Non-Gaussianity in multiple three-form field inflation

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In this work, we present a method for implementing the δN formalism to study the primordial non-Gaussianity produced in multiple three-form field inflation. Using a dual description relating three-form fields to non-canonical scalar fields, and employing existing results, we produce expressions for the bispectrum of the curvature perturbation in terms of three-form quantities. We study the bispectrum generated in a two three-form field inflationary scenario for a particular potential which for suitable values of the parameters was found in earlier work to give values of the spectral index and ratio of tensor to scalar perturbations compatible with current bounds. We calculate the reduced bispectrum for this model, finding an amplitude in equilateral and orthogonal configurations of $\mathcal{O}(1)$ and in the squeezed limit of $\mathcal{O}(10^{-3})$. We confirm, therefore, that this three-form inflationary scenario is compatible with present observational constraints.

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I. INTRODUCTION

Inflation is a successful paradigm which solves the horizon and flatness problems [1]. Measurements of the Cosmic Microwave Background (CMB) anisotropies confirm that the primordial density perturbations are close to scale invariant, adiabatic and Gaussian [2–4]. This is as expected from the simplest models of inflation and confirms inflation as our favoured theory for the origin of structure. The results from Plank 2015 [2], and the BICEP2/Keck Array and Planck joint analysis [5] also severely constrain the amplitude of gravitational waves produced by inflation, with the latest bounds on the tilt of the scalar power spectrum (n_s) and the tensor to scalar ratio (r) given by [3]

$$n_s = 0.968 \pm 0.006 \quad \text{and} \quad r_{0.002} < 0.09 \quad \text{at } 95\% \text{ CL}\,. \tag{1}$$

Despite its observational successes, however, there are a considerable variety of different models of inflation which can be motivated theoretically [6, 7]. These include multifield models, and models with non-canonical scalar fields. The degeneracy of the predictions from various models of inflation is an ongoing problem for cosmologists. One way to probe further the nature of inflation, is to study the statistics of the perturbations it produces beyond the two point correlation function [8], starting with the three-point function. This is parametrised in Fourier space by the bipsectrum [9–14], a function of the amplitude of three wavevectors that sum to zero as a consequence of momentum conservation. Although most canonical single field models of inflation produce an unobservably small bispectrum, multi-field and non-canonical models in particular can produce levels in tension with present or detectable by future probes. The former produces a bispectrum of close to the “local shape". This is a function of three wavenumbers which peaks in the squeezed limit where two wavenumbers are much larger than the third. The latter produces a bispectrum of “equilateral shape" which tends to zero in the squeezed limit, but peaks when all three wavenumbers are similar in size (see e.g. [14, 15] for reviews). A third shape is often considered which peaks on folded triangles, where two wavenumbers are approximately half of the third, and can be produced by models with non Bunch-Davis initial conditions [14]. Planck 2015 has put constraints on these shapes. Introducing three parameters $f_{\rm NL}^{\rm loc}$, $f_{\rm NL}^{\rm equi}$

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and $f_{\rm NL}^{\rm ortho}$, which parametrise the overall amplitude of a local, equilateral and orthonormal shape template for the bispectrum, Planck 2015 tells us that

$$f_{\rm NL}^{\rm local} = 0.8 \pm 5.0\,, \quad f_{\rm NL}^{\rm equi} = -4 \pm 43\,, \quad f_{\rm NL}^{\rm ortho} = -26 \pm 21, \tag{2}$$

at 68% CL (see Ref. [16]). These bounds are stringent, though it is too early to exclude non-canonical or multi-field inflationary models by means of Gaussianity (see e.g. [17–19].)

Despite the success of inflation driven by scalar fields, three-forms provide a viable alternative (and a viable model of dark energy) [20–27]. In this article we therefore study how to calculate the bispectrum in any multiple three-form inflationary scenario. To do so we develop a method to adapt the δN formalism [28–32] to the three-form setting. We then calculate the bispectrum generated in a concrete model with two three-forms. Inflationary scenarios with two three-forms were proposed in [26], and shown under a suitable choice of the three-form potential and initial conditions to satisfy the Planck data concerning the power spectrum and tensor to scalar ratio. Here we compute the bispectrum for the successful example considered in that paper and check it is also consistent with the latest observational constraints.

The plan of the paper is as follows. In section II we briefly summarize the $\mathbb{N}$ three-forms inflationary model studied in Ref. [26]. Subsequently, in section III we discuss the bispectrum and describe a procedure to adapt the δN formalism [33] to multiple three-forms to calculate it. We explain a numerical method for calculating derivatives of the unperturbed number of e-foldings with respect to the unperturbed three-form field values at sound horizon crossing, and show how these derivatives can be related to those of a dual scalar field description. In turn these can be used in combination with existing results to compute the bispectrum. We stress that although our method utilises the dual scalar field description, it is not possible in general to simply pass to that description and work solely with a scalar field model. In section IV we consider an explicit example from Ref. [26] which provides a power-spectrum compatible with Planck constraints and compute the bispectrum in that model. We quantify and compare the momentum dependent contribution and momentum independent contributions of the reduced bispectrum and plot the shape of the bispectrum. We conclude in section V.

II. MULTIPLE THREE-FORM MODEL

In this section, we briefly present the model with $\mathbb{N}$ three-form fields introduced in [26]. We take a flat Friedmann-Lemaï¿œtre-Robertson-Walker (FLRW) cosmology, described with the metric $ds^2 = -dt^2 + a^2(t)d\boldsymbol{x}^2$, where $a(t)$ is the scale factor with t cosmic time. The general action for two three-form fields minimally coupled to Einstein gravity can be written as[1]

$$S = -\int d^4x\sqrt{-g}\left[\frac{1}{2}R - \sum_{I=1}^{\mathbb{N}}\left(\frac{1}{48}F_I^2 + V(A_I^2)\right)\right]\,, \tag{3}$$

where $A^{(I)}_{\beta\gamma\delta}$ is the $I^{\rm th}$ three-form field and squared quantities indicate contraction of all the indices. The strength tensor of the three-form is given by[2]

$$F^{(I)}_{\alpha\beta\gamma\delta} \equiv 4\nabla_{[\alpha}A^{(I)}_{\beta\gamma\delta]}, \tag{4}$$

where antisymmetrisation is denoted by square brackets. As we have assumed a homogeneous and isotropic universe, the three-form fields depend only on time and hence only the space like components will be dynamical. Therefore the nonzero components are given by [21]

$$A^{(I)}_{ijk} = a^3(t)\epsilon_{ijk}\chi_I(t) \quad \Rightarrow A_I^2 = 6\chi_I^2, \tag{5}$$

where $\chi_I(t)$ is a comoving field associated to the nth three-form field and ϵ_{ijk} is the standard three dimensional Levi-Civita symbol.

