

Comparison of different strategies for using fossil calibrations to generate the time prior in Bayesian molecular clock dating

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Abstract

Fossil calibrations are the utmost source of information for resolving the distances between molecular sequences into estimates of absolute times and absolute rates in molecular clock dating analysis. The quality of calibrations is thus expected to have a major impact on divergence time estimates even if a huge amount of molecular data is available. In Bayesian molecular clock dating, fossil calibration information is incorporated in the analysis through the prior on divergence times (the time prior). Here, we evaluate three strategies for converting fossil calibrations (in the form of minimum- and maximum-age bounds) into the prior on times, which differ according to whether they borrow information from the maximum age of ancestral nodes and minimum age of descendent nodes to form constraints for any given node on the phylogeny. We study a simple example that is analytically tractable, and analyse two real datasets (one of 10 primate species and another of 48 seed plant species) using three Bayesian dating programs: MCMCTree, MrBayes and BEAST2. We examine how different calibration strategies, the birth-death process, and automatic truncation (to enforce the constraint that ancestral nodes are older than descendent nodes) interact to determine the time prior. In general, truncation has a great impact on calibrations so that the effective priors on the calibration node ages after the truncation can be very different from the user-specified calibration densities. The different strategies for generating the effective prior also had considerable impact, leading to very different marginal effective priors. Arbitrary parameters used to implement minimum-bound calibrations were found to have a strong impact upon the prior and posterior of the divergence times. Our results highlight the importance of inspecting the joint time prior used by the dating program before any Bayesian dating analysis.

Keywords: Bayesian inference, molecular clock dating, divergence times, fossil calibration, time prior.

33 1. Introduction

34 Bayesian inference has become the methodology of choice for molecular clock dating of species
35 divergences because it provides a natural framework for incorporating different sources of
36 information (e.g., from fossils and molecules) (dos Reis, et al. 2016). In a Bayesian dating analysis,
37 one would ideally summarize the relevant prior evidence about species divergence times (say, from
38 the fossil record, geological events, etc.) in a multidimensional joint prior of ages for all nodes on the
39 phylogeny (called the time prior). However, specifying high-dimensional priors with complex
40 correlation structures is a notoriously difficult task, and furthermore, our knowledge of the fossil
41 evidence and of how it informs the species divergence times is very imprecise. The current practice is
42 for the paleontologist to specify minimum- and maximum-age constraints on certain nodes on the tree
43 based on the fossil evidence (Thorne, et al. 1998; Kishino, et al. 2001; Benton, et al. 2009; Ho and
44 Phillips 2009). Such *user-specified* fossil calibrations are then used by the Bayesian dating program to
45 construct the time prior, with the distribution of the ages of non-calibration nodes supplanted by a
46 branching-process model (e.g., a birth-death process) (Yang and Rannala 2006). The user-specified
47 calibration densities are assigned to single nodes on the tree and often do not satisfy the requirement
48 that any ancestral node should be older than its descendants, and thus the dating software must
49 ‘truncate’ the calibration densities to satisfy this constraint. We refer to the resulting prior of node
50 ages used by the dating software as the *effective prior*, and this may be very different from the original
51 user-specified calibration densities (Inoue, et al. 2010; Warnock, et al. 2015). Furthermore, Bayesian
52 dating programs such as MultiDivTime (Thorne, et al. 1998), MCMCTree (Yang 2007), BEAST2
53 (Bouckaert, et al. 2014) and MrBayes (Ronquist, et al. 2012b) use different procedures to combine
54 calibration densities with the birth-death process model to generate the time prior, so that different
55 programs may produce very different time priors from the same user-specified fossil calibrations
56 (Inoue, et al. 2010).

57 Thus, users of dating software are encouraged to run the Markov Chain Monte Carlo (MCMC)
58 algorithm without molecular data to generate the time prior used by the program and to inspect it to
59 ensure that it is a reasonable representation of the fossil evidence. A cross-validation method for
60 assessing the quality of calibrations, based on the consistency between fossils and between fossils and
61 molecules, has also been proposed (Near, et al. 2005). This was noted to sometimes lead to the
62 selection of calibrations of poor reliability (Marshall 2008; Benton, et al. 2009; Warnock, et al. 2015).
63 The problem appears to be partly due to the fact that fossil-calibration constraints provided by the
64 paleontologist are “over-interpreted” by the Bayesian dating program. For example, when fossil
65 evidence suggests that the age of a clade is between 50Ma and 100Ma, the dating software may
66 incorporate that information by assigning a uniform distribution, $t \sim U(50, 100)$, implying, for
67 example, $P\{50 < t < 60\} = P\{90 < t < 100\}$. Such probabilistic statements about the true age may not
68 be intended by the paleontologist. However minimum and maximum bounds alone, in the form of 50

69 $< t < 100$, are insufficient to permit a Bayesian dating analysis: a full statistical distribution for the
70 true age has to be specified.

71 The way that the fossil-based bounds on node ages are converted into statistical distributions in a
72 dating analysis may thus have an important impact on the posterior time estimates. Consider the
73 unbalanced 5-species phylogeny of Figure 1. Suppose that fossil evidence suggests that the age of
74 node 4 should be at least 10 Myrs, while the age of the root is at most 100 Ma, with $t_4 > 10$ and $t_1 <$
75 100 (Figure 1). Three simple strategies appear possible to construct the calibration densities. In
76 strategy 1 (st1), we apply a minimum-bound calibration on t_4 , by using a decay function from 10 Ma
77 to ∞ (such as the offset-exponential), while the age of the root may be assigned a uniform distribution
78 $t_1 \sim U(0, 100)$. Ages of the non-calibration nodes (t_2 and t_3) have densities specified by the birth-death
79 process. In strategy 2 (st2), we propagate the minimum and maximum bounds to all calibration nodes:
80 the root acquires the minimum bound from node 4, while node 4 inherits the maximum age of the
81 root, so that both nodes have joint bounds: $t_4 \sim U(10, 100)$, and $t_1 \sim U(10, 100)$. In strategy 3 (st3), we
82 propagate the minimum and maximum bounds to all nodes on the phylogeny, so that $t_i \sim U(10, 100)$
83 for $i = 1, 2, 3$ and 4. In all three strategies, the dating program will automatically apply a truncation so
84 that $t_4 < t_3 < t_2 < t_1$. Different programs use different procedures to perform the truncation and to
85 combine the calibration densities with the branching process model (Inoue, et al. 2010). As a result
86 the three strategies should lead to different time priors, and the different programs will also differ
87 even for the same strategy. For simple cases, it is possible to calculate analytically the resulting
88 marginal priors for the node ages after truncation. However, for large phylogenies with dozens of
89 fossil calibrations, analytical calculation is impossible, and the user needs to estimate the prior by
90 running the Bayesian MCMC program without sequence data.

91 Here we study how the different calibration strategies affect the time prior and the posterior time
92 estimates. We examine two approaches used by Bayesian dating programs to combine calibration
93 densities with the branching process to form a prior density for all node ages (the time prior): the
94 *conditional construction* used by MCMCTree (Yang and Rannala 2006) and the *multiplicative*
95 *construction* used by BEAST (Bouckaert, et al. 2014) and MrBayes (Ronquist, et al. 2012b) (see
96 Heled and Drummond 2015). We study a simple example that is analytically tractable, and then
97 analyze two real datasets: one of 10 primate species, and another of 48 seed plant species. We show
98 that the different calibration strategies as well as truncation have significant impacts on the time prior
99 and the resulting posterior time estimates. We discuss the implications of our results and give
100 recommendations for the construction of reasonable time priors.

101 **2. Material and Methods**

102 **2.1. Fossil calibrations and the time prior**

103 We consider three types of constraints on a node age based on the fossil evidence: minimum-age
104 bound, maximum-age bound, and joint (maximum- and minimum-age) bounds (Figure 2). These are
105 implemented in different Bayesian dating programs using different approaches.

106 **Minimum-age calibrations (Figure 2a).** In MCMCTree, a minimum bound is represented using
107 a truncated Cauchy distribution, denoted $L(t_L, p, c, p_L)$ (Inoue, et al. 2010). Here t_L is the minimum
108 age bound, p determines how far the mode of the distribution is from the minimum, c determines how
109 sharply the distribution decays to zero, and p_L is the left tail probability (i.e. the probability that the
110 minimum bound is violated). Smaller values of p and c give a more concentrated calibration density,
111 with a higher probability that the true age is close to the minimum age. For example, $p = 0.1$ means
112 the mode of the distribution is at $(1 + p)t_L = 1.1t_L$. Here we used $p = 0.1$, $c = 0.1$, and $p_L = 0.01$.

113 In MrBayes and BEAST2, minimum bounds are represented using an offset-exponential
114 distribution (Heled and Drummond 2012; Ronquist, et al. 2012b; Bouckaert, et al. 2014). If y has an
115 exponential distribution with rate parameter θ or mean $1/\theta$, then $t = y + t_L$ has an offset-exponential
116 distribution with parameters θ and t_L , with mean $\theta^{-1} + t_L$. A large θ means that the true age is likely to
117 be close to t_L . In this study, we used $\theta = 10/t_L$, so that the mean of the distribution is $1.1 t_L$.

118 **Maximum-age calibrations (Figure 2b).** Maximum bounds are represented by a uniform
119 distribution $U \sim (0, t_U)$, where t_U is the maximum age. Bounds are hard (with zero probability for any
120 ages outside the interval) in BEAST2 and MrBayes, and soft in MCMCTree, with p_U to be the error
121 probability that the bound is violated.

