The Dynamics of Exploitation and Class in Accumulation Economies*

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Abstract

This paper analyses the equilibrium dynamics of exploitation and class in accumulation economies with population growth, technical change, and bargaining by adopting a novel computational approach. First, the determinants of the emergence and persistence of exploitation and class are investigated, and the role of labour-saving technical change and, even more importantly, power is highlighted. Second, it is shown that the concept of exploitation provides the foundations for a logically coherent and empirically relevant analysis of inequalities and class relations in advanced capitalist economies. An index that identifies the exploitation level, or intensity of each individual can be defined and its distribution studied using the standard tools developed in the theory of inequality measurement.

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1 Introduction

One of the key tenets of Marxian economics is the idea that exploitation and class are defining features of capitalist economies. This raises two issues. First, the existence of a logically coherent and empirically meaningful definition of exploitation and class. Second, the economic mechanisms that lead to the emergence and persistence of exploitation and class.

The first issue has received a lot of attention in the literature. The received view is that no logically consistent and empirically meaningful definition exists which captures the key positive and normative insights of the Marxian theory of exploitation and class in general economies. This view has been questioned in a series of recent papers by Yoshihara and Veneziani [34, 35, 27]. In these contributions, the concept of exploitation is analysed using a novel, general axiomatic approach which allows one to rigorously capture the normative and positive foundations of exploitation theory. Contrary to the received wisdom, it is shown that there exists a nonempty class of logically consistent definitions - conceptually related to the so-called "New Interpretation" (Duménil [7]; Foley [11]; Mohun [15]; Duménil, Foley, and Lévy [9]) - that satisfy a set of desirable properties in general convex economies with heterogeneous agents - including the existence of a robust relation between profits and exploitation, as well as between class and exploitation.

These contributions, however, focus on one-period economies with no savings and accumulation, and have relatively little to say about the second issue in the opening paragraph, namely the dynamics of class, exploitation, and profits. This is not a minor issue. In his seminal theory, Roemer [19] has proved that exploitation and classes emerge as the equilibrium outcome of differential ownership of the means of production in competitive economies with optimising agents when capital is scarce, leading him to conclude that the normative relevance of exploitation reduces to an exclusive emphasis on asset inequalities.

Yet Roemer's results have been derived in one-period models whereas capitalism, according to Marx, is an inherently dynamic system geared towards capital accumulation and so one may argue that class and exploitation should be analysed in a dynamic framework. In dynamic accumulation economies, it is not difficult to show that capital may become abundant, leading profits and exploitation to disappear (Devine and Dymski [6]). Perhaps more surprisingly, Veneziani [22, 23] and Veneziani and Yoshihara [24] have proved that if savings are allowed in a dynamic capitalist economy, then asset inequalities are necessary for exploitation to emerge, but alone they are not sufficient for it to persist even if agents do *not* accumulate in equilibrium.

These results cast doubts on the claim that asset inequalities are necessary and sufficient for the emergence and the persistence of exploitative relations, and raise the issue of the determinants of exploitation and class.

This paper adopts the conceptual approach to exploitation proposed by Yoshihara and Veneziani [34, 35, 27] in order to study the dynamics of asset inequalities, exploitation and classes. We significantly generalise Roemer's [18, 19] accumulation economies with maximising agents in order to incorporate nonstationary prices, population growth, time-varying consumption norms, technical change, and distributive conflict. We analyse - both formally and computationally - the dynamic equilibrium trajectories of the economies and their class and exploitation structures, and generalise some fundamental insights originally proved by Roemer [19], including the so-called *Class-Exploitation Correspondence Principle* (henceforth, CECP).

To be specific, we consider three main models exploring different mechanisms for the emergence and persistence of exploitation and class. We start off by analysing a basic economy with constant consumption, population, and technology: this benchmark scenario confirms the insights of the previous literature by showing that accumulation leads exploitation to disappear because the economy eventually becomes labour constrained.

We then extend the model to consider technical innovations, population growth, and consumption dynamics. Empirically, the long-run evolution of capitalist economies has indeed been characterised by an increase in (average) consumption opportunities and by a tendential expansion of technical knowledge, leading to a progressive increase in labour productivity (Flaschel et al. [10]). Theoretically, our analysis confirms that labour-saving technical progress may play a crucial role in making exploitation persistent by guaranteeing the persistent abundance of labour (Skillman [20]). In other words, the capitalists' control of investment and innovation decisions can make exploitation persistent by maintaining labour unemployment over time.

Although many actual capitalist economies have indeed gone through long spells of labour unemployment, one may argue with Roemer [19] that a general theory of exploitation and classes should not crucially depend - either positively or normatively - on structural imbalances in factor markets. Exploitation and class are characteristics of capitalist relations of production and full employment does not make capitalist economies non-exploitative. Therefore we analyse an extension of the basic model with population growth, technical change and accumulation in which full employment occurs in every period and distribution is determined using a general Nash bargaining procedure with the bargaining power of each agent endogenously determined as a function of their ownership of the means of production and class solidarity.

The results are quite striking: technical change and population growth are not sufficient to make exploitation persistent unless capitalists are sufficiently powerful and class solidarity among propertyless workers is sufficiently weak, *even if the economy never becomes labour constrained*. Capitalist power is an essential determinant of the persistence of exploitation and class.

In all economies, we analyse the evolution of the structure of exploitative relations. By deriving a robust correspondence between class and exploitation status, the CECP yields relevant normative insights on capitalist economies. Yet, the CECP draws a rather partial, coarse picture of the structure of exploitative relations: two economies with similar numbers of agents belonging to each class and each exploitation category may still be very different. Based on Yoshihara and Veneziani [34, 35, 27], we propose a novel index of the level, or intensity of exploitation for individual agents, whose distribution provides a finer and more comprehensive picture of exploitative relations. The analysis of its distribution yields relevant normative insights, and it raises some interesting issues that are conceptually analogous to those discussed in the literature on the measurement of income inequality.

Another contribution of the paper is methodological. Given the complexity of the economies considered, the paper adopts a novel computational approach to Marxian exploitation theory. Pioneering work applying computational methods to Marxian theory includes Wright [31, 32, 33], Cogliano [4], and Cogliano and Jiang [5]. But the latter contributions focus on Marxian price and value theory and the circuit of capital rather than exploitation and class. More related to our work is an unpublished paper by Takamasu [21], which adopts a computational approach to study class formation in accumulation economies. Yet this paper does not analyse exploitation and it only considers a very basic scenario with constant technology, population and consumption. Moreover, there is no explicit analysis of agents' maximising decisions or of the equilibrium conditions.

By moving beyond the straightjacket of analytical solutions, a computational approach allows us to study the equilibrium determination of exploitation status and the Class-Exploitation Correspondence Principle, and trace the co-evolution of exploitation and class over time in complex economies with endogenous technical change, population growth, norm-based consumption dynamics, and generalised N-agent bargaining. The results obtained are robust to changes in the specification of technology, population, preferences and endowments, but also of some of the behavioural assumptions.

The structure of the paper is as follows. Section 2 describes the general framework. Section 3 analyses the benchmark economy with stationary technology, population and consumption. Section 4 derives some key theoretical results concerning class and exploitation in the basic model. Section 5 presents the index capturing the level of exploitation of each agent. Section 6 analyses the dynamics of the basic model computationally. Section 7 extends the analysis to economies with exogenous technical change and population dynamics, whereas section 8 focuses on the role of bargaining and power in economies with full employment, endogenous technical change and population growth. Section 9 presents the robustness checks. Section 10 concludes.

2 The framework

Consider a dynamic extension of Roemer's [19] accumulating economy with a labour market and only one good produced and consumed.[1] In every period $t = 1, 2, \ldots$, let $\mathcal{N}_t$ denote the set of agents with cardinality N_t and generic element ν. At the beginning of each production period t, there is a finite set, $\mathcal{P}_t$, of *Leontief production techniques* (A_t, L_t), where $0 < A_t < 1$ and $L_t > 0$, and all agents have access to the techniques in $\mathcal{P}_t$.

In every period t, agents have identical preferences but possess potentially different endowments of labour, l_{t-1}^{ν}, and capital, ω_{t-1}^{ν}, inherited from previous periods. The distribution of endowments at the beginning of t is given by $\Pi_{t-1} = \left(l_{t-1}^{\nu}\right)_{\nu \in \mathcal{N}_t} \in \mathbb{R}_{++}^{N_t}$ and $\Omega_{t-1} = \left(\omega_{t-1}^{\nu}\right)_{\nu \in \mathcal{N}_t} \in \mathbb{R}_{+}^{N_t}$. In every t, each agent $\nu \in \mathcal{N}_t$ is therefore completely identified by a duplet $\left(l_{t-1}^{\nu}, \omega_{t-1}^{\nu}\right) \in \mathbb{R}_{+}^{2}$. An agent $\nu \in \mathcal{N}_t$ endowed with $\left(l_{t-1}^{\nu}, \omega_{t-1}^{\nu}\right)$ can engage in three types of production activity: she can sell her labour power z_t^{ν}; she can hire others to operate a technique $(A_t, L_t) \in \mathcal{P}_t$ at the level y_t^{ν}; or she can work on her own to operate $(A_t, L_t) \in \mathcal{P}_t$ at the level x_t^{ν}.

Following Roemer [18, 19], we assume that production takes time and current choices are constrained by past events. To be precise, wages are paid ex post and $w_t \in \mathbb{R}_+$ denotes the nominal wage rate at the end of t, but every agent must be able to lay out in advance the operating costs for the activities she chooses to operate using her wealth W_{t-1}^{ν}. Letting $p_t \in \mathbb{R}_+$ denote the price of the produced commodity at the end of t and beginning of $t+1$, the market value of agent ν's endowment - her wealth - is $W_{t-1}^{\nu} = p_{t-1}\omega_{t-1}^{\nu}$. The wealth that is not used for production activities can be invested to purchase goods to sell at the end of the period, δ_t^{ν}.

[1] Given our focus on the dynamics of exploitation and class, the one-good assumption yields no loss of generality. The model can be extended to include n commodities, albeit at the cost of a significant increase in technicalities and computational intensity.

Our main behavioural assumption postulates that agents wish to accumulate as much as possible, subject to consuming $b_t \in \mathbb{R}_{++}$ per unit of labour performed, $\Lambda_t^\nu \equiv L_t x_t^\nu + z_t^\nu$. Within every period t, we consider b_t as a constant parameter, identifying a socially-determined basic consumption standard, but we do allow for the possibility that b_t changes endogenously over time incorporating evolving social norms, culture, and so on.[2]

This modelling choice is motivated by our focus on the dynamics of exploitation and class in capitalist economies characterised by a drive to accumulate, rather than on consumer choices. Theoretically, it is consistent with the classical-Marxian tradition where consumption is largely the product of social norms, rather than utility-maximising behaviour, and it allows us to analyse the persistence of class and exploitation abstracting from heterogeneous individual consumption behaviour. Unlike in many accumulation models in the Marxian tradition, however, the introduction of a consumption standard raises some interesting theoretical and technical issues, as it imposes a relevant and oft-neglected constraint on the set of equilibria.

3 The basic model

In this section, we analyse the *basic model*, which is characterised by stationary population, technology, preferences, consumption norms, and labour endowments. We focus on the basic model for analytical clarity and because it provides a theoretical benchmark and starting point for our analysis. However, the framework, concepts, and definitions can be easily extended and the results derived continue to hold in more general economies (as confirmed also by the simulations).

Let $\mathcal{N}_t = \mathcal{N}$, $\mathcal{P}_t = \mathcal{P} = \{(A, L)\}$, $b_t = b$, and $\left(l_{t-1}^\nu\right)_{\nu\in\mathcal{N}} = (l^\nu)_{\nu\in\mathcal{N}}$ for all t, and suppose that the economy can produce a surplus: $(1 - bL) > A$ or, equivalently, $1 - vb > 0$, where $v = L(1 - A)^{-1}$ denotes the embodied labour value. In every t, given (p_{t-1}, p_t, w_t), every agent $\nu \in \mathcal{N}$ chooses $\xi_t^\nu \equiv (x_t^\nu; y_t^\nu; z_t^\nu; \delta_t^\nu)$ to maximise her wealth subject to purchasing b per unit of labour performed (1) and to the constraints set by her capital (2) and labour (3) endowments. Formally, every ν solves the following programme MP_t^ν:

$$\max_{\xi_t^\nu \in \mathbb{R}_+^4} p_t \omega_t^\nu$$

[2]Observe that b_t does not specify an *absolute* level of subsistence and agent ν can always reach the consumption standard $\Lambda_t^\nu b_t$ if she does not work at all. Our treatment of subsistence is a generalisation of Roemer's [19] accumulating economies in which the subsistence level is set equal to zero at *all* levels of Λ_t^ν.

subject to

$$
\begin{aligned}
p_t x_t^\nu + [p_t - w_t L]\, y_t^\nu + w_t z_t^\nu + p_t \delta_t^\nu &= p_t b \Lambda_t^\nu + p_t \omega_t^\nu && (1)\\
p_{t-1} A x_t^\nu + p_{t-1} A y_t^\nu + p_{t-1} \delta_t^\nu &= p_{t-1} \omega_{t-1}^\nu, && (2)\\
L x_t^\nu + z_t^\nu &\leqq l^\nu. && (3)
\end{aligned}
$$

Let $\mathcal{A}^\nu(p_{t-1}, p_t, w_t)$ be the set of actions ξ_t^ν that solve MP_t^ν at prices (p_{t-1}, p_t, w_t). Let $V_t^\nu\left(p_{t-1}\omega_{t-1}^\nu; (p_{t-1}, p_t, w_t)\right) \equiv \max p_t \omega_t^\nu$ be the value of MP_t^ν. Let $(p, w) \equiv \{(p_t, w_t)\}_{t=1,\ldots}$ and let $(x^\nu; y^\nu; z^\nu; \delta^\nu) \equiv \xi^\nu = \{\xi_t^\nu\}_{t=1,\ldots}$. A *basic accumulation economy* is defined by agents $\mathcal{N}$, technology (A, L), consumption bundle b, labour endowments Π, and initial capital endowments Ω_0; and is denoted as $E(\mathcal{N}; (A, L); b; \Pi, \Omega_0)$, or, as a shorthand notation, E_0. Let $x_t \equiv \sum_{\nu \in \mathcal{N}} x_t^\nu$, and likewise for y_t, z_t, δ_t, ω_t, Λ_t, and l. Based on Roemer [19], the equilibrium notion can be defined.

Definition 1: A *reproducible solution* (RS) for $E(\mathcal{N}; (A, L); b; \Pi, \Omega_0)$ is a vector (p, w) and associated actions $(\xi^\nu)_{\nu \in \mathcal{N}}$, such that at all t:

(a) $\xi_t^\nu \in \mathcal{A}^\nu(p_{t-1}, p_t, w_t)$, for all $\nu \in \mathcal{N}$ (individual optimality);

(b) $A(x_t + y_t) + \delta_t \leqq \omega_{t-1}$ (capital market);

(c) $L y_t = z_t$ (labour market);

(d) $(x_t + y_t) + \delta_t \geqq b\Lambda_t + \omega_t$ (goods market).

At a RS, in every period: (a) all agents optimise; (b) aggregate capital is sufficient for production plans; (c) the labour market clears; (d) aggregate supply is sufficient for consumption and accumulation plans. E_0 can thus be interpreted either as a sequence of generations living for one period or as an infinitely-lived economy analysed in a sequence of temporary equilibria.

It may be argued that the concept of RS imposes stringent requirements on agents' rationality and expectation formation. For, agents trade in the good and labour market at the beginning of each period based on expectations of prices that will form at the end of the period, and in equilibrium these expectations are exactly correct. Two points should be made here to motivate the focus on RS's. First, formally, because the RS is a *temporary* equilibrium notion, it imposes much less stringent rationality and consistency requirements than standard *intertemporal* optimisation models. Indeed, as shown below, provided agents correctly expect a nonnegative profit rate and a real wage above the mimum standard to emerge at the end of the period, their choices will be optimal even if their expectations turn out not to be perfectly accurate. Second, theoretically, our purpose is to analyse the dynamic equilibrium trajectories of Marxian economies and their class and exploitation structures as defined by Roemer [18, 19]. It is therefore appropriate, at least as a first step, to adopt a theoretical framework as close as possible to Roemer's, including - crucially - his Marxian equilibrium concept.

For any (p, w), the profit rate at t is $\pi_t = \frac{p_t - p_{t-1}A - w_t L}{p_{t-1}A}$. Given the structure of the economy, we shall focus on equilibria with strictly positive prices, so that the profit rate is well defined at all t.[3] By constraints (1)-(2), it immediately follows that at any RS, only (p_t, w_t) matter for individual choices at all t and so we can take the produced commodity as the numéraire, setting $p_t = 1$, all t.[4] Let the normalised price vector be denoted as $(\mathbf{1}, \widehat{w})$, where $\mathbf{1} = (1, 1, ...)$ and, at any t, $\widehat{w}_t$ is the real wage rate and $\pi_t = \frac{1-A-\widehat{w}_t L}{A}$. In what follows, with a slight abuse in notation, in the analysis of individual choices at t, we shall simply refer to the price vector $(1, \widehat{w}_t)$.

Given the previous observations, by constraints (1)-(2), it follows that at any RS, for all $\nu \in \mathcal{N}$ and all t, the following equation must hold

$$\omega_t^\nu = [1 - A - \widehat{w}_t L] (x_t^\nu + y_t^\nu) + (\widehat{w}_t - b) (L x_t^\nu + z_t^\nu) + \omega_{t-1}^\nu. \tag{4}$$

Equation (4) has a number of implications.[5] First, it is immediate to prove that at any RS, if $\omega_{t-1} > 0$, then $\widehat{w}_t \geqq b$ and $\pi_t \geqq 0$, all t. Next, Lemma 1 proves that if the profit rate is strictly positive, then all wealth is used productively and if the wage rate is above the minimum standard b, then the labour constraint (3) binds, for all agents at the solution to MP_t^ν.