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[1] We work in the units of Planck mass $M_{\rm Pl} = 1$.

[2] Throughout this letter, the Latin index I will be used to refer the number of the quantity (e.g., the three-form field) or the $I^{\rm th}$ quantity/field. The other Latin indices, which take the values $i, j = 1, 2, 3...$, will indicate the three dimensional quantities; whereas the Greek indices will be used to denote four-dimensional quantities and they stand for $\mu, \nu = 0, 1, 2, 3$.

In general, any p-form in $d-$dimensions has a dual of $(d-p)-$form [22, 24]. In our case three-form field (A) and its field tensor 4-form (F) are dual to a vector and a scalar field respectively which can be expressed as [24]

$$F_{\mu\nu\rho\sigma} = -\epsilon_{\mu\nu\rho\sigma}\phi, \qquad A_{\mu\nu\rho} = \epsilon_{\alpha\mu\nu\rho}B^{\alpha}\,, \tag{6}$$

where $\epsilon_{\mu\nu\rho\sigma}$is an antisymmetric tensor.

The corresponding action for the scalar field dual representation of the $\mathbb{N}$ three-forms is [24, 26]

$$S = -\int d^4x\sqrt{-g}\left[\frac{1}{2}R + P\left(X, \phi_I\right)\right]\,, \tag{7}$$

where

$$P\left(X, \phi_I\right) = \sum_{I=1}^{\mathbb{N}}\left(\chi_I V_{I,\chi_I} - V\left(\chi_I\right) - \frac{\phi_I^2}{2}\right)\,, \tag{8}$$

with $X = -\frac{1}{2}G^{IJ}\left(\phi\right)\partial_\mu\phi_I\partial^\mu\phi_J$. In this model the field metric is $G^{IJ}\left(\phi\right) = \delta^{IJ}$, therefore we have $X = \sum X_I$. The three-from fields still present on the right hand side of Eq. (8) should be viewed as functions of the kinetic terms X_I though the inverse of the relation

$$X_I = \frac{1}{2}V_{,\chi_I}^2\,. \tag{9}$$

Considering the many studies of non-canonical scalar fields in cosmology, it might be tempting to think that given a three-form theory the best way to proceed would be to simply pass to the dual scalar field theory and work solely with scalar field quantities. Beginning from a set of massive three-form fields, however, it is not analytically tractable to write the dual scalar field theory except for very particular potentials. This is because of the difficulty in inverting Eq. (9). Yet, in a similar manner to that advocated in Ref. [24] for the single field case, we will see that we can still make use of the dual theory indirectly.

For a background unperturbed FLRW cosmology, we can use the dualities defined in Eq. (6) to write the following relation between a three-form field and its dual scalar field

$$\phi_I = \dot{\chi}_I + 3H\chi_I. \tag{10}$$

Moreover from action (3) the background Klein-Gordon equations for the $\mathbb{N}$ three-form fields read

$$\ddot{\chi}_I + 3H\dot{\chi}_I + 3\dot{H}\chi_I + V_{,\chi_I} = 0. \tag{11}$$

The Friedmann equations are

$$H^2 = \frac{1}{6}\left[\sum_{I=1}^{\mathbb{N}}\left(\dot{\chi}_I + 3H\chi_I\right)^2 + 2V\right]\,, \tag{12}$$

and

$$\dot{H} = -\frac{1}{2}\left[\sum_{I=1}^{\mathbb{N}}V_{,\chi_I}\chi_I\right]\,. \tag{13}$$

We express the field equations of motion (11) in terms of e-folding time, $N = \ln a(t)$, as

$$H^2\chi_I'' + \left(3H^2 + \dot{H}\right)\chi_I' + 3\dot{H}\chi_I + V_{,\chi_I} = 0\,, \tag{14}$$

where $\chi_I' \equiv d\chi_I/dN$. And the Hubble parameter from Eq. (12) can be expressed as

$$H^2 = \frac{V\left(\chi_I\right)}{3\left(1 - \sum_I w_I^2\right)}\,. \tag{15}$$

where $w_I = \frac{\chi_I' + 3\chi_I}{\sqrt{6}}$.

In subsequent sections, our strategy in using the duality to calculate non-Gaussianities will be to use equations derived for multiple scalar fields, but write the quantities involved in terms of three-form fields. In particular, we will need the following derivatives for two three-forms which we compute here for later use

$$P_{,X} \equiv \sum_I P_{,X_I} = \sum_I P_{,\chi_I}\left(\frac{\partial \chi_I}{\partial X_I}\right) = \sum_I \frac{\chi_I}{V_{\chi_I}} . \tag{16}$$

And similarly

$$P_{,X_I X_I} = \frac{1}{V_{,\chi_I\chi_I} V_{,\chi_I}^2} - \frac{\chi_I}{V_{,\chi_I}^3} . \tag{17}$$

$$P_{,X_I X_I X_I} = -\frac{V_{,\chi_I\chi_I\chi_I}}{V_{,\chi_I\chi_I}^3 V_{,\chi_I}^2} + \frac{3\chi_I}{V_{,\chi_I}^5} - \frac{3}{V_{,\chi_I}^4 V_{,\chi_I\chi_I}} . \tag{18}$$

$$P_{,I} = -\phi_I = -\sqrt{6} H w_I . \tag{19}$$

III. NON-GAUSSIANITY AND THE δN FORMALISM

A. The δN formalism

The δN formalism is based on the separate universe assumption [28–32, 34] and provides a powerful tool to evaluate the superhorizon evolution of the curvature perturbation. In the case of multiple three-forms, however, the direct implementation of the δN formalism would be cumbersome. Using the formal relation between three-forms and their scalar field duals [24, 26], however, one can indirectly implement the δN formalism while still employing only three-form quantities that are easy to calculate.

