Essays on Volatility in Open Economy Macroeconomics

Thesis submitted in partial fulfillment of the requirements of the degree of Doctor of Philosophy (Ph.D.) in Economics

by

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Statement of Originality

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Summary

The following Ph.D. thesis is the collection of three self-contained chapters, written as part of a research agenda that analyses volatility in open economy macroeconomics. Emerging markets in general, and Latin American economies in particular, are major producers of commodities that range from oil to mining and agricultural goods. Countries such as Argentina, Chile and Peru are net exporters of commodities and the production of the latter takes a representative share of their gross domestic product. Thus, small open economies are highly dependent on commodity prices, and their behaviour holds a very close relationship with these countries' national income. Commodity prices are -however- of a very volatile nature, and this volatility can have an impact on the macroeconomic performance of commodity-producing countries.

The terms of trade -the relative price of total exports in terms of total imports- is a more general concept used in open economy macroeconomics that is closely related to commodity prices. Chapters 1 and 2 in this thesis study the effects of the volatility of terms of trade in a small open economy using a dynamic stochastic general equilibrium model within the real business cycles framework. In the first chapter, a focus is placed on the link between the terms of trade and total factor productivity, especially in economies that are not diversified and cluster their production around a particular commodity. The second chapter allows for an importable goods sector in addition to the exportable goods counterpart. Under this

framework, the chapter emphasizes on the different dynamics that take place between producing sectors when shocks to the terms of trade occur.

Chapter 3 takes a slightly different approach, as it analyses the effects of commodity price volatility on the performance of a number of large companies in Latin America, by using a range of financial indicators such as leverage, liquidity and capital accumulation. This chapter is a good empirical complement to chapters 1 and 2, as it moves from terms of trade to commodities, and from aggregate economies to firm-level data. A short summary of each of the chapters is shown below.

Chapter 1: Total factor productivity, terms of trade and time-varying volatility in a small open economy, shows that changes in the volatility of total factor productivity in a small open economy have a negative effect on consumption, investment and output. I show this by solving a dynamic, stochastic general equilibrium model driven by a process of stochastic volatility. This process entails two types of shocks: a shock to productivity and a shock to the volatility around productivity. The model can replicate the stylized facts of small open economies and shows that productivity shocks are expansionary while volatility shocks are contractionary. When the economy is oriented towards the production of an exportable commodity, productivity shocks are closely related to the terms of trade, such that the economy is driven by shocks to the terms of trade and its volatility.

Chapter 2: Terms of trade shocks, time-varying volatility and intersectoral dynamics in a small open economy, shows that shocks to the volatility of the terms of trade have a negative long-run effect on output, consumption and investment in a commodity-producing small open economy. I show this by solving a dynamic, stochastic general equilibrium model with two producing sectors driven by a stochastic volatility process on the terms of trade. The model can replicate the negative relationship between output and the terms of trade observed in the Chilean economy, as well as the Obstfeld-Razin-Svensson effects by which favorable terms of trade are associated with a deficit in the trade balance. Terms of trade shocks lead to a reallocation of resources towards the exportables sector, but volatility shocks have a negative long-run effect in the aggregate economy.

Chapter 3: Commodity price volatility and investment dynamics: a firm-level study, shows that commodity price volatility affects the investment decisions of firms across sectors in small open economies. Using an unbalanced panel data of more than 800 firms across nine different industries in Argentina, Brazil, Chile, Colombia, Mexico, Peru and Venezuela, I find that favorable prices in strategic sectors such as energy, agriculture and mining lead to an increase in firm investment. Likewise, energy and agriculture price volatility also contribute to investment positively. Other firm-related factors, such as consumer demand, the size of the firm, as well as its leverage and liquidity are significant contributors to investment behaviour.

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Chapter 1

Total factor productivity, terms of trade and time-varying volatility in a small open economy

1.1 Introduction

This paper shows that changes in the volatility of total factor productivity in a small open economy have a negative effect on consumption, investment and output. The effect of volatility is independent of whether actual productivity changes or not. While shocks to total factor productivity are expansionary, meaning that consumption, investment, hours increased and output grow, shocks to the volatility of total factor productivity are instead contractionary. This is shown using a real business cycle model for a small open economy that is able to match the stylized facts of the Canadian economy. In particular, it replicates its consumption-smoothing nature, the volatile nature of investment, the correlation of consumption and investment with output, and the persistence of consumption and output.

In this model, there is only one good produced that is completely exported, while the representative household imports a final good for consumption. Thus, total factor productivity in this model can be interpreted as the terms of trade, since the exported commodity is expressed in terms of the final (imported) good. This fact is very useful since in the case of small open economies, including the Canadian economy, their production matrix tends to center around the production of a handful of primary goods for exports, while their imports are made of final consumption goods.

The model is built in a way such that total factor productivity follows stochastic volatility, a process originally used to model volatile variables in finance. Under stochastic volatility, the standard deviation of a productivity shock is time-varying, following an autoregresive process driven by a stochastic shock itself. Thus, there are two types of shocks in the model: a shock affecting productivity directly and a shock affecting the volatility around productivity. The ability of the model to replicate the stylized facts mentioned above is robust to changes in certain parameters of the model, such as the coefficient of relative risk aversion, the persistence of productivity and volatility shocks, and the degree of dispersion of productivity and volatility shocks.

The model is solved using perturbation methods. Specifically, a third order approximation to the policy function that accounts for shocks to the volatility around productivity shocks, which are not accounted for in a first- or second order approximation to the policy function.

Section 2.2 studies the RBC models related to the model used in this paper, reviews the literature on volatility in macroeconomics, and analyses the relationship between total factor productivity and the terms of trade. Section 1.3 describes the stylized facts concerning real business cycles in small open economies. Section 2.4 describes the model, the values of the parameters used for its calibration, and the perturbation methods applied to find its solution. Section 2.5 shows the results of the paper in terms of the second moments simulated by the model and the impulse responses from both productivity and volatility shocks, followed by a number of robustness checks to the model. Section 2.6 concludes.

1.2 Literature review

The paper is at the junction of three streams of literature: real-business cycle models for small open economies; uncertainty and volatility in macroeconomics; and papers linking terms of trade and total factor productivity in the case of small open economies.

The model used in this paper is based on the RBC literature for small open economies started by Mendoza (1991) –the seminal and standard neoclassical growth model for a small open economy[1]– and followed by Neumeyer and Perri (2005). The properties of both models are summarised in Schmitt-Grohé and Uribe (2003). The model consists of a country that produces one tradable good and has access to international financial markets that supply a foreign asset. The interest rate paid on this asset is decomposed into an international rate and a country risk spread, and business cycle fluctuations are driven by shocks to total factor productivity. Mendoza (1991) eliminates the unit root dynamics of the model by using an endogenous discount factor (Uzawa-type preferences), but Schmitt-Grohé and Uribe (2003) find that there are alternative ways to find a stationary steady state in a SOE-RBC model that throw similar second moments and impulse responses at business-cycle frequencies. These alternatives include the use of a debt-elastic interest rate and portfolio adjustment costs.

Fernandez-Villaverde et al. (2011) follow the steps of Fernandez-Villaverde and Rubio-Ramirez (2007) and Justiniano and Primiceri (2008), who study time-varying volatility in macroeconomics using stochastic volatility, and use the framework of Mendoza (1991) and Neumeyer and Perri (2005) to focus on the volatility of the real interest rate at which small open economies borrow to explain business cycle fluctuations. They model interest rates under stochastic volatility, such that there are two types of shocks in the model: a shock to the interest rate itself and a shock to its volatility.[2]

Finally, Mendoza (1991) makes the link between the terms of trade and total factor productivity, as the only good produced in the model is a tradable commodity, while the final good consumed is imported and taken as the numeraire. Thus, the price of

[1] Neoclassical growth theory outlines how steady economic growth can be accomplished thanks to three driving forces: labor (population growth), capital (capital accumulation) and productivity (technological progress). See Solow (1956) and Swan (1956). The model is a contribution to the closed-economy RBC models of Kydland and Prescott (1982), Long and Plosser (1983) and King et al. (1988).

[2] See Shephard (2005) for a detailed introduction to stochastic volatility.

the exported good is given relative to the final imported good, such that the technology shock is in fact equal to the terms of trade. This link was later mentioned in Easterly et al. (2001), who argue that to explain volatility in small open economies, terms of trade shocks can have the same effect as technology shocks. Kohli (2004) says that improvements in the terms of trade are similar to technological progress: after a positive shock to the terms of trade, a country can import more goods in exchange for what it exports (or export less in exchange for what it imports). Kehoe and Ruhl (2008) say that international trade can be seen as a production technology, where imports are inputs and the outputs are exports. Imports are transformed into exports at the rate of the price of exports relative to the price of imports (terms of trade). In that same spirit, Garcia-Cicco et al. (2010) point out that the disturbance in their model is not limited to exogenous changes in technology but include other disturbances that may affect total factor productivity, such as shocks to the terms of trade.

1.3 Stylized facts

1.3.1 Real business cycles

Table 1.3.1: Business cycles in small open economies

	Volatility (s.d.)				Correlation with y			Persistence (autocorrelation)			
Country	σ_y	σ_c/σ_y	σ_i/σ_y	σ_{tb}	$\rho_{c,y}$	$\rho_{i,y}$	$\rho_{tb,y}$	$\rho_{y_{t,t-1}}$	$\rho_{c_{t,t-1}}$	$\rho_{i_{t,t-1}}$	$\rho_{tb_{t,t-1}}$
Developing											
Argentina	5.57	1.05	2.80	2.80	0.95	0.93	-0.88	0.55	0.57	0.57	0.52
Chile	4.78	1.31	2.69	2.09	0.91	0.85	-0.73	0.62	0.67	0.53	0.50
Developed											
Australia	1.54	0.70	3.16	0.96	0.52	0.64	-0.16	0.49	0.49	0.54	0.52
Canada	1.95	0.85	2.53	1.11	0.65	0.74	0.04	0.60	0.80	0.57	0.74

Data obtained from the World Bank's World Development Indicators; the frequency is annual from 1960 to 2017 and all variables are expressed in real per capita terms (see appendix 2.7.1 for details). Output y, consumption c, and investment i are detrended in logs using the Hodrick-Prescott filter with smoothness parameter $\lambda = 100$. The trade balance (as a share of output) tb is detrended in levels using the same filter and smoothness parameter.

Table 2.3.2 shows the stylized facts of the business cycles in Argentina, Australia, Canada and Chile. The frequency is annual between 1960 and 2017 and all variables are expressed in per capita terms.[3] These four countries are good examples of small open economies; countries open to international financial markets that are too small to have any influence on foreign interest rates.[4] There is however, some heterogeneity within these countries: Argentina and Chile belong to a group of developing small open economies while Australia and Canada are already developed small open economies.

Volatilities for each variable are measured by their standard deviation. Thus, the volatility of output is given by σ_y and the volatility of trade balance (as a share of output) is σ_{tb}. The ratios σ_c/σ_y and σ_i/σ_y depict the volatility of consumption and investment with respect to the volatility of output, respectively. The relationship of consumption, investment and the trade balance/output ratio with output is measured by the correlations $\rho_{c,y}, \rho_{i,y}$ and $\rho_{tb,y}$, respectively. The persistence of output, consumption, investment and the trade balance-to-output ratio is measured by the first-lag autocorrelations $\rho_{y_{t,t-1}}, \rho_{c_{t,t-1}}, \rho_{i_{t,t-1}}$ and $\rho_{tb_{t,t-1}}$, respectively.

First, it is observed that business cycles in developing small open economies are more volatile than in developed ones. The volatility of output, measured by its standard deviation σ_y, is much higher in Argentina and Chile (5.57 and 4.78 percent, respectively) than in Australia and Canada (1.54 and 1.95 percent, respectively). Likewise, the volatility of the trade-balance-to-output ratio σ_{tb} is about twice as large in the developing countries than in the developed ones: 2.80 and 2.09 in Argentina and Chile, and 0.96 and 1.11 in Australia and Canada.

Second, consumption smoothing is stronger in developed small open economies.

[3]See appendix 2.7.1 for the data description.

[4]The definition of a small open economy is that of a country in which agents borrow and lend in international financial markets at an interest rate already determined by these markets. See Guerron-Quintana (2013).

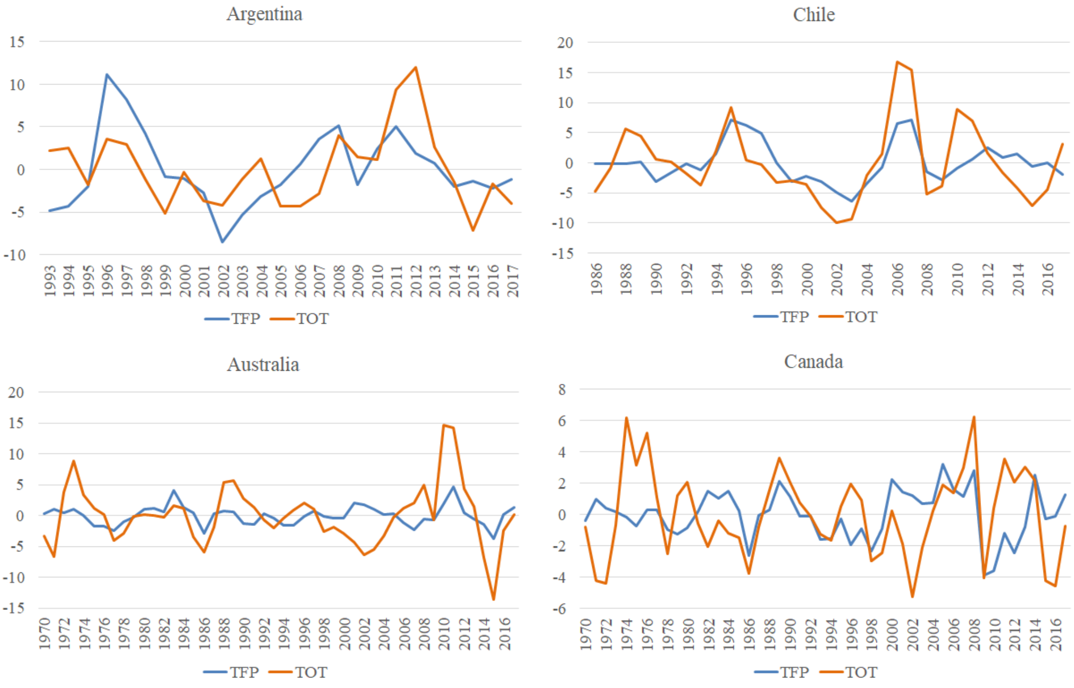
In Argentina and Chile, consumption is more volatile than output, as seen in the consumption-to-output volatility ratio σ_c/σ_y, where values are above unity (1.05 and 1.31, respectively). Meanwhile, this ratio is below unity in developed small open economies, meaning that consumption is less volatile than output in these countries: the value of σ_c/σ_y is 0.70 in Australia and 0.85 in Canada.

Third, investment is much more volatile with respect to output in small open economies. In all four cases, investment is about three times as volatile as output. In Argentina and Chile, the investment-to-output volatility ratio σ_i/σ_y has a value of 2.80 and 2.69, respectively. In Australia and Canada, investment volatility is 3.16 and 2.53 times higher than output volatility, respectively.

Fourth, the trade balance is countercyclical in developing small open economies, but acyclical in developed small open economies. In particular, there is a strong negative relationship between the trade balance and output in the first group of countries, while there is no clear relationship between these variables in the latter group. While the correlation between the trade balance and output is -0.88 and -0.73 in Argentina and Chile, the correlation between these variables in Australia is negative but very low (-0.16) and slightly positive in Canada (0.04).

Fifth, business cycles are moderately persistent. The first-lag autocorrelation of cyclical output $\rho_{y_{t,t-1}}$, consumption $\rho_{c_{t,t-1}}$, investment $\rho_{i_{t,t-1}}$ and the trade-balance to output ratio $\rho_{tb/y_{t,t-1}}$ in all countries show values that are -in general- around 0.50, suggesting that even though exogenous shocks do not subtantially affect aggregate variables overtime, they are still vulnerable to volatility.

Figure 1.3.1: Total factor productivity and terms of trade



Total factor productivity series (blue) obtained from the database of the Federal Reserve Bank of St. Louis; the Terms of trade (orange) are obtained from the OECD for Australia, Canada and Chile, and the World Bank for Argentina. The series have an annual frequency and have been detrended using a Hodrick-Prescott filter with a coefficient $\lambda = 100$. The period of availability is from 1993 in Argentina, 1986 in Chile, and 1970 in Australia and Canada; all series end in 2017. The two series are have a correlation coefficient $\rho = 0.43$ in Argentina, $\rho = 0.69$ in Chile, $\rho = 0.44$ in Australia and $\rho = 0.21$ in Canada.

1.3.2 Total factor productivity and terms of trade

Figure 1.3.1 shows the cyclical component of total factor productivity and the terms of trade in Argentina, Chile, Australia and Canada. It can be observed that there is a stronger relationship between both variables in developing small open economies than in the developed counterparts. The reason is because the economies in the former group of countries have a higher dependency on commodity prices. For instance, the correlation coeffcient between total factor productivity and terms of trade in Chile is $\rho = 0.69$, as the country is a major producer of copper and its thus dependent on international prices. On the other side of the spectrum, the more diversified Canadian economy shows a value $\rho = 0.21$ for the same coefficient. It is possible to see, however, similar patterns of behavior in these series for Canada during specific periods. These include the fall and recovery observed in the mid-1980s associated with the oil market uncertainty, the negative trend of the first half of the 1990s, and the sharp drop and further recovery in the late 2000s, in the context of the global financial crisis.

As mentioned in section 2.2, the literature shows the existence of a link between total factor productivity and the terms of trade under certain conditions. These conditions include a strong dependency on commodity exports. In the context of the model (see section 2.4), total factor productivity is indeed interpreted as the terms of trade because the small open economy produces only one commodity and imports a final consumption good from abroad, such that the final good -the exported commodity- is expressed in terms of the (imported) consumption good.

This paper focuses on the particular case of Canada, as it aims to make a direct comparison to the results seen in the baseline model of Mendoza (1991) and Schmitt-Grohé and Uribe (2003) which calibrate data for this country. Table 1.3.2 shows that in Canada, commodity exports are about half the total amount of ex-

Table 1.3.2: Commodities as a share of total exports (2013-2017)

Argentina	69 %
Australia	84 %
Canada	**48 %**
Chile	86 %
France	19 %
Germany	11 %
Italy	16 %
Switzerland	16 %

Source: State of Commodity Dependence 2019 UNCTAD (2019).

ports (48 percent on average between 2013 and 2017). Even though this is a small number compared to developing small open economies such as Argentina and Chile, where commodity exports represented 69 and 86 percent of total exports, or Australia -a developed small open economy- where these exports rose to 84 percent, it is an important amount with respect to other open economies. In particular, commodity exports in advanced European economies such as France, Germany or Italy represent less than 20 percent of total exports. This puts into perspective that Canada is a relatively commodity dependent economy where the terms of trade are then of relevance.

To look into this further, table 1.3.3 shows the main Canadian exports and imports by sector in 2017. First, exports of mineral products and metals –primary commodities– amount to more than 32 percent of total exports. Meanwhile, most imports such as machines and transportation goods are final goods of consumption. Thus, this composition of trade in Canada fits to that of the model that I use in this paper. Therefore, the shocks that are analyzed in this framework can be interpreted -to some extent- as terms of trade shocks. Since oil is the main commodity of export in Canada (see appendix 1.7.2), we can assume shocks affecting productivity in the model (see next section) as being shocks affecting oil prices and its volatility.

Table 1.3.3: Canada – Commodities as a share of total exports and imports(2013-2017)

Exports		Imports	
Mineral products	24%	Machines	24%
Transportation	19%	Transportation	20%
Machines	10%	Mineral products	9.5%
Metals	8.4%	Chemical products	9%
Chemical products	6.7%	Metals	7%
Vegetables	5.8%	Plastics	5.0%
Plastics	4.2%	Foodstuffs	4.7%
Paper goods	3.9%	Textiles	3.9%
Foodstuffs	3.7%	Miscellaneous	3.1%
Animal products	3.1%	Vegatable products	2.7%

Source: Observatory of Economic Complexity, (Simoes and Hidalgo, 2011).

1.4 Model

The model follows the real-business-cycle framework for a small open economy (SOE-RBC) of Mendoza (1991) and Schmitt-Grohé and Uribe (2003), where the economy produces only one good: a completely-exported commodity. Households consume a final imported good, where consumption is smoothed intertemporally thanks to access to an incomplete market of foreign debt with an interest rate exogenously determined abroad. A stationary equilibrium for the holdings of this debt is reached through a country-risk premium on the interest rate that increases with the amount of debt outstanding. Foreign assets finance trade imbalances. The process for total factor productivity driving fluctuations in the economy is modelled under a stochastic volatility framework. The economy is subject to two types of shocks: a shock to total factor productivity and a shock to its volatility (i.e. a shock to the standard deviation of the TFP shock). Preferences are time separable and generate a labor supply independent from wealth.

1.4.1 Structure

Households.- The economy is populated by an infinite number of identical households that maximize lifetime utility:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, h_t) \tag{1.4.1}$$

where E_0 is the expectations operator conditional on information available in period 0. Utility is given by a concave function, increasing in consumption of a final good c_t and decreasing in hours worked h_t:

$$U(c_t, h_t) = \frac{\left[c_t - \frac{h_t^{1+\omega}}{1+\omega}\right]^{1-\gamma} - 1}{1-\gamma} \tag{1.4.2}$$

where γ is the inverse intertemporal elasticity of substitution[5] and ω is the inverse of the Frisch elasticity of labor supply. The preferences in equation 1.4.2 are known as the Greenwood-Hercowitz-Huffman preferences (Greenwood et al., 1988), where the labor supply (the marginal rate of substitution between consumption and leisure) is independent of the level of consumption, such that there is no income effect on the supply of labor.[6]

The period-by-period budget constraint faced by the representative household is given by:

$$c_t + i_t + \Phi(k_{t+1} - k_t) + (1 + r_{t-1})d_{t-1} = y_t + d_t \tag{1.4.3}$$

where i_t denotes investment in physical capital k_t. The latter evolves according to the law of motion:

$$k_{t+1} = (1-\delta)k_t + i_t \tag{1.4.4}$$

[5] Also known as the coefficient of relative risk aversion.

[6] These preferences have been used extensively in the SOE-RBC literature (see Mendoza (1991) and Correia et al. (1995)). They are nested in the more general Jaimovich-Rebelo preferences (Jaimovich and Rebelo, 2009) also used in open economy macroeconomics (see appendix 1.7.3).

with depreciation rate of capital δ. Capital adjustment costs have the quadratic function:

$$\Phi(k_{t+1} - k_t) = \frac{\phi}{2}(k_{t+1} - k_t)^2$$

which satisfies conditions $\Phi(0) = \Phi'(0)$ and $\Phi''(0) > 0$. These ensure that in the steady state, adjustment costs are zero and the relative price of capital goods in terms of consumption is unity.

Financial markets.- The household has access to asset d_t in incomplete international capital markets, meaning that payments contingent on realizations of productivity A_t are not allowed. Access to this market is subject to a no-Ponzi constraint:

$$\lim_{j \to \infty} E_t \frac{d_{t+j}}{\Pi_{s=0}^{j}(1 + r_s)} \leq 0 \tag{1.4.5}$$

The interest rate r_t paid for holding these assets is the result of the sum of a risk-free interest rate r^* and country-risk premium given by function $\psi(\cdot)$ increasing in the aggregate average level of foreign debt outstanding, following Senhadji (1994):

$$r_t = r^* + \psi\Big(e^{d_t - \bar{d}} - 1\Big) \tag{1.4.6}$$

where ψ is a constant parameter and $\bar{d}$ is the steady state level of foreign debt, taken as given. The inclusion of function $\psi(\cdot)$ has the role of inducing stationarity to the model.[7]

Production.- The economy produces a commodity y_t using a linearly homogeneous function:

$$y_t = A_t F(k_t, h_t) \tag{1.4.7}$$

[7]See Schmitt-Grohé and Uribe (2003) for details.

where $F(k_t, h_t)$ follows a Cobb-Douglas technology:

$$F(k_t, h_t) = k_t^{\alpha} h_t^{1-\alpha}$$

and $\alpha \in (0, 1)$ is the income share of capital in output. There is a unitary elasticity of substitution between capital and labor.

Stochastic process.- A_t follows an autoregresive process:

$$\log A_t = \rho_A \log A_{t-1} + \sigma_t \varepsilon_t \qquad \varepsilon_t \sim \log N\Big(-\frac{\sigma_t^2}{2}, \sigma_t^2\Big) \tag{1.4.8}$$

where $\rho_A < 1$. The standard deviation σ_t of innovation ε_t is not constant, but rather time-varying, following an autoregresive process itself:

$$\log \sigma_t = (1 - \rho_\sigma) \log \bar{\sigma} + \rho_\sigma \log \sigma_{t-1} + \eta u_t \qquad u_t \sim N(0, 1) \tag{1.4.9}$$

where $\rho_\sigma < 1$, $\bar{\sigma}$ is the long-run standard deviation of A_t, and η is the standard deviation of innovation u_t, which is constant over time. Equations 1.4.8 and 1.4.9 jointly form the process known as stochastic volatility, described in Shephard (2005) among others.

As A_t has innovations that are log-normal, implying a non-symmetric distribution, then a second order approximation around the deterministic steady state does not capture effects of volatility from innovations. Instead, these effects will be captured under a third order approximation around the non-stochastic steady state.[8]

Optimality conditions.- The Lagrangian for the maximization problem of the

[8] See Andreasen (2012).

household is:

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \Big\{ U(c_t, h_t) + \lambda_t [A_t F(k_t, h_t) + (1-\delta)k_t + ... \tag{1.4.10}$$

$$d_t - c_t - (1 + r_{t-1})d_{t-1} - k_{t+1} - \Phi(k_{t+1} - k_t)] \Big\}$$

where λ_t is the Lagrange multiplier associated to the sequential budget constraint 1.4.3 and $U(c_t, h_t)$ is given by 1.4.2. The first order conditions of 1.4.1 with respect to consumption c_t, hours worked h_t, investment i_t and foreign debt d_t are then given by:

$$\lambda_t = \beta(1 + r_t)E_t\lambda_{t+1} \tag{1.4.11}$$

$$U_c(c_t, h_t) = \lambda_t \tag{1.4.12}$$

$$-U_h(c_t, h_t) = \lambda_t A_t F_h(k_t, h_t) \tag{1.4.13}$$

$$\lambda_t[1 + \phi(k_{t+1} - k_t)] = \beta E_t \lambda_{t+1} \Big[A_{t+1} F_k(k_t, h_t) + 1 - \delta + \phi(k_{t+2} - k_{t+1}) \Big] \tag{1.4.14}$$

Competitive equilibrium.- A competitive equilibrium is a set of processes of endogenous variables $\{c_t, i_t, h_t, k_{t+1}, d_t, y_t, r_t, \lambda_t\}_{t=0}^{\infty}$ satisfying the household's first order conditions 2.4.6 to 2.4.9, equilibrium conditions 1.4.3, 1.4.4, 1.4.6 and the no-Ponzi constraint 1.4.5 holding with equality, given initial conditions A_0, d_{-1} and k_0 and the stochastic processes for A_t and σ_t.

Finally, the equilibrium process for the trade balance tb_t is defined as:

$$tb_t \equiv y_t - c_t - i_t - \Phi(k_{t+1} - k_t) \tag{1.4.15}$$

1.4.2 Calibration of the model parameters

Table 2.4.1 shows the values used for the parameters in the model. These are calibrated in part following the baseline SOE-RBC literature of Mendoza (1991) and

Table 1.4.1: Baseline parameters

Parameter	Value	Description	Source/target
β	$1/(1+r^*)$	Subjective discount factor	
r^*	0.05	Risk-free interest rate	U.S. Treasury Bill interest rate
γ	2	Risk aversion	Mendoza (1991)
α	0.33	Capital share in production	Mendoza (1991)
δ	0.1	Depreciation rate	Mendoza (1991)
ω	0.455	Inverse Frisch elasticity	Mendoza (1991)
ρ_A	0.42	Autocorrelation TFP	Schmitt-Grohé and Uribe (2003)
$\bar{\sigma}$	0.0129	Standard deviation TFP	Schmitt-Grohé and Uribe (2003)
ρ_A^{tot}	0.4117	Autocorrelation TOT	AR(1) estimation
$\bar{\sigma}^{tot}$	0.0306	Standard deviation TOT	AR(1) estimation
ρ_σ	0.95	Persistence volatility shock	Fernandez-Villaverde and Rubio-Ramirez (2010)
η	0.1	Std.dev. volatility shock	Fernandez-Villaverde and Rubio-Ramirez (2010)
$\bar{d}$	0.935	Long-run debt level	Trade balance-to-output ratio
ϕ	0.027	Capital adjustment cost	Investment-to-output volatility ratio
ψ	0.029	Debt-elasticity parameter	Consumption-to-output volatility ratio

The table for the estimation of the AR(1) process for the Canadian terms of trade are found in appendix 1.7.6.