122 **Joint (minimum- and maximum-age) calibrations (Figure 2c).** Joint bounds are represented by
123 a uniform distribution $U(t_L, t_U)$ in all three programs. Again, bounds are hard in BEAST2 and
124 MrBayes, and soft in MCMCTree, which assigns p_L and p_U as the error probabilities for violations of
125 the bounds (Yang and Rannala 2006). We use $p_L = 0.01$ and $p_U = 0.05$.

126 **2.2 Calibration strategies to generate the time prior**

127 The calibration strategies are different ways of generating the effective prior given the fossil
128 bounds on the calibration nodes on the phylogeny. We consider three strategies.

129 **Calibration strategy st1:** Minimum and maximum constraints were applied to calibration nodes
130 as given, without propagating onto other nodes.

131 **Calibration strategy st2:** Minimum and maximum constraints are propagated onto all calibration
132 nodes, so that every calibration node has joint minimum and maximum bounds, represented by a
133 uniform distribution. In other words, if a calibration node lacks a minimum bound, the minimum
134 bound of its oldest descendent node is used, and if a calibration node lacks a maximum bound, the
135 maximum bound of its youngest ancestor is used.

136 **Calibration strategy st3.** This is like st2 but minimum and maximum bounds are propagated
 137 onto all interior nodes on the phylogeny, so that every node has a pair of joint bounds. Note that in
 138 st2, every calibration node has a pair of bounds while in st3, every interior node has a pair of bounds.

139 The rooted tree topology was fixed in all analyses. This is a requirement for MCMCTree and we
 140 did the same for BEAST2 and MrBayes to avoid the confounding effects of alternative phylogenies.
 141 A constraint on the root is required in MCMCTree (Yang and Rannala, 2006) and MrBayes
 142 (Ronquist, et al. 2012b). BEAST2 does not require a constraint on the root, one or more calibrations
 143 on internal nodes may be sufficient (Heled and Drummond 2012, 2015).

144 The Bayesian analysis requires a prior on the ages of all nodes on the tree. The birth-death
 145 branching process is used to provide the prior distribution for the non-calibration nodes, which is
 146 combined with the effective prior for the calibration nodes after the truncation, to generate the time
 147 prior. Two procedures have been used to achieve this in the current dating programs.

148 In MCMCTree, the so-called *conditional construction* is used (Yang and Rannala 2006). Let t_C be
 149 the ages of the calibration nodes, and $t_{\bar{C}}$ be the ages of the non-calibration nodes. In the example of
 150 Figure 1, $t_C = \{t_1, t_4\}$ while $t_{\bar{C}} = \{t_2, t_3\}$. The conditional construction gives the density of all node
 151 ages as

$$152 \quad f(t_C, t_{\bar{C}}) = f(t_C) \cdot f_{\text{BDS}}(t_{\bar{C}} | t_C), \quad (1)$$

153 where $f(t_C)$ is the effective prior on the ages of the calibration nodes, given by the user-specified
 154 calibration densities after truncation, while $f_{\text{BDS}}(t_{\bar{C}} | t_C)$ is the conditional density of the non-
 155 calibration nodes given the calibration node ages, specified by the birth-death-sampling (BDS)
 156 process (Yang and Rannala 1997).

157 Both BEAST2 and MrBayes use the so-called *multiplicative construction*, in which the birth-
 158 death process density for all node ages is multiplied with the densities for the calibration nodes to
 159 generate the time prior (Heled and Drummond, 2012; Heled and Drummond, 2015).

$$160 \quad f(t_C, t_{\bar{C}}) \propto f(t_C) \cdot f_{\text{BDS}}(t_C, t_{\bar{C}}) = f(t_C) \cdot [f_{\text{BDS}}(t_C) \cdot f_{\text{BDS}}(t_{\bar{C}} | t_C)], \quad (2)$$

161 Here t_C is the density of node ages for the calibration nodes based on the user-specified calibration
 162 densities (with suitable truncation so that ancestors are older than the descendants), and $f_{\text{BDS}}(t_C)$ is the
 163 marginal density of the node ages for the calibration nodes as specified by the birth-death-sampling
 164 process, while $f_{\text{BDS}}(t_{\bar{C}} | t_C)$ is the conditional density of the ages of the non-calibration nodes given
 165 the ages of the calibration nodes as specified by the birth-death-sampling process. As the density of t_C
 166 occurs twice in eq. 2, this density is mathematically incorrect and “does not follow the rules of
 167 probability calculus” (Heled and Drummond 2012). Here we treat both constructions as heuristic
 168 methods for converting user-specified constraints into the time prior.

169 **2.3. Analysis of a simple example with five species**

170 We use a simple and analytically tractable case of five species (Figure 1) to explore the different
 171 approaches to constructing the time prior (the prior for all node ages). Nodes 1 and 4 are calibration
 172 nodes, with the fossil constraints $t_1 < 100$ Myrs and $t_4 > 10$, while t_2 and t_3 are non-calibration nodes,
 173 for which the densities are provided by a branching process such as the birth-death-sampling process.
 174 As the birth-death process has no beginning and no ending, it is necessary to condition the process
 175 either on the time of origin, or the age of the root, or on the number of sampled extant species (N)
 176 (Yang and Rannala 1997). Here we condition on both the number of sampled extant species and the
 177 age of the root, as in Yang and Rannala (1997). Let λ be the per-lineage birth (speciation) rate, μ the
 178 per-lineage death (extinction) rate, and ρ the sampling fraction. We fix the parameters in the model at
 179 $\lambda = \mu = 1$ and $\rho = 0$, so that the ages of the nonroot nodes are order statistics from a uniform kernel
 180 (Yang and Rannala 1997). In other words, given the root age t_1 , node ages t_2, t_3 and t_4 can be
 181 generated by sampling three independent random variables from $U(0, t_1)$ and then ordering them. The
 182 joint distribution is

183
$$f_{\text{BDS}}(t_4, t_3, t_2 | t_1) = \frac{3!}{t_1^3}, \quad 0 < t_4 < t_3 < t_2 < t_1. \quad (3)$$

184 This is equivalent to the Dirichlet time prior used by Thorne et al. (1998).

185 **Calibration strategy 1 (st1).** We consider the *conditional* construction used by MCMCTree first
 186 (Yang and Rannala 2006). The calibration density for t_1 (the root age) is the uniform distribution

187
$$f_C(t_1) = 1/t_U, \quad 0 < t_1 < t_U, \quad (4)$$

188 with $t_U = 100$, while that for t_4 is the offset-exponential

189
$$f_C(t_4) = \theta e^{-\theta(t_4 - t_L)}, \quad t_L < t_4 < \infty, \quad (5)$$

190 where $t_L = 10$ and we choose $\theta = 1/t_L$ so that the mean is $2t_L = 20$ Ma.

191 Multiplying those user-specified calibration densities and removing the unfeasible region (where
 192 $t_4 > t_1$) by truncation leads to the effective prior used by the dating program

193
$$f_C(t_1, t_4) = \frac{1}{k_1} \theta e^{-\theta(t_4 - t_L)} \cdot \frac{1}{t_U}, \quad t_L < t_4 < t_1 < t_U, \quad (6)$$

194 where $k_1 = \int_L^{t_U} \int_{t_L}^{t_1} \theta e^{-\theta(t_4 - t_L)} \cdot \frac{1}{t_U} dt_4 dt_1 = 0.80001$ is a normalizing constant, to ensure that $f_C(t_1, t_4)$

195 integrates to 1.

196 Under the birth-death-sampling process model, with $\lambda = \mu = 1$ and $\rho = 0$, the joint density for t_2
 197 and t_3 , conditioned on the calibration node ages (t_1 and t_4), is given by the fact that t_2 and t_3 are order
 198 statistics from $U(t_4, t_1)$, with density

199
$$f_{\text{BDS}}(t_2, t_3 | t_1, t_4) = 2/(t_1 - t_4)^2, \quad t_4 < t_3 < t_2 < t_1. \quad (7)$$

200 The effective time prior or the joint density for all node ages is thus

201 $f(t_1, t_2, t_3, t_4) = f_C(t_1, t_4) f_{\text{BDS}}(t_2, t_3 | t_1, t_4) = \frac{1}{k_1} \theta e^{-\theta(t_4 - t_L)} \cdot \frac{1}{t_U} \times \frac{2}{(t_1 - t_4)^2}, \quad t_L < t_4 < t_3 < t_2 < t_1 < t_U, \quad (8)$

202 where k_1 is the normalizing constant defined below equation (6).

203 The marginal prior densities of the calibration node ages (t_1 and t_4) can be obtained by integration.

204 $f(t_4) = \int_{t_L}^{t_U} f_C(t_1, t_4) dt_1 = \frac{1}{k_1 t_U} \theta e^{-\theta(t_4 - t_L)} \cdot (t_U - t_4), \quad t_L < t_4 < t_U, \quad (9)$

205 $f(t_1) = \int_{t_L}^{t_1} f_C(t_1, t_4) dt_4 = \frac{1}{k_1 t_U} \left[1 - e^{-\theta(t_1 - t_L)} \right], \quad t_L < t_1 < t_U, \quad (10)$

206 Note that eq. 9 can also be derived by integrating out t_1, t_2, t_3 from $f(t_1, t_2, t_3, t_4)$, and eq. 10 can be
 207 derived by integrating out t_2, t_3, t_4 from $f(t_1, t_2, t_3, t_4)$. Figure 3a (st1) shows the user-specified
 208 calibration densities and the effective (marginal) priors after the truncation.