The δN formalism allows the evolution of the curvature perturbation to be calculated, on scales larger than the horizon scale where one can neglect spatial gradients, using only the evolution of unperturbed "separate universes". The central result is that the difference in the number of e-folds that occurs from different positions on an initial flat slice of space-time to a final uniform density slice, when compared with some fiducial value, is related to the curvature perturbation. Writing the number of e-foldings as a function of the initial and final time on the relevant hypersurfaces,

$$N\left(t,\, t_i,\, x\right) = \int_{t_i}^{t} dt' H\left(t',\, x\right) , \tag{20}$$

the primordial curvature perturbation can be expressed as

$$\zeta\left(t, x\right) = N\left(t,\, t_i,\, x\right) - N_0\left(t,\, t_i\right) , \tag{21}$$

where $N_0\left(t,\, t_i\right) = \int_{t_i}^{t} dt' H_0\left(t'\right)$. Taking $t_i = t_*$, the time corresponding to the modes exiting the horizon ($kc_s = aH$), the curvature perturbation on superhorizon scales can be written in terms of partial derivatives of N with respect to the unperturbed scalar field values at horizon exit, while holding the initial and final hypersurface constant. More precisely

$$\zeta\left(t,\, x\right) = \sum_I N_{,I}\left(t\right) \delta\phi_*^I(x) + \frac{1}{2}\sum_{IJ} N_{,IJ}\left(t\right) \delta\phi_*^I\left(x\right) \delta\phi_*^J\left(x\right) + \cdots , \tag{22}$$

where $N_{,I} = \frac{\partial N}{\partial \phi_I^*}$. In momentum space we have

$$\zeta(k) = N_{,I}\delta\phi_*^I(k) + \frac{1}{2} N_{,IJ}\left[\delta\phi_*^I \star \delta\phi_*^J\right](k) + \cdots , \tag{23}$$

where $\star$ indicates a convolution.

B. The bispectrum

In Fourier space the two and three point functions are defined, respectively, by

$$\langle \zeta\left(\mathbf{k_1}\right) \zeta\left(\mathbf{k_2}\right)\rangle = \left(2\pi\right)^3 \delta^3\left(\mathbf{k_1} + \mathbf{k_2}\right) P_\zeta\left(k_1\right) , \tag{24}$$

$$\langle \zeta\left(\mathbf{k_1}\right) \zeta\left(\mathbf{k_2}\right) \zeta\left(\mathbf{k_3}\right)\rangle = \left(2\pi\right)^3 \delta^3\left(\mathbf{k_1} + \mathbf{k_2} + \mathbf{k_3}\right) \mathcal{B}_\zeta\left(k_1, k_2, k_3\right) , \tag{25}$$

where $P_\zeta(k)$ is the power spectrum, and $B_\zeta(k_1, k_2, k_3)$ the bispectrum. Often the bispectrum is normalised to form the reduced bispectrum $f_{\rm NL}(k_1, k_2, k_3)$

$$B_\zeta(k_1, k_2, k_3) = \frac{6}{5} f_{\rm NL}(k_1, k_2, k_3) \Big[P_\zeta(k_1) P_\zeta(k_2) + P_\zeta(k_2) P_\zeta(k_3) + P_\zeta(k_3) P_\zeta(k_1) \Big] , \tag{26}$$

C. Calculating the bispectrum with δN

The power spectrum and bispectrum of field fluctuations at horizon crossing follow from the two and three-point correlations of these perturbations as

$$\langle \delta\phi^I_*(\mathbf{k_1}) \delta\phi^J_*(\mathbf{k_2}) \rangle = (2\pi)^3 G^{IJ} \frac{2\pi^2}{k^3} \mathcal{P}^* \delta(\mathbf{k_1} + \mathbf{k_2}) \tag{27}$$

$$\langle \delta\phi^I_*(\mathbf{k_1}) \delta\phi^J_*(\mathbf{k_2}) \delta\phi^K_*(\mathbf{k_3}) \rangle = (2\pi)^3 \frac{4\pi^4}{\Pi_i k_i^3} \mathcal{P}^{*2} A^{IJK}(k_1, k_2, k_3) \delta(\mathbf{k_1} + \mathbf{k_2} + \mathbf{k_2}) , \tag{28}$$

where $\mathcal{P} = Pk^3/(2\pi^2)$. Employing the δN expansion one finds that

$$P_\zeta(k) = N_I N_I P^* \tag{29}$$

and

$$f_{\rm NL} = f^{(3)}_{\rm NL} + f^{(4)}_{\rm NL} + \cdots , \tag{30}$$

where

$$\begin{aligned} f^{(3)}_{\rm NL} &= \frac{5}{6} \frac{N_{,I} N_{,J} N_{,K} A^{IJK}}{\left(G^{IJ} N_{,I} N_{,J}\right)^2 \sum_i k_i^3}, \\ f^{(4)}_{\rm NL} &= \frac{5}{6} \frac{G^{IK} G^{JL} N_{,I} N_{,J} N_{,KL}}{\left(G^{IJ} N_{,I} N_{,J}\right)^2}, \end{aligned} \tag{31}$$

Here $f^{(3)}_{\rm NL}$ is momentum dependent, whereas $f^{(4)}_{\rm NL}$ is momentum independent (which is the definition of local $f_{\rm NL}$) [3]. In general, the dominant contribution, $f^{(3)}_{\rm NL}$ or $f^{(4)}_{\rm NL}$, is model dependent. For example, in the case of multiple canonical scalar fields inflation, $f^{(4)}_{\rm NL}$ can become significant . In contrast, for non-canonical models, $f^{(3)}_{\rm NL}$ can become large.