Schmitt-Grohé and Uribe (2003), as well as the work on volatility in macroeconomics from Fernandez-Villaverde and Rubio-Ramirez (2010). The objective is to match the model's simulated moments with the stylized facts observed in the data for the Canadian economy. I study Canada, a developed small open economy, in order to compare my results to those from the baseline work of Mendoza (1991) and Schmitt-Grohé and Uribe (2003). The time unit of the model is a year.

Following the baseline model of Mendoza (1991), the coefficient for relative risk aversion γ is equal to 2. The share of capital α in the Cobb-Douglas production function is one third. The depreciation rate for capital δ is set at 10 percent. The inverse Frisch elasticity of labour supply ω is 0.455, while the risk-free interest rate r^* is set to 5 percent, assumed as the long-run value of the U.S. Treasury Bill rate.

The stochastic volatility process of equations 1.4.8 and 1.4.9 is calibrated twice. First, I follow the baseline calibration of Schmitt-Grohé and Uribe (2003) in which the coefficient for persistence of total factor productivity ρ_A equals 0.42 and the standard deviation of the productivity shock $\bar{\sigma}$ is 0.01. In the second calibration, I estimate an AR(1) process for the Canadian terms of trade using the data described in section 1.3 and the results are shown in appendix 1.7.6. The value of the autoregresive coefficient is very similar to the calibrated counterpart: $\rho_A^{tot} = 0.4117$, reinforcing the idea of a very close relationship between total factor productivity and the terms of trade. In terms of the standard deviation, the terms of trade estimation gives a higher value for $\bar{\sigma}^{tot}$ equal to 0.0306.

The autocorrelation coefficient of the volatility process ρ_σ is 0.95, while the standard deviation of the shock to volatility η is 0.10. The latter two values are taken from Fernandez-Villaverde and Rubio-Ramirez (2010), who state that these values generate volatility dynamics similar to those observed in U.S. data.[9]

The long-run level of debt $\bar{d} = 0.935$ is set to match the average value of the trade balance as a share of output in Canada between 1960 and 2017 equal to 3 percent.[10] The parameter for capital adjustment cost $\phi = 0.027$ is set to match volatility ratio of investment to output σ_i/σ_y, while the parameter ψ that controls the elasticity of debt is set at 0.03 to match the volatility ratio of consumption to output σ_c/σ_y.

1.4.3 Solution

To solve the model, I obtain a linear (Taylor series) approximation of the policy function around its non-stochastic steady state.[11] The convenience of perturbation methods -as this solution approach is also known- over other methods to solve DSGE models has been described in Aruoba et al. (2006) and Fernandez-Villaverde et al.

[9] Although Canada is considered a small open economy in this paper while the U.S. is a large economy, the proximity and similarities between the two countries leads to assume that they share a similar volatility process.

[10] See appendix 1.7.5.

[11] See appendix 1.8.

(2011). In short, other methods such as value function iteration or projection methods are slower to run as the number of state variables increases.

From 1.4.8 and 1.4.9, it can be seen that business cycles in the model are driven by two different shocks that affect the dynamics of total factor productivity differently: ε_t is a shock that affects the level of A_t, while u_t is a shock that affects the standard deviation of ε_t. Thus, ε_t is an innovation to A_t (a level shock), while u_t is an innovation to the volatility of productivity (a volatility shock). One of the objectives of this work is to study the effects of changes in the volatility of productivity A_t even when its long-run mean remains constant. In other words, the study of the effects of a volatility shock u_t when the level shock ε_t is fixed at zero.

In a linear (first order) approximation to the policy function, volatility shocks do not play any role as there is certainty equivalence: the approximation to the unconditional means of endogenous variables coincides with their values in the non-stochastic steady state, missing all of the dynamics induced by volatility. Thus, the first-order approximated policy function depends exclusively on the level shock ε_t and does not include the volatility shock u_t. A second-order approximation to the policy function is able to capture important effects of stochastic volatility but only indirectly through the joint interaction with the level shock, via the cross-product term $\varepsilon_t u_t$. That means that if ε_t is zero (the case when productivity does not suffer from any shock), there is no volatility effect on the policy function. In a third-order approximation, however, the volatility shock u_t enters the policy function as an independent argument, meaning that even if there are no level shocks to productivity, volatility shocks can be analyzed separately.[12]

[12] There is extensive literature that studies this in detail. To name a few, Schmitt-Grohé and Uribe (2004) studies the properties of moving from a first- to a second-order approximation to the policy function, while Andreasen (2012) does a similar analysis for the case of a third-order approximation. See appendix 1.8.

1.5 Results

1.5.1 Simulated second moments

Table 1.5.1 shows the empirical moments of the data along with the second moments generated from the model using the baseline calibration (Model I) as well as the estimated terms-of-trade parameters (Model II). Given the similar values in both cases, the simulated moments for both models are also similar. The volatility ranking among the aggregate variables is well replicated: investment is the most volatile variable, followed by output, consumption and the trade balance as the least volatile variable. In Model I, these volatilities are given by standard deviations $\sigma_i = 9.26, \sigma_y = 3.66, \sigma_c = 3.11$ and $\sigma_{tb} = 1.15$. In Model II, volatilities are higher because of the higher standard deviation of the shock, but the ordering of volatilities remains: $\sigma_i = 20.09, \sigma_y = 7.72, \sigma_c = 6.54$ and $\sigma_{tb} = 2.39$.

As mentioned previously, the model is calibrated to match two particular targets which are the relative volatility of consumption and investment respect to output. In Model I, the relative consumption volatility is below unity ($\sigma_c/\sigma_y = 0.85$), showing the consumption-smoothing nature of households in the small open economy. Meanwhile, the investment volatility is more than twice as high as output volatility ($\sigma_i/\sigma_y = 2.53$), a feature that -as shown in section 1.3- is not only seen in developed small open economies but also in developing ones. The values for the volatility ratios in Model II are practically the same.

The model is then able to replicate other main features of the small open economy business cycles.[13] In terms of the correlations with output, the model is good at showing a strong positive relationship between consumption and output. However, while the data shows a value of $\rho_{c,y}$ of 0.65, this correlation is much higher in the model ($\rho_{c,y} = 0.97$). In the case of investment, its correlation with output $\rho_{i,y}$ is well

[13] Given the similarities, results for 'model I' hereafter.

Table 1.5.1: Empirical and simulated second moments

	Data	Model I	Model II
Volatilities			
σ_y	1.95	3.66	7.72
σ_c	1.66	3.11	6.54
σ_i	4.93	9.26	20.09
σ_{tb}	1.11	1.15	2.39
Volatility ratios			
σ_c/σ_y	0.85	0.85	0.85
σ_i/σ_y	2.53	2.53	2.59
σ_{tb}/σ_y	0.57	0.31	0.31
Correlation with output			
$\rho_{c,y}$	0.65	0.97	0.96
$\rho_{i,y}$	0.74	0.79	0.78
$\rho_{tb,y}$	0.04	-0.27	-0.27
Autocorrelations			
$\rho_{y_{t,t-1}}$	0.60	0.69	0.67
$\rho_{c_{t,t-1}}$	0.80	0.80	0.78
$\rho_{i_{t,t-1}}$	0.57	0.11	0.10
$\rho_{tb_{t,t-1}}$	0.74	0.04	0.05

The empirical second moments are based on Canadian data from 1960 to 2017 (consumption data from 1970 to 2017) displayed in table 2.3.2 of section 1.3. Model I shows results for the model using parameters for ρ_A and $\bar{\sigma}$ used in Schmitt-Grohé and Uribe (2003). Model II uses values from an AR(1) estimation of the terms of trade for Canada (see appendix 1.7.6).

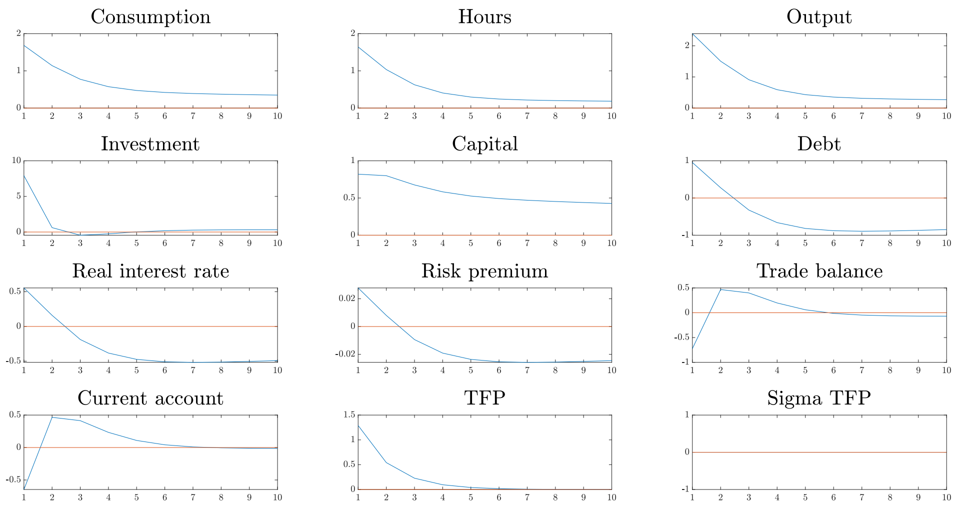
replicated, with values of 0.74 and 0.79 in the data and the model, respectively. The model is also able to explain the acyclical relationship between the trade balance and output: while the data shows that this relationship is practically non-existant ($\rho_{tb,y} = 0.04$), the model shows a low, negative relationship ($\rho_{tb,y} = -0.27$).

Finally, the model replicates the persistence of consumption ($\rho_{c_t,c_{t-1}} = 0.80$) and is close at explaining the persistence of investment ($\rho_{y_t,y_{t-1}} = 0.60$ in the data and 0.69 in the model). In terms of the persistence of investment and the trade balance, the model is not capable of replicating their empirical values: while the data shows a persistence in both aggregate variables with values above 0.50 ($\rho_{i_{t,t-1}} = 0.57$ and $\rho_{tb_{t,t-1}} = 0.74$), the model predicts that these variables are not persistent, with auto-correlation values closer to zero ($\rho_{i_{t,t-1}} = 0.11$ and $\rho_{tb_{t,t-1}} = 0.04$, respectively). The reason for the low persistence of investment is that given the baseline calibration, total factor productivity shocks are not persistent, reason for which investment is not very high and invesment variability is low (more in section 1.5.2). In the case of the trade balance, the given coefficient for relative this aversion given in the calibration implies a relative low level of risk and thus lower quantity of debt purchased. Since debt and the trade balance are directly related (see section 2.4), a low volatility of debt implies a relatively low fluctuation of the trade balance (more in section 1.5.2).

1.5.2 Impulse response functions

TFP shocks

Figure 1.5.1: Impulse responses - one s.d. shock to TFP



As mentioned in section 1.4.3, the dynamics of the model are driven by two shocks: a direct shock ε_t to total factor productivity, and a shock u_t to the volatility of TFP. Figure 1.5.1 shows the effects of a one standard-deviation shock to TFP on consumption, hours worked, output, investment, capital, debt, the real interest rate, the country risk premium, the trade balance and the current account.[14] The shock

[14]Appendix 1.7.7 shows the results for the model using the estimated parameters for the terms of trade (Model II).

of one standard-deviation is equal to a 1.29% deviation in TFP from its steady-state level, which dissipates only after approximately 6 periods, as it is seen in the lower centre graph of the figure. It is important to note that the volatility σ_t ("Sigma TFP" in the figure) does not show any change over time because the shock ε_t affects only the level of TFP and not its volatility.

Consumption grows in about 1.7% in the period of the shock, and it returns to its steady-state level very slowly because total future income will be slightly higher and households adjust consumption to this shock through the Euler equation. Hours of labor increase 1.6% because the shock to total factor productivity affects the marginal product of labor positively, and since there is no income effect on the labor supply, the substitution effects dominate on the household's labor supply decisions.

Investment will increase in the period of the shock in almost 8% with respect to its level in the steady state, due to the rise in the marginal product of capital. This means that capital will increase, although in less than one percent due to depreciation and capital adjustment costs. In this baseline calibration of the model, the TFP shock has a low persistence ($\rho_A = 0.42$) such that the increase in the marginal product of capital is known by the agents to be short-lived. Thus, investment does not prolong over time and falls quickly to its initial level after one period. This is because households know that the productivity increase is only temporary, and the marginal product of capital is not persistent enough to motivate further investment.

The increase of both consumption and investment in the period of the shock causes the trade balance to deteriorate, as domestic output is not sufficient to satisfy the increase in absorption. The rapid fall of investment, however, leads to an overshooting in the trade balance in subsequent periods, before it returns to its steady state level. In addition, output continues to grow after the shock, not only because of the productivity shock, but also due to changes in capital coming from the rise in invesment after the shock, lagged due to the law of motion of capital.

The productivity shock also affects the household's savings decisions. In particular, these increase near proportionally with the shock, in almost 1 percent. However, because the positive investment effects are quickly reversed and since consumption slowly returns to its steady state level, the foreign debt holdings fall and household sell these holdings in international markets.

The effects described above and observed in figure 1.5.1 are well known in the literature. The following section analyzes what occurs to these macroeconomic variables when the volatility of total factor productivity is affected.

Volatility shocks

Figure 1.5.2: Impulse responses - one s.d. shock to TFP volatility

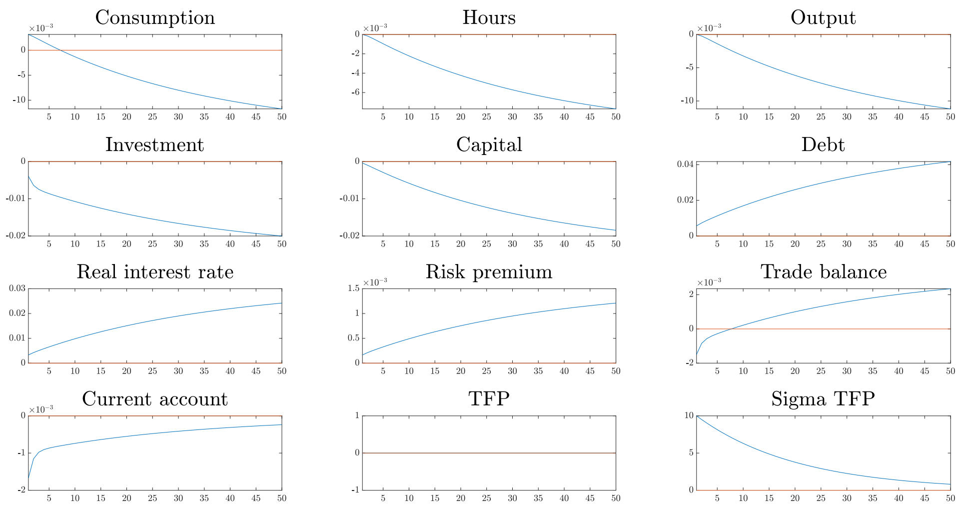


Figure 1.5.2 shows the effects of a one standard-deviation shock to the innovation to volatility of TFP: a one standard-deviation shock to u_t in equation 1.4.9 equal to an increase of almost 10% in σ_t ('Sigma TFP'). In a similar fashion to the previous

case, the level of total factor productivity ("TFP" in the figure) remains constant under this shock. This is because the volatility shock does not affect TFP directly, but instead it affects the degree of dispersion around productivity shocks, such that the dynamic responses observed in figure 1.5.2 are linked exclusively to changes in volatility around productivity, and not to changes in productivity itself.

The effect of TFP volatility on the economy is relatively different from the so-called level TFP shock. First, although the one standard-deviation shock to u_t leads to a slight increase in consumption, this jump is then reversed after a number of periods and then consumption shows a steady decline. This means that higher volatility of TFP has a negative and persistent effect on consumption. This can be explained by the effect known in the literature as precautionary savings: the higher degree of volatility affects the consumption decisions of households who decide to consume less and save more for the future. This can be seen as households accumulate debt over time to insure themselves against uncertainty coming from volatility, leading to an increase in both the country risk premium and the real interest rate (figure 1.5.2).

Three observations are made from the analysis of this so-called volatility shock in the model. First, the effects of volatility shock u_t are qualitatively the opposite to those from productivity shock ε_t. While a productivity shock leads to increases in consumption, investment and output, a volatility shock leads to a long-run fall in consumption, investment, capital, and output. The reason for this can be attributed to precautionary savings decisions. The higher degree of volatility affects the consumption decisions of households who decide to consume less and save more for the future. This can be seen as households accumulate debt over time to insure themselves against uncertainty coming from volatility, leading to an increase in both the country risk premium and the real interest rate.

Second, the effects of the shock on the macroeconomic variables are very persistent. As mentioned in the previous paragraph, the fall in consumption, investment and output takes place over the long-run. This is evidently imbedded to the model calibration, where the autoregressive process for σ_t is highly persistent, but it explains that volatility and uncertainty have a stronger impact on household decisions than what a productivity shock does.

Third, the quantitative effects of the volatility shock are very small. The fall in consumption reaches about 0.001% but only after several periods. The change in the labor supply suffers from a similar dynamic. The effects on investment and capital, however, are more significant: they have a fall of about 0.02% but only 50 periods after the shock. A similar thing occurs to output, which falls in 0.01% in a similar period. The small quantitative impact of the volatility shock is of interest considering that the one standard-deviation shock to u_t implies an increase in σ_t of almost 10 percent. By comparison, the one standard-deviation productivity shock ε_t represented a 1.3 percent increase and had more significant effects on the variables studied.

Robustness checks

How much are volatility shocks helping to explain the second moments observed in a small open economy? The results thus far shown in this section have described the second moments simulated by the model and the dynamic responses in the small open economy after one-standard deviation shocks to productivity and volatility. In order to understand the underpinnings of the model even more, this section conducts a series of exercises to see what are the effects of certain changes in the value of the model parameters. In particular, this section studies the variations in the model coming from changes in the parameters measuring risk aversion, the persistence of productivity and volatility shocks, and the degree of dispersion of productivity and volatility shocks. These results are shown in table 1.5.2.

Table 1.5.2: Robustness check simulated moments

	$u_t = 0$	$\gamma = 5$	$\gamma = 10$	$\rho_\sigma = 0.25$	$\eta = 0.50$	$\rho_A = 0.90$	$\bar{\sigma} = 0.05$
Volatilities							
σ_y	4.77	3.95	4.33	3.33	10.78	12.43	14.96
σ_c	3.03	3.47	3.95	2.83	9.22	11.83	12.67
σ_i	2.62	9.59	10.01	8.45	26.98	23.06	41.68
σ_{tb}	1.03	1.20	1.25	1.05	3.47	3.73	4.80
Volatility ratios							
σ_c/σ_y	0.64	0.88	0.91	0.85	0.86	0.95	0.85
σ_i/σ_y	0.55	2.43	2.31	2.54	2.50	1.86	2.79
σ_{tb}/σ_y	0.22	0.30	0.29	0.32	0.32	0.30	0.32
Corr. with output							
$\rho_{c,y}$	0.97	0.97	0.97	0.97	0.97	0.99	0.97
$\rho_{i,y}$	0.79	0.81	0.83	0.79	0.78	0.67	0.76
$\rho_{tb,y}$	-0.26	-0.32	-0.39	-0.27	-0.25	-0.01	-0.26
Autocorrelations							
$\rho_{y_{t,t-1}}$	0.69	0.73	0.78	0.68	0.69	0.97	0.68
$\rho_{c_{t,t-1}}$	0.80	0.84	0.88	0.79	0.80	0.98	0.79
$\rho_{i_{t,t-1}}$	0.12	0.16	0.24	0.11	0.08	0.51	0.10
$\rho_{tb_{t,t-1}}$	0.06	0.12	0.20	0.04	0.04	0.28	0.04

$u_t = 0$ is the model solved when the volatility shock is switched off. $\gamma = 5$ and $\gamma = 10$ show results when the model uses higher coefficients of relative risk aversion. $\rho_\sigma = 0.25$ is for the model under a low persistence in the volatility autoregresive process. $\eta = 0.5$ is for the model with a higher standard deviation in the volatility shock. $\rho_A = 0.9$ is for a model with a high persistence in the productivity shock.

i) Risk aversion:

The first exercise deals with an increase in the coefficient of relative risk aversion from its baseline calibration of $\gamma = 2$. If households are less prone to risk, for which we can assume higher risk-aversion coefficients ($\gamma = 5$ or 10), the volatility ranking of macroeconomic variables does not change. That is, investment remains the most volatile variable in the model, followed by consumption, investment and the trade balance/output ratio. In addition, the consumption-smoothing nature of the agents does not change -since consumption is less volatile than output- but it is much weaker as σ_c/σ_y grows to 0.88 and 0.91 under γ values of 5 and 10, respectively. The new coefficient values do not affect the correlation of consumption and output $\rho_{c,y}$ as it is already very high (0.97) with respect to the data (0.65). However, as γ grows to 5 and 10, consumption and output increase in persistence: $\rho_{y_{t,t-1}}$ goes from

0.69 to 0.73 and 0.78, values that distance themselves from that of the data (0.60).

Regarding the dynamics of the model after a volatility shock under higher risk aversion, figures 2.7.3 and 2.7.4 in the appendix show the impulse responses under these scenarios. When $\gamma = 5$, there is an overall negative effect on the small open economy affecting consumption, investment and output. However, differing from the baseline case, the is a quick overshoot where consumption increases as well as output and investment. Even though this effect might seem counterintuitive to some extent, a number of reasons could explain this increase in economic activity even after higher volatility. One of these reasons are the so-called real option effect: a firm that installs capital today that could be sold in the future (put option) may benefit from volatility as the future sell price of capital might be higher. Thus, the value of the put option obtaned increases with higher volatility, leading to a rise in investment. The response may also be explained by what is known as the Oi-Hartmann-Abel effect.[15] This effect states that a higher variance of productivity increases investment, hiring and output because optimal capital and labor choices are convex in productivity.[16] One final remark about these impulse responses is related to their magnitudes. While the baseline model provided quantitatively weak responses to volatility shocks, they increase -although slightly- under higher risk aversion, meaning that agents in the economy are more aware of the shocks and their effects.

ii) Productivity:

What happens when there are changes in the AR(1) process for σ_t? Given that the baseline model assumes that $\rho_\sigma = 0.95$ and $\eta = 0.1$ following Fernandez-Villaverde and Rubio-Ramirez (2010), this section analyzes two counterfactual scenarios: one in which the autoregresive nature of σ_t is much lower, and another where the degree

[15] Oi (1961), Hartman (1972) and Abel (1983).

[16] See Born and Pfeifer (2014).

of dispersion of shocks to u_t is higher. In particular, when $\rho_\sigma = 0.25$, the overall volatility in the model is lower, as it can be seen in the values for the standard deviaitons of output, consumption, investment and the trade balance. This could be seen as an improvement in the model since these volatilities are closer to those seen in the data, while other features such the volatility ranking and volatility ratios remain unchanged, as well as correlations and persistence. The impulse responses shown in figure 1.7.5 show that these are persistent but quantitatively low, as in the baseline model.

When the magnitude of the standard deviation to volatility shocks is increased from $\bar{\sigma} = 0.01$ in the baseline model to $\bar{\sigma} = 0.05$, an opposite effect occurs with respect to the previous case: since the dispersion of volatility shocks rises, volatilities in the model increase considerably. In fact, investment volatility grows from 9.26% in the baseline model to 26.98% (see 1.5.2 for the rest of the numbers). However, the rest of the model features remain the same in terms of volatility ratios, output correlation and persistence. Likewise, the responses of consumption, investment and output are similar to the baseline model, showing a long-run decline that are slightly stronger in quantitative terms.

iii) Volatility:

More than the autoregresive process to total factor productivity, it is perhaps more interesting to analyze the robustness of the autoregressive process to volatility. In that sense, I study the effects of a change in ρ_σ and η. When there is a drop in the autoregresive coefficient from $\rho_\sigma = 0.9$ to $\rho_\sigma = 0.25$ overall volatility in the model falls, but not significantly nor affecting the volatility rank of variables. The volatility ratios are also robust to the change in ρ_σ, while output correlations do not vary significantly. Lastly, persistence parameters and the model dynamics are also robust to changes in ρ_σ.

When the dispersion of shocks to u_t is stronger, going from $\eta = 0.1$ to 0.5, the overall volatility of the model rises but the simulated second moments and model dynamics are robust. This shows that the model presented in section 2.4 makes a good approximation as describing the volatile nature of the small open economy.

One consideration should be made regarding the robustness checks described above and the dynamic responses after a level and volatility shock. Given the nature of stochastic volatility, it would be expected that changes affecting equation 1.4.9 do not trickle down to the impulse responses from a shock to productivity (the left-hand side in figures 2.7.3 to 1.7.8 in appendix 2.7.7). Indeed, changes to the coefficient of risk aversion γ do not affect the overall dynamics of the model after a productivity shock. In a similar fashion, figures 1.7.7 and 1.7.6 show that the impulse responses after a productivity shock are identical even after changes to the parameters of the autoregresive process σ_t. Meanwhile, figure 1.7.7 shows that since the persistence of the productivity shock is affected, then indeed the impulse responses differ with respect to the baseline model. Finally, 1.7.8 shows that a positive change to $\bar{\sigma}$ does not change the nature of the impulse responses but it does affect their magnitudes, which is to be expected as the dispersion of the productivity shocks grows but all other factors are kept fixed.

1.6 Conclusion

This paper has studied a real business cycle model for a small open economy in order to analyze the effects of shocks to the volatility of total factor productivity. The model is driven by a stochastic volatility process, reason for which the model must be solved using a third order approximation to the policy function, in order to analyze productivity and volatility shocks separately. Shocks to the volatility of total factor productivity are contractionary, meaning that an increase in the degree of dispersion around shocks to total factor productivity negatively affect consumption, hours worked, investment and output, although these negative effects are small. The explanation for this behavior comes from precautionary savings motives of the representative households, who are more uncertain about the future, which hampers investment. This differs from the expansionary productivity shock that lead to higher consumption, investment and output.

However, there are some possible shortcomings within the model's framework. First, the model's assumption of the production of a single commodity may be more suitable for countries with a less diversified production matrix, such as an oil-producing country in the Middle East or a commodity-dependent economy in Latin America. At the same time, a model comprising two or more producing sectors would allow for a more specific analysis of the terms of trade. In particular, the production of an exportable commodity and an importable good allows for an explicit definition of the terms of trade, separately from total factor productivity. Thus, different shocks on these two variables and their volatilities can be studied one at a time. These caveats should be taken into account for future work.

1.7 Appendix

1.7.1 Data

The data for Argentina, Australia, Canada and Chile was obtained from the World Development Indicators of the World Bank, available online: `https://databank.worldbank.org`. The series used are *"GDP (constant LCU)"* for output, *"final consumption expenditure (constant LCU)"* for consumption and *"gross fixed capital formation (constant LCU)"* for investment. The difference between *"exports of goods and services (constant LCU)"* and *"imports of goods and services (constant LCU)"* is calculated to obtain the trade balance.[17] The frequency of the data is annual from 1960 to 2017, except for Canada's consumption series which dates from 1970 to 2017. Data on the total population of each country is also downloaded from the same source to obtain the variables in per capita terms.[18]

1.7.2 Canadian exports according to product

Table 1.7.1: Canadian exports by good (share of total exports)

Crude petroleum	14.0 %
Large sized cars	6.0 %
Medium sized cars	4.3 %
Oils petroleum	3.0 %
Natural gas (gas state)	2.1 %
Gold	1.8 %
Small sized cars	1.5 %
Coal	1.4 %
Colza seeds	1.4 %
Medicaments	1.2 %

Source: Observatory of Economic Complexity, (Simoes and Hidalgo, 2011).

1.7.3 GHH and other household preferences

Greenwood–Hercowitz–Huffman (GHH) preferences (Greenwood et al., 1988) are a special case of the Jaimovich-Rebelo preferences (Jaimovich and Rebelo, 2009) that parameterize the strength of short-run wealth effects on labor supply. Given consumption c_t and hours worked h_t in period t, the representative household's utility function has the form:

$$U(c_t, h_t) = \frac{\left(c_t - b_t \frac{h_t^{1+\omega}}{1+\omega}\right)^{1-\gamma} - 1}{1-\gamma} \tag{1.7.1}$$

[17]LCU are local currency units.