209 In the *multiplicative* construction used by BEAST and MrBayes, the densities for the calibration
 210 nodes of eqs. 4 and 5 are multiplied with the joint density of the ages of all non-root nodes from the
 211 birth-death-sampling process (eq. 3) to give

212 $f(t_1, t_2, t_3, t_4) \neq \frac{1}{k_2} \times f_C(t_1) \cdot f_C(t_4) \cdot f_{\text{BDS}}(t_2, t_3, t_4 | t_1)$
 $= \frac{1}{k_2} \times \frac{1}{t_U} \cdot \theta e^{-\theta(t_4 - t_L)} \cdot \frac{3!}{t_1^3}, \quad t_L < t_4 < t_3 < t_2 < t_1, \quad (11)$

213 where $k_2 = \int_{t_L}^{t_U} \int_{t_L}^{t_1} \int_{t_L}^{t_2} \int_{t_L}^{t_3} \frac{1}{t_U} \cdot \theta e^{-\theta(t_4 - t_L)} \cdot \frac{3!}{t_1^3} dt_4 dt_3 dt_2 dt_1 = 0.0174371$ is a normalizing constant. Note that
 214 eq. 11 does not make mathematical sense as two different densities occur for t_4 , one in $f_C(t_4)$ and the
 215 other in $f_{\text{BDS}}(t_2, t_3, t_4 | t_1)$. The marginal (effective) priors for the calibration node ages (t_1 and t_4) can be
 216 obtained by integration

217 $f(t_1) = \int_{t_L}^{t_1} \int_{t_L}^{t_2} \int_{t_L}^{t_3} f(t_1, t_2, t_3, t_4) dt_4 dt_3 dt_2 = \frac{3!}{k_2 t_U t_1^3} \left[\frac{(t_1 - t_L)^2}{2} + \frac{t_L - t_1}{\theta} + \frac{1 - e^{-\theta(t_1 - t_L)}}{\theta^2} \right],$
 $f(t_4) = \int_{t_4}^{t_U} \int_{t_3}^{t_U} \int_{t_2}^{t_U} f(t_1, t_2, t_3, t_4) dt_1 dt_2 dt_3 = \frac{3\theta e^{-\theta(t_4 - t_L)}}{k_2 t_U} \left[\log \frac{t_U}{t_4} - 1.5 + \frac{2t_4}{t_U} - \frac{t_4^2}{2t_U^2} \right], \quad (12)$

218 with $t_L < t_1 < t_U$ and $t_L < t_4 < t_U$. Figure 3b (st1) shows the user-specified calibration densities and the
 219 effective (marginal) priors after the truncation.

220 **Calibration strategy 2 (st2).** The minimum and maximum bounds are propagated onto all
 221 calibration nodes so that the calibration densities are

222 $f_C(t_1) = 1/(t_U - t_L), \quad t_L < t_1 < t_U,$
 $f_C(t_4) = 1/(t_U - t_L), \quad t_L < t_4 < t_U. \quad (13)$

223 We first consider the *conditional* construction. After truncation, the effective joint prior for t_1 and
 224 t_4 becomes, in contrast to eq. 6,

225 $f_C(t_1, t_4) = 2/(t_U - t_L)^2, \quad t_L < t_4 < t_1 < t_U. \quad (14)$

226 This is multiplied with the birth-death-sampling process density for the non-calibration nodes of eq. 7
 227 to give the time prior as

$$228 \quad f(t_1, t_2, t_3, t_4) = f_C(t_1, t_4) \cdot f_{\text{BDS}}(t_2, t_3 | t_1, t_4) = \frac{2}{(t_U - t_L)^2} \times \frac{2}{(t_1 - t_4)^2}, \quad t_L < t_4 < t_3 < t_2 < t_1 < t_U. \quad (15)$$

229 The marginal densities for the calibration node ages are

$$230 \quad \begin{aligned} f(t_1) &= \int_{t_L}^{t_1} f_C(t_1, t_4) dt_4 = \frac{2}{(t_U - t_L)^2} \times (t_1 - t_L), \quad t_L < t_1 < t_U, \\ f(t_4) &= \int_{t_4}^{t_U} f_C(t_1, t_4) dt_1 = \frac{2}{(t_U - t_L)^2} \times (t_U - t_4), \quad t_L < t_4 < t_U. \end{aligned} \quad (16)$$

231 Figure 3a (st2) shows the densities.

232 With the *multiplicative* construction, the time prior is given by multiplying the calibration
 233 densities (eq. 13) with the birth-death-sampling density for the noncalibration nodes (eq. 3) and then
 234 applying truncation

$$235 \quad f(t_1, t_2, t_3, t_4) = \frac{1}{k_3} \times \frac{1}{(t_U - t_L)^2} \cdot \frac{3!}{t_1^3}, \quad t_L < t_4 < t_3 < t_2 < t_1 < t_U, \quad (17)$$

236 where $k_3 = \int_{t_L}^{t_U} \int_{t_L}^{t_1} \int_{t_L}^{t_2} \int_{t_L}^{t_3} \frac{1}{(t_U - t_L)^2} \cdot \frac{3!}{t_1^3} dt_4 dt_3 dt_2 dt_1 = 0.00530524$ is a normalizing constant, calculated
 237 numerically. The marginal (effective) priors for the calibration nodes (t_1 and t_4) are then

$$238 \quad \begin{aligned} f(t_1) &= \int_{t_L}^{t_1} \int_{t_L}^{t_2} \int_{t_L}^{t_3} f(t_1, t_2, t_3, t_4) dt_4 dt_3 dt_2 = \frac{3}{k_3 (t_U - t_L)^2} \left[\log \frac{t_U}{t_1} - 1.5 + \frac{2t_4}{t_U} - \frac{t_4^2}{2t_U^2} \right], \\ f(t_4) &= \int_{t_4}^{t_U} \int_{t_3}^{t_U} \int_{t_2}^{t_U} f(t_1, t_2, t_3, t_4) dt_1 dt_2 dt_3 = \frac{(t_1 - t_L)^3}{k_3 t_1^3 (t_U - t_L)^2}, \end{aligned} \quad (18)$$

239 with $t_L < t_1 < t_U$ and $t_L < t_4 < t_U$. Figure 3b (st2) shows the user-specified calibration densities and the
 240 effective (marginal) priors after the truncation.

241 **Calibration strategy 3 (st3).** The minimum and maximum bounds are propagated onto all nodes
 242 on the phylogeny, so that every node has joint bounds: $f_C(t_i) = 1/(t_U - t_L)$, $t_L < t_i < t_U$, for $i = 1, 2, 3, 4$.
 243 With the *conditional* construction, the birth-death-sampling model plays no role in the construction of
 244 the time prior since all nodes have calibration information. After truncation, the effective time prior is

$$245 \quad f(t_1, t_2, t_3, t_4) = \frac{4!}{(t_U - t_L)^4}, \quad t_L < t_4 < t_3 < t_2 < t_1 < t_U. \quad (19)$$

246 Since t_4 is the smallest of four independent and identically distributed (i.i.d.) random variables and t_1
 247 is the largest, their marginal densities are given by the distribution of order statistics

$$248 \quad \begin{aligned} f(t_1) &= 4 \cdot \left(\frac{t_1 - t_L}{t_U - t_L} \right)^3 \cdot \frac{1}{t_U - t_L}, \quad t_L < t_1 < t_U, \\ f(t_4) &= 4 \cdot \left(\frac{t_U - t_4}{t_U - t_L} \right)^3 \cdot \frac{1}{t_U - t_L}, \quad t_L < t_4 < t_U. \end{aligned} \quad (20)$$

249 Figure 3a (st3) shows the densities. Truncation now has a strong effect.

250 With the *multiplicative* construction, two options seem possible. The first is to ignore the birth-
 251 death process density since all the node ages have calibration with this strategy. This is then
 252 equivalent to the conditional construction of MCMCTree. The second is to multiply the calibration
 253 densities (eq. 19) with the birth-death-sampling density of eq. (3), followed by a truncation to give

$$254 \quad f(t_1, t_2, t_3, t_4) = \frac{1}{k_4} \frac{4!}{(t_U - t_L)^4} \times \frac{3!}{t_1^3}, \quad t_L < t_4 < t_3 < t_2 < t_1 < t_U, \quad (21)$$

255 where the normalizing constant $k_4 = \int_{t_L}^{t_U} \int_{t_L}^{t_1} \int_{t_L}^{t_2} \int_{t_L}^{t_3} \frac{4!}{(t_U - t_L)^4} \times \frac{3!}{t_1^3} dt_4 dt_3 dt_2 dt_1 = 0.000015719$. The marginal
 256 priors for t_1 and t_4 are then

$$257 \quad f(t_1) = \int_{t_L}^{t_1} \int_{t_L}^{t_2} \int_{t_L}^{t_3} f(t_1, t_2, t_3, t_4) dt_4 dt_3 dt_2 = \frac{4! \times 3}{k_4 (t_U - t_L)^4} \left[\log \frac{t_U}{t_1} - 1.5 + \frac{2t_4}{t_U} - \frac{2t_4^2}{2t_U^2} \right], \quad (22)$$

$$f(t_4) = \int_{t_4}^{t_U} \int_{t_3}^{t_U} \int_{t_2}^{t_U} f(t_1, t_2, t_3, t_4) dt_1 dt_2 dt_3 = \frac{4! \times (t_1 - t_L)^3}{k_4 t_1^3 (t_U - t_L)^4},$$

258 with $t_L < t_1 < t_U$ and $t_L < t_4 < t_U$. Figure 3b (st3) shows the user-specified calibration densities and the
 259 effective (marginal) priors after the truncation.

