) theory on $S^1$ is given by 5D MSYM by showing that they both lead to the same DLCQ of the (2,0) theory on a finite circle. This crucially assumed the $S$-duality property of the 5D theory, which is explicit when defined in terms of deconstruction.

A common feature of all these proposals is that they do not require any new states that do not appear in 5D MSYM to describe the (2,0) theory. This is the central observation in [4,5] and therefore this is compatible with both [1,2] and [3]. Conversely, these proposals provide some support to the claim that although 5D MSYM is perturbatively divergent [6] and power counting nonrenormalizable, it should not simply be viewed as the low-energy effective theory of the (2,0) theory on $S^1$ in the Wilsonian sense, meaning that some heavy states have been integrated out. Rather, the spectrum and interactions are those of the (2,0) theory. The proposals of [1–3] offer alternative methods for computing physical quantities beyond the techniques of traditional perturbative quantum field theory.

In addition there are still other ways that may be used to define the (2,0) theory starting from the conformal field theory of an arbitrary number of M2-branes [36]. For example, one can consider a large number of M2-branes that are blown up via a Myers effect into M5-branes wrapped on an $S^3$ of finite radius. It was argued in [37] that the resulting fluctuations of the M5-branes are given by 5D MSYM on $S^2$, where $S^3$ is viewed as a Hopf fibration over $S^2$. Furthermore 5D MSYM on a three-torus of finite size can be obtained from cubic arrays of M2-branes [38]. In principle all these proposals give definitions of 5D MSYM and the (2,0) theory on $S^1$. It remains to be seen if they are equivalent.

Finally, we should also mention some other recent proposals for the (2,0) theory which we did not discuss. One very interesting proposal is [39] which considers the (2,0) theory on $\mathbb{R} \times S^5$. Realizing $S^5$ as a Hopf bundle over $\mathbb{C}P^2$ one can then perform an $\mathbb{Z}_k$ orbifold that acts on $U(1)$ fiber. In the large $k$ limit one therefore finds the (2,0) theory is given by 5D MSYM on $\mathbb{R} \times \mathbb{C}P^2$. In this scenario the radius of $\mathbb{C}P^2$ determines the scale $g^2_{\mathrm{YM}}$ and $1/k$ plays the role of a dimensionless coupling constant. Furthermore, since $k$ is discrete, there is hope that perturbation theory about large $k$ is finite allowing one to extrapolate to small $k$.

In addition, there have been other recent conjectures for the (2,0) theory that focus on novel Lagrangian descriptions for the self-dual tensor directly in six dimensions [40] or five-dimensional models that include the KK towers of states [41,42]. It would be interesting to understand the relation of these papers to the conjectures we discuss here. In particular, since the proposals discussed here capture at least a significant portion of the dynamics of the (2,0) theory, these other proposals should be related to them in some concrete way.

## ACKNOWLEDGMENTS

We would like to thank T. Banks, J. Drummond, C. Grojean, A. Karch, G. Moore, A. Royston, D. Shih and especially O. Aharony for discussions and comments. C. P. is supported by the U.S. Department of Energy under Grant No. DE-FG02-96ER40959. M. S. S. is supported by an EURYI award of the European Science Foundation.

## APPENDIX

Here we collect our conventions for the gamma matrices used in Sec. II C. The Spin(1,4) gamma matrices are given by

$$\gamma^\mu = \left\{ \begin{pmatrix} 0 & i\sigma^m \\ i\bar{\sigma}^m & 0 \end{pmatrix}, \begin{pmatrix} -\mathbb{1}_{2\times 2} & 0 \\ 0 & \mathbb{1}_{2\times 2} \end{pmatrix} \right\}, \qquad C_5 = \begin{pmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{pmatrix}, \tag{A1}$$

with $m = 0, \ldots, 3$ and $\sigma^m = \{\mathbb{1}, \sigma^i\}$ and $\bar{\sigma}^m = \{\mathbb{1}, -\sigma^i\}$. They satisfy

$$\begin{aligned} &\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu} \qquad \gamma^0(\gamma^\mu)^\dagger\gamma^0 = \gamma^\mu \\ &\gamma^0\gamma^1\gamma^2\gamma^3\gamma^4 = -i \qquad C_5\gamma^\mu C_5^{-1} = (\gamma^\mu)^T \\ &(C_5\gamma^\mu)^T = -C_5\gamma^\mu \qquad C_5^T = -C_5. \end{aligned} \tag{A2}$$