[18]In RBC models, the basic units of analysis are consumers, firms and the government. These models then make predictions for the representative agent's level of income and its consumption and savings decisions, and the representative firm's investment and production decisions. Thus, when comparing the predictions of the model with the observed data, it makes sense to consider time series on *per capita* aggregate activity, spending and trade. In addition, since the focus of study is to understand fluctuations at business cycle frequencies, we must then filter our data by detrending it and analyzing it in terms of its cyclical components.

where $b_t = c_t^{\xi} b_{t-1}^{1-\xi}$. Agents internalize the dynamics of b_t in the maximization problem. The presence of b_t makes preferences non time-separable in consumption and hours worked. When $\xi \to 0$, b_t turns into a constant given the absence of exogenous growth with $b_{t-1} = b_t$, and equation 1.7.1 equals equation 1.4.2 of section 2.4, where the wealth effect on the labor supply disappears. On the other hand when $\xi = 1$, then $b_t = c_t$ such that preferences are of the King-Plosser-Rebelo type (King et al., 1988):

$$U(c_t, h_t) = \frac{\left(c_t\left(1 - \frac{h_t^{1+\omega}}{1+\omega}\right)\right)^{1-\gamma} - 1}{1-\gamma} \tag{1.7.2}$$

1.7.4 Deterministic steady state

2.4.6 at the steady state gives:

$$1 = \beta\left[1 + r^* + \psi_1(e^{d-\bar{d}} - 1)\right] \tag{1.7.3}$$

Assuming $\beta(1 + r^*) = 1$ implies:

$$d = \bar{d}, \tag{1.7.4}$$

2.4.8 at the steady state is:

$$1 = \beta\left[\alpha\left(\frac{k}{h}\right)^{\alpha-1} + 1 - \delta\right], \tag{1.7.5}$$

this leads to the capital-to-labor ratio κ:

$$\kappa \equiv \frac{k}{h} = \left(\frac{\beta^{-1} - 1 + \delta}{\alpha}\right)^{1/(\alpha-1)} \tag{1.7.6}$$

Using 1.7.6 to eliminate the capital-labor ratio from 2.4.8 evaluated at the steady state, one obtains the level of hours worked at the steady state:

$$h = \left[(1-\alpha)\kappa^{\alpha}\right]^{1/(\omega-1)} \tag{1.7.7}$$

Then, the level of capital k at the steady state is:

$$k = \kappa h \tag{1.7.8}$$

Lastly, to find steady-state consumption, combine 1.4.3 with 1.4.6 at the steady state to obtain:

$$c = -r^* \bar{d} + \kappa^\alpha h - \delta k \tag{1.7.9}$$

1.7.5 Long-run equilibrium level of debt

From the aggregate resource constraint:

$$d_t = [1+r^*+\psi(e^{d_t-\bar{d}}-1)]d_{t-1}+c_t+k_{t+1}-(1-\delta)k_t+\Phi(k_{t+1}-k_t)-A_tF(k_t,h_t) \tag{1.7.10}$$

and the definition for the trade balance:

$$tb_t = y_t - c_t - i_t - \Phi(k_{t+1} - k_t) \tag{1.7.11}$$

The steady state gives:

$$tb = r^* \bar{d} \tag{1.7.12}$$

Where it is implied that the small open economy should have a trade balance surplus to service its foreign debt. Dividing both sides by output and solving for $\bar{d}$:

$$\bar{d} = \frac{tb/y}{r^*} y \tag{1.7.13}$$

At the steady state, output is given by:[19]

$$y = [(1-\alpha)\kappa^{\alpha\omega}]^{\frac{1}{\omega-1}} \tag{1.7.14}$$

[19] See Schmitt-Grohé and Uribe (2017).

Table 2.4.1 shows the calibrated parameters for α, r^* and δ. When substituted into equation 1.7.6 yields $\kappa = 3.244$. Using this value in equation 1.7.14 along the value given for ω, then $y = 1.508$. Given that the trade balance to output ratio is 3.1%, $tb/y = 0.031$. These two values are substituted into equation 1.7.13 and thus $\bar{d} = 0.935$.

1.7.6 Terms of trade AR(1) estimation

Dependent Variable: Terms of trade - Canada
Method: ARMA Maximum Likelihood (OPG - BHHH)
Sample: 1970 2017
Included observations: 48
Convergence achieved after 5 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
AR(1)	0.411786	0.132242	3.113877	0.0032
SIGMASQ	0.000760	0.000157	4.830024	0.0000

R-squared	0.175694	Mean dependent var	$-4.17E-06$
Adjusted R-squared	0.157775	S.D. dependent var	0.030689
S.E. of regression	0.028165	Akaike info criterion	−4.256861
Sum squared resid	0.036489	Schwarz criterion	−4.178895
Log likelihood	104.1647	Hannan-Quinn criter.	−4.227398
Durbin-Watson stat	1.753437		

Inverted AR Roots	.41

1.7.7 Model II - Impulse response functions after a level shock and a volatility shock

Figure 1.7.1: Level shock

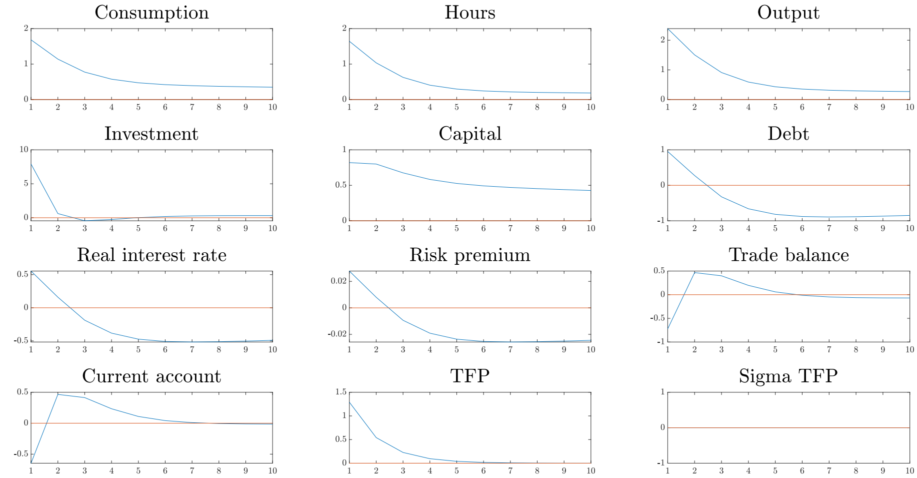
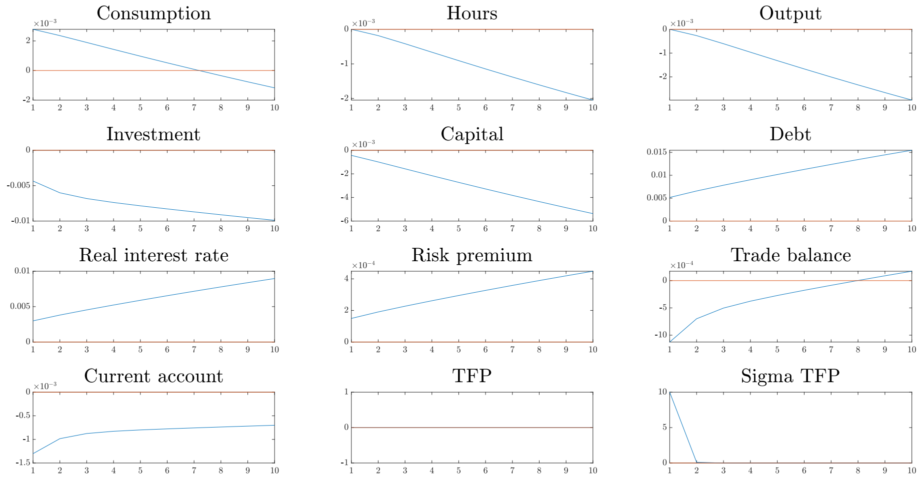


Figure 1.7.2: Volatility shock



1.7.8 Robustness check: impulse response functions after a level shock (left-hand side) and volatility shock (right-hand side)

Figure 1.7.3: Coefficient of relative risk aversion $\gamma = 5$

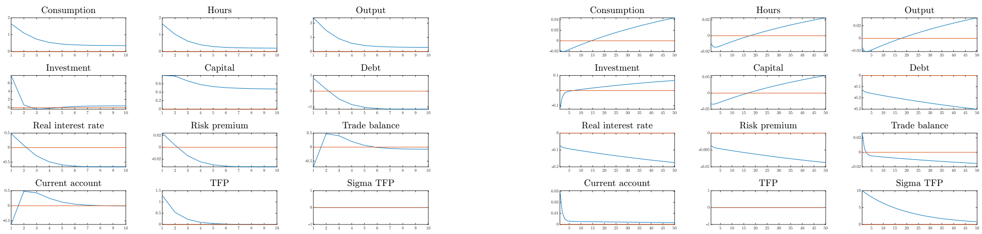


Figure 1.7.4: Coefficient of relative risk aversion $\gamma = 10$

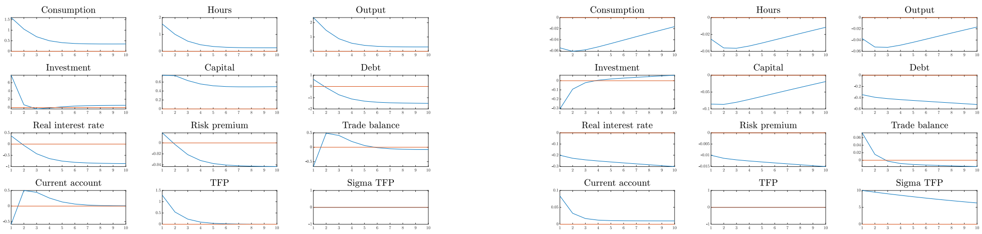


Figure 1.7.5: Persistence of volatility shock $\rho_\sigma = 0.25$

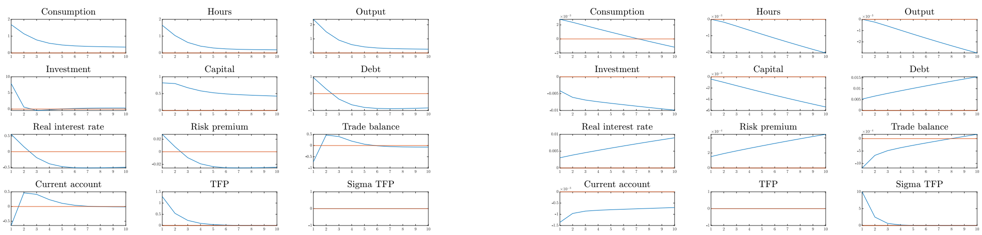


Figure 1.7.6: Coefficient of dispersion to volatility shocks $\eta = 0.5$

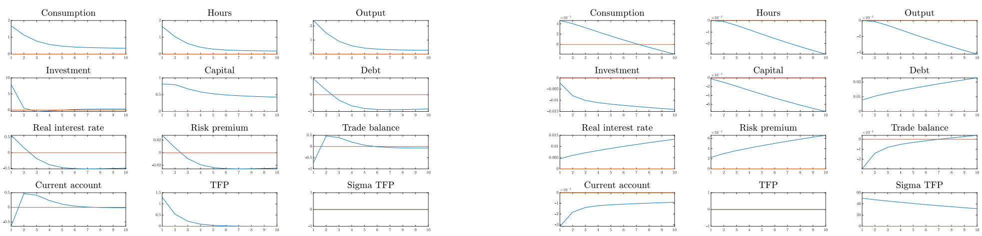


Figure 1.7.7: Persistence of productivity shock $\rho_A = 0.9$

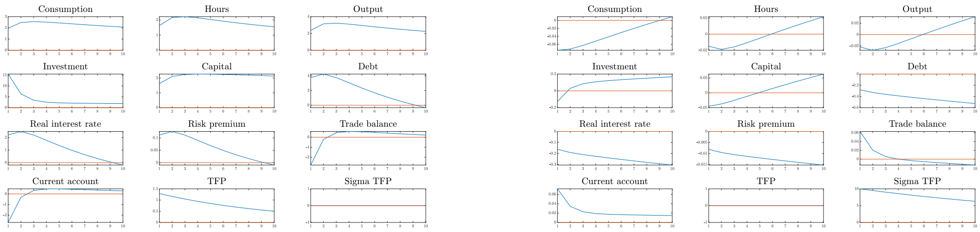
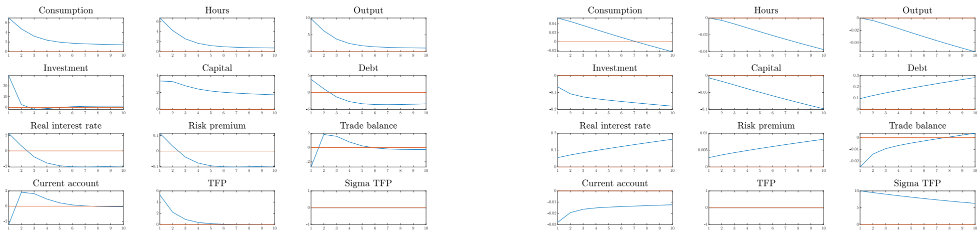


Figure 1.7.8: Coefficient of dispersion to productivity shocks $\bar{\sigma} = 0.05$



1.8 Solution of a DSGE model by perturbation methods

There is more than one method for finding the solution to a DSGE model. One such method is known as Dynamic Programming or Value Function Iteration (Bellman, 1957). Likewise, projection methods are another commonly used approach that is well explained by Judd (1998). The method implemented in this paper is a linear approximation to the policy function of the model. Linear approximations to the policy function are also known as perturbation methods, described by Collard and Juillard (2001a) and Schmitt-Grohé and Uribe (2004).

Perturbation methods build Taylor series approximations to the solution of a DSGE model around its deterministic steady state, solving the functional equation problem $\mathcal{H}(d) = 0$ by specifying a Taylor series expansion to the unknown function d in terms of the state variables of the model.

The equilibrium conditions for the general case of a model are:

$$E_t\mathcal{H}(y, y', x, x') = 0 \tag{1.8.1}$$

where E_t is the mathematical expectations operator; y and x are vectors of control and state variables in the current period, respectively and of sizes $n_y \times 1$ and $n_x \times 1$. Given $n = n_x + n_y$, function $\mathcal{H}$ maps $R^{n_y} \times R^{n_y} \times R^{n_x} \times R^{n_x}$ into R^n. Vectors y' and x' contain the control and state variables for the following period. The vector of states x is made of endogenous state variables and exogenous state variables (e.g. productivity shocks):

$$x = [x_1; x_2]' \tag{1.8.2}$$

The exogenous stochastic variables follow the process:

$$x_2' = c(x_2) + \sigma\eta_\epsilon\epsilon' \tag{1.8.3}$$

where x_2 and ϵ are of order $n_\epsilon \times 1$. The solution of the model is given by the following set of decision rules for the control variables and the state variables:

$$y = g(x, \sigma) \tag{1.8.4}$$

$$x' = h(x, \sigma) + \sigma\eta\epsilon' \tag{1.8.5}$$

where g maps $R^{n_x} \times R^+$ into R^{n_y} and h maps $R^{n_x} \times R^+$ into R^{n_x}. Equation 1.8.4 shows that controls depend on current states, while equation 1.8.5 shows that states in the future depend on states in the current period, as well as on future innovations.

Matrix η is of order $n_x \times n_\epsilon$ and given by:

$$\eta = \begin{bmatrix} \emptyset \\ \\ \eta_\epsilon \end{bmatrix}$$

$\emptyset$ rows are the states in the current period determining endogenous states in the future. η_ϵ are exogenous states in the following period depending on current states and future innovations.

The goal of perturbation methods is to find an approximation of functions g and h around the deterministic steady state $x_t = \bar{x}$ and $\sigma = 0$ by a Taylor series expansion. The steady state is defined as vectors $(\bar{x}, \bar{y})$ such that:

$$\mathcal{H}(\bar{y}, \bar{y}, \bar{x}, \bar{x}) = 0 \tag{1.8.6}$$

Plug in the solutions 1.8.4 and 1.8.5 to the equilibrium conditions of equation 1.8.1 to get:

$$F(x,\sigma) \equiv E_t\mathcal{H}\Big(g(x,\sigma), g(h(x,\sigma) + \sigma\eta\epsilon', \sigma), x, h(x,\sigma) + \sigma\eta\epsilon'\Big) = 0 \tag{1.8.7}$$

Since $F(x,\sigma) = 0$ for any x and σ, any derivatives of F must also be zero:

$$F_{x_i^k \sigma^j}(x,\sigma) = 0 \tag{1.8.8}$$

With this information, the **first order approximation** of $g(x,\sigma)$ and $h(x,\sigma)$ around $x = \bar{x}$ and $\sigma = 0$ is:

$$g(x,\sigma) = g(\bar{x},0) + g_x(\bar{x},0)(x-\bar{x})' + g_\sigma(\bar{x},0)\sigma \tag{1.8.9}$$

$$h(x,\sigma) = h(\bar{x},0) + h_x(\bar{x},0)(x-\bar{x})' + h_\sigma(\bar{x},0)\sigma \tag{1.8.10}$$

It is necessary to find $g_x(\bar{x},0), g_\sigma(\bar{x},0), h_x(\bar{x},0)$ and $h_\sigma(\bar{x},0)$. Taking the condition of equation 1.8.8 into account, g_x and h_x are found as the solution to the system:

$$[F_x(\bar{x},0)]^i_j = [\mathcal{H}_{y'}]^i_\alpha [g_x]^\alpha_\beta [h_x]^\beta_j + [\mathcal{H}_y]^i_\alpha [g_x]^\alpha_j + [\mathcal{H}_{x'}]^i_\beta [h_x]^\beta_j + [\mathcal{H}_x]^i_j = 0 \tag{1.8.11}$$

where $i = 1,...,n; j,\beta = 1,...,n_x; \alpha = 1,...,n_y$. The derivatives of $\mathcal{H}$ evaluated at $(y,y',x,x') = (\bar{y},\bar{y},\bar{x},\bar{x})$ are known. The system is then made of $n \times n_x$ quadratic equations in the $n \times n_x$ unknowns given by the elements of g_x and h_x. The system can be solved with a quadratic equation solver, such as the one from Blanchard and Khan (1985), Uhlig (1995), Klein (2000) or Sims (2002).

Likewise, g_σ and h_σ are found as the solution to the system:

$$\begin{aligned}[F_\sigma(\bar{x},0)]^i &= E_t([\mathcal{H}_{y'}]^i_\alpha [g_x]^\alpha_\beta [h_\sigma]^\beta + [\mathcal{H}_{y'}]^i_\alpha [g_x]^\alpha_\beta [\eta]^\beta_\phi [\epsilon']^\phi + [\mathcal{H}_{y'}]^i_\alpha [g_\sigma]^\alpha + [\mathcal{H}'_x]^i_\beta [h_\sigma]^\beta + [\mathcal{H}_{x'}]^i_\beta [\eta]^\beta_\phi [\epsilon']^\phi) \\ &= [\mathcal{H}_{y'}]^i_\alpha [g_x]^\alpha_\beta [h_\sigma]^\beta + [\mathcal{H}_{y'}]^i_\alpha [g_\sigma]^\alpha + [\mathcal{H}_y]^i_\alpha [g_\sigma]^\alpha + [\mathcal{H}_{x'}]^i_\beta [h_\sigma]^\beta \\ &= 0 \end{aligned} \tag{1.8.12}$$

where $i = 1,...,n; \alpha = 1,...,n_y; \beta = 1,...,n_x; \phi = 1,...,n_\epsilon$. In this order of approximation, it is not needed to correct the constant term of the approximation to the policy function for the size of the variance of the shocks because of certainty equivalence. This means that in a first order approximation, the expected values of x and y are equal to their values in the non-stochastic steady state $\bar{x}$ and $\bar{y}$, respectively.

The **second order approximation** of $g(x,\sigma)$ and $h(x,\sigma)$ around $x = \bar{x}$ and $\sigma = 0$ is:

$$[g(x,\sigma)]^i = [g(\bar{x},0)]^i + [g_x(\bar{x},0)]^i_a[(x-\bar{x})]_a + [g_\sigma(\bar{x},0)]^i\sigma$$
$$+ \frac{1}{2}[g_{xx}(\bar{x},0)]^i_{ab}[(x-\bar{x})]_a[(x-\bar{x})]_b + \frac{1}{2}[g_{x\sigma}(\bar{x},0)]^i_a[(x-\bar{x})]_a\sigma$$
$$+ \frac{1}{2}[g_{\sigma x}(\bar{x},0)]^i_a[(x-\bar{x})]_a\sigma + \frac{1}{2}[g_{\sigma\sigma}(\bar{x},0)]^i\sigma\sigma \quad (1.8.13)$$

$$[h(x,\sigma)]^j = [h(\bar{x},0)]^j + [h_x(\bar{x},0)]^j_a[(x-\bar{x})]_a + [h_\sigma(\bar{x},0)]^j\sigma$$
$$+ \frac{1}{2}[h_{xx}(\bar{x},0)]^j_{ab}[(x-\bar{x})]_a[(x-\bar{x})]_b + \frac{1}{2}[h_{x\sigma}(\bar{x},0)]^j_a[(x-\bar{x})]_a\sigma$$
$$+ \frac{1}{2}[h_{\sigma x}(\bar{x},0)]^j_a[(x-\bar{x})]_a\sigma + \frac{1}{2}[h_{\sigma\sigma}(\bar{x},0)]^j\sigma\sigma \quad (1.8.14)$$

where $i = 1, ..., n_y; a, b = 1, ..., n_x$ and $j = 1, ..., n_x$. In these two approximations, the terms $[g_{xx}], [g_{x\sigma}], [g_{\sigma x}], [g_{\sigma\sigma}], [h_{xx}, h_{x\sigma}], [h_{\sigma x}], [h_{\sigma\sigma}]$ are unknown. They are found by taking the derivative of $F(x,\sigma)$ with respect to x and σ twice and evaluating them at $x = \bar{x}$ and $\sigma = 0$, where these derivatives must be equal to zero. In particular g_{xx} and h_{xx} are found through the derivative $[F_{xx}(\bar{x},0)]^i_{jk}$; $g_{\sigma\sigma}$ and $h_{\sigma\sigma}$ are found as the solution to the system coming from $[F_{\sigma\sigma}(\bar{x},0)]^i$; and the cross derivatives $g_{x\sigma}$ and $h_{x\sigma}$ are found as a solution to the system of equations coming from $[F_{\sigma x}(\bar{x},0)]^i_j$.[20]

In this order of approximation, the coefficients of the policy function on the linear terms in the state vector do not depend on the size of the variance of the shocks.

Finally, the **third order approximation** of $g(x,\sigma)$ and $h(x,\sigma)$ around $x = \bar{x}$ and $\sigma = 0$, including the non-zero first and second order terms, is:

$$[g(x,\sigma)]^i = [g(\bar{x},0)]^i + [g_x(\bar{x},0)]^i_a[(x-\bar{x})]_a + \frac{1}{2}[g_{xx}(\bar{x},0)]^i_{ab}[(x-\bar{x})]_a[(x-\bar{x})]_b$$
$$+ \frac{1}{2}[g_{\sigma\sigma}(\bar{x},0)]^i\sigma\sigma + \frac{1}{6}[g_{xxx}(\bar{x},0)]^i_{abc}[(x-\bar{x})]_a[(x-\bar{x})]_b[(x-\bar{x})]_c$$
$$+ \frac{3}{6}[g_{\sigma\sigma x}(\bar{x},0)]^i_c\sigma\sigma[(x-\bar{x})]_c + \frac{3}{6}[g_{\sigma xx}(\bar{x},0)]^i_{bc}\sigma[(x-\bar{x})]_b[(x-\bar{x})]_c$$

[20] See Schmitt-Grohé and Uribe (2004) for the description of these equations which are omitted here for convenience.

$$+\frac{1}{6}[g_{\sigma\sigma\sigma}(\bar{x},0)]^i\sigma\sigma\sigma \quad (1.8.15)$$

$$\begin{aligned}[h(x,\sigma)]^j = [h(\bar{x},0)]^j + [h_x(\bar{x},0)]^j_a[(x-\bar{x})]_a + \frac{1}{2}[h_{xx}(\bar{x},0)]^j_{ab}[(x-\bar{x})]_a[(x-\bar{x})]_b \\ + \frac{1}{2}[h_{\sigma\sigma}(\bar{x},0)]^j\sigma\sigma + \frac{1}{6}[h_{xxx}(\bar{x},0)]^j_{abc}[(x-\bar{x})]_a[(x-\bar{x})]_b[(x-\bar{x})]_c \\ + \frac{3}{6}[h_{\sigma\sigma x}(\bar{x},0)]^j_c\sigma\sigma[(x-\bar{x})]_c + \frac{3}{6}[h_{\sigma xx}(\bar{x},0)]^j_{bc}\sigma[(x-\bar{x})]_b[(x-\bar{x})]_c \\ + \frac{1}{6}[h_{\sigma\sigma\sigma}(\bar{x},0)]^j\sigma\sigma\sigma \quad (1.8.16)\end{aligned}$$

where $i = 1, ..., n_y; a, b, c = 1, ..., n_x$ and $j = 1, ..., n_x$.

The terms $[g_{xx}], [g_{x\sigma}], [g_{\sigma x}], [g_{\sigma\sigma}],\ [h_{xx}, h_{x\sigma}], [h_{\sigma x}], [h_{\sigma\sigma}]$ are unknown and are found by solving the systems of equations shown in the appendix of Andreasen (2012). In this case, $g_{\sigma xx}, h_{\sigma xx}, g_{\sigma\sigma x}$ and $h_{\sigma\sigma x}$ do not depend on the third moments of the innovations. Meanwhile, if all innovations have symmetric distributions, then $g_{\sigma\sigma\sigma}$ and $h_{\sigma\sigma\sigma}$ are equal to zero.

1.8.1 Application: the case of a simple asset pricing model

The following application of the solution of a DSGE model has been used extensively in the literature as a simple example of a model that can be solved either analytically or using perturbation methods.[21] It is the Lucas asset pricing model in which a representative agent maximises lifetime utility based on consumption C_t:

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{C_t^\theta}{\theta} \quad (1.8.17)$$

subject to:

$$p_t e_{t+1} + C_t = p_t e_t + d_t e_t \quad (1.8.18)$$

[21] See Burnside (1998), Collard and Juillard (2001a), Schmitt-Grohé and Uribe (2004) and Andreasen (2012)

where e_t is an endowment of "trees" owned by the representative household in period t at price p_t. The dividends d_t follow the stochastic process:

$$d_{t+1} = \exp(x_{t+1})d_t \tag{1.8.19}$$

where $\exp(x_t)$ is the rate of growth of dividends which follows the AR(1) process:

$$x_{t+1} = (1-\rho)\bar{x} + \rho x_t + \sigma\eta\varepsilon_{t+1} \tag{1.8.20}$$

with $\varepsilon_{t+1} \sim N(0,1)$.

The optimality conditions of the household's problem are the budget constraint, a borrowing limit to prevent Ponzi schemes and the Euler equation:

$$p_t C_t^{\theta-1} = \beta E_t \left[C_{t+1}^{\theta-1}(p_{t+1} + d_{t+1}) \right] \tag{1.8.21}$$

In equilibrium, $C_t = d_t$ and $e_t = 1$. If the price-to-dividend ratio is defined as $y_t = p_t/d_t$ then:

$$y_t = \beta E_t \Big[\exp(\theta x_{t+1})(1 + y_{t+1}) \Big] \tag{1.8.22}$$

The analytical solution is given by:[22]

$$y_t \equiv g(x_t, \sigma) = \sum_{t=0}^{\infty} \beta^i \exp(a_i + b_i(x_t - \bar{x})) \tag{1.8.23}$$

where a_i and b_i are defined:

$$a_i = \theta\bar{x}i + \frac{\theta^2\sigma^2\eta^2}{2(1-\rho)^2}\left[i - \frac{2\rho(1-\rho^i)}{1-\rho} + \frac{\rho^2(1-\rho^{2i})}{1-\rho^2} \right] \tag{1.8.24}$$

$$b_i = \frac{\theta\rho(1-\rho^i)}{1-\rho} \tag{1.8.25}$$

[22]See Burnside (1998) for the algebraic procedure.