Lemma 1: Let $\left((\mathbf{1}, \widehat{w}), (\xi^\nu)_{\nu \in \mathcal{N}}\right)$ be a RS for E_0. At any t: if $\pi_t > 0$, then $A(x_t^\nu + y_t^\nu) = \omega_{t-1}^\nu$, all $\nu \in \mathcal{N}$; and if $\widehat{w}_t > b$, then $L x_t^\nu + z_t^\nu = l^\nu$, all $\nu \in \mathcal{N}$.

Next, it is possible to derive an explicit expression for the value of MP_t^ν and for the growth rate of capital, g_t^ν, for all agents.

Lemma 2: Let $\left((\mathbf{1}, \widehat{w}), (\xi^\nu)_{\nu \in \mathcal{N}}\right)$ be a RS for E_0. Then $V_t^\nu\left(\omega_{t-1}^\nu; (1, \widehat{w}_t)\right) = (1 + \pi_t)\, \omega_{t-1}^\nu + (\widehat{w}_t - b)\, l^\nu$, and $g_t^\nu = \pi_t + (\widehat{w}_t - b) \frac{l^\nu}{\omega_{t-1}^\nu}$, for all $\nu \in \mathcal{N}$.

Lemma 2 has some interesting implications concerning the dynamics of accumulation. Let $\pi^{\max} \equiv \frac{1-A-bL}{A}$. Firstly, at all t, the aggregate growth

[3] It immediately follows from MP_t^ν that if there is some t' such that $p_{t'} = 0$, then at any RS it must be $p_t = 0$ for all $t > t'$.

[4] Differences in beginning-of-period prices, p_{t-1}, and end-of-period prices, p_t, are inconsequential for agents' choices. At the beginning of t, given p_{t-1} and the expected (p_t, w_t), for every unit of wealth stored to be sold at the end of t one foregoes A^{-1} units of output produced at the end of t. Therefore one will invest productively (rather than storing the good) provided $(p_t - w_t L)\, A^{-1} \geqq p_t$: beginning of period prices do not enter the decision.

[5] Lemmas 1 and 2 follow immediately from equation (4) and so their proofs are omitted.

rate of the economy is $g_t = \pi_t + (\widehat{w}_t - b) \frac{l}{\omega_{t-1}}$. Hence, if $l = LA^{-1}\omega_{t-1}$, then $g_t = \pi^{\max}$, and if $\widehat{w}_t = b$, then $g_t^{\nu} = g_t = \pi^{\max}$, for all $\nu \in \mathcal{N}$ such that $\omega_{t-1}^{\nu} > 0$. Secondly, if $\widehat{w}_t > b$, then for any $\nu, \mu \in \mathcal{N}$, $g_t^{\nu} > g_t^{\mu}$ if and only if $\frac{l^{\nu}}{\omega_{t-1}^{\nu}} > \frac{l^{\mu}}{\omega_{t-1}^{\mu}}$. Finally, if $\pi_t = 0$ then $g_t^{\nu} = \frac{(1-vb)}{v}\frac{l^{\nu}}{\omega_{t-1}^{\nu}}$, for all $\nu \in \mathcal{N}$ such that $\omega_{t-1}^{\nu} > 0$, and $g_t = \frac{(1-vb)}{v}\frac{l}{\omega_{t-1}}$. Therefore, if there exists $t' \geqq 1$ such that $\pi_t = 0$ for all $t \geqq t'$, then the growth rate of the basic economy decreases over time and tends asymptotically to zero.

Lemma 3 derives a useful property of the set of solutions of MP_t^{ν}.

Lemma 3: Let $(\mathbf{1}, \widehat{w})$ be a given price vector such that $\pi_t \geqq 0$ and $\widehat{w}_t \geqq b$, some t. If ξ_t^{ν} solves MP_t^{ν}, then $\xi_t'^{\nu} \in \mathbb{R}_+^4$ also solves MP_t^{ν} whenever $x_t'^{\nu} + y_t'^{\nu} = x_t^{\nu} + y_t^{\nu}$ and $z_t'^{\nu} - Ly_t'^{\nu} = z_t^{\nu} - Ly_t^{\nu}$.

Proof: It is easy to check that $\xi_t'^{\nu}$ satisfies constraints (1)-(2). Moreover, labour performed is the same in ξ_t^{ν} and $\xi_t'^{\nu}$, since $L(x_t'^{\nu} + y_t'^{\nu}) + (z_t'^{\nu} - Ly_t'^{\nu}) = L(x_t^{\nu} + y_t^{\nu}) + (z_t^{\nu} - Ly_t^{\nu})$. Then the result follows from equation (4).■

Lemma 3 implies that if $(x_t^{\nu}; y_t^{\nu}; z_t^{\nu}; \delta_t^{\nu})$ solves MP_t^{ν}, then there is another vector $(0; y_t'^{\nu}; z_t'^{\nu}; \delta_t^{\nu})$ which solves MP_t^{ν}. In the simulations, this allows us to select one of the many potential solutions of MP_t^{ν} by setting $x_t^{\nu} = 0$ for all $\nu \in \mathcal{N}$.[6]

Theorem 1 characterises the equilibria of the economy.

Theorem 1: Let $\left((\mathbf{1}, \widehat{w}), (\xi^{\nu})_{\nu \in \mathcal{N}}\right)$ be a RS for E_0. At any t:

(i) If $\pi_t > 0$ and $\widehat{w}_t > b$, then $l = LA^{-1}\omega_{t-1}$. Furthermore, for any $(1, \widehat{w}_t')$ such that $\pi_t' \geqq 0$ and $\widehat{w}_t' \geqq b$, $(\xi_t^{\nu})_{\nu \in \mathcal{N}}$ also satisfies conditions (a)-(d) of Definition 1;

(ii) If $l > LA^{-1}\omega_{t-1} > 0$ then $\widehat{w}_t = b$;

(iii) If $l < LA^{-1}\omega_{t-1}$ then $\pi_t = 0$.

Proof: *Part (i).* By Lemma 1, $A(x_t^{\nu} + y_t^{\nu}) = \omega_{t-1}^{\nu}$ and $Lx_t^{\nu} + z_t^{\nu} = l^{\nu}$, for all $\nu \in \mathcal{N}$. Therefore, $A(x_t + y_t) = \omega_{t-1}$ and, by Definition 1(c), $L(x_t + y_t) = Lx_t + z_t = l$. Since $(x_t + y_t) = A^{-1}\omega_{t-1}$, we have $L(x_t + y_t) = LA^{-1}\omega_{t-1} = l$. To prove the second part of the statement, take any $(1, \widehat{w}_t')$ such that $\pi_t' \geqq 0$ and $\widehat{w}_t' \geqq b$. Then, it is immediate to show that ξ_t^{ν} solves MP_t^{ν} at $(1, \widehat{w}_t')$ for all ν and $(\xi_t^{\nu})_{\nu \in \mathcal{N}}$ satisfies conditions (b)-(d) of Definition 1 by assumption.

[6] It is important to stress that this choice has no implications whatsoever for the analysis of exploitation and class, because - as shown in section 4 - the exploitation and class status of agents do *not* depend on the specific solution to MP_t^{ν} considered.

Part (ii). By contradiction. Suppose that $\widehat{w}_t > b$. Then, for all $\nu \in \mathcal{N}$, by (2), $A(x_t^\nu + y_t^\nu) \leqq \omega_{t-1}^\nu$ and by Lemma 1, $Lx_t^\nu + z_t^\nu = l^\nu$. But, since $l > LA^{-1}\omega_{t-1}$, $Ly_t < z_t$ holds, contradicting Definition 1(c). Hence $\widehat{w}_t = b$.

Part (iii). By contradiction. Suppose that $\pi_t > 0$. Then, for all $\nu \in \mathcal{N}$, by (3), $Lx_t^\nu + z_t^\nu \leqq l^\nu$ and by Lemma 1, $A(x_t^\nu + y_t^\nu) = \omega_{t-1}^\nu$. But, since $l < LA^{-1}\omega_{t-1}$, $Ly_t > z_t$ holds, contradicting Definition 1(c). Hence $\pi_t = 0$.■

Theorem 1 defines the theoretical framework for the analysis of the dynamics of the economy. Although it only identifies necessary conditions for the existence of a RS, it does shed some light on how to construct the dynamic general equilibria. Consider part (ii) of the proof. Suppose $l > LA^{-1}\omega_{t-1}$, some t. If $\widehat{w}_t = b$, then $\pi_t > 0$ and labour performed does not produce any net income for accumulation, and for all $\nu \in \mathcal{N}$, any $(0; y_t^\nu; z_t^\nu; 0)$ with $Ay_t^\nu = \omega_{t-1}^\nu$ solves MP_t^ν. Therefore since $Ay_t = \omega_{t-1}$ and $l > LA^{-1}\omega_{t-1}$, we can choose a suitable profile $(z_t^\nu)_{\nu \in \mathcal{N}}$ such that $Ly_t = z_t$ and all conditions of Definition 1 are satisfied at t.

Consider part (iii) of the proof. Suppose $l < LA^{-1}\omega_{t-1}$, some t. If $\pi_t = 0$, then $\widehat{w}_t > b$ and capital holders are indifferent between using their wealth productively and just carrying it for sale at the end of the period, and for all $\nu \in \mathcal{N}$, any $(0; y_t^\nu; z_t^\nu; \delta_t^\nu)$ with $z_t^\nu = l^\nu$ solves MP_t^ν. Therefore since $z_t = l$ and $l < LA^{-1}\omega_{t-1}$, we can choose a suitable profile $(y_t^\nu)_{\nu \in \mathcal{N}}$ such that $Ly_t = z_t$ and all conditions of Definition 1 are satisfied at t.

4 Exploitation and Class in the Accumulation Economy

The concept of exploitation in the accumulation economy can now be introduced. In what follows, exploitation status is defined in every period t: this is a natural assumption if the model describes a series of one-period economies, otherwise it reflects a focus on *within period* exploitation.[7] Unlike in subsistence economies, focusing on the bundle consumed by an agent may be misleading as both poor and rich agents consume b per unit of labour expended, but their *potential* consumption is very different. Definition 2 identifies exploitation status in terms of the bundles of goods that an agent *can* purchase with her income. More precisely, for all $\nu \in \mathcal{N}$ and all (p_{t-1}, p_t, w_t), let c_t^ν satisfy $p_t c_t^\nu = V_t^\nu \left(W_{t-1}^\nu; (p_{t-1}, p_t, w_t)\right) + p_t b \Lambda_t^\nu - p_t \omega_{t-1}^\nu$. Then

[7] For a discussion of *within period* and *whole life* exploitation, see Veneziani [22, 23].

Definition 2 [Roemer [19]]: Agent ν is *exploited* at t if and only if $\Lambda_t^\nu > vc_t^\nu$; she is an *exploiter* if and only if $\Lambda_t^\nu < vc_t^\nu$; and she is *neither exploited nor an exploiter* if and only if $\Lambda_t^\nu = vc_t^\nu$.

Theorem 2 characterises the exploitation status of every agent, based on their wealth per unit of labour performed $\frac{W_{t-1}^\nu}{\Lambda_t^\nu}$:

Theorem 2: Let $\left((\mathbf{1}, \widehat{w}), (\xi^\nu)_{\nu\in\mathcal{N}}\right)$ be a RS for E_0. At any t, if $\pi_t > 0$:
(i) agent ν is an exploiter $\Leftrightarrow \frac{W_{t-1}^\nu}{\Lambda_t^\nu} > \frac{1}{\pi_t}\frac{[1-\widehat{w}_t v]}{v}$;
(ii) agent ν is exploited $\Leftrightarrow \frac{W_{t-1}^\nu}{\Lambda_t^\nu} < \frac{1}{\pi_t}\frac{[1-\widehat{w}_t v]}{v}$;
(iii) agent ν is neither exploited nor an exploiter $\Leftrightarrow \frac{W_{t-1}^\nu}{\Lambda_t^\nu} = \frac{1}{\pi_t}\frac{[1-\widehat{w}_t v]}{v}$.

Proof: By Lemma 2, $V_t^\nu\left(W_{t-1}^\nu; (1, \widehat{w}_t)\right) = (1+\pi_t) W_{t-1}^\nu + (\widehat{w}_t - b)\, l^\nu$, which in turn implies that $c_t^\nu = \pi_t W_{t-1}^\nu + (\widehat{w}_t - b)\, l^\nu + b\Lambda_t^\nu$, for all t and all $\nu \in \mathcal{N}$. Therefore for any $\widehat{w}_t \geqq b$, using Lemma 1, $c_t^\nu = \pi_t W_{t-1}^\nu + \widehat{w}_t \Lambda_t^\nu$, for all t and all $\nu \in \mathcal{N}$. But then agent ν is an exploiter if and only if $v\left(\pi_t W_{t-1}^\nu + \widehat{w}_t\Lambda_t^\nu\right) > \Lambda_t^\nu$, and the first part of the statement follows from simple algebraic manipulations. The other two parts follow in like manner.■

Theorem 2 generalises analogous results by Roemer [19], as it characterises the exploitation status of all agents also in economies with unemployed labour. If $\Lambda_t^\nu = l^\nu$, all $\nu \in \mathcal{N}$, then by Theorem 2 exploitation status is determined by the ratio of capital and labour *endowments* as in Roemer [19]. If the economy is characterised by unemployed labour, however, $\Lambda_t^\nu < l^\nu$ for at least some $\nu \in \mathcal{N}$ and exploitation status is determined by the ratio of the capital endowment *and labour performed*, $\frac{W_{t-1}^\nu}{\Lambda_t^\nu}$.

Observe that Theorem 2 holds if $\pi_t > 0$. If $\pi_t = 0$ then $\widehat{w}_t = (1/v) > b$ and $\Lambda_t^\nu = vc_t^\nu$ for all $\nu \in \mathcal{N}$ and no exploitation exists in the economy. This correspondence between profits and exploitation is a standard result in Marxian theory (for a discussion, see Veneziani and Yoshihara [27]).

Following Roemer [19], *classes* can be defined based on the way in which agents relate to the means of production.[8] Let (a_1, a_2, a_3) be a vector where $a_i \in \{+, 0\}$, $i = 1, 2, 3$, where "+" means a positive entry. In every t, agent ν is said to be a member of class (a_1, a_2, a_3), if there is $\xi_t^\nu = (x_t^\nu; y_t^\nu; z_t^\nu; \delta_t^\nu) \in \mathcal{A}^\nu(1, \widehat{w}_t)$ such that $(x_t^\nu; y_t^\nu; z_t^\nu)$ has the form (a_1, a_2, a_3). The notation $(+, +, 0)$ implies, for instance, that an agent works in her own 'shop' and hires others to work for her; $(+, 0, +)$ implies that an agent works both in her own 'shop'

[8]Again, observe that we focus on *within period* classes (see Veneziani [22, 23]).

and for others; and so on. Although there are eight conceivable classes, only the following four are theoretically relevant.

$$
\begin{aligned}
C_t^1 &= \left\{\nu \in \mathcal{N} \mid \mathcal{A}^\nu\left(1, \widehat{w}_t\right) \text{ has a solution of the form } (+,+,0) \setminus (+,0,0)\right\}, \\
C_t^2 &= \left\{\nu \in \mathcal{N} \mid \mathcal{A}^\nu\left(1, \widehat{w}_t\right) \text{ has a solution of the form } (+,0,0)\right\}, \\
C_t^3 &= \left\{\nu \in \mathcal{N} \mid \mathcal{A}^\nu\left(1, \widehat{w}_t\right) \text{ has a solution of the form } (+,0,+) \setminus (+,0,0)\right\}, \\
C_t^4 &= \left\{\nu \in \mathcal{N} \mid \mathcal{A}^\nu\left(1, \widehat{w}_t\right) \text{ has a solution of the form } (0,0,+)\right\}.
\end{aligned}
$$

The notation $(a_1, a_2, a_3) \setminus (a_1', a_2', a_3')$ means that agent ν is a member of class (a_1, a_2, a_3) but not of class (a_1', a_2', a_3').

Theorem 3 proves that $C_t^1 - C_t^4$ represent a partition of the set of agents.

Theorem 3: Let $\left((\mathbf{1}, \widehat{w}), (\xi^\nu)_{\nu \in \mathcal{N}}\right)$ be a RS for E_0. At any t, if $\pi_t > 0$:

$$
\begin{aligned}
&\text{(i) } \nu \in (+,+,0)\setminus(+,0,0) \Leftrightarrow Ly_t^\nu > z_t^\nu \text{ for all } \xi_t^\nu \in \mathcal{A}^\nu\left(1, \widehat{w}_t\right); \\
&\text{(ii) } \nu \in (+,0,0) \Leftrightarrow Ly_t^\nu = z_t^\nu \text{ for some } \xi_t^\nu \in \mathcal{A}^\nu\left(1, \widehat{w}_t\right); \\
&\text{(iii) } \nu \in (+,0,+)\setminus(+,0,0) \Leftrightarrow Ly_t^\nu < z_t^\nu \text{ for all } \xi_t^\nu \in \mathcal{A}^\nu\left(1, \widehat{w}_t\right); \\
&\text{(iv) } \nu \in (0,0,+) \Leftrightarrow W_{t-1}^\nu = 0.
\end{aligned}
$$

Proof: 1. If $\pi_t > 0$, then by Lemma 1, for all $\nu \in \mathcal{N}$, $A\left(x_t^\nu + y_t^\nu\right) = \omega_{t-1}^\nu$ for all $\xi_t^\nu \in \mathcal{A}^\nu\left(1, \widehat{w}_t\right)$. Therefore $\nu \in (0,0,+)$ implies $W_{t-1}^\nu = 0$. Conversely, it is easy to see that for all $\widehat{w}_t \geqq b$, if $W_{t-1}^\nu = 0$, then $\nu \in (0,0,+)$.