260 The calibration densities and the effective time priors generated by the conditional and the
 261 multiplicative constructions using the three calibration strategies are plotted in Figure 3. From Figure
 262 3a it is apparent that with the conditional construction strategy st1 generates marginal priors that are
 263 closest to the original calibration densities. This is because the youngest node is calibrated with an
 264 offset-exponential distribution with a relatively short tail, and so truncation between the two
 265 calibration densities is minimal. In Strategy st2 the youngest node inherits the maximum age
 266 constraint from the root. This strategy avoids the choice of arbitrary parameters in the Cauchy or
 267 shifted-exponential calibrations. In this case truncation is more severe, and the marginal prior
 268 densities differ substantially from the calibration densities. In strategy st3, the inclusion of two
 269 additional calibration densities for t_3 and t_2 increases the truncation effect, and the result is that the
 270 marginal priors on t_4 and t_1 are pushed apart. The multiplicative construction is shown in Figure 3b.
 271 Strategy st1 generates marginal priors that are closest to the original calibration densities, while
 272 truncation has a major impact in strategies st2 and st3, so that the marginal prior densities differ
 273 substantially from the calibration. St2 and st3 generate nearly identical prior densities. Overall Figure
 274 3 shows that the conditional and the multiplicative constructions, as well as the different calibration
 275 strategies, generate very different effective time priors.

276 **2.4. Analysis of the primate dataset**

277 We used eight mitochondrial coding genes (Cyt B, CO1, CO2, CO3, ND2, ND3, ND4 and ND4L)
 278 and the mitochondrial 12S and 16S ribosomal RNA (rRNA) genes from nine primate species and an
 279 outgroup (*Tupaia belangeri*) (Figure 4a) (GenBank accession numbers in Table S1). We partitioned
 280 the data into three partitions: (1) 1st and 2nd codon positions; (2) 3rd codon positions and (3) rRNA

281 genes. The final alignment had 9,361 base pairs. The data were analyzed using the three dating
282 programs (MCMCTree, BEAST2, and MrBayes), under the independent-rates model to construct the
283 prior of the rates. The time unit is set at 100 Myrs. The sequence likelihood was calculated under the
284 HKY+ Γ_5 substitution model (Hasegawa, et al. 1985; Yang 1994), with separate substitution-rate
285 parameters assigned and estimated for each partition.

286 There are nine fossil calibrations (Table 1) (dos Reis, et al. 2012), five of which are joint
287 minimum and maximum bounds, while the other four are minimum bounds only. We implemented
288 calibration strategies st1 and st2 in the programs MCMCTree, BEAST2, and MrBayes. As all nine
289 interior nodes have calibration information, st3 is equivalent to st2. Bounds are soft in MCMCTree,
290 and hard in BEAST2 and MrBayes. Minimum bounds are implemented using the truncated Cauchy
291 distribution in MCMCTree and the offset-exponential distribution in BEAST2 and MrBayes.

292 In MCMCTree, the parameters of the birth-death-sampling process are fixed at $\lambda = \mu = 1$, and $\rho =$
293 0. These specify a uniform kernel. The independent-rates model (IR) assumes that the rates for
294 branches are independent variables from the lognormal distribution, specified by the mean of the rate
295 (η) and the variance of the log rate σ^2 (which determines the extent of rate variation across branches)
296 (Rannala and Yang 2007). The mean rate is assigned a gamma prior $\eta \sim G(2, 2)$ with mean $2/2 = 1.0$
297 substitutions per site per time unit (100MY) or 10^{-8} substitutions per site per year, and the rate drift
298 parameter is assigned another gamma prior, $\sigma^2 \sim G(1, 10)$, with mean 0.1.

299 Both BEAST2 and MrBayes assign hyperpriors to implement the birth-death-sampling model: the
300 net diversification rate $\lambda - \mu \sim U(0, 1)$ and the relative extinction rate $\mu/\lambda \sim U(0, 1)$ (Stadler 2010;
301 Hohna, et al. 2011). In MrBayes the sampling probability (ρ) is fixed at 0.02.

302 In BEAST2 we specified a *Relaxed Clock Log Normal* (ucln) model, which assumes that the
303 substitution rates for branches are independent variables from a lognormal distribution (Drummond, et
304 al. 2006). The lognormal distribution is parametrized using the mean and the standard deviation. The
305 mean (*uclnMean.c*) was assigned a gamma hyperprior $G(2, 0.5)$ with mean 1.0, and the standard
306 deviation (*uclnStdev.c*) was assigned a gamma hyperprior $G(2, 0.05)$ with mean 0.1.

307 In MrBayes we used the *Independent Gamma Rate* (IGR) model in where the rates for branches
308 are independent variables from a gamma distribution (Lepage, et al. 2007). The gamma model is
309 parametrized using two parameters: the mean and variance. The mean is assigned a lognormal
310 hyperprior $LN(-0.125, 0.5)$, with the mean $\exp\{-0.125 + 0.5^2/2\} = 1.0$. The variance (*Igrvarpr*) is
311 assigned an exponential prior with mean 0.1.

312 The MCMC sampling settings were determined through pilot runs and differed among the
313 programs. We ran each program at least twice, and checked for convergence by comparing the
314 posterior mean estimates between runs and by plotting the time series traces of the samples. We then
315 merged the samples from the runs before summarizing the posterior. For MCMCTree, two runs were
316 performed, each consisting of 2×10^6 iterations after a burn-in of 4×10^4 iterations and sampling every

317 200, resulting in a total of 2×10^4 samples from the two runs. For MrBayes, two runs were performed,
318 each consisting of 2×10^6 iterations, sampling every 100, with the burn-in set to 25% of samples,
319 resulting in a total of 3×10^4 samples from the two runs. For BEAST2 we performed three runs, each
320 consisting of 10^7 iterations, sampling every 1000. The burn-in was set to 30% of samples, resulting in
321 a total of 21,000 samples from all three runs.

322 **2.5. Analysis of the seed plant dataset**

323 We used five plastid genes (*atpB*, *matK*, *NdhF*, *rbcL*, and *rps4*) and two nuclear RNA genes (*18s*
324 and *26s*) for 48 seed plant species (GenBank accession numbers in Table S2) from Barba-Montoya et
325 al. (Submitted). The tree topology of Figure 4b is fixed. The sequence alignment had 13,211 base
326 pairs, with 26% missing data. We treated the data as three partitions: (1) 1st and 2nd codon positions for
327 plastid genes; (2) 3rd positions for plastid genes and (3) nuclear RNA genes. The data were analyzed
328 using the three programs (MCMCTree, BEAST2, and MrBayes), with similar settings as in the
329 analysis of the primate dataset, but some modifications were necessary to accommodate the
330 differences in the time scale and in the rate. The sequence likelihood was calculated under the
331 HKY+ Γ_5 substitution model (Hasegawa, et al. 1985; Yang 1994), with separate substitution-rate
332 parameters assigned and estimated for each partition. In MCMCTree the approximate likelihood
333 method (Thorne, et al. 1998; dos Reis and Yang 2011) is used to calculate the sequence likelihood,
334 using the maximum likelihood estimates of branch lengths and the Hessian matrix. In BEAST2 and
335 MrBayes the sequence likelihood was calculated exactly.

336 There are 15 fossil calibrations on the tree (Figure 4b) (Barba-Montoya, et al. Submitted) Among
337 them seven are joint minimum and maximum bounds and eight are minimum bounds (Table 2). The
338 time unit is set to 100 Myrs. The calibration information is implemented in the three programs using
339 the three strategies as described earlier.

340 In MCMCTree, the parameters of the birth-death-sampling process are fixed at $\lambda = \mu = 1$, and $\rho =$
341 0. We used the independent-rates (IR) model, with the overall rate assigned a gamma hyperprior $G(2,$
342 30) with mean $2/30 = 0.067$ substitutions per site per 100MY, and with the variance of the log-rate
343 assigned a gamma hyperprior $\sigma^2 \sim G(2, 20)$ with mean 0.1. Two runs were performed, each
344 consisting of 10^6 iterations after a burn-in of 40,000 iterations and sampling every 200. The combined
345 sample of 10,000 samples was used to summarize.

346 In the BEAST2 and MrBayes analyses, hyperpriors are assigned to parameters in the birth-death-
347 sampling model: $\lambda - \mu \sim U(0, 1)$ and $\mu/\lambda \sim U(0, 1)$ (Stadler, 2010; Höhna et al., 2012). In MrBayes,
348 the sampling probability (ρ) is fixed at 0.0002.

349 In BEAST2 we specified the uclD model. The mean of the lognormal (*uclDMean.c*) was assigned
350 a gamma hyperprior $G(2, 0.0335)$ with mean 0.067, and the standard deviation of the lognormal
351 (*uclDStdev.c*) was assigned a gamma hyperprior $G(2, 0.05)$ with mean 0.1. Three runs were

352 performed, each consisting of 10^7 iterations, sampling every 1000. The burn-in was set to 30% of
353 samples, resulting in a total of 21,000 samples from the posterior from the three runs.