When evaluating the following derivatives: $a_i = \theta \bar{x} i$. Then one obtains the following first, second and third order derivatives around the deterministic steady state $g(x, \sigma) = (\bar{x}, 0)$ in the same fashion as equations 1.8.9, 1.8.10, 1.8.13, 1.8.14, 1.8.15 and 1.8.16:

$$g_x(\bar{x}, 0) =$$

$$\sum_{i=1}^{\infty} \beta^i \exp\left[a_i + b_i(x_t - \bar{x})\right] b_i$$

$$= \sum_{i=1}^{\infty} \beta^i \exp\left[\theta \bar{x} i\right] b_i \quad (1.8.26)$$

$$g_\sigma(\bar{x}, 0) =$$

$$\sum_{i=1}^{\infty} \beta^i \exp\left[a_i + b_i(x_t - \bar{x})\right] \frac{\theta^2 \sigma \eta^2}{(1-\rho)^2} \left[i - \frac{2\rho(1-\rho^i)}{1-\rho} + \frac{\rho^2(1-\rho^{2i})}{1-\rho^2}\right]$$

$$= 0 \quad (1.8.27)$$

$$g_{xx}(\bar{x}, 0) =$$

$$\sum_{i=1}^{\infty} \beta^i \exp\left[a_i + b_i(x_t - \bar{x})\right] b_i^2$$

$$= \sum_{i=1}^{\infty} \beta^i \exp\left[\theta \bar{x} i\right] b_i^2 \quad (1.8.28)$$

$$g_{x\sigma}(\bar{x}, 0) =$$

$$\sum_{i=1}^{\infty} \beta^i \exp\left[a_i + b_i(x_t - \bar{x})\right] \frac{2\theta^2 \sigma \eta^2}{2(1-\rho)^2} \left[i - \frac{2\rho(1-\rho^i)}{1-\rho} + \frac{\rho^2(1-\rho^{2i})}{1-\rho^2}\right] b_i$$

$$= 0 \quad (1.8.29)$$

$$g_{\sigma\sigma}(\bar{x},0) =$$

$$\sum_{i=1}^{\infty}\beta^i \exp\left[a_i + b_i(x_t-\bar{x})\right]\left(\frac{\theta^2\sigma\eta^2}{(1-\rho)^2}\left[i-\frac{2\rho(1-\rho^i)}{1-\rho}+\frac{\rho^2(1-\rho^{2i})}{1-\rho^2}\right]\right)^2$$
$$+\sum_{i=1}^{\infty}\beta^i \exp\left[a_i + b_i(x_t-\bar{x})\right]\frac{\theta^2\eta^2}{(1-\rho)^2}\left[i-\frac{2\rho(1-\rho^i)}{1-\rho}+\frac{\rho^2(1-\rho^{2i})}{1-\rho^2}\right]$$
$$=\sum_{i=1}^{\infty}\beta^i \exp\left[a_i + b_i(x_t-\bar{x})\right]\frac{\theta^2\eta^2}{(1-\rho)^2}\left[i-\frac{2\rho(1-\rho^i)}{1-\rho}+\frac{\rho^2(1-\rho^{2i})}{1-\rho^2}\right]$$
$$=\sum_{i=1}^{\infty}\beta^i \exp\left[\theta\bar{x}i\right]\frac{\theta^2\eta^2}{(1-\rho)^2}\left[i-\frac{2\rho(1-\rho^i)}{1-\rho}+\frac{\rho^2(1-\rho^{2i})}{1-\rho^2}\right] \quad (1.8.30)$$

$$g_{xxx}(\bar{x},0) =$$

$$\sum_{i=1}^{\infty}\beta^i \exp\left[a_i + b_i(x_t-\bar{x})\right] b_i^3$$
$$=\sum_{i=1}^{\infty}\beta^i \exp\left[\theta\bar{x}i\right] b_i^3 \quad (1.8.31)$$

$$g_{\sigma xx}(\bar{x},0) =$$

$$\sum_{i=1}^{\infty}\beta^i \exp\left[a_i + b_i(x_t-\bar{x})\right] b_i^2\frac{\theta^2\sigma\eta^2}{(1-\rho)^2}\left[i-\frac{2\rho(1-\rho^i)}{1-\rho}+\frac{\rho^2(1-\rho^{2i})}{1-\rho^2}\right]$$
$$=0 \quad (1.8.32)$$

$$g_{x\sigma\sigma}(\bar{x},0) =$$

$$\sum_{i=1}^{\infty}\beta^i \exp\left[a_i + b_i(x_t-\bar{x})\right]\frac{\theta^2\eta^2}{(1-\rho)^2}\left[i-\frac{2\rho(1-\rho^i)}{1-\rho}+\frac{\rho^2(1-\rho^{2i})}{1-\rho^2}\right] b_i \quad (1.8.33)$$

$$g_{\sigma\sigma\sigma}(\bar{x},0) =$$

$$\sum_{i=1}^{\infty}\beta^i \exp\left[a_i + b_i(x_t-\bar{x})\right]\frac{\theta^2\eta^2}{(1-\rho)^2}\left[i-\frac{2\rho(1-\rho^i)}{1-\rho}+\frac{\rho^2(1-\rho^{2i})}{1-\rho^2}\right]$$

$$\times \frac{\theta^2 \sigma \eta^2}{(1-\rho)^2} \left[i - \frac{2\rho(1-\rho^i)}{1-\rho} + \frac{\rho^2(1-\rho^{2i})}{1-\rho^2} \right]$$

$$= 0 \quad (1.8.34)$$

From these terms, it can be observed that $g_{xx}, g_{\sigma\sigma}, g_{xxx}$ and $g_{x\sigma\sigma}$ are different from zero, meaning that second and third order approximations do have an effect on the policy rule, differing from the certainty equivalence of a first order approximation.

Chapter 2

Terms of trade shocks, time-varying volatility and intersectoral dynamics in a small open economy

2.1 Introduction

This paper shows that shocks to the volatility of the terms of trade have a negative long-run effect on output, consumption and investment in a commodity-producing small open economy. The effect of terms of trade volatility on the economy is independent of the level of the terms of trade.

A simple structural vector autoregression shows that a shock to the terms of trade leads to a trade balance deficit, showing consistency with the Obstfeld-Razin-Svensson effects. Meanwhile, shocks to terms of trade volatility lead to a short-lived increase in aggregate investment. These results are validated with a real business cycles model for a small open economy featuring two producing sectors and stochastic volatility, such that the economy is driven by two types of shocks: a shock to the terms of trade and a shock to the volatility of the terms of trade. The model is solved using perturbation methods that allow to exploit information contained in higher-order moments of the distribution of shocks. The model is solved up to a third-order approximation of the policy function so that the volatility shock enters this function as an independent argument.

The model is able to replicate the ranking of persistence in the variables in the economy, the negative correlation between output and the trade balance, and the Obstfeld-Razin-Svensson effects, which are the negative relationship between the trade balance and the terms of trade. After a terms of trade shock, the economy grows and shifts towards the exportables sector, taking advantage of the favorable export prices. When the volatility shock hits, the effects are contractionary after a short-lived shift of the economy towards the importables sector to take advantage of its relative stability with respect to the exportables sector. To study the importance of volatility shocks in the model, a variance decomposition shows that the inclusion of volatility shocks in the model is important as it explains a good amount of the volatility of the variables contained in the model.

The paper is organized as follows: section 2.2 provides a review of the literature to put this work in context. Section 2.3 outlines the nature of terms of trade in Chile and the main features of Chilean business cycles to motivate this work. Section 2.4 describes of the model, the calibration of its parameters and the solution method used. Section 2.5 shows the results of the model in terms of its simulated moments, impulse response functions and variance decomposition, as well as some robustness checks. Section 2.6 concludes.

2.2 Literature Review

This paper is written at the junction of the RBC literature for small open economies (SOE-RBC) and the work on volatility and uncertainty in macroeconomics. The model used is closely based on Schmitt-Grohé and Uribe (2018), a two sector model for a small open economy with the main characteristics of the baseline SOE-RBC framework, in which the economy is driven by shocks to total factor productivity (Mendoza, 1991) and real interest rates (Schmitt-Grohé and Uribe, 2003). Following McCallum (1989), which suggests to include the terms of trade as a source of shocks in an RBC model, Mendoza (1991) makes a link between total factor productivity and the terms of trade, since the only good produced in the model is a fully exported commodity expressed in terms of a final imported good.

The model has similarities with Kim and Loungani (1992), as they build a two-sector model that includes energy as a productive input and its relative price as an exogenous process for a source of shocks. There are also close links with Mendoza (1995), which builds a three-sector model of exportable, importable and non-tradable goods, where the relative price of exportables with respect to importables

-the terms of trade- is exogenous and therefore a source of business cycles. Similarly, Kose (2002) builds a two-sector open economy model that produces an exportable primary good and a final non-tradable good, where the shocks come from the relative price of capital goods, intermediate inputs and the interest rate. In more recent literature, Garcia-Cicco et al. (2010) build an RBC model to explain business cycle fluctuations in emerging countries by analyzing effects that can be interpreted as coming from shocks to the terms of trade. In addition, Drechsel and Tenreyro (2018) build a two-sector model where on sector produces a particular commodity in which positive shocks to commodity prices have a positive effect on macroeconomic variables.

The SOE-RBC literature above studies the effects of shocks to the conditional *mean* of the terms of trade. However, interest has recently been given to the effects of temporary shocks to the conditional *second moment* of economic variables for the analysis of uncertainty and time-varying volatility in macroeconomics. The work on volatility and uncertainty in macroeconomics has grown in recent years, with Justiniano and Primiceri (2008), Bloom (2009) and Fernandez-Villaverde et al. (2011)as good sources of reference in the literature.[1]

In the context of the recent commodity price boom, many reports were made to analyse the relationship between commodity prices and the macroeconomy in commodity-producing countries. These reports studied: the syncronised behaviour of commodity prices with industrial production and global economic activity (IMF, 2012a); the higher GDP growth in these countries in periods of high commodity prices (1970s and 2000s), and lower growth in periods of adverse prices (1980s and 1990s) (IMF, 2012b); and the drop in commodity prices of 2014 accompanied by declines in real GDP growth rates in commodity-exporting countries (IMF, 2015).

[1] See Chapter 1.

Fernandez et al. (2015) document this relationship in the case of Latin American countries, including Chile, via empirical methods and in a business cycle model.

2.3 Empirical facts

2.3.1 Terms of trade in the Chilean economy

Table 2.3.1: Commodity dependency indicators in Chile

Merchandise and Commodity export dependence	**1995**	**2013-2017**	**2017**
Merchandise export value (millions of U.S. dollars)	15,901	69,770	69,229
Merchandise export concentration by product (HH index*)	0.3054	0.3267	0.3354
Commodity export value (millions of U.S. dollars)	13,825	59,363	60,285
Commodity exports (share of total merchandise exports)	87	86	87
Commodity exports (share of GDP)	18.4	22.8	22.0
Total natural resources rents (share of GDP)	8.0	13.2	10.5
Exports by commodity group (as share of exports)	87	86	87
Agricultural commodities	37	30	30
Fuels	-	1	1
Ores, metals	49	55	56
Three leading commodity exports (as share of total exports)	48	57	58
Copper	30	27	25
Copper ores and concentrates	12	22	25
Fruits and nuts	6	8	8

Source: State of Commodity Dependence 2019 UNCTAD (2019). The Herfindahl-Hirschmann (HH) index is a measure of the degree of product concentration that takes values between 0 (no concentration) and 1 (complete concentration). See UNCTAD (2019) for details.

Chile is a country with a high dependency on commodities, as they represent more than 80 percent of the country's total merchandise exports.[2] Table 2.3.1 shows that between 2013 and 2017, commodities represented 86 percent of total exports, a value that has not changed for more than twenty years. Of these commodities, copper is by far the most important one, as half of the Chile's total exports in 2017 came from this good in its various forms: copper ore, refined copper, raw copper, etc. Most of the remaining commodity exports are in the form of agricultural goods such as fruits (grapes, apples), fish and poultry, foodstuffs, etc., representing 30 percent of total exports. Once again, this composition has remained stable over

[2]The United Nations Conference on Trade and Development set the threshold of commodity dependence at 60 percent of commodity exports with respect to total exports, and 80 percent for high commodity dependence. See UNCTAD (2017).

time. Table 2.3.1 also shows the relevance of Chilean commodities in its aggregate economy: commodity exports represent more than 20 percent of gross domestic product, meaning that shocks affecting commodities can have a relevant effect on total output. Given these statistics, and the definition of the terms of trade as the ratio of the price of exports and the price of imports, terms of trade are expected to have a strong relevance on the Chilean business cycles.

2.3.2 Real business cycles in Chile

Table 2.3.2: Real business cycles stylized facts in Chile

Persistence	
$\rho_y(1)$	0.6251
$\rho_c(1)$	0.6778
$\rho_i(1)$	0.5290
$\rho_{tb}(1)$	0.4989
$\rho_{tot}(1)$	0.3003
Volatility ratio (w.r.t. output)	
σ_c/σ_y	1.3125
σ_i/σ_y	2.6916
σ_{tb}/σ_y	0.4377
σ_{tot}/σ_y	2.2157
Volatility ratio (w.r.t. terms of trade)	
σ_y/σ_{tot}	0.4513
σ_c/σ_{tot}	0.5924
σ_i/σ_{tot}	1.2148
σ_{tb}/σ_{tot}	0.1976
Correlation with output	
$\rho_{c,y}$	0.9085
$\rho_{i,y}$	0.8523
$\rho_{tb,y}$	-0.7334
$\rho_{tot,y}$	0.3175
Correlation with terms of trade	
$\rho_{y,tot}$	0.3175
$\rho_{c,tot}$	0.2957
$\rho_{i,tot}$	0.4023
$\rho_{tb,tot}$	-0.4638

Source: World Bank's World Development Indicators. The frequency is annual from 1960 to 2017 and all variables are expressed in real per capita terms (see appendix 2.7.1 for details). Output y, consumption c, and investment i are detrended in logs using the Hodrick-Prescott filter with smoothness parameter $\lambda = 100$. The trade balance (as a share of output) tb is detrended in levels using the same filter and smoothness parameter.

Table 2.3.2 shows the stylized facts observed in the business cycles of the Chilean economy. The moments shown on this table come from the cyclical components of the main macroeconomic series of Chile: output, consumption, investment, the trade balance and the terms of trade.[3] From these stylized facts we can observe the following:

i) High persistence of output and consumption, and moderate persistence of investment and the trade balance. The persistence of each variable, measured by its autocorrelation, shows there is a moderate persistence in output and consumption, while investment and the trade balance are less persistent with autocorrelations near 0.50. The terms of trade, meanwhile, has an autocorrelation of only 0.30, indicating not only a low persistence but also giving an indication of its volatile nature.

ii) Consumption is more volatile than output. The ratios of volatilities of all macroeconomic variables -measured by their standard deviations- with respect to the standard deviation of output y shows first of all, that there is a low degree of consumption smoothing, since the ratio $\sigma_c/\sigma_y = 1.3125$ shows that consumption is more volatile than output. Investment is almost three times as volatile as output, following a conventional features in advanced economies' business cycles, while the trade balance shows more stability with respect to output, being only half as volatile. The terms of trade, our series of interest, is twice as volatile as output with $\sigma_{tot}/\sigma_y = 2.2157$, which is once again proof of its volatile nature.

iii) The terms of trade are volatile. This volatile nature of the terms of trade is once again shown through the volatility ratios with respect to the terms of trade, which show that only investment is more volatile than the terms of trade, since $\sigma_i/\sigma_{tot} = 1.2148$. Output and consumption, meanwhile, are about half as volatile as

[3]See the appendix 2.7.1 for details on the construction of these series.

the terms of trade with relative ratios of 0.4513 and 0.5924, respectively, while the trade balance shows a very low relative volatility with respect to the terms of trade, $\sigma_{tb}/\sigma_{tot} = 0.1976$.

iv) A negative relationship between the trade balance and output. This is shown by the negative correlation between these two variables, which implies that higher economic activity leads to higher aggregate demand. This increase in aggregate demand entails higher consumption of domestic goods, but also of imported ones, leading to a negative balance of trade.

v) A negative relationship between the trade balance and the terms of trade. This relates to the fact that after an initial positive income effect from favourable terms of trade, the quantity of exports to the rest of the world decreases as they become more expensive for other countries. At the same time, imports increase as the domestic economy becomes richer from the revenues received, leading to a negative result in the balance of trade. The negative relationship between the terms of trade and the trade balance mentioned above has been studied extensively in the literature of open economy macroeconomics. It is described in the literature as the Obstfeld-Razin-Svensson (ORS) effects (Obstfeld (1982) and Svensson and Razin (1983)). These effects tend to occur over time, in contrast to the short-run Harberger-Laursen-Metzler (HLM) effects of Harberger (1950) and Laursen and Metzler (1950), by which there is a positive relationship between the terms of trade and the trade balance because favourable terms of trade coming from –for example– an increase in the price of exportable goods, lead to an increase in revenues from exports, and thus a positive balance of trade.

2.3.3 Terms of trade volatility shocks: a VAR model

In this section we study what the data shows when the Chilean economy is subject to two types of shocks: a shock to the terms of trade, and a shock to the volatility of the terms of trade. In a similar fashion as Schmitt-Grohé and Uribe (2018), the empirical exercise consists of the following systems:

$$A_0 X_t^{tot} = A_1 X_{t-1}^{tot} + \mu_t \tag{2.3.1}$$

$$A_0 X_t^{\sigma} = A_1 X_{t-1}^{\sigma} + \mu_t \tag{2.3.2}$$

where:

$$X_t^{tot} = \begin{bmatrix} tot_t & tb_t & y_t & c_t & i_t \end{bmatrix}'$$

and

$$X_t^{\sigma} = \begin{bmatrix} \sigma_t^{tot} & tb_t & y_t & c_t & i_t \end{bmatrix}'$$

where σ_t^{tot} is identified as the standard deviation of the terms of trade, calculated on an annual basis using the quarterly data described in appendix 2.7.1. The rest of the variables are, as mentioned in the same appendix, expressed in terms of their cyclical components using a Hodrick-Prescott filter with a conventional smoothing parameter.

The model differs from Schmitt-Grohé and Uribe (2018) in that it excludes the real exchange rate from X_t^{tot}, as I aim to focus more on terms of trade and its volatility (more on this in section 2.4). For that reason, this version also includes an additional matrix X_t^{σ} to study the effects of terms of trade volatility on the macroeconomy.

The ordering of the variables is as observed in the matrices above with A_0 being lower triangular; X_t^σ differs from X_t^{tot} in that the latter includes the terms of trade tot_t as a variable preceding the trade balance tb_t, output y_t, consumption c_t and investment i_t, while X_t^σ has the volatility of the terms of trade σ_t^{tot} in front of the mentioned variables.

Impulse response functions:

Figure 2.3.1 shows the responses of the variables in X_t^{tot} to a two standard-deviation shock to the terms of trade tot_t. The effect of this shock on investment i_t is positive and statistically significant for about two periods after the shock. The interpretation is straightforward as favourable terms of trade increase confidence in the economy and lead to a rise in investment. This translates into a positive, statistically significant effect on output. Even though the effect of the shock on consumption is not relevant, there is a statistically significant effect on the trade balance, which falls for two consecutive periods after the shock and remains negative thereafter. This shows that a positive terms of trade shock has an income effect that boosts the consumption of foreign goods, increasing imports more than any positive effect on exports. These effects fall in the category of the Obstfeld-Razin-Svensson effects for Chile, contrary to what is found for most small open economies in Schmitt-Grohé and Uribe (2018), which is closer to the so-called Harberger-Laursen-Metzler effects.

However, this work is interested in the effects of changes in the volatility of the terms of trade on aggregate variables. In that sense, figure 2.3.2 shows the effects of a two standard-deviation shock to the volatility of the terms of trade σ_t^{tot} in X_t^σ. The effect of this shock on investment is statistically significant for one period (year) after the shock. The effect does not appear to be statistically significant to all vari-

Figure 2.3.1: Response to a 2 s.d. shock to terms of trade

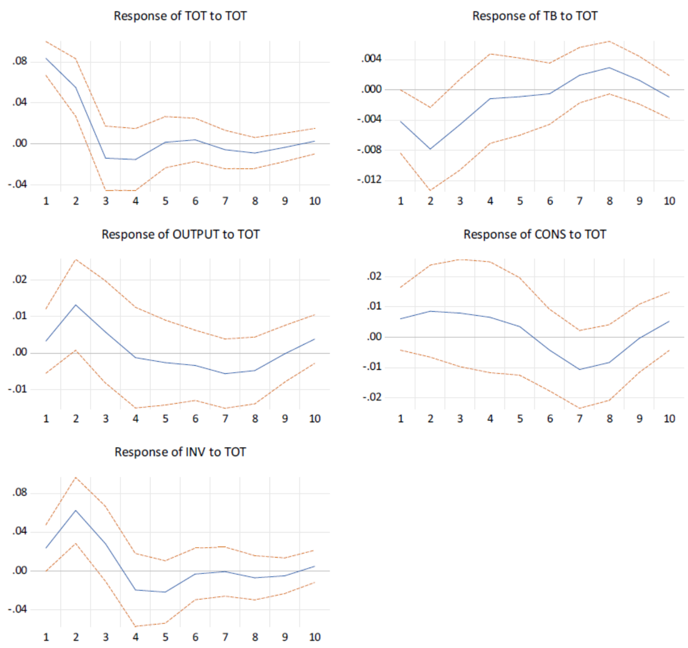
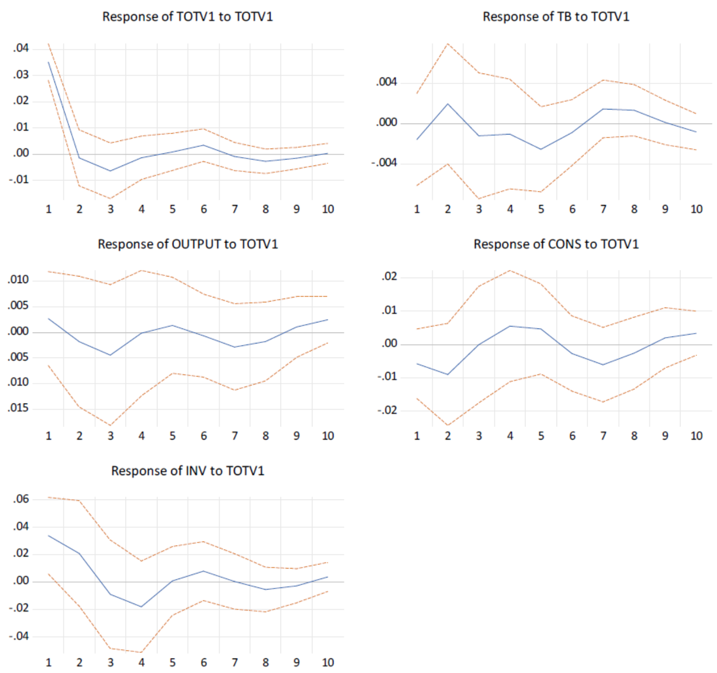


Figure 2.3.2: Response to a 2 s.d. shock to terms of trade volatility



ables of the model. The only effect appears to be on investment (a positive effect) although only for one period after the shock. Given the ordering of the variables in the system, the effect of the shock to terms of trade volatility might include changes in the other variables. Another reason has been explained in the literature through the so-called Oi-Hartman-Abel affect, by which growth occurs under uncertain times because firms increase investment to exploit good outcomes in order to hedge against bad times.[4]

Variance decomposition:

Appendix 2.7.2 shows the variance decomposition of both VAR systems X_t^{tot} and X_t^{σ}. The variance decomposition of X_t^{tot} shows that most of the forecast error variance of the terms of trade is explained by the variable itself. Ten periods after a shock to tot_t, three quarters of the forecast error variance of the terms of trade are explained by the terms of trade, 12 percent by output and about 7 percent by consumption. Investment and the trade balance explain only around 3 and 1 percent of the forecast error variance, respectively. These results are in line to those found for multiple countries in Schmitt-Grohé and Uribe (2018), where macroeconomic variables do not explain much of the forecast error variance of the terms of trade. For the case of the terms of trade volatility, the results quite similar: 70 percent of the forecast error variance of σ_t^{tot} is explained by itself. However, investment now explains 11 percent of the forecast error variance, followed by output (10 percent), consumption (7 percent) and the trade balance (1 percent).

The question on whether terms of trade volatility affects a small open economy such as Chile is addressed in the following section through a dynamic stochastic general equilibrium model.

[4]See Oi (1961), Hartman (1972) and Abel (1983).

2.4 Model

This section studies a SOE-RBC two-sector model of tradable goods as in Schmitt-Grohé and Uribe (2018), in which one sector produces an exportable good x and the other produces an importable good m. The commodity x is mostly (but not completely) exported abroad, while the importable good m is partly imported from the rest of the world but is also domestically produced. Both goods x and m are used in the production of a final good used for consumption and investment. Unlike the three-sector version of Mendoza (1995), this model excludes the non-tradables sector to focus only on the two tradable sectors x and m. This is done for simplicity at the expense of losing an important variable: the real exchange rate. However, the focus of the paper is on two main variables only: the terms of trade –exportables and importables– and its volatility.[5]

Households maximize lifetime utility through consumption of the final good and leisure from working in the exportable- and importable-good producing sectors. Households have access to international financial markets where the price of financial assets is determined. The terms of trade are defined as the ratio between the price of exportable good x and importable good m. As a small open economy, this ratio is assumed to be determined exogenously, and in this particular case, it follows a stochastic volatility process, being subject to stochastic shocks to its first- and second moments.

[5] A particular case in which all of commodities x are exported, while all importable goods m are imported would imply an unrealistic situation in which overall trade (the total sum of exports and imports) represents 200 percent of GDP. In reality, according to Schmitt-Grohé and Uribe (2017), this rate is around 40 percent in emerging countries.

2.4.1 Households

The economy is populated by a large number of identical households that maximize lifetime utility:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, h_t^x, h_t^m) \tag{2.4.1}$$

where utility is a function of i) consumption c_t of a final good -a composite of an exportable commodity x and an importable good m, ii) labor h_t^x provided by the household to the exportable good producing sector, and iii) labor h_t^m provided to the importable good industry. $U(c_t, h_t^m, h_t^x)$ takes the preference form of Greenwood et al. (1988) which eliminates the income effect on the supply of labour:

$$U(c_t, h_t^x, h_t^m) = \frac{\left[c_t - \frac{(h_t^x)^{\omega_x}}{\omega_x} - \frac{(h_t^m)^{\omega_m}}{\omega_m}\right]^{1-\gamma} - 1}{1-\gamma} \tag{2.4.2}$$

where γ is the inverse intertemporal elasticity of substituion and ω_x and ω_m are equal to one plus the Frisch elasticity of labour supply. Households maximize lifetime utility 2.4.2 subject to the sequential budget constraint:

$$\begin{aligned} c_t + i_t^x + i_t^m + \Phi^x(k_{t+1}^x - k_t^x) + \Phi^m(k_{t+1}^m - k_t^m) + d_t \\ = \frac{d_{t+1}}{1+r_t} + w_t^x h_t^x + u_t^x k_t^x + w_t^m h_t^m + u_t^m k_t^m \end{aligned} \tag{2.4.3}$$

where i_t^x and i_t^m are investments in capital k_t^x and k_t^m, which follow the laws of motion:

$$k_{t+1}^x = i_t^x + (1-\delta)k_t^x \tag{2.4.4}$$

$$k_{t+1}^m = i_t^m + (1-\delta)k_t^m \tag{2.4.5}$$

and $\Phi^x(\cdot)$ and $\Phi^m(\cdot)$ are capital adjustment costs:

$$\Phi^x(k_{t+1}^x - k_t^x) = \frac{\phi}{2}(k_{t+1}^x - k_t^x)^2$$

$$\Phi^m(k_{t+1}^m - k_t^m) = \frac{\phi}{2}(k_{t+1}^m - k_t^m)^2$$

d_t is a one-period non-contingent international financial asset, w_t^x and w_t^m are wage rates in each producing sector, and u_t^x and u_t^m are the rental prices of capital in each producing sector. Consumption, investment, wages, rental rates, debt and capital adjustment costs are all expressed in units of consumption.