2. Consider agents with $W_{t-1}^\nu > 0$. By the convexity of MP_t^ν, if $Ly_t^\nu > z_t^\nu$ for some $\xi_t^\nu \in \mathcal{A}^\nu\left(1, \widehat{w}_t\right)$ and $Ly_t^{\prime\nu} < z_t^{\prime\nu}$ for some $\xi_t^{\prime\nu} \in \mathcal{A}^\nu\left(1, \widehat{w}_t\right)$, then there is $\xi_t^{\prime\prime\nu} \in \mathcal{A}^\nu\left(1, \widehat{w}_t\right)$ such that $Ly_t^{\prime\prime\nu} = z_t^{\prime\prime\nu}$. Therefore, for all agents with $W_{t-1}^\nu > 0$: either $Ly_t^\nu > z_t^\nu$ for all $\xi_t^\nu \in \mathcal{A}^\nu\left(1, \widehat{w}_t\right)$; or $Ly_t^\nu < z_t^\nu$ for all $\xi_t^\nu \in \mathcal{A}^\nu\left(1, \widehat{w}_t\right)$; or $Ly_t^\nu = z_t^\nu$ for some $\xi_t^\nu \in \mathcal{A}^\nu\left(1, \widehat{w}_t\right)$. The latter are mutually exclusive and exhaustive cases.

3. *Part (i).* Suppose $Ly_t^\nu > z_t^\nu$ for all $\xi_t^\nu \in \mathcal{A}^\nu\left(1, \widehat{w}_t\right)$. Consider two cases:

Case 1: $\widehat{w}_t > b$. By Lemma 1, at all $\xi_t^\nu \in \mathcal{A}^\nu\left(1, \widehat{w}_t\right)$, it must be $A\left(x_t^\nu + y_t^\nu\right) = \omega_{t-1}^\nu$ and $Lx_t^\nu + z_t^\nu = l^\nu$. From the first equation it follows that $Lx_t^\nu + Ly_t^\nu = LA^{-1}\omega_{t-1}^\nu$ and so we have $LA^{-1}\omega_{t-1}^\nu - Ly_t^\nu = l^\nu - z_t^\nu$. Since $Ly_t^\nu > z_t^\nu$ for all $\xi_t^\nu \in \mathcal{A}^\nu\left(1, \widehat{w}_t\right)$, then $LA^{-1}\omega_{t-1}^\nu > l^\nu$.

Consider $\xi_t^{\prime\nu} = (x_t^{\prime\nu}; y_t^{\prime\nu}; 0; 0)$ such that $Lx_t^{\prime\nu} = l^\nu$, and $x_t^{\prime\nu} + y_t^{\prime\nu} = x_t^\nu + y_t^\nu$. Note that $y_t^{\prime\nu} = A^{-1}\omega_{t-1}^\nu - L^{-1}l^\nu > 0$ and so noting that $z_t^{\prime\nu} - Ly_t^{\prime\nu} = z_t^\nu - Ly_t^\nu$, by Lemma 3 it follows that $\xi_t^{\prime\nu} \in \mathcal{A}^\nu\left(1, \widehat{w}_t\right)$. Hence, $\nu \in (+,+,0)$.

It remains to show that $\nu \notin (+,0,0)$. Suppose, by way of contradiction, that there is $\xi_t^\nu = (x_t^\nu; 0; 0; 0) \in \mathcal{A}^\nu\left(1, \widehat{w}_t\right)$. Since $\pi_t > 0$, then by Lemma 1, $Ax_t^\nu = \omega_{t-1}^\nu$ and so $Lx_t^\nu = LA^{-1}\omega_{t-1}^\nu > l^\nu$, a contradiction.

Case 2: $\widehat{w}_t = b$. In this case, any ξ_t^ν such that $\delta_t^\nu = 0$, $A(x_t^\nu + y_t^\nu) = \omega_{t-1}^\nu$, and $Lx_t^\nu + z_t^\nu \leqq l^\nu$, solves MP_t^ν. Therefore, it is immediate to see that $\nu \in (+,+,0)$. Further, $\xi_t^\nu = \left(0; A^{-1}\omega_{t-1}^\nu; l^\nu; 0\right) \in \mathcal{A}^\nu(1, \widehat{w}_t)$, and therefore $Ly_t^\nu > z_t^\nu$ for all $\xi_t^\nu \in \mathcal{A}^\nu(1, \widehat{w}_t)$ implies $LA^{-1}\omega_{t-1}^\nu > l^\nu$. Hence the same argument as in Case 1 can be used to prove $\nu \notin (+,0,0)$.

4. *Parts (ii) and (iii)* are proved similarly.■

Theorem 3 characterises the class structure of the accumulating economy, based on the way in which agents relate to the means of production. An immediate implication of Theorem 3 is that the class status of each agent is related to her productive endowments.

Corollary 1: Let $\left((\mathbf{1}, \widehat{w}), (\xi^\nu)_{\nu \in \mathcal{N}}\right)$ be a RS for E_0. Consider any t with $\pi_t > 0$. Then, $\nu \in C_t^1 \Leftrightarrow LA^{-1}\omega_{t-1}^\nu > l^\nu$ and $\nu \in C_t^4 \Leftrightarrow W_{t-1}^\nu = 0$. Furthermore, if $\widehat{w}_t > b$, then $\nu \in C_t^2 \Leftrightarrow LA^{-1}\omega_{t-1}^\nu = l^\nu$ and $\nu \in C_t^3 \Leftrightarrow LA^{-1}\omega_{t-1}^\nu < l^\nu$; whereas if $\widehat{w}_t = b$, then $\nu \in C_t^2 \Leftrightarrow LA^{-1}\omega_{t-1}^\nu \leqq l^\nu$ and $C_t^3 = \varnothing$.

A fundamental insight of Marxian exploitation theory is the existence of a tight relation between class positions and exploitation status. This is formalised in the next principle.[9]

Class-Exploitation Correspondence Principle (CECP) [Roemer [19]]**:** *Given any economy E_0, at any RS and any t, if $\pi_t > 0$:*

(A) every member of C_t^1 is an exploiter.

(B) every member of $C_t^3 \cup C_t^4$ such that $\Lambda_t^\nu > 0$ is exploited.

The next result proves that the CECP holds in the accumulation economies considered in this paper.

Theorem 4 (**CECP**): Let $\left((\mathbf{1}, \widehat{w}), (\xi^\nu)_{\nu \in \mathcal{N}}\right)$ be a RS for E_0. At any t, such that $\pi_t > 0$, if $\nu \in C_t^1$ then ν is an exploiter and if $\nu \in C_t^3 \cup C_t^4$ with $\Lambda_t^\nu > 0$ then ν is exploited. Furthermore, if $\widehat{w}_t > b$ then:

$$\begin{aligned} \nu &\in C_t^1 \Leftrightarrow \nu \text{ is an exploiter;} \\ \nu &\in C_t^2 \Leftrightarrow \nu \text{ is neither exploited nor an exploiter;} \\ \nu &\in C_t^3 \cup C_t^4 \Leftrightarrow \nu \text{ is exploited.} \end{aligned}$$

Proof: 1. If $\widehat{w}_t > b$, then the result follows immediately from Corollary 1 and Theorem 2, noting that by Lemma 1, $\Lambda_t^\nu = l^\nu$ and $\frac{1}{\pi_t}\frac{[1-\widehat{w}_t v]}{v} = \frac{A}{L}$.

[9] In part (B), we impose the condition that agents in the lower classes spend some of their time working. This is a theoretically appropriate restriction since the exploitation status of agents who do not engage in any economic activities is unclear.

2. Suppose that $\widehat{w}_t = b$. By Corollary 1, $\nu \in C_t^3 \cup C_t^4 \Leftrightarrow W_{t-1}^\nu = 0$. Hence by Theorem 2, if $\Lambda_t^\nu > 0$, then ν is exploited. Further, by Corollary 1, $\nu \in C_t^1 \Leftrightarrow LA^{-1}\omega_{t-1}^\nu > l^\nu$. Noting that $\frac{W_{t-1}^\nu}{\Lambda_t^\nu} \geqq \frac{W_{t-1}^\nu}{l^\nu} > \frac{A}{L} = \frac{1}{\pi_t}\frac{[1-\widehat{w}_t v]}{v}$, the result follows from Theorem 2.■

Theorem 4 confirms the standard Marxist insight that agents in the upper classes are exploiters and agents in the lower classes are exploited.

5 An index of exploitation

The relation between class and exploitation status derived in the previous section provides some interesting normative insights on the structural injustices characterising capitalist economies, as Roemer [19] has forcefully argued. Yet, an exclusive focus on classes and on the sets of exploiters and exploited agents, as well as on the CECP, yields a rather partial, coarse picture of the structure of exploitative relations: two economies with similar numbers of agents belonging to each class and each exploitation category may still be very different. Based on Definition 2, it is possible to extend the normative reach of the concept of exploitation and provide a finer and more comprehensive picture of exploitative relations. For Definition 2 allows us to move beyond a purely aggregate analysis and explore the exploitation status of every agent. This immediately raises the issue of the measurement of the *intensity of exploitation*, both at the individual and at the aggregate level. It is certainly desirable to have a notion of exploitation that allows us to make statements such as "agent A is less exploited than agent B", or "Economy C is more exploitative than economy D", or "Economy E is becoming increasingly exploitative over time".

Definition 2 states that exploitation status is determined according to whether $\Lambda_t^\nu \gtrless vc_t^\nu$. Therefore a natural index of the intensity of exploitation of any agent $\nu \in \mathcal{N}_t$ in period t is:

$$e_t^\nu = \frac{\Lambda_t^\nu}{vc_t^\nu},$$

where $e_t^\nu \in [0, \infty)$, and e_t^ν can be interpreted as the rate of labour supplied relative to the labour necessary to obtain one unit of consumption. From this perspective, exploitative relations are equivalent to inequalities in labour hours supplied to earn one unit of income (measured in the labour numéraire) and the notion of exploitation is normatively relevant in that

e_t^ν can be interpreted as a well-being index capturing access to some fundamental primary goods, namely economic resources (or commodities) and free hours. The profile $(e_t^\nu)_{\nu \in \mathcal{N}_t}$ then measures the distribution of access to resources and free hours and so of opportunity for well-being.

Agent ν is exploited if and only if $e_t^\nu > 1$, but, assuming e_t^ν to be a meaningful cardinal and interpersonally comparable measure, a much richer analysis of exploitative relations is possible. For example, one can say that the greater e_t^ν the more exploited ν is and, for any ν,$\mu \in \mathcal{N}_t$, if $e_t^\nu > e_t^\mu > 1$ then ν is more exploited than μ. And similarly for exploiters. More generally, we can analyse the distribution of e_t^ν at a given point in time, as well as its evolution over time, and ask questions about the exploitation structure of the economy. A more polarised distribution of e_t^ν, for instance, suggests an increase in the intensity of exploitation. Interestingly, the mathematical structure of the profile $(e_t^\nu)_{\nu \in \mathcal{N}_t}$ is similar to that of income or wealth distributions, and so the measurement of the aggregate degree of exploitation raises similar issues as the measurement of income or wealth inequalities. Below, we provide a complete description of the (dynamics of the) distribution of e_t^ν. We also trace the evolution of the associated Gini index, interpreted as one example of a summary measure of the aggregate degree of exploitation.

6 The dynamics of the basic model

This section analyses the basic model computationally. The aim is to illustrate the relevance of the theoretical results derived above and to rigorously describe the dynamics of exploitation and class in the benchmark case. This section also provides the basic formal and computational framework for the analysis of more complex economies below.

The simulation begins with data on $(\mathcal{N}; (A, L); b; \Pi, \Omega_0)$. The benchmark set of parameters is: $N = 100$, $A = 0.5$, $L = 0.25$, $b = 1.9$, and $l^\nu = 1$, for all $\nu \in \mathcal{N}$.[10] *Aggregate* initial capital is $\omega_0 = 25$, so that $l > LA^{-1}\omega_0$ holds and the economy is initially capital constrained, starting far from the knife-edge condition $l = LA^{-1}\omega_{t-1}$. This is important in order to examine the existence and evolution of exploitation and classes over time: by Theorem 1, the knife-edge condition identifies a ceiling to capital accumulation as it provides an upper bound to the aggregate amount of capital compatible with the existence of positive profits and exploitation.

[10]Therefore $v_t b_t = vb < 1$ all t and the productivity condition is satisfied throughout the simulation. The same holds in all of the models below.

The *distribution* of aggregate capital is determined in order to mimic the empirical wealth distribution in the U.S. (Allegretto [3]). In all of our simulations, ω_0 is distributed such that there are five groups of agents. The first group comprises 50-70% of the total population and each ν in this group is assigned $\omega_0^\nu = 0$.[11] The top 1% of agents are assigned 40% of ω_0, the next 4% are assigned 30% of ω_0, the next 15% are assigned 20%, and the remaining 10% of ω_0 is distributed to whatever agents remain (10%-30% of N).

This parameterisation represents our benchmark in all simulations, unless otherwise stated.[12] It is worth stressing at the outset that the values chosen are empirically reasonable, but - as shown in section 9 below - our key insights are robust to different choices of the initial values of the key parameters.

Concerning agents, by Lemma 3, we restrict the computational analysis to solutions of MP_t^ν for all ν and t of the form $(0; y_t^\nu; z_t^\nu; \delta_t^\nu)$. To be specific, at any t, we set $\xi_t^\nu = \left(0; A^{-1}\omega_{t-1}^\nu; \frac{LA^{-1}\omega_{t-1}}{l} l^\nu; 0\right)$, $\xi_t^\nu = \left(0; \frac{l}{LA^{-1}\omega_{t-1}} A^{-1}\omega_{t-1}^\nu; l^\nu; \left(1 - \frac{l}{LA^{-1}\omega_{t-1}}\right)\omega_{t-1}^\nu\right)$, or $\xi_t^\nu = \left(0; A^{-1}\omega_{t-1}^\nu; l^\nu; 0\right)$, for all ν, depending on whether the economy is capital constrained, labour constrained, or on the knife-edge. In all of our simulations, this specification of agents' optimal choices guarantees that the conditions in Definition 1 are always satisfied.[13]

The simulation runs for $T = 50$ periods. The simulation first checks whether the economy is capital constrained, labour constrained, or on the knife-edge and updates $\widehat{w}_t$ accordingly. Once $\widehat{w}_t$ is determined, π_t is known and the agents then solve MP_t^ν. The agents' endowments are updated according to equation (4) and the simulation then repeats as necessary.[14]

The results of the simulation over T can be found in Figures 1-3. Figure 1 reports the aggregate activity levels $(x_t, y_t, z_t, \delta_t)$, aggregate net output $(1 - A)\, y_t$, net output per capita $(1 - A)\, y_t/N$, wealth W_{t-1}, the growth rate of capital g_t, $\widehat{w}_t$ and b, and π_t. In all panels, the dashed vertical line denotes the period in which the economy becomes labour constrained.

[11] The number of agents assigned to this group is randomly drawn from a uniform distribution.

[12] The procedure to determine the initial wealth distribution is the same in all simulations. While the initial distributions differ across models due to different starting points in relation to the knife-edge and to the randomness built into the procedure, the differences are sufficiently small that the results are unaffected and can be compared across models.

[13] See section 1 of the Addendum. As already noted, the focus on these solutions has no implications for any of the results or conclusions in the paper. This is because the exploitation and class status of agents, the distribution of the exploitation intensity index, income and wealth, as well as the the dynamics of consumption, technical change, and accumulation do not depend on the specific solution to the optimisation problem analysed.

[14] All simulations are done using *Mathematica* version 10.