354 In MrBayes we used the *Independent Gamma Rate* (IGR) model. The mean of the gamma was
355 assigned a lognormal hyperprior $\text{LN}(-2.79, 0.5^2)$, with the mean $\exp\{-2.79 + 0.5^2/2\} = 0.07$, and the
356 variance of the gamma is assigned an exponential hyperprior with mean 0.1. Four runs were
357 performed, each consisting of 1.5×10^6 iterations, sampling every 100. The burn-in was set to 33.3%
358 of samples, resulting in a total of 4×10^4 samples from all four runs.

359 **3. Results**

360 **3.1. Analysis of a simple example with five species**

361 Figure 5 shows the results from analysing this example using the three different dating programs.
362 In MCMCTree (Figure 5a) the calibration density used for t_4 in strategy st1, is the Cauchy distribution
363 (shifted-exponential) with parameters $t_L=10$, $p = 0.2$, $c = 0.5$ and $p_L = 0.0001$. We fix the parameters
364 in the birth-death-sampling model at $\lambda = \mu = 1$ and $\rho = 0$ in all strategies. The prior densities generated
365 by the three calibration strategies using MCMCTree (Figure 5a, st1, st2, st3) are almost identical to
366 those from the conditional construction (Figure 3a, st1, st2, st3).

367 To examine the implementation in MrBayes and BEAST (Figures 5b, c, and d) we fix the
368 parameters in the birth-death-sampling model at $\lambda = \mu = 1$ and $\rho = 0$. To avoid numerical problems,
369 we used $\lambda = 1.001$, $\mu = 0.999$ and $\rho = 0.0001$. In MrBayes the net diversification rate $\lambda - \mu$ is fixed at
370 0.002, the relative extinction rate μ / λ is fixed at 0.998 and the sampling probability (ρ) is fixed at
371 0.0001. In BEAST1 and BEAST2 we use for the net diversification rate $\lambda - \mu$ a uniform distribution
372 $U(0.00199, 0.00201)$ and for the relative extinction rate μ / λ $U(0.99799, 0.99801)$. In BEAST1 we use
373 $U(0.000099, 0.000101)$ for the sampling probability (ρ). None of these programs generated identical
374 results to the multiplicative construction. The prior densities generated by MrBayes and BEAST1
375 were similar but not identical. Precise reasons for the discrepancies between the analytical example,
376 BEAST1 and MrBayes are unknown. One possible reason is that BEAST1 and MrBayes may not be
377 conditioning the birth-death-sampling age density on both root (t_1) and the number of sampled species
378 (N). Here we emphasize the large differences in the prior generated by the conditional and
379 multiplicative constructions and the priors from the three calibration strategies.

380 **3.2. Analysis of the primate dataset**

381 The calibration densities and the effective time priors generated by the three programs using
382 calibration strategies st1 and st2 are plotted in Figures 6 and 7. The posterior distributions of
383 divergence times are shown in Figures 7 and 8.

384 First, we note that with both st1 and st2, the user-specified calibration densities are very different
385 from the marginal densities for the node ages in the effective time prior. This difference is mainly

386 caused by the truncation to enforce the constraint that ancestors are older than descendants. In
387 particular, the root age assigned a pair of bounds represented by the uniform distribution, and in the
388 time prior, the density is pushed towards the maximum. Node 18 is a descendent to many other
389 interior nodes but is ancestral to none, so that its density is pushed towards the minimum. The patterns
390 for other nodes are more complex. Second, strategy st2, which uses uniform bounds for all interior
391 nodes, show much greater truncation effect so that the user-specified calibration densities and the
392 marginal prior densities are even more different than under strategy st1. Third, the differences in the
393 prior of node ages are transferred to the differences in the posterior. For example, the prior favoured
394 much older age for the root under st2 than under st1 for all three programs (Figure 7a, b, c, node 11),
395 and this pattern persisted in the posterior.

396 Lastly, the three dating programs produced similar priors and posteriors (Figures 7 and 8),
397 although MCMCTree produced slightly older time estimates and wider intervals, especially for old
398 nodes such as the root.

399

400 ***3.4. Analysis of seed plant dataset***

401 The calibration densities and the effective time priors generated by the three programs using the
402 three calibration strategies are plotted in Figures 9 and 10. The posterior distributions of node ages are
403 shown in Figures 10 and 11. We see similar patterns to those in the analysis of the primate dataset.
404 First, there are large differences between calibration densities specified by the user on one hand and
405 the (marginal) effective prior densities used by the dating software on the other. The difference is
406 particularly pronounced for nodes with wide joint bounds as the effective prior used by the dating
407 software is much narrower. Furthermore, truncation pushes the ages of old nodes such as the root
408 towards the user-specified maximum bound, or even outside the maximum bound in the case of
409 MCMCTREE, which allows bound violation due to its use of soft bounds (e.g., Figure 10a, b, and c,
410 node 49). At the same time, truncation has the effect of pushing the ages of younger nodes towards
411 the minimum bound in the prior (e.g., Figure 10a, b, and c, nodes 86, 88, and 89).

412 Second, as in the case of the primate dataset, the posterior of the node ages is sensitive to the
413 prior, and differences in the time prior are directly transferred to differences in the posterior. For
414 example, nodes 77 and 78 are older under st2 than under st1 and even older under st3, and exactly the
415 same trend persists in the posterior (Figure 10a, b, c). This pattern holds for all three dating programs.

416 Third, strategies st2 and st3 showed greater truncation effects so that the user-specified calibration
417 densities and the marginal prior densities are even more different than under st1. The large differences
418 in the priors of the three strategies persisted in the posterior. The time estimates tended to be older
419 under st2 than under st1, while st3 produced the oldest time estimates (Figures 10 and 11). For
420 example, the posterior mean estimated using st1 suggests that the eudicots (node 57) originated
421 around 155 Ma, but using st3 the posterior mean was around 195 Ma, with a difference of 40 Myrs.

422 The origin of monocots (node 78) was dated to ~136 Ma under st1 in BEAST2 and MrBayes and 150
423 Ma in MCMCTree, but using st3 the posterior mean for this node was around 190 Ma, with again a
424 difference of ~40 Myrs. These differences in the posterior reflect the differences in the time prior
425 generated under the three strategies (Figures 10 and 11).

426 Differences in posterior time estimates exist among the three dating programs, reflecting their
427 different procedures to construct the time prior using the same fossil-calibration information (Figures
428 9 and 10). BEAST2 produced slightly younger estimates of root age (node 49) and MCMCTree
429 produced narrower intervals than BEAST2 and MrBayes. The differences among the dating programs
430 in both the prior and the posterior are the smallest for calibration strategy st3. This is because with st3
431 all nodes on the phylogeny were calibrated, so that the birth-death-sampling process plays no or little
432 role in specifying the time prior.

433 **4. Discussion**

434 In a conventional Bayesian analysis, the posterior distribution of the parameters converge to a
435 point mass (the true value of the parameter) and the prior becomes less and less important when the
436 amount of data approaches infinity. Bayesian molecular clock dating is an unconventional estimation
437 problem in the sense that such convergence to truth does not occur (Yang and Rannala 2006). If the
438 amount of molecular data increases and the fossil calibration information is fixed, the posterior will
439 not converge to a point or to the true node ages, and furthermore the prior will continue to exert a
440 large impact on the posterior. Even if we use whole genomes in the dating analysis so that sequence
441 distances and branch lengths are estimated with virtually no random sampling errors, fossil
442 calibrations and the time prior constructed using the fossil calibrations will remain important to the
443 posterior time estimates. The fundamental difficulty faced by the dating analysis is the confounding
444 effect of time and rate in sequence comparisons: molecular data provide information about the genetic
445 distances, and only fossil calibrations (or dated geological events) can resolve the distances into
446 absolute times and absolute rates. The asymptotic dynamics of the dating problem has been
447 characterized in the infinite-sites theory (Yang and Rannala 2006; Rannala and Yang 2007; dos Reis
448 and Yang 2013; Zhu, et al. 2015).

449 Our analyses highlight the fact that the different dating programs such as MrBayes, BEAST, and
450 MCMCTree use different and somewhat arbitrary procedures to construct the prior on divergence
451 times and the resulting time priors may be very different among the programs even if exactly the same
452 fossil calibration information is specified. We suggest that the user should be aware of such
453 differences and always inspect the time prior by running the program without using the sequence data.
454 The differences in the time prior may and may not have a large impact on the posterior time estimates,
455 depending on the number, nature and locations of the fossil calibrations on the phylogeny, the amount
456 of sequence data, and the seriousness of the violation of the clock, among other things. Similarly it is
457 not possible to make a general recommendation as to which procedure is more appropriate for all

458 datasets (perhaps beyond the fact that the ‘multiplicative construction’ is a mathematical mistake and
459 should be avoided). A procedure that produces time priors that better match the original calibration
460 densities should make it easier for the user to summarize the fossil evidence, but we note that such a
461 requirement may not be achievable because truncation can have a very large effect so that the
462 effective priors are very different from the calibration densities whatever procedure is followed to
463 convert the calibration densities to the effective time prior. In the future, we see probabilistic
464 modeling and statistical analysis of fossil data (including both fossil presence/absence data and
465 morphological measurements) as an important approach to summarizing the fossil evidence to
466 generate distributions of divergence times for use as molecular clock calibrations (Tavaré, et al. 2002;
467 Wilkinson, et al. 2011; Ronquist, et al. 2012a; Bracken-Grissom, et al. 2014; Heath, et al. 2014). For
468 the present, we suggest that the palaeontologist should take a proactive role in constructing calibration
469 densities, by making subjective judgments regarding the quality of the fossil and its placement on the
470 phylogeny. We also encourage the use of the error probabilities in soft-bound calibrations as an
471 approach to represent the uncertainties in the soft maximum bounds. It should be stressed that
472 decisions will be made arbitrarily by the computer program if not subjectively by the palaeontologist.
473 Given that in many cases the resulting time prior can be quite counterintuitively different to the
474 calibration densities, we cannot emphasize enough how important it is for the user to explicitly
475 calculate the time prior by running the MCMC analysis without data.