The Lagrangian for the household's maximization problem is:

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ U(c_t, h_t^x, h_t^m) + \lambda_t \Bigg[\frac{d_{t+1}}{1+r_t} + w_t^x h_t^x + w_t^m h_t^m$$
$$+ u_t^x k_t^x + u_t^m k_t^m - c_t - i_t^x - i_t^m - \Phi^x(k_{t+1}^x - k_t^x) - \Phi^m(k_{t+1}^m - k_t^m) - d_t \Bigg] \Bigg\}$$

where λ_t is the Lagrange multiplier associated to the sequential budget constraint 2.4.3; the utility function $U(c_t, h_t^x, h_t^m)$ is given by 2.4.2. The first order conditions of the Lagrangian with respect to consumption, labour in both sectors, foreign debt, and capital in both sectors are given by:

$$U_c(c_t, h_t^x, h_t^m) = \lambda_t \tag{2.4.6}$$

$$U_{h^x}(c_t, h_t^x, h_t^m) = \lambda_t w_t^x \tag{2.4.7}$$

$$U_{h^m}(c_t, h_t^x, h_t^m) = \lambda_t w_t^m \tag{2.4.8}$$

$$\lambda_t = \beta(1+r_t)E_t\lambda_{t+1} \tag{2.4.9}$$

$$\lambda_t \Big[1 + \Phi_x'(k_{t+1}^x - k_t^x) \Big] = \beta E_t \lambda_{t+1} \Big[u_{t+1}^x + 1 - \delta + \Phi_x'(k_{t+2}^x - k_{t+1}^x) \Big] \tag{2.4.10}$$

$$\lambda_t \Big[1 + \Phi_m'(k_{t+1}^m - k_t^m) \Big] = \beta E_t \lambda_{t+1} \Big[u_{t+1}^m + 1 - \delta + \Phi_m'(k_{t+2}^m - k_{t+1}^m) \Big] \tag{2.4.11}$$

2.4.2 Production of goods x and m

The exportable and importable goods are each produced using capital and labor under a Cobb-Douglas type production function which is increasing, concave and homogeneous of degree one:

$$y_t^x = A^x (k_t^x)^{\alpha_x} (h_t^x)^{1-\alpha_x} \tag{2.4.12}$$

$$y_t^m = A^m (k_t^m)^{\alpha_m} (h_t^m)^{1-\alpha_m} \tag{2.4.13}$$

where y_t^x and y_t^m are the outputs in each sector, A^x and A^m stand for productivity parameters, and α_x and α_m are the shares of capital in production. Firms operate in a competitive market of factors of production and goods, such that profits in each sector are:

$$p_t^x y_t^x - w_t^x h_t^x - u_t^x k_t^x$$
$$p_t^m y_t^m - w_t^m h_t^m - u_t^m k_t^m$$

where p_t^x and p_t^m denote the prices of exportable and importable goods, respectively, in terms of the final good. Profit maximization leads to first order conditions:

$$u_t^x = p_t^x A^x \alpha (k_t^x)^{\alpha-1} (h_t^x)^{1-\alpha} \tag{2.4.14}$$

$$w_t^x = p_t^x (1-\alpha) A^x \left(\frac{k_t^x}{h_t^x}\right)^{\alpha} \tag{2.4.15}$$

$$u_t^m = p_t^m A^m \alpha (k_t^m)^{\alpha-1} (h_t^m)^{1-\alpha} \tag{2.4.16}$$

$$w_t^m = p_t^m (1-\alpha) A^m \left(\frac{k_t^m}{h_t^m}\right)^{\alpha} \tag{2.4.17}$$

2.4.3 Production of final goods

The final good produced, taken as the numeraire, is an aggregation of exportable and importable goods through a constant elasticity of substitution (CES) function that is increasing, concave and homogeneous of degree one:

$$F(a_t^x, a_t^m) = \left[(1-\chi)(a_t^x)^{1-\frac{1}{\mu}} + \chi (a_t^m)^{1-\frac{1}{\mu}}\right]^{\frac{1}{1-\frac{1}{\mu}}}$$

where a_t^x and a_t^m are the absorptions of goods x and m in the production of the final good, respectively. χ measures the weight of a_t^m in the final good production, and μ measures the elasticity of absorption between exportable and importable goods.

Similar to the exportable and importable good firms, the final good firm operates in a competitive market, such that profits are given by:

$$F(a_t^x, a_t^m) - p_t^x a_t^x - p_t^m a_t^m$$

where again, p_t^x and p_t^m denote the prices of exportable and importable goods in terms of the final good. Profit maximization gives first order conditions:

$$F_x(a_t^x, a_t^m) = p_t^x \tag{2.4.18}$$

$$F_m(a_t^x, a_t^m) = p_t^m \tag{2.4.19}$$

2.4.4 Equilibrium

In equilibrium, aggregate demand (including capital adjustment costs) equals aggregate supply:

$$c_t + i_t^x + i_t^m + \Phi^x(k_{t+1}^x - k_t^x) + \Phi^m(k_{t+1}^m - k_t^m) = F(a_t^x, a_t^m) \tag{2.4.20}$$

The evolution of external debt holdings is given by:

$$\frac{d_{t+1}}{1+r_t} = d_t + m_t - x_t \tag{2.4.21}$$

Exports x_t are the difference between domestic production and absorption of exportable goods, while imports m_t are the difference between domestic absorption and production of importable goods:

$$x_t = p_t^x (y_t^x - a_t^x) \tag{2.4.22}$$

$$m_t = p_t^m (a_t^m - y_t^m) \tag{2.4.23}$$

where p_t^x and p_t^m are the prices of exportable and importable goods in terms of the final good. To ensure stationarity, the interest rate on assets faced by domestic agents r_t is a function of the risk-free real interest rate r^* and a household-specific interest rate premium given by function $\psi(d_t)$ that increases with the aggregate level of foreign debt d_t, such that as debt increases, so does the interest rate households face in financial markets, affecting the household's optimal decisions:[6]

$$r_t = r^* + \psi\Big(e^{d_t - \bar{d}} - 1\Big) \tag{2.4.24}$$

where $\bar{d}$ is the level of debt in the steady state and ψ is a debt-elasticity parameter.

2.4.5 Terms of trade

The terms of trade are defined as the ratio between the price of exportables x and the price of importables m:

$$tot_t = \frac{p_t^x}{p_t^m} \tag{2.4.25}$$

This variable follows the autoregresive law of motion:

$$\log(tot_t) = \rho \log(tot_{t-1}) + \sigma_t^{tot} \varepsilon_t \qquad \varepsilon_t \sim \log N\Bigg(-\frac{(\sigma_t^{tot})^2}{2}, (\sigma_t^{tot})^2 \Bigg) \tag{2.4.26}$$

where σ_t^{tot} is a time-dependent standard deviation of an independent and identically distributed shock ε_t. This standard deviation also follows an autoregresive process:

[6]See Schmitt-Grohé and Uribe (2003) for other mechanisms to attain stationarity in a SOE-RBC model.

$$\log(\sigma_t^{tot}) = (1-\rho_\sigma)\log(\bar{\sigma}) + \rho_\sigma \log(\sigma_{t-1}^{tot}) + \eta u_t \qquad u_t \sim N(0,1) \tag{2.4.27}$$

where $\bar{\sigma}$ is the unconditional standard deviation of tot_t and η measures the standard deviation of an independent identically distributed shock u_t.

Equations 2.4.26 and 2.4.27 depict the terms of trade under a process of stochastic volatility, where tot_t is driven by two shocks: a shock ε_t that affects the (level of the) terms of trade directly (level shock), and a shock u_t that affects the standard deviation of ε_t, thus the volatility of the level shock. In that sense, u_t is deemed as a volatility shock.

Stochastic volatility is prefered over other time-varying volatility models such as the Generalized Autoregressive Conditional Heteroscedasticity (GARCH) process, which assumes that the variance of a variable is a function of its own past and a squared scaled innovation.[7] Under such framework, there is only one shock driving the dynamics of both the variable itself and its volatility: when there is a large innovation, there is a large volatility in the next period. Thus, we cannot separate a volatility shock from a level shock, and higher volatilities are triggered only by large level innovations. The interconnection of levels and volatility precludes the use of GARCH models to assess the effects of volatility independently from the effects of level shocks in a DSGE model.

A competitive equilibrium is a set of processes $c_t, h_t^m, h_t^x, d_{t+1}, i_t^m, i_t^x, k_{t+1}^m, k_{t+1}^x, a_t^m, a_t^x, p_t^m, p_t^x, y_t^m, y_t^x, w_t^m, w_t^x, u_t^x, u_t^m, r_t, \lambda_t, m_t$ and x_t satisfying equations 2.4.4 to 2.4.24, given initial conditions k_0^m, k_0^x, d_0 and the stochastic processes for tot_t and σ_t^{tot} of equations 2.4.26 and 2.4.27.

[7] The variance in a GARCH process is represented by: $\sigma_t^2 = \omega + \alpha(\sigma_{t-1}\varepsilon_{t-1})^2 + \beta\sigma_{t-1}^2$

2.4.6 Calibration of the model parameters

Table 2.4.1: Model parameters

Parameter	Description	Value	Source/target
Calibration			
β	Subjective discount factor	0.909	$1/(1+r^*)$
r^*	Real interest rate	0.10	U.S. Bill rate plus Chilean premium
δ	Depreciation rate	0.10	Mendoza (1991)
γ	Inverse intertemporal elasticity of substitution	2	Mendoza (1991)
ω_x	One plus Frisch elasticity labour supply x-sector	1.455	Mendoza (1991)
ω_m	One plus Frisch elasticity labour supply m-sector	1.455	Mendoza (1991)
α_x	Capital share in production x-sector	0.39	Fernandez et al. (2015)
α_m	Capital share in production m-sector	0.39	Fernandez et al. (2015)
ϕ_x	Capital adjustment cost x-sector	0.77	Correlation trade balance/output
ϕ_m	Capital adjustment cost m-sector	0.68	Correlation trade balance/terms of trade
ψ	Debt-elasticity	2.1	Terms of trade persistence
A^m	Productivity m-sector	1.000	Schmitt-Grohe and Uribe (2017)
μ	Sector elasticity of substitution	1.000	Schmitt-Grohe and Uribe (2017)
A^x	Productivity x-sector	0.951	Analytical steady-state
$\bar{d}$	Steady-state quantity of debt	0.107	Analytical steady-state
χ	Weight of m-sector in final good aggregation	0.214	Analytical steady-state
Stochastic volatility			
ρ	Autocorrelation terms-of-trade	0.2974	Chilean terms of trade
$\bar{\sigma}$	Standard deviation terms-of-trade	0.1056	Chilean terms of trade
ρ_σ	Autocorr. terms-of-trade volatility	0.3338	
η	Std. Dev. terms-of-trade volatility	0.0512	

Table 2.4.1 shows the values of the parameters in the model. These are calibrated in order to match the simulated moments from the model with selected targets chosen from stylized facts observed in the data. The time unit of the model is a year, as the paper focuses on the long-run effects of terms-of-trade volatility shocks, leaving price rigidities and short-run effects aside. In addition, this paper wants consistence with other relevant work in international macroeconomics that worked with annual data, such as Mendoza (1991), Mendoza (1995) and Garcia-Cicco et al. (2010), among others. This also allows us to work with a longer timespan than what would be possible under a monthly or quarterly model, allowing to distance this paper from a related work from Pfeifer et al. (2012) that uses quarterly data and therefore a much shorter timespan for calibration compared the one used herein.

The subjective discount factor β is equal to the inverse real interest rate $1/(1+r^*)$, where r^* is set at 10 percent, taking 5 percent as the long-run value of the U.S. 3-month Treasury Bill and the other 5 percent as the Chilean country risk premium, based on the long-run average return of the Chilean U.S. Dollar denominated Government Bond.[8] The depreciation rate for capital δ is set at an annual rate of 10 percent. The inverse elasticity of intertemporal substitution γ is equal to 2, and the value of one plus the inverse of the intertemporal elasticity of substitution in labor supply in both sectors ω_x and ω_m is equal to 1.455, following Mendoza (1991). The share of capital in the Cobb-Douglas production function in both sectors α_x and α_m is symmetrical and equal to 0.39, a value taken from Fernandez et al. (2015).

The parameters for capital adjustment costs in the export and import sector ϕ_x and ϕ_m are calibrated to match two targeted moments: the negative relationship between the trade balance and output and the negative relationship between the trade balance and the terms of trade. These values are $\phi_x = 0.77$ and $\phi_m = 0.68$, which in addition to helping match the targeted empirical moments, they also provide flexibility to investment in both exportable and importable producing sectors. The parameter of debt elasticity is $\psi = 2.1$ to match the persistence coefficient of the terms of trade. Lastly, the parameter for productivity in the importable-good sector A^m is set at unity following Schmitt-Grohé and Uribe (2017).

$A^x, \bar{d}$ and χ are determined after solving the model analytically for the deterministic steady-state.[9] In that context, $A^x = 0.951$, meaning that the productivity of exportables is slightly lower than the productivity of importables. The level of long-run debt is $\bar{d} = 0.107$ and the weight of the absorption of importables in the final good is $\chi = 0.214$, meaning that the importance of exportables in aggregate production is high, as about 80 percent of total output comes from that sector. The

[8]The long-run U.S. 3-month Treasury Bill rate is taken as its average value between 1960 and 2017, while the long-run return of the Chilean Government Bond is measured as its average value between 2004 and 2017. Source: Federal Reserve Bank of St. Louis and Bloomberg.

[9]See appendix 2.7.4 for the procedure to find the values of these parameters in the steady state.

coefficient for the elasticity of substitution between producing sectors is $\mu = 1$. This choice is based on the annual frequency of the model and on the fact that when $\mu = 1$, χ is interpreted as the share of importable-good absorption in the final good production.[10]

I calibrate the coefficients in 2.4.26 by estimating a one-lag autoregresive process of the terms of trade, with results shown in appendix 2.7.5. The value for the autocorrelation of the terms of trade $\rho = 0.2974$ is taken from the coefficient of the first lag of the regression, which is statistically significant, while $\bar{\sigma} = 0.1043$ is taken from the standard deviation of the dependent variable. The residuals of this regression display heteroskedasticity, as it is seen in the various tests performed and shown in appendix 2.7.6, proving that tot_t features a time-varying volatility.[11] The coefficients in 2.4.27 take values $\rho_\sigma = 0.3338$ and $\eta = 0.0512$, respectively, based on the estimation of an autoregresive process for σ_t^{tot} using the data on terms of trade volatility used for the SVAR of section 2.3.3.

2.4.7 Solution method

The model is solved using perturbation methods, implementing a third-order approximation to the policy function. Perturbations consist of Taylor series expansions of the policy function, such that they can exploit information contained in the higher-order moments of the distribution of the shocks in the model.[12]

The model described in section 2.4 is driven by two types of shocks: a shock ε_t

[10] See Schmitt-Grohé and Uribe (2017) for details on the determination of μ at different frequencies, and appendix 2.7.4 for the interpretation of χ when $\mu = 1$.

[11] Five different tests of homoskedasticity are implemented. The Breusch-Pagan-Godfrey and White tests reject the null hypothesis of homoskedasticity at the 1 percent significance level, while the Glejser rejects the null hypothesis at the 5 percent significance level. Only the ARCH and Harvey tests do not reject the null hypothesis at 12 and 43 percent significance levels, respectively. See appendix 2.7.6.

[12] See Judd (1996), Judd (1998), Collard and Juillard (2001a) and Collard and Juillard (2001b) for perturbation methods theory. This method does not suffer from the curse of dimensionality, meaning there are no computational problems even if the number of state variables increases.

to the terms of trade and a shock u_t to the volatility of the terms of trade (i.e. a shock to the standard deviation of ε_t). In this context, the Taylor series approximation of the policy function is implemented up to the third-order. The reason is that in a first-order approximation to the policy function, there is certainty equivalence: the approximation to the unconditional means of endogenous variables coincides with their values in the non-stochastic steady state, missing all of the dynamics induced by volatility. In particular, the first-order approximated policy function depends exclusively on shock ε_t and does not include the volatility shock u_t.

A second-order approximation to the policy function can capture the effects of the volatility shock u_t (Schmitt-Grohé and Uribe, 2004). However, these effects are captured only indirectly through the joint interaction with the level shock via the cross-product term $\varepsilon_t u_t$. That is, the *product* of the two innovations appear in the policy function, meaning that if ε_t is zero (the case when terms of trade do not suffer from any shock), there is no volatility effect on the policy function.

In a third-order approximation, however, the volatility shock u_t enters the policy function as an independent argument, meaning that even if there are no shocks to the first-moment ($\varepsilon_t = 0$), shocks to the second-moment can be analyzed separately. In other words, innovations to volatility play a role by themselves in the policy function.

2.5 Results

2.5.1 Simulation of moments

Table 2.5.1 shows the empirical and simulated moments of the Chilean business cycles. The left-hand side of the table reproduces the empirical moments obtained from Chilean data, already shown in table 2.3.2. The right-hand side shows the

Table 2.5.1: Empirical and simulated moments

	Data	Model
Persistence (autocorrelation)		
$\rho_{y_{t,t-1}}$	0.6251	0.6300
$\rho_{c_{t,t-1}}$	0.6778	0.7986
$\rho_{i_{t,t-1}}$	0.5290	0.3271
$\rho_{tb_{t,t-1}}$	0.4989	0.4661
$\rho_{tot_{t,t-1}}$	0.3003	0.3041
Volatility ratio (w.r.t output)		
σ_c/σ_y	1.3125	0.6649
σ_i/σ_y	2.6916	0.4009
σ_{tb}/σ_y	0.4377	0.5863
σ_{tot}/σ_y	2.2157	10.8163
Volatility ratio (w.r.t terms of trade)		
σ_y/σ_{tot}	0.1219	0.0925
σ_c/σ_{tot}	0.0754	0.0615
σ_i/σ_{tot}	0.0337	0.0371
σ_{tb}/σ_{tot}	0.0143	0.0542
Correlation with output		
$\rho_{c,y}$	0.9085	0.9682
$\rho_{i,y}$	0.8523	0.9327
$\rho_{tb,y}$	-0.7334	-0.7362
$\rho_{tot,y}$	0.3175	0.7913
Correlation with terms of trade		
$\rho_{y,tot}$	0.3175	0.7913
$\rho_{c,tot}$	0.2957	0.6256
$\rho_{i,tot}$	0.4023	0.9388
$\rho_{tb,tot}$	-0.4638	-0.4695

The empirical volatility ratios with respect to the terms of trade comes from the results of the variance decomposition of the VAR estimated in section 2.3, with results shown in appendix 2.7.2

simulated series obtained from the model's solution, where shocks ε_t and u_t are the drivers of the model.

The first group of indicators show the persistence of the variables, measured by their one-lag autocorrelation. The model does a good job replicating the model's ranking of persistence among macroeconomic variables. In particular, consumption is the most persistent variable in the model ($\rho_{c_{t,t-1}} = 0.7986$) followed by output ($\rho_{y_{t,t-1}} = 0.6300$). Meanwhile, investment displays a more volatile nature in comparison to the other variables mentioned above ($\rho_{i_{t,t-1}} = 0.3271$) followed by the trade balance ($\rho_{tb_{t,t-1}} = 0.3005$). Then, the model is calibrated so that the persistence of the terms of trade ($\rho_{tot_{t,t-1}} = 0.3041$) matches that of the data ($\rho_{tot_{t,t-1}} = 0.3003$).

The second group of indicators in table 2.5.1 shows the volatility of consumption, investment, the trade balance and the terms of trade with respect to output volatility. It can be seen from the data that consumption volatility in Chile is higher than that of output ($\sigma_c/\sigma_y = 1.3125$). The low consumption smoothing of households -as consumption fluctuates more than output- is a feature typical of developing small open economies (see Chapter 1). Unfortunately, this is not captured in the model, as consumption appears to be less volatile ($\sigma_c/\sigma_y = 0.6649$). Likewise, the model's investment volatility ratio ($\sigma_i/\sigma_y = 0.4009$) is much less volatile than the data ($\sigma_i/\sigma_y = 2.6916$), even with very low values for the capital adjustment cost parameters ϕ_x and ϕ_m.

In terms of the third group of indicators, which measure the volatility of output, consumption, investment and the trade balance with respect to the volatility of the terms of trade, I use the results from the variance decomposition of the VAR estimated in section 2.3 as the empirical counterpart, with results shown in appendix 2.7.2. In particular, I use the results coming from ten periods after the shock to the terms of trade. The model does a good job replicating a low relative volatility of output ($\sigma_y/\sigma_{tot} = 0.0925$), consumption ($\sigma_c/\sigma_{tot} = 0.0615$), investment ($\sigma_i/\sigma_{tot} = 0.0371$) and the trade balance ($\sigma_{tb}/\sigma_{tot} = 0.0542$).

As previously mentioned, one of the calibration targets of the model is the countercyclical relationship between the trade balance and output. The highly negative correlation between these two variables ($\rho_{tb,y} = -0.7362$) shows that in small open economies, higher economic activity leads to an increase of consumption of imported goods that exceeds the increase of exports coming from a terms of trade shock. In that sense, we can say that the price-effect of a terms of trade shock is offset by the demand-effect. Then, the model is good at featuring the highly procyclical nature of consumption ($\rho_{c,y} = 0.9682$) and investment ($\rho_{i,y} = 0.9387$) with correlations with output that are very close to the empirical counterparts. It also matches the positive correlation of output with the terms of trade ($\rho_{tot,y} = 0.7913$), although this correlation is much lower in the data.

The other target of the model is the negative relationship between the terms of trade and the balance of trade: the so-called Obstfeld-Razin-Svensson (ORS) effect discussed in section 2.3. The model makes a good replication of this correlation ($\rho_{tb,tot} = -0.4695$): high terms of trade are associated to higher imports since households are relatively richer. This is stronger than the income effect that positively affects the value exports in the short-run. The prevalence of this effect over the Harberger-Laursen-Metzler counterpart can be explained by the fact that, since the frequency of our data is annual, most short-run effects coming from terms of trade level- and volatility shocks are discarded in favor of long-run effects. Finally, the model is able to replicate a positive relationship between the terms of trade and output ($\rho_{y,tot} = -0.7913$), consumption ($\rho_{c,tot} = -0.6256$) and investment ($\rho_{i,tot} = -0.9388$). However, these simulated correlations are higher than those found in the data.

2.5.2 Impulse response functions

Terms of trade shocks

Figure 2.5.1 shows the impulse response functions after a shock of two standard deviations to the terms of trade ε_t in equation 2.4.26. This is equal to about an increase 11 percent in the terms of trade with respect to its long-run mean. As it can be seen from the figure, the volatility of the terms of trade does not change, as it is only the level of the terms of trade that is affected.

This shock has a direct and positive effect on the economy: the components of aggregate demand in the model –i.e. consumption and investment- grow by about 3 percent and 13 percent, respectively. Output also increases by about 5 percent, showing that a shock to the terms of trade has a direct positive impact on both aggregate demand and aggregate supply.

The responses seen in figure 2.5.1 are better explained by looking at figure 2.5.2, which shows the effects of the terms of trade shock at the sectoral level. Growth is channeled through a shift in production from the importables sector ym towards the exportables sector yx, growing by more than 5 percent while the importables sector ym falls by 1.5 percent after two periods. From the weight parameter χ, it is expected that any impact on the production of exportables will have stronger consequences on the aggregate economy than any effects from the importables sector.

The rise of aggregate investment is also explained by the impact on investment in the exportables sector ix, which grows by 40 percent after the shock while investment in importables im falls by almost 10 percent. Demand for factors of production in exportables grows, such that wages wx and the rental price of capital in the ex-

Figure 2.5.1: Impulse responses from a two s.d. terms of trade level shock

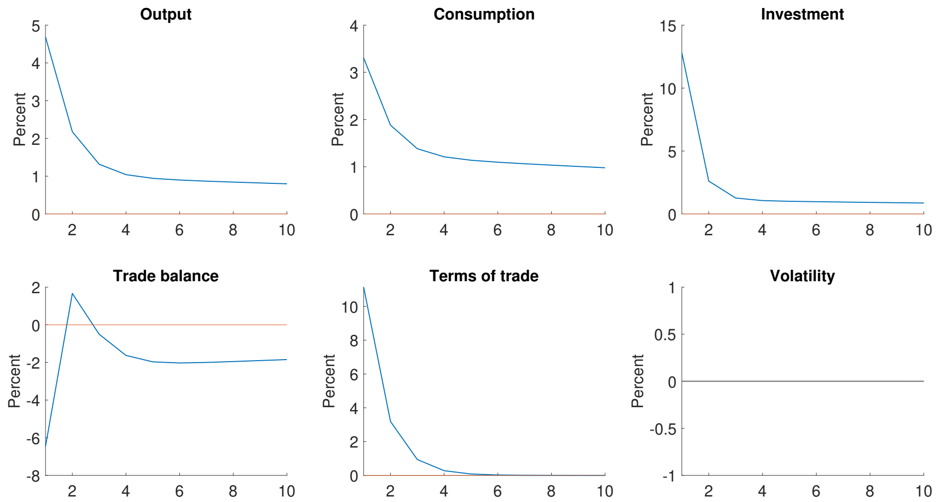
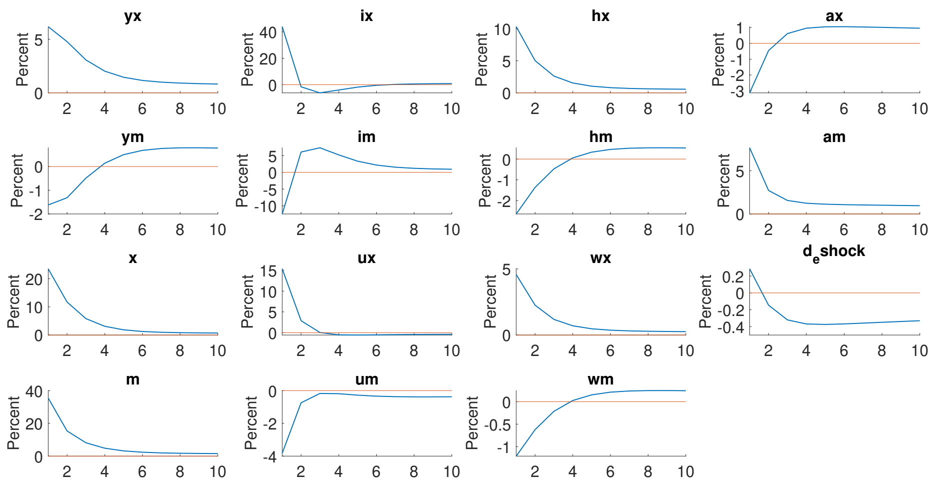


Figure 2.5.2: Impulse responses from a two s.d. terms of trade level shock



portables sector ux grow by about 5 and 15 percent respectively, as a result of the shock. Meanwhile, wages and capital rental rates in the importable-good sector (wm and um respectively) have an initial drop after the shock, as production preferences shift towards exportables.

The shock increases the cost of exportables as inputs in the production of the final good. Thus, absorption of exportables ax falls by 3 percent in favor of importables am, which grow by more than 5 percent after the shock. Since the production of exportables has grown but absorption has fallen, there is an increase in exports x of about 20 percent. Because importables have become relatively cheaper, absorption of importables am grows, although production of importables falls as resources are now destined towards the more profitable exportable-good sector. Thus, imports m grow by 40 percent. As a result, the terms of trade shock leads to a fall in the trade balance of 6 percent, seen in the lower-left graph of figure 2.5.1.

The response of the trade balance over time shows its volatile nature, which is consistent with the cyclical properties shown in table 2.3.2; in particular, its low autocorrelation ($\rho_{tb} = 0.3041$). Then, the balance of trade deficit observed in the long-run is consistent with the Obstfeld-Razin-Svensson effects, by which favorable terms of trade lead to growth in imports and ultimately a deficit on the trade balance. This is also found in the data through the negative correlation of the trade balance and the terms of trade ($\rho_{tb,tot} = -0.4695$). Imports grow leading to the deficit in the trade balance because the terms of trade shock allows households to increase consumption without the need to finance it with debt. Thus, they become creditors and reduce their debt holdings as seen in figure 2.5.2.