Figure 1: Summary results - Basic model

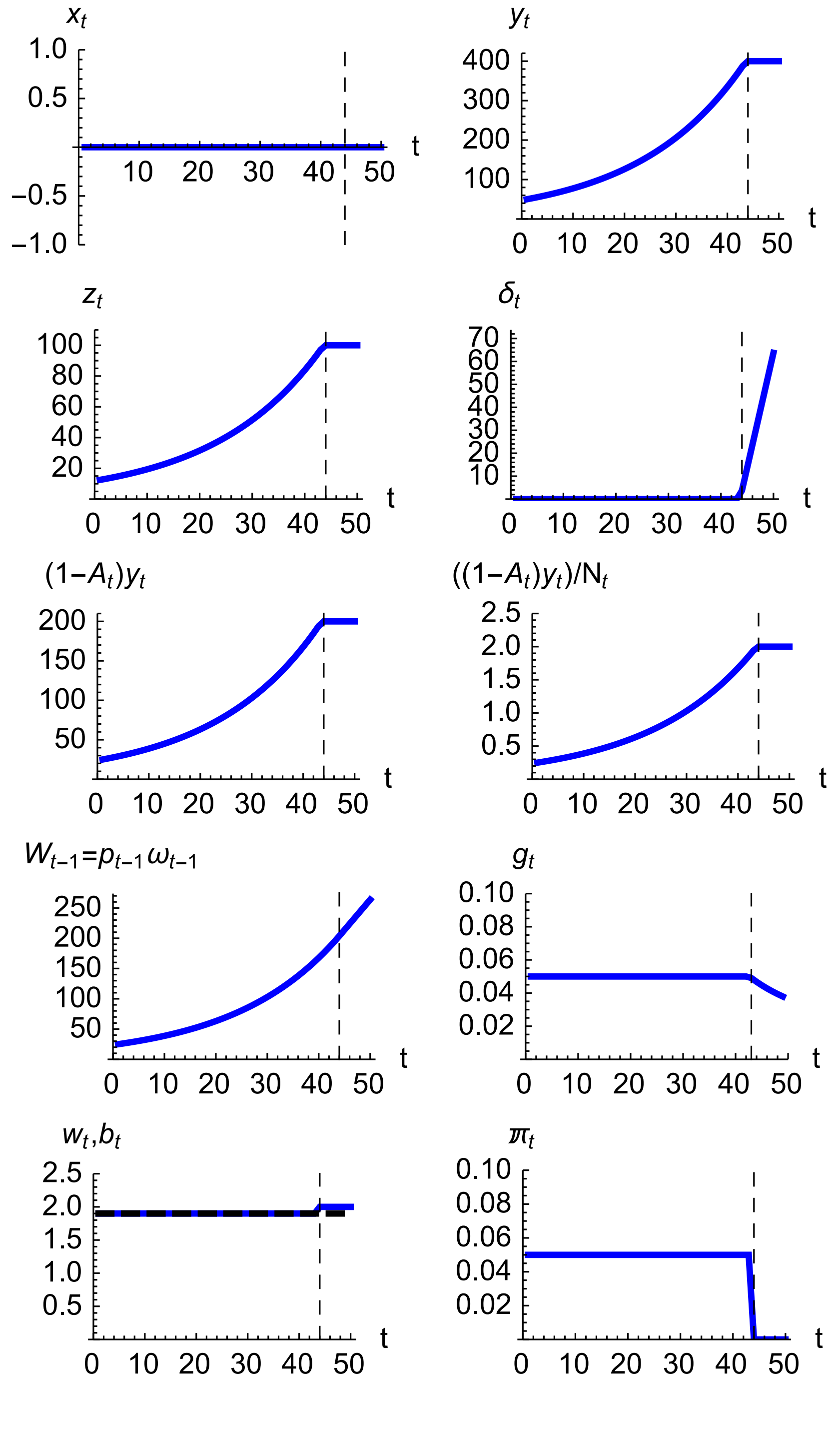


Figure 2(a) reports the dynamics of exploitation by providing a headcount of the agents who are exploiters, exploited, or neither. Clearly, the exploitation status of agents is constant while the economy is capital constrained and exploitation ceases to exist once it becomes labour constrained. Figure 2(b) captures the class composition of the economy. The pattern of C_t^1 and C_t^2 is interesting and reveals the relation between endowments and class status: at the beginning of the simulation, a relatively low level of capital implies that for a number of agents labour demand (Ly_t^ν) is lower than labour endowment (l^ν), placing them into C_t^2. As accumulation progresses, however, their labour demand grows and they eventually join C_t^1.[15] Figure 2(c) illustrates Theorem 4 and shows the existence of a robust correspondence between class and exploitation status (until the economy becomes labour constrained).

Figure 3 provides a complete description of the distribution of the exploitation intensity index, e_t^ν, over the course of the simulation. Prior to the economy becoming labour constrained, the distribution of e_t^ν is constant over time: there is no tendency for exploitation to diminish. When the economy becomes labour constrained, profits and exploitation disappear, and one can observe that $e_t^\nu = 1$, all $\nu \in \mathcal{N}$. Interestingly, agents in the lower classes are exploited to somewhat different degrees. All agents in C_t^4 are exploited with equal intensity, since they all have zero endowments and work the same amount. The exploitation status of agents in C_t^2 is not obvious instead: some of them are indeed exploited (albeit less than members of C_t^4) but agents at the upper end of C_t^2 are exploiters (albeit less than members of C_t^1).

Figure 3 displays a relatively low dispersion of the exploitation index. This is due to the fact that, unlike in actual economies, all agents perform the same amount of labour and, in the capital constrained phase, the given parameterisation (in particular the rather high value of b) yields a low profit rate. Different values of the parameters, or a heterogeneous allocation of labour (perhaps inversely proportional to wealth, in order to reflect class differences) lead to a much higher dispersion (see section 9 below).[16]

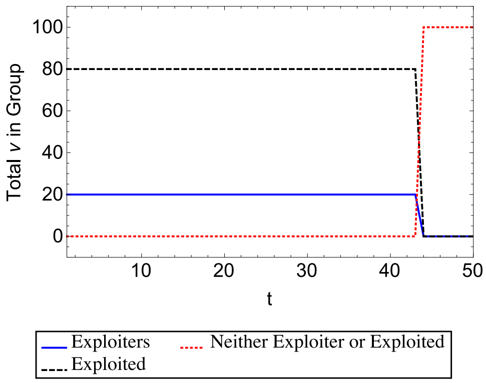
Figure 4(a) shows the Gini coefficient of wealth. The index remains constant as long as exploitation exists, because all agents in the middle and upper classes accumulate at the same rate, and so their relative positions in the wealth distribution remain unchanged, even though they become increasingly wealthier than propertyless agents in C_t^4. Once the economy becomes labour constrained and exploitation ceases, the Gini coefficient monotonically

[15] In Figure 2(b), and in all similar figures below, the class composition of the economy is shown only for the periods t with $\pi_t > 0$, consistently with Theorem 3 and Corollary 1.

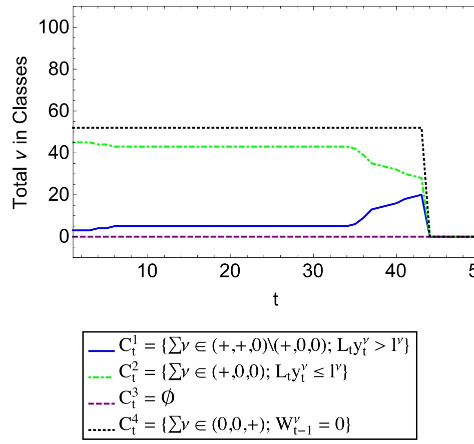
[16] The Gini coefficient of $(e_t^\nu)_{v\in\mathcal{N}}$ is constant and equal to 0.0284712 until the economy becomes labour constrained, when it drops to zero.

Figure 2: Class and exploitation status - Basic model

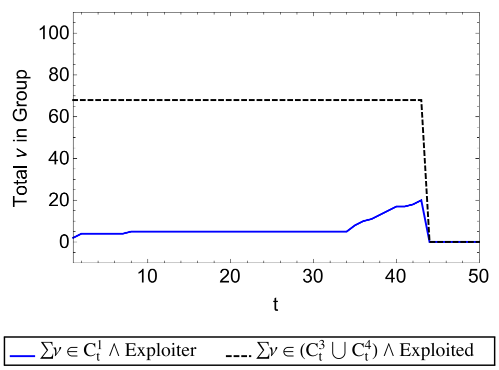
(a) Exploitation status



(b) Class status



(c) CECP



decreases and asymptotically approaches zero. The same pattern emerges in Figure 4(b) where the whole wealth distribution is shown for select t (before *and* after the end of exploitation).

In summary, the results of the basic model confirm that exploitation and classes are meaningful concepts to analyse the economic and social structure of capitalist economies, and the distribution of the exploitation intensity index yields interesting normative insights. They also confirm, however, that in an accumulation economy, absent any countervailing forces, exploitation disappears over time as profits vanish. Our next step, therefore, is to extend the basic model in order to explore the mechanisms that could lead to the persistence of exploitation in the presence of accumulation.

Figure 3: Exploitation intensity index - Basic model

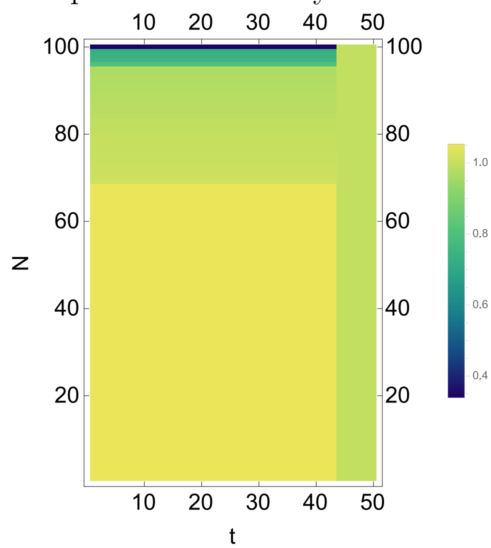
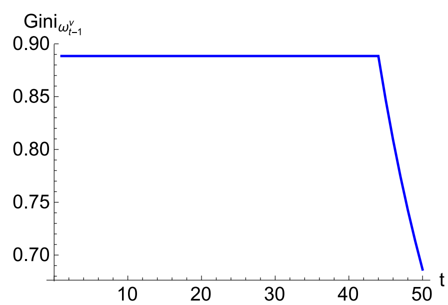
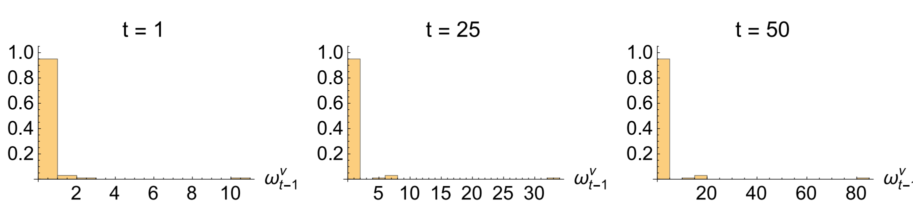


Figure 4: Distribution of wealth - Basic model

(a) Gini coefficient of wealth



(b) Distribution of wealth for select t (relative frequency)



7 On the persistent abundance of labour

In this section, we extend the basic model in order to incorporate technical change and a variable population. This choice reflects both empirical and theoretical concerns. Empirically, the long-run evolution of capitalist economies has indeed been characterised by a tendential expansion of technical knowledge, leading to a progressive increase in labour productivity (see Flaschel et al. [10]), and by significant population movements.

Theoretically, a fundamental feature of capitalism as a dynamic system is its constant tendency to revolutionise production, and it is certainly restrictive to assume technology to remain constant over time. Furthermore, in classical-Marxian approaches, population growth and, especially, labour-saving technical change have traditionally been considered two of the key mechanisms for maintaining labour abundant relative to capital.

Because we are interested in the effect of technical progress and population flows on the dynamics of exploitation and classes, in this section we keep our analysis as simple as possible and assume that technical knowledge and population grow at exogenously given, historically plausible, rates. As for technical change, we focus on labour-saving innovations that reduce L_t at a constant rate.

In this section, we allow b_t to vary over time, assuming that it keeps pace with the growth rate of the economy. This is empirically reasonable as the long-run evolution of capitalist economies has indeed been characterised by an increase in (average) consumption opportunities and consumption norms have evolved over time. It also reflects the Marxian insights on the social nature of consumption and the idea that consumption norms depend on the general level of development of the economy.[17]

To be specific, we assume that consumption norms grow at the same rate as aggregate capital - our proxy for the level of development of the economy. This allows the economy to settle on a steady growth path but it is important to emphasise that none of our insights on profits, exploitation and class depends on this specification. For example, all of our key conclusions continue to hold if consumption norms depend on labour productivity, rather than wealth,[18] or indeed, if consumption norms do not change at all.

[17] The co-evolution of accumulation and workers' consumption is sometimes considered to be one of the defining features of historical trajectories of capitalist economies à la Marx (Duménil and Levy [8], p.206).

[18] We are grateful to an anonymous referee, for suggesting labour productivity as an alternative proxy for the level of development of the economy.

7.1 Simulation results

In our simulations, we start with A_t, L_t, b_t, and N_t at the same values as in section 6: $A_0 = 0.5$, $L_0 = 0.25$, $b_0 = 1.9$, and $N_0 = 100$. As in the basic model $l_t^\nu = 1$ for all $\nu \in \mathcal{N}_t$ and all t. The initial distribution of endowments Ω_0 is handled as in section 6, with $\omega_0 = 25$, so that $l > L_1 A_1^{-1}\omega_0$ and the economy is capital constrained at the start of the simulation.

As the simulation unfolds, L_t is reduced by 2 percent during each t, while A_t remains constant, so that labour productivity increases constantly. This parameterisation is very close to the OECD estimates of the average annual growth rate of productivity for advanced countries during 1971-2014 and is generally consistent with empirical evidence on the long-run dynamics of labour productivity in the U.S. and in other high income countries (see, for example, Wolff [29],[30]). The growth rate of population, N_t, is chosen to be roughly 1 percent,[19] consistent with the UN Population Division estimates of the average annual population growth rates for upper middle- and high-income countries since World War II. As we argue in section 9, however, our results are robust to alternative choices of both growth rates.

Finally, the evolution of consumption norms closely maps the development of the economy. To be precise, the growth rate of b_t is proportional to the rate of accumulation:

$$\frac{b_t - b_{t-1}}{b_{t-1}} = \phi \frac{\omega_{t-1} - \omega_{t-2}}{\omega_{t-2}}.$$

In this section, we assume that $\phi = 1$. We discuss the more general case in section 9 below.

The simulation occurs in the following order: (1) initialisation, $t = 1$; (2) b_t is updated; (3) N_t is updated; (4) $\widehat{w}_t$ is determined; (5) L_t and π_t are updated; (6) agents solve MP_t^ν; and (7) the sequence (2)-(6) is repeated for T, with $T = 500$.

The results are depicted in Figures 5-9(b). Figure 5 reports the same information as Figure 1 for the basic model. Some differences between Figures 1 and 5 emerge, notably concerning the dynamics of net output, wealth, and labour performed. One striking feature of the model is the extremely rapid convergence to a steady growth path. Thus, net output and wealth grow over the course of the simulation (as driven by y_t), but z_t remains fairly steady, starting at 12.5 and settling at 12.8894 after $t = 4$. This is because at all t, $z_t = L_t A^{-1}\omega_{t-1}$ and g_t quickly settles at a constant level such that

[19] The population growth rate is approximate because the population must be rounded to the nearest integer.

$1 + \frac{L_{t+1} - L_t}{L_t} = \frac{1}{1+g_t}$, all t. On the steady growth path the economy remains capital constrained. Therefore $\widehat{w}_t = b_t$, all t, and there is a secular increase in consumption (and wages) from $b_0 = 1.9$ to $b^{\max} = 46,801.9$, and, as expected from Lemma 2, π_t settles at $\pi_t = \pi_t^{\max} = g_t = 0.0204082$ for $t > 5$. Figure 6 shows the dynamics of A_t, L_t, and labour values.

As predicted, labour-saving technical change and population growth allow exploitation and classes to persist by maintaining capital scarcity relative to labour, and therefore a positive profit rate despite the increase in the consumption norm, b_t, and so in the equilibrium real wage rate.

Figures 7(a)-7(b) clearly show the persistence of exploitation and classes.[20] Indeed, they portray an extreme *polarisation* of exploitation and class status. As accumulation proceeds, propertyless agents arrive in the economy and drive the proportion of exploited agents and the share of agents in C_t^4 up, while the number of agents in C_t^1 and C_t^2 remains constant, and so their proportion decreases, together with the share of exploiters. Remarkably, the economy displays no class mobility: because $\widehat{w}_t = b_t$ for all t, agents who start in C_t^4 remain in C_t^4, and because L_t decreases and y_t^ν increases such that $L_t y_t^\nu$ remains constant, agents who begin in C_t^1 and C_t^2 also remain in their respective classes for all t. Figure 7(c) confirms that the CECP continues to hold in the more general economy considered here.

Figures 8(a)-8(b) provide information about the distribution of the exploitation intensity index e_t^ν which reinforces these conclusions. Given the significant growth in population over $T = 500$, Figure 8(a) shows the distribution of e_t^ν by *percentile* for all t. Agents with $e_t^\nu < 1$ can be seen during the first half of the simulation, but their numbers decline as the population grows and a consistently larger percentage of agents exhibit $e_t^\nu > 1$. Figure 8(b) shows, interestingly, that the Gini of e_t^ν decreases over time. This is not because exploitation is disappearing from the economy; it is an artifact of the way in which Gini coefficients are constructed. The ever-increasing number of agents with $\omega_{t-1}^\nu = 0$ leads to the vast majority of agents having identical, *high* values of e_t^ν, which lowers the Gini of e_t^ν. Given the clear pattern of increasing polarisation emerging from Figures 7(a)-7(b) and 8(a), this suggests that the Gini coefficient may not be the best index of the degree of exploitation in the economy, and it raises the interesting issue of the appropriate aggregate measure of exploitative relations.

The wealth distribution displays a similar pattern. Figure 9(a) shows that

[20] Unlike in the previous model, given the changing population over time, in this section (and in the next) the charts reporting class and exploitation status show the *percentage* of the population at all t belonging to a given class or exploitation classification.