For general multifield non-canonical models in slow-roll (which is the situation relevant to our models), utilising the In-In formalism to calculate the statistics of the scalar field perturbations on flat hypersurfaces at horizon crossing it was found that

$$P_* = \frac{H^2}{2k^3 P_{,X}}, \tag{32}$$

and that [37]

$$A_{IJK} = \frac{1}{4} \sqrt{\frac{P_{,X}}{2}} \tilde{A}_{IJK}, \tag{33}$$

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[3] Technically these results are valid only when there is not a large hierarchy between the three wavenumbers of the bispectrum and they can all be assumed to cross the horizon at roughly the same time. This provides a good approximation even for large hierarchies as long as there is not a significant evolution between the horizon crossing times of the three modes (see Refs. [35, 36] for a full discussion)

with

$$
\begin{aligned}
\tilde{A}^{IJK} =& G^{IJ}\epsilon^{K}\frac{u}{\epsilon}\left[\frac{4k_1^2k_2^2k_3^2}{K^3} - 2\left(\mathbf{k_1}.\mathbf{k_2}\right)k_3^2\left(\frac{1}{K}+\frac{k_1+k_2}{K^2}+\frac{2k_1k_2}{K^3}\right)\right] \\
& - G^{IJ}\epsilon^{K}\left[6\frac{k_1^2k_2^2}{K}+2\frac{k_1^2k_2^2\left(k_3+2k_2\right)}{K^2}+k_3k_2^2-k_3^3\right] \\
& + G^{IJ}\left[\left(3\frac{u}{\epsilon}+4u+4\right)\tilde{\epsilon}^{K}+\tilde{\epsilon}^{K}_{,X}\frac{12H^2}{P_{,X}}\right]\times \\
& \left[-\frac{k_1^2k_2^2}{K}-\frac{k_1^2k_2^2k_3}{K^2}+\left(\mathbf{k_1}.\mathbf{k_2}\right)\left(-K+\frac{\sum\limits_{i>j}k_ik_j}{K}+\frac{k_1k_2k_3}{K^2}\right)\right] \\
& + \frac{\epsilon^{IJ}}{\epsilon}\epsilon^{K}\left(\frac{2\lambda}{H^2\epsilon^2}-\frac{u}{\epsilon}\right)\frac{4k_1^2k_2^2k_3^2}{K^3} + \text{perms.}\,,
\end{aligned}
\tag{34}
$$

where $K = k_1 + k_2 + k_3$, and the Hubble parameter H, the sound speed squared $\left(c_s^2\right)$, and slow-roll parameters $\left(\epsilon, \epsilon^I, ..., \text{etc.}\right)$ are evaluated at sound horizon exit $c_s k = aH$. Expressions for c_s^2, u and λ are given in Ref. [37] for non-Canonical models[4]. In this work, we express all of these parameters in terms of three-form quantities using Eqs. (8) and (10). First u is defined as

$$
u \equiv \frac{1}{c_s^2} - 1\,,
\tag{35}
$$

where the effective speed sound is given by

$$
c_s^2 = \frac{P_{,X}}{2XP_{,XX}+P_{,X}} = \frac{\sum\limits_I \frac{\chi_I}{V_{,\chi_I}}}{\sum\limits_I V^{-1}_{,\chi_I\chi_I}}\,.
\tag{36}
$$

We also define λ, such that

$$
\lambda = X^2P_{,XX} + \frac{2}{3}X^3P_{,XXX} = -\sum_I \frac{V^3_{,\chi_I}V_{\chi_I\chi_I\chi_I}}{12V^3_{,\chi_I\chi_I}}\,.
\tag{37}
$$

The various slow-roll quantities are defined by

$$
\epsilon \equiv -\frac{\dot{H}}{H^2} = \frac{3}{2}\frac{\sum\limits_I \chi_I V_{,\chi_I}}{V}\left(1-\sum_I w_I^2\right)\,,
\tag{38}
$$

$$
\epsilon^{IJ} = \frac{P_{,X}\dot{\phi}^I\dot{\phi}^J}{2H^2} = \frac{P_{,X}\sqrt{X_IX_J}}{2H^2} = \epsilon^I\epsilon^J\,,
\tag{39}
$$

where

$$
\epsilon^I = \sqrt{\frac{X_IP_{,X}}{2H^2}} = \sqrt{\frac{3V^2_{,\chi_I}}{4V}\left(\sum_I\frac{\chi_I}{V_{,\chi_I}}\right)\left(1-\sum_I w_I^2\right)}\,,
\tag{40}
$$

$$
\tilde{\epsilon}_I = -\frac{P_{,I}}{3\sqrt{2P_{,X}}H^2} = \frac{\sqrt{6}w_I}{3\sqrt{2\sum\limits_I\frac{\chi_I}{V_{,\chi_I}}}H}\,.
\tag{41}
$$

[4] We have corrected typos in the first and third lines of Eq. (34) that were present in Ref. [37].

Using the Friedmann equation in Eq. (12) we obtain

$$\begin{aligned}\tilde{\epsilon}^{I}_{,X} &= -\frac{P_{,XI}}{3\sqrt{2P_{,X}}H^2} + P_{,I}\left[\frac{2XP_{,XX}+P_{,X}}{9\sqrt{2P_{,X}}H^4} + \frac{P_{,XX}}{6\sqrt{2}P_{,X}^{3/2}H^2}\right], \\ &= -\sqrt{6}Hw_I\left[\frac{\sum_I V^{-1}_{,\chi_I\chi_I}}{\sqrt{2\sum_I \frac{\chi_I}{V_{,\chi_I}}V}} + \frac{\sum_I\left(V^{-1}_{,\chi_I\chi_I}V^{-2}_{,\chi_I} - \chi_I V^{-3}_{,\chi_I}\right)}{3\sqrt{2}\left(\sum_I \frac{\chi_I}{V_{,\chi_I}}\right)^{3/2}V}\right]\left(1-\sum_I w_I^2\right).\end{aligned} \tag{42}$$

Note that the dual scalar field action in Eq. (8) satisfies $P_{,XI}=0$.

In the squeezed limit i.e., $k_2 \to 0$, it can be seen from Eq. (34) that $f_{\rm NL}^{(3)}$ reduces to the order of slow-roll parameters. Therefore $f_{\rm NL}^{(4)}$ is expected to be dominant in this limit if non-Gaussianity is significant.

D. The δN for two three-forms

The crucial step, when it comes to computing $f_{\rm NL}$, is the calculation of the derivatives of N with respect to the fields at the sound horizon crossing. In general $N_{,I}$ and $N_{,IJ}$ evolve on superhorizon scales and except in a few cases (see e.g Ref [38]) the analytical computation of these quantities is not tractable. For this reason we do our computations numerically using a method that will be explained in section IV.