Figure 2.5.3: Impulse responses from a two s.d. terms of trade volatility shock

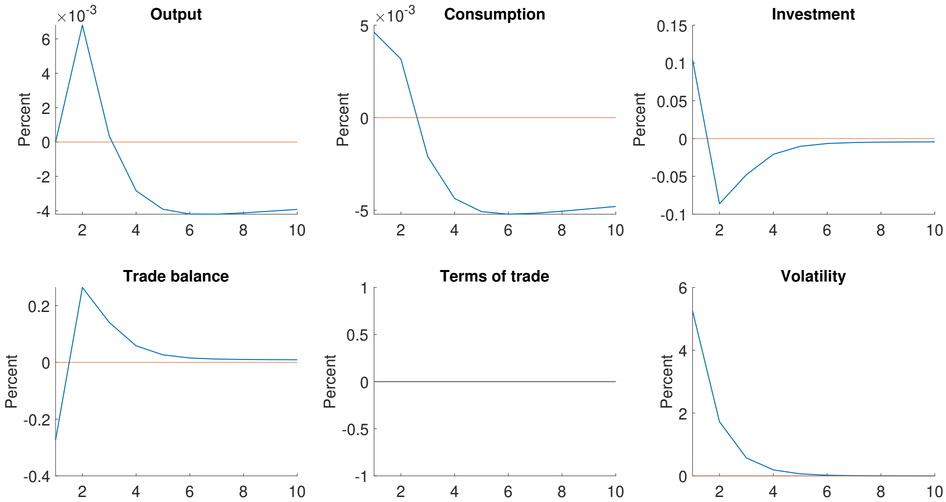
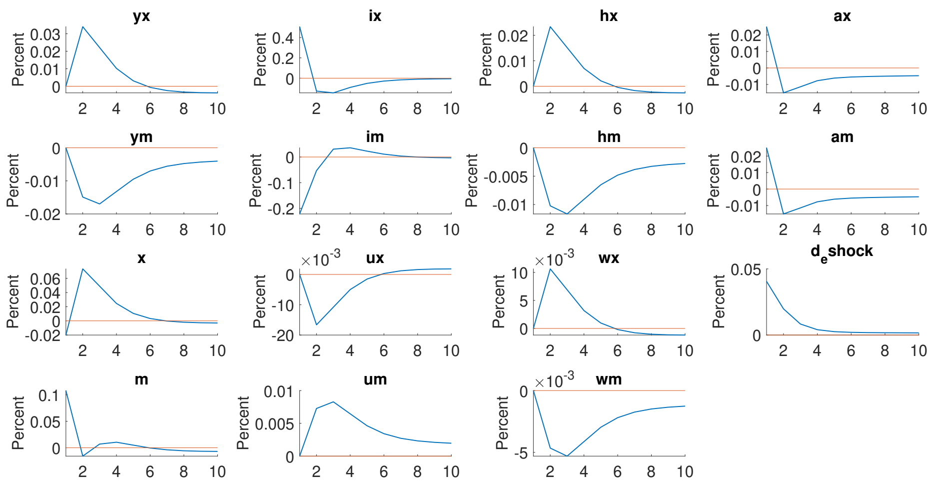


Figure 2.5.4: Impulse responses from a two s.d. terms of trade volatility shock



Volatility shocks

The objective of this paper is to analyze how the results above differ when there is an increase in the *volatility* of the terms of trade. Or in terms of the model, what happens to the macroeconomic variables when there is a shock to u_t in equation 2.4.27. Figure 2.5.3 shows the response of output, consumption, investment and the balance of trade after a two standard deviation shock to u_t, equal to about a 5 percent increase in the volatility of the terms of trade. As it is clear, there is no reaction in the terms of trade because the shock is only affecting the volatility component of the series. Instead, the standard deviation (the magnitude) of the shock to the level of terms of trade is affected. The ability to differenciate between these two shocks is the advantage of stochastic volatility with respect to the GARCH model mentioned in section 2.4.

Initially, the effects of a volatility shock appear to be similar to those coming from a terms of trade shock: consumption and investment grow, leading to an increase in output and a deficit in the trade balance. However, these positive effects are quickly reversed: after the initial rise, consumption falls persistently over time after the volatility shock, as seen in figure 2.5.3. To understand the mechanism behind this drop, recall that 2.4.2 is a concave constant relative-risk aversion utility function, such that the marginal utility of consumption $U_c(c_t, h_t^x, h_t^m)$ is convex. The Euler equation coming from the solution of the household's problem (see section 2.4) is:

$$\frac{1}{1+r_t} = \beta \cdot \frac{E_t \cdot U_c(c_{t+1}, h_{t+1}^x, h_{t+1}^m)}{U_c(c_t, h_t^x, h_t^m)} \tag{2.5.1}$$

Higher volatility in the terms-of-trade does not affect current marginal utility of consumption $U_c(c_t, h_t^x, h_t^m)$, but it will affect future *expected* marginal utility $E_tU_c(c_{t+1}, h_{t+1}^x, h_{t+1}^m)$. Because of the convexity of $U_c(c_t, h_t^x, h_t^m)$ and by Jensen's inequality, higher uncertainty leads to a more disperse (higher) expected marginal utility $E_tU_c(c_{t+1}, h_{t+1}^x, h_{t+1}^m)$. The reason is that a volatility shock leads to changes in the distribution of the shock to the terms of trade ε_t, such that output in each sector will grow (or fall) by a higher margin and $E_tU_c(c_{t+1}, h_{t+1}^x, h_{t+1}^m)$ will rise. A higher marginal utility of consumption in the future deters households from consuming today and rather they increase savings to buffer for the uncertain income coming from more volatile terms of trade, as it can be seen in figure 2.5.4, where debt holdings d_t increase after the volatility shock.

Meanwhile, the behavior of aggregate investment is explained by the rise of investment in the exportables sector ix, which is higher than the fall of investment in importables im. However, the positive effect on investment is short-lived: even though investment in importables recovers over time, there is a sudden drop of investment in the exportables sector. From the weight parameter χ, by which the share of the exportables sector in aggregate production is around 80 percent, it is expected that changes in ix will have a stronger effect on the economy than im, which is what occurs as seen in figure 2.5.3.

From figure 2.5.4, sectoral output is affected by the investment dynamics of the model. After the volatility shock, and with a one-period lag, output in the exportables sector rises while production of importables falls. This effect is similar to what happens under a terms of trade shock. The difference resides in the fact that -if only volatility is driving the model- ix starts a steady decline that becomes contractionary after six periods. Production of importables im, meanwhile, never reaches expansionary values, so that aggregate output is contractionary three period after a volatility shock, as seen in figure 2.5.3.

How do these results compare to the data? Comparing the impulse responses of figures 2.5.3 and 2.5.4 with those from figure 2.3.2, it is seen that in both cases, the effects of a volatility shock are quantitatively small. In the case of investment, a volatility shock leads to a volatile reaction in the model that can be also seen in the data. In the empirical VAR, a shock to terms of trade volatility leads to an initial rise in investment followed by a drop and then oscilation near the steady-state. The fluctuating nature of the trade balance can also be seen, where in both cases, a volatility shock has an initial negative effect on the trade balance but sees a recovery shortly after. Appendix 2.7.3 shows the impulse response functions when the SVAR of section 2.3.3 is re-estimated using sectoral data. In particular, two additional SVARs are estimated: a SVAR where aggregate output and aggregate investment is replaced by output and investment in the exportables sector, and a SVAR where aggregate output and aggregate investment is replaced by output and investment in the importables sector.[13] The impulse responses from these systems are shown in figure 2.7.1. The SVAR of the exportables sector replicates the fall in the trade balance and the rise of consumption following a terms of trade shock. More importantly, it shows the fall in output and consumption, as well as the balance of trade surplus following a terms of trade volatility shock. Figure 2.7.2 shows the responses of the SVAR including the importables sector.

2.5.3 Variance decomposition

As it was seen in the previous section, the effects from volatility shocks differ from the effects of terms of trade shocks. This goes in line with most of the literature on uncertainty, where the latter deters consumption, investment and hiring through various channels.[14] However, the effects of terms of trade volatility shocks appear to be very small. Figure 2.5.3 shows that even though a volatility shock implies a

[13] See appendix 2.7.3.

[14] See for example, Bloom (2009).

Table 2.5.2: Volatility of model simulated moments under different order approximations

Vol./Approx.	1st-order	2nd-order	Baseline model 3rd-order
σ_y	1.700	1.971	2.171
		(16.0)	(10.1)
σ_c	1.147	1.288	1.443
		(12.3)	(12.0)
σ_i	0.694	0.814	0.870
		(17.3)	(6.9)
σ_{tb}	1.305	1.291	1.273
		(-1.1)	(-1.4)
σ_{tot}	22.248	22.767	23.479
		(2.3)	(3.1)

The table displays the volatilities of output, consumption, investment, the trade balance and terms of trade measured by their standard deviations (in percentage) when the model is solved under a first, second and third order approximation of the policy function. The third order approximation to the policy function is the baseline model. Percentage changes in volatility, from one solution to the next, are shown in parentheses.

change in the standard deviation of the terms of trade in almost 5 percent, the effects on consumption, investment and output are of less than one percent. This seems to match the results coming from the estimations of the VAR system presented in section 2.3.3, where the effects of a shock to the volatility of the terms of trade did not show statistical significance in output and consumption.

Given the importance of the terms of trade in a commodity-exporting country such as Chile, how much of the Chilean business cycle fluctuations are explained by the terms of trade and how much by the volatility of the terms of trade? Because of the non-linear nature of the model, a conventional variance decomposition of the kind observed in a linear model cannot be made. For that reason, the approach to this question is to compare the implied volatility of the variables from the baseline model driven by both a level and volatility shocks ε_t and u_t, respectively, with simulated volatilities from a model driven only by a terms of trade (level) shock ε_t, such that $u_t = 0$.

Table 2.5.2 shows the volatility -measured by the standard deviation- of the main variables of the model under three different approximations to the policy function. Besides the third order approximation of the policy function from the baseline model, the results from the first and second order approximations are shown. It can be observed that in the model, a second and third-order approximation to the policy function contribute a good amount of volatility: when the policy function (the model solution) is approximated up to a second-order, such that volatility enters the model via a combination of ε_t and u_t ((Schmitt-Grohé and Uribe, 2004)), output volatility σ_y grows 16.0 percent with respect to the model solved with a linear approximation. Likewise, consumption volatility σ_c grows 12.3 percent, and the volatility of investment grows 17.3 percent. Under a second order approximation, however, volatility shocks still do not enter the policy function independently. With the solution method of the baseline model (third order approximation), volatility in the model continues to increase: output volatility σ_y grows by another 10.1 percent, consumption volatility σ_c grows 12.0 percent, and investment volatility grows σ_i grows 6.9 percent. This shows that volatility shocks u_t are very important to explain volatility in the model independently of ε_t.

Table 2.5.3: Volatility of model simulated moments under one or both shocks

	Baseline model		
Shocks	ε_t, u_t	ε_t	**Change (%)**
σ_y	2.171	2.045	(6.2)
σ_c	1.443	1.351	(6.8)
σ_i	0.870	0.829	(5.0)
σ_{tb}	1.273	1.275	(-0.2)
σ_{tot}	23.479	22.543	(4.2)

The baseline model includes both level ε_t and volatility u_t shocks. The last column measures the change of results (volatilities) in percentage terms between the baseline model and the model with level shock ε_t only.

To see the importance of modeling terms of trade tot_t under stochastic volatility, table 2.5.3 shows the moments obtained from the model solution when volatility shocks are shut down -i.e. when $u_t = 0$. By solving the model including the volatility shock, the standard deviation of output grows by 6.2 percent, consumption volatility grows 6.8 percent, and investment volatility increases 5.0 percent. This is important because ignoring the volatile nature of the variables in the model would affect the match of its simulated second moments with the empirical counterpart.

2.5.4 Robustness check

Table 2.5.4: Simulated second moments

	Baseline	$\gamma = 5$	$\gamma = 10$	$\mu = 0.5$	$\mu = 1.5$
Persistence (autocorrelation)					
$\rho_{y_{t,t-1}}$	0.6300	0.7319	0.4881	0.6403	0.6217
$\rho_{c_{t,t-1}}$	0.7986	0.8692	0.4917	0.8032	0.7943
$\rho_{i_{t,t-1}}$	0.3271	0.4569	0.4874	0.4012	0.2824
$\rho_{tb_{t,t-1}}$	0.4661	0.6800	0.0853	0.7912	0.2869
$\rho_{tot_{t,t-1}}$	0.3041	0.3041	0.3041	0.3041	0.3041
Volatility ratio (w.r.t. output)					
σ_c/σ_y	0.6649	0.6889	0.4232	0.6697	0.6624
σ_i/σ_y	0.4009	0.3796	0.5895	0.3877	0.4102
σ_{tb}/σ_y	0.5863	0.5628	0.8271	0.2936	0.8535
σ_{tot}/σ_y	10.8163	9.294	11.7472	6.3213	13.7186
Volatility ratio (w.r.t. terms of trade)					
σ_y/σ_{tot}	0.0925	0.1076	0.0851	0.1582	0.0729
σ_c/σ_{tot}	0.0615	0.0741	0.0360	0.1059	0.0483
σ_i/σ_{tot}	0.0371	0.0408	0.0502	0.0613	0.0299
σ_{tb}/σ_{tot}	0.0542	0.0606	0.0704	0.0464	0.0622
Correlation with output					
$\rho_{c,y}$	0.9682	0.9748	1.0000	0.9708	0.9668
$\rho_{i,y}$	0.9327	0.9461	0.9734	0.9442	0.9236
$\rho_{tb,y}$	-0.7362	-0.7674	-0.0677	-0.8493	-0.6489
$\rho_{tot,y}$	0.7913	0.6712	0.9460	0.7894	0.7886
Correlation with terms of trade					
$\rho_{y,tot}$	0.7913	0.6712	0.9460	0.7894	0.7886
$\rho_{c,tot}$	0.6256	0.5002	0.9455	0.6265	0.6228
$\rho_{i,tot}$	0.9388	0.8567	0.9025	0.9384	0.9338
$\rho_{tb,tot}$	-0.4695	-0.3309	-0.0164	-0.4627	-0.4601

The baseline column shows the results of the baseline model already shown in table 2.5.1, where $\gamma = 2$ and $\mu = 1$. The rest of the columns show the simulated second moments when the coefficient of relative risk aversion takes values $\gamma = 2$ and $\gamma = 5$; and the coefficient for intersectoral elasticity of subtitution takes values $\mu = 0.5$ and $\mu = 1.5$.

In this section, we analyze the results of the model when solved under different parameters. For instance, the focus is set on: i) the effects from changes in the coefficient of relative risk aversion and ii) the effects from changes in the intersector elasticity of substitution. Table 2.5.4 shows the simulated second moments in the economy under the different changes to the model parameters.

In Neumeyer and Perri (2005), the relative risk aversion coefficient is set at $\gamma = 5$, to emphasize on the nature of developing small open economies. Under this risk aversion coefficient, the model does not lose the properties shown by the simulated second moments: the persistence of output and the rest of the variables holds much relationship with their empirical counterparts. The relationships between consumption, investment and the trade balance with output remain the same. In particular, the stylized fact of a negative correlation between the trade balance and output remains, while the Obstfeld-Razin-Svensson effects are also observed by the negative correlation between the trade balance and the terms of trade.

However, increasing the coefficient to $\gamma = 10$ leads to poorer results. As agents become more risk averse, the properties of the simulated economy change: the economy becomes more volatile, as the persistence of most variables in the model is reduced; there is virtually no relationship between output and the trade balance, implying that because of risk, agents reduce their consumption abd therefore imports. In addition, the Obstfeld-Razin-Svensson effects seen both in the data and the baseline calibration disappear. In terms of the model dynamics, figures 2.7.3 and 2.7.4 in appendix 2.7.7 show the responses of the model after a two standard deviation shock to the terms of trade and its volatility, with results that are qualitatively similar to the baseline model.

The baseline model assumes a constant elasticity of substitution between producing sectors, such that $\mu = 1$. In this exercise, I analyze the cases in which there is less elasticity of substitution between sectors, as well as more elasticity. In the first case, $\mu = 0.5$, the features of the model do not change considerably. These are:

the persistence coefficients, the negative correlation between the trade balance and output, and the ORS effects (negative correlation between the trade balance and the terms of trade). Similarly, under higher sectoral elasticity of substitution, $\mu = 1.5$, the features of the simulated moments do not change significantly, and the impulse responses from figures 2.7.5 and 2.7.6 show that the model dynamics after volatility shocks are robust to the different degrees of intersector elasticity.

2.6 Conclusion

This paper studied a two-sector real business cycle model for a small open economy to study the macroeconomic effects of shocks to the volatility of the terms of trade. In the model, the economy produces an exportable- and an importable good. The terms of trade are defined as the relative price between exportable- and importable goods, and the terms of trade follow a process of stochastic volatility. The model is solved using a third order approximation of the policy function, to differentitate between shock to the terms of trade and shocks to the volatility of the terms of trade. The model predicts that terms of trade shocks lead to an increase of exports as a terms of trade shock implies a more favorable prices of exportables. At the same time, there is a transition from the use of exportable goods in the production of the final good towards the use of importable goods, as the latter inputs become relatively cheaper. Volatility shocks have an overall negative effect on output, consumption and investment, despite having an increase after the shock. The initial rise coming from the favorable terms of trade is offset by the higher uncertainty generated by the volatility shock, which affects consumption and investment.

The model used in this paper has some limitations that could be taken into account for further work. The two-sector model, including an exportable and importable sector, leaves non-tradable goods aside. The inclusion of a non-tradable sector would imply the presence of the real exchange rate in the model. Then, certain phenomena such as the Dutch disease -the effect of output on the non-tradable sector after an oil price boom- could be studied in the context of higher volatility in prices. Besides the different robustness checks implemented in this work, analyzing the role of sectoral elasticity and risk aversion, other shocks may be included in this paper's model, such as the real interest rate or an autoregresive process for total factor productivity.

2.7 Appendix

2.7.1 Data

The data on Chilean macroeconomic aggregates was obtained from the World Bank's World Development Indicators database, available online at `https://databank.worldbank.org`. The series used are:

1. "GDP (constant LCU)" for gross domestic product.
2. "Final consumption expenditure (constant LCU)" for consumption.
3. "Gross fixed capital formation (constant LCU)" for investment.
4. "Exports of goods and services (constant LCU)" and "Imports of goods and services (constant LCU)" for the trade balance.
5. "Population, total".

LCU stands for local currency unit. In order to express the variables in per capita terms, all series are divided by the total population. The frequency of the data is annual from 1960 to 2017.

The series for the terms of trade are obtained from the database of Bennett and Valdes (2001) for the period M1:1965-M12:1999, and from the Central Bank of Chile for the period Q1:2000-Q4:2017. A joint quarterly series is then created for the period Q1:1965-Q4:2017, using the three-month mean of the monthly terms of trade of the Bennett and Valdes (2001) database. Then, two annual series are created for the period 1965-2017: a terms-of-trade series from the mean of the quarterly terms of trade; and a terms-of-trade volatility series from the standard deviation of the quarterly terms of trade observed each year.

The cyclical components of these variables with respect to their long-run trend are obtained using a Hodrick-Prescott filter with smoothing parameter $\lambda = 100$.

2.7.2 Variance decomposition of the VARs estimated in the empirical section

Table 2.7.1: **Variance decomposition - VAR including terms of trade** tot_t

Period	Terms of trade (tot_t)	Trade balance (tb_t)	Output (y_t)	Consumption (c_t)	Investment (i_t)
1	100.00000	0.00000	0.00000	0.00000	0.00000
2	85.18088	1.31243	4.25578	7.32532	1.92556
3	82.51861	1.33161	6.74113	7.08495	2.32368
4	78.54214	1.25888	10.67676	7.26392	2.25829
5	77.10960	1.32898	11.35089	7.62228	2.58824
6	76.86025	1.38217	11.49271	7.67989	2.58496
7	76.05345	1.41501	11.98881	7.62207	2.92065
8	75.56528	1.43053	12.16276	7.53405	3.30735
9	75.47319	1.43322	12.20839	7.51755	3.36763
10	75.45156	1.43577	12.19890	7.54200	3.37175

Ordering of variables for the Cholesky decomposition: $tot_t, tb_t, y_t, c_t, i_t$.

Table 2.7.2: **Variance decomposition - VAR including terms of trade volatility** σ_t^{tot}

Period	TOT-volatility (σ_t^{tot})	Trade balance (tb_t)	Output (y_t)	Consumption (c_t)	Investment (i_t)
1	100.00000	0.00000	0.00000	0.00000	0.00000
2	87.07308	0.22574	3.77428	8.73379	0.19310
3	78.56480	0.48192	5.32475	7.65453	7.97399
4	77.69468	0.55747	5.84575	7.78125	8.12084
5	73.80151	0.79869	8.30759	7.57795	9.51425
6	71.89692	0.90576	9.14977	7.45463	10.59291
7	71.82310	0.92672	9.14092	7.47645	10.63280
8	71.07822	1.14807	9.46154	7.45932	10.85283
9	70.23076	1.28641	10.05384	7.42621	11.00278
10	69.99107	1.29517	10.26660	7.41172	11.03543

Ordering of variables for the Cholesky decomposition: $\sigma_t^{tot}, tb_t, y_t, c_t, i_t$.

2.7.3 Extended VARs for output and investment by sector

Figures 2.7.1 and 2.7.2 are impulse responses functions to two-standard-deviation shocks to the terms of trade and terms of trade volatility in the context of VARs of a similar fashion of those estimated in section 2.3.3, with the exception that aggregate output y_t and investment i_t in the systems are replaced by sectoral output and investment y_t^x, y_t^m, i_t^x and i_t^m, such that the estimated VARs are:

$$X_t^{tot} = \begin{bmatrix} tot_t & tb_t & y_t^x & c_t & i_t^x \end{bmatrix}'$$

$$X_t^{\sigma} = \begin{bmatrix} \sigma_t^{tot} & tb_t & y_t^x & c_t & i_t^x \end{bmatrix}'$$

$$X_t^{tot} = \begin{bmatrix} tot_t & tb_t & y_t^m & c_t & i_t^m \end{bmatrix}'$$

$$X_t^{\sigma} = \begin{bmatrix} \sigma_t^{tot} & tb_t & y_t^m & c_t & i_t^m \end{bmatrix}'$$

Sectoral output is Gross domestic product by sector at annual frequency between 1960 and 2017. Investment is Gross fixed capital formation by sector at annual frequency from 1996 to 2017. Exportables data is taken from the mining sector, while importables are obtained from agriculture, fishing and manufacturing. The source is the national accounts data from the Central Bank of Chile.

Figure 2.7.1: Exportables data

Figure 2.7.2: Importables data

2.7.4 Model

Equilibrium and first order conditions at the steady state

$$\lambda = \left(c - \frac{(h^m)^{\omega_m}}{\omega_m} - \frac{(h^x)^{\omega_x}}{\omega_x} \right)^{-\gamma} \tag{2.7.1}$$

$$w^x = (h^x)^{\omega_x - 1} \tag{2.7.2}$$

$$w^m = (h^m)^{\omega_m - 1} \tag{2.7.3}$$

$$\beta = \frac{1}{1+r} \tag{2.7.4}$$

$$u^x = \frac{1}{\beta} - 1 + \delta \tag{2.7.5}$$

$$u^m = \frac{1}{\beta} - 1 + \delta \tag{2.7.6}$$

$$i^x = \delta k^x \tag{2.7.7}$$

$$i^m = \delta k^m \tag{2.7.8}$$

$$F_m(a^x, a^m) = p^m \tag{2.7.9}$$

$$\frac{F_x(a^x, a^m)}{F_m(a^x, a^m)} = tot \tag{2.7.10}$$

$$y^m = A^m (k^m)^{\alpha_m} (h^m)^{1-\alpha_m} \tag{2.7.11}$$

$$y^x = A^x (k^x)^{\alpha_x} (h^x)^{1-\alpha_x} \tag{2.7.12}$$

$$u^m = p^m A^m \alpha (k^m)^{\alpha - 1} (h^m)^{1-\alpha} \tag{2.7.13}$$

$$\frac{1-\alpha}{\alpha} \frac{k^m}{h^m} = \frac{w^m}{u^m} \tag{2.7.14}$$

$$u^x = p^x A^x \alpha (k^x)^{\alpha - 1} (h^x)^{1-\alpha} \tag{2.7.15}$$

$$w^x = p^x (1-\alpha) A^x \left(\frac{k^x}{h^x} \right)^{\alpha} \tag{2.7.16}$$

$$c + i^x + i^m = F(a^m, a^x) \tag{2.7.17}$$

$$m = p^m (a^m - y^m) \tag{2.7.18}$$

$$x = p^x (y^x - a^x) \tag{2.7.19}$$

$$x - m = \frac{rd}{1+r} \tag{2.7.20}$$

$$r = r^* + \psi (e^{d - \bar{d}} - 1) \tag{2.7.21}$$

$$tot = \frac{p^x}{p^m} \tag{2.7.22}$$

$$tot = \overline{tot} \tag{2.7.23}$$

$$\sigma^{tot} = \bar{\sigma} \tag{2.7.24}$$

Calibration for the structural parameters $A^x, \bar{d}$ and χ

Define the ratio of production of exportable and importable goods as:

$$\Gamma = \frac{p^x y^x}{p^m y^m} \tag{2.7.25}$$

To find the productivity of the exportable-good sector A^x, define A as the ratio of sectoral productivities:

$$A \equiv A^x / A^m \tag{2.7.26}$$

Then, pre-multiplying equations 2.7.11 and 2.7.12 by p^x and p^m, respectively and then taking the ratio we obtain:

$$\Gamma = totAK^{\alpha} H^{1-\alpha} \tag{2.7.27}$$

where:

$$K \equiv \frac{k^x}{k^m} \tag{2.7.28}$$

$$H \equiv \frac{h^x}{h^m} \tag{2.7.29}$$

2.7.13 and 2.7.15 imply:

$$K = \Gamma \tag{2.7.30}$$

2.7.14 and 2.7.16 imply:

$$\frac{\Gamma}{H} = W \tag{2.7.31}$$

where $W \equiv w^x / w^m$. 2.7.3 and 2.7.4 imply:

$$H^{\omega - 1} = W \tag{2.7.32}$$

Combining 2.7.31 and 2.7.32:

$$\frac{\Gamma}{H} = H^{\omega-1} \tag{2.7.33}$$

Thus, we have the following definitions:

$$H = \Gamma^{1/\omega} \tag{2.7.34}$$

$$W = \Gamma^{(\omega-1)/\omega} \tag{2.7.35}$$

Using these expressions in 2.7.27 and solving for A:

$$A = \frac{\Gamma^{(1-\alpha)(\omega-1)/\omega}}{tot} \tag{2.7.36}$$

where $\alpha = 0.39$, $\omega = 1.455$ and $tot = 1$. $\Gamma = 0.768$ from the ratio of Chile's production of exportable and importable goods.[15] Combining 2.7.26 and 2.7.36 gives $A^x = 0.9509$.

To find the steady state of d, start with 2.7.20:

$$x - m = \frac{rd}{1+r} \tag{2.7.37}$$

and define the balance of trade as a share of output:

$$s_{tb} = \frac{x - m}{p^x y^x + p^m y^m} \tag{2.7.38}$$

Dividing and multiplying the left-hand side of 2.7.20 by y (where $y = p^x y^x + p^m y^m$) we get:

$$s_{tb} \cdot y = \frac{rd}{1+r} \tag{2.7.39}$$

[15]Source: Central Bank of Chile. GDP by sector between 1960 and 2017. Exportable good production is considered as the real output in the mining sector ("Minería"), while the importable good production is the real output of the tradable sector excluding mining: the sum of agriculture, fishing and manufacturing ("Agropecuario-silvícola", "Pesca" and "Industria Manufacturera").

Solving for d:

$$d = \frac{s_{tb} \cdot y(1+r)}{r} \tag{2.7.40}$$

The value for d equals 0.1073. Then, by 2.7.21, $d = \bar{d}$.

To find the value of the weight χ of the importable-good absorption in total output, the ratio of first order conditions 2.4.18 and 2.4.19 from the final-good firm's profit maximization is:

$$\frac{F_x(a^x, a^m)}{F_m(a^x, a^m)} = tot \tag{2.7.41}$$

then:

$$\left(\frac{a^x}{a^m}\right)^{-1/\mu} = \frac{\chi}{1-\chi} tot \tag{2.7.42}$$

Let $\Upsilon \equiv p^x a^x/(p^m a^m)$ be defined as the steady-state exportable-to-importable absorption ratio, such that:

$$\left(\frac{\Upsilon}{tot}\right)^{-1/\mu} = \frac{\chi}{1-\chi} tot \tag{2.7.43}$$

Solving for χ:

$$\chi = \frac{(\Upsilon tot^{\mu-1})^{-1/\mu}}{1 + (\Upsilon tot^{\mu-1})^{-1/\mu}} \tag{2.7.44}$$

To find the value of Υ, use the definition mentioned above:

$$\Upsilon = \frac{p^x a^x}{p^m a^m} \tag{2.7.45}$$

Then, using 2.7.18 and 2.7.19:

$$\Upsilon = \frac{p^x y^x - x}{p^m y^m + m} \tag{2.7.46}$$

dividing each term by output y, one obtains:

$$\Upsilon = \frac{\frac{\Gamma}{1+\Gamma} - s_x}{\frac{1}{1+\Gamma} + s_x - s_{tb}} \qquad (2.7.47)$$

where s_x is the share of exports in total output:

$$s_x = \frac{x}{p^x y^x + p^m y^m} \qquad (2.7.48)$$

and s_{tb} is the trade-balance as a share of output defined in 2.7.38. Then, $s_x = 0.2338$ and $s_{tb} = 0.0636$, based on empirical data observed for Chile in the period of analysis.[16]

Given the values for Γ, s_x and s_{tb}, one obtains $\Upsilon = 0.2726$. Then, from 2.7.44 and given $\mu = 1$, $\chi = 0.2142$ is obtained.