Figure 5: Summary results - Model with exogenous technical change and population growth

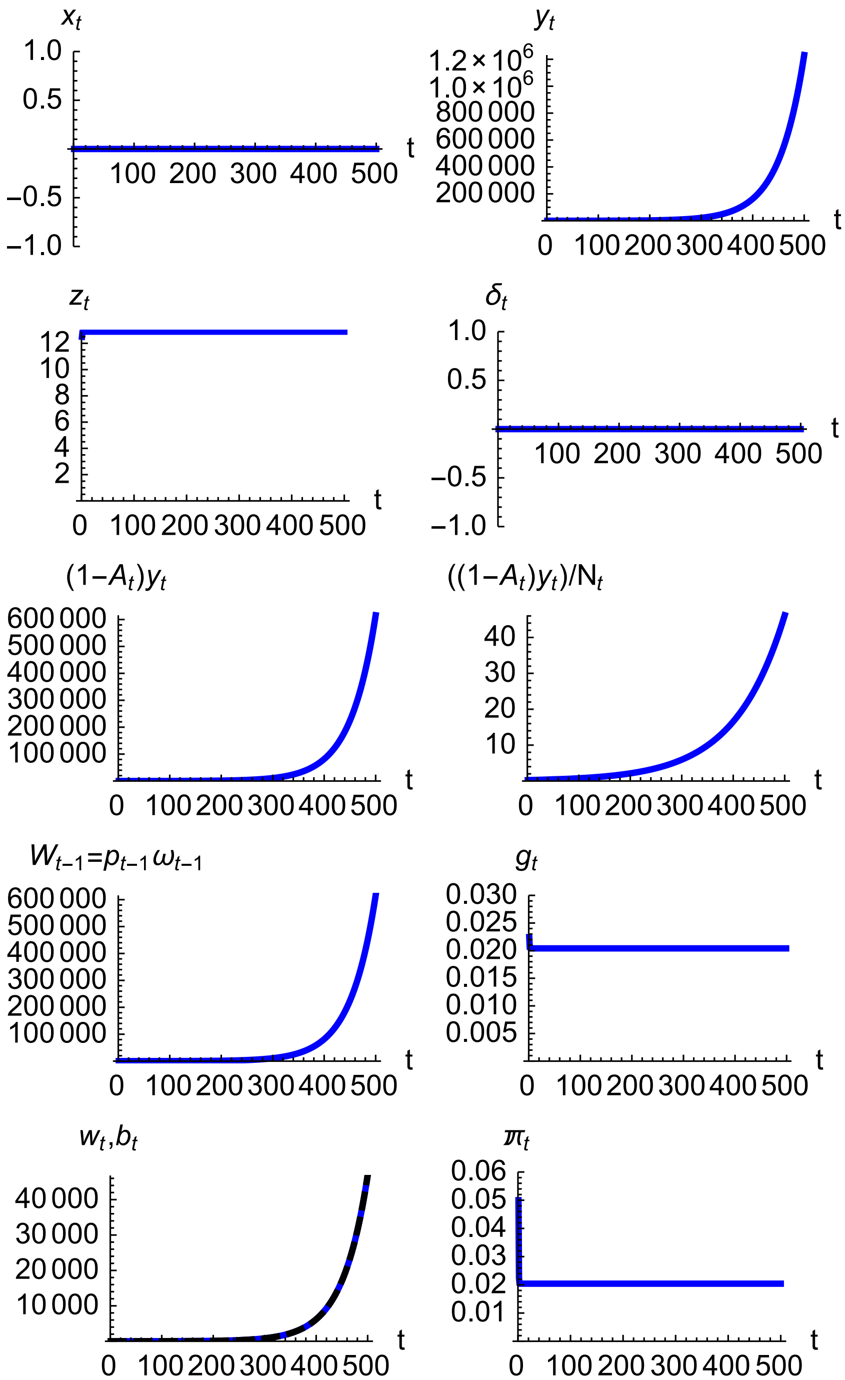
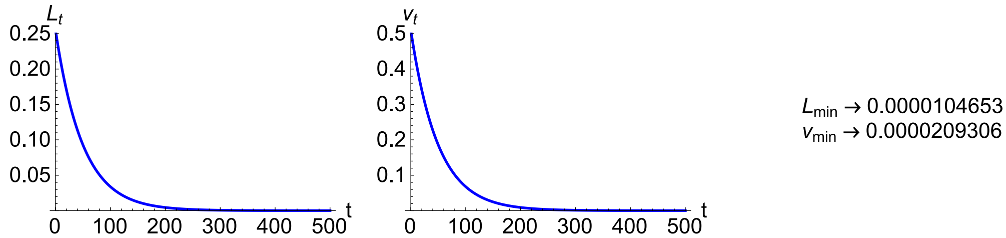


Figure 6: Technology and labour values - Model with exogenous technical change and population growth



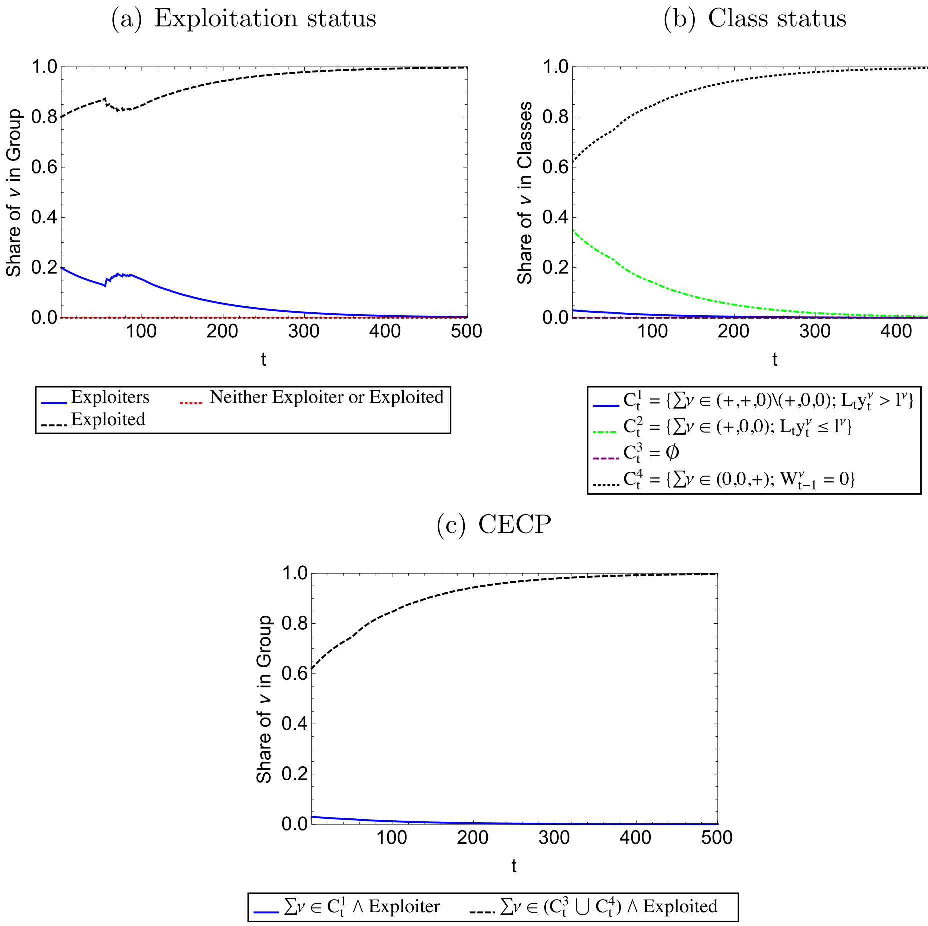
the Gini coefficient of wealth increases monotonically and asymptotically approaches one, with a final value of 0.999126. The extreme polarisation is also evident when looking at snapshots of the whole distribution of wealth for select t as in Figure 9(b). The severe initial wealth inequalities are exacerbated over time until the percentage of agents with $\omega_{t-1}^{\nu} > 0$ is barely visible.

In summary, the results support the claim that labour-saving technical change and population dynamics can contribute to explain the persistence of exploitation in accumulation economies (Skillman [20]). Their key role in this context is to make capital persistently scarce relative to labour. Labour unemployment puts downward pressure on the real wage, keeping it at the minimum socially acceptable level, and this implies that the vast majority of agents remain propertyless. Even though the economic conditions of those who have 'nothing to lose but their chains' improve over time, the class and exploitation structure of the economy remains invariant - indeed, the economy becomes increasingly polarised.

This is certainly an important insight but two important limitations of the model should be noted which suggest to interpret the results with caution. First, given the nature of capitalist economies, and certainly from a Marxian perspective, it is not entirely satisfactory to suppose technological progress to be completely independent from prices and profitability. For profit maximising behaviour leads capitalist firms to constantly search for cost-reducing innovations, especially if they face the pressure of rising wage costs. And a similar concern holds for population dynamics, which tends to be responsive to economic conditions, especially (but not exclusively) in the long run and if migration flows are taken into account.

Second, the results in this section are driven by a specific mechanism to determine distributive variables: population growth and technical change play a key role in creating labour unemployment which in turn forces the

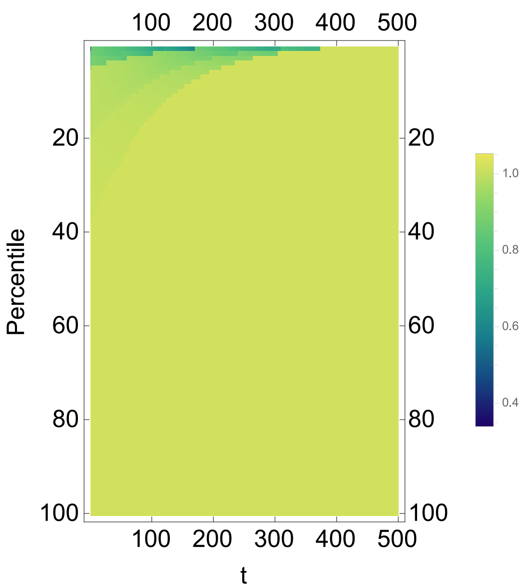
Figure 7: Class and exploitation status - Model with exogenous technical change and population growth



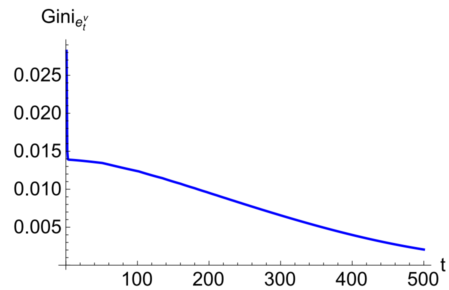
real wage down to the (socially determined and time-evolving) consumption norm. This would seem to suggest that labour unemployment is a necessary determinant of the persistence of exploitative relations. Yet it would be important, both normatively and theoretically, to analyse exploitation and class in economies with full employment. Marx's theory of exploitation does not crucially depend on the existence of unemployment and full employment does not make capitalist economies non-exploitative. Further, given the extremely skewed asset distribution, the assumption of a perfectly competitive labour market seems objectionable. With the richest portion of the population holding the vast majority of wealth and employing an increasing mass of propertyless agents, - and the issues of power and class solidarity that

Figure 8: Exploitation intensity index - Model with exogenous technical change and population growth

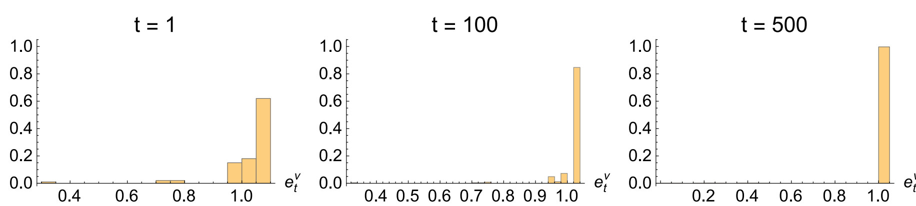
(a) Exploitation intensity across agents



(b) Gini coefficient of e_t^ν



(c) Distribution of e_t^ν for select t (relative frequency)

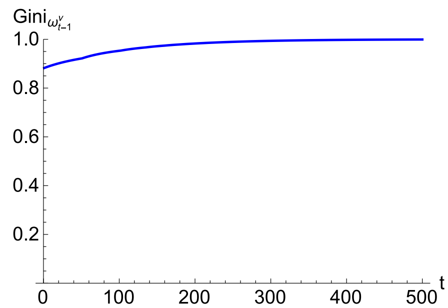


this polarised wealth distribution raises, - it seems natural to analyse a more complex model for the determination of the key distributive variables.

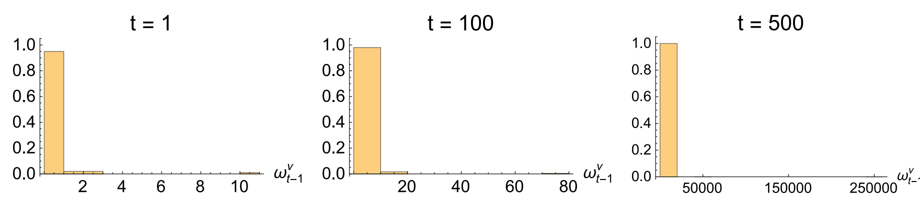
In the next section, we extend our analysis to explore the interplay of accumulation, population growth, and technical change in more general economies with full employment and generalised bargaining.

Figure 9: Distribution of wealth - Model with exogenous technical change and population growth

(a) Gini coefficient of wealth



(b) Distribution of wealth for select t (relative frequency)



8 Endogenous technical change with collective bargaining on wages and profits

In this section, we analyse exploitation and class in capitalist economies with technical change and population growth, by focusing on dynamic paths characterised by the full employment of labour. The key point to note is that whenever the economy is at the knife-edge, with $L_t A_t^{-1} \omega_{t-1} = l_t$, the equilibrium distribution is generally indeterminate.[21] This allows us to explore the effects of bargaining on exploitation and class.

[21] As pointed out by an anonymous referee, this exemplifies an indeterminacy that is generic to the sequential temporary equilibria of linear economies. As Mandler [13, 14] has shown, a linear sequence economy will often (and not by mere fluke) evolve toward factor proportions such that all resource constraints are binding, rendering the temporary equilibrium of its later periods indeterminate.

8.1 The bargaining model

Consider any period t such that $L_t A_t^{-1} \omega_{t-1} = l_t$. For given technology and population, we assume that income distribution is determined by collective bargaining at the beginning of t, before the production process starts. To be specific, given $p_t = 1$, the bargaining game $S(A_t, L_t)$ is defined as[22]

$$S(A_t, L_t) \equiv \cup_{\widehat{w}_t \in \left[b, \frac{1}{v_t}\right]} \left\{ (s_t^\nu)_{\nu \in \mathcal{N}_t} \mid s_t^\nu \leqq V_t^\nu \left(\omega_{t-1}^\nu; (1, \widehat{w}_t) \right) \quad (\forall \nu \in \mathcal{N}_t) \right\},$$

where $V_t^\nu \left(\omega_{t-1}^\nu; (1, \widehat{w}_t) \right) = (1 + \pi_t) \omega_{t-1}^\nu + (\widehat{w}_t - b) l_t^\nu$, all $\nu \in \mathcal{N}_t$, and so the set of Pareto-optimal payoffs is identified by varying $\widehat{w}_t$. Let $\sigma_t \equiv (\sigma_t^\nu)_{\nu \in \mathcal{N}_t} \in \mathbb{R}_+^{N_t}$ with $\sum_{\nu \in \mathcal{N}_t} \sigma_t^\nu = 1$ be a profile of weights capturing the agents' *bargaining power*. The Nash bargaining solution $N^{(\sigma_t)}(S(A_t, L_t))$ is:[23]

$$N^{(\sigma_t)}(S(A_t, L_t)) = \arg \max_{\widehat{w}_t \in \left[b, \frac{1}{v_t}\right]} \prod_{\nu \in \mathcal{N}_t} \left[V_t^\nu \left(\omega_{t-1}^\nu; (1, \widehat{w}_t) \right) \right]^{\sigma_t^\nu}.$$

There are many conceivable ways of capturing agents' bargaining power. Different specifications yield different dynamics of income distributions and exploitative relations. In what follows, we consider a large set of possibilities by specifying the bargaining power of each agent $\nu \in \mathcal{N}_t$ as follows:

$$\sigma_t^\nu \equiv (1 - \epsilon) \frac{\omega_{t-1}^\nu}{\omega_{t-1}} + \epsilon \frac{N_t^\nu}{N_t}, \text{ some } \epsilon \in [0, 1], \tag{5}$$

where $N_t^\nu \geq 1$ is the number of agents who possess the same wealth as ν. Formally, $N_t^\nu \equiv \# \left\{ \nu' \in \mathcal{N}_t \mid \omega_{t-1}^{\nu'} = \omega_{t-1}^\nu \right\}$.

Equation (5) incorporates some important insights of Marxian theory. For it states that the bargaining power of each agent $\nu \in \mathcal{N}_t$ depends partly on the ownership of means of production (more precisely, on their share of aggregate capital) and partly on the number of agents who share the same objective condition as ν (in terms of capital ownership). Thus, bargaining power derives either from economic resources, or from the ability of agents in a similar condition to act collectively.

Equation (5) is very general. The weight ϵ allows us to capture the polar cases of economies with weak solidarity and low levels of organisation

[22] In this section, we focus on the bargaining procedure and assume again a constant b.

[23] We adopt the Nash solution as a technically convenient reduced-form approach to the bargaining problem. As is common in the literature on non-cooperative implementation of bargaining solutions, the Nash solution should be understood as the equilibrium outcome of some underlying noncooperative bargaining procedure.

($\epsilon \approx 0$), and economies in which the power of capital and economic resources is mitigated by collective action ($\epsilon \approx 1$), as well as any intermediate scenarios. If $\epsilon = 0$, then the bargaining power of each agent is proportional to her share of aggregate capital: capitalists have all the bargaining power while propertyless agents have no influence on the determination of wages and profits. If $\epsilon = 1$, then - given a very skewed distribution of wealth - the richer segments of the population have virtually no bargaining power: propertyless agents constitute the majority of the population and play the main role in determining bargaining outcomes.[24]

Let $\widehat{w}_t^{N(\sigma_t)} \in N^{(\sigma_t)}(S(A_t, L_t))$: every $\nu \in \mathcal{N}_t$ solves MP_t^ν given the price vector $\left(1, \widehat{w}_t^{N(\sigma_t)}\right)$, and given $((A_t, L_t), b)$. Because $L_t A_t^{-1} \omega_{t-1} = l_t$, it is immediate to show that conditions (a)-(d) of Definition 1 are satisfied.