First of all we must rewrite the derivatives in terms of three-forms. Here we do this explicitly for two three-forms. The same procedure can be extended trivially to $\mathbb{N}$ three-form fields. We can infer the following relations from Eqs. (10) and (15) relating two three-forms to the two non-canonical scalar fields

$$\phi_1 = \sqrt{6}Hw_1 \equiv \phi_1\left(\chi_1,\chi_2,w_1,w_2\right), \tag{43}$$

$$\phi_2 = \sqrt{6}Hw_2 \equiv \phi_2\left(\chi_1,\chi_2,w_1,w_2\right), \tag{44}$$

It is highly non-trivial to invert the relations in Eqs. (43) and (44). While the fields are slowly rolling, one can verify that the approximation $w_I \approx \sqrt{\frac{3}{2}}\chi_I$ is accurately satisfied (see Ref.[26]). As a consequence, we express the N derivatives $N_{,I}$ and $N_{,IJ}$ in terms of the two three-forms χ_1, χ_2 as

$$\frac{\partial N}{\partial\phi_1^*} = \frac{\partial N}{\partial\chi_1^*}\frac{\partial\chi_1^*}{\partial\phi_1^*} + \frac{\partial N}{\partial\chi_2^*}\frac{\partial\chi_2^*}{\partial\phi_1^*}, \tag{45}$$

$$\begin{aligned}\frac{\partial^2 N}{\partial\phi_1^*\partial\phi_2^*} =& \frac{\partial N}{\partial\chi_1^*}\frac{\partial^2\chi_1^*}{\partial\phi_1^*\partial\phi_2^*} + \frac{\partial N}{\partial\chi_2^*}\frac{\partial^2\chi_2^*}{\partial\phi_1^*\partial\phi_2^*} + \frac{\partial^2 N}{\partial\chi_1^{*2}}\frac{\partial\chi_1^*}{\partial\phi_1^*}\frac{\partial\chi_1^*}{\partial\phi_2^*} \\ &+ \frac{\partial^2 N}{\partial\chi_2^{*2}}\frac{\partial\chi_2^*}{\partial\phi_1^*}\frac{\partial\chi_2^*}{\partial\phi_2^*} + \frac{\partial^2 N}{\partial\chi_1^*\partial\chi_2^*}\frac{\partial\chi_1^*}{\partial\phi_1^*}\frac{\partial\chi_2^*}{\partial\phi_2^*} + \frac{\partial^2 N}{\partial\chi_1^*\partial\chi_2^*}\frac{\partial\chi_1^*}{\partial\phi_2^*}\frac{\partial\chi_2^*}{\partial\phi_1^*},\end{aligned} \tag{46}$$

$$\frac{\partial^2 N}{\partial\phi_1^{*2}} = \frac{\partial N}{\partial\chi_1^*}\frac{\partial^2\chi_1^*}{\partial\phi_1^{*2}} + \frac{\partial N}{\partial\chi_2^*}\frac{\partial^2\chi_2^*}{\partial\phi_1^{*2}} + \frac{\partial^2 N}{\partial\chi_1^{*2}}\left(\frac{\partial\chi_1^*}{\partial\phi_1^*}\right)^2 + \frac{\partial^2 N}{\partial\chi_2^{*2}}\left(\frac{\partial\chi_2^*}{\partial\phi_1^*}\right)^2 + 2\frac{\partial^2 N}{\partial\chi_1^*\partial\chi_2^*}\frac{\partial\chi_1^*}{\partial\phi_1^*}\frac{\partial\chi_2^*}{\partial\phi_1^*}. \tag{47}$$

derivatives of ϕ_2. These equations define the relations among the N derivatives ($N_{,I}$ and $N_{,IJ}$) with respect to scalar field ϕ_I^* to the N derivatives with respect to three-form fields at horizon crossing $\frac{\partial N}{\partial\chi_1^*}$, $\frac{\partial N}{\partial\chi_2^*}$, $\frac{\partial^2 N}{\partial\chi_1^*\partial\chi_2^*}$, $\frac{\partial^2 N}{\partial\chi_1^{*2}}$, $\frac{\partial^2 N}{\partial\chi_2^{*2}}$. In other words, we have indirectly transported the δN formalism from scalar fields to three-form fields. However, we still need to calculate the derivatives of the three-form fields with respect to the dual scalar fields. For this purpose we differentiate the relations (43) and (44) keeping in mind that ϕ_1 and ϕ_2 are independent fields. Then we have that

$$\frac{d\phi_1}{d\phi_1} = \frac{1}{\sqrt{6}w_1}\frac{\partial H}{\partial\phi_1} + \frac{1}{\sqrt{6}H}\frac{\partial w_1}{\partial\phi_1} = 1. \tag{48}$$

$$\frac{d\phi_1}{d\phi_2} = \frac{1}{\sqrt{6}w_1}\frac{\partial H}{\partial\phi_2} + \frac{1}{\sqrt{6}H}\frac{\partial w_1}{\partial\phi_2} = 0. \tag{49}$$

$$\frac{d\phi_2}{d\phi_1} = \frac{1}{\sqrt{6}w_2}\frac{\partial H}{\partial \phi_2} + \frac{1}{\sqrt{6}H}\frac{\partial w_2}{\partial \phi_2} = 1\,. \tag{50}$$

$$\frac{d\phi_2}{d\phi_2} = \frac{1}{\sqrt{6}w_2}\frac{\partial H}{\partial \phi_1} + \frac{1}{\sqrt{6}H}\frac{\partial w_2}{\partial \phi_1} = 0\,. \tag{51}$$