476 In this paper, we have focused on divergence time estimation when fossil calibration information
477 is available on certain nodes on the tree, a procedure called *node calibration*. Recently *tip-calibration*
478 methods have been developed, which analyze fossil data jointly with molecular data, in the so-called
479 fossilized birth-death process model (Heath, et al. 2014; Zhang, et al. 2016). Morphological characters
480 for both extant and extinct (fossil) species can be incorporated into a joint analysis with the molecular
481 data for extant species (Ronquist, et al. 2012a; O'Reilly, et al. 2015). The dates for the fossil species
482 provide the calibration information that resolves the morphological distances into absolute times and
483 rates, which are propagated to the other nodes on the phylogeny represented by the molecular data.
484 While the approach shows great promise, it has its own set of challenges (dos Reis et al., 2016;
485 Ronquist et al., 2016). First, morphological characters, driven by natural selection and adaptation to
486 environment and occasionally undergoing convergent evolution, rarely evolving in a clock-like
487 fashion (Kimura 1983). Second, morphological characters may be strongly correlated. Thus current
488 models (Lewis 2011), which ignore the correlation, are overstating the information content in the data.
489 Third, without constraints on the interior nodes, the Bayesian dating analysis tends to be very
490 sensitive to the birth-death-sampling process used to specify the time prior. Changing the parameters
491 in the branching process may change the shape of the tree (reflected in the relative of internal versus
492 external branch lengths), leading to drastically different posterior time estimates (Drummond and
493 Stadler 2016; Ronquist, et al. 2016; Zhang, et al. 2016). We believe that both node calibrations and tip
494 calibrations will have a major role to play in the foreseeable future (O'Reilly, et al. 2015).

495 **Author contributions**

496 M.d.R, and Z.Y. conceived the project and designed the analysis. J.B.-M. prepared the data sets
497 and carried out the real data analysis. M.d.R carried out the theoretical 5-species analysis. All authors
498 contributed to the interpretation of results and worked on the manuscript.

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504 **Appendix A. Supplementary material**

505 The molecular sequence alignments and five phylogenetic trees in Newick format with fossil
506 calibrations have been deposited in Figshare: <https://figshare.com/s/2d1ac059646932e74525>
507 Supplementary data associated with this article can be found, in the online version, at....

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609 Figure Legends

610 **Figure 1.** A five-species phylogeny used in the analytical example of fossil calibration strategies.

611 **Figure 2.** Probability densities for describing uncertainties in fossil calibrations: (a) soft minimum
612 bound represented by a shifted-exponential distribution specified as $t_L = 20, p = 0.1, c = 0.1, p_L =$
613 0.01 ; (b) soft maximum bound specified as “ $t_U = 80, p_R = 0.05$ ”; and (c) soft lower and upper bound,
614 specified as “ $t_L = 20, t_U = 80, p_L = 0.01, p_U = 0.05$ ”. Black solid lines represent calibration densities.
615 Red dashed lines represent a) minimum age (t_L), b) maximum age (t_U) and c) both (t_L, t_U).

616 **Figure 3.** User-specified calibrations and effective priors for node ages t_1 and t_4 under three
617 calibration strategies (st1, st2, st3) in a simple example of five species (Figure 1), generated using the
618 (a) conditional and (b) the multiplicative construction. Dashed lines represent the user-specified
619 calibration densities, while dotted lines represent the effective prior densities.

620 **Figure 4.** Phylogenies for a) 10 primate species, and b) 48 seed plant species. Calibration nodes
621 are indicated by black solid circles.

622 **Figure 5.** User-specified calibrations and effective priors for node ages t_1 and t_4 under three
623 calibration strategies (st1, st2, st3) in a simple example of five species (Figure 1), generated using a)
624 MCMCTree; b) MrBayes; c) BEAST1 and d) BEAST2. Dashed lines represent the user-specified
625 calibration densities, while dotted lines represent the effective prior densities.

626 **Figure 6.** Means and 95% CIs in the time prior (the prior for node ages) on the primate phylogeny
627 (Figure 5a) generated using calibration strategies st1 and st2 and three dating programs: MCMCTree,
628 BEAST2 and MrBayes.

629 **Figure 7.** User-specified calibration densities (dashed lines), effective time priors (dotted lines),
630 and the posterior (solid lines) for the primate dataset, under calibration strategies st1 (red) and st2
631 (black), implemented in MCMCTree, BEAST2 and MrBayes.

632 **Figure 8.** Timetrees showing posterior divergence time estimates for the primates. The branches
633 are drawn to reflect the posterior means of node ages and the bars represent 95% HPD intervals. The
634 dataset was analysed using MCMCTree, MrBayes and BEAST2 under the independent-rates model,
635 using calibration strategies st1 and st2.

636 **Figure 9.** Means and 95% CIs in the time prior for node ages on the seed plant phylogeny (Figure
637 5b) generated using three calibration strategies (st1-3) and three dating programs: MCMCTree,
638 BEAST2 and MrBayes. Calibration nodes are highlighted in red.

639 **Figure 10.** User-specified calibration densities (dashed lines), effective time priors (dotted lines),
640 and the posterior (solid lines) for the seed plant dataset, under calibration strategies st1 (red), st2
641 (black), and st3 (blue), implemented in MCMCTree, BEAST2 and MrBayes. Only the 15 calibration
642 nodes are used in the plots.

643 **Figure 11.** Timetrees showing posterior divergence time estimates for major seed plant groups.
644 The branches are drawn to reflect the posterior means of node ages and the bars represent 95% HPD
645 intervals. The dataset was analysed using MCMCTree, MrBayes and BEAST2 under the
646 independent-rates model, using three calibration strategies: st1, st2, and st3.

647

648 **Tables**

649

650 **Table 1.** Primate fossil calibrations used in this study

Node	Clade	Minimum (Ma)	Maximum (Ma)
11	Scandentia-Primates	61.5 († <i>Carpolestidae</i>)	130 (absence of placentals)
12	Primates (<i>Otolemur</i> -Human)	55.6 († <i>Aliatlasius</i>)	—
13	Haplorhini (<i>Tarsius</i> -Human)	45 († <i>Tarsius</i>)	—
14	Anthropoidea (<i>Callithrix</i> -Human)	33.7 († <i>Catopithecus</i>)	—
15	Catarrhini (<i>Macaca</i> -Human)	23.5 († <i>Proconsul</i>)	34 (absence of hominoids)
16	Hominidae (<i>Pongo</i> -Human)	11.2 († <i>Sivapithecus</i>)	33.7 (absence of pongines)
17	Ponginae (<i>Gorilla</i> - <i>Pan</i> /Human)	7.25 († <i>Chorapithecus</i>)	—
18	Homininae (<i>Pan</i> -Human)	5.7 († <i>Orrorin</i>)	10 (absence of hominines)
19	Lorisoidea (<i>Otolemur</i> - <i>Microcebus</i>)	33.7 († <i>Karanisia</i>)	55.6 (absence of strepsirrhines)

Note: All calibrations are derived from dos Reis et al. (2012). Fossil taxa are indicated by a dagger (†) before their names.

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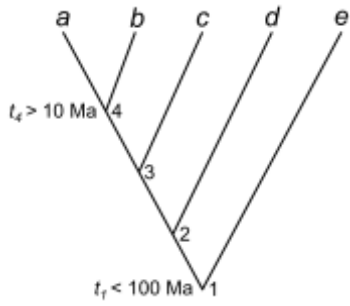
653 **Table 2.** Seed plant fossil calibrations used in this study

Node	Clade	Minimum divergence time (Ma)	Maximum divergence time (Ma)
49	Spermatophytes (<i>Ginkgo-Quercus</i>)	308.14 († <i>Cordaites iowensis</i>)	365.63 (base of Vco zone which contains the first seeds)
50	Angiosperms (<i>Amborella-Quercus</i>)	125.9 (tricolpate pollen)	247.3 (sediments below the oldest occurrence of angiosperm like pollen which are devoid of such pollen)
57	Eudicots without Ceratophyllm (<i>Nandina-Quercus</i>)	119.6 († <i>Hyracantha decussata</i>)	—
65*	No name (<i>Arabidopsis-Quercus</i>)	82.8 († <i>Paleocclusia chevalieri</i> and † <i>Dressiantha bicarpellata</i>)	127.2 (oldest potential age of tricolpate pollen)
70	Vitales (<i>Vitis-Leea</i>)	65.6 († <i>Indovitis chitaleyae</i>)	—
76	Cornales (<i>Petalonix-Cornus</i>)	85.8 († <i>Tylerianthus crossmanensis</i>)	—
77	Proteales (<i>Nelumbo-Platanus</i>)	107.59 († <i>Sapindopsis variabilis</i> , † <i>Aquia brookensis</i> and † <i>Palatonocarpus brookensis</i>)	—
78	Monocots (<i>Acorus-Musa</i>)	112.6 († <i>Liliacidites</i>)	—
84	Chloroanthaeles (<i>Cloranthus-Hedyosmum</i>)	92.8 († <i>Pennipolis</i>)	—
86	No name (<i>Trimenia-Illicium</i>)	107.59 († <i>Anacostia virginensis</i>)	—
88	Cabombaceae (<i>Cabomba-Numphaea</i>)	111 († <i>Phuricarpellatia peltata</i>)	—
89	Acrogymnospermae (<i>Ginkgo-Pinus</i>)	308.14 († <i>Cordaites iowensis</i>)	365.7 (base of Vco zone which contains the first seeds)
90	Conifers (<i>Pinus-Metasequoia</i>)	147 († <i>Rissikia media</i>)	312.38 (sediments bearing † <i>Cordaites iowensis</i>)
92	Gnetales (<i>Gnetum-Welwitschia</i>)	119.6 († <i>Eoantha zherkihini</i>)	312.38 (sediments bearing † <i>Cordaites iowensis</i>)
94	No name (<i>Ginkgo-Cycas</i>)	264.7 († <i>Crossozamia</i>)	365.63 (base of Vco zone which contains the first seeds)

Note: Calibrations are derived from Barba-Montoya et al. (submitted) and (*) from Clarke et al. (2011). Fossil taxa are indicated by a dagger (†) before their names.