At the end of the period, both population and technological knowledge are updated.

8.2 Population

We assume that economic growth drives population growth - for example, by determining population flows in or out of the economy, or as part of a general Malthusian mechanism. Formally, in each t, if the economy is at a knife-edge equilibrium, then population in period $t+1$, l_{t+1}, will be

$$l_{t+1} = \left[(1 - bL_t) A_t^{-1}\right] l_t,$$

where $\left[(1 - bL_t) A_t^{-1}\right]$ corresponds to the growth rate of capital at t and can be interpreted, in a classical-Marxian fashion, as the *natural rate of population growth*. For it is reasonable to assume that in a situation of full utilisation of capital and labour, the growth rate of population adjusts to the rate of capital accumulation.[25]

[24] Although wealth influences bargaining power, equation (5) does not imply that wealthier agents' obtain higher payoffs. First, in general, bargaining power does not depend *only* on wealth and below we also consider scenarios in which wealth has no direct effect on bargaining power ($\epsilon = 1$). Second, wealth enters also the definition of an agent's bargaining group and N_t^ν, and given a skewed wealth distribution, N_t^ν (and thus bargaining power) is actually decreasing in wealth.

[25] If one allows for unemployed labour or capital, one could assume that if the economy is capital constrained at t, then $l_{t+1} < \left[(1 - bL_t) A_t^{-1}\right] l_t$; while if it is labour constrained then $l_{t+1} > \left[(1 - bL_t) A_t^{-1}\right] l_t$. In the former case, labour unemployment and a real wage rate equal to the minimum consumption standard, imply that labour supply grows slowly. In the latter case, a buoyant labour market and a high real wage, induce a high growth rate of the population.

8.3 Technical progress

We assume that when the profit rate falls below a certain threshold, capitalists increase their efforts to innovate and introduce new production techniques. Formally, let $\pi_t^{N(\sigma_t)} = \frac{1-A_t-\widehat{w}_t^{N(\sigma_t)}L_t}{A_t}$ be the profit rate resulting from the bargaining process. Recall that $\mathcal{P}_t$ denotes the set of Leontief production techniques (A_t, L_t) available to agents at t. Let (A', L') denote a new technique emerging from the innovation process. We assume that there exists a value of the profit rate $\pi^* \in \mathbb{R}_+$ that represents the capitalists' minimum profitability benchmark (which depends on economic, institutional and even cultural factors) such that if $\pi_t^{N(\sigma_t)} \geqq \pi^*$, then $\mathcal{P}_{t+1} = \mathcal{P}_t$, and no innovation occurs. If, instead, $\pi_t^{N(\sigma_t)} < \pi^*$ then capitalists redouble their innovation and R&D efforts, a new technique (A', L') is generated with probability $\lambda \leqq 1$ and $\mathcal{P}_{t+1} = \mathcal{P}_t \cup (A', L')$. The new technique (A', L') has $A' = g_A A_t$ and $L' = g_L L_t$, $g_A, g_L \in \mathbb{R}^2_+$, and is adopted in period $t+1$ only if it yields the maximum profit rate.

This formulation of technical progress is both theoretically appropriate and empirically reasonable. Theoretically, our model incorporates key insights from both classical-Marxian and evolutionary analyses of technical change, in that the innovation process is fundamentally *profit-driven*, and innovations are both *discontinuous* and *local*.

Technical progress is *profit-driven* because only profitable changes are adopted. This is a defining feature of the classical-Marxian framework, as Duménil and Levy [8] have argued, but it is also a key assumption in the Schumpeterian literature (see, for example, the classic papers by Nelson et al. [16] and Aghion and Howitt [1]). But the innovation process is linked to the trajectory of the profit rate also because significant declines in profitability spur innovation activities and thus tend to yield changes in production processes. This is consistent with standard Marxian insights, whereby "a declining profit rate will lead at some point to a structural crisis, and 'something' will happen with respect to technical change" (Duménil and Levy [8], p.203). But the Schumpeterian literature also emphasises the strongly countercyclical nature of R&D investments both theoretically (Aghion and Howitt [1]; Wälde [28]) and empirically (Aghion et al. [2]).[26]

The *discontinuous* nature of technical change incorporates a Schumpeterian view of innovation as a jerky process (Nelson et al. [16]; Wälde [28]). Formally, our modelling of technical change can be interpreted either as the reduced form - and limit point - of a more complex stochastic process whereby

[26]We thank Peter H. Matthews for alerting us to this literature.

the likelihood of (profitable) innovations increases with R&D efforts, and the latter increase as profitability decreases. But it can also be seen as incorporating satisficing behaviour conceptually analogous to that formalised by Nelson et al. [16] in their classic evolutionary model of technical change in which "Firms with positive capital in the current state retain the production technique of that state, with probability one, if their currently calculated gross return on capital exceeds 0.16. ... Firms that do not make a gross return of 0.16 undergo a probabilistic technique-change process." ([16], p.95).

We also follow the classical-Marxian and evolutionary literature in assuming that innovations are *local*: agents do not have a global scan of alternatives and search around existing processes (see, for example, Nelson et al. [16]; Duménil and Levy [8]). Therefore when innovations occur, they yield relatively small changes in technical coefficients. Consistently with this approach, we assume that the parameters g_A, g_L are relatively close to one.

In the simulations, we set $g_A = g_L = g < 1$. This simplifies the procedure by allowing us to focus on innovations that are unambiguously cost-reducing and are therefore adopted by all agents. Further, our assumptions on technical change and population growth imply that if the economy is at a full employment equilibrium in t, there will be neither excess capital nor excess labour in $t+1$. This should be interpreted as a schematic representation (due to the linear structure of the model) of a self-correcting mechanism, or as part of a balanced growth path. However, it is important to stress that, as discussed in section 9 below, all of our results are robust to alternative choices of the parameters - indeed, to altogether different specifications of technical progress.

8.4 Simulation Results

The simulation begins with the same $N_0 = 100$ and the same distribution of wealth as in all previous models, and we set $A_0 = 0.9$, $L_0 = 1$, and $b = 0.08$.

For the technical change routine the threshold profit rate is $\pi^* = 0.03$, $g_A = g_L = g$, and the likelihood of discovering a new technique when appropriate is $\lambda = 1$. When technical change takes place, the value of g is randomly drawn from a uniform distribution from 0.9 to 0.96. As concerns the bargaining parameter ϵ in equation (5), below we consider three different scenarios, namely the two polar cases with capitalist dominance ($\epsilon = 0$) and class solidarity ($\epsilon = 1$), and the intermediate case with $\epsilon \in (0,1)$.

The simulation occurs in the following order: (1) determine $(\sigma_t^\nu)_{\nu \in \mathcal{N}_t}$; (2) determine $\widehat{w}_t^{N(\sigma_t^\nu)}$ and $\pi_t^{N(\sigma_t)}$; (3) agents solve MP_t^ν; (4) determine population l_{t+1} to balance the knife-edge condition;[27] (5) update A_{t+1} and L_{t+1} if appropriate; (6) repeat steps (2)-(5) for T.

8.4.1 Capitalist dominance

Consider first the case where economic resources, and specifically ownership of the means of production, are the key determinant of bargaining power and $\epsilon = 0$. Figure 10 reports the summary results of the aggregate activity levels $(y_t, x_t, z_t, \delta_t)$, net output $(1 - A_t)y_t$, net output per capita $(1 - A_t)y_t/N_t$, wealth W_{t-1}, the growth rate of capital g_t, and the distributive variables $\left(\widehat{w}_t^{N(\sigma_t^\nu)}, \pi_t^{N(\sigma_t)}\right)$. Two features stand out. First, the economy rapidly settles on a steady growth path with a constant growth rate equal to the profit rate and constant net output per capita. Second, the bargaining power of capitalists is such as to guarantee that the profit rate remains positive and at a rather high level, while $\widehat{w}_t^{N(\sigma_t^\nu)} = b$ for all t, *even though the economy never becomes capital constrained.* Figure 11 displays A_t, L_t, and v_t.

Figures 12-13(b) describe the exploitation and class status of agents. They show a clear pattern of increasing polarisation in the economy. As time goes by, an ever increasing proportion of the agents are exploited by an ever decreasing minority of exploiters. The fraction of agents in the lower classes constantly rises while that of agents in the upper classes falls. As in section 7.1, and for similar reasons, the Gini coefficient of the exploitation index displays a marked downward trend and tends to zero as the simulation proceeds. This reinforces the doubts on its use as an index of the aggregate degree of exploitation, at least in a dynamic context with variable population.

Figure 14 reports the Gini coefficient of wealth and the distribution of wealth for select t, which also show a pattern of increasing inequality.

8.4.2 Class solidarity

Consider next the case where bargaining power is determined solely by class solidarity and $\epsilon = 1$. The results are presented in Figures 15 and 16. Accumulation proceeds at a fast and accelerating pace. Given their numerical preponderance, the power of class solidarity in wage bargaining favours

[27] In order to ensure that $l_t = L_t A_t^{-1} \omega_{t-1}$ holds for all t, N_t is determined by rounding $L_t A_t^{-1} \omega_{t-1}$ up to the nearest integer. Therefore ω_{t-1} is also adjusted upward to maintain the knife-edge. Any additional capital is added to the endowment of the wealthiest agent in order to avoid random changes in the behaviour of the simulation. Any new agents enter the simulation with $\omega_{t-1}^\nu = 0$. This is reasonable as a first approximation, or if one interprets population growth mainly as the product of migration flows.

Figure 10: Summary results - Bargaining model with $\epsilon = 0$

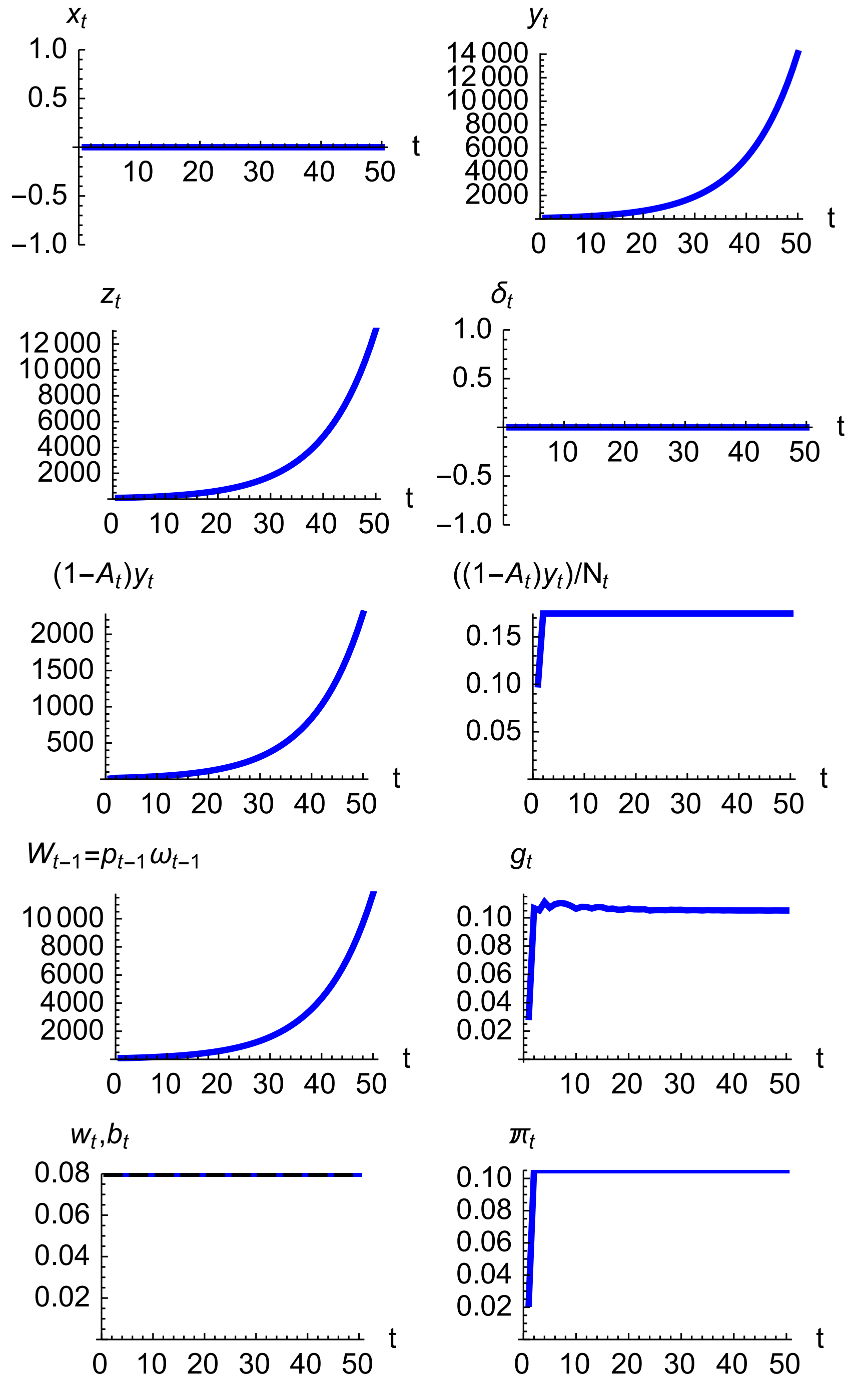
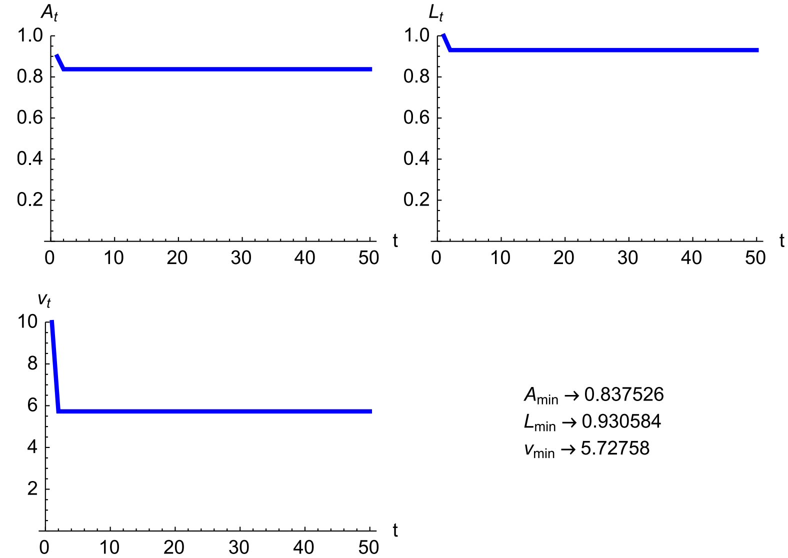


Figure 11: Technology and labour values - Bargaining model with $\epsilon = 0$



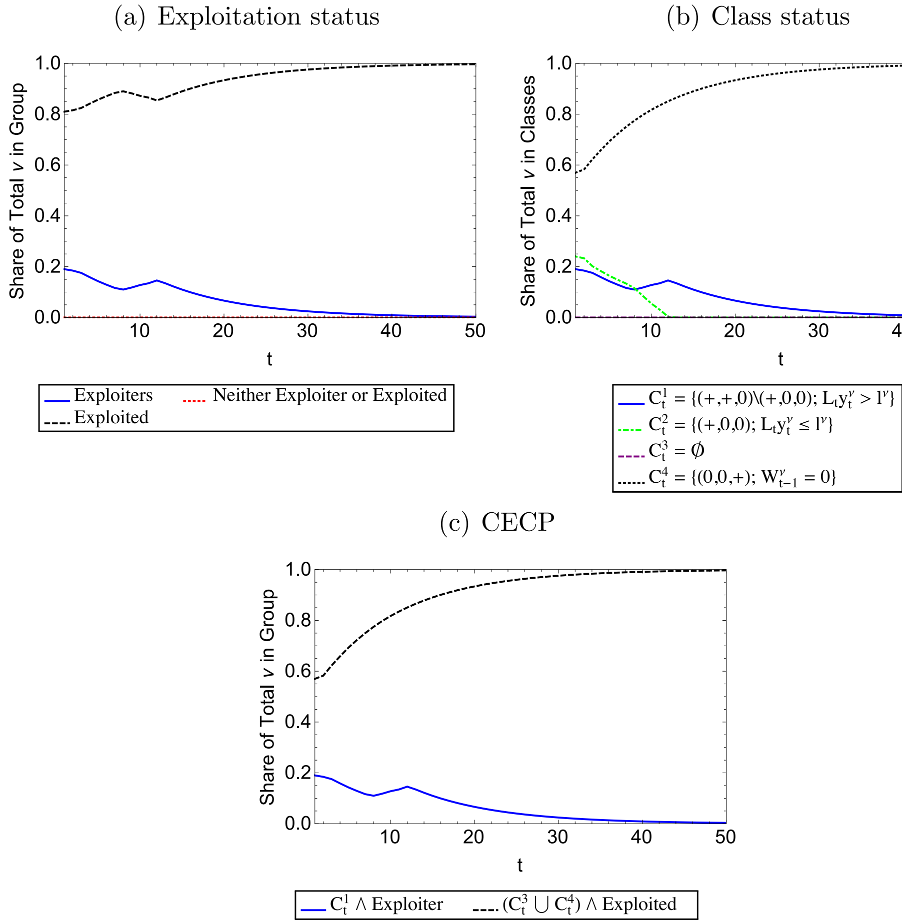
propertyless agents who set $\widehat{w}_t^{N(\sigma_t^\nu)} = \frac{1}{v_t}$ and $\pi_t^{N(\sigma_t)} = 0$ for all $t \geq 1$. Thus, working class solidarity immediately eliminates exploitation and the benefits of accumulation (and productivity increases) are distributed in the form of increased wages. The figures reporting exploitation and class status are not shown as they do not convey much information. The Gini coefficient of $(e_t^\nu)_{\nu \in \mathcal{N}_t}$ is obviously equal to zero throughout the simulation.