Solving Eqs. (48)-(51) for a potential of the form $V = V(\chi_1) + V(\chi_2)$, we obtain

$$\begin{aligned}\frac{\partial\chi_1}{\partial\phi_1} =& \frac{\chi_2 V_{,\chi_2} + H^2\left(6 - 9\chi_1^2\right)}{3H\left(6H^2 + \chi_1 V_{,\chi_1} + \chi_2 V_{,\chi_2}\right)}\\ \frac{\partial\chi_1}{\partial\phi_2} =& -\frac{\chi_1\left(V_{,\chi_2} + 9H^2\chi_2\right)}{3H\left(6H^2 + \chi_1 V_{,\chi_1} + \chi_2 V_{,\chi_2}\right)}\end{aligned} \tag{52}$$

$$\begin{aligned}\frac{\partial^2\chi_1}{\partial\phi_1^2} =& \frac{-1}{9H^2\left(6H^2 + \chi_1 V_{,\chi_1} + \chi_2 V_{,\chi_2}\right)^3}\{\chi_1 V_{,\chi_1}^2\left[\chi_2\left(\chi_2 V_{,\chi_2\chi_2} + 2V_{,\chi_2}\right) + H^2\left(9\chi_1^2 - 6\right)\right]\\ & - 2V_{,\chi_1}\left[-3H^2\chi_2\left(3V_{,\chi_2\chi_2}\chi_1^2\chi_2 + 6V_{,\chi_2}\chi_1^2 + 4V_{,\chi_2}\right) - V_{,\chi_2}^2\chi_2^2 + 18H^4\left(3\chi_1^2 - 2\right)\right]\\ & + \chi_1 V_{,\chi_1\chi_1}\left(\chi_2 V_{,\chi_2} + H^2\left(6 - 9\chi_1^2\right)\right)^2\\ & - 9\chi_1 H^2\left(-3H^2\chi_2\left(3V_{,\chi_2\chi_2}\chi_1^2\chi_2 + 12V_{,\chi_2}\right) - 3V_{,\chi_2}^2\chi_2^2 + 54H^4\left(3\chi_1^2 - 2\right)\right)\}\,.\end{aligned} \tag{53}$$

$$\begin{aligned}\frac{\partial^2\chi_1}{\partial\phi_2^2} =& \frac{-1}{9H^2\left(6H^2 + \chi_1 V_{,\chi_1} + \chi_2 V_{,\chi_2}\right)^3}\{\chi_1\left[18V_{,\chi_2}H^2\chi_2\left(V_{,\chi_1\chi_1}\chi_1^2 - 18H^2\right) - 2V_{,\chi_2}^3\chi_2\right]\\ & + \chi_1 V_{,\chi_2}^2\left[\chi_1\left(\chi_1 V_{,\chi_1\chi_1} - 2V_{,\chi_1}\right) - 3H^2\left(3\chi_2^2 + 10\right)\right] + \chi_1 V_{,\chi_2\chi_2}\left[V_{,\chi_1}\chi_1 + H^2\left(6 - 9\chi_2^2\right)\right]^2\\ & + 9\chi_1 H^2\left[3H^2\chi_1\left(3V_{,\chi_1\chi_1}\chi_1\chi_2^2 + 4V_{,\chi_1}\right) + V_{,\chi_1}^2\chi_1^2 - 18H^4\left(9\chi_2^2 - 2\right)\right]\}\,.\end{aligned} \tag{54}$$

$$\begin{aligned}\frac{\partial^2\chi_1}{\partial\phi_1\partial\phi_2} =& \frac{1}{9H^2\left(6H^2 + \chi_1 V_{,\chi_1} + \chi_2 V_{,\chi_2}\right)^3}\{-V_{,\chi_2}^3\chi_2^2 + V_{,\chi_2}^2\chi_2\left[V_{,\chi_1\chi_1}\chi_1^2 + 3H^2\left(-4 + 3\chi_1^2 - 3\chi_2^2\right)\right]\\ & + V_{,\chi_2}\left[3H^2\chi_1\left(V_{,\chi_1\chi_1}\chi_1\left(-3\chi_1^2 + 3\chi_2^2 + 2\right) + 3V_{,\chi_1}\left(\chi_1^2 - \chi_2^2 + 2\right)\right) + V_{,\chi_1}^2\chi_1^2\right]\\ & + 36V_{,\chi_2}H^4\left(6\chi_1^2 - 3\chi_2^2 - 1\right) + \chi_2\left(V_{,\chi_2\chi_2}V_{,\chi_1}^2\chi_1^2 + 3V_{,\chi_2\chi_2}V_{,\chi_1}H^2\chi_1\left(3\chi_1^2 - 3\chi_2^2 + 2\right)\right) +\\ & \chi_2\left[162H^6\left(9\chi_1^2 - 2\right) + 27H^4\chi_1\left(\chi_1\left(-3V_{,\chi_1\chi_1}\chi_1^2 + 2V_{,\chi_1\chi_1} - 3V_{,\chi_2\chi_2}\chi_2^2 + 2V_{,\chi_2\chi_2}\right) + 4V_{,\chi_1}\right)\right]\}\,.\end{aligned} \tag{55}$$

The remaining derivatives can be obtained from these by interchanging $1 \leftrightarrow 2$. Following Eqs. (45)-(47) the quantities obtained in Eqs. (52)-(55) are to be evaluated at $kc_s = aH$. However, the derivatives of N with respect to the three-form fields evolve on superhorizon scales.

IV. TWO THREE-FORMS NON-GAUSSIANITY AND OBSERVATIONAL DATA

In this section, we aim to update the observational status of two three-form inflation [26] by means of calculating the reduced bispectrum $f_{\rm NL}$. In Ref. [26], various dynamics of two three-form fields in the context of inflation were explored. In this model, there exists two kinds of inflationary dynamics, namely, type I and type II solutions. In type I cases the trajectories in field space are straight lines, and the role of isocurvature perturbations during inflation is negligible, whereas the type II solutions have curved trajectories in field space where the isocurvature perturbations are expected to play a role. In Ref. [26], type II solutions with tuned initial conditions were shown to be consistent with Planck 2013 data predicting scalar spectral index $n_s \sim 0.967$ and the tensor to scalar ratio $r \sim 0.0422$. Also the running of the spectral index $\left(\frac{dn_s}{dlnk}\right)$ in this model is negligible and compatible with the observational data. The three-form dynamics which give rise to these consistent predictions is plotted in the left panel of Fig. 1. We have taken the same initial conditions and the parameter values [5] as in [26].

---

[5] The initial conditions considered in Fig. 1 correspond to the values of three-form fields at horizon crossing, whereas initial conditions in Ref. [26] were taken at pre-slow-roll regime. Nevertheless, we study the same inflationary trajectory which was proved to be compatible with the Planck 2013 data in Ref. [26].