Proof: weight χ of importable-good absorption in total output

The ratio of first order conditions 2.4.18 and 2.4.19 gives:

$$\frac{a_t^x}{a_t^m} = \left(\frac{1-\chi}{\chi}\right)^{\mu} \left(\frac{p_t^x}{p_t^m}\right)^{-\mu} \qquad (2.7.49)$$

Taking logs:

$$\log\left(\frac{a_t^x}{a_t^m}\right) = \mu \log\left(\frac{1-\chi}{\chi}\right) - \mu \log\left(\frac{p_t^x}{p_t^m}\right) \qquad (2.7.50)$$

Then, the change in the ratio of absorption of exportable and importable goods when there is a change in the terms of trade is given by:

$$\frac{\Delta \log(a_t^x / a_t^m)}{\Delta \log(p_t^x / p_t^m)} = -\mu \qquad (2.7.51)$$

[16]Source: Central Bank of Chile.

By 2.7.51, a one per cent change in the terms of trade makes the absorption of exportables (relative to importables) fall by μ per cent. As $\mu \to 1$, the final good aggregator converges to a Cobb-Douglas form with share parameter χ:

$$\lim_{\mu \to 1} F(a_t^x, a_t^m) = (a_t^m)^\chi (a_t^x)^{1-\chi} \tag{2.7.52}$$

Under such form, χ is the share of a_t^m in aggregate production.

2.7.5 Terms-of-trade and terms-of-trade volatility AR(1) estimations

Table 2.7.3: Terms of trade AR(1)

Dependent Variable: TOT
Method: Least Squares
Sample (adjusted): 1966 2017
Included observations: 52 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.002385	0.014115	0.168954	0.8665
TOT(-1)	0.297446	0.133595	2.226469	0.0305

R-squared	0.090201	Mean dependent var	0.002116
Adjusted R-squared	0.072005	S.D. dependent var	0.105653
S.E. of regression	0.101778	Akaike info criterion	−1.694341
Sum squared resid	0.517939	Schwarz criterion	−1.619293
Log likelihood	46.05287	Hannan-Quinn criter.	−1.665570
F-statistic	4.957164	Durbin-Watson stat	1.745327
Prob(F-statistic)	0.030518		

Table 2.7.4: Terms of trade volatility AR(1)

Dependent Variable: TOTV
Method: Least Squares
Sample (adjusted): 1966 2018
Included observations: 53 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.031516	0.009146	3.445938	0.0011
TOTV(-1)	0.333818	0.131919	2.530473	0.0145

R-squared	0.111549	Mean dependent var	0.047274
Adjusted R-squared	0.094129	S.D. dependent var	0.051232
S.E. of regression	0.048761	Akaike info criterion	−3.166751
Sum squared resid	0.121261	Schwarz criterion	−3.092400
Log likelihood	85.91889	Hannan-Quinn criter.	−3.138159
F-statistic	6.403292	Durbin-Watson stat	1.934927
Prob(F-statistic)	0.014520		

2.7.6 Heteroskedasticity test for residuals of terms-of-trade AR(1)

Null hypothesis: Homoskedasticity			
Heteroskedasticity Test: ARCH			
F-statistic	2.495620	Prob. F(1,49)	0.1206
Obs*R-squared	2.471601	Prob. Chi-Square(1)	0.1159
Heteroskedasticity Test: Breusch-Pagan-Godfrey			
F-statistic	7.973210	Prob. F(1,50)	0.0068
Obs*R-squared	7.151699	Prob. Chi-Square(1)	0.0075
Scaled explained SS	7.146781	Prob. Chi-Square(1)	0.0075
Heteroskedasticity Test: Glejser			
F-statistic	5.243674	Prob. F(1,50)	0.0263
Obs*R-squared	4.935787	Prob. Chi-Square(1)	0.0263
Scaled explained SS	5.260795	Prob. Chi-Square(1)	0.0218
Heteroskedasticity Test: Harvey			
F-statistic	0.626862	Prob. F(1,50)	0.4322
Obs*R-squared	0.643865	Prob. Chi-Square(1)	0.4223
Scaled explained SS	0.607238	Prob. Chi-Square(1)	0.4358
Heteroskedasticity Test: White			
F-statistic	5.357523	Prob. F(2,49)	0.0079
Obs*R-squared	9.330688	Prob. Chi-Square(2)	0.0094
Scaled explained SS	9.324272	Prob. Chi-Square(2)	0.0094

2.7.7 Robustness check: impulse response functions

Figure 2.7.3: Coefficient of relative risk aversion $\gamma = 5$; responses to level shock (left-hand side) and volatility shock (right-hand side)

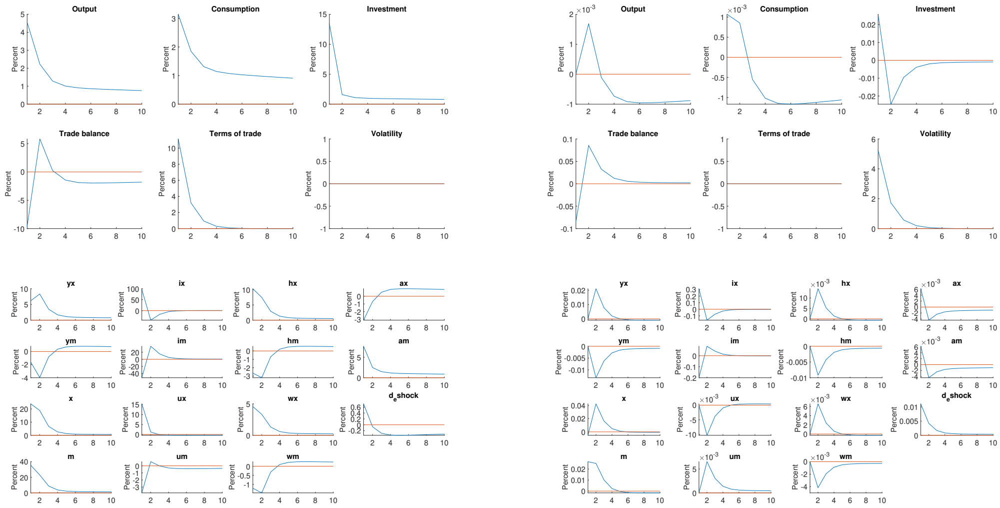


Figure 2.7.4: Coefficient of relative risk aversion $\gamma = 10$; responses to level shock (left-hand side) and volatility shock (right-hand side)

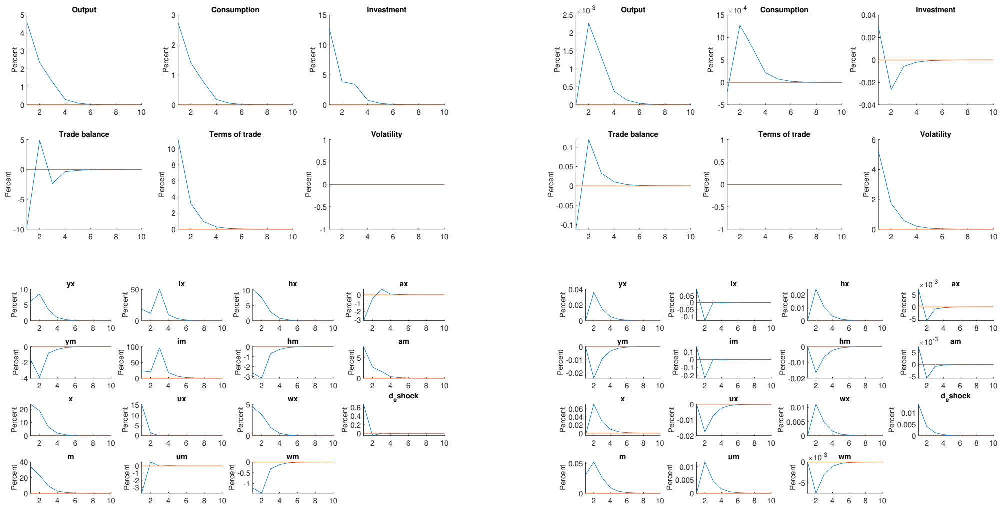


Figure 2.7.5: Coefficient of sector elasticity of substitution $\mu = 0.5$; responses to level shock (left-hand side) and volatility shock (right-hand side)

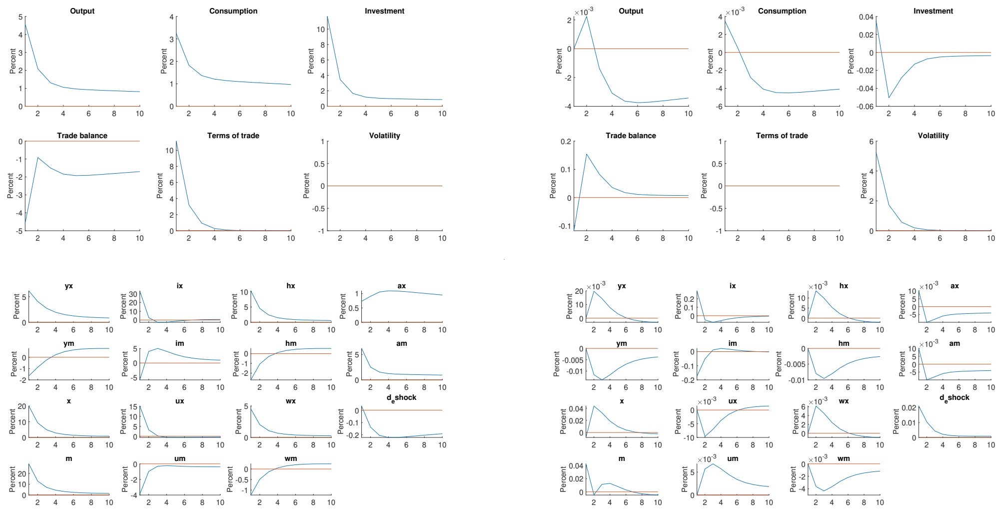
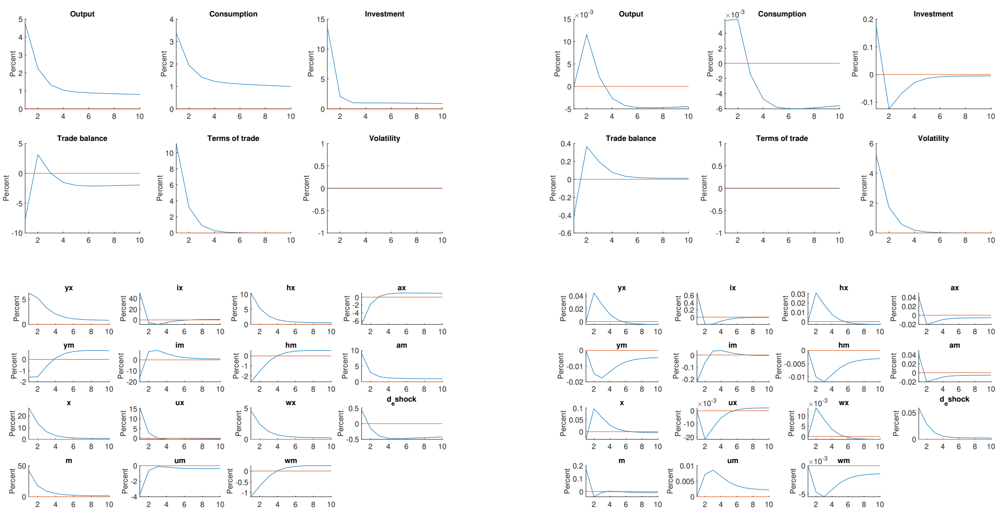


Figure 2.7.6: Coefficient of sector elasticity of substitution $\mu = 1.5$; responses to level shock (left-hand side) and volatility shock (right-hand side)



Chapter 3

Commodity price volatility and investment dynamics: a firm-level study

3.1 Introduction

The real business cycles literature has shown that uncertainty and volatility have significant effects on the macroeconomy through changes in the consumption and investment decisions of households and firms. These changes are normally triggered by shocks to total factor productivity. In turn, recent literature has found that the latter is mostly driven by the behavior of a small number of large firms, meaning that firm-level behavior is a key determinant of real business cycles.

In Latin America, oil and commodities have a relevant impact on the macroeconomy, as most countries are major producers of different types of these primary goods, ranging from oil in Colombia and Venezuela, to copper in Chile and Peru and to maize in Argentina and soy in Brazil. Prices for these commodities are determined in world markets, so producing companies have little or no power over them, and thus must make production and investment decisions based on these external factors.

In recent years, commodity prices have seen major changes both at their levels and at their volatility. During the so-called commodity super-cycle of the 2000s, prices in the energy, mining and agricultural sectors had dramatic increases, driven largely by the surging demand of emerging economies. However, the financial crisis of 2008 brought prices to an abrupt fall, bringing volatility and uncertainty to the markets and to the underlying commodity-producing economies.

The objective of this paper is to study the investment behaviour of firms in commodity-producing economies in Latin America in the context of the commodity price super-cycle. For that purporse, I work with a database of firms at a quarterly frequency obtained from Compustat, from which I extract and generate a number of indicative variables such as firm-size, leverage, liquidity, and the firms' level of sales. With this unbalanced panel data, I estimate the firm investment levels as a function of the mentioned indicators, in addition to measures of quarterly average commodity prices and quarterly commodity price volatility.

Section 3.2 does a review of the literature related to this work. Section 3.3 describes the data used in terms of a description of the firms included in the sample, the financial indicators constructed, and the behaviour of the indices of oil and commodity prices. Section 3.4 shows the empirical framework implemented to describe the drivers of investment at the firm level. Section 3.5 concludes.

3.2 Literature review

This paper is written in the context of the literature studying the role of firms explaining macroeconomic dynamics. A seminal work comes from Gabaix (2011), which finds that the behaviour of a small number of firms explains a large part of the U.S. business cycle fluctuations. In real business cycle models, total factor productivity is the component of output growth that is not determined by capital or labor. In that sense, the findings of Gabaix (2011) make a major contribution in trying to fill that void.[1] Thus, it is important to analyse the different factors determining the investment behaviour of firms, which in turn help to determine business cycle fluctuations.

Dixit and Pindyck (1994) provide a theoretical approach to the capital investment decisions of firms, dwelling on such concepts as the irreversibility of investments and the uncertainty under which these take place. In terms of the effects of uncertainty on investment, the literature goes back to Hartman (1972), Caballero (1991), and Leahy and Whited (1996) with the latter showing some of the stylized facts around this topic. Abel (1983) and Abel and Eberly (1994) validate the fact that higher

[1] In growth accounting, the concept of the Solow residual is defined as the unknown source of economic fluctuations explained by total factor productivity (Solow, 1957).

uncertainty increases the investment of competitive firms under constant returns to scale. Guiso and Parigi (1999) study the effects of uncertainty of demand on investment behaviour. Bloom (2007) studies how uncertainty affects research and development. An important work comes from Bloom et al. (2007), which focuses on the role of uncertainty on investment dynamics, showing that higher uncertainty affects the way investment responds to demand shocks through an increase in real option values, which leads to firms becoming more cautious when making investment decisions.

Following Abel and Eberly (1994), Bertola and Caballero (1994) and Abel and Eberly (1996) focus on investment when reversibility is difficult. Manso (2008) studies investment reversibility and its link with the agency cost of debt. In relation to debt, Ottonello and Winberry (2018) study the investment channel of monetary policy and how this channel is affected by the heterogeneity across firms. Firms with low debt burdens are more responsive to monetary shocks, while firms with high levels of debt cannot modify their investment decisions after changes in monetary policy. Ahn et al. (2006) study the link between leverage and investment in diversified firms, where the effect is stronger in some sectors with respect to others. Khan et al. (2016) work with a model of heterogenous firms to show that larger and collateralized firms have higher levels of investment than those which are smaller and have less collateral. Similarly, Crouzet and Mehrotra (2018) make a difference between small and large firms and business cycle fluctations, finding that large firms are less sensitive to business fluctuations than the rest. Gertler and Gilchrist (1994) focus on manufacturing firms and how they are affected by monetary policy. Along the same lines, Jeenas (2018) studies how the effects of monetary policy on investment under different degrees of liquidity in the balance sheet of the firms, as well as their leverage.

Finally, the paper also fits in the literature that studies the effect of oil and commodity price shocks on the macroeconomy. For the impact on the U.S. economy, it is important to see the work of Kilian (2008b), Kilian (2008a) and Kilian (2009). Lee and Ni (2002), study the dynamic effects of oil price shocks using industry level data, while Fukunaga and Sudo (2010) do a similar job giving a comparison between the economies of the U.S. and Japan.

3.3 Data

3.3.1 The firms

Table 3.3.1: Summary of firms by country and sector

	ARG	**BRA**	**CHL**	**COL**	**ECU**	**MEX**	**PER**	**VEN**	**TOTAL**
Agriculture, forestry, fishing	3	6	10	0	0	1	4	0	**24**
Mining	2	9	8	1	0	4	17	0	**41**
Construction	5	11	5	3	0	8	1	0	**33**
Manufacturing	23	96	44	11	1	36	26	8	**245**
Oil and gas	3	3	0	1	0	1	2	0	**10**
Machinery and equipment	17	88	24	6	1	23	20	8	**187**
Transport and comunication	18	113	49	10	0	27	15	4	**236**
Wholesale and retail	4	26	16	6	1	20	6	0	**79**
Finance	18	111	56	19	4	56	30	15	**309**
Services	2	33	24	3	0	10	3	0	**75**
Other (conglomerates)	1	2	5	0	0	2	1	0	**11**
TOTAL	**96**	**498**	**241**	**60**	**7**	**188**	**125**	**35**	**1250**

Source: Compustat. Data collected quarterly for the period 1995Q4 - 2018Q4. Countries included are Argentina (ARG), Brazil (BRA), Chile (CHL), Colombia (COL), Ecuador (ECU), Mexico (MEX), Peru (PER) and Venezuela (VEN).

I study an unbalanced panel of 1,250 firms from eight Latin American economies across eleven different industries at a quarterly frequency from 1995Q4 to 2018Q4. The data is obtained from Compustat, a source of financial, statistical and market information on publicly listed companies produced by credit-rating agency Standard and Poor's.

Table 3.3.1 summarizes the number of firms used in the sample according to countries and industries. The firms used in the sample come from Argentina (96), Brazil (498), Chile (241), Colombia (60), Ecuador (7), Mexico (188), Peru (125) and Venezuela (35). The sectors in which the firms operate are agriculture, forestry and fishing (24); mining (41); construction (33); manufacturing (245); oil and gas (10); machinery and equipment (187); transportation, communications, electric gas and sanitary services (236); wholesale and retail (79); finance (309); services (75) and others (11).[2]

The largest group of firms is clustered in finance (309), which is relevant as this is the sector necessary to channel funds to firms for their investment activities. The largest non-financial sectors in the database are manufacturing (245), transport and comunication (236) and machinery and equipment (187). Manufacturing is important in the production of final consumption goods, while machinery and equipment is directly related to the purchase of capital goods. The number of firms in agriculture, forestry and fishing (24), mining (41), and oil and gas (10) is small but still relevant due to the size of these firms in the economy, as markets in the production of primary goods tend to be less competitive and more concentrated to either public or privately owned companies.

3.3.2 Financial indicators

The database contains information on the financial position of firms, explained by the following measures: total assets, cash and short-term investments, debt in current liabilities, long-term debt, sales/turnover, capital expenditure and property, plant and equipment. From this information, the following financial indicators are built for the i firms in the sample for each period t available:

1. Investment $\Delta log(K_{i,t})$, measured as the log-difference in capital, where capital is given by property, plant and equipment.

[2]Economic sectors are defined by their Standard Industry Classification (SIC) code (see appendix 3.6.1).

2. The growth of sales $\Delta\log(Y_{i,t})$, measured by the log-difference of sales.

3. Leverage $V_{i,t}$, measured as the ratio of the sum of current liabilities and long-term debt to total assets.

4. Liquidity $Q_{i,t}$, measured as the ratio of cash to total assets.

Table 3.3.2: Firm-level data summary of statistics

						Quantiles			
Indicator		**Obs**	**Mean**	**S.D.**	**Min**	**.25**	**Med**	**.75**	**Max**
Sales growth:	$\Delta\log(Y_t)$	27,593	2.42	15.66	-48.04	-5.48	2.75	10.97	47.00
Investment:	$\Delta\log(K_t)$	27,593	1.85	5.01	-11.14	-0.88	0.80	3.59	25.22
Leverage:	V_t	27,594	28.24	13.01	3.86	18.11	27.77	37.61	58.66
Liquidity:	Q_t	27,595	7.64	6.20	0.45	2.68	5.96	10.99	27.45

Source: Compustat database. All variables are in percentages. To avoid outliers, all variables are winsorized at 10 percent, removing observations at the 5th and 95th percentiles. See Gabaix (2011) for details.

Table 3.3.2 shows the descriptive statistics of the data. After winsorizing the data to remove outliers, there are over 27,500 observations from which the following statistics are observed:

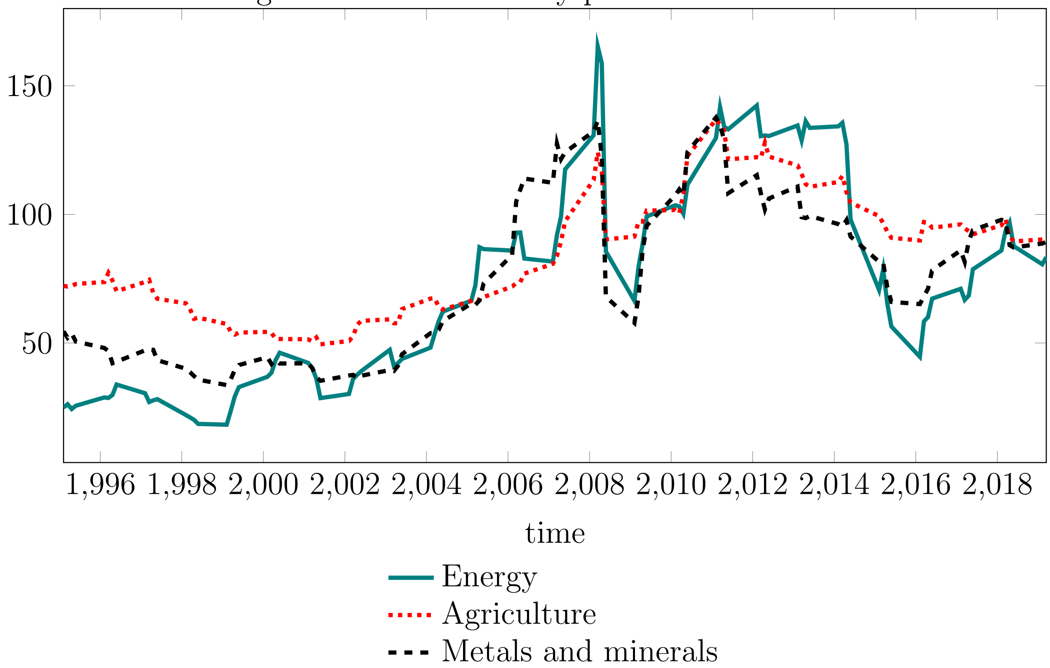
1. Sales grow at an average rate of 2.42 percent each quarter, with a standard deviation of 15.66 percent.

2. Investment -the quarterly growth of property plant and equipment- is on average equal to 1.85 percent with a standard deviation of 5 percent.

3. Leverage -the ratio of debt to total assets- is equal to 28 percent. This value is reasonable and similar to that found in U.S. data.

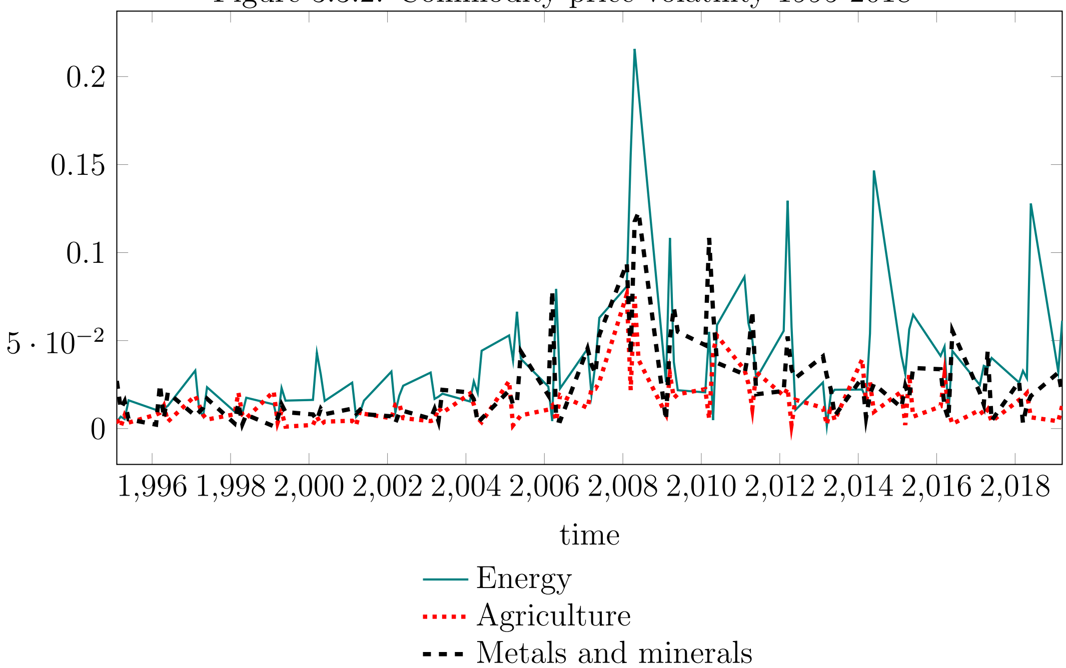
4. Liquidity, defined as the ratio of total assets to current liabilities, is 7.64 on average.

Figure 3.3.1: Commodity price indices 1995-2018



Source: World Bank Commodities Price Data. Frequencies are quarterly. The series are the three-month average of the monthly indices for energy, agriculture and metals and minerals from 1995M1 to 208M12 (base 2010M6=100). The series are in real terms using the U.S. Consumer Price Index from the Federal Reserve of St. Louis Economic Data.

Figure 3.3.2: Commodity price volatility 1995-2018



Source: World Bank Commodities Price Data. Frequencies are quarterly. The series are the three-month standard deviation of the monthly indices for energy, agriculture and metals and minerals from 1995M1 to 208M12.

3.3.3 Commodity price indices

For commodity price indices, I use information from the World Bank.[3] I choose the main indices for energy, non-energy, agriculture and metals and minerals. Based on the main exports and imports in Latin American countries, I also choose the series for crude oil, coffee, soybeans, maize, sugar, aluminum, iron ore, copper and gold. The original indices have a monthly frequency from 1995M1 to 2018M12, with base 2010M6=100. I bring them to real terms using the United States Consumer Price Index obtained from the Federal Reserve Bank of St. Louis Economic Data.[4] The series are then brought to quarterly terms both in their mean and standard deviation. All variables are stationary at the level, except for gold that is first-difference stationary (See appendix 3.6.3).

Figure 3.3.1 displays the behavior of three main commodity price indices: energy, agriculture and metals and minerals (see appendix 3.6.2 for a detailed description of the indices' composition). Meanwhile, figure 3.3.2 shows the volatility of commodity prices, measured by the indices' three-month standard deviation. From the figures, two stylized facts are singled out. First, there is a significant spike in volatility in 2008: for the case of energy, the standard deviation rises to a little more than 20 percent in the third quarter of that year, four times more than the average around 5 percent observed for the period before. Similar cases follow for agriculture and mining. Second, average volatility remained high after 2008: the standard deviation of the price indices for energy, agriculture and mining behave differently after 2008, reaching higher peaks and becoming less predictable than they were before the start of the commodity supercycle.

To understand this in more detail, figures 3.6.1 and 3.6.2 in the appendix respec-

[3] World Bank Commodities Price database (The Pink Sheet): `https://databank.worldbank.org`. See appendix 3.6.2 for description.

[4] `https://fred.stlouisfed.org/series/CPIAUCSL`.

tively show the three-month average and standard deviation of the indices for oil, coffee, soybeans, maize, sugar, aluminum, iron ore, copper and gold. In the case of oil, this commodity holds a close relationship to the energy index shown in figure 3.3.1 (after all, oil represents 85 percent of the energy index). After relative stability in the late 1990s, there is a rapid surge in oil prices that stops in 2008Q3, when a much more volatile period begins. The first graph in figure 3.6.2 shows this clearly: while the pre-financial-crisis standard deviation of oil was 3,28, this value increased to 5,18 on average after 2008Q4. A similar case to oil is copper: the standard deviation between 1995Q2 and 2008Q3 was 2,68 on average, rising to 3,78 for the period afterwards. For agricultural commodities such as coffee, soybeans, maize and sugar, prices are much more volatile throughout the period of study, as seen in figure 3.6.1. However, time-varying volatility is observed in the pre- and post-crisis periods. With the exception of coffee, the other commodities show a higher average volatility after 2008Q3: 3,43 for soybeans, 4,36 for maize, and 5,98 for sugar compared to 3,41, 3,97 and 3,79 respectively for the previous period. In the case of gold, the only series that shows a unit root, volatility is also time-varying. Its average three-month standard deviation equals 0,98 for the pre-crisis period, increasing to 2,60 thereafter.