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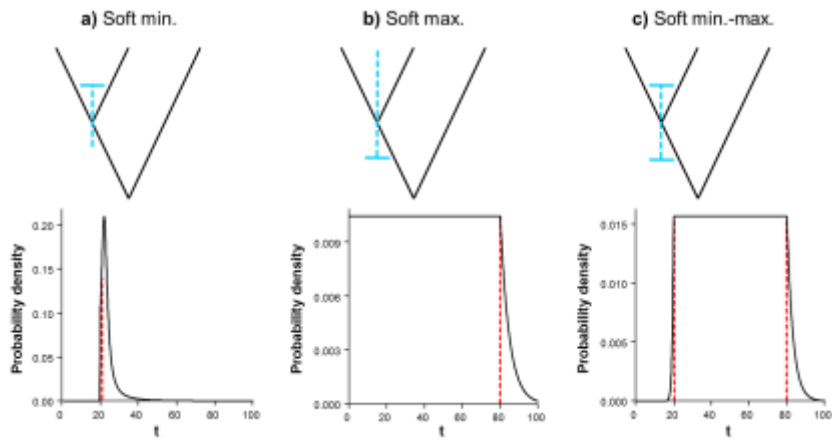
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Figure 1

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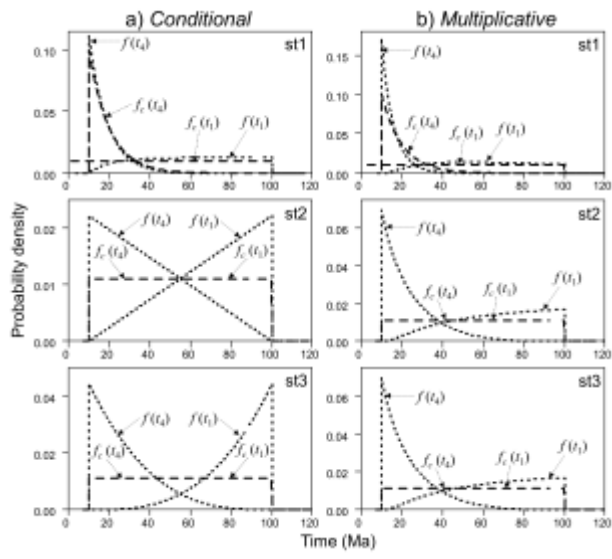
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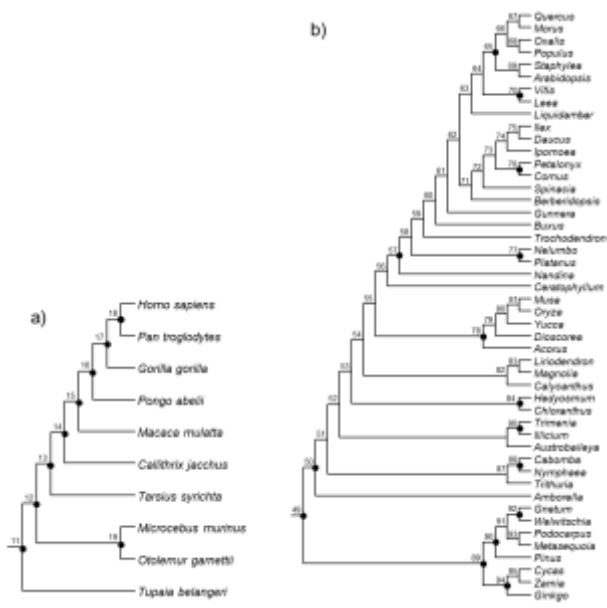
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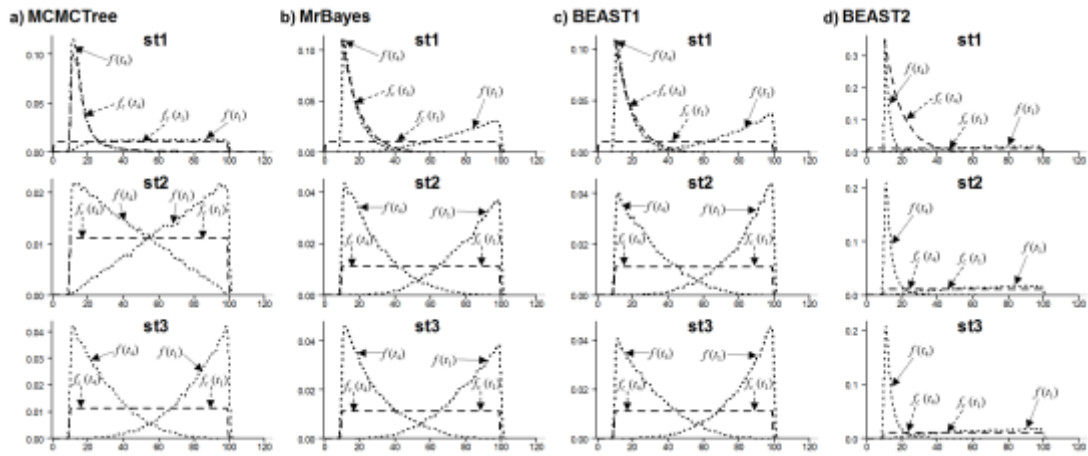
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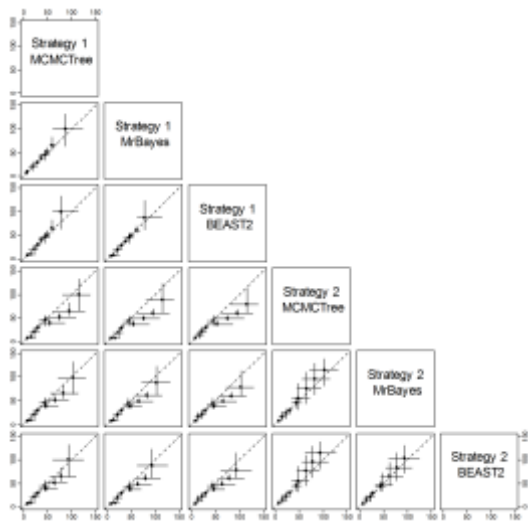
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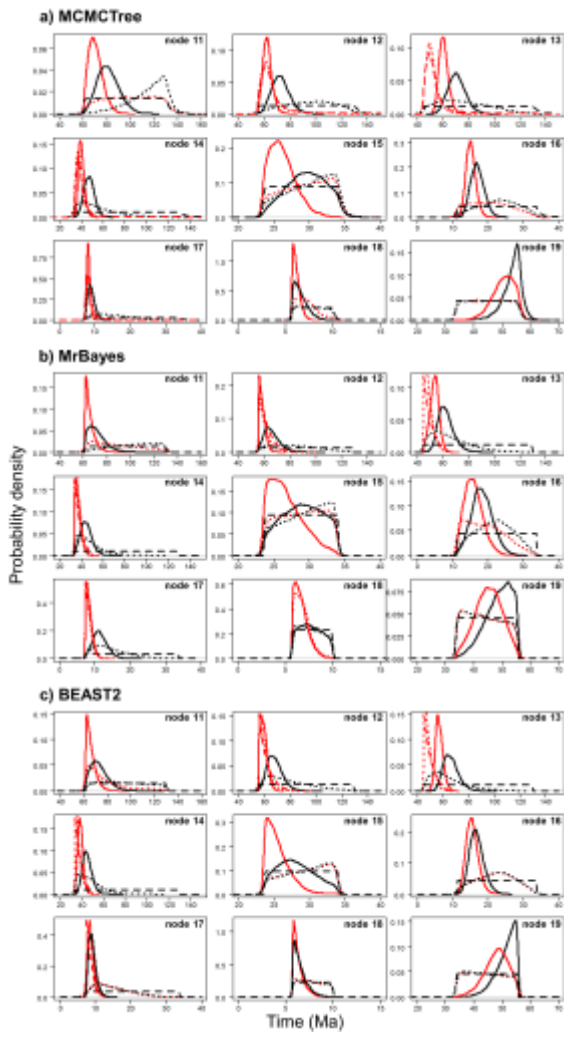
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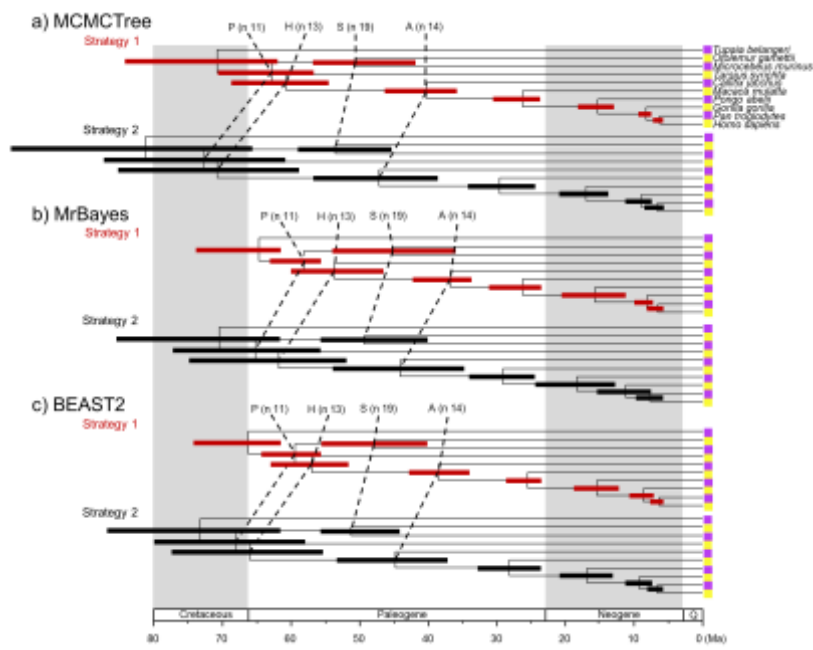
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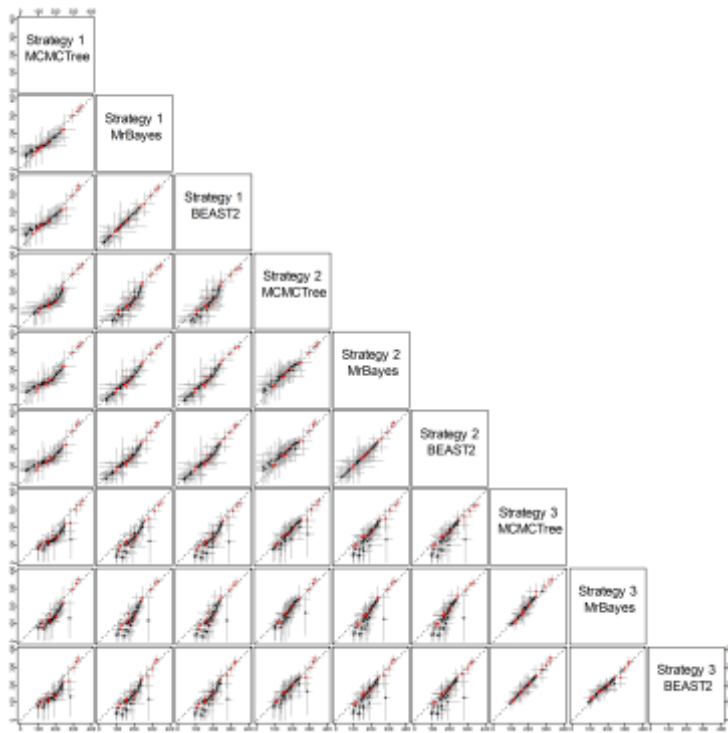
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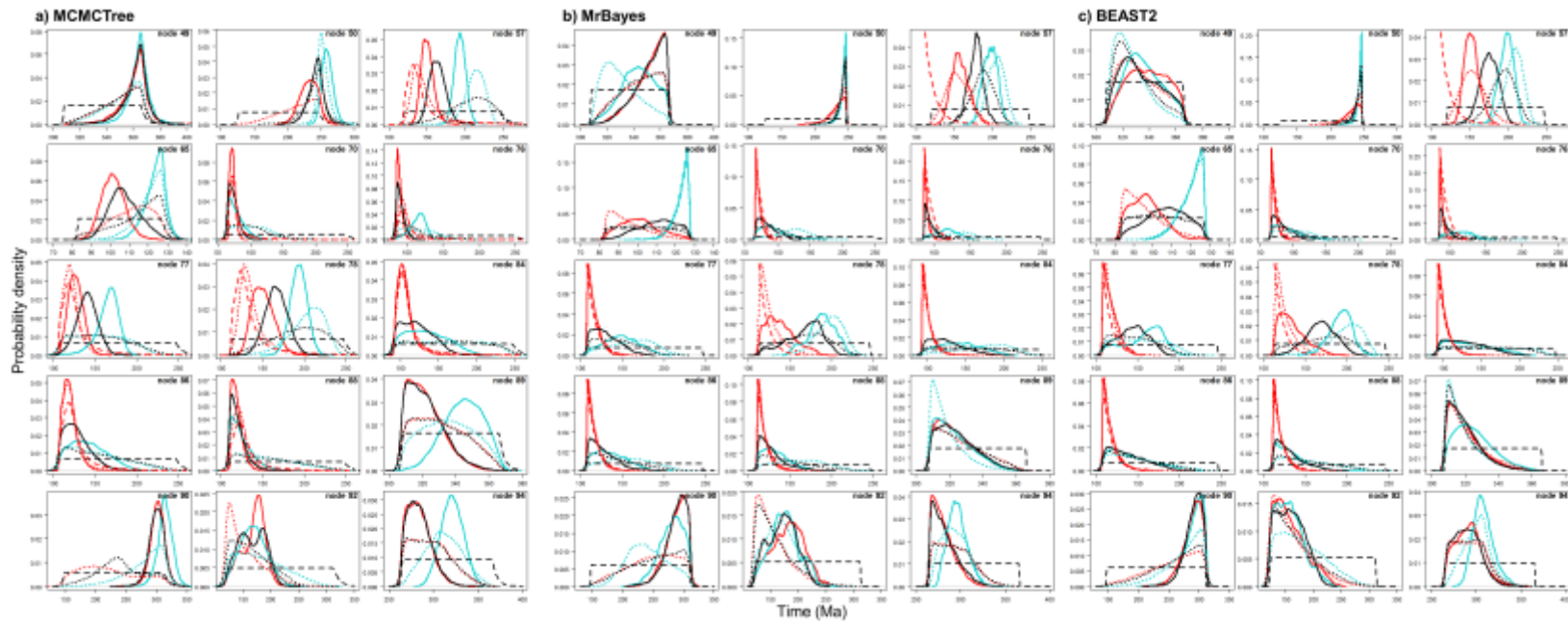