The Gini coefficient of wealth displays an interesting behaviour over time and provides a rather different picture, compared to the Gini of $(e_t^\nu)_{\nu \in \mathcal{N}_t}$. It initially decreases, reflecting the reduction in inequalities due to the zero profit rate and the accumulation of wealth by all agents (since $\widehat{w}_t > b$ all t). This trend is then reversed as growth accelerates and the number of new, propertyless agents joining the economy in each t grows closer to (and eventually surpasses) the number of existing agents with a positive (and possibly quite large) amount of wealth accumulated over time.

8.4.3 Mixed scenarios

In this section, we consider economies in which $\epsilon \in (0, 1)$ and the bargaining power of agents derives *both* from the ownership of the means of production *and* from class solidarity and collective action. The fundamental, and rather striking result of the computational analysis is that class solidarity plays a dominant role in the bargaining process and tends to drive the dynamics of

Figure 12: Class and exploitation status - Bargaining model with $\epsilon = 0$

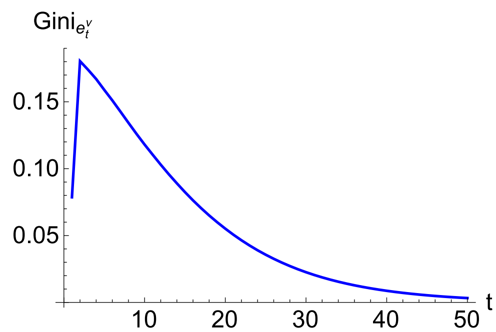


the economy. For instance, for any $\epsilon \geq 0.5$, the economy behaves in a manner qualitatively similar to the model with $\epsilon = 1$: the bargaining process leads to $\widehat{w}_t^{N(\sigma_t^\nu)} = \frac{1}{v_t}$ and $\pi_t^{N(\sigma_t)} = 0$ for all t.

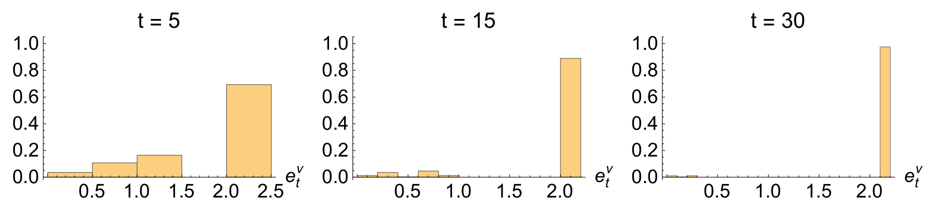
Only very small values of ϵ - in the range of 0.001 to 0.005 - allow for behaviour along the knife-edge at which $\pi_t^{N(\sigma_t)} > 0$ and $\widehat{w}_t^{N(\sigma_t^\nu)} > b$, but only for a small number of periods. For $\epsilon \in [0.001, 0.005]$, the economy displays a very interesting cyclical behaviour: the bargaining power of propertyless agents drives profits initially to zero but, once technical change occurs, the profit rate starts to increase. However, as population also grows, the number of new (propertyless) agents added in every t is large and their class solidarity begins to outweigh the power of relatively wealthy agents. This leads the profit rate to decrease and even though - when $\pi_t^{N(\sigma_t)}$ falls below π^* - other

Figure 13: Information on e_t^ν - Bargaining model with $\epsilon = 0$

(a) Gini coefficient of e_t^ν



(b) Distribution of e_t^ν for select t (relative frequency)



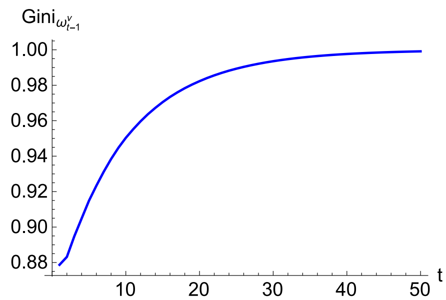
technical innovations are introduced, they only allow for a short-lived recovery of profitability. The economy then enters a phase during which profits and exploitation disappear for any remaining t.

Below, we show the results of the simulation for $\epsilon = 0.002$. Figure 18 reports the summary results of the main aggregate and distributive variables. As in the previous case, the combination of population growth and wage bargaining leads to fast and accelerating accumulation. The bottom right panel captures the cycle in the profit rate described above, where $\pi_t^{N(\sigma_t)} = 0$ for all $t \geq 12$. Figure 19 displays A_t, L_t, and v_t.

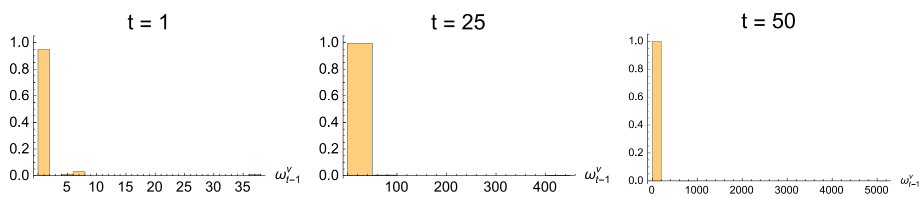
Figures 20(a)-20(c) show the exploitation and class status of agents. Figure 20(a) captures the lack of exploitation at the start of the simulation, its reemergence for a while, and its eventual disappearance. Figure 20(b) reports the dynamics of classes, which is richer than in previous models. Whenever $\widehat{w}_t^{N(\sigma_t^\nu)} > b$, all agents in C_t^4 accumulate and join C_{t+1}^2, and are replaced in C_{t+1}^4 by the newly arrived agents with $\omega_t^\nu = 0$. Hence, the economy shows some upward mobility. As accumulation progresses, wage bargaining mediates the movement of some agents between C_t^1 and C_t^2, but the continued

Figure 14: Distribution of wealth - Bargaining model with $\epsilon = 0$

(a) Gini coefficient of wealth



(b) Distribution of wealth for select t (relative frequency)



expansion of C_t^4 and the faster pace of accumulation induce a downward trend in the size of C_t^2 and an upward trend in that of C_t^1. The growth of C_t^4 (despite upward social mobility) reflects the arrival of an increasingly larger number of propertyless agents. Figure 20(c) captures the short period of time for which capitalists are able to convert their ownership of the means of production into economic advantage and the CECP applies, after which the correspondence between class and exploitation status breaks down.

Figures 21(a) shows the Gini coefficient for e_t^{ν}, which mirrors the cycle in the profit rate with exploitation intensity eventually converging to one for all agents once $\pi_t^{N(\sigma_t)} = 0$. Snapshots of the distribution of e_t^{ν} for select t are provided in Figure 21(b) to show the progression of exploitation intensity to uniformity as accumulation progresses. The interaction between varying distributional patterns and the changing composition of the population over time leads to a pattern of the Gini coefficient for wealth (Figure 22(a)) that is on the whole qualitatively similar to that of the model with class solidarity. The basic intuition is the same and also reflected in the distribution of wealth for select t shown in Figure 22(b).

Figure 15: Summary results - Bargaining model with $\epsilon = 1$

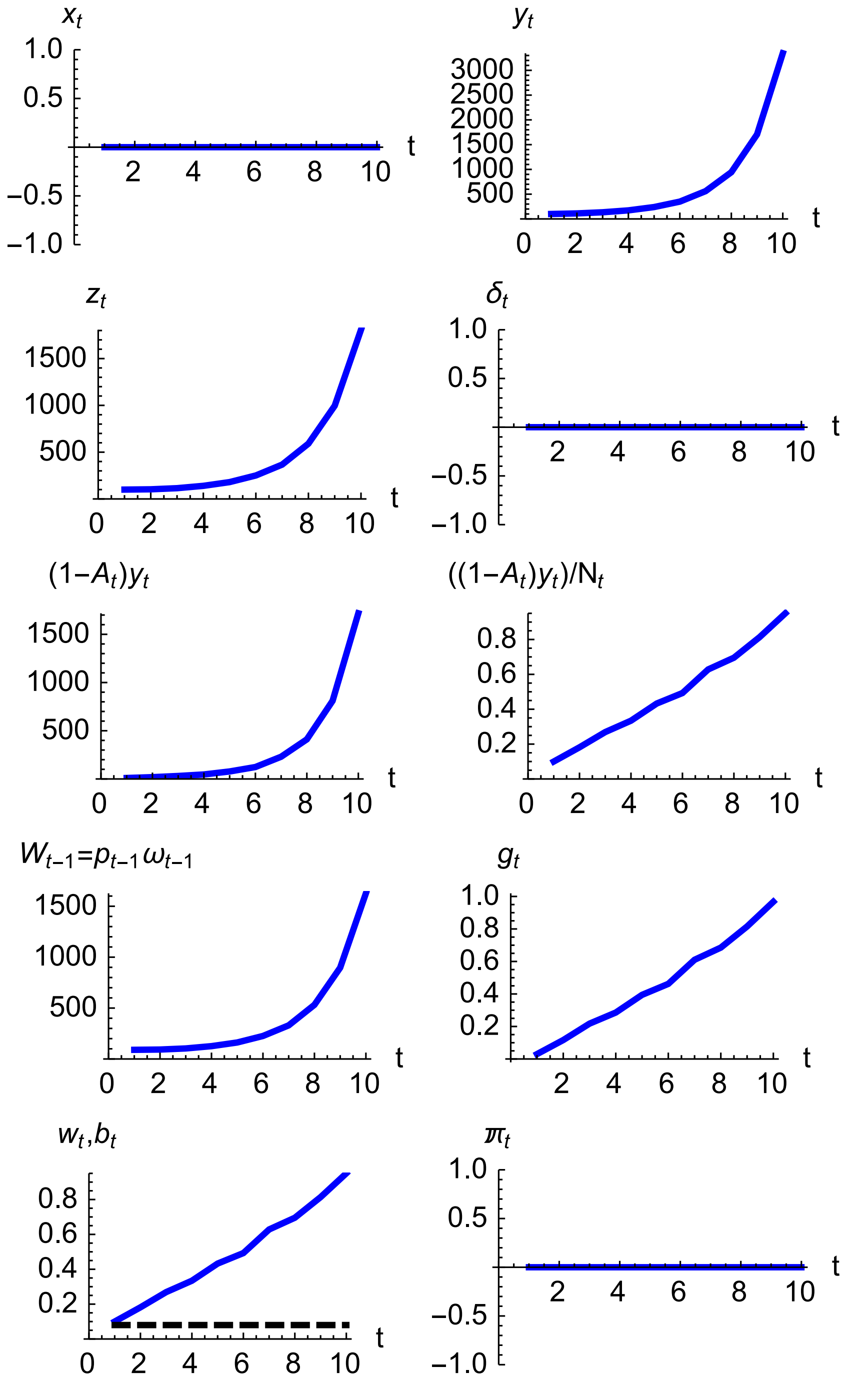
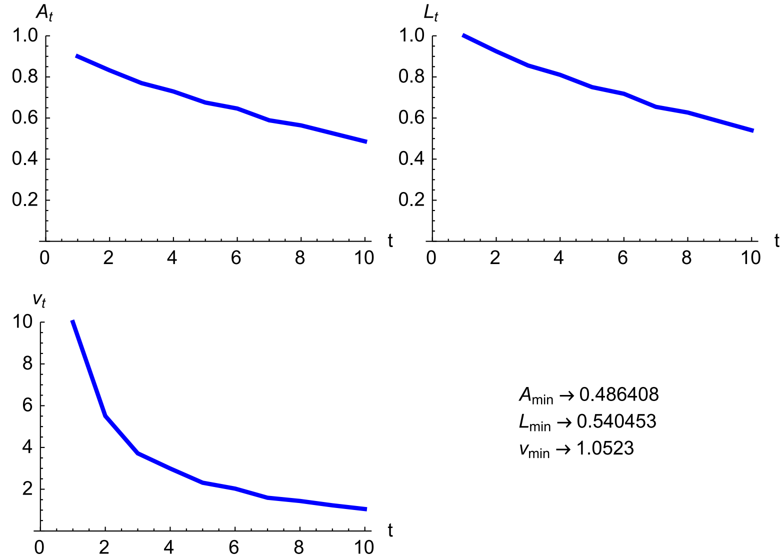


Figure 16: Technology and labour values - Bargaining model with $\epsilon = 1$



In closing this section, it is worth noting that a comparison of the dynamic paths of the three models suggests that the class solidarity regime is best and the capitalist dominance regime is worst in terms of several indicators, including growth rates, capital accumulation, labour productivity, and per capita consumption. Of course, this may depend on the exact specification of our model. Yet, this insight is fundamentally consistent with the fact that capitalist economies with a strong concentration of power in the hands of capitalists tend to settle on dynamic paths with high profit rates and low growth rates (at least relative to their potential), as forcefully shown by Piketty [17]. Indeed, in terms of our model, the trajectory of advanced capitalist economies in the last forty years (the neo-liberal era) might be explained as the product of a shift from a kind of mixed regime with a strong working class to a capitalist dominance regime.

9 Robustness

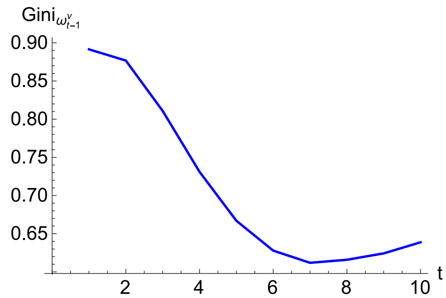
We have analysed many variations of our models in order to assess the robustness of our results. In this section, we briefly summarise the main points.[28]

First, we have considered alternative specifications of the initial values of

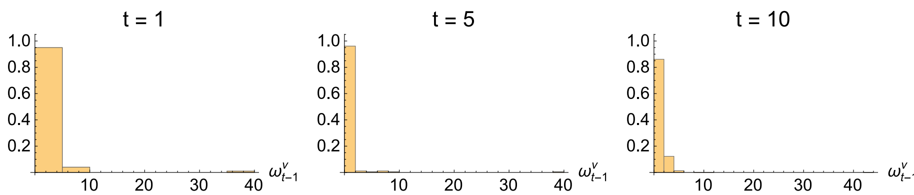
[28]Results are available from the authors upon request. (See the additional results reported in sections 2-5 of the Addendum.)

Figure 17: Distribution of wealth - Bargaining model with $\epsilon = 1$

(a) Gini coefficient of wealth



(b) Distribution of wealth for select t (relative frequency)



the key parameters of all models. Different values of the initial population, $\mathcal{N}_0$, technology, (A_0, L_0), consumption bundle, b_0, and aggregate capital, ω_0, make hardly any difference to our conclusions.[29] Different distributions of the initial aggregate capital, ω_0, also have no impact on the results, provided capital is unequally distributed and there are some propertyless agents.

Changes in these parameters may affect the quantitative features of the economies (e.g., the size of the various classes, the period in which the economy becomes labour constrained, and so on) but the qualitative patterns of class and exploitation status, as well as the basic summary results, remain essentially unchanged. It is worth noting, however, that different values of the initial parameters yield different distributions of e_t^{ν}. For example, a simulation of the basic economy with the same population and asset distribution as in section 6, and $(A = 0.5, L = 0.25, b = 0.5)$ yields a distribution of e_t^{ν} in the range [0.03306, 4] with a Gini coefficient of 0.26568 - quite close to the

[29] Subject, obviously, to the economy remaining capital constrained or on the knife edge, depending on the model.