3.4 Empirical evaluation

3.4.1 The baseline model

To analyze the dynamics of investment at the firm-level, and how they are affected by commodity prices and commodity price volatility, I start with the specification shown in equation 3.4.1 -based on Bloom et al. (2007)- that studies the relationship between firm investment and demand growth:

$$\Delta logK_{i,t} = \beta \Delta logY_{i,t} + \theta(logY_{i,t-1} - logK_{i,t-1}) + A_i + B_t + C_j + D_n + \Gamma_{dt} + \upsilon_{i,t} \quad (3.4.1)$$

Investment of firm i in period t is the dependent variable $\Delta logK_{i,t}$, explained by the change in demand (measured by sales) $\Delta logY_{i,t}$ in the same period, an error correction term that measures the lagged difference between demand and capital $(logY_{i,t-1} - logK_{i,t-1})$, and a series of coefficients that account for firm fixed effects A_i, time fixed effects B_t, country fixed effects C_j, sector (division) fixed effects D_j, and sector-time fixed effects Γ_{dt}.

Table 3.4.1 shows the results of coming from the estimation of equation 3.4.1. Results show that there is a direct relationship between the growth of sales and investment, as the coefficients for $\Delta logY_{i,t}$ are positive and statistically significant in all specifications of the model, using different combinations of fixed effects for firms, years, countries, sectors and a combination of sector-year. In all cases, a one percent increase in sales leads to growth of about three percent in capital (plant, property and equipment). Meanwhile, the error correction term between sales and capital has a negative coefficient in all specifications of the model, but it is not statistically significant.

Given that these results indicate a relationship between demand growth and invest-

Table 3.4.1: Determinants of firm investment - Baseline model

Dependent variable: $\Delta logK_{i,t}$ Model:	(1)	(2)	(3)	(4)	(5)	(6)
$\Delta logY_{i,t}$	0.0295***	0.0285***	0.0303***	0.0305***	0.0295***	0.0295***
	(14.42)	(14.11)	(14.94)	(15.02)	(14.73)	(14.76)
$logY_{i,t-1} - logK_{i,t-1}$	-0.0002	-0.0004	-0.0001	-0.0001	-0.0003	-0.0003
	(-0.68)	(-1.25)	(-0.58)	(-0.57)	(-1.19)	(-1.01)
Firm FE	Yes	Yes	Yes	Yes	Yes	No
Time (year) FE	No	Yes	No	No	Yes	No
Country FE	No	No	Yes	No	Yes	No
Sector FE	No	No	No	Yes	Yes	No
Sector-year FE	No	No	No	No	No	Yes
N	23,249	23,249	23,249	23,249	23,249	23,249
R^2(overall)	0.0113	0.0421	0.0149	0.0152	0.0515	0.0660

Note: t-statistics are in parentheses. Significance: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

ment, I then make a similar estimation to explain firm investment including two very important financial indicators of the firm: leverage and liquidity. The estimation is set out as in equation 3.4.2:

$$\Delta logK_{i,t} = \beta\Delta logY_{i,t} + \gamma V_{t-j} + \theta Q_{t-j} + A_i + \upsilon_{i,t} \tag{3.4.2}$$

Leverage $V_{i,t}$ measures the degree of indebtment in the firm, while liquidity $Q_{i,t}$ indicates the amount of cash of the firm, with which it is capable of meeting its short-term obligations.

Table 3.4.2: Determinants of firm investment - Leverage and liquidity at different lags

Dependent variable: $\Delta logK_{i,t}$				
Lag:	j=0	j=1	j=2	j=4
$\Delta logY_{i,t}$	0.0274***	0.0284***	0.0282***	0.0297***
	(12.93)	(13.22)	(13.03)	(14.04)
V_{t-j} (leverage)	-0.0211***	0.0012	-0.0051*	-0.0011
	(-5.24)	(0.51)	(-2.02)	(-0.44)
Q_{t-j} (liquidity)	0.0604***	0.0008	0.0129*	0.0055
	(7.55)	(0.16)	(2.40)	(1.04)
Firm FE	Yes	Yes	Yes	Yes
N	21,365	21,167	21,129	21,156
R^2(overall)	0.0150	0.0109	0.0110	0.0115

Note: t-statistics are in parentheses. Significance: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 3.4.2 shows the estimation results for equation 3.4.2 at lags 0, 1, 2, and 4 to show contemporaneous effects of leverage and liquidity on investment, as well as quarterly, semi-annual and annually lagged effects using firm fixed effects. It can be seen that these two financial indicators have a statistically significant effect on firm investment contemporaneously and after two quarters. The positive sign for the liquidity coefficient and the negative sign for leverage shows that more liquid firms can afford to increase their amount of capital over time, while indebted firms are more compromised and must therefore reduce their levels of investment.

Table 3.4.3 replicates the results of the estimations for equation 3.4.2 when $j = 2$, and studies the case under different fixed effects for time, country, sector and sector-year, to show that the effects are consistent under the different model especifications. The specification with firm fixed effects gives statistically significant coefficients (column 1), as well as the model specification with firm and country fixed effects (column 3) and the model with firm and sector fixed effects (column 4). The model with firm and sector-year fixed effects (column 6) gives statistically significant coefficients for demand $\Delta logY_{i,t}$ and lagged liquidity Q_{t-2}. Results from this table show that lagged liquidity Q_{t-2} and leverage V_{t-2} have opposite effects on investment for the firms: firms with more liquidity will have more investment while those with higher leverage will invest less.

Table 3.4.3: Determinants of firm investment - Leverage and liquidity

Dependent variable: $\Delta logK_{i,t}$ Model:	(1)	(2)	(3)	(4)	(5)	(6)
$\Delta logY_{i,t}$	0.0282***	0.0271***	0.0292***	0.0293***	0.0283***	0.0283***
	(13.03)	(12.71)	(13.57)	(13.66)	(13.33)	(13.34)
V_{t-2} (leverage)	-0.0051*	-0.0031	-0.0050*	-0.0050*	-0.0031	-0.0035
	(-2.02)	(-1.27)	(-1.99)	(-2.00)	(-1.28)	(-1.41)
Q_{t-2} (liquidity)	0.0129*	0.0095	0.0140*	0.0142*	0.0103	0.0107*
	(2.40)	(1.80)	(2.62)	(2.67)	(1.94)	(2.03)
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes
Time (year) FE	No	Yes	No	No	Yes	No
Country FE	No	No	Yes	No	Yes	No
Sector FE	No	No	No	Yes	Yes	No
Sector-year FE	No	No	No	No	No	Yes
N	21,129	21,129	21,129	21,129	21,129	21,129
R^2(overall)	0.0110	0.0386	0.0139	0.0147	0.0470	0.0614

Note: t-statistics are in parentheses. Significance: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

3.4.2 Commodity price volatility

The previous section shows the main determinants of firm investment in the sample including Latin American firms. These are demand growth and two financial indicators: leverage and liquidity. In this section, I analyse whether commodity prices have any influence on these determinants. As it was observed above, commodity prices follow a volatile process that accentuated during and after the financial crisis of 2008. Equation 3.4.3 continues with the analysis started and observed in equations 3.4.1 and 3.4.2:

$$\Delta logK_{i,t} = \beta_1 \Delta logY_{i,t} + \beta_2(\Delta logY_{i,t})^2 + \beta_3 V_{i,t-2} + \beta_4 Q_{i,t-2} + \beta_5(logY_{i,t-1} - logK_{i,t-1})$$
$$+ \sum_c \gamma_c SD^c_{i,t-1} + \sum_c \gamma_c \Delta SD^c_{i,t} + \sum_c \gamma_c (SD^c_{i,t} * \Delta logY_{i,t}) + \sum_c \gamma_c (SD^c_{i,t} * V_{i,t-2})$$
$$+ \sum_c \gamma_c (SD^c_{i,t} * Q_{i,t-2}) + A_i + B_t + C_j + D_n + \Gamma_{dt} + \upsilon_{i,t} \quad (3.4.3)$$

Namely, investment of firm i in period t $\Delta logK_{i,t}$ depends on a linear and a quadratic component of sales growth $\Delta logY_{i,t}$ and $(\Delta logY_{i,t})^2$, respectively; the two financial indicators: leverage $V_{i,t-2}$ and liquidity $Q_{i,t-2}$; the error correction term equal to the difference between sales and capital $(logY_{i,t-1} - logK_{i,t-1})$, the one-period lag of the price volatility of a chosen commodity $SD_{i,t-1}$, the difference in commodity price volatility $\Delta SD_{i,t}$, a number of series measuring commodity price volatility and their interactions with demand growth $(SD_{i,t} * \Delta logY_{i,t})$, leverage $(SD_{i,t} * V_{i,t-2})$ and liquidity $(SD_{i,t} * Q_{i,t-2})$, and the group of fixed effects included in the estimation of equation 3.4.1.

Table 3.4.4 shows the results of the estimation of equation 3.4.3. Columns 1 to 6 show the results under different sets of fixed effects: firm, time(year), country, sector and sector-year. It can be observed that both the linear and quadratic component of sales growth are positive and statistically significant at all the model specifications, showing that demand has a strong and non-linear relationship with firm investment. In terms of the financial indicators, the level of debt held by firms -measured by leverage- leads to a fall of investment. This negative relationship is statistically significant for two specifications of the model (3 and 5), showing that firms under financial strain are less keen to invest in capital, as it was shown in the previous section. In terms liquidity, results do not show any statistically significant effect on investment. This suggests that the amount of cash accumulated by firms does not have an effect on their investment decisions. Although it is known that liquidity is important for firms during times of economic downturn, a better position of this indicator does not lead to new investment decisions in the data set. Likewise, the error correction term measuring the difference between sales and capital, does not offer any explanation on investment in the evaluation of equation 3.4.3, as the coefficients estimated are not statistically significant. However, they do show a negative value as in the case of model 3.4.1.

The following set of coefficients are related to commodity price volatility for three particular indices: energy, agriculture and metals and minerals (details on the composition of these indices is described in section 3.3.3 and appendix 3.6.2). Commodity price volatility is measured by its standard deviation $SD_{i,t}$, its one-period difference $\Delta SD_{i,t}$ and its interaction with the change of sales, lagged leverage and lagged liquidity ($SD_{i,t} * \Delta logY_{i,t}, SD_{i,t} * V_{i,t-2}$ and $SD_{i,t} * Q_{i,t-2}$, respectively).

For the case of energy, volatility in the price of this commodity does not appear to have an effect on firm investment in Latin America. Similarly, the interaction of energy price volatility with firm financial indicators does not give any statistically significant effects on investment. This is surprising given the importance of oil and gas in Latin American economies. Instead, other commodity price volatilities do seem to have an effect on firm investment.

For the case of agriculture, the change in volatility $\Delta SD_{i,t}$ is statistically significant in all model specifications and has a negative coefficient, meaning that changes in agricultural price volatility leads to lower investment in Latin American firms. The interaction of price volatility in agriculture with growth in sales is positive and statistically significant, differing from the case of energy prices and show no significant interaction with sales.

Table 3.4.4: Determinants of firm investment - Commodity price volatility

Dependent variable: $\Delta logK_{i,t}$ Model:	(1)	(2)	(3)	(4)	(5)	(6)
$\Delta logY_{i,t}$	0.0184***	0.0193***	0.0192***	0.0190***	0.0199***	0.0202***
	(4.41)	(4.65)	(4.65)	(4.60)	(4.81)	(4.91)
$(\Delta logY_{i,t})^2$	0.0482***	0.0433***	0.0449***	0.0465***	0.0409***	0.0414***
	(4.75)	(4.31)	(4.59)	(4.74)	(4.22)	(4.27)
V_{t-2} (leverage)	-0.0095	-0.0091	-0.0099*	-0.0098	-0.0108*	-0.0096
	(-1.88)	(-1.82)	(-1.97)	(-1.95)	(-2.16)	(-1.94)
Q_{t-2} (liquidity)	-0.0088	-0.0073	-0.0073	-0.0065	-0.0090	-0.0078
	(-0.81)	(-0.67)	(-0.67)	(-0.60)	(-0.83)	(-0.72)
$logY_{i,t-1} - logK_{i,t-1}$	-0.0003	-0.0003	-0.0003	-0.0002	-0.0004	-0.0003
	(-0.70)	(-0.84)	(-0.70)	(-0.64)	(-0.98)	(-0.89)
Energy						
$SD_{i,t-1}$	0.0005	0.0005	0.0005	0.0005	0.0004	0.0005
	(1.90)	(1.57)	(1.87)	(1.89)	(1.45)	(1.56)
$\Delta SD_{i,t}$	0.0001	0.0002	0.0001	0.0000	0.0002	0.0002
	(0.52)	(1.64)	(0.46)	(0.44)	(1.54)	(1.58)
$SD_{i,t} * \Delta logY_{i,t}$	0.0002	0.0001	0.0004	0.0004	0.0003	0.0002
	(0.35)	(0.15)	(0.57)	(0.63)	(0.48)	(0.32)
$SD_{i,t} * V_{i,t-2}$	-0.0004	-0.0003	-0.0003	-0.0004	-0.0003	-0.0003
	(-0.47)	(-0.38)	(-0.44)	(-0.45)	(-0.35)	(-0.36)
$SD_{i,t} * Q_{i,t-2}$	0.0006	0.0003	0.0005	0.0004	0.0006	0.0003
	(0.33)	(0.21)	(0.29)	(0.27)	(0.36)	(0.16)
Agriculture						
$SD_{i,t-1}$	0.0013	0.0000	0.0012	0.0012	-0.0004	-0.0002
	(1.48)	(-0.01)	(1.39)	(1.45)	(-0.37)	(-0.17)
$\Delta SD_{i,t}$	-0.0016***	-0.0009*	-0.0016***	-0.0016***	-0.0009*	-0.0009*
	(-4.87)	(-2.08)	(-4.91)	(-4.90)	(-2.01)	(-2.06)
$SD_{i,t} * \Delta logY_{i,t}$	0.0058***	0.0048*	0.0055**	0.0057**	0.0045*	0.0045*
	(3.08)	(2.55)	(2.97)	(3.10)	(2.40)	(2.45)
$SD_{i,t} * V_{i,t-2}$	0.0010	0.0016	0.0014	0.0014	0.0029	0.0021
	(0.43)	(0.71)	(0.62)	(0.61)	(1.31)	(0.93)
$SD_{i,t} * Q_{i,t-2}$	0.0084	0.0078	0.0081	0.0077	0.0071	0.0077
	(1.80)	(1.69)	(1.76)	(1.66)	(1.56)	(1.69)
Metals and minerals						
$SD_{i,t-1}$	0.0006	0.0007	0.0006	0.0006	0.0007	0.0006
	(1.40)	(1.22)	(1.42)	(1.42)	(1.22)	(1.11)
$\Delta SD_{i,t}$	-0.0007***	-0.0011***	-0.0007***	-0.0007***	-0.0010***	-0.0011***
	(-3.88)	(-4.48)	(-3.85)	(-3.84)	(-4.16)	(-4.34)
$SD_{i,t} * \Delta logY_{i,t}$	-0.0011	-0.0006	-0.0011	-0.0011	-0.0003	-0.0005
	(-1.02)	(-0.54)	(-0.99)	(-1.04)	(-0.29)	(-0.44)
$SD_{i,t} * V_{i,t-2}$	0.0026*	0.0023	0.0025	0.0025*	0.0018	0.0021
	(2.08)	(1.80)	(1.95)	(1.96)	(1.42)	(1.68)
$SD_{i,t} * Q_{i,t-2}$	-0.0024	-0.0018	-0.0022	-0.0022	-0.0014	-0.0014
	(-0.91)	(-0.69)	(-0.87)	(-0.84)	(-0.56)	(-0.56)
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes
Time (year) FE	No	Yes	No	No	Yes	No
Country FE	No	No	Yes	No	Yes	No
Sector FE	No	No	No	Yes	Yes	No
Sector-year FE	No	No	No	No	No	Yes
N	17,931	17,931	17,931	17,931	17,931	17,931
R^2(overall)	0.0256	0.0475	0.0299	0.0300	0.0751	0.0577

Note: t-statistics are in parentheses.

In metals and minerals, the relationship is similar to that of agriculture: $\Delta SD_{i,t}$ and negative coefficients that are statistically significant in all specifications of the model. In addition, while there is no interaction with sales, there is a significant interaction with lagged leverage for the case of models with firm fixed effects and firm and sector fixed effects (columns 1 and 4, respectively).

3.5 Conclusion

This paper studies the dynamics of investment at the firm level for a group of emerging economies in Latin America, where the importance of primary goods is relevant. The commodity sectors I focus on are energy, agriculture and mining, as they representative of the countries of study. The data shows that prices in these sectors suffered from a boom between the early 2000s and the financial crisis of 2008, where they suffered a considerate drop and later recovery. The data also shows that the volatility in commodity prices changed as a result: before 2008, commodity prices showed much more stability than after.

In that context, a sample of 1,250 firms is chosen comprising more than ten industries across eight different countries in Latin America. The data obtained from Compustat provides information on capital, debt, assets, etc. from which I construct a number of variables such as investment, sales growth, leverage and liquidity. This information is used in an econometric estimation where investment at the firm level is explained by these demand and financial indicators interacting with commodity price volatility, measured as the standard deviation of selected indices. Results show that demand growth and liquidity explain firm investment, while leverage has a negative effect. There does not appear to be a direct effect of energy price volatility on firm investment, although volatility in the price of agricultural and mining commodities does seem to have a negative effect on commodities.

3.6 Appendix

3.6.1 Standard Industry Classification code filtering

The filtering of firms according to industry is implemented according to the following SIC codes: Agricultural, forestry and fishing (100 to 999); Mining (1000 to 1499); Construction (1500 to 1799); Manufacturing (2000 to 2899); Oil (2900 to 2999); Machinery and equipment (3000 to 3999); Transport and comunication (4000 to 4999); Wholesale and retail (5000 to 5999); Finance (6000 to 6999); Services (7000 to 8999) and Others (9900 to 9999).

3.6.2 World Bank Commodity Price Data

There are three major indices in the commodity database describing three sectors: energy, non-energy and precious metals. The energy index is built using prices from three commodities (percentage weights in parenthesis): crude oil (84.6), natural gas (10.8) and coal (4.7). The non-energy index is divided into three sub-indices: agriculture (64.9), metals and minerals (31.6) and fertilizers (3.6). The agriculture index is composed of: food (40.0), raw materials (16.5), and beverages (8.4). Food is divided into vegetable oils and meals (16.3), grains (11.3) and other food (12.4). Vegetable oils and meals is built from soybeans (4.0), soybean oil (2.1), soybean meal (4.3), palm oil (4.9), coconut oil (0.5) and groundnut oil (0.5). Grains is built from maize (4.6), rice (3.4), wheat (2.8) and barley (0.5). Other food is built from sugar (3.9), (meat) beef (2.7), (meat) chicken (2.4), bananas (1.9) and oranges (1.4). Beverages is built using prices of coffee (3.8), cocoa (3.1) and tea (1.5). The raw materials index is made from timber (8.6) -in turn made of sawnwood (6.7) and logs (1.9)- and rubber (3.7), tobacco (2.3) and cotton (1.9). The metals and minerals index is built using prices of copper (12.1), aluminum (8.4), iron ore (6.0), nickel (2.5), zinc (1.3), tin (0.7) and lead (0.6). The fertilizers index is built from nitrogenous (1.5), phosphate (0.8), potassium (0.7) and phosphate rock (0.6). The precious metals index is built using prices for gold (77.8), silver (18.9) and platinum (3.3). See World Bank for details.[5]

3.6.3 Augmented Dickey-Fuller unit root tests for commodity price series

Table 3.6.1: Augmented Dickey-Fuller unit root tests for commodity price series

	Z(t) Test Statistic	p-value
Index		
Energy	-1.772	0.0398**
Non-energy	-1.387	0.0843*
Agriculture	-1.421	0.0793*
Minerals & metals	-1.586	0.0581*
Precious metals	-0.814	0.2090
Commodity		
Oil	-1.792	0.0382**
Coffee	-2.417	0.0088***
Soybeans	-1.882	0.0315**
Maize	-1.960	0.0265**
Sugar	-2.316	0.0114**
Aluminum	-2.488	0.0073***
Iron ore	-1.835	0.0348**
Copper	-1.499	0.0686*
Gold	-0.599	0.2753

Number of observations per series: 97. Time series include a drift and no lags. Z(t) has t-distribution with critical values: -2.366 (1%), -1.661 (5%), and -1.291 (10%). The series for Precious metals and Gold are first-difference stationary.

[5] https://www.worldbank.org/en/research/commodity-markets

Figure 3.6.1: Commodity price indices 1995-2018

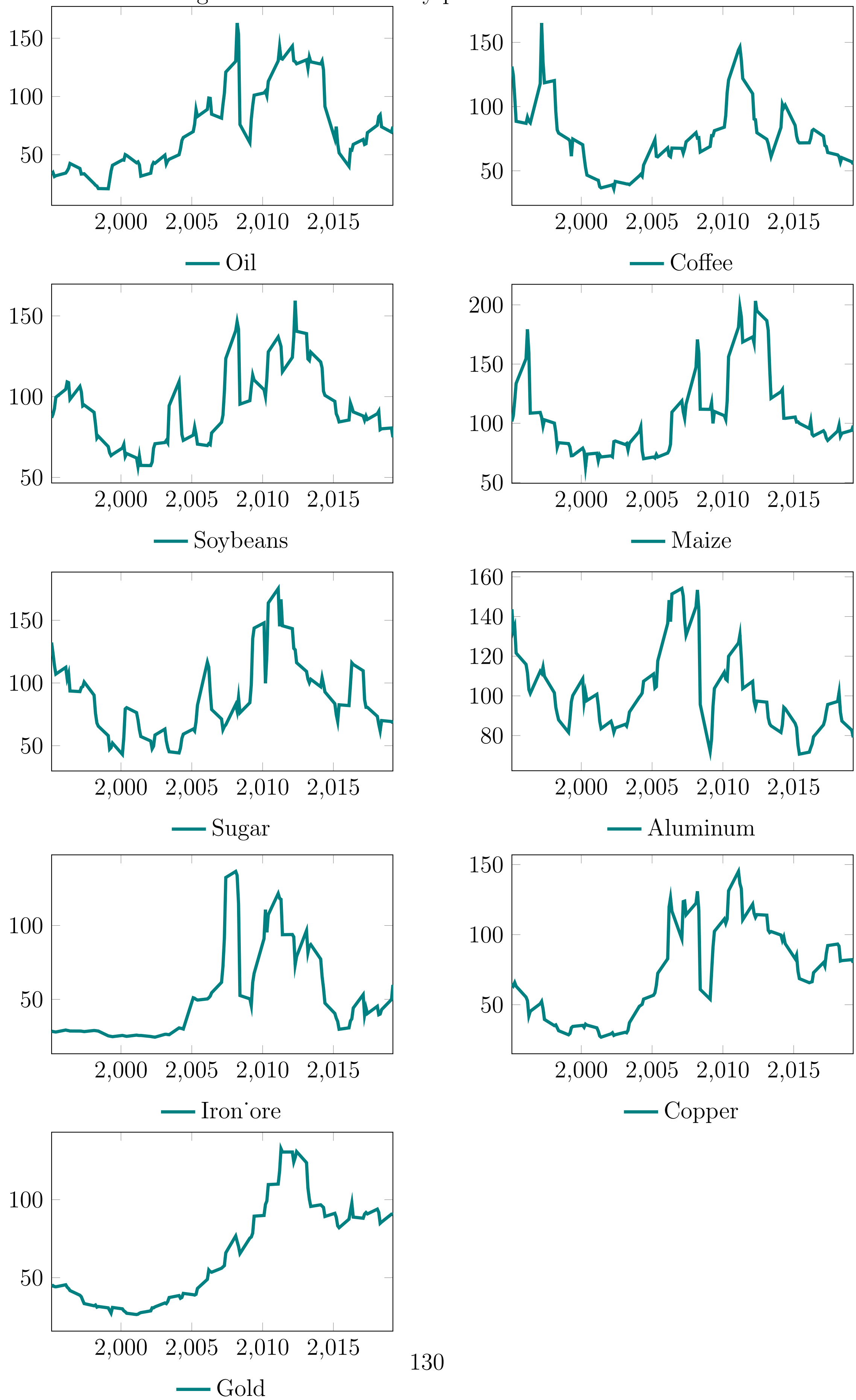
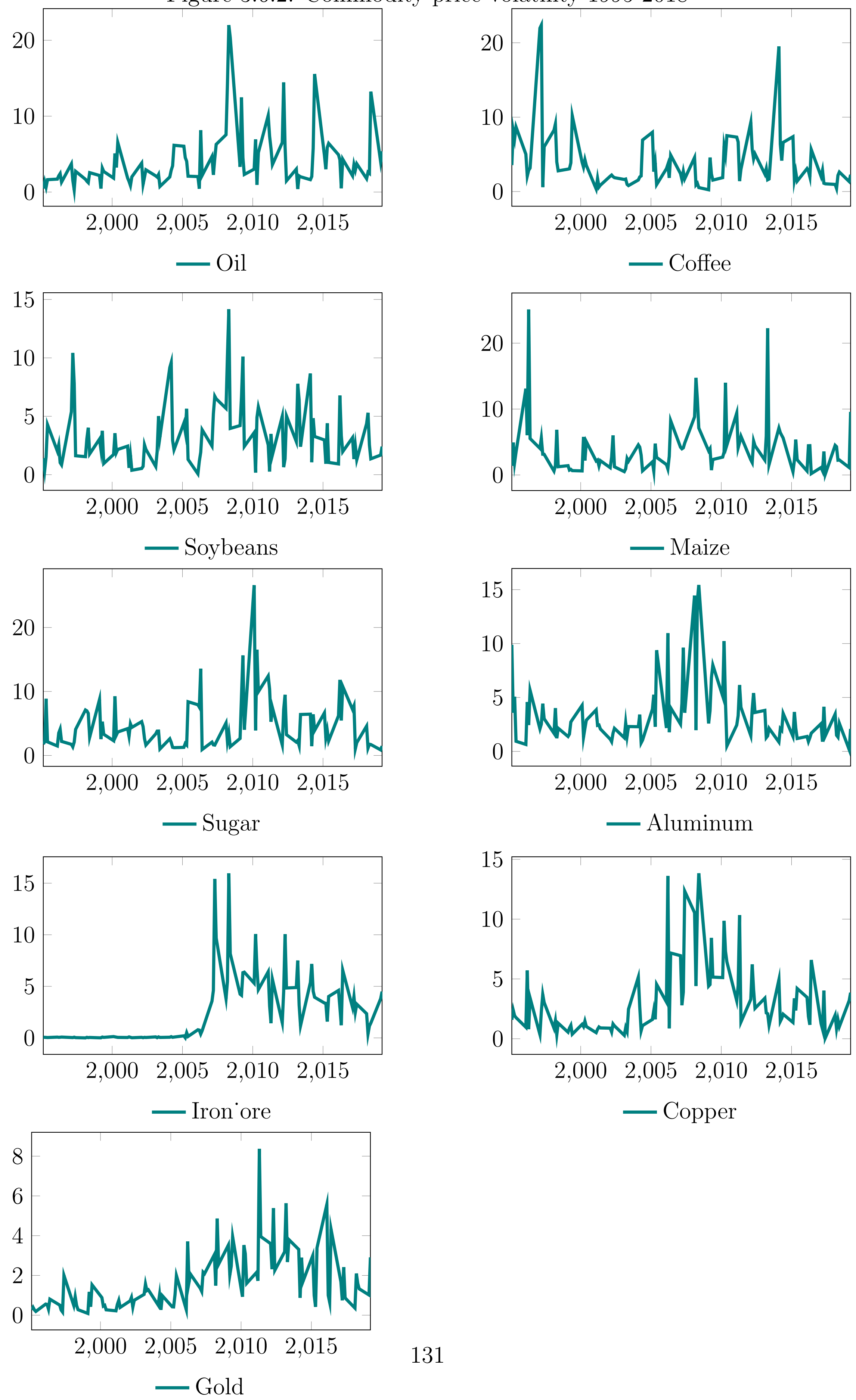


Figure 3.6.2: Commodity price volatility 1995-2018



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