On the other hand, the Hermitian Spin(5) gamma matrices are given by

$$\lambda^1 = \begin{pmatrix} \sigma^2 & 0 \\ 0 & -\sigma^2 \end{pmatrix}, \qquad \lambda^2 = \begin{pmatrix} -\sigma^1 & 0 \\ 0 & \sigma^1 \end{pmatrix}, \qquad \lambda^3 = \begin{pmatrix} 0 & \mathbb{1}_{2\times 2} \\ \mathbb{1}_{2\times 2} & 0 \end{pmatrix}, \qquad \lambda^4 = \begin{pmatrix} 0 & -i\mathbb{1}_{2\times 2} \\ i\mathbb{1}_{2\times 2} & 0 \end{pmatrix},$$
$$\lambda^5 = \begin{pmatrix} -\sigma^3 & 0 \\ 0 & \sigma^3 \end{pmatrix}, \qquad K = \begin{pmatrix} 0 & i\sigma^2 \\ i\sigma^2 & 0 \end{pmatrix} \tag{A3}$$

and satisfy

$$\{\lambda^I, \lambda^J\} = 2\delta^{IJ}, \qquad (\lambda^I)^\dagger = \lambda^I, \qquad \lambda^1\lambda^2\lambda^3\lambda^4\lambda^5 = 1 \qquad K^T = -K, \qquad (K\lambda^I)^T = -K\lambda^I. \tag{A4}$$

The conjugate fermions are defined as

$$\bar{\psi}_i \equiv \psi_i^\dagger \gamma^0 \tag{A5}$$

and satisfy the symplectic Majorana condition

$$\bar{\psi}_i = \psi_j^T C_5 K_i^j. \tag{A6}$$

We also give the superfield expansion, used in Sec. II D. In Wess-Zumino gauge, one has that

$$\begin{aligned} V^{(i)} &= -\theta\sigma^m\bar{\theta}A_m^{(i)} + i\theta^2\bar{\theta}\bar{\lambda}^{(i)} - i\bar{\theta}^2\theta\lambda^{(i)} + \frac{1}{2}\theta^2\bar{\theta}^2 D^{(i)} \\ W_\alpha^{(i)} &= -i\lambda_\alpha^{(i)} + \theta_\alpha D^{(i)} - \frac{i}{2}(\sigma^m\bar{\sigma}^n\theta)_\alpha F_{mn}^{(i)} + \theta^2(\sigma^m D_m\bar{\lambda}^{(i)})_\alpha \\ \Phi^{(i)} &= \Phi^{(i)} + i\theta\sigma^m\bar{\theta}\partial_m\Phi^{(i)} - \frac{1}{4}\theta^2\bar{\theta}^2\Phi^{(i)} + \sqrt{2}\theta\chi^{(i)} - \frac{i}{\sqrt{2}}\theta^2\partial_m\chi^{(i)}\sigma^m\bar{\theta} + \theta^2 F_{\Phi^{(i)}} \end{aligned} \tag{A7}$$

for the vector multiplets, while

$$\begin{aligned} Q^{(i)} &= Q^{(i)} + i\theta\sigma^m\bar{\theta}\partial_m Q^{(i)} - \frac{1}{4}\theta^2\bar{\theta}^2\Box Q^{(i)} + \sqrt{2}\theta\psi^{(i)} - \frac{i}{\sqrt{2}}\theta^2\partial_m\psi^{(i)}\sigma^m\bar{\theta} + \theta^2 F_{Q^{(i)}} \\ \tilde{Q}^{(i)\dagger} &= \tilde{Q}^{(i)\dagger} - i\theta\sigma^m\bar{\theta}\partial_m\tilde{Q}^{(i)\dagger} - \frac{1}{4}\theta^2\bar{\theta}^2\Box\tilde{Q}^{(i)\dagger} + \sqrt{2}\bar{\theta}\bar{\tilde{\psi}}^{(i)} + \frac{i}{\sqrt{2}}\bar{\theta}^2\theta\sigma^m\partial_m\bar{\tilde{\psi}}^{(i)} + \bar{\theta}^2 F_{\tilde{Q}^{(i)}}^\dagger \end{aligned} \tag{A8}$$

for the hypermultiplets, where we are using the same letter for the chiral superfields as for their scalar components in the hope that no confusion will arise.