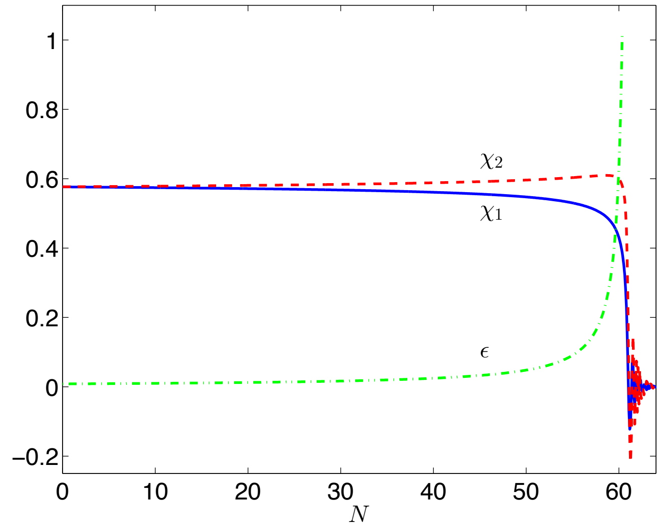

Figure 1: The numerical solutions of (14) for $\chi_1(N)$ (solid line) and $\chi_2(N)$ (dashed line). The dash-dotted line corresponds to the slow-roll parameter $\epsilon(N)$ and $\epsilon = 1$ indicates the end of inflation at $N = 60.35$. We considered the potentials $V_1 = V_{10}\left(\chi_1^2 + b_1\chi_1^4\right)$ and $V_2 = V_{20}\left(\chi_2^2 + b_2\chi_2^4\right)$ with $V_{10} = 1$, $V_{20} = 0.93$, $b_{1,2} = -0.35$ and taken initial conditions $\chi_1(0) \approx 0.5763$, $\chi_2(0) \approx 0.5766$, $\chi_1'(0) = -0.000224$, $\chi_2'(0) = 0.00014$.

The observational prediction of non-Gaussianity for multi-field inflation is deeply associated with the evolution of isocurvature perturbations. In the single field inflation the statistics of the curvature pertrubation evaluated at horizon exit can be confronted with the observation. This is because the curvature perturbation is conserved on superhorizon scales if the system is adiabatic [31, 39, 40]. Whereas for multi-field models, the statistics evolve on superhorizon scales and non-Gaussianity can be generated as a consequence of the presence of isocurvature perturbations. This can happen in two regimes, namely, (i) during inflation [41–44], (ii) after inflation such as in the curvaton model [45–56]. In general the statistics will continue to evolve until all isocurvature perturbations decay, the so called adiabatic limit [43]. We evaluate $f_{\rm NL}$ at the end of inflation, this will be a good approximation as long as reheating proceeds quickly, and curvaton type effects do not occur.

To calculate $f_{\rm NL}$ given in Eq. (30), we need to compute the N derivatives with respect to the initial conditions of three-form fields defined in Eqs. (45-47). To compute these numerically, we define the following discrete derivatives which can in principle be extended to any number of fields,

$$
\begin{aligned}
N_{,\chi_1^*} &= \frac{N\left(\chi_1^* + \Delta\chi_1\,,\, \chi_2^*\right) - N\left(\chi_1^* - \Delta\chi_1\,,\, \chi_2^*\right)}{2\Delta\chi_1}, \\
N_{,\chi_1^*\chi_1^*} &= \frac{N\left(\chi_1^* + \Delta\chi_1\,,\, \chi_2^*\right) - 2N\left(\chi_1^*\right) + N\left(\chi_1^* + \Delta\chi_1\,,\, \chi_2^*\right)}{\Delta\chi_1^2}, \\
N_{,\chi_1^*\chi_2^*} &= \left[N\left(\chi_1^* + \Delta\chi_1\,,\, \chi_2^* + \Delta\chi_2\right) - N\left(\chi_1^* + \Delta\chi_1\,,\, \chi_2^* - \Delta\chi_2\right) - \right. \\
&\quad \left. N\left(\chi_1^* - \Delta\chi_1\,,\, \chi_2^* + \Delta\chi_2\right) + N\left(\chi_1^* - \Delta\chi_1\,,\, \chi_2^* - \Delta\chi_2\right)\right]\left(4\Delta\chi_1^2\right)^{-1},
\end{aligned}
\tag{56}
$$

and similarly we can obtain the remaining derivatives by interchanging $1 \leftrightarrow 2$. In the above expression, $N(\chi_1, \chi_2)$ is the number of e-foldings which occurs starting at initial conditions $\{\chi_1^*, \chi_2^*\}$ and ending at a given final energy density. This final energy density is defined by the condition that $N(\chi_1, \chi_2) = 60.35$ at the point $\epsilon = 1$. That is the central point in the finite difference represents a trajectory that undergoes 60 e-folds of inflation, from the initial field value until inflation ends, and the density at that time is used as the final density for all the other points in the difference scheme. These other points will therefore represent slightly different amounts of inflation, and we note that their associated trajectories will not end exactly at the point $\epsilon = 1$. In our numerical results we take $\Delta\chi_I \sim 10^{-5}$. Using the N derivatives calculated from (56) and evaluating the amplitude given by Eq. (33), we compute $f_{\rm NL}$ in (30). We obtain the momentum independent contribution $f_{NL}^{(4)}$ in (31) to be very small $\mathcal{O}\left(10^{-3}\right)$. In Fig. 2 we plot the total $f_{\rm NL}$ versus N for squeezed ($k_2 \ll k_1 = k_3$), equilateral ($k_1 = k_2 = k_3$) and orthogonal ($k_1 = 2k_2 = 2k_3$) triangles.