Figure 18: Summary results - Bargaining model with $\epsilon = 0.002$

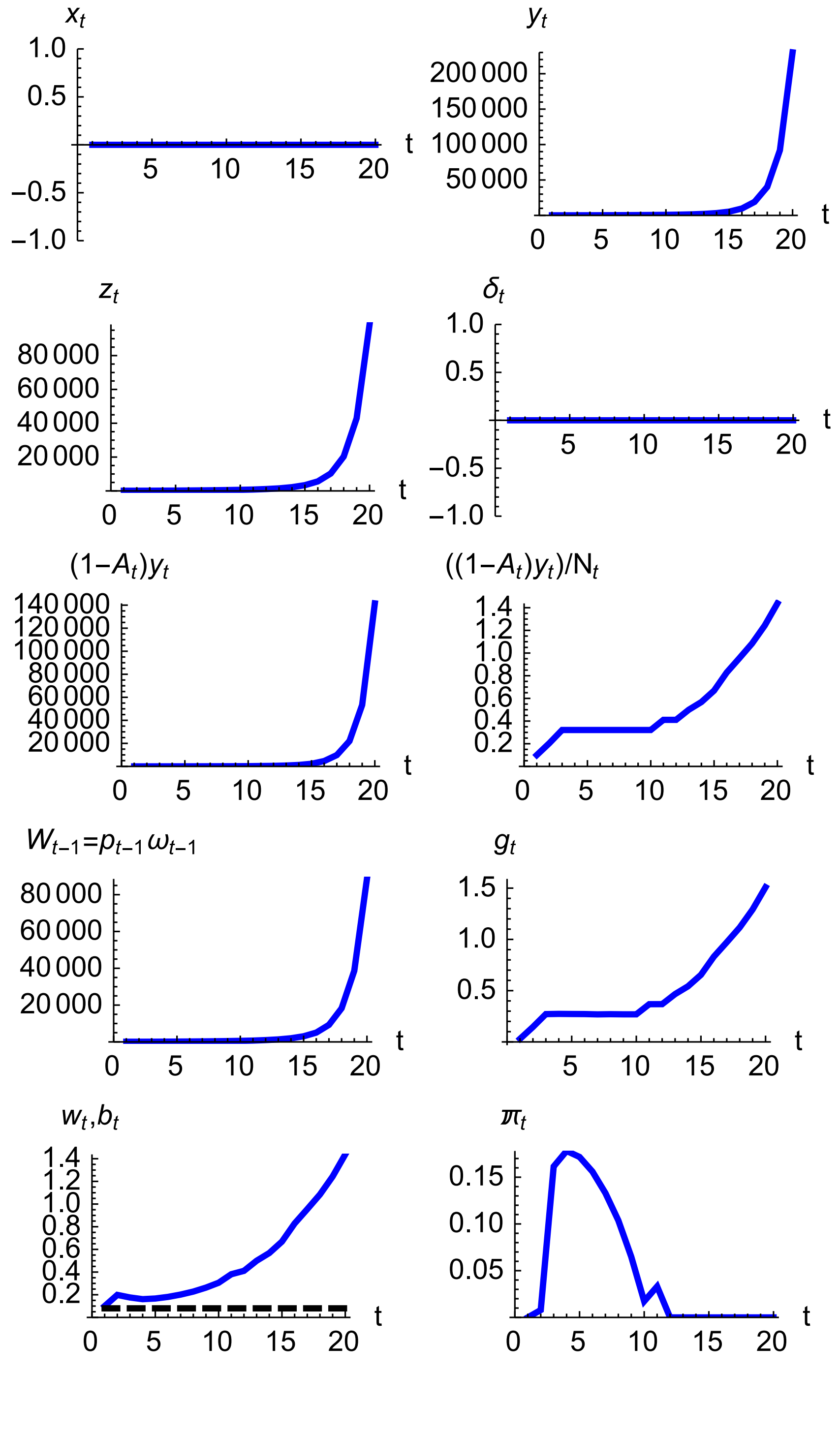
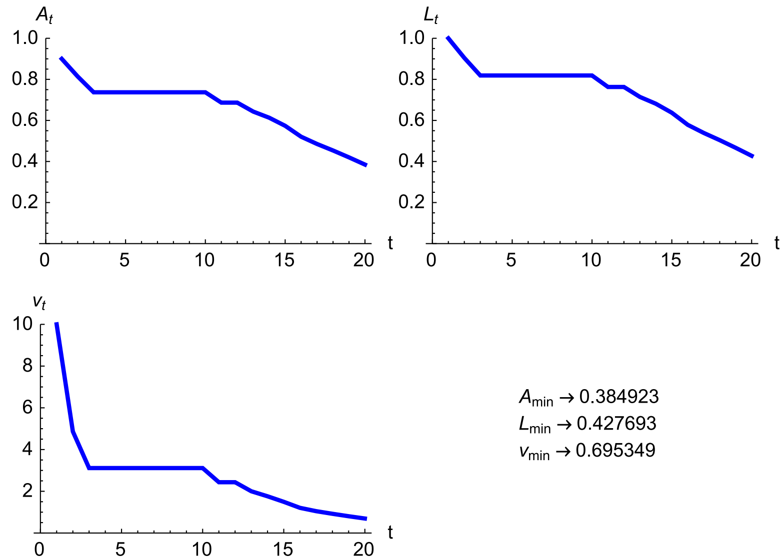


Figure 19: Technology and labour values - Bargaining model with $\epsilon = 0.002$

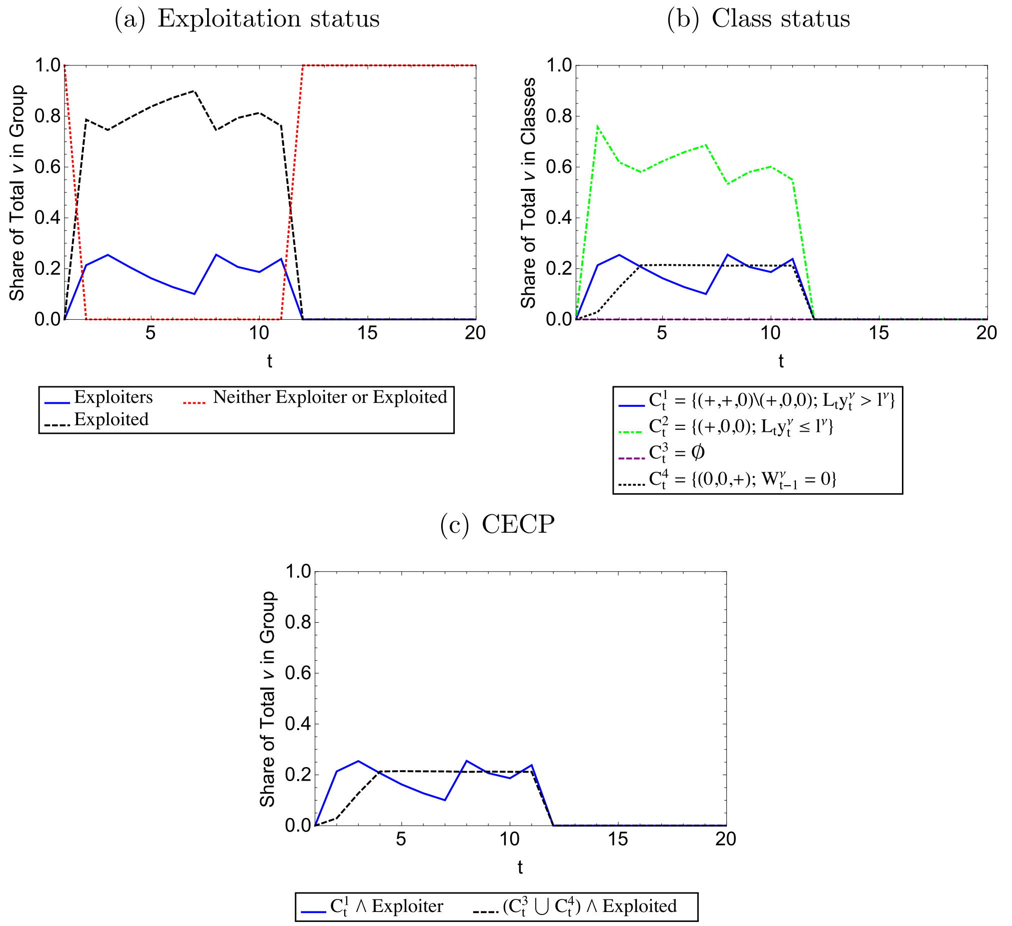


Gini coefficient of income in actual capitalist economies.

Second, we have analysed the robustness of the main results on class structure and on the relation between exploitation and class status by adopting an alternative definition of classes. Corollary 1 proves that class status is determined for each agent by the difference between actual labour demand and *potential* labour supply - i.e., the agent's labour endowment. Yet one may argue that class status should be determined based on agents' *actual* position in the labour market, and so on their actual labour supply. All of our main insights continue to hold if this alternative approach is adopted.

Third, the results in section 7 are also strongly robust to alternative specifications of the model and for a wide range of the key parameters, including the growth rates of technical progress and population. None of the key insights change if one considers economies characterised by either technical progress or population growth only; or if consumption norms depend on other state variables, such as labour productivity, rather than wealth; or indeed, if b_t remains constant. Nor does the introduction of endogenous capital-using labour-saving technical progress change the main conclusions, but the economy displays an interesting phenomenon of persistent "exploitation cycles". As accumulation progresses with a given technique (A_t, L_t), exploitation tends to decrease as e_t^ν tends to 1 for all ν. However, when a new technique is introduced, profitability and inequality in exploitation intensity are restored, driving a wedge between the lower and upper classes,

Figure 20: Class and exploitation status - Bargaining model with $\epsilon = 0.002$

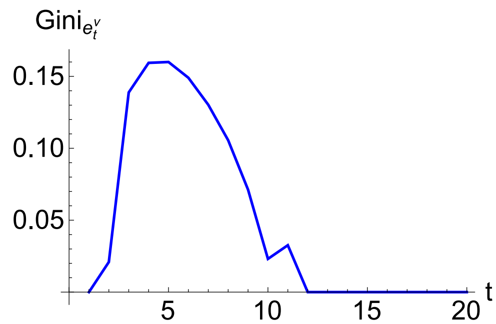


and the pattern of accumulation and exploitation resumes until another production technique is introduced.

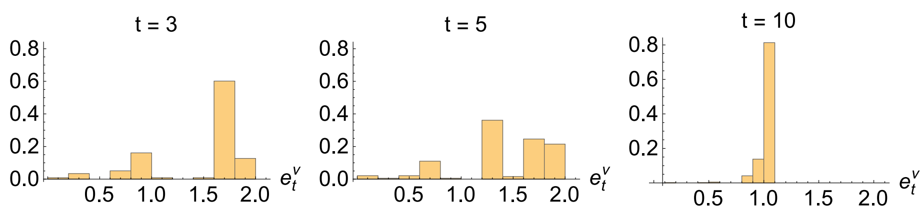
Fourth, section 8 already reports the results for a large set of values of the bargaining parameter ϵ. But the key insights are robust to changes in various other features of the model. For example, the main results remain essentially unchanged if we relax the assumption that the likelihood of discovering a new technique when appropriate is exactly equal to one; or if we allow technical change to take place at a faster pace by increasing the range of g; or even if we allow for more general nonnegative values of g_A, g_L. Indeed, the main results are robust to alternative specifications of the innovation process. For example, the introduction of exogenous labour-saving technical change along the lines of section 7 makes virtually no difference.

Figure 21: Information on e_t^ν - Bargaining model with $\epsilon = 0.002$

(a) Gini coefficient of e_t^ν



(b) Distribution of e_t^ν for select t (relative frequency)

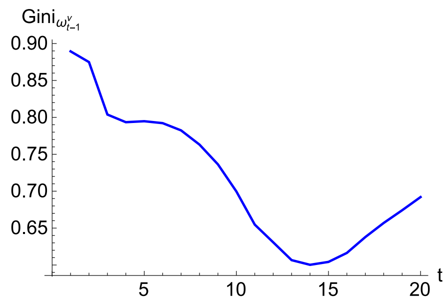


Perhaps more interestingly, in equation (5), instead of defining N_t^ν as the number of agents who possess *exactly* the same amount of wealth as ν, one may adopt a broader categorisation and define N_t^ν as the number of agents with wealth *within the same interval* as ν, or who belong to the *same class* as ν. These alternative specifications do not change the results significantly given the rather unequal wealth distribution, and given our specification of population growth. For in all of these scenarios the bargaining power of the poorest segments of the working class remains unchanged, and it increases over time as more propertyless agents appear in the economy.

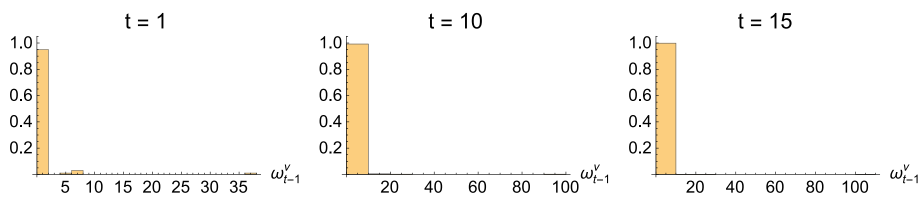
Finally, the key insight on the fundamental relevance of capitalist power for the persistence of exploitation in economies characterised by full employment remains valid even if one changes both the structure of bargaining *and* the innovation process. To be specific, we have analysed models with labour-saving technical progress and consumption co-evolving with aggregate capital so as to maintain the economy at the knife-edge and three different ways of determining the real wage. First, we suppose that the bargaining power of

Figure 22: Distribution of wealth - Bargaining model with $\epsilon = 0.002$

(a) Gini coefficient of wealth



(b) Distribution of wealth for select t (relative frequency)



the two classes is constant and $\widehat{w}_t$ is simply set at the midpoint between b_t and the maximum value $\frac{1}{v_t}$ corresponding to $\pi_t = 0$. Next, we capture the uncertainty that usually characterises real bargaining processes by assuming that in every t, $\widehat{w}_t$ is randomly selected within the interval $[b_t, \frac{1}{v_t}]$. In the third scenario, we incorporate the intuition that a more buoyant economy increases workers' bargaining power and assume that the real wage is increasing in the rate of accumulation, g_t.[30] In all scenarios, exploitation decreases over time and tends to disappear in the long run. Interestingly, however, the mechanism driving this result is different from that in section 8: the profit rate quickly converges to the level consistent with a steady growth path and remains persistently positive, but since $\widehat{w}_t > b_t$, then consistently with Lemma 2, poorer agents accumulate faster than wealthier ones, so that wealth levels converge, eliminating the wealth differences necessary to exploitation.[31]

[30]Formally, in every period t, $\widehat{w}_t = k_t(1/v_t) + (1 - k_t)b_t$ where $k_t = g_t/(1 + g_t)$.

[31]We are grateful to an anonymous referee for suggesting to explore this mechanism.

10 Conclusions

This paper analyses the equilibrium dynamics of exploitation and class in general accumulation economies with population growth, technical change, and bargaining by adopting a novel computational approach. Two sets of results emerge from the analysis. First, in capitalist economies characterised by a drive to accumulate, labour-saving technical change plays a key role in guaranteeing the persistence of exploitation and class by making labour persistently abundant relative to capital. Nonetheless, and perhaps strikingly, in competitive economies characterised by full employment and bargaining over distribution, labour-saving technical change and population growth are not sufficient to generate persistent class and exploitative relations *even if the economy never becomes labour constrained*. The model forcefully highlights the importance of power, and specifically bargaining power, as one of the possible determinants of the persistence of classes and exploitation. It is only when economic resources, and specifically the ownership of the means of production, give a significant advantage in distributive conflict - compared to class solidarity among propertyless agents - that exploitative relations persist over time as an equilibrium feature of capitalist economies.

Second, far from being metaphysical, the concept of exploitation provides the foundations for a logically coherent and empirically relevant analysis of inequalities and class relations in advanced capitalist economies. An index that identifies the exploitation level, or intensity of each individual, e_t^ν, can be defined and its empirical distribution studied using the standard tools of the theory of inequality measurement. In our analysis, we have focused on the Gini coefficient for illustrative purposes as it is one of the most widely used indices of dispersion, but - as our simulations show - the Gini coefficient does not necessarily convey clear normative information about the distribution of e_t^ν. An interesting open question concerns precisely the appropriate aggregate index of the intensity of exploitation.[32]

It would be interesting to extend our analysis to economies with n goods, more general technologies, and even more complex bargaining procedures. A particularly intriguing question, for example, concerns the relation between power and technical change, and their joint relevance for class and exploitation. For the use of economic resources in distributive conflict is only one dimension of capitalist power. Control over the means of production implies that capitalists can also determine - at least to some extent - the direction and

[32]The answer to this question depends on the normative insights that the notion exploitation is meant to capture. This issue is analysed in [26].

nature of technical change. In line with the recent literature on power-biased technical change (e.g., Guy and Skott [12]), one could analyse innovations that allow capitalists - either directly or indirectly via changes in the structure of production - to alter power relations, for example, by affecting class solidarity and increasing the relative weight of economic resources, ϵ.[33]

Another important question concerns the implications of heterogeneity in labour skills and in consumption/leisure trade-offs. On the one hand, as already noted, the assumption that agents have identical skills and preferences, and perform the same amount of labour leads to less extreme inequalities in exploitation intensity than are observed in empirical income or wealth distributions. We interpret our results as identifying a lower bound to exploitative relations, and as such they seem quite remarkable. But it would be interesting to analyse the effects of heterogeneity on the distribution of $(e_t^\nu)_{\nu \in \mathcal{N}_t}$.

On the other hand, as is well known, the introduction of heterogeneous labour poses major problems to the standard definition of labour embodied v_t. In this respect, it is important to stress that if the approach developed in Veneziani and Yoshihara [27, 25] is adopted, then the definition of the exploitation intensity index can be extended to economies with n goods, general technologies, and heterogeneous labour, preferences and skills, and exploitative relations can be analysed empirically based on available data.

Finally, in section 8, we have analysed the Nash solution as a technically convenient, reduced-form approach to the bargaining problem. Following the literature on non-cooperative implementation of bargaining solutions, the Nash solution should be seen as the equilibrium outcome of some (unspecified) underlying non-cooperative bargaining procedure. This also provides a justification for our focus on a bargaining-theoretic determination of distribution only at the knife-edge: we interpret our analysis as a metaphor of the effects of changes in the bargaining power of workers induced by labour market conditions. Our model in section 8 can be understood as a special case of a more general bargaining model in which unemployment drives workers' bargaining power to zero. From this perspective, the wage rate is always determined via some (noncooperative) bargaining (both in and outside of the knife-edge), but when labour is abundant bargaining yields a wage rate equal to the competitive equilibrium level. It would therefore be quite interesting to extend our analysis by explicitly considering an underlying n-agent non-cooperative bargaining problem in which workers' bargaining power is a more general (possibly, smoothly decreasing) function of the unemployment

[33] Or, perhaps more subtly, using a divide-and-conquer strategy that changes the relevant N_t^ν for propertyless workers.

rate.

Although it does not provide answers to these questions, this paper provides a conceptual and analytical framework to tackle them.

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