[1] O. Aharony, M. Berkooz, S. Kachru, N. Seiberg, and E. Silverstein, Adv. Theor. Math. Phys. **1**, 148 (1998).
[2] O. Aharony, M. Berkooz, and N. Seiberg, Adv. Theor. Math. Phys. **2**, 119 (1998).
[3] N. Arkani-Hamed, A. G. Cohen, D. B. Kaplan, A. Karch, and L. Motl, J. High Energy Phys. 01 (2003) 083.
[4] M. R. Douglas, J. High Energy Phys. 02 (2011) 011.
[5] N. Lambert, C. Papageorgakis, and M. Schmidt-Sommerfeld, J. High Energy Phys. 01 (2011) 083.
[6] Z. Bern, J. J. Carrasco, L. J. Dixon, M. R. Douglas, M. von Hippel, and H. Johansson, Phys. Rev. D **87**, 025018 (2013).
[7] Y. Tachikawa, J. High Energy Phys. 11 (2011) 123.
[8] J. Kallen and M. Zabzine, J. High Energy Phys. 05 (2012) 125.
[9] K. Hosomichi, R.-K. Seong, and S. Terashima, Nucl. Phys. **B865**, 376 (2012).
[10] J. Kallen, J. Qiu, and M. Zabzine, J. High Energy Phys. 08 (2012) 157.
[11] H.-C. Kim and S. Kim, J. High Energy Phys. 05 (2013) 144.
[12] J. Kallen, J. Minahan, A. Nedelin, and M. Zabzine, J. High Energy Phys. 10 (2012) 184.
[13] A. Gustavsson, Phys. Lett. B **706**, 225 (2011).
[14] D. Bak and A. Gustavsson, J. High Energy Phys. 12 (2012) 099.
[15] H.-C. Kim, J. Kim, and S. Kim, arXiv:1211.0144.
[16] N. Arkani-Hamed, A. G. Cohen, and H. Georgi, Phys. Rev. Lett. **86**, 4757 (2001).
[17] S. Catterall, D. B. Kaplan, and M. Unsal, Phys. Rep. **484**, 71 (2009).
[18] L. Alvarez-Gaume and S. Hassan, Fortschr. Phys. **45**, 159 (1997).
[19] C. Csaki, J. Erlich, C. Grojean, and G. D. Kribs, Phys. Rev. D **65**, 015003 (2001).
[20] K. Hashimoto, T.-S. Tai, and S. Terashima, J. High Energy Phys. 04 (2009) 025.
[21] Y. Honma and S. Zhang, Prog. Theor. Phys. **123**, 449 (2010).
[22] M. Rozali, Phys. Lett. B **400**, 260 (1997).

[23] M. Berkooz, M. Rozali, and N. Seiberg, Phys. Lett. B **408**, 105 (1997).
[24] T. Banks, W. Fischler, S. Shenker, and L. Susskind, Phys. Rev. D **55**, 5112 (1997).
[25] L. Susskind, arXiv:hep-th/9704080.
[26] N. Seiberg, Phys. Rev. Lett. **79**, 3577 (1997).
[27] N. S. Manton, Phys. Lett. **110B**, 54 (1982).
[28] N. Lambert and C. Papageorgakis, J. High Energy Phys. 08 (2010) 083.
[29] N. Lambert and P. Richmond, J. High Energy Phys. 02 (2012) 013.
[30] E. Witten, arXiv:0905.2720.
[31] J. P. Allen and D. J. Smith, J. High Energy Phys. 02 (2013) 113.
[32] D. Stuart, Commun. Math. Phys. **166**, 149 (1994).
[33] N. Manton and T. Samols, Phys. Lett. B **215**, 559 (1988).
[34] P. Irwin, arXiv:hep-th/0004054.
[35] O. J. Ganor and S. Sethi, J. High Energy Phys. 01 (1998) 007.
[36] O. Aharony, O. Bergman, D. L. Jafferis, and J. Maldacena, J. High Energy Phys. 10 (2008) 091.
[37] N. Lambert, H. Nastase, and C. Papageorgakis, Phys. Rev. D **85**, 066002 (2012).
[38] I. Jeon, N. Lambert, and P. Richmond, J. High Energy Phys. 11 (2012) 100.
[39] H.-C. Kim and K. Lee, arXiv:1210.0853.
[40] C.-S. Chu and S.-L. Ko, J. High Energy Phys. 05 (2012) 028.
[41] P.-M. Ho, K.-W. Huang, and Y. Matsuo, J. High Energy Phys. 07 (2011) 021.
[42] F. Bonetti, T. W. Grimm, and S. Hohenegger, J. High Energy Phys. 05 (2013) 129.