It is convenient to express the reduced bispectrum in terms of the following independent variables [57, 58]

$$
\alpha = \frac{k_2 - k_3}{k} \quad , \quad \beta = \frac{k - k_1}{k} \quad \text{where} \quad k = \frac{k_1 + k_2 + k_3}{2}\,, \tag{57}
$$

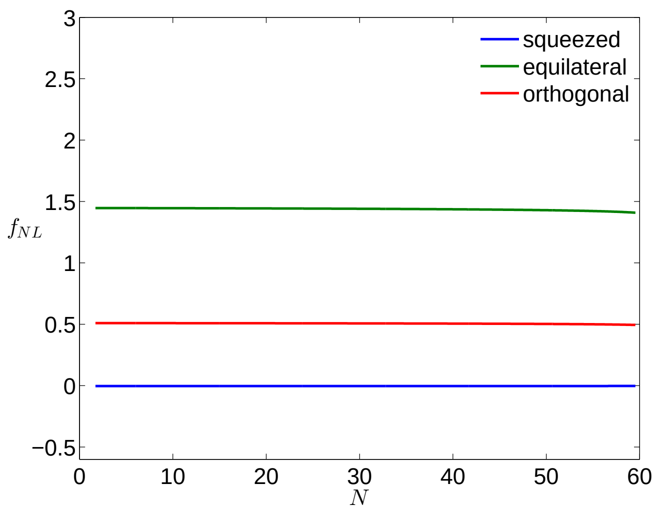

Figure 2: In this plot we depict $f_{\rm NL}$ against N for squeezed $(k_2 \ll k_1 = k_3)$ equilateral $(k_1 = k_2 = k_3)$ and orthogonal $(k_1 = 2k_2 = 2k_3)$ configurations. We considered the potentials $V_1 = V_{10}\left(\chi_1^2 + b_1\chi_1^4\right)$ and $V_2 = V_{20}\left(\chi_2^2 + b_2\chi_2^4\right)$ with $V_{10} = 1$, $V_{20} = 0.93$, $b_{1,2} = -0.35$ and taken initial conditions $\chi_1(0) \approx 0.5763$, $\chi_2(0) \approx 0.5766$, $\chi_1'(0) = -0.000224$, $\chi_2'(0) = 0.00014$.

where $0 \leq \beta \leq 1$ and, $-(1-\beta) \leq \alpha \leq (1-\beta)$. In Fig. 3 we depict the shape of a slice through the reduced bispectrum $f_{\rm NL}(k_1, k_2, k_3)$ at $N = 60$ using these variables. The bispectrum shape reveals details about the dominant interaction contributions [59]. In general, the presence of a signal in the 'squeezed' limit represents the interaction of the long wavelength mode, which already exited the horizon, with the short wavelength modes still being within the horizon. This can happen in the case where more than one light scalar field drives the period of inflation. When, instead, we observe a peak in the 'equilateral' limit, the dominant interaction between the fields occurs when the modes are exiting the horizon at the same time during inflation. This is taken to be the distinctive feature of models with non-canonical kinetic term (or) models involving higher derivative interactions [14]. In the case of multiple non-canonical scalar field inflation (which is effectively happening in the two three-forms inflation scenario), it is possible that we would encounter a mixture of shapes [14, 59]. Though in the example we explored there is no significant signal in the squeezed limit.

V. CONCLUSIONS

In this article we presented a generic framework to compute primordial non-Gaussianity in the case of multiple three-form fields inflation. We followed the δN formalism which is a well known method to study the evolution of curvature perturbations on superhorizon scales in the case of multiple scalar fields. Due to the fact that the three-form fields are dual to non-canonical scalar fields, which was shown in [24], we developed an indirect methodology to implement δN formalism to three-form fields. For a specific case of two three-form fields, we derived a relation between

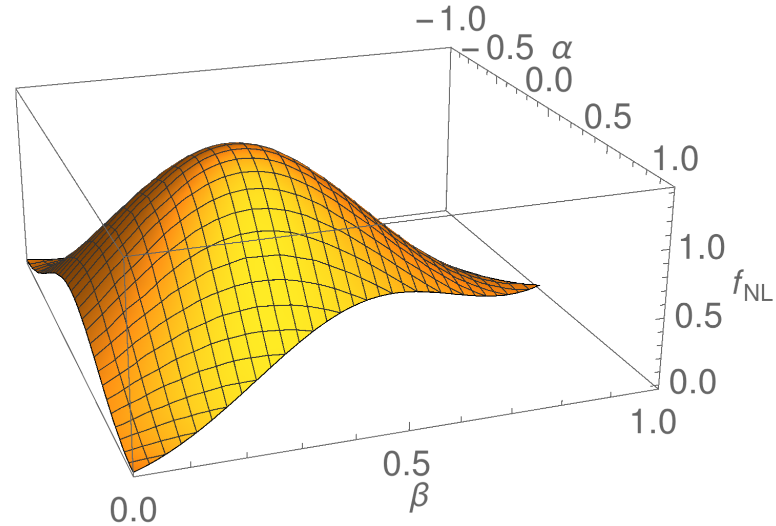

Figure 3: Graphical representation of NG shape $f_{\rm NL}\,(\alpha,\,\beta)$. We considered the potentials $V_1 = V_{10}\left(\chi_1^2 + b_1\chi_1^4\right)$ and $V_2 = V_{20}\left(\chi_2^2 + b_2\chi_2^4\right)$ with $V_{10} = 1$, $V_{20} = 0.93$, $b_{1,2} = -0.35$ and taken initial conditions $\chi_1\,(0) \approx 0.5763$, $\chi_2\,(0) \approx 0.5766$, $\chi_1'\,(0) = -0.000224$, $\chi_2'\,(0) = 0.00014$.

the derivatives of N with respect to unperturbed values of scalar field duals at horizon exit $c_s k = aH$ and the N derivatives with respect to three-form fields. We employed a numerical finite difference approach for this purpose. We computed the bispectrum at horizon exit for the two three-form field case using known expressions for three-point field space correlations for a general multi-scalar field model. Then using the N derivatives we determined the complete superhorizon evolution of $f_{\rm NL}$ for squeezed, equilateral and orthogonal configurations until the end of inflation. We considered a suitable choice of potentials and specific values of model parameters that were consistent with $n_s \sim 0.967$ and $r \sim 0.0422$ [26]. We obtained the corresponding $f_{\rm NL}$ predictions for the two three-form inflationary model as $f_{\rm NL}^{\rm sq} \sim -2.6 \times 10^{-3}$, $f_{\rm NL}^{\rm eq} \sim 1.409$, $f_{\rm NL}^{\rm orth} \sim 0.495$. Therefore, the model is well within the observational bounds of Planck 2015 data but in principal could be falsifiable with the future probes. It would be interesting to study the trispectrum in this model which we defer for future investigations.

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