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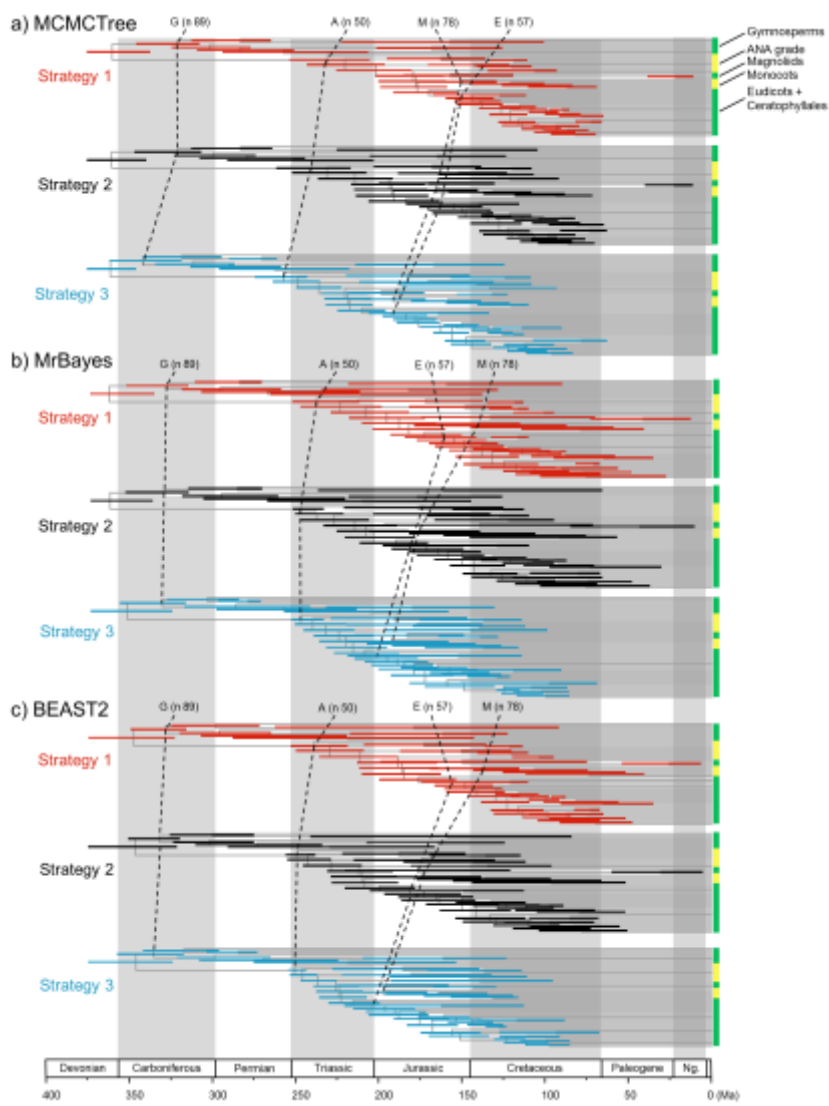


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707 Figure 10

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712 Figure 11

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